ASSINGMENT

1.

Sol. S-1 → Assume a variable p to be an arbitary number

S-2 → Consider the string $0^p 1^p \in L$

S-3 → $w = xyz$

$|xyz| = 2p \geq p$

Now only way to break this string in xyz triplet, such that $|xy| \leq p$ and $y \neq \varepsilon$ is by choosing $y = 0^k$ for some ~~$1 \leq p \leq k$~~ $1 \leq k \leq p$

S-4 → $xy^iz = 0^k 0^{p-k} 1^p \notin L$

$\therefore xy^iz \notin L$

~~contradicti~~ contradiction so not regular

S-2 → $w = xyz$ with $|xy| \leq P$, $|y| \geq 1$

S-3 → Suppose $y = 0^a$, $0 < a \leq P$ then by pumping lemma we have —

S-4 → $xyz = 0^{P-a} \, 0^{ai} \, 1^{P+P!} \in L$ for all $i \geq 0$

$i = \left(1 + \frac{P!}{a}\right)$ and you get

S-5 → $xyz = 0^{P-a} \, 0^{a\left(1 + \frac{P!}{a}\right)} \, 1^{P+P!}$

$= 0^{P-a} \, 0^{a} \, 0^{a\left(\frac{P!}{a}\right)} \, 1^{P+P!}$

$= 0^{P+P!} \, 1^{P+P!} \notin L$

Contradiction so not regular

S-2 → Consider the string $1^{p^2} \in L$

S-3 → $|1^{p^2}| > p$

Now only way to break this string in xyz triplet, such that $|xy| \leq p$ and $y \notin \varepsilon$ is to choose $y = 1^k$ for some $1 \leq k \leq p$

S-4 → $xy^iz = 1^1 1^{2k} 1^{p^2 - l - k}$ which is not a perfect square

$\therefore$ contradiction

Hence L is not regular

$\underline{S-2}$ $\rightarrow$ $w = xyz$

Length of string $|xyz| = P$

$\underline{S-3}$ $\rightarrow$ for $|xy^iz|$

$|y| \geq 1$

Let $i = P+1$

$$|xy^{P+1}z| = |xyz| + |y^P|$$

$$= P + P(|y|)$$

$$= P(1 + |y|)$$

$$|xy^iz| = P(1+|y|)$$

$\therefore$ $|xy^iz| \notin L$

i.e. contradiction

So it is not regular

S-2 → Let a be an arbitary number

S-3 → $w = xyz$

Length of string $|xyz| = p+q \geq a$

S-4 → By choosing $y = 0^K$ for some $1 \leq K \leq a$

S-5 → for $|xy^iz|$ $\qquad |y| \geq 1$

Let $i \geq 0$

$xy^iz = 1^K 1^{P-K} 0^q \notin L$

$xy^iz \notin L$

i.e. contradiction

So, it is not regular

S-2 → Let string length = P

So, by pumping lemma,

$w = xyz$ condition 1) $|y| > 0$

2) $|xy| \leq P$

3) $xy^i z \in L$

S-3 → Let $w = 0^P 1 0^P$ $|w| \geq P$ and $w \in L$

S-4 → According to condition 2 x and y and composed of only 0's

S-5 → By condition 1, it follows that $y = 0^K$ for some $K > 0$

S-6 → According to 3, we can take $i = 0$ and the resulting string will still be in L.

$xy^0 z \in L$

$\therefore$ L is not regular

7.

Sol.

S-1 → Let $L_p = a^i b^j c^k$ where $n = i$, $m = j$, $n + m = k$

S-2 → Let $L_p' = \{a^1 b^j c^j : j > 0\}$ is not regular

$L_p' = L_p \cap L(ab^*c^*)$

If L_p was regular then L' would be regular too.

S-3 → If we can prove that L_p can be pumped so it will be proved that L_p is not regular by converse of pumping lemma.

S-4 → Let pumping length = 2

for $i = 0$, $j = 0$, $k \geq 0$: $s = c^k$

set $x = \varepsilon$ $y = c$ $z = c^{k-1}$

for every $i \geq 0$ xy^iz is in L_p

for every $i \geq 0$, xy^iz is in L_p

S-6 → for $i = 0$, and any $j \geq 1$: $w = ab^jc^j$

set $x = \varepsilon$ $y = a$; $z = b^jc^j$

for every $i \geq 0$, xy^iz is in L_p

S-7 → for $i = 2$ and any $j, k \geq 0$; $w = aab^jc^k$

set $x = \varepsilon$, $y = aa$, $z = b^jc^k$

for every $i \geq 0$, xy^iz is in L_p

S-8 → for $i \geq 3$ and any $j, k \geq 0$: $w = aaa^{i-2}b^jc^k$

set $x = \varepsilon$, $y = a$, $z = aa^{i-2}b^jc^k$

for every $i \geq 0$, xy^iz is in L_p

as L_p can be pumped

$\therefore$ L_p is not regular

$L_p' = a\ b\ c$ is not regular

$$L_p' = L_p \cap L(a^* b^* c^*)$$

If L_p was regular than L_p' would be regular too

If we can prove that L_p can be pumped so it will be proved that L_p is not regular by converse of pumping lemma.

Let pumping length $= 2$

<u>S-1</u> → for $i = 0$, $j = 0$, $k = 0$; $w = \epsilon$

set $x = \varepsilon$, $y = \varepsilon$, $z = \varepsilon$

for every $i \geq 0$ $xy^i z$ is in L_p

<u>S-2</u> → for $i = 1$, $j \geq 1$: $w = a b^j c^{2j}$

set $x = \varepsilon$ $y = a$, $z = b^j c^{2j}$

for every $i \geq 0$, $xy^i z$ is in L_p

S-3 → for $i = 2$ and any $j, k \geq 0$: $w = aab^j c^{2j}$

set $x = \varepsilon$, $y = aa$, $z = b^j c^{2j}$

for every $i \geq 0$, xy^iz is in L_p

S-4 → for $i = 3$ and any $j, k \geq 0$: $w = aaa^{i-2}b^j c^{2j}$

set $x = \varepsilon$, $y = a$, $z = aa^{i-2}b^j c^k$

for every $i \geq 0$, xy^iz is in L_p

as L_p can be pumped

$\therefore$ L_p is not regular