3

AC Fundamentals and Single-phase Circuits

TOPICS DISCUSSED

- Concept of dc and ac
- Concept of frequency
- Time period
- Instantaneous value
- Average value and maximum value of an alternating quantity
- Sinusoidal and non-sinusoidal wave forms
- Concept of root mean square value
- Concept of phase and phase difference
- Single-phase ac circuits
- Series–parallel circuit containing resistance inductance and capacitance
- Concept of apparent power
- Real power and reactive power
- Resonance in ac circuits

Now days electricity is generated in the form of ac (alternating current). The generated electricity is transmitted, distributed, and mostly utilized in the form of ac. In this chapter, the fundamental concepts of ac and ac circuits have been discussed.

3.1.1 Introduction

We have known that current drawn from a battery is unidirectional. The polarities of the battery are marked +ve and −ve. When a particular load, say a lamp (represented by its resistance) is connected across the two terminals of the battery, current flows through the lamp in a particular direction. The magnitude of current as well as its direction remains constant with respect to time as long as the battery voltage remains constant. Such a current is known as steady or constant-value direct current.

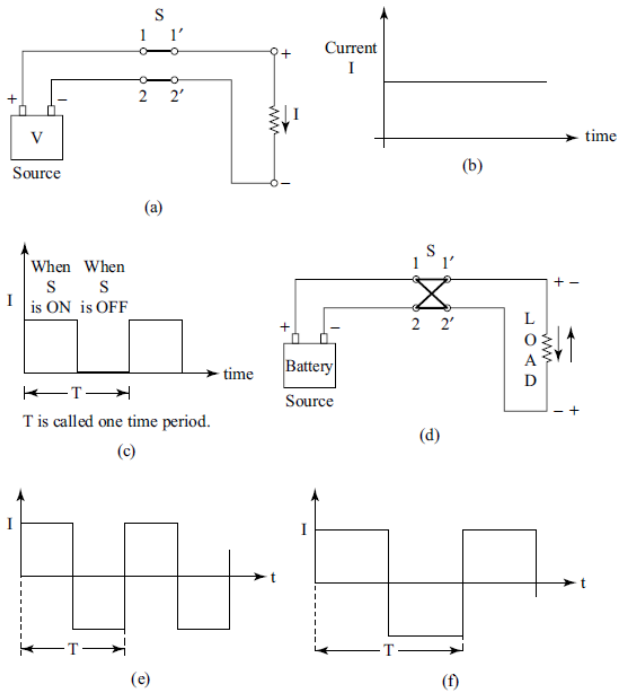


Fig 3.1 Concept of dc and ac illustrated. (a) A battery connected to a resistive load; (b) direct current of constant magnitude; (c) direct current of variable magnitude; (d) a battery connected to a resistive load through a reversing switch; (e) alternating current of square–wave shape; (f) alternating current of rectangular wave shape

When the direction of current through a circuit continuously changes, such a current is called alternating current. The polarities of the ac supply source changes alternately and causes alternating current to flow through the load connected across the terminals. Fig. 3.1 (a) shows dc flowing through the load when connected across the battery terminals. Since the magnitude is assumed constant, it is represented through a graph as shown in Fig. 3.1 (b). Fig. 3.1 (c) to load terminal 1′. It can be noted that if the period of switching *ON* in both the directions in kept constant, the load current will be alternating in nature and its wave shape will be square or rectangular type as shown in Fig. 3.1 (e) and 3.1 (f).

To generate ac from an available dc source, we need an automatic switching arrangement. This is achieved by using electronic circuitry as in the case of inverters used as emergency lighting arrangement.

However, alternating current on a large scale is made available by using ac generators installed in power houses. AC generators are driven by turbines (gas, steam, water). Turbines are used to create a relative motion between a set of magnets and a set of coils. The rate of change of magnetic flux linkages or the rate of cutting of flux by the conductors of the coils causes EMF to be induced in the coil windings. The relative motion between the coils and the magnets producing a magnetic field can be created by making one system rotating with respect to the other. For example, we may have a stationary magnetic field system and inside the magnetic field we can place the coils which will be rotated by a prime mover (i.e., a turbine).

Alternately, the coils could be kept stationary and a set of magnets could be made rotating, thus causing EMF to be induced in the coils. We shall see the nature of EMF induced when we create a relative motion between a set of coils and a magnetic field. For simplicity we will consider only one coil rotating in a magnetic field created by a North and a South Pole.

3.1.2 Generation of Alternating Voltage in an Elementary Generator

In Fig. 3.2 (a) is shown a coil having a few turns rotated in a magnetic field. If ϕ is the flux produced in Webers in the magnetic field and N is the number of turns of the coil, the flux linkage by the coil, i.e., the amount of flux passing through the coil will be ($N\phi$) Webers. When the coil rotates, there is a change in the flux linkage. The induced EMF ‘e’ is the rate of change of flux linkage which can be expressed as

$$e = -d/dt\,(N\phi) = -N\,d\phi/dt \text{ V}$$

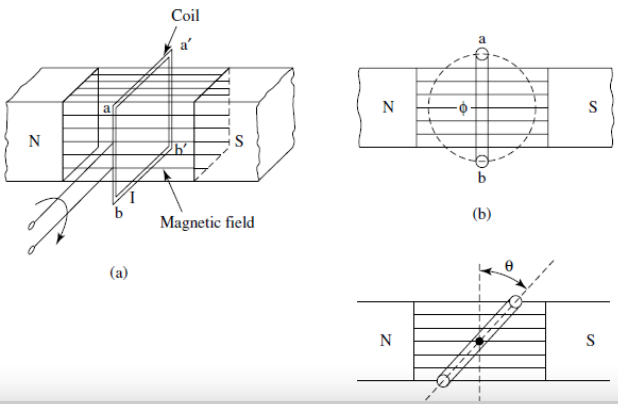


Figure 3.2 EMF induced in a coil rotated in a magnetic field. (a) Maximum flux linkage but minimum rate of change of flux linkage; (b) same as in (a); (c) the coil has rotated by an angle q from its vertical position increasing the rate of change of flux linkage

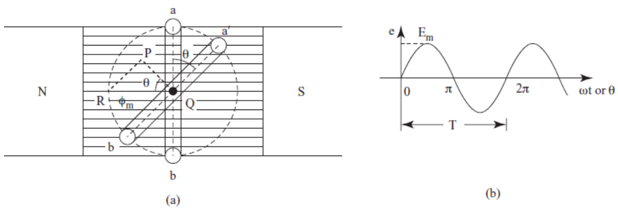


Figure 3.3 Sinusoidal EMF induced in a coil rotating in a uniform magnetic field. (a) Coil rotating; (b) wave shape of the induced EMF

It may be seen from Fig. 3.2 (a) and 3.2 (b) that flux ϕ is perpendicular to the coil. When the coil rotates through an angle, say θ, from its vertical axis, as shown in Fig. 3.2 (c), the component of flux ϕ which then becomes perpendicular to the plane of the coil is $\phi_m \cos$ θ. If θ is taken as ωt, $\phi = \phi_m \cos \theta t$.

In position a′b′ of the coil ab, PQ is the component of flux $\phi = \phi_m$, i.e., RQ that will link the coil. From Fig. 3.3 (a),

$$PQ = RQ \cos\theta = \phi_m \cos\theta = \phi_m \cos\omega t$$

Induced EMF,
$$e = -N\, d\phi/dt = -N\, d/dt\,(\phi_m \cos\omega t)$$
$$= N\omega\phi_m \sin\omega t$$

or,
$$e = E_m \sin\omega t \tag{3.1}$$

where,
$$E_m = N\omega\phi_m = N\, 2\pi f \phi_m\, V$$

It is seen from eq. (3.1) that the induced EMF is sinusoidal in nature when the coil rotates in a uniform magnetic field as has been shown in Fig. 3.3 (b). For the initial position of rotation, i.e., when the coil plane is vertical to the direction of the flux, the EMF induced is minimum because a little change in angle θ does not cause much change in the flux linkage, or cutting of flux by the conductor is minimum. In the horizontal position of the coil, any small change in the coil angle causes a large change in the flux linkage or the cutting of flux by the conductor is maximum, and hence the induced EMF is the highest at that position.

3.1.3 Concept of Frequency, Cycle, Time Period, Instantaneous Value, Average Value, and Maximum Value

One set of positive values and the subsequent one set of negative values of an alternating quantity constitute a **cycle**. The time taken for the generation of one cycle of EMF or flow of current caused due to such an EMF is called the **time period, T**. The total number of cycles of EMF or current produced per second is called the

In 1 second the cycle produced is 1/T

Since f is the cycle produced per second,

$$f = 1/T \tag{3.2}$$

The value of an alternating quantity at any instant of time is called its **instantaneous value**. Such values are expressed in small lettering like e, i, etc. For sinusoidal waves, we may write

$$e = E_m \text{ Sin } \theta$$

and $\quad i = I_m \text{ Sin } \theta$

at $\quad \theta = 0°, e = E_m \text{ Sin } 0° = 0$

at $\quad \theta = 90°, e = E_m \text{ Sin } 90° = E_m$

E_m is called the maximum value which occurs at θ = 90°, i.e., when the plane of the rotating coil is parallel to the magnetic field.

3.1.4 Sinusoidal and Non-sinusoidal Wave Forms

We have seen earlier that when a coil rotates in a uniform magnetic field the EMF induced in the coil is sinusoidal in nature. The wave shape of an alternating voltage or current produced in an ac generator having uniform flux distribution is also sinusoidal in nature. However, an alternating quantity may be non-sinusoidal also. Any non-sinusoidal wave can be seen as consisting of a number of sinusoidal waves of different frequencies. Such component sine waves of a non-sinusoidal wave are called harmonic waves. In Fig. 3.4 (a) have been shown non-sinusoidal waves. In Fig. 3.4 (b) have been shown non-sinusoidal waves and their corresponding component sine waves. The component sine wave having the same frequency as the original wave is called fundamental wave and the sine waves of higher frequencies are called harmonic waves or simply harmonics.

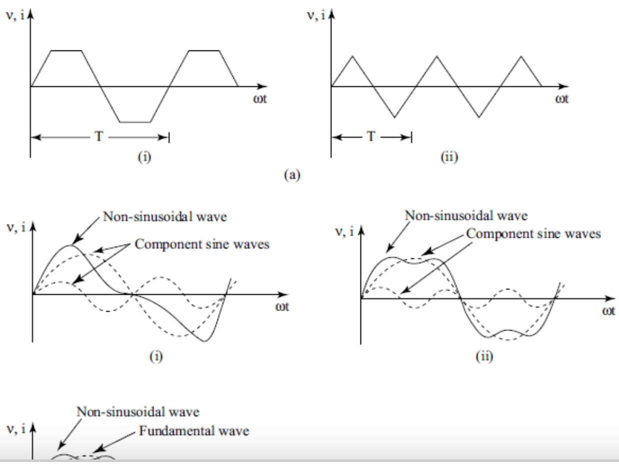


Figure 3.4 Non-sinusoidal waves and harmonics

In Fig. 3.4 (a) have been shown a trapezoidal and a triangular-type non-sinusoidal wave. In Fig. 3.4 b (i) has been shown a non-sinusoidal wave which is the sum of two component sine waves of different frequencies. One has the same frequency as the non-sinusoidal wave. This is called the fundamental. The other harmonic wave has twice the frequency as the fundamental. This is called the second harmonic. The non-sinusoidal wave shown in 3.4 b (ii) and (iii) are composed of a fundamental wave and a third harmonic. A third harmonic wave has three times the frequency as the fundamental wave. The number of harmonics present in an alternating non-sinusoidal quantity will depend upon the complexity of the wave shape. A symmetrical wave is the one whose positive half is identical to its negative half. Whether a wave is symmetrical or not can be tested by lifting the negative half and shifting it to the positive half axis and placing it just over the positive half. If both the half waves match each other, the wave shape is symmetrical. When generators are built physically symmetrical, the EMF wave shape induced in the coils in such machines are symmetrical in nature. A symmetrical wave will contain fundamental and odd harmonics only. The presence of even harmonics, i.e., 2nd harmonic, 4th harmonic, etc. will be there in non-symmetrical, non-sinusoidal ac waves.

3.1.5 Concept of Average Value and Root Mean Square (RMS) Value of an Alternating Quantity

For a symmetrical alternating voltage or current wave, the positive half is identical to the negative half, and hence the average value of the quantity for a complete cycle is zero. In earlier days the usefulness of such ac was questioned and only dc was considered effective. However, it was observed that when ac is allowed to pass through a resistance element, heat is produced. The question that arose was that if the average value of an alternating quantity is zero, why then was it producing heat. The concept of effective value was then brought in from the point of view of heat equivalence.

Average value

The average value of an alternating quantity is the sum of all its values divided by the total number of values. A waveform has continuous variable values with repeat to time, t or angle θ where θ = ωt. The pattern of wave repeats after every cycle. The sum of all the values in a cycle is determined by the integration of its values over a period of time. A full cycle is formed in 2π radians or in T seconds where T is the time period. A symmetrical wave is one where the positive half cycle is exactly the same as the negative half cycle. If we integrate the values for a complete cycle and take its average over one cycle, the quantity becomes equal to zero. The average value if calculated over a complete cycle would become zero.

$$V_{av} = \frac{1}{2\pi}\int_0^{2\pi} v\, d\theta = 0$$

Average value of a sinewave or any other symmetrical wave over a complete cycle is zero

For half-wave or full-wave rectified waves we need to calculate the average value. When we intend to calculate the average value of such waves, we calculate the average value for one-half cycle.

The average value is calculated as

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} v\, d\theta$$

or,

$$V_{av} = \frac{2}{T}\int_0^{T/2} v\, dt$$

Effective or RMS value

The effective value or RMS value of an alternating quantity is determined by considering equivalent heating effect.

The effective value of an alternating quantity (say current) is that the value of dc current which when flowing through a given circuit element (say a resistance element) for a given time will produce the same amount of heat as produced by the alternating current when flowing through the same circuit element for the same time.

Let I be the equivalent effective value of the ac flowing through a resistance element R for a time t, then the amount of heat produced, H is expressed as

$$H \propto I^2 Rt = K\, I^2\, Rt \text{ Calories} \tag{3.3}$$

Now, let the alternating current i, be passed through the same resistance R for the same time t, as shown in Fig. 3.5. Current i has been shown divided into n intervals and the magnitudes are i_1, i_2, i_3, etc. Heat produced in t seconds by the ac is equal to the sum of heat produced in n intervals of time during the time t. This can be expressed as

$$\begin{aligned} H &\propto i_1^2 R\, t/n + i_2^2 R\, t/n + i_3^2 R\, t/n + \cdots + i_n^2 R\, t/n \\ &\propto \frac{(i_1^2 + i_2^2 + i_3^2 + \cdots + i_n^2)}{n} Rt \\ H &= K\frac{(i_1^2 + i_2^2 + i_3^2 + \cdots + i_n^2)}{n} Rt \end{aligned} \tag{3.4}$$

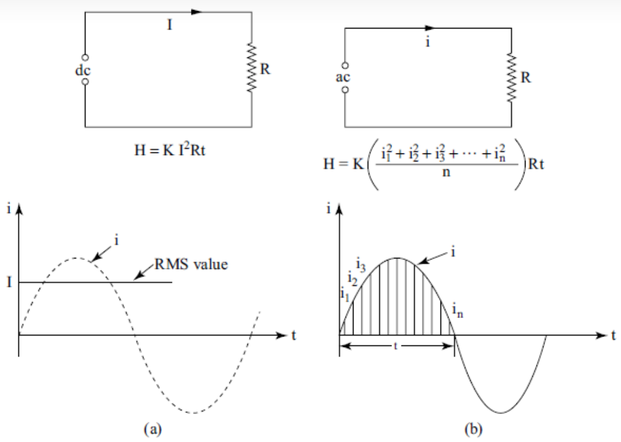


Figure 3.5 RMS value of an alternating current illustrated

Equating expressions (3.3) and (3.4),

$$I^2 Rt = \frac{(i_1^2 + i_2^2 + i_3^2 + \cdots + i_n^2)}{n} Rt$$

or,

$$I = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \cdots + i_n^2}{n}}$$

Thus, the effective value is equal to the square root of the mean of the squares of instantaneous values of the alternating quantities. Alternately, this can be read as square mean root value or root mean square value, i.e., RMS value. While expressing alternating quantities we always use RMS values and write in capital letters as E, I, V, etc. To further clarify the concept of the effective or RMS value let us find out the value of dc current, I which gives the same amount of heating as that of ac when it flows through a resistance of value, say, R. The alternating quantity is represented as, say, $i = I_m \sin \omega t$.

The power dissipated in R by the dc current,

$$P_{av} = I^2 R \text{ watts} \qquad \text{(i)}$$

The instantaneous value of power dissipated in R by the ac current,

$$P = i^2 R = I_m^2 \; \text{Sin}^2 \omega t \times R$$

$$= \frac{I_m^2 R}{2}(1 - \cos 2\omega t)$$

The average value of the second term is zero as it is a cosine function varying with time.

The average value of power, P for the ac current,

$$P_{av} = \frac{I_m^2 R}{2} \qquad \text{(ii)}$$

Equating the power dissipated due to dc current, I and the ac. current, i we can get the effective value as

$$I^2 R = \frac{I_m^2 R}{2}$$

or,

$$I = \frac{I_m}{\sqrt{2}} = 0.707\ I_m$$

The effective or RMS value of an alternating quantity is either for one-half of a cycle or for a full cycle as

$$I = \sqrt{\frac{1}{T}\int_0^T i^2 dt} \quad \text{or} \quad I = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2 dt}$$

The RMS values of an alternating quantity of any type of wave shapes can be calculated using analytical methods.

3.1.6 Analytical Method of Calculation of RMS Value, Average Value, and Form Factor

Suppose we have a sinusoidal alternating current, we have to first square it, then take its mean over one cycle or half cycle, and then take the square root (note that RMS value is calculated by making reverse operation i.e., first square, then take mean and then take square root).

Square of the current $i = I_m \sin\theta$ is $I_m^2 \sin^2\theta$

Its mean over one cycle is calculated by integrating it from 0 to 2π and dividing by the time period of 2π as follows:

Mean of square $= 1/2\pi \int_0^{2\pi} I_m^2 \sin^2\theta\ d\theta$

RMS value, $I = \sqrt{1/2\pi \int_0^{2\pi} I_m^2 \sin^2\theta\, d\theta}$

$$I = \sqrt{\frac{I_m^2}{4\pi} \int_0^{2\pi} (1 - \cos 2\theta)\, d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \times 2\pi}$$

$$= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

Therefore, $$I = \frac{I_m}{\sqrt{2}} = 0.707\, I_m \tag{3.5}$$

i.e.,

$$\text{RMS value} = \frac{\text{Maximum value}}{\sqrt{2}} \quad \text{(for a sinusoidal wave)}$$

If we calculate the RMS value for half cycle, it can be seen that we will get the same value by calculating as

$$I = \sqrt{\frac{1}{\pi} \int_0^{\pi} I_m^2 \sin^2\theta\, d\theta} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

Average value

Average value of a sinusoidally varying quantity over one cycle is zero because for the first half cycle current flows in the positive direction and for the second half cycle same current flows in the negative direction, i.e., in the opposite direction.

Average value has to be calculated by considering one-half cycle as

$$I_{av} = \frac{1}{\pi} \int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi} \int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi} [-\cos\theta]_0^{\pi}$$

$$= -\frac{I_m}{\pi} [-1 - 1]$$

$$= \frac{2\, I_m}{\pi} = 0.637\, I_m$$

Similarly average value for sinusoidal voltage, $V_{av} = 0.637\, V_m$

Therefore, average value $\frac{2\, I_m}{\pi}$ or $= \frac{2\, V_m}{\pi}$ (3.6)

Form factor

As the name suggests, form factor is an indicator of the shape or the form of the ac wave. It is the ratio of the RMS value to the average value of an alternating quantity. For a sinusoidal varying quantity, the form factor K_f is

$$K_f = \frac{\text{RMS value}}{\text{Average value}} = \frac{0.707\, I_m}{0.637\, I_m} = 1.11 \quad (3.7)$$

The sharper the wave shape, the more will be the value of the form factor. For example, for a triangular wave, form factor will be more than 1.11 and for a rectangular wave form factor will be less than 1.11 (in fact, its value will be 1). The peak or crest factor is the ratio of peak or maximum value to its rms value.

It is obvious that by knowing the value of the form factor, the RMS value can be calculated if the average or mean value is known.

3.1.7 RMS and Average Values of Half-wave-rectified Alternating Quantity

A half-rectified sine wave is shown in Fig. 3.6. A half-wave-rectified quantity, whether voltage or current will have its one half cycle blocked by using a diode rectifier as shown. Since the diode allows current to flow in one direction only, current through the load resistance will flow, in one direction only. During the negative half cycle of the input voltage the diode will block current flow and hence no voltage will be appearing across the load during all negative half cycles. For half-wave-rectified current or voltage, we have to consider the current or voltage which is available for the positive half cycles and average it for the complete cycle. For a complete cycle, i.e., from 0 to 2π, current flows only from 0 to π. To calculate the RMS value we have to square the current, take its sum from 0 to π and then take the average for the whole cycle, i.e., from 0 to 2π. Thus

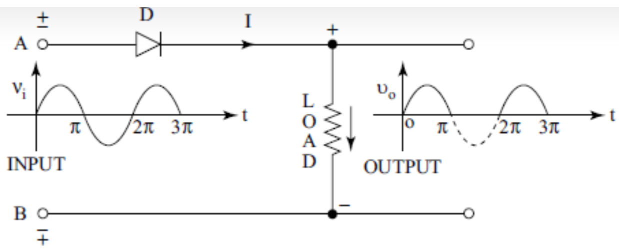


Figure 3.6 Half-rectified sine wave

$$I = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \quad \sin^2\theta \, d\theta} = \sqrt{\frac{I_m^2}{4\pi}\int_0^{\pi}(I - \cos 2\theta)\,d\theta}$$

$$[\text{since } 2\sin^2\theta = 1 - \cos 2\theta]$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \times \pi} = \frac{I_m}{2}$$

Thus, the RMS value of a full sine wave is $\frac{I_m}{\sqrt{2}}$ and for a half wave, $\frac{I_m}{2}$

Average value of current for half sine wave is

$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} i\,d\theta = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta\,d\theta$$

$$= \frac{I_m}{2\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{2\pi} \times 2 = \frac{I_m}{\pi}$$

Note that for a complete sine wave, the average value was calculated as and for a half-rectified sine wave, the average value has been calculated as

$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} i\,d\theta = \frac{I_m}{\pi}. \tag{3.8}$$

Obviously, we note that for a half-rectified wave, the average value is half of that of a full sine wave.

Form factor for a half sine wave quantity is

$$K_f = \frac{\text{RMS value}}{\text{Average value}} = \frac{I_m}{2} \times \frac{\pi}{I_m}$$

$$= \frac{\pi}{2} = \frac{3.14}{2} = 1.57$$

3.1.8 Concept of Phase and Phase Difference

The position of a coil or a set of coils forming a winding with respect to some axis of reference is called its *phase*. If three coils are placed at different angles with respect to the reference axis, there exists a *space phase difference* between these three coils AA′, BB′, and CC′,. When EMFs will be induced in these coils due to the cutting of the magnetic flux or due to change in flux linkages, the EMFs will have similar *time phase difference* between them as shown in Fig. 3.7.

A magnet has been shown rotating in the anticlockwise direction. Maximum flux will be cut by the coil AA′ at time, t = 0. Hence, maximum voltage will be in the coil AA′ at time, t = 0 as has been shown as v_A in Fig. 3.7 Maximum flux will be cut by the coil BB′ after an elapse of angle 30°, i.e., by the time the rotating magnet rotates by an angle of 30°. Similarly, maximum flux will be cut by the coil CC′ after an elapse of time represented by 60°.

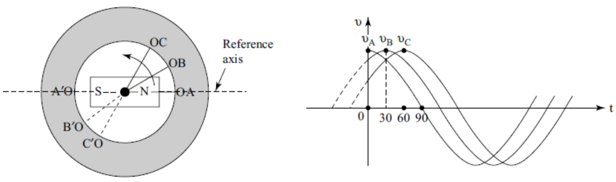


Figure 3.7 Concept of phase and phase difference illustrated

The voltage waves in coil AA′, BB′, and CC′ will, therefore, have a time phase difference of 30°. (30° corresponds to the time taken by the rotating magnet to rotate by 30°). Since voltage v_A is appearing

Such phase difference may exist between the voltage and current in an electrical circuit. If current in a circuit changes in accordance with the voltage, i.e., when the voltage is at its maximum value, the current is also at its maximum value, and when the voltage starts increasing in the positive direction from its zero value, the current also starts increasing in the positive direction from its zero value; then, the voltage and current are said to be in phase as shown in Fig. 3.8 (a). Note that the magnitudes of voltage and current may be different. In Fig. 3.8 (b) is shown current, i lagging the voltage by 90°, i.e., by an angle $\pi/2$. The expressions for voltage and current as shown in Fig. 3.8 (a) can be written as

$v = V_m \sin\omega t$

$i = I_m \sin(\omega t + 0) = I_m \sin\omega t$

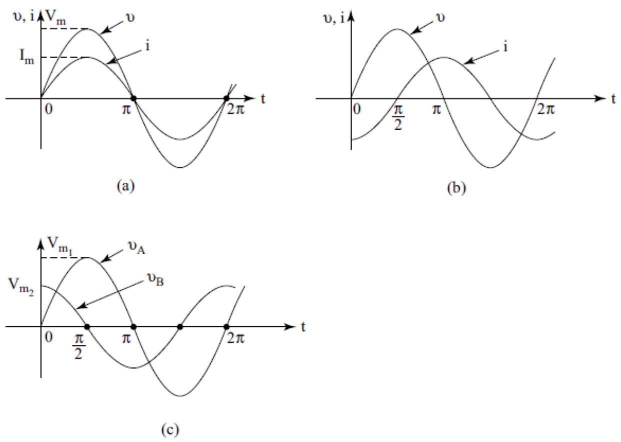


Figure 3.8 Phase and phase difference between voltage and current

The voltage and current shown in Fig. 3.8 (b) can be represented as

$v = V_m \sin \omega t$

$i = I_m \sin(\omega t - \pi/2)$

If current is leading the voltage by $\pi/2$ degrees, we will represent the current, i as

$i = I_m \sin (\omega t + \pi/2)$

If two voltages n_A and n_B are represented as in Fig. 3.8 (c), they can be expressed as

$v = V_{m1} \sin \omega t$

$v = V_{m2} \sin(\omega t + \Pi/2)$

v_A appears. However, if v_B is taken as the reference voltage we can express v_B and v_A as

$v_B = V_m \sin \omega t$

$v_A = V_m \sin (\omega t - \omega/2)$

Example 3.1 An alternating voltage of 100 sin 314 t is applied to a half-wave diode rectifier which is in series with a resistance of 20 Ω. What is the RMS value of the current drawn from the supply source?

Solution:

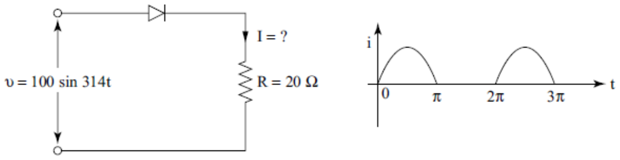


Figure 3.9 Circuit diagram of example 3.1

We have, $v = V_m \sin \omega t = 100 \sin 314\, t$

$V_m = 100$ V

For a full sine wave, RMS value

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

For half-rectified wave

$$V_{rms} = \frac{V_m}{2}$$

Therefore,

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{2 \times 20} = \frac{100}{2 \times 20} = 2.5\ \text{A}$$

Example 3.2 An alternating sinusoidal voltage of $v = 150 \sin 100\pi t$ is

Solution:

v = 150 sin 100 ωt

standard form, $v = V_m \sin \omega t$

The maximum value of voltage, $V_m = 150$

ω = 100 π = 2 πf

frequency, f = 50 Hz

The circuit in the question is a half-wave-rectified one.

For half sine wave, the RMS value

$$V_{rms} = \frac{V_m}{2} \text{ and } V_{av} = \frac{V_m}{\pi}$$

Therefore, $$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{2R} = \frac{150}{2 \times 50} = 1.5 \text{ A}$$

$$I_{av} = \frac{V_{av}}{R} = \frac{V_m}{\pi R} = \frac{150}{3.14 \times 50} = 0.95 \text{ A}$$

Form factor, $$K_f = \frac{I_{rms}}{I_{av}} = \frac{1.5}{0.95} = 1.57$$

Example 3.3 Calculate the RMS value, average value and form factor of a half-rectified square voltage shown in Fig. 3.10.

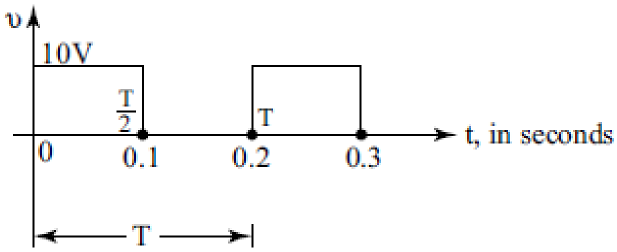


Figure 3.10 Circuit diagram of example 3.3

Solution:

For half-rectified wave, $$V_{av} = \frac{1}{T}\int_0^{T/2} v\, dt$$

Here, T = 0.2, n =10 V

Therefore

$$V_{av} = \frac{1}{0.2}\int_0^{0.1} 10\,dt = \frac{1}{0.2}[10t]_0^{0.1} = \frac{1}{0.2} \times 10 \times 0.1 = 5\text{ V}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^{T/2} v^2\,dt} = \sqrt{\frac{1}{0.2}\int_0^{0.1} 10^2\,dt}$$

$$= \sqrt{\frac{1}{0.2}\left[[10]^2\,t\right]_0^{0.1}} = \sqrt{\frac{1}{0.2}[100 \times 0.1]}$$

$$= \sqrt{\frac{10}{0.2}} = \sqrt{50} = 7.09\text{ V}$$

Form factor, $$K_f = \frac{V_{rms}}{V_{av}} = \frac{7.09}{5} = 1.4$$

Example 3.4 Calculate the RMS value and average value of the elevated saw-tooth-type current wave shown in Fig. 3.11

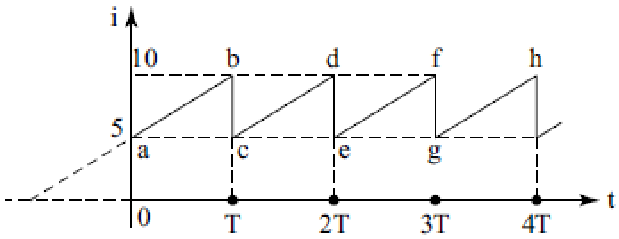


Figure 3.11 Saw-tooth wave of example 3.4

Solution:

It can be seen from the wave shape that 0abcT makes one cycle. The same pattern is being repeated for each time period of T. The equation for the line ab is of the form y = mx + c. Here the slope m is bc/ac, i.e., equal to 5/T. The value of c is 5 and y is represented by i and x by t.

Therefore, the equation of the line ab is

$$i = \frac{5t}{T} + 5$$

$$I_{av} = \frac{1}{T}\int_0^T i\,dt = \frac{1}{T}\int_0^T \left[\frac{5t}{T} + 5\right] dt$$

$$= \frac{1}{T}\left[\frac{5t^2}{2T} + 5t\right]_0^T$$

$$= \frac{1}{T}\left[\frac{5T^2}{2T} + 5T\right] = \frac{1}{T} \times 7.5T = 7.5\text{ A}$$

(In this case by actual observation of the wave shape as shown in Fig. 3.10, the average value can also be determined)

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T \left(\frac{5t}{T} + 5\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T \left(\frac{25t^2}{T^2} + 5^2 + 2 \times \frac{5t}{T} \times 5\right) dt}$$

$$= \sqrt{\frac{1}{T}\left[\frac{25t^3}{3T^2} + 25T + \frac{50t^2}{2T}\right]_0^T}$$

$$= \sqrt{\frac{1}{T}\left[\frac{25T}{3} + 25T + 25T\right]}$$

$$= \sqrt{\frac{175}{3}} = 7.68\text{ A}$$

Example 3.5 Find the average value, RMS value and form factor of the saw-tooth current wave shown in Fig. 3.12

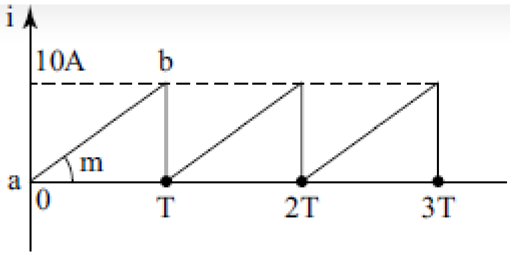


Figure 3.12 Diagram for example 3.5

Solution:

The equation of the line ab is of the form

y = mx

Here, y = i, m = 10/T, and x = t

Therefore, we can write

$$i = \frac{10t}{T}$$

RMS value of i,

$$I = \sqrt{\frac{1}{T}\int_0^T \left(\frac{10t}{T}\right)^2 dt}$$

$$= \sqrt{\frac{1}{T}\int_0^T \frac{100t^2}{T^2} dt} = \sqrt{\frac{1}{T}\left[\frac{100}{T^2}\,\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{100/3} = 5.78\,\text{A}$$

Average value of a right-angle triangle is half of its height, i.e., equal to

I_{av} = 10/2 = 5A

Form factor

RMS value 5.78

The students are to note that the form factor of a saw-tooth wave has been calculated as 1.15 whereas for a sine wave the value was 1.11. Since a saw-tooth wave is stiffer than a sine wave, its form factor is higher than that of a sine wave.

3.2 SINGLE-PHASE AC CIRCUITS

A resistance, an inductance, and a capacitance are the basic elements of an ac circuit. These elements are connected in series and parallel combinations to form an actual circuit. Circuits may include any two or three elements. For example, we may have one resistance and one inductance connected in series across an ac supply source. We may have one resistance connected in series with one inductance and one capacitance in parallel. Accordingly circuits are named as L-R circuits, L-R-C circuits, etc. We will take up few series circuits, few parallel circuits, and some series–parallel circuits and calculate the main current, branch currents, power, power factor, etc. Before this, we will discuss the behaviour of R, L, and C in ac circuits

3.2.1 Behaviour of R, L, and C in AC Circuits

In this section we will study the relationship of applied voltage and current in an ac circuit involving only a resistance, an inductance, and a capacitance. When a resistance is connected across an ac supply we call it a purely resistive circuit. Similarly an inductance coil connected across an ac supply is called a purely inductive circuit and a capacitance connected across an ac supply is called a purely capacitive circuit. We shall study the phase relationship between the applied voltage and current flowing in each case under steady-state condition.

AC applied across a pure resistor

When we say a pure resistance we assume that the resistance wire does not have any inductance or capacitance. Fig. 3.13 shows a pure resistance connected across an ac supply. The voltage and current wave forms as well as the phasor diagram showing the positions of voltage and current have been shown. The instantaneous value of

Voltage, v of the source is $v = V_m \text{ Sin } \omega t$

Where, V_m is the maximum value of the voltage in Volts; $\omega = 2\pi f$ rad/sec; and f is the frequency of supply voltage in cycles per second.

The current flowing though the circuit will be

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

At $$\omega t = \frac{\pi}{2},\ i = \frac{V_m}{R}$$

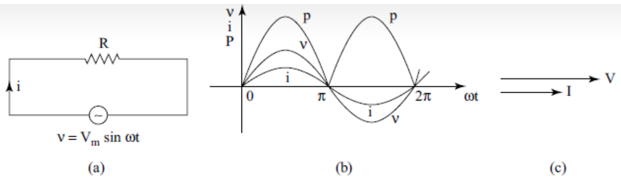


Figure 3.13 (a) Resistive circuit with a sinusoidal voltage source; (b) voltage and current wave shapes; (c) phasor diagram

The maximum value of i is I_m

Therefore,

$$I_m = \frac{V_m}{R}$$

Thus, I can be written as

$$i = \frac{V_m}{R}\sin\omega t$$

or,

$$i = I_m \sin\omega t$$

The steady-state response of the circuit is also sinusoidal of the same frequency of the voltage applied. As shown in Fig. 3.13 (b), both voltage and current wave shapes are sinusoidal and their frequency is also the same. Since current is proportional to the voltage all the time, the two wave forms are in phase with each other.

The phasor diagram is drawn with the RMS values of the time-varying quantities. As shown in Fig. 3.13 (c), V and I are the RMS values of voltage and current. They have been shown in phase. For the sake of clarity only, the two phasors have been shown with a gap between them.

In a purely resistive circuit, current and voltage are in phase. Power is the product of voltage and current. The product, P = VI has been calculated for all instants of time and has been shown in Fig. 3.13 (b). Power in a resistive circuit is taken as the average power which is

Power in a resistive circuit,

$$P = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\theta \, I_m \sin\theta \, d\theta \qquad [\theta = \omega t]$$

$$= \frac{1}{2\pi} V_m I_m \int_0^{2\pi} \sin^2\theta \, d\theta$$

Power factor is the cosine of the phase angle between voltage and current.

In a resistive circuit the phase difference between voltage and current is zero, i.e., they are in phase. So the phase angle $\theta = 0$.

Power factor, $\quad P\,f = \cos\theta = \cos 0° = 1$

AC applied across a pure inductor

A pure inductor means that the resistance of the inductor coil is assumed to be zero. The coil has only inductance, L. Such an inductor is connected across a sinusoidally varying voltage, $v = V_m \sin wt$ as has been shown in Fig. 3.14 (a).

As a result of application of voltage, v an alternating current, i will flow through the circuit. This alternating current will produce an alternating magnetic field around the inductor. This alternating or changing field flux will produce an EMF in the coil:

$$e = L\frac{di}{dt}$$

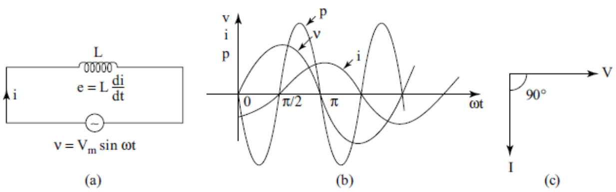


Figure 3.14 (a) Inductive circuit with a sinusoidal voltage input; (b) wave shapes of voltage, current, and power; (c) phasor diagram

This EMF will oppose the voltage applied (remember Lenz's law). Therefore, we can write

$$v = e = L\frac{di}{dt}$$

or, $$L\,di = v\,dt = V_m \sin \omega t\, dt$$

or, $$di = \frac{V_m}{L} \sin \omega t\, dt$$

Integrating

$$i = \frac{V_m}{L}\int \sin \omega t\, dt$$

$$= \frac{V_m}{\omega L}(-\cos \omega t)$$

or, $$i = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

or, $$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

where $$I_m = \frac{V_m}{\omega L}$$

Thus, we observe that in a purely inductive circuit

$v = V_m \sin \omega t$

and $$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$= I_m \sin (\omega t - 90°)$

The current, i is also sinusoidal but lagging behind, v by 90°. The voltage and current wave shapes have been shown in Fig. 3.14 (b). The instantaneous power, p is the product of v and i. The wave shape of instantaneous power has also been shown in the figure. The phasor diagram of RMS values of v and i has been shown in Fig. 3.14 (c). In a purely inductive circuit current, I lags the voltage, V by $\frac{\pi}{2}$ degrees, i.e., 90°.

Power factor, $\cos \phi = \cos 90° = 0$

Average power $$P = \frac{1}{2\pi}\int_0^{2\pi} V_m \operatorname{Sin} \omega t\; I_m \operatorname{Sin}(\omega t - \pi/2)\, d\omega t$$

$$= \frac{V_m I_m}{2\pi}\int_0^{2\pi} \operatorname{Sin} \omega t \operatorname{Sin}(\omega t - \pi/2)\, d\omega t$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} 2 \operatorname{Sin} \omega t \operatorname{Sin}(\omega t - \pi/2)\, d\omega t$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} (0 - \operatorname{Sin} 2\omega t)\, d\omega t$$

$$= 0$$

Average power in a purely inductive circuit, $P = 0$ (3.10)

Hence, the average power absorbed by a pure inductor is zero.

The opposition offered by an inductor to the flow of current is X_L which is equal to ωL This is called the inductive reactance and is expressed in Ohms. Inductance, L is expressed in Henry.

As mentioned earlier, the values of alternating quantities are expressed in terms of their effective or RMS values rather than their maximum values.

Therefore,

$$I_m = \frac{V_m}{\omega L} \text{ can be written as}$$

$$\frac{I_m}{\sqrt{2}} = \frac{V_m/\sqrt{2}}{\omega L}$$

or, $$I = \frac{V}{X_L}$$

or, $$V = I X_L$$

If V is taken as the reference axis, we can represent V as a phasor and represent as $V\angle 0°$.

Since current, I is lagging voltage, V by 90°, we represent the current as $I\angle -90° \text{ or } -jI$ for a purely inductive circuit. Again, If I taken as the reference axis, then I and V can be represented as $I\angle 0°$, and $V\angle +90° \text{ or } +jV$, respectively, as shown in Fig. 3.15.

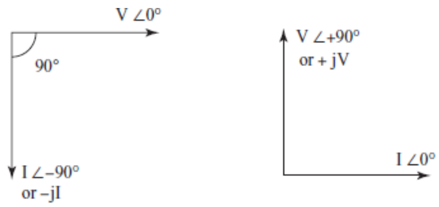


Figure 3.15 Phasor diagram of V and I in a purely inductive circuit

Note that j is an operator which indicates rotation of a phasor by 90° in the anti clockwise direction from the reference axis.

Now let us examine why the power absorbed by a pure inductive circuit is zero. We refer back to Fig. 3.14 (b) where it is seen that for one half cycle power is negative and for the next half cycle power is positive. The average value for a complete cycle, the power consumed is zero. Positive power indicates that power is drawn by the circuit from the supply source. When current rises in the circuit, energy is required to establish a magnetic field around the inductor coil. This energy is supplied by the source and is stored in the magnetic field. As the current starts reducing, the magnetic field colfrom the source to the inductor and back to the source is called reactive power which will also be discussed in a separate section.

AC applied across a pure capacitor

A sinusoidal voltage source has been shown connected across a pure capacitor in Fig. 3.16 (a). When current starts flowing, the capacitor starts getting charged. The charge, q of the capacitor in terms of capacitance of the capacitor, C and supply voltage, v is expressed as

q = Cv

Current, i is the rate of flow of charge. Therefore,

$$i = \frac{dq}{dt}$$

$$= C\frac{dv}{dt}$$

$$= C\frac{d}{dt} V_m \sin \omega t$$

$$= \omega C V_m \cos \omega t$$

or, $$i = \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

or, $$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

where $$I_m = \omega C V_m = \frac{V_m}{1/\omega C} = \frac{V_m}{X_c}$$

Hence in a pure capacitive circuit, v = V_m sin ωt and current

$i = I_m \sin(\omega t + \frac{\pi}{2})$. Current leads the voltage by 90°.

$X_C = \frac{1}{\omega C}$ is called the capacitive reactance of the capacitor.

To express in terms of RMS values,

We take $\frac{I_m}{\sqrt{2}} = I$ and $\frac{V_m}{\sqrt{2}} = V$

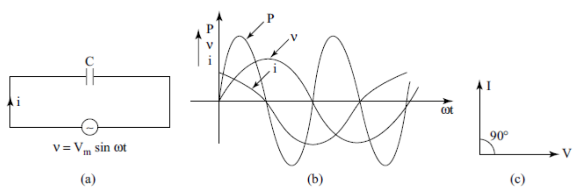


Figure 3.16 (a) Pure capacitive circuit; (b) wave shapes of voltage, current, and power; (c) phasor diagram

$$I_m = \frac{V_m}{X_C} \text{ can be written as}$$

$$I = \frac{V}{X_C}$$

X_C is the opposition offered by the capacitor to the flow of current and is called capacitive reactance.

Like an inductor, in a capacitor also the average power absorbed for a complete cycle is zero. When voltage is applied, the capacitor starts getting charged, energy gets stored in the capacitor in the form of electro-static field. When the applied voltage starts falling from its maximum value, the energy starts getting returned to the supply. This way, the power is absorbed from and then returned to the supply source The net power absorbed by a pure capacitor is zero. Since current leads the voltage by 90°, the power factor of the circuit is

P.f = $\cos\phi$ = cos 90° = 0

The average or net power in a pure capacitor circuit can be calculated as

$$P = \frac{1}{2\pi}\int_0^{2\pi} p\ d\omega t = \frac{1}{2\pi}\int_0^{2\pi} vi\ d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t\ I_m \sin(\omega t + \pi/2)\, d\omega t$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} 2\sin\omega t \sin(\omega t + \pi/2)\, d\omega t$$

or, $$P = \frac{V_m I_m}{4\pi}\int_0^{2\pi} \sin 2\omega t\, d\omega t$$

$$= 0$$

Average power in a purely capacitive circuit, $P = 0$ (3.11)

Hence, it is proved that the average power obsorbed by a pure capacitor is zero.

Example 3.6 An inductor of 0.5 H is connected across a 230 V, 50 Hz supply. Write the equations for instantaneous values of voltage and current.

Solution:

$$V = 230\text{ V}, V_m = \sqrt{2}\,V = 1.414 \times 230 = 324\text{ V}$$

$$X_L = \omega L = 2\pi fL = 2 \times 3.14 \times 50 \times 0.5\ \Omega = 157\ \Omega$$

$$I = \frac{V}{\ } = \frac{230}{\ } = \frac{230}{\ } = 1.46\text{ A}$$

The equations are

$$V = V_m \sin \omega t = 324 \sin \omega t = 324 \sin 2\pi ft = 324 \sin 314t$$

and
$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) = 2.06 \sin\left(314t - \frac{\pi}{2}\right)$$

Example 3.7 A 230 V, 50 Hz sinusoidal supply is connected across a (i) resistance of 25 Ω; (ii) inductance of 0.5 H; (iii) capacitance of 100 μF. Write the expressions for instantaneous current in each case.

Solution:

given
$$V = 230\text{V}$$
$$V_m = \sqrt{2}\,V = 1.414 \times 230 = 324.3\,\text{V}$$
$$\omega = 2\pi f = 2 \times 3.14 \times 50 = 314\ \text{rad/sec}$$

Voltage equation is

$$v = V_m \sin \omega t$$

or,
$$v = 324.3 \sin 314t$$

Inductive reactance,
$$X_L = \omega L = 314 \times 0.5 = 157\ \Omega$$

Capacitive reactance,
$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 100 \times 10^{-6}}$$

$$= \frac{10^{-6}}{314 \times 100} = 32.2\ \Omega$$

When the voltage is applied across a 25 Ω resistor, the current will be

$$i = \frac{V_m}{R} \sin \omega t = \frac{324.3}{25} \sin 314t$$

or,
$$i = 12.97 \sin 314t\ \text{A}$$

Current through the inductor is

$$i = \frac{V_m}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= \frac{324.3}{157}\sin\left(314t - \frac{\pi}{2}\right)$$

or,

$$i = 2.06 \sin (314t - 90°) \text{ A}$$

Current through the capacitor is

$$i = \frac{V_m}{X_c}\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= \frac{324.3}{32.2}\sin (314t + 90°)$$

or,

$$i = 10.07 \sin (314t + 90°) \text{ A}$$

Exapmle 3.8 An alternating voltage of RMS value 100 V, 50 Hz is applied separately across a resistance of 10 Ω, an inductor of 100 mH, and a capacitor of 100 μF. Calculate the current flow in each case. Also draw and explain the phasor diagrams.

Solution:

$$R = 10\ \Omega$$

$$X_L = \omega L = 2\pi f L = 2 \times 3.14 \times 50 \times 100 \times 10^{-3}\ \Omega$$

$$= 31.4\ \Omega$$

$$\mathbf{X}_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 50 \times 100 \times 10^{-6}}$$

$$= \frac{10^6}{314 \times 100} = 31.8\ \Omega$$

Current through R,

$$= \frac{100}{10} = 10 \text{ A}$$

Current through L,

$$= \frac{100}{X_L} = \frac{100}{31.4} = 3.18 \text{ A}$$

Current through C,

$$= \frac{100}{X_C} = \frac{100}{31.8} = 3.1 \text{ A}$$

We know that in a resistive circuit current is in phase with the applied voltage; in a purely inductive circuit current lags the voltage by 90°; and in a purely capacitive circuit current leads the voltage by 90°. The phasor diagrams have been shown in Fig. 3.17.

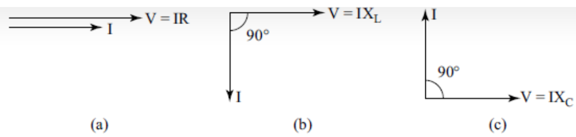


Figure 3.17 Phasor diagrams (a) resistive circuit; (b) purely inductive circuit; (c) purely capacitive circuit

3.2.2 L–R Series Circuit

Let us consider a resistance element and an inductor connected in series as shown in Fig. 3.18. A voltage, V of frequency, f is applied across the whole circuit. The voltage drop across the resistance is V_R and across the inductor is V_L. Current flowing through the circuit is I.

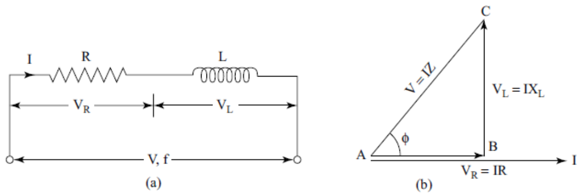


Figure 3.18 (a) R–L series circuit; (b) phasor diagram

$V_R = IR$, $V_L = IX_L$ where $X_L = \omega L = 2\pi fL$

We have to add V_R and V_L to get V. But these are to be added vectorially as they are all not in phase, i.e., these vectors are not along the same direction. To draw the current and voltage phasor we take the current I as the reference phasor as shown in Fig. 3.18 (b), since current I is common to V_R and V_L, i.e., since the same current is flowing through both resistance and inductance. We have, therefore, chosen I as the reference phasor. Voltage drop across the resistance and the current flowing through it are in phase. This is because, as we have seen earlier that in a resistive circuit, voltage and current are in phase. The current flowing through an inductor lags the voltage across it by 90°. That is to say, voltage drop across L, i.e., V_L will lead the current by 90°. Again $V_L = IX_L$ and $X_L = \omega L$. The vector sum of V_R and V_L is equal to V. The angle between V and I is called the power factor angle ϕ . Power factor is cosϕ . Considering the triangle ABC we can express

$$V^2 = V_R^2 + V_L^2$$

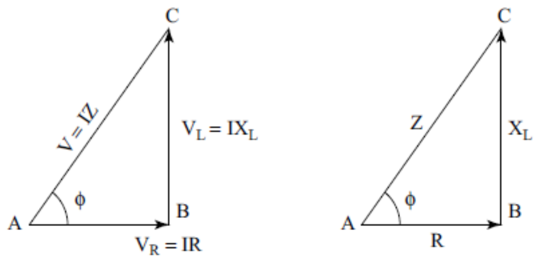
or, $$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

or, $$I = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{Z} \text{ or, } V = IZ$$

where $$Z = \sqrt{R^2 + X_L^2}$$

Z is called the impedance of the total circuit. Triangle ABC in Fig. 3.18 (b) is also called the impendance triangle which is redrawn as in Fig. 3.19. From the impedance triangle



(a) (b)

Figure 3.19 Impedance triangle for R–L circuit

$$Z = \sqrt{R^2 + X_L^2}$$

or, $$Z = R + jX_L$$

Where j indicates rotation by 90° in anti-clockwise direction.

$$\cos\phi = \frac{R}{Z}$$

Or, $$Z\cos\phi = R$$

and $$Z\sin\phi = X_L$$

Fig. 3.19 (a) is the same as Fig. 3.19 (b). The current, I has been kept aside which is common to all the sides. Impedance Z can be represented as the vector sum of R and X_L. since IX_L is leading I by 90° and R is in phase with I, we can write

$$Z = R + jX_L, \text{ and } \cos\phi = \frac{R}{Z}, \tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R}$$

$$\phi = \tan^{-1}\frac{X_L}{R}, \cos\phi = \frac{R}{Z}$$

Apparent power (S)

It is defined as the product of the RMS value of voltage (V) and current (I). It is denoted by S

$$\text{Apparent power,} \quad S = VI = \text{Voltage} \times \text{Current}$$

Unit of apparent power is VA or kVA. (3.12)

Real or true power or active power (P or W)

It is the power which is actually dissipated in the circuit resistance. (watt-full power)

$$\text{Active power, } P = \text{Apparent power} \times \text{power factor} \quad (3.13)$$

$$\text{Or,} \quad P = VI\cos\phi \text{ Watts or kW}$$

Reactive power (Q)

It is the power developed in the inductive reactance of the circuit. (watt-less power)

$$Q = I^2 X_L = I^2 Z \sin\phi = I\,(ZI)\sin\phi$$

$$\text{or,} \quad Q = VI\sin\phi \text{ VAR} \quad (3.14)$$

These three powers are shown in the power triangle of Fig. 3.21 (b) from where it can be seen that

$$S^2 = P^2 + Q^2$$
$$S = \sqrt{P^2 + Q^2}$$
$$\text{kVA} = \sqrt{(\text{kW})^2 + (\text{kVAR})^2} \quad (3.15)$$

3.2.4 Power in an AC Circuit

Let us now develop a general expression for power in an ac circuit by considering the instantaneous values of voltage and current. A sinusoidal voltage n is expressed as

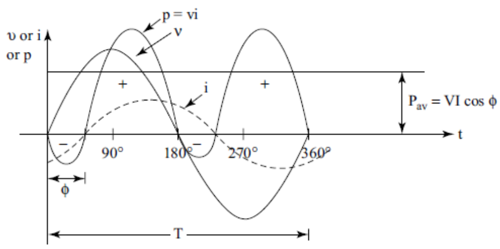


Figure 3.20 Wave forms of voltage, current, and power in an R–L series circuit

In a circuit when current is lagging the voltage by an angle ϕ , current i is expressed as

$$i = I_m \sin(\omega t - \phi)$$

Sinusoidal waveforms of voltage and current are shown in Fig. 3.20. It is seen that the current wave is lagging the voltage wave by an angle, ϕ which is the power factor angle.

Fig. 3.20 clearly shows that current in an R–L circuit lags voltage by an angle ϕ , which is called the power factor angle.

The expression for the voltage and current in series R–L circuit is,

$$v = V_m \sin \omega t$$
$$i = I_m \sin(\omega t - \phi), \text{ as I lags V}$$

The power is product of instantaneous values of voltage and current,

$$\begin{aligned} p &= v \times i \\ &= V_m \sin \omega t \times I_m \sin(\omega t - \phi) \\ &= \frac{1}{2} V_m\ I_m [2 \sin \omega t \sin(\omega t - \phi)] \\ &= \frac{1}{2} V_m\ I_m [\cos \phi - \cos(2\omega t - \phi)] \\ &= \frac{1}{2} V_m\ I_m \cos \phi - \frac{1}{2} V_m\ I_m \cos(2\omega t - \phi) \end{aligned}$$

The average power over a complete cycle is calculated as

$$\begin{aligned} P_{av} &= \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{2} V_m\ I_m [\cos \phi - \cos(2\omega t - \phi)] d\omega t \\ &= \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{2} V_m\ I_m \cos \phi\ d\omega t - \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{2} V_m\ I_m \cos(2\omega t - \phi) d\omega t \end{aligned}$$

Now, the second term is a cosine term whose average value over a complete cycle is zero.

Hence, the average power consumed is

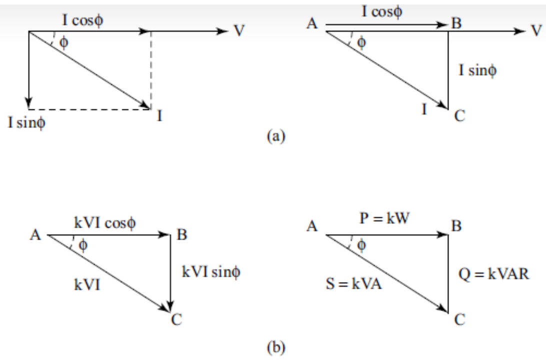


Figure 3.21 power triangle diagram

$$P_{av} = V_{rms} \times I_{rms} \cos\phi = VI\cos\phi \text{ W}$$

Power factor

It may be defined as the cosine of the phase angle between the voltage and current; cos ϕ is known as power factor. Power factor can also be expressed as the ratio, R/Z = resistance/impedance = cos ϕ

In Fig. 3.21, the power triangle diagram has been developed from the simple voltage–current relationship in an R–L series circuit. First we have shown I laggingV by the power factor angle ϕ. The in-phase component of I is I cos ϕ and quadratuse component is I sin ϕ as have been shown in Fig. 3.21 (a).

Multiplying all the sides of the triangle ABC by KV (kilo-volt), we can draw the power triangle as in Fig. 3.21 (b)

$$\begin{aligned} \text{kVA} \cos\phi &= \text{kW} \\ \text{kVA} \sin\phi &= \text{kVAR} \end{aligned}$$

In the power triangle diagram, if θ is taken as zero, i.e., if the circuit is resistive, reactive power, Q becomes zero. If the circuit is having pure inductance or capacitance, ϕ = 90, active power, P becomes zero. Reactive power will be present whenever there is inductance or capacitance in the circuit. Inductors and capacitors are energy-storing and energy-releasing devices in the form of magnetic and electric fields, respectively, and are of importance in the field of electrical engineering.

3.2.5 R—C Series Circuit

Consider a circuit consisting of a pure resistance R and connected in series with a pure capacitor C across an ac supply of frequency f as

1. Drop across pure resistance $V_R = I \times R$
2. Drop across pure capacitance $V_C = I \times X_C$

Where $X_C = \frac{1}{2\pi fC}$ and I, V_R, V_C are the RMS values

The phasor diagram for such a circuit can be drawn by taking the current as a reference phasor represented by OA as shown in Fig. 3.23. The voltage drop V_R across the resistance is in phase with current and is represented by OB. The voltage drop across the capacitance V_C lags the current by 90° and is represented by BC. The phasor OC is the phasor sum of two voltages V_R and V_C. Hence, the OC represents the applied voltages. Thus, in a capacitive circuit, current leads the voltages by an angle ϕ . The same phasor diagram can be drawn by taking voltage, V as the reference vector as shown in Fig. 3.23 (b).

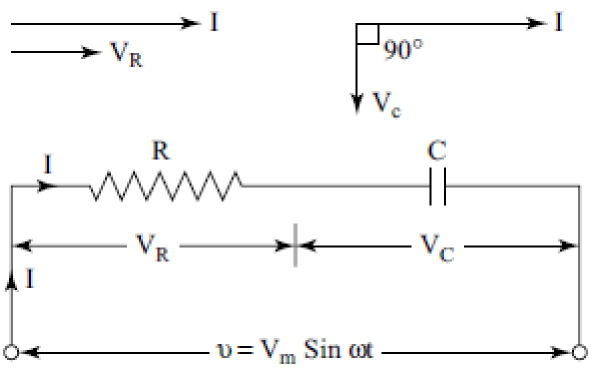


Figure 3.22 R–C series circuit

In Fig. 3.23 (b), we have drawn V as the reference vector. Then current, I has been shown leading V by an angle ϕ . The voltage drop across the resistance, V_R = IR has been drawn in phase with I. The voltage drop across the capacitance $V_C = IX_C$ has been drawn lagging I by 90° (V_C lagging I is the same as I leading V_C). The length of V_R and V_C are such that they make an angle of 90°.

In an R–C series circuit, I leads V by an angle ϕ or supply voltage V lags current I by an angle θ as shown in the phasor diagram in Fig. 3.23 (b).

$$\tan\phi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R}$$

$$\phi = \tan^{-1}\frac{X_C}{R}$$

Applied voltage,

$$V = \sqrt{V_R^2 + V_C^2}$$

$$= \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2}$$

$$V = IZ$$

where $Z = \sqrt{R^2 + X_C^2}$ = impedance of the circuit

Voltage and current wave shapes of this circuit are shown in Fig. 3.24, which shows that the current in a capacitive circuit leads the voltage by an angle ϕ , which is called the power factor angle.

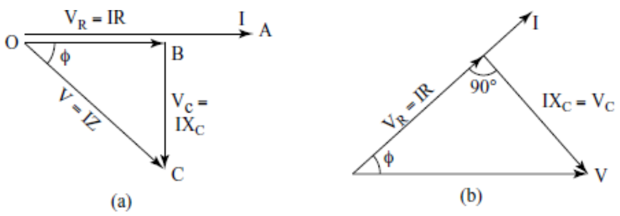


Figure 3.23 Phasor diagrams of R–C series circuit

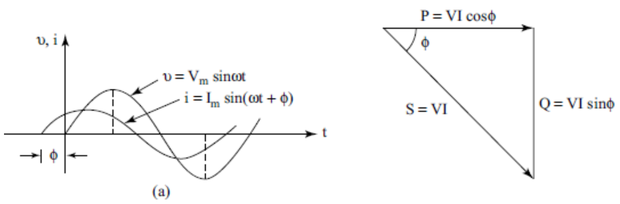


Figure 3.24 Wave forms of voltage and current and their phase relationship in an R–C series circuit

Power and power triangle

The expression for voltage and current is

$$v = V_m \operatorname{Sin} \omega t$$

$$i = I_m \operatorname{Sin}(\omega t + \phi) \text{ as I leads V}$$

Power is the product of voltage and current. The instantaneous power is

$$
\begin{aligned}
P &= v \times i \\
&= V_m \operatorname{Sin} \omega t \times I_m \operatorname{Sin}(\omega t + \phi) \\
&= \frac{1}{2} V_m\ I_m\ [2 \sin \omega t \sin(\omega t + \phi)] \\
&= \frac{1}{2} V_m\ I_m [\cos(-\phi) - \cos(2\omega t + \phi)] \\
&= \frac{1}{2} V_m\ I_m\ \cos\phi - \frac{V_m I_m}{2} \cos(2\omega t + \phi)
\end{aligned}
$$

$$\text{as} \cos(-\phi) = \cos\phi$$

The second term is a cosine term whose average value over a complete cycle is zero. Hence, average power consumed by the circuit is

$$P_{av} = \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$P_{av} = V_{rms}\ I_{rms}\ \cos\phi = VI \cos\phi\ W$$

The power triangle has been shown in Fig. 3.24 (b).

Thus, various powers are

Apparent power S = VI Volt Amperes or VA

Active power P = VI cos ϕ W

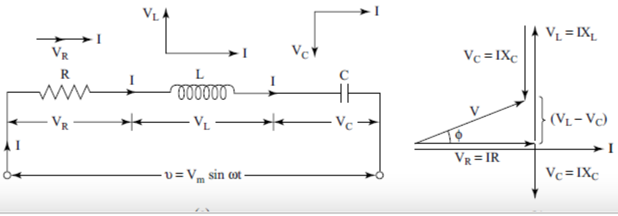
Reactive power Q = VI sin ϕ VAR

where cos ϕ =Power factor of the circuit.

Note: Power factor, cosf is lagging for an inductive circuit and is leading for a capacitive circuit.

3.2.6 R–L–C Series Circuit

Consider a circuit consisting of resistance R, inductance L, and capacitance C connected in series with each other across an ac supply. The circuit has been shown in Fig. 3.25.



The circuit draws a current I. Due to flow of current I, there are voltage drops across R, L, and C which are given by

1. drop across resistance R is $V_R = IR$
2. drop across inductance L is $V_L = IX_L$
3. drop across capacitance C is $V_C = IX_C$

where I, V_R, V_L, and V_C are the RMS values.

The phasor diagram depends on the magnitude of V_L and V_C, which obviously depends upon X_L and X_C. Let us consider the different cases.

1. When $\mathbf{X_L} > \mathbf{X_C}$, i.e., when inductive reactance is more than the capacitive reactance.

   The circuit will effectively be inductive in nature. When $X_L > X_C$, obviously, IX_L, i.e., V_L is greater than IX_C, i.e., V_C. So the resultant of V_L and V_C will be $V_L - V_C$ so that V is the phasor sum of V_R and $(V_L - V_C)$. The phasor sum of V_R and $(V_L - V_C)$ gives the resultant supply voltage V. This is shown in Fig. 3.25 (b) and again redrawn as in Fig. 3.26.

Applied voltage is

$$OB = \sqrt{OA^2 + AB^2}$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

or,

$$V = IZ$$

where

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\tan\phi = \frac{(X_L - X_C)}{R}, \quad \phi = \tan^{-1}\frac{(X_L - X_C)}{R}$$

Note when $X_L > X_C$, the R–L–C series circuit will effectively be an inductive circuit where current I will lag the voltage V as has been shown in the phasor diagram of Fig. 3.26.

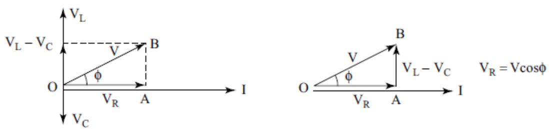


Figure 3.26 Phasor diagram of current and voltage drops in an R–L–C circuit where $X_L > X_C$

will lead (V_C - V_L). The phasor sum of V_R and (V_C - V_L) gives the resultant supply voltage V. This is shown in Fig. 3.27.

Applied voltage represented by OB $= \sqrt{OA^2 + AB^2}$

$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$= \sqrt{(IR)^2 + (IX_C - IX_L)^2}$$

$$= I\sqrt{R^2 + (X_C - X_L)^2}$$

or, $V = IZ$

where $Z = \sqrt{R^2 + (X_C - X_L)^2}$

Phase angle, $\phi = \tan^{-1}\left(\dfrac{X_C - X_L}{R}\right)$

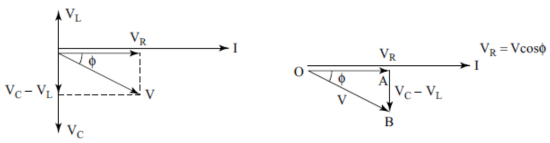


Figure 3.27 Phasor diagram of an R–L–C series circuit when $X_L < X_C$

3. **When XL = XC**

When $X_L = X_C$, obviously, $V_L = V_C$. So, V_L and V_C will cancel each other and their resultant will be zero. So, V_R = V. In such a case the overall circuit will behave like a purely resistive circuit. The phasor diagram is shown in Fig. 3.28. The impedance of the circuit will be minimum, i.e., equal to R.

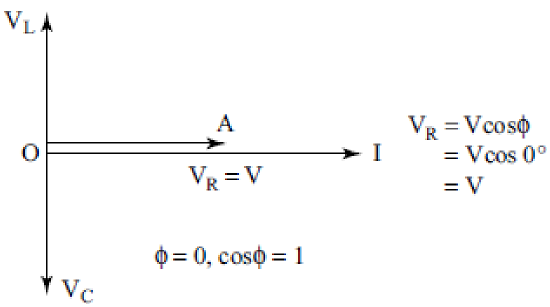


Figure 3.28 Phasor diagram of an R–L–C series circuit when $X_L = X_C$

Power and power triangle

The average power consumed by the circuit is

P_{av} = Average power consumed by R + Average power consumed by

But $V_R = V \cos \phi$ in all the cases.

Therefore, $P = VI \cos \phi$ W.

Thus, for any condition, that is when $X_L > X_C$ or $X_L < X_C$ or $X_L = X_C$, power can be expressed as P = Voltage × Component of I in phase with $V = VI \cos \phi$

Note that when $X_L = X_C$, the component of I in phase with V is I only because $I \cos \phi = I$ (as $\cos \phi = 1$).

3.2.7 AC Parallel Circuits

Parallel circuits are formed by two or more series circuits connected to a common source of supply. The parallel brances may include a single element or a combination of elements in series.

Methods for solving ac parallel circuits:

The following three methods are available for solving ac parallel circuits:

1. phasor or vector method
2. admittance method
3. using vector algebra (symbolic method or j-operator method)

These methods are explained with examples as follows.

1. Phasor or vector method

A parallel circuit consisting of three branches has been shown in Fig. 3.29. Branch 1 consists of R_1, L_1, and C_1 in series. Branch 2 is resistive and capacitive and branch 3 is resistive and inductive. Let the current be I_1, I_2, and I_3 in the branch 1, 2, and 3, respectively. The total current drawn by the circuit is the phasor sum of I_1, I_2, and I_3.

Branch 1

Impedance of branch 1, $= \sqrt{(R_1)^2 + (X_{L1} - X_{C1})^2} = Z_1$

Current $I_1 = V/Z_1$

Phase difference of this current with respect to the applied voltage is given by $\phi_1 = \tan^{-1} \dfrac{(X_{L1} - X_{C1})}{R_1}$

This current will lag the applied voltage by an angle ϕ_1, if $X_{L1} > X_{C1}$.

In case $X_{C1} > X_{L1}$, then I_1, will lead V.

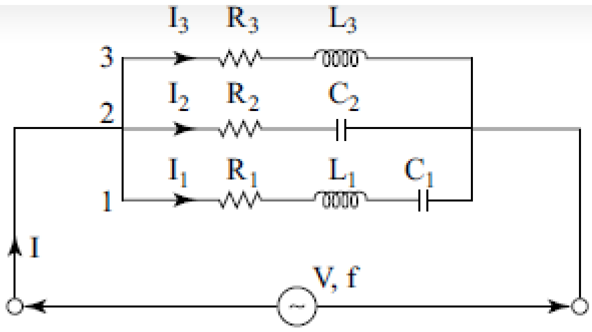


Figure 3.29 AC parallel circuit

Branch 2

Capacitive branch (I_2 leads V)

Impedance of branch 2, $Z_2 = \sqrt{(R_2)^2 + (X_{C2})^2}$

Current $I_2 = V/Z_2$

The branch current I_2 leads the applied voltage V, by an angle ϕ_2, given by

$$\phi_2 = \tan^{-1}\frac{(X_{C2})}{R_2}$$

Branch 3

Inductive branch 3, $Z_3 = \sqrt{(R_3)^2 + (X_{L3})^2}$

Current $I_3 = V/Z_3$

This current will lag the applied voltage by an angle ϕ_3,

$$\phi_3 = \tan^{-1}\frac{X_{L3}}{R_3}$$

Choose a current scale and draw to the scale the current vectors with the voltage as the reference axis. Add vectorially any two currents, say I_1 and I_2. The vector sum of I_1 and I_2 is OE as shown in

An alternate method is to show the three currents with the voltage as the horizontal reference axis as shown in Fig. 3.30 (b). Calculate the sum of the horizonal components and vertical components of the currents and then determine the resultant.

The branch currents with their phase angles with respect to V has been shown (not to the scale) separately in Fig. 3.30 (b).

The resultant current I can be found out by resolving the branch currents I_1, I_2, and I_3 into their X and Y components as shown in Fig. 3.30 (b).

X component of I_1 (OL) = $I_1 \cos \phi_1$

X component of I_2 (OM) = $I_2 \cos \phi_2$

X component of I_3 (ON) = $I_3 \cos \phi_3$

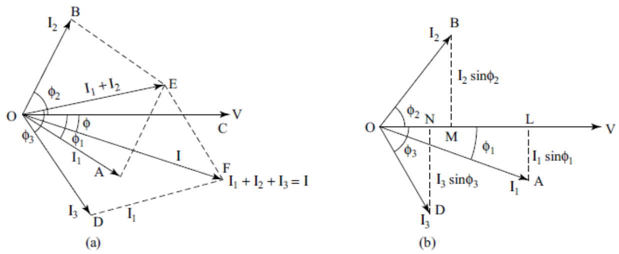


Figure 3.30 Phasor diagrams of parallel circuit shown in Fig. 3.29

Sum of X component (active component) of branch currents = $I_1 \cos \phi_1 + I_2 \cos \phi_2 + I_3 \cos \phi_3$

Y component of I_1 (AL) = $-I_1 \sin \phi_1$

Y component of I_2 (BM) = $+I_2 \sin \phi_2$

Y component of I_3 (ON) = $-I_3 \sin \phi_3$

Sum of Y component (reactive component) of branch currents = $-I_1 \sin \phi_1 + I_2 \sin \phi_2 - I_3 \sin \phi_3$

Active component of resultant current I = $I \cos \phi$

Reactive component of resultant current I = $I \sin \phi$

Active and reactive components of resultant current must be equal to the sum of active and reactive components of branch currents.

$\therefore$

$$I\cos\phi = I_1\cos\phi_1 + I_2\cos\phi_2 + I_3\cos\phi_3$$
$$I\sin\phi = -I_1\sin\phi_1 + I_2\sin\phi_2 - I_3\sin\phi_3$$

Resultant current

$$I = \sqrt{(I\cos\phi)^2 + (I\sin\phi)^2}$$
$$= \sqrt{(I_1\cos\phi_1 + I_2\cos\phi_2 + I_3\cos\phi_3)^2 + (-I_1\sin\phi_1 + I_2\sin\phi_2 - I_3\sin\phi_3)^2}$$
$$\tan\phi = \frac{I\sin\phi}{I\cos\phi}$$
$$\phi = \tan^{-1}\frac{(-I_1\sin\phi_1 + I_2\sin\phi_2 - I_3\sin\phi_3)}{(I_1\cos\phi_1 + I_2\cos\phi_2 + I_3\cos\phi_3)}$$

Resultant current lags the applied voltage if ϕ is —ve, and leads the voltage in case ϕ is +ve.

Power factor of the circuit as a whole is

$$\cos\phi = \frac{I_1\cos\phi_1 + I_2\cos\phi_2 + I_3\cos\phi_3}{I}$$
$$= \frac{\text{sum of active components of branch currents}}{\text{resultant current}}$$

2. Admittance method

Concept of Admittance Method: Admittance is defined as the reciprocal of the impedance. It is denoted by Y and is measured in unit mho or siemens.

Components of admittance:

If the circuit contains R and L, $Z = R + jX_L$;

If the circuit contains R and C, $Z = R - jX_C$.

Considering X_L and X_C as X we can write $Z = R \pm jX$.

Consider an impedance as given by

$$Z = R \pm jX$$

Positive sign is for an inductive circuit and negative sign is for a capacitive circuit.

Admittance $$Y = \frac{1}{Z} = \frac{1}{R \pm jX}$$

Rationalizing the above expression,

$$Y = \frac{R \mp jX}{(R \pm jX)(R \mp jX)}$$

$$= \frac{R \mp jX}{R^2 + X^2}$$

$$= \frac{R}{R^2 + X^2} \mp j\frac{X}{R^2 + X^2}$$

$$= \frac{R}{Z^2} \mp j\,\frac{X}{Z^2}$$

$$Y = G \mp jB$$

Where G = Conductance $= \frac{R}{Z^2}$ mho

and B = Susceptance $= \frac{X}{Z^2}$ mho

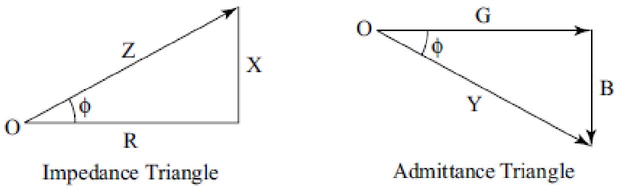


Fig 3.31 Impedance and admittance triangles

B is negative if the circuit is inductive and B is positive if the circuit is capacitive. The impedance triangle and admittance triangle for the circuit have been shown in Fig. 3.31.

Application of admittance method

Consider a parallel circuit consisting of two branches 1 and 2. Branch 1 has R_1 and L_1 series while Branch 2 has R_2 and l_2 series, respectively. The voltage applied to the circuit is V Volts as shown in Fig. 3.32.

Total conductance is found by adding the conductances of two branches. Similarly, the total susceptance is found by algebraically adding the individual susceptance of different branches.

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad I_1 = \frac{V}{Z_1} = VY_1$$

$$Y = Y_1 + Y_2 \qquad I_2 = \frac{V}{Z_2} = VY_2$$

Total susceptance $B = (-B_1) + B_2$

Total current $I = VY$

Power factor $\cos\phi = G/Y$

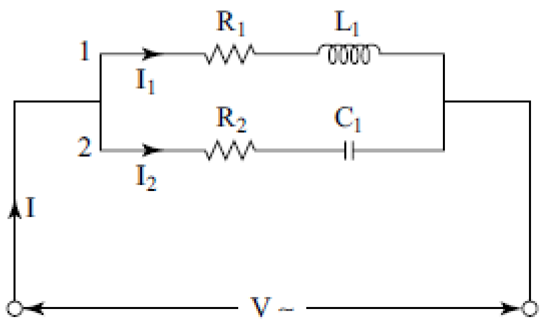


Figure 3.32 Parallel circuit

It is quite clear that this method requires calculations which are time consuming. To illustrate this method we will take one example.

Example 3.9 Two impedences Z_1 and Z_2 are connected in parallel across a 230 V, 50 Hz supply. The impelance, Z_1 consists of a resistance of 14 Ω and an inductance of 16 mH. The impedance, Z_2 consists of a resistance of 18 Ω and an inductance of 32 mH. Calculate the branch currents, line current, and total power factor. Draw the phasor diagram showing the voltage and currents.

Solution:

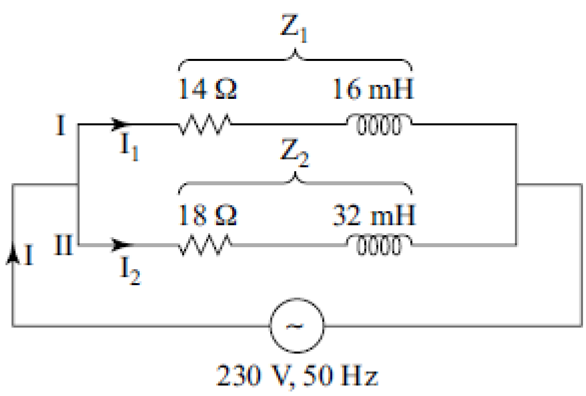


Figure 3.33

Let $R_1 = 14\ \Omega,\ X_L = \omega L_1 = 2\pi f L_1 = 2 \times 3.14 \times 50 \times 16 \times 10^{-3} = 5\ \Omega$

The phase angles of Z_1 and Z_2 are calculated from the impedance triangles as

$$\phi_1 = \tan^{-1}\frac{X_{L1}}{R_1} = \tan^{-1}\frac{5}{14} = 19.6°$$

$$\phi_2 = \tan^{-1}\frac{X_{L2}}{R_2} = \tan^{-1}\frac{10}{18} = 29°$$

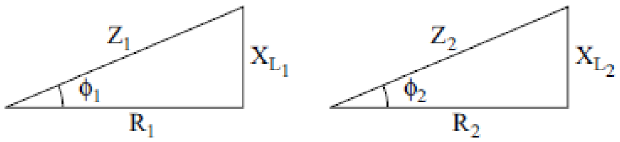


Figure 3.34 (a)

Thus, $Z_1 = 14.9\angle 19.6°$ and $Z_2 = 20.6\angle 29°$

Admittance of branch I is Y_1 and admittance of branch II is Y_2

$$Y_1 = \frac{1}{Z_1} = \frac{1}{14.9\angle 19.6°} = 0.067\angle -19.6°$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{20.6\angle 29°} = 0.0485\angle -29°$$

Taking voltage, V as the reference axis,

$$I_1 = \frac{V}{Z_1} = VY_1 = 230\angle 0 \times 0.067\angle -19.6° = 15.41\angle -19.6° \text{ A}$$

$$I_2 = \frac{\mathbf{V}}{Z_2} = VY_2 = 230\angle 0° \times 0.0485\angle -29° = 11.15\angle -29° \text{ A}$$

The phasor diagram showing V, I_1, I_2 has been shown in Fig. 3.34 (b). The sum of I_1 and I_2 gives total current, I. The cos of angle between V and I gives the value of total power factor

Figure 3.34 (b)

Taking the cosine and sine components of the branch currents and the line current

Substituting values

Current, I can also be calculated as

Example 3.10 In Fig. 3.35 is shown a parallel circuit, an inductance L and a parallel R connected across 200 V, 50 Hz ac supply. Calculate:

(a) the current drawn from the supply; (b) apparent power; (c) real power; (d) reactive power.

Figure 3.35

Solution:

Resistance of resistive branch, R = 40 Ω

Inductive reactance of inductive branch.

Figure 3.36

2. Apparent power, $S = V \times I = 200 \times 11.18 = 2.236$ kVA
3. Real power, $P = V I \cos\phi = V I_R = 200 \times 5 = 1.0$ kW
4. Reactive power, $Q = VI \sin\phi = V \times I_L = 200 \times 10 = 2.0$ kVAR

Example 3.11 The parallel circuit shown in the Fig. 3.37 is connected across a single-phase 100 V, 50 Hz ac supply. Calculate:

1. the branch currents
2. the total current
3. the supply power factor
4. the active and reactive power supplied by the source.

Figure 3.37

Solution:

It is assumed that the students are aware of the method of representation of a complex number in the forms of a + ib or a + jb. However, this has been explained in the next section.

3. Use of phasor algebra

Alternating quantities like voltage, current, etc., can be represented either in the polar form or in the rectangular form on real and imaginary axis. In Fig. 3.38 is shown a voltage, V represented in the complex plane.

The voltage, V can be represented as . This is called the polar form of representation. Voltage V can also be represented as $V = a + jb = V\cos\phi + jV\sin\phi$. This is called the rectangular form of representation using a j operator.

Significance of operator j

The operator j used in the above expression indicates a real operation. This operation when applied to a phasor, indicates the rotation of that phasor in the counter-clockwise direction through 90°

the anticlockwise direction by an angle 90° and as such its position now is along the Y-axis. If the operator j is again applied to phasor jA, it turns in the counter-clockwise direction through another 90°, thus giving a phasor j^2A which is equal and opposite to the phasor A, i.e., equal to −A. See Fig. 3.39 (a).

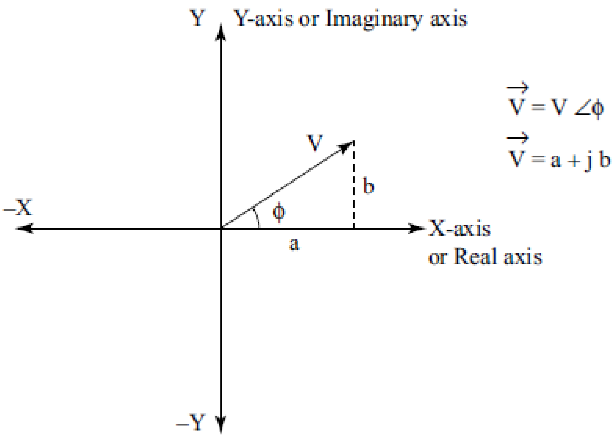


Figure 3.38 Representation of a phasor

Thus, j^2 can be seen as equal to –1. Therefore, the value of j becomes equal to $\sqrt{-1}$.

Hence,

$$j = +\sqrt{-1},\ 90^\circ \text{ CCW rotation from OX-axis}$$

$$j^2 = j \times j = (\sqrt{-1})^2 = -1,\ 180^\circ \text{ CCW rotation from OX-axis}$$

$$j^3 = (\sqrt{-1})^3 = -\sqrt{-1},\ 270^\circ \text{ CCW rotation from OX-axis}$$

and

$$j^4 = (\sqrt{-1})^4 = (-1)^2 = 1,\ 360^\circ \text{ CCW rotation from OX-axis}$$

From above, it is concluded that j is an operator rather than a real number. However, it represents a phasor along the Y-axis, whereas the real number is represented along the X-axis.

As shown in Fig. 3.39 (b), phasor OB can be represented as $5\angle\phi$ in the polar form. In the rectangular form OB is represented as 4 + j3

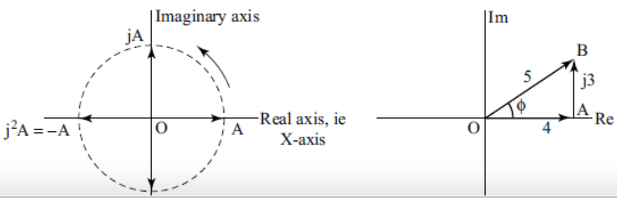


Figure 3.39 Use of operator j to represent a phasor

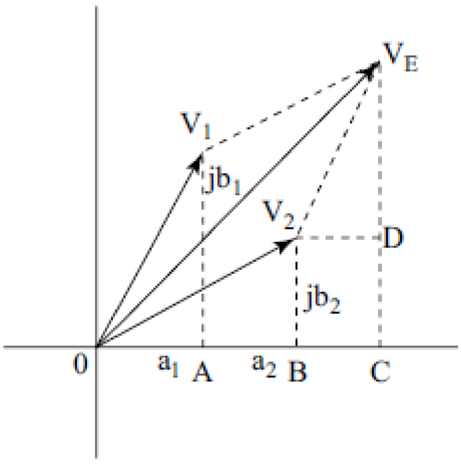


Figure 3.40 Addition of phasor quantities

$$\overrightarrow{OB} = \left|\sqrt{OA^2 + AB^2}\right| \underline{\left|\tan^{-1}\frac{AB}{OA}\right.}$$

$$= \sqrt{4^2 + 3^2} \underline{\left|\tan^{-1}\frac{3}{4}\right.}$$

$$= 5\underline{|37°} = 5\cos 37° + j\,5\sin 37°$$

$$= 5 \times 0.8 + j5 \times 0.6 = 4 + j3$$

Addition and subtraction of phasor quantities

Refer to Fig. 3.40.

Let $V_1 = a_1 + jb_1$ and $V_2 = a_2 + jb_2$

Addition: $V = V_1 + V_2$

$$= (a_1 + jb_1) + (a_2 + jb_2)$$

$$= (a_1 + a_2) + j(b_1 + b_2)$$

Multiplication

Division

angles are subtracted algebraically

Example 3.12 A coil having a resistance of 5 Ω and inductance of 30 mH in series are connected across a 230 V, 50 Hz supply. Calculate current, power factor, and power consumed.

Solution:

Current I is lagging the voltage, V by 62°. Power factor =cos ϕ

Here, p.f. = cos 62° = 0.47 lagging. The phasor diagram along with its circuit has been shown in Fig. 3.41.

Figure 3.41

Example 3.13 For the R–L–C series circuit shown in Fig. 3.42 (a), calculate current, power factor, and power consumed.

Figure 3.42 (a)

Here, X_L is greater than X_C. Therefore, the circuit reactance will be $X_L – X_C$.

This shows that magnitude of current is 10.75 A and the current lags the voltage by 45.3°.

Example 3.14 Two coils having impedance Z_1 and Z_2 are connected in series across a 230 V, 50 Hz power supply as shown in Fig. 3.42 (b).

Figure 3.42 (b)

The voltage drop across Z_1 is equal to Calculate the value of Z_2.

Solution:

Since this is a series circuit, the current flowing through the circuit is the same.

The circuit current can be calculated by using any of the following relations

since the same current will flow through Z_2,

Note that the impedance of an R–L circuit is R + j X_L and impedance

Example 3.15 An alternating voltage, V = (160 + j170)V is connected across an L–R series circuit. A current of I = (12 – j5) A flows through the circuit. Calculate impedance, power factor, and power consumed. Draw the phasor diagram.

Solution:

The series circuit consists of a resistance of 4.24 Ω and an inductive reactance of 11.28 Ω. The phasor diagram is drawn by considering a reference axis. Let x-axis be the reference axis. The voltage applied has a magnitude of 233 V and is making 46.8° with the reference axis in the positive direction, i.e., the anticlockwise direction. Current flowing is 19.2 A lagging the reference axis by 22.6° as shown in Fig. 3.43. The angle between phasor V and phasor I is 69.4°. This is the power factor angle.

Figure 3.43

If supply frequency is taken as 50 Hz, the value of L can be calculated from X_L.

Example 3.16 A sinusoidal voltage of v = 325 sin 314t when applied across an L–R series circuit causes a current of *i* = 14.14 sin (314t – 60°) flowing through the circuit. Calculate the value of L and R of the circuit. Also calculate power consumed.

Solution:

Example 3.17 A variable resistance R and an inductance L of value 100 mH in series are connected across at 50 Hz supply. Calculate at what value of R the voltage across the inductor will be half the supply voltage.

Solution:

Figure 3.44

We have to find R for which

Squaring,

Example 3.18 A voltage of v = 100 sin (314t + 0) is applied across a resistance and inductance in series. A current of 10 sin (314t – π/6) flows through the circuit. Calculate the value of R and L of the circuit. Also calculate power and power factor.

Solution:

Current, I is lagging V by 30°.

Example 3.19 The expression of applied voltage and current flowing through an ac series L–R circuit are

Calculate for the circuit (i) power factor; (ii) average power; (iii) impedance; (iv) R and L

Solution:

We will compare the voltage, n and current, i with the standard form

Figure 3.45

We have represented in Fig. 3.45 the voltage and current with respect to a common reference axis. The voltage, V is leading the reference axis by π/3°, i.e., 60°, while current I is leading the reference axis by 30°. The phase angle between V and I is 30°. The current in the circuit lags the voltage by 30°.

Power factor, cos ϕ = cos 30° = 0.866 lagging

Expressing in the rectangular form,

Example 3.20 In an L–R–C series circuit the voltage drops across the resistor, inductor, and capacitor are 20 V, 60 V, and 30 V, respectively. Calculate the magnitude of the applied voltage and the power factor of the circuit.

Solution:

In Fig. 3.47 the voltage drops across the circuit components and their phase relationship have been drawn. Since it is a series circuit, it is always convenient to take current as the reference axis. The voltage drop across the resistor and the current are in phase. Voltage across the inductor will lead the current, and voltage across the capacitor will lag the current. The circuit diagram and the phasor diagram have been shown.

From triangle ABC

Since current I is lagging V by an angle ϕ = 56.6°, the power factor is taken as lagging power factor.

Example 3.21 In the circuit shown in Fig. 3.48, calculate the value of R and C.

Figure 3.48

Solution:

Current I is leading V by degrees.

Taking V as the reference axis,

In an L–R–C series circuit, if current I is leading the voltage V, we have to consider the circuit as leading p.f circuit. This means the capacitive reactance is more than the inductive reactance (i.e., the circuit is effectively an R–C circuit. We will draw the phasor diagram by taking current on the reference axis. Here we see that V is lagging I by the power factor angle. That is, I is leading V by an angle ϕ. The phasor diagram taking I as the reference axis has been shown in Fig. 3.49

Figure 3.49

If we take V as the reference axis,

Power factor = cos ϕ = cos 30° = 0.866 leading.

In Fig. 3.50, AB = IR = V_R has been drawn in the direction of current I. I(X_c – X_L) is effectively a voltage drop which is capacitive in nature. I will lead I(X_c – X_L), or we can say that I(X_c – X_L) will lag I by 90°. BC has been shown lagging AB by 90°. The sum of AB and BC is AC which the total voltage, V and V = IZ. By taking away I from all the sides of the triangle ABC, the impedance triangle has be drawn.

Fig 3.50

Example 3.22 A resistance of 15 W and an inductance of 100 mH are connected in parallel across at 230 V, 50 Hz supply. Calculate the branch currents, line current, and power factor. Also calculate the power consumed in the circuit.

Solution:

The circuit diagram and the phasor diagram have been shown in Fig. 3.51. We note that in a parallel circuit the voltage applied across the branches is the same. The current in the resistive branch is in phase with the voltage while current in the inductive branch lags the voltage by 90°. The phasor sum of the branch currents gives us the total line current. Since in a parallel circuit voltage, V is common to the parallel branches, we generally take V as the reference axis while drawing the phasor diagram. Current through the resistive branch, I_R has been drawn in phase with V. Current through the inductive branch, I_L is lagging V by 90°. The sum of I_R and I_L gives I as has been shown in Fig. 3.51.

Fig 3.51

Now using the given values, calculations are made as follows.

Since the line current I is lagging the voltage V by 25°, the power factor is mentioned as lagging. The students should note that while mentioning power factor, it is essential to indicate whether the same is lagging or leading.

Example 3.23 For the circuit shown in Fig. 3.52 calculate the total current drawn from the supply. Also calculate the power and power factor of the circuit.

Fig 3052

Solution:

For branch I, the impedance Z_1 is calculated as

Similarly for branch II,

The phasor diagram representing the branch currents and the line current with respect to the supply voltage has been shown in Fig. 3.53. The line current lags the applied voltage by an angle, ϕ = 53 °.

Figure 3.53

Example 3.24 Two impedances $Z_1 = 10 + j12$ and $Z_2 = 12 - j10$ are connected in parallel across a 230 V, 50 HZ supply. Calculate the current, power factor, and power consumed.

Solution:

The two impedances are of the form, $Z_1 = R_1 + jX_L$ and $Z_2 = R_2 - jX_C$

Z_1 is composed of a resistor and an inductor while Z_2 is composed of a resistor and a capacitor.

Fig 3.54

We may calculate, and and then add I_1 and I_2 to get I. Alternately, we may find the equivalent impedance of the circuit, Z and then find,

Current, I lags voltage, V by 15°

Power factor = cos ϕ = cos 15° = 0.96 lagging

Magnitude of current, I = 20.68 A

Supply voltage, V = 230 V

Example 3.25 For the circuit shown in Fig. 3.55 calculate the total current, power, and power factor of the whole circuit. Also calculate the reactive power and apparent power of the circuit. Draw the phasor diagram.

Fig 3.55

Total Current, $I = 9.8$ A

The voltage V is making an angle of +30° with the reference axis as shown in Fig. 3.56. Current I_2 is making 82° with the reference axis; current I_1 is making −22.2° with the reference axis. The resultant of I_1 and I_2 is I. Current, I is making an angle of −7.8° with the reference axis. The phase difference between V and I is 37.8° as has been shown in Fig. 3.47.

Figure 3.56

Therefore, power factor, $\cos \phi = \cos 37.8° = 0.79$ lagging

Example 3.26 Three impedances, Z_1, Z_2, Z_3 are connected in parallel across a 230 V, 50 HZ supply. The values are given

. Calculate the total admittance, equivalent impedance, total current, power factor, and power consumed by the whole circuit.

Solution:

Fig 3.57

Example 3.27 For the circuit shown in Fig. 3.58 calculate the current in each branch and total current by the admittance method. Also calculate power and power factor of the total circuit.

Figure 3.58

3.2.8 AC Series—Parallel Circuits

Consider the series–parallel circuit consisting of three branches A, B, and C as shown. In Fig. 3.59.

Figure 3.59

Example 3.28 Determine the total current drawn from the supply by the series–parallel circuit shown in Fig. 3.60. Also calculate the power factor of the circuit.

Figure 3.60

Power factor, cos = cos 22° = 0.92 lagging

Example 3.29 What should be the value of R for which a current of 25 A will flow through it in the circuit shown in Fig. 3.61.

Figure 3.61

Solution:

Example 3.30 In the series–parallel circuit shown in Fig. 3.62, the parallel branches A and B are in series with branch C. The impedances are $Z_A = (4 + j3)\ \Omega$, $Z_B = (10 - j7)\ \Omega$ and $Z_C = (6 + j5)\ \Omega$. If the

Figure 3.62

Using current divider rule,

Phase angle between applied voltage V and line current I is – 32.2°

Hence, power factor of the whole circuit = cos = cos 32.2°

= 0.846 lagging

Voltage drop across series branch C, $V_C = I_C Z_C$

Voltage drop across parallel branches, $V_A = V_B = I_C \cdot Z_{AB}$

Note that voltage across the parallel branches is also equal to $I_A Z_A$ or $I_B Z_B$.

The complete phasor diagram is shown in Fig. 3.63.

Figure 3.63 Phasor diagram representing voltages and currents in the circuit of Fig. 3.55

Example 3.31 In the circuit shown in Fig. 3.64, determine the voltage at 50 Hz to be applied across terminals AB in order that a current of 10 A flows in the capacitor.

Figure 3.64

Solution:

Current in the capacitive branch,

Voltage drop across the parallel branch

Example 3.32 In a series–parallel circuit shown in Fig. 3.65, the parallel branches A, B, and C are in series with branch D. Calculate:

1. the impedance of the overall circuit,
2. current taken by the circuit, and
3. power consumed by each branch and the total power consumed.

Figure 3.65

Solution:

(i) Impedance of branch A,

$$Z_A = 2 + j0 = 2\ \Omega$$

Impedance of branch B,

Impedance of branch C, $Z_C = (2 - j2)\ \Omega$

Equivalent impedance of the parallel circuit,

Impedance of branch D, $Z_D = 1 + j1$

Total impedance of the overall circuit Z, = $Z_P + Z_D = 1.136 - j\,0.118 + 1 + j\,1 = 2.136 + j\,0.882$

(iii) $I_D = 47.6$ A

$R_D = 1\ \Omega$

Power consumed by branch $D = I_D^2 R_D$

$= (47.6)^2 \times 1$

$= 2265.8$ W

Voltage drop across terminals PQ = IZ_P

3.3 RESONANCE IN AC CIRCUITS

In ac circuits resonance occurs when two independent energy storing devices are capable of interchanging energy from one another. For example, inductance and capacitance are the two energy-storing devices or elements of an ac circuit, which may create a condition of resonance.

Resonance occurs in other systems also, like in a mechanical system, where mass and spring are the two energy-storing elements and they may create a condition of resonance. Mass stores energy when in motion and a spring stores energy when it is elongated or compressed.

An electric circuit generally consists of circuit elements like resistance, inductance, and capacitance. The voltage and frequency are generally constant at the supply terminal. However, in electronic communication circuits, the supply voltage may have variable frequency. When frequency is variable, the inductive and capacitive reactance of the circuit elements will change The current in the circuit will depend upon the values of X_L and X_C and that of R. A condition may occur at a particular frequency that the impedance offered to the flow of current is maximum or minimum. The circuit elements, namely the inductance element and the capacitance element are often connected in series or in parallel. It will be interesting and also useful to study the effect of varying input frequency on the circuit condition when these elements are connected in series or in parallel.

3.3.1 Resonance in AC Series Circuit

Let us consider a series circuit consisting of a resistor, an inductor, and a capacitor as shown in Fig. 3.66 (a). The supply voltage is constant but its frequency is variable.

Figure 3.66 Resonance in R–L–C series circuit: (a) circuit diagram; (b) variation of R, X_L, X_C with frequency; (c) variation of impedance

The impedance of the circuit,

As the frequency is changing, both X_L and X_C will change. Inductive reactance X_L will increase as the frequency, f is increasing while the capacitive reactance, X_C will decrease with increasing frequency. The value of R is independent of frequency. The variation of R, X_L, and X_C with variation of frequency, f has been shown in Fig. 3.66 (b). It may be noted that inductive reactance is jX_L and capacitive reactance is $-jX_c$, i.e., vectorially they should be shown in opposite directions. However, in Fig. 3.66 (b) we have shown their magnitudes only. At a frequency f_0, it is seen that the magnitude of X_L is equal to X_C as the two curves cut at point P. Since X_L and X_C are vectorially jX_L and $-jX_c$, the two reactances will cancel each other when frequency is f_0. At f_0 the impedance of the series R–L–C circuit is equal to R which is the minimum value of Z. In Fig. 3.66 (c), X_L is represented as jX_L and X_C is represented as $-jX_C$. The graph of $X = X_L - X_C$ has also been drawn. The total impedance graph of Z shows that at $f = f_0$, $Z = R$, i.e., at f_0 the circuit offers minimum impedance, and hence maximum current will flow through the circuit.

At minimum value of Z, the current in the circuit will be maximum as $I = V/R$. This condition of the circuit when X_L equals X_C, $Z = R$, current is maximum and is called the resonant condition and the frequency, f_0 at which resonance occurs is called the resonant frequency. At resonance, since X_L equals X_C, we can write

At resonance, frequency is f_0, current $I_0 = V/R$, power factor is unity, voltage drops across R, L, C are respectively, V_R, V_L, and V_C and supply voltage V is equal to the voltage drop across the resistance V_R.

Since at resonance, current is maximum and is very high, power dissipation $I_0^2 R$ is maximum and the rate of energy storage in the inductor and the capacitor is maximum and they are equal. The value of R is usually small (this is the resistance of the inductive coil), and hence voltage drop across it, i.e., V_R is also small as compared to the voltage drops across L and C. Voltage drops V_L and V_C are higher than V_R. However, as $V_L = V_C$ and they are in phase opposition as shown in Fig. 3.67 (b), the net voltage across L and C in series, V_X is equal to zero. Thus, the supply voltage will be equal to V_R.

Students will find it interesting to note that under the resonant condition the voltage across C or L will be many times more than the supply voltage. The power which is dissipated in the resistor is called active power. The energy which is stored in the inductor and the capacitor are due to reactive power. The energy stored in the inductor and the capacitor oscillates between them and the circuit as a whole appears to be a resistive only. The variation of circuit current as the frequency changes at different values of circuit resistance have been shown in Fig. 3.68 (a).

As can be noticed from the Fig. 3.68, at lower values of R, i.e., when $R = R_1$ the sharpness of the current curve is increased. At the resonant

Quality factor

The ratio of the reactive power to the resistive power is called the quality factor. Quality factor is also defined as the ratio of voltage drop appearing across the inductor or the capacitor to the supply voltage. Thus,

Figure 3.67 Resonance condition in R–L–C series circuit: (a) circuit diagram; (b) phasor diagram

We had earlier calculated f_0 as

Figure 3.68 (a) Resonance curves for two values of resistance; (b) bandwidth of a resonant circuit

In terms of X_C

Higher the ratio of reactive power to active power or higher the ratio of voltage across L and C to the supply voltage, the higher is the value of quality factor. Higher quality factor means sharper is the resonant factor curve and better is the ability of the network to accept current or power signals.

Bandwidth

Bandwidth is the range of frequencies for which the power delivered to the resistor is equal to half the power delivered to the resistor at resonance. As can be seen from Fig. 3.68 (b), the range of frequencies is $(f_2 - f_1)$ and the corresponding current is i.e., 0.707 I_0

Therefore, the range of frequencies within which current does not drop below 0.707 times the maximum value of current, i.e., I_0 is called the bandwidth. See Fig. 3.68 (b). The frequencies f_1 and f_2 are often called the lower and upper cut-off frequencies. Bandwidth, $BW = f_2 - f_1$.

From Fig. 3.68 (b) it is seen that at both f_1 and f_2, the power delivered to R is equal to half the power delivered to R at resonance.

Using equation (3.18) we can write

From the above, the two values of frequencies, i.e., ω_1 and ω_2 are

Normally, is very small as compared to in a resonant circuit.

Therefore,

f_2 and f_1 are the higher and lower bandwidth frequencies. Let us now calculate the value of $f_0 / (f_2 - f_1)$

Quality factor Q as we have seen earlier is

From eq. (3.20), it is seen that if the quality factor is high, bandwidth will be narrow. The circuit will, therefore, allow only a narrow band of signal frequencies which are close to the resonant frequency. Such a circuit is, therefore, highly selective in allowing signals to pass through. High-quality-factor resonant circuits are also called ‘*tuned circuits*’, which will be studied in detail in electronic circuit design.

Example 3.33 An R–L–C series circuit has R = 10 Ω, L = 0.1 H, and C = 8 μf.

Calculate the following:

1. resonant frequency;
2. Q-factor of the circuit at resonance;
3. half-power frequencies and bandwidth.

Solution:

(a) We know that the condition for the series resonance is

$X_L = X_C$

Substituting values

(b)

Substituting values

(c) Half-power frequencies are the frequencies corresponding to current which is 0.707 of the resonant current. They are f_1 and f_2, i.e., the lower, and upper frequencies forming the bandwidth.

Example 3.34 A circuit of R = 4 Ω, L = 0.5 H, and a variable capacitance C in series is connected across a 100 V, 50 Hz supply. Calculate:

1. the value of capacitance for which resonance will occur;
2. the voltage across the capacitor at resonance and the Q-factor of the circuit

Solution:

(a) Applying condition for series resonance

(b) Resonant Current,

Voltage across the capacitor,

The students should note that a very high voltage of 3925 V is appearing across the capacitor while the supply voltage is only 100 V.

Thus, the voltage multiplication across the capacitor as also across the inductor is 39.25 times at resonance.

Example 3.35 A resistor, a variable iron-core inductor, and a capacitor are connected across a 230 V, 50 Hz supply. By varying the position of the iron core inside the inductor coil, its inductance is changed. Maximum current of 1.5 A was obtained in the circuit by changing the inductance of the coil. At that time the voltage across the capacitor was measured as 600 V. Calculate the values of circuit parameters.

Solution:

We know that the maximum current flows at resonance when X_L = X_C and Z = R

Example 3.36 An inductor, a variable capacitor, and a resistor are connected in series across a constant voltage, 100 Hz power supply. When the capacitor value is fixed at 100 μF, the current reaches its maximum value. Current gets reduced to half its maximum value when the capacitor value is 200 μF. Calculate the values of circuit parameters and the Q-factor of the circuit.

Solution:

Let resonant frequency be f_0.

Maximum value of current,

At a frequency of 100 Hz, $C = 200 \times 10^{-6}$ F, current is reduced to half, i.e., impedance becomes equal to twice its value at resonance, i.e., equals 2R.

Example 3.37 An inductive coil of resistance 10 Ω and inductance 20 mH are connected in series with a capacitor of 10 μF. Calculate the frequency at which the circuit will resonate. If a voltage of 50 V at resonant frequency was applied across the circuit, calculate the voltage across the circuit components and the Q-factor.

Solution:

$R = 10\ \Omega$, $L = 20 \times 10^{-3}$ H, $C = 10 \times 10^{-6}$ F

at resonance, $X_L = X_C$

from which we get resonance frequency

At resonance, impedance of the circuit is equal to R. Therefore, the maximum current that will flow is equal to

To calculate the voltage drop across the circuit components, we calculate X_L and X_C at the resonance frequency first.

voltage drop across L, $V_L = I_0 X_L = 5 \times 44.7 = 223.5V$

voltage drop across R, $V_R = I_0 R = 5 \times 10 = 50V$

Note that the voltage drop across R is the same as the supply voltage, i.e., 50 V. Voltage drop across the capacitor should be the same as the voltage drop across inductive reactance X_L. Let us calculate V_C.

Substituting values

Example 3.38 A coil of inductance 1 mH and resistance 50 Ω connected in series with a capacitor is fed from a constant voltage, variable frequency supply source. If the maximum current of 5 A flows at a frequency of 50 Hz, calculate the value of C and the applied voltage.

Solution:

Resonant frequency,

At resonance, $X_L = X_C$, $Z = R$

Voltage drop across R = Supply voltage = $I_m R = I_0 R$

Thus, the applied voltage = 5 × 50 = 250V

Example 3.39 An inductive coil has a resistance of 2.5 Ω and an inductive reactance of 25 Ω. This coil is connected in series with a variable capacitance and a voltage of 200 V at 50 Hz is applied across the series circuit. Calculate the value of C at which the current in the circuit will be maximum. Also calculate the power factor, impedance, and current in the circuit under that condition.

Solution:

When current is maximum in an R, L, C series circuit, the circuit is under the resonance condition. At resonance, $X_L = X_C$ and $Z = R$.

Here

At resonance, $X_L = X_C$, $Z = R$. The circuit behaves like a resistive circuit. Therefore, the power factor = 1. Impedance, $Z = R$, and current is maximum.

Let us consider an inductive coil and a capacitor in parallel connected across a constant voltage variable frequency supply source as shown in Fig. 3.69 (a). Practically, both the capacitor and the inductor will have some losses which should be represented by a small resistance in series. Here we assume the capacitor as a lossless one while the inductor coil has some resistance which has been shown separately. The phasor diagram of voltage and current components have been shown in Fig. 3.69 (b). The line current I is equal to the in-phase component of I_L with the voltage V, i.e.,

. At resonance, the current through the capacitor I_C is balanced by as shown. Thus, the reactive component of line current which is the phasor sum of I_C and is zero. The condition for resonance is

The condition of resonance is $X_L X_C = Z^2_L$

To calculate resonance frequency, f_0 we take, $X_L X_C = Z^2_L = R^2 + X^2_L$

This is the resonance frequency of a parallel L and C circuit.

If we consider the value of R as negligible, then the resonance frequency is

This value is the same as calculated for a series resonance circuit. The line current I is equal to which is the minimum current occurring at resonance. If the value of R is reduced, the cosine component of I_L will get reduced. When R is made equal to zero, will be zero and the whole of I_L will be reactive or wattless component and will be equal and opposite to I_C.

is in phase with V when resonance occurs. The circuit impedance, Z_0 is calculated as

From the condition of resonance

Z_0 is known as the equivalent impedance or '*dynamic impedance*' of the parallel resonant circuit. It can be noticed that
is in phase with the supply voltage. This shows that the circuit behaves like a resistive circuit only since the reactive component currents cancel each other. The impedance Z_0 is therefore resistive only. Since current is minimum, impedance of the circuit, Z_0 = L/CR is maximum under the resonant condition. Since current at resonance is minimum, a parallel resonant circuit is often referred to as a *'rejector circuit'* meaning that a parallel resonant circuit tends to reject current at resonant frequency.

It may be noted that the current drawn from the supply at resonance, i.e., is minimum. The current circulating through the capacitor and the inductor, i.e., I_C which is equal to is very high (I_C >> I or Since I_C is many times more than I, we can say that parallel resonance is a case of current resonance. Here we recall that series resonance is a case of voltage resonance as voltage across the capacitor or the inductor is many times higher than the supply voltage

Q-factor of parallel circuit

The ratio of circulating current between the two parallel branches, i.e., the capacitor and the inductor to the circuit line current is called the Q-factor or current magnification factor of the parallel circuit. Thus,

The effect of variation of frequency on circuit impedance and current has been shown in Fig. 3.70. At resonant frequency f_0, the impedance is maximum, i.e., equal to The current at resonance is the minimum and is equal to I_0 where

Figure 3.70 Variation of impedance and current in a parallel resonant circuit

Table 3.1 Comparison Between Series and Parallel Resonance

Parameters	Series Circuit	Parallel Circuit
Current at resonance	Maximum, I_0 = V/R (Acceptor type)	Minimum, I_0 = (Rejector type)
Impedance at resonance	Minimum, Z_0 = R	Maximum, Z_0 = L/CR
Power factor at resonance	Unity	Unity
Resonant frequency		
Magnification element	Voltage	Current
Magnification factor or Q-factor	Q-factor	Q-factor =

Bandwidth of parallel resonant circuit

Bandwidth of a parallel resonant circuit is determined the same way as in the case of the series resonant circuit. Bandwidth is the range of frequencies ($f_2 - f_1$) where the power dissipated is half of the power dissipated at the resonant frequency.

The critical parameters of series and parallel resonant circuits have been compared and shown in Table 3.1

We have seen that resonance in ac series and parallel circuits can take place at a particular frequency when a constant voltage, variable frequency supply source is applied across the circuit. The frequency at which resonance occurs are

If we neglect the small value of R, f_0 for series and parallel resonance is the same.

If we have a constant frequency supply source, a resonance condition can also be achieved if we change the value of L or C creating a condition when

Example 3.40 An inductive coil has a resistance of 10 Ω and inductance of 100 mH. This coil is connected in parallel with a capacitor of 20 μF. A variable frequency power at 100 V is applied across this parallel circuit. Calculate the frequency at which the circuit will reasonate. Also calculate the Q-factor, dynamic impedance of the circuit, and resonant current.

Solution:

Resonant frequency,

Substituting the given values,

Dynamic impedance,

Current at resonance,

Example 3.41 An inductive coil of resistance 5 Ω and inductive reactance 10 Ω is connected across a voltage of 230 V at 50 Hz. Calculate the value of the capacitor which when connected in parallel with the coil will bring down the magnitude of the circuit current to a minimum. Draw the phasor diagram.

Solution:

Figure 3.71

Before a capacitor is connected, current flowing through the inductor, I_L is

If a capacitor is now connected in parallel, it must draw a current I_C which will lead V by 90°. The magnitude of I_C must be equal to $I_L$sin so that these two currents cancel each other. In such a case, the resultant current, I is the in-phase current, i.e., $I_L$cos .

This is the minimum current drawn by the circuit and is called the resonant current, I_0.

Example 3.42 An inductor having a resistance of 4 Ω and an inductance of 20 mH are connected across a 230 V, 50 Hz supply. What value of capacitance should be connected in parallel to the inductor to produce a resonance condition? What will be the value of the resonant current?

Solution:

For resonance, the current drawn by the capacitor in parallel must be equal to

Resonant current for parallel resonance is the minimum current which is the in-phase component, i.e., .

The phasor diagram representing the resonant condition has been shown in Fig. 3.72.

Figure 3.72

Example 3.43 Calculate the value of R_1 in the circuit given in Fig. 3.73 such that the circuit will resonate.

Figure 3.73

We know that at resonance the impedance of the circuit will be resistive only. We will calculate the value of impedance in a complex form and equate its imaginary part to zero to determine the value of R_1. Here, $Z = R_1 + j6$ and $Z_2 = 10 - j4$

During resonance, the imaginary part of Z will be zero.

Therfore,

3.4 REVIEW QUESTIONS

A. Short Answer Type Questions

1. Explain frequency, time period, instantaneous value, maximum value, and average value for a sinusoidal voltage.

2. What do you understand by harmonic waves of a non-sinusoidal wave?

3. Why do we use RMS value instead of average value for an alternating quantity?

4. Show that for a sinusoidal voltage, RMS value is 0.707 times its maximum value.

5. What is the value of form factor for a sine wave? What is the significance of the value of form factor for an alternating quantity?

6. The form factors for different kinds of voltage wave shapes have been calculated as 1.0, 1.11, and 1.15. Is it possible to predict the type of the voltage wave shapes?

7. What is inductive reactance, capacitive reactance, and impedance of an L–R–C circuit?

8. What is meant by power factor of an ac circuit? What is its minimum value and its maximum value?

9. Prove that average power in an ac circuit is ϕ where V is the RMS value of voltage, I is the RMS value of current, and ϕ the power factor.

10. What is the significance of very low (poor) power factor of a circuit?

11. A resistance R, an inductance L, and a capacitance C are connected in series across an alternating voltage, V. A current I flows

the impedance triangle.

13. Show that current in a pure inductive circuit lags the voltage by 90°.

14. What is the power factor of a purely resistive circuit, purely inductive circuit, and purely capacitive circuit?

15. State the condition for maximum current in an L–R–C series circuit.

16. State the condition for series resonance in an L–R–C circuit.

17. State the condition for parallel resonance. How do we calculate the value of capacitor to be shunted to create a resonant condition?

18. What is the value of resonant frequency in the case of series resonance and in the case of parallel resonance?

19. What is meant by Q-factor of a series resonant circuit? What does Q-factor signify?

20. What do you mean by bandwidth in a series circuit?

21. A resonant circuit with high Q-factor is also called a tuned circuits. Explain why.

22. Write the expression for resonant frequency, Q-factor, and dynamic impedance for a parallel resonant circuit.

23. Explain what is meant by phase and phase difference of alternating quantities.

24. A sinusoidal current is expressed as i = 100 sin 314t. What is the maximum value, RMS value, frequency, and time period of the alternating current?

25. A sinusoidal voltage, when connected across an ac series circuit produces a current, i=20 sin(314t – 30°). What is the power factor of the circuit? Draw the phasor diagram.

26. Define apparent power, active power, and reactive power of an ac circuit.

27. Define the terms impedance, inductive reactance, capacitive reactance, admittance, active power, reactive power, and power factor for an ac circuit.

28. Two are connected in series. What is the value of equivalent impedance?

29. Two impedances are connected in parallel. What is the equivalent impedance?

30. An impedance of 10 + j10 is connected across a voltage of V. What is the magnitude of current and the value of power factor?

31. Two impedances

B. Numerical Problems

33. Calculate the RMS value of an alternating current $i = 20(1 + \sin\theta)$.

[Ans 24.5 A]

34. Calculate the RMS value of a half-wave-rectified voltage of maximum value of 100 V.

[Ans 50 V]

35. An alternating voltage is expressed as $v = 141.1 \sin 314t$. What is the RMS value, time period, and frequency?

[Ans 100 V, 20 msec, 50 Hz]

36. An alternating current of frequency 50 Hz has its maximum value of 5 A and lagging the voltage by 30°. Write the equation for the current.

[Ans $i = 5 \sin(314t - 30°)$]

37. An alternating voltage is expressed as, $v = 100 \sin 314t$. Determine the time taken for the voltage to reach half its maximum value, time counted from $t = 0$. At what time will voltage reach its maximum value?

[Ans $t = 1.66$ msec; $t = 5$ msec]

38. Determine the average value of the voltage wave form shown in Fig. 3.74.

[Ans 7.5 V]

Figure 3.74

39. An alternating voltage is defined as:

$v = 100 \sin\theta$ V $\quad 0 < \theta < \pi$

[Ans 50 V]

40. Find the RMS value of the sinusoidal voltage waveform shown in Fig. 3.75.

[Ans 47.6 V]

Figure 3.75

41. Find the RMS value of the voltage wave shown in Fig. 3.76.

[Ans 20.4 V]

Figure 3.76

42. A resistance of 50 Ω, an inductance of 0.1 H, and a capacitance of 50 mf are connected in series across a 230 V, 50 Hz supply. Calculate (i) the value of impedance; (ii) current flowing; (iii) power factor; (iv) power consumed.

[Ans Z = 59.5 Ω, I = 3.86 A, P.f. = 0.84 leading; P = 746.7 W]

43. In an R–L–C series circuit, the voltage across R is 160 V, across L is 240 V, and power consumed is 1000 W when a voltage of 200 V at 50 Hz is applied across the circuit. Calculate the value of the capacitor and the current flowing through the circuit.

[Ans C = 165.8 μF, 6.25 A]

44. A impedance of is connected across a 230 V, 50 Hz supply. Calculate the values of circuit elements, current, and power factor.

[Ans R = 25, C = 73.54 μF, I = 4.6, P.f. = 0.5 leading]

45. A coil of resistance 5 Ω and inductance 20 mH is connected across a voltage of v = 230 sin 314t. Write an expression for the current flowing through the circuit.

46. A resistance of 50 Ω and a capacitor of 100 mf are connected in series. The supply voltage to the circuit is 200 V at 50 Hz. Calculate the voltage across the resistor and across the capacitor. Also calculate current and power factor.

[Ans V_R = 168.7 V, V_C = 107 V, I = 3.37 A, P.f. = 0.84 leading]

47. In an R–L–C series circuit a maximum current of 0.5 A is obtained by varying the value of inductance L. The supply voltage is fixed at 230 V, 50 Hz. When maximum current flows through the circuit, the voltage measured across the capacitor is 350 V. What are the values of the circuit parameters?

[Ans R = 460 A, L = 2.229 H, C = 4.549 µF]

48. A 200 V, a variable frequency supply is connected across an L–R–C series circuit with R = 10 Ω, L = 10 mH, and C = 1 mF. Calculate the frequency at which reasonance will occur. Also calculate the Q-factor and bandwidth.

[Ans f = 1591.5 Hz, Q-factor = 10, Bandwidth = 159 H_Z]

49. A coil of R = 10 Ω, L = 0.023 H connected in parallel with another coil of R = 5 Ω, L = 0.035 H. The combination is connected across at 200 V, 50 Hz supply. Calculate the current drawn from the supply and the power factor.

[Ans I = 31.4 A, P.f. = 0.63 lagging]

50. A coil of resistance 20 Ω and inductance of 300 mH is connected in parallel with a capacitance of 200 µF. The combination is connected across 200 V, variable frequency power supply. At what frequency will the parallel circuit resonate and what would be the current at resonance?

[Ans 20.5 Hz, 2.66 A]

51. Calculate the value of R in the circuit shown in figure below such that the circuit will resonate.

Figure 3.77

[Ans R = 6 Ω.]

[Ans f_1 =117 KHz, f_2=.19 KHz]

C. Multiple Choice Questions

1. The voltage and current in an ac circuit is represented by $v = v_m \sin(\omega t + 30°)$ and $i = I_m \sin(\omega t - 45°)$. The power factor angle of the circuit is
   1. 15°
   2. 75°
   3. 45°
   4. 30°.
2. A current is represented by i = 100 sin (314t – 30°) A. The RMS value of the current and the frequency are, respectively,
   1. 100 A and 314 Hz
   2. 100 A and 50 Hz
   3. 70.7 A and 314 Hz
   4. 70.7 A and 50 Hz.
3. A current of 10 A is flowing through a circuit. The power factor is 0.5 lagging. The instantaneous value of the current can be written as
   1. i = 10 sin 60° A
   2. i = 10 sin (wt – 30°)A
   3. i = 14.14 sin (wt – 60°)
   4. i = 14.14 sin (wt + 60°).
4. In a purely inductive circuit
   1. current lags the voltage by 90°
   2. current leads the voltage by 90°
   3. voltage lags the current by 90°
   4. current lags the voltage by 180°.
5. Form factor of an ac wave indicates
   1. low sharp or steep the wave shape is
   2. low flat the wave shape is
   3. low symmetrical the wave shape is
   4. the degree of its conformily to sinusoidal form.
6. Power consumed by a pure inductor is
   1. infinite
   2. very high
   3. zero
   4. very small.
7. If form factor of a sinusoidal wave is 1.11, then the form factor of a triangular wave will
   1. also be 1.11
   2. be less than 1.11
   3. be more than 1.11
   4. be = 1.
8. A voltage of v = 100 sin (314t – 30°) is connected across a 10 Ω resistor. The power dissipated in the circuit will be

9. The average value of a sinusoidal current is
    1.
    2.
    3.
    4. (d) .
10. Form factor of an alternating wave form is the ratio of
    1. RMS value and average value
    2. average value and RMS value
    3. maximum value and average value
    4. maximum value and RMS value.
11. A capacitance of C Farad is connected to a 230 V, 50 Hz supply. The value of capacitive reactance is
    1. 314 C Ω
    2. Ω
    3. 628 C Ω
    4. Ω.
12. The form factor of a square wave is
    1. 1.11
    2. 1.0
    3. 0
    4. 1.414.
13. Two sinusoidal waves are represented as v_1 = 100 sin (ωt + 30°) and v_2 200 sin (ωt – 60°). The phasor relationship between the voltages can be expressed as
    1. v_1 lags v_2 by 90°
    2. v_2 lags v_1 by 90°
    3. v_1 leads v_2 by 30°
    4. v_2 lags v_1 by 60°.
14. Inductive reactance of a coil of 0.1 H at 50 Hz is
    1. 31.4 Ω
    2. 62.8 Ω
    3. 314 Ω
    4. 5 Ω.
15. The power factor of a purely resistive circuit is
    1. 1.0
    2. 0
    3. 0.1
    4. 0.5.
16. A sinusoidal voltage is represented as n = 141.4 sin . The RMS value, frequency and phase angle are respectively
    1. 141.4, 628, 60°
    2. 100, 100, –60°
    3. 141.4, 50, 60°
    4. 141.4, 100, 60°.

Answers to Multiple Choice Questions

5. (a)
6. (c)
7. (c)
8. (c)
9. (a)
10. (a)
11. (b)
12. (b)
13. (b)
14. (a)
15. (a)
16. (b)

D. Multiple Choice Questions

(On single-phase ac circuits)

1. In an R–L series circuit, the power factor of the circuit is increased if
   1. X_L, the inductive reactance is increased
   2. X_L, the inductive reactance is decreased
   3. R, resistance is decreased
   4. the supply frequency is increased.
2. The power factor of an R–L circuit can be expressed as
   1.
   2.
   3.
   4. .
3. An R–L series circuit consists of R = 3 Ω and X_L = 4 Ω. The impedance of the circuit is
   1. Z = 7 Ω
   2. Z = 1 Ω
   3. Z = 5 Ω
   4. Ω.
4. The impedance of a circuit is The value of resistance and inductive reactance of the circuit, respectively, are
   1. 10 Ω and 17.32 Ω
   2. 17.32 Ω and 10 Ω
   3. 10 Ω and 10 Ω
   4. 10 Ω and 8.66 Ω.
5. The impedance q an R–L series circuit is 628 when the supply frequency is 50 Hz. The value of inductance, L is
   1. 314 H
   2. 1 H
   3. 2 H
   4. 628 H.
6. An impedance of z = 6 + j8 Ω is connected across a 200 V, 50 Hz supply. The power factor of the circuit is
   1. 0.6 lagging

1. 10 A
2. 20 A
3. 14–28 A
4. 2 A.

8. An R–L series circuit has an impedance of 10 + j10 W. The power factor angle of the circuit is

    1. 30° lagging
    2. 30° leading
    3. 45° leading
    4. 45° lagging.

9. An R–C series circuit has a resistance of 10 Ω and a capacitive reactance of 10 Ω. What will be the phase difference between the voltage and current in the circuit?

    1. Current will lead the voltage by 90°
    2. Current will lag the voltage by 90°
    3. Current will lead the voltage by 45°
    4. Current will lag the voltage by 45°.

10. The impedance of an R–L series circuit is (50 + j100) Ω at 50 Hz. When the supply frequency is 100 Hz, the value of impedance will be

    1. (50 + j 1000) Ω
    2. (50 + j 200) Ω
    3. (100 + j 100) Ω
    4. (100 + j 200) Ω.

11. A voltage of v = 10 sin (314t + 15°) is applied across an R–L–C series circuit, where R = 5 Ω, X_L = 15 Ω, and X_C = 10 Ω. The current flowing the circuit will be

    1. 0.33 A
    2. 1 A
    3. 1.414 A
    4. 2 A.

12. The resonant frequency in R–L–C series circuit is

    1.
    2.
    3.
    4. .

13. A series RLC circuit has R = 50 Ω, L = 50 μH, C = 2 ΩF. The Q-factor of the circuit is

    1. 0.1
    2. 1
    3. 10
    4. 2.

14. When a parallel circuit is in resonance, which of the following of the circuit is maximum?

    1. Current
    2. Impedance
    3. Admittance
    4. Power factor.

3. $X_C = X_L$
4. .

16. In an R–L–C series circuit, if the frequency is made more than the resonant frequency the circuit will effectively be

    1. inductive
    2. capacitive
    3. resistive
    4. oscillatory.

17. Two impedances, Z1 = 4 + j4 W and Z2 = 4 – j4 Ω are connected in parallel. Their equivalent impedance is

    1. 8 + j8 Ω
    2. 4 + j0 Ω
    3. 8 – j8 Ω
    4. 8 + j0 Ω.

18. When an inductance, L and a resistance, R are connected in parallel across an ac supply, the current drawn by the two parallel branches will be out of phase by

    1. (a) 0°
    2. (B) 90°
    3. (C) 180°
    4. (d) 45°.

19. When an inductance, L and a capacitance, C are connected in parallel across an ac supply, the current drawn by the two parallel branches will be out of phase by

    1. (a) 0°
    2. (90°
    3. 180°
    4. 45°.

20. In an R–L circuit, XL = R. The power factor angle q the circuit is

    1. 30°
    2. 45°
    3. 60°
    4. 0°.

21. In a series resonant circuit, a change in supply voltage will cause a change in

    1. the current drawn
    2. the Q-factor q the circuit
    3. the bandwidth of the circuit
    4. the resonant frequency.

22. Which of the following conditions is true for both series and parallel resonance?

    1. impedance is minimum
    2. power factor is unity
    3. power factor is zero
    4. power is low.

23. The bandwidth of a series R–L–C circuit is

24. The product of voltage and current in an ac circuit is called

   1. active power
   2. apparent power
   3. average power
   4. reactive power.

25. In a series resonance circuit

   1. L = C
   2. L = R
   3. $X_L = X_C$
   4. R = L = C.

Answers to Multiple Choice Questions

(On single-phase ac circuits)

1. (b)
2. (a)
3. (c)
4. (b)
5. (b)
6. (a)
7. (b)
8. (d)
9. (c)
10. (b)
11. (b)
12. (c)
13. (a)
14. (b)
15. (b)
16. (a)
17. (b)
18. (b)
19. (c)
20. (b)
21. (a)
22. (b)
23. (b)
24. (b)
25. (c)