Online Class 04-02-2021

Slot: B21+B22+B23

Probability, Statistics and Reliability

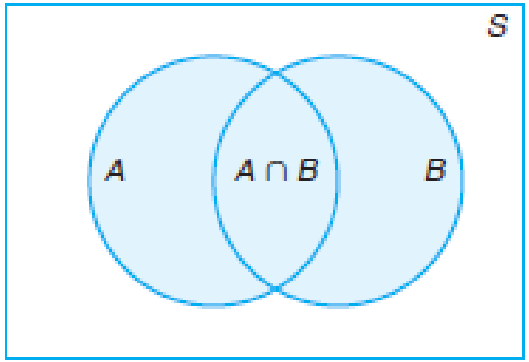
: MAT3003

Topic: Addition Theorem, and Multiplication Theorem

Theorems of Probability

- Addition Theorem
- Multiplication Theorem

ADDITION THEOREM (OR ADDITION RULE)



If A and B are two events, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

For three events A, B, and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

If A and B are mutually exclusive, then

$$P(A \cup B) = P(A) + P(B).$$

If $A_1, A_2, \ldots, A_n$ are mutually exclusive, then

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n).$$

Question 1

John is going to graduate from an industrial engineering department in a university by the end of the semester. After being interviewed at two companies he likes, he assesses that his probability of getting an offer from company *A* is 0.8, and his probability of getting an offer from company *B* is 0.6.

If he believes that the probability that he will get offers from both companies is 0.5, what is the probability that he will get at least one offer from these two companies?

Solution

Using the additive rule, we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.8 + 0.6 - 0.5 = 0.9.$$

QUESTIONS ON ADDITION THEOREM

Question: A card is drawn from a pack of 52 cards. What is the probability of getting either a king or a queen?

Solution: There are 4kings and 4 queens in a pack of 52 cards.

The probability of drawing a king card is P (K) = $\frac{4}{52}$

and the probability of drawing a queen card is P (Q) = $\frac{4}{52}$

Since, both the events are mutually exclusive, the probability that the card drawn is either a king or a queen is

P (K or Q) = P (K) + P (Q)

$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

Question : A perfect die is tossed. What is the probability of throwing 3 or 5?

Solution : The probability of throwing 3 is P (A) = $\frac{1}{6}$

The probability of throwing 5 is P (B) = $\frac{1}{6}$

P (A or B) = P (A) + P (B)

$$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$$

Question :A card is drawn at random from a pack of cards. Find the probability that the drawn card is either a club or an ace of diamond.

Solution :Probability of drawing a club P (A) = $\frac{13}{52}$

Probability of drawing an ace of diamond P (B) = $\frac{1}{52}$

P (A or B) = P (A) + P (B)

$$\frac{13}{52} + \frac{1}{52} = \frac{14}{52} = \frac{7}{26}$$

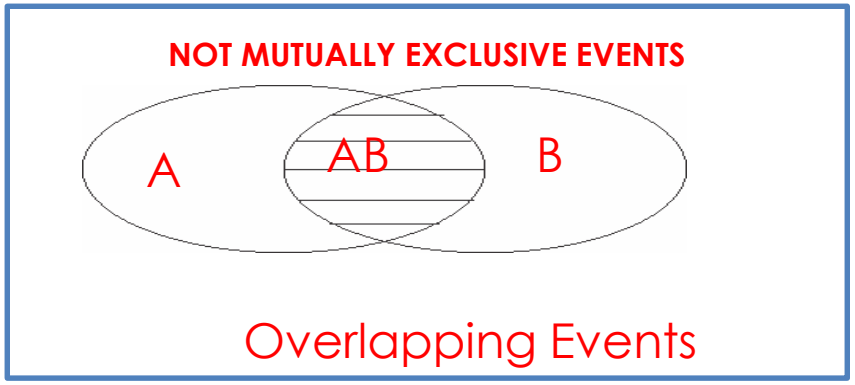
ADDITION THEOREM FOR NOT MUTUALLY EXCLUSIVE EVENTS

Two or more events are known as partially overlapping if part of one event and part of another event occur together. Thus, when the events are not mutually exclusive the addition theorem has to be modified.

Symbolically,

P (A or B or Both) = P (A) + P (B) – P (AB)

The following figure illustrates this point:



QUESTIONS ON MODIFIED ADDITION THEOREM

Question: A card is drawn at random from a well shuffled pack of cards. What is the probability that it is either a spade or a king?

Solution: The Probability of drawing a spade P (A) $= \frac{13}{52}$

The Probability of drawing a King P (B) $= \frac{4}{52}$

The Probability of drawing a King of Spade P (AB) $= \frac{1}{52}$

P (A or B or Both) = P (A) + P (B) – P (AB)

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52} = \frac{4}{13}$$

MULTIPLICATION THEOREM OF PROBABILITY

If two events A and B are independent then the probability that both of them will occur is equal to the product of their individual probabilities.

$$P(AB) = P(A \cap B) = P(A \text{ and } B) = P(A).\ P(B)$$

Question:

From a pack of 52 cards, two cards are drawn at random one after the other with replacement. What is the probability that both cards are kings?

Solution:

The probability of drawing a king

$$P(A) = 4/52$$

The probability of drawing again the king after replacement

$$P(B) = 4/52$$

Since the two events are independent, the probability of drawing two kings is:

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

Question:

A bag contains 4 red balls, 3 white balls and 5 black balls. Two balls are drawn one after the other with replacement. Find the probability that first is red and the second is black.

Solution:

Probability of red ball in the first draw = 4/12

The probability of a black ball in the second draw = 5/12

Since the events are independent, the probability that first is red and the second are black will be:

$$P(1R).P(1B) = \frac{4}{12} \times \frac{5}{12} = \frac{20}{144} = \frac{10}{72} = \frac{5}{36}$$

Permutations and Combinations

Factorial Formula for Permutations

The number of **permutations**, or *arrangements*, of n distinct things taken r at a time, where $r \leq n$, can be calculated as

$$_nP_r = \frac{n!}{(n-r)!}.$$

Factorial Formula for Combinations

The number of **combinations**, or *subsets*, of n distinct things taken r at a time, where $r \leq n$, can be calculated as

$$_nC_r = \frac{_nP_r}{r!} = \frac{n!}{r!(n-r)!}.$$

Counting Techniques

Permutations	Combinations
The number of ways to **arrange** things.	The number of ways to **choose** things.
Order matters	**Order does not matter**
${}_nP_r = n!/(n-r)!$	${}_nC_r = n!/[\ r!(n-r)!]$
Clue words: arrangement, schedule, order	Clue words: group, sample, selection

Permutations and Combinations

Evaluate each problem.

a) ${}_5P_3$

$$\frac{5!}{(5-3)!}$$

$$\frac{5!}{2!}$$

$$5 \cdot 4 \cdot 3$$

60

b) ${}_5C_3$

$$\frac{5!}{3!\,(5-3)!}$$

$$\frac{5!}{3!\,2!}$$

$$\frac{5 \cdot 4}{2}$$

10

c) ${}_6P_6$

$$\frac{6!}{(6-6)!}$$

$$\frac{6!}{0!}$$

$$\frac{720}{1}$$

720

d) ${}_6C_6$

$$\frac{6!}{6!\,(6-6)!}$$

$$\frac{6!}{6!\,0!}$$

$$\frac{6!}{6!\,1}$$

1

Question:

- Four students while entering a room found that seven chairs were lying vacant. In how many ways could the seats be occupied?

Solution

$$7P_4 = \frac{7!}{(7-4)!} = 840$$

Question:

- If two coins are tossed. In how many ways can they fall?

Solution

First coin	Second coin
H	H
T	T
H	T
T	H

THE TWO COINS MAY FALL TOGETHER IN 4 WAYS

Practice Questions

Q.1. Suppose it is known that the probability that the component survives for more than 6000 hours is 0.42. Suppose also that the probability that the component survives no longer than 4000 hours is 0.04.

a) What is the probability that the life of the component is less than or equal to 6000 hours

b) What is the probability that the life of the component is greater than 4000 hours.

Q.2. If 5 of a company's 12 delivery trucks do not meet emission standard and 4 of the 12 trucks are randomly picked for inspection. What is the probability that none of them meets emission standards?

Thank you