Vetores de $\mathbb{R}^n$ $\quad$ $\mathbb{R}^2 : (x,y) = v, \vec{v}$

$\mathbb{R}^3 : (x,y,z)$

$x \in \mathbb{R}^n \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \left(\in \mathbb{R}^{n \times 1}\right)$

Matrizes do $\mathbb{R}^{m \times n}$, $A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$

Produto interno

e.g. $\vec{v} \circ \vec{w}$ ou $\langle \vec{v}, \vec{w} \rangle = v_1 \cdot w_1 + v_2 \cdot w_2 + \ldots + v_n \cdot w_n$

$$\underbrace{v^T w}_{1\times n \;\; n\times 1} = \sum_{i=1}^{n} v_i w_i$$

$$\alpha v + \beta w = \begin{bmatrix} \alpha v_1 + \beta w_1 \\ \vdots \\ \alpha v_n + \beta w_n \end{bmatrix} \qquad v = (v_1, \ldots, v_n)^T$$

$$y = Ax, \quad A \in \mathbb{R}^{m \times n}, \; x \in \mathbb{R}^n, \; y \in \mathbb{R}^m$$

$$y_i = \sum_{j=1}^{n} a_{ij} x_j, \quad i = 1, \ldots, m$$

Linha i de A é $a_i \in \mathbb{R}^n$, $i = 1, \ldots, m$

$$A = \begin{bmatrix} a_1^T \\ \vdots \\ a_m^T \end{bmatrix} \qquad a_{ij} = (a_i)_j$$

Coluna j de A é $A_j \in \mathbb{R}^m$, $j = 1, \ldots, n$

$$A = [A_1 \,\vdots\, A_2 \,\vdots\, \cdots \,\vdots\, A_n]$$

Fatoração LU

$$PAQ^T = LU \qquad L = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ * & \ddots & 1 \end{bmatrix} \quad U = \begin{bmatrix} \ddots & * \\ 0 & \ddots \end{bmatrix}$$

pivoteamento parcial $\Rightarrow Q = I$

sem pivot. $P = I$ e $Q = I$.

$$PA = LU$$

transposta $\Rightarrow$ $A^T P^T = U^T L^T$

$$P^T P = I$$

$Ax = b$ | $A^T y = c$

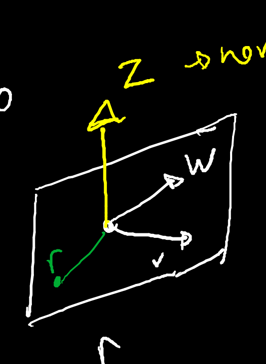
$PAx = Pb$ | $A^T P^T P y = c$

$LUx = Pb$ | $U^T L^T (Py) = c$

$\begin{cases} Ly = Pb \\ Ux = y \end{cases}$ | $\begin{cases} U^T w = c \\ L^T z = w \\ y = P^T z \end{cases}$

Hiperplano

$\mathbb{R}^3$



$v \times w$ paralelo a z

$$z = v \times w$$

$$r - r_0 \perp z$$

Plano: $r - r_0 \perp z \Rightarrow (r - r_0)^T z = 0$ (fixo, fixo)

$\hookrightarrow (r_1, r_2, r_3)^T$

$$r^T z = \underbrace{r_0^T z}_{\text{cte}}$$

Hiperplano: $a^T x = b$, $a \in \mathbb{R}^n$, $b \in \mathbb{R}$

todo x t.q. ↗

Semiespaço: $\boxed{a^T x \geq b}$

$f : \mathbb{R}^n \to \mathbb{R} \qquad f(x)$

$\hookrightarrow x = (x_1, \ldots, x_n)^T$

$$\nabla f(x) = \begin{bmatrix} \partial f / \partial x_1 \\ \vdots \\ \partial f / \partial x_n \end{bmatrix}$$

$$f(x) = x_1 x_4 + e^{x_2 + 2x_3} - \ln\left(x_1^2 + x_3^2 + 1\right)$$

$$\nabla f(x) = \begin{bmatrix} x_4 - \dfrac{2x_1}{x_1^2 + x_3^2 + 1} \\ e^{x_2 + 2x_3} \\ 2e^{x_2 + 2x_3} - \dfrac{2x_3}{x_1^2 + x_3^2 + 1} \\ x_1 \end{bmatrix}$$

Derivada direcional de f diferenciável

$$\lim_{t \to 0^+} \frac{f(x + td) - f(x)}{t} = \nabla f(x)^T d$$

Função linear: $f(x) = c^T x$, $c \in \mathbb{R}^n$

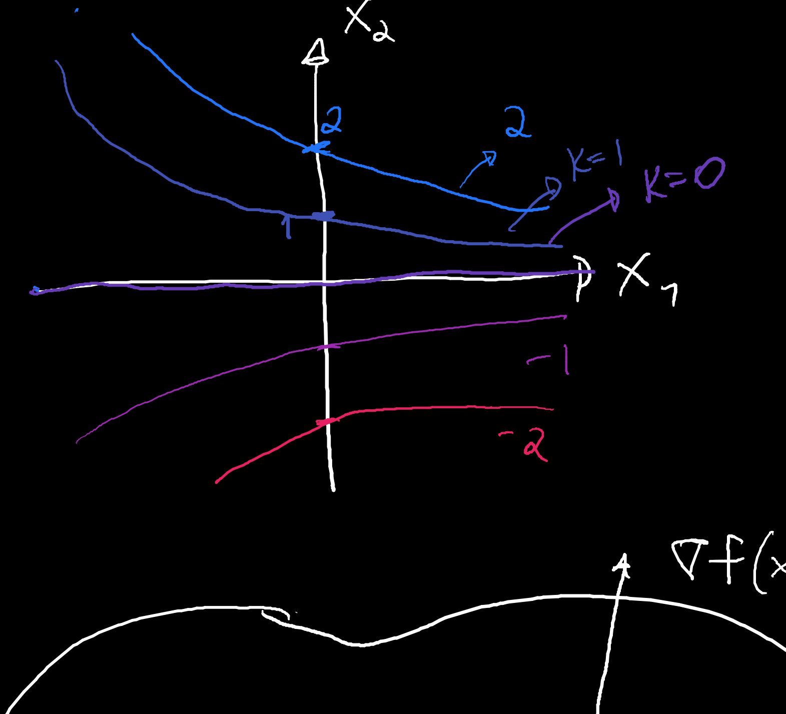
$$\nabla f(x) = c.$$

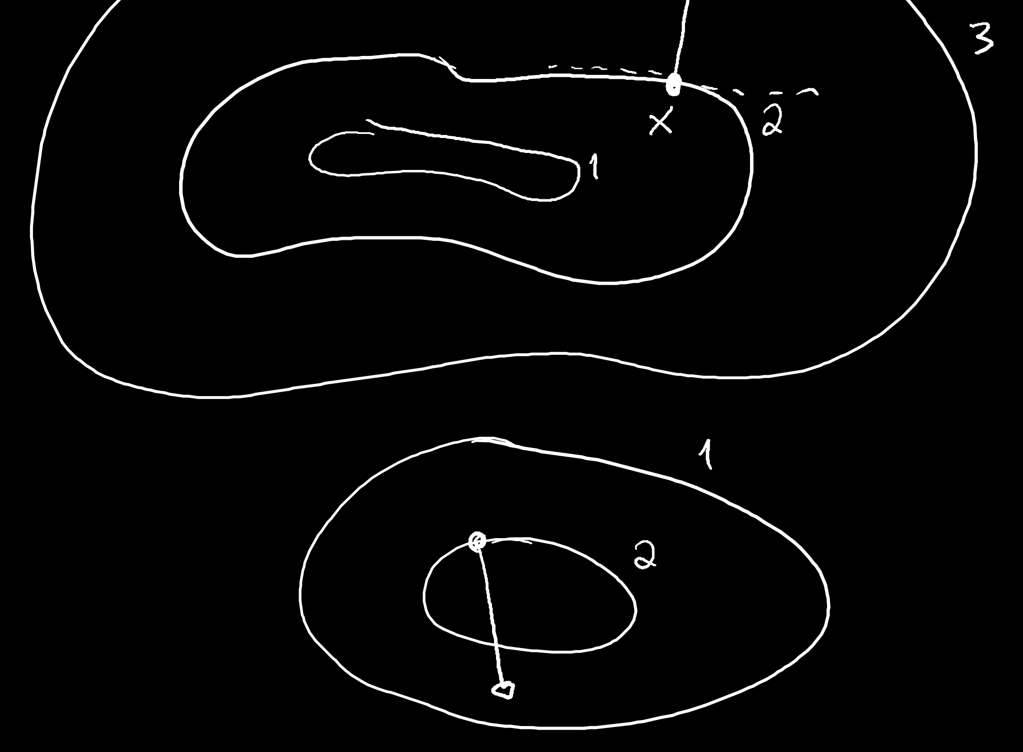
Curvas de nível $\quad f : \mathbb{R}^2 \to \mathbb{R}$

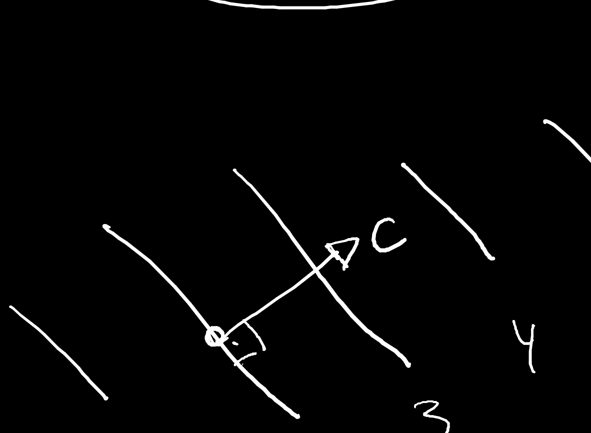
Nível $K \in \mathbb{R}$ é o conj. $\boxed{f(x) = K}$

$$f(x) = x_2 e^{x_1}$$

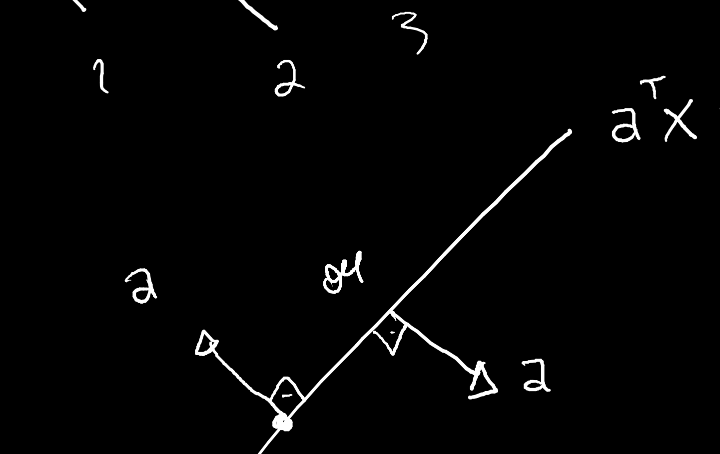
$$f(x) = K \Rightarrow \boxed{x_2 e^{x_1} = K} \Rightarrow x_2 = K \cdot e^{-x_1}$$





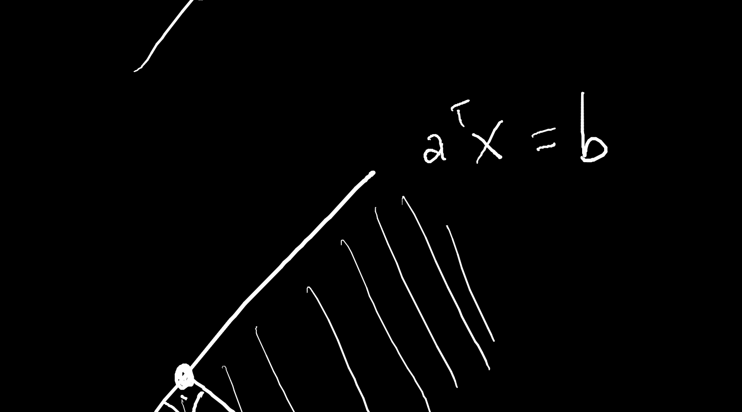


$c^T x$



$a^T x = b$

$a_1 x_1 + a_2 x_2 = b$



$a^T x \geq b$