8

Three-phase Induction Motors

TOPICS DISCUSSED

- Basic principle of a three-phase induction motor
- Constructional details
- Rotating magnetic field
- Synchronous speed
- Slip speed
- Rotor-equivalent circuit
- Rotor current
- Rotor frequency
- Power-flow diagram
- Torque
- Torque-slip characteristics
- Effect of change of rotor circuit resistance on torque-slip characteristics
- Starting torque
- Condition for maximum torque

- Losses
- Efficiency
- Applications

8.1 INTRODUCTION

Three-phase induction motors are used in many industrial applications such as a drive motor. These motors are very rugged, and hence there is virtually no maintenance required. Only three-phase supply is required for the stator. No supply is to be provided to the rotor. The rotor is energized due to electromagnetic induction.

As the name suggests, a three-phase induction motor will have three windings placed in stator slots 120° apart connected either in star or in delta formation. Three-phase supply is provided to these three windings. Due to electromagnetic induction, EMF will be induced into the rotor winding, and if the rotor winding is closed, current will flow through the rotor winding. The interaction between the field produced, due to current flow in the stator windings, when fed from a three-phase supply, and the current-carrying rotor conductors will produce a torque which will rotate the rotor. This is the basic principle of an induction motor. We will now discuss the constructional details and the principle of working of a three-phase induction motor in detail.

8.2 CONSTRUCTIONAL DETAILS

The main parts of any rotating electrical machine, as we already know, are the stator and the rotor. The stator is a hollow cylindrical structure while the rotor is a solid cylindrical body which is placed inside the stator supported at the two ends by two end shields. A small air gap is maintained between the stator and the rotor so that the rotor can rotate freely. The rotor shaft is held at the two ends by two bearings so that the frictional loss is minimum.

Fig. 8.1 (a) shows the stator and the rotor with two end shields from two sides to be brought nearer after placing the rotor inside the stator. When the end shields are fitted to the stator from two sides with the rotor shaft passing through the bearings, the rotor will rest on the bearings and the rotor will remain separated from the stator by a small air gap. The three-phase windings are made on the stator. The windings, made of a number of coils, are placed in slots in the stator. Three-phase winding consists of three identical windings separated from each other by 120° in space. Here, each phase winding has been shown made of three coils only. In actual practice, there will be more coils used per phase. As shown is Fig. 8.2, R–R′ is one winding, Y–Y′ is the second winding, and B–B′ is the third winding. The axes of the three windings are separated from each other by 120°. The three windings have been shown separated making 120° with each other in Fig. 8.2 (b). The three-phase windings have been shown connected in star formation by joining the end terminals R′, Y′, B′ together. Now three-phase supply can be connected to the three open terminals R, Y, B.

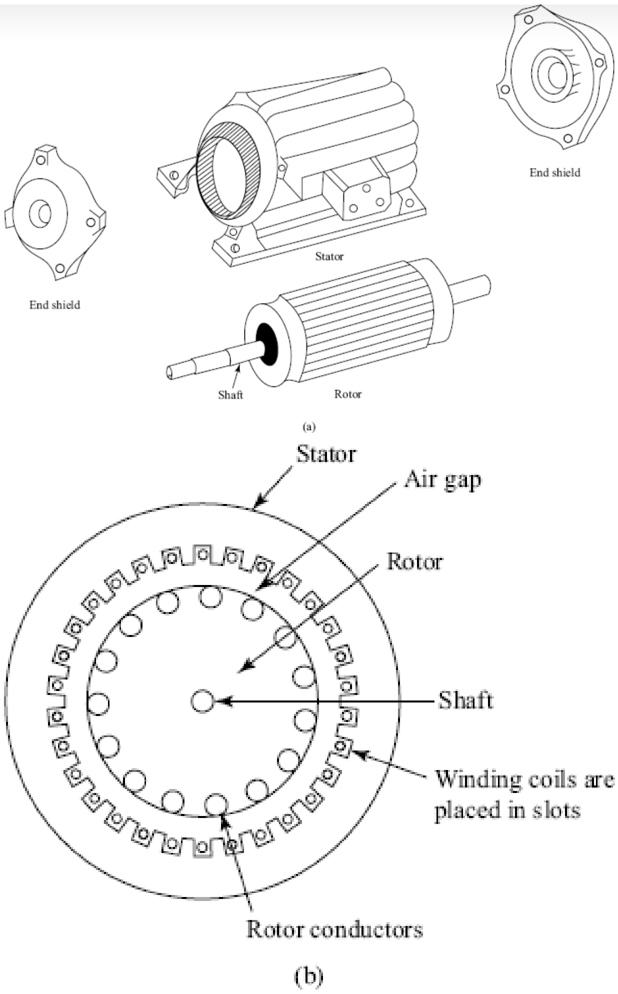


Figure 8.1 Constructional details of a three-phase induction motor: (a) stator, rotor, and end shields in isometric view; (b) cross-sectional view of stator and rotor

The rotor of an induction motor is of two types, namely, *squirrel-cage type* or *slip-ring type*. In squirrel-cage type, the rotor winding is made of bars inserted in slots made on the rotor surface. The bars are pushed into the slots and are connected from both sides through conducting rings. The connection of the rotor bars with the help of *end rings* has been shown in Fig. 8.3 (a).

Fig. 8.3 (b) shows the three-phase stator windings connected to a three-phase supply with the rotor closed on itself. In Fig. 8.3 (c) is shown the slip-ring-type rotor where the rotor winding is also made in the same way as the stator winding but the open terminals of the windings are connected permanently to three slip rings mounted on the rotor shaft. Extra resistance can be connected in the rotor circuit through brush and slip-ring arrangement. The rotor along with the slip rings mounted on its shaft is free to rotate, while brushes and the extra resistance are stationary.

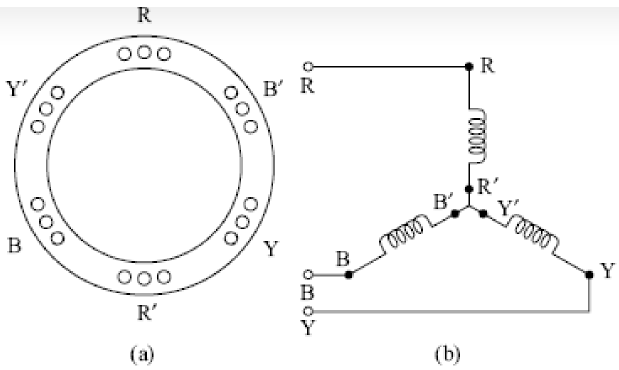


Figure 8.2 (a) Simple three-phase winding placed on stator slots; (b) stator windings connected in star

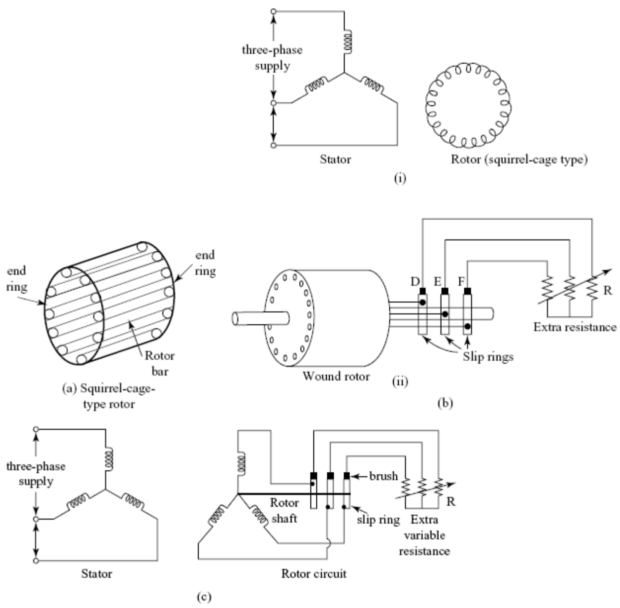


Figure 8.3 (a) Squirrel-cage-type rotor; (b) stator and rotor circuits (i) squirrel-cage type, (ii) slip-ring type; (c) slip-ring-type induction motor-stator and rotor circuits

By connecting extra resistance in the rotor circuit during starting, very high starting torque can be developed in slip-ring motors. For squirrel-cage rotor, it is not possible to add any extra resistance in the rotor as the circuit is closed by itself permanently and no terminals are brought out.

8.3 WINDINGS AND POLE FORMATION

Let us now understand how windings can be made for different number of poles. Fig. 8.4 shows a simple two-pole and four-pole stator winding. For simplicity, winding for only one phase has been shown.

two poles will be formed as has been shown. If ac supply is given, the polarities will change, the direction of current through the coil will change, and hence the positions of North and South poles will change continuously in every half cycle of power supply. That is to say, when ac supply is applied, a two-pole alternating magnetic field will be produced whose axis will lie along the horizontal direction as shown. Since for a sinusoidal ac supply both magnitude as well as direction of current will change, the magnetic fields produced will have its magnitude as well as direction changing continuously along a fixed axis.

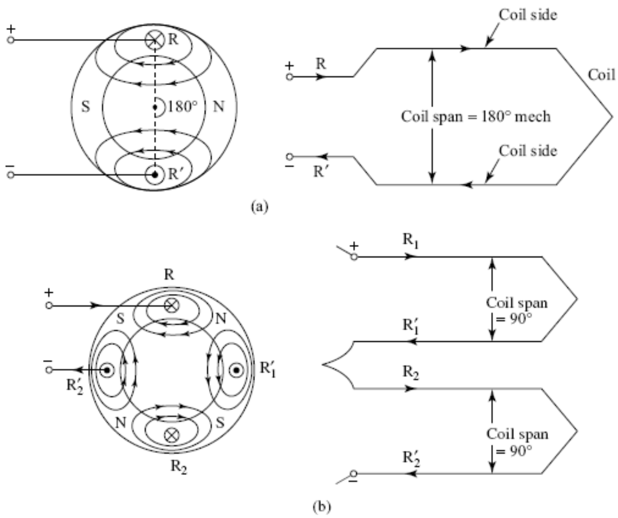


Figure 8.4 Simplified two-pole and four-pole stator winding

In Fig. 8.4 (b), a simple four-pole stator winding has been shown. Here the coil span has been reduced to 90° mechanical. Two coils connected in series has formed the winding. Four poles are formed with current and flux directions as shown. You can now easily draw a simple six-pole or an eight-pole stator winding with reduced coil spans. In a three-phase induction motor, three separate windings, each wound for two-pole, four-pole, or any even number of poles as required, are made. These windings are connected either in star or in delta and three-phase supply is applied to the windings to produce a resultant magnetic field. The resultant magnetic field is the sum total of the magnetic fields produced by the three winding ampere turns. We will soon see that the resultant magnetic field, when a three-phase supply is applied to a three-phase stator winding, is a rotating magnetic field. A rotating magnetic field is one whose axis goes on rotating continuously when supply is given. That is, the position of North and South poles goes on shifting, as time passes, at a fast speed. The speed N_s depends upon frequency of power supply f, and the number of poles P, for which the winding has been made as

If $f = 50$ Hz, $P = 2$,

$$N_s = \frac{120 \times 50}{2} = 3000 \text{ rpm}$$
$$= 50 \text{ rps}$$

8.4 PRODUCTION OF ROTATING MAGNETIC FIELD

Now let us actually see how the field rotates when a three-phase supply is connected to a three-phase stator winding. For the sake of understanding, we will consider only three consecutive instants of time of the three-phase supply voltage, show the direction of current flowing through each of the stator windings, and then draw the resultant magnetic field produced. In a three-phase supply three separate sinusoidal voltages having a displacement of 120° with respect to time is available. We will represent a three-phase supply and assume that these are connected to a two-pole three-phase stator winding as shown in Fig. 8.5 (b).

When three-phase supply is connected to R, Y, and B terminals of the stator which are connected in star, current flowing through the phases will be as follows:

at time t_1 R-phase: zero; y-phase: −ve; B-phase: +ve

at time t_2 R-phase: +ve; y-phase: −ve; B-phase: −ve

at time t_3 R-phase: zero; y-phase: +ve; B-phase: −ve

Positive current in a phase, say R-phase, will be shown as entering through R and leaving through R′, and for negative current direction will be just reverse. Positive current will be represented by a cross and negative current will be represented by a dot. Accordingly, all the current directions have been shown and the resultant field drawn. It is observed that for half cycle of current flow, i.e., in time t_1 to t_3, the magnetic-field axis has rotated by 180°. For one complete cycle of current flow, the resultant magnetic field produced will rotate by 360° i.e., one revolution. As current continues to flow through the three windings, the magnetic-field axis will go on rotating at a speed given by

$$N_s = \frac{120f}{P}$$

If frequency is high, N_s will be high and if number of poles, P is high, N_s will be low.

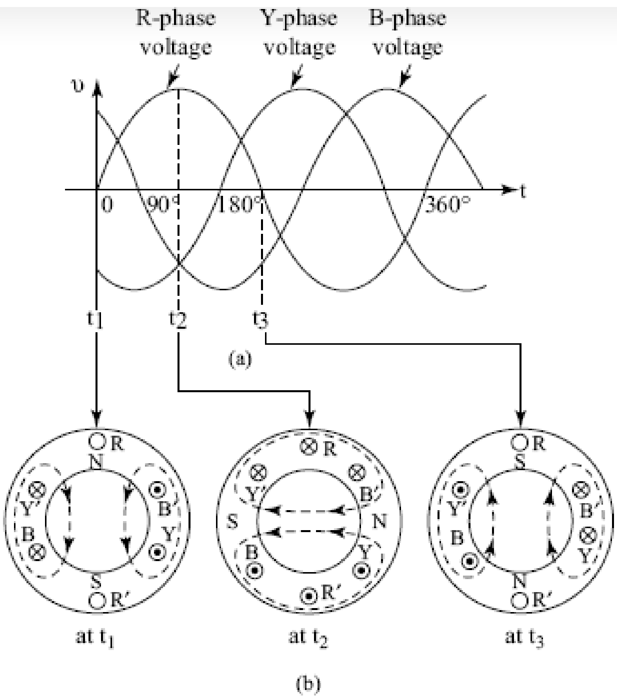


Figure 8.5 (a) Three-phase supply; (b) two-pole, three-phase stator winding has been drawn three times to show the field produced at time t_1, t_2, and t_3, respectively

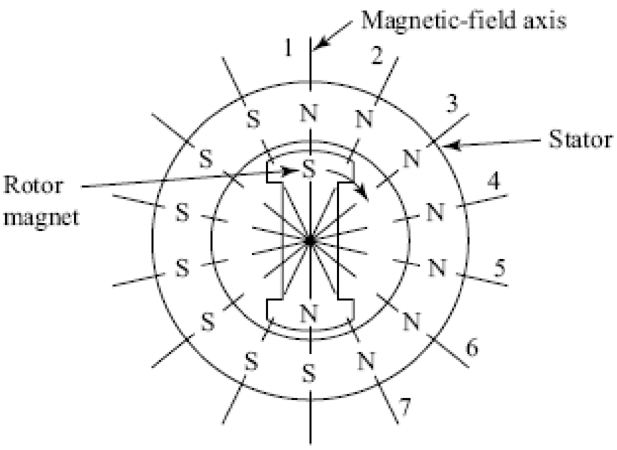


Figure 8.6 Two magnetic fields always try to align with each other

Now let us consider an electromagnet, or a permanent magnet, which is free to rotate, placed inside the rotating magnetic field as shown in Fig. 8.6.

The magnetic-field axis rotates in the clockwise direction and shifts its position from 1 to 2, 3, 4..., and so on. The position of S-pole will be opposite to the position of N-pole, as the poles rotate.

therefore this speed is called synchronous speed. If the rotor is an electromagnet with dc supply given to its windings, the rotor will rotate at synchronous speed when three-phase supply is applied to the stator winding. Three-phase supply to three-phase stator windings produce a magnetic field which rotates at a constant speed. The rotor, when energized or excited by passing current through its windings, becomes an electromagnet. The rotor, magnet, which is free to rotate, aligns itself with the rotating magnetic field. It will continue to rotate at the same speed as the rotating magnetic field. Such a motor will be called a three-phase synchronous motor.

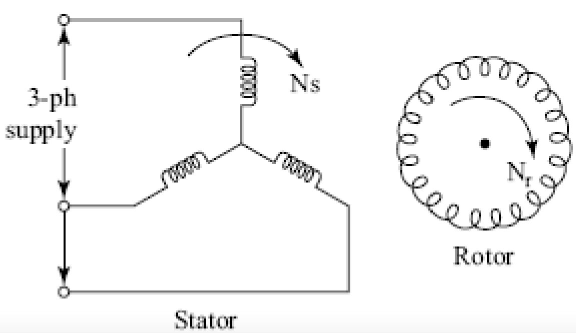
In a three-phase induction motor, however, only three-phase supply is applied across the stator windings. No supply is provided to the rotor winding. The rotor is made a closed winding either directly as in the case of squirrel-cage winding; or, through extra resistance inserted in the rotor circuit as in the case of slip-ring motors. The rotor gets excited due to electromagnetic induction. We will now study the principle of working of a three-phase induction motor.

8.5 PRINCIPLE OF WORKING

In a three-phase induction motor, the stator is wound with a three-phase winding for P number of poles. The poles for which the winding is made could be 2, 4, 6, 8, … etc. The rotor which is placed inside the stator is either squirrel-cage type or slip-ring type. In both cases, current flowing through the three-phase stator winding produces a rotating magnetic field which will be rotating at a speed, N_s where

$$N_s = \frac{120f}{P}$$

For a 50 Hz supply and P = 2, N_s is 3000 rpm. The rotating field will be rotating continuously at a very high speed. The rotating flux will cut the stationary rotor windings at that speed. Due to this cutting of flux, EMF will be induced in the rotor winding. As the rotor circuit has been made to be a closed winding, current will flow through the rotor-winding conductors. Thus the rotor circuit gets excited due to electromagnetic induction effect. Because of interaction between the current-carrying rotor conductors and the rotating magnetic field, torque will be developed in the rotor, which will rotate the rotor in the same direction as the rotating magnetic field at a speed N_r.



The rotor will attain a speed N_r which is somewhat less than the speed of the rotating magnetic field, N_s. Although the rotor will try to attain a speed of N_s, it will never be able to attain that speed, because if it does, there will be no relative velocity between the rotating field and the speed of the rotor, no EMF induced in the rotor, no current flow in the rotor conductors, no torque developed, and no rotation of the rotor. That is why an induction motor cannot run at synchronous speed, N_s as it is to be excited by electromagnetic induction, which is possible only if there exists a relative velocity between the rotating magnetic field and the rotor.

The difference between the speed of the rotating magnetic field, N_s and the rotor speed N_r is the slip S. Slip is usually expressed as the percentage of N_s, thus,

slip, $$S = \frac{N_s - N_r}{N_s}$$

or, $$SN_s = N_s - N_r \text{ or, } N_r = (1 - S)\,N_s \qquad (8.2)$$

Percentage Slip $$= \frac{N_s - N_r}{N_s} \times 100$$

Slip of a three-phase induction motor is generally 3 to 4 per cent. For example, when a 400 V, 3-ph, 50 Hz supply is connected to a four-pole three-phase induction motor, the speed of the rotating field will be 1500 rpm. The rotor will rotate at a speed less than 1500 rpm, may be, say 1440 rpm. In such a case slip S is

$$S = \frac{(1500 - 1440) \times 100}{1500}$$
$$= 4 \text{ per cent}$$

Changing the direction of rotation

It has been observed that if the sequence of the three-phase supply connected to the three-phase stator windings is changed, the direction of the rotating field produced will change, i.e., the field which was rotating in the clockwise direction will now rotate in the anti-clockwise direction. This has been shown in Fig. 8.8. As shown in Fig. 8.8 (b), the connections to R- and Y-phase supply to the stator windings have been interchanged but the connection to phase B remains same as in Fig. 8.8 (a). The direction of rotation of the rotating magnetic field will reverse, and hence the motor will rotate in the opposite direction. Thus, to change the direction of rotation of a three-phase induction motor we have to simply interchange the con-

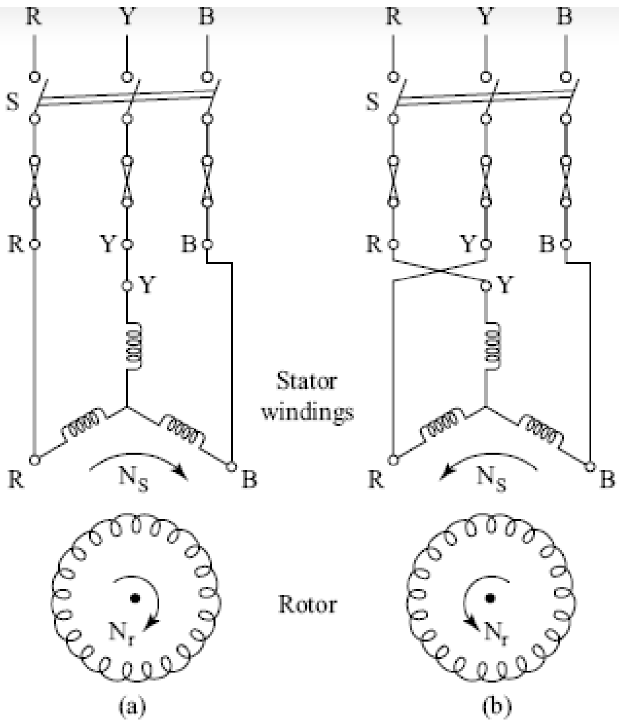


Figure 8.8 Method of changing the direction of rotation of a three-phase induction motor

8.6 ROTOR-INDUCED EMF, ROTOR FREQUENCY, ROTOR CURRENT

The frequency of supply to the stator is f Hz. The EMF induced in the rotor when the rotor is rotating at a speed N_r is due to the difference of speed of the rotating field and the rotor speed, i.e., $N_s - N_r$.

When the rotor is at standstill, i.e., not yet rotating, the difference between speed of rotation of the stator field and the rotor is $N_s - 0 = N_s$. The EMF induced in the rotor will be maximum and will have a frequency f. When the rotor attains a speed of N_r, the frequency of the induced EMF will get reduced, and let this rotor frequency be called, f_r. Thus, when the relative speed is N_s, the frequency of the induced EMF in the rotor is f. When the relative speed is $(N_s - N_r)$, the frequency is f_r. The relationship between f and f_r is found as follows:

when relative speed is N_s, frequency is f.

When relative speed is $(N_s - N_r)$, the frequency, f_r is

$$\frac{f(N_s - N_r)}{N_s} = \frac{N_s - N_r}{N_s} \times f$$

$$= Sf$$

therefore, $$f_r = Sf \tag{8.3}$$

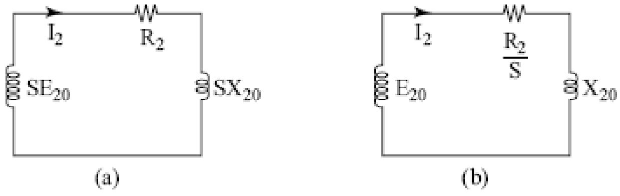


Figure 8.9 Rotor circuit when the rotor rotates at a slip, S

The rotor reactance at standstill, $X_{20} = 2\pi fL$.

The rotor reactance X_2 when the rotor is rotating will be corresponding to the rotor frequency f_r. Thus,

$$X_2 = 2\pi f_r L = 2\pi SfL = S2\pi fL = SX_{20}$$

That is, rotor reactance under running condition is equal to slip times the rotor reactance at standstill.

The resistance of the rotor circuit is R_2. R_2 is independent of rotor speed. Resistance does not change with speed. Rotor current is I_2. The rotor circuit when the rotor is rotating is represented as shown in Fig. 8.9 (a).

The EMF induced in the rotor when it is rotating is SE_{20}, the rotor-circuit resistance is R_2, the rotor-circuit reactance is SX_{20}, and the rotor current is I_2.

The current in the rotor circuit is

$$I_2 = \frac{S\,E_{20}}{\sqrt{R_2^2 + S^2 X_{20}^2}} \qquad (8.4)$$

Dividing both numerator and denominator by S,

$$I_2 = \frac{E_{20}}{\sqrt{(R_2/S)^2 + X_{20}^2}}$$

The equivalent circuit representation of the rotor for the above equation has been shown in Fig. 8.9 (b). This is the condition of the circuit when the rotor is at standstill with E_{20} as the EMF induced, I_2 as current flowing, X_{20} as the rotor-circuit reactance, and R_2/S the rotor-circuit resistance. But we know that the rotor-circuit resistance is R_2 and not R_2/S.

$$\frac{R_2}{S} = R_2 + \frac{R_2}{S} - R_2 \text{ (By adding and subtracting } R_2\text{)}$$

$$= R_2 + R_2\left(\frac{1}{S} - 1\right)$$

$$= R_2 + R_2\left(\frac{1-S}{S}\right)$$

In $\frac{R_2}{S}$ we see two resistances, viz rotor-circuit resistance R_2 and another resistance $R_2\left(\frac{1-S}{S}\right)$. The question that arises is what is the significance of resistance $R_2\left(\frac{1-S}{S}\right)$. The power lost in this resistance will be equal to $I_2^2 R_2\left(\frac{1-S}{S}\right)$. This power is the electrical equivalent of the mechanical load on the motor shaft.

This power is, therefore, called fictitious (not real but equivalent to) electrical load representing the mechanical power output of the motor. In an induction motor the input power is $\sqrt{3}$ V_L I_L cos ϕ. The output is mechanical whose electrical-equivalent power has been found as above. Not all the input power is converted into mechanical power: some power gets lost in the conversion process.

Example 8.1 A four-pole, three-phase induction when supplied with 400 V, 50 Hz supply rotates at a slip of 4 per cent. What is the speed of the motor?

Solution:

$$N_s = \frac{120f}{p} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$S = \frac{N_s - N_r}{N_s} \quad \text{Substituting values, } 0.04 = \frac{1500 - N_r}{1500}$$

or, $N_r = 1500 - 60 = 1440 \text{ rpm}$

8.7 LOSSES IN INDUCTION MOTORS

The input power to an induction motor is taken from the supply mains. For a three-phase induction motor the input power is 3 V_{Ph} I_{ph} cos ϕ, $\sqrt{3}$ V_L I_L cos ϕ, where V_{ph}, I_{ph} are the phase values and V_L, I_L are the line values. This electrical power input is converted

When the rotor rotates against the wind (air) friction, power is lost as windage loss and bearing-friction loss and brush-friction loss (in case of slip-ring motors only). Iron loss is the sum of hysteresis loss and eddy curent loss. Hysteresis and eddy current losses depend upon the frequency and flux density. The frequency of the induced EMF and current in the rotor is very small (f_r = sf). Therefore, rotor core loss is considered negligible. Stator core loss is constant at all loads since the supply voltage and frequency are normally constant. I^2R losses in the stator and rotor windings are variable, i.e., they vary with change of load (i.e., load current).

Thus, the various losses are

1. stator copper loss;
2. rotor copper loss;
3. iron loss in stator;
4. iron loss in the rotor (very small);
5. air-friction loss due to rotation of the rotor;
6. bearing-friction loss;
7. brush- and slip-ring-friction loss (in case of slip-ring motors only).

8.8 POWER FLOW DIAGRAM

The flow of power in an induction motor from stator to rotor is depicted in the form of a diagram as in Fig. 8.10. In the figure we have considered rotor core loss as negligible. Power transferred from stator to rotor is through the rotating magnetic field. The rotating magnetic field is rotating at synchronous speed, N_S. Therefore, the power transferred $= \frac{2\pi TN_r}{60}$ W. The same power is transferred to the rotor through the magnetic field and there is no loss in the air gap in this transfer. Therefore, stator output is taken as equal to rotor input. When the rotor rotates at a speed N_r, the power developed by the rotor is $= \frac{2\pi TN_r}{60}$ W. The difference between rotor input and rotor power developed is expressed as

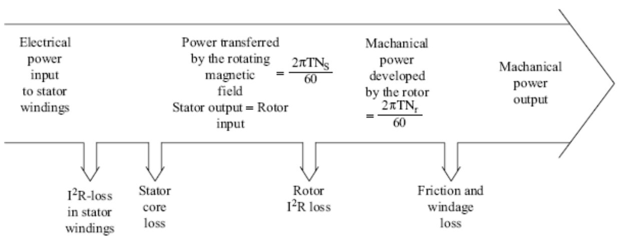


Figure 8.10 Power flow diagram of an induction motor

$$\frac{2\pi TN_s}{60} - \frac{2\pi TN_r}{60} = \text{Rotor } I^2R\text{-loss (neglecting rotor core loss)}$$

When three-phase supply is applied to the three-phase stator windings of an induction motor, as shown in Fig. 8.11, a rotating magnetic field is produced which rotates at synchronous speed, N_s where

$$N_s = \frac{120f}{P}$$

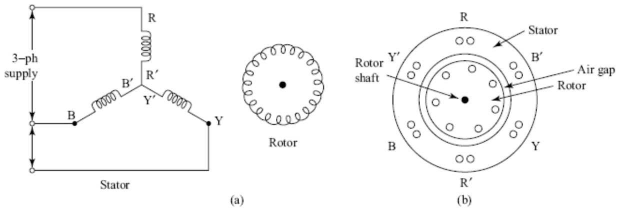


Figure 8.11 (a) Stator and rotor windings of a three-phase induction motor; (b) simplified corss-sectional view

Through this rotating magnetic field, power is transferred from stator to the rotor via the air gap. The power transferred from stator to rotor through the magnetic field in equal to $\frac{2\pi TN_s}{60}$. This is the input power to the rotor since there in no power loss in the air gap. Torque will be developed in the rotor which will cause the rotor to rotate at a speed N_r. The power developed by the rotor is $\frac{2\pi TN_r}{60}$.

The difference between the input power and the power developed is the loss in the rotor. In the rotor there will be copper loss and core loss.

Therefore,

$$\frac{2\pi TN_s}{60} - \frac{2\pi TN_r}{60} = \text{Rotor copper loss} + \text{Rotor core loss}$$

The core loss (sum of Hysteresis loss and eddy current loss) in the rotor is negligible as the frequency of the induced EMF in the core will have frequency, f_r = Sf. f_r is small, and hence core loss can be considered negligible.

Thus,

Dividing both sides by rotor input

$$\frac{(2\pi TN_s/60)-(2\pi TN_r/60)}{(2\pi TN_s/60)}=\frac{\text{Rotor copper loss}}{\text{Rotor input}}$$

$$\frac{N_s-N_r}{N_s}=\frac{\text{Rotor copper loss}}{\text{Rotor input}}$$

$\therefore$ Rotor copper loss = Slip $\times$ Rotor input (8.5)

The above is an important relationship which can also be used to arrive at the expression for torque as has been shown below.

$$\text{Rotor input}\times\text{Slip}=I_2^2R_2$$

Putting the value of I_2 from eq. (8.4)

$$\frac{2\pi TN_s}{60}\times S=\left[\frac{SE_{20}}{\sqrt{R_2^2+S^2X_{20}^2}}\right]^2R_2$$

$$\frac{2\pi TN_s}{60}\times S=\frac{S^2E_{20}^2}{R_2^2+S^2X_{20}^2}R_2$$

or,

$$T=\frac{60}{2\pi N_s}\,\frac{SE_{20}^2R_2}{R_2^2+S^2X_{20}^2}$$

If the supply voltage, V is constant, then the induced EMF E_{20} in the rotor at standstill will be constant (like in a transformer)

Therefore, $T=K\dfrac{SR_2}{R_2^2+S^2X_{20}^2}$, where $K=\dfrac{60\times E_{20}^2}{2\pi N_s}$ (8.6)

The value of S varies from 0 to 1. Slip is 0 if the rotor is able to rotate at synchronous speed, N_s. At the moment of start, rotor speed N_r is zero, and hence slip is 1 or 100 per cent. Rotor resistance R_2 is much smaller than the standstill rotor reactance, X_{20}.

Using the expression for torque, we can draw the torque versus slip (or speed) characteristic of the three-phase induction motor.

8.10 STARTING TORQUE

From the torque equation with S = 1

starting torque, $$T_s = \frac{KR_2}{R_2^2 + X_{20}^2} \tag{8.7}$$

The value of starting torque will depend upon the rotor circuit parameters, i.e., R_2 and X_{20}.

The value of X_{20} is generally higher than R_2. Let us assume $R_2 = 1\ \Omega$ and $X_{20} = 8\ \Omega$.

Let us, for the sake of interest, study the value of T_s if R_2 is increased (which can be done by including an extra resistance in the rotor circuit while starting the motor).

With $R_2 = 1$ and $X_{20} = 8$, and using K to be having a value, say 100

$$T_s = \frac{100 \times 1}{1 + 8^2} = \frac{100}{65} = 1.54$$

Now let us make $R_2 = 2\ \Omega$, Then T_s is

$$T_s = \frac{100 \times 2}{2^2 + 8^2} = \frac{200}{68} = 2.95$$

We further increase R_2 and make $R_2 = 4\ \Omega$

then, $$T_s = \frac{100 \times 4}{4^2 + 8^2} = \frac{400}{80} = 5$$

Thus, we see that if rotor-circuit resistance is increased, we can achieve higher starting torque developed by the motor which is a requirement in many applications.

8.11 CONDITION FOR MAXIMUM TORQUE

Condition for maximum torque can be found out by maximizing the expression for torque, i.e., by differentiating T with respect to slip, S and equating to zero. Let us consider the torque equation which is

$$T = K \frac{SR_2}{R_2^2 + S^2 X_{20}^2}$$

or, $$T = K \frac{R_2}{(R_2^2 / S) + S X_{20}^2}$$

$$\frac{dT}{dS}\left(\frac{R_2^2}{S} + S X_{20}^2\right) = 0$$

or, $$\frac{dT}{dS}\left(R_2^2 S^{-1} + S X_{20}^2\right) = 0$$

or, $$-R_2^2 S^{-2} + X_{20}^2 = 0$$

or, $$-\frac{R_2^2}{S^2} + X_{20}^2 = 0$$

or, $$R_2^2 = S^2 X_{20}^2$$

or, $$R_2 = S X_{20}, \quad \text{or} \quad S = \frac{R_2}{X_{20}}$$

Thus, the condition for maximum torque is established. Maximum torque in an induction motor will occur at a slip at which,

$$S = \frac{R_2}{X_{20}} \qquad (8.8)$$

If we take R_2 = 1 Ω and X_{20} = 8 Ω, maximum torque will occur when the slip is $\frac{1}{8}$, i.e., 0.125.

If R_2 = 2 Ω and X_{20} = 8 Ω, maximum torque will occur when the slip is $\frac{1}{4}$, i.e., 0.25.

When value of R_2 equals the value of X_{20}, i.e., when R_2 = 8 Ω, and X_{20} = 8 Ω, maximum torque will occur at S = 1, i.e., at starting.

Example 8.2 A four-pole, three-phase induction motor is supplied with 400 V, 50 Hz supply. The rotor-circuit resistance is 2 Ω and standstill rotor-circuit reactance is 8 Ω. Calculate the speed at which maximum torque will be developed.

Solution:

Given, f = 50 Hz, P = 4. R_2 = 2 Ω, X_{20} = 8 Ω

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500\,\text{rpm}$$

Condition for maximum torque is given by

$R_2 = SX_{20}$

at this slip, rotor speed, $N_r = (1 - S)N_s$

$= (1 - 0.25)1500$

$= 1125$ rpm

The value of maximum torque

We will put the condition for maximum torque, i.e., $R_2 = S X_{20}$ in the torque equation as

$$T = K \frac{S R_2}{R_2^2 + S^2 X_{20}^2}$$

putting $R_2 = SX_{20}$, Maximum torque T_m is

$$T_m = \frac{K.S.S X_{20}}{S^2 X_{20}^2 + S^2 X_{20}^2}$$

or,

$$T_m = \frac{K S^2 X_{20}}{2 S^2 X_{20}^2}$$

or,

$$T_m = \frac{K}{2 X_{20}} \tag{8.9}$$

Thus, we see that the value of maximum torque, T_m is independent of rotor resistance R_2 but the slip at which maximum torque is developed changes with value of R_2.

8.12 TORQUE–SLIP CHARACTERISTIC

The variation of torque with slip can be studied using the torque equation. The shape of the torque–slip characteristic is predicted as follows:

Let us write the torque equation derived earlier. Referring to (8.7), we have

$$T = \frac{K S R_2}{R_2^2 + S^2 X_{20}^2}$$

Let us assume $R_2 = 1\ \Omega$ and $X_{20} = 8\ \Omega$.

The value of S changes from 0 to 1. When the value of S is very

$$T = \frac{K}{R_2} S$$

or, $$T \propto S$$

and for larger values of S, say 0.2, 0.3, 0.4, etc., R_2^2 is smaller than $S^2 X_{20}^2$, and hence we can write

$$T = \frac{K S R_2}{S^2 X_{20}^2} = \frac{K R_2}{X_{20}^2} \frac{1}{S}$$

or, $$T \propto \frac{1}{S}$$

Thus, we see that for lower values of slip, torque is directly proportional to slip, and for higher values of slip, torque is inversely proportional to slip. We further calculate the values of torque at S = 0 and S = 1. Also, we calculate the value of maximum torque and the slip at which maximum torque occurs. These values will help us draw the torque–slip characteristic.

At S = 0, T = 0

With $$K = 100,\ T_m = \frac{K}{2 X_{20}} = \frac{100}{2 X 8} = 6.25$$

The value of slip at which T_m occurs is

$$S = \frac{R_2}{X_{20}} = \frac{1}{8} = 0.125$$

and the value of starting torque at S = 1 is

$$T_s = \frac{K R_2}{R_2^2 + X_{20}^2} = \frac{100 \times 1}{1^2 + 8^2} = \frac{100}{65} = 1.54$$

We will now draw the torque–slip characteristic using the above values.

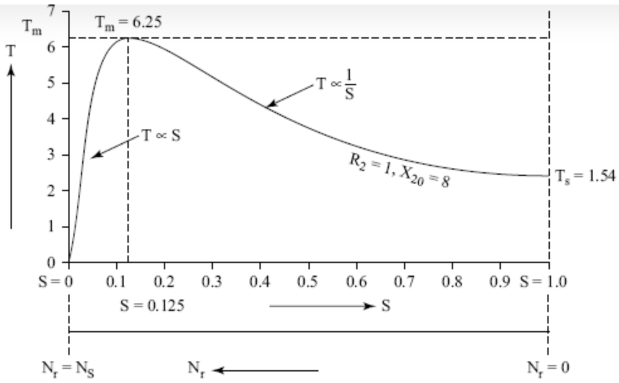


Figure 8.12 Torque–slip or torque–speed characteristic of an induction motor

Since just at start, speed is zero, and slip is 1, we can represent S = 1 as $N_r = 0$. Again when S = 0, N_r must be equal to N_s since

$$S = \frac{N_s - N_r}{N_s}$$

Thus, the torque–slip characteristic can also be shown as torque–speed characteristic as has been shown in Fig. 8.12.

8.13 VARIATION OF TORQUE–SLIP CHARACTERISTIC WITH CHANGE IN ROTOR–CIRCUIT RESISTANCE

We will now study the effect of variation of rotor-circuit resistance on the torque-slip characteristics of an induction motor.

We have known that starting torque, maximum torque, and the slip, S at which maximum torque occurs changes with change of rotor-circuit resistance, the reactance remaining unchanged. The basic shape of the torque–slip charactesic however remains the same. We will draw four torque–slip characteristic of a three-phase induction motor for

1. $R_2 = 1\ \Omega$, $X_{20} = 8\ \Omega$
2. $R_2 = 2\ \Omega$, $X_{20} = 8\ \Omega$
3. $R_2 = 4\ \Omega$, $X_{20} = 8\ \Omega$
4. $R_2 = 8\ \Omega$, $X_{20} = 8\ \Omega$

Firstly, we calculate T_s, T_m, S at which T_m occurs.

$$T_{s1} = \frac{KR_2}{R_2^2 + X_{20}^2} = \frac{100 \times 1}{1^2 + 8^2} = 1.54 \quad \text{(assuming K = 100)}$$

$$T_{m1} = \frac{K}{2\,X_{20}} = \frac{100}{2 \times 8} = 6.25$$

1. $$S_{m1} = \frac{R_2}{X_{20}} = \frac{1}{8} = 0.125$$

$$T_{s2} = \frac{K R_2}{R_2^2 + X_{20}^2} = \frac{100 \times 2}{2^2 + 8^2} = \frac{200}{68} = 2.95$$

$$T_{m2} = \frac{K}{2 \times 20} = \frac{100}{2 \times 8} = 6.25$$

$$S_{m2} = \frac{R_2}{X_{20}} = \frac{2}{8} = 0.25$$

2.

$$T_{s3} = \frac{K R_2}{R_2^2 + X_{20}^2} = \frac{100 \times 6}{4^2 + 8^2} = 5.0$$

$$T_{m3} = 6.25$$

$$S_{m3} = \frac{R_2}{X_{20}} = \frac{4}{8} = 0.5$$

3.

$$T_{s4} = \frac{KR_2}{R_2^2 + X_{20}^2} = \frac{100 \times 8}{8^2 + 8^2} = 6.25$$

$$T_{m4} = 6.25$$

$$S_{m4} = \frac{R_2}{X_{20}} = \frac{8}{8} = 1.0$$

4.

To draw the torque–slip characteristics, we will take the value of T_{s1}, T_{m1}, and S_{m1} in each case. For example, with R_2 = 1 Ω, X_{20} = 8 Ω, T_{s1} = 1.54. S_{m1} = 0.125 and T_{m1} = 6.25 and with T = 0 at S = 0, we can draw the T–S characteristic with its slope on $T \alpha S$ for lower values of slip and $T \alpha \frac{1}{S}$ for higher values of slip.

We can now draw the four characteristics as shown in Fig. 8.13.

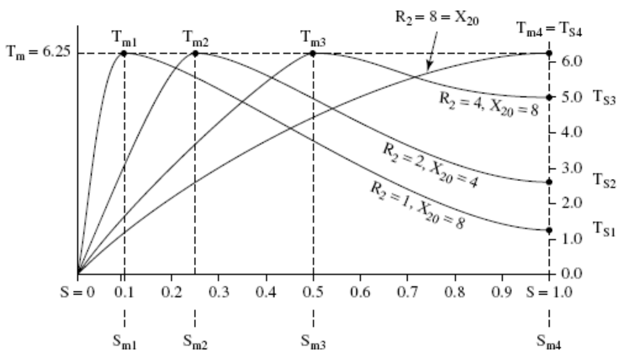


Figure 8.13 Torque–slip characteristics with increasing rotor-circuit resistance

By observing the characteristics, we can conclude that

3. starting torque becomes equal to maximum torque when rotor-circuit resistance is made equal to rotor-circuit reactance;
4. slip at which maximum torque is developed changes with change of rotor-circuit resistance;
5. increasing rotor-circuit resistance beyond the value of rotor-circuit reactance will reduce the starting torque (this can be verified by calculating T_s at R_2 = 12 Ω (say) and X_{20} = 8 Ω).

8.14 STARTING OF INDUCTION MOTORS

There is similarity between a transformer and an induction motor. An induction motor is like a short circuited transformer. This has been shown in Fig. 8.14. The only difference is that the secondary, i.e., the rotor in an induction motor is cylindrical in shape and is free to rotate while a transformer is totally a static device. The secondary winding, i.e., the rotor winding of an induction motor is closed, i.e., shorted. The similarity between an induction motor and a short-circuited transformer has been shown in Fig. 8.14 (a and b).

With the secondary winding of a transformer short circuited, if full voltage is applied across the primary windings, very high current will flow through the windings. As in the case of transformers, when full voltage applied across stator terminals of an induction motor and the rotor is stationary, very high current will flow through the rotor and stator windings. If this high current is allowed to flow for a considerable time the motor windings will be burnt out. However, as the motor picks up speed, the EMF induced in it will be SE_{20} so that

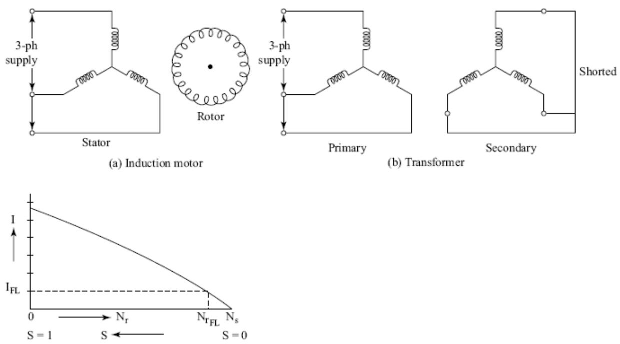


Figure 8.14 (a) Induction motor; (b) transformer with its secondary windings shorted; (c) high current drawn by an induction motor if started with full voltage

$$I_2 = \frac{SE_{20}}{\sqrt{R_2^2 + S^2 X_{20}^2}}$$

As speed increases, i.e., slip decreases, I_2 will go on decreasing; accordingly the current drawn from the supply will also be gradually be as high as six-times its full-load current. Thus, it becomes necessary to limit the starting current of an induction motor.

Starting of three-phase induction motors by applying full voltage directly to the stator windings is restricted to small motors upto 5 kW rating. If large motors are started this way, heavy current will be drawn (usually six to eight times the rated current) by the motors. This will not only be harmful to the motors in the long run but will also create heavy voltage drop in the electrical distribution lines, which will disturb the working of other electrical gadgets and machines connected to the line. Higher the rating of the motor, higher will be the disturbance of the line voltage. Starting of motors upto 5 kW rating may be done by applying full voltage. This is called direct-on-line or DOL starting of motors. Reduced voltage starting should be done for larger motors by using a three-phase auto-transformer or by connecting the stator windings of the motor first in star formation and giving the supply and as the motor picks up sufficient speed, connecting the windings in delta formation.

Thus, there are three types of starters used in starting of three-phase induction motors. They are

1. direct-on-line starters;
2. star–delta starters;
3. auto-transformer starters.

8.14.1 Direct-on-Line Starting

Small induction motors upto the rating of 5 kW are allowed to be started direct-on-line by the electricity boards. For large motors, starters have to be used.

The simplest method of starting a three-phase induction motor is to connect it to a three-phase supply through a switch as shown in Fig. 8.15 (a). However, in case of short-circuit or overload condition due to any wrong connection or excessive loading, the motor will not be protected. So, in a DOL starter provision for overload protection and short-circuit protection must be made. For short-circuit protection, cartridge-type fuses are used. These are connected in series with the lines supplying power to the motor. For overload protection, a thermal overload relay is used. The contact of the relay will open when, due to overload current, the relay will get heated up.

The switching of supply to the motor is done through a contactor and not by a manual switch. Push-button switches are provided for starting and stopping the motor. When the START push button is pressed, the contactor coil gets energized, its main contacts close, and the motor starts running. For stopping the motor the OFF push button is to be pressed due to which the contactor coil will not receive supply and will be de-energized. The contacts of the contractor will open, thereby stopping the motor. In case of over load also the contactor will get de energized due to opening of the contact of the overload relay. The working of the DOL starter is further explained using Fig. 8.15 (b) as follows.

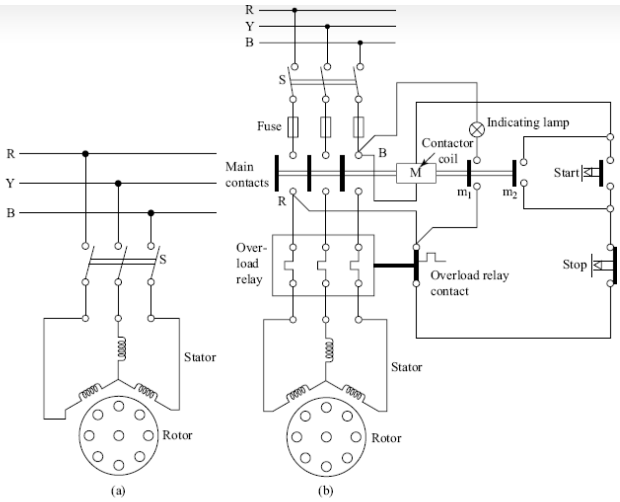


Figure 8.15 DOL starting of three-phase induction motors: (a) direct connections to supply through a switch but without any overload or short-circuit protection; (b) push-button-operated DOL starter with overload and short-circuit protection

When the start push button is pressed the contactor coil, M gets energized as the coil gets connected to R-phase and B-phase. The main contacts as well as the auxiliary contacts m_1 and m_2 will close. The motor will get full supply and start rotating. Since the auxiliary contact m_2, is closed, pressure on the start push button can be released and the contactor will continue to remain energized. Indicating lamp will get supply through contact m_1, and will glow, showing that the motor is running. To switch off the motor the stop push button is to be pressed which will de-energize the contactor coil thereby opening its contacts and making power supply to the motor disconnected. In case the motor is overloaded, current drawn will increase beyond the rated capacity, the overload relay contact will open, the contactor coil will get de-energised and, as a consequence, the motor will get disconnected from the supply.

8.14.2 Manual Star–Delta Starter

Fig. 8.16 Shows a manual star–delta starter where the switch S is placed in start position to connect the stator winding terminals A_2, B_2, C_2 in star and connecting the other ends of the windings, i.e., A_1, B_1 and C_1 to the supply. The motor will start running with the stator windings star connected and with full supply voltage applied to them. Once the motor picks up speed, the switch is placed in run position and the stator windings will get connected in delta and the same supply voltage applied to them. This way the current drawn by the motor from the supply lines during starting is reduced to one-third the value of the current that would have flown if the windings were delta connected during starting period.

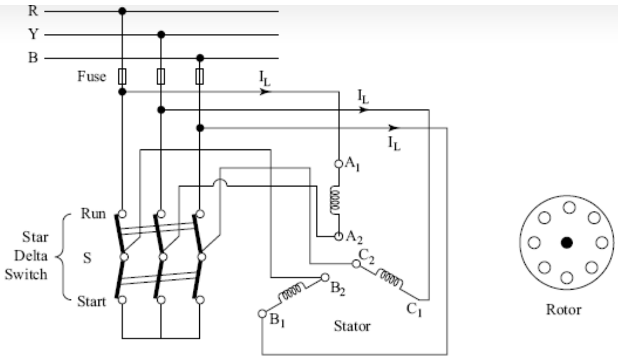


Figure 8.16 Manual star–delta starter

This can be understood from the following calculations.

When windings are star connected, the line and phase quantities are represented as shown in Fig. 8.17 (a).

Here,

$$I'_{Ph} = \frac{V}{\sqrt{3}\, Z_{ph}} = I'_L$$

When windings are delta connected, the relationships of line and phase quantities are shown as in Fig. 8.17 (b).

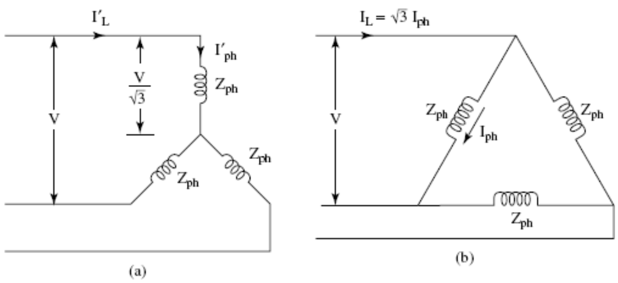


Figure 8.17 (a) Stator windings star connected during starting; (b) stator windings delta connected in running condition

Here,

$$I_{Ph} = \frac{V}{Z_{Ph}}$$

$$I_L = \sqrt{3}\, I_{Ph} = \frac{\sqrt{3}V}{Z_{Ph}}$$

$$\frac{I'_L}{I_L} = \frac{I'_{Ph}}{\sqrt{3}\, I_{Ph}} = \frac{V}{\sqrt{3} Z_{Ph}} \times \frac{Z_{Ph}}{\sqrt{3} V} = \frac{1}{3}$$

or, $$I'_L = \frac{1}{3} I_L$$

Thus, current drawn during starting is reduced to one-third if the windings are first star connected and then delta connected. Note that in the manual star–delta starter shown in Fig. 8.16, over load protection device has not been shown.

Star–delta starters are available in automatic form. The windings are first connected in star before full voltage is applied. After the rotor picks up sufficient speed, a time-delay relay (TDR) operates and then the windings get connected in delta. Such starters are called push-button-operated star–delta starters. The operating time of the TDR can be adjusted according to the time taken by the rotor to pick up sufficient speed.

In auto-transformer starters, reduced voltage is applied to the stator windings at starting with the help of a three-phase auto-transformer.

8.15 SPEED CONTROL OF INDUCTION MOTORS

Different types of electric motors are manufactured for use as drives in various kinds of industrial applications. The selection criterion for use are their ruggedness, cost, ease of use, supply requirement, and control of speed. Induction motors have become universally accepted as a first choice because of their satisfying all the above-mentioned requirements. Because of availability of power electronic control devices, speed control of induction motors has become easy now. The basic methods of speed control of induction motors will be discussed. The electronic method of speed control is beyond the scope of this book. The details of use of electronic control are dealt with in a separate subject of power electronics.

We have known that the slip of an induction motor is expressed as

$$S = \frac{N_s - N_r}{N_s}$$

or, $$SN_s = N_s - N_r$$

or, $$N_r = N_s(1 - S)$$

or, $$N_r = \frac{120f}{P}(1 - S)$$

where N_r is the speed of the motor, f is the frequency of supply, and P is the number of poles of the motor.

This equation indicates that the speed N_r of the motor can be changed by any of the following methods:

4. in addition, speed of slip-ring induction motors can be changed by changing the rotor-circuit resistance.

These methods are described in brief as follows:

(a) *Control of speed by changing supply frequency*

By changing the supply frequency, the speed of the motor can be increased or decreased smoothly. However, the supply frequency available from the electricity supply authority is at 50 Hz which is fixed. Frequency conversion equipment is, therefore, needed for speed control of motors. Variable frequency supply can be obtained from a separate motor generator set, rotary convertors, or solid-state electronic devices. The frequency changing device should change frequency and applied voltage simultaneously as a direct ratio. If frequency is increased, the supply voltage must also be increased and if frequency is decreased, supply voltage must also be decreased proportionally. This will keep the torque developed constant and the operating efficiency high.

(b) *Control of speed by pole changing*

Three techniques, viz (i) use of separate stator windings wound for two different number of poles; (ii) use of consequent pole technique, and (iii) use of pole-amplitude modulation can be used.

Instead of one winding for, say eight poles on the stator we may use two separate windings insulated from each other, say one for eight poles and the other for 10 poles. The synchronous speed corresponding to P = 8 and P = 10 will be 750 rpm and 600 rpm, respectively. The rotor speed corresponding to these synchronous speeds will be somewhat less than these synchronous speeds. Thus, we will get two rotor speeds by having a switching arrangement of power supply to the two stator windings as per our speed requirement. It may be noted that the rotor poles are by induction effect, and hence will be the same as the stator number of poles. If the stator number of poles are increased, the rotor poles will also increase. The number of poles of the stator and rotor must be the same.

In the consequent pole technique, terminals are brought out from the stator winding and by proper switching arrangement the number of poles formed by the stator current is changed. The technique is illustrated in Fig. 8.18. For, simplicity, only two coils have been shown forming the stator winding. Through switching arrangement, supply to the windings are changed as shown in Fig. 8.18 (a) and (b).

The direction of flux around the current-carrying conductors have been shown. The effective number of poles when supply is given at terminal P and taken out from terminal Q is 4. When supply is given at R and taken out from P and Q, the number of poles formed is 2. Thus, we will get two speeds corresponding to P = 4 and P = 2 by changing the power supply points.

Pole-amplitude modulation technique involves reversing the connection of one-half of the windings. It is possible to obtain a different ratio of pole formation, and hence of the rotor speed.

(c) *Speed control by changing the slip*

Slip of the motor, and hence the rotor speed can be changed by

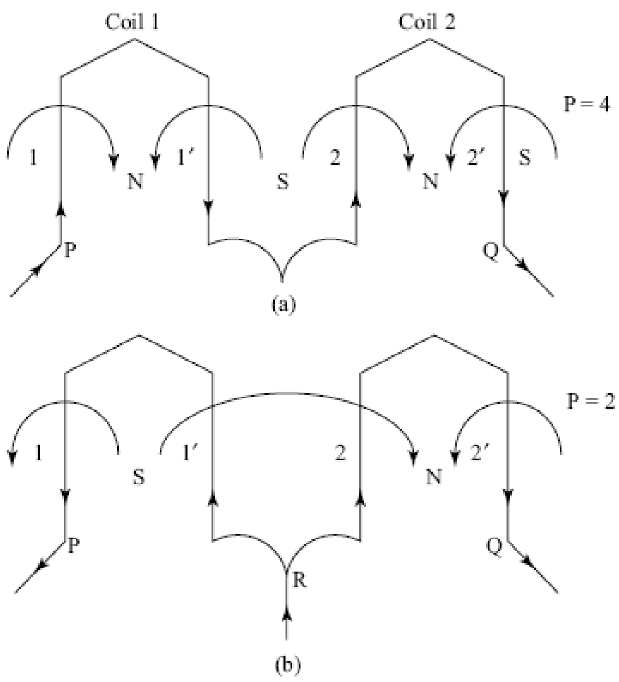


Figure 8.18 Speed control by using the consequent pole technique: (a) four-pole formation; (b) two-pole formation

(d) *Speed control by changing rotor-circuit resistance*

In slip-ring induction motors, it is possible to add extra resistance in the rotor circuit. To obtain high starting torque and also to control speed when the motor is running, the extra resistance connected in the rotor circuit is changed. Higher the rotor-circuit resistance, lower will be the speed obtained. However, starting torque will increase with increase of rotor-circuit resistance.

8.16 DETERMINATION OF EFFICIENCY

Efficiency of an induction motor can be determined by loading the motor and measuring the mechanical output and the electrical input. By converting the input and output in either electrical unit or in mechanical unit, the efficiency of the motor can be determined by taking the ratio of output and input. Small induction motors can be tested by this method where it is possible to load the machine in the laboratory.

In testing of large motors, the indirect method is adopted where the losses of the motor are determined from some tests and the efficiency is calculated as

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}}$$

tional losses are constant at all loads as long as supply voltage and frequency are constant and the speed of rotation does not change much as the load on the motor varies. Copper loss in the stator and rotor varies with load. As the load on the motor changes, the stator and rotor currents vary and the copper losses in the windings also vary.

Two tests are performed on the motor to determine the constant losses and variable losses so as to determine the efficiency of the motor. These tests are described as follows.

8.16.1 No-load Test

The motor is run on no load with full voltage applied across its stator terminals as shown in Fig. 8.19. Two single-phase wattmeters are connected to measure the three-phase power input. Since with no load connected on the motor shaft, the input power is completely wasted as loss, the sum of the wattmeter reading can be considered equal to the various losses. Therefore,

$W_1 + W_2 = I^2R$ loss in the windings at no load

+ Iron loss in the core

+ Friction and windage loss due to rotation of the rotor

By subtracting the no-load I^2R loss from the input power, the constant losses can be calculated. Note that I^2R loss in the rotor at no load is very small. Thus, we will consider only the I^2R loss in the stator windings.

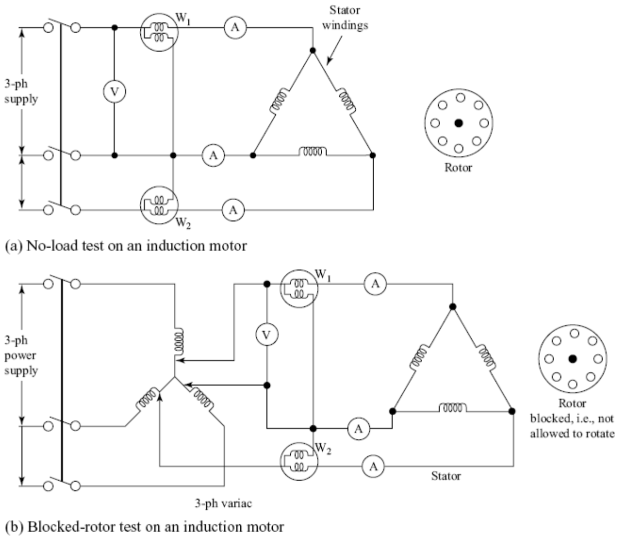


Figure 8.19 Circuits for no-load and blocked-rotor tests

variac is adjusted so that full-load-rated current flows through the windings. The core loss which is proportional to this low input voltage is small. There is no friction and windage loss as the rotor is not allowed to rotate. The sum of the wattmeter readings can approximately be taken as equal to full-load I^2R loss in the windings.

Thus, we observe that the no-load test and the blocked-rotor test together provide us the account for losses that would take place when the motor is fully loaded with full voltage applied across its terminals. By knowing the losses, we will be able to calculate the efficiency of the motor.

In these two tests, we have created a loading condition that would happen when the motor is actually loaded. That is why this method of finding out the efficiency without actually applying load on the motor shaft is called an indirect method.

8.17 APPLICATIONS OF INDUCTION MOTORS

Around 90 per cent of the electrical motors used in industry and domestic appliances are either three-phase induction motors or single-phase induction motors. This is because induction motors are rugged in construction requiring hardly any maintenance, that they are comparatively cheap, and require supply only to the stator. No supply is required to be given to the rotor. The rotor gets excited by virtue of electromagnetic induction. Further, there is no requirement of brush, slip rings, or commutator. However, slip-ring-type induction motors where extra resistance is added to the rotor circuit are used in applications where high starting torque is required.

Three-phase induction motors are used as drive motors in pumps, lifts, cranes, hoists, lifts, compressors, large capacity exhaust fans, driving lathe machines, crushers, in oil extracting mills, textile and paper mills, etc.

8.18 SOLVED NUMERICAL PROBLEMS

Example 8.3 A 3 hp, three-phase, four-pole, 400 V, 50 Hz induction motor runs at 1440 rpm. What will be the frequency of the rotor-induced EMF?

Solution:

Synchronous speed, $$N_s = \frac{120f}{p} = \frac{120 \times 50}{4} = 1500\,\text{rpm}$$

Rotor speed, N_r = 1440 rpm

Slip, $$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1400}{1500} = 0.04$$

Stator supply frequency, f = 50 Hz

Rotor induced EMF frequency, f_r = Sf

Example 8.4 The frequency of the rotor-induced EMF of 400 V, three-phase, 50 Hz, six-pole induction motor is 2 Hz. Calculate the speed of the motor.

Solution:

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Rotor frequency, $f_r = Sf$

or, $2 = S \times 50$

or, $S = 0.04$

Again, $S = \frac{N_s - N_r}{N_s}$

Substituting values $0.04 = \frac{1000 - N_r}{1000}$

or, $N_r = 960$ rpm

Example 8.5 A slip-ring-type three-phase induction motor rotates at a speed of 1440 rpm when a 400 V, 50 Hz is applied across the stator terminals. What will be the frequency of the rotor-induced EMF?

Solution:

We know the synchronous speed of the rotating magnetic field produced is expressed as

$$N_s = \frac{120f}{P}$$

Here, f = 50 Hz but number of poles of the stator winding has not been mentioned. The number of poles can be 2, 4, 6, 8, etc.

for $P = 2, \quad N_s = \frac{120 \times 50}{2} = 3000 \text{ rpm}$

for $P = 4, \quad N_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$

for $P = 6, \quad N_s = \frac{120 \times 50}{6} = 1000 \text{ rpm}$

for $P = 8, \quad N_s = \frac{120 \times 50}{8} = 750 \text{ rpm}$

speed corresponding to this rotor speed must, therefore, be 1500 rpm Thus, N_r = 1440 rpm and N_s = 1500 rpm.

Slip,
$$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1440}{1500} = 0.04$$

Rotor frequency, $f_r = S \times f = 0.04 \times 50 = 2$ Hz

Example 8.6 A six-pole, three-phase synchronous generator driven at 1000 rpm supplies power to an induction motor which runs at a speed of 1440 rpm on full load. Calculate the percentage slip of the motor and the number of poles of the motor.

Solution:

The synchronous generator is rotated at a synchronous speed of 1000 rpm by a prime mover. The synchronous speed N_s is expressed as

$$N_s = \frac{120f}{P}$$ where f is the frequency of the generated EMF

or,
$$f = \frac{N_s \times P}{120}$$
$$= \frac{1000 \times 6}{120} = 50\,\text{Hz}$$

This synchronous generator now supplies power to the induction motor at a frequency of 50 Hz. The synchronous speed of the rotating magnetic field produced in the motor is

$$N_s = \frac{120 \times f}{p} = \frac{120 \times 50}{2} = 3000\,\text{rpm (for } P = 2)$$
and
$$N_s = \frac{120 \times 50}{4} = 1500\,\text{rpm (for } P = 4)$$

The motor speed is 1440 rpm, which should be slightly less than the synchronous speed. Logically, the number of poles of the motor must be 4.

Percentageslip,
$$S = \frac{N_s - N_r}{N_s} \times 100 = \frac{(1500 - 1440) \times 100}{1500} = 4 \text{ per cent}$$

Example 8.7 A 400 V, 50 Hz, three-phase induction motor is rotating at 960 rpm on full load. Calculate the following for the motor:

Number of poles; full-load slip; frequency of rotor-induced EMF; speed of the rotor magnetic field with respect to the rotor.

f = 50 Hz
P = 2, 4, 6, etc.

$$N_s = \frac{120\ f}{p} = \frac{120 \times 50}{2} = 3000\ \text{rpm}\ (\text{for}\ P = 2)$$
$$= \frac{120 \times 50}{4} = 1500\ \text{rpm}\ (\text{for}\ P = 4)$$

Rotor speed is somewhat less than the synchronous speed N_S. Logically, here N_s can only be 1500 rpm, when N_S = 1500 rpm, P = 4.

Full load slip,
$$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1440}{1500} = 0.04$$

Frequency of rotor induced EMF $f_r = S \times f = 0.04 \times 50$

= 2 Hz

Speed of rotor field with respect to rotor, N is

$$N = \frac{120 \times f_r}{P} = \frac{120 \times 2}{4} = 60\ \text{rpm}$$

The rotor rotates at a speed of 1440 rpm. This means the speed of the rotor with respect to stator, which is stationary, is 1440 rpm.

The speed of the rotor field with respect to the rotor is 60 rpm.

Therefore, the speed of the rotor field with respect to the stator is 1440 + 60 = 1500 rpm. And, the speed of the rotating magnetic field produced by the stator rotates at synchronous speed, N_S with respect to the stator. In this case the speed of rotating field produced by the stator is 1500 rpm. Thus, we see that both the magnetic fields of the stator and rotor are stationary with respect to each other, which is, of course, the essential condition for production of torque.

Example 8.8 A three-phase, four-pole, 50 Hz induction motor rotates at a full-load speed of 1470 rpm. The EMF measured between the slip-ring terminals when the rotor is not rotating is 200 V. The rotor windings are star connected and has resistance and stand-still reactance per phase of 0.1 Ω and 1.0 Ω, respectively. Calculate the rotor current on full load.

Solution:

$$N_r = 1470\ \text{rpm}$$

$$N_s = \frac{120f}{} = \frac{120 \times 50}{} = 1500\ \text{rpm}$$

Rotor-induced EMF between the slip rings at standstill, E_{20} = 200 V.

As the rotor windings are star connected,

E_{20} per phase $$= \frac{200}{\sqrt{3}} = 115.4\ \text{V}$$

When the rotor is rotating at a speed of 1470 rpm the rotor-induced EMF per phase, E_2 is

$E_2 = S\ E_{20} = 0.02 \times 115.4 = 2.3$ V

Rotor current when the rotor is rotating at 1470 rpm,

$$I_2 = \frac{S\,E_{20}}{Z_2} = \frac{S\,E_{20}}{\sqrt{R_2^2 + (SX_{20})^2}}$$

Substituting values

$$I_2 = \frac{2.3}{\sqrt{(0.1)^2 + (0.02 \times 1)^2}} = \frac{2.3}{0.102} = 22.5\ \text{A}$$

Example 8.9 A 15 hp, three-phase, four-pole, 50 Hz induction motor has full-load speed of 1455 rpm. The friction and windage loss of the motor at this speed is 600 W. Calculate the rotor copper loss.

Solution:

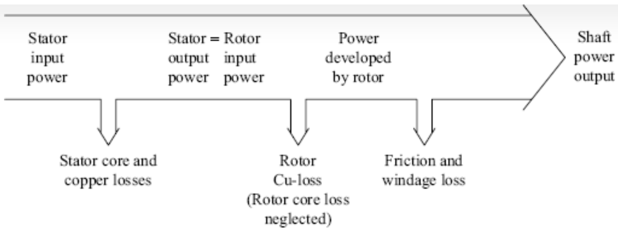
$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500\ \text{rpm}$$

Slip, $$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1455}{1500} = 0.03$$

Motor output available at the shaft = 15 hp

= 15 × 735.5 W
= 11032 W

Now, let us look at the power flow diagram



Power developed by rotor = Shaft power output + Friction and Windage losses

= 11032 + 600

= 11632 W

We know the relation

Rotor copper loss = S × Rotor input   (See eq. 8.3)

(Slip × Rotor input) is lost as rotor copper loss

The remaining power, i.e., (1–S) Rotor input, is developed as the rotor power.

Therefore,

Power developed by rotor = (1–S) Rotor input

$$= \frac{(1-S)\ \text{Rotor copper loss}}{S}$$

Thus,

$$\text{Rotor copper loss} = \frac{S}{(1-S)} \times \text{Rotor power developed}$$

$$= \frac{0.3 \times 11632}{(1-0.03)}\ \text{W}$$

$$= 360\ \text{W}$$

Example 8.10 A 10 hp, four-pole, 50 Hz, three-phase induction motor has friction and windage loss of 3 per cent of output. Calculate at full load the rotor copper loss, rotor input for a full-load slip of 4 per cent. If at this load stator loss is 6 per cent of the input power, calculate the efficiency.

Solution:

Output power = 10 hp = 10 × 735.5 W

= 7355 W

and slip, S = 0.04

Friction and windage loss = 0.03 × 7355 W

Power developed = 7355 − 220.6 = 7134.4 W

Rotor copper loss = S × Rotor input, (i)

Power developed = (1–S) Rotor input (ii)

From (i) and (ii),

$$\text{Power developed} = (1-S)\frac{\text{Rotor copper loss}}{S}$$

or,
$$\text{Rotor copper loss} = \frac{S}{(1-S)}\text{Power developed}$$

Substituting values,
$$\text{Rotor copper loss} = \frac{0.04}{0.96} \times 7134.4 \text{ W}$$
$$= 297.3 \text{ W}$$

Rotor input − Rotor copper loss = Power developed

Therefore, Rotor input = Power developed + Rotor copper loss

= 7134.4 + 297.3

= 7431.7 W

Let input power be = X W

Out of this, 0.06 X W is wasted as stator losses. Stator output = Rotor input = 0.94 X W. Equating with actual values, 0.94 X = 7431.7 W

or, X = 7906 W

$$\text{Percentage Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{7355 \times 100}{7906} = 93 \text{ per cent}$$

Example 8.11 A four-pole 50 Hz, three-phase induction motor has rotor resistance of 0.5 Ω phase. The maximum torque occurs at a speed or 1470 rpm. Calculate the ratio of starting torque to maximum torque.

Solution:

The synchronous speed,

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Rotor speed, N_r = 1200 rpm

$$S = \frac{N_s - N_r}{N_s} = \frac{1500 - 1470}{1500} = 0.02$$

Thus, $X_{20} = \frac{R_2}{S} = \frac{0.5}{0.02} = 25\,\Omega$

For constant supply voltage, the expression for torque, T is given as

$$T = \frac{K\,SR_2}{R_2^2 + S^2X_{20}^2}$$

Value of maximimum torque T_m is

$$T_m = \frac{K}{2X_{20}}$$

At starting, S = 1, Starting torque T_{st} is

$$T_{st} = \frac{K\,R_2}{R_2^2 + X_{20}^2}$$

Substituting values,

$$\frac{T_{st}}{T_m} = \frac{K\,R_2}{R_2^2 + X_{20}^2} \times \frac{2\,X_{20}}{K} = \frac{2\,X_{20}R_2}{R_2^2 + X_{20}^2}$$

$$\frac{T_{st}}{T_m} = \frac{2 \times 25 \times 0.5}{(.5)^2 + (25)^2} = 0.04$$

That is, the starting torque is only 4 per cent of the maximum torque.

Example 8.12 No-load test and blocked-rotor test were performed on a 10 hp, four-pole, 400 V, 50 Hz, three-phase induction motor to determine its efficiency. The test data are given as follows:

no-load test: V = 400 V, I = 6 A, P = 300 W

blocked-rotor test: V = 40 V, I = 24 A, P = 700 W

Calculate the efficiency of the motor on full load.

Solution:

Losses under blocked-rotor test is considered equal to I^2R losses in

$$3\, I^2 R'_e = 700 \text{ W}$$

$$R'_e = \frac{700}{3 \times 24 \times 24} = 0.4 \ \Omega$$

At no load, $I^2 R$ loss in the windings

$= 3\, I_0^2\, R'_e = 3 \times 6^2 \times 0.4$

= 43.2 W

No-load power input = 300 W

Core loss + Frictional and Windage loss = 300 – 43.2

= 256.8 W

Output = 10 hp = 10 × 735.5 = 7355 W

Efficiency, $$\eta = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{10 \times 735.5 \times 100}{10 \times 735.5 + 700 + 256.8}$$
$$= 88.5 \text{ per cent}$$

Example 8.13 A three-phase, 20-pole slip-ring induction motor runs at 291 rpm when connected to a 50 Hz supply. Calculate slip for full-load torque if the total rotor-circuit resistance is doubled. Assume $R_2 >> SX_2$.

Solution:

slip, $$N_s = \frac{120f}{P} = \frac{120 \times 50}{20} = 300 \text{ rpm}$$
$$S = \frac{N_s - N_r}{N_s} \times 100 = \frac{(300 - 291)}{300} \times 100 = 3 \text{ per cent}$$

Full-load torque equation is,

$$T = \frac{K S E_{20}^2 R_2}{R_2^2 + S^2 X_{20}^2}$$

$$\text{If } R^2 >> S^2 X_{20}^2$$

$$T = \frac{K S E_{20}^2 R_2}{} = \frac{K S E_{20}^2}{}$$

$R_2 \propto S$

If R_2 is doubled, S will be doubled. The slip at doubled R_2 will be 6 per cent.

8.19 REVIEW QUESTIONS

A. Short Answer Type Questions

1. Explain the principle of working of a three-phase induction motor.
2. Explain why a three-phase induction motor can not run at synchronous speed.
3. What is meant by slip of an induction motor? What is the value of slip at starting and at synchronous speed?
4. How can you change the direction of rotation of a three-phase induction motor? What is meant by phase sequence of power supply?
5. Explain the purpose of making two types of rotor construction for three-phase induction motors.
6. Write an expression for torque developed by an induction motor and write the meaning of each term.
7. Draw the torque-speed characteristic of a three-phase induction motor and explain its shape.
8. Show that stator magnetic field and the field produced by the rotor mmf are stationary with respect to each other.
9. What is the condition for maximum torque developed at starting?
10. Show how a rotating magnetic field is produced when a three-phase supply is connected across a three-phase winding.
11. What is the expression for maximum torque developed in a three-phase induction motor?
12. Draw the power-flow diagram in an induction motor.
13. What are the various losses in an induction motor? State the factors on which they depend.
14. Explain how efficiency of an induction motor can be determined by performing no-load test and blocked-rotor test.
15. Explain why a starter is required to start a large three-phase induction motor.
16. Draw a push-button-operated direct-on-line starter for an induction motor.
17. Draw the connection diagram for a manual star–delta starter.
18. Show the effect of variation of rotor-circuit resistance on the torque-speed characteristic of an induction motor.
19. What is the limit of increasing the rotor-circuit resistance for achieving a high starting torque? Explain your answer.
20. If the applied voltage is reduced to half, what will be the reduction in torque developed?
21. Prove that in an induction motor the rotor copper loss is slip times the rotor input.
22. Show how the maximum torque developed in an induction motor is independent of rotor-circuit resistance.
23. How can you determine experimentally, the full-load copper loss of an induction motor?
24. Establish the relation rotor frequency, $f_r = S \times f$.
25. Mention various applications of three-phase induction

27. Explain why the rotor-circuit reactance of an induction motor varies with speed?
28. Establish the relation, $X_2 = S X_{20}$ where S is the slip, X_2 is the rotor-circuit reactance under running condition, and X_{20} is the rotor-circuit reactance when the rotor is at rest.
29. Establish the similarity between a transformer and an induction motor.
30. At what slip will torque developed by an induction motor be maximum when rotor resistance equals half the rotor reactance at standstill.
31. In an induction motor, slip is always positive, why?
32. Explain why an induction motor draws heavy current at starting when started on full voltage.

B. Numerical Problems

33. A four-pole, three-phase, 50 Hz induction motor rotates at a speed of 1440 rpm. Calculate its slip in percentage. Also calculate the frequency of the induced EMF in the rotor circuit.

[Ans 4 per cent; 2 Hz]

34. A six-pole, three-phase, 400 V, 50 Hz induction motor is running at a speed of 940 rpm. Calculate its slip.

[Ans 6 per cent]

35. A four-pole, three-phase, 400 V, 50 Hz induction motor develops an induced EMF in the rotor of 2 Hz. What is the speed of the motor?

[Ans 1440 rpm]

36. A four-pole, three-phase induction motor in connected to a 50 Hz supply. Calculate synchronous speed; the rotor speed when slip is 4 per cent, and the rotor frequency when the rotor is running at 1425 rpm.

[Ans 1500 rpm, 1440 rpm, 2.5 Hz]

37. The input power to the rotor of a 400 V, 50 Hz, six-pole, three-phase induction motor is 75 kW. The frequency of the rotor induced EMF is 2 Hz. Calculate slip; rotor speed, and power developed by the rotor; rotor I^2R loss.

[Ans 0.04, 960 rpm, 72 kW; 1 kW]

38. A 400 V, 50 Hz, six-pole, three-phase induction motor running at 975 rpm draws 40 kW from the mains. The stator loss is 1kW. The friction and windage loss is 2 kW. Calculate (i) slip; (ii) I^2R loss

[Ans 2.5 per cent; 975 W; 49 hp; 90 per cent]

C. Multiple Choice Questions

1. For production of a rotating magnetic field using stationary windings we must connect
    1. a single-phase supply to a single-phase winding
    2. a three-phase supply to a three-phase winding
    3. a single-phase supply to a two-phase winding
    4. either a single-phase winding or a two-phase winding to a single-phase supply.
2. When a three-phase 50 Hz, 400 V supply is applied across a four-pole, three-phase winding, a rotating magnetic field is produced which is rotating at
    1. 200 rpm
    2. 1600 rpm
    3. 1500 rpm
    4. 3000 rpm.
3. A four-pole, three-phase induction motor is rotating at 1440 rpm when a 400 V, 50 Hz supply is applied to its stator terminals. The slip of the motor expressed in percentage is
    1. 4 per cent
    2. 2 per cent
    3. 5 per cent
    4. 6 per cent.
4. The synchronous speed N_S, frequency f, and number of poles, P are related as
    1. $N_s = \frac{120P}{f}$
    2. $N_s = \frac{120f}{P}$
    3. $N_s = \frac{fP}{120}$
    4. $\frac{Pf}{60}$.
5. When an induction motor is yet to start, i.e., at standstill, its slip is
    1. 0
    2. 1
    3. infinity
    4. near to 0.
6. The speed of a three-phase induction motor when supplied with three-phase, 400 V, 50 Hz supply is 1440 rpm. The number of poles for which the windings are made must be
    1. 2
    2. 4
    3. 6
    4. 8.
7. A three-phase induction motor develops a torque of 500 Nm at normal supply voltage. If the supply voltage is reduced to half, what will be the torque developed?
    1. 250 Nm
    2. 125 Nm
    3. will remain constant at 500 Nm
    4. 62.5 Nm.

2. $S = \dfrac{Ns - Nr}{Nr}$
3. $\dfrac{Ns - Nr}{Ns}$
4. $\dfrac{Nr}{Ns - Nr}$.

9. A four-pole, 50 Hz, 400 V, three-phase induction motor is running at 1440 rpm. The frequency of rotor-induced EMF is
    1. 2 Hz
    2. 4 Hz
    3. 1 Hz
    4. 50 Hz.
10. A four-pole, three-phase induction motor when fed from a 400 V, 50 Hz supply, runs at 1440 rpm. The frequency of EMF induced in the rotor is
    1. 2 Hz
    2. 45 Hz
    3. 50 Hz
    4. 3 Hz.
11. If an induction motor by some means is rotated at synchronous speed, then
    1. the EMF induced in the rotor will be maximum
    2. the EMF induced in the rotor will be zero
    3. the torque developed by the rotor will be maximum
    4. the frequency of induced EMF will be slip times the supply frequency.
12. Increase in the rotor-circuit resistance of an induction motor will
    1. increase the starting torque
    2. decrease the starting torque
    3. increase the maximum torque developed
    4. decrease the maximum torque developed.
13. Torque developed by a three-phase induction motor depends on
    1. the supply voltage
    2. the rotor-circuit resistance
    3. the slip of the rotor
    4. all the above.
14. A large capacity three-phase induction motor is started using a star–delta starter instead of starting direct-on-line. The starting current
    1. is increased three times
    2. remains constant
    3. is reduced to one-third its value
    4. is reduced to half its value.
15. With rotor-circuit resistance, R_2 of 2 Ω, the maximum torque of an induction motor is developed at 10 percent slip. When the maximum torque is developed at 20 percent slip, the value of R_2 will be
    1. 1 Ω
    2. 2 Ω
    3. 0.5 Ω
    4. 4 Ω.
16. The speed of a three-phase slip-ring-type induction motor can be controlled by

17. The four no-load speeds of three-phase induction motors operating on 400 V, 50 Hz supply are 576 rpm, 720 rpm, 1440 rpm, and 2880 rpm. The number of poles of the motor, respectively, are
    1. 8, 6, 4, 2
    2. 10, 8, 4, 2
    3. 12, 8, 4, 2
    4. 16, 8, 4, 2.
18. The power factor of a three-phase induction motor on no load is found to be 0.15 lagging. When the motor is fully loaded its power factor would be around
    1. 0.05 lagging
    2. 0.15 lagging
    3. 0.85 lagging
    4. 0.85 leading.
19. The three supply terminals R, Y, B when connected, respectively, to the three stator terminals a, b, c of a three-phase induction motor, the motor rotates in the clockwise direction. The motor will rotate in the reverse direction if
    1. R is connected to c, Y is connected to a, and B is connected to b
    2. R is connected to b, Y is connected to a, connection of B to c remains unchanged
    3. the supply voltage is reduced to half its value
    4. the frequency of supply is gradually reduced.
20. A three-phase induction motor has standstill rotor-circuit resistance, X_{20} of 8 Ω and resistance of 2 Ω. The maximum torque will be developed at starting if the circuit parameters are
    1. R_2 = 2 Ω, X_{20} = 4 Ω
    2. R_2 = 4 Ω, X_{20} = 8 Ω
    3. R_2 = 8 Ω, X_{20} = 8 Ω
    4. R_2 = 16 Ω, X_{20} = 8 Ω.

Answers to Multiple Choice Questions

1. (b)
2. (c)
3. (a)
4. (b)
5. (b)
6. (b)
7. (b)
8. (c)
9. (a)
10. (d)
11. (b)
12. (a)
13. (d)
14. (c)
15. (d)
16. (d)
17. (b)
18. (c)
19. (b)
20. (c)