Inter-conversion (CFG & PDA)

CFG to PDA Conversion

CFL

$$\text{CFG} \underset{}{\overset{\text{Conversion}}{\rightleftarrows}} \text{PDA}$$

⇒ CFG → PDA:

Let $L = L(G)$ where $G = (V, \Sigma, P, S)$ is a CFG, we construct a PDA (A) with empty stack as

$$A = (q, \Sigma, \{V \cup \Sigma\}, \delta, q, S, \phi)$$

Rule1: $\delta(q, \epsilon, A) = \{ (q, \alpha) \mid A \rightarrow \alpha \text{ is in } P \}$

Rule2: $\delta(q, \beta, \beta) = \{ (q, \epsilon) \}$ for every $\beta \in \Sigma$

CFG to PDA Conversion

Example:

$S \to 0BB$

$B \to 0S \mid 1S \mid 0$

Also test for string 010^4

R1: $\delta(q, \epsilon, S) = (q, 0BB)$

R2: $\delta(q, \epsilon, B) = (q, 0S)$

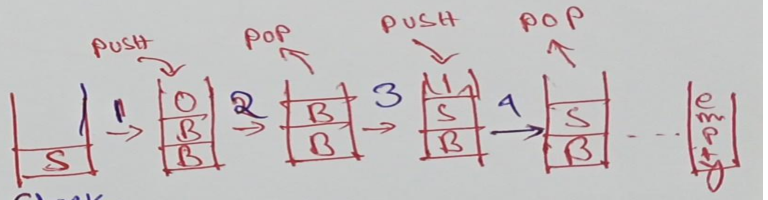
R3: $\delta(q, \epsilon, B) = (q, 1S)$

R4: $\delta(q, \epsilon, B) = (q, 0)$

R5: $\delta(q, 0, 0) = (q, \epsilon)$

R6: $\delta(q, 1, 1) = (q, \epsilon)$

$\Rightarrow$ testing of string 010^4



Stack

1. $(q, 010^4, S)$ [R1]
2. $(q, 010^4, 0BB)$ [by R5]
3. $(q, 10^4, BB)$ [by R3]
4. $(q, 10^4, 1SB)$ [by R6]
5. $(q, 0^4, SB)$ [by R_1]
6. $(q, 0^4, 0BBB)$ [by R_5]
7. $(q, 0^3, BBB)$ [by R_4]
8. $(q, 0^3, 0BB)$ [by R5]
9. $(q, 0^2, BB)$ [by R4]

$(q, 0^2, 0B)$ 10.

$\downarrow$ by R_5 and R_4 (11, 12)

(q, ϵ, ϵ) 13.

PDA to CFG

⇒ The productions (P) are produced by following the moves of PDA as follows

1) S productions are given by $S \rightarrow [q_0 \, z_0 \, q]$ for every $q \in Q$

2) Each erasing move, i.e. $\delta(q, a, z) = (q', \Lambda)$, induces production $[q, z, q'] \longrightarrow a$

$$\delta(q, a, z) = (q', \epsilon)$$

$$[q, z, q'] \Longrightarrow a$$

3) Non-erasing moves:

$$\delta(q, a, z) = (q_1, z_1 z_2 z_m)$$

$$[q \; z \; q'] \rightarrow a \, [q_1, z_1, q_2] \, [q_2 \, z_2 \, q_3] \, [q_3 \, z_3 \, q_4] \; \ldots \; [q_m \, z_m \, q']$$

where $q', q_1, q_2, q_3 \ldots q_m$ can be any state in Q

$\boxed{2^m \rightarrow \text{combinations}}$

PDA to CFG

Example: $A = (\{q_0, q_1\}, \{a, b\}, \{Z_0 Z\}, \delta, q_0, Z_0, \phi)$

$$\delta(q_0, b, Z_0) = (q_0, ZZ_0)$$

$$\delta(q_0, \epsilon, Z_0) = (q_0, \epsilon)$$

$$\delta(q_0, b, Z) = (q_0, ZZ)$$

$$\delta(q_0, a, Z) = (q_1, Z)$$

$$\delta(q_1, b, Z) = (q_1, \epsilon)$$

$$\delta(q_1, a, Z_0) = (q_0, Z_0)$$

For starting symbol (i)

$S \to [q_0 Z_0 q_0] \to A$ $\qquad S \to A \longrightarrow P_1$

$S \to [q_0 Z_0 q_1] \to B$ $\qquad S \to B \longrightarrow P_2$

PDA → CFG

$\delta(q_0, b, Z_0) = (q_0, Z Z_0)$

$2^2 = 4$ combination

$[q_0\ Z_0\ \ ] \rightarrow b\,[q_0\ Z\ \ ][\ \ Z_0\ \ ]$

$[q_0\ Z_0\ \ ] \rightarrow b\,[q_0\ Z\ \ ][\ \ Z_0\ \ ]$

$[q_0\ Z_0\ \ ] \rightarrow b\,[q_0\ Z\ \ ][\ \ Z_0\ \ ]$

$[q_0\ Z_0\ \ ] \rightarrow b\,[q_0\ Z\ \ ][\ \ Z_0\ \ ]$

⇓

$Q = \{q_0, q_1\}$

$[q_0\ Z_0\ q_0] \rightarrow b\,[q_0\ Z\ q_0]\,[q_0\ Z_0\ q_0]$ — P_3

$[q_0\ Z_0\ q_0] \rightarrow b\,[q_0\ Z\ q_1]\,[q_1\ Z_0\ q_0]$ — P_4

$[q_0\ Z_0\ q_1] \rightarrow b\,[q_0\ Z\ q_0]\,[q_0\ Z_0\ q_1]$ — P_5

$[q_0\ Z_0\ q_1] \rightarrow b\,[q_0\ Z\ q_1]\,[q_1\ Z_0\ q_1]$ — P_6

⇒ $\delta(q_0, \epsilon, Z_0) = (q_0, \epsilon)$

$[q_0\ Z_0\ q_0] \rightarrow \epsilon$ — P_7

PDA → CFG

$\Rightarrow \delta(q_0, b, Z) = (q_0, ZZ)$ $\quad 2^2 = 4$ combinations

? $P_8 \rightarrow P_{11}$

$\Rightarrow \delta(q_0, a, Z) = (q_1, Z)$ $\quad 2^1 = 2$ combination

$[q_0 Z q_0] \rightarrow a[q_1 Z q_0]$ — P_{12}

$[q_0 Z q_1] \rightarrow a[q_1 Z q_1]$ — P_{13}

$\Rightarrow \delta(q_1, b, Z) = (q_1, \epsilon)$

$[q_1 Z q_1] \rightarrow b$ — P_{14}

$\Rightarrow \delta(q_1, a, Z_0) = (q_0, Z_0)$ $\quad 2^1$

$[q_1 Z_0 q_0] \rightarrow a[q_0 Z_0 q_0]$ — P_{15}

$[q_1 Z_0 q_1] \rightarrow a[q_0 Z_0 q_1]$ — P_{16}

PDA $\rightarrow$ CFG

$\Rightarrow \delta(q_0, b, Z) = (q_0, ZZ)$

$[q_0 Z q_0] \rightarrow b [q_0 Z q_0] [q_0 Z q_0]$ — P_8

$[q_0 Z q_0] \rightarrow b [q_0 Z q_1] [q_1 Z q_0]$ — P_9

$[q_0 Z q_1] \rightarrow b [q_0 Z q_0] [q_0 Z q_1]$ — P_{10}

$[q_0 Z q_1] \rightarrow b [q_0 Z q_1] [q_1 Z q_1]$ — P_{11}

States (Assumed)

$[q_0 Z_0 q_0] = A$

$[q_0 Z q_0] = D$

$[q_0 Z_0 q_1] = B$

⋮

$S \rightarrow A$

$S \rightarrow B$

$S \rightarrow A \mid B$

$P_8 \longrightarrow A \rightarrow b A A$