Module-1

Question Bank

Q.1: Find limits of the function $\lim_{(x,y)\to(\frac{\pi}{2},0)} \frac{cosy+1}{y-sinx}$.

Ans: -2

Q.2: Is $f(x,y) = \begin{cases} \frac{x^2}{\sqrt{x^2+y^2}} & ,(x,y) \neq (0,0) \\ 3 & , \quad (x,y) = (0,0) \end{cases}$

is continuous at the origin and redefine if necessary to make it continuous at (0,0).

Ans:- Discontinues

Q.3: Use the limit definition of partial derivative to compute the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial x}$ of the function at the point $(-2,3)$

$$f(x,y) = \sqrt{2x+3y-1}$$

Ans:- $\boldsymbol{f_x(-2,3) = \frac{1}{2},\ f_y(-2,3) = 3/4}$

Q.4: Express $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ as functions of u and v both by using the Chain Rule and by expressing *z* directly in terms of *u* and *v* before differentiating. Then (b) evaluate $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ a at the given point *(u, v)*, $z = 4e^x logy$, $x = log(ucosv)$, $y = usinv$; $(u,v) = (2,\pi/4)$.

Ans:- $\boldsymbol{(a)\frac{\partial z}{\partial u} = 4cosv\ log(usinv) + 4cosv}$,

$$\boldsymbol{\frac{\partial z}{\partial v} = -4u\ sinv\ log(usinv) + \frac{4ucos^2v}{sinv}}$$

(b) $\boldsymbol{\frac{\partial z}{\partial u} = \sqrt{2}(log2+2),\ \frac{\partial z}{\partial v} = -2\sqrt{2}(log2-2)}$

Q.5: The lengths *a*, *b*, and *c* of the edges of a rectangular box are changing with time. At the instant in question, *a* = 1m, *b* = 2 m, *c* = 3 m, $\frac{da}{dt} = \frac{db}{dt} = 1\frac{m}{sec}$ and $\frac{dc}{dt} = -3\ m/sec$. At what rates are the box's volume *V* and surface area S changing at that instant? Are the box's interior diagonals increasing in length or decreasing?

Ans:- Change in volume 3 and change in surface area is 5.

Q.6: Find all second order derivatives of

$$f(x,y) = x^2y + cosy + ysiny$$

Ans:- $\boldsymbol{f_{xx} = 2y, f_{yy} = cosy - ysiny, f_{xy} = 2x}$

Q.7: Use Taylor's formula for $f(x,y) = \sin(x^2+y^2)$ at the origin to find quadratic and cubic approximations of *f* near the origin.

Ans:- $\boldsymbol{Quadratic}: x^2 + y^2$; $\boldsymbol{cubic}: x^2 + y^2$

Q.8: If the base radius & height of a cone are measured as 4 & 8 inches with a possible error 0.04 and 0.08 inches res. calculate the percentage error in calculating the volume of the cone.

Ans:- 3%

Q.9: Use Lagrange's multiplier method. the dimensions of a rectangular box of maximum capacity whose surface area is given when box is closed at top.

Ans:- $\boldsymbol{x = y = z = \sqrt{\frac{S}{6}}}$

Q.10: Discuss the extremes values of the function $f(x, y) = x^3 + y^3 - 3axy$

Ans:- Maximum if $a < 0$ and Minimum if $a > 0$ at (a,a).

Q.11: A rectangular box is placed on xy-plane whose vertex is at origin. Find the maximum volume of the box if the vertex facing to the origin lies on the plane $6x + 4y + 3z = 24$.

Ans:-64/9

Q.12: Find an equation for the plane that is tangent to the given surface $z = \sqrt{y - x}$ at the given point (1,2,1).

Ans:- .x-y+2z-1=0

Q.13: Find the limit of the function

$$\lim_{\substack{(x,y)\to(0,0) \\ x\neq y}} \frac{x - y + 2\sqrt{x} - 2\sqrt{y}}{\sqrt{x} - \sqrt{y}}$$

Ans:- 2

Q.14: Verify that $f_{xy} = f_{yx}$ for $e^x + xlogy + ylogx$

Q.15: Express $\frac{dw}{dt}$ in terms of t both by using (a) Chain Rule and expressing w in terms of t and differentiating w with respect to t, (b) evaluate $\frac{dw}{dt}$ at given value of t.

$$w = 2ye^x - \ln z, \quad x = \ln(t^2 + 1), \quad y = \tan^{-1} t, \quad z = e^t;$$
$$t = 1$$

Ans.- **(a)** $\frac{dw}{dt} = 4t \tan^{-1} t + 1$, **(b)** $\frac{dw}{dt}(1) = \pi + 1$

Q.16: Find all the local maxima, local minima and saddle point of the function

$$f(x,y) = x^3 - y^3 - 2xy + 6$$

Ans:- $\boldsymbol{f(0,0), saddle\ point};\ \boldsymbol{f(-\frac{2}{3},\frac{2}{3}) = \frac{170}{27}}$ **local maximum**

Q.17: Find the minimum distance from the surface $x^2 - y^2 - z^2 = 1$ to the origin.

Ans:- 1

Q.18: Use Taylor's formula for $f(x,y) = e^x \log(1+y)$ at the origin to find quadratic and cubic approximations of *f* near the origin.

Ans:- Quadratic: $\boldsymbol{y + \frac{1}{2}(2xy - y^2)}$**, Cubic:** $\boldsymbol{y + \frac{1}{2}(2xy - y^2) + \frac{1}{6}(3x^2y - 3xy^2 + 2y^3)}$