Pumping Lemma for RE

Pumping Lemma (For Regular Languages)

» Pumping Lemma is used to prove that a language is NOT REGULAR.

» It cannot be used to prove that a Language is Regular.

If A is a Regular Language, then A has a pumping Length 'p' such that any string 'S' where $|S| \geq p$ may be divided into 3 parts $S = xyz$ such that the following Conditions must be true.

1) $xy^iz \in A$ for every $i \geq 0$

2) $|y| > 0$

3) $|xy| \leq p$

Pumping Lemma (For Regular Expression)

⇒ To prove that a language is not Regular using Pumping Lemma, follow the below steps:

(we prove using Contradiction)

→ Assume that A is Regular

→ It has to have a pumping Length (say p)

→ All strings longer than p can be pumped $|S| \geq p$

→ Now find a string 'S' in A such that $|S| \geq p$

→ Divide S into xyz

→ Show that $xy^iz \notin A$ for some i

→ Then Consider all ways that S can be divided into xyz.

→ Show that none of these can satisfy all the 3 pumping conditions at the same time

→ S can not be pumped = Contradiction

Pumping Lemma (For Regular Languages)-Example

⇒ Using pumping Lemma prove that the language $A = \{a^n b^n \mid n \geq 0\}$ is not Regular

Proof

→ Assume that A is Regular

→ It need to have a pumping length, say P

→ Choose string, $S = a^P b^P$

x y z

→ For example, P = 7 , ⇒ S = aaaaaaa bbbbbbb

→ All the possible ways to divide S are as follows.

Case 1: The y is in the 'a' part

a a a a a a a b b b b b b b

x y z

→ $xy^iz \Rightarrow xy^2z$ X

aa aaaa aaaaab bbbbbbb

11 ≠ 7

Case 2: The y is in the 'b' part

a a a a a a a b b b b b b b

x y z

→ $xy^iz \Rightarrow xy^2z$ X

aaaaaaabb bbbbbbbbb

7 ≠ 11

Case 3: The y is in the 'a' and 'b' part

a a a a a a a b b b b b b b

x y z

→ $xy^iz \Rightarrow xy^2z$ X

aaaaa aabb aabb bbbbb

not following $a^n b^n$

Check $|xy| \leq P$, P = 7

Pumping Lemma (FOR RL) - Example

⇒ Using pumping Lemma prove that the language $A = \{YY \mid Y \in \{0,1\}^*\}$ is Not Regular

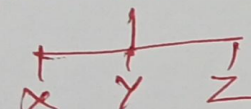
01,01

Proof

→ Assume that A is Regular

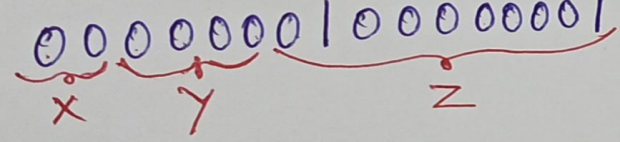
→ Pumping length say p

→ $S = 0^p 1\ 0^p 1$



p=7

0000000100000001



$xy^iz \Rightarrow xy^2z$

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$\notin A$

$|y| > 0$ ✓

$|xy| \leq p = 7$ ✓

Therefore: A is not Regular