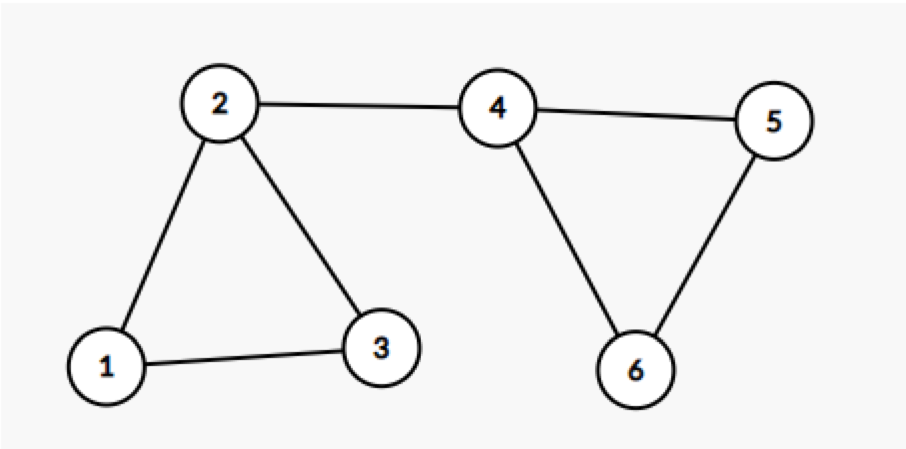
Bridge Trees

Bridge component of a graph

Given an undirected graph G, a bridge component of G is defined as the maximal connected subgraph which does not contain any bridge. Hence the bridge components of a graph are basically connected graphs(components) having cycles in which there exist at least 2 simple paths connecting every pair of vertices in that component or a pair of nodes connected by an edge.

For Example:



In the above graph, there are **2** bridge components in the graph connected by the bridge 2-4:

1. (1, 2, 3)
2. (4, 5, 6)

Bridge Tree

Given an undirected connected graph G, the **Bridge Tree** of G is defined as the graph obtained by shrinking each of its bridge components into a single node connected by bridge edges. The graph obtained will be a **'tree'** where each node will represent a bridge component of G and an edge representing one of the bridges of G.

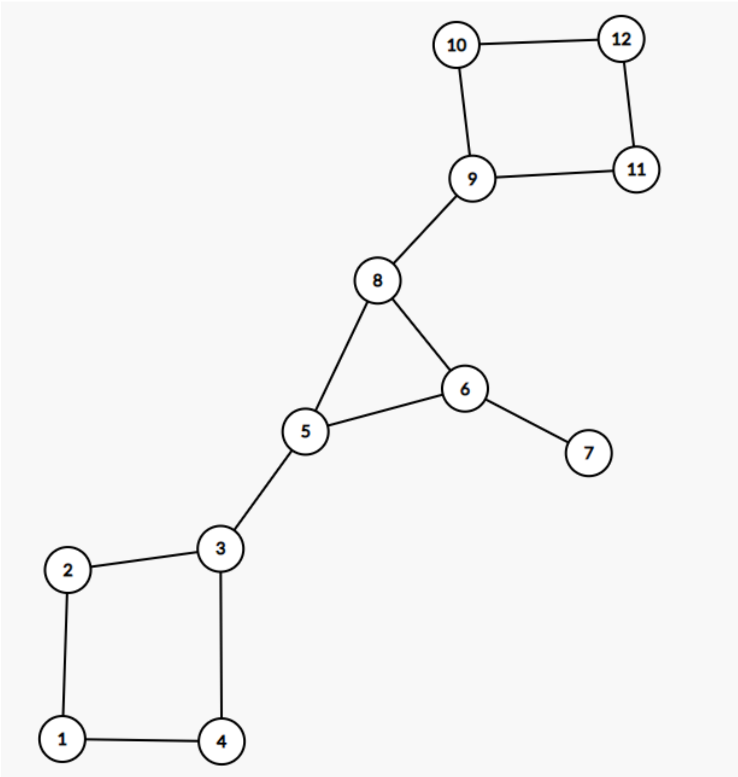
The claim for the obtained graph(Bridge Tree) to be necessarily a tree can be proved by contradiction. Let us assume that the graph obtained(Bridge Tree) is not a tree i.e the graph will have at least one cycle, by definition the edges of the obtained graph represent the bridge edges of the original graph, however deleting any edge of the cycle will not disconnect the graph. Hence by contradiction, the obtained graph(Bridge Tree) is a 'tree'.

For Example:

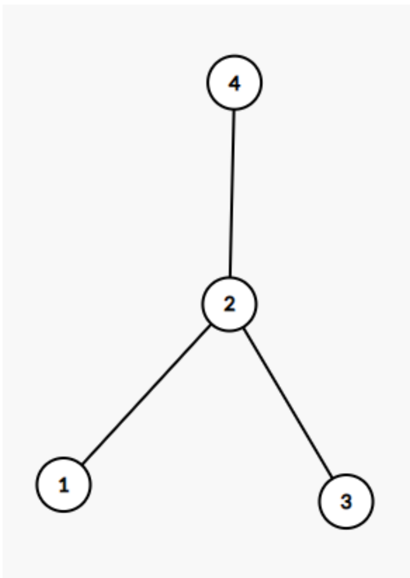
Let us assume we are given the following graph with different bridge components as:

1. (1, 2, 3, 4)
2. (5, 6, 8)
3. (6, 7)

4. (9, 10, 11, 12)



Converting each of the bridge components of a graph into a single node, we get the following graph which is known as the **Bridge Tree**.



In the above graph, the nodes representing the bridge components are:
1 => (1, 2, 3, 4)
2 => (5, 6, 8)
3 => (6, 7)

4 => (9, 10, 11, 12)

The bridge tree of a graph having N nodes can have at most N nodes and N - 1 edges i.e the initial graph is a tree itself as in a tree every edge is a bridge.