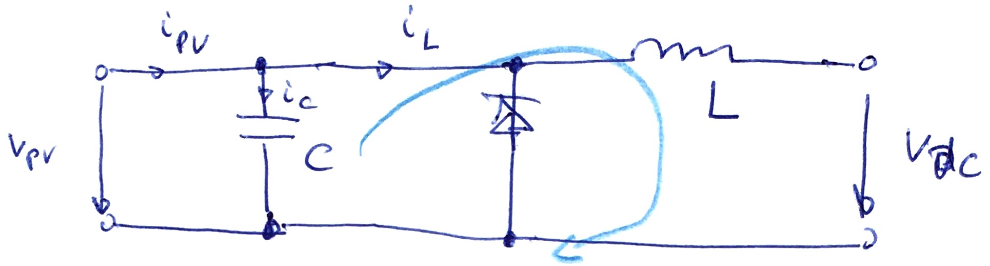
4.4.2021
5.4.2021

State-space averaging of Buck Converter

(without Losses)

Q : ON



Node $\Sigma i = 0$: $i_{pv} - i_c - i_L = 0$

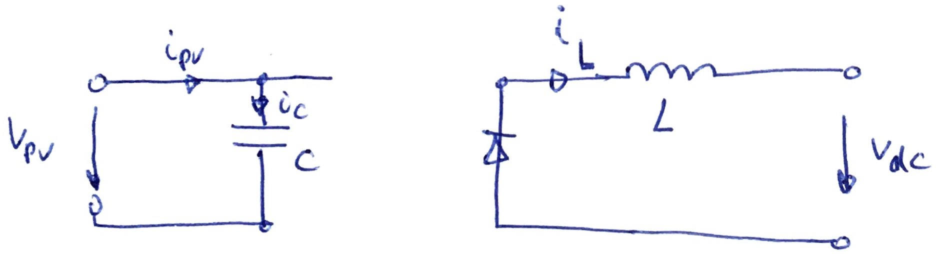
$$i_c = C\frac{dV_c}{dt} = C\frac{dV_{pv}}{dt} \qquad V_c = V_{pv}$$

$$i_{pv} - C\frac{dV_{pv}}{dt} - i_L = 0$$

$$C\frac{dV_{pv}}{dt} = i_{pv} - i_L \qquad i_{pv} = f_{son}(V_{pv})$$

$$\dot{x}_1 = -\frac{1}{C}x_2 + \frac{1}{C}i_{pv} \qquad V_{pv} = x_1 \quad i_L = x_2$$

Q : OFF



$$i_{pv} - C\frac{dV_{pv}}{dt} = 0 \qquad \frac{dV_{pv}}{dt} = \frac{1}{C}i_{pv}$$

Q : ON

Mesh $\curvearrowright$ $\Sigma V = 0$

$$-V_{PV} + L\frac{di_L}{dt} + V_{dC} = 0$$

$$\frac{di_L}{dt} = \frac{1}{L}(V_{PV} - V_{dC})$$

$$\dot{x}_2 = \frac{1}{L}x_1 - \frac{1}{L}V_{dC}$$

Q : OFF

Mesh $\Sigma V = 0$

$\curvearrowright$

$$L\frac{di_L}{dt} + V_{dC} = 0$$

$$\frac{di_L}{dt} = -\frac{1}{L}V_{dC}$$

State-space form Q : ON

$$\dot{x}_1 = -\frac{1}{C}x_2 + \frac{1}{C}i_{PV} \qquad \text{with } i_{PV} = f_{SDM}(\underbrace{V_{PV}}_{x_1}, S)$$

$$\dot{x}_2 = \frac{1}{L}x_1 - \frac{1}{L}V_{dC}$$

$$\dot{\underline{x}} = \begin{pmatrix} \frac{1}{C} \frac{f_{SDM}(x_1, S)}{x_1} & -1/C \\ 1/L & 0 \end{pmatrix} \underline{x} + \begin{pmatrix} 0 \\ -1/L \, V_{dc} \end{pmatrix}$$

State-space for Q: off

$$\dot{\underline{x}} = \begin{pmatrix} \frac{1}{C} \frac{f_{SDM}(x_1, S)}{x_1} & 0 \\ 0 & 0 \end{pmatrix} \underline{x} + \begin{pmatrix} 0 \\ -1/L \, V_{dc} \end{pmatrix}$$

Joint state-space form for $Q \in \{ON, OFF\}$

D is denoted as duty cycle and the control variable

$$\boxed{\dot{\underline{x}} = \begin{pmatrix} 1/C \frac{f_{SDM}(x_1, S)}{x_1} & -\frac{1}{C} D \\ \frac{1}{L} D & 0 \end{pmatrix} \underline{x} + \begin{pmatrix} 0 \\ -1/L \, V_{dc} \end{pmatrix} \quad (1)}$$

$x_0 = (V_{OC,STC}, 0)^T$ $\qquad$ $x_0 = x(t=0)$

☐ Taylor Linearization at one equilibrium point

x_1:

$$0 = \frac{1}{C} \frac{f_{SDM}(x_1, S)}{x_1} x_1 - \frac{1}{C} D_c x_2$$

$$f_{SDM}(x_1, S) = D_c x_2$$

example $D_c = 0{,}8$ $\rightarrow$ $f_{SDM}(x_{1c}, S)\Big|_{x_{1c}, S_{PTC}} = 0{,}8\, x_{2c}$

$$i_{PV_c} = 0{,}8\, i_{Lc}$$

general $\boxed{i_{PVc} = D_c\, i_{Lc}}$

x_2:

$$0 = \frac{1}{L} D_c x_1 - \frac{1}{L} V_{dc}$$

with $x_1 = V_{PV}$

$$D_c V_{PV,C} = V_{dc,C}$$

$$\boxed{V_{PV,C} = \frac{1}{D_c} V_{dc,C}}$$

$\Delta u = D - D_c$

$\Delta x_1 = x_1 - x_{1,c}$

$\Delta x_2 = x_2 - x_{2,c}$

- Non Linear model

$$\boxed{\dot{\underline{x}} = \underline{f}(x_1, x_2, D)\Big|_S}$$

$(2) \hat{=} (1)$

- Taylor Lin.

$$\Delta\dot{\underline{x}} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix}\Bigg|_{c,s} \Delta\underline{x} + \begin{pmatrix} \frac{\partial f_1}{\partial D} \\ \frac{\partial f_2}{\partial D} \end{pmatrix}\Bigg|_{c,s} \Delta u$$

$$\left.\begin{aligned} \dot{x}_1 &= f_1 = \frac{1}{C} f_{SDM}(x_1, S) - \frac{1}{C} D\, x_2 \\ \dot{x}_2 &= f_2 = \frac{1}{L} D x_1 - \frac{1}{L} V_{dc}\Big|_c \end{aligned}\right\} \hat{=} (1)$$

$$\Delta\dot{\underline{x}} = \begin{pmatrix} \frac{1}{C}\frac{\partial f_{SDM}}{\partial x_1}(x_1, S)\Big|_{x_{1,c}} & -\frac{1}{C} D_c \\ \frac{1}{L} D_c & 0 \end{pmatrix} \Delta\underline{x} + \begin{pmatrix} -\frac{1}{C}\underbrace{x_{2,c}}_{I_{L,c}} \\ \frac{1}{L}\underbrace{x_{1,c}}_{V_{PV,c}} \end{pmatrix} \Delta u \qquad (2)$$

⇝ entspricht dem Modell (6.18) in Xiao, PV Power Systems, Wiley