Regular Expression

Regular Expression

⇒ Regular Expressions are used for representing certain sets of strings in an algebraic fashion.

① Any terminal symbol i.e. symbols $\in \Sigma$ including Λ and ϕ are regular expressions. $a, b, c, \ldots \Lambda, \phi$

② The union of two regular expressions is also a regular expression. $R_1, R_2 \to (R_1 + R_2)$

③ The concatenation of two regular expressions is also a regular expression. $R_1, R_2 \to (R_1 \cdot R_2)$

④ The iteration (or closure) of a regular expression is also a regular expression. $R \to R^*$ $a^* = \Lambda, a, aa \ldots$

⑤ The regular expression over Σ are precisely those obtained recursively by the application of the above rules once or several times.

Regular Expression - Examples

⇒ Describe the following sets as Regular Expressions

1) $\{0, 1, 2\}$ 0 or 1 or 2

   $R = 0 + 1 + 2$

2) $\{\Lambda, ab\}$

   $R = \Lambda\, ab$

3) $\{abb, a, b, bba\}$ abb or a or b or bba

   $R = abb + a + b + bba$

4) $\{\Lambda, 0, 00, 000, ---\}$ closure of 0

   $R = 0^*$

5) $\{1, 11, 111, 1111, ----\}$

   $R = 1^+$

Identities of Regular Expression

1) $\phi + R = R$

2) $\phi R + R\phi = \phi$

3) $\epsilon R = R\epsilon = R$

4) $\epsilon^* = \epsilon$ and $\phi^* = \epsilon$

5) $R + R = R$

6) $R^* R^* = R^*$

7) $RR^* = R^* R$

8) $(R^*)^* = R^*$

9) $\epsilon + RR^* = \epsilon + R^* R = R^*$

$R^+ \cup \epsilon = R^*$

10) $(PQ)^* P = P(QP)^*$

11) $(P+Q)^* = (P^* Q^*)^* = (P^* + Q^*)^*$

12) $(P+Q)R = PR + QR$ and

$R(P+Q) = RP + RQ$

ARDEN'S THEOREM

⇒ If P and Q are Two Regular Expressions over Σ, and if P does not Contain ϵ, then the following equation in R given by $\boxed{R = Q + RP}$ has a unique solution i.e. $\boxed{R = QP^*}$

$$R = Q + RP \longrightarrow (1)$$

$$= Q + QP^*P$$

$$= Q(\epsilon + P^*P)$$

$$= QP^*$$

$$R = QP^*$$

$$[\epsilon + R^*R = R^*]$$

Proved that $R = QP^*$ is a ~~soutu~~ solution.

⇒ Prove that $R = QP^*$ is a unique solution.

$$R = Q + RP$$

$$= Q + [Q + RP]P$$

$$= Q + QP + RP^2$$

$$= Q + QP + [Q + RP]P^2$$

$$= Q + QP + QP^2 + RP^3$$

$$\vdots$$

$$= Q + QP + QP^2 + \dots QP^n + RP^{n+1}$$

$$[R = QP^*]$$

$$= Q + QP + QP^2 + \dots QP^n + QP^*P^{n+1}$$

$$= Q[\epsilon + P + P^2 + \dots P^n + P^*P^{n+1}]$$

$$R = QP^*$$

An Example Proof Using Identities of R E

⇒ Prove that $(1+00^*1)+(1+00^*1)(0+10^*1)^*(0+10^*1)$ is equal to $0^*1(0+10^*1)^*$

$$LHS = (1+00^*1)+(1+00^*1)(0+10^*1)^*(0+10^*1)$$

$$= (1+00^*1)[\epsilon+(0+10^*1)^*(0+10^*1)]$$

$\epsilon+R^*R = R^*$

$$= (1+00^*1)(0+10^*1)^*$$

$$= (\epsilon.1+00^*1)(0+10^*1)^*$$

$\epsilon.R = R$

$$= (\epsilon+00^*)1(0+10^*1)^*$$

$\epsilon+R^*R = R^*$

$$= 0^*1(0+10^*1)^* = \underline{\underline{RHS}}$$

Designing RE - Examples

⇒ Design RE for the following languages over $\{a, b\}$

1) Language accepting strings of length exactly 2
2) Language accepting strings of length atleast 2
3) Language accepting strings of length atmost 2

Soln

1) $L_1 = \{aa, ab, ba, bb\}$

$$R = aa + ab + ba + bb$$
$$= a(a+b) + b(a+b)$$
$$= (a+b)(a+b)$$

2) $L_2 = \{aa, ab, ba, bb, aaa, \ldots\}$

$$R = (a+b)(a+b)(a+b)^*$$

$* = 0, 1, 2, 3 \ldots$

3) $L_3 = \{\epsilon, a, b, aa, ab, ba, bb\}$

$$R = \epsilon + a + b + aa + ab + ba + bb$$
$$= (\epsilon + a + b)(\epsilon + a + b)$$