

Project Evaluation and Review Techniques (PERT)

PERT

Each activity will have three time estimates

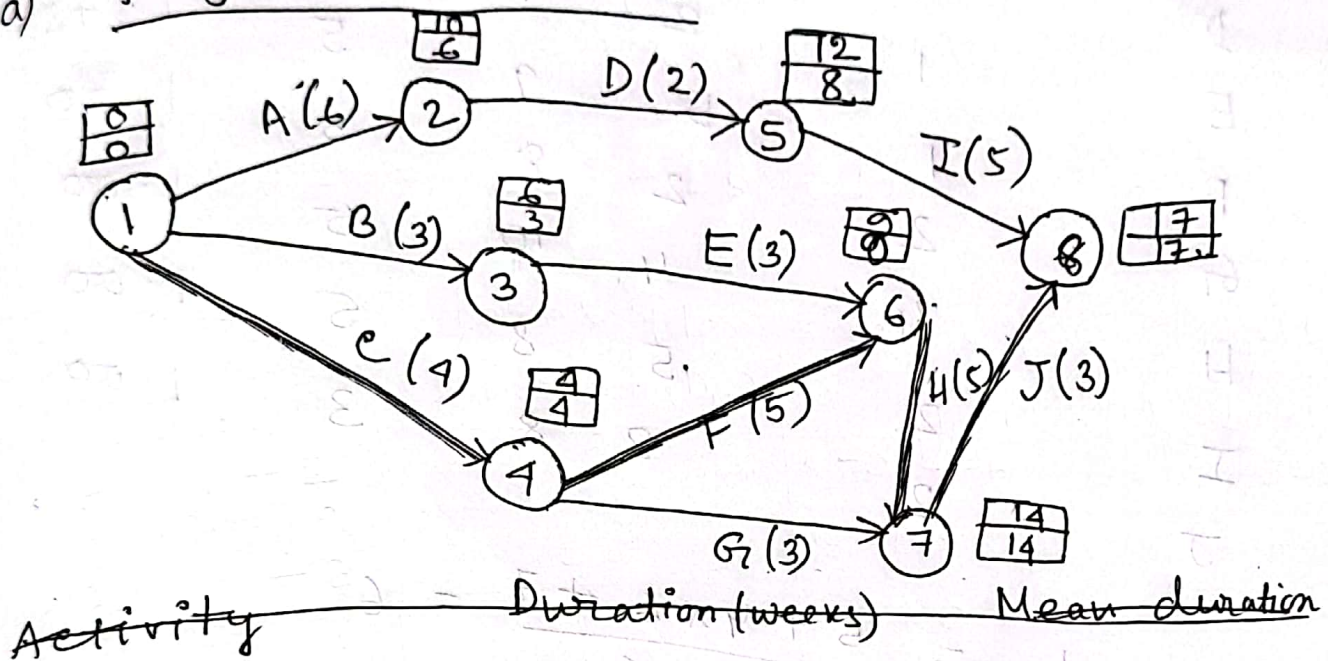
- Optimistic time
- Most Likely time
- Pessimistic time

Consider the following table summarizing the details of a project:

Activity	Predecessors	Durations (weeks)		
		O	m	P
A	-	5	6	7
B	-	1	3	5
C	-	1	4	7
D	A	1	2	3
E	B	1	2	9
F	C	1	5	9
G	C	2	2	8
H	E, F	4	4	10
I	D	2	5	8
J	H, G	2	2	8

- Construct the project network.
- Find the expected duration and variance of each activity.
- Find the critical path and expected projected completion time.
- What is the probability of completing the project on or before 22 weeks?

a) Project Network



Mean duration, $t_e = \text{expected duration}$

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Variance $\sigma^2 = \left(\frac{t_p - t_o}{6} \right)^2$

Earliest start time, $ES_i = \max (ES_i + D_{ij})$

Latest completion time, $LC_i = \min (LC_j - D_{ij})$

Critical path conditions

1) $ES_i = LC_i$ 2) $ES_j = LC_j$

3) $ES_j - ES_i = LC_j - LC_i = D_{ij}$

b)

Activity	Duration (weeks)			Mean duration	Variance
	O	m	p		
A	5	6	7	6	0.44
B	1	3	5	3	0.44
C	1	4	7	4	1.00
D	1	2	3	2	0.11
E	1	2	9	3	1.78
F	1	5	9	5	1.78
G	2	2	8	3	1.00
H	4	4	10	5	1.00
I	2	5	8	5	1.00
J	2	2	8	3	1.00

For A, $t_e = \frac{5 + 4 \times 6 + 7}{6} = 6$

$\sigma^2 = \left(\frac{7-5}{6} \right)^2 = \frac{1}{9} = 0.11$

c) Critical path:
 1 → A → 6 → 7 → 8
 C → F → H → J
 Project completion time = 24 + 5 + 5 + 3 = 17 weeks

Activities
on Critical path

Mean
duration

Variance

C

4

1.00

F

5

1.78

H

5

1.00

J

3

1.00

$$\sigma = \sqrt{4.78} = 2.19$$

$$\frac{1.00}{4.78}$$

Probable

Probability of completing the project

$$P(x \leq 22) =$$

$$P\left[\frac{x - \mu}{\sigma} \leq \frac{22 - 17}{\sqrt{4.78}}\right]$$

$$= P\left[Z \leq \frac{22 - 17}{2.19}\right]$$

$$= P(Z \leq 2.28)$$

$$= 0.9887$$

This value is obtained from standard normal distribution table. Therefore the probability of completing the project on or before 22 weeks is 0.9887.
i.e. 98.87%.