

# Hidden Markov Models & Conditional Random Fields

Andrei Barbu

School of Electrical and Computer Engineering  
Purdue University

March 17, 2013



# History

- ▶ Developed by Markov in 1906
- ▶ Markov was a disciple of Chebyshev along with Lyapunov
- ▶ Introduced for no practical reason, except maybe to spite Nekrasov due to the dispute over the Weak Law of Large Numbers
- ▶ Within 1 year it was being used to clear up issues in thermodynamics

# History

- ▶ Developed by Markov in 1906
- ▶ Markov was a disciple of Chebyshev along with Lyapunov
- ▶ Introduced for no practical reason, except maybe to spite Nekrasov due to the dispute over the Weak Law of Large Numbers
- ▶ Within 1 year it was being used to clear up issues in thermodynamics

# History

- ▶ Developed by Markov in 1906
- ▶ Markov was a disciple of Chebyshev along with Lyapunov
- ▶ Introduced for no practical reason, except maybe to spite Nekrasov due to the dispute over the Weak Law of Large Numbers
- ▶ Within 1 year it was being used to clear up issues in thermodynamics

# History

- ▶ Developed by Markov in 1906
- ▶ Markov was a disciple of Chebyshev along with Lyapunov
- ▶ Introduced for no practical reason, except maybe to spite Nekrasov due to the dispute over the Weak Law of Large Numbers
- ▶ Within 1 year it was being used to clear up issues in thermodynamics

# Markov Models

- ▶ Process with the Markov property:  $P(s_{t+1}|s_t, \dots, s_0) = P(s_{t+1}|s_t)$
- ▶ Process with discrete time

Two processes



# Markov Models

- ▶ Process with the Markov property:  $P(s_{t+1}|s_t, \dots, s_0) = P(s_{t+1}|s_t)$
- ▶ Process with discrete time

Two processes



# Markov Models

- ▶ Process with the Markov property:  $P(s_{t+1}|s_t, \dots, s_0) = P(s_{t+1}|s_t)$
- ▶ Process with discrete time

## Two processes

- ▶ 
$$\begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}$$

- ▶ 
$$\sum_j p_{ji} = 1$$

- ▶ 
$$\begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} \begin{bmatrix} N_0(t) \\ N_1(t) \end{bmatrix} = \begin{bmatrix} N_0(t+1) \\ N_1(t+1) \end{bmatrix}$$



# Markov Models

- ▶ Process with the Markov property:  $P(s_{t+1}|s_t, \dots, s_0) = P(s_{t+1}|s_t)$
- ▶ Process with discrete time

## Two processes

- ▶  $\begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}$
- ▶  $\sum_j p_{ji} = 1$
- ▶  $\begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} \begin{bmatrix} N_0(t) \\ N_1(t) \end{bmatrix} = \begin{bmatrix} N_0(t+1) \\ N_1(t+1) \end{bmatrix}$

# Markov Models

- ▶ Process with the Markov property:  $P(s_{t+1}|s_t, \dots, s_0) = P(s_{t+1}|s_t)$
- ▶ Process with discrete time

## Two processes

- ▶  $\begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}$
- ▶  $\sum_i p_{ji} = 1$
- ▶  $\begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} \begin{bmatrix} N_0(t) \\ N_1(t) \end{bmatrix} = \begin{bmatrix} N_0(t+1) \\ N_1(t+1) \end{bmatrix}$

# Markov Models

- ▶ Process with the Markov property:  $P(s_{t+1}|s_t, \dots, s_0) = P(s_{t+1}|s_t)$
- ▶ Process with discrete time

## Two processes

- ▶  $\begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}$
- ▶  $\sum_j p_{ji} = 1$
- ▶  $\begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} \begin{bmatrix} N_0(t) \\ N_1(t) \end{bmatrix} = \begin{bmatrix} N_0(t+1) \\ N_1(t+1) \end{bmatrix}$