ASSIGNMENT - 2

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Q.1 ⇒ The time to failure in hours of an important piece of electronic equipment used in a manufactured DVD player has the density function.

$$f(x) = \begin{cases} \frac{1}{2000} \exp(-x/2000), & x \geq 0 \\ & x < 0 \end{cases}$$

a) Find $F(x)$

b) Determine the P that the Component (and thus the DVD Player) Lasts more than 1000 hours before the Component needs to be replaced.

c) Determine the Prob- that the Component fails before 2000 hours.

Solve →

$$F(x) = P(X \leq x)$$

$$= \int_{-\infty}^{x} f(t)dt$$

$$= \int_{0}^{x} \frac{1}{2000} e^{-\frac{t}{200}} dt + \int_{-\infty}^{0} 0 dt$$

$$= \left(-e^{-t/2000}\right)\Big|_{0}^{x} + 0$$

$$= -e^{-x/200} + e^{0}$$

$$= 1 - e^{-x/2000} \text{ for all } x \geq 0$$

On the other hand f is a zero function for $x < 0$ and its integral on a ~~segment~~ is 0 so.

Date:

Page No:

$F(x) = 0$ for all $x < 0$

b) $P(x > 1000) = 1 - P(x \leq 1000)$

$= 1 - F(1000)$

$= 1 - (1 - e^{-1000/2000})$

$= 1 - 1 + e^{-1/2}$

$= 0.6065$

c) $P(x < 2000) = P(x \leq 2000)$

$= F(2000)$

$= 1 - e^{-2000/2000}$

$= 1 - 1/e$

$= 0.6321$ Ans

Q.2 => A Cereal manufacturer is aware that the weight at the product in the box varies slightly from box to box. in fact, Considerable, historical data have allowed the determination of the density function that describes the probability structure for the weight (in ounces). Letting that describes the P. structure for the weight. Letting x be the random Variable weight, in ounces, the density can be described as:

$$f(x) = \begin{cases} 2/5, & 23.75 \leq x \leq 26.25 \\ 0, & \text{otherwise} \end{cases}$$

a) Verify that this is a valid density function.

b) Determine P that the weight is smaller than 24 ounces.

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Date:
Page No:

Sol :- 4) To be a Valid density function it has to be

$$\int_{\infty}^{\infty} f(x)\,dx = 1$$

$$\text{so} \quad = -\int_{-\infty}^{\infty} f(x)\,dx = \int_{23.75}^{26.25} \frac{2}{5}\,dx$$

$$= \frac{2}{5} x \Big|_{23.75}^{26.25} \Rightarrow \frac{2}{5}(26.25 - 23.75)$$

$$= \frac{2}{5} \times 2.5 = 1$$

therefore, $f(x)$ is Valid density fumition

5) $$P(x < 24) = \int_{-\infty}^{24} f(x)\,dx \Rightarrow \int_{23.75}^{24} \frac{2}{5}\,dx$$

$$= \frac{2}{5} x \Big|_{23.75}^{24} \Rightarrow \frac{2}{5}(24 - 23.75)$$

$$= \frac{2}{5} \times 0.25 \Rightarrow 0.1 \text{ Ans.}$$

Q.3 :) An important factor in Solid missile fuel is the particle size distribution. significant Problems occur if the partice sizes are too Large From Production date.
in the past, it has been determined that the particle size (in micrometers) distribution is characterised by

Scanned with CamScanner

Date:

Page No:

$$f(x) = \begin{cases} 3x^{-4}, & x > 1 \\ 0, & \text{otherwise} \end{cases}$$

a) Verify that this is a valid density function.

b) Evaluate $F(x)$

c) What is the probability that a random particle from the manufactured fuel exceeds 4 micrometers.

Solve: Given $f(x) = \begin{cases} 3x^{-4}, & ; x \geq 1 \\ 0 & \text{Otherwise} \end{cases}$

a) $\int_1^{\infty} f(x)\,dx = 1$

$\int_1^{\infty} 3x^{-4}\,dx \Rightarrow \left[\frac{-1}{x^3}\right]_1^{\infty} = 1$

$\therefore f(x)$ is a valid density function.

b) $F(x) = \int_x f(a)\,dx$

$f(x) = 3\int_1^x x^{-4}\,dx = \left[\frac{-1}{x^3}\right]_1^x = 1 - x^{-3}$

c) $P(x>4) = 1 - P(x<4)$

$= 1 - \left[\frac{-1}{x^3}\right]_1^4$

$= 1 + \left[\frac{1}{64} - 1\right]$

$= 0.0156$ Ans

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Date:
Page No:

Q.4 ⇒ Based om extemsive testimg., it is determimed by the mamfactures of a Washing machime that the time Y (im Years) before a major repair is required is characterized by the probability density function

$$f(y) = \begin{cases} 1/4\, e^{-y/4} & , y \geq 0 \\ 0 & , \text{Otherwise} \end{cases}$$

a) Critics would certainly considers the product a bargain if it is umlikely to require a major repair before the sixth year. Comment on this by determining $P(Y>6)$

b) What is the Pro. that a major repair occures im the first years.

Solve : a) $P(Y>6)$

$$P(Y>6) = 1 - P(Y \leq 6)$$

$$= 1 - \int_{-\infty}^{6} f(y)\,dy$$

$$= 1 - \int_{0}^{6} \frac{1}{4} e^{-y/4}\,dy$$

$$= 1 - \left[-e^{-y/4}\right)\Big|_0^6$$

$$= 1 - \left(-e^{-6/4} + e^{0}\right)$$

$$= 1 + e^{-3/2} - 1$$

$$= 0.2231$$

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b) $P(Y \le 1)$

$$P(Y \le 1) = \int_{-\infty}^{1} f(y)\,dy$$

$$= \int_{0}^{1} \frac{1}{4} e^{-y/4}\,dy$$

$$= \left(-e^{-y/4}\right)\Big|_{0}^{1}$$

$$= \left(-e^{-1/4} + e^{0}\right)$$

$$= 1 - e^{-1/4}$$

$$= 0.2212 \quad \text{Ans.}$$

Q.5 ⇒ The proportion of the budget for a certain type of industrial company that is allotted to environmental and pollution control is coming under scrutiny. ~~A~~ data collection project determines that the distribution of these proportions is given by :

$$f(y) = \begin{cases} 5(1-y)^4, & 0 < y \le 1 \\ 0, & \text{otherwis} \end{cases}$$

a) Verify that the above is a valid density function

b) What is the prob. that a company chosen at random expends less than 10% of its budget on environmental and pollution controls.

c) What is the Pro that a Company Selected at random Spends more than 50% at its budget on —"—

Scanned with CamScanner

Date:
Page No:

Soln (a) $\int_0^1 f(y)\,dy = 1$

$$\int_0^1 5(1-y)^4\,dy = 5\left.\frac{(1-y)^5}{(-1)5}\right|_0^1$$

$$= -(y-1)^5\Big|_0^1$$

$$\Rightarrow \int f(y)\,dy = 1$$

Density function is a valid function

(b) $P(y<0.1) = 5\int_0^{0.1}(1-y)^4\,dy$

$$= \left.\frac{5}{-5}(1-y)^5\right|_0^{0.1}$$

$$P(y<0.1) = 1-(0.9)^5 = 0.4095$$

(c) $P(y>0.5) = \int_{0.5}^1 5(1-y)^4\,dy$

$$P(y>0.5) = (-1)(1-y)^5\Big|_{0.5}^1$$

$$= (0.5)^5$$

$$= 0.0313 \text{ Ans}$$

Q(1) Suppose a certain type of small data processing firm is so specialized that some have difficulty making a profit in their first year of operation, The P. density function that characterize the properfime Y that mak a profit is given by.

$$f(y) = \begin{cases} k y^4 (1-y)^3 & 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

a) What is the value of μ that renders the above a valid density function

b) Find the Pro that at most 50% of the firms mare a profit in the first year

c) Find the Pro that at last 50% of the firms more a profit in the first year

Sol a) if f is a valid density function it must ~~satisfy~~ $\int_{-\infty}^{\infty} f(y)\, dy = 1$

$$1 = \int_{-\infty}^{\infty} f(y)\, dy$$

$$= \int_0^1 k y^4 (1-y)^3 dy$$

$$= k \int_0^1 y^4 (1-3y+3y^2-y^3) dy$$

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$$= k\int_0^1 \left(y^4 - 3y^5 + 3y^6 - y^7\right) dy$$

$$= k\left(\frac{1}{5}y^5 - \frac{1}{2}y^6 + \frac{3}{7}y^7 - \frac{1}{8}y^8\right)\Big|_0^1$$

$$= k\left(\frac{1}{5} - \frac{1}{2} + \frac{3}{7} - \frac{1}{8}\right)$$

$$= \frac{k}{280} = \boxed{k = 280}$$

b) $P(Y \leq 0.5)$

$$P(Y < 0.5) = P(0 < Y < 0.5)$$

$$= \int_0^{0.5} 280\, y^4 (1 - y^3)\, dy$$

$$= 280\int_0^{0.5} y^4 \left(1 - 3y + 3y^2 - y^3\right) dy$$

$$= 280\int_0^{0.5} y^4 - 3y^5 + 3y^6 - y^7 \,]\, dy$$

$$= 280\left(\frac{1}{5}y^5 - \frac{1}{2}y^6 + \frac{3}{7}y^7 - \frac{1}{8}y^8\right]\Big|_0^{0.5}$$

$$= 280\left(\frac{1}{5}\times 0.5^5 - \frac{1}{2}\times 0.5^6 + \frac{3}{7}\times 0.5^7 - \frac{1}{8}\times 0.5^8\right]$$

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$$-\frac{93}{256}$$

(c) $P(Y > 0.8)$

$$P(Y > 0.8) = 1 - P(Y < 0.8)$$

$$= 1 - P(0 \le y \le 0.8)$$

$$= 1 - \int_0^{0.8} 280\, y^4 (1-y)^3\, dy$$

$$= 1 - 280 \int_0^{0.8} y^4 (1 - 3y + 3y^2 - y^3)\, dy$$

$$= 1 - 280 \int_0^{0.8} (y^4 - 3y^5 + 3y^6 - y^7)\, dy$$

$$= 1 - 280 \left(\frac{1}{5} y^5 - \frac{1}{2} y^6 + \frac{3}{7} y^7 - \frac{1}{8} y^8 \right) \Big]_0^{0.8}$$

$$= 1 - 280 \left(\frac{1}{5} \times 0.8^5 - \frac{1}{2} 0.8^6 + \frac{3}{7} \times 0.8^7 - \frac{1}{8} \times 0.8^8 \right]$$

$$= 0.0563 \text{ Ans.}$$

Scanned with CamScanner

Date:

Page No:

Q.7⇒ A Used-car dealership has found that the length of time before a major repair is required on the cars it sells is normally distributed with a mean equal to 10 months and a standard deviation of 3 months. If the dealer wants only 5% of the cars to fail before the end of the guarantee period, for how many month should the cars be guaranted

Solve- given μ = Mean = 10

σ = standard deviation = 3

$$P(x < x_0) = 5\% = 0.05$$

We note that the Prob. 0.05 lies exactly in the middle b/w 0.0495 and 0.0505

The Prob. is 0.494 lies in the row −1.6 and in the column .04 of the normal pro. table and thus the corresponding Z-score is then −1.6 + 0.4 = 1.64

The prob is 0.0505 lies in the row −1.6 and in the coloumn .05 of the normal probability table and —'—

−1.6 − 10.5 = 1.65.

Since the prob 0.05 lies exactly in the middle b/w 0.0495 and 0.0505 we expect the Z score Z_0 to lies exactly in the middle b/w the corresponding Z-scores −1.64 and −1.65, which is this

−1.64

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Date:
Page No:

$$Z_0 = -1.645$$

The z-score is the observed value decreased by the mean divided by the standard deviation

$$Z_0 = \frac{x - \mu}{\sigma} = \frac{x_0 - 10}{3}$$

$$\therefore Z_0 = -1.645$$

$$\frac{x_0 - 10}{3} = -1.645$$

Multiply each side by 3.

$$x_0 - 10 = -4.935$$

$$\boxed{x_0 = 5.065 \text{ moths}}$$ Ans.

Q.8) A home security system is designed to have a 99% reliability rate. Suppose that nine homes equipped with this system experience an attempted burglary find the Prob. of these events

a) At least one of the alarms is triggered.
b) More than seven of the alarms are triggered
c) Eight or fewer alarms are triggered.

Sol^n: (a) $P(X=0) = C_0^9 \; 0.99^0 \; (1-0.99)^9$

$$= \frac{9!}{0!\,(9-0)!} \times 0.99^0 \cdot 0.01^9 \approx 0$$

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Date:
Page No:

$$P(X \geq 1) = 1 - P(X=0)$$
$$= 1 - 0$$
$$= 1$$

(b) at $\mu = 8, 9$;

$$P(X=8) = {}^{9}C_{8} \times 0.99^{8} \cdot (1-0.99)^{9-8}$$

$$= \frac{9!}{8!(9-8)!} \times 0.99^{8}\ 0.01$$

$$\approx 0.0830$$

$$P(X=9) = {}^{9}C_{9} \times 0.99^{9} \times (1-0.99)^{9-9}$$

$$= \frac{9!}{9!\,(9-9)!} \times 0.99^{9} \times 0.01^{0}$$

$$\approx 0.9135$$

Since it is not possible to obtains two differens numbers of successes on the same simulation

addition rule for mutually exclusive events

$$P(x > 7) = P(X=8) + P(X=9)$$
$$= 0.830 + 0.9135$$
$$= 0.9966$$

c) $P(x \leq 8) = 1 - P(x=9)$
$$= 1 - 0.9135 \Rightarrow 0.0865 \text{ Ans}$$

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