

Knuth Morris Pratt Algorithm(KMP ALgorithm)

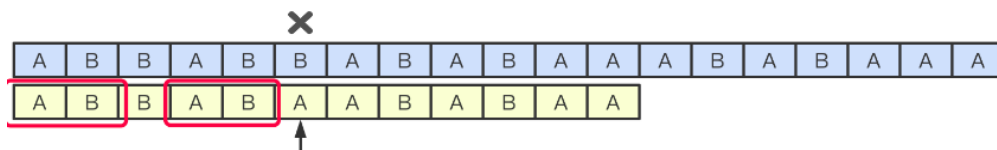
We first define the prefix function of the string - The prefix function of a string s is an array lps of length same as that of s such that $lps[i]$ stores the information about the string $s[0..i]$. It stores the length of the maximum prefix that is also the suffix of the substring $s[0..i]$.

For example :

For the pattern "AAAABAA",

$lps[]$ is $[0, 1, 2, 0, 1, 2, 3]$

$lps[0]$ is 0 by definition. The longest prefix that is also the suffix of string $s[0..1]$ which is AA is 1. (Note that we are only considering the proper prefix and suffix). Similarly, For the whole string AAABAAA it is 3, hence the $lps[6]$ is 3.



Algorithm for Computing the LPS array.

- We compute the prefix values $lps[i]$ in a loop by iterating from 1 to $n - 1$.
- To calculate the current value $lps[i]$ we set the variable j denoting the length of best suffix for $i - 1$. So $j = lps[i - 1]$.
- Test if the suffix of length $j + 1$ is also a prefix by comparing $s[j]$ with $s[i]$. If they are equal then we assign $lps[i] = j + 1$ else reduce $j = lps[j - 1]$.
- If we have reached $j = 0$ we assign $lps[i] = 0$ and continue to the next iteration .

```
function PrefixArray(s)
    n = s.length;
    // initialize to all zeroes
    lps = array[n];

    for i from 1 to n - 1
        j = lps[i-1];
        // update j untill s[i] becomes equal to s[j]
        while j greater than 0 && s[i] no equal to s[j]
            j = lps[j-1];

        // if extra character matches increase j
        if s[i] equal to s[j]
            j += 1;

        // update lps[i]
```

```
lps[i] = j;

// return the array
return lps
```

Time Complexity: $O(N)$, where N is the length of the pattern.

Algorithm for searching the pattern.

Now consider a new string $S' = \text{pattern} + \# + \text{text}$ where $+$ denotes the concatenation operator. Now, what is the condition that pattern appears at position $[i - M + 1 \dots i]$ in the string text. The $\text{lps}[i]$ should be equal to M for the corresponding position of i in S' . Note that $\text{lps}[i]$ cannot be larger than M because of the $\#$ character.

- Create $S' = \text{pattern} + \# + \text{text}$
- Compute the lps array of S'
- For each i from $2*M$ to $M + N$ check the value of $\text{lps}[i]$.
- If it is equal to M then we have found an occurrence at the position $i - 2*M$ in the string text.

```
function StringSearchKMP(text, pattern)
    // construct the new string
    S' = pattern + '#' + text

    // compute its prefix array
    lps = PrefixArray(S')
    N = text.length
    M = pattern.length

    for i from 2*M to M + N
        // longest prefix match is equal to the length of pattern
        if lps[i] == M
            // print the corresponding position
            print the occurrence i - 2*M

    return
```

Time Complexity: $O(N + M)$, where N is the total length of the pattern and M is the length of the pattern we need to search.