Reg. No.:

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TERM END EXAMINATION (TEE) – NOV./DEC. 2018

Programme	**B.Tech.**	**Semester**	**Fall 2018-19**
Course Name	**Calculus and Laplace Transforms**	**Course Code**	**MAT1001**
Faculty Name	**Dr. Anant Kant Shukla**	**Slot / Class No**	**B2+LB2+TB2/1076**
Time	**3 Hrs.**	**Max. Marks**	**100**

Answer ALL the Questions

Q. No.		**Question Description**	Marks
		PART A – (60 Marks)	
1	(a)	Find the absolute extreme values of the temperature $T(x,y) = 4x^2 + 9y^2 - 8x - 12y + 4$ which is defined on a rectangle $x = 0, x = 2, y = 0, y = 3$.	12
		OR	
	(b)	(i) Expand $e^x \sin y$ in ascending powers of $(x - 0)$ and $(y - \frac{\pi}{2})$ up to three terms by using Taylor's formula.	6
		(ii) About how accurately may, the volume of a right circular cylinder be calculated if its radius and height are in error by 0.5%.	6
2	(a)	Evaluate $\iint_R (x - y^2)\cos^2(x + y)dx\, dy$ where R is the region in xy −plane with vertices $(\pi, 0), (2\pi, \pi)$ and $(0, \pi)$. Use the transformation $u = x + y$ and $v = x - y$ to find its value.	12
		OR	
	(b)	(i) Write the transformation to convert Cartesian co-ordinate system to cylindrical and spherical polar co-ordinates. Also find the associated Jacobian of the transformation for both the cases.	6
		(ii) Evaluate the volume of the region bounded above by the surface $z = e^{-(x^2+y^2)}$ and bounded below by the region $x^2 + y^2 = 1$ in second quadrant of the xy −plane.	6
3	(a)	Verify the Green's theorem for $\vec{F} = [y - \sin x, \cos x]$ where C is the boundary of a triangle with vertices $(0,0), \left(\frac{\pi}{2}, 0\right), (\frac{\pi}{2}, 1)$ in xy −plane.	12

OR

(b) (i) Evaluate $\oint_C \vec{r}.d\vec{r}$ where the notation $\vec{r}$ and C have their usual meanings. 4

(ii) Check whether the vector function $\vec{F} = [\sin y + z, x\cos y - z, x - y]$ is irrotational. If so, find the corresponding potential. 8

4 (a) Find a general solution of the differential equation $y''(x) + 16\,y(x) = 32\sec 2x$. 12

OR

(b) (i) A drop of liquid evaporates at a rate proportional to its surface area. If the radius initially is 4 mm and it reduced to 2 mm in first 5 minutes then find the radius of the drop as a function of time. 6

(ii) Solve $(x-4)y'(x) + 3\,y(x) - 12(x-4)^3 = 0$. 6

5 (a) Find the solution of the differential equation 12

$y''(t) + 5y'(t) + 6y(t) = 1 - u(t-3) - u(t-5)$ with initial conditions

$y(0) = 0, y'(0) = 0$.

OR

(b) (i) Find $L\{\sinh t\}$ by using the first shifting theorem. 6

(ii) Find the value of the integral $\int_0^\infty t\,e^{-3t}\cos 2t\ dt$ by using Laplace transform. 6

PART B – (40 Marks)

6 Check whether the function $f(x,y) = \begin{cases} \frac{xy}{x^2+2y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ 8

is differentiable. What about its partial derivatives at origin with respect to x and y?

7 Evaluate $\int_0^2 \int_{\frac{y^2}{2}}^{y} \frac{y}{(2-x)(2x-y^2)^{\frac{1}{2}}}\ dx\ dy$. 8

8 Find the directional derivative of $f(x,y) = x^2y^3 + xy$ at $(2,1)$ in the direction of a unit vector which makes an angle of $60°$ with the positive x −axis in the xy −plane. 8

9 Solve $(3x^2 + 2e^y)dx + (2xe^y + 3y^2)dy = 0, y(0) = 1$. 8

10 Find $L^{-1}\left\{\frac{4s+16}{(s^2+8s+25)^2}\right\}$. 8

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Faculty Name	**Dr. Anant Kant Shukla**	**Slot / Class No**	**C2+LC2+TC2/1077**
Time	**3 Hrs.**	**Max. Marks**	**100**

Answer ALL the Questions

Q. No.		Question Description	Marks
		PART A – (60 Marks)	
1	(a)	An aquarium with rectangular sides and bottom (with no top) is to hold 32 litres of water. Find its dimensions so that it will use the least amount of material.	12
		OR	
	(b)	(i) Find the equation of the tangent plane to the surface $x^2 + y^2 + z - 16 = 0$ at the point $(1,3,6)$.	6
		(ii) Check whether the $\lim_{(x,y)\to(0,1)} \tan^{-1}\left(\frac{y}{x}\right)$ exist or not. If it exists then find the value of it.	6
2	(a)	Evaluate $\int_0^3 \int_0^4 \int_{\frac{y}{2}}^{\frac{y}{2}+1} \left(\frac{2x-y}{2} + \frac{z}{3}\right) dx\, dy\, dz$ by using the transformation $u = \frac{2x-y}{2}, v = \frac{y}{2}, w = \frac{z}{3}$.	12
		OR	
	(b)	(i) If $f(x,y) = 200(y+1)$ gives the density of population of a plane region on earth where x and y are measured in miles. Find the number of people in the region bounded by the curves $x = y^2$ and $x = 2 - y^2$.	6
		(ii) Evaluate $\iint_R e^{x^2}\, dx\, dy$, where the region R is given by $2y \le x \le 2, 0 \le y \le 1$.	6
3	(a)	Verify the Gauss Divergence theorem if $\vec{F} = [4xz, -y^2, yz]$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.	12

OR

(b) (i) Evaluate $\int_{(0,0)}^{(2,1)}[(10x^4 - 2xy^3)dx - 3x^2y^2dy]$ along the path $x^4 - 6xy^3 = 4y^2$. 6

(ii) Write the standard parametric representation of cone, circular cylinder and sphere. 6

4 (a) Find the solution of the differential equation $y''(x) - 2y'(x) + y(x) = x^{1.5}\, e^x$ by the method of variation of parameters. 12

OR

(b) (i) A particle moves on a straight line so that its acceleration is equal to four times of its velocity. At time $t = 0$ its displacement from the origin is 2 feet and its velocity is 3 feet/sec. Find an approximation to the time when the displacement is 10 feet. 6

(ii) Solve $(y - y^2x^2 \sin x)dx + x\, dy = 0$. 6

5 (a) Solve $y''(t) + 4\, y(t) = f(t)$ with $y(0) = 1, y'(0) = 0$ where $f(t) = \begin{cases} 0, & 0 \le t < 1 \\ 1, & 1 \le t < 2 \\ 0, & t \ge 2 \end{cases}$ by using Laplace transform. 12

OR

(b) (i) Find $L\{e^{-2t} \cos t \;\; \cos 2t\}$. 6

(ii) Evaluate $L^{-1}\{\frac{2as}{(s^2+a^2)^2} + \frac{a}{(s-a)^2}\}$. 6

PART B – (40 Marks)

6 If $f(u, v, w)$ is a differentiable function and $u = x - y, v = y - z$ and $w = z - x$ then find the value of $f_x + f_y + f_z$. 8

7 Evaluate $\iint_R \sqrt{x^2 + y^2}\, dx\, dy$ where R is the region in the xy −plane bounded by the circles $x^2 + y^2 = 16$ and $x^2 + y^2 = 25$. 8

8 Calculate the flux of the vector function $\vec{F} = [y^3, x^3, z^3]$ across the surface $S: x^2 + 4y^2 = 4, x \ge 0, y \ge 0, 0 \le z \le h$ where h be a constant. 8

9 Solve $y''(x) - 4y'(x) - 5\, y(x) = c^{-3x} + e^x$ by using the method of undetermined coefficients. 8

10 Find inverse Laplace transform of $\frac{s^2+2s-9}{(s^2+2s+17)(s^2+2s-24)}$. 8

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