

Working of Binary Indexed Tree

Let the input array be A and the Binary Indexed array be BIT.

- **getSum():** returns the sum of the subarray [0, ..., x] from BIT[] arr.

Algorithm:

1. Initialize the sum as 0, and the curIdx as x+1.
2. Do the following if curIdx is greater than 0
 - a. Add the BIT[curIdx] to the sum.
 - b. Update curIdx as

$$\text{parentIdx} = \text{curIdx} - (\text{curIdx} \& (-\text{curIdx}))$$
$$\text{curIdx} = \text{parentIdx}$$

- c. Repeat step 2.
3. Return sum.

- **update():** updates the BIT array if we change the original array as $\text{arr}[\text{curIdx}] += \text{val}$.

Algorithm:

1. Initialize curIdx as x+1.
2. Do the following if curIdx is less than or equal to n
 - a. Add the value to BIT[curIdx] i.e,

$$\text{BIT}[\text{curIdx}] += \text{val}$$

- b. Update curIdx to child index where childIndex is calculated as.

$$\text{childIndex} = \text{curIdx} + (\text{curIdx} \& (\text{curIdx}))$$
$$\text{curIdx} = \text{childIndex}$$

- c. Repeat step 2.
3. Return sum.

Implementation

- **Update()**

```
function update(n, curIdx, val, BIT[]){
    curIdx = curIdx + 1

    while curIdx <= n
        BIT[curIdx] = BIT[curIdx] + val
        childIndex = curIdx + (curIdx & (-curIdx))
        curIdx = childIndex
}
```

- **construction()**

Initially, all values in BIT[] are equal to 0. Then, we call the update() function for each element of the given array A to construct the Fenwick Tree.

```
function construction(n, A[]) {  
  
    // initialize a new array BIT[] equal to size of array A + 1  
    i = 1  
    while i <= n  
        BIT[i] = 0  
        i++  
  
    i = 0  
    // call update function for every element of the arr  
    while i <= n  
        update(n, i, A[i], BIT)  
        i++  
  
}
```

- **getSum()**

```
function getSum(n, curIdx, BIT[]){  
    sum = 0  
  
    while curIdx > 0  
        sum = sum + BIT[curIdx]  
        parentIndex = curIdx - (curIdx & (-curIdx))  
        curIdx = parentIndex  
  
    return sum  
}
```

Time Complexity: The maximum number of set bits in the binary representation of a number n is $O(\text{Log}n)$. Therefore, we traverse at-most $O(\text{Log}n)$ nodes in both getSum() and update() operations. The time complexity of the construction is $O(n\text{Log}n)$ as it calls update() for all n elements.

Now for calculating the sum in some specific range (l,r), we can say that

$$\text{rangeSum}(l, r) = \text{getSum}(r) - \text{getSum}(l-1).$$