Ex: Find all the basic feasible solution of the system and classify.

$$2x_1 + 6x_2 + 3x_3 + x_4 = 3$$

$$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2,$$

$$x_i \geq 0, \ i = 1,2,3,4.$$

Soln (Hints): $m = 2$, $n = 4$, The total no of basic solns are ${}^4C_2 = 6$.

(I) $x_3 = x_4 = 0$ (non basic), then $x_1 = 0$, $x_2 = 1/2$
(degenerate basic soln)

(II) $x_2 = x_4 = 0$ (non basic) then $x_1 = -0.6$, $x_3 = 1.4$
(infeasible basic soln)

(III) $x_2 = x_3 = 0$ (non basic), then
$x_1 = 2.6667$, $x_4 = -2.3333$
(infeasible basic soln)

(IV) $x_4 = x_1 = 0$ (non basic) Then
$x_2 = 0.5$, $x_3 = 0$
(degenerate basic soln)

(V) $x_1 = x_3 = 0$ (non basic) Then
$x_2 = 0.5$, $x_4 = 0$
(degenerate basic soln)

(VI) $x_1 = x_2 = 0$ (non basic)
$x_3 = 1.1429$, $x_4 = -0.4286$
(infeasible basic soln)

Ex: Show that the feasible soln $x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 2$ to the system $x_1 + x_2 - 3x_3 = 2;$
$x_1 + x_2 + x_3 = 2;$
$2x_1 + 4x_2 + 3x_3 - x_4 = 4,$
$x_1, x_2, x_3, x_4 \geq 0$
is not basic.

Graphical Solution of LPP

For finding the graphical solution of LPP, we will consider the LPP of two variables only. If we go beyond two variables, then visualization will be improper. Let us have a look at the 2-dimensional LPP formulation.

Formulation of 2-dimensional LPP

Min / Max $c_1 x_1 + c_2 x_2$

Subject to $a_{11} x_1 + a_{12} x_2 \leq b_1$

$a_{21} x_1 + a_{22} x_2 \leq b_2$

$- - -$

$a_{m1} x_1 + a_{m2} x_2 \leq b_m$.

$x_1, x_2, b_1, b_2 .. \geq 0$.

There are four possibilities in solution of LPP.

Solution of LPP

- Unique Solution
- Multiple Solution
- Unbounded Solution
- Infeasible Solution

Solution approach:

There are few steps for graphical solution of LPP. The following approach is called search approach

Step 1 → Consider each constraint ~~as~~ inequality as equality and plot the lines graphically. Then plot the halfspaces corresponding to each inequality.

Step 2 → Intersection of all constraints will be feasible region. So this step is to identify the feasible region.

Step 3 → This step is to find out the extreme points of the feasible region.

Step 4 → Compute the value of the objective function at each extreme point.

Step 5 → Select the extreme point which optimizes the value of the objective function.

The next approach is called ISO profit (for maximization problem) or ISO-cost (for minimization problem) approach.

Upto step 2, it is similar to search approach.

Step 3 → Choose a convenient profit line of the objective function (for maximization problem) or cost line (for minimization problem) so that it lies within the feasible region. The line takes the form

$c^T x = k, \; k \in R$

Step 4 → Move this profit line parallel to itself away from the origin (for maximization problem) or move the cost line parallel to itself close to the origin (for minimization problem)

Step 5 → Identify the optimum solution as the co-ordinates of the extreme point of feasible region touched by the highest possible ISO profit line or lowest possible ISO cost line.

Example 1: Max $Z = x_1 + 3x_2$

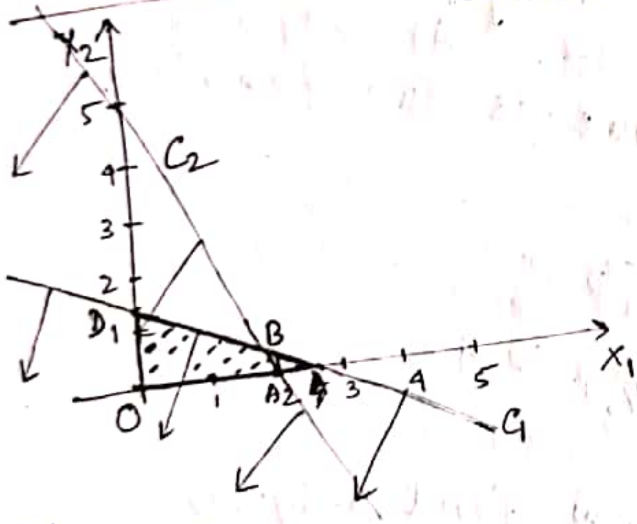
Subject to $3x_1 + 6x_2 \leq 8$ — (C_1)

$5x_1 + 2x_2 \leq 10$ — (C_2)

$x_1, x_2 \geq 0$.

Search approach: Consider $\frac{x_1}{8/3} + \frac{x_2}{4/3} = 1$ — (1)

$\frac{x_1}{2} + \frac{x_2}{5} = 1$ — (2)



Feasible region: OABD.

O: (0,0).

A: (2) intersects X_1 at A.
A = (2,0)

B: intersection of (1) and (2)
B: $\left(\frac{11}{6}, \frac{5}{12}\right)$.

D: (1) intersects X_2 at D.
D = $(0, 4/3)$

$Z|_O = 0$.

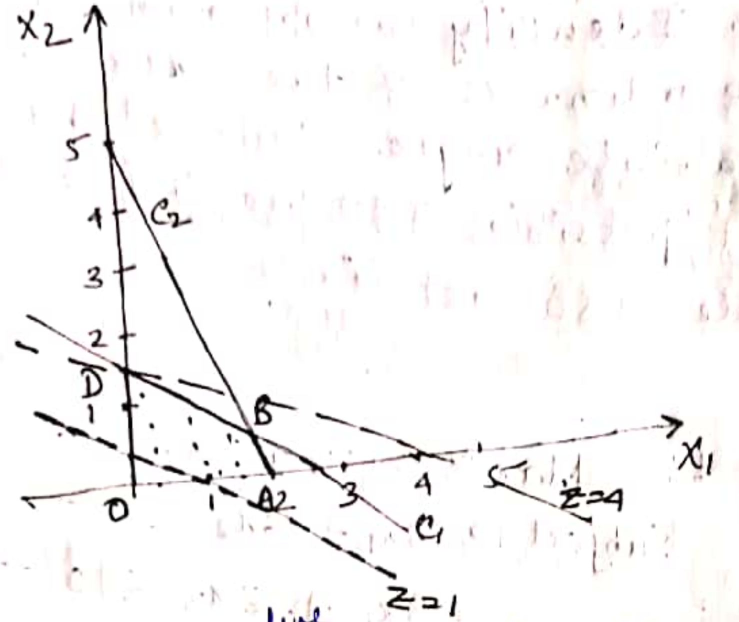
$Z|_{A(2,0)} = 2$

$Z|_{B\left(\frac{11}{6}, 5/12\right)} = 37/12$

$Z|_{D(0, 4/3)} = 4$

Therefore D is the optimal solution of LPP and corresponding maximum value of D is $z|_{(0, 4/3)} = 4$.

ISO profit line approach



Draw the profit line Z=1 within the feasible region i.e. $x_1 + 3x_2 = 1$. Move this line away from the origin parallel to itself. At the highest point D, it touches the feasible region.

So $Z_{max} = Z_D$

i.e. $Z(0, 4/3) = 4$.

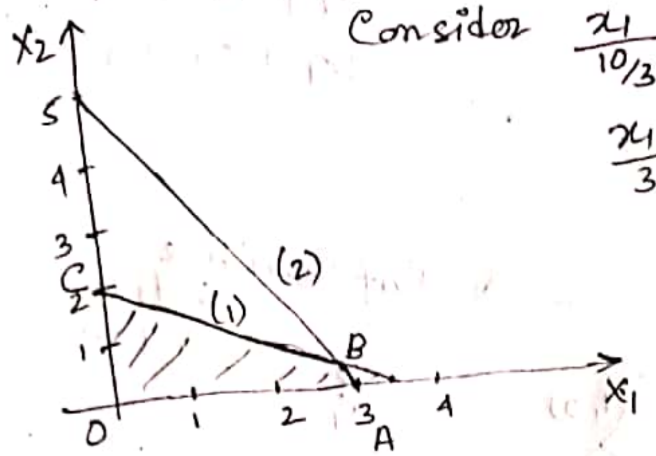
In this approach, we find unique solution of LPP.

Example 2:

$$\text{Max } z = 6x_1 + 10x_2$$

Subject to $3x_1 + 5x_2 \le 10$

$5x_1 + 3x_2 \le 15$, $x_1, x_2 \ge 0$.



Consider $\frac{x_1}{10/3} + \frac{x_2}{2} = 1 \to (1)$

$\frac{x_1}{3} + \frac{x_2}{5} = 1 \to (2)$

O $(0,0)$

A $(3,0)$

B $\left(\frac{45}{16}, \frac{5}{16}\right)$

C $(0,2)$

$z|_{(0,0)} = 0$. $z|_{(3,0)} = 18$.

$z|_{\left(\frac{45}{16}, \frac{5}{16}\right)} = 20$ $z|_{(0,2)} = 20$

So maximum value is attained at B and C. This LPP has more than one solution. Since, the feasible region is the convex set, the line joining B and C will give ~~you~~ maximum ~~number~~ value of objective function. So, this LPP has infinite no of optimum solution.

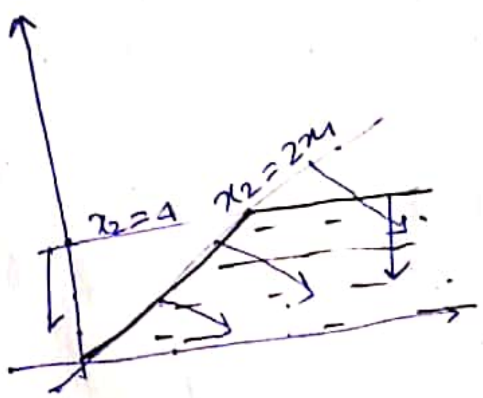
Example 3:

$$\text{Max } z = 3x_1 + 8x_2$$

Subject to $2x_1 - x_2 \ge 0$

$x_2 \le 4$

$x_1, x_2 \ge 0$.



Consider

$x_2 = 2x_1$ —(1)

$x_2 = 4 \to (2)$

Unbounded

Sol^n.

Ex : 4

Max $z = x_1 + x_2$

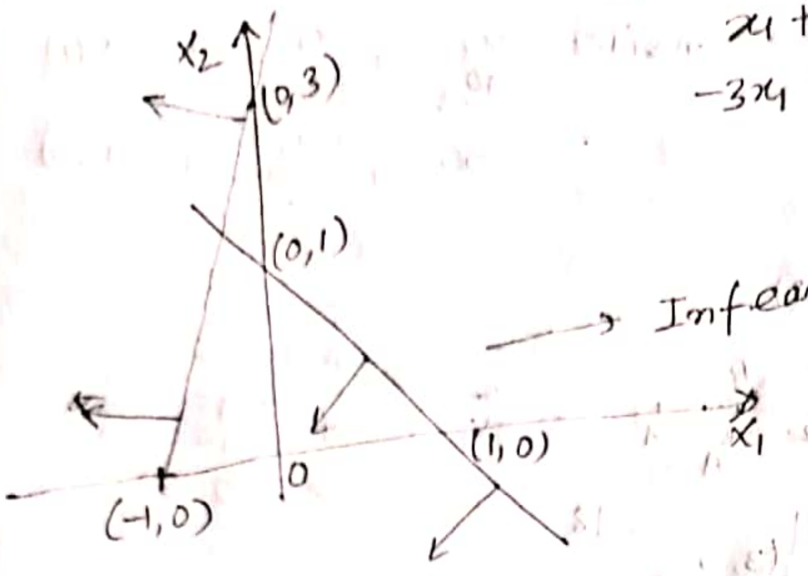
Subject to $x_1 + x_2 \leq 1$.

$-3x_1 + x_2 \geq 3$, $x_1, x_2 \geq 0$.

Consider

$x_1 + x_2 = 1$

$-3x_1 + x_2 = 3$



→ Infeasible solution.

Practice Set .

(1) Max $z = 5x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 5$

$3x_1 + 8x_2 \leq 24$

$x_1, x_2 \geq 0$ (Ans: $z_{max} = 25$)

(2) Max $z = 3x_1 + 8x_2$

Subject to $x_1 + x_2 \leq 5$

$3x_1 + 8x_2 \leq 24$

$x_1, x_2 \geq 0$.