

Pumping Lemma for CFL

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⇒ Pumping Lemma (for CFL) is used to prove that a language is NOT Context Free.

If A is a CFL, then, A has a Pumping Length ' p ' such that any string ' S ', where $|S| \geq p$ may be divided into 5 pieces $S = UVXYZ$ such that the following conditions must be true:

- 1) UV^iXYZ is in A for every $i \geq 0$
- 2) $|VY| > 0$
- 3) $|VXY| \leq p$

⇒ Following are the steps to prove a language is not CFL: [Using Contradiction]

- ① Assume that A is Context free
- ② It has to have a pumping length (say p)
- ③ All string longer than p can be pumped $|S| \geq p$
- ④ now find a string ' S ' in A such that $|S| \geq p$
- ⑤ Divide S into $UVXYZ$
- ⑥ Show that $UV^iXYZ \notin A$ for some i
- ⑦ Then consider the ways that S can be divided into $UVXYZ$
- ⑧ Show that none of these can satisfy all the 3 pumping conditions at the same time.
- ⑨ S can not be pumped == CONTRADICTION

Example - 1

⇒ Show that $L = \{a^N b^N c^N \mid N \geq 0\}$ is not Context Free.

- ① Assume that L is Context Free
- ② L must have a pumping length (say P)
- ③ Now we take a string S such that $S = a^P b^P c^P$
- ④ We divide S into parts $UVXYZ$

Es. $P=4$ So, $S = a^4 b^4 c^4$

Case I: V and Y each contain only one type of symbol.

$a a a a b b b b c c c c$
 $\underbrace{\quad}_U \underbrace{\quad}_V \underbrace{\quad}_X \underbrace{\quad}_Y \underbrace{\quad}_Z$

$$UV^i X Y^i Z \quad [i=2]$$
$$UV^2 X Y^2 Z$$

$a a a a a b b b b c c c c c$
 $a^5 b^4 c^5 \notin L$

Case II: Either V or Y has more than one kind of symbols.

$a a a a b b b b c c c c$
 $\underbrace{\quad}_U \underbrace{\quad}_V \underbrace{\quad}_X \underbrace{\quad}_Y \underbrace{\quad}_Z$

$$UV^i X Y^i Z \quad [i=2]$$
$$UV^2 X Y^2 Z$$

$a a a a b b a a b b b b c c c c$
 $\notin L$

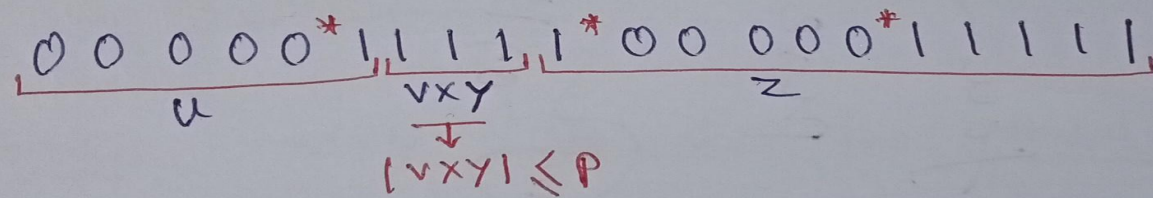
Example - 2

Show that $L = \{w^p w \mid w \in \{0,1\}^*\}$ is NOT Context Free.

- Assume that L is Context Free
- L must have a pumping length (say p)
- Now we ~~take~~ take a string S such that $S = 0^p 1 0^p 1^p$
- we divide S into parts $uvxyz$

B8. $p = 5$ so, $S = 0^5 1 0^5 1^5$

Case 1: vxy does not straddle a boundary
↳ doesn't cross the boundary.



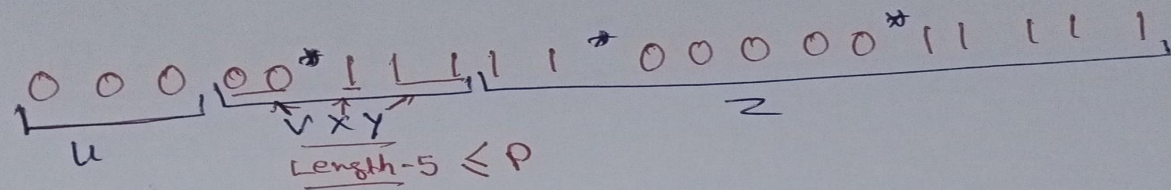
$$uv^i xy^i z \Rightarrow uv^2 xy^2 z$$

000001111110000011111

$0^5 1 0^5 1$ $\notin L$

Example 2

Case 2a: \sqrt{xy} straddles the first boundary

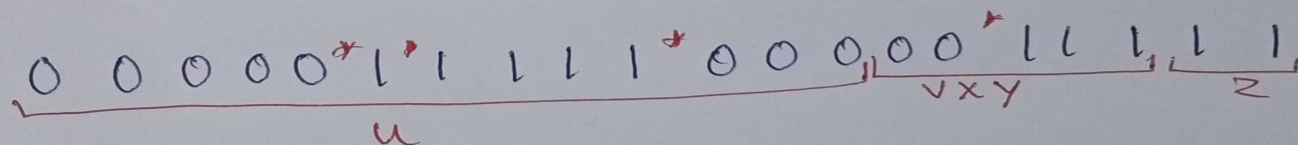


$$Uv^2xy^2z$$

0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 1 1 1 1

$\underline{0101} \notin L$

Case 2b: \sqrt{xy} straddles the third boundary



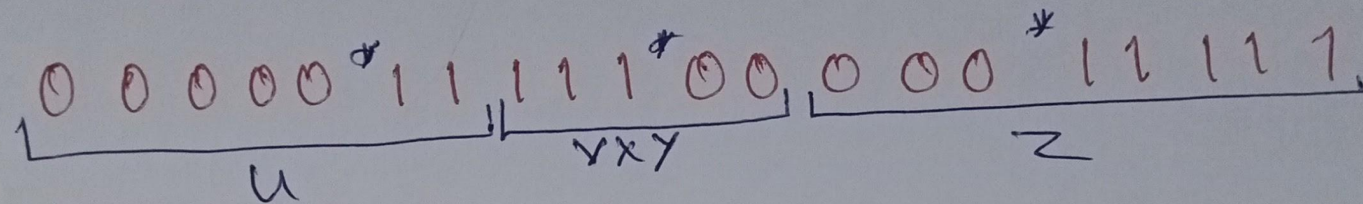
$$Uv^{\sim}xy^{\sim}z$$

0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1 1

$\underline{0101} \notin L$

Example 2

Case 3: vxy straddles the midpoint



uv^nxv^nz

0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1

$\underbrace{0^5 1^7}_{L} \underbrace{0^7 1^5}_{L} \notin L$

L is not Context Free.