

~~NAME~~ LAXMI PARMAR - 19BEC10044

Q.1 \Rightarrow The time to failure in hours of an important piece of electronic equipment used in a manufactured DVD player has the density function.

$$f(x) = \begin{cases} \frac{1}{2000} \exp(-x/2000), & x \geq 0 \\ 0, & x < 0 \end{cases}$$

a) Find $F(x)$

b) Determine the P. that the component (and thus the DVD player) lasts more than 1000 hours before the component needs to be replaced.

c) Determine the Prob. that the component fails before 2000 hours.

Solve $\rightarrow F(x) = P(X \leq x)$

$$= \int_{-\infty}^x f(t) dt$$

$$= \int_0^x \frac{1}{2000} e^{-t/2000} dt + \int_{-\infty}^0 0 dt$$

$$= \left(-e^{-t/2000} \right) \Big|_0^x + 0$$

$$= -e^{-x/2000} + e^0$$

$$= 1 - e^{-x/2000} \text{ for all } x \geq 0$$

On the other hand, it is a zero function for $x < 0$ and its integral on a segment is 0 so.

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$$F(x) = 0 \quad \text{for all } x < 0$$

$$\begin{aligned} b) \quad P(x > 1000) &= 1 - P(x \leq 1000) \\ &= 1 - F(1000) \\ &= 1 - (1 - e^{-1000/2000}) \\ &= 1 - 1 + e^{-1/2} \\ &= 0.6065 \end{aligned}$$

$$\begin{aligned} c) \quad P(x < 2000) &= P(x \leq 2000) \\ &= F(2000) \\ &= 1 - e^{-2000/2000} \\ &= 1 - 1/e \\ &= 0.6391 \quad \text{Ans} \end{aligned}$$

Q.2 \Rightarrow A cereal manufacturer is aware that the weight of the product in the box varies slightly from box to box. In fact, considerable historical data have allowed the determination of the density function that describes the probability structure for the weight (in ounces). Letting that describes the p. structure for the weight. Letting x be the random variable weight, in ounces, the density can be described as:

$$f(x) = \begin{cases} 2/5, & 23.75 \leq x \leq 26.25 \\ 0, & \text{otherwise} \end{cases}$$

- Verify that this is a valid density function.
- Determine P that the weight is smaller than 24 ounces.

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Sol: - 4) To be a Valid density function it has to be

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\text{So } \int_{-\infty}^{\infty} f(x) dx = \int_{23.75}^{26.25} \frac{2}{5} dx$$

$$= \frac{2}{5} x \Big|_{23.75}^{26.25} \Rightarrow \frac{2}{5} (26.25 - 23.75)$$

$$= \frac{2}{5} \times 2.5 = 1$$

therefore, $f(x)$ is valid density function

$$5) P(x < 24) = \int_{-\infty}^{24} f(x) dx \Rightarrow \int_{23.75}^{24} \frac{2}{5} dx$$

$$= \frac{2}{5} x \Big|_{23.75}^{24} \Rightarrow \frac{2}{5} (24 - 23.75)$$

$$= \frac{2}{5} \times 0.25 \Rightarrow 0.1 \text{ Ans}$$

Q.3:) An important factor in Solid missile fuel is the particle size distribution. Significant Problems occur if the particle sizes are 100 Large From Production date.

In the past, it has been determined that the particle size (in micrometers) distribution is characterised by

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$$f(x) = \begin{cases} 3x^{-4}, & x > 1 \\ 0, & \text{otherwise} \end{cases}$$

- a) Verify that this is a valid density function.
 b) Evaluate $F(x)$
 c) What is the probability that a random particle from the manufactured fuel exceeds 4 micrometres.

Solve Given $f(x) = \begin{cases} 3x^{-4}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$

a) $\int_1^{\infty} f(x) dx = 1$

$$\int_1^{\infty} 3x^{-4} dx = \left[\frac{-1}{x^3} \right]_1^{\infty} = 1$$

$\therefore f(x)$ is a valid density function.

b) $F(x) = \int_x^{\infty} f(a) da$

$$f(x) = 3 \int_1^x x^{-4} dx = \left[\frac{-1}{x^3} \right]_1^x = 1 - x^{-3}$$

c) $P(x > 4) = 1 - P(x < 4)$
 $= 1 - \left[\frac{-1}{x^3} \right]_1^4$
 $= 1 + \left[\frac{1}{64} - 1 \right]$
 $= 0.0156 \text{ Ans}$

Q.4 → Based on extensive testing, it is determined by the manufactures of a washing machine that the time γ (in years) before a major repair is required is characterized by the probability density function

$$f(\gamma) = \begin{cases} 1/4 e^{-\gamma/4} & , \gamma \geq 0 \\ 0 & , \text{otherwise} \end{cases}$$

- a) Critics would certainly consider the product a bargain if it is unlikely to require a major repair before the sixth year. Comment on this by determining $P(\gamma > 6)$
- b) What is the Pro. that a major repair occurs in the first years.

Solve : a) $P(\gamma > 6)$

$$P(\gamma > 6) = 1 - P(\gamma \leq 6)$$

$$= 1 - \int_{-\infty}^6 f(\gamma) d\gamma$$

$$= 1 - \int_0^6 \frac{1}{4} e^{-\gamma/4} d\gamma$$

$$= 1 - \left[-e^{-\gamma/4} \right]_0^6$$

$$= 1 - (-e^{-6/4} + e^0)$$

$$= 1 + e^{-3/2} - 1$$

$$= 0.2231$$

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b) $P(Y \leq 1)$

$$\begin{aligned}
 P(Y \leq 1) &= \int_{-\infty}^1 f(y) dy \\
 &= \int_0^1 \frac{1}{4} e^{-y/4} dy \\
 &= \left(-e^{-y/4} \right) \Big|_0^1 \\
 &= (-e^{-1/4} + e^0) \\
 &= 1 - e^{-1/4} \\
 &= 0.2212 \text{ Ans.}
 \end{aligned}$$

Q.5) The proportion of the budget for a certain type of industrial company that is allotted to environmental and pollution control is running under scrutiny. A data collection project determines that the distribution of these proportions is given by,

$$f(y) = \begin{cases} 5(1-y)^4, & 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

- Verify that the above is a valid density function
- What is the prob. that a company chosen at random expends less than 10% of its budget on environmental and pollution controls.
- What is the Prob that a company selected at random spends more than 50% of its budget on —

Soln

$$4) \int_0^1 f(y) dy = 1$$

$$\int_0^1 5(1-y)^4 dy = 5(1-y)^5 \Big|_0^1$$

$$= - (y-1)^5 \Big|_0^1$$

$$= \int_0^1 f(y) dy = 1$$

Density function is a valid function

$$(b) P(y < 0.1) = 5 \int_0^{0.1} (1-y)^4 dy$$

$$= \frac{5}{-5} (1-y)^5 \Big|_0^{0.1}$$

$$P(y < 0.1) = 1 - (0.9)^5 = 0.4095$$

$$(c) P(y > 0.5) = \int_{0.5}^1 5(1-y)^4 dy$$

$$P(y > 0.5) = (-1) (1-y)^5 \Big|_{0.5}^1 =$$

$$= (0.5)^5$$

$$= 0.03125 \text{ Am}$$

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Q.5) Suppose a certain type of small data processing firm is so specialized that some have difficulty making a profit in their first year of operation. The p.d.f. density function that characterizes the proportion y that make a profit is given by

$$f(y) = \begin{cases} k y^4 (1-y)^3 & 0 \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

- What is the value of k that renders the above a valid density function
- Find the Prob that at most 50% of the firms make a profit in the first year
- Find the Prob that at least 50% of the firms make a profit in the first year

sol 1) if f is a valid density function it must satisfy $\int_{-\infty}^{\infty} f(y) dy = 1$

$$1 = \int_{-\infty}^{\infty} f(y) dy$$

$$= \int_0^1 k y^4 (1-y)^3 dy$$

$$= k \int_0^1 y^4 (1 - 3y + 3y^2 - y^3) dy$$

$$= k \int_0^1 (y^4 - 3y^5 + 3y^6 - y^7) dy$$

$$= k \left(\frac{1}{5} y^5 - \frac{1}{2} y^6 + \frac{3}{7} y^7 - \frac{1}{8} y^8 \right) \Big|_0^1$$

$$= k \left(\frac{1}{5} - \frac{1}{2} + \frac{3}{7} - \frac{1}{8} \right)$$

$$= \frac{k}{280} \quad \boxed{k = 280}$$

b) $P(Y \leq 0.5)$

$$P(Y < 0.5) = P(0 < Y < 0.5)$$

$$= \int_0^{0.5} 280 y^4 (1 - y^3) dy$$

$$= 280 \int_0^{0.5} y^4 (1 - 3y + 3y^2 - y^3) dy$$

$$= 280 \int_0^{0.5} [y^4 - 3y^5 + 3y^6 - y^7] dy$$

$$= 280 \left(\frac{1}{5} y^5 - \frac{1}{2} y^6 + \frac{3}{7} y^7 - \frac{1}{8} y^8 \right) \Big|_0^{0.5}$$

$$= 280 \left(\frac{1}{5} \times 0.5^5 - \frac{1}{2} \times 0.5^6 + \frac{3}{7} \times 0.5^7 - \frac{1}{8} \times 0.5^8 \right)$$

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$$= \frac{93}{256}$$

$$(c) P(Y > 0.8)$$

$$P(Y > 0.8) = 1 - P(Y \leq 0.8)$$

$$= 1 - P(0 \leq Y \leq 0.8)$$

$$= 1 - \int_0^{0.8} 280 y^4 (1-y)^3 dy$$

$$= 1 - 280 \int_0^{0.8} y^4 (1 - 3y + 3y^2 - y^3) dy$$

$$= 1 - 280 \int_0^{0.8} (y^4 - 3y^5 + 3y^6 - y^7) dy$$

$$= 1 - 280 \left(\frac{1}{5} y^5 - \frac{3}{2} y^6 + \frac{3}{7} y^7 - \frac{1}{8} y^8 \right) \Big|_0^{0.8}$$

$$= 1 - 280 \left(\frac{1}{5} \times 0.8^5 - \frac{3}{2} \times 0.8^6 + \frac{3}{7} \times 0.8^7 - \frac{1}{8} \times 0.8^8 \right)$$

$$= 0.0563 \text{ Ans.}$$

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Q.7) A Used-car dealership has found that the length of time before a major repair is required on the cars it sells is normally distributed with a mean equal to 10 months and a standard deviation of 3 months. If the dealer wants only 5% of the cars to fail before the end of the guarantee period, for how many months should the cars be guaranteed?

Solve - Given $\mu = \text{Mean} = 10$
 $\sigma = \text{standard deviation} = 3$

$$P(M < 40) = 5\% = 0.05$$

We note that the Prob. 0.05 lies exactly in the middle b/w 0.0495 and 0.0505

The Prob. is 0.494 lies in the row -1.6 and in the column .04 of the normal prob. table and thus the corresponding Z-score is then $-1.6 + 0.4 = -1.64$

The prob. is 0.0505 lies in the row -1.6 and in the column .05 of the normal probability table and $-1.6 + 0.5 = -1.65$

Since the prob. 0.05 lies exactly in the middle b/w 0.0495 and 0.0505 we expect the Z score Z_0 to lie exactly in the middle b/w the corresponding Z-scores -1.64 and -1.65, which is that -1.64

$$Z_0 = -1.645$$

The Z -score is the observed value decreased by the mean, divided by the standard deviation

$$Z_0 = \frac{x_0 - \mu}{\sigma} = \frac{x_0 - 10}{3}$$

$$\therefore Z_0 = -1.645$$

$$\frac{x_0 - 10}{3} = -1.645$$

Multiply each side by 3.

$$x_0 - 10 = -4.935$$

$$x_0 = 5.065 \text{ moths} \quad \text{Ans.}$$

Q8) A home security system is designed to have a 99% reliability rate. Suppose that nine homes equipped with this system experience an attempted burglary. Find the Prob. of these events

- At least one of the alarms is triggered.
- More than seven of the alarms are triggered.
- Eight or fewer alarms are triggered.

$$\begin{aligned} \text{Sol}^n: (a) \quad P(X=0) &= C_0^9 \cdot 0.99^0 \cdot (1-0.99)^9 \\ &= \frac{9!}{0!(9-0)!} \times 0.99^0 \cdot 0.01^9 \approx 0 \end{aligned}$$

$$\begin{aligned}
 P(X \geq 1) &= 1 - P(X=0) \\
 &= 1 - 0 \\
 &= 1
 \end{aligned}$$

(b) $n = 9$;

$$\begin{aligned}
 P(X=8) &= {}^9C_8 \times 0.99^8 \times (1-0.99)^{9-8} \\
 &= \frac{9!}{8!(9-8)!} \times 0.99^8 \times 0.01 \\
 &= 0.0830
 \end{aligned}$$

$$\begin{aligned}
 P(X=9) &= {}^9C_9 \times 0.99^9 \times (1-0.99)^{9-9} \\
 &= \frac{9!}{9!(9-9)!} \times 0.99^9 \times 0.01^0 \\
 &\approx 0.9135
 \end{aligned}$$

Since it is not possible to obtain two different numbers of successes on the same simulation

addition rule for mutually exclusive events

$$\begin{aligned}
 P(X \geq 7) &= P(X=8) + P(X=9) \\
 &= 0.0830 + 0.9135 \\
 &= 0.9965
 \end{aligned}$$

$$\begin{aligned}
 c) P(X \leq 8) &= 1 - P(X=9) \\
 &= 1 - 0.9135 \Rightarrow 0.0865 \text{ Ans}
 \end{aligned}$$