Find Laplace Transform of the following functions:

1. $f(t) = \begin{cases} -1, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$

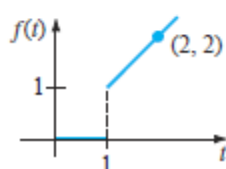2. $f(t) = \begin{cases} 4, & 0 \le t < 2 \\ 0, & t \ge 2 \end{cases}$

3. $f(t) = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$

4. $f(t) = \begin{cases} 2t + 1, & 0 \le t < 1 \\ 0, & t \ge 1 \end{cases}$

5. $f(t) = \begin{cases} \sin t, & 0 \le t < \\ 0, & t \ge \end{cases}$

6. $f(t) = \begin{cases} 0, & 0 \le t < /2 \\ \cos t, & t \ge /2 \end{cases}$

7.

Find the inverse Laplace Transform of the following:

1. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^3}\right\}$

2. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^4}\right\}$

3. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2} - \dfrac{48}{s^5}\right\}$

4. $\mathscr{L}^{-1}\left\{\left(\dfrac{2}{s} - \dfrac{1}{s^3}\right)^2\right\}$

5. $\mathscr{L}^{-1}\left\{\dfrac{(s + 1)^3}{s^4}\right\}$

6. $\mathscr{L}^{-1}\left\{\dfrac{(s + 2)^2}{s^3}\right\}$

7. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2} - \dfrac{1}{s} + \dfrac{1}{s - 2}\right\}$

8. $\mathscr{L}^{-1}\left\{\dfrac{4}{s} + \dfrac{6}{s^5} - \dfrac{1}{s + 8}\right\}$

9. $\mathscr{L}^{-1}\left\{\dfrac{1}{4s + 1}\right\}$

10. $\mathscr{L}^{-1}\left\{\dfrac{1}{5s - 2}\right\}$

11. $\mathscr{L}^{-1}\left\{\dfrac{5}{s^2 + 49}\right\}$

12. $\mathscr{L}^{-1}\left\{\dfrac{10s}{s^2 + 16}\right\}$

13. $\mathscr{L}^{-1}\left\{\dfrac{4s}{4s^2 + 1}\right\}$

14. $\mathscr{L}^{-1}\left\{\dfrac{1}{4s^2 + 1}\right\}$

15. $\mathscr{L}^{-1}\left\{\dfrac{2s - 6}{s^2 + 9}\right\}$

16. $\mathscr{L}^{-1}\left\{\dfrac{s + 1}{s^2 + 2}\right\}$

17. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2 + 3s}\right\}$

18. $\mathcal{L}^{-1}\left\{\dfrac{s + 1}{s^2 - 4s}\right\}$

19. $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2 + 2s - 3}\right\}$

20. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2 + s - 20}\right\}$

21. $\mathcal{L}^{-1}\left\{\dfrac{0.9s}{(s - 0.1)(s + 0.2)}\right\}$

22. $\mathcal{L}^{-1}\left\{\dfrac{s - 3}{(s - \sqrt{3})(s + \sqrt{3})}\right\}$

23. $\mathcal{L}^{-1}\left\{\dfrac{s}{(s - 2)(s - 3)(s - 6)}\right\}$

24. $\mathcal{L}^{-1}\left\{\dfrac{s^2 + 1}{s(s - 1)(s + 1)(s - 2)}\right\}$

25. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^3 + 5s}\right\}$

26. $\mathcal{L}^{-1}\left\{\dfrac{s}{(s + 2)(s^2 + 4)}\right\}$

27. $\mathcal{L}^{-1}\left\{\dfrac{2s - 4}{(s^2 + s)(s^2 + 1)}\right\}$

28. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^4 - 9}\right\}$

29. $\mathcal{L}^{-1}\left\{\dfrac{1}{(s^2 + 1)(s^2 + 4)}\right\}$

30. $\mathcal{L}^{-1}\left\{\dfrac{6s + 3}{s^4 + 5s^2 + 4}\right\}$

Solve the following initial value problems by using Laplace transform:

31. $\dfrac{dy}{dt} - y = 1, \quad y(0) = 0$

32. $2\dfrac{dy}{dt} + y = 0, \quad y(0) = -3$

33. $y' + 6y = e^{4t}, \quad y(0) = 2$

34. $y' - y = 2\cos 5t, \quad y(0) = 0$

35. $y'' + 5y' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0$

36. $y'' - 4y' = 6e^{3t} - 3e^{-t}, \quad y(0) = 1, \quad y'(0) = -1$

37. $y'' + y = \sqrt{2}\sin\sqrt{2}t, \quad y(0) = 10, \quad y'(0) = 0$

38. $y'' + 9y = e^{t}, \quad y(0) = 0, \quad y'(0) = 0$

39. $2y''' + 3y'' - 3y' - 2y = e^{-t}, \quad y(0) = 0, \quad y'(0) = 0,$
$y''(0) = 1$

40. $y''' + 2y'' - y' - 2y = \sin 3t, \quad y(0) = 0, \quad y'(0) = 0,$
$y''(0) = 1$

Find Laplace Transform of the following functions:

1. $\mathcal{L}\{te^{-10t}\}$

2. $\mathcal{L}\{t^3 e^t\}$

3. $\mathcal{L}\{t\cos 2t\}$

4. $\mathcal{L}\{t\sinh 3t\}$

5. $\mathcal{L}\{t^2 \sinh t\}$

6. $\mathcal{L}\{t^2 \cos t\}$

7. $\mathcal{L}\{te^{2t}\sin 6t\}$

8. $\mathcal{L}\{te^{-3t}\cos 3t\}$

Find the inverse Laplace Transform of the following:

11. $\mathcal{L}^{-1}\left\{\dfrac{1}{(s+2)^3}\right\}$ 12. $\mathcal{L}^{-1}\left\{\dfrac{1}{(s-1)^4}\right\}$

13. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2-6s+10}\right\}$ 14. $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2+2s+5}\right\}$

15. $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+4s+5}\right\}$ 16. $\mathcal{L}^{-1}\left\{\dfrac{2s+5}{s^2+6s+34}\right\}$

17. $\mathcal{L}^{-1}\left\{\dfrac{s}{(s+1)^2}\right\}$ 18. $\mathcal{L}^{-1}\left\{\dfrac{5s}{(s-2)^2}\right\}$

19. $\mathcal{L}^{-1}\left\{\dfrac{2s-1}{s^2(s+1)^3}\right\}$ 20. $\mathcal{L}^{-1}\left\{\dfrac{(s+1)^2}{(s+2)^4}\right\}$

Solve the following initial value problems by using Laplace transform:

9. $y' + y = t\sin t$, $y(0) = 0$

10. $y' - y = te^t\sin t$, $y(0) = 0$

11. $y'' + 9y = \cos 3t$, $y(0) = 2$, $y'(0) = 5$

12. $y'' + y = \sin t$, $y(0) = 1$, $y'(0) = -1$

13. $y'' + 16y = f(t)$, $y(0) = 0$, $y'(0) = 1$, where

$$f(t) = \begin{cases} \cos 4t, & 0 \le t < \\ 0, & t \ge \end{cases}$$

14. $y'' + y = f(t)$, $y(0) = 1$, $y'(0) = 0$, where

$$f(t) = \begin{cases} 1, & 0 \le t < /2 \\ \sin t, & t \ge /2 \end{cases}$$

21. $y' + 4y = e^{-4t}$, $y(0) = 2$

22. $y' - y = 1 + te^t$, $y(0) = 0$

23. $y'' + 2y' + y = 0$, $y(0) = 1, y'(0) = 1$

24. $y'' - 4y' + 4y = t^3e^{2t}$, $y(0) = 0, y'(0) = 0$

25. $y'' - 6y' + 9y = t$, $y(0) = 0, y'(0) = 1$

26. $y'' - 4y' + 4y = t^3$, $y(0) = 1, y'(0) = 0$

27. $y'' - 6y' + 13y = 0$, $y(0) = 0, y'(0) = -3$

28. $2y'' + 20y' + 51y = 0$, $y(0) = 2, y'(0) = 0$

29. $y'' - y' = e^t\cos t$, $y(0) = 0, y'(0) = 0$

30. $y'' - 2y' + 5y = 1 + t$, $y(0) = 0, y'(0) = 4$

Find the Laplace Transform of the following functions

19. $\mathscr{L}\{1 * t^3\}$

20. $\mathscr{L}\{t^2 * te^t\}$

21. $\mathscr{L}\{e^{-t} * e^t \cos t\}$

22. $\mathscr{L}\{e^{2t} * \sin t\}$

23. $\mathscr{L}\left\{\int_0^t e^\tau \, d\tau\right\}$

24. $\mathscr{L}\left\{\int_0^t \cos \tau \, d\tau\right\}$

25. $\mathscr{L}\left\{\int_0^t e^{-\tau} \cos \tau \, d\tau\right\}$

26. $\mathscr{L}\left\{\int_0^t \tau \sin \tau \, d\tau\right\}$

27. $\mathscr{L}\left\{\int_0^t \tau e^{t-\tau} \, d\tau\right\}$

28. $\mathscr{L}\left\{\int_0^t \sin \tau \cos (t - \tau) \, d\tau\right\}$

29. $\mathscr{L}\left\{t \int_0^t \sin\tau \, d\tau\right\}$

30. $\mathscr{L}\left\{t \int_0^t \tau e^{-\tau} \, d\tau\right\}$

Find the inverse Laplace Transform of the followings:

31. $\mathscr{L}^{-1}\left\{\dfrac{1}{s(s - 1)}\right\}$

32. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^2(s - 1)}\right\}$

33. $\mathscr{L}^{-1}\left\{\dfrac{1}{s^3(s - 1)}\right\}$

34. $\mathscr{L}^{-1}\left\{\dfrac{1}{s(s - a)^2}\right\}$