Course Objectives:

Linear algebra is one of the most important subjects of pure mathematics and has many applications in electrical, communications and computer science. This course aims at introducing students to the fundamental concepts of linear algebra by starting with linear equations and culminating in abstract vector spaces and linear transformations.

Course Outcomes:

By the end of the course, the students will be able to

1. solve systems of linear equations
2. understand the concepts of vector spaces and subspaces, basis and dimensions, linear transformations and inner product spaces and their matrix representations
3. use Gram-Schmidt process to obtain orthonormal basis,
4. find the change of basis matrix with respect to two bases of a vector space.

Linear Equations and Matrices Introduction - Gaussian elimination and Gauss Jordan methods - Block matrices - Elementary matrices - permutation matrix - inverse matrices - LDU factorization - Applications to electrical networks and cryptography.

Vector Spaces and Subspaces Vector spaces and subspaces - Linear Independence, Basis and Dimension - Row, Column and Null spaces - Rank and Nullity - Bases for subspaces - Invertibility - Application: Interpolation and Wronskian.

Linear Transformations Definition and Examples - properties - The Range and Kernel - Invertible linear transformations - Isomorphism - Application: Computer graphics - Matrices of linear transformations - Vector space of linear transformations - change of bases - similarity.

Inner Product Spaces Inner products - The lengths and angles of vectors - Matrix representations of inner products - Orthogonal projections - Gram-Schmidt orthogonalization.

Applications of Inner Product Spaces QR factorization - Singular Value Decomposition - Projection - orthogonal projections - relations of fundamental subspaces - Least square solutions - Orthogonal projection matrices.

Text Book(s):

1. Linear Algebra by Jin Ho Kwak and Sungpyo Hong, Second edition, Springer, 2004.
2. Linear Algebra with applications by Steven J. Leon, 8th Edition, Pearson, 2010.

Reference Book(s):

1. Elementary Linear Algebra by Stephen Andrilli and David Hecker, 4th edition, Academic Press, 2010.
2. Introduction to Linear Algebra by Gilbert Strang, 4th edition, Wellesley-Cambridge Press, 2011.
3. Introductory Linear Algebra - An applied first course by Bernard Kolman and David R. Hill, 9th Edition, Pearson education, 2011.
4. Linear Algebra A Modern Introduction by David Poole, 2nd edition, Thomson Learning, 2006

Problem 1.1.1

Apply Gauss elimination method to solve the equation $x + 4y - z = -5$, $x + y - 6z = -12$, $3x - y - z = 4$.

$$\begin{bmatrix} 1 & 4 & -1 \\ 1 & 1 & -6 \\ 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -12 \\ 4 \end{bmatrix}$$

Its augmented matrix is

$$C = \begin{bmatrix} A & : & B \end{bmatrix} = \begin{bmatrix} 1 & 4 & -1 & : & -5 \\ 1 & 1 & -6 & : & -12 \\ 3 & -1 & -1 & : & 4 \end{bmatrix}$$

Applying operations:

$$R_2 \Rightarrow R_2 - R_1,$$
$$R_3 \Rightarrow R_3 - 3R_1.$$

$$C \sim \begin{bmatrix} 1 & 4 & -1 & : & -5 \\ 0 & -3 & -5 & : & -7 \\ 0 & -13 & 2 & : & 19 \end{bmatrix}$$

Applying operation $R_3 \Rightarrow 3R_3 - 13R_2$

$$C \sim \begin{bmatrix} 1 & 4 & -1 & : & -5 \\ 0 & -3 & -5 & : & -7 \\ 0 & 0 & 71 & : & 148 \end{bmatrix}$$

which can be written as

$$\begin{bmatrix} 1 & 4 & -1 \\ 0 & -3 & -5 \\ 0 & 0 & 71 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ -7 \\ 148 \end{bmatrix}$$

$$\begin{aligned} x + 4y - z &= -5 \qquad (1) \\ -3y - 5z &= -7 \qquad (2) \\ 71z &= 148 \qquad (3) \end{aligned}$$

$$z = \frac{148}{71}$$

From equation (2),

$$y = \frac{-1}{3}\left[-7 + 5\left(\frac{148}{71}\right)\right] = \frac{-81}{71}$$

From equation (1),

$$x = \frac{117}{71}$$

$$x = \frac{117}{71}; y = \frac{-81}{71}; z = \frac{148}{71}$$

Example 1.1.2

Using Gauss elimination method find the solutions of $x + 4y - z = -5$, $x + y - 6z = -12$, $3x - y - z = 4$.

Ans:

$$x = \frac{117}{71}$$
$$y = \frac{-81}{71}$$
$$z = \frac{148}{71}$$