Graphs

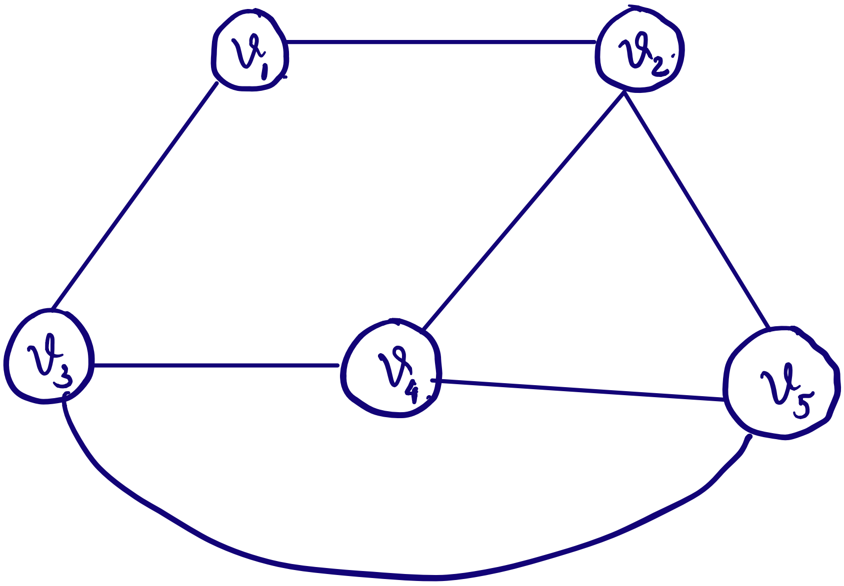
A graph G, consists of two sets, V and E.

V is a finite, non-empty set of vertices

E is a set of pairs of vertices; these pairs are called edges.

Representation : $G = (V, E)$

Example:-



$V = \{ v_1, v_2, v_3, v_4, v_5 \}$

$E = \{ (v_1, v_2), (v_1, v_3), (v_2, v_4), (v_2, v_5),$

$(v_3, v_4), (v_3, v_5), (v_4, v_5)\}$

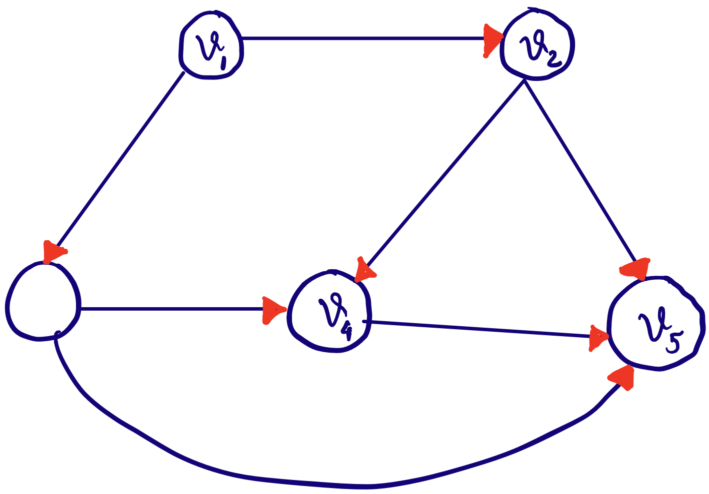
Observation:-

Given V (vertices set), then $E \subseteq V \times V$

In above example,

Graph is undirected graph because we have not made any distinction between an edge (v_1, v_2) and (v_2, v_1) or (v_2, v_4) and (v_4, v_2), etc. So in undirected graph, pairs of vertices are unordered.

In directed graph, see the example below



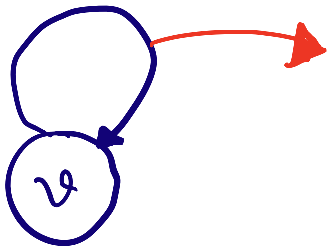
Edges (v_1, v_2) and (v_2, v_1) are different.

So in directed graph these pairs are ordered pairs.

In above graph there is an edge from v_1 to v_2 but not from v_2 to v_1

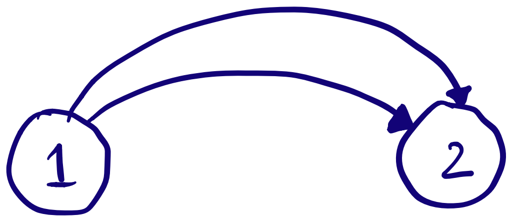
For simplicity, we put following restrictions on the graphs.

(i) A graph may not have an edge from a vertex v, back to itself.

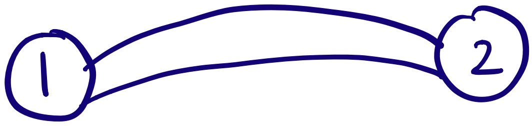


self loop or self edges.

(ii) A graph may not have multiple occurences of the same edge. If we remove this restriction, we get multigraph.



or



undirected graph

Maximum no. of unordered pairs (u, v) is a graph $G=(V,E)$ with n vertices (i.e., $|V| = n$) is $^{n}C_2 = \frac{n(n-1)}{2}$

If an undirected graph $G=(V,E)$ with $|V| = n$, with exactly $n(n-1)/2$ edges is said to be complete graph.

For directed graph $G=(V,E)$ such that $|V|=n$, maximum number of edges possible = $n(n-1)$

If $(u,v) \in E$ then vertices 'u' and 'v' are adjacent vertices.

Path : A path from vertex v to u in the graph G is a sequence of vertices

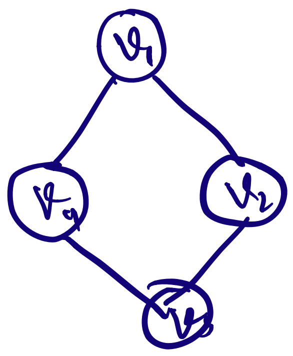
$$v, v_1, v_2, v_3, \ldots, v_n, u.$$

such that

$$(v, v_1), (v_1, v_2), (v_2, v_3) \ldots, (v_n, u) \in E(G)$$

The length of the path is no. of edges on it.

A Simple path is a path in which all the vertices except possibly the first and last are distinct.



v_1, v_2, v_3, v_4, v_1

A cycle is a simple path in which first and last vertices are same.

Graph Representation

→ Adjacency matrix Representation

→ Adjacency list Representation

$G = (V, E)$ such that $|V| = n$. $(n \geq 1)$

Adjacency matrix of G is a two dimensional matrix of size $n \times n$.

$A_{n \times n}$ with property.

$$A[i][j] = 1 \text{ iff } (i, j) \in E$$
$$= 0 \text{ otherwise.}$$