Bipartite Graphs

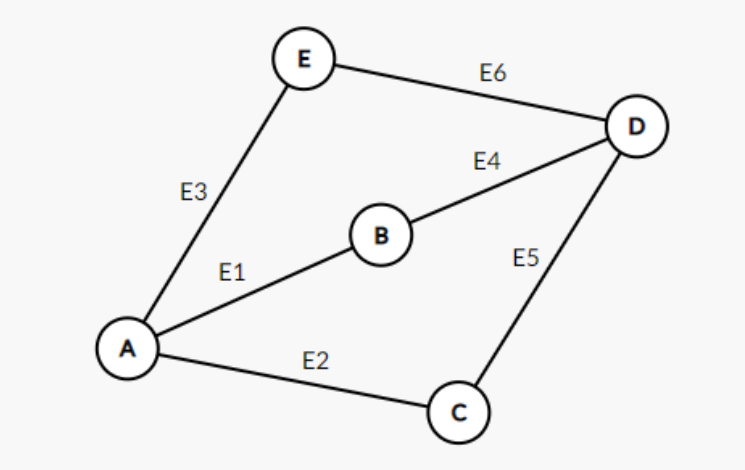
In bipartite graphs, we divide the graphs into two disjoint sets, and each edge connects a vertex from one set to a vertex in another subset.

A simple graph G = (V, E) is called a bipartite graph if its vertices can be divided into two disjoint sets $V = V_1 \cup V_2$, such that every edge has the form e = (a, b) where a belongs to V_1 and b belongs to V_2. One important condition is that no vertices belonging to the same set are connected i.e. no two vertices in V1(or V2) are connected

Properties of Bipartite Graphs

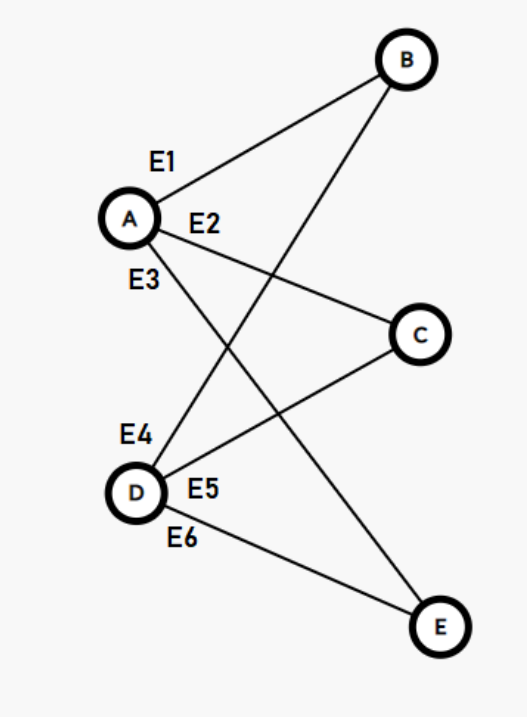
- A Graph is called bipartite if and only if the given graph does not have an odd length cycle
- A complete bipartite graph $K_{m.n.}$ is a bipartite graph that has each vertex from one set adjacent to each vertex from another set.

Consider the graph G as shown below



Now, to satisfy the definition of a bipartite graph, the vertex set needs to be partitioned into two sets such that each edge joins the two vertex sets.

Is it possible to partition the vertex set of G so that it satisfies the bipartite graph definition? Let's find out:



This is the same graph G, just with a different representation. Here, we partition the vertex set V = (A, B, C, D, E)into two disjoint vertex sets V_1 = (A, D) and V_2 = (B, C, E) .

Also, we can see the edges from the edge set E = (E_1, E_2, E_3, E_4, E_5) follow the definition of a bipartite graph. Each edge has one endpoint in V_1 and another endpoint in V_2. There are no edges whose both endpoints belong to V_1 or V_2.

Therefore, we can conclude that graph G is a bipartite graph.