Problem 2.7.6

Find similar matrix for $A = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

Find Matrix Eigenvalues ...

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} (1-\lambda) & -2 & 0 \\ 0 & (2-\lambda) & 0 \\ 0 & 0 & (-2-\lambda) \end{bmatrix} = 0$$

$$\begin{aligned}
(1-\lambda)((2-\lambda)\times(-2-\lambda) - 0\times 0) - (-2)(0\times(-2-\lambda) - 0\times 0) + 0(0\times 0 - (2-\lambda)\times 0) &= 0 \\
(1-\lambda)((-4+\lambda 2) - 0) + 2(0-0) + 0(0-0) &= 0 \\
(1-\lambda)(-4+\lambda 2) + 2(0) + 0(0) &= 0 \\
(-4+4\lambda+\lambda 2-\lambda 3) + 0 + 0 &= 0 \\
(-\lambda 3+\lambda 2+4\lambda-4) &= 0 \\
-(\lambda-1)(\lambda-2)(\lambda+2) &= 0 \\
(\lambda-1) = 0 or (\lambda-2) = 0 or (\lambda+2) &= 0
\end{aligned}$$

$\therefore$ The eigenvalues of the matrix A are given by $\lambda = -2, 1, 2,$

Eigen vector for $\lambda = -2$

$$v_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Eigen vector for $\lambda = 1$

$$v_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Eigen vector for $\lambda = 2$

$$v_3 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 1 & -2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$P^{-1} = \frac{1}{|P|} adj(P)$$

To find $|P|$:

$$|A| = \begin{vmatrix} 0 & 1 & -2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

$$= 0 \times \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} - 1 \times \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} - 2 \times \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix}$$

$$= 0 \times (0 \times 0 - 1 \times 0) - 1 \times (0 \times 0 - 1 \times 1) - 2 \times (0 \times 0 - 0 \times 1)$$

$$= 0 \times (0 + 0) - 1 \times (0 - 1) - 2 \times (0 + 0)$$

$$= 0 \times (0) - 1 \times (-1) - 2 \times (0)$$

$$= 0 + 1 + 0$$

$$= 1$$

To find adjoint of P

$$adj(P) = adj \begin{bmatrix} 0 & 1 & -2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} +\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} & +\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} \\ -\begin{vmatrix} 1 & -2 \\ 0 & 0 \end{vmatrix} & +\begin{vmatrix} 0 & -2 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \\ +\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \end{bmatrix}^T$$

$$adj(P) = \begin{bmatrix} +(0\times 0 - 1\times 0) & -(0\times 0 - 1\times 1) & +(0\times 0 - 0\times 1) \\ -(1\times 0 - (-2)\times 0) & +(0\times 0 - (-2)\times 1) & -(0\times 0 - 1\times 1) \\ +(1\times 1 - (-2)\times 0) & -(0\times 1 - (-2)\times 0) & +(0\times 0 - 1\times 0) \end{bmatrix}^T$$

$$= \begin{bmatrix} +(0+0) & -(0-1) & +(0+0) \\ -(0+0) & +(0+2) & -(0-1) \\ +(1+0) & -(0+0) & +(0+0) \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P^{-1} = \frac{1}{|P|} adj(P) = \frac{1}{1} \times \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Since,

$$B = P^{-1}AP = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & -2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & -2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 0 \\ 0 & 2 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 & -2 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

To verify the solution:

$$trace(A) = trace\begin{bmatrix} 1 & -2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix} = 1 + 2 + (-2) = 1$$

$$trace(B) = trace\begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = (-2) + 1 + 2 = 1$$

Eigen values of A=$-2, 1, 2$
Eigen values of $B = -2, 1, 2$

$$|A| = \begin{vmatrix} 1 & -2 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -2 \end{vmatrix} = 1 \times \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} - (-2) \times \begin{bmatrix} 0 & 0 \\ 0 & -2 \end{bmatrix} + 0 \times \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix}$$

$$= 1 \times (2 \times (-2) - 0 \times 0) + 2 \times (0 \times (-2) - 0 \times 0) + 0 \times (0 \times 0 - 2 \times 0)$$

$$= 1 \times (-4 + 0) + 2 \times (0 + 0) + 0 \times (0 + 0)$$

$$= 1 \times (-4) + 2 \times (0) + 0 \times (0)$$

$$= -4 + 0 + 0 = -4$$

$$|B| = \begin{vmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{vmatrix} = -2 \times \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} + 0 \times \begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} + 0 \times \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$= -2 \times (1 \times 2 - 0 \times 0) + 0 \times (0 \times 2 - 0 \times 0) + 0 \times (0 \times 0 - 1 \times 0)$$

$$= -2 \times (2 + 0) + 0 \times (0 + 0) + 0 \times (0 + 0)$$

$$= -2 \times (2) + 0 \times (0) + 0 \times (0)$$

$$= -4 + 0 + 0 = -4$$