Example on Gomorian Cutting plane method

1) Max $z = x_1 + 3x_2$

s.t. $3x_1 + 6x_2 \le 8$

$5x_1 + 2x_2 \le 10$

$x_1, x_2 \ge 0$ and +ve integers.

$\Rightarrow$ Max $z = x_1 + 3x_2 + 0 \cdot x_3 + 0x_4$

s.t. $3x_1 + 6x_2 + x_3 = 8$

$5x_1 + 2x_2 + x_4 = 10$

$x_1, x_2, x_3, x_4 \ge 0$

C_B	B	x_B	c_j / b	1 / y_1	3 / y_2	0 / y_3	0 / y_4	x_{Br}/y_{rj}
0	y_3	x_3	8	3	$\boxed{6}$	1	0	8/6 →
0	y_4	x_4	10	5	2	0	1	10/2
			$z_j - c_j$	−1	−3	0	0	

$\uparrow$ $x_2 \rightarrow$ entering, $x_3 \rightarrow$ leaving

C_B	B	x_B	c_j / b	1 / y_1	3 / y_2	0 / y_3	0 / y_4	x_{Br}/y_{rj}
3	y_2	x_2	4/3	1/2	1	1/6	0	
0	y_4	x_4	22/3	4	0	−1/3	1	
			$z_j - c_j$	1/2	0	1/2	0	

$z_j - c_j \ge 0 \ \forall j$

$x_2 = 4/3$ ↳ does not satisfy integer constraint.

$x_{Bi} \rightarrow (x_2, x_4) \rightarrow \left(\frac{4}{3}, \frac{22}{3}\right)$

f_{BK} = fractional part = 1/3

$$f_{BK} - \sum_{l=m+1}^{n} f_{Kl}\, x_l \le 0.$$

$f_{BK} = f_{10}$, $f_{10} = 1/3$, $f_{11} = 1/2$, $f_{13} = 1/6$.

$$f_{10} - (f_{11} x_1 + f_{13} x_3) \le 0$$

$$\frac{1}{3} - \left(\frac{1}{2} x_1 + \frac{1}{6} x_3\right) \le 0$$

$$\Rightarrow -\frac{x_1}{2} - \frac{x_3}{6} + s_1 = -\frac{1}{3}$$

↳ s_1 is the gomorian slack variable.

			c_j	1	3	0	0	0
c_B	B	x_B	b	y_1	y_2	y_3	y_4	S_1
3	y_2	x_2	$4/3$	$1/2$	1	$1/6$	0	0
0	y_4	x_4	$22/3$	4	0	$-1/3$	1	0
0	S_1	S_1	$-1/3$	$\boxed{-1/2}$	0	$-1/6$	0	1 $\rightarrow$
			$Z_j - c_j$	$1/2$ $\uparrow$	0	$1/2$	0	0

$Z_j - c_j \geq 0$ But sol^n is infeasible.

$S_1 = -1/3$

We use dual simplex method

$\text{Max}\left\{ \frac{1/2}{-1/2}, \frac{1/2}{-1/6} \right\} = \text{Max}\{-1, -3\}$

$= -1$: $x_1 \rightarrow$ entering variable

$S_1 \rightarrow$ leaving variable,

			c_j	1	3	0	0	0
c_B	B	x_B	b	y_1	y_2	y_3	y_4	S_1
3	y_2	x_2	1	0	1	0	0	1
0	y_4	x_4	$14/3$	0	0	$-5/3$	1	8
1	y_1	x_1	$2/3$	1	0	$1/3$	0	-2
			$Z_j - c_j$	0	0	$1/3$	0	1

$Z_j - c_j \geq 0$, But $x_1 = 2/3$

$x_4 = 14/3$

Fractional part of x_1 & $x_4 = \frac{2}{3}$

$f_{10} = 2/3$, $f_{13} = 1/3$

$\frac{2}{3} - \frac{1}{3} x_3 \leq 0$

$\Rightarrow -\frac{1}{3} x_3 + S_2 = -\frac{2}{3}$

C_B	B	x_B	c_j / b	1 / y_1	3 / y_2	0 / y_3	0 / y_4	0 / s_2
3	y_2	x_2	1	0	1	0	0	0
0	y_4	x_4	14/3	0	0	-5/3	1	0
1	y_1	x_1	2/3	1	0	1/3	0	0
0	s_2	s_2	-2/3	0	0	$\boxed{-1/3}$	0	1
	$Z_j - c_j$			0	0	1/3	0	0

$S_2 = -2/3$

$S_2 \to$ outgoing vector

$x_3 \to$ entering vector

Dual simplex to be used.

C_B	B	x_B	c_j / b	1 / y_1	3 / y_2	0 / y_3	0 / y_4	0 / s_2
3	y_2	x_2	1	0	1	0	0	-5
0	y_4	x_4	4/3	0	0	0	0	1
1	y_1	x_1	0	1	0	0	0	-3
0	y_3	x_3	2	0	0	1	0	1
	$Z_j - c_j$			0	0	0	0	1

$Z_j - c_j \geq 0$

Optimum sol^n $x_1^* = 0$, $x_2^* = 1$

$Z_{max} = 3$

Ex 2. The owner of a Garment shop makes 2 types of shirt : A shirt and B shirt. He makes a profit Rs 1 and Rs 4 ~~for~~ per shirt on A shirt and B shirt resp. He has two tailors, X and Y for stitching the shirts. Tailors X and Y devote 7 hours and 15 hours resp per day. Both type of shirts are stitched by both the tailors. Tailor X and

Tailor 4 spend 2 hours and 5 hours resp. to stitch A shirt and 1 hour and 4 hours resp in stitching B shirt. How many ~~types~~ shirts of both ~~shirt~~ type should be stitched in order to maximize daily profit ?

Formulation →

$x_1 \rightarrow$ no. of A shirts

$x_2 \rightarrow$ no of B shirts

$$\text{Max } z = \cancel{20}\, x_1 + 4x_2$$

$$\text{s.t. } \quad 2x_1 + 4x_2 \le 7$$

$$5x_1 + 4x_2 \le 15$$

$x_1, x_2 \geq 0$ and integers

$$\text{Max } Z = x_1 + 4x_2$$

$$\text{s.t. } \quad 2x_1 + 4x_2 + x_3 = 7$$

$$5x_1 + 4x_2 + x_4 = 15$$

$$x_1, x_2, x_3, x_4 \geq 0$$

		C_j		1	4	0	0	
C_B	B	x_B	b	y_1	y_2	y_3	y_4	x_{Br}/y_{rj}
0	y_3	x_3	7	2	4	1	0	$7/4 \rightarrow$
0	y_4	x_4	15	5	4	0	1	$15/4$
		$Z_j - c_j$		-1	-4	0	0	

↑

$x_2 \rightarrow$ entering variable

$x_3 \rightarrow$ leaving variable

		C_j		1	4	0	0	
C_B	B	x_B	b	y_1	y_2	y_3	y_4	x_{Br}/y_{rj}
	y_2	x_2 ~~x_4~~	$7/4$	$1/2$	1	$1/4$	0	
	y_4	x_4 8	8	3	0	-1	1	
		$Z_j - c_j$		1	0	1	0	

$Z_j - c_j \geq 0 \quad x_2 = 7/4$

fractional part of $7/4 = 3/4$

$f_{10} = 3/4,\ f_{11} = 1/2,\ f_{13} = 1/4$;

$$\frac{3}{4} - \left(\frac{1}{2}x_1 + \frac{1}{4}x_3\right) \le 0.$$

$$-\frac{1}{2}x_1 - \frac{1}{4}x_3 + S_1 = -3/4.$$

			c_j	1	4	0	0	0	
c_B	B	x_B	b	y_1	y_2	y_3	y_4	S_1	
4	y_2	x_2	$7/4$	$1/2$	1	$1/4$	0	0	
0	y_4	x_4	8	3	0	-1	1	0	
0	S_1	S_1	$-3/4$	$\boxed{-1/2}$	0	$-1/4$	0	1	$\rightarrow$
			$z_j - c_j$	1	0	1	0	0	
				$\uparrow$					

$z_j - c_j \ge 0$, $S_1 = -3/4$. We will use dual simplex method

$S_1 \rightarrow$ leaving variable, $x_1 \rightarrow$ entering variable

			c_j	1	4	0	0	0
c_B	B	x_B	b	y_1	y_2	y_3	y_4	S_1
4	y_2	x_2	1	0	1	0	1	1
0	y_4	x_4	$7/2$	0	0	$1/2$	0	6
0	y_1	x_1	$3/2$	1	0	$1/2$	0	-2
			$z_j - c_j$	0	0	$1/2$	4	2

$z_j - c_j \ge 0 \ \forall j$

We fractional part of x_1 and $x_4 = 1/2$

We choose x_4

$f_{20} = 1/2,\quad f_{23} = 1/2.$

$$\frac{1}{2} - \frac{1}{2}x_3 \le 0$$

$$-\frac{1}{2}x_3 + S_2 = -\frac{1}{2}.$$

C_B	B	x_B	C_j / b	1 / y_1	4 / y_2	0 / y_3	0 / y_4	0 / S_2
4	y_2	x_2	1	0	1	0	1	0
0	y_4	x_4	7/2	0	0	1/2	0	0
1	y_1	x_1	3/2	1	0	1/2	0	0
0	S_2	S_2	$-1/2$	0	0	$\boxed{-1/2}$	0	1 →
		$Z_j - C_j$		0	0	1/2 ↑	4	0

$S_2 = -1/2$. Dual simplex. $y_2 \to$ leaving variable

$x_3 \to$ entering variable

C_B	B	x_B	C_j / b	1 / y_1	4 / y_2	0 / y_3	0 / y_4	0 / S_2
4	y_2	x_2	1	0	1	0	1	0
0	y_4	x_4	3	0	0	0	0	1
1	y_1	x_1	1	1	0	0	0	1
0	y_3	x_3	1	0	0	1	0	-2
		$Z_j - C_j$		0	0	0	4	1

$Z_j - C_j \geq 0 \ \forall j$.

$x_1^* = 1, \ x_2^* = 1$

$Z_{max} = 5$.

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