Two phase Method

Big M method involves a high penalty parameter M > 0, which is sufficient large, for the artificial variable. When we have more number of variables or ~~cont~~ constraints in practical life, it becomes an issue what should be the possible value of M, during implementation. So it becomes computational difficult. To overcome this particular method, there is another artificial variable technique: called Two phase method.

Two phase simplex method consists of two phases:

Phase 1:

(i) Convert each of the constraints ~~with~~ into equality constraints using slack, surplus and artificial variables and write the LPP in standard form.

(ii) We assume a new auxiliary objective function constructed as:

$$\text{Max } z^* = 0 \cdot x_1 + 0 \cdot x_2 + \cdots + 0 \cdot x_n - 1 \cdot x_{a_1} - 1 \cdot x_{a_2} - \cdots - 1 \cdot x_{a_m}$$

(-1) is the price added for each of the artificial variables $x_{a_1}, x_{a_2}, \cdots x_{a_m}$. 0 price is assigned to each of the variable $x_1, x_2, \cdots x_n$ including slack and surplus variables.

$\therefore$ Max $z^* = 0$, if all the artificial variables are zero.

or Max $z^* < 0$, if atleast one artificial variable is positive.

(iii) Apply simplex algorithm to solve LPP.

(iv) Suppose $z_j - c_j \geq 0$ at the end of phase I. Three cases may arise.

(a) Max $z^* = 0 \Rightarrow$ all the artificial variable may disappear from basis and we get a BFS.

(b) Max $z^* = 0 \Rightarrow$ one or more artificial variable may appear in the basis with zero value. We have a BFS of the problem but there may have redundancy in the original constraint equation.

(c) Max $z^* < 0 \Rightarrow$ One or more artificial variable appear in the final basis with positive value. No BFS of the original problem.

* BFS: Basic feasible solution.

Phase 2:

When iteration of phase 1 ends with either (IV) - (a) or (IV) - (b) conclusion, then we go to phase 2 to obtain the optimum value of the ~~p~~ objection function.

Assign actual coeffecients of the variables including slack and surplus variables and zero coeffecient value to any artificial variables present in the basis of the last table of phase 1. Also remove the artificial variables which are not present in the last table of in the basis of the last table of phase-1.

Then apply simplex algorithm to obtain ~~b~~ the optimum solution.

Example 1. Solve the following ~~simplex~~ LPP ~~Method~~ using Two Phase Simplex method.

$$\text{Max } Z = 2x_1 + x_2 + 3x_3$$

Subject to

$$x_1 + x_2 + 2x_3 \le 5$$

$$2x_1 + 3x_2 + 4x_3 = 12$$

$$x_1, x_2, x_3 \ge 0.$$

→ Introducing slack and artificial variables and converting the LPP to standard form, we get

$$\text{Max } Z = 2x_1 + x_2 + x_3 + 0\cdot x_4 + 0\cdot x_5$$

s.t. $x_1 + x_2 + 2x_3 + x_4 + 0x_5 = 5$

$2x_1 + 3x_2 + 4x_3 + 0x_4 + x_5 = 12,$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

where x_4 : slack variable.

x_5 : Artificial variable.

Phase 1 :

Construct the auxiliary objective function

$$\text{Max } Z_A = 0\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 + 0\cdot x_4 - 1\cdot x_5$$

Table-1

C_B	B	x_B	C_j / b	0 / a_1	0 / a_2	0 / a_3	0 / a_4	-1 / a_5	Min ratio
-1	a_5	x_5	12	2	3	4	0	1	$12/4=3$
0	a_4	x_4	5	1	1	$\boxed{2}$	1	0	$5/2=2.5$ →
		$Z_j - C_j$		-2	-3	-4 ↑	0	0	

x_4 → leaving variable

x_3 → entering variable

Table-2

C_B	B	x_B	C_j / b	0 / a_1	0 / a_2	0 / a_3	0 / a_4	-1 / a_5	Min ratio
-1	a_5	x_5	2	0	$\boxed{1}$	0	-2	1	$2/1=2$
0	a_3	x_3	5/2	1/2	1/2	1	1/2	0	$5/2 / 1/2 = 5$
		$Z_j - C_j$		0	-1 ↑	0	2	0	

x_5 → leaving variable.

x_2 → entering variable.

Table-3

c_B	B	x_B	c_j / b	0 / a_1	0 / a_2	0 / a_3	0 / a_4	-1 / a_5
0	a_2	x_2	2	0	1	0	-2	1
0	a_3	x_3	$\frac{3}{2}$	$\frac{1}{2}$	0	1	$\frac{3}{2}$	$-\frac{1}{2}$
			$z_j - c_j$	0	0	0	0	1

$\therefore$ $z_j - c_j \geq 0$ $\forall j$. No artificial variable presents in the basis. Next we will go to phase 2. Since a_5 is not present in basis, we will not write a_5 in phase 2.

Phase 2

Table-1

c_B	B	x_B	c_j / b	2 / a_1	1 / a_2	3 / a_3	0 / a_4	Min ratio
1	a_2	x_2	2	0	1	0	-2	—
3	a_3	x_3	3/2	$\boxed{1/2}$	0	1	3/2	3 $\rightarrow$
			$z_j - c_j$	$-1/2$	0	0	5/2	

$x_1 \rightarrow$ entering variable

$x_3 \rightarrow$ leaving variable

Table-2

c_B	B	x_B	c_j / b	2 / a_1	1 / a_2	3 / a_3	0 / a_4
1	a_2	x_2	2	0	1	0	2
2	a_1	x_1	3	1	0	2	3
			$z_j - c_j$	0	0	1	8

$z_j - c_j \geq 0$ $\forall j$.

$\therefore$ $x_1^* = 3$, $x_2^* = 2$, $x_3^* = 0$

$$Z_{max} = 2 \times 3 + 1 \cdot 2 + 3 \cdot 0 = 8.$$

Example 2: Max $Z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$.

→ Introducing slack, surplus and artificial variables in LPP and converting into standard form, we get

$$\text{Max } Z = 3x_1 + 2x_2 + 0 \cdot x_3 + 0 \cdot x_4 + 0 \cdot x_5$$

S.t. $2x_1 + x_2 + x_3 = 2$

$3x_1 + 4x_2 - x_4 + x_5 = 12$

$x_1, x_2, x_3, x_4, x_5 \geq 0$.

x_3 → slack variables

x_4 → surplus variables, x_5 → artificial variable

Phase 1 $\quad Z_A = 0x_1 + 0\, x_2 + 0 \cdot x_3 + 0 \cdot x_4 - 1 \cdot x_5$

Table-1

		c_j		0	0	0	0	-1	Min
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5	Ratio
-1	a_5	x_5	12	3	4	0	-1	1	12/4=3
0	a_3	x_3	2	2	$\boxed{1}$	1	0	0	2/1=2 →
	$Z_j - C_j$			-3	-4	0	1	0	
					↑				

x_3 → leaving variable

x_2 → entering variable

		c_j		0	0	0	0	-1
C_B	B	x_B	b	a_1	a_2	a_3	a_4	a_5
-1	a_5	x_5	4	-5	0	-1	-1	1
0	a_2	x_2	2	2	1	1	0	0
		$Z_j - c_j$		5	0	4	1	0

$Z_j - c_j \geq 0 \ \forall j$. But the artificial variable x_5 is present in basis at positive level i.e. $x_5 = 4$. So the problem has no feasible solution.

Problem Set: Use two phase method to solve the following problem:

(1)

Max $Z = 3x_1 - x_2$

S.t. $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 2$

$x_1 \leq 4$

$x_1, x_2 \geq 0$

(Ans: $x_1^* = 2$, $x_2^* = 0$
$Z_{max} = 6$)

(2)

Max $Z = 2x_1 + x_2 + x_3$

S.t. $4x_1 + 6x_2 + 3x_3 \leq 8$

$3x_1 - 6x_2 - 4x_3 \leq 1$

$2x_1 + 3x_2 - 5x_3 \geq 4$

$x_1, x_2, x_3 \geq 0$

[Ans: $x_1^* = \frac{9}{7}$, $x_2^* = \frac{10}{21}$,
$x_3^* = 0$, $Z_{max} = \frac{64}{21}$]

(3) Max $Z = 5x_1 + 3x_2$

Subject to $2x_1 + x_2 \leq 1$

$3x_1 + 4x_2 \geq 12$

$x_1, x_2 \geq 0$. [No feasible solution]