Online Class 03-02-2021

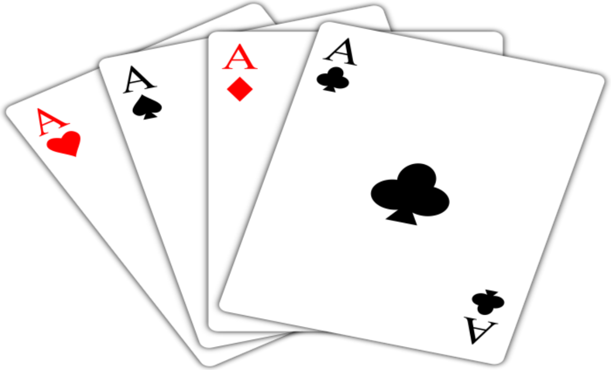
Slot: B21+B22+B23

Probability, Statistics and Reliability

: MAT3003

Topic: Introduction to Probability Concepts,

Random Experiments, Events



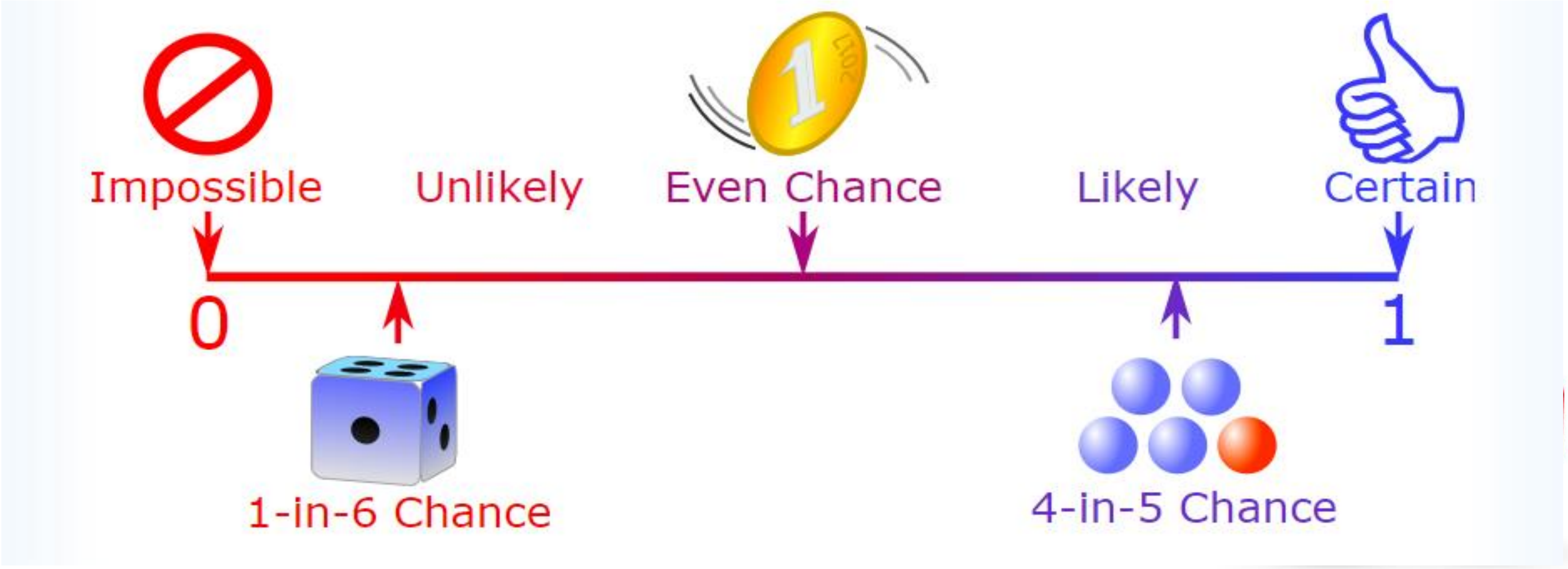
PROBABILITY

Consider the following statement

- The chance of India winning a cricket match against Pakistan is good
- The chance that it will rain on 15th August is pretty high
- The chance that price of share issued by Tata group would go up in the next two years is very high

Probability is basically the chance of something happening. In probability theory, we provide a numerical value for the degree of belief against the chance of occurrence of an event.

Probability is nothing but the quantitative measure of uncertainty **(chance)**.



Probability

- Chance of happening and not happening of an event

Probability = $\dfrac{Number\,of\;\;favourable\,cases}{Total\,number\,of\;\;cases}$

Some Important Terms and Concepts

Random Experiment

- A random experiment is a process whose outcome is uncertain.
- The result may be any one of the various possible outcomes but may not be same every time.
- Eg: if an unbaised dice is thrown it will not always fall with any particular number up. Any of the six numbers on the dice can come up.

Properties of random experiment

An experiment is called random experiment if it satisfies the following **two** conditions :

1. It has **more** than one possible outcome

2. It is **not** possible to predict the outcome in advance

Sample Space

- A set of all possible outcomes from an experiment is called sample space.
- Eg: **Rolling a die**
- There are six possible outcomes and the sample space consists of six elements:
- {1, 2, 3, 4, 5, 6}.

Answer this!

1. Two coins are tossed once. find a sample space.

2. Find the sample space associated with the experiment of rolling a pair of dice.

Trial and Event

- The performance of a random experiment is called a ***trial*** and the outcome an ***event***.
- Eg: Throwing of a dice would be called a trials and the result falling of any one of six numbers 1,2, 3, 4, 5, 6 an event.

Consider the experiment of tossing a coin two times and associated sample space is **S= {HH, HT, TH, TT}**.

Now suppose we are interested in those outcomes which respond to the occurrence of exactly one head.
Then what are those outcome!

We see that **HT** and **TH** are the only elements of **S** corresponding to the occurrence of this happening (event). Those two elements form the set **E= (HT, TH).**
The set **E** is a subset of sample space **S**.

Consider the experiment of tossing a coin two times, with sample space

S= {HH, HT, TH, TT}.

Now suppose we are interested in those outcomes which respond to the occurrence of exactly one head.

Then what are those outcome!

Answer: **E= (HT, TH).**

The set **E** is a subset of sample space **S**.

Definition: **Event** is a set of outcomes of an experiment to which a probability is assigned.

Answer this!

Find event for the random experiment of tossing two coins-

Number of tails is ***exactly*** two

Number of tails is *at least* one

Number of heads is ***at most*** one

Second toss is ***not*** head

Number of tails is ***almost*** two

Number of tails is ***more*** than 2

Simple event

- An event consisting of only one possible outcome is called simple event.
- Eg: in tossing a dice the chance of getting 3 is a single event (because 3 occurs in the dice only once)

Compound event

- An event consisting of more than one possible outcomes is called compound event.
- Eg: in a dice the chance of getting an odd number is a compound event (because odd numbers are more than one i.e. 1, 3 and 5)

Exhaustive Cases or Events

- It is the total number of all the possible outcomes of an experiment.
- For eg: in the throw of a single dice the exhaustive cases are 6. however if 2 dice are thrown simultaneously the exhaustive number of cases would be (6x6=36)

Favourable cases

- The number of outcomes which result in the happening of a desired event.
- Eg: in a single throw of a dice the number of favourable cases for getting an odd number are three , i.e 1, 3 and 5

Mutually Exclusive Events

- If in an experiment the occurrence of an event prevents or rules out the happening of all other events in the same experiment, then these events are said to be **mutually exclusive events.**
- **Example:** In tossing a coin, the events head and tail are mutually exclusive, because if outcome is head, then possibility of getting a tail in the same trial is ruled out.

Equally Likely Events

- Events are said to be equally likely if the chance of their happening is equal.
- Example: In a throw of an unbiased dice, the coming up of 1, 2, 3, 4, 5, 6 is equally likely.

Independent and Dependent Events

- Two or more events are said to be independent if the happening of any one does not depend on the happening of the other.
- Events which are ***not independent*** are called dependent events

Example: If we draw a card from a pack of well shuffled cards and again draw a card from the rest of pack of cards (consisting 51 cards), then the second draw is dependent on the first.

But if on the other hand we draw a second card from the pack by replacing the card drawn the second draw is known as independent of the first.

Questions

1. What is the probability of getting an even number in a single throw with a dice?
2. What is the probability of getting tail in a throw of a coin?
3. A bag contains 6 white balls, 9 black balls. What is the probability of drawing a black ball?

Question:

An unbiased cubic dice is thrown. Determine the probability of getting:

(a) 6
(b) An even number
(c) An odd number
(d) A multiple of 2
(e) A multiple of 3

Question:

One card is drawn from a well shuffled pack of 52 playing cards. What is the probability that it is a -

(a) King
(b) King of red color
(c) King of heart
(d) Numeric card
(e) Numeric card bearing a multiple of 2
(f) Red numeric card bearing a multiple of 2
(g) Black odd numeric card

Question:

Find the probability of getting a red ace when a card is drawn at random from an ordinary deck of cards.

Solution

Since there are 52 cards and there are 2 red aces, namely, the ace of hearts and the ace of diamonds, $P(\text{red ace}) = \frac{2}{52} = \frac{1}{26}$.

A card is drawn from an ordinary deck. Find these probabilities.

a. Of getting a jack

b. Of getting the 6 of clubs (i.e., a 6 and a club)

c. Of getting a 3 or a diamond

d. Of getting a 3 or a 6

Solution

a. Refer to the sample space in Figure 4–2. There are 4 jacks so there are 4 outcomes in event E and 52 possible outcomes in the sample space. Hence,

$$P(\text{jack}) = \tfrac{4}{52} = \tfrac{1}{13}$$

b. Since there is only one 6 of clubs in event E, the probability of getting a 6 of clubs is

$$P(6 \text{ of clubs}) = \tfrac{1}{52}$$

c. There are four 3s and 13 diamonds, but the 3 of diamonds is counted twice in this listing. Hence, there are 16 possibilities of drawing a 3 or a diamond, so

$$P(3 \text{ or diamond}) = \tfrac{16}{52} = \tfrac{4}{13}$$

This is an example of the inclusive or.

d. Since there are four 3s and four 6s,

$$P(3 \text{ or } 6) = \tfrac{8}{52} = \tfrac{2}{13}$$

This is an example of the exclusive or.

When a single die is rolled, what is the probability of getting a number less than 7?

Solution

Since all outcomes—1, 2, 3, 4, 5, and 6—are less than 7, the probability is

$$P(\text{number less than 7}) = \frac{6}{6} = 1$$

The event of getting a number less than 7 is certain.

In a sample of 50 people, 21 had type O blood, 22 had type A blood, 5 had type B blood, and 2 had type AB blood. Set up a frequency distribution and find the following probabilities.

a. A person has type O blood.

b. A person has type A or type B blood.

c. A person has neither type A nor type O blood.

d. A person does not have type AB blood.

Solution

Type	Frequency
A	22
B	5
AB	2
O	21
Total	50

a. $P(\text{O}) = \dfrac{f}{n} = \dfrac{21}{50}$

b. $P(\text{A or B}) = \dfrac{22}{50} + \dfrac{5}{50} = \dfrac{27}{50}$

(Add the frequencies of the two classes.)

c. $P(\text{neither A nor O}) = \dfrac{5}{50} + \dfrac{2}{50} = \dfrac{7}{50}$

(Neither A nor O means that a person has either type B or type AB blood.)

d. $P(\text{not AB}) = 1 - P(\text{AB}) = 1 - \dfrac{2}{50} = \dfrac{48}{50} = \dfrac{24}{25}$

(Find the probability of not AB by subtracting the probability of type AB from 1.)

Thank you