

Definition 1.7.1 (LDU factorization)

$$A = LUD$$

Remark 1.7.2

$$\begin{bmatrix} * & 6th & 5th \\ 1st & * & 4th \\ 2nd & 3rd & * \end{bmatrix} \text{ and } \begin{bmatrix} * & 12th & 11th & 9th \\ 1st & * & 10th & 8th \\ 2nd & 5th & 6th & * \end{bmatrix}$$

Problem 1.7.3

Find LDU of $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix}$.

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

$$R_2 \Rightarrow -3R_1 + R_2$$

$$R_3 \Rightarrow (-1)R_1 + R_3$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & -2 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix} = E_1 A$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$R_2 \Rightarrow \frac{2}{3}R_3 + R_2$$

$$R_1 \Rightarrow \frac{-1}{3}R_3 + R_1$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix} = E_2 E_1 A$$

$$E_2 = \begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \Rightarrow \frac{1}{2}R_2 + R_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= E_3 E_2 E_1 A = D$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & \frac{-1}{2} & \frac{1}{3} \\ 0 & 1 & \frac{-2}{3} \\ 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\therefore A = LUD = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{-1}{2} & \frac{1}{3} \\ 0 & 1 & \frac{-2}{3} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$