4

Three-phase System

TOPICS DISCUSSED

- Advantages of a three-phase system
- Three-phase windings and their connections
- Active and reactive power
- Measurement of power in three-phase circuits

4.1 INTRODUCTION

Generation, transmission, and distribution of electricity is done by three-phase electrical networks consisting of generators, transformers, and transmission and distribution lines forming the power system. In a three-phase system we have three independent voltages induced in the three windings of the generator. To understand the difference between a single-phase voltage and a three-phase voltage, let us consider how these voltages are generated in ac generators. We have known that EMF is induced in a coil if it cuts lines of force. In Fig. 4.1 we have placed one coil in slots of a hollow cylindrical stator core. A two-pole magnet is rotated at a particular speed by some means. The flux lines will cut the conductors and EMF will be induced in the coil. Since North and South poles' flux will cut the conductors alternately, an alternating single-phase voltage will be induced in the coil.

ive motion between the magnetic field and the conductor. The magnitude of the induced EMF will depend upon the number of coils, the strength of the magnetic field, and the speed of rotation of the magnet. The frequency of the induced EMF will depend upon the number of magnetic poles confronted by the coils per revolution. Normally, the number of magnetic poles and the speed of rotation of the magnetic poles by a drive, usually a turbine, is kept constant. The number of coils and number of turns used in each coil are kept as per design and are constant once the machine is constructed. That is why the magnitude of the induced EMF and its frequency is constant. Thus, we get a single-phase voltage from the single-phase winding which can be used to supply an electric circuit comprising resistance, inductance, and capacitance elements.

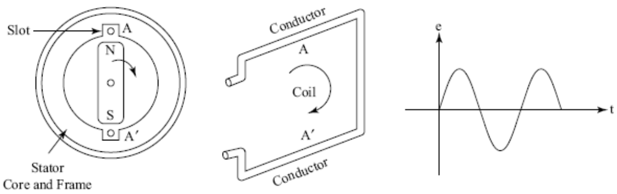

Figure 4.1 Generation of single-phase voltage

In a three-phase system we will have three-phase voltages induced in the three-phase windings of the generator. In all generating stations three-phase generators are installed.

4.2 ADVANTAGES OF THREE-PHASE SYSTEMS

A three-phase system has a number of advantages over a single-phase system. Some of these are mentioned as follows:

1. the output of a three-phase machine generating electricity is more than the output of a single-phase machine of the same size;
2. the most commonly used three-phase induction motors are self starting. For single-phase motors, as will be explained in a separate chapter, a separate starting winding is required;
3. electrical power transmission from the generating station to the places of use is done by transmission lines. It has been seen that three-phase power transmission is more economical than single-phase power transmission;
4. the power factor of three-phase systems is better than that of the single-phase systems;
5. single-phase supply can also be obtained from a three-phase supply;
6. the instantaneous power in a single-phase system is fluctuating with time giving rise to noisy performance of single-phase motors. The power output of a symmetrical three-phase system is steady;
7. for rectification of ac into dc, the dc output voltage becomes less fluctuating if the number of phases is increased.

Thus, we see that from generation, transmission, distribution, and utilization points of view, three-phase systems are preferred over

Due to a number of practical considerations, generators are built to generate poly-phase voltages. Commercial generators are built to generate three-phase voltages. In three-phase generators, three separate windings are made. Windings are made of coils. These windings are placed in stator slots at an angle of 120˚ apart as shown in Fig. 4.2 RR′ is one phase winding.

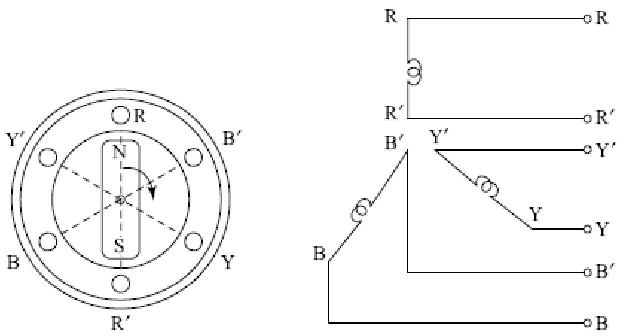

Figure 4.2 Generation of three-phase voltages

YY′ is the second phase winding and BB′ is the third phase winding. The three-phase windings are placed at an angular distance of 120°. For simplicity only one coil per phase has been shown. In practice a number of coils connected in series makes one phase winding.

When the magnetic poles are rotated by a prime mover (say a turbine), the magnetic flux of North and South poles will cut the windings in sequence. For clockwise rotation, flux will be cut by coil RR′ first, then by coil YY′, and then by coil BB′. Therefore, EMF will be induced in these coils in sequence. There will be a time phase difference between the EMFs induced in these coils (windings). The time phase difference will be 120°. In terms of time, the phase difference will be the time taken by the magnetic poles to rotate by 120°, i.e., one-third of a revolution. Thus, across the three-phase windings we will get three voltages which are equal in magnitude and frequency but having a time phase difference of 120° between them, as shown in Fig. 4.3

The equation of voltages are

$$e_R = E_m \sin \omega t \qquad (4.1)$$

$$e_Y = E_m \sin (\omega t - 120°) \qquad (4.2)$$

$$e_B = E_m \sin (\omega t - 240°) \qquad (4.3)$$

Resultant EMF = $E_m \sin \omega t + E_m \sin (\omega t - 120°) + E_m \sin (\omega t - 240°) = 0$

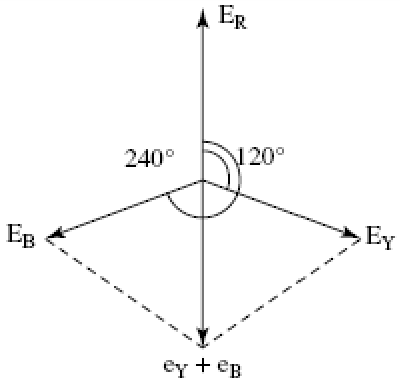

Figure 4.3 Three-phase voltages displaced in time phase by 120°

Since the three voltages are equal in magnitude but displaced in time phase by 120˚, their phasor sum is zero as shown in Fig. 4.3

Three-phase supply is required for large-capacity electrical loads. These loads could be three-phase motors used in industrial, commercial, agricultural, and other sectors. For example, the water pump used for irrigation purpose is invariably a three-phase-motor-driven pump requiring a three-phase supply. So, like three-phase supply, we will have three-phase loads. Three-phase supply will be supplying electrical power to three-phase loads. A number of terms are used in connection with three-phase supply and three-phase loads. These are described as follows. Further the three-phase windings can be connected together in the form of star or delta. The voltage between the two-phase windings and current flowing through the phase windings, and the supply line will be different in star and delta connections. These will be studied in detail.

By now we must have realized that by phase we mean a winding. A phase difference between two windings is the physical angular displacement between them. In a three-phase winding, the phase difference between the windings is 120°. Phase sequence is the order in which maximum voltage is induced in the windings. For example, if the magnetic field cuts the conductors of the phase RR′ first and then cuts the conductors of phase YY′, and lastly cuts the conductors of phase BB′, then EMF will be induced in all the phases of equal magnitude but their maximum value will appear in a sequence RYB. Then we call the phase sequence of EMF as RYB. If the magnet system of Fig. 4.2 rotates in the anticlockwise direction, the phase sequence of EMF induced in the three phases will be RBY.

Let us consider an elementary three-phase generator as shown in Fig. 4.4 (a). Three-phase windings are placed in slots in the stator. For simplicity, only one coil per phase has been used. R-R′ is one coil making R-phase winding. Y-Y′ is another coil forming Y-phase winding. B-B′ is one coil forming B-phase winding. These three-phase windings are placed in the stator slots at an angle of 120° in space. For simplicity only one coil per phase has been shown. In actual practice, a number of coils are connected together to form each in a sequence. The emfs induced in these coils are sinusoidal in nature because of the nature of flux distribution. The voltage induced in the three-phase windings will be identical in nature but they will be displaced in time-phase by 120° as has been shown. The order in which the phase voltages attain their maximum value or peak value, V_m is called the phase sequence. If the rotor rotates in the clockwise direction, voltages in the phases will be induced in the sequence, V_R, V_Y, V_B. If the rotor poles rotate in the opposite direction the phase sequence of the induced voltage will change from RYB to RBY.

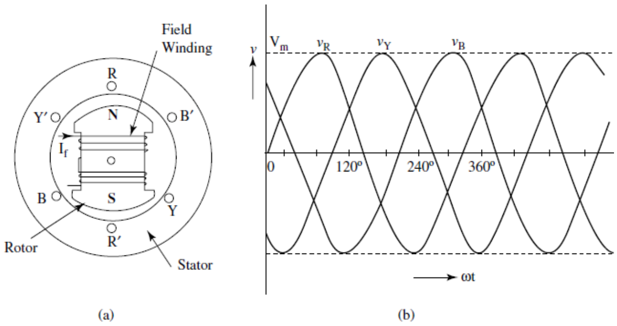

Figure 4.4 (a) Three-phase two-pole generator; (b) three-phase voltages induced in the windings

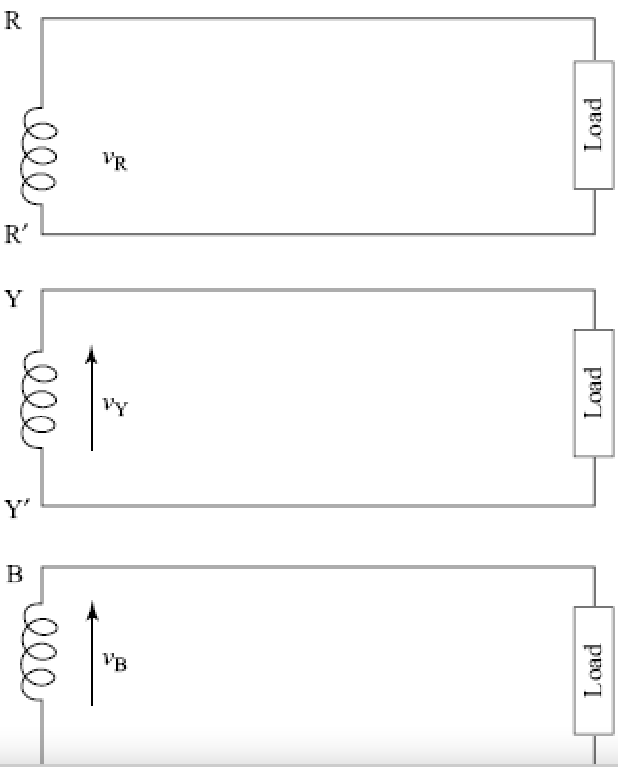

Figure 4.5 The three-phase windings are connected to the load independently through six wires

The phase voltages should supply power to electrical loads for which the two end terminals of each phase are to be connected to loads as shown in Fig. 4.5 Six wires are required to be taken out from the generator to the load. Instead of taking out six wires from the generator and connecting them to the loads separately as shown in Fig. 4.5, the three-phase windings are connected either in star or in delta so that only three wires are to be taken out from the generator to the load. The loads are also connected either in star or in delta. In the case of the star connection a fourth wire may be taken out from the neutral point.

4.4 TERMS USED IN THREE-PHASE SYSTEMS AND CIRCUITS

These are some of the terms used while describing a three-phase system. These are as follows:

- *Balanced supply*: a set of three sinusoidal voltages (or currents) that are equal in magnitude but has a phase difference of 120°, constitute a balanced three-phase voltage (or current) system.
- *Unbalanced supply*: a three-phase system is said to be unbalanced when either of the three-phase voltages are unequal in magnitude or the phase angle between the three phases is not equal to 120°.
- *Balanced load:* if the load impedances of the three phases are identical in magnitude as well as phase angle, then the load is said to be balanced. It implies that the load has the same value of resistance R and reactance X_L and/or X_C in each phase.
- *Unbalanced load:* if the load impedances of the three phases are neither identical in magnitude nor in phase angle then the load is said to be unbalanced.
- *Single phasing:* when one phase of the three-phase supply is not available then the condition is called single phasing.
- *Phase sequence:* the order in which the maximum value of voltages of each phase appear is called the phase sequence. It can be RYB or RBY.
- *Coil:* a coil is made of conducting wire, say copper, having an insulation cover. A coil can be of a single turn or many number of turns. Normally a coil will have a number of turns. A single turn of a coil will have two conductors on its two sides called coil sides
- *Winding:* a number of coils are used to make one winding. Normally the winding coils are connected in series. One winding forms one phase.
- *Symmetrical system:* in a symmetrical three-phase system the magnitude of three-phase voltages is the same but there is a time phase difference of 120^0 between the voltages.

4.5 THREE-PHASE WINDING CONNECTIONS

A three-phase generator will have three-phase windings. These phase windings can be connected in two ways:

The star connection is formed by connecting the starting or finishing ends of all the three windings together. A fourth conductor which is taken out of the star point is called the neutral point. The remaining three ends are brought out for connection to load. These ends are generally referred to as R–Y–B, to which load is to be connected. The star connection is shown in Fig. 4.6 (a). This is a three-phase, four-wire star-connected system. If no neutral conductor is taken out from the system it gives rise to a three-phase, three-wire star-connected system.

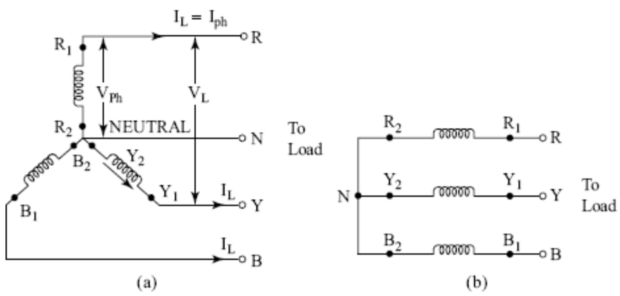

Figure 4.6 Star connection of phase windings: (a) three-phase four-wire system; (b) three-phase three-wire system

The current flowing through each line conductor is called line current, I_L. In the star connection the line current is also the phase current. Similarly, voltage across each phase is called phase voltage, V_{Ph}. Voltage across any two line conductors is called line voltage, V_L. When a balanced three-phase load is connected across the supply terminals R,Y,B currents will flow through the circuit. The sum of these currents, i.e., I_R, I_Y, and I_B will be zero. The neutral wire connected between the supply neutral point and the load neutral point will carry no current for a balanced system.

4.5.2 Delta Connection

The delta connection is formed by connecting the end of one winding to the starting end of the other and connections are continued to form a closed loop. In this case, the current flowing through each line conductor is called line current I_L and the current flowing through each phase winding is called phase current, I_{Ph}. However, we find that the phase voltage is the same as the line voltage in a delta connection. The delta connection of windings has been shown in Fig. 4.7

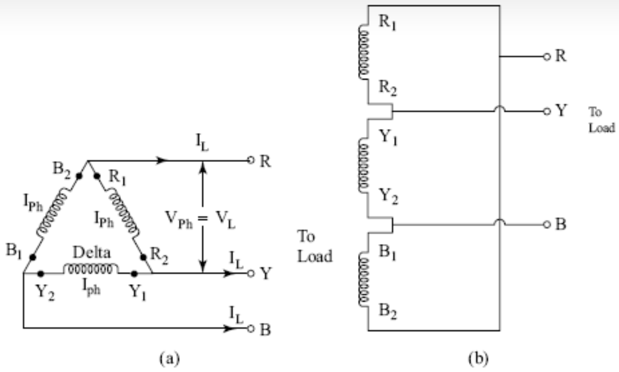

Figure 4.7 Delta connection of three-phase windings: (a) three-phase three-wire delta connection; (b) connection scheme of windings forming a delta

4.5.3 Relationship of Line and Phase Voltages, and Currents in a Star-connected System

Consider the balanced star-connected system as shown in Fig. 4.8

Suppose load is inductive and, therefore, current will lag the applied voltage by angle Φ. Consider a balanced system so that the magnitude of current and voltage of each phase will be the same.

i.e., Phase voltages, $V_R = V_Y = V_B = V_{Ph}$

Line current, $I_R = I_Y = I_B = I_L$

Line voltage, $V_L = V_{RY} = V_{YB} = V_{BR}$

Phase current, $I_{Ph} = I_R = I_Y = I_B$

$I_L = I_{PH}$ for star connection as the same phase current passes through the lines to the load.

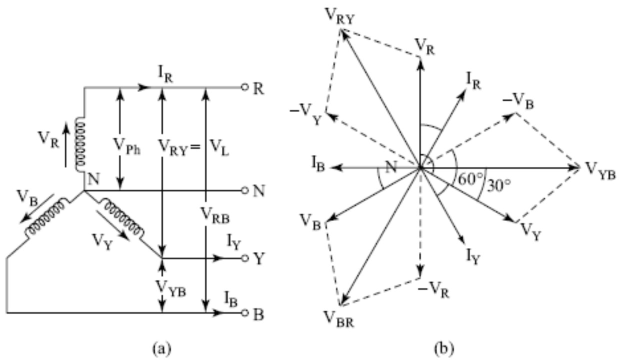

Figure 4.8 (a) Balanced star-connected system; (b) phasor diagram

Similarly $V_{YB} = V_{YN} + (-V_{BN})$

and $V_{BR} = V_{BN} + (-V_{RN})$

The procedure for drawing the phasor diagram of Fig. 4.8 (b) is as follows.

Draw three phasors V_R, V_Y, and V_B representing the phase voltages. These voltages are of equal magnitude but displaced by 120°. The line voltage phasors, V_{RY}, V_{YB}, V_{BR} are drawn by vectorially adding the phase voltages. For example, to draw line voltage V_{RY} we have to add the phase voltages as

$$V_{RY} = V_{RN} + V_{NY} = V_{RN} + (-V_{YN})$$

The phasor V_{YN} is obtained by reversing V_{NY}. V_{RY} is obtained by vectorially adding V_{RN} and V_{YN} as has been shown in Fig. 4.8 (b). Similarly the other line voltages have been drawn. The phase currents I_R, I_B, and I_Y have been shown lagging the phase voltages by the power factor angle ϕ.

From the phasor diagram shown in Fig. 4.6 (b), the phase angle between phasors V_R and $(-V_Y)$ is 60°.

$\therefore$

$$V_{RY} = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60^\circ}$$

$$V_{RY} = V_L = \sqrt{V_{Ph}^2 + V_{Ph}^2 + 2V_{Ph} V_{Ph} \times \frac{1}{2}}$$

$$V_L = \sqrt{3}\, V_{Ph}$$

$$V_L = \sqrt{3}\, V_{Ph} \qquad (4.4)$$

Thus, for the star-connected system

Line Voltage = $\sqrt{3}$ × Phase voltage

Line Current = Phase current

Power: Power output per phase = $V_{Ph} I_{Ph} \cos \phi$

Total power output = $3 V_{Ph} I_{Ph} \cos \phi$

$$= 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos \phi$$

$$P = \sqrt{3}\, V_L\, I_L \cos \phi \qquad (4.5)$$

Consider the balanced delta-connected system as shown in Fig. 4.9

In a delta-connected system the voltage aeros the winding, i.e., the phases is the same as that across the line terminals. However, the current through the phases is not the same as through the supply lines.

Therefore, in the case of the delta-connected circuit, phase voltage is equal to the line voltage, but line current is not equal to phase current.

Line voltage	$V_L = V_{RY} = V_{YB} = V_{BR}$
Line current	$I_L = I_R = I_Y = I_B$
Phase voltage	$V_{Ph} = V_{RY} = V_{YB} = V_{BR}$
Phase current	$I_{Ph} = I_{RY} = I_{YB} = I_{BR}$ (4.6)

$\therefore V_{Ph} = V_L$ for delta-connected load. (4.6)

In Fig. 4.9 (a) is shown a three-phase delta-connected supply system connected to a three-phase delta-connected load. The line currents are I_R, I_Y, and I_B, respectively. The phase currents are I_{RY}, I_{YB}, and I_{BR}. The phasor diagram in Fig. 4.9 (b) has been developed by first showing the three-phase voltages V_{YB}, V_{BR}, and V_{RY} of equal magnitude but displaced by 120° from each other. Then the phase currents I_{YB}, I_{BR}, and I_{RY} have been shown lagging respective phase voltages by power factor angle Φ. The line currents are drawn by applying KCL at the nodes R, Y, and B and adding the phasors, as has been shown.

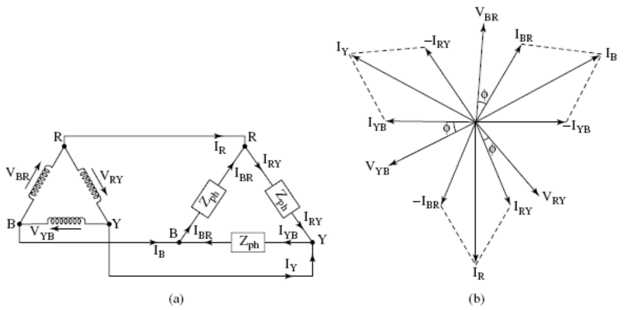

Figure 4.9 Delta-connected system: (a) a three-phase delta-connected load supplied from a delta-connected supply source; (b) phasor diagram of voltages and currents

To derive the relation between I_L and I_{Ph}, apply KCL at node R, as shown in Fig. 4.9 (b)

$$I_R + I_{BR} = I_{RY}$$

$\therefore$ $I_R = I_{RY} - I_{BR}$

$I_B = I_{BR} - I_{YB}$

Since phase angle between phase currents I_{RY} and $-I_{BR}$ is 60°

$$\therefore \quad I_R = \sqrt{I_{RY}^2 + I_{BR}^2 + 2I_{RY}\, I_{BR} \cos 60°}$$

$$I_R = I_L = \sqrt{I_{Ph}^2 + I_{Ph}^2 + 2I_{Ph}\, I_{Ph} \times \frac{1}{2}}$$

$$I_L = \sqrt{3} \times I_{ph}$$

Thus, for a three-phase delta-connected system,

Line current = $\sqrt{3}$ × Phase current

Line voltage = Phase voltage

Power: Power output per phase = $V_{Ph}\, I_{Ph} \cos \phi$

Total power output = $3V_{Ph}\, I_{Ph} \cos \phi$

$$= 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cos \phi$$

$$\sqrt{3} V_L\, I_L \cos \phi$$

Power = $\sqrt{3}$ × Line voltage × Line current × Power factor

For both star-connected and delta-connected systems, the total power P is

$$P = \sqrt{3}\, V_L\, I_L \cos \phi \tag{4.7}$$

If per phase power is P_h and total power is P_T,

then $P_T = 3 P_h$ (4.8)

4.6 ACTIVE AND REACTIVE POWER

The in-phase component of I_{Ph} along V has been shown in Fig. 4.10 as $I_{Ph} \cos \phi$ and the perpendicular component as $I_{Ph} \sin \phi$. If we multiply all the sides of the triangle ABC by V_{Ph}, the triangle becomes a power triangle where AB = $V_{Ph}\, I_{Ph} \cos \phi$ is called the *active power*, BC = $V_{Ph}\, I_{Ph} \sin \phi$ is called the *reactive power*, and $V_{Ph}\, I_{Ph}$ is called the *apparent power*.

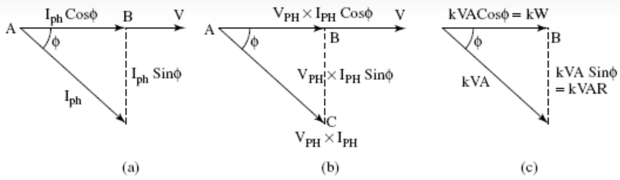

Figure 4.10 Relationship between active power, reactive power, and apparent power

Apparent power = kVA (4.9)

Apparent power × cos Φ = Active power = kW (4.10)

Apparent power × sin Φ = Reactive power = kVAR (4.11)

Multiplying all the sides of the power triangle by 10^3, i.e., expressing the power in terms of 'kilo', the power triangle is redrawn as has been shown in Fig. 4.8 (c) kVA is kilo Volt Ampere.

kVA cos Φ = kW (kilo Watt) (4.12)

kVA sin Φ = kVAR (kilo Var) (4.13)

4.7 COMPARISON BETWEEN STAR CONNECTION AND DELTA CONNECTION

As mentioned earlier, the three windings of a generator can be connected either in star or in delta. Same is the case with transformers. Three-phase electrical loads and the windings of three-phase motors can also be connected in star or in delta.

The relationship between voltages, currents and their phase relationship along with some other related factors have been compared and are presented in a tabular form.

Star Connection	Delta Connection
1. Line current is the same as phase current, i.e., $I_L = I_{Ph}$	1. Line current is $\sqrt{3}$ × the phase current, i.e., $I_L = \sqrt{3} I_{Ph}$
2. Line voltage is $\sqrt{3}$ the phase voltage, i.e., $V_L = \sqrt{3} V_{Ph}$	2. Line voltage is the same as phase voltage, i.e., $V_L = V_{ph}$
3. Total power = $\sqrt{3} V_L I_L \cos\phi$	3. Total power = $\sqrt{3} V_L I_L \cos\phi$
4. Per phase power = $V_{Ph} I_{Ph} \cos\phi$	4. Per phase power = $V_{Ph} I_{Ph} \cos\phi$
5. Three-phase three-wire and three-phase four-wire systems are possible	5. Three-phase three-wire system is possible
6. Line voltages lead the respective phase voltages by 30°	6. Line currents lag the respective phase currents by 30°

Example 4.1 A 400 V, three-phase, 50 Hz power supply is applied

Solution:

The load is delta connected. Hence

$$V_{Ph} = V_L = 400 \text{ V}$$

$$Z_{Ph} = R + jX = 6 + j8 = \sqrt{6^2 + 8^2} \angle \tan^{-1}\frac{8}{6} = 10\angle 53^\circ \ \Omega$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{400\angle 0^\circ}{10\angle 53^\circ} = 40\angle -53^\circ \text{ A}$$

Power factor, $\cos\phi = \cos 53^\circ = 0.6$ lagging

$$I_L = \sqrt{3}\, I_{Ph} = 1.732 \times 40 = 69.28 \text{ A}$$

Power factor, $\cos\phi = \frac{R}{Z} = \frac{6}{10} = 0.6$ lagging

$\sin\phi = 0.8$

Active Power $= \sqrt{3}\, V_L\, I_L \cos\phi = 1.732 \times 400 \times 69.28 \times 0.6$

$= 28798 \text{ W} = 28.798 \text{ kW}$

$\simeq 28.8 \text{ kW}$

Reactive Power $= \sqrt{3}\, V_L\, I_L \sin\phi = 1.732 \times 400 \times 69.28 \times 0.8$

$= 38397 \text{ VAR} = 38.397 \text{ kVAR}$

$\simeq 38.4 \text{ kVAR}$

Apparent Power $= 3\, V_{Ph}\, I_{Ph} = \sqrt{3}\, V_L\, I_L$

$= 1.732 \times 400 \times 69.28$

$= 47997 \text{ VA} = 47.997 \text{ kVA}$

$\simeq 48 \text{ kVA}$

The power triangle is shown in Fig. 4.11

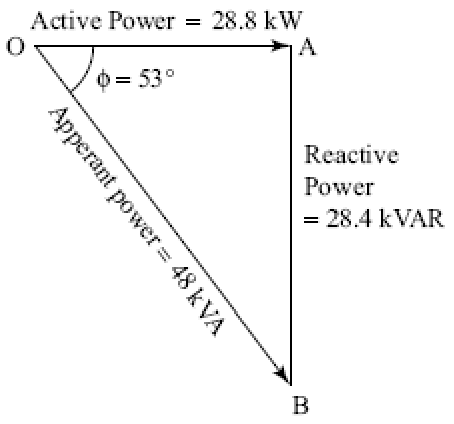

Figure 4.11

Apparent Power in

$$\text{kVA} = \sqrt{(\text{Active Power})^2 + (\text{Reactive Power})^2}$$

$$= \sqrt{(28.8)^2 + (38.4)^2}$$
$$= \sqrt{829.44 + 1474.56}$$
$$= \sqrt{2304} = 48$$

Example 4.2 A balanced star-connected load of (8 + j6) Ω per phase is connected to a balanced three-phase, 400 V supply. Find the line current, power factor, power, and total volt-amperes.

Solution:

Phase voltage, $V_P = \frac{\text{Line Voltage, } V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\,\text{V}$

Impedence per phase, $Z_P = \sqrt{R^2 + X_L^2} = \sqrt{8^2 + 6^2} = 10\,\Omega$

Phase current, $I_P = \frac{V_P}{Z_P} = \frac{231}{10} = 23.1\ \text{A}$

Line current $I_L = I_P = 23.1\ \text{A}$

Power factor $\cos\phi = \frac{R}{Z} = \frac{8}{10} = 0.8\,(\text{lagging})$

Total power, $P = \sqrt{3}\,V_L\,I_L\cos\phi$

$$= \sqrt{3} \times 400 \times 23.1 \times 0.8$$

$$= 12{,}800\ \text{W}$$

Total volt amperes $= \sqrt{3}\,V_L\,I_L$

$$= \sqrt{3} \times 400 \times 23.1 = 16{,}000\ \text{VA}$$

Example 4.3 A three-phase four-wire supply system has a line voltage of 400 V. Three non-inductive loads of 16 kW, 8 kW, and 12 kW are connected between R, Y, and B phases and the neutral, respectively. Calculate the current flowing through the neutral wire.

Solution:

Loads connected between the different phases and the neutral are of 16 kW, 8 kW, and 12 kW, respectively.

The current through the neutral wire line is the phasor sum of all the line currents. We will first calculate the line currents and then add them vectorially.

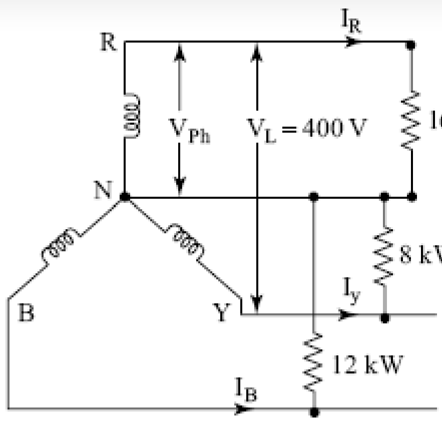

Figure 4.12

For star connection,
$$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{VB} = 231\text{ V}$$

The three-phase voltages are equal but have a phase difference of 120° between them.

$$V_R = 231\angle 0°,\ V_Y = 231\angle -120°, \text{and } V_B = 231\angle -240°$$

$$I_R = \frac{10 \times 1000}{V_R} = \frac{16 \times 1000}{231\angle 0} = 69.3\angle 0°$$

$$I_Y = \frac{8 \times 1000}{V_Y} = \frac{8 \times 1000}{231\angle -120°} = 34.6\angle 120°$$

$$I_B = \frac{12 \times 1000}{V_B} = \frac{12 \times 1000}{231\angle -240°} = 52\angle 240°$$

Current through the neutral wire, I_N is

$$\begin{aligned}
I_N &= I_R + I_Y + I_B \\
&= 69.3\angle 0° + 34.6\angle 120° + 52\angle 240° \\
&= 69.3(\cos 0° + j\ \sin 0°) + 34.6(\cos 120° + j\ \sin 120°) \\
&\quad + 52(\cos 240° + j\ \sin 240°) \\
&= 69.3(1 + j0) + 34.6(-0.5 + j0.866) \\
&\quad + 52(-0.5 + j0.866) \\
I_N &= 69.3 - 17.3 + j\,30 - 26 + j\,45 \\
&= 26 + j\,75
\end{aligned}$$

$$|I_N| = \sqrt{26^2 + 75^2} = \sqrt{6301}$$

V, 50 Hz, three-phase supply is connected across the load. Calculate phase voltage, phase current, power factor, power consumed per phase, and the total power consumed by the load.

Solution:

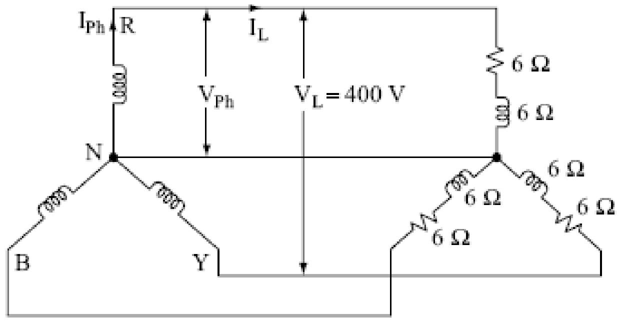

Figure 4.13

$$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\ \text{V}$$

$$Z_{Ph} = 6 + j\,6\,\Omega = 8.48\ \underline{|45^\circ}\ \Omega$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{231\underline{|0^\circ}}{8.48\underline{|45^\circ}} = 27.2\underline{|-45^\circ}\ \text{A}$$

$$I_{Ph} = I_L = 27.2\ \text{A}$$

Angle between V_{Ph} and I_{Ph} is 45°.

Power factor = cos 45° = 0.7 lagging

Power absorbed by each phase of the load = $V_{Ph}\ I_{Ph}\ \cos\phi$

= 231 × 27.2 × 0.7

= 4398 W

(Total power consumed = 3 × 4398 = 13194 W

Example 4.5 A 400 V, 50 Hz, three-phase supply is provided to a three-phase star-connected load. Each phase of the load absorbs a power of 2000 W. The load power factor is 0.8 lagging. Calculate the total power supplied to the load; the line current.

Solution:

Power consumed by each phase = 2000 W

Power consumed by all the three phases = 3 × 2000 W

$$V_L = 400 \text{ V}, \; V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

V

Power consumed per phase = $V_{Ph} I_{Ph} \cos \phi$

$$V_{Ph} I_{Ph} \cos \phi = 2000$$

$$I_L = I_{Ph} = \frac{2000}{231 \times 0.8} = 10.82 \text{ A}$$

Example 4.6 A 400 V, 50 Hz, three-phase supply is provided to a three-phase delta-connected load. The resistance and inductance of each phase of the load is 8 W and 0.04 H, respectively. Calculate the phase current and the line current drawn by the load. Also calculate the total power consumed.

Solution:

The impedance of load per phase,

$Z_{Ph} = R + j\,\omega L = 8 + j\,2\pi \times 50 \times 0.04$

$= 8 + j\,12.56\ \Omega.$

Since the load is delta connected, $V_{Ph} = V_L = 400$ V

$$I_{ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{400}{8 + j\,12.56} = \frac{400\angle 0}{14.89\angle \tan^{-1}(12.56/8)} = 26.86 \angle -58^\circ;$$

$$I_L = \sqrt{3}\; I_{ph} = 1.73 \times 26.86 = 36 \text{ A}$$

Total power consumed $= 3\, V_{Ph} I_{Ph} \cos \phi$

$= 3 \times 400 \times 26.86 \cos \phi$ W

$= 3 \times 400 \times 26.86 \times 0.52 = 1736$ W

Example 4.7 A three-phase star-connected load consumes a total of 12 kW at a power factor of 0.8 lagging when connected to a 400 V, three-phase, 50 Hz power supply. Calculate the resistance and inductance of load per phase.

Solution:

Total power consumed = 12 kW

Per phase power consumed = 4 kW

So, $V_{Ph} I_{Ph} \cos \phi = 4000$ W

$$I_{ph} = \frac{4000}{V_{ph} \cos\phi}$$

or,

$$I_{Ph} = \frac{4000}{(400/\sqrt{3}) \times 0.8} = 21.6 \text{ A}$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}}$$

$$Z_{Ph} = \frac{V_{Ph}}{I_{Ph}} = \frac{231}{21.6} = 10.7\ \Omega. \text{ Power factor, } \cos\phi = 0.8,\ \sin\phi = 0.6$$

$$R = Z_{Ph} \cos\phi = 10.7 \times 0.8 = 8.56\ \Omega$$

$$X = Z_{Ph} \sin\phi = 10.7 \times 0.6 = 6.42\ \Omega$$

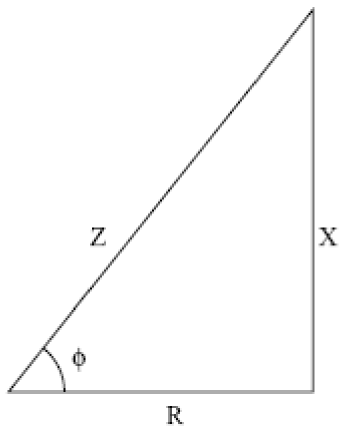

Figure 4.14

$$X = 6.42$$
$$2\pi fL = 6.42$$
$$L = \frac{6.42}{2 \times 3.14 \times 50} = 20.4 \times 10^{-3} \text{ H}$$
$$= 20.4 \text{ mH}$$

Example 4.8 A balanced three-phase star-connected load of 8 + j6 Ω per phase is supplied by a 400 V, 50 Hz supply. Calculate the line current, power factor, active, and reactive power.

Solution:

$$V_L = 400 \text{ V, For star connection, } V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}}$$

$$Z \quad = 8 + j6 = \sqrt{8^2 + 6^2}\,\big|\tan^{-1}\frac{6}{\ } = 10\big|37^\circ$$

For star connection,

$I_{Ph} = I_L = 23.1$ A

Angle of lag of I_{Ph} with V_{Ph} is 37°

Power factor $= \cos \phi = \cos 37° = 0.8$ lagging

Active power $= \sqrt{3}\, V_L\, I_L \cos \phi$

$= 1.732 \times 400 \times 23.1 \times 0.8$

$= 12802$ W $= 12.802$ kW

Reactive power $= \sqrt{3}\, V_L\, I_L \sin \phi$

$= 1.732 \times 400 \times 23.1 \times 0.6$

$= 9602$ VAR $= 9.602$ kVAR

Example 4.9 A delta-connected three-phase motor load is supplied from a 400 V, thee-phase, 50 Hz supply system. The line current drawn is 21 A. The input power is 11 kW. What will be the line current and power factor when the motor windings are delta connected?

Solution:

Line voltage, $V_L = 400$ V, $V_{Ph} = V_L$ for delta connection

Line current, $I_L = \sqrt{3}\, I_{Ph} = 21\text{A}$

Impedance of each winding,

$$Z_{Ph} = \frac{V_{Ph}}{I_{Ph}} = \frac{400}{I_L/\sqrt{3}} = \frac{400 \times \sqrt{3}}{21} = 33\ \Omega$$

$$P = \sqrt{3}\, V_L\, I_L \cos\phi = \sqrt{3} \times 400 \times 21 \times \cos\phi$$

or, $$\cos\phi = \frac{P}{\sqrt{3} \times 400 \times 21} = \frac{11 \times 1000}{\sqrt{3} \times 400 \times 21} = 0.756$$

When the motor windings are star connected

$$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{1.732} = 231\ \text{V}$$

Z_{Ph} will remain the same as the same windings are connected in star

In star connection, the line current is the same as phase current, hence, $I_L = I_{Ph} = 7$ A.

Power factor, depends on the circuit parameters

$$\cos\phi = \frac{R}{Z}$$

Since both R and Z remain unchanged, the power factor will remain the same at 0.756. Students may note that line current in a star connection is one-third of the line current in a delta connection.

Example 4.10 A balanced star-connected load of 4 + j6 Ω per phase is connected across a 400 V, three-phase, 50 Hz supply. Calculate line current, phase current, line voltage, phase voltage, power factor, total power, and reactive power.

Solution:

$$Z/_{Ph} = 4 + j6 = 7.21\underline{|56^\circ}$$

For star connection, $$V_{Ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{231\underline{|0}}{7.21\underline{|56^\circ}} = 32\underline{|-56^\circ}$$

Power factor, $\cos\phi = \cos 56° = 0.56$ lagging

$I_{Ph} = 32$ A

$I_{Ph} = I_L = 32$ A

$V_L = 400$ V

$V_{Ph} = 231$ V

Total Power $$= \sqrt{3} V_L I_L \cos\phi$$

$= 3 V_{Ph} I_{Ph} \cos\phi$

$= 3 \times 231 \times 32 \times 0.56$

Total reactive power $= \sqrt{3}\, V_L\, I_L \sin\phi$

$= 3\, V_{Ph}\, I_{Ph} \sin\phi$

$= 3 \times 231 \times 32 \times 0.83$

$= 18406$ VAR

$= 18.406$ kVAR

4.8 MEASUREMENT OF POWER IN THREE-PHASE CIRCUITS

We have known that in dc circuits power is measured as the product of voltage and current, i.e., power, P = VI. DC power can be measured using a voltmeter and an ammeter. In ac circuits power, $P = VI \cos\phi$. In three-phase ac circuits, total power is three times the power per phase. Wattmeter is an instrument used for measurement of power in ac circuits. Wattmeters are available as single-phase wattmeters and three-phase wattmeters. Single-phase wattmeters can be used to measure three-phase power. In case of star-connected balanced load with neutral connection, only one single-phase wattmeter can be used to measure the three-phase power. The three-phase power is three times the single-phase power. For unbalanced three-phase loads, i.e., if the currents in the three phases are not the same, two wattmeters are to be used to measure the three-phase power. These methods are described in the following sections.

4.8.1 One-Wattmeter Method

In this method, only one single-phase wattmeter can be used to measure the total three-phase power. In this method, the current coil (CC) of the wattmeter is connected in series with any phase and the pressure coil (PC) is connected between that phase and the neutral as shown in Fig. 4.15 One-wattmeter method has a demerit that even a slight degree of unbalance in the load produces a large error in the measurement. In this method one wattmeter will measure only the power of one phase. Hence, total power is taken as three times the wattmeter reading.

∴ Total Power $= 3 \times V_{ph}\, I_{ph} \cos\phi$

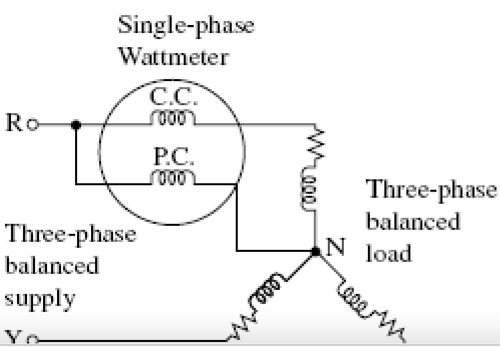

Figure 4.15 One-wattmeter method of measuring power of a star-connected balanced three-phase load

4.8.2 Two-Wattmeter Method

This method requires only two wattmeters to measure three-phase power for balanced as well as unbalanced loads. In this method two wattmeters are connected in two phases and their pressure coils are connected to the remaining third phase as has been shown in Fig. 4.16

This method of measurement is useful for balanced and unbalanced loads.

Let us consider the measurement of three-phase power of a star-connected load using two single-phase wattmeters as has been shown in Fig. 4.17(a). We will calculate the power measured by the two wattmeters separately. Let W_1 and W_2 respectively be the two wattmeter readings. Current flowing through the current coil of wattmeter W_1 is I_R. The voltage appearing across its pressure coil is V_{RB}. The wattmeter reading will be equal to, $W_1 = V_{RB}\, I_R$. cos of angle between V_{RB} and I_R. Similarly, the wattmeter reading W_2 will be equal to, $W_2 = V_{YB}\, I_B$ cos of angle between V_{YB} and I_B. We will now draw the phasor diagram, and calculate W_1 and W_2.

From the phasor diagram as shown in Fig. 4.17 (b),

$$W_1 = V_{RB}\, I_L \cos(30 - \phi) = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30 - \phi) = V_L I_L \cos(30 - \phi) \qquad (4.14)$$

And $$W_2 = V_{YB}\, I_Y \cos(30 + \phi) = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30 + \phi) = V_L I_L \cos(30 + \phi) \qquad (4.15)$$

We know that the total power in a three-phase circuit is $3V_{ph}\, I_{ph} \cos \phi$ or equal to $\sqrt{3} V_L\, I_L \cos \phi$

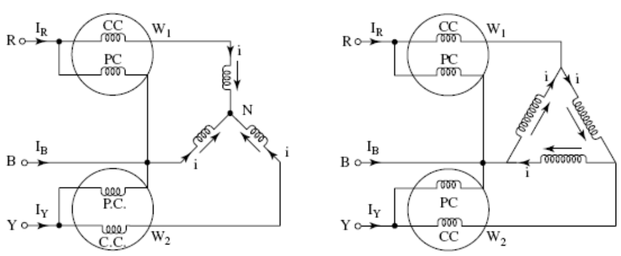

Figure 4.16 Two-wattmeter method of measuring power for star- and delta-connected load

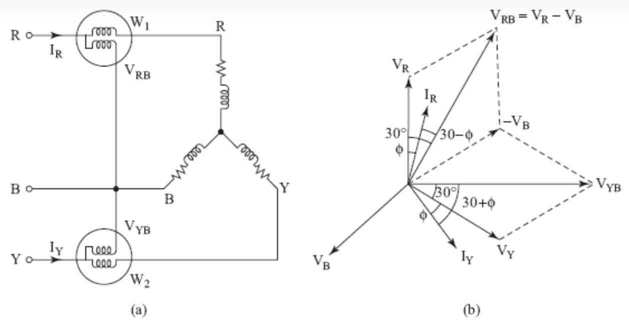

Figure 4.17 (a) Measurement of three-phase power using two single-phase wattmetters; (b) phasor diagram

Let us add the two wattmeter readings, i.e., W_1 and W_2.

$$\begin{aligned}
W_1 + W_2 &= \sqrt{3}\,V_{ph}\,I_{ph}\cos(30-\phi) + \sqrt{3}\,V_{ph}\,I_{ph}\cos(30+\phi) \\
&= \sqrt{3}\,V_{ph}\,I_{ph}\,[\cos(30-\phi) + \cos(30+\phi)] \\
&= \sqrt{3}\,V_{ph}\,I_{ph}\,2\cos\phi\cos 30^\circ \\
&= \sqrt{3}\,V_{ph}\,I_{ph}\,2\cos\phi\frac{\sqrt{3}}{2} \\
&= 3V_{ph}\,I_{ph}\cos\phi
\end{aligned}$$

or,

$$W_1 + W_2 = \sqrt{3}\,V_L\,I_L\cos\phi \tag{4.16}$$

Thus, it is proved that the sum of the two wattmeter readings is equal to the three-phase power.

Now, let us see what we get when the two wattmeter readings are subtracted from each other

$$\begin{aligned}
W_1 - W_2 &= \sqrt{3}\,V_{ph}\,I_{ph}\,[\cos(30^\circ-\varphi) - \cos(30^\circ+\varphi)] \\
&= \sqrt{3}\,V_{ph}\,I_{ph}\,2\sin\varphi\,\sin 30^\circ
\end{aligned}$$

or,

$$\sqrt{3}(W_1 - W_2) = 3\,V_{ph}\,I_{ph}\sin\varphi$$

or,

$$\sqrt{3}(W_1 - W_2) = \sqrt{3}\,V_L\,I_L\sin\varphi \tag{4.17}$$

Dividing eq. (4.17) by eq. (4.16)

$$\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = \frac{\sqrt{3}\,V_L\,I_L\sin\phi}{\sqrt{3}\,V_L\,I_L\cos\phi} = \tan\phi$$

or,

$$\varphi = \tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2}$$

Power factor,

$$\cos\varphi = \cos\tan^{-1}\frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \tag{4.18}$$

Thus, from the two wattmeter readings, we can calculate the total active and reactive powers and the power factor of the circuit.

$$W_1 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30-\phi)$$
$$W_2 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30+\phi)$$

We will consider a power factor of unity, 0.5, less than 0.5, and 0 and study the effect on the wattmeter readings.

1. At unity power factor i.e., when $\cos\phi = 1$ i.e., $\phi = 0$

   $$W_1 = \sqrt{3}\, V_{ph}\, I_{ph} \cos 30°$$
   $$W_2 = \sqrt{3}\, V_{ph}\, I_{ph} \cos 30°$$

   Thus at Power factor = 1, both the wattmeter readings will be positive and of equal value.

2. At 0.5 power factor, i.e., cos ϕ = 0.5 i.e., ϕ = 60°.

   $$W_1 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(-30°)$$
   $$= \sqrt{3}\, V_{ph}\, I_{ph} \cos 30°$$
   $$W_2 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30° + 60°) = 0$$

   Thus, at power factor equal to 0.5, one of the wattmeters will give zero reading.

3. When the power factor is less than 0.5, i.e., when ϕ > 60°. Let us observe the wattmeter readings.

   $$W_1 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30-\phi)$$
   $$W_2 = \sqrt{3}\, V_{ph}\, I_{ph} \cos(30+\phi)$$

   When ϕ > 60, W_1 will give positive readings but W_2 will give a negative reading.

   Thus, for power factor less than 0.5, i.e., for ϕ > 60°, one of

$W_1 = V_L\, I_L \cos(30° - 90°) = V_L\, I_L \cos 60°$
$W_2 = V_L\, I_L \cos(30° + 90°) = -V_L\, I_L \sin 30°$

Both the wattmeters show equal but opposite readings. Hence, the total power consumed will be zero.

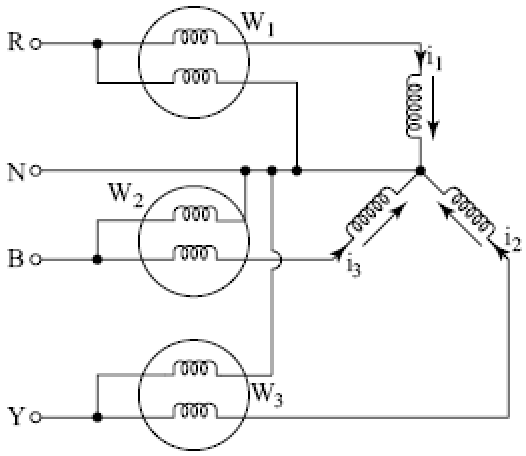

Figure 4.18 Measurement of three-phase balanced or unbalanced power using three single-phase wattmeters

4.8.3 Three-Wattmeter Method

In this method, three wattmeters are used to measure three-phase power. Three wattmeters are connected in each phase as has been shown in Fig. 4.18, and their pressure coils are connected between each phase and the neutral. This method is valid for the measurement of three-phase power for balanced and unbalanced loads. The main drawback of this method is the requirement of three wattmeters.

Example 4.11 In the two-wattmeter method of power measurement for a three-phase load, the readings of the wattmeter are 1000 W and 550 W. What is the power factor of the load?

Solution:

$W_1 = 1000$ W

$W_2 = 1000$ W
$W_2 = 550$ W

Power factor of load,

$$= \cos \tan^{-1} \sqrt{3} \frac{1000 - 550}{1000 + 550}$$

$$= \cos 26.695°$$

$$= 0.893 \text{ lagging}$$

Example 4.12 In the measurement of three-phase power by the two-wattmeter method, for a certain load, one of the wattmeters reads 20 kW and the other 5 kW after the current coil connection of one of the wattmeters has been reversed. Calculate the power and power factor of the load

Solution:

W_1 = 20 kW

W_2 = – 5 kW

P = W_1 + W_2 = 20 – 5 = 15 kW

Power factor of the load $= \cos \tan^{-1} \frac{W_1 - W_2}{W_1 + W_2} \sqrt{3}$

$$= \cos \tan^{-1} \frac{20 - (-5)}{20 + (-5)} \sqrt{3}$$

$$= 0.3273 \text{ lagging}$$

Example 4.13 Draw the connection diagram for measurement of power in a three-phase star-connected load using the two-wattmeter method. In one such a measurement the load connected was 30 kW at 0.7 pf lagging. Find the reading of each wattmeter.

Solution:

The connection diagram for measurement of power in a three-phase Y-connected load using the two-wattmeter method has been shown in Fig. 4.16 We know that the reading of the two wattmeters will be

ϕ V_{ph} I_{ph} cos (30 – ϕ) and $\sqrt{3}$ V_{ph} I_{ph} cos (30 + ϕ), respectively. For

star connection, $\sqrt{3}$ V_{ph} = V_L and I_{ph} = I_L.

The total load,	$P = 30$ kW
Power factor,	$\cos\phi = 0.7$ lagging
Phase angle,	$\phi = \cos^{-1}(0.7) = 45.57°$ lagging

So,
$$V_L I_L = \frac{P \text{ in kW} \times 1000}{\sqrt{3}\cos\phi}$$
$$= \frac{30 \times 1000}{\sqrt{3} \times 0.7} = 24743.6 \text{ VA}$$

Reading of wattmeter,
$$W_1 = V_L I_L \cos(30 - \phi)$$
$$= 24743.6 \cos(30 - 45.57°)$$
$$= 23.835 \text{ kW}$$

Reading of wattmeter,
$$W_2 = V_L I_L \cos(30 + \phi)$$
$$= 24743.6 \cos(30 + 45.57°)$$
$$= 6.165 \text{ kW}$$

To check, total power,
$$P = W_1 + W_2 = 23.835 + 6.165$$
$$= 30 \text{ kW}$$

Example 4.14 A three-phase balanced load connected across a 3Φ, 400 V ac supply draws a line current of 10 A. Two wattmeters are used to measure input power. The ratio of two wattmeter readings is 2:1. Find the readings of the two wattmeters.

Solution:

Let the ratio of wattmeter readings be X, i.e., $\dfrac{W_2}{W_1} = X$

and
$$\tan\phi = \sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

Now we will divide both numerator and dinominator by W_1. Then tan Φ will be

$$= \sqrt{3}\left(\frac{1 - W_2/W_1}{1 + W_2/W_1}\right) = \sqrt{3}\left(\frac{1 - X}{1 - X}\right)$$

and power factor,
$$\cos\phi = \frac{1}{\sec\phi} = \frac{1}{\sqrt{1 + \tan^2\phi}}$$
$$= \frac{1}{\sqrt{1 + 3[(1 - X)/(1 + X)]^2}}$$

Substituting
$$\frac{W_2}{W_1} = \frac{1}{2} = 0.5$$
$$\cos\phi = \frac{1}{\sqrt{1 + 3(1 - 0.5/1 + 0.5)}} = 0.866$$

or,
$$\phi = \cos^{-1}(0.866)$$
$$= 30°$$

Wattmeter reading, $W_1 = V_L I_L \cos(30° - 30°) = 400 \times 10 \times \cos 0° =$ 4000 W

Wattmeter reading, $W_2 = V_L I_L \cos(30° + 30°) = 400 \times 10 \times \cos 60° =$

three-phase, three-wire balanced supply system. The power input to the load is measured by the two-wattmeter method and the two wattmeters read 3 kW and 1 kW, respectively. Determine the values of R and L connected in each phase.

Solution:

Reading of wattmeter 1, W_1 = 3 kW

Reading of wattmeter 2, W_2 = 1 kW

Total power $P = W_1 + W_2 = 3 + 1 = 4$ kW

Power factor of the circuit, $\cos\phi = \cos\tan^{-1}\frac{W_1 - W_2}{W_1 + W_2}\sqrt{3}$

$$= \cos\tan^{-1}\frac{3-1}{3+1}\sqrt{3}$$
$$= \cos 40.89$$
$$= 0.7559 \text{ lagging}$$

Line current, $I_L = \frac{P}{\sqrt{3}\,V_L \cos\phi}$

$$= \frac{4 \times 1000}{\sqrt{3} \times 400 \times 0.7559} = 7.64 \text{ A} = \text{Phase current, } I_p$$

Impedance of the circuit per phase $Z = \frac{V_P}{I_P} = \frac{400\sqrt{3}}{7.64}$

$$= 30.237\ \Omega$$

∴ $R = Z\cos\phi = 30.237 \times 0.7559 = 22.856\ \Omega$

Reactance per phase, $X_L = \sqrt{Z^2 - R^2}$

$$= \sqrt{(30.237)^2 - (22.856)^2}$$
$$= 19.796\ \Omega$$

Inductance per phase, $L = \frac{X_L}{2\pi f}$

$$= \frac{19.796}{2\pi \times 50}$$
$$= 0.063 \text{ H}$$
$$= 63 \text{ mH}$$

Example 4.16 The power input to a three-phase motor is measured by two single-phase wattmeters. The total input power has been measured as equal to 15 kW and the power factor calculated as 0.5. What have been the readings of the two wattmeters?

Solution:

Total power = $W_1 + W_2$ = 15 kW

We have to calculate W_1 and W_2

When we measure three-phase power by the two-wattmeter method, the readings of the two wattmeters are

$$W_1 = V_L I_L \cos(30 - \phi)$$

and $$W_2 = V_L I_L \cos(30 + \phi)$$

$$\cos\phi = 0.5, \phi = 60°$$

$$\sqrt{3} V_L I_L \cos\phi = 15 \text{ kW}$$

$$V_L I_L = \frac{15}{\sqrt{3} \times 0.5} = 17.3 \text{ kVA}$$

$$W_1 = V_L I_L \cos(30 - \phi)$$

$$= 17.3 \cos(30 - 60°)$$

$$= 17.3 \times 0.866 = 15 \text{ kW}$$

$$W_2 = V_L I_L \cos(30 + \phi)$$

$$= 17.3 \cos 90°$$

$$= 17.3 \times 0$$

$$= 0$$

Total power $$= W_1 + W_2 = 15 + 0 = 15 \text{ kW}$$

Thus, at a load power factor of 0.5, one of the wattmeters has given zero reading. This has been explained earlier under the effect of change of power factor on wattmeter readings.

4.9 REVIEW QUESTIONS

A. Short Answer Type Questions

1. What is the difference between a single-phase winding and a three-phase winding ?
2. Draw wave shapes of a three-phase supply.
3. What is the difference between a balanced load and an unbalanced load ?
4. Show that the phasor sum of the three-phase balanced voltages is zero.
5. What do you mean by phase sequence of three-phase voltages?
6. Distinguish between star connection and delta connection of three-phase windings.
7. Derive the relationship between line current, line voltage, phase current, and phase voltage in case of star and delta connection of three-phase windings.
8. Prove that the power in a three-phase circuit is equal to $\sqrt{3}$ $V_L I_L \cos\phi$
9. Distinguish between active power and reactive power in a three-phase system.
10. What is the significance of low power factor of any load on the system?
11. Draw the circuit diagram for measurement of three-phase power with two single-phase wattmeters.
12. At what value of load power factor the reading of one of the wattmeters, in the two-wattmeter method of measurement of three-phase power, will be zero?
13. What are the advantages of the three-phase system over the single-phase system?
14. Write the relationship between phase voltage and current in a delta-connected load.
15. Draw the connection diagram for three-phase resistive–

B. Numerical Problems

1. Three coils having same resistance and inductance are connected in star. A three-phase 400 V supply is connected across the three coils. The power consumed by each coil is 800 W and the load power factor is 0.8 lagging. What is the total power consumed by the coils. If now the coils are connected in delta across the supply what would be the total power consumed? Also calculate the line current when delta connected.

   [Ans 2400 W, 7200 W, 13 A]

2. A balanced three-phase star-connected load supplied from a 400 V, 50 Hz, three-phase supply system. The current drawn by each phase is $20\angle{-60^\circ}$ A. Calculate the line current, phase voltage, and total power consumed.

   [Ans 20 A, 230.94 V, 6928 W]

3. A delta-connected load has a resistance of 15 Ω and inductance of 0.03 H per phase. The supply voltage is 400 V, 50 Hz. Calculate line current, phase current, phase voltage, and total power consumed.

   [Ans 39.1 A, 22.5 A, 400 V, 22.94 kW]

4. Three identical coils of resistance 20 Ω and inductance 500 mH are connected first in star and then in delta across a 400 V, 50 Hz power supply. Calculate phase current, line current, phase voltage, and power consumed per phase.

   [Ans 1.46 A, 1.46 A, 230.94 V, 42.6 W; 2.53 A, 4.38 A, 400 V, 127.8 W]

5. Calculate the phase current and line current of a delta-connected load drawing 75 kW at 0.8 power factor from a 440 V, three-phase supply.

   [Ans 71 A, 122.97 A]

6. The power consumed by a three-phase balanced load has been measured by two single-phase wattmeters. The readings of the two wattmeters are 8.2 kW and 7.5 kW. Calculate the total power consumed and the load power factor.

   [Ans 15.7 kW, 0.997 lagging]

7. In the measurement of three-phase power by two single-

[Ans 0.75]

8. Three identical coils are connected in star across a three-phase 415 V, 50 Hz supply. The total power drawn is 3 kW at a power factor of 0.3. Calculate the resistance and inductance of each coil.

   [Ans 5.16 Ω, 52.3 mH]

9. Two single-phase wattmeters are used to measure three-phase power. The readings of the two wattmeters are 2000 W and 400 W, respectively. Calculate the power factor of the circuit. What would be the power factor if the reading of the second wattmeter is negative?

   [Ans 0.65, 0.36]

10. Three identical coils each having a resistance of 10 Ω and inductive reactance of 10 Ω are first connected in star and then connected in delta across a 400 V, 50 Hz power supply. Calculate in each case the line current and the readings of two wattmeters connected for the measurement of power.

    [Ans 16.33 A, 49 A, 6309 W and 1690 W, 18931 W and 5072 W]

C. Multiple Choice Questions

1. Three-phase system is used
   1. For transmission of electrical power
   2. For generation of electrical power
   3. For distribution of electrical power
   4. For generation, transmission, and distribution of electrical power.
2. In a three-phase system the phase sequence indicates.
   1. The amplitude of voltages
   2. The order in which the voltages obtain their maximum values
   3. The phase difference between the three voltages
   4. The frequency in which the phase voltages are changing.
3. In a star-connected system the relationship between the phase and line quantities are
   1. $V_{ph} = V_L$
   2. $V_{ph} = \sqrt{3} V_L$
   3. $3I_{ph} = I_L$
   4. $I_{ph} = I_L$.
4. In a delta-connected system the relationship between the phase and line quantities are

2. $V_{ph} = \sqrt{3} V_L$
3. $I_{ph} = I_L$
4. $I_L = \sqrt{3} I_{ph}$.

5. Line currents drawn by a three-phase star-connected bnalanced load is 12 A when connected to a balanced three-phase four-wire system. The neutral current will be

   1. 36 A
   2. 4 A
   3. 0 A
   4. 3 A.

6. Power in a balanced three-phase system circuit is

   1. $3 V_p I_p \cos \phi$
   2. $3 V_L I_L \cos \phi$
   3. $\sqrt{3} V_p I_p \cos \phi$
   4. $V_p I_p \cos \phi$.

7. Reactive power of a three-phase circuit is

   1. $\sqrt{3} V_p I_p \sin \phi$
   2. $\sqrt{3} V_L I_L \sin \phi$
   3. $\sqrt{3} V_L I_L \cos \phi$
   4. $\sqrt{3} V_L I_L$.

8. power in a single-phase ac circuit can be expressed as

   1. $\sqrt{3} V_p I_p \cos \phi$
   2. $3 V_p I_p \cos \phi$
   3. $V_p I_p \cos \phi$
   4. $V_p I_p \cos \phi$.

9. One single-phase wattmeter can be used to measure power in a three-phase circuit when

   1. The load is balanced
   2. The load is delta connected and is balanced
   3. The load is balanced, star connected, and the neutral wire is available
   4. The load is balanced and star connected.

10. A balanced three-phase sinusoidal power supply means

   1. Three sinusoidal voltages of the same frequency and

3. Three sinusoidal voltages of some frequency and maximum value with no time phase displacement between them
4. Three sinusoidal voltages of any value but having a time phase displacement of 120^{o} between them.

11. An unbalanced three-phase supply system will have

    1. Three unequal voltages
    2. Three voltages having unequal time phase displacement between them
    3. Three voltages of unequal magnitude and angular displacement among them
    4. All of the above.

12. In the two-wattmeter method of measuring three-phase power, the reading of the two wattmeters will be equal when the power factor of the circuit is

    1. 0
    2. 1
    3. 0.5
    4. 0.866.

13. In the two-wattmeter method of measuring three-phase power, the reading of one of the wattmeters can be negative when the power factor angle is

    1. More than 60°
    2. Less than 60°
    3. More than 30°
    4. Less than 30°.

14. Four equal resistance of 100 Ω each connected in delta is supplied from 400 V three-phase star-connected supply, the line current drawn will be

    1. 12 A
    2. 4 A
    3. 6.928 A
    4. 13.856 A.

Answers to Multiple Choice Questions

1. (d)
2. (b)
3. (d)
4. (d)
5. (e)
6. (a)
7. (b)
8. (c)
9. (c)
10. (a)
11. (d)
12. (b)
13. (a)
14. (c)