# Operational Research.

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Q) $2x_1 + 6x_2 + 3x_3 + x_4 = 3$
$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$

Comparing the above system with $AX = B$, we get,

$A = \begin{pmatrix} 2 & 6 & 3 & 1 \\ 6 & 4 & 4 & 6 \end{pmatrix}$ $B = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

There are 4 variables, 2 equations, so the max$^m$ no. of solutions are

$^4C_2 = \frac{4!}{2! \cdot 2!} = 6$

$rank(A) = 2 = rank(A/B)$

We construct from $A = (a_1 \; a_2 \; a_3 \; a_4)$

$B_1 = (a_1 \; a_2) = \begin{pmatrix} 2 & 6 \\ 6 & 4 \end{pmatrix}$

$B_2 = (a_1 \; a_3) = \begin{pmatrix} 2 & 3 \\ 6 & 4 \end{pmatrix}$

$B_3 = (a_1 \quad a_4) = \begin{pmatrix} 2 & 1 \\ 6 & 6 \end{pmatrix}$

$B_4 = (a_2 \quad a_3) = \begin{pmatrix} 6 & 3 \\ 4 & 4 \end{pmatrix}$

$B_5 = (a_2 \quad a_4) = \begin{pmatrix} 6 & 1 \\ 4 & 6 \end{pmatrix}$

$B_6 = (a_3 \quad a_4) = \begin{pmatrix} 3 & 1 \\ 4 & 6 \end{pmatrix}$

So, the basic solution $x_{Bi}$ will take the form $X_{Bi} = B_i^{-1} b$.

Inverse of $X = \frac{1}{|X|} \times adj(X)$

$X_{B_1} = \frac{1}{14}\begin{bmatrix} 0 \\ 7 \end{bmatrix}$ $\quad X_{B_4} = \frac{1}{12}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$

$X_{B_2} = \frac{1}{10}\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ $\quad X_{B_5} = \frac{1}{32}\begin{bmatrix} 10 \\ 9 \end{bmatrix}$

$X_{B_3} = \frac{1}{6}\begin{bmatrix} 6 \\ 1 \end{bmatrix}$ $\quad X_{B_6} = \frac{1}{14}\begin{bmatrix} 10 \\ 3 \end{bmatrix}$

Since the basic solution $X_{B_1}$ corresponds to $(a_1, a_2)$. So $a_3$ and $a_4$ will be 0.

$\therefore$ One Solution can be $\frac{1}{4}\begin{bmatrix} 0 \\ -7 \\ 0 \\ 0 \end{bmatrix}$

For $X_{B_2}$, it corresponds to $(a_1, a_3)$. So $a_2$ and $a_4$ will be 0.

$\therefore$ One Solution can be $\frac{1}{10}\begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \end{bmatrix}$

For $X_{B_3}$, it corresponds to $(a_1, a_4)$. So, $a_2$ and $a_3$ will be 0.

$\therefore$ One Solution can be $\frac{1}{6}\begin{bmatrix} 6 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

For $X_{B_4}$, it corresponds to $(a_2, a_3)$. So, $a_1$ and $a_4$ will be 0.

$\therefore$ One Solution can be $\frac{1}{12}\begin{bmatrix} 0 \\ 4 \\ 3 \\ 0 \end{bmatrix}$

For $X_{B5}$, it corresponds to $(a_2, a_4)$.
So, $a_1$ and $a_3$ will be 0.

$\therefore$ One solution can be $\frac{1}{32}\begin{bmatrix} 0 \\ 10 \\ 0 \\ 9 \end{bmatrix}$.

For $X_{B6}$, it corresponds to $(a_3, a_4)$.
So, $a_1$ and $a_2$ will be 0.

$\therefore$ One solution can be $\frac{1}{14}\begin{bmatrix} 0 \\ 0 \\ 10 \\ 9 \end{bmatrix}$.

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