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Probability, Statistics and Reliability (MAT3003)

SLOT: B21 + B22 + B23

MODULE - 4

Topic: Hypothesis Testing: Type-I Error, Type-II Error, and Critical Region

Reference: Probability and Statistics for Engineers & Scientists by R. E. Walpole, Pearson (2012).

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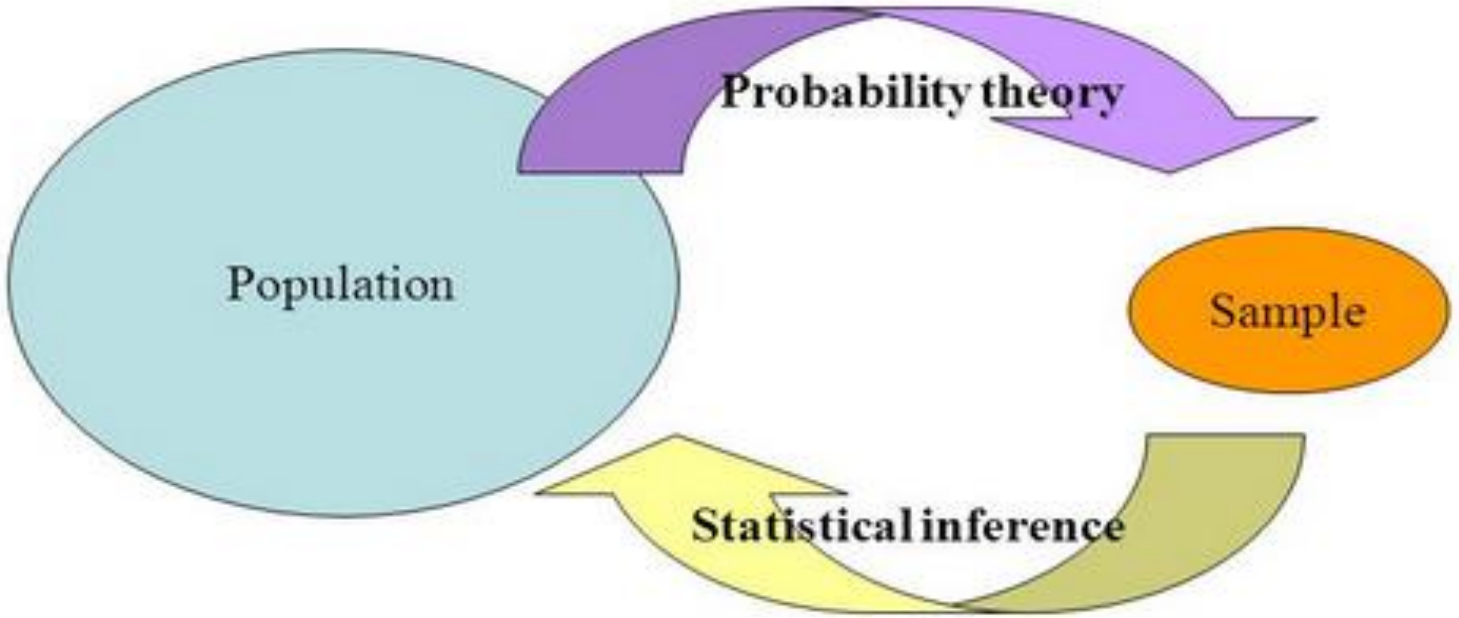
- **Definitions: Statistical Inference**
- **Hypothesis Testing, Types of Hypothesis**
- **Level of Significance, Level of Confidence**
- **Type –I and Type- II Errors**
- **Critical Region**

Definition

Statistical inference

To draw conclusions about the population of study from observations (a sample) obtained from the population.

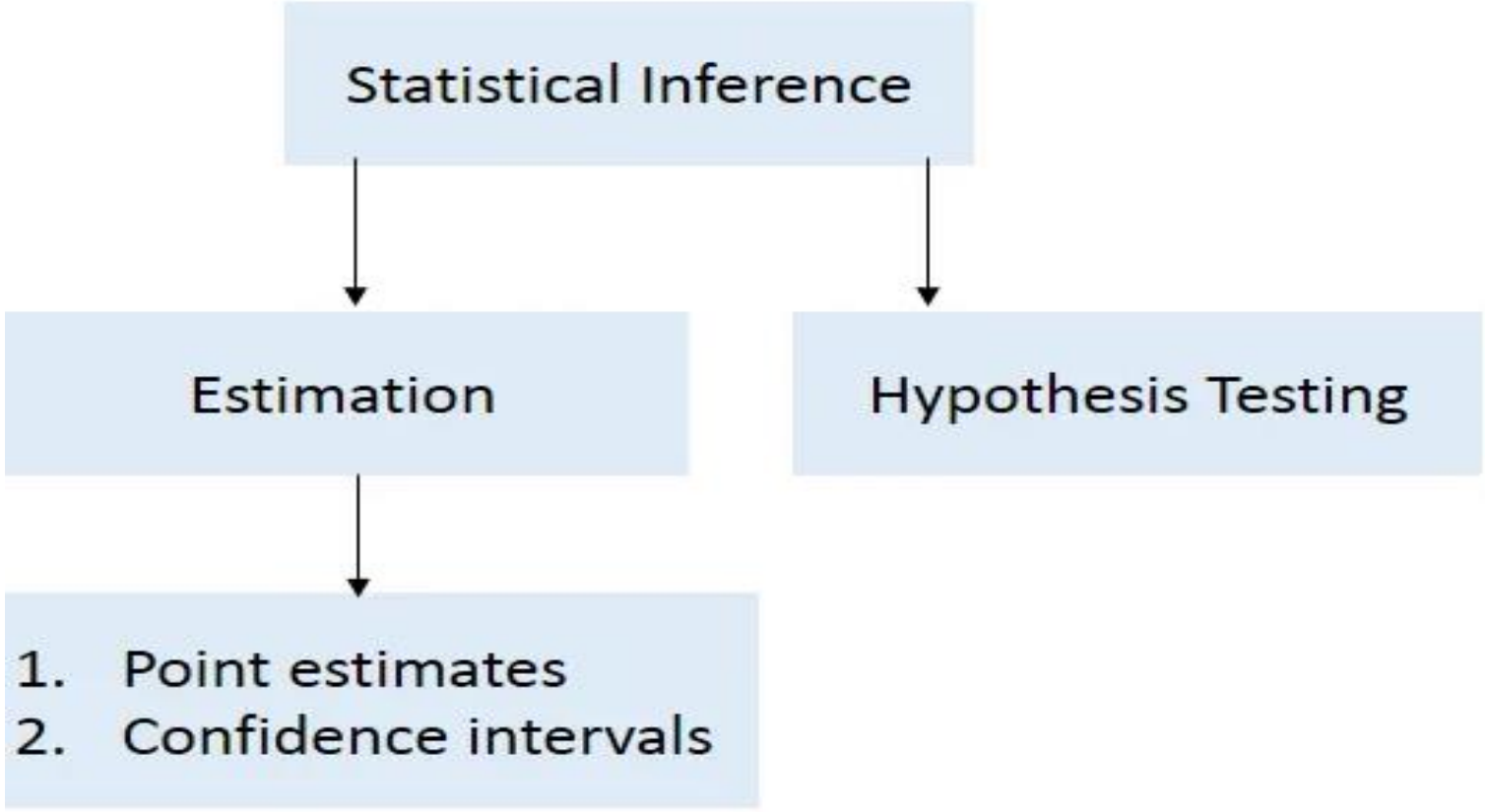


Reference: Probability and Statistics for Engineers & Scientists by R. E. Walpole, Pearson (2012).

Example

- Before the Loksabha or Vidhan sabha election process starts or just before the declaration of election results, the print media and electronic media conduct exit poll to predict the election result.
- In this process all voters are not included in the survey, only a portion of voter's i.e., sample is included to infer about the population.

Types of Statistical Inference



Reference: Probability and Statistics for Engineers & Scientists by R. E. Walpole, Pearson (2012).

- There are two types of statistical inference
  - Estimation
  - Hypothesis Testing

- Estimation is appropriate when we want to estimate a population parameter.

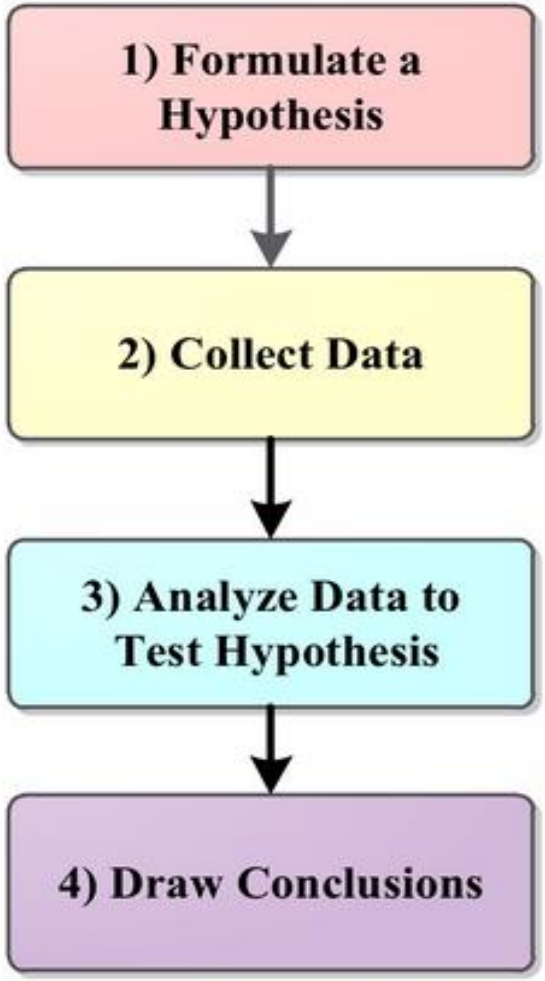
- Hypothesis testing is appropriate when we want to assess some claim about a population based on the evidence provided by a sample.

What is a Hypothesis?

- In Statistics, a hypothesis is defined as a statement, which may or may not be true about the population parameter or about the probability distribution of the parameter that we wish to validate on the basis of sample information.

Hypothesis Testing (or Testing of Hypothesis)

- *Hypothesis testing* is a statistical method that is used in making statistical decisions using experimental data.



Types of Hypothesis

- Two Types:

1. Null Hypothesis, which is denoted by H_0
2. Alternative hypothesis, which is denoted by H_1 or H_a

Null Hypothesis

In hypothesis testing, decision-maker (DM) should not be motivated by prospects of profit or loss resulting from the acceptance or rejection of the hypothesis.

The DM should be completely impartial and should have no brief for any party or company nor should allow his personal views to influence the decision i.e., H_0: $\mu=\mu_0$

Alternative Hypothesis

- Any hypothesis which is complementary to the null hypothesis is called an alternative hypothesis, denoted by H_1. For example, if we want to test the null hypothesis that the population has a specified mean μ_0, i.e., H_0: $\mu=\mu_0$ then the alternative hypothesis should be:

(i) H_1: $\mu \neq \mu_0$ (i.e., $\mu > \mu_0$ OR $\mu < \mu_0$)

(ii) H_1: $\mu > \mu_0$

(iii) H_1: $\mu < \mu_0$

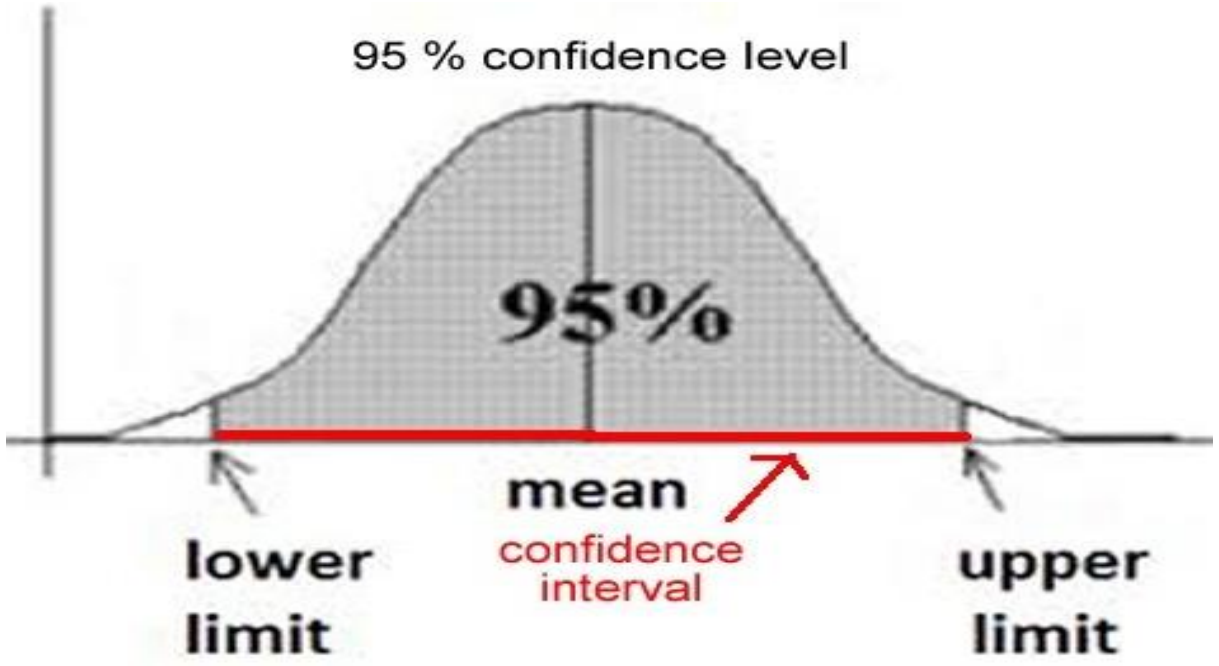
The alternative hypothesis in (i) is known as a two tailed alternative and the alternatives in (ii) and (iii) are known as right tailed and left tailed alternatives respectively.

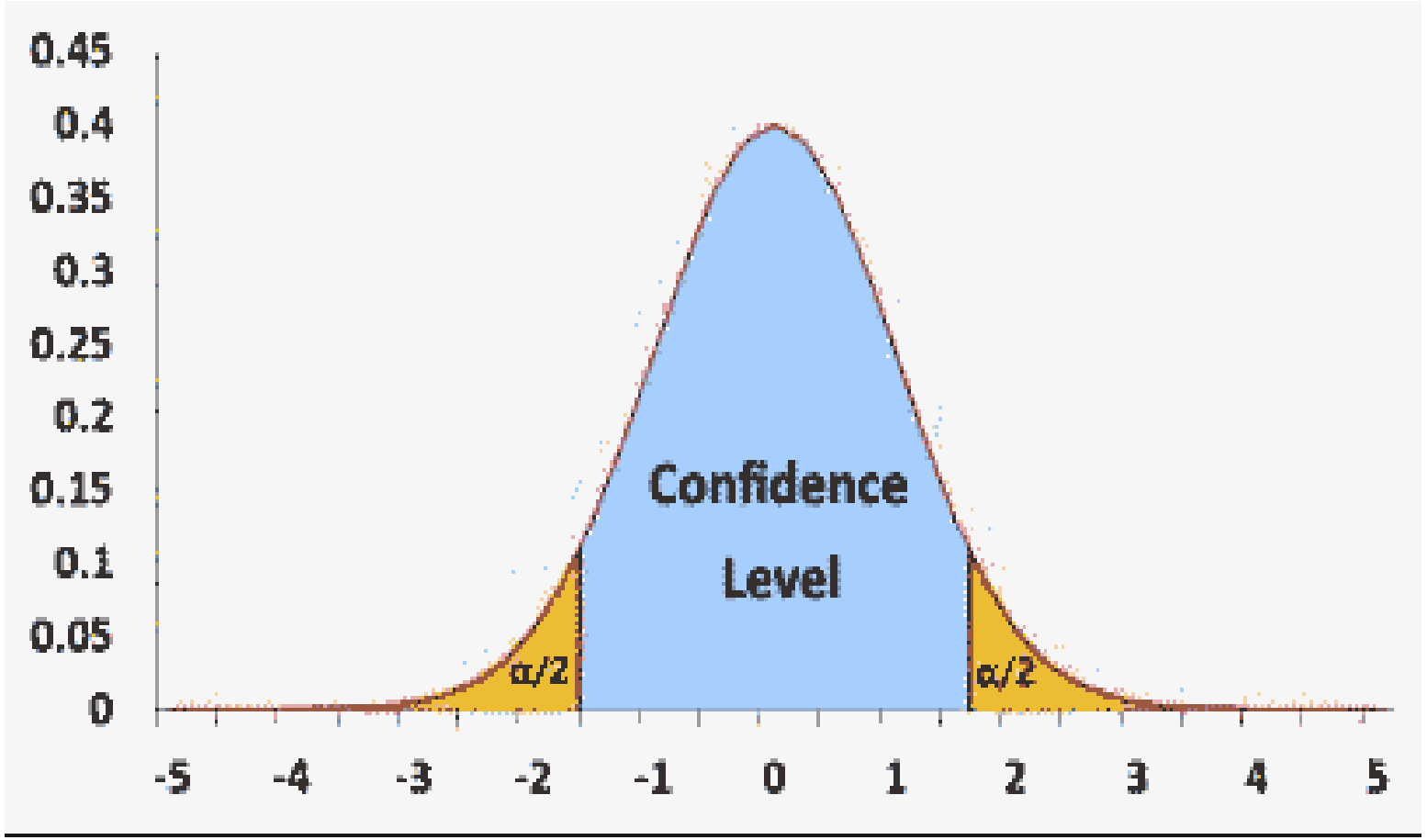
Level of Significance (α)

- The **significance level**, also denoted by α, is the probability of rejecting the null hypothesis when it is true.
- For example, a **significance level** of 0.05 indicates a 5% risk of concluding that a difference exists when there is no actual difference.
- Generally the level of significance is taken to be 5% or 1%.

Level of Confidence

- The **confidence level** tells you how sure you can be. It is expressed as a percentage and represents how often the true percentage of the population who would pick an answer lies within the **confidence** interval.
- The confidence with which a person rejects a null hypothesis depends upon the level of significance adopted. These may, hence, sometime be termed as **levels of confidence**.
- The 95% **confidence level** means you can be 95% certain; the 99% **confidence level** means you can be 99% certain.


95 % confidence level
95%
mean
confidence
interval
lower
limit
upper
limit


0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
Confidence
Level
α/2
α/2
-5
-4
-3
-2
-1
0
1
2
3
4
5

Meaning of Levels of Confidence

Level	Amount of Confidence	Interpretation
0.05	95%	If the experiment is repeated a 100 times, only on five occasions the obtained mean will fall outside the limited $\mu \pm 1.96$ SE
0.01	99%	If the experiment is repeated a 100 times, only on one occasions the obtained mean will fall outside the limited $\mu \pm 2.58$ SE

Steps in Hypothesis Testing

Step 1. Set up null hypothesis.

Step 2. Set up alternative hypothesis.

Step 3. Choose the level of significance.

Step 4. Test statistics.

Step 5. Interpret the result.

Type I and Type II Errors

- In testing any hypothesis, we get only two results: either we accept or we reject it.

  Hence, four cases may arise.

- Case 1: The hypothesis is true but test rejects it (Type –I error).
- Case 2: The hypothesis is false but test accepts it (Type –II error).
- Case 3: Null hypothesis is true, and the test accepts it (No Error).
- Case 3: Null hypothesis is false, and the test rejects it (No Error).

Type –I Error

- **Type –I Error:** In a hypothesis test, a type-I error occurs when the null hypothesis is rejected when it is in fact true. i.e., H_0 is wrongly rejected.
- Example: In a clinical trial of a new drug, the null hypothesis might be that the new drug is no better, on average, than the current drug. That is, there is no difference between the two drugs on average.
- α = Probability of type I error

  = Probability of rejecting H_0 when H_0 is true.

  = P(reject H_0/ H_0 is true).

Type –II Error

- **Type –II error**: In a hypothesis test, a type –II error occurs when the null hypothesis, H_0, is not rejected when it is in fact false.
- For example, in a clinical trial of a new drug, the null hypothesis might be that the new drug is no better, on average, than the current drug.
- β = Probability of type II error

  = Probability of accepting H_0 when H_1 is true.

  = P(accept H_0/ H_1 is true).

The following table from Harnett helps to illustrate the different types of error.

		Decision from Sample	
True statement		Reject H_0	Accept H_0
	H_0 True	Wrong (Type I Error)	Correct
	H_0 False (Or H_1 True)	Correct	Wrong (Type II Error)

Question 1

- If a medicine is administered to a few patients of a particular disease to cure them and the medicine is curing the disease. However, it is claimed that it has no effect or has an adverse effect, and hence it is discontinued. What type is this error?

Ans. This is Type I Error.

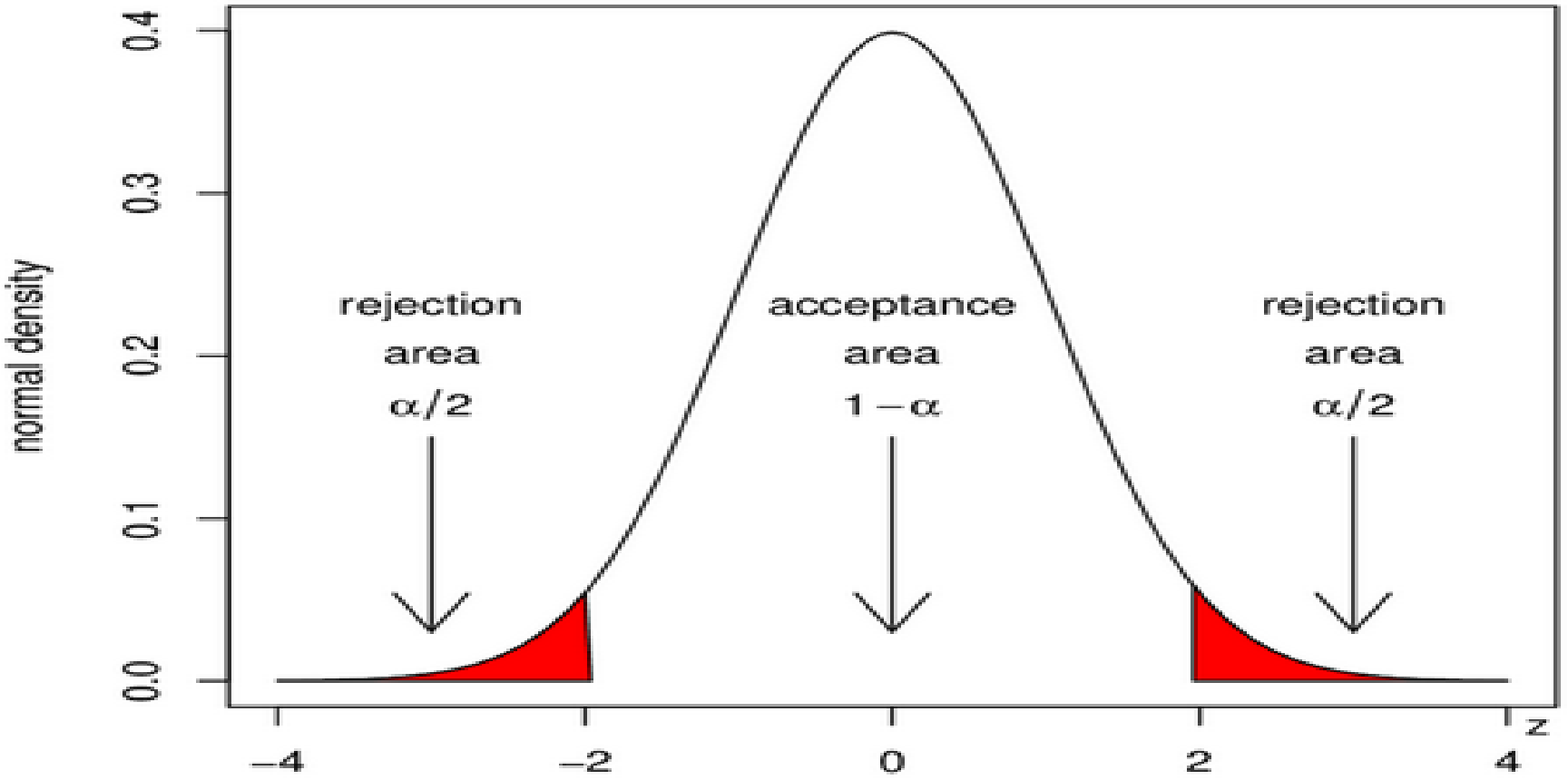
Question 2

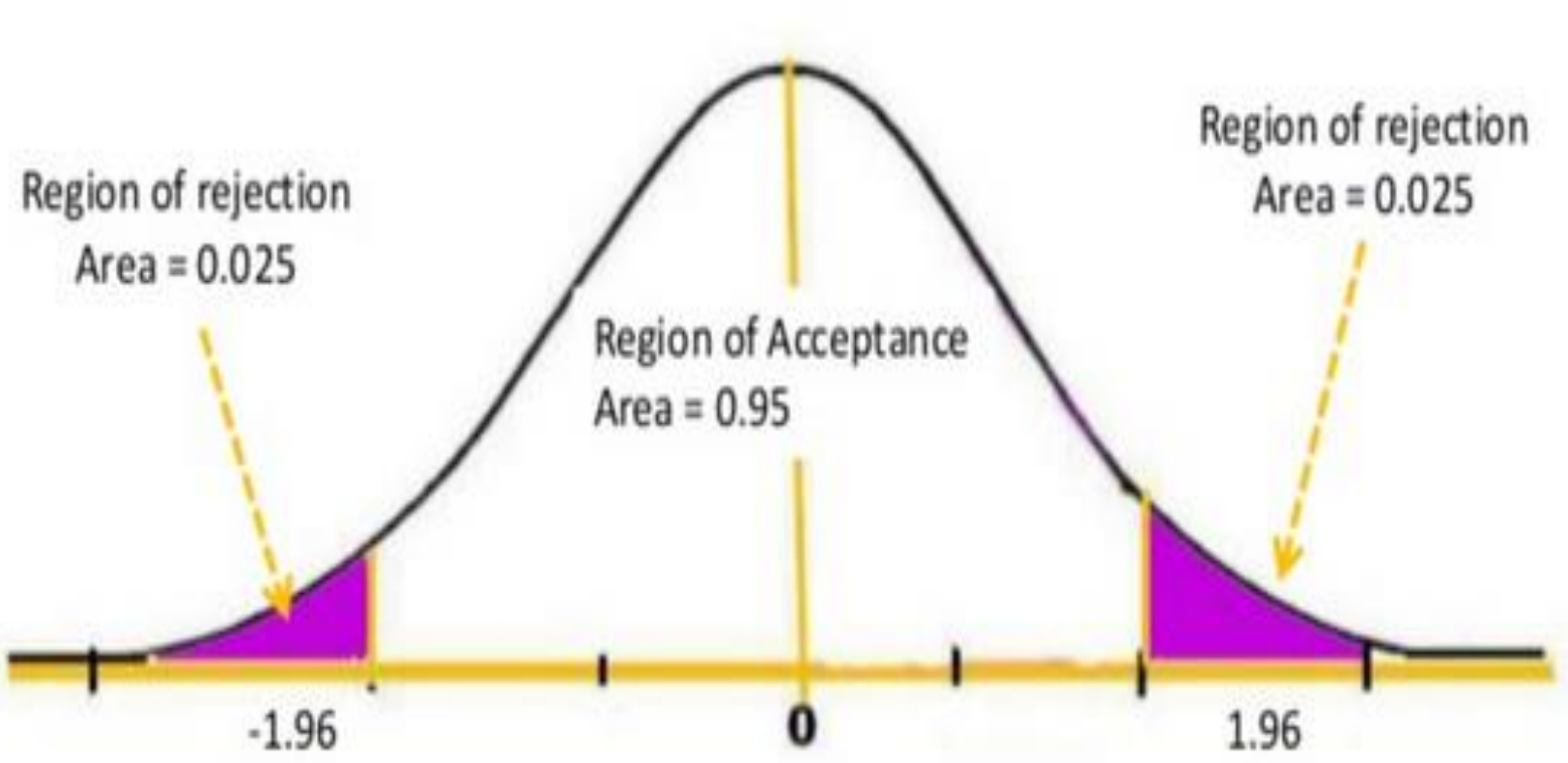
- Continuing to Question 1, on the contrary, the medicine has adverse effect and is claimed to have good effect and the treatment is continued. What type is this error?

Ans. This is Type II Error.

Important Remark

- $P(x \in W/H_0) = \alpha =$ *Level of Significance* of the test.
- $P(x \in W/H_1) = 1 - \beta$ is called the *Power of the test*.
- $P(x \in \bar{W}/H_0) = 1 - \alpha$
- $P(x \in \bar{W}/H_1) = \beta$
- Here, W is called the *Critical Region (or Region of Rejection, or Rejection Area)*.
- $\bar{W}$ is the complement of W, called the *Acceptance Area*.


normal density
0.0
0.1
0.2
0.3
0.4
rejection
area
α/2
acceptance
area
1 − α
rejection
area
α/2
−4
−2
0
2
4
z


Region of rejection
Area = 0.025
Region of Acceptance
Area = 0.95
Region of rejection
Area = 0.025
-1.96
0
1.96

Question 3

Let p be the probability that a coin will fall head in a single toss in order to test

$$H_0\colon\ p = 1/2 \text{ against } H_1\colon\ p = 3/4.$$

The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained.

Obtain the probability of type I error, type II error and power of the test.

Solution

Here H_0: p=1/2 against H_1: p=3/4

Let X be the number of heads in n tosses of a coin then $X \sim B(n,p)$, therefore

$$P(X=x) = {}_{n}C_{x}\, p^{x} q^{n-x} = {}_{5}C_{x}\, p^{x} q^{5-x}$$

Critical region is given by : $W=\{x: x \geq 4\} \Rightarrow \bar{W} = \{x: x \leq 3\}$

- α=Probability of the type I error $=P[X \geq 4)/H_0]$

$$=P(x=4/p=1/2)+P(x=5/p=1/2)$$

$$= {}_{5}C_{4}\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{5-4} + {}_{5}C_{5}\left(\frac{1}{2}\right)^{5}\left(\frac{1}{2}\right)^{5-5} = 3/16$$

- β= Probability of the type II error = $P(x \in \bar{W}/H_1)$

$$=1- P(x \in W/H_1)$$

$$=P[X \leq 3/H_1]=1- P[X \geq 4)/H_1]$$

$$=1- P(x=4/p=3/4)+P(x=5/p=3/4) = 47/128$$

- Power of the test $= 1-\beta = 81/128$

Question -4 (For Students)

- If $x \geq 1$ is the critical region for testing H_0: θ=2 against the alternative hypothesis H_1: θ=1, on the basis of the single observation from the population

$$f(x, \theta) = \theta e^{-\theta x}; 0 \leq x < \infty$$

Find the values of Type I and Type II errors.

Practice Questions

THANK YOU

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