2–8 DOUBLE INTEGRALS

Describe the region of integration and evaluate.

2. $\int_0^2 \int_x^{2x} (x + y)^2 \, dy \, dx$

3. $\int_0^3 \int_{-y}^{y} (x^2 + y^2) \, dx \, dy$

4. Prob. 3, order reversed.

5. $\int_0^1 \int_{x^2}^{x} (1 - 2xy) \, dy \, dx$

6. $\int_0^2 \int_0^{y} \sinh (x + y) \, dx \, dy$

7. Prob. 6, order reversed.

8. $\int_0^{\pi/4} \int_0^{\cos y} x^2 \sin y \, dx \, dy$

In Exercises 11–16, integrate f over the given region.

11. **Quadrilateral** $f(x, y) = x/y$ over the region in the first quadrant bounded by the lines $y = x, y = 2x, x = 1, x = 2$

12. **Square** $f(x, y) = 1/(xy)$ over the square $1 \le x \le 2$, $1 \le y \le 2$

13. **Triangle** $f(x, y) = x^2 + y^2$ over the triangular region with vertices (0, 0), (1, 0), and (0, 1)

14. **Rectangle** $f(x, y) = y \cos xy$ over the rectangle $0 \le x \le \pi$, $0 \le y \le 1$

15. **Triangle** $f(u, v) = v - \sqrt{u}$ over the triangular region cut from the first quadrant of the uv-plane by the line $u + v = 1$

16. **Curved region** $f(s, t) = e^s \ln t$ over the region in the first quadrant of the st-plane that lies above the curve $s = \ln t$ from $t = 1$ to $t = 2$

Volume Beneath a Surface $z = f(x, y)$

41. Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and below by the triangle enclosed by the lines $y = x, x = 0$, and $x + y = 2$ in the xy-plane.

42. Find the volume of the solid that is bounded above by the cylinder $z = x^2$ and below by the region enclosed by the parabola $y = 2 - x^2$ and the line $y = x$ in the xy-plane.

43. Find the volume of the solid whose base is the region in the xy-plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $z = x + 4$.

44. Find the volume of the solid in the first octant bounded by the coordinate planes, the cylinder $x^2 + y^2 = 4$, and the plane $z + y = 3$.

Ans(41): 4/3

Ans(42):63/20

Ans(43):625/12

Ans(44):(9π-8)/3

In Exercises 31–40, sketch the region of integration, reverse the order of integration, and evaluate the integral.

31. $\displaystyle\int_0^{\pi}\int_x^{\pi}\frac{\sin y}{y}\,dy\,dx$

32. $\displaystyle\int_0^{2}\int_x^{2}2y^2\sin xy\,dy\,dx$

33. $\displaystyle\int_0^{1}\int_y^{1}x^2e^{xy}\,dx\,dy$

34. $\displaystyle\int_0^{2}\int_0^{4-x^2}\frac{xe^{2y}}{4-y}\,dy\,dx$

35. $\displaystyle\int_0^{2\sqrt{\ln 3}}\int_{y/2}^{\sqrt{\ln 3}}e^{x^2}\,dx\,dy$

36. $\displaystyle\int_0^{3}\int_{\sqrt{x/3}}^{1}e^{y^3}\,dy\,dx$

37. $\displaystyle\int_0^{1/16}\int_{y^{1/4}}^{1/2}\cos\left(16\pi x^5\right)\,dx\,dy$

38. $\displaystyle\int_0^{8}\int_{\sqrt[3]{x}}^{2}\frac{dy\,dx}{y^4+1}$

39. **Square region** $\iint_R (y-2x^2)\,dA$ where R is the region bounded by the square $|x|+|y|=1$

40. **Triangular region** $\iint_R xy\,dA$ where R is the region bounded by the lines $y=x, y=2x$, and $x+y=2$

In Exercises 1–16, change the Cartesian integral into an equivalent polar integral. Then evaluate the polar integral.

1. $\displaystyle\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}dy\,dx$

2. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}dy\,dx$

3. $\displaystyle\int_0^{1}\int_0^{\sqrt{1-y^2}}(x^2+y^2)\,dx\,dy$

4. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}(x^2+y^2)\,dy\,dx$

5. $\displaystyle\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}dy\,dx$

6. $\displaystyle\int_0^{2}\int_0^{\sqrt{4-y^2}}(x^2+y^2)\,dx\,dy$

7. $\displaystyle\int_0^{6}\int_0^{y}x\,dx\,dy$

8. $\displaystyle\int_0^{2}\int_0^{x}y\,dy\,dx$

9. $\displaystyle\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0}\frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx$

10. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{0}\frac{4\sqrt{x^2+y^2}}{1+x^2+y^2}\,dx\,dy$

11. $\displaystyle\int_0^{\ln 2}\int_0^{\sqrt{(\ln 2)^2-y^2}}e^{\sqrt{x^2+y^2}}\,dx\,dy$

12. $\displaystyle\int_0^{1}\int_0^{\sqrt{1-x^2}}e^{-(x^2+y^2)}\,dy\,dx$

13. $\displaystyle\int_0^{2}\int_0^{\sqrt{1-(x-1)^2}}\frac{x+y}{x^2+y^2}\,dy\,dx$

14. $\displaystyle\int_0^{2}\int_{-\sqrt{1-(y-1)^2}}^{0}xy^2\,dx\,dy$

15. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\ln\left(x^2+y^2+1\right)\,dx\,dy$

16. $\displaystyle\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\frac{2}{(1+x^2+y^2)^2}\,dy\,dx$

Evaluate the integrals in Exercises 7–20.

7. $\displaystyle\int_0^1\int_0^1\int_0^1 (x^2 + y^2 + z^2)\, dz\, dy\, dx$

8. $\displaystyle\int_0^{\sqrt{2}}\int_0^{3y}\int_{x^2+3y^2}^{8-x^2-y^2} dz\, dx\, dy$

9. $\displaystyle\int_1^e\int_1^e\int_1^e \frac{1}{xyz}\, dx\, dy\, dz$

10. $\displaystyle\int_0^1\int_0^{3-3x}\int_0^{3-3x-y} dz\, dy\, dx$

11. $\displaystyle\int_0^1\int_0^{\pi}\int_0^{\pi} y \sin z\, dx\, dy\, dz$

12. $\displaystyle\int_{-1}^1\int_{-1}^1\int_{-1}^1 (x + y + z)\, dy\, dx\, dz$

13. $\displaystyle\int_0^3\int_0^{\sqrt{9-x^2}}\int_0^{\sqrt{9-x^2}} dz\, dy\, dx$

14. $\displaystyle\int_0^2\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_0^{2x+y} dz\, dx\, dy$

15. $\displaystyle\int_0^1\int_0^{2-x}\int_0^{2-x-y} dz\, dy\, dx$

16. $\displaystyle\int_0^1\int_0^{1-x^2}\int_3^{4-x^2-y} x\, dz\, dy\, dx$

17. $\displaystyle\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} \cos(u + v + w)\, du\, dv\, dw$ (uvw-space)

18. $\displaystyle\int_1^e\int_1^e\int_1^e \ln r \ln s \ln t\, dt\, dr\, ds$ (rst-space)

19. $\displaystyle\int_0^{\pi/4}\int_0^{\ln\sec v}\int_{-\infty}^{2t} e^x\, dx\, dt\, dv$ (tvx-space)