

Solⁿ 1:- Here there is two can C_1 and C_2 namely
~~Info(D)~~ = "yes" and "no"

$$C_1 = \text{Yes} = 9 \quad ; \quad C_2 = \text{No} = 5.$$

So,

$$\text{Info}(D) = - \sum p_i \log_2(p_i)$$

$$\text{Info}(D) = -\frac{9}{14} \log_2\left(\frac{9}{14}\right) - \frac{5}{14} \log_2\left(\frac{5}{14}\right)$$

$$\boxed{\text{Info}(D) = \cancel{0.409} \quad 0.940}$$

1) Information required for age attribute

$$\text{Info}_A(D) = \sum_{j=1}^n \frac{|D_j|}{D} \times \text{Info}(D)$$

Age attribute has three distribution
Youth, Middle Aged and Senior
respectively.

So,

$$\text{Info}_A(D) = \frac{5}{14} \times \left(-\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right) \right) +$$
$$\frac{4}{14} \times \left(-\frac{4}{4} \log_2\left(\frac{4}{4}\right) \right) + \frac{5}{14} \times \left(-\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) \right)$$

$$\text{Info}_A(D) = 0.347 + 0 + 0.347$$

$$\boxed{\text{Info}_A(D) = 0.694}$$

Age attribute

2) Information required for Income attribute

$$\text{Info}_I(D) = \sum_{j=1}^N \frac{|D_j|}{D} \times \text{Info}(D)$$

~~Info_I~~(D): Income has three distribution
high, Medium and Low respectively
So,

$$\text{Info}_I(D) = \frac{4}{14} \times \left(-\frac{2}{4} \log_2 \left(\frac{2}{4} \right) - \frac{2}{4} \log_2 \left(\frac{2}{4} \right) \right) +$$

$$\frac{6}{14} \times \left(-\frac{4}{6} \log_2 \left(\frac{4}{6} \right) - \frac{2}{6} \log_2 \left(\frac{2}{6} \right) \right) +$$

$$\frac{4}{14} \times \left(-\frac{3}{4} \log_2 \left(\frac{3}{4} \right) - \frac{1}{4} \log_2 \left(\frac{1}{4} \right) \right)$$

$$\boxed{\text{Info}_I(D) = 0.912}$$

3) Student information required

$$Info_S(D) = \sum_{j=1}^J \frac{|D_j|}{D} \times Info(D)$$

Student has two distribution "Yes" and "No" respectively,
so,

$$Info_S(D) = \sum_{j=1}^J \frac{|D_j|}{D} \times Info(D)$$

$$Info_S(D) = \frac{7}{14} \times \left(-\frac{3}{7} \log_2 \left(\frac{3}{7} \right) - \frac{4}{7} \log_2 \left(\frac{4}{7} \right) \right) +$$

$$\frac{7}{14} \times \left(-\frac{6}{7} \log_2 \left(\frac{6}{7} \right) - \frac{1}{7} \log_2 \left(\frac{1}{7} \right) \right)$$

Info

$$Info_S(D) = 0.789$$

4) Credit-rating information required.

$$\text{Info}_C(D) = \sum_{j=1}^x \frac{|D_j|}{D} \times \text{Info}_j(D)$$

Credit-rating has two distribution "fair" and "excellent" respectively.
So,

$$\begin{aligned}\text{Info}_C(D) &= \frac{8}{14} \times \left(-\frac{6}{8} \log_2 \left(\frac{6}{8} \right) - \frac{2}{8} \log_2 \left(\frac{2}{8} \right) \right) \\ &\quad + \frac{6}{14} \times \left(-\frac{3}{6} \log_2 \left(\frac{3}{6} \right) - \frac{3}{6} \log_2 \left(\frac{3}{6} \right) \right)\end{aligned}$$

$$\text{Info}_C(D) = 0.464 + 0.429.$$

$$\boxed{\text{Info}_C(D) = 0.893}$$

Now the gain of all attributes

$$\begin{aligned}\text{Gain}(\text{age}) &= \text{Info}(D) - \text{Info}_A(D) \\ \text{Gain}(\text{age}) &= 0.940 - 0.694\end{aligned}$$

$$\boxed{\text{Gain}(\text{age}) = 0.246}$$

$$2) \text{ Gain (income)} = \text{Info}(D) - \text{Info}_2(D)$$

$$= 0.940 - 0.912$$

$$\boxed{\text{Gain (income)} = 0.028}$$

$$3) \text{ Gain (student)} = \text{Info}(D) - \text{Info}_c(D)$$

$$= 0.940 - 0.789$$

$$\boxed{\text{Gain (student)} = 0.151}$$

$$4) \text{ Gain (Credit-rating)} = \text{Info}(D) - \text{Info}_{cc}(D)$$

$$\text{Gain (credit-rating)} = 0.940 - 0.893$$

$$\boxed{\text{Gain (credit-rating)} = 0.047}$$

on comparing all the gain's of all 4 attributes "age" is the perfect root node.

Decision-Tree

