Integer Linear Programming Problem

- Linear programming problem so far uses variables which are positive real numbers.
- Always this is not true. For example if a variable represents no of chairs, no of vehicles, no of books, no of persons, etc.
- In an LPP, where all the variables are restricted as integers is called pure IPP.
- If some of the variables are restricted as integers, then it is called mixed IPP.

Structure of general IPP

$$\text{Max } Z = cx$$
$$\text{S.t. } Ax = b, \; x \geq 0$$
$$\text{and } x_j \in X \text{ are integers.}$$

Solution method of IPP

(i) Gomory's cutting plane method.

(ii) Branch and Bound method.

Gomory's cutting plane method :

This method was developed by R. E. Gomory. This method is based on:

'Introducing new constraints (or cuts) to the problem which removes non integer optimal solution but does not affect the feasible integer solutions."

In this method, we first find the optimal solution of the given IPP by simplex method disregarding the integer condition of the variables.

Following situations may occur:

i) If values of all variables are integer in the optimal solution.
⇒ current solution will be desired optimum integer solution.

ii) Otherwise the problem requires some modification. We introduce a secondary constraint (Gomory's cut) that reduces some non-integer values but does not eliminate any feasible integer solution.

iii) The optimal solution of the modified problem is obtained by standard algorithm. In this solution, if all the variables are integers, then procedure stops.

iv) Otherwise another secondary constraint is added to IPP and the process is repeated.

Let the following table give optimal non integer sol^n

		c_j	c_1	c_2	...	c_k	...	c_m	0	...	0
C_B	B	x_B	b	y_1	y_2	...	y_k	...	y_m	y_{m+1}	... y_n
c_1	y_1	x_1	b_1	1	0	...	0	...	0	$y_{1,m+1}$	... $y_{1,n}$
c_2	y_2	x_2	b_2	0	1	...	0	...	0	$y_{2,m+1}$	... $y_{2,n}$
⋮		⋮									
c_k	y_k	x_k	b_k	0	0	...	1	...	0	$y_{k,m+1}$	... $y_{k,n}$
⋮											
c_m	y_m	x_m	b_m	0	0	...	0	...	1	$y_{m,m+1}$	... $y_{m,n}$
		$z_j - c_j$		0	0	...		...	0	z_{m+1}	... z_n

$x_1, \dots, x_m$ are basic variables.

$x_{m+1}, \dots, x_n$ are non basic variables

We assume that kth basic variable corresponds to non-integer sol^n / c

$x_{Bk} = 0 \cdot x_1 + 0 \cdot x_2 + \dots + 1 \cdot x_k + \dots - 0 \cdot x_m + (y_{k,m+1}) x_{m+1} + \dots + (y_{k,n}) x_n$

$$x_{Bk} = x_k + \sum_{l=m+1}^{n} (y_{k,l}) x_l \longrightarrow (1)$$

$$x_k = x_{Bk} - \sum_{l=m+1}^{n} (y_{k,l}) x_l$$

Let $x_{Bk} = \underbrace{I_{Bk}}_{\text{integer part}} + \underbrace{f_{Bk}}_{\text{fractional part}}$

$y_{k,l} = I_{k,l} + f_{kl}$

$f_{Bk}, f_{kl} \in [0,1)$

From (1)

$$x_k = (I_{B,k} + f_{B,k}) - \sum_{l=m+1}^{n} x_l (I_{kl} + f_{kl})$$

$$f_{Bk} - \sum_{l=m+1}^{n} x_l f_{kl} = x_k - (I_{Bk} - \sum_{l=m+1}^{n} x_l f_{kl}) \quad (2)$$

Since $0 < f_{Bk} < 1$

$$\sum_{l=m+1}^{n} x_l f_{kl} \geq 0 \quad \&$$

$$f_{Bk} - \sum_{l=m+1}^{n} x_l f_{kl} \leq f_{Bk} \leq 1$$

$$f_{Bk} - \sum_{l=m+1}^{n} f_{kl} x_l \leq 0$$

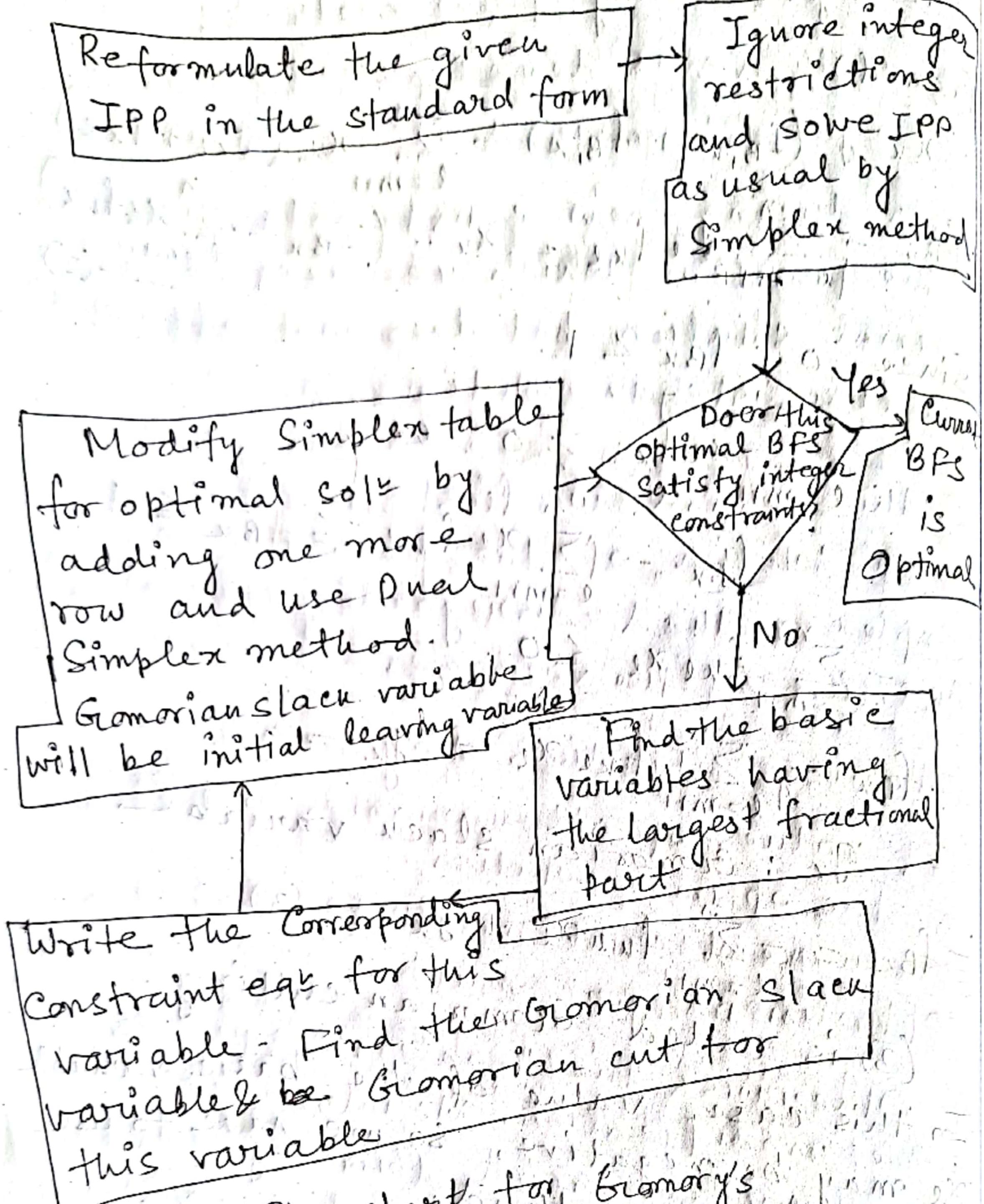
$$f_{Bk} - \sum_{l=m+1}^{n} f_{kl} x_l + g_k = 0$$

g_k : Gromorian slack variable.

$$-f_{Bk} = -\sum_{l=m+1}^{n} f_{kl} x_l + g_k \quad (3)$$

(3) is called Gromorian cutting.

In this case, value of the basic variable x_l may be negative, so we have to use dual simplex method.



Flow chart for Gomory's pure IPP