Saurabh Singh

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Class-Test

Q>1 Minimize:

$Z = 10x_1 + 6x_2 + 2x_3$,

Given: $-x_1 + x_2 + x_3 \geq 1$

$3x_1 + x_2 - x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$

In standard form: $Z = -10x_1 + 6x_2 + 2x_3 + 0 \cdot x_4 + 0 \cdot x_5$

$\therefore B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ & $B^{-1} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, a_4 & a_5 as Basis vector

$b = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\therefore X_B = B^{-1}b = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

Also, $C_B = 0$ & $Z_j - C_j = -C_j \geq 0$, $j = 1, 2, 3$

Using Dual-Simplex method:

		C_j		-10	-6	-2	0	0
C_B	B	X_B	$B^{-1}b$	y_1	y_2	y_3	y_4	y_5
0	a_4	x_4	-1	1	-1	-1	1	0
0	a_5	x_5	-2	-3	-1	1	0	1
$Z_j - C_j$				10	6	2	0	0

$\therefore \ x_B = \text{Min}\{-1, -2\} = -2$

$\therefore$ a_5 is leaving vector

$\frac{Z_K - C_K}{y_{rK}} = \text{Max}\left\{\frac{10}{-3}, \frac{6}{-1}\right\} = \frac{-10}{3}$

$\therefore$ $K = 1$ & a_1 is entering vector.

Transformed table

C_B	B	X_B	C_j / b	-10 / y_1	-6 / y_2	-2 / y_3	0 / y_4	0 / y_5
0	a_4	x_4	$-5/3$	0	$-4/3$	$-2/3$	1	$1/3$
-10	a_1	x_1	$2/3$	1	$1/3$	$-1/3$	0	$-1/3$
$Z_j - C_j$				0	$8/3$	$16/3$	0	$10/3$

Here, x_4 is infeasible. Basic variable!

& $\frac{Z_K - C_K}{y_K} = \text{Max}\left\{\frac{8/3}{-4/3}, \frac{16/3}{-2/3}\right\} = -2$

$\Rightarrow$ a_2 is the entering vector

Now, Transformed table is -

C_B	B	X_B	C_j / b	-10 / y_1	-6 / y_2	-2 / y_3	0 / y_4	0 / y_5
-6	a_2	x_2	$5/4$	0	1	$1/2$	$-3/4$	$-1/4$
-10	a_1	x_1	$1/4$	1	0	$-1/2$	$1/4$	$-1/4$
$Z_j - C_j$				0	0	4	2	4

Here, feasible solution is obtained as

$x_1 = 1/4, \; x_2 = 5/4, \; x_3 = 0$

& $z_{min} = 10$

A>2

To

From

	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

Row minimization:

	A	B	C	D	E
A	∞	1	4	6	0
B	4	∞	1	6	0
C	4	3	∞	0	3
D	8	0	2	∞	1
E	0	2	1	7	∞

Column - Minimization:

	A	B	C	D	E
A	∞	1	3	6	0
B	4	∞	0	6	0
C	4	9	∞	0	0
D	8	0	1	∞	4
E	0	2	0	7	∞

$N = n$, $5 = 5 \times 5$

∞	1	3	8	[0]
4	∞	[0]	6	0
4	3	∞	[0]	3
8	[0]	1	∞	1
[0]	2	0	7	∞

A - E - A, B - C - D - B

Cost = 1 + 3 + 4 + 4 + 1 = 13

As per sequence from above assignment, indicates to produce A, then E & then A without producing product B, C & D. It violates restriction of producing each product.

Now, we have to examine matrix for Best sol^n. assigning with C15 to C12 & C42 to C45

∞	[1]	3	6	0
4	∞	[0]	6	0
4	3	∞	[0]	3
8	0	1	∞	[1]
[0]	2	0	7	∞

Now, sequence A - B - C - D - E - A

$\therefore$ Cost $= 2 + 3 + 4 + 5 + 1 = 15$

Here, the cost is increased By Rs. 2.

A>3

	Fodder 1	Fodder 2
Nutrient A	2	1
B	2	3
C	1	1

Cost of fodder 1 is ₹ 3 per unit & that of fodder 2 ₹ 2.

Let x unit of fodder 1 & y units of fodder 2.

$\therefore$ total cost : $z = 3x + 2y$

Nutrient	Fodder 1	Fodder 2	Minimum requirements
A	2	1	14
B	2	3	22
C	1	1	1

$\therefore$ constraints:

$$2x + y \geq 14$$
$$2x + 3y \geq 22$$
$$x + y \geq 1,$$

$$x \geq 0,$$
$$y \geq 0$$

$\therefore$ LPP will be:

Minimize: $z := 3x + 2y$

$2x + y \geq 14$, $2x + 3y \geq 22$, $x + y \geq 1$, $x \geq 0, y \geq 0$

A>4

No. of Supply constraints : 3
~~No. of~~ of Demand constraints = 3

	D_1	D_2	D_3	supply
S_1	6	8	4	14
S_2	4	9	3	12
S_3	1	2	6	5
Demand	6	10	15	

In 1st row, Smallest transporation cost is 4 in cell S_1D_3

allocation in this cell is min (14, 15) = 14

~~Now~~ so table will be

	D_1	D_2	D_3	Supply
~~S_1~~	6	8	14	0
S_2	4	9	3	12
S_3	1	2	6	5
Demand	6	10	1	

in 2nd row, smallest transportation cost is 3 in S_2D_3

∴ Allocation in the cell will be min (12, 1) = 1

Now, Table will be!

	D_1	D_2	D_3	Supply
S_1	6	8	14	0
S_2	4	9	1	11
S_3	1	2	6	5
Demand	6	10	0	

In 2nd row, smallest transportation cost is 4 in cell S_2D_1

$\therefore$ allocation to the cell will be $\min(11, 6) = 6$

	D_1	D_2	D_3	supply
S_1	~~6~~	8	14	0
S_2	6	9	1	5
S_3	1	2	6	5
Demand	0	10	0	

Similarly! 1 S_2D_2 will be allocated by 5.

& In 3rd row, smallest transportation cost is 2 in S_3D_2,

so it will be allocated by $\min(5, 5) = 5$

Initial feasible solution Table

	D_1	D_2	D_3	supply
S_1	6	8	14	14
S_2	6	5	1	12
S_3	1	5	6	5
Demand	6	10	15	31/31

Minimum total transportation cost $= 4 \times 14 + 4 \times 6 + 9 \times 5 + 3 \times 1 + 2 \times 5 = 138$

No. of allocated cells $= 5 = 3 + 3 - 1 = 5$

$\therefore$ solution is non-degenerate!