

First Order Differential Equations

1). Separable form

$$1. \frac{dy}{dx} = \sin 5x$$

$$2. \frac{dy}{dx} = (x + 1)^2$$

$$3. dx + e^{3x} dy = 0$$

$$4. dy - (y - 1)^2 dx = 0$$

$$5. x \frac{dy}{dx} = 4y$$

$$6. \frac{dy}{dx} + 2xy^2 = 0$$

$$7. \frac{dy}{dx} = e^{3x+2y}$$

$$8. e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$$

$$9. y \ln x \frac{dx}{dy} = \left(\frac{y+1}{x} \right)^2$$

$$10. \frac{dy}{dx} = \left(\frac{2y+3}{4x+5} \right)^2$$

$$11. \csc y dx + \sec^2 x dy = 0$$

$$12. \sin 3x dx + 2y \cos^3 3x dy = 0$$

$$13. (e^y + 1)^2 e^{-y} dx + (e^x + 1)^3 e^{-x} dy = 0$$

$$14. x(1 + y^2)^{1/2} dx = y(1 + x^2)^{1/2} dy$$

2).Linear Differential Equations of order one: $\frac{dy(x)}{dx} + P(x)y(x) = Q(x)$

1. $\frac{dy}{dx} = 5y$

2. $\frac{dy}{dx} + 2y = 0$

3. $\frac{dy}{dx} + y = e^{3x}$

4. $3 \frac{dy}{dx} + 12y = 4$

5. $y' + 3x^2y = x^2$

6. $y' + 2xy = x^3$

7. $x^2y' + xy = 1$

8. $y' = 2y + x^2 + 5$

9. $x \frac{dy}{dx} - y = x^2 \sin x$

10. $x \frac{dy}{dx} + 2y = 3$

11. $x \frac{dy}{dx} + 4y = x^3 - x$

12. $(1 + x) \frac{dy}{dx} - xy = x + x^2$

13. $x^2y' + x(x + 2)y = e^x$

14. $xy' + (1 + x)y = e^{-x} \sin 2x$

15. $y dx - 4(x + y^6) dy = 0$

16. $y dx = (ye^y - 2x) dy$

17. $\cos x \frac{dy}{dx} + (\sin x)y = 1$

3).

Bernoulli's Equation The differential equation

$$\frac{dy}{dx} + P(x)y = f(x)y^n,$$

In Problems 15–20 solve the given differential equation by using an appropriate substitution.

15. $x \frac{dy}{dx} + y = \frac{1}{y^2}$ 16. $\frac{dy}{dx} - y = e^x y^2$

17. $\frac{dy}{dx} = y(xy^3 - 1)$ 18. $x \frac{dy}{dx} - (1 + x)y = xy^2$

19. $t^2 \frac{dy}{dt} + y^2 = ty$ 20. $3(1 + t^2) \frac{dy}{dt} = 2ty(y^3 - 1)$

In Problems 21 and 22 solve the given initial-value problem.

21. $x^2 \frac{dy}{dx} - 2xy = 3y^4, \quad y(1) = \frac{1}{2}$

22. $y^{1/2} \frac{dy}{dx} + y^{3/2} = 1, \quad y(0) = 4$

4). Exact Differential Equations: Check whether the following equations are exact or not and hence get the solution

1. $(2x - 1) dx + (3y + 7) dy = 0$

2. $(2x + y) dx - (x + 6y) dy = 0$

3. $(5x + 4y) dx + (4x - 8y^3) dy = 0$

4. $(\sin y - y \sin x) dx + (\cos x + x \cos y - y) dy = 0$

5. $(2xy^2 - 3) dx + (2x^2y + 4) dy = 0$

9. $(x - y^3 + y^2 \sin x) dx = (3xy^2 + 2y \cos x) dy$

10. $(x^3 + y^3) dx + 3xy^2 dy = 0$

In Problems 21–26 solve the given initial-value problem.

21. $(x + y)^2 dx + (2xy + x^2 - 1) dy = 0, \quad y(1) = 1$

22. $(e^x + y) dx + (2 + x + ye^y) dy = 0, \quad y(0) = 1$

23. $(4y + 2t - 5) dt + (6y + 4t - 1) dy = 0, \quad y(-1) = 2$

24. $\left(\frac{3y^2 - t^2}{y^5} \frac{dy}{dt} + \frac{t}{2y^4} \right) = 0, \quad y(1) = 1$

25. $(y^2 \cos x - 3x^2y - 2x) dx + (2y \sin x - x^3 + \ln y) dy = 0, \quad y(0) = e$

5). Homogeneous DE: $\left(\frac{dy}{dx} = f\left(\frac{y}{x}\right) \right)$

1. $(x - y) dx + x dy = 0$ 2. $(x + y) dx + x dy = 0$

3. $x dx + (y - 2x) dy = 0$ 4. $y dx = 2(x + y) dy$

5. $(y^2 + yx) dx - x^2 dy = 0$

6. $(y^2 + yx) dx + x^2 dy = 0$

7. $\frac{dy}{dx} = \frac{y - x}{y + x}$

8. $\frac{dy}{dx} = \frac{x + 3y}{3x + y}$

9. $-y dx + (x + \sqrt{xy}) dy = 0$

10. $x \frac{dy}{dx} = y + \sqrt{x^2 - y^2}, \quad x > 0$

Second Order Linear Differential Equations

In Problems 1–14 find the general solution of the given second-order differential equation.

1. $4y'' + y' = 0$
2. $y'' - 36y = 0$
3. $y'' - y' - 6y = 0$
4. $y'' - 3y' + 2y = 0$
5. $y'' + 8y' + 16y = 0$
6. $y'' - 10y' + 25y = 0$
7. $12y'' - 5y' - 2y = 0$
8. $y'' + 4y' - y = 0$
9. $y'' + 9y = 0$
10. $3y'' + y = 0$
11. $y'' - 4y' + 5y = 0$
12. $2y'' + 2y' + y = 0$
13. $3y'' + 2y' + y = 0$
14. $2y'' - 3y' + 4y = 0$

Method of Undetermined Coefficients: (Method for solving-Second Order Non-homogeneous Linear DEs): $y''(x) + a y'(x) + b y(x) = g(x)$, a and b are constants.

$g(x)$	Trial Particular Solutions Form of y_p
1. 1 (any constant)	A
2. $5x + 7$	$Ax + B$
3. $3x^2 - 2$	$Ax^2 + Bx + C$
4. $x^3 - x + 1$	$Ax^3 + Bx^2 + Cx + E$
5. $\sin 4x$	$A \cos 4x + B \sin 4x$
6. $\cos 4x$	$A \cos 4x + B \sin 4x$
7. e^{5x}	Ae^{5x}
8. $(9x - 2)e^{5x}$	$(Ax + B)e^{5x}$
9. $x^2 e^{5x}$	$(Ax^2 + Bx + C)e^{5x}$
10. $e^{3x} \sin 4x$	$Ae^{3x} \cos 4x + Be^{3x} \sin 4x$
11. $5x^2 \sin 4x$	$(Ax^2 + Bx + C) \cos 4x + (Ex^2 + Fx + G) \sin 4x$
12. $xe^{3x} \cos 4x$	$(Ax + B)e^{3x} \cos 4x + (Cx + E)e^{3x} \sin 4x$

Find general solutions of the following Des:

1. $y'' + 3y' + 2y = 6$

2. $4y'' + 9y = 15$

3. $y'' - 10y' + 25y = 30x + 3$

4. $y'' + y' - 6y = 2x$

5. $\frac{1}{4}y'' + y' + y = x^2 - 2x$

6. $y'' - 8y' + 20y = 100x^2 - 26xe^x$

7. $y'' + 3y = -48x^2e^{3x}$

8. $4y'' - 4y' - 3y = \cos 2x$

9. $y'' - y' = -3$

10. $y'' + 2y' = 2x + 5 - e^{-2x}$

11. $y'' - y' + \frac{1}{4}y = 3 + e^{x/2}$

12. $y'' - 16y = 2e^{4x}$

13. $y'' + 4y = 3 \sin 2x$

14. $y'' - 4y = (x^2 - 3) \sin 2x$

15. $y'' + y = 2x \sin x$

Method of Variation of Parameters: Method to find solution of the following class of DEs

$$y'' + P(x)y' + Q(x)y = f(x)$$

In Problems 1–18 solve each differential equation by variation of parameters.

1. $y'' + y = \sec x$

2. $y'' + y = \tan x$

3. $y'' + y = \sin x$

4. $y'' + y = \sec \theta \tan \theta$

5. $y'' + y = \cos^2 x$

6. $y'' + y = \sec^2 x$

7. $y'' - y = \cosh x$

8. $y'' - y = \sinh 2x$

9. $y'' - 4y = \frac{e^{2x}}{x}$

10. $y'' - 9y = \frac{9x}{e^{3x}}$

11. $y'' + 3y' + 2y = \frac{1}{1 + e^x}$

12. $y'' - 2y' + y = \frac{e^x}{1 + x^2}$

13. $y'' + 3y' + 2y = \sin e^x$

14. $y'' - 2y' + y = e^t \arctan t$

15. $y'' + 2y' + y = e^{-t} \ln t$

16. $2y'' + 2y' + y = 4\sqrt{x}$

17. $3y'' - 6y' + 6y = e^x \sec x$

18. $4y'' - 4y' + y = e^{x/2} \sqrt{1 - x^2}$