# ELEMENTS OF PROGRAMMING EXERCISES AND SOLUTIONS

ASEF AHMED

## 1. Foundations

1.1. **Categories of Ideas: Entity, Species, Genus.** No exercises.

1.2. **Values.**

**Lemma 1.1.** *If a value type is uniquely represented, equality implies representational equality.*

*Solution.* Suppose a value type $\mathtt{T}$ is uniquely represented. Denote by $\mathtt{v}, \mathtt{v}' : \mathtt{T}$ as equal values of $\mathtt{T}$. By unique representation, $\mathtt{v}, \mathtt{v}'$ each correspond uniquely to the abstract entities $\mathtt{E}, \mathtt{E}'$, and by equality of values, these entities must also be equal. Hence the data $\mathtt{D}, \mathtt{D}'$ for $\mathtt{v}, \mathtt{v}'$ are identical, and so $\mathtt{v}, \mathtt{v}'$ are representationally equal. □

**Lemma 1.2.** *If a value type is not ambiguous, representational equality implies equality.*

*Solution.* Suppose a value type $\mathtt{T}$ is not ambiguous. Denote by $\mathtt{v}, \mathtt{v}' : \mathtt{T}$ as representationally equal values of $\mathtt{T}$. As $\mathtt{T}$ is not ambiguous, $\mathtt{v}, \mathtt{v}'$ must each have at most one interpretation, and by representational equality, the data $\mathtt{D}, \mathtt{D}'$ for the values are identical. Hence the values $\mathtt{v}, \mathtt{v}'$ must represent the same abstract entity $\mathtt{E}$, and so they are equal. □