Dual Simplex Method :

In the simplex method, we start with an initial BFS and maintain the feasiblity of the basic solⁿ ($x_B \geq 0$) at each iteration. The basis is changed untill the optimality condition is satisfied.

In dual simplex method, the optimality condition is checked at each iteration and basis is changed untill feasibility condition of the basic solution is satisfied.

Hence dual simplex method is like a mirror image of simplex method.

Dual simplex method solves an LPP with negative element in 'b' rows; but it does not require artificial variables. This reduces a lot of labour and other difficulties to handle artificial variable.

The set of basic solutions of $Ax = b$ with $z_j - c_j \geq 0 \; \forall j$ depends only on the vectors a_j and c_j and not on the requirement vector b.

Here although a basic solution will be optimal for which $z_j - c_j \geq 0 \; \forall j$ but a basic solution may not be feasible for which $z_j - c_j \geq 0 \; \forall j$.

Dual simplex method gives an algorithm in which we start with a Basic Optimal solution of the simplex for which the optimality ($z_j - c_j \geq 0$) condition is satisfied but feasibility conditions ($X_B \geq 0$) are not satisfied. Then it decreases the number of negative variabl at each iteration while maintaining the optimality. Optimal solution (if exists) can be obtained within finite number of steps.

<u>Drawback</u>

The dual simplex procedure can be applied only for those problems which form an optimal and infeasible soln at the initial stage.

Since sometimes it is difficult to obtai an initial basic solution for which the optimality condition is satisfied, this method may not be used for general purpose LPP.

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Steps in dual Simplex Method:

1. If the given problem is a minimization type, reduce it to maximization type.

2. Convert ~~,~~ '$\geq$' type inequality constraint into '$\leq$' type by multiplying (-1).

3. ~~At~~ Introduce slack variables to form basis vector of the problem and construct usual simplex table.

Note that initial basic solution need not be feasible (i.e. X_{Bi} may be negative)

4. Compute net evaluation $z_j - c_j$

(I) If $z_j - c_j \geq 0 \ \forall j$, and $X_{Bi} \geq 0 \ \forall i$ Corresponding solution is optimal and basic solution.

(II) If atleast one $z_j - c_j < 0 \Rightarrow$ is not Dual Simplex method applicable.

(III) [Because dual simplex needs optimal soln from initial table and moves towards the feasible soln)

(III) If $z_j - c_j \geq 0 \ \forall j$ and atleast one $X_{Bi} < 0$, go to next step.

5. The vector to be removed from basis is determined first is by selecting most -ve x_{B_i} i.e.

$$x_{Br} = \underset{i}{\mathrm{Min}} \{ x_{Bi},\ x_{Bi} < 0 \}$$

so that a_r leaves the basis and x_{Br} becomes zero.

6. Check the nature of $y_{rj}\ \forall j$

(i) If $y_{rj} \geq 0\ \forall j \Rightarrow$ No feasible solⁿ.

(ii) If $y_{rj} < 0$ for atleast one j, then compute $\frac{z_k - c_k}{y_{rk}} = \mathrm{Max} \left\{ \frac{z_j - c_j}{y_{rj}},\ y_{rj} < 0 \right\}$

corresponding column vector a_k enters the basis B.

The usual simplex transformation formula is used for the ~~to~~ transformation until a basic solution is obtained for which $x_B \geq 0$ and finally we get the optimal solution.

Example 1: Solve the following LPP using Dual Simplex method.

$$\mathrm{Min}\ z = x_1 + x_2$$

S.t. $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0$

Converting it into maximization type and Introducing slack variables; we obtain

$$\text{Max } z^* = -x_1 - x_2 + 0x_3 + 0x_4$$

$$\text{s.t. } -2x_1 - x_2 + x_3 = -4$$

$$-x_1 - 7x_2 + x_4 = -7$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

x_3, x_4 forms initial basis.

			c_j	-1	-1	0	0	
C_B	B	x_B	b	a_1	a_2	a_3	a_4	
0	a_3	x_3	-4	-2	-1	1	0	
0	a_4	x_4	-7	-1	$\boxed{-7}$	0	1	$\rightarrow$
	$z_j - c_j$			1	1	0	0	
					$\uparrow$			

$z_j - c_j \geq 0 \ \forall j.$

$x_{B_1} = x_3 = -4$

$x_{B_2} = x_4 = -7$

$x_{Br} = \min \{x_3, x_4 \,\&\, x_i < 0\}$

$= \min \{-4, -7\} = -7 \rightarrow x_4$

$\therefore x_4$ leaves basis.

$$\max_j \left\{ \frac{z_j - c_j}{y_{2j}}, \ y_{2j} < 0 \right\}$$

$$= \max \left\{ \frac{z_1 - c_1}{y_{21}}, \frac{z_2 - c_2}{y_{22}} \right\}$$

$$= \max \left\{ \frac{1}{-1}, \frac{1}{-7} \right\} = -\frac{1}{7} \rightarrow a_2$$

$\therefore x_2$ enters ~~to~~ basis

C_B	B	x_B	c_j / b	-1 / a_1	-1 / a_2	0 / a_3	0 / a_4
0	a_3	x_3	$\boxed{-3}$	$\boxed{-13/7}$	0	1	$-1/7$
-1	a_2	x_2	1	$1/7$	1	0	$-1/7$
		$z_j - c_j$		$6/7$	0	0	$1/7$

$z_j - c_j \geq 0 \ \forall j$

$\text{Min}\{x_2, x_3\} = \text{Min}\{-3, 1\} = -3$

x_3 leave the basis

$$\underset{j}{\text{Max}}\left\{\frac{z_j - c_j}{y_{1j}}, y_{1j} < 0\right\} = \underset{j}{\text{Max}}\left\{\frac{6/7}{-13/7}, \frac{1/7}{-1/7}\right\}$$

$$= -\frac{6}{13}.$$

x_1 enters the basis.

C_B	B	x_B	c_j / b	a_1	a_2	a_3	a_4
-1	a_1	x_1	$21/13$	1	0	$-7/13$	$1/13$
-1	a_2	x_2	$10/13$	0	1	$1/13$	$-2/13$
		$z_j - c_j$		0	0	$6/13$	$1/13$

$z_j - c_j \geq 0 \ \forall j$ and $x_{Bi} \geq 0 \ \forall i$.

Optimal solution is

$x_1^* = 21/13, \quad x_2^* = 10/13.$

$$z^*_{max} = -21/13 - 10/13 = -\frac{31}{13}$$

$$z_{min} = -z^*_{max} = \frac{31}{13}.$$