Multitape TM and Decidability & Undecidability

Multitape Turing machine

Theorem: Every multitape TM has an equivalent Single Tape TM.

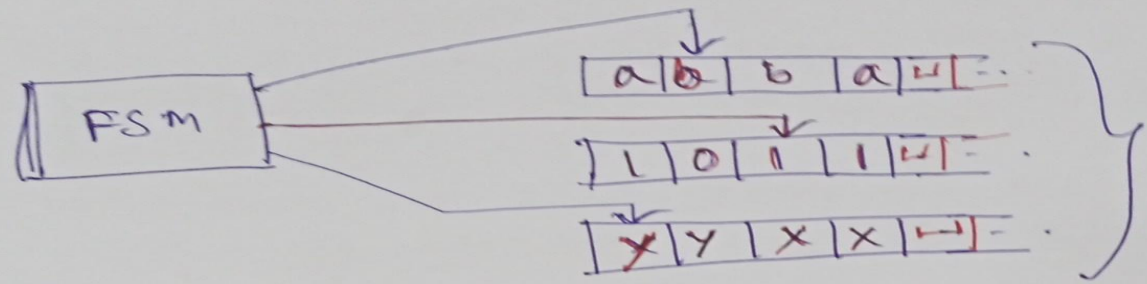
Proof: Given a multitape TM show how to build a single tape TM

→ Need to store all tapes on a single tape
Show the data representation

→ Each tape has a tape head.
Show how to store that info

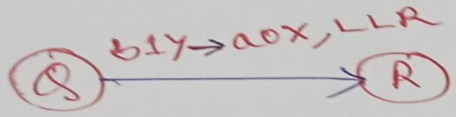
→ Need to transform a move in the Multitape TM into one or more moves in the Single Tape TM.

Multitape TM



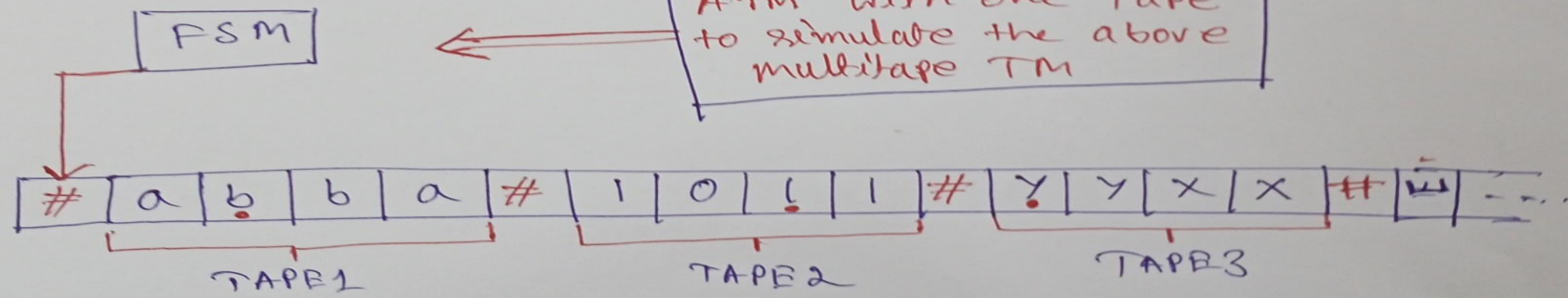
Several Tapes. Each has its own TAPE HEAD.

An Example with K = 3 Tapes TM.



Single TAPE TM

A TM with one Tape to simulate the above multitape TM



→ '#' is used to seperate the TAPE's.

→ Add "dots" to show where Head "K" is [Marking process]

→ To simulate a transition from state q, we must scan our Tape to see which symbols are under the K Tape heads.

Single Tape TM

→ Once we determine this and are ready to make the transition, we must scan across the tape again to update the cells (1) and move the dots (2).

→ Whenever one head moves off the right end, we must shift our tape so we can insert a ␣.

Decidability and Undecidability

Recursive Language:

→ A language L is said to be recursive if there exists a TM which will accept all the strings in 'L' and reject all the strings not in 'L'.

→ The TM will halt every time and give an answer (accepted or rejected) for each and every string input.

[Always HALT/STOP]

Recursively Enumerable Language:

→ A language 'L' is said to be a recursively enumerable language if there exists a TM which will accept (and therefore HALT) for all the input strings which are in 'L'.

→ But may or may not HALT/STOP for all input strings which are not in 'L'.

Decidability and Undecidability

Decidable Language:

A language 'L' is decidable if it is a recursive language and vice-versa.

Partially Decidable Language:

A language 'L' is partially decidable if 'L' is a recursively enumerable language.

Undecidable language →

→ If a language is not partially decidable then that language is undecidable

→ A language is undecidable if it is not decidable.

→ NO TM for undecidable languages.

Summary:

Recursive Language	TM always HALT/STOP
Recursively enumerable Languages	TM may or may not HALT.
Decidable Languages	Recursive language
Partially Decidable Language	Recursively Enumerable language
UNDECIDABLE	NO TM for that Language.