3.4 Transiting planets

A transits of a planet in front of its parent star occurs if the line of sight is very close to the orbital plane. The transit probability is thereby much enhanced for planets with small separations. The unexpected presence of hot Jupiters made transit observation after the detection of 51 Peg b an attractive and very successful planet search technique. Transit data form now a cornerstone for extra-solar planet research.

Transits have the following basic characteristics (see Fig. 3.7):

- Transits occur periodically,
- the stellar intensity is reduced by an amount which is proportional to the size of the planet,
- the planet occults during the transit different regions of the star, what may introduce during the transit photometric and spectroscopic features due to distinct surface structures of the star,
- during the transit some light passes through the outermost atmosphere of the planet what may introduce measurable absorption effects and allow an investigation of the uppermost atmosphere of the planet.

Secondary eclipses occur for most transiting planets about half an orbital period before or after the transit. If the emission of the planet is strong and the sensitivity of the measurement high then one can measure the drop in intensity when the planet goes into secondary eclipse.

Orbital phase curves may also be detected, because the hemisphere facing the star is expected to be brighter than the planet's night side. This requires that the measuring sensitivity is at least accurate enough for secondary eclipse observations.

Figure 3.7: Light curve features for transiting planets.

3.4.1 Approximations for basic transit parameters

We discuss here first approximate transit parameters for planets on a circular orbit and for central transits. This simplified treatment illustrates the basic transit properties. More detailed derivations and dependencies of the transit parameters are described in the following sections.

The transit depth is the fractional reduction of the stellar intensity $\Delta I/I$ due to the planet transit. Assuming a homogeneous stellar disk and treating the planet like a black disk yields

$$\frac{\Delta I}{I} \approx \frac{R_P^2}{R_S^2}\,. \tag{3.14}$$

The transit depth is equal to the ratio of the cross sections of the planet to the star. This measurement yields the absolute radius R_P of the planet, if R_S of the star is known. If also the mass of the planet is known, then one gets the mean density $\bar{\rho}$ of the planet, which is a very important parameter for the planet characterization.

For a Jupiter – Sun system the transit depth is about 1 %, and for an Earth – Sun system about $1.0 \cdot 10^{-4}$ (see Table 3.7).

The transit duration of a planet on a circular orbit across the center of the star is

$$\Delta t \approx \frac{P}{2\pi a}\, 2R_S\,. \tag{3.15}$$

where R_S is the stellar radius. This is an upper limit for circular orbits because non-central transits are shorter. This formulation defines the transit duration from the mid-ingress to the mid-egress phase.

We can express the transit duration as fraction of the orbital period:

$$\frac{\Delta t}{P} \approx \frac{R_S}{\pi a}\,.$$

Table 3.7: Transit properties of solar system planets for an “outside” observer. P is the orbital period, Δt the absolute and $\Delta t/P$ the relative transit duration, $\Delta I/I$ the transit depth, p_{trans} the transit probability, and i the orbit inclination

planet	P [yr]	Δt [hr]	$\Delta t/P$	$\Delta I/I$	p_{trans}	i
Mercury	0.241	8.1	$38 \cdot 10^{-4}$	$1.2 \cdot 10^{-5}$	1.19 %	6.33
Venus	0.615	11.0	$20 \cdot 10^{-4}$	$7.6 \cdot 10^{-5}$	0.65 %	2.16
Earth	1.000	13.0	$15 \cdot 10^{-4}$	$8.4 \cdot 10^{-5}$	0.47 %	1.65
Mars	1.880	16.0	$9.7 \cdot 10^{-4}$	$2.4 \cdot 10^{-5}$	0.31 %	1.71
Jupiter	11.86	29.6	$2.9 \cdot 10^{-4}$	1.01 %	0.089 %	0.39
Saturn	29.5	40.1	$1.5 \cdot 10^{-4}$	0.75 %	0.049 %	0.87
Uranus	84.0	57.0	$0.77 \cdot 10^{-4}$	0.135 %	0.024 %	1.09
Neptune	164.8	71.3	$0.49 \cdot 10^{-4}$	0.127 %	0.015 %	0.72

The transit probability describes the chance that a planet shows period transits. For systems with circular orbits a transit will occur for

$$a|\cos i| < R_S\,.$$

This condition considers only full transits and grazing transits where at least half of the planet is in front of the star. For the integration we need to consider that all inclinations in the range $i = 90° \pm \theta$ are considered and this needs to be normalized to a random distribution of orbit orientations according to:

$$p_{\rm trans} = \frac{\int_{90-\theta}^{90+\theta} \sin\vartheta d\vartheta}{\int_0^{180} \sin\vartheta d\vartheta} = \frac{-\cos\vartheta|_{90-\theta}^{90+\theta}}{-\cos\vartheta|_0^{180}} = \cos(90° - \theta)\,,$$

or with the relations for $\cos i$ follows the approximate transit probability:

$$p_{\rm trans} \approx \frac{R_S}{a}\,. \tag{3.16}$$

3.4.2 Detailed transit geometry

The basic transit geometry, which is equivalent to the classical eclipse description of binary stars, is illustrated in Fig. 3.8. The description for the transit is also valid for the secondary eclipse of the planet behind the star.

Figure 3.8: Transit times.

– Four **transit times** or eclipses times are defined: the start and end of the ingress phase t_i and t_{ii} and the start and end of the egress phase t_{iii} and t_{iv} which are also called the 1$^{\rm st}$, 2$^{\rm nd}$, 3$^{\rm rd}$ and 4$^{\rm th}$ contact. In addition there is of course the mid-eclipse time $t_{\rm ecl}$.

- For the **transit duration** or eclipse duration one distinguishes between the total (transit or eclipse) phase which is the time span where at least some parts of one object is behind the other object while the full eclipse phase is the time span where one object is located completely in front or behind the other object.

$$\Delta t_{\rm tot} = t_{iv} - t_i \quad \text{and} \quad \Delta t_{\rm full} = t_{iii} - t_{ii}$$

- The **impact parameter** b is the minimum projected distance of the center of the two objects at mid-eclipse.
- A **partial transit** or a partial eclipse occur for $R_S - R_P < b < R_S + R_P$ without full phase $\Delta t_{\rm full} = 0$.

The transit timing can be calculated from the radii of the star R_S and the planet R_P and the orbital parameters. For circular orbits we can write for the transit times relative to mid-eclipse $t_{\rm ecl} = 0$:

$$t_{iv} = -t_i = \frac{P}{2\pi a}\sqrt{(R_S + R_P)^2 - b^2} \quad \text{for} \quad b \leq R_S + R_P \tag{3.17}$$

and

$$t_{iii} = -t_{ii} = \frac{P}{2\pi a}\sqrt{(R_S - R_P)^2 - b^2} \quad \text{for} \quad b \leq R_S - R_P\,. \tag{3.18}$$

The total and full transit duration are twice these values, respectively. From the full and and total transit duration one can derive the impact parameter (square the equation above and solve for b^2, if the ratio between the radii R_P/R_S is known, e.g. from the transit depths:

$$b^2 = R_S^2\,\frac{(1 - R_P/R_S)^2 - (\Delta t_{\rm full}/\Delta t_{\rm tot})^2(1 + R_P/R_S)^2}{1 - (\Delta t_{\rm full}/\Delta t_{\rm tot})^2}\,. \tag{3.19}$$

For a central transit there is $(\Delta t_{\rm full}/\Delta t_{\rm tot}) = (R_S - R_P)/(R_S + R_P)$ and b becomes zero as it should be.

For small planet radius $R_P \ll R_S$ one can simplify this further and it results for the impact parameter

$$b^2 \approx R_S^2\Big(\frac{R_S - R_P}{R_S}\frac{\Delta t_{\rm tot}}{\Delta t_{\rm full}}\Big)$$

Summary: With the measurement of the different eclipse times and the radius ratio one can determine the impact parameter b and derive an accurate estimate on the orbit inclination.

For the simplified treatment given above the following approximations are used, which are for many cases very reasonable:

- The relative trajectory of the transiting or eclipsed object is treated as a straight line what is a good approximation for a large separation $d \gg R_S$.
- The relative transverse motion is considered to be constant during the entire transit, what is reasonable for a large separation $d \gg R_S$.
- For many cases it will be possible to use circular orbits as good approximation. If eccentricity is non-negligible then one can correct the timing-formula with the following correction factors:

$$f_{\rm corr} \approx \frac{1 - \epsilon^2}{1 \pm \epsilon \sin\omega}\,, \tag{3.20}$$

  where $\pm$ depends on whether this factor is used for the the transit or eclipse times or whether it is used for the determination of the impact parameter b.

3.4.3 Transit and eclipse light curve

The combined flux of the star and the planet $F(t)$ can be described for the different phases as the sum from the star and the planet $F(t) = F_S(t) + F_P(t)$. These fluxes can vary due to the following reasons:

- $F_S(t)$ and $F_P(t)$ are the intrinsic flux of the star and the planet which can vary with time,
- in addition parts of the star are occulted during the transit phase which can be described by a transit attenuation function $\delta_t(t)$. This yields then the observed stellar flux
$$F'_S(t) = F_S(t)(1 - \delta_t(t))\,.$$
- Similarly we can describe the observed flux from the planet which is the intrinsic flux which may be attenuated $\delta_e(t)$ during secondary eclipse by the star
$$F'_P(t) = F_P(t)(1 - \delta_e(t))\,.$$

We are not interested in the intrinsic variability of the star and normalize the total flux to the intrinsic stellar flux:

$$f(t) = \frac{F'_S(t) + F'_P(t)}{F_S(t)} = 1 - \delta_t(t) + \frac{F_P(t)}{F_S(t)}(1 - \delta_e(t))\,. \tag{3.21}$$

The individual terms can be characterized as follows:

- the attenuation by the transiting planet $\delta_t(t)$ is in Eq. 3.21 a large term of the order 1 % for a giant planet transiting a solar-type star. In many cases only this "transit" term must be considered. Outside a transit there is $\delta_t(t) = 0$.
- the intrinsic flux of the planet normalized to the stellar flux $F_P(t)/F_S(t)$ is usually a small term < 0.1 % and hard to measure. In this ratio the time dependence of the stellar brightness can be neglected $F_S(t) \approx \bar{F_S}$ because the relative phase effects in the planet flux are expected to be much larger. However, strong intrinsic variability of the star on short timescales can introduce problems with the normalization and introduces significant uncertainties in the leading "1-term" which can be larger than $F_P(t)/\bar{F_S}$.
- the attenuation of the planet flux during secondary eclipse is $\delta_e(t) = 0$ out of secondary eclipse and $\delta_e(t) = 1$ during the full eclipse phase. Only during ingress and egress a time dependence is expected which can be approximated with a linear interpolation or another simple fit.

3.4.4 Limb darkening

It is well known from the sun that the solar disk has no uniform surface brightness. The sun is brighter in the center and the surface brightness drops towards the limb. This effect is universal for stars. This implies for the attenuation during a planetary transit:

- $\delta_t > R_P^2/R_S^2$ when the planet is in front of the center of the star,
- $\delta_t < R_P^2/R_S^2$ when the planet is in front of the outer regions of the star.

Thus the transit light curve is not just a simple trapezium defined by the transit times t_i to t_{iv} and a constant transit depth δ_t, but a rounded transit curve depending on the exact transit trajectory and the surface brightness distribution of the star. This level of detail must be taken into account for an accurate derivation of radius ratio R_P/R_S.
Different limb darkening laws are used for the description of stars. A frequently used function has the following radial dependence:

$$\frac{I(r)}{I_0} = 1 - a(1-\mu(r)) - b(1-\mu(r))^2 \,. \tag{3.22}$$

The used term $\mu(r) = \cos\theta = 1 - \sqrt{1-(r/R_s)^2}$ in this quadratic fit equation describes the emissivity of the atmosphere as function of the angle θ between surface normal and direction of the emission. For the center of the disk $r = 0$, there is $\mu = \cos\theta = 1$ and $I(r = 0) = I_0$. For the limb $r = R_S$ there is $\mu = 0$ and $I(R_S)/I_o = 1 - a - b$. The emitted intensity $I(\mu)$ is a typical result of radiative transfer calculation for stellar atmospheres. The fit formula given above is only one of many different fit models used in the literature.

Some general properties of stellar limb darkening are:

- the limb darkening law depends on the spectral type of the star,
- for a given star the limb darkening is a function of wavelengths, for solar type stars it is stronger for short wavelengths,
- the transit light curves provide currently the best measurements for the limb darkening of stars.

One of the slides shows the wavelength dependence of the limb darkening as function of wavelength measured for the transit of HD 209458 b with the spectroscopic mode of HST.

Small scale stellar surface structure. The transit depth $\delta_t(t)$ is a measure of the intensity of the hidden region on the stellar photosphere. Small scale inhomogeneities on the stellar structure can therefore produce bumps in the transit light curve if they are hidden by the planet on its trajectory in front of the star.

- the transit intensity is higher (the attenuation less deep) if the planet is in front of a dark spot,
- the transit intensity is lower (more attenuation) if the planet is in front of a bright region.

Different examples for surface structure were found in well observed transit light curves, like for the active star HD 189733. For extra-solar planet research it is important to be aware of this effect so that it can be taken into account in the transit light curve analysis.

3.5 Observational data for transiting planets

First transit observations. The first successful transit observations were made in 1999 for HD 209458 b with a small telescope and a simple camera system by a group led by David Charbonneau. They had a systematic program of follow-up photometry of all close-in giant planets which were detected by the RV-method. The detected transit depth for this object is about $\delta_t \approx 1.5$ %. The fact, that planetary transits can be detected with simple equipment demonstrated that this technique has a huge potential for extra-solar planet research.

3.5.1 Requirements for transit photometry.

Transit photometry requires observations with a high photometric accuracy during at least the duration of the transit which lasts for close-in planets typically a few hours. For different science goals the following rough photometric sensitivities $\Delta I/I$ must be achieved (3σ detection):

- $\Delta I/I \approx 1$ % during several hours for the detection of transits of giant planets around solar type stars,
- $\Delta I/I \approx 10^{-3}$ during several hours for the detection of the secondary eclipse of hot Jupiters in the mid-IR wavelength range $\lambda > 3\,\mu$m; if this is precision can be obtained for about a full period then one can also measure the phase curve of the planet,
- $\Delta I/I \approx 10^{-4}$ during several hours for the detection of the transits of terrestrial planets around solar type stars,
- $\Delta I/I \approx 10^{-4}$ during several hours in narrow band or spectroscopic mode for the measurement of the spectral dependence of the transit depths of giant planets.
- $\Delta I/I \approx 10^{-6}$ for the measurement of the transit spectrum of an Earth like planet around a solar-type star.

Ground based observations are limited by the atmospheric turbulence which produces photometric fluctuation at a level of about $\Delta I/I = 0.2 - 0.5$ %. This limit depends not on telescope size and therefore one can measure the 1 %-transits of giant planets around bright stars ≈ 10 mag with small 10 cm telescopes, and around faint stars ≈ 20 mag with large 8 m telescopes.

Space observations have the huge advantage that there is no disturbing atmosphere. The achievable measuring precision is then limited by instrumental effects and the photon noise. Systematic effects can easily reach a level of more than 0.1 % if the space telescope is not designed for high precision photometry or if there are unforeseen disturbing effects. On the other side it has been demonstrated with HST and the KEPLER satellite that a precision of $\Delta I/I = 10^{-4}$ can be achieved, and new instruments with a higher precision are currently built or planned.

Photon noise is a severe limit for small space instruments, because it is so expensive to build large telescope in space. The photon noise is given according to the Poisson statistics by

$$\sigma = \sqrt{N}$$

where N is the number of collected photons for the transit measurement. This means that a 3σ measuring precision of $\Delta I/I = 10^{-4}$ requires about $N \approx 10^9$ photons per hour. This can be achieved for about a 13 mag star with the KEPLER space telescope collecting all photons from 400 - 900 nm.

3.5.2 Results from the transit search programs from the ground.

There are two major types of transit searches form the ground, the RV follow-up and the wide field transit search.

RV follow-up: The follow-up search for transits of planets detected by the RV-method is very useful. The chance of making a successful detection is quite high because the presence of a planet and its approximate mass and therefore size is known. Further the transit search can be well scheduled because the epoch of a possible transit follows also from the RV curve.

Well known transiting systems which were detected by follow-up observations are:

- HD 209458 b, the first transiting extra-solar planet discovered. It is a hot Jupiter on a 3.5 day period around a bright ($m_V = 7.5$ mag) G0 V star. This is one of the brightest, and best studied transiting systems.
- HD 189733 b, a hot Jupiter on a 2.2 day period around a nearby (19.5 pc), bright ($m_V = 7.7$ mag) and quite active K2 V star. This is one of the brightest and best studied transiting systems.
- HD 80606 b is a $M_P = 4M_J$ hot Jupiter planet on an $P = 111$ days orbit with the extreme eccentricity of $\epsilon = 0.93$ around a G5 V star. The irradiation for this planet changes by a factor of 800 from apostron to periastron.

Wide field search for close-in planets. With small wide field cameras stars of magnitude 8 to 12 mag can be monitored for periodic transits by giant planets. The HAT and WASP search programs found up to now more than 100 transiting planets. Most of the found objects are giant planets on short orbits, the majority with $P = 2$ to 5 days. The found planets tend to have large radii, between 1.1 to 1.3 R_J. Such large radii are not expected for intrinsically cold (=old) giant planets. The large radii are suspected to be induced by the strong irradiation and possibly also by tidal effect. Large, giant planets on short orbits are of course the most easy objects to detect and therefore this sample is strongly biased in this direction.

The transit search surveys confirms strongly the predominance of orbital periods of $3 - 5$ days for giant planets. This must be a particularly stable configuration.

The search for transits around M-stars is a very interesting variant for ground-based planet searches. Because the radii of M-stars are only 0.1 to 0.4 $R_\odot$ transits of smaller planets produce a transit signal of more than 0.1 %. A well known example for a successful transit detection of GJ 1214 b, a Neptune-sized planet with $R_P = 2.7R_E$ on a 1.6 day orbit. Because the star is a M4.5 V star with a mass of 0.15 $M_\odot$, and only a radius of 0.21 $R_\odot$ also a Neptune sized planets is detectable from the ground. The mean density of this planet is 1.9 g/cm^3 again similar to the ice giants in the solar system. The equilibrium temperature is about 450 K.

3.5.3 Result from the follow-up observation with space telescopes

Space telescopes are the ideal follow-up instruments for transit and secondary eclipse observations. Because these instrument are in space a higher precision than with ground based observations is achievable. Follow-up observations provide successful measurements because the best observing epoch can be selected and the very valuable observing time can be invested optimally. Most important follow-up space instruments for transit measurements are HST and SPITZER. Some results from these observations will be discussed in the section on hot jupiters.

HST follow-up. A few examples of HST transit observations are shown in some of the slides. Key characteristics of HST transit observations are:

- HST achieves the highest quality transit data with low resolution spectroscopy in the visual. This mode provides transit light curves for the 300 - 900 nm range which an accuracy up to 10^{-4}.
- HST is with 2.5 m diameter a large space telescope which can also observe planetary transits for fainter stars.
- HST is a versatile observatory with many different observing modes from the far-UV to the near-IR.
- A disadvantage of HST is the high pressure factor for observing time. Therefore one needs to have a very strong science case to obtain observing time.
- Because HST orbits Earth in about 2 hours the transit observations are always interrupted. During each orbit the spacecraft goes from the night side to the day side and after an integration of about 1 hour there is always a gap of one hour. The day - night changes introduce also strong thermal effects which cause difficulties for accurate photometric measurements.

SPITZER follow up . Two typical results from SPITZER secondary eclipse and phase curve observations will be discussed in chapter on hot jupiters. Key characteristics of the SPITZER spacecraft are:

- The SPITZER spacecraft is an 85 cm telescope which provides photometric measurements in the mid-IR, at wavelength of about 3.6 μm, and 4.5 μm, 5.8 μm, and 8 μm and longer wavelength. After the consumption of the cryogen the two short wave channels 3.6 μm, and 4.5 μm can still be used for transit studies.
- the expected planet signal is large in the SPITZER range and therefore relatively easy to detect,
- SPITZER is located at the Lagrange point 4 what allows uninterrupted observations during several hours.
- a big disadvantage of SPITZER is, that it was not designed for high precision photometry. Despite this it reaches a rather good sensitivity but requires many instrumental corrections which induce quite large uncertainties.

The most important results are the measurements of the thermal emission from close-in planets via secondary eclipse observations and even phase curve measurements. This will be discussed in the Section on hot jupiters.

3.5.4 The KEPLER satellite mission

The KEPLER satellite produced with a systematic transit search a scientific revolution for the study of statistical properties of extra-solar planets. It found more than 2000 planet candidates most of which will be confirmed in the coming years. Thanks to this mission we have now a good understanding on the statistics of planets with periods of less than about 1 year.
The key characteristics of the KEPLER mission are:

- KEPLER is a modified Schmidt telescope with a entrance corrector lens of 0.95 m aperture and a 1.4 m diameter f/1 primary mirror,
- the instrument has a large field of view of $10.5^\circ \times 10.5^\circ$ or 115 square degrees,
- the satellite is located on an Earth trailing orbit and can therefore point continuously at the same sky region in the constellations Cygnus and Lyra,
- the detector system monitors the brightness of about 150'000 main sequence stars in the brightness range 11 – 15 mag,
- the system is sensitive enough to detect a single transit of an Earth-sized planet ($\Delta I/I \approx 10^{-4}$) in front of a 12 mag G2 V star.
- KEPLER is working now without major interruptions since May 2009 or more than 1400 days.

Confirmation of planet candidates. The main problem of the KEPLER mission is the verification of transit candidates. There are many variable stars in the field which produce photometric signals which look like transits. Several test must be applied to verify the presence of a real planet transit:

- the transit signal must be periodic,
- the light curve should look like a transit,
- the photo-center should remain stable between transit and out-of transit phases, to exclude the presence of blended background or foreground target, e.g. an eclipsing binary, which may introduce a transit like disturbance (so called false positive),
- a mass determination using the RV-method or transit timing variation measurements can confirm the presence of a planetary mass object.

Basic detection statistics. The following numbers were derived based on 16 months of Kepler measurements (Batalha et al. 2013, ApJS 204, 24):

- about 190'000 stellar light-curves were measured, about 130'000 during the entire period,
- about 5000 stars show periodic, transit like light curves,
- about 2300 viable planet transit candidates were identified,
- about 400 systems show transits from multiple planets, giving about 900 detected planet candidates in multiple systems,
- about 200 planets are firmly confirmed with mass determinations and many more will follow in the coming years,
- the richest system found so far is Kepler-11 with 6 transiting planets.

3.5.5 Main results from the KEPLER mission

The Kepler mission is not yet completed and only preliminary results are available. However, they give already a very good impression about statistical properties of extra-solar planets with $R_P > 2\,\mathrm{R}_E$ and $P < 50$ days (see Howard et al. 2012, ApJS 201, 15).

Frequency of extra-solar planets. The frequency of extra-solar planet can be estimated taking into account the transit probability p_{trans} which is a function of stellar radius and orbital separation. A study selecting planets with $P < 50$ days and $R_P > 2\ \mathrm{R}_E$ (the sample of easy to detect short period, "large" planets) obtained the following planet to star ratios N_P/N_S:

- there are about $N_P/N_S = 0.18$ planets per star with $R_p > 2\mathrm{R}_E$ and $P < 50$ days.
- and about $N_P/N_S = 0.01$ giant planet per star with $P < 50$ days,
- planets with small radii are much more frequent indicating that Earth-sized planets $R_P \approx 1\,\mathrm{R}_E$ and a period $P < 50$ days could be present around every third or second star or even more frequently.

Note, that our solar system with its 8 planets is a system which does not qualify to be counted in this sample ($P < 50$ days). The RV-results from the HARPS instrument are in agreement with these statistics from KEPLER.

Period distribution. The period distribution of planets follows from the transit detection rates, taking the strong period dependence of the transit probability into account. For long periods the number of detections gets small and the correction factors large. Therefore, the KEPLER transit survey provides good results for planets on orbits with short periods but not for planet with orbits > 1 year. The main findings are:

- planets with very short periods $P < 2$ days are very rare, less than 0.1 % of all stars have such a planet,
- hot Jupiters ($R_P > 8\,\mathrm{R}_E$) with $P < 10$ days occur around 0.5 % of all stars, and the same number applies also for the period range $P = 10 - 50$ days.
- There is, like in the RV-data, a pile up of Jupiters around $P = 3 - 5$ days and a clear minimum for $P = 5 - 10$ days.
- planets with radii intermediate between Neptune and Jupiter $R_P = 4 - 8\,\mathrm{R}_E$ have essentially the same frequency and period dependence like the giant planets, but there lacks the clear period minimum at $P = 5 - 10$ days in the distribution.
- small planets $R_P = 2-4\,\mathrm{R}_E$ have a similar frequency for very short periods $P = 2-4$ days like giant planets, but they are 5 to 10 times more frequent than giant planets for $P > 10$ days.

Frequency of planets for different stellar types. The KEPLER sample is large enough to investigate the planet occurrence as function of stellar type. In one of the slides the results are plotted for planets with $R_P > 2\,\mathrm{R}_E$ and $P < 50$ days. The main points are:

- for small planets, $R_P = 2 - 4\,\mathrm{R}_E$, there is a very strong dependence of the planet occurrence with stellar type with a planet to star ratio $N_P/N_S \approx 0.25$ for K and early M dwarfs (1 Neptune-sized planet with $P < 50$ days per 4 stars)

- for G-stars (the majority of the objects in the KEPLER sample) the derived planet to star ratios for small planets $R_P = 2 - 4\,\mathrm{R}_E$ is about $N_P/N_S \approx 0.15$, and for F stars about $N_P/N_S \approx 0.10$.
- larger planets $R_P > 4\,\mathrm{R}_E$ show no preference for stellar types, they seem to occur with equal ratios $N_P/N_S \approx 0.02$ around F, K, G, and M dwarfs.

Multiple planets. Thanks to the many multi-planet detections by KEPLER one can investigate the properties of multiple systems. First of all, systems with multiple transiting planets are frequent among the 1405 stars with transits (Fabricki et al. 2013, ApJ 768, 14):

- for 1044 stars (74 %) only one transiting planet was detected,
- for 242 stars (17 %) transits of two planets were detected,
- 85 stars (6 %) show transits of three planets,
- 25 stars (1.8 %) show four planets with transits,
- 8 stars (0.6 %) show five planets,
- 1 star shows six planets (Kepler-11).

These are of course only lower limits since the analysis of more data and more studies will reveal more transiting planets. Form these statistics one can infer:

- systems with multiple transiting systems are frequent, indicating first that multiple planetary systems are frequent and second that the orbits are often close to coplanar,
- for close-in Jupiters there are essentially no additional second planet with transits found indicating that there is rarely a second co-planar planet in these systems,
- in systems with multiple transiting planets the innermost planet tends to be a small planet,
- period ratios between planets are larger than $P_{\mathrm{out}}/P_{\mathrm{in}} > 1.25$ defining an observational limit for the "planet packing" density.
- planets are typically not in orbital resonance, but they prefer period ratios which are just a bit larger than 3:2 or 2:1 and avoid ratios just below this value.

3.6 The empirical mass-radius relation for planets

Transiting planets with radius determinations and mass determination can be used for empirical mass radius $M_P - R_P$ and radius-density $R_P - \bar{\rho}$ diagrams. Such diagrams are shown in one of the slides. Ground-based data provide mainly a sample of giant planets with short periods while KEPLER provides data for smaller planets.

For the giant planets the masses are determined with the RV method. Masses for some KEPLER planets are derived based on transit timing variations (see Section 3.6. Errors in the mass and radius determinations are small for the giant planets while the errors for the masses for low mass planets are quite large. Additional, but smaller error sources are the uncertainties in the mass and radius estimates for the host stars. The sample was devided into lower mass planets $M_P < 150\,\mathrm{M_E} = 0.5\,\mathrm{M}_J$ and higher mass planets $M_P > 0.5\,\mathrm{M}_J$. There is also a systematic difference between planets with higher and lower incident flux, where the border line is just the median irradiation value. This median value corresponds to an irradiation per unit area which is 800 times stronger than for Earth.

The following properties for the mass M_P, radius R_P and mean density $\bar{\rho}$ can be derived (see slide):

- giant planets with masses $> 100\,\mathrm{M}_E$ have without exception radii in the range $R_P = 10 - 20\,\mathrm{R}_E = 1 - 2\,\mathrm{R}_J$,
- the strongly irradiated giant planet have clearly a larger radius,
- lower mass planets $M_P < 150\,\mathrm{M}_E$ show a strong radius-mass dependence which can be described roughly by the relation

$$\frac{R_P}{R_E} \approx \Big(\frac{M_P}{M_E}\Big)^{0.5},$$

- the scatter around this curve is large since there are $10\,\mathrm{M}_E$ planets with small radii $R_P \approx 1.5\,\mathrm{R}_E$ and such with large radii $R_P \approx 6\,\mathrm{R}_E$,
- strongly irradiated Neptune-mass planets tend to be small,
- the density for giant planets increases from about $\bar{\rho} = 0.3\,\mathrm{g/cm^3}$ at $M_P \approx 0.3\,\mathrm{M}_J$ to $\bar{\rho} = 10\,\mathrm{g/cm^3}$ for $M_P \approx 10\,\mathrm{M}_J$,
- for low mass planets there is a huge spread in density which can be as low as $\bar{\rho} \approx 0.4\,\mathrm{g/cm^3}$ or as high as $\bar{\rho} = 10\,\mathrm{g/cm^3}$ for planets with $M_P = 10\,\mathrm{M}_E$
- clearly, the strongly irradiated low mass planet are the ones with very high densities.

Interpretation. The available data can be interpreted as follows. Giant planets become not larger with higher mass because their matter is simply more compressed if the pressure increases. The enhanced radii due to strong irradiation is an important process for close-in planets. Lower mass planets show a strong variety of densities. This might point to the fact that irradiation and evaporation has a strong impact on these planets. Low mass planets with mean densities of $\bar{\rho} \approx 10\,\mathrm{g/cm^{-3}}$ could be large rocky / iron cores which have lost their lower density H envelope. Of course it would be interesting to compare the currently available data with planets at larger distances, which are only slightly affected by irradiation. This may be possible in the future.