2

DC Networks and Network Theorems

TOPICS DISCUSSED

- Circuit and circuit elements
- Voltage and current sources
- Series and parallel circuits; Kirchhoff's laws
- Superposition theorem
- Thevenin's theorem
- Norton's theorem
- Millman's theorem
- Maximum power transfer theorem
- Star-delta transformation of resistances
- Transients in R–L and R–C circuits

2.1 INTRODUCTION

A network is an interconnection of elements, components, input signals, and output signals. The networks are of two types, viz active network and passive network. An active network contains one or more sources of supply whereas a passive network does not contain dissipated in a resistor is I^2Rt where I is the current flowing, R is the resistance value, and t is the time. Energy stored in a capacitor is $\frac{1}{2}CV^2$ where C is the capacitance of the capacitor and V is the potential across it. For an inductor, the energy stored is 1/2 L I^2 where L is the inductance and I is the current flowing through it.

The formulas used to calculate the value of R, L, and C are:

$$R = \rho\frac{\ell}{a};$$

[ρ is the resistivity, ℓ is the length and *a* is the area of cross section of the wire]

$$L = \frac{\mu N^2 A}{\ell};$$

[μ is the permeability, N is the number of turns, A is the area of the coil ℓ the length of the flux path]

$$C = \frac{\varepsilon A}{d};$$

[ε is the permittivity of the material between the two plates, A is the area of each plate, and d is the distance between the plates.

Analysis of networks or circuits involve calculation with respect to finding out current flowing through an element, voltage across a component, power dissipated or stored in a circuit component, etc.

Laws and theorems have been introduced to make the task of network analysis simpler. To solve a particular network problem, a number of alternative methods or theorems can be applied. Experience will guide us as to which one will be the quickest or easiest method to apply. In this chapter the circuit laws and theorems, voltage sources, various methods of connection of circuit components and their transformations, etc. will be discussed. Only dc networks will be taken up in this chapter.

2.2 DC NETWORK TERMINOLOGIES, VOLTAGE, AND CURRENT SOURCES

Before discussing various laws and theorems, certain terminologies related to dc networks are described first.

2.2.1 Network Terminologies

While discussing network theorems, laws, and electrical and electronic circuits, one often comes across the following terms.

1. **Circuit:** A conducting path through which an electric current either flows or is intended to flow is called a circuit.
2. **Electric network:** A combination of various circuit elements, connected in any manner, is called an electric network.
3. **Linear circuit:** The circuit whose parameters are constant, i.e., they do not change with application of voltage or current is called a linear circuit.
4. **Non linear circuit:** The circuit whose parameters change with the application of voltage or current is called a non linear circuit.
5. **Circuit parameters**: The various elements of an electric

7. **Unilateral circuit:** A unilateral circuit is one whose properties or characteristics change with the direction of its operation. E.g., diode rectifier.
8. **Active network:** An active network is one which containts one or more sources of EMF.
9. **Passive network:** A passive network is one which does not contain any source of EMF.
10. **Node:** A node is a junction in a circuit where two or more circuit elements are connected together.
11. **Branch:** The part of a network which lies between two junctions is called a branch.
12. **Loop:** A loop is a closed path in a network formed by a number of connected branches.
13. **Mesh:** Any path which contains no other paths within it is called a mesh. Thus, a loop contains meshes but a mesh does not contain a loop.
14. **Lumped circuit:** The circuits in which circuit elements can be represented mutually independent and not interconnected.

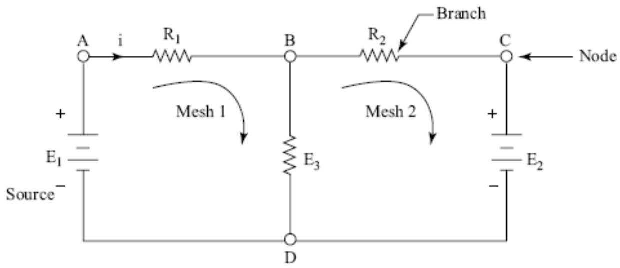


Figure 2.1 Different parts of an electric circuit

For convenience, the nodes are labelled by latters.

For example in Fig. 2.1,

No. of nodes, N = 4 (i.e., A, B, C, D)

No. of branches, B = 5 (i.e., AB, BC, BD, CD, AD)

Independent meshes, M = B – N + 1

= 5 – 4 + 1 = 2 (i.e., ABDA, BCDB)

No. of loop = 3 (i.e., ABDA, BCDB and ABCDA). It is seen that a loop ABCDA encloses two meshes, i.e., mesh 1 and mesh 2.

2.2.2 Voltage and Current Sources

A source is a device which converts mechanical, thermal, chemical or some other form of energy into electrical energy. There are two types of sources: voltage sources and current sources.

Voltage source

voltage source is shown in Fig. 2.2 (a). In an ideal voltage source the terminal voltage is independent of the load resistance, R_L connected. Whatever is the voltage of the source, the same voltage is available across the load terminals of R_L, i.e., $V_L = V_S$ under loading condition as shown in Fig. 2.2 (b). There is no drop of voltage in the source supplying current to the load. The internal resistance of the source is therefore, zero.

In a practical voltage source, there will be a drop in voltage available across the load due to voltage drop in the resistance of the source itself when a load is connected as shown in Fig. 2.2 (c).

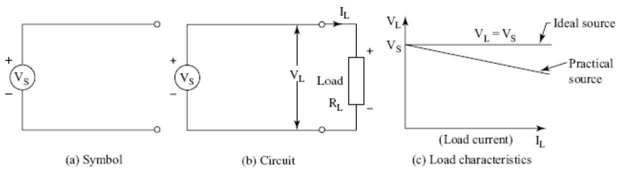


Figure 2.2 Voltage source and its characteristics

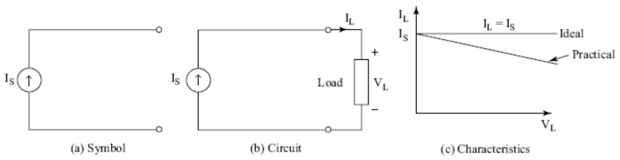


Figure 2.3 Current source and its characteristics

Current source

In certain applications a constant current flow through the circuit is required. When the load resistance is connected between the output terminals, a constant current I_L will flow through the load.

The examples of current sources are photo electric cells, collector current in transistors, etc. The symbol of current source is shown in Fig. 2.3

Practical voltage and current sources

A practical voltage source like a battery has the drooping load characteristics due to some internal resistance. A voltage source has small internal resistance in series while a current source has some high internal *resistance* in parallel.

For ideal voltage source $R_{se} = 0$

For ideal current source $R_{sh} = \infty$

A practical voltage source is shown as an ideal voltage source in series with a resistance. This resistance is called the internal resistance of the source as has been shown in Fig. 2.4 (a). A practical current source is shown as an ideal current source in parallel with its

V_L (open circuit), i.e., V_L (OC) = V_S that is, when the load R_L is removed, the circuit becomes an open circuit and the voltage across the source becomes the same as the voltage across the load terminals.

When the load is short circuited, the short-circuit current, I_L (SC) = V_S/R_{SE}.

In the same way, from Fig. 2.4 (b), we can write

$$V_L(OC) = I_{Sh} R_{Sh}$$

and $$I_L(SC) = I_S$$

In source transformation as discussed in section 2.2.3, we shall use the equivalence of open-circuit voltage and short-circuit current.

Independent and dependent sources

The magnitude of an independent source does not depend upon the current in the circuit or voltage across any other element in the circuit. The magnitude of a dependent source gets changed due to some other current or voltage in the circuit. An independent source is represented by a circle while a dependent source is represented by a diamond-shaped symbol. Dependent voltage sources are also called controlled sources.

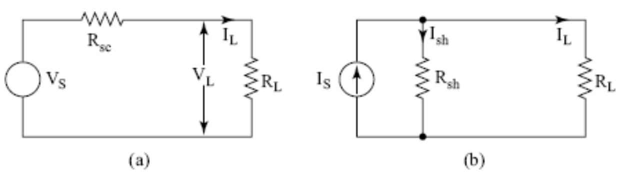


Figure 2.4 Representation as (a) practical voltage source (b) practical current source

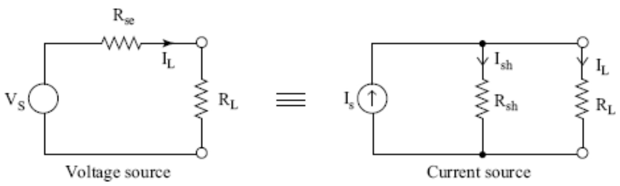


Figure 2.5 Equivalent current source

There are four kinds of dependent sources:

- voltage-controlled voltage source (vcvs)

Dependent voltage sources find applications in electronic circuits and devices.

2.2.3 Source Transformation

A voltage source can be represented as a current source. Similarly a current source can be represented as a voltage source. This often helps the solutions of circuit problems.

Voltage source into current source and current source into voltage source

A voltage source is equivalent to a current source and vice-versa if they produce equal values of I_L and V_L when connected to the load R_L. They should also provide the same open-circuit voltage and short-circuit current.

If voltage source is converted into current source as in Fig. 2.5, we consider the short circuit current equivalence then $I_s = \frac{V_s}{R_{se}}$.

[Short circuit current in the two equivalent circuits are respectively V_S/R_{Se} and I_S]

If current source converted into voltage source, as in Fig. 2.6, we consider the open-circuit voltage equivalence, then, $V_s = I_s R_{sh}$

A few examples will further clarify this concept.

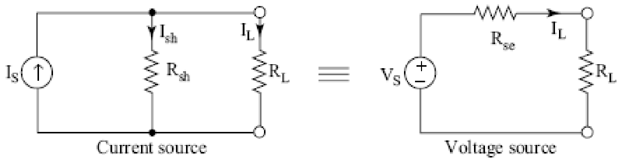


Figure 2.6 Equivalent voltage source

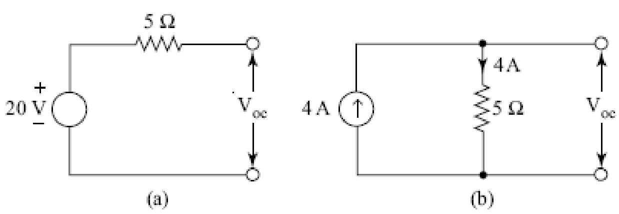


Figure 2.7 Conversion of a voltage source into a current source

Examples 2.1 Convert a voltage source of 20 volts with internal resistance of 5 Ω into an equivalent current source.

Solution:

The internal resistance will be the same as R_{se}

The condition for equivalence is checked from the following conditions viz v_{oc} should be same and I_{sc} should also be same.

In Fig. 2.7 (a), V_{oc} = 20 V.

In Fig. 2.7 (b), V_{oc} = 4 A × 5 Ω = 20 V.

I_{sc} in Fig. 2.7 (a), 4 A.

I_{sc} in Fig. 2.7 (b), 4 A.

This two circuits are equivalent because the open circuit voltage and short circuit current are the same in both the circuits.

Example 2.2 Convert a current source of 100 A with internal resistance of 10 Ω into an equivalent voltage source.

Solution:

Here

$$I = 100\ \text{A},\ R_{sh} = 10\ \Omega$$

For an equivalent voltage source

$$V = I \times R_{sh} = 100 \times 10 = 1000\ \text{V}$$
$$R_{sh} = R_{se} = 10\ \Omega \text{ in series}$$

The open circuit voltage and short circuit current are the same in the two equivalent circuits as shown in Fig. 2.8 (a) and 2.8 (b), respectively.

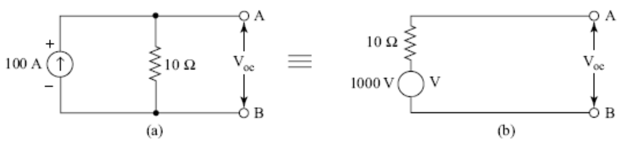


Figure 2.8 Conversion of a current source into an equivalent voltage source

2.3 SERIES–PARALLEL CIRCUITS

Resistances, capacitances, and inductances are often connected in series, in parallel, or a combination of series and parallel. We need to calculate the division of voltage and currents in such circuits as shown in Fig. 2.9. The circuit is called a series circuit.

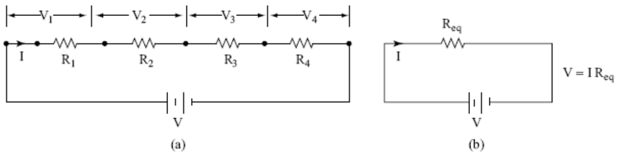


Figure 2.9 DC series circuit

The voltage drops across the resistances are V_1, V_2, V_3, and V_4, respectively. Since the same current is flowing through all the resistances, we can write

$$V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3, \text{ and } V_4 = IR_4$$

Again, the total voltage, V applied is equal to the sum of the voltage drops across the resistances,

Thus we can write

$$V = V_1 + V_2 + V_3 + V_4$$

To find the value of equivalent resistance of a number of resistances connected in series, we equate the voltage, V of the two equivalent in units as shown in Fig. 2.9 (a) and Fig. 2.9 (b) as

or, $$I\,R_{eq} = IR_1 + IR_2 + IR_3 + IR_4$$

or, $$R_{eq} = R_1 + R_2 + R_3 + R_4$$

Assuming R_{eq} as equal to R,

$$R = R_1 + R_2 + R_3 + R_4. \qquad (2.1)$$

Thus, when resistances are connected in series, the total equivalent resistance appearing across the supply can be taken as equal to the sum of the individual resistances.

2.3.2 Parallel Circuits

When a number of resistors are connected in such a way that both nected in parallel. The total current drawn from the battery is I. This current gets divided into I_1, I_2, I_3 such that $I = I_1 + I_2 + I_3$. As voltage V is appearing across each of these three resistors, applying Ohm's law we write

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \qquad \text{(i)}$$

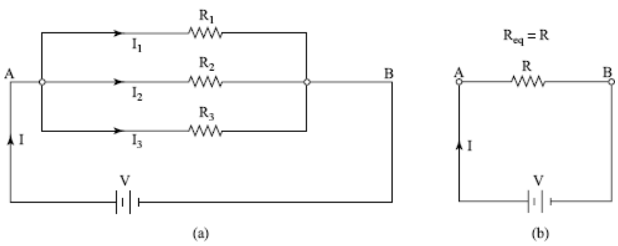


Figure 2.10 Parallel connection of resistors

Let the equivalent resistance of the three resistors connected in parallel across terminals A and B be R as shown in Fig. 2.10 (b). Then,

$$I = \frac{V}{R} \qquad \text{(ii)}$$

From (i) and (ii),

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

or,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general, if there are n resistors connected in parallel, the equivalent resistance R is expressed as

Figure 2.11 shows a number of resistors connected in series–parallel combinations. Here, two parallel branches and one resistance, all connected in series have been shown. To determine the equivalent resistance across the end terminals of the entire circuit, we first calculate the equivalent resistance of parallel branches and then put them in series along with any individual resistance already connected in series.

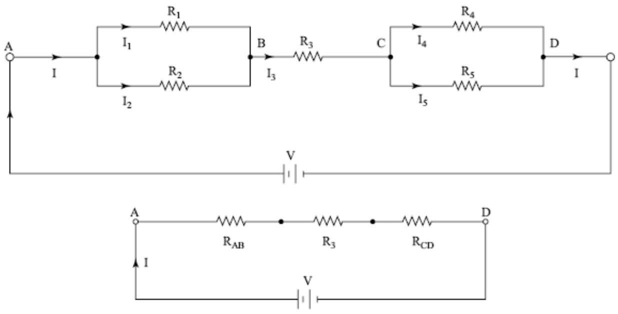


Figure 2.11 DC series–parallel circuit

$$\frac{1}{R_{AB}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

or,

$$R_{AB} = \frac{R_1 R_2}{R_1 + R_2}$$

and

$$\frac{1}{R_{CD}} = \frac{1}{R_4} + \frac{1}{R_5}$$

$$R_{CD} = \frac{R_4 R_5}{R_4 + R_5}$$

Total resistance, R = Series combination of R_{AB} + R_{BC} + R_{CD}

$$R = \frac{R_1 R_2}{\;} + R + \frac{R_4 R_5}{\;}$$

In any electrical circuit we will find a number of such resistances connected in series–parallel combinations.

2.4 VOLTAGE AND CURRENT DIVIDER RULES

2.4.1 Voltage Divider Rule

For easy calculation of voltage drop across resistors in a series circuit, a voltage divider rule is used which is illustrated in Fig. 2.12.

$$I = \frac{V}{R_1 + R_2 + R_3}$$

$$V_1 = I\,R_1 = \frac{V}{R_1 + R_2 + R_3} \times R_1 = \frac{V}{R_T} \times R_1$$

where

$$R_T = R(\text{Total}) = R_1 + R_2 + R_3$$

Similarly,

$$V_2 = I\,R_2 = \frac{V}{R_T} \times R_2$$

and

$$V_3 = I\,R_3 = \frac{V}{R_T} \times R_3$$

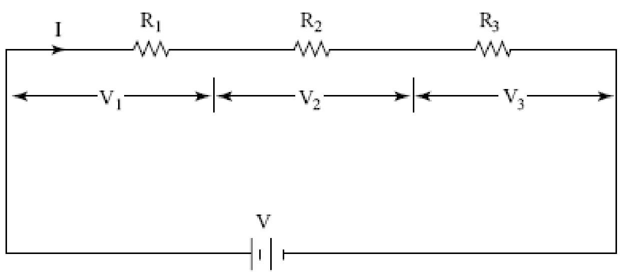


Figure 2.12 Voltage divider rule

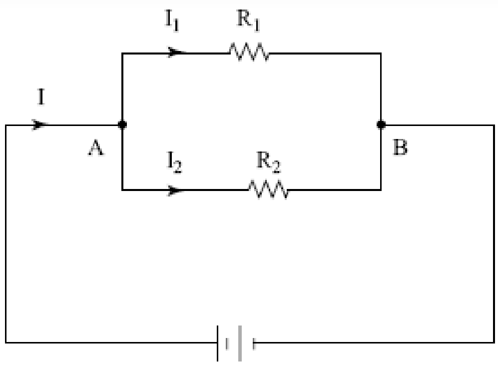


Figure 2.13 Current divider rule

Thus the voltage divider rule states that voltage drop across any resistor in a series circuit is proportional to the ratio of its resistance to the total resistance of the series circuit.

2.4.2 Current Divider Rule

Current divider rule is used in parallel circuits to find the branch currents if the total current is known. To illustrate, this rule is applied to two parallel branches as in Fig. 2.13.

$$V_{AB} = I_1R_1 = I_2R_2$$

and

$$I = I_1 + I_2$$
$$I_1R_1 = (I - I_1)R_2$$

$$I_1(R_1 + R_2) = IR_2$$

or,

$$I_1 = I\frac{R_2}{R_1 + R_2} \qquad \text{(i)}$$

And,

$$I_2 = I - I_1 = I - I\frac{R_2}{R_1 + R_2}$$

$$= I\left[1 - \frac{R_2}{R_1 + R_2}\right]$$

or,

$$I_2 = I\frac{R_1}{R_1 + R_2} \qquad \text{(ii)}$$

Thus, in a parallel circuit of two resistances, current through one branch is equal to line current multiplied by the ratio of resistance of the other branch divided by the total resistance as have been shown in (i) and (ii).

Example 2.3 Calculate the current flowing through the various resistances in the circuit shown in Fig. 2.14.

Solution:

The circuit is reduced to a simple circuit through the following step: across terminals A and B, the 4 Ω resistor is connected in parallel with two 2 Ω resistor in series. Thus, we have two 4 Ω resistors connected in parallel across terminal AB as has been shown in Fig. 2.15 (b). In Fig. 2.15 (c) is shown the equivalent to two 4 Ω resistances in parallel. In Fig. 2.15 (d) is shown the total resistance 4 Ω connected across the 12 V supply. The current is, therefore, 3 A. This 3 A will get divided equally in the two parallel branches as can be seen from Fig. 2.15 (b) and (a).

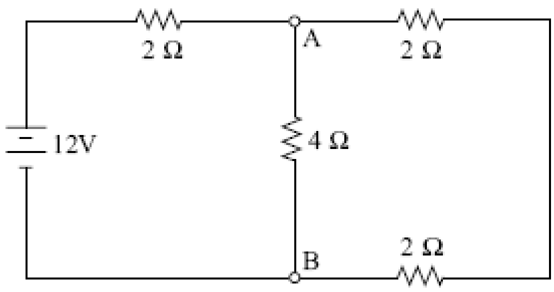


Figure 2.14

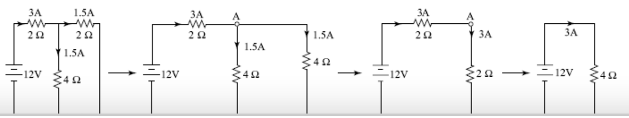


Figure 2.15

In Fig. 2.15 (d), current is $\dfrac{12\ \text{V}}{4\ \Omega} = 3\ \text{A}.$

In Fig. 2.15 (c) current is 3 A as it is a series circuit.

In Fig. 2.15 (b) current 3 A gets divided equally.

Example 2.4 Calculate the current supplied by the battery in the network shown in Fig. 2.16.

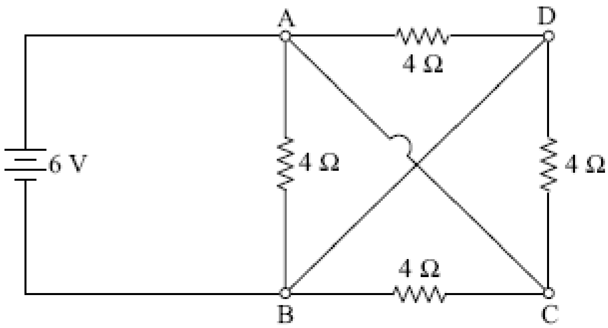


Figure 2.16

Solution:

Points A and C are joined together. Similarly points B and D are joined together. Thus, we can first bring point A and C together and the circuit will look like as has been shown in Fig. 2.17.

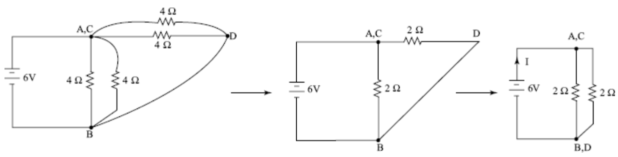


Figure 2.17

The circuit is further simplified by paralleling the two 4 Ω resistances between A and B and another two 4 Ω resistances between A and D. Now by bringing D and B together and paralleling the two 2 Ω resistors, the current I is calculate as 6 A.

Example 2.5 Calculate the resistance between the terminals P and Q of the network shown in Fig. 2.18.

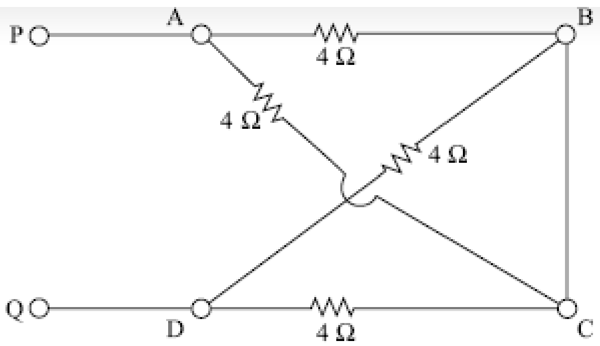


Figure 2.18

Solution:

Let us bring points B and C together. Then we get the same circuit of Fig. 2.18 modified as has been shown in Fig. 2.19. The successive reduction of the circuit has been shown in steps in Fig. 2.19 (a), (b), (c), and (d).

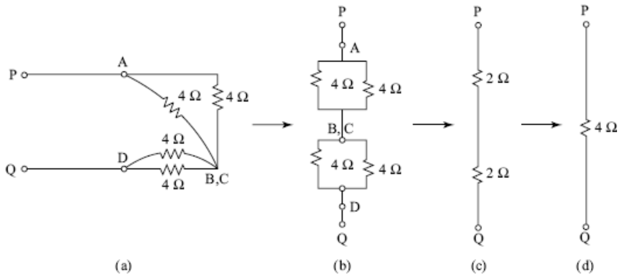


Figure 2.19

2.5 KIRCHHOFF'S LAWS

Two laws given by Gustav Robert Kirchhoff (1824–1887) are very useful in writing network equations. These laws are known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL). These laws do not depend upon, whether the circuit is made of resistance, inductance or capacitance, or a combination of them.

2.5.1 Kirchhoff's Current Law

This law is applied at any node of an electric network. This law states that the algebraic sum of currents meeting at a junction or a node in a circuit is zero. KCL can be expressed mathematically as

$$\sum_{j=1}^{n} I_j = 0$$

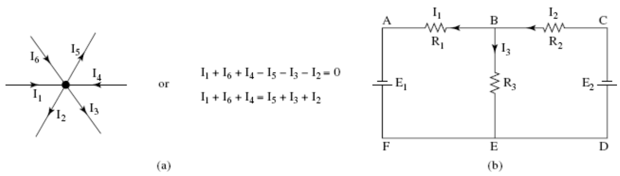


Figure 2.20 (a) Application of Kirchhoff's current law; (b) circuit for application of KVL

By observing Fig. 2.20, we can state KCL in another form:

The sum of current flowing towards a junction or a node is equal to the sum of currents flowing out of the junction.

The current entering the junction has been taken as positive while the currents leaving the junction have been taken as negative. That is to say there is no accumulation of current in a junction.

2.5.2 Kirchhoff's Voltage Law

This law is applicable to any closed loop in a circuit.

KVL states that at any instant of time the algebraic sum of voltages in a closed loop is zero.

In applying KVL in a loop or a mesh a proper sign must be assigned to the voltage drop in a branch and the source of voltage present in a mesh. For this, a positive sign may be assigned to the rise in voltage and a negative sign may be assigned to the fall or drop in voltage.

KVL can be expressed mathematically as

$$\sum_{j=1}^{n} V_j = 0$$

where V_j represents the voltages of all the branches in a mesh or a loop, i.e., in the jth element around the closed loop having n elements.

Let us apply KCL and KVL in a circuit shown in Fig. 2.20 (b). The current flowing through the branches have been shown.

Applying KCL at node B, we can write

$$I_1 + I_3 = I_2 \qquad \text{(i)}$$

$$+ I_1 R_1 - I_3 R_3 - E_1 = 0$$

or,

$$I_1 R_1 - (I_2 - I_1) R_3 - E_1 = 0 \qquad \text{(ii)}$$

The students need to note that while we move in the direction of the flow of current, the voltage across the circuit element is taken as negative. While we move from the negative terminal of the source of EMF to the positive terminal, the voltage is taken as positive. That is why we had taken voltage drop across the branch AB as $+I_1R_1$ and across BE as $-I_3R_3$. Since we were moving from the positive terminal of the battery towards its negative terminal while going round the mesh we had considered it as voltage drop and assigned a negative sign.

Using this convention, for the mesh CBEDC, applying KVL we can write

$$- I_2 R_2 - I_3 R_3 - E_2 = 0$$

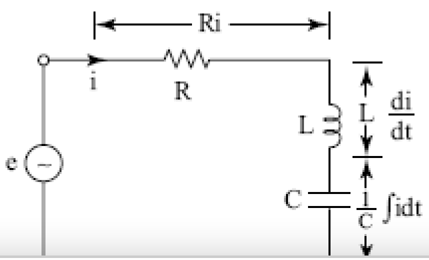
or,

$$-I_2 R_2 - (I_2 - I_1) R_3 - E_2 = 0 \qquad \text{(iii)}$$

In the two equations, i.e., in (ii) and (iii), if the values of R_1, R_2, R_3, E_1, and E_2, are known, we can calculate the branch currents by solving these equations.

Students need to note that Kirchhoff's laws are applicable to both dc and ac circuits.

Let us apply KVL in a circuit consisting of a resistance, an inductance, and a capacitance connected across a voltage source as has been shown in Fig. 2.21. We will equate the voltage rise with the voltage drops.



The voltage equation is

$$e = Ri + L\frac{di}{dt} + \frac{1}{C}\int idt \qquad \text{(iv)}$$

While solving network problems using Kirchhoff's laws we frame a number of simultaneous equations. These equations are solved to determine the currents in various branches in a circuit. We will discuss solving of simultaneous equations by the method of determinants or Cramer's Rule.

2.5.3 Solution of Simultaneous Equations Using Cramer's Rule

Let the three simultaneous equations written for a network problem be of the form

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}$$

Where x, y, z are the three variables.

We can determine the values of x, y, and z using Cramer's rule as

$$x = \frac{\Delta_a}{\Delta} \quad \text{Where,} \quad \Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

and

$$\Delta_a = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

Similarly,

$$\Delta \quad \begin{vmatrix} a_1 & d_1 & c_1 \end{vmatrix}$$

and

$$Z = \frac{\Delta_c}{\Delta}$$

and

$$\Delta_c = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

The determinants expressed as Δ, Δ_a, Δ_b, and Δ_c are to be evaluated so as to find x, y, and z.

2.5.4 Method of Evaluating Determinant

1. Let us write two simultaneous equations as

$$\begin{aligned} a_1x + b_1y &= m \\ a_2x + b_2y &= n \end{aligned}$$

Where x and y are the variables.

The common determinant Δ is evaluated as

$$\Delta = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

The determinant for x, i.e., Δ_a is evaluated as

$$\Delta_a = \begin{vmatrix} m & b_1 \\ n & b_2 \end{vmatrix} = mb_2 - nb_1$$

The determinant for y, i.e., Δ_b is evaluated as

$$\Delta_b = \begin{vmatrix} a_1 & m \\ a_2 & n \end{vmatrix} = a_1 n - a_2 m$$

$$x = \frac{\Delta_a}{\Delta} \text{ and } y = \frac{\Delta_b}{\Delta}$$

2. If there are more than two simultaneous equations, say three, the method of evaluating the determinants would be as illustrated below. Let the common determinant be

$$\Delta = \begin{vmatrix} 5 & 100 & 10 \\ 7 & -50 & -2 \\ 3 & -50 & -3 \end{vmatrix}$$

The procedure followed is like this. Select any row and the first column. Multiply each element in the row or the column by its minor and by + sign or − sign and then add the product. The multiplication by + sign or − sign is decided by a factor $(-1)^{j+k}$ where the minor of the element is appearing in row j and column k. Therefore, the determinant Δ given above is calculated as

$$\begin{aligned}\Delta &= (5) \times \begin{vmatrix} -50 & -2 \\ -50 & -3 \end{vmatrix} - (100) \times \begin{vmatrix} 7 & -2 \\ 3 & -3 \end{vmatrix} + (10) \begin{vmatrix} 7 & -50 \\ 3 & -50 \end{vmatrix} \\ &= 5(150 - 100) - 100(-21 + 6) + 10(-350 + 150) \\ &= 5 \times 50 - 100(-15) + 10(-200) \\ &= 250 + 1500 - 2000 \\ &= -250\end{aligned}$$

It can be observed that multiplication of the minor is done alternately as positive and negative starting from the first row or the first column. Now we will take up a few numerical problems to calculate branch currents in an electric network.

Example 2.6 Use KCL and KVL to calculate the branch currents in the circuit shown in Fig. 2.22.

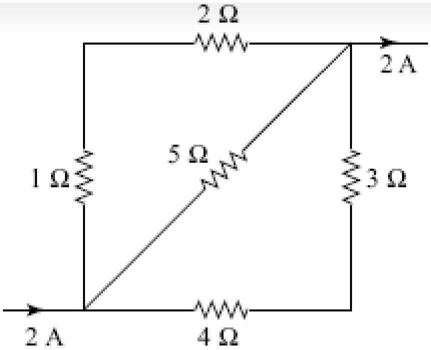


Figure 2.22

Solution:

We first indicate the branch currents applying KCL as shown in Fig. 2.23.

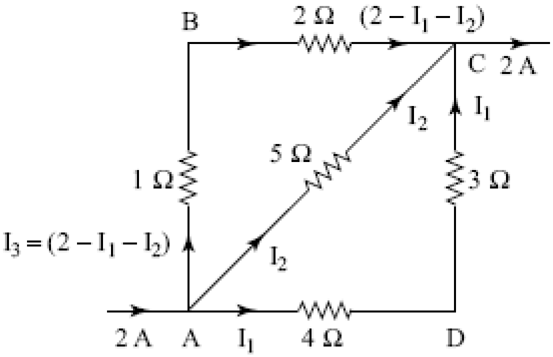


Figure 2.23

We have assumed I_1 and I_2 flowing respectively through branches AD and AC. Since 2 A is entering the node A, 2 − (I_1 + I_2) must be coming out through the branch AB. Similarly, applying KCL at node C we see that the sum of currents entering the node is equal to the current coming out of the node. Thus current distribution in the various branches is perfectly done.

Now, we will apply KVL to the loop ABCA and loop ACDA.

From loop ABCA we can write the voltage equation as

$$-1(2 - I_1 - I_2) - 2(2 - I_1 - I_2) + 5\,I_2 = 0$$

or,

$$3\,I_1 + 8\,I_2 = 6 \qquad \text{(i)}$$

$$-5I_2 + 3I_1 + 4I_1 = 0$$

or, $$7I_1 - 5I_2 = 0 \qquad \text{(ii)}$$

To solve eqs. (i) and (ii), multiply eq. (i) by 7 and (ii) by 3 and subtract as

$$21\,I_1 + 56\,I_2 = 42 \qquad \text{(i)}$$

$$21\,I_1 - 15\,I_2 = 0 \qquad \text{(ii)}$$

From which

$$I_2 = \frac{42}{71}\,A$$

$$I_1 = \frac{30}{71}\,A$$

and

$$I_3 = 2 - (I_1 + I_2) = 2 - \left(\frac{30}{71} + \frac{42}{71}\right)$$

$$= \frac{70}{71}\,A$$

sum of

$$I_1 + I_2 + I_3 = 2\,A$$

Example 2.7 Calculate applying Kirchhoff's laws the current flowing through the 8 Ω resistor in the circuit shown in Fig. 2.24.

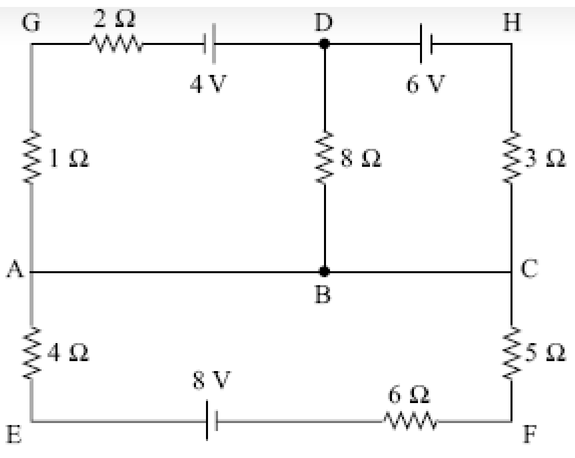


Figure 2.24

Solution:

By observing the given circuit we see that nodes A, B, C are at the same potential and they can be joined together so that the circuit will be like shown in Fig. 2.25.

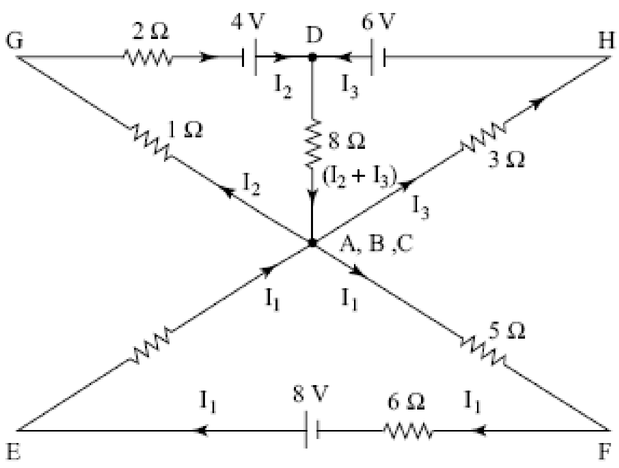


Figure 2.25

In the loop EA FE, current I_1 will flow. No current from this loop will flow to the other two loops. Current flowing from E to A is to be the same as the current flowing from A to F.

The distribution of currents in loop GDAG and HDAH have been shown. By applying KVL in these loops we write:

for loop GDAG

for loop HDAH

$$6 - 8(I_2 + I_3) - 3I_3 = 0$$

or, $$8I_2 + 11I_3 = 6 \qquad \text{(ii)}$$

Solving eqs. (i) and (ii)

$$I_3 = 0.6 \text{ A and } I_2 = -0.07 \text{ A}$$

$$\begin{aligned} \text{Current through the } 8\ \Omega \text{ resistor} &= I_2 + I_3 \\ &= -0.07 + 0.6 \\ &= 0.53 \text{ A} \end{aligned}$$

So far, we have assumed branch currents in a network and applying KVL written the voltage equations. From the loop equations, we have calculated the branch currents. Two other methods, namely Maxwell's mesh current method and Node voltage method, are described in the following sections.

2.6 MAXWELL'S MESH CURRENT METHOD

A mesh is a smallest loop in a network. KVL is applied to each mesh in terms of mesh currents instead of branch currents. As a convention, mesh currents are assumed to be flowing in the clockwise direction without branching out at the junctions. Applying KVL, the voltage equations are framed. By knowing the mesh currents, the branch currents can be determined. The procedure followed is explained through an example. Let us calculate the current flowing through the branches in the circuit given in Fig. 2.26.

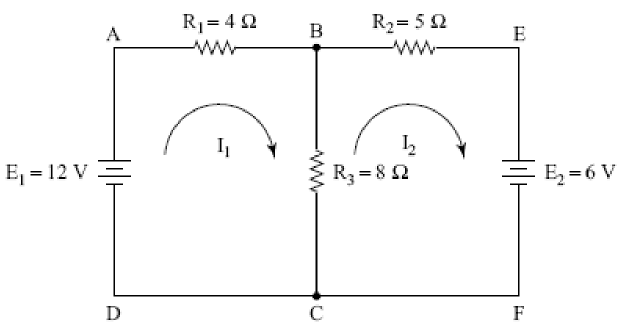


Figure 2.26

downward direction while I_2 is flowing in the upward direction.

We will now write the voltage equations for the two loops applying KVL and then solve the equations. If the value of any mesh currents is calculated as negative, we will take the direction of that mesh current opposite to the assumed clockwise direction.

For loop DABCD, the voltage equation is

$$12 - 4 I_1 - 8(I_1 - I_2) = 0$$

or, $$3 I_1 - 2I_2 = 3 \quad \text{(i)}$$

For loop BEFCB, the voltage equation is

$$-5 I_2 - 6 - (I_2 - I_1)\, 8 = 0$$

or, $$8 I_1 - 13 I_2 = 6 \quad \text{(ii)}$$

solving eqs. (i) and (ii), we get

$$I_1 = 1.17\text{ A},\ I_2 = 0.26\text{ A}$$

and current flowing through R_3 is $(I_1 - I_2) = 0.91$ A

I_1 is flowing through R_1, I_2 is flowing through R_2 and $(I_1 - I_2)$ is flowing through R_3.

Example 2.8 Using the mesh current method calculate the current flowing through the resistors in the circuit shown in Fig. 2.27.

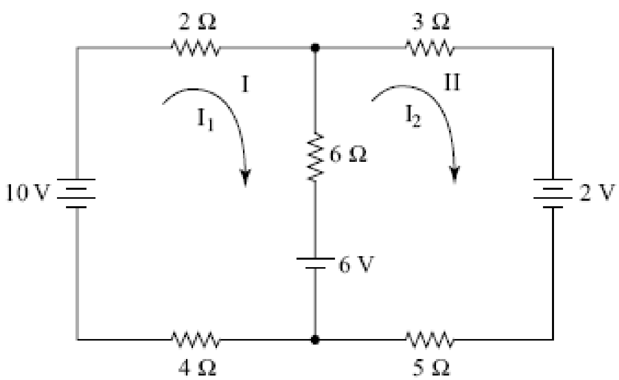


Figure 2.27

Solution:

Applying KVL in mesh I,

Applying KVL in mesh II,

$$-3I_2 - 2 - 5I_2 + 6 + 6(I_1 - I_2) = 0$$

or, $$-6I_1 + 14I_2 = 4 \qquad \text{(ii)}$$

Adding eqs. (i) and (ii)

$$11\,I_2 = 6$$

or, $$I_2 = \frac{6}{11}\,A$$

and $$I_1 = \frac{2+3I_2}{6} = \frac{2+3\times 6/11}{6} = \frac{20}{33}\,A$$

Current through the 6 Ω resistor is $(I_1 - I_2)$ which is equal to $\frac{2}{33}$ A.

Example 2.9 A network with three meshes has been shown in Fig. 2.28. Applying Maxwell's mesh current method determine the value of the unknown voltage, V for which the mesh current, I_1 will be zero.

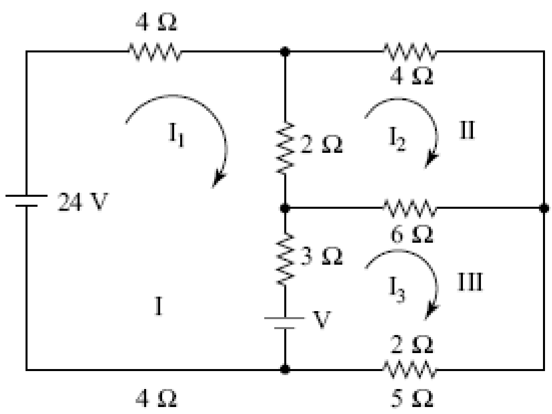


Figure 2.28

Solution:

Applying KVL in mesh I, II, and III respectively, we get

$$-4I_1 - 2(I_1 - I_2) - 3(I_1 - I_3) - V + 24 = 0$$

or, $$9I_1 - 2I_2 - 3I_3 = 24 - V \qquad \text{(i)}$$

$$-6(I_3 - I_2) - 2I_3 + V - 3(I_3 - I_1) = 0$$

or,

$$3I_1 + 6I_2 - 11I_3 = -V \qquad \text{(iii)}$$

From (i), (ii), and (iii) the determinants Δ and Δ_a or Δ_1 are

$$\Delta = \begin{vmatrix} 9 & -2 & -3 \\ 1 & -6 & 3 \\ 3 & 6 & -11 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} 24 - v & -2 & -3 \\ 0 & -6 & 3 \\ -v & 6 & -11 \end{vmatrix}$$

According to Cramer's rule,

$$I_1 = \frac{\Delta_1}{\Delta}, \quad I_2 = \frac{\Delta_2}{\Delta} \quad I_3 = \frac{\Delta_3}{\Delta}$$

Here condition is that I_1 must be zero.

$$I_1 = \frac{\Delta_1}{\Delta} = 0. \text{ So, } \Delta_1 \text{ must be zero}$$

Thus, we equate Δ_1 to zero.

$$\begin{vmatrix} 24 - v & -2 & -3 \\ 0 & -6 & 3 \\ -v & 6 & -11 \end{vmatrix} = 0$$

or,

$$(24 - v)\begin{vmatrix} -6 & 3 \\ 6 & -11 \end{vmatrix} - (-2)\begin{vmatrix} 0 & 3 \\ -v & -11 \end{vmatrix} - 3\begin{vmatrix} 0 & -6 \\ -v & 6 \end{vmatrix} = 0$$

$$(24 - V)[66 - 18] + 2[0 - (-3V)] - 3[0 - 6V] = 0$$

or,

$$(24 - V)\,48 + 6V + 18V = 0$$

or,

$$24 \times 48 - 48V + 24V = 0$$

or,

$$24V = 24 \times 48$$

or,

$$V = 48 \text{ Volts}$$

circuit analysis is suitable where a network has a number of loops, and hence a large number of simultaneous equations are to be solved. The procedure for the node voltage method is explained through an example.

Example 2.10 For the circuit shown in Fig. 2.29 determine the voltages at nodes B and C and calculate the current through the 8 Ω resistor.

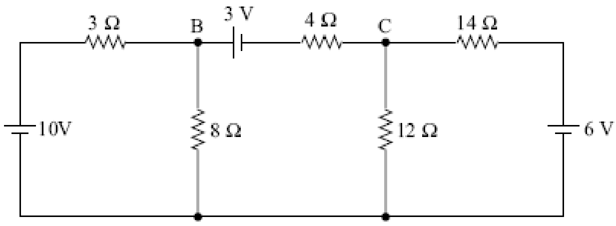


Figure 2.29

Solution:

We will take one reference node at zero potential. Generally the node at which maximum branches are meeting is taken as the reference node. Let R is the reference node as shown in Fig. 2.30. The reference node will be called ground node or zero potential node.

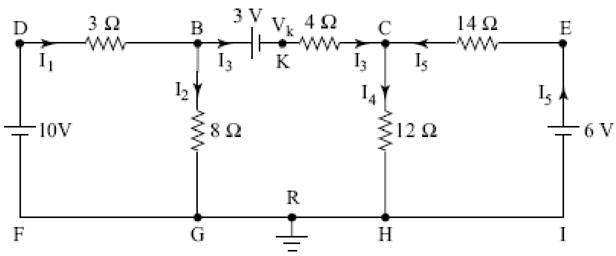


Figure 2.30

Points F, G, R, H, I are at zero reference potential. Let us now assign potential at all nodes with respect to the reference node. Let V_D, V_B, V_C, V_E are the potentials at points D, B, C, and E, respectively. Let us also assume unknown currents I_I, I_2, I_3, I_4, and I_5 flowing through the branches.

Applying Ohm's law currents I_1, I_2, I_3, I_4, and I_5 are expressed as

$$\frac{V_D - V_B}{3} = I_1; \quad \frac{V_B}{8} = I_2$$

To find I_3, we assume potential at point k as V_K. We can write, $V_K + 3 = V_B$

and $$I_3 = \frac{V_K - V_C}{4} = \frac{V_B - 3 - V_C}{4}$$

Applying KCL at node B,

$$I_1 = I_2 + I_3$$

or, $$\frac{10 - V_B}{3} = \frac{V_B}{8} + \frac{V_B - V_C - 3}{4}$$

or, $$17\,V_B - 6\,V_C = 98 \qquad \text{(i)}$$

$$I_4 = I_3 + I_5$$

or, $$\frac{V_C}{12} = \frac{6 - V_C}{14} + \frac{V_B - V_C - 3}{4}$$

or, $$21\,V_B - 34\,V_C = 27 \qquad \text{(ii)}$$

Solving eqs. (i) and (ii), we get

$$V_B = 7.01\text{ V};\ V_C = 3.537\text{ V}$$

and current in 8 Ω resistor,

$$I_2 = \frac{V_B}{8} = \frac{7.01}{8} = 0.88\text{ A}$$

More problems using this method have been solved separately.

2.8 NETWORK THEOREMS

We described earlier the mesh current and nodal voltage analysis of circuit problems. The procedure involves solving of number of equations depending upon the complexity of the network. Many networks require only restricted analysis, e.g., finding current through a particular resistor or finding the value of load resistance at which maximum power will be transferred from the source to the load.

calculations. The circuit theorems being discussed in this chapter are as follows:

1. Superposition theorem
2. Thevenin's theorem
3. Norton's theorem
4. Millman's theorem
5. Maximum power transfer theorem

In addition, circuit simplification using the star-delta transformation method has also been discussed with plenty of examples.

2.8.1 Superposition Theorem

An electrical circuit may contain more than one source of supply. The sources of supply may be a voltage source or a current source. In solving of circuit problems having multiple sources of supply, the effect of each source is calculated separately and the combined effect of all the sources are taken into consideration. This is the essence of the superposition theorem.

The superposition theorem states that in a linear network containing more than one source, the current flowing in any branch is the algebraic sum of currents that would have been produced by each source taken separately, with all the other sources replaced by their respective internal resistances. In case the internal resistance of a source is not provided, the voltage sources will be short circuited and current sources will be open circuited.

The procedure for solving circuit problems using the above stated superposition theorem is illustrated through a few examples.

Example 2.11 Using the superposition theorem find the value of current, I_{BD} in the circuit shown in Fig. 2.31.

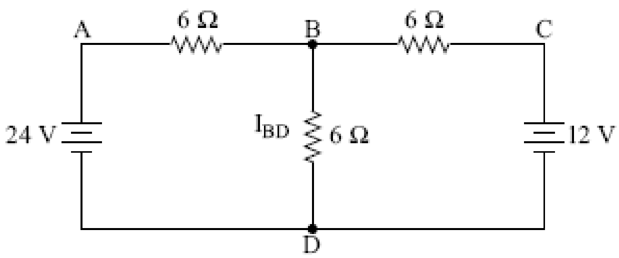


Figure 2.31

Solution:

We shall consider each source separately and calculate the current flowing through the branch BD. First the 24 V source is taken by short circuiting the 12 V source as shown in Fig. 2.32.

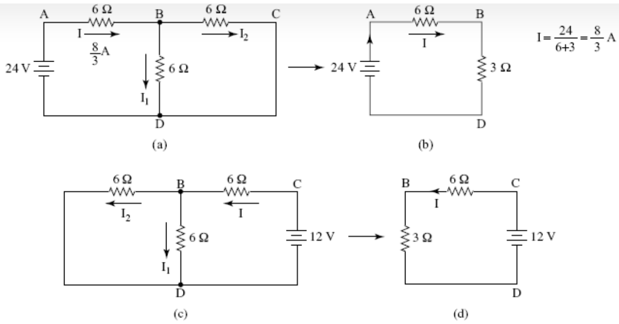


Figure 2.32

The current flowing from the battery is calculated $I = \frac{8}{3} A$ as shown in Fig. 2.32 (b). This $I = \frac{8}{3} A$ gets divided into two parts as I_1 and I_2. Current through the resistor across BD is I_1. To find I_1 we can use the current division rule as

$$I_1 = I \frac{R_2}{R_1 + R_2} = \frac{8}{3} \times \frac{6}{6+6} = \frac{4}{3} A$$

Now, consider the 12 V source and short circuit the 24 V source as shown in Fig. 2.32 (c). The current supplied by the 12 V source is calculated as

$$I = \frac{12}{6+3}$$

$$= \frac{4}{3} A$$

The total current I due to 12 V supply has been calculated as $\frac{4}{3}$ A. This current gets divided into I_1 and I_2 as has been shown in Fig. 2.32 (a). Current I_1 is calculated using the current division rule as

$$I_1 = I \times \frac{R_2}{R_1 + R_2} = \frac{4}{3} \times \frac{6}{6+6} = \frac{2}{3} A$$

$$I_{BD} = I_1 \text{ due to 24 V source} + I_1 \text{ due to the 12 V source}$$

$$= \frac{4}{3} + \frac{2}{3} = 2\text{ A}, \text{ flowing from node B towards node D}$$

Example 2.12 For the circuit shown in Fig. 2.32, calculate the current, I using the superposition theorem.

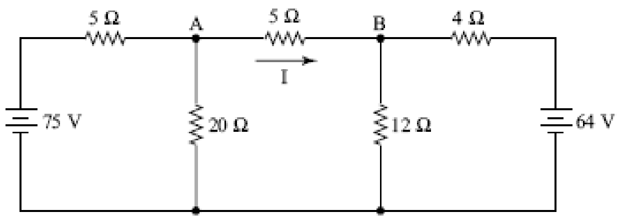


Figure 2.33

Solution:

We will consider the 75 V source first and short circuit the 64 V source. The current supplied by the 75 V source will be calculated. From the total current, current flowing through the resistor across terminals A and B will be calculated. The steps are illustrated in Fig. 2.34. When 64 V is short circuited, the 12 Ω and 4 Ω resistors get connected in parallel.

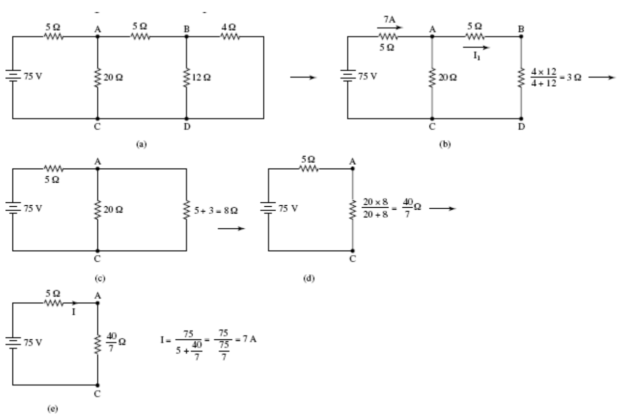


Figure 2.34

From Fig. 2.34 (e), the battery current calculated has been 7A and the current through the 5 Ω resistor across terminals A and B is cal-

$$I_1 = 7 \times \frac{20}{20+5+3} = 5A$$

This 5A through the resistor is due to the voltage source of 75 V. Now, we will calculate the current through the same resistor due to the other voltage source. We will short circuit the 75 V source and proceed as follows.

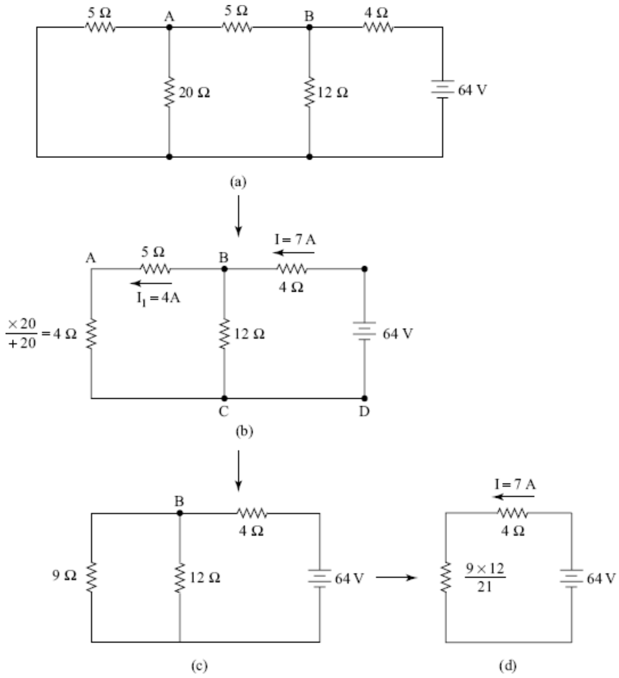


Figure 2.35

Current,

$$I = \frac{64}{4+108/21} = \frac{64}{84+108/21} = \frac{64 \times 21}{192} = 7A$$

Current I_1 through parallel circuit BAC in Fig. 2.35 (b), is calculated as

$$I_1 = I \times \frac{12}{5+4} = 7 \times \frac{12}{9+12} = 4\ A$$

$I_{AB} = 5 - 4 = 1$ A when the effect of both the voltage sources are superimposed.

Example 2.13 Determine current through a 8 W resistor in the network shown in Fig. 2.36 using the superposition theorem.

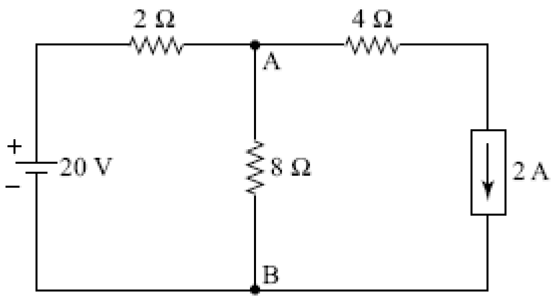
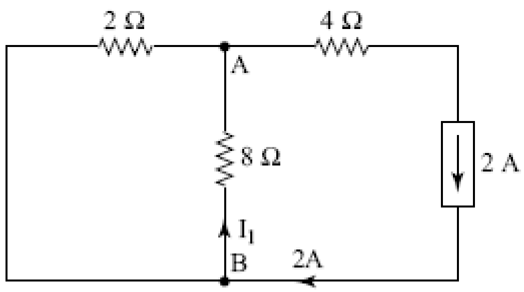


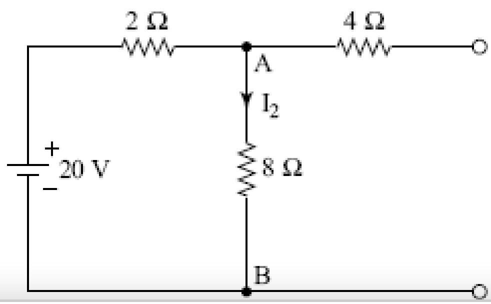
Figure 2.36

Solution:

Step 1: First the effect of current source will be considered. The voltage source is replaced by a short circuit. Using the current division rule, we determine current I_1 flowing from B to A as

$$I_1 = 2 \times \frac{2}{2+8}$$
$$= 0.4 \text{ A}$$





Step 2: Now if only the voltage source is considered, the current source has to be open circuited as shown in Fig. 2.37. The current flowing through the 8 Ω resistor is determined as

$$I_2 = \frac{20}{2+8} = 2\text{ A}$$

Step 3: Hence total current in the 8 W resistor from A to B is resistor,

$$\begin{aligned} I &= -I_1 + I_2 \\ &= -0.4 + 2 = 1.6\text{ A} \end{aligned}$$

Example 2.14 Two batteries are connected in parallel, each represented by an emf along with its internal resistance. A load resistance of 6 W is connected across the batteries. Calculate the current through each battery and through the load.

Solution:

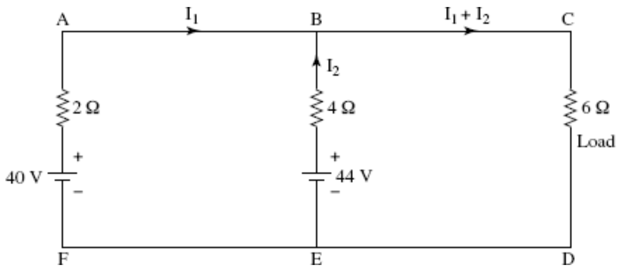


Figure 2.38

The circuit diagram of two batteries supplying a common load has been shown in Fig. 2.38.

Apply KVL to mesh ABEFA and BCDEB after arbitrarily showing the branch current directions. Current through the batteries are I_1 and I_2 and through the load is $I_1 + I_2$, respectively, as has been shown.

From mesh ABEFA,

$$\begin{aligned} 40 - 2I_1 + 4\,(I_2) - 44 = 0 \\ -2I_1 + 4I_2 = -40 + 44 \end{aligned}$$

From mesh BCDEB

$$-6(I_1 + I_2) + 44 - 4I_2 = 0$$

$$-6I_1 - 10I_2 = -44 \text{ or, } 6I_1 + 10I_2 = 44 \qquad \text{(ii)}$$

Students are once again reminded that while going through the loop or the mesh, the following sign connection has been followed.

1. Moving from negative terminal towards positive terminal of a battery is considered positive voltage, i.e., as voltage rise
2. Voltage drop in the resistor is taken as negative when moving in the direction of current flow.

Solving eq (i) and (ii) we get,

$$\begin{array}{r} \cancel{6I_1} - 12\,I_2 = -12 \\ \cancel{6I_1} + 10\,I_2 = 44 \\ \hline -22\,I_2 = -56 \end{array}$$

$$I_2 = \frac{28}{11}\,\text{A}$$

$$I_1 = \frac{34}{11}\,\text{A}$$

Total current through the load

$$= I_1 + I_2 = \frac{34}{11} + \frac{28}{11} = \frac{62}{11}\,\text{A}$$

Example 2.15 Calculate the current through the galvanometer in the bridge circuit shown in Fig. 2.39 (a)

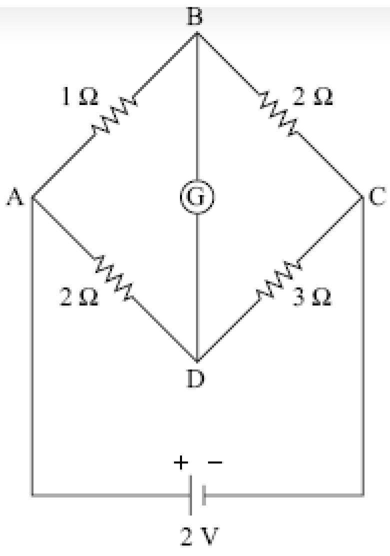


Figure 2.39 (a)

Solution:

Step 1:– We apply mesh analysis in ABDA, BCDB, ABCA

In mesh ABDA

$$-I_1 - 4I_3 + 2I_2 = 0 \qquad \text{(i)}$$

In mesh BCDB

$$-2(I_1 - I_3) + 3(I_2 + I_3) + 4I_3 = 0$$

or, $$-2I_1 + 2I_3 + 3I_2 + 3I_3 + 4I_3 = 0$$

or, $$-2I_1 + 3I_2 + 9I_3 = 0 \qquad \text{(ii)}$$

In mesh ABCA

$$-I_1 - 2(I_1 - I_3) + 2 = 0$$

or, $$-3I_1 + 2I_3 = -2 \qquad \text{(iii)}$$

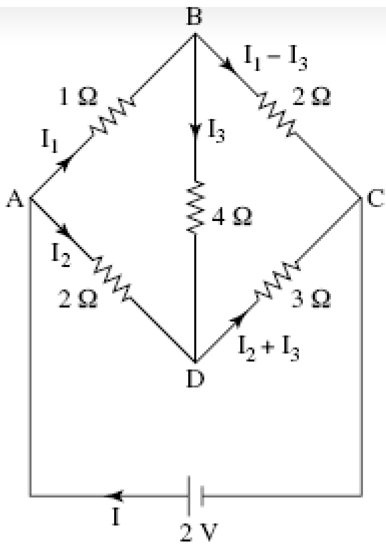


Figure 2.39 (b)

The three equations are written again as

$$-I_1 + 2I_2 - 4I_3 = 0 \quad \text{(i)}$$
$$-2I_1 + 3I_2 + 9I_3 = 0 \quad \text{(ii)}$$
$$-3I_1 + 2I_3 = -2 \quad \text{(iii)}$$

$$\begin{bmatrix} -1 & 2 & -4 \\ -2 & 3 & 9 \\ -3 & 0 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} -1 & 2 & -4 \\ -2 & 3 & 9 \\ -3 & 0 & 2 \end{bmatrix}$$

$$= -1[6-0] - 2[-4+27] - 4[0+9]$$
$$= -88$$

$$\Delta_1 = \begin{bmatrix} 0 & 2 & -4 \\ 0 & 3 & 9 \\ -2 & 0 & 2 \end{bmatrix}$$

$$= 0[6-0] - 2[0+18] - 4[0+6]$$
$$= -60$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\cancel{-}60}{\cancel{-}88} = \frac{30}{44}\text{ A}$$

Similarly,

Similarly

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{1}{44}\ A$$

Current through galvanomeneter

$$= I_3 = \frac{1}{44}\ A$$

Example 2.16 Applying KCL, determine current I_S in the electric circuit to make V_0 = 16 V in the network shown in Fig. 2.40.

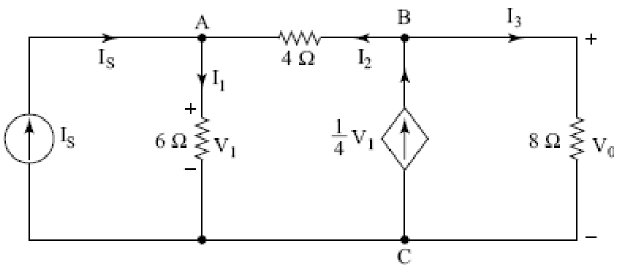


Figure 2.40

Solution:

Now applying Kirchhoff's current law to nodes A and B we have

$$I_1 = I_2 + I_S \qquad \text{(i)}$$

and $$I_2 + I_3 = \frac{V_1}{4} \qquad \text{(ii)}$$

also voltage of node B = V_0 =16 V

Voltage across AC + voltage across AB = voltage at node B.

$$V_1 + 4I_2 = 16\ V \qquad \text{(iii)}$$

$$I_1 = \frac{V_1}{6} \qquad \text{(iv)}$$

$V_1 = 12\ \text{V} \quad I_1 = 2\ \text{A} \quad I_2 = 1\ \text{A} \quad I_S = I_1 - I_2 = 2 - 1 = 1\ \text{A}$

Therefore, $I_S = 1\ \text{A} \quad I_3 = \frac{V_1}{4} - I_2 = 3 - 1 = 2\ \text{A}.$

Example 2.17 Two batteries A and B are connected in parellel to a load of 10 W. Battery A has an EMF of 12 V and an internal resistance of 2 W and Battery B has an EMF of 10 V and internal resistance of 1 W. Using nodal analysis to determine the current supplied by each battery and the load current.

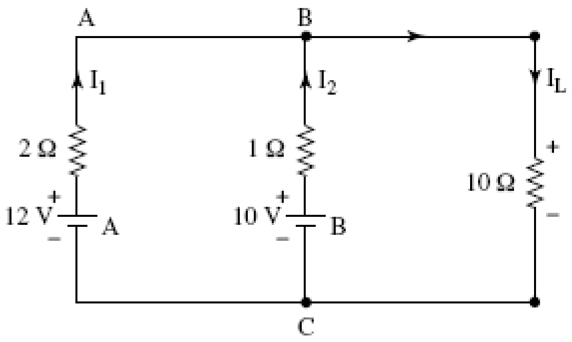


Figure 2.41

Solution:

The circuit diagram of two batteries supplying a load has been shown in Fig. 2.41. Taking node C as a reference node and potential of nodes A and B be V_A and V_B and current distribution

for node $A = I_1 = \dfrac{12 - V_A}{2}$

for node $B = I_2 = \dfrac{10 - V_B}{1}$

$$I_L = \frac{V_B}{10}$$

at node B, using KCL,

$$I_L = I_1 + I_2$$

$$\frac{V_B}{10} = \frac{12 - V_A}{2} + \frac{10 - V_B}{1}$$

$$V_A = V_B = 10\ \text{V}$$

Current supplied by battery	$I_1 = \dfrac{12 - V_A}{2} = 1\ A$
Current supplied by battery	$B = I_2 = \dfrac{10 - 10}{1} = 0$
Load current	$I_L = \dfrac{V_B}{10} = 1A$

Example 2.18 For the circuit shown in Fig. 2.42, find voltages of nodes B and C and determine current through the 8 W resistor.

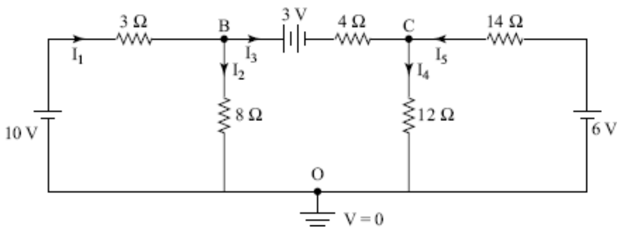


Figure 2.42

Let the reference point be at 0 which is taken at zero potential. By applying KCL at node B we get

$$I_1 = I_2 + I_3$$

or,

$$\frac{10 - V_B}{3} = \frac{V_B}{8} + \frac{V_B - 3 - V_C}{4}$$

$$17V_B - 6V_C = 98 \qquad \text{(i)}$$

$$I_4 = I_3 + I_5$$

$$\frac{V_C}{12} = \frac{V_B - V_C - 3}{4} + \frac{6 - V_C}{14}$$

$$21V_B - 34V_C = 27 \qquad \text{(ii)}$$

Solving eq. (i) and (ii), we get,

Voltage of node	B, $V_B = 7.0133$ V
Voltage of node	C, $V_C = 3.5376$ V

Example 2.19 Two batteries of EMF 2.05 V and 2.15 V having internal resistances of 0.05 Ω and 0.04 Ω, respectively are connected togather in parallel to supply a load resistance of 1 Ω. Calculate using the superposition theorem, current supplied by each battery and also the load current

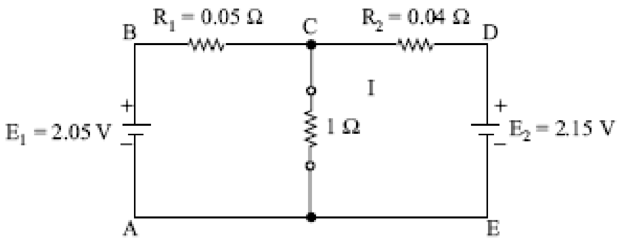


Figure 2.43

Solution:

First we will consider the effect of voltage source E_1 by short circuiting E_2. The currents will be calculated by considering the series parallel connections of resistances as in Fig. 2.44. Thus,

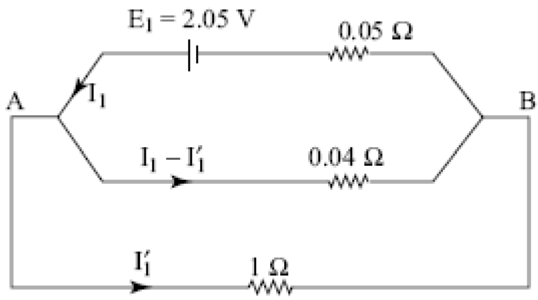


Figure 2.44

$$I_1 = \frac{2.05}{0.05 + 0.04 \times 1/1.04} = \frac{2.05}{0.085} = 23.2\,\mathrm{A}$$

Using current divider rule,

$$I_1' = I_1 \times \frac{0.04}{0.04 + 1} = 23.2 \times \frac{0.04}{1.04} = 0.892\,\mathrm{A} \text{ from A to B.}$$

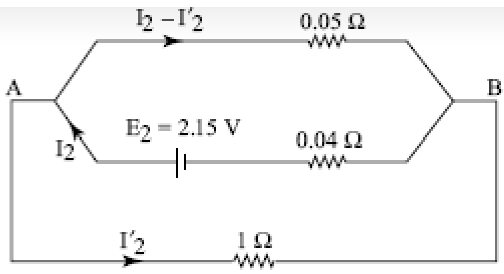


Figure 2.45

$$
\begin{aligned}
I_2 &= \frac{2.15}{0.04 + 0.05 \times 1/1.05} \\
&= 24.54 \text{ A}
\end{aligned}
$$

current through the 1 Ω resistor,

$$
\begin{aligned}
I'_2 &= I_2 \times \frac{0.05}{0.05+1} \\
&= 24.54 \times \frac{0.05}{1.05} \\
&= 1.169\text{A} \qquad \text{from A to B}
\end{aligned}
$$

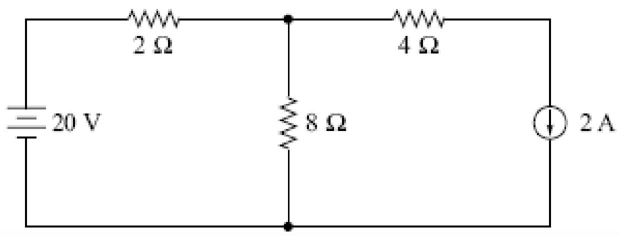
By superimposing the effect of two voltage sources, the current through the 1 Ω resistor, I is calculated as

$$I = I'_1 + I'_2 = 0.892 + 1.169$$

or,

$$I = 2.061\text{A}$$

Example 2.20 Determine the current through the 8 Ω resistor in the network shown in Fig. 2.46. Use the superposition theorem.



Solution:

First remove one of the sources, say the voltage source and calculate the current flow through the 8 Ω resistor.

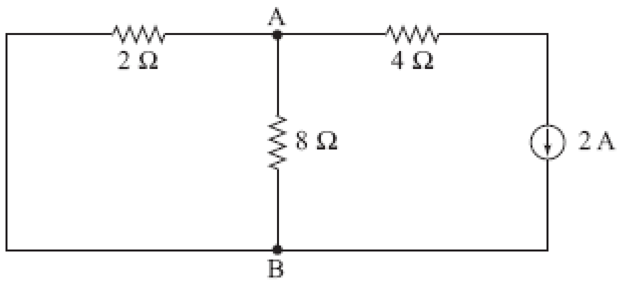


Figure 2.47

Current of 2 A will be divided into two parts, one part going through the 8 Ω resistor from B to A and the other part going through the 2 Ω resistor. Current I_1' going through the 8 W resistor is calculated as

$$I_1' = I \times \frac{2}{2+8} = 2 \times \frac{2}{10} = 0.4\text{A} \text{ from B to A}$$

Now, we will consider the voltage source, keeping the current source open circuited and find the current through the 8 Ω resistor. Using Ohm's law, the current through the 8 Ω resistor is calculated as

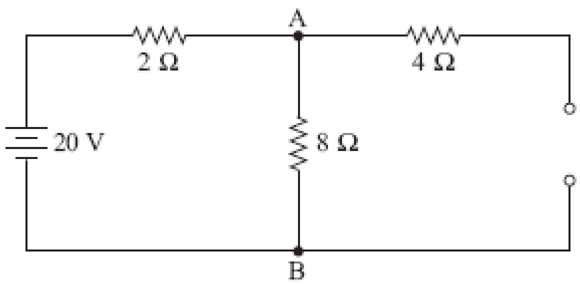


Figure 2.48

$$I_1'' = \frac{20}{2+8} = 2\text{A} \text{ from A to B}$$

$$I = -I_1' + I_1'' = -0.4 + 2 = 1.6\,A \text{ from A to B}$$

2.8.2 Thevenin's Theorem

Application of this theorem often comes useful when we want to determine the current flowing through any branch or component of a network. We can conveniently determine the current through any component when it is required that the component be replaced. The use of kirchchoffs laws to calculate the branch current for the changed value of a resistor becomes time consuming as we have to repeat the calculations.

Here, the whole circuit across the terminals of the resistor, through which current flow is to be calculated, is converted into a voltage source with an internal resistance. The voltage is the open circuit voltage of the network across the terminals and internal resistance is the equivalent resistance of the whole circuit across the open-circuited terminals by replacing the voltage sources by their internal resistances. The current through the resistor R, is

$$I = \frac{V_{OC}}{R_{eq} + R} \qquad (2.3)$$

Where V_{OC} is the open-circuit voltage across the terminals of resistor R (when R is removed from the circuit); R_{eq} is the equivalent circuit resistance across the terminals of R.

The theorem will be stated a little later after a specific problem is solved applying the procedure mentioned. Let us consider a circuit as in Fig. 2.49.

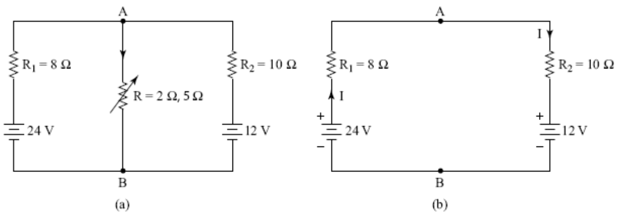


Figure 2.49

To calculate the current through the variable resistor R, the first step would be to take away the resistor R and calculate the V_{OC} across terminals A and B. Applying KVL in the circuit of Fig. 2.49 (b)

$$+24 - 8\,I - 10\,I - 12 = 0$$

or, $\quad I = 0.67\,A$

$$V_{oc} = 24 - 8 \times 0.67 = 18.64\ \text{V}$$

The same result will be arrived at when we approach from A to B.

Now we calculate R_{eq} across terminals A and B by short circuiting the voltage sources or by replacing them by their internal resistances. Across terminals AB we find two resistances of 8 Ω and 10 Ω connected in parallel. R_{eq} across terminals A and

$$= \frac{8 \times 10}{8 + 10} = 4.44\ \Omega$$

Thus, Thevenin's equivalent circuit is represented as in Fig. 2.50

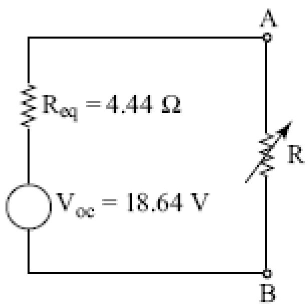


Figure 2.50

Current through R with its variable values i.e., for R = 2Ω and R = 5Ω are calculated as

$$I = \frac{V_{OC}}{R_{eq} + R} = \frac{18.64}{4.44 + 2} = 2.89\,\text{A}$$

and $$I = \frac{V_{OC}}{R_{eq} + R} = \frac{18.64}{4.44 + 5} = 1.97\,\text{A}$$

Now, we are in a position to state Thevenin's Theorem as

Any two terminals of an electrical network consisting of active and passive elements (i.e., voltage sources and resistors) can be replaced by an equivalent voltage source and an equivalent series resistance. The voltage source is the open-circuit voltage between the terminals caused by the active network. The series resistance is the equivalent resistance of the whole circuit across the terminals looking to the circuit from the two terminals with all the sources of EMF short circuited.

The steps involved in applying Thevenin's theorem in calculating current in a circuit component are as follows:

been removed. This is called V_{OC}.

3. Calculate the equivalent resistance of the whole circuit across the terminals after replacing the sources of EMFs by their internal resistances (or by simply short–circuiting them if internal resistance is not provided or not known) and by keeping the current sources open–circuited (i.e., considering having infinite resistance).
4. Draw the Thevenins equivalent circuit with V_{OC} as the voltage source, R_{eq} as the internal resistance of the voltage source, and R is the load resistance connected across the voltage source.
5. Calculate the current through R using the relation.

$$I = \frac{V_{oc}}{R_{eq} + R}$$

Example 2.22 Using Thevenin's theorem calculate the range of current flowing through the resistance R when its value is varied from 6 Ω to 36 Ω.

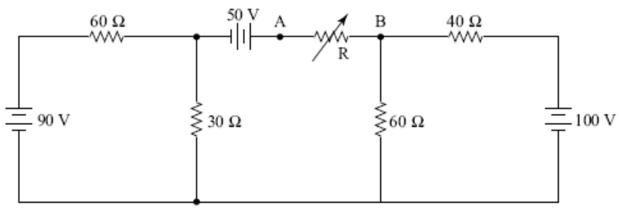


Figure 2.51

Solution:

The open circuit voltage, V_{OC} across the terminals A and B by removing the resistance R is calculated as

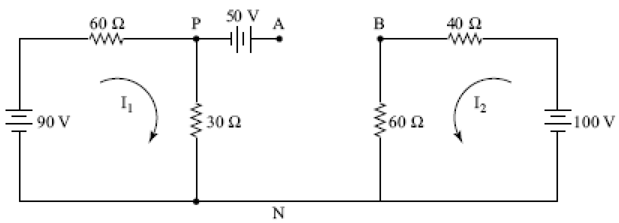


Figure 2.52

Voltage drop across the 30 W resistor = 30 × 1 = 30 V. The potential of point P with respect to N is + 30 V. The potential of point A with respect of point P is +50 volts. Therefore, the to potential of point A with respect to point N is +30 +50 = +80 V.

Voltage drop across the 60 Ω resistor in the right-hand-side loop is calculated as

$$I_2 = \frac{100}{40+60} = 1\text{ A}$$
$$V_{NB} = 60 \times 1 = 60\text{ V}$$

Potential of point B with respect to point N is + 60 V and that of A w.r.t. N = $30I_1$ + 50 = +80V.

Now we observe that the potential of point A with respect to point N is +80 V and the potential of point B with respect to point N is +60 V. Therefore, the potential of point A with respect to point B, i.e., V_{OC} becomes +20 V, point A being at higher a potential than point B.

To calculate R_{eq} we redraw the circuit by short circuiting the sources of EMFs as in Fig. 2.53.

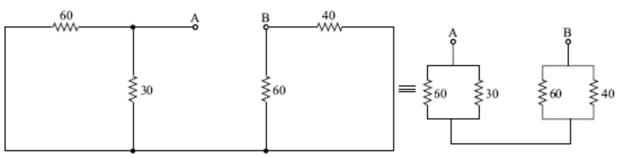
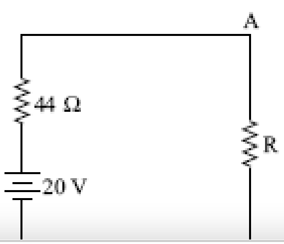


Figure 2.53

$$R_{eq} = \frac{60 \times 30}{60+30} + \frac{60 \times 40}{60+40} = 44\ \Omega$$

Thevenin's equivalent circuit is drawn as in Fig. 2.54



$$I = \frac{20}{44+6} \quad \text{when } R = 6\ \Omega$$

$$= 0.4\ A$$

$$I = \frac{20}{44+36} \text{ when } R = 36$$

$$= 0.25\ A$$

The value of current through the resistor R will vary from 0.4 A to 0.25 A when its value is changed from 6 W to 36 W keeping the other circuit conditions unchanged.

Example 2.22 Using Thevenin's theorem calculate the current flowing through the load resistance R_L connected across the terminals A and B as shown.

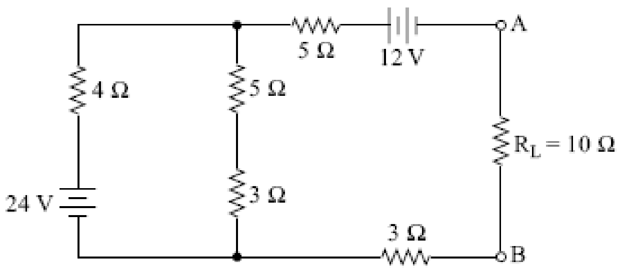


Figure 2.55

Solution:

First we remove the load resistance R_L from the circuit and calculate the open circuit voltage, V_{OC} across its terminals A and B as,

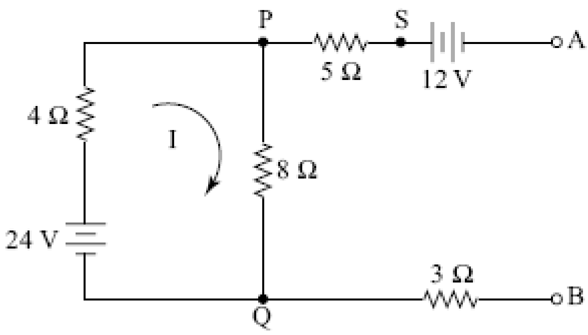


Figure 2.56

$$I = \frac{24}{4+8} = 2\ \text{A}$$

voltage drop across PQ is equal to IR = 2 × 8 = 16 V. Terminal P is at higher potential than terminal Q. No current is flowing through the 3 Ω and 5 Ω resistors in the circuit because these are open circuited. The potential of A with respect to B is calculated starting from point B as

$$\begin{aligned} V_{AB} &= V_{BQ} + V_{QP} + V_{PS} - V_{SA} \\ &= 0 + 16\ \text{V} + 0 - 12\ \text{V} \\ &= +4\ \text{V} \\ &= V_{OC} \end{aligned}$$

For calculating R_{eq}, we short circuit the sources of EMFs as

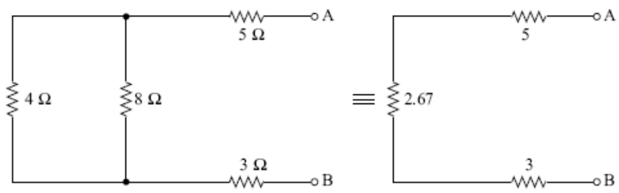
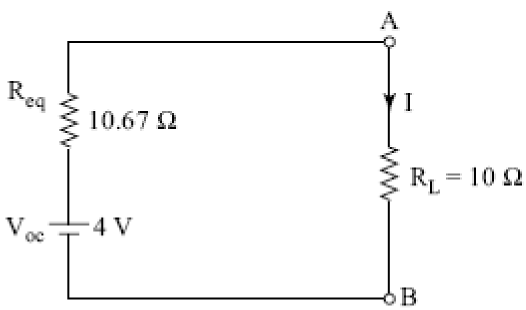


Figure 2.57

$$R_{eq} = R_{AB} = 10.67\ \Omega$$

Thevenin's equivalent circuit and current through the load resistor is calculated as



$$I = \frac{V_{OC}}{R_{eq} + R_L}$$

$$= \frac{4}{10.67 + 10} = 0.193\,A$$

Example 2.23 Use Thevenin's theorem to calculate the current flowing through the 5 Ω resistor in the circuit shown in Fig. 2.59.

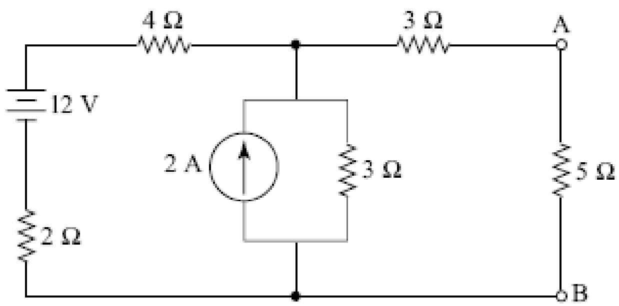


Figure 2.59

Solution:

We shall convert the current source into its equivalent voltage source and also take away the 5 Ω resistor from the terminals A and B. We will calculate the V_{oc} across terminals AB.

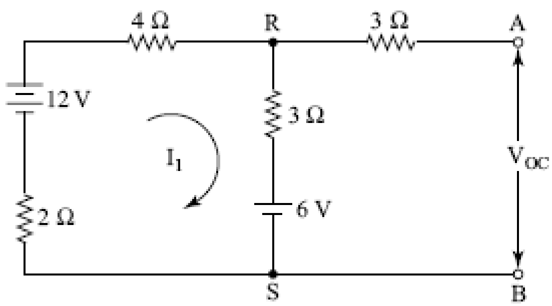


Figure 2.60

Using kirchhoff's voltage equation in the loop as shown above we write

Voltage across point S and R is calculated by moving upwords from point S to point R.

Considering the voltage rise we get,

$$+6\text{V} + 3 \times \frac{2}{3}\text{V} = +8 \text{ volts}$$

Since there is no current flowing in the 3 Ω resistor between point R and A, the potential across point A with repeat to B is 8 Volts, point A being at higher potential than point B.

R_{eq} is calculated as

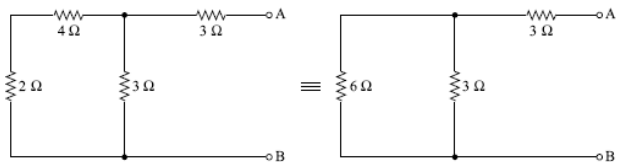


Figure 2.61

$$R_{AB} = R_{eq} = 3 + \frac{3 \times 6}{9}$$
$$= 5\ \Omega$$

Thevenins equivalent circuit is

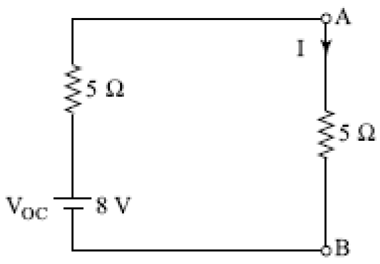


Figure 2.62

$$I = \frac{V_{OC}}{R_{eq} + R} = \frac{8}{5+5}$$
$$= 0.8 \text{ Amps}$$

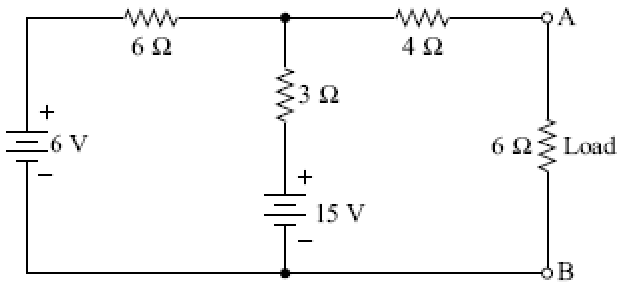


Figure 2.63

Solution:

Step 1:– Remove load resistance through which current is required to be calculated.

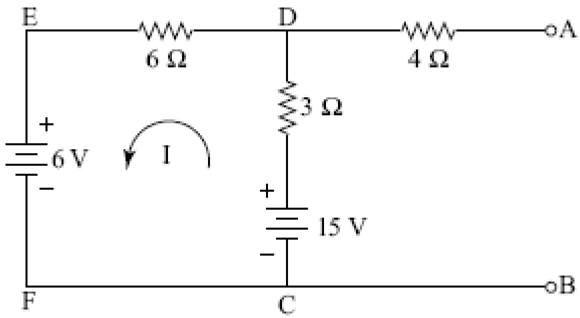


Figure 2.64

Applying KVL in the loop CDEFC,

$$15 - 3I - 6I - 6 = 0$$

or, $$9I = 9$$

or, $$I = 1$$

voltage across CD

$$= 15 - 3I$$
$$= 15 - 3 \times 1$$
$$= 12\ V$$

$$R_{th} = R_{eq} = 4 + \frac{3 \times 6}{3+6} = 6\ \Omega$$

Current through the load, $I = \frac{V_{oc}}{R_{eq} + R} = \frac{12}{6+6} = 1\,A$

Example 2.25 The resistance of the various arms of a Wheatstone bridge are shown in Fig. 2.65. The battery has an EMF of 2 V and negligible internal resistance. Using Thevenin's theorem, determine the value and direction of the current in the galvanometer circuit BD.

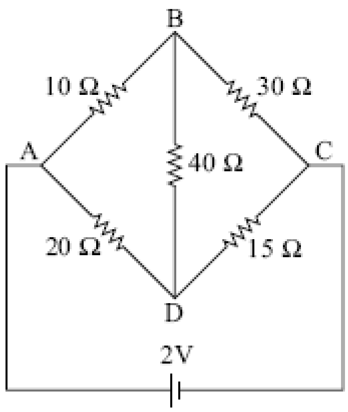
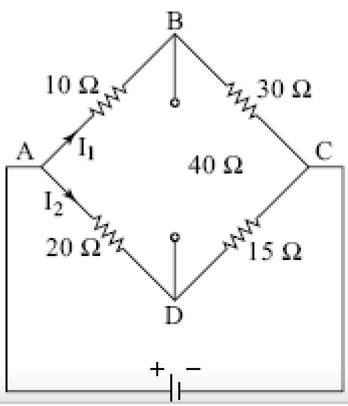


Figure 2.65

Solution:

Remove the load resistance of 40 Ω between terminals B and D. Calculate V_{BD} as V_{OC}.



$$V_{BC} = 30I_1 = \frac{30 \times V}{30+10} \therefore V_{BC} = \frac{30 \times 2}{30+10} = 1.5\,V$$

$$V_{DC} = 15I_2 = \frac{15 \times V}{20+15} \therefore V_{DC} = \frac{15 \times 2}{20+15} = \frac{30}{35} \text{ or } \frac{6}{7}\,V$$

PD between terminals B and D

$$= V_B - V_D$$

$$= 1.5 - \frac{6}{7} = \frac{4.5}{7}\,V$$

The equivalent resistance, R_{eq} which is also called R_{th} is calculated by short circuiting the voltage source and rearranging the circuit components as shown

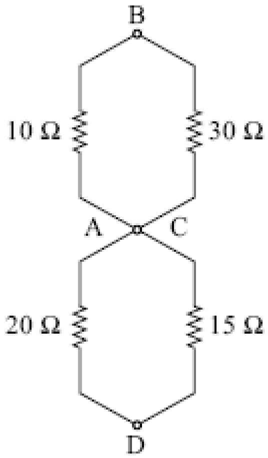


Figure 2.67

$$R_{th} = R_{eq} = \frac{10 \times 30}{10+30} + \frac{20 \times 15}{20+15} = \frac{225}{14}\,\Omega$$

$$I_L = \frac{V_{th}}{R_{th}+R_L} = \frac{4.5/7}{225/14+40} = \frac{9}{785}A = 11.45\,mA$$

Since point B is at a higher potential than point D, I_L of 11.45 mA will flow from B to D.

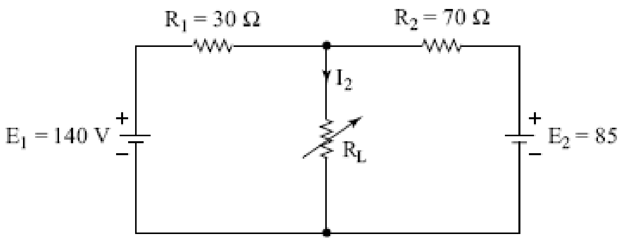


Figure 2.68

Solution:

Step 1:– Remove load resistance and draw the circuit as shown. Calculate the current flowing in the circuit applying KVL in the loop AB-CDEA as,

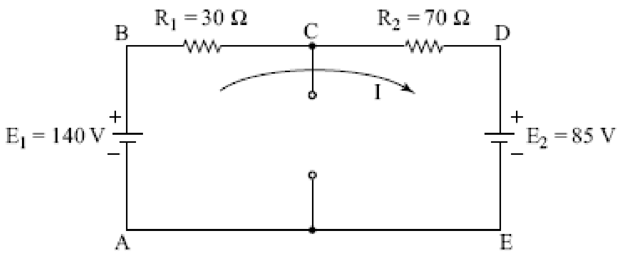


Figure 2.69

$$R_{eq} = \frac{30 \times 70}{30 + 70} = 21\ \Omega$$

$$140 - 30I - 70I - 85 = 0$$

$$+100I = +55$$

$$I = \frac{55}{100} = 0.55\ \text{A}$$

$$V_{oc} = V_{th} = E_1 - IR_1$$

$$= 140 - 0.55 \times 30 = 123.5\ \text{V}$$

For, $R_L = 5$

$$I = \frac{V_{oc}}{R_{eq} + R_L} = \frac{123.5}{21 + 5} = 4.75\ \text{A}$$

for $R_L = 15\ \Omega$, $I = \dfrac{123.5}{21 + 15} = 3.43\ \text{A}$

for $R_L = 50\ \Omega$, $I = \dfrac{123.5}{21 + 50} = 1.74\ \text{A}$

Example 2.27 Calculate current through a 1,000 Ω resistor connec-

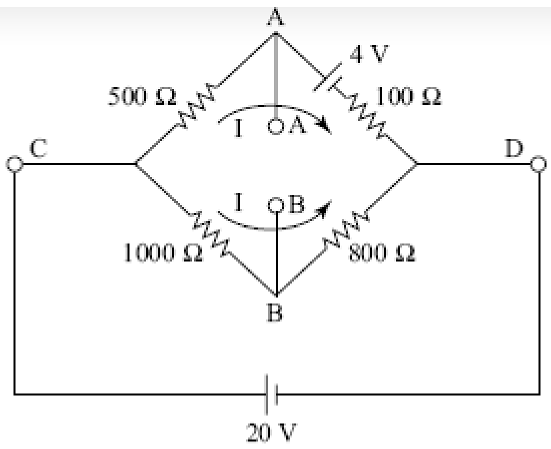


Figure 2.70(a)

Solution:

$$V_{CB} = \frac{20 \times 1000}{1000 + 800} = -11.11\,V$$

Applying KVL in DC AD,

$$20 - 500\,I - 4 - 100\,I = 0$$

or,

$$I = \frac{16}{600}\,A$$

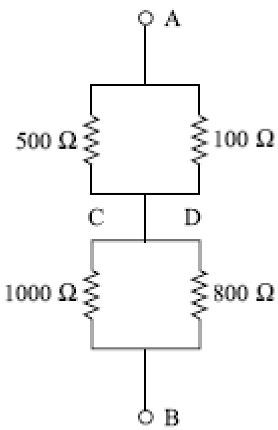


Figure 2.70(b)

$$V_{CA} = -\frac{16}{600} \times 500 - \frac{80}{60}$$

$$= -13.33\ \text{V}$$

$$V_{CB} = -11.11\ \text{V and}$$

$$V_{CA} = -13.33\ \text{V}$$

Point B is at higher potential than point A.

i.e.,

$$V_{BA} = 2.22\ \text{V}$$

$$V_{OC} = 2.22\ \text{V}$$

R_{eq} across terminals A and B is calculated considering 500 Ω and 100 Ω resistance in parallel plus 1000 Ω and 800 Ω resistances in parallel as shown in Fig. 2.70 (b).

$$R_{eq} = \frac{500 \times 100}{500 + 100} + \frac{1000 \times 800}{1000 + 800} = \frac{9500}{18}\ \Omega$$

Current

$$I = \frac{Voc}{R_{eq} + R_L} = \frac{2.22}{9500/18 + 1000}\ \text{A}$$

$$= \frac{2.22}{9500/18 + 1000}\ \text{mA}$$

$$= 1.5\ \text{mA}$$

Example 2.28 Calculate using Thevenin's theorem the current flowing through the 5 Ω resistor connected across the terminals A and B as shown.

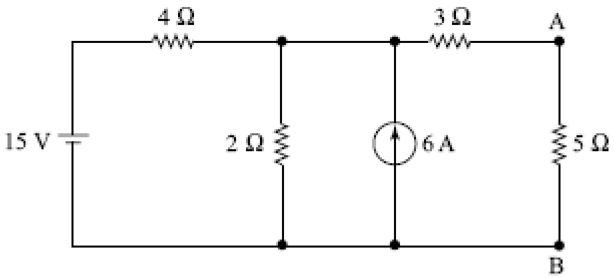


Figure 2.71

Solution:

Remove the load resistance of 5 Ω connected between the terminals source. The equivalent resistance will be equal to 3 Ω plus the parallel combination of 2 Ω and 3 Ω resistors. This comes to equal to 4.33 Ω.

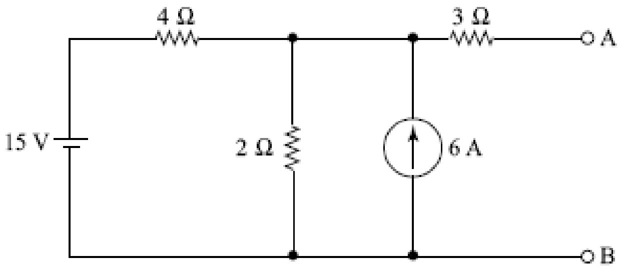


Figure 2.72

Now we will convert the current source into an equivalent voltage source.

Converting current source of 6 A connected across the 2 Ω resistor, the equivalent voltage source is represented as shown.

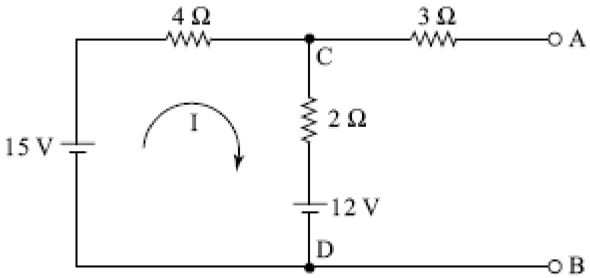


Figure 2.73

Now, we will calculate the open circuit voltage V_{oc} across terminals A and B. Applying KVL in the loop,

$$15 - 4I - 2I - 12 = 0$$

or, $$-6I = -3$$

or, $$I = 0.5\text{ A}$$

Since no current flows through the 3 Ω resistor, $V_{AB} = V_{OC} = V_{CD}$.

$V_{CD} = 12 + 2I$

Point C is at a higher potential than point D. Hence point A is at a higher potential than point B. Current through the load resistor will flow from A to B.

$$I = \frac{V_{AB}}{R_{eq} + R_L}$$

$$= \frac{13}{4.33 + 5}$$

Current through the load resistor $= 1.39$ A

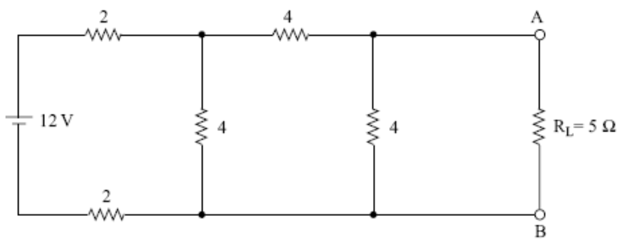


Figure 2.74

2.8.3 Norton's Theorem

We have seen earlier that in applying Thevenin's theorem, a network is converted into a voltage source and an equivalent series resistance connected across two terminals of any resistance through which current has to be calculated.

In applying Norton's theorem, a network is converted into a constant current source and a parallel resistance across the terminals of the resistance through which current has to be calculated. The Norton's theorem is stated as follows:

Any two terminal networks consisting of voltage sources and resistances can be converted into a constant current source and a parallel resistance. The magnitude of the constant current is equal to the current which will flow if the two terminals are short circuited and the parallel resistance is the equivalent resistance of the whole network viewed from the open-circuited terminals after all the voltage and current sources are replaced by their internal resistances.

To understand the application of the theorem let us consider a simple circuit as shown is Fig. 2.74.

Applying Norton's theorem, let us calculate the current that would flow through the load resistance, $R_L = 5\ \Omega$ as in Fig. 2.74. The first step is to remove the load resistance and then short circuit the terminals AB as shown in Fig. 2.75, and calculate I_{SC}. This is done as in parallel across terminals EF, their equivalent resistance become 2 Ω. Thus the total current, I supplied by the battery becomes

$$I = \frac{12}{2+2+2} = 2\ \text{A}$$

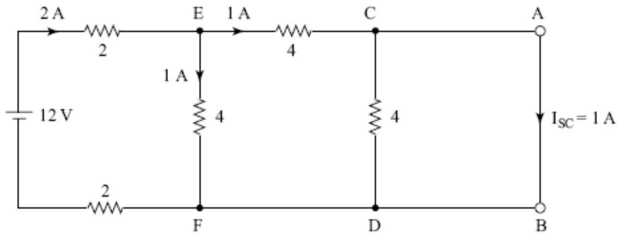


Figure 2.75

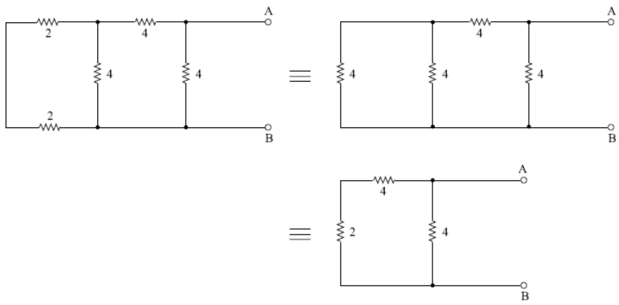


Figure 2.76

At point E, this I = 2 A gets divided equally; 1 A going in branch EF and 1 A to branch EC (CD being shorted). At point C, 1 A current will flow through the short-circuited path provided between terminals A and B. Therefore, I_{SC} = 1 A.

Now, the resistance of the network viewed from the terminals AB when the battery is short circuited is

$$R_{AB} = \frac{6 \times 4}{10} = 2.4\ \Omega$$

(see also Fig. 2.76)

Norton's equivalent circuit is shown in Fig. 2.77. Now, using the current divider rule, the current through the load resistance is calculated as,

$$I_L = I\frac{2.4}{2.4+5}$$

$$= \frac{1.0 \times 2.4}{7.4} = 0.324\ A$$

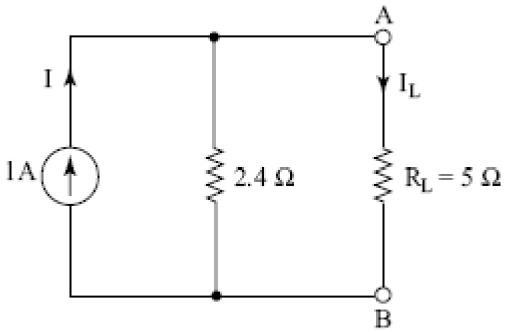


Figure 2.77

Example 2.29 Using Norton's theorem calculate the current flowing through the load resistance connected across the terminals A and B as shown in Fig. 2.78. Also apply Thevenin's theorem to calculate the same.

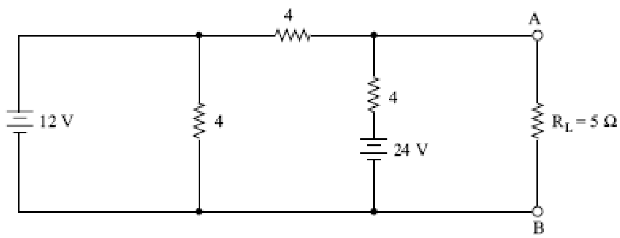


Figure 2.78

Solution:

Apply Norton's theroem

The first step is to remove R_L and short circuit the terminals A and B, and calculate I_{SC} due to the two voltage sources.

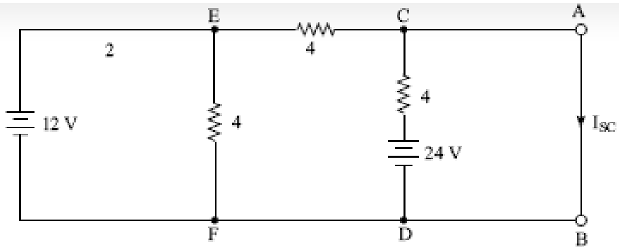


Figure 2.79

When terminals A and B are shorted, terminals C and D are also shorted. Two 4 Ω resistances are seen connected in parallel across terminals E and F. The current from the battery of 12 V is

$$I = \frac{12}{2} = 6\ \text{A}$$

This 6 A will get divided into 3 A each at branch EF and E C A B D F. Thus, I_{SC} due to the 12 V battery source is 3 A. I_{SC} due to the 24 V battery is calculated by considering the loop DC ABD which is 24/4 = 6 A. Thus, total I_{SC} due to both the sources of EMF is 3 + 6 = 9 Amps. The equivalent resistance of the network across terminals A and B is calculated as (after short circuiting the sources of EMFs).

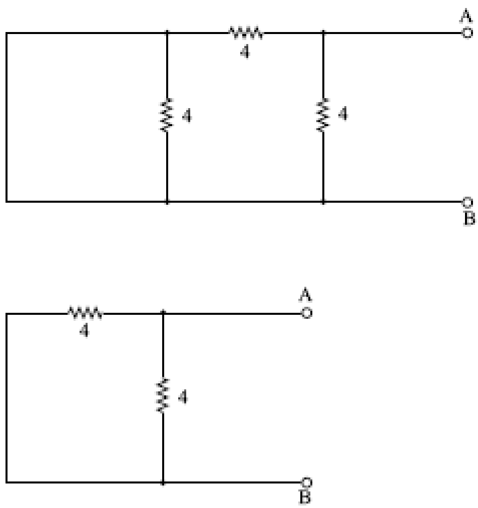


Figure 2.80

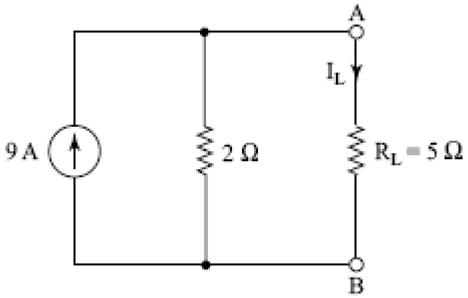


Figure 2.81

$$I_L = 9 \times \frac{2}{2+5} = \frac{18}{7}\ \text{A}$$

Apply Thevenin's theorem

$$R_{eq} = R_{AB} = R_{Th} = 2\ \Omega$$

Let us calculate V_{OC} across terminals A and B,

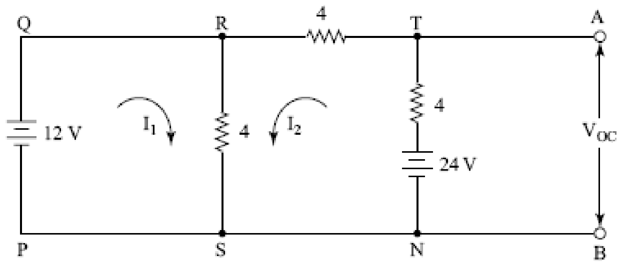


Figure 2.82

From loop PQRS, applying KVL,

$$+12 - (I_1 + I_2)\,4 = 0$$

or,
$$(I_1 + I_2)\ \frac{12}{4} = 3\ \text{A}$$

Similarly from loop N T R S,

$$+24 - 4I_2 - 4I_2 - 4(I_1 + I_2) = 0$$

or,
$$8\,I_2 = 24 - 4(I_1 + I_2)$$

Substituting the value of $(I_1 + I_2)$

$$I_2 = \frac{24 - 4 \times 3}{8} = \frac{12}{8} = \frac{3}{2}\ \text{A}$$

$$V_{OC} = V_{NT} = 24 - 4I_2 = 24 - 4 \times \frac{3}{2}\ 18\ \text{V}$$

According to Thevenin's,

$$I_L = \frac{V_{OC}}{R_{Th} + R_L} = \frac{18}{2+5} = \frac{18}{7}\ \text{A}$$

Thus, we get the same value of current through the load resistance. In circuit solutions, experience will tell you as to which theorem is more suitable for which kind of circuit solution.

Example 2.30 Apply Norton's theorem to determine the current flowing through the resistance of 6 Ω connected across the terminals. A and B. Also calculate the potential of point A. What will be the current through the 6 Ω resistor across AK. Solve this problem using Thevenin's theorem also.

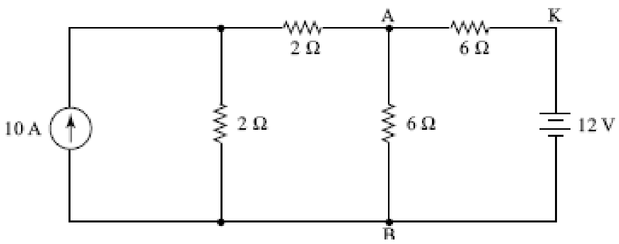


Figure 2.83

Solution:

After removing the 6 Ω resistor across terminals A and B, we short circuit the terminals and determine I_{sc} due to the current source and also due to the voltage source and then add them to find their combined effect. Terminals A and B have been shown short circuited separately for each of the voltage sources.

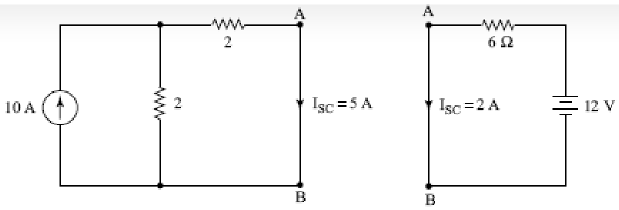


Figure 2.84

$$\text{Total } I_{sc} = 5 + 2 = 7 \text{ A}$$

Equivalent resistance across A and B is determined by open circuiting the current source and short circuiting the voltage source.

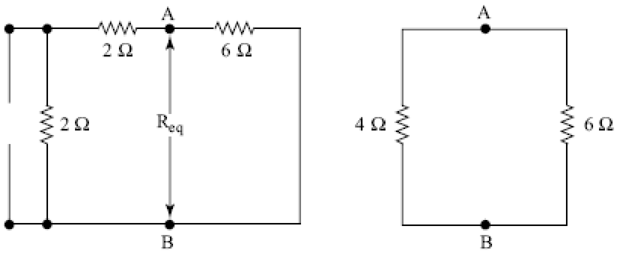
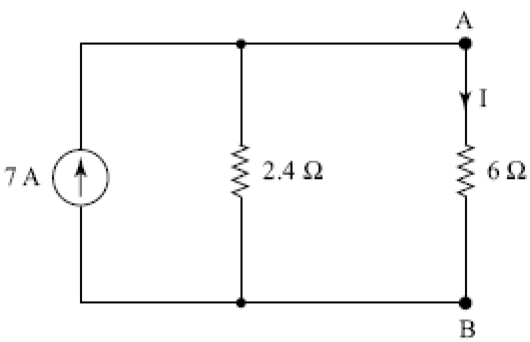


Figure 2.85

$$R_{eq} = R_{AB} = \frac{6 \times 4}{6 + 4} = 2.4\ \Omega$$

Norton's equivalent circuit is shown in Fig. 2.86. Using the current divider rule current, I is calculated as



$$I = \frac{7 \times 2.4}{2.4 + 6} = 2A$$

The potential of A with respect to B = 6 × 2 = 12 V

Since potential across A and B is 12 V and the potential of point K with respect to point B is again 12 V, no current will flow through the resistor connected between terminals A and K.

Now, let us apply Thevenin's theorem

We convert the current source into its equivalent voltage source of the network and redraw it as

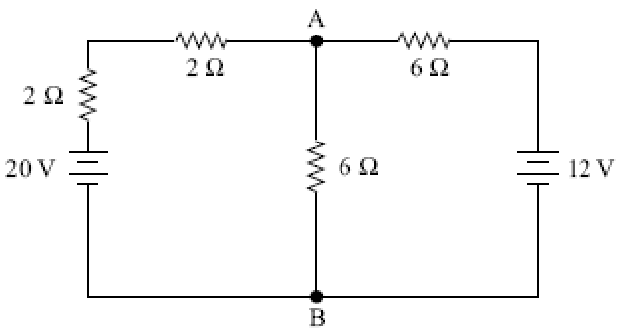


Figure 2.87

Remove the 6 Ω resistor across AB and determine V_{OC},

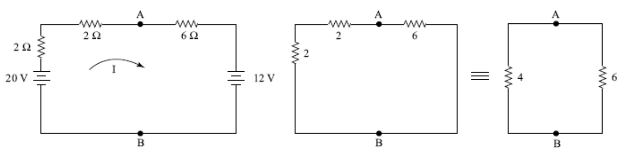


Figure 2.88

We write,

$$+20\text{ V} - 2I - 2I - 6I - 12\text{ V} = 0$$

or,

$$10\,I = 8\text{ V}$$
$$I = 0.8\text{ A}$$
$$V_{OC} = +12 + 6 \times 0.8 = 16.8\text{ V}$$

Thevenin's equivalent circuit is

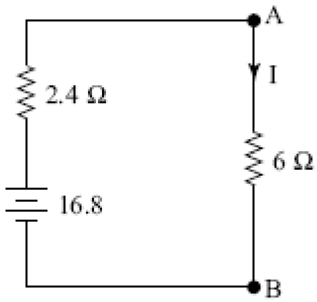


Figure 2.89

$$I = \frac{16.8}{R_{Th} + R} = \frac{16.8}{2.4 + 6} = 2\ A$$

Example 2.31 By applying Thevenin's as well as Norton's theorem show that current flowing through the 16 Ω resistance in the following network is 0.5 A.

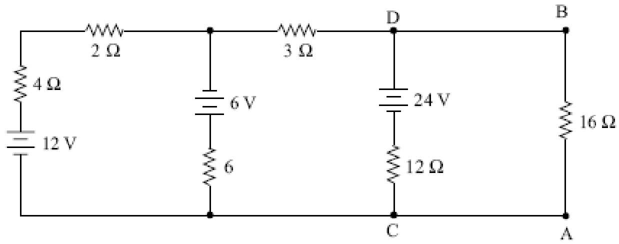
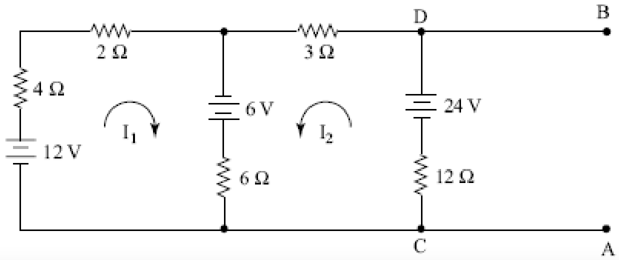


Figure 2.90

Apply Thevenin's theorem

Remove the 16 Ω resistor and calculate V_{OC}



Kirchhoff's voltage equations are

For loop I,

$$12 - 6I_1 + 6 - 6(I_1 + I_2) = 0$$

or, $$6(2I_1 + I_2) = 18$$

or, $$2I_1 + I_2 = 3 \qquad \text{(i)}$$

For loop II,

$$24 - 3I_2 + 6 - (I_1 + I_2)6 - 12I_2 = 0$$

or, $$21I_2 + 6\,I_1 = 30$$

or, $$7I_2 + 2\,I_1 = 10 \qquad \text{(ii)}$$

$$2I_1 + 7I_2 = 10 \qquad \text{(iii)}$$

$$2I_1 + I_2 = 3 \qquad \text{(i)}$$

Subtracting,

$$6I_2 = 7$$

or, $$I_2 = \frac{7}{6}\text{ A}$$

From (i)

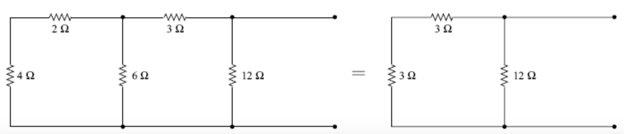
$$2I_1 = 3 - I_2 = 3 - \frac{7}{6} = \frac{11}{6}$$

$$I_1 = \frac{11}{12}\text{ A}$$

$$V_{CD} = V_{OC} = -12\,I_2 + 24$$

$$= -12 \times \frac{7}{6} + 24 = -14 + 24 = 10\text{ V}$$

R_{th} is calculated as



$$R_{th} = \frac{6 \times 12}{6+12} = 4\ \Omega$$

Current through the 16 Ω resistor is

$$I = \frac{V_{OC}}{R_{th} + 16} = \frac{10}{4+16} = \frac{10}{20} = 0.5\ \text{A}$$

Now, we will apply Norton's theorem to calculate the current.

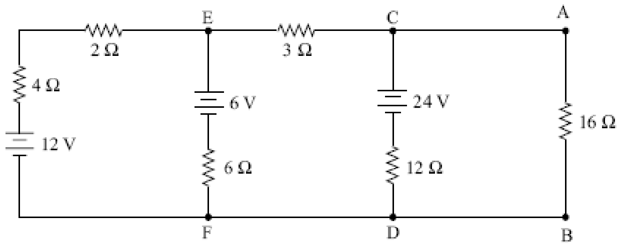


Figure 2.93

First, we will convert the voltage sources into equivalent current sources so that the circuit becomes

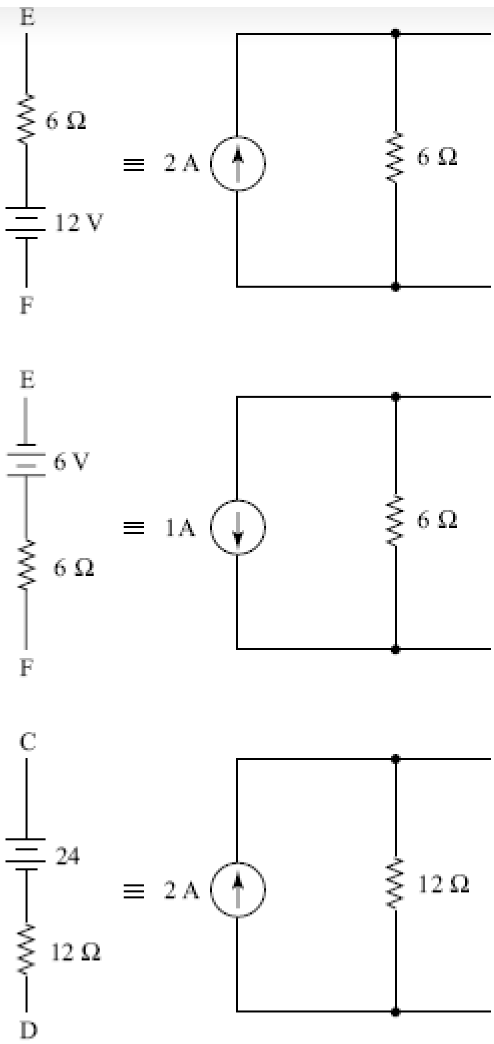


Figure 2.94

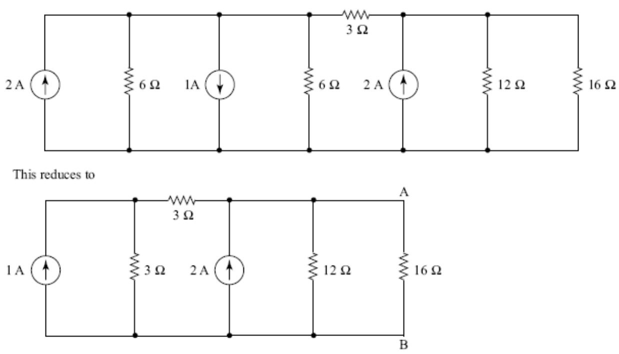
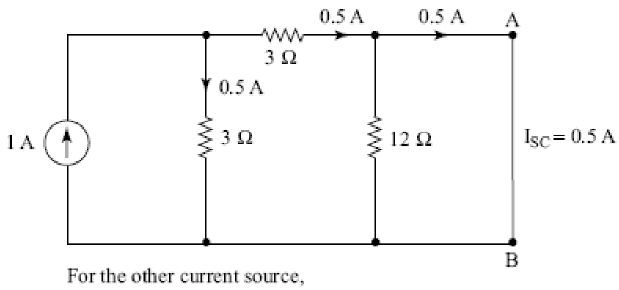


Figure 2.95



For the other current source,

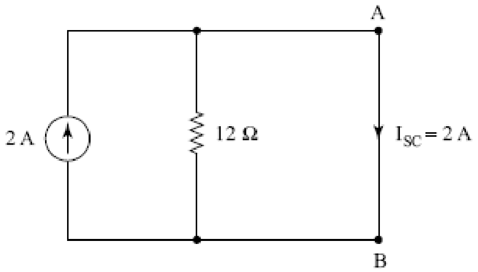


Figure 2.96

Total $\quad I_{SC} = 0.5 + 2 = 2.5$ A

The equivalent resistance is calculated by open circuiting the current sources as

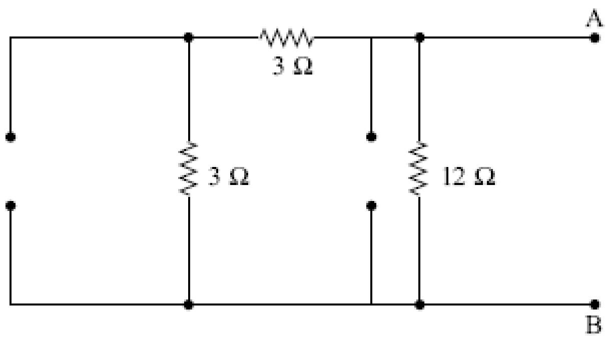


Figure 2.97

$$R_{eq} = \frac{6 \times 12}{6 + 12} = 4\,\Omega$$

Norton’s equivalent circuit is

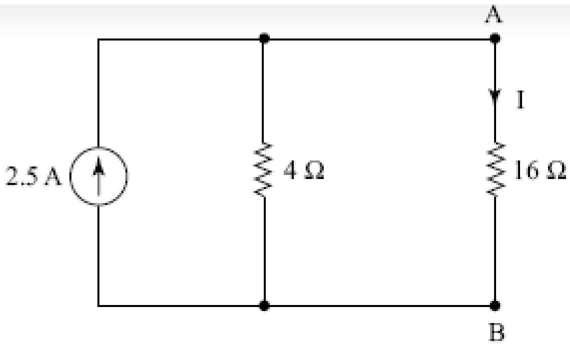


Figure 2.98

$$I = \frac{2.5 \times 4}{4 + 16}$$

or,

$$I = \frac{2.5}{5} = 0.5\ \text{A}$$

Thus, the current through the 16 Ω resistor calculated using Thevenin's theorem and Norton's theorem is the same.

2.8.4 Millman's Theorem

When a number of voltage sources form parallel branches, the common voltage across their terminals can be found out by applying Millman's theorem. To understand the theorem let us consider a circuit having three parallel branches as shown in Fig. 2.99 (a). By converting the voltage sources into equivalent current sources, the circuit will be as shown in Fig. 2.99 (b) and (c)

The voltage across terminals A and B, i.e., V_{AB} can be calculated as

$$V_{AB} = I_T R_{eq} = \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right) \times \frac{1}{[(1/R_1) + (1/R_2) + (1/R_3)]} \tag{2.4}$$

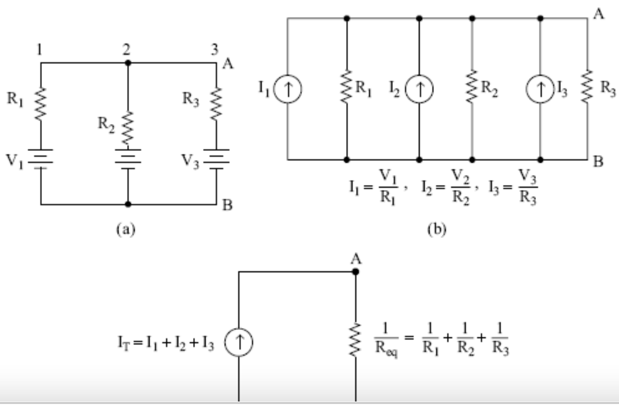


Figure 2.99

2.8.5 Maximum Power Transfer Theorem

Power is supplied from a source to a load. Figure 2.100 shows a generator with internal resistance, R_i supplying power to a load resistance, R_L. Maximum power transfer theorem tells us at what load (i.e., at what value of R_L) maximum power will be transferred from the source to the load.

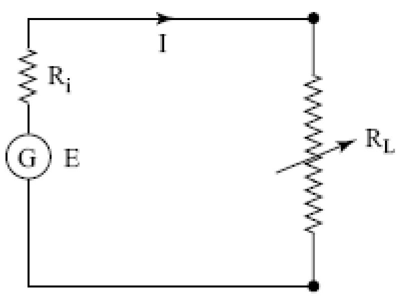
Let us arrive at the condition of maximum power transfer through the following calculations and thereafter state the related theorem.

Current, $$I = \frac{E}{R_i + R_L}$$

Let power consumed by or delivered to the load be P_L. Since power = I^2R,

$$P_L = I^2R_L = \left(\frac{E}{R_i + R_L}\right)^2 R_L$$

Since R_L is variable, for determining the condition for maximum power transfer from the source to the load, we will differentiate P_L with respect to R_L.



(d)

Figure 2.100

i.e., we will make $$\frac{dP_L}{dR_L} = 0$$

Thus, $$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[\frac{E^2R_L}{(R_L+R_i)^2}\right] = 0$$ or, $$\frac{d}{dR_L}E^2\left[\frac{R_L}{(R_L+R_i)^2}\right] = 0$$

or, $$\frac{d}{dR_L}E^2[R_L(R_L+R_i)^{-2}] = 0$$

or, $$E^2\left[\frac{1}{(R_L+R_i)^2} - \frac{2R_L}{(R_L+R_i)^3}\right] = 0$$

Thus, maximum power transfer will occur when the value of the load resistance is equal to the internal resistance of the source.

The maximum power transfer theorem is stated as follows:

In a dc network maximum power will be consumed by the load or maximum power will be transferred from the source to the load when the load resistance becomes equal to the internal resistance of the network as viewed from the load terminals.

The value of maximum power when $R_L = R_i$ is calculated as

$$P_L(\max) = \frac{E^2 R_L}{(R_L + R_L)^2} \quad (\text{since } R_i = R_L)$$

$$= \frac{E^2}{4R_L} = \frac{E^2}{4R_i} \qquad (2.5)$$

When a complex dc network is to be analysed for maximum power transfer, the circuit can first be converted into a voltage source with one internal resistance by applying Thevenin's theorem.

Let us take a few specific problems to understand this theorem.

Example 2.32 A 12 V battery is supplying power to a resistive load R_L through a network as shown in Fig. 2.101. Calculate at what value of R_L power transferred to the load will be maximum and what would be the value of that maximum power.

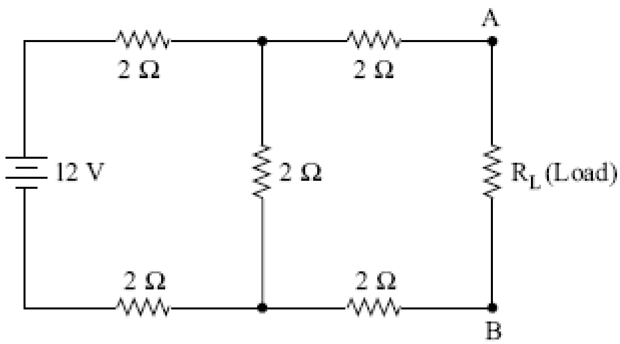


Figure 2.101

Solution:

Let us convert this circuit into a Thevenin's equivalent circuit through the following steps:

open circuit voltage V_{AB} is calculated as

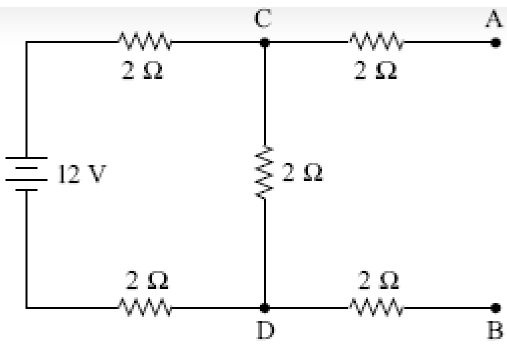


Figure 2.102

$$I = \frac{12}{2+2+2} = \frac{12}{6} = 2\ A$$

Voltage drop across terminals C and D = 2 × 2

= 4 V

Voltage across terminals C and D is the same as the voltage across terminals A and B since no current is flowing beyond terminals C and D when the load resistance has been removed.

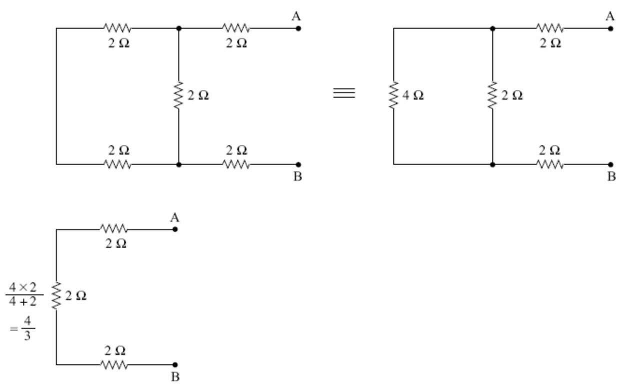


Figure 2.103

The equivalent resistance of the circuit as viewed from the load end, after short circuiting the voltage source is calculated as,

$$R_{eq} = 2+2+\frac{4}{3} = \frac{16}{3}$$

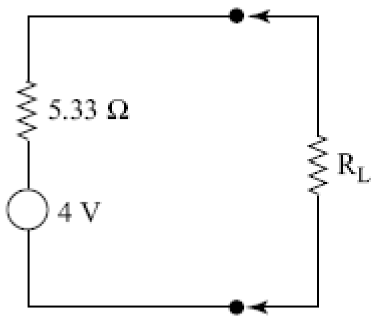


Figure 2.104

For maximum power transfer, $R_L = R_i$

Therefore, $R_L = 5.33\ \Omega$

Value of maximum power

$$P_L(\max) = \frac{E^2}{4R_L} = \frac{4^2}{4 \times 5.33} = \frac{4}{5.33}\ \text{Watts}$$

$$= 0.75\ \text{Watts}$$

Example 2.33 Calculate the value of load resistance, R_L for which maximum power will flow to the load. Also calculate the maximum power transfer efficiency, i.e., the power transmission efficiency when maximum power is transferred.

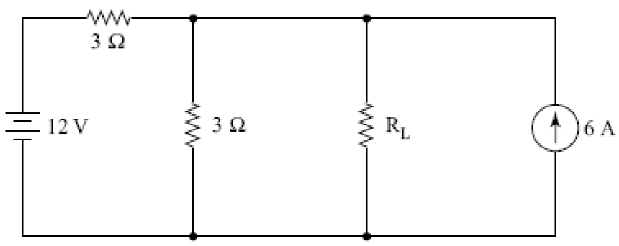
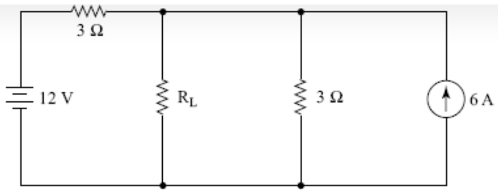


Figure 2.105



By converting the current source into equivalent voltage source

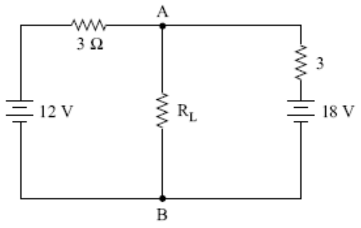
or, 

Figure 2.106

Open circuit voltage across terminals A and B is calculated as

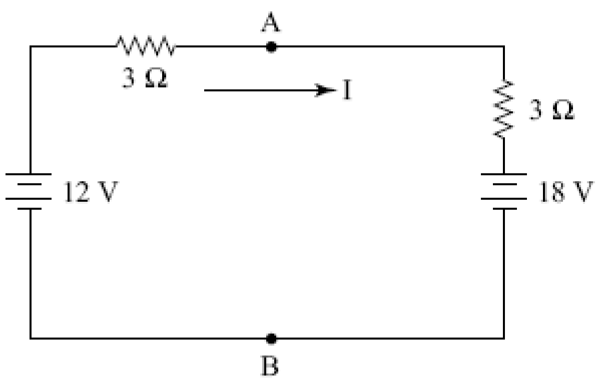


Figure 2.107

Applying KVL,

$$+12 - 3\,I - 3\,I - 18\text{ V} = 0$$
$$-6\text{V} = 6\,I$$
$$I = -1\text{ A}$$

V_{OC} across terminals A and B = 18 V – 3 × 1
= 15 V

R_{eq} across terminals A and B by short circuiting voltage sources is,

$$R_{eq} = \frac{3}{2}\ \Omega$$

Ω

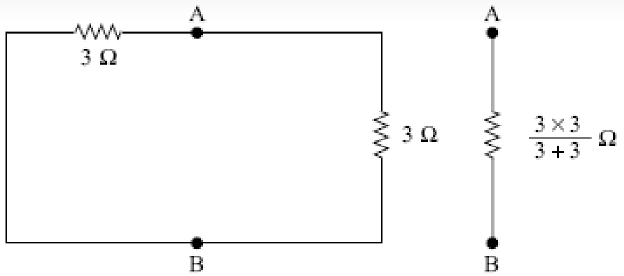


Figure 2.108

Thevenin's equivalent circuit across R_L is

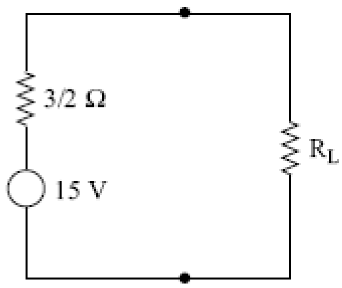


Figure 2.109

For maximum power transfer, $R_L = \frac{3}{2}\Omega = 1.5\,\Omega$

Current, $I = \frac{15}{1.5+1.5} \quad [\because R_L = R_i]$

$= 5\text{ A}$

Maximum power $= I^2R_L = 5^2 \times 1.5 = 37.5\text{ Watts}$

Power transfer efficiency, $\eta = \frac{\text{Power supplied to the load}}{\text{Total power supplied by the source}}$

$$= \frac{I^2R_L}{I^2R_i + I^2R_L} = \frac{R_L}{R_i + R_L}$$

$$= \frac{1}{1 + (R_i / R_L)}$$

when $R_i = R_L$,

$$\%\eta = \frac{1}{1+1} \times 100 = 50 \text{ per cent}$$

This shows that for maximum power transfer conditions, the power transfer efficiency is 50 per cent only.

The maximum power transferred from the source to the load and also the power transfer efficiency are important in practical

Example 2.34 Calculate the value of load resistance, R_L for which maximum power will be transferred from the source to the load in the following circuit.

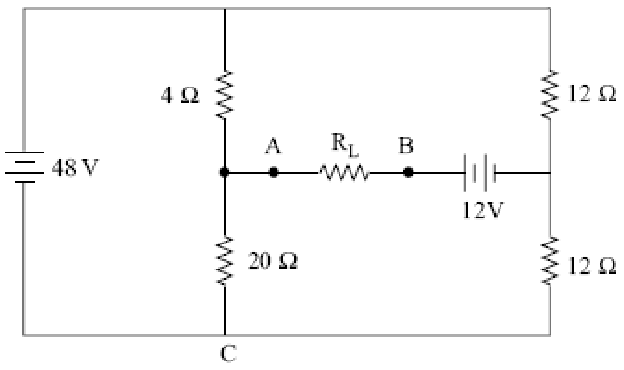


Figure 2.110

Solution:

We will apply Thevenin's theorem to reduce the circuit into a single voltage source connected across the load terminals A and B.

After removing R_L from terminals A and B we will calculate the open-circuit voltage across these terminals as

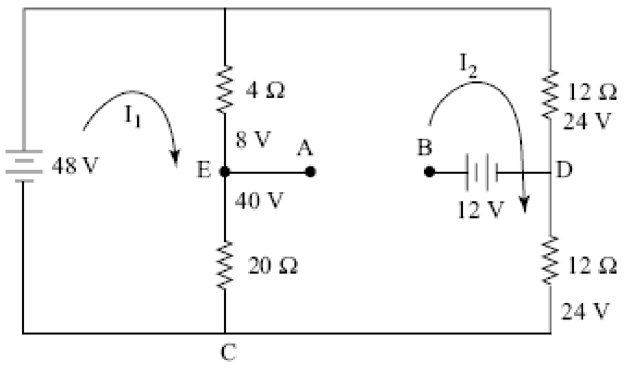


Figure 2.111

$$I_1 = \frac{48}{4+20} = 2\text{ A and } I_2 = \frac{48}{12+12} = 2\text{ A}$$

Voltage drops across the 4 Ω resistor is 8 V, 20 Ω resistor is 40 V, Potential difference between point C and D is 24 V. Point D is at a higher potential than C.

Potential of B with respect to C is 24 V + 12 V = 36 V.

Since the potential of point E or A is 40 V with respect to point C and the potential of point B with respect to point C is 36 V, point A is at a higher potential than point B. The potential difference between points A and B is 4 V. This is called V_{OC}.

Now let us calculate the equivalent resistance of the circuit across terminals A and B after short circuiting the sources of EMFs.

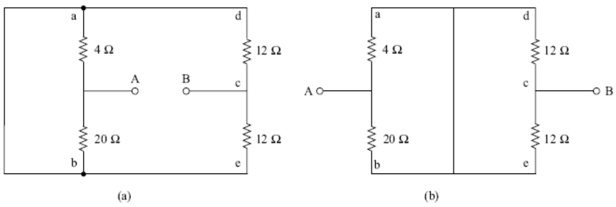


Figure 2.112

Rearranging

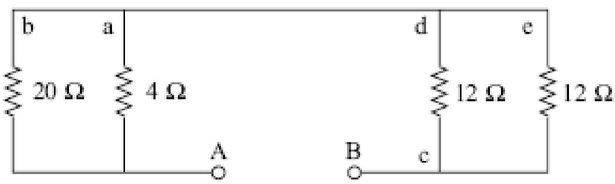


Figure 2.113

Note that ABDE are all connected together.

$$R_{\text{thevenin}} = \frac{4 \times 20}{4 + 20} + \frac{12 \times 12}{12 + 12}$$
$$= 3.33 + 6 = 9.33\ \Omega$$

Thevenin's equivalent circuit is, therefore,

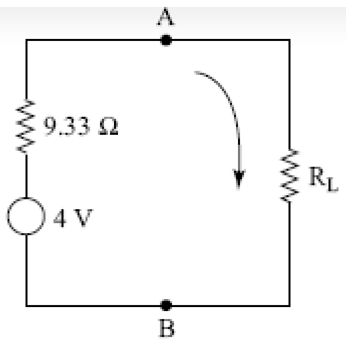


Figure 2.114

For maximum power transfer, $R_L = R_i$

$\therefore$ $R_L = 9.33\ \Omega$

Current, $I = \dfrac{4}{9.33 + 9.33}$

$= 0.214$ A

Maximum power transferred

$P_L = I^2 R_L$

$= (0.214)^2 \times 9.33$ Watts

$= 0.42$ Watts

P_L (max) can also be calculated using the relation, P_L (max)

$$= \frac{E^2}{4R_L} = \frac{4^2}{4 \times 9.33} = 0.42 \text{ Watts}$$

Example 2.35 Calculate the value of load resistance R_L for which maximum power transfer will occur from source to load. Also calculate the value of maximum power and power transfer efficiency.

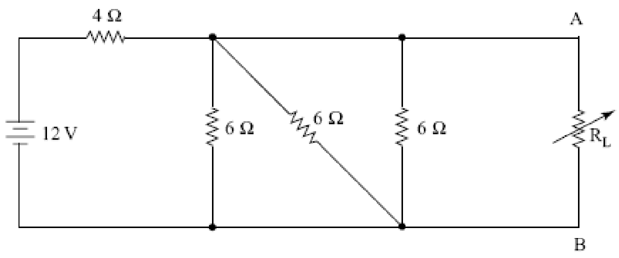


Figure 2.115

Solution:

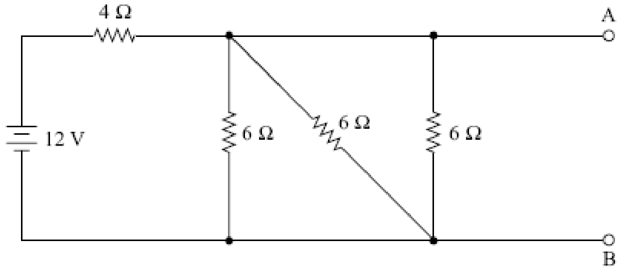


Figure 2.116

Since all the three 6 Ω resistors are in parallel, the above circuit gets reduced to

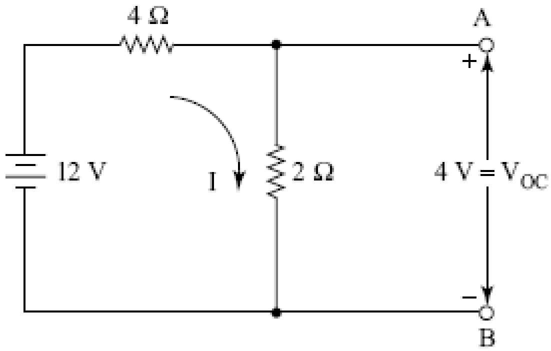


Figure 2.117

$$I = \frac{12}{4+2} = 2\text{ A}$$

Voltage across AB, i.e., $V_{OC} = IR = 2 \times 2 = 4$ V

Equivalent resistance R_{Th} is calculated as,

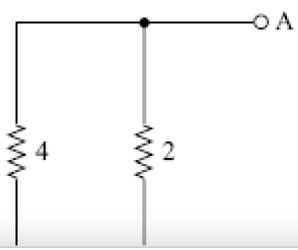


Figure 2.118

$$R_{Th} = \frac{4\times 2}{4+2} = \frac{4}{3}\ \Omega$$

Thus Thevenin's equivalent circuit is

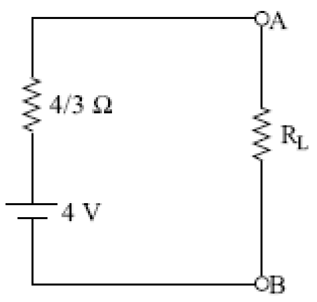


Figure 2.119

For maximum power transfer, R_L = R_i

$$R_L = \frac{4}{3}\Omega$$

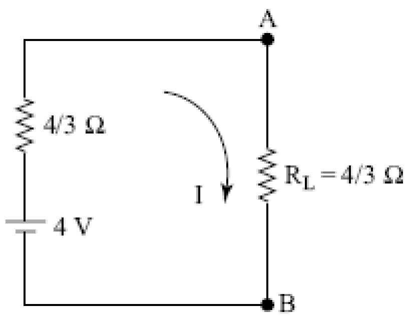


Figure 2.120

Current through the circuit, $I = \dfrac{4}{4/3+4/3} = 1.5\,A$

Power transferred or consumed = $I^2 R_L$

$$= (1.5)^2 \times \frac{4}{3}$$

$$= 2.25 \times \frac{4}{3} \text{ Watts}$$

$$= 3 \text{ Watts}$$

Transfer Efficiency, $\eta = \frac{R_L}{R_L + R_i} = \frac{R_i}{2R_i} = \frac{1}{2}$

Percent Efficiency = 50 per cent.

2.9 STAR–DELTA TRANSFORMATION

Electrical network problems can be simplified by converting three resistances forming a delta to corresponding three resistances forming an equivalent star between the three terminals of the network. Similarly, resistances in the star formation can be converted into equivalent delta. Let us take a simple example as in Fig. 2.121 (a). Suppose we want to calculate the current supplied by the voltage source to the network. As such we have to write the equations for the three loops using Kirchhoff's laws and solve these equations to find the total current supplied by the battery. However, simply by transformation of three resistances in delta to three resistances forming an equivalent star, the circuit is simplified and solution of the circuit becomes very easy. This process of transformation of a delta to star and simplication of the solution of the problem is illustrated in Fig. 2.121.

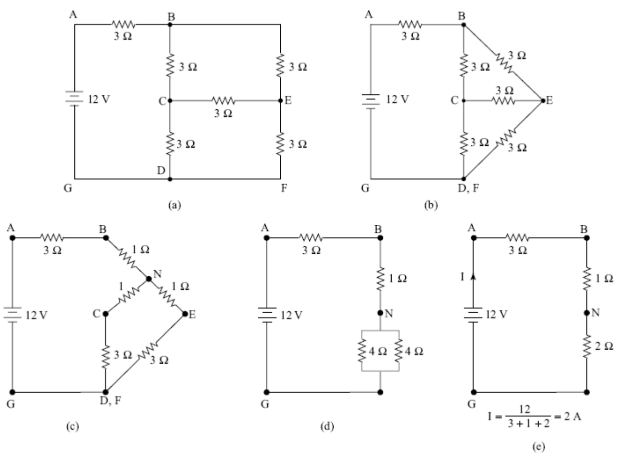


Figure 2.121

It can be seen from Fig. 2.121 (a) and (b) that between terminals B, C, and E three resistances of 3 Ω each are forming a delta. This delta is converted into an equivalent star between the same terminals replacing the 3 Ω resistances by equivalent 1 Ω resistances as shown in the battery current is calculated as shown in Fig. 2.121 (d) and (e). This shows that star to delta and delta to star transformation of resistances is advantageous in solving electrical circuit problems.

2.9.1 Transforming Relations for Delta to Star

Let us consider three resistances R_{12}, R_{23}, and R_{31} connected in delta formation between the terminals A, B, and C. Let their equivalent star-forming resistances between the same terminals be R_1, R_2, and R_3 as shown in Fig. 2.122. These two arrangements of resistances can be said to be equivalent if the resistance measured between any two terminals is the same in both the arrangements.

If we measure resistance between terminals A and B, from Fig. 2.122 (a) we will get R_{12} and a series combination of R_{23} and R_{31} in parallel, i.e.,

$$R_{AB} = \frac{R_{12}\,(R_{23} + R_{31})}{R_{12} + (R_{23} + R_{31})}$$

From Fig. 2.122 (b) we get across terminals A and B, R_1 and R_2 in series, terminal C being open and not connected. Therefore,

$$R_{AB} = R_1 + R_2$$

For the purpose of equivalence we can write

$$R_1 + R_2 = \frac{R_{12}\,R_{23} + R_{12}R_{31}}{R_{12} + R_{23} + R_{31}} \qquad (1)$$

In the same way between terminals B and C, the equivalence can be expressed as

$$R_2 + R_3 = \frac{R_{23}\,(R_{31} + R_{12})}{R_{23} + (R_{31} + R_{12})} \qquad (2)$$

Between terminals C and A, the equivalence can be expressed as

$$R_1 + R_3 = \frac{R_{31}\,(R_{23} + R_{12})}{R_{31} + (R_{23} + R_{12})} \qquad (3)$$

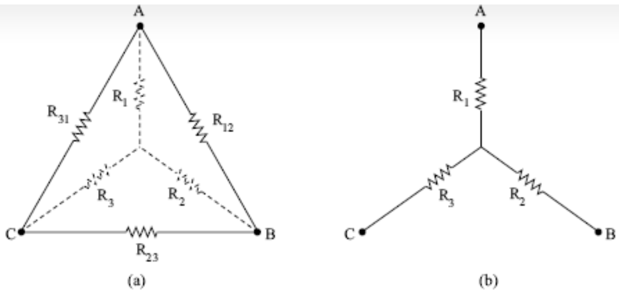


Figure 2.122

Subtracting eq. (2) from eq. (1)

$$R_1 - R_3 = \frac{R_{12} R_{31} - R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \tag{4}$$

Adding eq. (4) with eq. (3)

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

This way by solving eqs. (1), (2), and (3), R_1, R_2, R_3 can be found.

Thus, when delta-connected resistances are changed to an equivalent star-forming resistances, their values are:

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \tag{5}$$

$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}} \tag{6}$$

$$R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \tag{7}$$

To remember this expressions of R_1, R_2, and R_3 let us look at Fig. 2.122 (a) again. The star equivalence of delta-forming resistances can be shown through dotted lines. The value of R_1 is equal to the product of the two resistances of delta touching point A, i.e., R_{12} and

Now let us consider star to delta transformation as shown in Fig. 135. The basic equations guiding the conversion from delta to star remains the same in this case also. As such we can use eqs. (1), (2), and (3). Solving eqs. (1), (2), and (3) we got eqs. (5), (6), and (7).

Multiplying eq. (5) by eq. (6),

$$R_1R_2 = \frac{R_{12}^2 R_{23} R_{31}}{(R_{12} + R_{23} + R_{31})^2} \qquad (8)$$

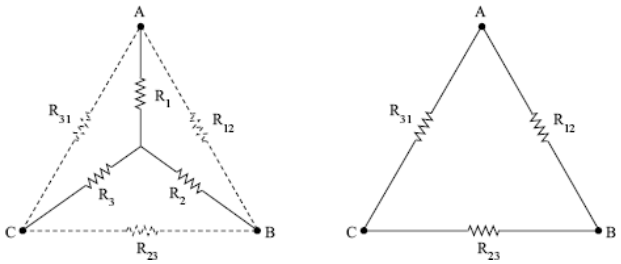


Figure 2.123

Multiplying eq. (5) by eq. (7),

$$R_1R_3 = \frac{R_{31}^2 R_{12} R_{23}}{(R_{12} + R_{23} + R_{31})^2} \qquad (9)$$

Multiplying eq. (6) by eq. (7),

$$R_2R_3 = \frac{R_{23}^2 R_{12} R_{31}}{(R_{12} + R_{23} + R_{31})^2} \qquad (10)$$

Now, adding eqs. (8), (9), and (10)

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} R_{23} R_{31}(R_{12} + R_{23} + R_{31})}{(R_{12} + R_{23} + R_{31})^2}$$

or, $$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$

Earlier we calculated $$R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$ [as in eq. (7)]

Therfore, $R_1R_2 + R_2R_3 + R_3R_1 = R_{12}R_3$

Dividing both sides by $$R_3, R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

Similarly R_{23} and R_{31} can be calculated.

Thus, from eqs. (5), (6), and (7), R_{12}, R_{23}, and R_{31} in terms of R_1, R_2,

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3} \quad (11)$$

$$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1} \quad (12)$$

$$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2} \quad (13)$$

Remembering of these expressions is easy as R_{12} is the sum of R_1 and R_2 plus product of R_1 and R_2 divided by the third resistor, i.e., R_3. (i.e., delta-equivalent resistance of one side is the sum of touching resistances plus product of the touching resistance dividing by the non-touching resistance).

Now let us solve a few problems of network simplification using star–delta transformation and series–parallel calculations.

Example 2.36 Calculate the equivalent resistance of the network across terminals P and Q.

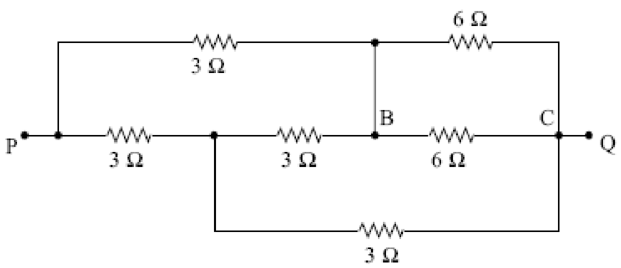


Figure 2.124

Solution:

Two 6 Ω resistors are in parallel. Their equivalent resistance is 3 Ω. So the circuit is redrawn as shown in Fig. 2.125. By pulling point Q downwords this circuit of 2.125 is drawn as in Fig. 2.126 (a).

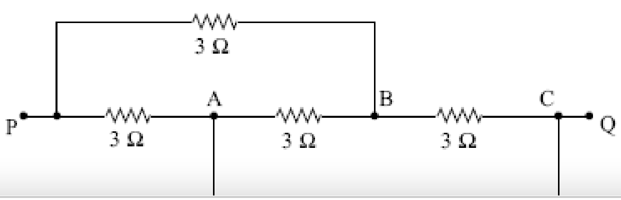


Figure 2.125

Between the terminals A, B, and C, the three resistances of 3 Ω each are connected in delta. This delta is now transformed into an equivalent star with the values calculated using the transformation relationship. The equivalent resistance in star are calculated as R_A, R_B, and R_C as shown in Fig. 2.126 (b).

$$R_A = \frac{R_{AB} R_{AC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$= \frac{3 \times 3}{3+3+3} = 1\,\Omega$$

$$R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC} + R_{CA}}$$

$$= \frac{3 \times 3}{3+3+3} = 1\,\Omega$$

$$R_C = \frac{R_{BC} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$= \frac{3 \times 3}{3+3+3} = 1\,\Omega$$

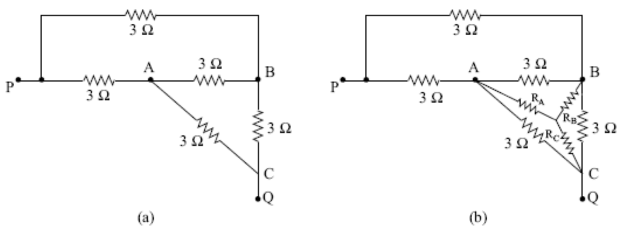


Figure 2.126

The circuit is drawn with the equivalent star as shown in Fig. 2.127.

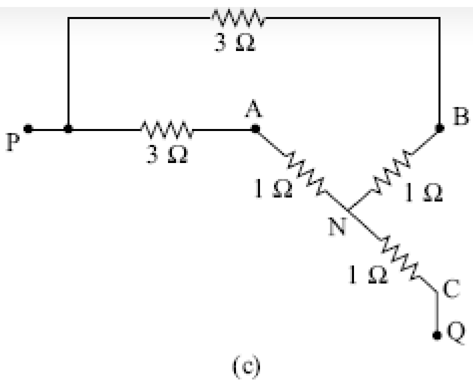


Figure 2.127

By considering series and parallel connection of resistances, the circuit is further simplified as in Fig. 140.

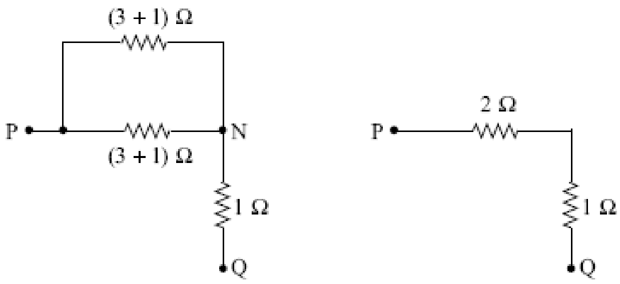


Figure 2.128

$R_{PQ} = 3\ \Omega$

Students should remember that the terminals between which the equivalent resistance has to be calculated have to be kept intact in the transformation process.

Example 2.37 Calculate the current, I supplied by the battery in the circuit shown in Fig. 2.129.

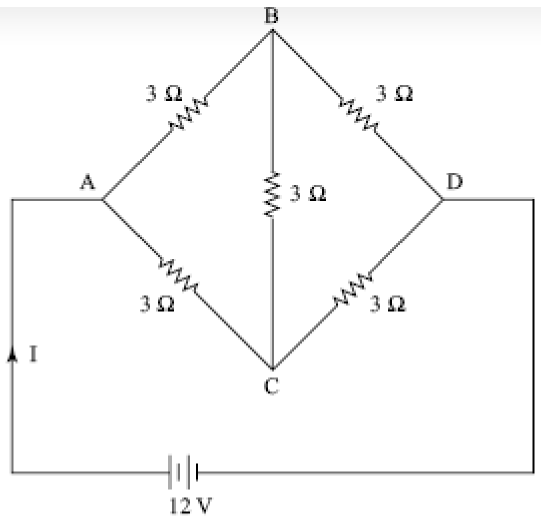


Figure 2.129

Solution:

Instead of applying Kirchhoff's laws and writing the loop equations, we will convert the delta between ABC or BCD into equivalent star and then make simplifications to calculate I as

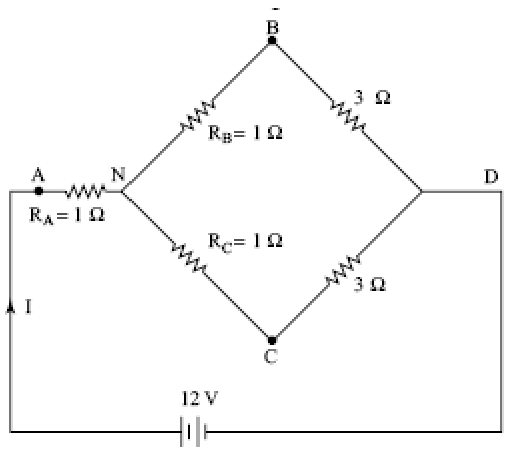


Figure 2.130

$$R_A = \frac{3 \times 3}{3+3+3} = 1\ \Omega$$

$$R_B = \frac{3 \times 3}{3+3+3} = 1\ \Omega$$

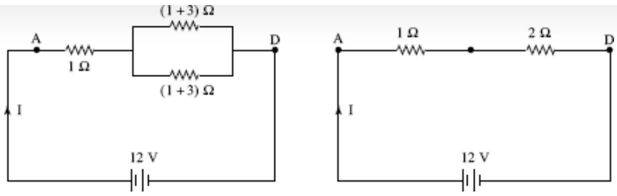


Figure 2.131

$$I=\frac{12}{2+1}=4\,A$$

Example 2.38 Six resistances each of value R W are connected as shown in Fig. 2.132. Calculate the equivalent resistance across the terminals B and C

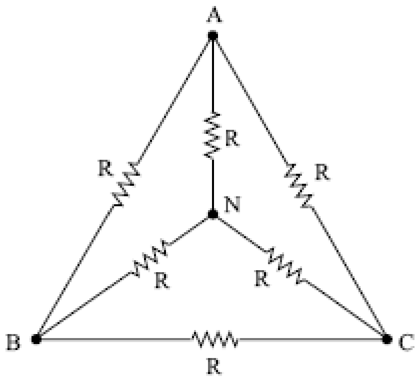


Figure 2.132

Solution:

Resistances between AN, AB, and AC form a star point at A. We will convert this star into an equivalent delta between the terminals B, N, and C as shown in Fig. 2.133.

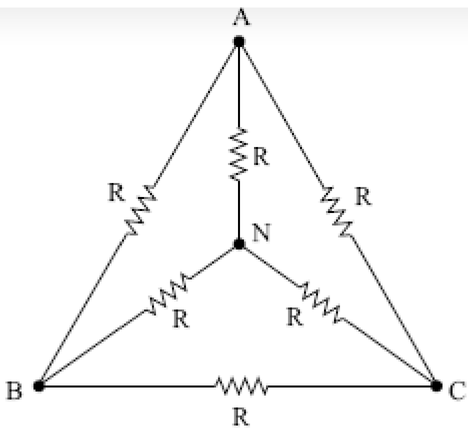


Figure 2.133

$$R_{NC} = R + R + \frac{R \times R}{R}$$
$$= 3R$$

Similarly,

R_{BN} = 3R

R_{BC} = 3R

This network is further simplified as

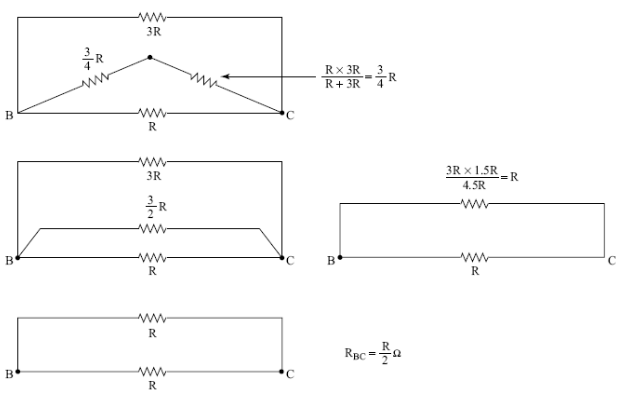


Figure 2.134

The students are advised to solve this problem by converting the three resistances forming star point at N into an equivalent delta touching points A, B, and C and then solve by considering series par-

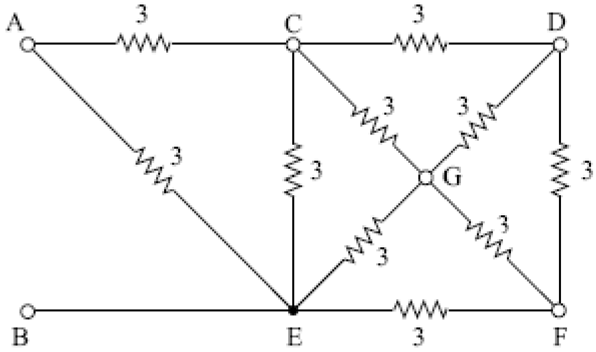


Figure 2.135

Solution:

Let us transform the delta–forming resistances between terminals CDG and EFG. The network will be simplied as

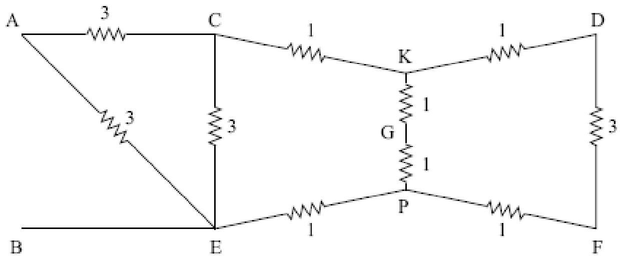
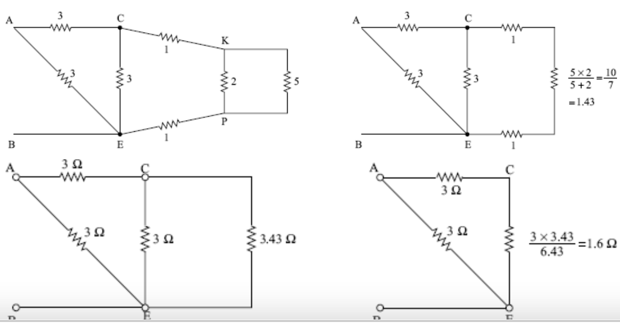


Figure 2.136

Between newly formed terminals K and P, resistances 1 Ω, 3 Ω, and 1 Ω are in series. They are connected in parallel with series combination of two 1 Ω resistors. Thus the network can successively be simplified as



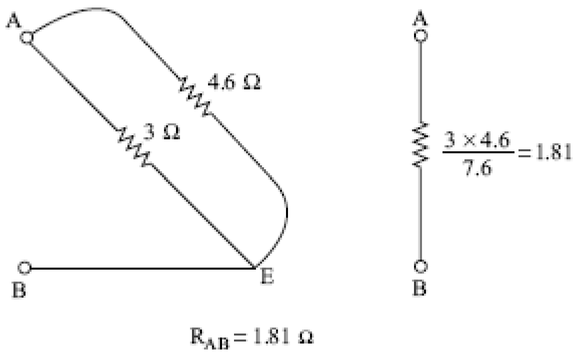


Figure 2.138

Example 2.40 Find the voltage drop across the 10 Ω resistor in the network shown in Fig. 2.139.

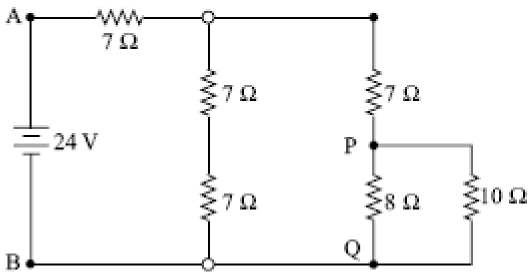


Figure 2.139

Solution:

Firstly we calculate the total current supplied by the battery by determining the equivalent resistance of the circuit across terminals AB. The total resistance of the circuit is calculated by successively reducing the circuit as shown in Fig. 2.140.

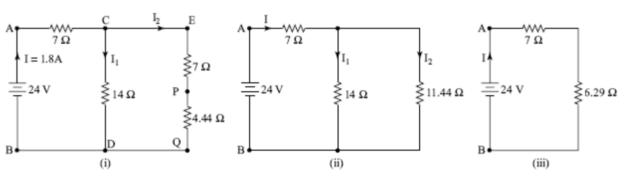


Figure 2.140

$$I = \frac{24}{13.29} = 1.8 \text{ A}$$

Working backwards, the branch currents are calculated. If total current supplied as calculated is 1.8 A, then current I_2 in Fig. 2.140 (ii) is calculated using the current division rule as

$$I_2 = I\frac{14}{14+11.44} = 1.8\frac{14}{25.44} = 0.99 \text{ A}$$

As the same current I_2 is flowing from P to Q, the voltage drop across PQ which is the same as voltage drop across the 10 Ω resistor is calculated as

$V_{PQ} = V_{10\,\Omega} = I_2 R = 0.99 \times 4.44 = 4.39$ V

The battery voltage of 24 V is dropped across the series resistance of 7 Ω, across the combination of 7 Ω and 4.44 Ω resistors as

$7 \times I + 7 \times I_2 + 4.44 \times I_2 = 7 \times 1.8 + 11.44 \times 0.99 = 24$ V

This problem can be solved in another way like, total current, I = 1.8 A.

Drop across AC = 7 × 1.8 = 12.6 V.

Voltage across CD = 24 − 12.6 = 11.4 V

Voltage across EQ = Voltage across CD = 11.4 V

Voltage across

$$PQ = \frac{V_{EQ} \times 4.44}{7+4.44} = \frac{11.4 \times 4.44}{7+4.44} = 4.39 \text{ V}$$

Example 2.40 Calculate the equivalent resistance between the terminals A and B of the network shown in Fig. 2.141.

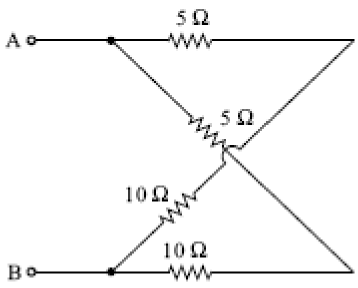


Figure 2.141

This is a simple series and parallel connection of resistors. The circuit is redrawn as

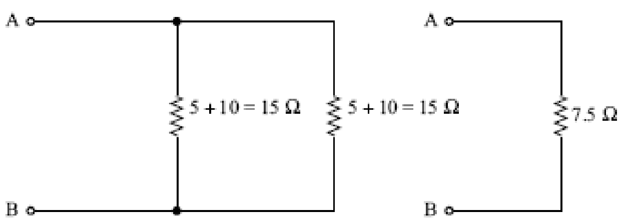


Figure 2.142

The equivalent resistance is calculated as 7.5 Ω.

Example 2.42 Calculate the total current supplied by the battery in the network shown in Fig. 2.143. All resistances shown are in Ohms.

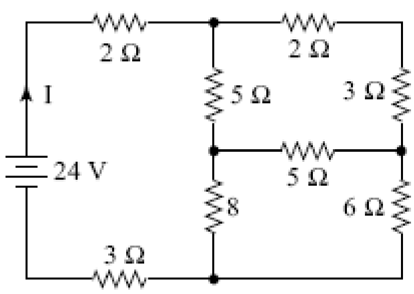
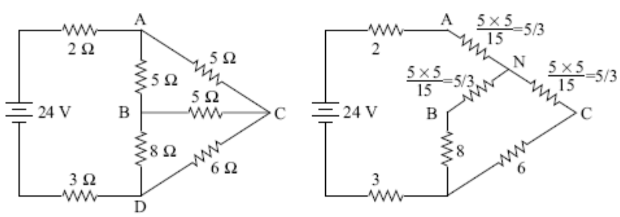


Figure 2.143

Solution:

The circuit is redrawn as shown in Fig. 2.144 after adding two series resistors. The three resistors of 5 Ω each forming a delta across terminals A, B, and C is converted into equivalent star across three terminals and then the circuit is further simplified through series and parallel operations as shown in Fig. 2.145.



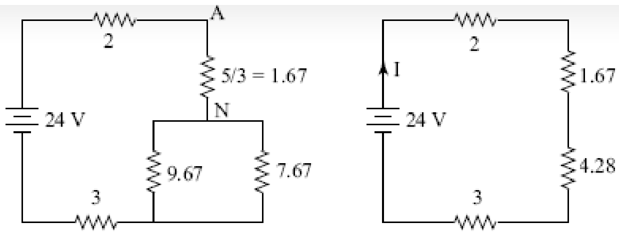


Figure 2.145

$$I = \frac{24\ \text{V}}{2+3+1.67+4.28} = 2.2\ \text{A}$$

Example 2.43 Four resistances are connected as shown in Fig. 2.146. Calculate the equivalent resistance across terminals A and B. What voltage is required to be applied across terminals AB so that potential drop across the terminals A and P is 25 V?

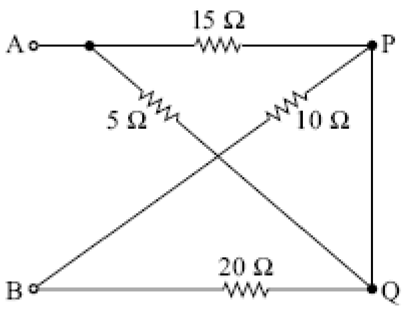


Figure 2.146

Solution:

By examining the given network, we see that terminals P and Q are at the same potential. The circuit is then redrawn as shown by connecting the 10 Ω resistor between B and Q. Similarly a 5 Ω resistor is shown connected between points A and P. The circuit is then simplified in steps as shown.

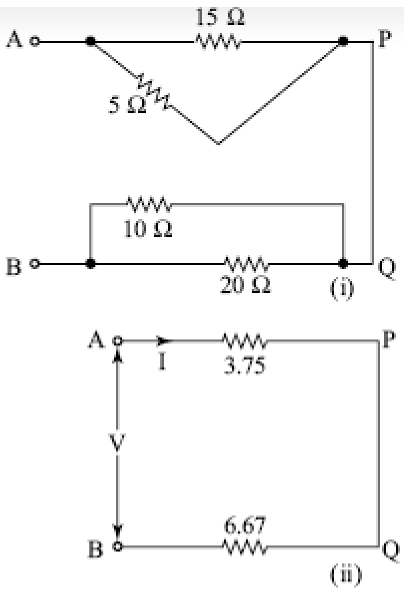


Figure 2.147

Equivalent resistance R_{AB} = 3.75 + 6.67 = 10.42 Ω.

Now let us calculate the supply voltage so that P.D across AP is 25 V.

Assuming V_{AP} = 25, from Fig. 2.147 (ii),

$$I = \frac{V_{AP}}{R_{AP}} = \frac{25}{3.75} = 6.66 \text{ A}$$

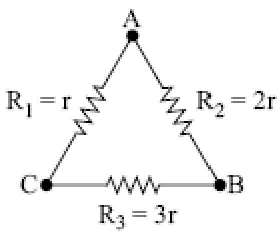
$$V_{BQ} = R_{BQ} \times I = 6.67 \times 6.66 = 44.46 \text{ V}$$

Therefore

$$\begin{aligned} V &= V_{AP} + V_{BQ} \\ &= 25 \text{ V} + 44.46 \text{ V} \\ &= 69.46 \text{ V} \end{aligned}$$

Example 2.44 Three Resistances r, 2r, 3r are connected in delta. Determine the resistance of an equivalent star connection.

Solution:



$$R_A = \frac{R_1 \times R_2}{R_1 + R_2 + R_3} = \frac{2r^{\cancel{2}}}{6\cancel{r}} = \frac{r}{3}$$

$$R_B = \frac{R_2 \times R_3}{R_1 + R_2 + R_3} = \frac{\cancel{6}r^{\cancel{2}}}{\cancel{6}\cancel{r}} = r$$

$$R_C = \frac{R_3 \times R_1}{R_1 + R_2 + R_3} = \frac{3r \times \cancel{r}}{6\cancel{r}} = \frac{r}{2}$$

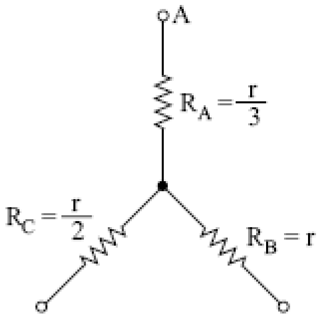


Figure 2.149

Example 2.45 Find the resistance between terminal XY of the bridge circuit shown in Fig. 2.150, by using delta–star conversion.

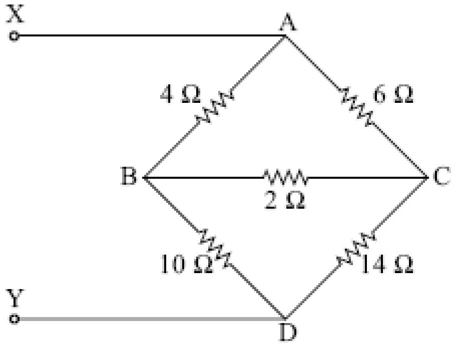


Figure 2.150

Solution:

Let us change the resistances forming a delta across terminals A, B, and C into equivalent star.

$$R_B = \frac{R_{AB} \times R_{BC}}{R_{AB} + R_{BC} + R_{AC}} = \frac{4 \times 2}{12} = \frac{2}{3}\ \Omega$$

$$R_C = \frac{R_{BC} \times R_{AC}}{R_{AB} + R_{BC} + R_{AC}} = \frac{2 \times 6}{12} = 1\ \Omega$$

The equivalent star–forming resistances are

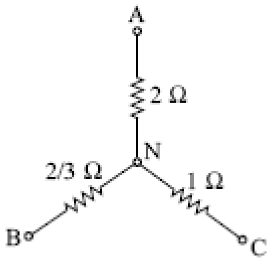


Figure 2.151

By replacing the delta resistance into equivalent star resistance, the circuit is drawn as in Fig. 2.152.

The resistances of the two parallel paths between N and D are 1 + 14 = 15 Ω and $\frac{2}{3} + 10 = \frac{32}{3}\ \Omega,$ Ω, respectively.

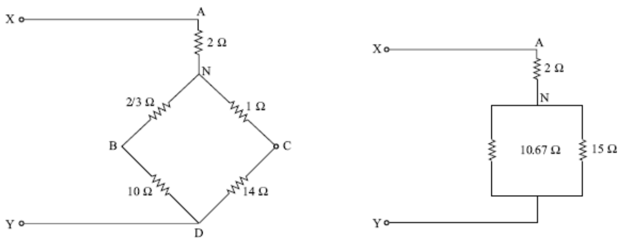


Figure 2.152

Total Resistance Network Terminal X and Y

$$= 2 + \frac{15 \times 10.67}{\ } = 8.23\ \Omega$$

Example 2.46 Find the resistance between terminals A and B in the electric circuit of Fig. 2.153 using Δ–Y transformation.

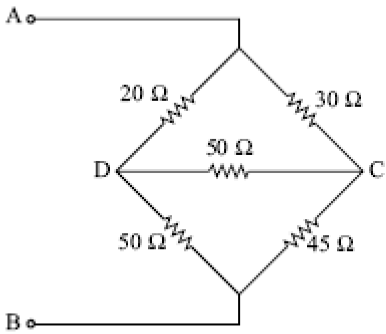


Figure 2.153

Solution:

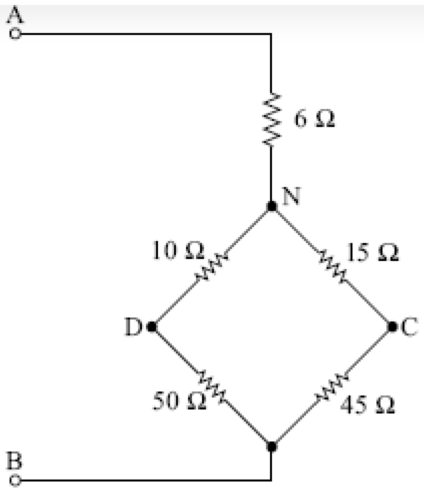
We convert the delta forming resistances between the terminals A, B, and C into an equivalent star. The resistances between the terminals A, B, and C and the star point N are R_{AN}, R_{CN}, and R_{DN}. These are calculated as

$$R_{AN} = \frac{20 \times 30}{100} = 6\ \Omega$$

$$R_{CN} = \frac{30 \times 50}{100} = 15\ \Omega$$

$$R_{DN} = \frac{20 \times 50}{100} = 10\ \Omega$$

After transformation of the delta into star, the circuit becomes as shown in Fig. 2.154.



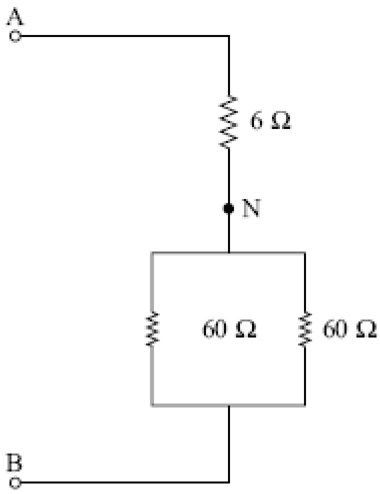


Figure 2.154

Total resistance,

$$R_{AB} = 6 + \frac{60 \times 60}{120}$$
$$= 36\ \Omega$$

Example 2.47 For the circuit shown in Fig. 2.155, calculate the current flowing through the 5 Ω resistor by using the nodal method.

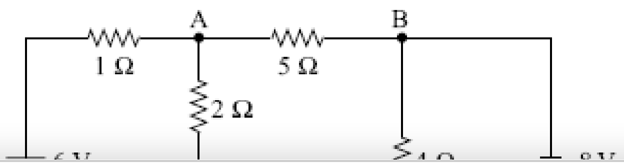


Figure 2.155

Solution:

Let V_A and V_B be the potentials at node A and node B, respectively. Let the reference node be at C. Let us assume current directions at node A as shown in Fig. 2.156.

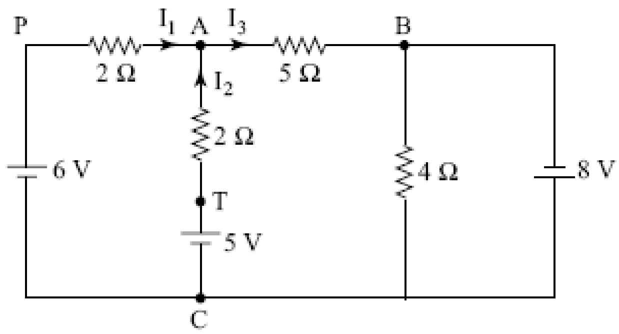


Figure 2.156

We will have incoming currents as equal to outgoing current, i.e.,

$$I_1 + I_2 = I_3 \tag{i}$$

Current, $I_1 = (V_P - V_A)/R = \frac{6 - V_A}{2}$ (ii)

Current, $I_2 = \frac{5 - V_A}{R} = \frac{5 - V_A}{2}$ [∵ potential of point T is +5V] (iii)

Current, $I_3 = \frac{V_A - V_B}{R} = \frac{V_A - (-8)}{5}$ (iv)

Note: Potential of point B with respect to C is –8 V.

Therefore, from (i), (ii), (iii) and (iv),

$$\frac{6 - V_A}{2} + \frac{5 - V_A}{2} - \frac{V_A + 8}{5} = 0$$

or, $$\frac{5(6 - V_A) + 5(5 - V_A) - 2(V_A + 8)}{10} = 0$$

or, $$V_A = \frac{39}{12} = 3.25 \text{ V}$$

Current through the 5 Ω resistor is I_3.

$$I_3 = \frac{V_A - V_B}{5} = \frac{3.25 - (-8)}{5}$$

$$= \frac{3.25 + 8}{5} = 2.25 \text{ A}$$

Example 2.48 Calculate the current flowing through the 8 Ω resistor by using nodal method in the network shown in Fig. 2.157.

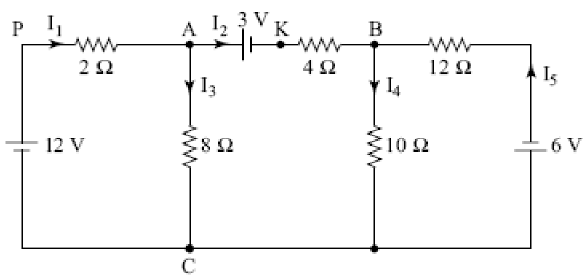


Figure 2.157

Solution:

Let V_A and V_B be the potential of nodes at A and B, respectively. Point C is considered as the reference node

$$I_1 = I_2 + I_3 \qquad \text{(i)}$$

$$I_1 = \frac{V_P - V_A}{2} = \frac{12 - V_A}{2}$$

$$I_3 = \frac{V_A}{8}$$

or, $$\frac{V_K - V_B}{4} = I_2 \text{ and } V_K + 3 = V_A$$

$$V_A - 3 - V_B = 4 I_2$$

or, $$I_2 = \frac{V_A - V_B - 3}{4}$$

Substituting in (i)

$$\frac{12 - V_A}{2} = \frac{V_A - V_B - 3}{4} + \frac{V_A}{8}$$

or, $$\frac{4(12 - V_A) - 2(V_A - V_B - 3) - V_A}{8} = 0$$

Considering currents at node B

or, $$7V_A - 2V_B - 56 = 0 \qquad \text{(ii)}$$

$$I_2 + I_5 = I_4$$

or, $$I_2 + I_5 - I_4 = 0$$

Substituting

$$\frac{V_A - V_B - 3}{4} + \frac{6 - V_B}{12} - \frac{V_B}{10} = 0$$

or, $$\frac{30(V_A - V_B - 3) + 10(6 - V_B) - 12V_B}{120} = 0$$

or, $$15V_A - 26\,V_B - 15 = 0 \qquad \text{(iii)}$$

Solving (ii) and (iii)

V_A = 9.38 V

Current through the 8 Ω resistor is I_3 and

$$I_3 = \frac{V_A}{8} = \frac{9.38}{8} = 1.17\,\text{A}$$

Example 2.49 Use nodal analysis to determine the current flowing through the various branches in the circuit shown in Fig. 2.158. All resistances shown are in Ohms.

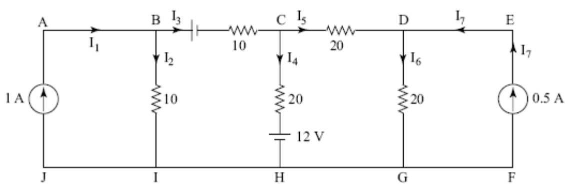


Figure 2.158

We have shown the current directions in the various branches of the circuit and will apply KCL of node B, node C, and node D, respectively. Let V_B, V_C, V_D be the voltages at these nodes.

We have

$I_1 = I_2 + I_3$

$$1 = \frac{V_B}{10} + \frac{V_B - V_C}{10}$$

or, $$2V_B - V_C - 10 = 0 \qquad \text{(i)}$$

Then

$I_3 = I_4 + I_5$ at node C

Putting values

$$\frac{V_B - V_C}{10} = \frac{V_C - 12}{20} + \frac{V_C - V_D}{20}$$

or, $$2V_B - 4V_C + V_D + 12 = 0 \qquad \text{(ii)}$$

Again

$I_5 + I_7 = I_6$

$$\frac{V_C - V_D}{20} + 0.5 = \frac{V_D}{20}$$

or, $$V_C - V_D + 10 = V_D$$

or, $$V_C - 2V_D + 10 = 0 \qquad \text{(iii)}$$

Solving Eqs. (i), (ii) and (iii), we get

$$V_B = 10.4\text{ V}, \quad V_C = 10.8\text{ V}, \quad V_D = 10.4\text{ V}$$

$$I_2 = \frac{V_B}{10} = \frac{10.4}{10} = 1.04\text{ A}$$

(direction of I_2 is opposite to that shown)

$$I_5 = \frac{V_C - V_D}{20} = \frac{10.8 - 10.4}{20} = \frac{0.4}{20} = 0.02 \text{ A}$$

$$I_6 = \frac{V_D}{20} = \frac{10.4}{20} = 0.52 \text{ A}$$

Again $\quad I_6 = I_7 + I_5 = 0.5 + 0.02 = 0.52 \text{ A}$

again $\quad I_2 = I_1 + I_3 = 1.0 + 0.04 = 1.04 \text{ A}$

$$I_4 + I_5 = I_3$$

$$\begin{aligned} I_4 &= I_3 - I_5 \\ &= 0.04 - 0.02 \\ &= 0.02 \text{ A} \end{aligned}$$

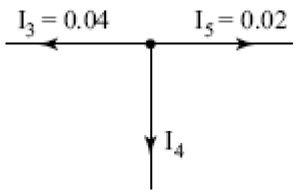


Figure 2.159

Example 2.50 Using nodal analysis calculate the current flowing through all the branches in the network shown in Fig. 2.160.

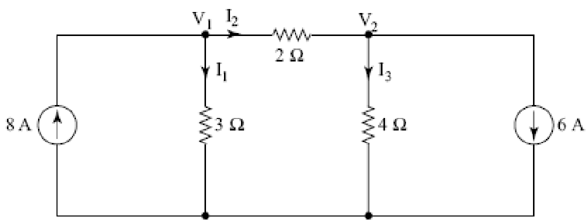


Figure 2.160

Solution:

Applying KCL, we can write

$8 = I_1 + I_2$

or, $\quad 8 = \frac{V_1}{3} + \frac{V_1 - V_2}{2}$

again $$I_2 = I_3 + 6$$

or, $$\frac{V_1 - V_2}{2} = \frac{V_2}{4} + 6$$

or, $$2V_1 - 3V_2 = 24 \qquad \text{(ii)}$$

from (ii)

$3V_2 = 2V_1 - 24$

$$V_2 = \frac{2V_1 - 24}{3}$$

Substituting V_2 in (i)

$$5V_1 - 3\left(\frac{2V_1 - 24}{3}\right) = 48$$

or, $$V_1 = 8\text{ V}$$

Putting value of V_1 in (i)

$$5V_1 - 3V_2 = 48$$

or, $$5 \times 8 - 3V_2 = 48$$

$$3V_2 = 40 - 48 = -8$$

$$V_2 = -\frac{8}{3}\text{ V}$$

$$I_2 = \frac{V_1 - V_2}{2} = \frac{8 - (-8/3)}{2} = 5.33\text{ A}$$

$$I_1 = \frac{V_1}{3} = \frac{8}{3} = 2.66\text{ A}$$

$$I_3 = \frac{V_2}{4} = -\frac{8}{3 \times 4} = -\frac{2}{3} = -0.67\text{ A}$$

To cross-check

$$I_2 = I_3 + 6 = -0.67 + 6 = 5.33\text{ A}$$

2.10 DC TRANSIENTS

2.10.1 Introduction

When a circuit containing inductance, capacitance, and resistance is switched on to a dc supply, the time taken for the current to attain a steady-state condition is called its transient response or transient time. Let a circuit contain a resistance and an inductance or a capacitance connected across a dc source of supply through a switch. When the switch is turned on, the current does not immediately reach its final value. Both the inductance and the capacitance are energy-storing elements. In an inductance, energy is stored in the form of a magnetic field, whereas in a capacitor energy is stored in the form of an electric field. Initially, the current flows at a high rate but as the energy-storing elements, i.e., either the inductor or the ca-the solution of differential equations and not algebraic equations. Transient response of an R–L circuit and an R–C circuit are discussed in this section.

2.10.2 Transient in R–L Circuit

In Fig. 2.161 is shown an inductor of L Henry connected in series with a resistance of R Ω. The combination is connected to a source of supply, V. When a two-way switch S connected to terminal 1, the circuit is on and when the switch is connected to terminal 2, the supply voltage is cut off and the R–L circuit gets short circuited. Let at time t = 0, the switch S be connected to terminal 1. The supply voltage V will be the sum of the voltage drop across the resistance and the voltage developed across the inductor. So we can write

$$V = Ri + L\frac{di}{dt} \quad \text{(i)}$$

or, $$\frac{V}{R} = i + \frac{L}{R}\frac{di}{dt}$$

or, $$\frac{V}{R} - i = \frac{L}{R}\frac{di}{dt}$$

or, $$\frac{R}{L}dt = \frac{di}{(V/R) - i}$$

Integrating, $$\frac{R}{L}t = \log\left(\frac{V}{R} - i\right) + K$$

at $$t = 0,\ i = 0$$

therefore, $$K = -\log\frac{V}{R}$$

Substituting $$\frac{R}{L}t = \log\frac{(V/R) - i}{(V/R)}$$

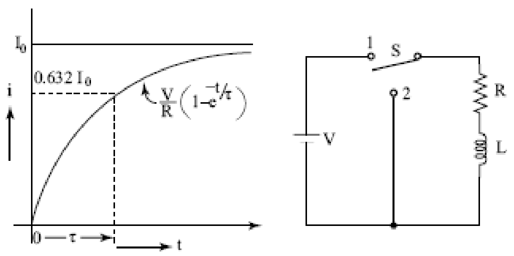


Figure 2.161 Rise in current in an R–L circuit

or, $$\frac{V}{R} - i = \frac{V}{R}e^{\frac{-R}{L}t}$$

or, $$i = \frac{V}{R}\left(1 - e^{\frac{-R}{L}t}\right)$$

or, $$i = \frac{V}{R}\left(1 - e^{\frac{-t}{\tau}}\right) \quad (2.6)$$

where $\tau = \dfrac{L}{R}$, called the time constant of the circuit.

At time $t = \infty$, the current is the steady-state current. Then

This current has two components, i.e.,

$i = I_0$ and $I_0 e^{-t/\tau}$

I_0 is the steady-state current and $I_0 e^{-t/\tau}$ is the transient component of the current which goes on decreasing exponentially with the passage of time. The rise in current in the R–L circuit when the switch is closed has been shown in Fig. 2.161. The rise in current in the circuit is initially rapid but gradually the rise becomes slower and finally comes to a steady-state value. Although theoretically speaking, the current would reach its steady-state value after infinite time, but practically this time is too small a time—a fraction of a second only. The time taken by the current to reach 63.2 per cent of its final value is called the *time constant of the circuit* where, $\tau = \dfrac{L}{R}$.

If we put $t = \tau = \dfrac{L}{R}$, the value of i will become 0.632 of I_0 as

$$i = I_0\left(1 - e^{\frac{-t}{\tau}}\right)$$

Put $\quad t = \tau$

then
$$\begin{aligned} i &= I_0\left(1 - e^{-1}\right) \\ &= I_0(1 - 0.368) \\ &= 0.632\, I_0 \end{aligned}$$

Thus, i = 63.2 per cent of I_0. This has been shown in Fig. 2.161.

Now let us consider what happens when the switch S in Fig. 2.161 is changed to position 2 as shown. We are of course assuming that the switch was in position 1 and current had attained its steady-state value of I_0 A. From position 1, the switch is moved to position 2. Applying KVL we can write the voltage equation as

$$L\frac{di}{dt} + Ri = 0$$

or, $$\frac{di}{i} = -\frac{R}{L}dt$$

Integrating with respect to time t

$$\log i = -\frac{R}{L}t + K$$

The value of K is determined by applying the initial conditions. Here

Therefore, $\log i = -\frac{R}{L}t + K$ (iii)

or, $\log I_0 = K$

or, $K = \log\frac{V}{R}$

Substituting the value of K in (iii)

$$\log i = -\frac{R}{L}t + \log\frac{V}{R}$$

or, $i = \frac{V}{R}e^{\frac{-R}{L}t} = I_0 e^{\frac{-R}{L}t} = I_0 e^{\frac{-t}{\tau}}$

where, $\tau = \frac{L}{R}$

The decay of current in the circuit has been shown in Fig. 2.162.

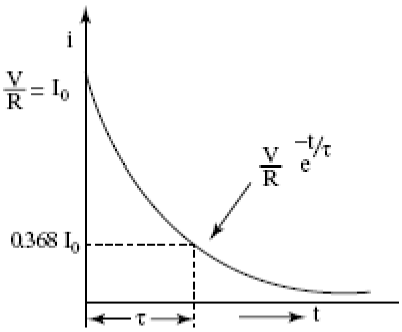


Figure 2.162 Decay in current in on R–L circuit

Like rise in current, the decay in current in an R–L circuit depends on the time constant $\tau = \frac{L}{R}$ seconds.

Time constant for decaying current is obtained by putting $t = \tau$ the expression for i as

$$i = I_0 e^{\frac{-t}{\tau}} = I_0 e^{-1} = 0.368\, I_0$$

As time is increased, the current goes on reducing. As for example the magnitude of decaying current at time $t = 5\tau$ will be

Example 2.51 A coil of resistance 5 Ω and inductance 0.1H is switched on to a 230 V, 50 Hz supply. Calculate the rate of rise of current at $t = 0$ and at $t = 2\tau$ where $\tau = L/R$. What is the steady-state value of the circuit current?

Solution:

For the rise in current

$$i = \frac{V}{R}\left(1 - e^{\frac{-R}{L}t}\right)$$

Substituting values, $$i = \frac{230}{5}\left(1 - e^{\frac{-5}{0.1}t}\right)$$

or, $$i = 46\left(1 - e^{-50t}\right)$$

The rate of change of current,

$$\frac{di}{dt} = 46 \times 50\, e^{-50t}$$
$$= 2300\, e^{-50t}$$

at $$t = 0$$
$$\frac{di}{dt} = 2300 \text{ A/sec}$$

at t equal to two times the time constant, $$\tau = 2\frac{L}{R}$$

$$t = 2\frac{L}{R} = 2\frac{0.1}{5} = 0.04 \text{ sec}$$
$$\frac{di}{dt} = 2300\, e^{-50 \times 0.04} = 2300\, e^{-2} \text{ A/sec}$$

The steady-state value, $$I_0 = \frac{V}{R} = \frac{230}{5} = 46 \text{ A}$$

The larger the time constant is, the more is the time taken by the current to rise or decay in a L–R circuit during transients. Current almost decays to zero at five times the time constant. Beyond this time the magnitude of current is less than one per cent of its steady-state value. When the circuit is switched on, energy is stored in the inductor in the form of a magnetic field and during switching off, the stored energy gets dissipated.

Example 2.52 A coil having an inductance of 1.4 H and a resistance

Solution:

We have,

$$i = I_0\left(1 - e^{\frac{-t}{\tau}}\right)$$
$$I_0 = \frac{V}{R} = \frac{12}{1} = 12\text{ A}$$
$$\tau = \frac{L}{R} = \frac{1.4}{1} = 1.4$$
$$t = 400\text{ ms} = 0.4\text{ sec}$$

Therefore,

$$i = 12\left(1 - e^{\frac{-0.4}{1.4}}\right)$$
$$= 12\left(1 - e^{-.285}\right)$$
$$= 3\text{ A}$$

For the second part of the problem we will use the expression for decaying current.

$$i = I_0 e^{-t/\tau}$$

We have to calculate *t* for *i* to become $I_0/2$

Therefore, $6 = 12e^{\frac{-t}{1.4}}$

or, $0.5 = e^{-t/1.4}$

or, $-\frac{t}{1.4} = \log 0.5$

Therefore $t = 0.75$ seconds

2.10.3 Transient in R–C Circuit

In Fig. 2.163 is shown a capacitor, C and a resistor, R connected in series across a dc voltage source, V through a two-way switch S. When the switch S is connected to position 1, the circuit is switched on. Current *i* will start flowing. Let the instantaneous voltage across the capacitor be v_c, and charge be q. Applying KVL, we can write

$$V = Ri + v_c \qquad \text{(i)}$$

and $C = \frac{q}{v_c}$

or, $q = Cv_c$

and $\int i\,dt = CVv_c$

or, $i = C\frac{dv_c}{dt}$

$$V = Ri + v_c$$

$$= RC\frac{dv_c}{dt} + v_c$$

or, $$V - v_c = RC\frac{dv_c}{dt}$$

or, $$\frac{dt}{RC} = \frac{dv_c}{V - v_c}$$

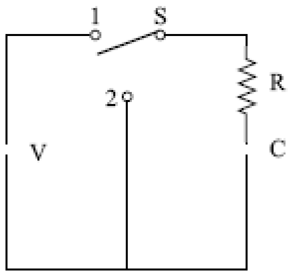


Figure 2.163 RC circuit supplied from a dc source through a two-way switch

Integrating with respect to *t*

$$\frac{t}{RC} = l_n\left(V - v_c\right) + K \qquad \text{(ii)}$$

To find the value of K we put the initial conditions. At time t = 0, the voltage across the capacitor $v_c = 0$.

Putting this value, we find the integration constant

$K = l_n V$

The eq. (ii) is written as

$$\frac{t}{RC} = -l_n\left(V - v_c\right) + l_n V$$

or, $$\frac{V - v_c}{V} = e^{\frac{-t}{RC}}$$

or, $$V - v_c = Ve^{\frac{-t}{RC}}$$

or, $$v_c = V\left(1 - e^{\frac{-t}{RC}}\right) \qquad (2.8)$$

$$= V\left(1 - e^{-t/\tau}\right)$$

$$V = Ri + v_c$$

or,
$$i = \frac{V}{R} - \frac{v_c}{R}$$
$$= \frac{V}{R} - \frac{V\left(1 - e^{-t/\tau}\right)}{R}$$
$$= \frac{V}{R} - \frac{V}{R} + \frac{V}{R}e^{-t/\tau}$$

or,
$$i = \frac{V}{R}e^{-t/\tau} = I_0 e^{-t/\tau} \tag{2.9}$$

Current I_0 is the initial charging current when the switch is just turned off. Because at that instant voltage across the capacitor, v_c = 0.

Therefore,

$$i = \frac{V}{R} = I_0$$

This is the initial circuit current, I_0 when the voltage across the capacitor is zero. As the capacitor gets charged, current flowing through the circuit which is, $i = \frac{V - v_c}{R}$ goes on decreasing from its initial value I_0.

Thus, when the circuit is switched on, while current in the circuit goes on decreasing, the voltage across the capacitor goes on increasing as shown in Fig. 2.164.

The time constant τ is defined as the time taken in seconds for the voltage across the capacitor, v_c to attain its final value, V if the rate of rise of voltage were the same as its initial value as indicated by the dotted line OC. The initial rate of rise of voltage across the capacitor is

$$\frac{BC}{OB} = \frac{V}{RC} = \frac{V}{\tau}$$

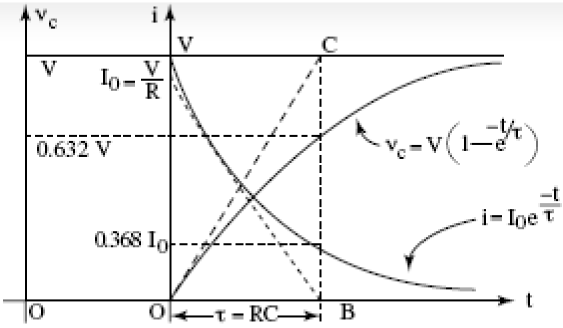


Figure 2.164 Transient current, i and capacitor voltage vc in a RC circuit

Now let us find out the value of the voltage across the capacitor at $t = \tau$ = RC.

Using the voltage equation

$$
\begin{aligned}
v_c &= V\left(1 - e^{\frac{-t}{\tau}}\right) \\
&= V\left(1 - e^{-1}\right) \qquad \text{as } t = \tau \\
&= V\left(1 - 0.368\right) \\
&= 0.632\ V
\end{aligned}
$$

Thus, *time constant can also be defined as the time taken in seconds for the voltage across the capacitor to reach 63.2 per cent of its final value as has been shown in the Fig. 2.164*.

The above explanation relates to charging of the capacitor upto a voltage V when the switch S is put in position 1. Now we will study the discharging of the capacitor when the switch is put in position 2. The capacitor voltage now at time t = 0 is equal to the supply voltage V. When the switch has been put in position 2, the supply voltage is cut off and the RC circuit is short circuited. The current flowing will be due to the voltage build up across the capacitor. The current flowing now will be $i = -\frac{v_c}{R}$ will flow in the opposite direction to the direction when the capacitor was being charged. That is why we have put a negative sign for *i*.

Again $\quad i = \frac{dq}{dt} = \frac{d}{dt}Cv_c = C\frac{dv_c}{dt}$

thus $\quad C\frac{dv_c}{dt} = -\frac{v_c}{R}$

or, $\quad \frac{dv_c}{dt} = \frac{v_c}{RC}$

or, $\quad -\frac{dv_c}{v_c} = \frac{dt}{RC}$

where K is the integration constant.

So, to find the value of K we will put the initial conditions, i.e., when t = 0, $v_c = V$

therefore, $$-l_n V = 0 + K$$

or, $$K = -l_n V$$

The eq. (v) is written as

$$-l_n v_c = \frac{t}{RC} - l_n V$$

or, $$\ln v_c - \ln V = \frac{t}{RC}$$

or, $$-v_c = Ve^{\frac{-t}{RC}} \quad \text{(iv)}$$

and $$i = -\frac{v_c}{R} = -\frac{V}{R}e^{\frac{-t}{RC}} = -I_0 e^{\frac{-t}{RC}} \quad (2.10)$$

The time constant, τ of the circuit is the time in seconds taken for the voltage across the charged capacitor to become zero if the initial rate of decay is maintained as shown by the dotted line AB. OB is the time constant, τ as has been shown.

If we calculate the value of v_c at $t = \tau$, we get

$$v_c = Ve^{\frac{-t}{\tau}} = Ve^{-1} \quad \text{for } t = \tau \quad (2.11)$$
$$= 0.368V$$

and $$i = I_0 e^{\frac{-t}{\tau}} = I_0 e^{-1} = 0.368 I_0 \quad (2.12)$$

Thus, for a time equal to time constant τ, both capacitor voltage v_c and the circuit current i are reduced to 36.8 per cent of their initial values with initial rate of decay remaining unchanged.

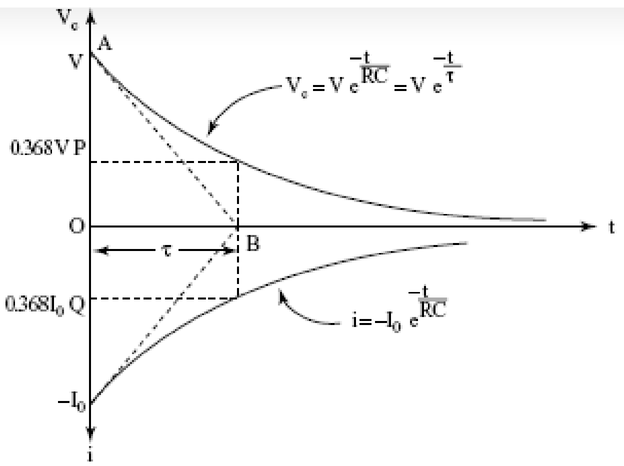


Figure 2.165 Transients in RC circuit, decay of voltage across the charged capacitor and the circuit current

Example 2.53 A capacitor of value 1 μF and a resistor of 5.45 MΩ are connected in series across a 220 V dc supply through a switch. Calculate the time by which the capacitor will be charged to 60 per cent of the supply voltage.

Solution:

$$R = 5.45 \times 10^6 \Omega, \quad C = 1 \times 10^{-6} F$$

Time constant,

$$\tau = RC = 5.45 \times 10^6 \times 1 \times 10^{-6} \text{ sec}$$

$$= 5.45 \text{ seconds}$$

$$v_c = V\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$0.6 \times 230 = 230\left(1 - e^{\frac{-t}{5.45}}\right)$$

$$0.6 = 1 - e^{\frac{-t}{5.45}}$$

or,

$$e^{-t/5.45} = 1 - 0.6 = 0.4$$

or,

$$e^{-t/5.45} = \frac{1}{0.4} = 2.5$$

or,

$$\frac{t}{5.45} = \ln 2.5 = 0.915$$

or,

$$t = 0.915 \times 5.45 = 4.98 \text{ seconds.}$$

Example 2.54 In the circuit shown in Fig. 2.166 the capacitor is fully charged when the switch is closed. Calculate the voltage across the fully charged capacitor. Also calculate the voltage across the capacitor and the current in the capacitor circuit 0.05 seconds after opening of the switch.

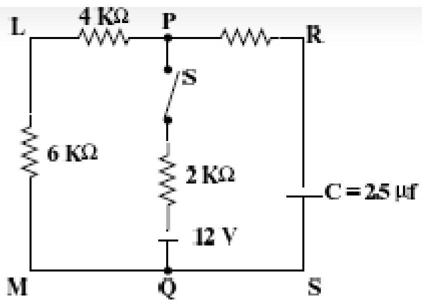


Figure 2.166

Solution:

When the capacitor is fully charged, no current will flow through the capacitor. The capacitor will be charged to a voltage equal to V_{PQ}. However, current, *I* flowing through the loop PLMQP will be

$$I = \frac{12}{(2+4+6)\times 10^3}\ \text{A}$$

Voltage across terminals P and Q, i.e., V_{PQ} is

V_{PQ} = IR drops across 4 kΩ and 6 kΩ resistors

$$I = \frac{12\times(4+6)\times 10^3}{12\times 10^3}\ \text{V}$$

$$= 10\ \text{V}$$

$$V_{PQ} = v_c = 10\ \text{V}$$

Now, when the switch is opened, at $t = 0$, $v_c = 10$ V. The capacitor will be getting discharged through the resistors 10 kΩ, 4 kΩ, and 6 kΩ in the loop RPLMQSR. The time constant of the circuit, $\tau = RC$.

$$\tau = RC = (10+4+6)\times 10^3 \times 2.5\times 10^{-6} = 20\times 10^3 \times 2.5\times 10^{-6}$$
$$= 0.05\ \text{seconds}$$

Let the capacitor voltage at $t = 0.05$ sec be v'_c and discharging current at $t = 0.05$ be i'.

$$v_c' = v_c e^{-t/\tau} = 10 e^{\frac{-0.05}{0.05}} = 10 \times e^{-1} = 10 \times 0.368 = 3.68V$$

Initial current at $t = 0$, $$I_0 = \frac{V_{AB}}{(4+6)\times 10^3} = \frac{10}{10\times 10^3} = 1\,\text{mA}$$

Current after 0.05 sec, $$i' = I_0 e^{-1} = 1 \times 0.368\,\text{mA} = 0.368\,\text{mA}$$

2.11 REVIEW QUESTIONS

A. Short Answer Type Questions

1. Define Ohm's law and state if there are any conditions.

2. Explain the concept of voltage and current source transformation with an example.

3. Give the concept of current, voltage, and resistance.

4. State the factors on which resistance of a wire depends. What is meant by resistivity of a conducting material?

5. Explain why silver is more conducting than copper.

6. Draw the V–I characteristics of a variable resistor whose value of resistance has been fixed at 5 Ω, 8 Ω, and 10 Ω.

7. Explain the effect of change of temperature on the resistance of most of the conducting materials.

8. What is meant by superconducting materials?

9. Prove that $R_t = R_0(1+\alpha_0 t)$ for a conducting material where R_t is the resistance at t°C, R_0 is the resistance at 0°C, α_0 is the temperature coefficient of resistance at 0°C and t is the rise in temperature.

10. If α_1 and α_2 are the temperature coefficients of resistance at t_1 and t_2 degrees, respectively, then prove that

$$\alpha_2 = \frac{\alpha_1}{1+\alpha_1(t_2 - t_1)}$$

11. Explain why resistance of most of the conducting materials increase with temperature.

12. From the definition of power as rate of doing work, show that power in an electric circuit

$$\text{P} = \text{VI Watts}$$

13. Distinguish between work, power, and energy.

14. Establish the relationship between resistance, current, voltage, power, and energy in an electric circuit.

15. Show that 1 kWh is equal to 860 kilo Calories.

17. State with example the current divider rule and the voltage divider rule as applicable to parallel circuits and series circuits, respectively.

18. State Kirchhoff's current law and Kirchhoff's voltage law.

19. What is Cramer's Rule?

20. State and explain Thevenin's theorem.

21. With a simple example show how, by applying Thevenin's theorem, current flowing through any branch of an electrical network can be calculated.

22. Write the steps of application of Thevenin's theorem.

23. State and explain Norton's theorem.

24. Distinguish between Thevenin's theorem and Norton's theorem.

25. What is maximum power transfer theorem? Prove the theorem.

26. Write the relationship of star–delta transformation of three resistors.

27. Distinguish between an ideal voltage source and a practical voltage source.

28. Write the conversion formula for delta to star conversion of three resistors.

29. What is the relationship between power, torque, and speed?

30. What are the limitations of Ohm's law? Is Ohm's law applicable in both dc and ac circuit?

31. State two fundamental laws of circuit analysis.

32. What is meant by time constant of an R–L circuit?

33. Derive equations that relate the resistance of a material at two different temperatures.

34. If n number of resistances each of value R are connected in parallel, then what will be the value of their equivalent resistance?

35. Derive the formula used in calculating the temperature coefficient of resistance at any temperature from its given value at any particular temperature.

B. Numerical Problems

36. A long copper wire has a resistance of 25 Ω at 40°C. Its resistance becomes 45 Ω when the temperature is 100°C. Calculate the value of its resistance at 0°C.

[Ans 11.7 Ω]

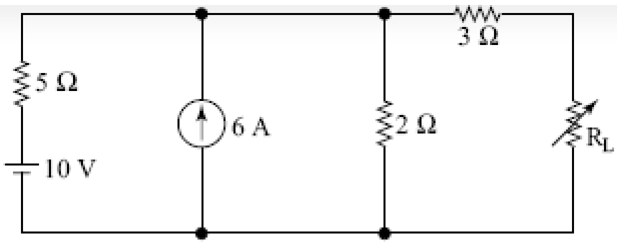


Figure 2.167

38. Calculate the value of R_L for which maximum power will be transferred from the source to the load in the network shown. Also calculate the value of maximum power transferred.

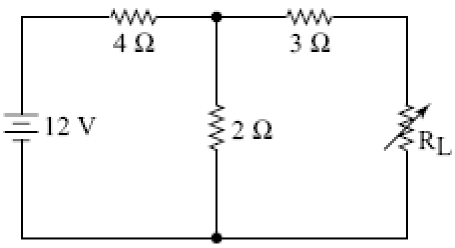


Figure 2.168

[Ans R_L = 7.33 Ω, P_{max} = 0.545 W]

39. Apply Norton's theorem to calculate the current through the 5 Ω resistor in the circuit shown. Also verify by applying Thevenin's theorem.

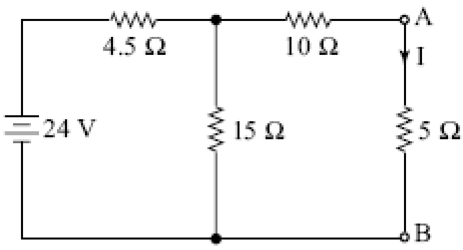


Figure 2.169

[Ans I = 1 A]

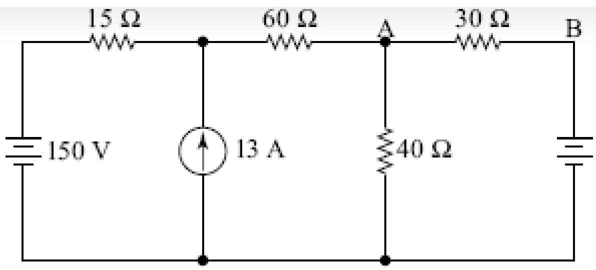


Figure 2.170

[Ans I_{AB} = 1.25 A]

41. By using the superposition theorem calculate the current flowing through the 10 Ω resistor in the network shown.

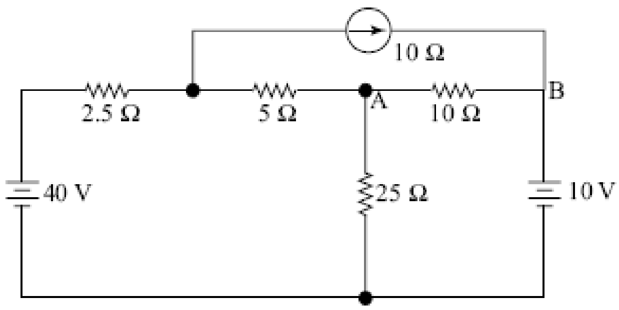


Figure 2.171

[Ans 0.054 A]

42. By applying Kirchhoff's laws or otherwise calculate the current flowing through the 6 Ω resistor in the network shown.

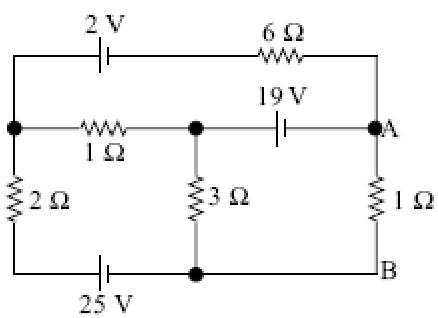


Figure 2.172

43. Three resistances of 25 Ω, 50 Ω, and 100 Ω are connected in parallel. If the total current drawn in 32 A, calculate the current drawn by each resistor.

[Ans 18.284 A, 9.144 A, 4.572 A]

44. Determine the current drawn from the battery in the circuit shown using Kirchhoff's laws.

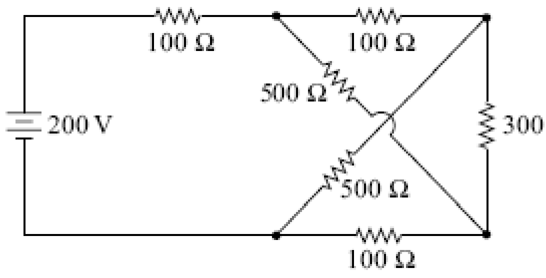


Figure 2.173

[Ans 0.6 A]

45. Use star–delta conversion of resistors to determine the current delivered by the battery in the network shown in Fig. 2.174.

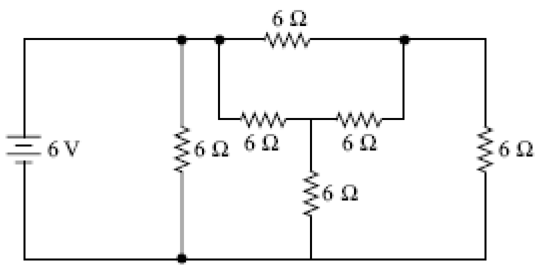


Figure 2.174

[Ans 2 A]

46. A winding wire made of copper has a resistance of 80 Ω at 15°C. Calculate its resistance at 50°C. Temperature coefficient of copper is 0.004/°C at 0°C.

[Ans 90.6 Ω]

[Ans 6.75 Ω]

48. Calculate the equivalent resistance of the network across the terminals A and B.

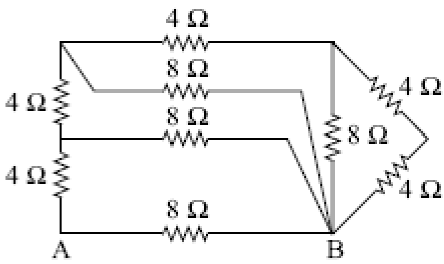


Figure 2.175

[Ans R_{AB} = 4 Ω]

49. Calculate the current supplied by the battery in the circuit shown in Fig. 2.176.

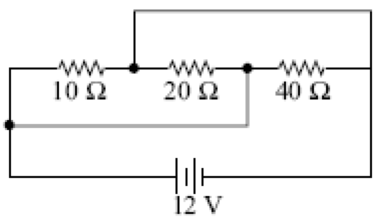
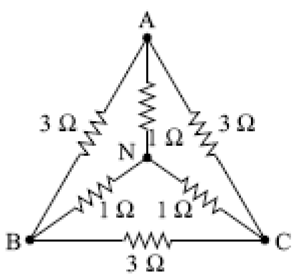


Figure 2.176

[Ans 2.099 A]

50. Calculate the equivalent resistance between the terminals A and B of the network shown. Also calculate R_{AN}.



[Ans R_{AB} = 1.0 Ω R_{AN} = 0.664 Ω]

51. Calculate the current supplied by the battery in the circuit shown. All resistances are in Ohms.

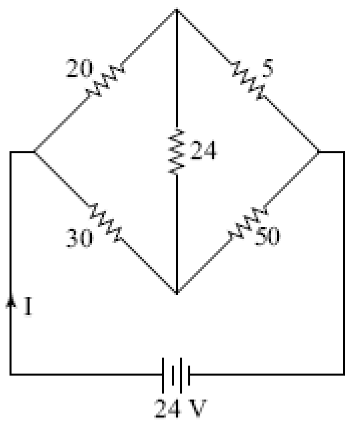


Figure 2.178

[Ans I = 1.35 A]

52. Using modal voltage analysis calculate the current flowing through the resistor connected across the terminals A and B as shown.

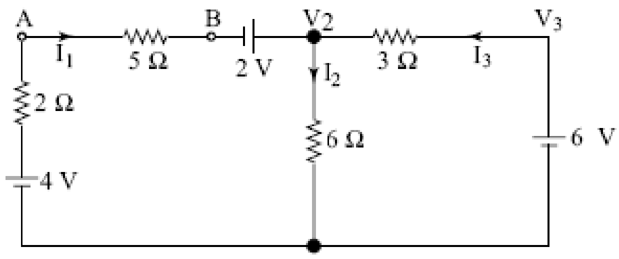


Figure 2.179

[Ans I_1 = 0.371 A]

53. Calculate the current flowing through the 2 Ω resistor connected across terminals A and B in the network shown by (i) applying Kirchhoff's laws; (ii) applying Thevenin's theorem; (iii) nodal voltage analysis. Compare the time taken by you in each case.

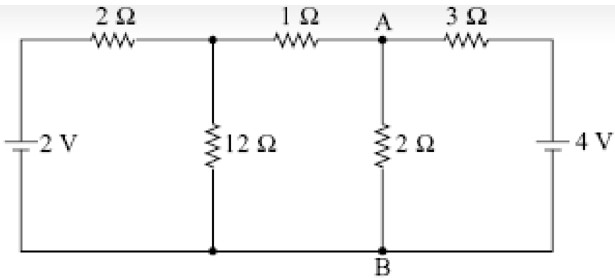


Figure 2.180

[Ans I = 0.817 A Applying Kirchhoff's laws takes the maximum time]

54. Calculate the current flowing through the 5 Ω resistor as shown in the network.

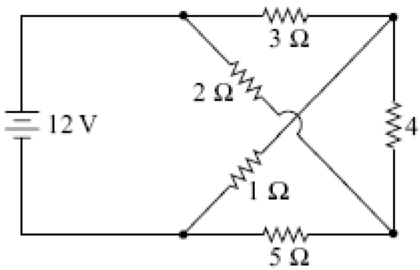


Figure 2.181

[Ans 0.663 A]

C. Multiple Choice Questions

1. Three resistances of equal value, R are connected such that they form a triangle having terminals A, B, and C. The equivalent value of the resistances across terminal A and B is equal to
   1. R/3
   2. 3/2 R
   3. $\frac{2}{3}R$.
   4. 3 R.
2. Four resistances of equal value, R are connected as shown. What is the equivalent resistance between the terminals A and B?

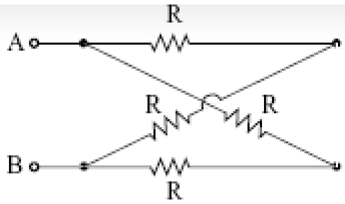


Figure 2.182

1. R/4
2. R/2
3. 4 R
4. R.

3. Four resistance of equal value, R are connected as shown in Fig. 2.183. What is the equivalent resistance between the terminals A and B?

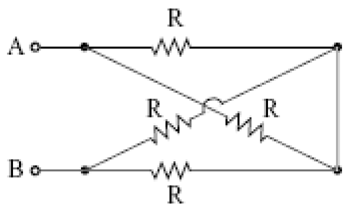


Figure 2.183

1. 0
2. 4 R
3. $\frac{R}{4}$.
4. R.

4. 4. What will be the equivalent resistance of the circuit between the terminals A and B?

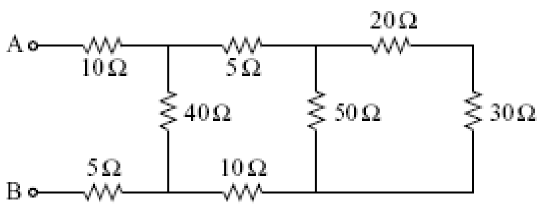


Figure 2.184

4. 80 Ω.

5. What are the values of R_1 and R_2 in the circuit shown?

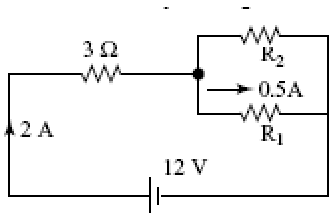


Figure 2.185

1. 12 Ω and 4 Ω
2. 4 Ω and 12 Ω
3. 6 Ω and 2 Ω
4. 2 Ω and 6 Ω.

6. The voltage applied across a 230 V, 60 W lamp is reduced to 115 V. What will be power consumed at this reduced voltage?

1. 60 W
2. 30 W
3. 120 W
4. 15 W.

7. Two resistances of equal value R and a wire of negligible resistances are connected in star formation as shown in Fig. 2.186. What will be the resistance between the terminals A and B when terminal C touches terminal A?

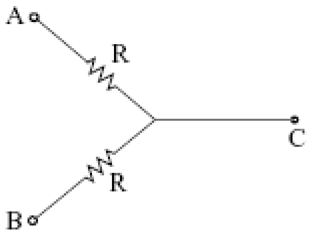


Figure 2.186

1. 2 R
2. $\frac{R}{2}$
3. R
4. 0.

4. resistivity.

9. Three resistances each of equal value R are connected in star formation. The equivalent delta will have three resistances of equal value which is

   1. $\frac{R}{3}$
   2. 3 R
   3. $\frac{2}{3}R$
   4. $\frac{R}{2}$.

10. Three resistance each of equal value R are connected in delta formation. The equivalent star will have three resistances of equal value which is

    1. $\frac{R}{3}$
    2. 3 R
    3. $\frac{2}{3}R$
    4. $\frac{R}{2}$.

11. A resistance of value R is connected across a voltage, V. What value of resistance should be connected in parallel with this resistance so that current drawn from the supply is doubled?

    1. R
    2. 2R
    3. $\frac{R}{2}$
    4. $\frac{R}{4}$.

12. A current of 10 A gets divided into two parallel paths having resistance of 2 Ω and 3 Ω, respectively. The current through the 2 Ω and 3 Ω resistance will be respectively

    1. 4 A and 6 A
    2. 6 A and 4 A
    3. 5 A and 5 A
    4. 2 A and 8 A.

13. 13. Eight, Ω resistances are connected in parallel across terminals A and B. What is the equivalent resistance across AB?

    1. 64 Ω
    2. 32 Ω
    3. 1 Ω
    4. 4 Ω.

1. 4 Ω and 6 Ω
2. 1 Ω and 9 Ω
3. 8 Ω and 2 Ω
4. 7 Ω and 3 Ω.

15. A wire of a particular length and cross-sectional area, a_1 is elongated to twice its length and the cross-sectional area gets reduced to a_2. Its resistance will increase by a factor of

1. $2\left(\frac{a_1}{a_2}\right)$
2. $2\left(\frac{a_2}{a_1}\right)$
3. $\frac{a_1}{a_2}$
4. $\frac{a_2}{a_1}$

16. In the circuit shown in Fig. 2.187, what are the values of currents I_1 and I_2?

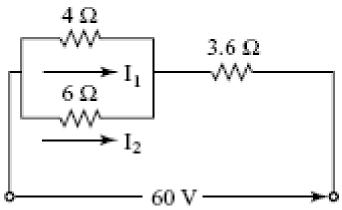


Figure 2.187

1. 4 A and 6 A
2. 6 A and 4 A
3. 10 A and 5 A
4. 2 A and 3 A.

17. Specific resistance of a conductor depends on

1. length of the conductor
2. area of cross sections of the conductor
3. resistance of the conductor
4. the nature of the material of the conductor.

18. In the circuit shown in Fig. 2.188, what is the value of R when the power dissipated in the 5 Ω resistor is 45 Ω?

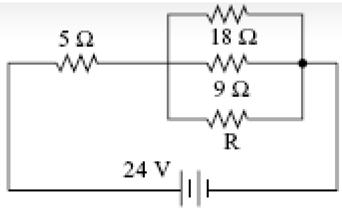


Figure 2.188

1. 9 Ω
2. 3 Ω
3. 6 Ω
4. 18 Ω.

19. The equivalent resistance of the circuit between the terminal A and B is

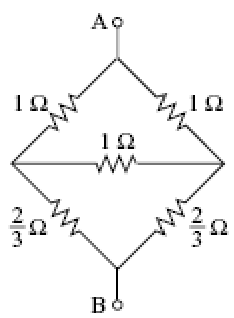


Figure 2.189

1. 1.66 Ω
2. 0.833 Ω
3. 2.66 Ω
4. 1.33 Ω.

20. The seem of two resistances connected in series across a supply voltage is 100 Ω. What are the values of the individual resistance if voltage drop across one of the resistors is 40 per cent of the supply voltage.

1. 20 Ω and 80 Ω
2. 50 Ω and 50 Ω
3. 40 Ω and 60 Ω
4. 10 Ω and 90 Ω.

21. Currents flowing through four resistances connected in parallel are 0.4 A, 0.3 A, 0.2 A, and 0.1 A, respectively. The equivalent resistance of the parallel circuit is 12 Ω. The value of resistances are

22. The resistance of a wire is 6 Ω. The wire is drawn such that its length increases three times. The resistance of the elongated wire will be (note volume remaining same, if length is increased, area of cross section will decrease proportionately)

   1. 18 Ω
   2. 36 Ω
   3. 54 Ω
   4. 6 Ω.

23. Resistivity of a conductor depends upon

   1. length of the conductor
   2. area of cross-section of the conductor
   3. type or nature of the material
   4. all the factors as in (a), (b), and (c).

24. A conductor of length l and diameter d has a resistance of R Ohms. The diameter of the conductor is halved and its length is doubled. What will be the value of resistance of the conductor?

   1. 2R Ω
   2. 4R Ω
   3. 8R Ω
   4. $\frac{R}{4}$ Ω.

25. Two resistances of value 12 Ω and 8 Ω are connected in parallel and the combination is connected in series with another resistance of value 5.2 Ω. This series parallel circuit is connected across a 100 V supply. The total current drawn will be

   1. 10 A
   2. 3.96 A
   3. 20 A
   4. 5 A.

26. With the increase in temperature

   1. the resistance of metal will increase and that of insulator will decrease
   2. the resistance of metal will decrease and that of insulator will increase
   3. resistance of both metal and insulator will increase
   4. resistance of both metal and insulator will decrease.

27. Two resistances when connected in parallel across a 6 V battery draws a total current of 6 A. When one of the resistances is disconnected, the current drawn becomes 3 A. The resistances are of values

   1. 1 Ω and 1 Ω
   2. 2 Ω and 2 Ω
   3. 6 Ω and 6 Ω
   4. 3 Ω and 2 Ω.

28. Two resistances, 40 Ω and 10 Ω are connected in parallel and the combination is connected in series with a 2 Ω resistor.

3. 20 A
4. 2.2 A.

29. Three resistance of 20 Ω each are connected in star. The resistance of each branch of the equivalent delta will be equal to

 1. 40 Ω
 2. 60 Ω
 3. 400 Ω
 4. 6.67 Ω.

30. Three resistances of 10 Ω each are connected in delta. The value of each of resistance of the equivalent star will be equal to

 1. 6.67 Ω
 2. 3.33 Ω
 3. 30 Ω
 4. 10 Ω.

31. The equivalent resistance of the network across terminals A and B, as shown in Fig. 2.190, will be

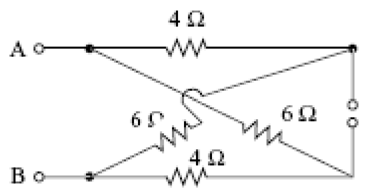


Figure 2.190

 1. 10 Ω
 2. 20 Ω
 3. 2.5 Ω
 4. 5 Ω.

32. In the circuit shown in Fig. 2.191, the voltage across terminals A and B is

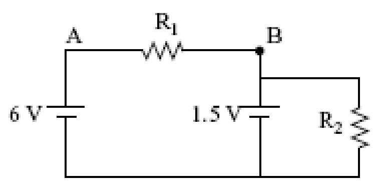


Figure 2.191

 1. (a) 4.5 V

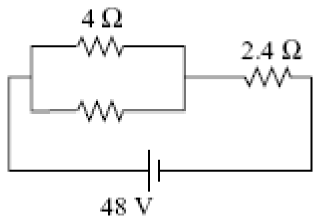


Figure 2.192

1. 24 V
2. 12 V
3. 48 V
4. 4.8 V.

34. The resistance between the terminals A and B of the circuit shown is

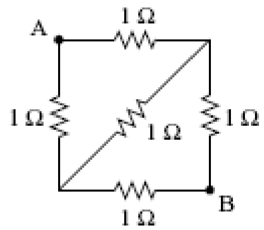


Figure 2.193

1. 1 Ω
2. 2 Ω
3. 5 Ω
4. 1.5 Ω.

35. The resistance between the terminals A and B of the circuit shown is

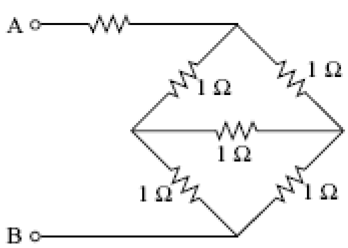


Figure 2.194

3. 6 Ω
4. 3 Ω.

Answers to Multiple Choice Questions

1. (c)
2. (d)
3. (d)
4. (b)
5. (a)
6. (d)
7. (c)
8. (a)
9. (b)
10. (a)
11. (a)
12. (b)
13. (c)
14. (a)
15. (a)
16. (b)
17. (d)
18. (c)
19. (b)
20. (c)
21. (a)
22. (c)
23. (c)
24. (c)
25. (a)
26. (a)
27. (b)
28. (a)
29. (b)
30. (b)
31. (d)
32. (d)
33. (a)
34. (a)
35. (b)