

Regular Languages

Regular Languages

⇒ A language is said to be a Regular Language if and only if some Finite State Machine ~~recognizes~~ recognizes it.

So what languages are not Regular?

The languages

- ⇒ Which are not recognized by any FSM.
- ⇒ Which requires memory.

⇒ When a language not recognized by any FSM?
Ans: When a language requires memory.

Because of,

- memory of FSM is very limited.
- It cannot store or count strings

Eg.

ababbababb
↑ ↑
↖ ↗
Store
required

$a^N b^N$
aaa bbb [count required]
aaaa bbbb

NOT REGULAR

Operations on Regular Languages

UNION

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

CONCATENATION

$$A \circ B = \{xy \mid x \in A \text{ and } y \in B\}$$

STAR

$$A^* = \{x_1 x_2 x_3 \dots x_k \mid k \geq 0 \text{ and each } x_i \in A\}$$

$$\text{Ex. } A = \{pq, r\}, \quad B = \{t, uv\}$$

$$A \cup B = \{pq, r, t, uv\}$$

$$A \circ B = \{pqt, pqur, rt, ruv\}$$

$$A^* = \{\epsilon, pq, r, pqr, rrpq, pqpq, rr, pqpqpq, rrr, \dots\}$$

Theorem 1: The class of Regular languages is closed Under UNION.

Theorem 2: The class of Regular languages is closed Under CONCATENATION.

DFA Exercise

⇒ For each of the following language, construct a DFA that accepts the language, in all cases alphabet is $\{0, 1\}$.

- ① $\{w : w \text{ is a binary string containing an odd number of 1's}\}$
- ② $\{w : w \text{ is a binary string containing 101 as substring}\}$
- ③ $\{w : \text{the length of } w \text{ is divisible by three}\}$
- ④ $\{w : 110 \text{ is not a substring of } w\}$
- ⑤ $\{w : w \text{ contains the substring } 1011\}$
- ⑥ $\{w : w \text{ contains an even number of 0's or exactly two 1's}\}$