

Operations Research

- Operations research is a scientific approach to decision making, which seeks to determine how best to design and operate a system, under conditions requiring the allocation of scarce resources.
- provides a set of algorithms that act as tools for effective problem solving and decision making.
- Extensive application in engineering, business and public systems.
- Used extensively by manufacturing and service industries in decision making.
- Origin during world war II when the British military asked scientists to analyze military problems.
- The application of mathematics and scientific methods to military applications was called operations research. Today it is also called management science.
- It is a scientific approach to decision making that seeks to determine how best to operate a system under conditions of allocating scarce resources.

Classification:

- Linear Programming (Optimization)
- Nonlinear Programming (Optimization)
- Queuing theory
- Reliability theory
- Game theory
- Network analysis
- Inventory

Operation research has this following subarea of study. The first one is also called the linear optimization problems, the second one is non linear optimization problems.

Queuing theory is that branch of operation research which studies the queues. E.g. When you go to bank to withdraw your money, you have to follow a queue. Similarly when you are putting up some jobs on a computer, then a queue is lined up, so this queuing theory studies the behaviour of queue. Reliability theory is also one of major areas of operations research which studies the reliability of a system whether it is a mechanical system, whether it is a building and so on.

Game theory is another very interesting area of operations research which talks about the decision making process when a game is played between two or more than two components. The network analysis is that study which talks about the study of networks, for example, when we have to design a network e.g. internet service providers or some other kind of network, then it is the study of network analysis. Inventory control is an important aspect when business has to deal with, i.e. when a commodity has to be stored, in some warehouse and the decision has to be made as to how much quantity of the commodity should be there in the inventory so that customers are not lost. At the same time, the cost of storing that commodity is minimized.

General Optimization problem :

Optimization is the key area of operations research since we encounter the optimization problem everywhere.

Let us define the mathematical definition of an Optimization problem.

Minimize (Maximize) $f(x)$

where $f: \mathbb{R}^n \rightarrow \mathbb{R}$, $X = (x_1, x_2, \dots, x_n)$

Subject to $X \in S \subseteq \mathbb{R}^n$, where S is defined by

$$g_k(x) \geq 0, \quad k = 1, 2, \dots, m.$$

$$h_j(x) = 0, \quad j = 1, 2, \dots, l.$$

$$a_i \leq x_i \leq b_i, \quad i = 1, 2, \dots, n.$$

Here f is called as the objective function which is a real valued function defined on \mathbb{R}^n space i.e. n dimensional Euclidean space. This is the function which has to be either maximized or minimized depending upon the problem in hand.

$X = (x_1, x_2, \dots, x_n)$ is the decision variable, this is the variable which has to be determined and these X values should belong to the set S where $S \subseteq \mathbb{R}^n$ is defined by the following conditions,

$$g_k(x) \geq 0, \quad k = 1, 2, \dots, m.$$

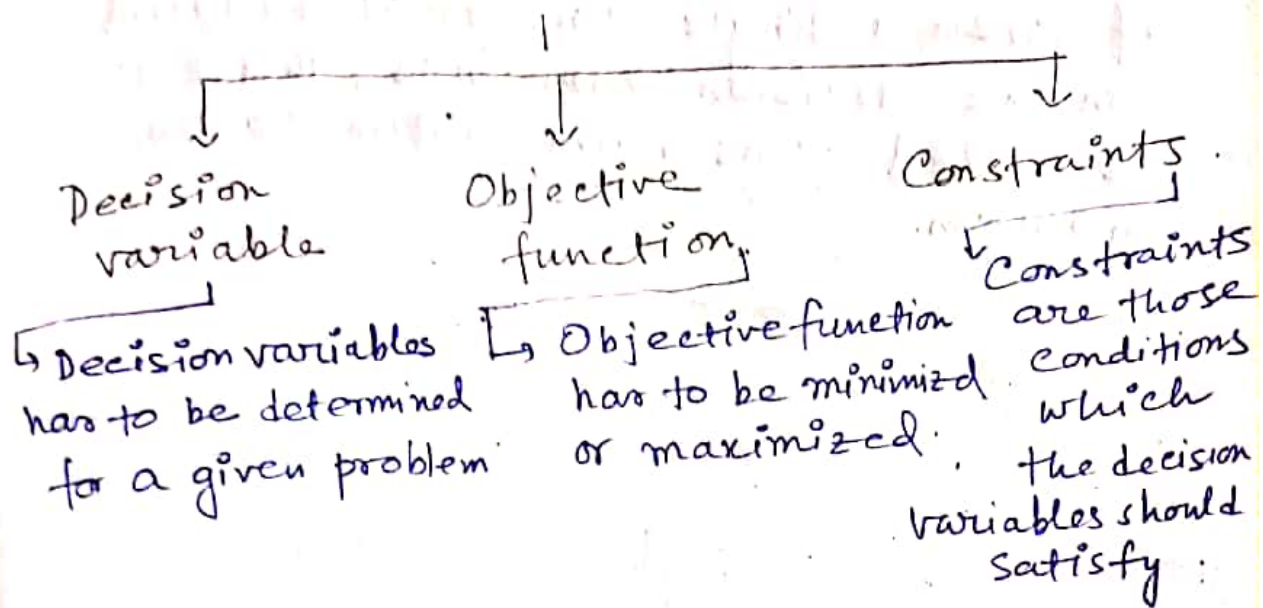
$$h_j(x) = 0, \quad j = 1, 2, \dots, l.$$

$$a_i \leq x_i \leq b_i, \quad i = 1, 2, \dots, n.$$

These g_1, g_2, \dots, g_m are called inequality constraints. h_1, h_2, \dots, h_l are called equality constraints. a_i, b_i are

called lower and upper bounds of the decision variable x_i 's, which has to be determined under the constraint set S .

Components of Optimization model



Classification of Optimization problems:

- Linear programming problems (LPP)
- Nonlinear programming problems (NLPP)
- Unconstrained Optimization problems
- Constrained Optimization problems.

The first one is the linear programming problem, a special case of optimization problems, in which all the functions involved i.e. f , g_k , h_j all are linear.

The second one is the nonlinear programming problem if any one of f , g_k , h_j is nonlinear.

There may be possible that the problem does not have any constraint. In such a situation, the problem is called as a unconstrained optimization problem.

If even a single constraint is imposed on the decision variables, then it is called constraint optimization problem.