Operational Research.
Assignment - 1

ABHISHEK SRIVASTAVA
19BCE10071

1) The equation can be written as:

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = b$$

Where, $a_1 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$, $a_2 \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, $a_3 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$,

$a_4 = \begin{pmatrix} -8 \\ -6 \end{pmatrix}$ and $b = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

No. of Square sub Matrices = ${}^4C_2 = 6$.

$B_1 = \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix}$, $B_2 = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}$, $B_3 = \begin{pmatrix} 4 & -8 \\ 3 & -6 \end{pmatrix}$

$B_4 = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$, $B_5 = \begin{pmatrix} 2 & -8 \\ 5 & -6 \end{pmatrix}$, $B_6 = \begin{pmatrix} 3 & -8 \\ 4 & -6 \end{pmatrix}$

det $B_1 = 14 \neq 0$, det $B_2 = 7 \neq 0$, det $B_3 = 0$
det $B_4 = -7 \neq 0$, det $B_5 = 28 \neq 0$, det $B_6 = 14 \neq 0$

Here 5 sub-matrices are non-singular

Ans: There are 5 basic solutions.

2) We write the System of Equations as:-

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 = b$$

where, $a_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $a_2 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$, $a_3 = \begin{pmatrix} -1 \\ 6 \end{pmatrix}$,

$a_4 = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$, $a_5 = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$.

There are ${}^4C_2 = 6$ sub matrices.

$B_1 = (a_1 a_2)$ $B_2 = (a_1 a_3)$ $B_3 = (a_1 a_4)$

$B_4 = (a_2 a_4)$ $B_5 = (a_2 a_3)$ $B_6 = (a_3 a_4)$

$\det B_1 = -7 \neq 0$, $\det B_2 \neq 6$, $\det B_i \neq 0$ $(i = 3, 4, 5, 6)$.

Now, $X_{B_1} = B_1^{-1} b = \frac{-1}{7}\begin{pmatrix} -2 & -3 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 8 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$X_{B_2} = B_2^{-1} b = \frac{1}{13}\begin{pmatrix} 6 & 1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 8 \\ -3 \end{pmatrix} = \begin{pmatrix} 45/13 \\ -14/13 \end{pmatrix}$

So we obtain one basic fasable solution as

$$(x_1, x_2, x_3, x_4) = (1, 2, 0, 0)$$

and one basic non-fasable solution as:-

$$(x_1, x_2, x_3, x_4) = \left(\frac{45}{13}, 0, \frac{-14}{13}, 0\right)$$

3) The given equations are linearly independent.

But the vectors $a_1 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and $a_3 = \begin{pmatrix} 5 \\ 10 \end{pmatrix}$ are not.

since, $B = (a_1 \ a_3) = \begin{pmatrix} 2 & 5 \\ 4 & 10 \end{pmatrix}$ and $\det B = 0$

$\therefore$ If we initially assume x_2 as a non-basic variable whose value is 0, the given equation reduce to $2x_1 + 5x_3 = 10$, and
$4x_1 + 10x_3 = 20$.

which represent the same equation having an infinite no. of solution but none are basic.

$\therefore$ For the given set of Eqns, a basic solution with x_2, a non-basic variable, does not exist.

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4) The given data represents LPP:

Max $z = 2x_1 + 2x_2 + 4x_3$

S.to
$$2x_1 + 3x_2 + x_3 \leq 300$$
$$x_1 + x_2 + 3x_3 \leq 300$$
$$x_1 + 3x_2 + x_3 \leq 240.$$

$$x_1, x_2, x_3 \geq 0$$

$\therefore$ option (b) is correct.

5) Suppose x_1, x_2, x_3, x_4 and x_5 be the quantities of 5 food respectively which constitutes the daily diet of each patient. Then the problem is to find x_1, x_2, x_3, x_4 and x_5 which will minimise the total cost.

$$Z = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4 + c_5x_5.$$

Subject to:
$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 \geq N_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 \geq N_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 \geq N_3$$
$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 \geq N_4$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

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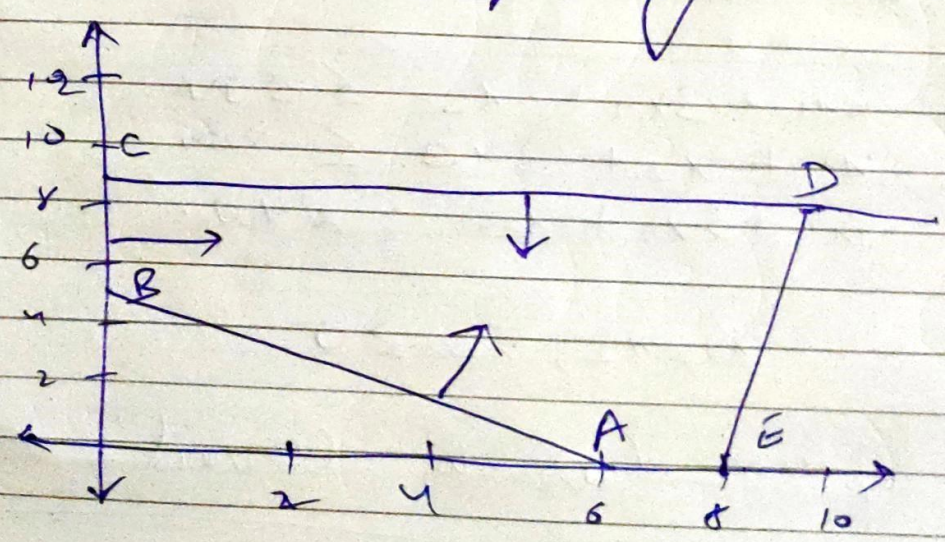
6) Graphical Solution.

Let $\overleftrightarrow{AD}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{DE}$ denote the straight lines given by the Equations $5x_1 + 6x_2 = 30$, $x_2 = 8$, and $9x_1 - 2x_2 = 72$ respectively:



The feasible region satisfied by all the constraints is the segment DE of the line $\overleftrightarrow{DE}$.

The extrem pts of convex regions are D(10, 9) and E(8, 0).

Now, $Z_1 = 32$ for $x_1 = 10, x_2 = 9$ at D.
$Z_2 = 40$ for $x_1 = 8, x_2 = 0$ at E.

$\therefore$ Max $Z = Z_2 = 40$ for $x_1 = 8, x_2 = 0$ at E.

$\therefore$ Option (b) is correct.

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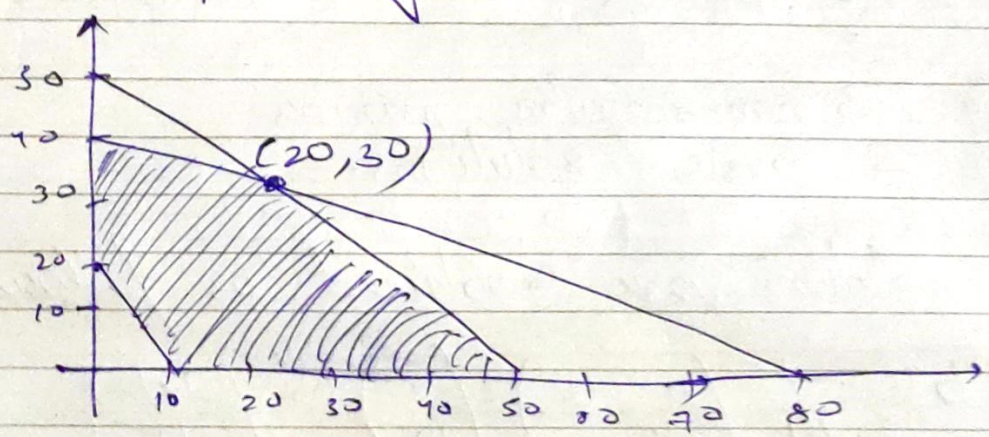
7) The satisfied regions are :-



The feasable corner pts are (10, 0), (50, 0), (20, 30), ~~and~~ (40) and (0, 20) and the values of the objective functions at the points are respectively. 40, 200, 170, 120 and 60.

$\therefore Z_{max} = 200$ at (50, 0)

Ans > a.

8) Here,

$$a_1 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, a_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, a_3 = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, a_4 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

and $b = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$

There can be atmost ${}^4C_2 = 6$ basic solutions.
The 6 square sub-matrices taking two at a time from a_1, a_2, a_3 and a_4 are:

$$B_1 = \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix} \quad B_2 = \begin{pmatrix} 2 & 3 \\ 4 & -1 \end{pmatrix} \quad B_3 = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$$

$$B_4 = \begin{pmatrix} -1 & 3 \\ -2 & -1 \end{pmatrix} \quad B_5 = \begin{pmatrix} -1 & 1 \\ -2 & 2 \end{pmatrix} \quad B_6 = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$\therefore$ $\det B_1 = 0$ $\quad \det B_2 = -14 \neq 0$ $\quad \det B_3 = 0$
$\det B_4 = 7 \neq 0$ $\quad \det B_5 = 0$ $\quad \det B_6 = 7 \neq 0$.

$\Rightarrow$ $\exists$ only 3 non-singular matrices.
$\Rightarrow$ $\exists$ only 3 basic solutions.

The basic solutions are obtained as follows:-

$$x_{B_2} = B_2^{-1} b = \frac{-1}{14} \begin{pmatrix} -1 & -3 \\ -4 & 2 \end{pmatrix} \begin{pmatrix} 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 18/7 \\ 2/7 \end{pmatrix}$$

$\Rightarrow$ Basic Solution $= \left(\frac{18}{7}, 0, \frac{2}{7}, 0 \right)$

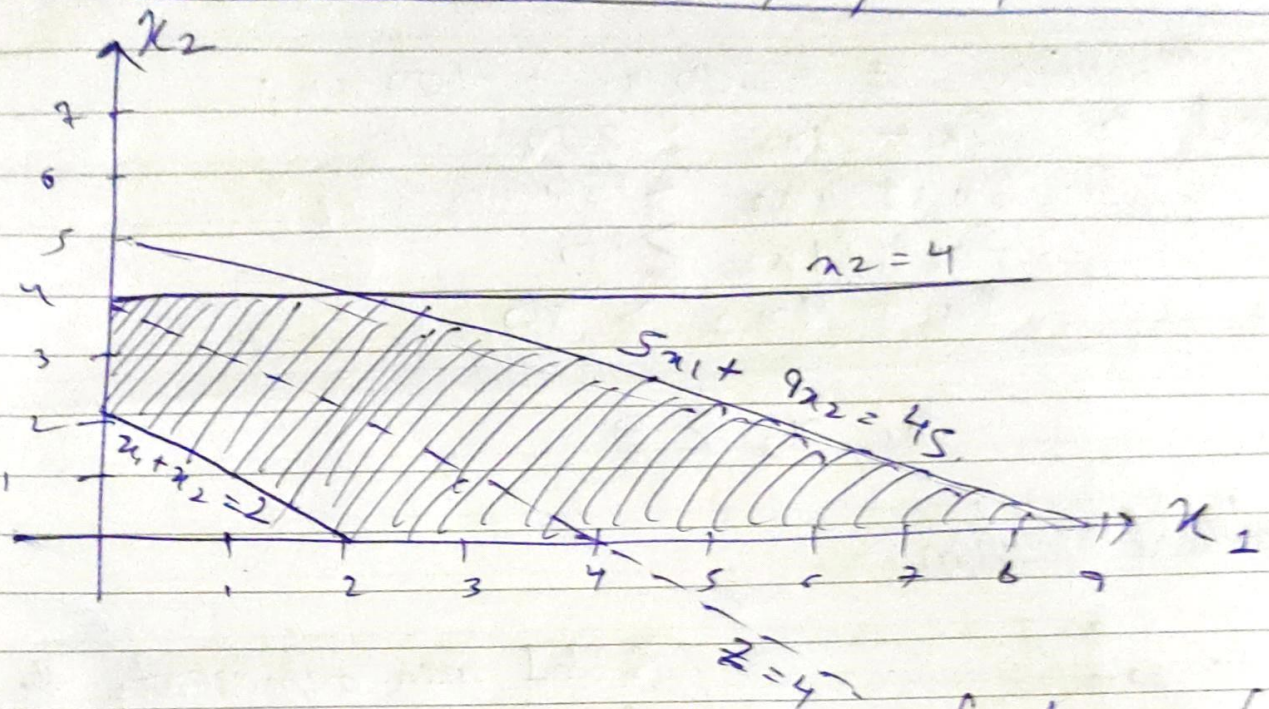
$$x_{B_4} = B_4^{-1} b = \frac{1}{7} \begin{pmatrix} -1 & -3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 6 \\ 10 \end{pmatrix} = \begin{pmatrix} -36/7 \\ 2/7 \end{pmatrix}$$

$\Rightarrow$ Basic Solution $= \left(0, \frac{-36}{7}, \frac{2}{7}, 0 \right)$.

$$x_{B6} = \frac{1}{7}\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 2/7 \\ 36/7 \end{pmatrix}$$

→ Basic Solution = $(0, 0, 2/7, 36/7)$.

9)



The line given by the objective function for $Z = 4$ when moved parallel to itself towards the ~~region~~ origin (as this is a minimization prob). coincides with the boundary line $x_1 + x_2 = 2$. and thus prob has ∞ no. of Solns.

$\therefore$ $Z_{min} = 2$ but the number of solutions is infinite.

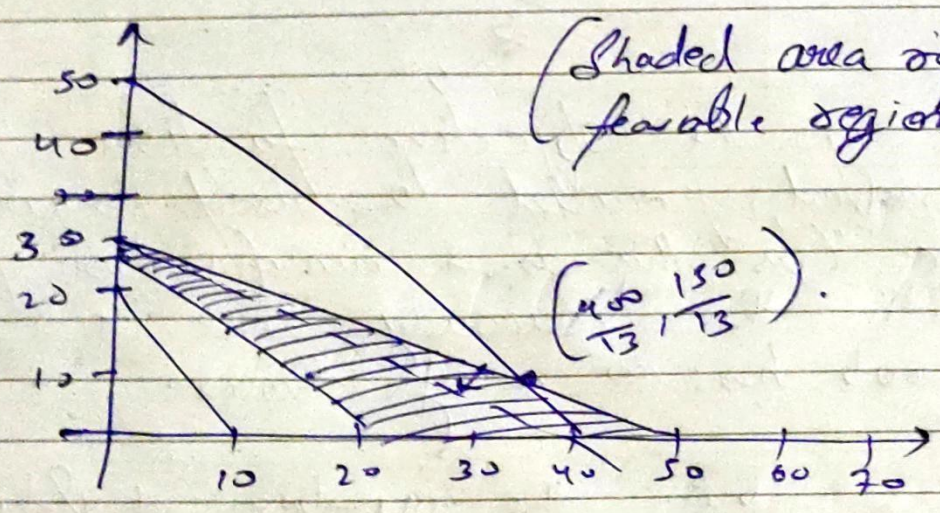
10) Let x_1 and x_2 productions runs be undertaken for the process 1 and 2 respectively, so that our aim is to:

$$\text{Minimize } Z = 300x_1 + 400x_2.$$

subject to
$$5x_1 + 4x_2 \leq 200$$
$$3x_1 + 5x_2 \leq 150$$
$$5x_1 + 4x_2 \geq 100$$
$$8x_1 + 4x_2 \geq 80$$

$$x_1, x_2 \geq 0$$

Graphical Solution :-



(Shaded area represents the feasible region.)

The line given by the objective func^n for $Z = 12000$ when moved parallel to itself away from origin finally encountes $\left(\frac{400}{13}, \frac{150}{13}\right)$ in the feasible region.

$$\therefore Z_{min} = 300 \times \frac{400}{13} + 400 \times \frac{150}{13} = \frac{180000}{13}.$$

The optimal Solution $\Rightarrow x_1 = \frac{400}{13}$, $x_2 = \frac{150}{13}$

Maximum profit = Rs $\frac{18000}{13}$

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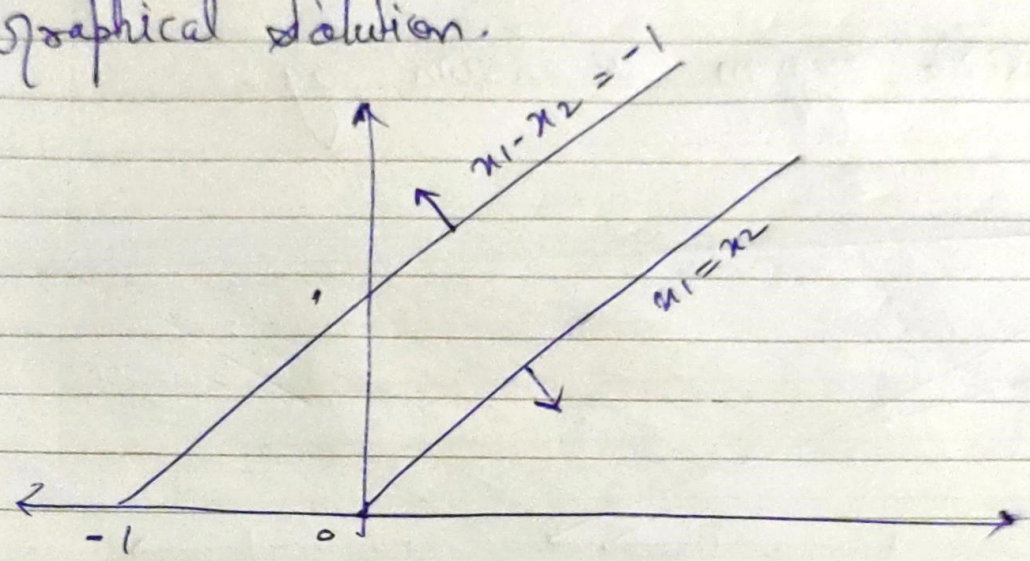
11) Graphical Solution.



Therefore, there is no feasible solution set defined by the constraints.

$\Rightarrow$ The given LPP has no feasible solutions.

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12) Let x_1 and x_2 be produced.

We formulate the given problem mathematically:-

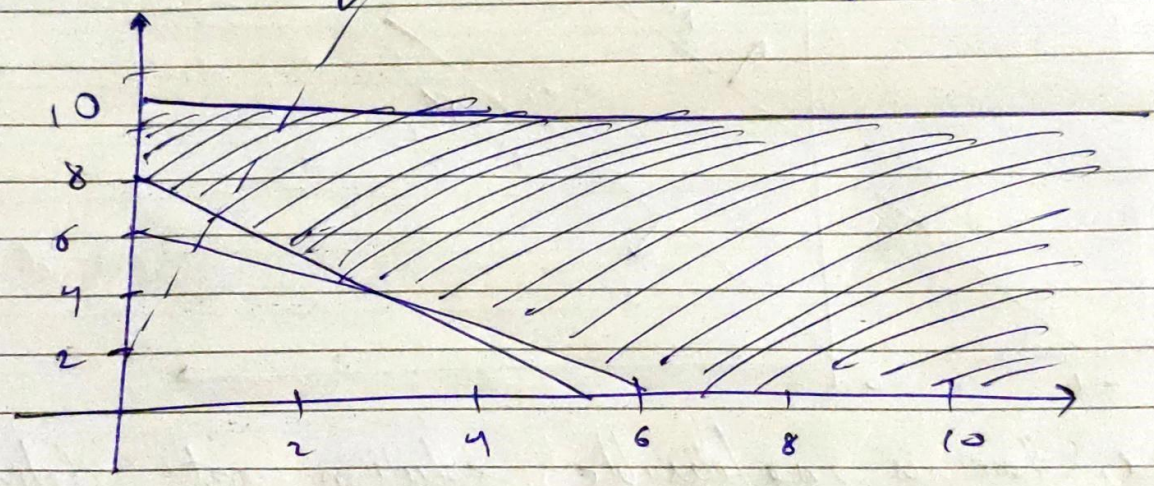
$$\text{Max } Z = 15x_1 + 12x_2$$

s.to :

$$6x_1 + 5x_2 \leq 3600$$
$$3x_1 + 5x_2 \leq 2500$$
$$8x_1 + 5x_2 \leq 4400$$

$$x_1, x_2 \geq 0.$$

13) The feasible region is shown by :-



In this case the region is not a closed polygon. The dotted line represents the objective function.

It is seen that there is no finite minimum value of Z within the region.

The problem have a Unbounded Solution.