

Tutorial-2

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①

Using Binomial Probability,

$$P(X=K) = \binom{n}{K} \cdot p^K \cdot q^{n-K} = \frac{n!}{K!(n-K)!} p^K (1-p)^{n-K}$$

Addition Rule:

$$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) \quad \text{--- ①}$$

Complement Rule

$$P(A^c) = P(\text{not } A) = 1 - P(A) \quad \text{--- ②}$$

Num of trials = 9 = n

Probability = $P = 99\% = 0.99$ ← ③

(a) At least 1 alarm is triggered

Evaluation the definition of B.P at $K=0$.

$$P(X=0) = \binom{9}{0} \times 0.99^0 \cdot (1-0.99)^{9-0} = \frac{9!}{0!(9-0)!} \cdot (0.99)^0 \cdot (0.01)^9$$

= 0

using ③

$$P(X \geq 1) = 1 - P(X=0) = 1$$

(b) Evaluate More than 7 alarms are triggered:

No. of trials = $n = 9$

$p = 0.99$.

$K = 8, 7$.

$$P(X=8) = {}^9C_8 \cdot (0.99)^8 \cdot (1-0.99)^{9-8} = 9 \times 0.083$$

$$P(X=9) = {}^9C_9 \cdot (0.99)^9 \cdot (1-0.99)^{9-9} = 0.9135$$

We will use addition Rule:-

$$\begin{aligned} P(X > 7) &= P(X=8) + P(X=9) \\ &= 0.083 + 0.9135 \\ &= 0.9965. \end{aligned}$$

$$(c) P(X=9) = {}^9C_9 \cdot (0.99)^9 \cdot (1-0.99)^{9-9} = 0.9135.$$

$$\begin{aligned} P(X \leq 8) &= 1 - P(X=9) \\ &= 1 - 0.9135 \\ &= 0.0865. \end{aligned}$$

2)

$n =$ No. of trials = 4.

$p =$ Probability = 30% = 0.3.

$$(a) P(X=4) = {}^4C_4 \cdot 0.3^4 \cdot (1-0.3)^{4-4} = 0.0081$$

$$(b) P(X=1) = {}^4C_1 \cdot 0.3^1 \cdot (1-0.3)^{4-1} = 0.4116$$

$$(c) P(X=0) = ({}^4C_0 \times 0.3^0) \times (1-0.3)^{4-0} = 0.2401$$

(3)

(4)

$n = \text{no. of trials} = 8$

$p = \text{probability} = 0.6$

$$(a) P(X=8) = {}^8C_8 \cdot (0.6)^8 \cdot (1-0.6)^{8-8} = 0.016796$$

(b) $P(X=8) = 0.016796$ is correct, because we note that row starting with 8 of the probability density function contains 0.016796.

(c) ~~Find~~ Probability $P(X \leq 7)$

$$P(X \leq 7) = 0.98320$$

(5) $n = \text{No. of trials} = 25$
 $p = \text{probability} = 40\% = 0.40$

(a) Mean (μ) = $n \times p$
 $= 25 \times 0.4 = 10$

Variance (σ) = npq
 $= np(1-p) = 25(0.4)(1-0.4)$
 $= 6$
 $\sigma = \sqrt{6} = 2.4495$

(b) Let us determine value of $\mu \pm 2\sigma$.

$$\mu - 2\sigma = 10 - 2(2.4495) = 5.1010$$

$$\mu + 2\sigma = 10 + 2(2.4495) = 14.8990$$

It contains value from 6 to 14 (Integers).

(c) $P(6 \leq x \leq 14) = P(x=6) + P(x=7) + P(x=8) + P(x=9)$
 $+ P(x=10) + P(x=11) + P(x=12) + P(x=13)$
 $+ P(x=14)$

~~≈ 0~~

$$= 0.9364$$

$$\textcircled{6} \quad \int_{-\infty}^{\infty} f(y) dy = 1$$

$$\begin{aligned} \Rightarrow 1 &= \int_{-\infty}^{\infty} f(y) dy = \int_0^1 K y^4 (1-y^3) dy \\ &= K \int_0^1 (y^4 - 3y^5 + 3y^6 - y^8) dy \\ &= K \left[\frac{1}{5} y^5 - \frac{1}{2} y^6 + \frac{3}{7} y^7 - \frac{1}{9} y^9 \right]_0^1 \\ &= K \left(\frac{1}{5} - \frac{1}{2} + \frac{3}{7} - \frac{1}{9} \right) = \frac{K}{280} \end{aligned}$$

$$(b) \quad P(Y < 0.6) = P(0 \leq Y \leq 0.6)$$

$$\begin{aligned} &= \int_0^{0.6} 280 y^4 (1-y^3) dy \\ &= 280 \left[\frac{1}{5} y^5 - \frac{1}{2} y^6 + \frac{3}{7} y^7 - \frac{1}{9} y^9 \right]_0^{0.6} \\ &= 280 \left(\frac{1}{5} \cdot 0.6^5 - \frac{1}{2} \cdot 0.6^6 + \frac{3}{7} \cdot 0.6^7 - \frac{1}{9} \cdot 0.6^9 \right) \end{aligned}$$

$$\begin{aligned} (c) \quad P(Y \geq 0.9) &= P(Y \leq 0.9) \\ &= 1 - P(0 \leq Y \leq 0.9) \end{aligned}$$

$$\begin{aligned} &= 1 - \int_0^{0.9} 280 y^4 (1-y^3) dy \\ &= 1 - 280 \left[\frac{1}{5} y^5 - \frac{1}{2} y^6 + \frac{3}{7} y^7 - \frac{1}{9} y^9 \right]_0^{0.9} \\ &= 1 - 280 \left(\frac{1}{5} \cdot 0.9^5 - \frac{1}{2} \cdot 0.9^6 + \frac{3}{7} \cdot 0.9^7 - \frac{1}{9} \cdot 0.9^9 \right) \end{aligned}$$

$$=$$