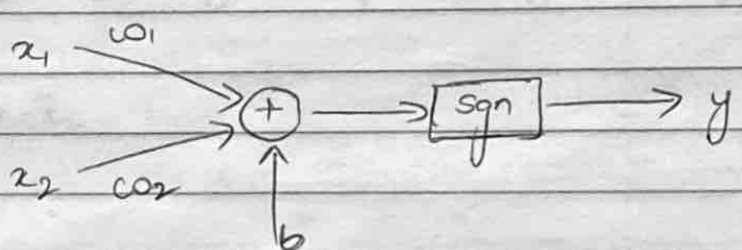
1. $f(x_1, x_2, x_3) = \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2$

$-1 \to$ FALSE $\quad 1 \to$ TRUE

$sgn(x)$ - activation function.

x_2	$\bar{x}_2$	x_1	$x_1 \bar{x}_2$
-1	1	1	1
-1	1	-1	-1
1	-1	-1	-1
1	-1	1	-1



$y = sgn(\omega_1 x_1 + \omega_2 x_2 + b)$

$y = 1$ if $\omega_1 x_1 + \omega_2 x_2 + b > 0$

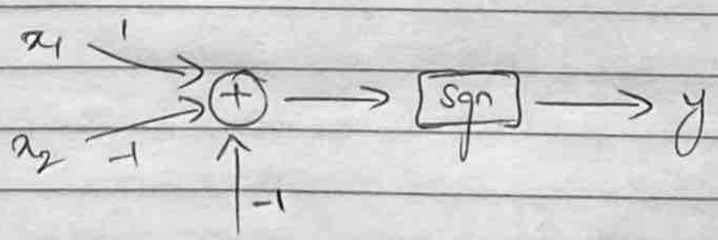
$(x_1, x_2) = (1, -1) \quad y = 1$

$\omega_1 - \omega_2 + b > 0$

~~let $\omega_1 = 1$ $\omega_2 = -1$ $b = 1$~~

~~$y = sgn(x_1 + x_2 + 1)$~~

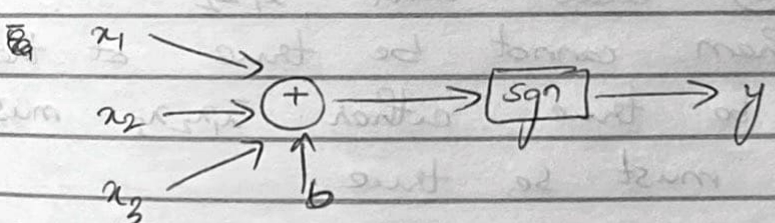
let $\omega_1 = 1 \quad \omega_2 = -1 \quad b = -1$



this network satisfies $x_1 \bar{x}_2$

$\bar{x}_1 x_2 x_3$

x_1	$\bar{x}_1$	x_2	x_3	$\bar{x}_1 x_2 x_3$
1	-1	1	1	-1
$\cancel{0}$	-1	$\cancel{0}$-1	1	-1
	-1	-1	-1	-1
+1	-1	1	-1	-1
-1	1	1	1	1
	1	-1	1	-1
			-1	-1
	1	-1	1	-1



$y = \text{sgn}(x_1 w_1 + w_2 x_2 + w_3 x_3 + b)$

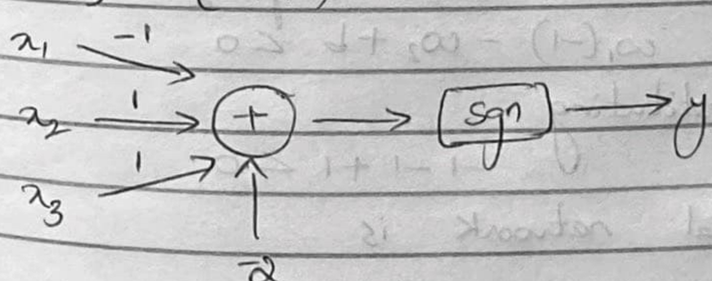
$= 1 \quad w_1 x_1 + w_2 x_2 + w_3 x_3 + b > 0$

$= -1 \quad w_1 x_1 + w_2 x_2 + w_3 x_3 + b < 0$

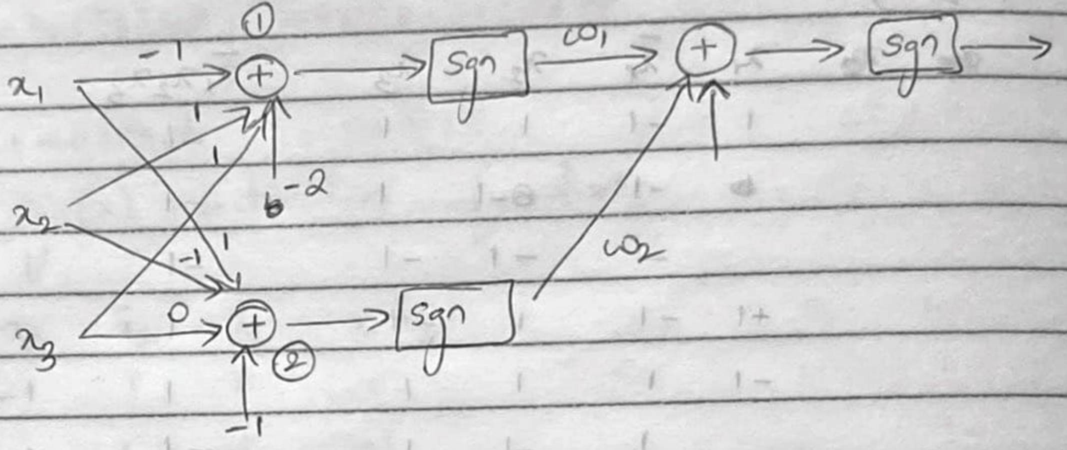
at $(x_1, x_2, x_3) = (-1, 1, 1) \quad y = 1$

$-w_1 + w_2 + w_3 + b > 0$

$w_1, w_2\ w_3 = (-1, 1, 1) \quad b = -2$



this network satisfies $\bar{x}_1 x_2 x_3$



upper part of the network (or) perceptron ① deals with $\bar{x}_1 x_2 x_3$

perceptron ② deals with $x_1 \bar{x}_2$

both of them cannot be true at the same time

so f to be true either $\bar{x}_1 x_2 x_3$ must be true (or) $x_1 \bar{x}_2$ must be true

$\omega_1(\text{output}_1) + \omega_2(\text{output}_2) + b > 0$

if either output_1 (or) output_2 is 1

let $\text{output}_1 = 1$ then $\text{output}_2 = -1$ as

$\bar{x}_1 x_2 x_3$ to be $x_2 = 1$ $\bar{x}_2 = -1$ hence $x_1 \bar{x}_2 = -1$

$\omega_1 - \omega_2 + b > 0$

$\omega_1 = +1$ $\omega_2 = +1$ $b = 1$

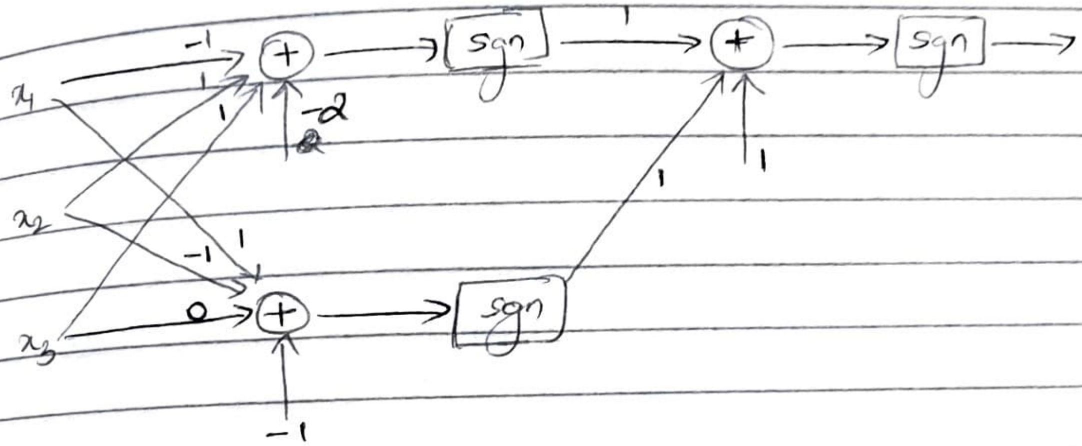
if $\text{output}_1 = -1$ and $\text{output}_2 = -1$

$\omega_1(-1) - \omega_2 + b < 0$

substituting

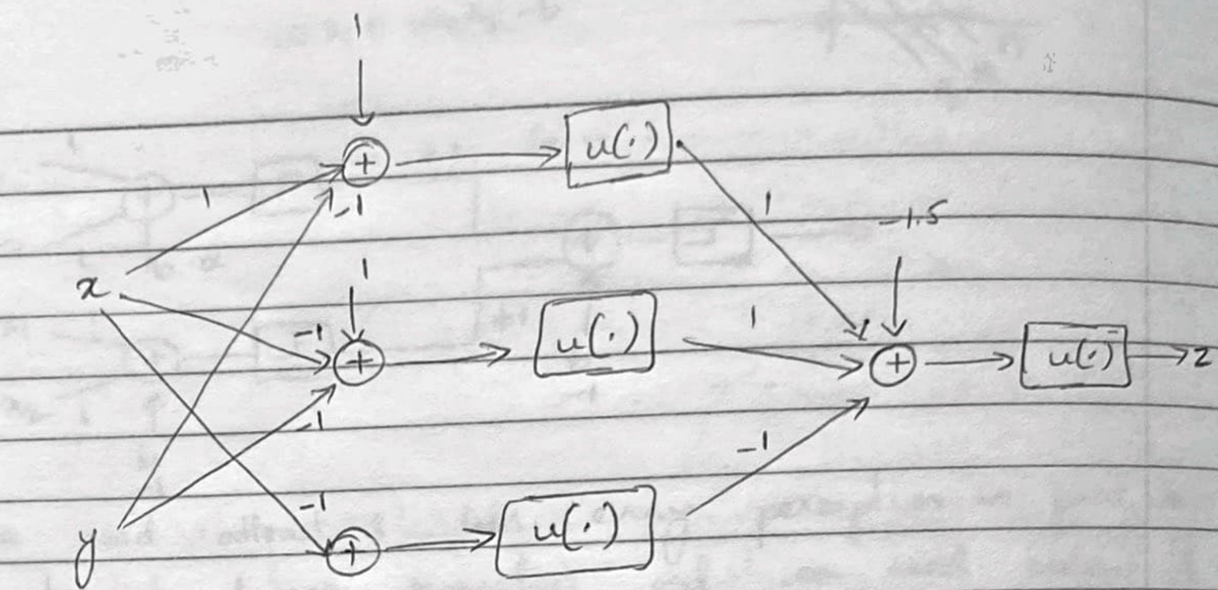
$-1 - 1 + 1 < 0$

hence final network is



x_1
x_2
x_3
sgn
sgn
sgn
-1
1
1
-2
0
-1
1
1
1
1
-1

2.



$$z = u\left(\underbrace{u(x-y+1) + u(-x-y+1) - u(-x) - 1.5}\right)$$

for this expression to be greater than zero.
such that z can be 1

$$u(x-y+1) + u(-x-y+1) - u(-x) - 1.5 \geq 0$$

$$z = \begin{cases} 1 & \text{if } u(x-y+1) + u(-x-y+1) - u(-x) - 1.5 \geq 0 \\ 0 & \text{else} \end{cases}$$

for this to be greater than zero.

$u(x-y+1)$ and $u(-x-y+1)$ should be one
and $u(-x)$ should be zero.

~~If u(-x) is zero~~
$u(-x)$ to be equal to zero x should be positive.

for $z=1$ x should be greater than zero.

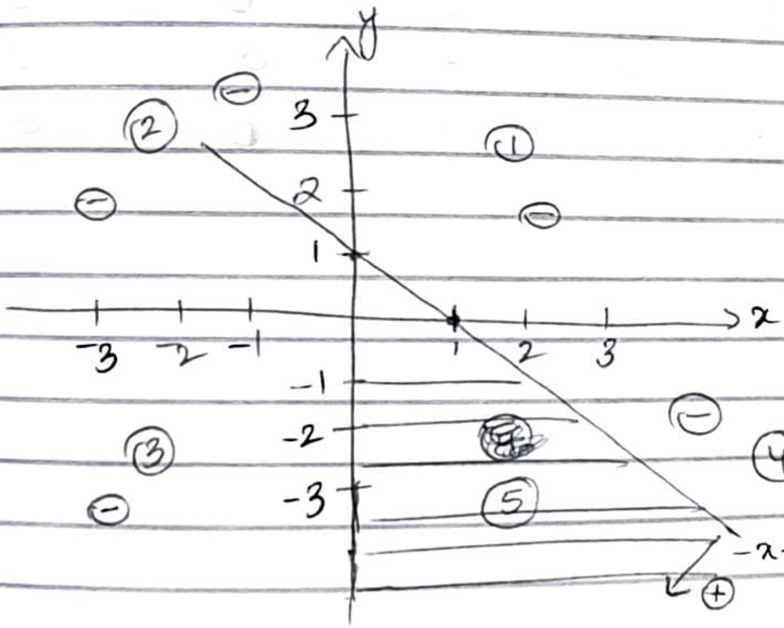
$-x-y+1=0$

$x+y-1=0$

and for $u(x-y+1)$ and $u(-x-y+1)$ to be greater 1
$x-y+1 \geq 0$ $\quad -x-y+1 \geq 0$ and $u(x)$

in order for this to happen
$x > y$ $\quad -y-x$ should be positive
$-x-y \geq 0$

$-x-y+1 \geq 0$ if both x and y are positive
this cannot happen.



in coordinate axes we can eliminate ①, ②, ③ regions for $z=1$

the only region left is ④
as said $x \geq 0$ for z to be 1
so in equation $-x-y+1 \geq 0$ $\quad x$ will be positive
so for $-x-y+1 \geq 0$ $\quad$ ~~$y \geq x+1$~~ y should be negative and $-y \geq x+1$
$-x-y+1=0$ line is drawn in the graph
~~$-x-y+1 \geq 0$ is the shaded region~~
$-x-y+1 \geq 0$ downward to the graph and $x=0$ is
as x cannot be less than zero. the region ⑤ is the region where
the boundary. so the region $z=1$