

Class - Test

Q-1) Minimise :

$$Z = 10x_1 + 6x_2 + 2x_3$$

Given

$$-x_1 + x_2 + x_3 \geq 1$$

$$3x_1 + x_2 - x_3 \geq 2$$

In standard form :

$$Z = -10x_1 + 6x_2 + 2x_3 + 0 \cdot x_4 + 0 \cdot x_5$$

$$\therefore B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \& \quad B^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\therefore x_B = B^{-1}b = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

Also, $C_B = 0 \times Z_j - C_j = -C_j \geq 0$, $j = 1, 2, 3$.

Using Dual-Simplex Method.

C_B	B	x_B	C_j $B^{-1}b$	y_1	y_2	y_3	y_4	y_5
0	a_4	x_4	-1	1	-1	-1	1	0
0	a_5	x_5	-2	-3	-1	1	0	1
				10	6	2	0	0

$$Z_j - C_j$$

$$\therefore X_B = \min \{-1, -2\} = -2$$

(2)

$\therefore a_5$ is leaving vector.

$$\frac{Z_k - C_k}{Y_{kk}} = \max \left\{ -\frac{10}{3}, -1 \right\} = -\frac{10}{3}$$

$\therefore k=1$ & a_1 is entering vector.

Transformed Table.

			θ	y_1	y_2	y_3	y_4	y_5
C_B	B	X_B	b	y_1	y_2	y_3	y_4	y_5
0	a_4	x_4	$4/3$	0	$-1/3$	$-2/3$	1	$1/3$
-10	a_1	x_1	$2/3$	1	$1/3$	$-1/3$	0	$-1/3$
			0	$8/3$	$16/3$	0	0	$12/3$

$$Z_j - C_j$$

Here, a_j is infeasible. Bivariable.

$$\delta \frac{Z_k - C_k}{y_k} = \max \left\{ \frac{8/3}{-4/3}, \frac{16/3}{-2/3} \right\} = -2$$

(3)

$\Rightarrow a_2$ is the entering vector.

Now, Transformation table is -

C_b	B	x_b	b	y_1	y_2	y_3	y_4	y_5
-6	a_2	x_2	5/4	0	1	1/2	-3/4	-1/4
-10	a_1	x_1	1/4	1	0	1/2	1/4	-1/4
				0	0	4	2	4

$$Z_j - C_j$$

Here feasible solution is obtained as:

$$x_1 = 1/4, x_2 = 5/4, x_3 = 0 \quad \& \quad Z_{\min} = 10$$

A \rightarrow 2

FROM

TO

	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	5	∞	5
E	1	3	2	8	∞

Row
Minimize

	A	B	C	D	E
A	∞	1	4	6	2
B	4	∞		6	0
C	4	3	∞	0	3
D	8	0	2	∞	1
E	0	2	1	7	∞

Column Minimization

(4)

	A	B	C	D	E
A	∞	4	3	6	9
B	4	∞	0	6	0
C	1	9	∞	0	0
D	8	0	1	∞	4
E	0	2	2	7	∞

$$N = n, S = 5 \times 5.$$

∞	1	3	6	[0]
4	∞	[0]	6	0
4	3	∞	[0]	3
8	[0]	1	∞	1
[0]	2	0	7	∞

$$A \Rightarrow E - A \Rightarrow B - C - D - 8$$

$$\begin{aligned} \text{Cost} &= 1 + 3 + 4 + 4 + 1 \\ &= 13 \end{aligned}$$

As per sequence from above assignment, indicates (5) to produce A, then E & then A without producing product B, C & D. It violates restriction of producing each product.

Now, we have to ~~ex~~ examine matrix for headsoln. assigning with C15 to C12 & R2 to C45.

∞	C1	3	6	0
4	∞	[0]	6	0
4	3	∞	[0]	3
8	0	1	∞	
[0]	3	0	7	∞

Now Sequence, A - B - C - D - E - A
 $\therefore \text{Cost} = 2 + 3 + 4 + 5 + 1 = 15.$

Here, the cost is increased by Rs. 2.

<u>A →</u>	fodder 1	fodder 2
Nutrient A	2	1
Nutrient B	2	3
Nutrient C	1	1

Cost of fodder 1 is ₹ 3 per unit & that of fodder 2 ₹ 2.

Let x unit of fodder 1 & y unit of fodder 2. ⑤
 \therefore total cost $z := 3x + 2y$

Nutrient	fodder 1	fodder 2	Minimum requirement.
A	2	1	14
B	2	3	22
C	1	1	1

\therefore Constraints :

$$\begin{aligned}
 2x + y &\geq 14 \\
 2x + 3y &\geq 22 \\
 x + y &\geq 1 ; \quad x \geq 0 ; y \geq 0
 \end{aligned}$$

\therefore LPP will be :

Minimise : $z := 3x + 2y$
 $2x + y \geq 14, 2x + 3y \geq 22, x + y \geq 1$
 $x \geq 0, y \geq 0.$

A → 4

No. of Supply Constraint = 3
No. of Demand constraint = 3

	D1	D2	D3	Supply
S1	6	8	4	14
S2	4	9	3	12
S3	1	2	6	15
Demand	6	10	15	

In 1st row:

Smallest transportation cost is 4 in cell S1D3.
allocation in this cell is $\min(14, 15) = 4$.

So, table will be:

S1	D1	D2	D3	Supply
S2	6	8	14	0
S2	4	9	3	12
S3	1	2	6	5
Demand	6	10	1	

In 2nd row, smallest transportation cost is 3 in S2D3.
∴ Allocation in the cell will be $\min(12, 1) = 1$

Now, table will be

	D1	D2	D3	Supply
S1	8	8	14	0
S2	4	9	1	11
S3	1	2	6	5
Demand	6	10	0	

(8)

In 2nd row, smallest transportation cost is 4 in cell S2D1. \therefore allocation to the cell will be $\min(11, 6)$

so,

	D1	D2	D3	Supply
S1	6	8	14	0
S2	6	9	1	5
S3	1	2	6	5
	0	10	0	

Similarly; S2D will be allocated by 5. In 3rd row, smallest transportation cost is $\min(5, 2) = 2$ in cell S3D2 so it will be allocated $\min(5, 2) = 2$.

Initial feasible Solution Table

(1)

	D 1	D 2	D 2	Supply
S1	6	8	14	14
S2	6	5	1	12
S3	1	5	6	5
	6	10	15	31/31

Minimum table transportation Cost

$$= 4 \times 14 + 4 \times 6 + 9 \times 5 + 3 \times 1 + 2 \times 5$$
$$= 138.$$

No. of allocated cells = $S = 3 + 3 - 1 = 5$

\therefore solution is ~~to~~ non-degenerate.