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Basic Concepts, Laws, and Principles

TOPICS DISCUSSED

- The need to study electrical and electronics engineering
- Behaviour of materials as conductors, semiconductors, and insulators
- Concept of current, resistance, potential, and potential difference
- Differences between electric field and magnetic field
- Ohm's law
- Effect of temperature on resistance
- Electromagnetism and electromagnetic induction
- Laws of electromagnetic induction
- Dynamically and statically induced EMF
- Self and mutual inductance
- Electrical circuit elements

1.1 INTRODUCTION

lighting, heating, and air-conditioning systems—all are examples of applications of electricity. Application of electricity is limitless and often extends beyond our imagination.

Electrical energy has been accepted as the form of energy which is clean and easy to transmit from one place to the other. All other forms of energy available in nature are, therefore, transformed into electrical energy and then transmitted to places where electricity is to be used for doing some work. Electrical engineering, therefore, has become a discipline, a branch of study which deals with generation, transmission, distribution, and utilization of electricity.

Electronics engineering is an offshoot of electrical engineering, which deals with the theory and use of electronic devices in which electrons are transported through vacuum, gas, or semiconductors. The motion of electrons in electronic devices like diodes, transistors, thyristors, etc. are controlled by electric fields. Modern computers and digital communication systems are advances of electronics. Introduction of very large scale integrated (VLSI) circuits has led to the miniaturization of all electronic systems.

Electrical and electronic engineering are, therefore, very exciting fields of study. A person who is unaware of the contribution of these fields of engineering and the basic concepts underlying the advancement, will only have to blame himself or herself for not taking any initiative in knowing the unknown.

In this chapter, we will introduce some basic concepts, laws, and principles which the students might have studied in physics. However, since these form the basis of understanding of the other chapters in this book, it will be good to study them again.

1.2 ATOMIC STRUCTURE AND ELECTRIC CHARGE

Several theories have been developed to explain the nature of electricity. The modern electron theory of matter, propounded by scientists Sir Earnest Rutherford and Niel Bohr considers every matter as electrical in nature. According to this atomic theory, every element is made up of atoms which are neutral in nature. The atom contains particles of electricity called electrons and protons. The number of electrons in an atom is equal to the number of protons.

The nucleus of an atom contains protons and neutrons. The neutrons carry no charge. The protons carry positive charge. The electrons revolve round the nucleus in elliptical orbits like the planets around the sun. The electrons carry negative charge. Since there are equal number of protons and electrons in an atom, an atom is basically neutral in nature.

If from a body consisting of neutral atoms, some electrons are removed, there will be a deficit of electrons in the body, and the body will attain positive charge. If neutral atoms of a body are supplied some extra electrons, the body will attain negative charge. Thus, we can say that the deficit or excess of electrons in a body is called charge.

Charge of an electron is very small. Coulomb is the unit of charge. The charge of an electron is only 1.602×10^{-19} Coulomb (C). Thus, we can say that the number of electrons per Coulomb is the reciprocal of 1.602×10^{-19} which equals approx. 6.28×10^{18} electrons.

Any charge is an example of static electricity because the electrons or protons are not in motion. You must have seen the effect of charged particles when you comb your hair with a plastic comb, the comb attracts some of your hair. The work of combing causes friction, producing charge of extra electrons and excess protons causing attraction.

Charge in motion is called electric current. Any charge has the potential of doing work, i.e., of moving another charge either by attraction or by repulsion. A charge is the result of separating electrons and protons. The charge of electrons or protons has potential because it likes to return back the work that was done to produce it.

1.3 CONDUCTORS, INSULATORS, AND SEMICONDUCTORS

The electrons in an atom revolve in different orbits or shells. The shells are named as K, L, M, N, etc. The number of electrons that should be in a filled inner shell is given by $2n^2$ where n is shell number 1, 2, 3, 4, etc. starting from the nearest one, i.e., first shell to the nucleus. If n = 1, the first shell will contain two electrons. If n = 2, the second shell will contain eight electrons. This way, the number of electrons in the shells are 2, 8, 18, 32, etc. The filled outermost shell should always contain a maximum number of eight electrons. The outermost shell of an atom may have less than eight electrons. As for example, copper has an atomic number of 29. This means, copper atom has 29 protons and 29 electrons. The protons are concentrated in the nucleus while the electrons are distributed in the K, L, M, and N shells as 2, 8, 18, and 1 electrons, respectively. The outermost shell of a copper atom has one electron only whereas this shell could have 8 electrons.

The position occupied by an electron in an orbit signifies its energy. There exists a force of attraction between the orbiting electron and the nucleus due to the opposite charge the of electron and the proton. The electrons in the inner orbits are closely bound to the nucleus than the electrons of the outer or outermost orbit. If the electron is far away from the nucleus, the force of attraction is weak, and hence the electrons of outermost orbit are often called free electrons. For example, a copper atom has only one atom in the last orbit which otherwise could have eight electrons.

In a copper wire consisting of large number of copper atoms, the atoms are held close together. The outermost electrons of atoms in the copper wire are not sure about which atom they belong to. They can move easily from one atom to the other in a random fashion. Such electrons which can move easily from one atom to the other in a random fashion are called free electrons. It is the movement of free electrons in a material like copper that constitutes flow of current. Here, of course, the net current flow will be zero as the movement of the free electrons is in random directions. When we apply a potential, which is nothing but a force, it will direct the flow of electrons in a particular direction, i.e., from a point of higher potential towards a point of lower potential. Thus, current flow is established between two points when there exists a potential difference between the points.

When in a material the electrons can move freely from one atom to another atom, the material is called a conductor. Silver, copper, gold, and aluminium are good conductors of electricity. In general, as carrier of electricity is to allow electric current to flow with the minimum of resistances, i.e., the minimum of opposition.

In a material where the outermost orbit of the atoms is completely filled, the material is called an insulator. Insulators like glass, rubber, mica, plastic, paper, air, etc. do not conduct electricity very easily. In the atoms of these materials, the electrons tend to stay in their own orbits. However, insulators can store electricity and can prevent flow of current through them. Insulating materials are used as dielectric in capacitors to store electric charge, i.e., electricity.

Carbon, silicon, and germanium having atomic numbers of 6, 14, and 32, respectively, are called semi conducting material. The number of electrons in the outermost orbit of their atoms is four instead of the maximum of eight. Thus, in the outermost orbit of a semiconductor material, there are four vacant positions for electrons. These vacant positions are called holes. In a material, the atoms are so close together that the electrons in the outermost orbit or shell behave as if they were orbiting in the outermost shells of two adjacent atoms producing a binding force between the atoms. In a semiconductor material the atoms forming a bonding, called covalent bonding, share their electrons in the outermost orbit, and thereby attain a stable state. The condition is like an insulator having all the eight positions in the outermost orbit filled by eight electrons. However, in semiconducting materials, with increase in temperature it is possible for some of the electrons to gain sufficient energy to break the covalent bonds and become free electrons, and cause the flow of current.

1.4 ELECTRIC FIELD AND MAGNETIC FIELD

When charges are separated, a space is created where forces are exerted on the charges. An electric field is such a space. Depending upon the polarity of the charges, the force is either attractive or repulsive. Therefore, we can say that static charges generate an electric field. An electric field influences the space surrounding it. Electric field strength is determined in terms of the force exerted on charges. A capacitor is a reservoir of charge. The two parallel plates of a capacitor, when connected to a voltage source, establishes an electric field between the plates. The positive terminal, or pole of the voltage source will draw electrons from plate 1 whereas the negative pole will push extra electrons on to plate 2. Voltage across the capacitor will rise. The capacitor gets charged equal to the voltage of the source. The capacitance of a capacitor is a measure of its ability to store charge. The capacitance of a capacitor is increased by the presence of a dielectric material between the two plates of the capacitor.

A current-carrying conductor or a coil produces magnetic field around it. The strength of the magnetic field produced depends on the magnitude of the current flowing through the conductor or the coil. There is presence of magnetic field around permanent magnets as well.

A magnet is a body which attracts iron, nickel, and cobalt. Permanent magnets retain their magnetic properties. Electromagnets are made from coils through which current is allowed to flow. Their magnetic properties will be present as long as current flows through the coil.

1.5.1 Electric Current

In any conducting material, the flow of electrons forms what is called current. Electrons have negative charge. Charge on an electron is very small. For this reason charge is expressed in terms of Coulomb. Charge of one Coulomb is equal to a charge of 6.28×10^{18} electrons. The excess or deficit of electrons in a body is called charge. Thus, electrical current is expressed as a flow of negative charge, i.e., electrons. Any substance like copper, aluminum, silver, etc. which has a large number of free electrons (i.e., loosly bound electrons in the outermost orbit of its atom) will permit the flow of electrons when electrical pressure in the form of EMF (electromotive force, i.e., voltage) is applied.

Since these materials conduct electricity, they are called conductors. They easily allow electric current to flow through them. The strength of current will depend upon the flow of charge per unit time. This is expressed as

$$\text{Current, } I = \frac{Q}{t} \tag{1.1}$$

where charge Q is measured in Coulomb and time, t in seconds. The unit of current, therefore, is Coulomb per second, when 1 C of charge flows in 1 s; the magnitude of current is called ampere, named after André-Marie Ampere.

Thus, 1 ampere of current is equivalent to the flow of charge of 1 Coulomb per second.

In earlier years, current was assumed to flow from positive to negative terminals. This convention is used even now although it is known that current is due to the movement of electrons from the negative to the positive terminal.

1.5.2 Resistance

Electrical resistance is the hindrance or opposition to the flow of electrons in a given material. It is measured in unit called ohm. Since current is the flow of electrons, resistance is the opposition offered by a material, to the flow of free electrons. Resistance, R, is directly proportional to the length of the material, and inversely proportional to the area of the cross section of the material, through which current flows. The resistance offered by conducting materials like copper and aluminum is low whereas resistance offered by some other conducting materials like nicrome, tungsten, etc. is very high. All these materials are called conducting materials. However, the values of resistivity of these materials are different. The resistance, R of a material is expressed as

$$R = \rho \frac{\ell}{A} \tag{1.2}$$

resistivity, i.e., 0.016 × 10^{-6} ohm-m. After silver, copper is most conducting. The resistivity or specific resistance of copper is somewhat more than that of silver, i.e., 0.018 × 10^{-6} ohm-m. That is to say, copper is less conducting than silver. We will see a little later why and how the value of resistance changes with temperature.

1.5.3 Potential and Potential Difference

EMF produces a force or pressure that causes the free electrons in a body to move in a particular direction. The unit of EMF is volt. EMF is also called electric potential. When a body is charged (i.e., either defficiency of electrons or excess of electrons is created), an amount of work is done. This work done is stored in the body in the form of potential energy. Such a charged body is capable of doing work by attracting or repelling other charges. *The ability of a charged body to do work in attracting or repelling charges is called its potential or electrical potential.* Work done to charge a body to 1 C is the measure of its potential expressed in volts:

$$\text{Volt} = \frac{\text{Work done in Joules}}{\text{Charge in Coulombs}} \tag{1.3}$$

When work done is 1 joule and charge moved is 1 C, the potential is called 1 volt. If we say that a point has a potential of 6 volts, it means that 6 Joules of work has been done in moving 1 C of charge to that point. In other words, we can say that every Coulomb of charge at that point has an energy of 6 Joules.

The potential difference of two points indicates the difference of charged condition of these points. Suppose point A has a potential of 6 volts, and point B has a potential of 3 volts. When the points A and B are joined together by a conducting wire, electrons will flow from point B to point A. We say that current flows from point A towards point B. The direction of current flow is taken from higher potential to lower potential while the flow of electrons are actually in the opposite direction. The flow of current from higher potential to lower potential is similar to the flow of water from a higher level to a lower level.

1.6 OHM'S LAW

George Simon Ohm found that the voltage, V between two terminals of a current-carrying conductor is directly proportional to the current, I flowing through it. The proportionality constant, R is the resistance of the conductor. Thus, according to Ohm's law

$$V = IR \text{ Or, } I = \frac{V}{R} \tag{1.4}$$

This relation will hold good provided the temperature and other physical conditions do not change.

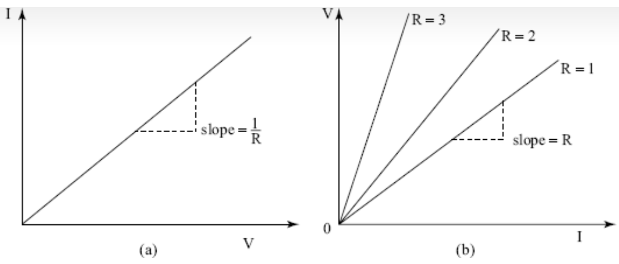


Figure 1.1 (a) Shows linear relationship between V and I; (b) V–I characteristics for different values of R

Ohm's law is not applicable to nonlinear devices like Zener diode, voltage regulators, etc. Ohm's law is expressed graphically on V and I-axies as a straight line passing through the origin as shown in Fig. 1.1 (a).

The relationship between V and I have been shown for different values of R in Fig. 1.1 (b). Here in V = RI, R indicates the slope of the line. The more the value of R is, the more will be the slope of the line as shown in Fig. 1.1 (b).

1.7 THE EFFECT OF TEMPERATURE ON RESISTANCE

Resistance of pure metals like copper, aluminum, etc. increases with increase in temperature. The variation of resistance with change in temperature has been shown as a linear relationship in Fig. 1.2.

The change in resistance due to change in temperature is found to be directly proportional to the initial resistance, i.e., $R_t - R_0 \propto R_0$. Resistance $(R_t - R_0)$ also varies directly as the temperature rise and this change also depends upon the nature of the material. Thus we can express the change in resistance as,

$$R_t - R_0 \propto R_0 t$$

or, $R_t - R_0 = \alpha_0 R_0 t$, where α_0 is called the temperature coefficient of resistance at 0°C.

$$\text{or, } R_t = R_0(1 + \alpha_0 t) \tag{1.5}$$

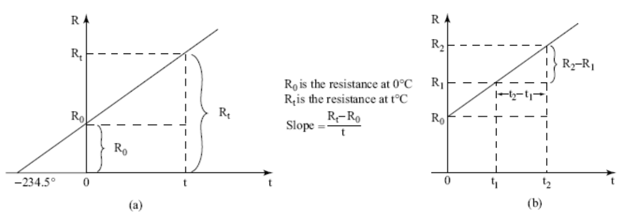


Figure 1.2 (a) Shows the variation of resistance with temperature; (b) resistances at two different temperatures

of the material continues to decrease with decrease in temperature below 0°C. If we go on decreasing the temperature to a very low value, the material attains a state of zero resistance. The material at that state becomes *superconducting*, i.e., conducting with no resistance at all.

Now suppose a conductor is heated from temperature t_1 to t_2. The resistance of the conductor at t_1 is R_1 and at t_2 is R_2 as has been shown in Fig. 1.2 (b).

Using eq. (1.5),

$$R_t = R_0(1+\alpha_0 t)$$

or, $$\alpha_0 = \frac{R_t - R_0}{R_0\, t}$$

or, $$\alpha_0 = \frac{(R_t - R_0)/t}{R_0} = \frac{\text{slope of resistance versus temp. graph}}{\text{original resistance}} \tag{1.6}$$

Using eq. (1.5), we can write

$$R_1 = R_0(1+\alpha_0 t_1)$$

and $$R_2 = R_0(1+\alpha_0 t_2)$$

From fig 1.2 (b) using the relation in (1.6), we can write

$$\alpha_1 = \frac{(R_2 - R_1)/(t_2 - t_1)}{R_1}$$

or, $$\alpha_1 R_1 (t_2 - t_1) = R_2 - R_1$$

or, $$R_2 = R_1 + \alpha_1 R_1 (t_2 - t_1)$$

or, $$R_2 = R_1[1 + \alpha_1(t_2 - t_1)] \tag{1.7}$$

Thus, if resistance at any temperature t_1 is known, the resistance at t_2 temperature can be calculated.

Calculation of α at different temperatures

We have seen,

$$\alpha_0 = \frac{\text{slope of resistance versus temp. graph}}{\text{Original resistance, } R_0}$$

If α_1 and α_2 are the temperature coefficients of resistance at t_1 and t_2 degrees, respectively, then

$$\alpha_1 = \frac{\text{slope of resistance versus temp. graph}}{R\ldots}$$

Thus, we can write,

$$\alpha_0 R_0 = \alpha_1 R_1 = \alpha_2 R_2 = \alpha_3 R_3 = \ \ldots \text{ and so on}$$

Therefore,

$$\alpha_1 = \frac{\alpha_0 R_0}{R_1} = \frac{\alpha_0 R_0}{R_0 (1 + \alpha_0 t_1)} = \frac{\alpha_0}{1 + \alpha_0 t_1} \tag{1.8}$$

$$\alpha_2 = \frac{\alpha_0 R_0}{R_2}$$

$$= \frac{\alpha_0 R_0}{R_0 (1 + \alpha_0 t_2)} = \frac{\alpha_0}{1 + \alpha_0 t_2}$$

and,

$$\alpha_2 R_2 = \alpha_1 R_1$$

or,

$$\alpha_2 = \frac{\alpha_1 R_1}{R_2} = \frac{\alpha_1 R_1}{R_1 [1 + \alpha_1 (t_2 - t_1)]}$$

or,

$$\alpha_2 = \frac{\alpha_1}{1 + \alpha_1 (t_2 - t_1)} \tag{1.9}$$

Temperature coefficient of resistance, α at 20°C and specific resistance ρ of certain material have been shown in Table 1.1.

Table 1.1 Temperature Coefficient and Specific Resistance of Different Materials

Material	Temp. coeff. of resistance α_{20}	Specific resistance ρ in micro–ohm
Silver	0.004	0.016
Copper	0.0039	0.018
Aluminium	0.0036	0.028
Iron	0.005	0.100
Brass	0.0015	0.070
Lead	0.0042	0.208
Tin	0.0046	0.110
Carbon	–0.00045	66.67

It is to be noted that carbon has a negative temperature coefficient of resistance. This means, the resistance of carbon decreases with increase in temperature.

By this time you must be wondering as to why resistance in most materials increases with increase in temperature while resistance in some decreases with increase in temperature.

flow of electrons is called resistance. At lower temperatures the vibration gets reduced, and hence the resistance.

1.8 WORK, POWER, AND ENERGY

1.8.1 Work

When a force is applied to a body causing it to move, and if a displacement, d is caused in the direction of the force, then

$$\text{Work done} = \text{Force} \times \text{Distance} \tag{1.10}$$

or,

$$W = F \times D$$

If force is in Newtons and d is in meters, then work done is expressed in Newton–meter which is called Joules.

1.8.2 Power

Power is the rate at which work is done, i.e., rate of doing work. Thus,

$$\text{Power, } P = \frac{\text{work done}}{\text{time}} = \frac{\text{Joules}}{\text{seconds}} \tag{1.11}$$

The unit of power is Joules/second which is also called Watt. When the amount of power is more, it is expressed in Kilowatt, i.e., kW.

$$1 \text{ kW} = 1 \times 10^3 \text{ W}$$

We have earlier seen in eq. (1.3), that electrical potential, V is expressed as

$$V = \frac{\text{work done}}{\text{charge}} = \frac{W}{Q}$$

or,

$$\text{Work done } = VQ = VIt \qquad \left[\because I = \frac{Q}{t}\right]$$

$$\text{Electrical Power, } P = \frac{\text{work done}}{t} = \frac{VIt}{t} = VI \text{ Watts} \tag{1.12}$$

Thus in a circuit if I is the current flowing, and V is the applied voltage across the terminals, power, P is expressed as

$$P = VI = V\frac{V}{R} = \frac{V^2}{R}$$

$$\text{Also, } P = VI = IR.I = I^2R$$

Thus electrical power can be expressed as

$$P = VI = \frac{V^2}{R} = I^2 R \text{ Watts} \quad (1.13)$$

Where V is in Volts,
I is in Amperes, and
R is in Ohms

1.8.3 Energy

Energy is defined as the capacity for doing work. The total work done in an electrical circuit is called electrical energy. When a voltage, V is applied, the charge, Q will flow so that

$$\begin{aligned}\text{Electrical energy} &= V \times Q \\ &= VIt \\ &= IRIt \\ &= I^2Rt \\ &= \frac{V^2}{R}t\end{aligned}$$

$$\text{or, Electrical energy} = \text{Power} \times \text{Time} \quad (1.14)$$

If power is in kW and time is in hour, the unit of energy will be in Kilowatt hour or kWh.

1.8.4 Units of Work, Power, and Energy

In SI unit, work done is the same as that of energy.

Mechanical work or energy

When a Force, F Newton acting on a body moves it in the direction of the force by a distance d meters:

Work done = F × D Nm or Joules

When a force F Newton is applied tangentially on a rotating body making a radius r meters, then

$$\begin{aligned}\text{Work done in 1 revolution} &= F \times 2\pi r \text{ (since distance moved is } 2\pi r) \\ &= 2\pi Fr \quad \text{Nm} \\ \text{Force} \times \text{Perpendicular distance} &= \text{Torque, i.e., } F \times r = T \\ \text{Work done in 1 revolution} &= 2\pi T \quad \text{Nm} \\ \therefore \quad \text{Work done in N revolutions/second} & \\ &= 2\pi TN \quad \text{Nm}\end{aligned}$$

If N is expressed in revolutions per minute (rpm)

$$\text{Work done} = \frac{2\pi TN}{60} \text{ Nm or Joules} \quad (1.15)$$

When a body of mass m kg is lifted to a height h meters against the

$$\text{Potential energy} = \text{Weight} \times \text{Height}$$
$$= \text{mgh Joules.} \qquad (1.16)$$

Kinetic energy of a body of mass m kg moving at a speed of v meters/sec² $= \frac{1}{2}mv^2$ Joules. (1.17)

Electrical energy

As mentioned earlier, work done in an electrical circuit is its energy.

$$\begin{aligned}\text{Electrical energy} &= \text{Applied voltage, V} \times \text{total flow of charge, Q}\\ &= VQ = VIt\\ &= IRIt\\ &= I^2Rt\\ &= \frac{V^2}{R}t\end{aligned}$$

$$\text{Electrical power} = \frac{\text{work done}}{\text{time}} = \frac{VIt}{t} = VI$$

$$\text{Electrical energy} = \text{Electrical power} \times \text{time} \qquad (1.18)$$

If electrical power is expressed is kW and time in hour, then

$$\text{Electrical energy} = \text{kWh} \qquad (1.19)$$

We will now convert kWh into Calories

$$\begin{aligned}1\ \text{kWh} &= 10^3 \times 60 \times 60\ \text{Watt second or Joules}\\ &= 36 \times 10^5\ \text{Joules}\end{aligned}$$

Since 1 Calorie = 4.2 Joules

$$1\ \text{kWh} = \frac{36 \times 10^5}{4.2} = 860 \times 10^3\ \text{Calories}$$

or, $$1\ \text{kWh} = 860\ \text{kiloCalories} \qquad (1.20)$$

Thermal energy

In SI unit* thermal energy is expressed in calories. One calorie indicates the amount of heat required to raise the temperature of 1 gm of water by 1°C. This heat is also called the specific heat. If m is the mass of the liquid, S is the specific heat, and t is the temperature rise required, then the amount of heat required, H is expressed as

$$\begin{aligned}H &= \text{mst calories}\\ &= 4.2 \times \text{mst Joules (since 1 cal} = 4.2\text{J)} \qquad (1.21)\end{aligned}$$

1 calorie = 4.2 Joules, has been established experimentally

Example 1.1 A copper wire has resistance of 0.85 ohms at 20°C. What will be its resistance at 40°C? Temperature coefficient of resist-

We know, $\alpha_1 = \dfrac{\alpha_0}{1 + \alpha_0 t_1}$

Here, $\alpha_{20} = \dfrac{0.004}{1+0.004\times 20} = 0.0037$

We know, $R_2 = R_1 [1+\alpha_1 (t_2 - t_1)]$

In this case, $R_{40} = R_{20}[1+\alpha_{20}(40-20)]$

$= 0.85\ [1+0.0037\times 20]$

$= 0.9129\ \Omega$

Example 1.2 The heating element of an electric heater made of nicrome wire has value of resistivity of 1×10^{-6} Ohm-m. The diameter of the wire is 0.2 mm. What length of this nicrome wire will make a resistance of 100 Ohms?

Solution:

We know, $R = \rho \dfrac{\ell}{a}$

or, $\ell = \dfrac{R.a}{\rho}$

given $R = 100\ \Omega$, $\rho = 1 \times 10^{-6}$ Ohm-m, d = 0.2 mm
area, $a = \pi d^2 = 3.14 \times (0.2 \times 10^{-3})^2 = 12.56 \times 10^{-8}\ m^2$

Substituting the values, length of wire, ℓ is

$$\ell = \frac{100\times 12.56\times 10^{-8}}{1\times 10^{-6}} = 12.56\,m$$

Example 1.3 It is required to raise the temperature of 12 kg of water in a container from 15°C to 40°C in 30 min through an immersion rod connected to a 230 V supply mains. Assuming an efficiency of operation as 80 per cent, calculate the current drawn by the heating element (immersion rod) from the supply. Also determine the rating of the immersion rod. Specific heat of water is 4.2 kiloJoules/kg/°C.

Solution:

Output or Energy spent in heating the water, H is

$$H = m\ s\ (t_2 - t_1)$$

Where m is the mass of water and s is the specific heat of water.

Here, $H = 12 \times 4.2 \times 10^3 \times (40 - 15)$ Joules

$= 126 \times 10^4$ Joules

We know, efficiency $= \dfrac{\text{Output}}{\text{Input}}$

$$= \frac{126 \times 10^4}{0.8}$$

$$= 157.5 \times 10^4 \text{ Joules}$$

The time of operation of the heater rod

$$= 30 \text{ min} = 30 \times 60 \text{ s}$$
$$= 1800 \text{ secs.}$$

The power rating of the heater

$$= \frac{\text{Energy input}}{\text{Time of operation}}$$

$$= \frac{157.5 \times 10^4 \text{ Joules}}{1800 \text{ seconds}}$$

$$= 870 \text{ Joules / sec}$$

$$= 870 \text{ Watts}$$

$$= 0.87 \text{ kW}$$

Current drawn from 230 V supply

P = VI = 870 Watts.

Therefore,

$$I = \frac{870}{230} = 3.78 \text{ A}$$

Example 1.4 A motor-driven water pump lifts 64 m^3 of water per minute to an overhead tank placed at a height of 20 metres. Calculate the power of the pump motor. Assume overall efficiency of the pump as 80 per cent.

Solution:

Work done/min = mgh Joules

m = 64 × 10^3 kg (1m^3 of water weights 1000 kg)

g = 9.81 m/sec^2

h = 20 m

Substituting values

$$\text{Work done/sec} = \frac{64 \times 10^3 \times 9.81 \times 20 \text{ Joules}}{60 \text{ seconds}}$$

$$= \frac{12.55 \times 10^6 \text{ Joules}}{60 \text{ seconds}}$$

Input power of the pump motor $= \dfrac{209.1}{0.8}$

= 261.3 KW

Example 1.5 A residential flat has the following average electrical consumptions per day:

1. 4 tube lights of 40 watts working for 5 hours per day;
2. 2 filament lamps of 60 watts working for 8 hours per day;
3. 1 water heater rated 2 kW working for 1 hour per day;
4. 1 water pump of 0.5 kW rating working for 3 hours per day.

Calculate the cost of energy per month if 1 kWh of energy (i.e., 1 unit of energy) costs ₹3.50.

Solution:

Total kilowatt hour consumption of each load for 30 days are calculated as:

$$\text{Tube lights} = \frac{4 \times 40 \times 5 \times 30}{1000} = 24\,\text{kWh}$$

$$\text{Filament lamps} = \frac{2 \times 60 \times 8 \times 30}{1000} = 28.8\,\text{kWh}$$

$$\text{Water heaters} = 1 \times 2 \times 1 \times 30 = 60\ \text{kWh}$$

$$\text{Water pump} = 1 \times 0.5 \times 3 \times 30 = 45\ \text{kWh}$$

Total kWh consumed per month

= 24 kWh + 28.8 kWh + 60 kWh + 45 kWh

= 157.5 kWh

One kWh of energy costs ₹3.50.

The total cost of energy per month = 157.5 × 3.50

= ₹551.25

Example 1.6 An electric kettle has to raise the temperature of 2 kg of water from 30°C to 100°C in 7 minutes. The kettle is having an efficiency of 80 per cent and is supplied from a 230 V supply. What should be the resistance of its heating element?

Solution:

$$m = 2\ \text{kg} = 2000\ \text{gms}$$
$$t_2 - t_1 = 100 - 30 = 70°C$$
$$\text{Specific heat of water} = 1$$
$$\text{Time of heating} = 7\ \text{minutes} = \frac{7}{60}\ \text{hours}$$

Output energy of the kettle = m s t

$$\begin{aligned}
&= 2000 \times 1 \times 70\ \text{Calories} \\
&= 140\ \text{kilo Calories} \\
&= \frac{140}{860}\ \text{kWh} \\
&= 0.1627\ \text{kWh}.\ [1\ \text{kWh} = 860\ \text{kCal}]
\end{aligned}$$

$$\text{Input energy} = \frac{\text{output energy}}{\text{efficiency}} = \frac{0.1627}{0.8} = 0.203\,\text{kWh}$$

$$\text{kW rating of the kettle} = \frac{0.203}{\text{time in hours}} = \frac{0.203 \times 60}{7} = 1.74\ \text{kW}$$

Supply voltage, V = 230 Volts.

Power, P = 1.74 kW = 1740 Watts.

V = 230 V

$$P = VI = V\frac{V}{R} = \frac{V^2}{R}\ \text{Watts}$$

or, $$R = \frac{V^2}{P} = \frac{230 \times 230}{1740} = 30.4\ \text{Ohms}$$

Example 1.7 Calculate the current flowing through a 60 W lamp on a 230 V supply when just switched on at an ambient temperature of 25°C. The operating temperature of the filament material is 2000°C and its temperature coefficient of resistance is 0.005 per degree C at 0°C.

Solution:

We know, power, $$W = VI = V\frac{V}{R} = \frac{V^2}{R}$$

Here, $W = 60\ \text{W},\ V = 230\ \text{V}$

$\therefore$ $$R = \frac{V^2}{W} = \frac{230 \times 230}{60} = 881.6\ \text{Ohms}$$

This resistance of the filament is at 2000°C. Let us call it R = 881.6

We know R_{2000}; we have to calculate R_{25} given α_0 = 0.005 ohm/°C.

We know, $$\alpha_1 = \frac{\alpha_0}{1+\alpha_0(t_1 - t_0)}$$

$\therefore$
$$\alpha_{25} = \frac{\alpha_0}{1+\alpha_0(25-0)} = \frac{0.005}{1+0.005(25)}$$
$$= 4.44 \times 10^{-3}\ /°C$$

We know the relation,

$$R_2 = R_1[1+\alpha_1(t_2 - t_1)]$$
$\therefore$
$$R_{2000} = R_{25}[1+\alpha_1(2000-25)]$$
$$881.6 = R_{25}[1+4.44\times10^{-3}\times1975]$$
$$R_{25} = 90.25\,\Omega$$

The current flowing through the 60 W lamp at the instant of switching will be corresponding to its resistance at 25°C.

$\therefore$
$$I = \frac{V}{R_{25}} = \frac{230}{90.25} = 2.55 \text{ Amps}$$

Example 1.8 A coil has a resistance of 18 Ω at 20°C and 20 Ω at 50°C. At what temperature will its resistance be 21 Ohms?

Solution:

$$R_{20} = 18,\ R_{50} = 20,\ R_t = 21 \text{ at what t?}$$

we know, $R_2 = R_1[1+\alpha_1(t_2 - t_1)]$

$\therefore$
$$R_{50} = R_{20}[1+\alpha_{20}(50-20)]$$

or,
$$20 = 18[1+\alpha_{20}(30)]$$

or,
$$\alpha_{20} = 3.7\times10^{-3}/°C$$

We can write,

substituting,

$$21 = 18\,[1 + 3.7 \times 10^{-3}(t_3 - 20)]$$

or, $$t_3 = 65°C.$$

Example 1.9 The resistance of a wire increases from 40 Ω at 20°C to 50 Ω at 70°C. Calculate the temp. coefficient of resistance at 0°C.

Solution:

given

$$R_{20} = 40\,\Omega,\; R_{70} = 50\,\Omega,\; \text{what is } \alpha_o?$$

$$R_2 = R_1[1 + \alpha_1(t_2 - t_1)]$$

$$50 = 40[1 + \alpha_1(70 - 20)]$$

$$\alpha_1 = 5 \times 10^{-3}/°C$$

$$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1}$$

or, $$5 \times 10^{-3} = \frac{\alpha_0}{1 + 20\,\alpha_0}$$

or, $$\alpha_0 = 5.55 \times 10^{-3}/°C$$

Example 1.10 A resistance element of cross-sectional area of 10 mm^2 and length 10 m draws a current of 4 A at 220 V supply at 20°C. Calculate the resistivity of the material. What current will be drawn when the temperature rises to 60°C? Assume α_{20} = 0.0003/°C.

Solution:

$a = 10\ mm^2$

$= 10 \times 10^{-6}\ m^2$

V = IR

or, $$R = \frac{V}{I} = \frac{220}{4} = 55\,\Omega$$

This resistance, we call as R_{20}

$$R = \rho \frac{\ell}{a}$$

or, $$R_{20} = \rho_{20} \frac{\ell}{a}$$

$$55 = \rho_{20} \frac{10}{10 \times 10^{-6}}$$

or, $$\rho_{60} = 55 \times 10^{-6} \text{ ohm-m}$$

given $$\rho_{20} = 0.0003 / °C$$

Since we have to calculate R_{60}, we have to P_{60}

$$\rho_{20} = \rho_{20}[1 + \alpha_{20}(60 - 20)]$$
$$= 55 \times 10^{-6} [1 + 0.0003 \times 40] = 55.66 \times 10^{-6} \Omega \text{ m}$$

Now,

$$R_{60} = \rho_{60} \frac{\ell}{a} = 55.66 \times 10^{-6} \frac{10}{10 \times 10^{-6}} = 55.66$$

Current,

$$I = \frac{V}{R_{60}} = \frac{220}{55.66} = 3.9525 \text{ Amps}$$

1.9 ELECTROMAGNETISM AND ELECTROMAGNETIC INDUCTION

1.9.1 Introduction

Electromagnetism is the study of interaction between electric current and magnetic field, and forces produced thereof. This section will include descriptions of magnetic field around current-carrying conductors, magnetic field produced by a current-carrying coil, force produced on a current-carrying conductor or a coil when placed in a magnetic field.

A Danish scientist, Oersted in the early nineteenth century discovered that there was a magnetic field around a current-carrying conductor. Lines of force in the form of concentric circles existed on a perpendicular plane around a current-carrying conductor. This meant, magnetism could be created by electric current. It was also observed that the direction of lines of force got changed when the direction of current flowing through the conductor was changed. A few years after the discovery of Oersted, Faraday, another scientist from England discovered that a magnetic field can create an electric current in a conductor. When there is a change in flux linkage in a conductor or a coil, EMF is induced in it. This phenomenon is credited to Faraday who established famous laws of electromagnetic induction. You will observe that most of the electrical machines and devices have been developed utilizing the observations and discoveries made as mentioned above.

1.9.2 Magnetic Field Around a Current-carrying Conductor

In Fig. 1.3 is shown a conductor carrying a current, I. Lines of force conductor have been shown. The cross at the centre of the conductor indicates that current is entering the conductor which is placed perpendicular to the plane of the paper. The lines of force in the form of concentric circles are on the plane of the paper. The direction of current through the conductor is reversed in Fig.1.3 (c). The dot at the centre of the conductor indicates that the current is coming towards the observer. The direction of the lines force around the conductor also get reversed.

The direction of flux lines around a current-carrying conductor is determined by applying the cork screw rule which is stated below.

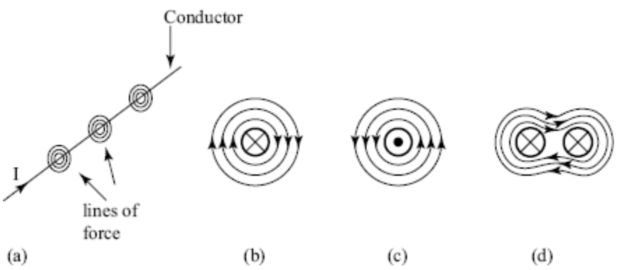


Figure 1.3 (a) A long current-carrying conductor; (b) cross-sectional view of a conductor with flux around it; (c) cross-sectional view of the conductor with the direction of current reversed; (d) resultant magnetic field produced by two current-carrying conductors

Cork Screw Rule: Consider a right hand screw held on one end of a current-carrying conductor and is rotated in the clockwise direction. If the advancement of the screw indicates the direction of current, the direction in which the screw is rotated will indicate the direction of the lines of force around the conductor.

In Fig. 1.3 (d) has been shown that two current-carrying conductors placed side by side produce a resultant magnetic field.

1.9.3 Magnetic Field Around a Coil

A coil is formed by winding a wire of certain cross section around a former (a hollow cylinder made of some non-magnetic material like bakelite, plastics, etc). Such a coil is often called a solenoid. When current is allowed to flow through such a coil, a magnetic field is produced by the coil. The direction of flux produced by a current-carrying coil is determined by applying the right-hand-grip rule. In Fig. 1.4 (a) is shown a current-carrying coil. If we hold the coil by our right hand in such a way that the four fingers bend towards the direction of the current flow through the coil turns, the thumb will indicate the direction of the resultant flux produced.

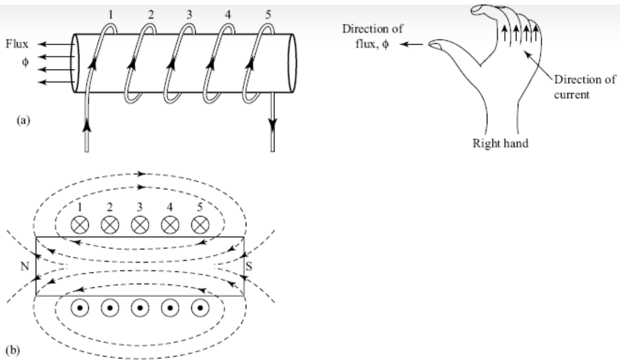
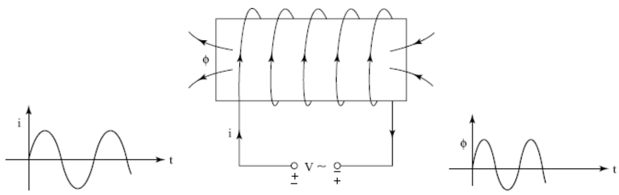


Figure 1.4 (a) Right-hand-grip rule applied to determine direction of flux produced by a current-carrying coil; (b) magnetic field produced by a current-carrying coil

The four fingers bend in the direction of current through the coil. The direction in which the thumb points is the direction of flux produced. In Fig. 1.4 (b), we have shown the cross-sectional view of the same coil. For the direction of current flow through the coil, cross-sections have been shown by putting cross and dot convention. The upper side of the coil turns 1, 2, 3, 4, 5 will indicate that current is entering while they will come out from the other side as shown in the bottom conductor cross-sections. By applying the cork screw rule also we can determine the direction of the resultant magnetic field and show the positions of North and South poles formed. If the direction of current flow through the coil is reversed, the direction of the magnetic lines of force will be opposite, and hence the positions of North and South poles will change.

If we apply some alternating voltage across the coil as shown in Fig. 1.5, the polarity of power supply will change in every half cycle of the applied voltage. If a sinusoidal ac supply is provided, both the magnitude as well as the direction of current flow will change. As a result, the magnitude of the magnetic field produced will change starting from zero value reaching its maximum value, then getting reduced again to zero, and then becoming negative. The direction of flux produced will change in every half cycle of current flow. Such a magnetic field whose magnitude as also its direction changes is called a pulsating alternating magnetic field. In case of dc supply, the magnetic field produced will be of constant magnitude and fixed polarity.



When a conductor carrying current is placed in a magnetic field it experiences a force. The force acts in a direction perpendicular to both the magnetic field and the current.

In Fig. 1.6 a conductor is shown placed perpendicular to the direction of magnetic field. Such a conductor in cross-sectional view has been shown by a small circle. The dot inside the small circle indicates that current is flowing towards the observer. The conductor will experience a force in the upward direction as has been shown. If the direction of current through the conductor is reversed, the force on the conductor will be in the downward direction.

The force on the conductor will depend upon the flux, ϕ or flux density, B $\left(B = \frac{\phi}{A}\right)$ w where A is the area of the magnetic poles. The force will also depend upon the effective length of the conductor in the magnetic field, i.e., ℓ on the magnitude of current flowing, i.e., I. The force developed is expressed as

$$F = BI\ \ell \text{ Newtons} \tag{1.22}$$

Here the current-carrying conductor and the magnetic fields are at right angles to each other. If, however, the conductor is inclined with the magnetic field by an angle θ, then the length of the conductor perpendicular to the magnetic field is to be considered as shown in Fig. 1.7. The length of the conductor perpendicular to the magnetic field is ℓ Sinθ. Thus, the general expression for force F is

$$F = BI\ \ell \text{ Sin}\theta \text{ Newton} \tag{1.23}$$

The direction of the force is determined by applying Fleming's left-hand-rule which is stated as:

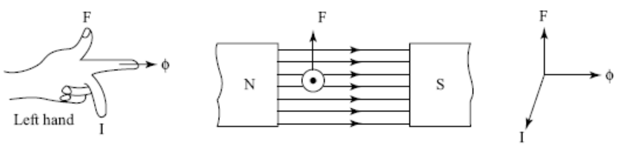


Figure 1.6 Force experienced by a conductor carrying current in a magnetic field

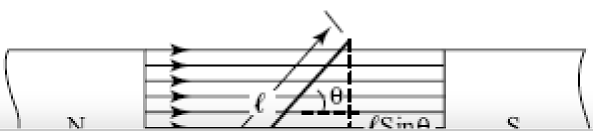


Figure 1.7 Force on a current-carrying conductor

Fleming's left-hand rule

The three fingers of the left hand are stretched as shown in Fig. 1.6. If the forefinger points towards the direction of the lines of force, and the middle finger points toward the current flowing through the conductor, then the thumb will point towards the direction of force experienced by the conductor.

1.9.5 A Current-carrying Coil Placed in a Magnetic Field

Now we will consider a coil placed in a magnetic field. A coil has two coil-sides which lie in the magnetic field. These coil sides are called conductors. Thus, a coil has two conductors. If a coil has two turns, the number of conductors will be four. See Fig. 1.8 (a and b). In Fig. 1.8 (c) has been shown a single turn coil placed in a magnetic field. The direction of current through the coil has also been shown. The direction of the magnetic field is from North pole to South pole. The direction of current in coil-side 'a' is upward, i.e., towards the observer. If we apply Fleming's left-hand rule, we find that coil-side 'a' will experience an upward' force. Similarly, by applying the same rule, we observe that coil-side 'á' will experience a downward force. The two forces acting simultaneously on the coil will develop a torque which will try to rotate the coil along an axis x-x′ in the clock-wise direction as has been shown in Fig. 1.8 (c). The coil will rotate by an angle of 90°. The North pole of the magnetic field produced by the current-carrying coil will face the stationary South pole as shown in Fig. 1.9.

The two magnetic fields get aligned as shown in Fig. 1.9 (b). If it is possible to change the direction of current in the coil when it changes its position from DD′ axis to XX′ axis, the coil will continue to develop torque in the clockwise direction. We will get continuous rotation of the coil. This is the basic principle of direct-current electric motor which will be discussed in detail in a separate chapter.

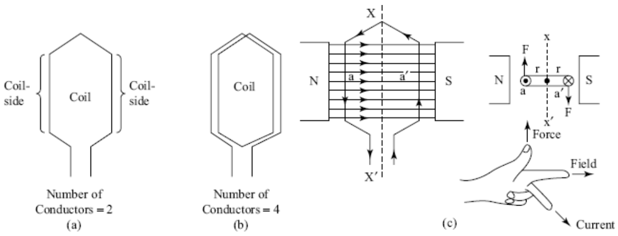


Figure 1.8 (a) A coil having one turn; (b) a coil having two turns; (c) a single turn coil carrying current is placed in a magnetic field; (d) the coil sides of the current-carrying coil in the magnetic field experience force

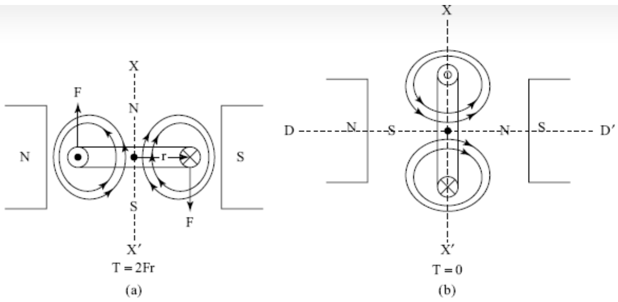


Figure 1.9 (a) A current-carrying coil in a magnetic field experiences a torque; (b) magnetic field produced by the current-carrying coil and the stationary magnetic field get aligned

1.10 LAWS OF ELECTROMAGNETIC INDUCTION

Faraday, on the basis of laboratory experiments, established that whenever there a is change in the magnetic flux linkage by a coil, EMF is induced in the coil. The magnitude of the EMF induced is proportional to the rate of change of flux linkages. Faraday's laws of electromagnetic induction are stated as:

First law: EMF is induced in a coil whenever magnetic field linking that coil is changed.

Second law: The magnitude of the induced EMF is proportional to the rate of change of flux linkage.

The rate of change of flux linkage is expressed as $N\frac{d\phi}{dt}$ where N is the number of turns of the coil linking the flux. Thus, the induced EMF, e is expressed as

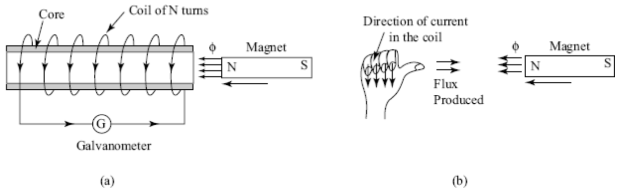
$$e = -N\frac{d\phi}{dt} \tag{1.24}$$

The minus sign is introduced in accordance with Lenz's law which is stated below.

Lenz's law: This law states that the induced EMF due to change of flux linkage by a coil will produce a current in the coil in such a direction that it will produce a magnetic field which will oppose the cause, that is the change in flux linkage.

The students may conduct an experiment in the laboratory, similar to that done by Faraday, which is explained below.

If the magnet shown in Fig. 1.10 (a) is quickly brought near the coil, there will be deflection in the galvanometer indicating EMF induced in the coil and current flow in the circuit. If the magnet is held stationary near the coil, although there is flux linking the coil, there will be no induced EMF since there is no change in the flux linkage. The induced EMF will be there only if there is increase or decrease



N is the number of turns of the ϕ the flux produced by the magnet.

Figure 1.10 Faraday's experiment on electromagnetic induction. (a) A magnet is suddenly brought near a coil; (b) determination of the direction of current produced in the coil

The direction of current flowing through the coil can be determined by applying the right-hand-grip rule. The rule is explained as follows.

Right-hand-grip rule

Hold the coil with your right hand with the thumb opposing the direction of movement of the magnet. The other four fingers will indicate the direction of current flow through the coil. This means that the current induced in the coil will produce flux in the direction of the thumb, thus opposing the flux producing the induced EMF in the coil. See Fig. 1.10 (b).

1.11 INDUCED EMF IN A COIL ROTATING IN A MAGNETIC FIELD

Now we will consider a coil rotated in a stationary magnetic field as shown in Fig. 1.11.

Here a coil, having two sides (conductors) is rotated in a uniform magnetic field as shown in Fig. 1.11. Because of the rotation of the coil in the magnetic field, flux linkage by the coil changes, i.e., the number of lines of force passing through the coil changes. Because of change of flux linkage, EMF is induced in the coil. The direction of the induced EMF in the conductors can be determined by applying *Fleming's right-hand rule* (FRHR).

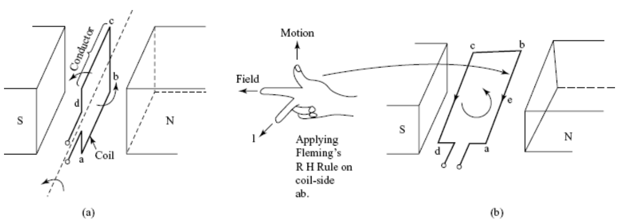


Figure 1.11 (a) EMF is induced in a coil when rotated in a magnetic field; (b) determination of direction of induced EMF

conductor. In Fig. 1.11 (b) is shown the direction of the induced EMF in coil-side ab of the rotating coil abcd. This coil side is shown going upwards. The magnetic field direction is from North pole to South pole. Hence, the direction of the induced EMF will be from b to a as determined by applying FRHR. The stronger the magnetic field is, the more will be the magnitude of EMF induced. The more the speed of rotation of the coil is, the more will be the magnitude of the EMF induced. This is because $\frac{d\phi}{dt}$ will increase if both ϕ as well as the rate of change of linkage of ϕ are changed. The magnitude of the EMF induced will also be directly proportional to the number of turns of the rotating coil, or the number of coils connected in series. The EMF induced can also be considered in terms of flux cut by a conductor (coil side) per second.

Here in Fig. 1.11, the number of poles is two. We can also have four poles, six poles, etc. When a conductor rotates in such magnetic field, it cuts the lines of force. The number of lines of force cut by a conductor in one revolution, when there are two poles, is 2ϕ Webers, where ϕ is the flux per pole. If there are say P number of poles, flux cut by a conductor in one revolution will be Pϕ Webers. If the coil makes 'n' revolutions per second, the time taken by a conductor to make one revolution will be 1/n seconds. Thus, flux cut per second will be the EMF induced, e which is

$$\text{Induced EMF, } e = \frac{\text{flux cut in 1 revolution in Wbs}}{\text{time taken in making 1 revolution in secs}}$$

or, $$e = \frac{P\phi}{1/n} \text{ Wb/sec}$$

or, $$e = P\phi n \text{ V} \qquad (1.25)$$

1.12 EMF INDUCED IN A CONDUCTOR

In terms of length of conductor, ℓ and velocity of the conductor, v in a magnetic field of flux density, B, the EMF induced in a conductor, e is calculated as

$$e = B\, l v \sin\theta \text{ V} \qquad (1.26)$$

To establish the above relation, let us consider a single conductor represented by a small circle (cross-sectional view) is moved in a magnetic field of strength B Wb/m^2 as shown in Fig. 1.12.

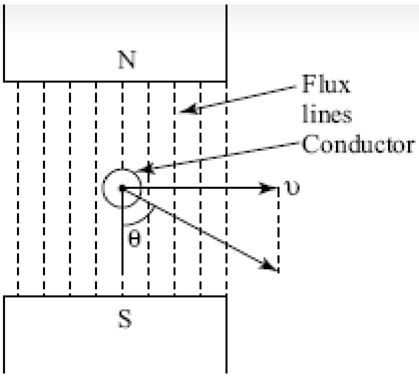


Figure 1.12 EMF induced in a conductor moving in a magnetic field

Let the conductor cut the flux at right angles by moving a distance dx meter. The area swept by the moving conductor is ℓ dx m^2. The flux density is B Wb/m^2.

$$\text{Flux cut} = \text{Flux density} \times \text{area}$$
$$= B\ \ell\ dx \text{ Webers}$$

The time taken to move a distance dx m is dt seconds.

Induced emf, e = Flux cut per second

$$\text{Therefore, } e = B\ \ell \frac{dx}{dt}\ V$$

Since $\frac{dx}{dt}$ the linear velocity v of the conductor,

$$e = Blv\ V \tag{1.27}$$

If the conductor moves in a direction making an angle θ with the direction of magnetic field as shown in Fig. 1.12, the induced EMF will be as stated earlier in eq. 1.26.

$$e = Blv \sin\theta\ V \tag{1.28}$$

1.13 DYNAMICALLY INDUCED EMF AND STATICALLY INDUCED EMF

When EMF is induced in a stationary coil by changing its flux linkage due to change in current flow through the coil, such emf is called statically induced EMF.

If a coil carries a current, flux is established around the coil. If the current is changed quickly, the flux linkage by the coil will change as shown in Fig. 1.13 (a).

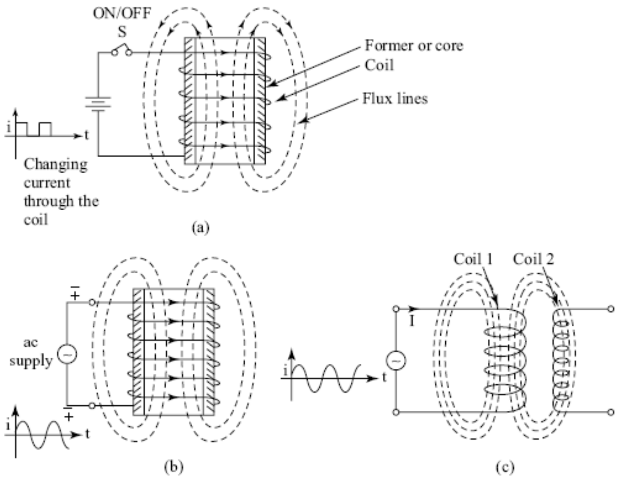


Figure 1.13 (a) Change in flux linkage in a coil due to switching ON and switching OFF of dc current; (b) change in flux linkage due to alternating current supply;(c) induced emf in coils 1 and 2 due to changing flux produced by alternating current flowing in coil 1

In Fig. 1.13 (a), a coil of certain number of turns is wound on a former, i.e., its core. Current is supplied from a battery by closing a switch S. If the switch is continuously turned on and off, flux linkage by the coil will change. The rate of change of the flux linkage will induce EMF in the coil.

A similar effect will be there if an ac supply is applied across the coil as shown in Fig. 1.13 (b). The direction of current in the coil is shown for the positive half cycle of the alternating current. The direction of current will change in every half cycle, and hence the direction of flux produced will change in every half cycle. The magnitude of current changes continuously since a sinusoidal current is flowing. This changing current will create a changing flux linkage, thereby inducing EMF in the coil in both the cases as shown in Fig. 1.13 (a) and (b). Note that in Fig. 1.13 (a), if the switch S is kept closed, a steady direct current, i.e., a constant current will flow through the coil. This constant current will produce a constant flux. There will be no change in flux linkage by the coil with respect to time, and hence no EMF will be induced in the coil. Thus, the **necessary condition for the production of induced EMF is that there should be a change in flux linkage and not merely flux linkage by a coil.**

As shown in Fig. 1.13 (c), when a second coil is brought near a coil producing changing flux, EMF will be induced in the second coil due to change in current in the first coil. This is called **mutually induced EMF.** In fact, EMF will be induced in both the coils as both the coils are linking a changing flux. However, in the second coil EMF is induced due to changing flux created by coil 1. The magnitude of the induced EMF will depend upon the rate of change of flux linkage and the number of turns of the individual coils. The induced EMF in the two coils, e_1 and e_2 will be

$$e_1 = -N_1 \frac{d\phi}{dt} \quad (1.29)$$

and

$$e_2 = -N_2 \frac{d\phi}{dt} \quad (1.30)$$

where N_1 and N_2 are the number of turns of coil 1 and coil 2, respectively.

You will study in a separate chapter how transformers are built utilizing the basic principle of mutually induced EMF.

1.15 SELF-INDUCTANCE OF A COIL

Consider a coil of N turns wound on a core of magnetic material. Let an alternating current i pass through the coil as shown in Fig. 1.14.

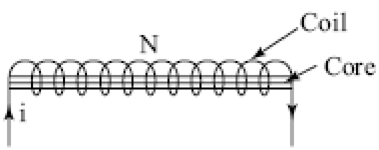


Figure 1.14 Inductance of a coil

The emf induced, e will be

$$e = -N\frac{d\phi}{dt}$$

$$\text{The flux, } \phi = B \times A = \mu HA = \mu \frac{Ni}{\ell} A = \frac{\mu NA}{\ell} i$$

where μ is the permeability of the core material; ℓ is the length of flux path; A is the area of the coil.substituting,

$$e = -N\frac{d}{dt}\left(\frac{\mu NA}{\ell}i\right)$$

$$= -\frac{\mu N^2 A}{\ell}\frac{di}{dt}$$

or, $$e = -L\frac{di}{dt} \qquad (1.31)$$

where $$L = \frac{\mu N^2 A}{\ell} \text{ Henry} \qquad (1.32)$$

L is called the coefficient of self inductance or simply self inductance of the coil.

Inductance of a coil is, therefore, dependent upon the permeability of the core material. If we put iron as the core material instead of any non-magnetic material, or air as the core, the inductance will increase many times. The (permcability), μ is expressed as

$\mu = \mu_o \mu_r$

where μ_r is the relative permeability and μ_o is the permeability of free space. The relative permeability of iron may be as high as 2000 times than that of air. Hence, an iron core coil may have an inductance value 2000 times more than that of an air-core one, other dimensions remaining the same. Again, inductance, L is inversely proportional to the length of the flux path and directly proportional to the area of the core material or the coil. Inductance is proportional to the square of the number of turns. To have an inductance of a large value, the number of turns should be high.

The inductance, L can be expressed in terms of the rate of change of the flux with respect to current flowing in the coil as

$$\text{Flux, } \phi = BA$$

$$= \mu HA$$

$$= \mu \frac{Ni}{l} A$$

For a small increment of di, let the increase of flux be dϕ. Therefore,

$$d\phi = \frac{\mu NA}{\ell} di$$

or, $$\frac{d\phi}{di} = \frac{\mu NA}{\ell}$$

or, $$N\frac{d\phi}{di} = \frac{\mu N^2 A}{\ell} = L$$

or, $$L = N\frac{d\phi}{di} \text{ Henry}$$

$\frac{d\phi}{di}$ be written as $\frac{\phi}{I}$

Therefore,

$$L = N\frac{\phi}{I} \text{ Henry}$$

Again,

$$L = \frac{\mu N^2 A}{\ell} = \frac{N^2}{\ell / \mu A} = \frac{N^2}{\text{Reluctance}} \quad (1.33)$$

where

$$\text{Reluctance} = \frac{\ell}{\mu A} \quad (1.34)$$

Remember that reluctance is the inverse of the permeability. Low reluctance will give rise to a high value of inductance. That is why in order to produce high value inductance, the number of turns should be high and the reluctance to the flux path should be low. The core should be made of high permeability material like iron.

Again,

$$L = N\frac{\phi}{I}$$

or,

$$LI = N\phi$$

Considering a small increase of i producing a small increase in ϕ as dϕ

$$Ldi = Nd\phi$$

or,

$$L\frac{di}{dt} = N\frac{d\phi}{dt} = e \quad (1.35)$$

Inductance is the property of a coil capable of inducing emf in itself due to changing current through it.

The formulae so far derived are

1. Force on a current-carrying conductor in a magnetic field

   F = BIℓ Newtons.

   If the conductor is inclined at an angle θ with the magnetic field,

   F = BIℓ Sinθ Newtons.

2. Induced EMF in a coil where there is change of flux linkage or change in current,

   $$e = N\frac{d\phi}{dt} \text{ V; } e = L\frac{di}{dt}$$

3. Induced EMF in a conductor rotating in a magnetic field,

   e = Pϕn V

4. Induced EMF in a conductor moving in a magnetic field in a perpendicular direction,

   $e = B\ell v$ V

   where B is the flux density in Wb/m^2, ℓ is the length of the conductor in m and v is the velocity in m/sec.

   If the conductor is moving at an angle of θ with the magnetic field, the induced EMF is

   $e = B\ell v \sin\theta$ V.

5. Induced EMF in a coil,

$$e = -N\frac{d\phi}{dt}$$

$$e = -L\frac{di}{dt}$$

$$L = \mu\frac{N^2 A}{\ell} = \frac{N^2}{\ell / \mu A} = \frac{N^2}{\text{Reluctance}}$$

$$L = N\frac{d\phi}{di} = N\frac{\phi}{I} \quad \text{(assuming linear magnetization)}$$

$$L = \frac{N\phi}{I}. \quad \text{If } \phi \text{ is equal to 1 Wb-turn and I is 1 ampere, then}$$

$$L = \frac{1\,\text{Wb-turn}}{1\,\text{Ampere}} = 1\,\text{Henry} \qquad (1.36)$$

Thus, we can say that a coil has an inductance of 1 Henry if a current of 1 Ampere flowing through the coil produces a flux linkage of 1 Wb-turn.

1.16 MUTUAL INDUCTANCE

Consider two coils having N_1 and N_2 number of turns placed near each other as shown in Fig. 1.15. Let a changing current, i_1, flow through coil 1. The flux produced by i_1 in N_1 is ϕ_1. Since coil 2 is placed near coil 1, a part of the flux produced by coil 1 will be linked by coil 2. Let flux ϕ_2 linked by coil 2 is $\phi_2 = K_1 \phi_1$ where $K_1 \leq 1$.

If magnetic coupling between the two coils is very tight, i.e., very good, the whole flux produced by coil 1 will link the coil 2, in which case the coefficient of the coupling K_1 will be 1. The induced EMF in coil 2 is e_2.

$$e_2 = N_2\frac{d\phi_2}{dt} = N_2\frac{d(K_1\phi_1)}{dt} = N_2 K_1\frac{d\phi_1}{dt} \qquad \text{(i)}$$

Flux,

$$\phi_1 = \frac{\text{mmf}}{\text{Reluctance}} = \frac{N_1 i_1}{\ell_1 / \mu A_1} = \frac{N_1 \mu A_1}{\ell_1} i_1 \qquad \text{(ii)}$$

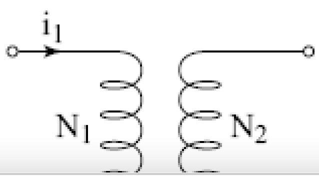


Figure 1.15 Mutual inductance of two coils

From (i) and (ii),

$$e_2 = N_2K_1\frac{d}{dt}\left(\frac{N_1\mu A_1 i_1}{\ell_1}\right) = \left(\frac{K_1N_1N_2\mu A_1}{\ell_1}\right)\frac{di_1}{dt}$$

or,

$$e_2 = M\frac{di_1}{dt}$$

where $M = \frac{K_1N_1N_2\mu A_1}{\ell_1}$, is called the mutual inductance of the two coils. (iii)

Similarly, if we calculate the induced EMF in coil 1, due to change in current in coil 2, we can find the induced EMF e_1 in coil 1 as

$$e_1 = \frac{K_2N_1N_2\mu A_2}{\ell_2}\frac{di_2}{dt}$$

where

$$M = \frac{K_2N_1N_2\mu A_2}{\ell_2} \quad \text{(iv)}$$

Now, multiplying the expression for M as in (iii) and (iv) above,

$$M^2 = \frac{K_1K_2N_1N_2N_1N_2\mu A_1\mu A_2}{\ell_1\ell_2}$$

or,

$$M^2 = K_1K_1\frac{\mu N_1^2A_1}{\ell_1}\frac{\mu N_2^2A_2}{\ell_2}$$

or,

$$M^2 = K_1K_2L_1L_2$$

Therefore,

$$M = \sqrt{K_1K_2}\sqrt{L_1L_2} = K\sqrt{L_1L_2} \quad (1.37)$$

where

$$K = \sqrt{K_1K_2}$$

Again from (iii),

$$M = K_1N_2\frac{N_1}{\ell_1/\mu A_1} \quad \text{(v)}$$

Flux, ϕ_1 from (ii) is,

$$\phi_1 = \frac{\text{mmf}}{\text{Reluctance}} = \frac{N_1I_1}{\ell_1/\mu A_1}$$

or,

$$\frac{\phi_1}{I_1} = \frac{N_1}{\ell_1/\mu A_1} \quad \text{(vi)}$$

From (iii) and (vi),

$$M = \frac{K_1 N_1 N_2 \ \mu A_1}{\ell_1}$$

$$= K_1 N_2 \frac{N_1}{(\ell_1/\mu A_1)}$$

$$M = K_1 N_2 \frac{\phi_1}{I_1}$$

$$M = N_2 \frac{\phi_2}{I_1} (\because \phi_2 = K_1 \phi_1)$$

Thus, $$M = \frac{N_2 \phi_2}{I_1} = \frac{\text{flux linkage in coil 2}}{\text{changing current in coil 1}} \quad (1.38)$$

From eq. (1.38) we can define the mutual inductance M between two coils as the flux linkage in one circuit due to change per unit of current in the other circuit.

Similarly, considering current change in the second coil

$$M = N_1 \frac{K_2 \phi_2}{I_2}$$

or, $$M = N_1 \frac{\phi_1}{I_2} (\because \phi_1 = K_2 \phi_2)$$

Thus, $$M = \frac{N_1 \phi_1}{I_2} = \frac{\text{flux linkage in coil 1}}{\text{changing current in coil 2}} \quad (1.39)$$

1.17 INDUCTANCE OF COILS CONNECTED IN SERIES HAVING A COMMON CORE

We have two coils having self-inductance L_1 and L_2 connected in series. In Fig. 1.16 (a), they produce flux in the same direction, and in Fig.1.16 (b), the connection is such that they produce flux in the opposite directions.

Since the two coils are connected in series, the same current flows through them.

If there is a change in current di amperes in time dt seconds, the EMF induced in coil 1 due to its self-inductance L_1 is

$$e_1 = -L_1 \frac{di}{dt} \quad \text{(i)}$$

Similarly, the EMF induced in coil 2 due to its self-inductance, L_2 is

$$e_2 = -L_2 \frac{di}{dt} \quad \text{(ii)}$$

$$e_{12} = -M\frac{di}{dt} \qquad \text{(iii)}$$

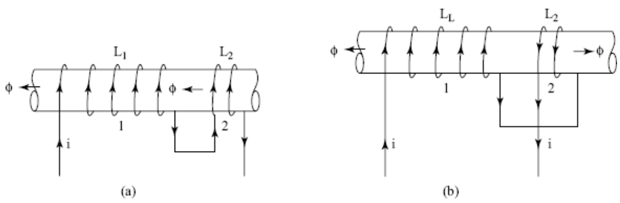


Figure 1.16 Coils connected in series in (a) commulatively; in (b) differentially

EMF induced in coil 2 due to change in current in coil 1 is

$$e_{21} = -M\frac{di}{dt} \qquad \text{(iv)}$$

Now let the total equivalent inductance of the single circuit comprising coil 1 and coil 2 as they are connected as in Fig. 1.16 (a) be L_e

The EMF induced in the whole circuit will, therefore, be

$$e = -L_e\frac{di}{dt} \qquad \text{(v)}$$

Thus, equating the expression for e in (iv) with the total EMFs as in (i), (ii), (iii), and (iv):

$$-L_1\frac{di}{dt} - L_2\frac{di}{dt} - M\frac{di}{dt} - M\frac{di}{dt} = -L_e\frac{di}{dt}$$

Therefore,

$$L_e = L_1 + L_2 + 2M \qquad \text{(vi)}$$

When the coils are differentially connected as in Fig. 1.16 (b), the EMF induced in coil 1 due to di in time dt in coil 2, i.e., $M\frac{di}{dt}$ in opposition to the EMF induced in coil 1 due to its self-inductance. Similar is the case of the EMF induced in coil 2 due to mutual inductance. Thus, for the differentially connected coil

Thus, the total inductance of an inductively coupled series-connected coil circuit can be expressed as

$$L_T = L_1 + L_2 \pm 2M \tag{1.40}$$

Dot convention is used to determine the sign of induced voltage $M\dfrac{di}{dt}$.

If we use dot convention, it will not be required to know the way the coils have been actually wound.

Example 1.11 The total inductance of two coils connected in series cumulatatively is 1.6 H and connected differentially is 0.0.4 H. The self inductance of one coil is 0.6 H. Calculate (a) the mutual inductance and (b) the coupling coefficient.

Solution:

We know, $\quad L_T = L_1 + L_2 \pm 2M$

Substituting the given values

$$L_1 + L_2 + 2M = 1.6 \quad \text{(i)}$$

and $\quad L_1 + L_2 - 2M = 0.4 \quad$ (ii)

From (i) and (ii)

$$4M = 1.2$$

or, $\quad M = 0.3\ \text{H}$

given, $\quad L_1 = 0.6$

Therefore, $\quad 0.6 + L_2 + 2 \times 0.3 = 1.6$

or, $\quad L_2 = 1.6 - 1.2 = 0.4\ \text{H}$

$$L_1 = 0.6\ \text{H},\ L_2 = 0.4\ \text{H},\ M = 0.3\ \text{H}$$

We know, $\quad M = K\sqrt{L_1 L_2}$

or, $\quad K = \dfrac{M}{\sqrt{L_1 L_2}}$

Substituting values, $\quad K = \dfrac{0.3}{\sqrt{0.6 \times 0.4}} = 0.612$

1.18 ENERGY STORED IN A MAGNETIC FIELD

Let us consider a coil supplied with an alternating voltage *v* due to which an alternating current flows through the coil. When current increases from its zero value, the magnetic field starts increasing and reaches its maximum value when current reaches its maximum value. When current starts decreasing, the field goes on decreasing and gradually becomes zero. Then, in the negative cycle if the curnetic field is stored and when the field collapses, the same energy is returned to the supply source. As such, no energy is consumed by the purely inductive coil. Therefore, energy stored is equal to the energy supplied.

Energy stored, $$W = vi\,dt$$

Induced EMF in the coil, $$e = -L\frac{di}{dt}$$

This induced EMF opposes the applied voltage from which it is produced. This is due to Lenz's law, so that

$$e = -v \text{ or, } v = -e$$

Thus,

$$W = (-e)i\,dt \quad \text{and} \quad e = -L\frac{di}{dt}$$

for a current change from 0 to I

$$\begin{aligned} E &= \int_0^I (-e)i\,dt \\ &= \int_0^I L\frac{di}{dt}i\,dt \\ &= \int_0^I Li\,di \\ &= L\frac{I^2}{2} = \frac{1}{2}LI^2 \text{ Joules} \end{aligned}$$

Energy stored in a coil of inductance L is

$$W = \frac{1}{2}LI^2 \text{ Joules} \tag{1.41}$$

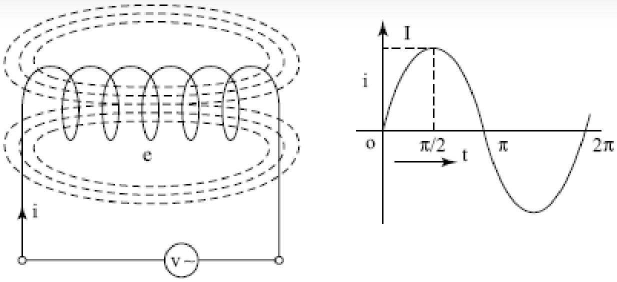


Figure 1.17 Magnetic field energy

Example 1.12 A conductor of length 0.5 m is placed in a magnetic field of strength 0.5 Wb/m^2. Calculate the force experienced by the conductor when a current of 50 A flows through it. If the force moves the conductor at a velocity of 20 m/sec, calculate the EMF induced in it.

Solution:

Force, F on a current-carrying conductor placed in a magnetic field is given as

F = B I ℓ Newton

Substituting the values,

F = 0.5 Wb/m^2 × 50 × 0.5 m
= 12.5 N

Induced EMF, e in a conductor moving in a magnetic field is given as

e = B l *v* V

Substituting the given values,

e = 0.5 Wb/m^2 × 0.5 m × 20 m/sec
= 5 Wb/sec = 5 V

Example 1.13 An iron-cored toroidal coil has 100 turns. The mean length of the flux path is 0.5 m and the cross-sectional area of the core is 10 cm^2. Calculate the inductance of the coil. Assume relative permeability of iron as 2000. Also calculate the induced emf in the coil when current of 5A is reversed in 10 ms.

Solution:

The expression for inductance in terms of its parameters is

$$L = \frac{\mu N^2 A}{\ell}$$

where,

$$\mu = \mu_o \mu_r$$

Current in the coil is changed from +5 A to −5 A in 10 × 10^{-3} secs.
Total change of current is 10 A.

$$L = \frac{4\pi \times 10^{-7} \times 2000 \times 100 \times 100 \times 10}{10000 \times 0.5}$$

$$= \frac{4\pi \times 10^{-4} \times 2 \times 10}{0.5}$$

$$= 16 \times 3.14 \times 10^{-3}\,H$$

$$= 50.24 \times 10^{-3}\,H$$

$$= 50.24\,mH$$

Induced EMF, $e = -L\dfrac{di}{dt}$

Average value of induced EMF, $E = L\,\dfrac{\text{change in current}}{\text{time taken}}$

$$E = L\frac{2I}{t}$$

Substituting values, $E = 50.24 \times 10^{-3}\,\dfrac{2 \times 5}{10 \times 10^{-3}}$

$$= 50.24\,V$$

Example 1.14 There is mutual magnetic coupling between two coils of number of turns 500 and 2000, respectively. Only 50% of the flux produced by the coil of 500 turns is linked with the coil of 1000 turns. Calculate the mutual inductance of the two coils. Also calculate the EMF induced in the coil of 1000 turns when current changes at the rate of 10A/second in the other coil. The self-inductance of the coil of 500 turns in 200 mH.

Solution:

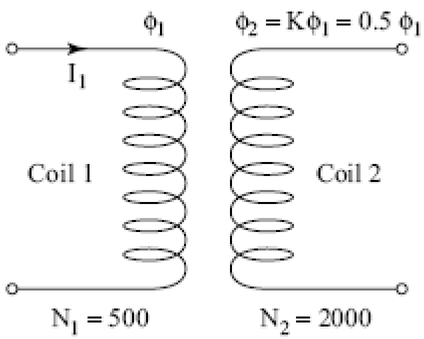


Figure 1.18

Mutual inductance,

$$M = \frac{\text{flux linkage in coil 2}}{\text{current change in coil 1}}$$

$$= N_2 \frac{K\phi_1}{I_1} = 2000 \times \frac{0.5 \times \phi_1}{I_1} = \frac{1000\phi_1}{I_1}$$

$$L_1 = \frac{N_1 \phi_1}{I_1}$$

or, $$\frac{\phi_1}{I_1} = \frac{L_1}{N_1} = \frac{200 \times 10^{-3}}{500}$$

$$M = 1000 \times \frac{\phi_1}{N_1} = \frac{1000 \times 200 \times 10^{-3}}{500}$$

or, $$M = 400 \times 10^{-3}\ \text{H}$$

Induced EMF in the second coil, e_2 is

$$e_2 = M \frac{di}{dt}$$
$$= 400 \times 10^{-3} \times 10$$
$$= 4\ \text{V}$$

Example 1.15 A current of 5 A flowing through a coil of 500 turns produces a flux of 1 mWb. Another coil is placed near this coil and current in this coil is suddenly reversed in 10 ms. As a result, the EMF induced in the second coil is measured as 50 V. Calculate self and mutual inductance of the coils assuming a coefficient of coupling as 60 per cent.

Solution:

$$e_2 = M \frac{di_1}{dt}$$

or, $$50 = M \frac{5+5}{10 \times 10^{-3}}$$ [+5 A current has been changed to −5 A]

or, $$M = \frac{50 \times 10 \times 10^{-3}}{10}$$

or, $$M = 50 \times 10^{-3}\ \text{H}$$

Self-inductance of coil 1 is

$$L_1 = \frac{N_1 \phi_1}{I_1} = \frac{500 \times 1 \times 10^{-3}}{5} = 10 \times 10^{-3}\ \text{H}$$

$$50 \times 10^{-3} = 0.6 \sqrt{10 \times 10^{-3} \times L_2}$$

or,
$$L_2 = 694.4 \times 10^{-3} \text{ H}$$

Example 1.16 Two coils of number of turns $N_1 = 1000$ and $N_2 = 400$, respectively, are placed near each other. They are magnetically coupled in such a way that 75 per cent of the flux produced by the one of 1000 turns links the other. A current of 6 A produces a flux of 0.8 mWb in N_1 and the same amount of current produces a flux of 0.5 mWb in the coil of N_2 turns. Determine L_1, L_2, M, and K for the coils.

Solution:

$$L_1 = N_1 \frac{\phi_1}{I_1} = 1000 \times \frac{0.8 \times 10^{-3}}{6} = 0.133 \text{ H}$$

$$L_2 = N_2 \frac{\phi_2}{I_2} = 400 \times \frac{0.5 \times 10^{-3}}{6} = 0.033 \text{ H}$$

$$M = N_2 \frac{K_1 \phi_1}{I_1} = 400 \times \frac{0.75 \times 0.8 \times 10^{-3}}{6}$$

$$= 0.04 \text{ H}$$

Using the relation, $M = K \sqrt{L_1 L_2}$

substituting values,

$$0.04 = K \sqrt{0.133 \times 0.033}$$

or,
$$K = \frac{0.04}{0.066} = 0.606$$

So,

Self-inductance of coil 1 = 0.133 H

Self-inductance of coil 2 = 0.033 H

Mutual inductance of the coils = 0.04 H

Coefficient of coupling = 0.606

1.19 ELECTRICAL CIRCUIT ELEMENTS

Resistors, inductors, and capacitors are the three basic circuit parameters or circuit components of any electrical network. Resistors can be wire-wound type or carbon-moulded type. When current

A resistor is an element that dissipates energy as heat when current flows through it.

Inductors are made of a coil having a number of turns. The core of the coil may be air or a magnetic material, which is placed inside the coil. When the coil is wound on an iron core, the inductor formed is called an iron-core inductor coil. Inductance of an inductor is directly proportional to the square of the number of turns of the coil used. Inductor stores energy because of current flowing through it.

A capacitor consists of two conductors or conducting plates between which a dielectric is placed. The capacitance of a capacitor is its ability to store electric charge. Different types of capacitors are available. They are named according to the dielectric placed between the conductors. Common types of capacitors are air, mica, paper, ceramic, etc.

1.19.1 Resistors

Wire-wound resistors are made of wires of constantan, manganin or nichrome wound on a ceramic tube. These resistances are available in ranges varying from a fraction of an ohm to thousands of ohms.

The power rating also varies from a fraction of a Watt to few kiloWatts. While specifying a resistance, both resistance value and power dissipating value must be mentioned. Electronic circuits require resistors of accurate values. The value of resistors used in electronic circuits is quite high, of the order of kilo ohms. Since carbon has high resistivity, carbon resistors are made with copper leads. Their power rating varies from a fraction of a Watt to several Watts. Color code is used to indicate the value of such resistors.

1.19.2 Inductors

The ability of a coil to induce EMF in itself when the current through it changes is called its inductance. The unit of inductance is Henry. 1 Henry of inductance causes 1 Volt to be induced when current changes at the rate of 1 Ampere per second:

$$e = L\frac{di}{dt}$$

or,

$$L = \frac{e}{di/dt}$$

where L is in Henry, e is in Volt, and $\frac{di}{dt}$ is in Ampere per second.

When steady direct current flows through an inductor, it will not affect the circuit as there is no change in current. Inductors are of two types viz air-core type and iron-core type. Inductors are also called chokes. Inductors are available in all current ranges. Air-core inductors are wound on bakelite or cardboard rods and are extensively used in electronic circuits in millihenry and microhenry ranges. High-value inductors are made of iron core. They are mainly

1.19.3 Capacitors

A capacitor, in its simplest form, consists of two thin parallel plates of conducting material separated by a dielectric material. A capacitor is capable of storing charge when a voltage is applied across the capacitor plates. If a voltage source, say a battery, is connected across the two plates of a parallel plate capacitor as shown in Fig. 1.19, electrons from the negative terminal of the voltage source accumulate on plate A of the capacitor. The other plate B loses electrons as it is connected to the positive terminal of the source of voltage. This way, the excess electrons produce negative charge on one side of the capacitor while the opposite side will have positive charge. The dielectric material placed in between the plates hold the charge because the free electrons cannot flow through an insulator (i.e., the dielectric material like air, paper, or mica). Storage of charge by a capacitor means that the charge remains in place even after the voltage source is disconnected. Capacitance of a capacitor is the ability to store charge. Charging and discharging are the two main effects of capacitors. When a voltage is applied, there is accumulation of charge in the capacitor and as a result voltage is built up across the terminals of the capacitor. This is called charging of the capacitor. The capacitor voltage becomes equal to the applied voltage when the capacitor is fully charged. The voltage across the capacitor remains even after the voltage source is disconnected. The capacitor discharges when a conducting path is provided across the plates without any applied voltage connected.

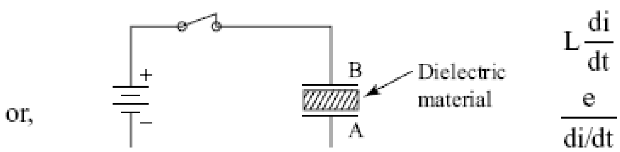


Figure 1.19 A capacitor stores charge in the dielectric material placed between the conducting plates

The more the charging voltage is, the more is the accumulation of charge in the capacitor. The amount of charge, Q stored in a capacitor is, therefore, proportional to the charging voltage, V. A capacitor with a large area of the parallel plates can store more charge. Capacitance of a capacitor also depends on the distance between the plates and the type of dielectric used between the plates. A large capacitor, obviously, will store more charge. Thus, we can write

$Q = CV$ Coulombs

where Q is the charge stored in Coulombs, V is the voltage applied across the plates, and C is the capacitance of the capacitor in Farads. The capacitance of a parallel plate capacitor is expressed as

$$C = \in \frac{A}{d} \tag{1.42}$$

The term absolute permittivity is expressed as

$\epsilon = \varepsilon_0 \varepsilon_r$

where ε_0 is the permittivity constant of vacuum and ε_r is the relative permittivity of the dielectric material placed between the two plates.

The value of ε_0 has been calculated experimentally as 8.85×10^{-12} Farad per meter.

Therefore, the capacitance of a parallel plate capacitor can be expressed as

$$C = \epsilon \frac{A}{d} = \varepsilon_r \frac{A}{d} \times 8.85 \times 10^{-12} \text{ Farad}$$

1.20 ENERGY STORED IN A CAPACITOR

We have known that when a capacitor is switched on to a dc supply, the charge q can be expressed as q = Cv, where at any instant q is the change, v is the potential difference across the capacitor plates, and C is the capacitance of the capacitor.

Potential difference of v volts across the capacitor means v Joules of work has to be done in transferring 1 Coulomb of change from one plate to the other. If a small charge dq is transferred then the work done dw can be expressed as

dw = vdq = Cvdv

The total work done in raising the potential of the capacitor to the supply voltage of V volt can be expressed as

$$\begin{aligned} W &= \int_0^V dW \\ &= \int_0^V Cv\,dv \\ &= C\left[\frac{\upsilon^2}{2}\right]_0^V \end{aligned}$$

$$W = \frac{1}{2}\ CV^2$$

This work done is stored in the electrostatic field set up between the plates of the capacitor in the form of energy. Thus, the energy stored, E is expressed as

$$\text{Energy stored} = \frac{1}{2}\ CV^2 \text{ Joules}$$

1

Example 1.17 The current through a 100 mH inductor charges from 0 to 200 mA in 4 μs. What is the value of the induced EMF in the inductor or the choke?

Solution:

$$e = L\frac{di}{dt} = 100 \times 10^{-3} \times \frac{200 \times 10^{-3}}{4 \times 10^{-6}}$$
$$= 5000\ \mathrm{V}$$
$$= 5\ \mathrm{kV}$$

It is observed that a high voltage is induced in the choke because of very fast change of current flow through it. In a tube light circuit, a high voltage is induced in the choke by the same method and is used to ionize the gas inside the tube light, and thus start the tube light.

Example 1.18 Self inductances of two coils are L_1 = 2 H and L_2 = 8 H. The coil L_1 produces a magnetic flux of 80 mWb of which only 60 μWb are linked with coil L_2. Calculate the mutual inductance of the two coils.

Solution:

The coefficient of coupling, K is given as

$$K = \frac{\text{mutual flux linkage between } L_1 \text{ and } L_2}{\text{flux produced by } L_1}$$
$$= \frac{60 \times 10^{-6}\ \mathrm{Wb}}{80 \times 10^{-6}\ \mathrm{Wb}} = 0.75$$

Mutual inductance M is calculated as

$$M = K\sqrt{L_1 L_2} = 0.75\sqrt{2 \times 8} = 3\,\mathrm{H}$$

Example 1.19 Calculate the capacitance of a capacitor made of two parallel plates of 3 m^2 having a distance between the plates of 1 cm. The dielectric is air between the plates.

$$C = \varepsilon_0 \varepsilon_r \frac{A}{d} = 8.85 \times 10^{-12} \times 1 \times \frac{3}{10^{-2}}\,\mathrm{F}$$

Note that although the area of the plates is large, the value of capacitance is very small. Instead of air as the dielectric, if we place mica or paper between the plates, capacitance will increase. If we also reduce the distance between the plates, the capacitance will increase.

Example 1.20 A 25 microfarad capacitor is switched on to a time varying voltage source. The voltage wave is such that voltage increases at the rate of 10 V per second. Calculate the charge accumulated in the capacitor at an elapse of 1 second and the amount of energy stored in the capacitor.

Solution:

$$\text{Charge, } q = CV = 25 \times 10^{-6} \times 10$$
$$= 250 \times 10^{-6} \text{ Coulomb}$$
$$\text{Energy stored, } W = \frac{1}{2} CV^2 = \frac{1}{2} \times 25 \times 10^{-6} \times 10^2$$
$$= 12.5 \times 10^{-4} \text{ Joules}$$

1.21 CAPACITOR IN PARALLEL AND IN SERIES

When we connect two capacitors in parallel, the plate areas are added. The total capacitance, therefore, gets added up. When capacitances C_1, C_2, C_3, etc. are connected in parallel, the total capacitance C_r becomes equal to

$C_r = C_1 + C_2 + C_3 + \ldots$

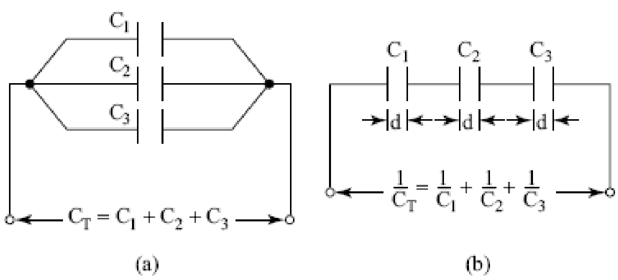


Figure 1.20 (a) Equivalent of capacitors connected in parallel; (b) equivalent of capacitors connected in series

This is shown in Fig. 1.20 (a).

Series connection of capacitors, as shown in Fig. 1.20 (b), is equivalent to increasing the effective distance between the plates or the thickness of the dielectric used. The combined capacitance is less than the individual value.

The value of a capacitor is always specified in either microfarad or picofarad. There are a variety of ways in which manufacturers indicate the value of a capacitor.

2. Charge in motion is called current. Explain with the help of atomic theory.
3. Distinguish between conductors, semiconductors, and insulators.
4. Distinguish between Work, Power, and Energy.
5. Differentiate between temperature coefficient of resistance and specific resistance.
6. Distinguish between an electric field and a magnetic field.
7. Define the following terms: Volt, Ampere, Ohm.
8. Explain why two parallel current-carrying conductors attract each other when current in them flow in the same direction.
9. State Fleming's Right-Hand Rule.
10. Explain that the EMF induced in a coil depends upon the flux and the speed of rotation of the coil.
11. Distinguish between statically induced EMF and dynamically induced EMF.
12. Explain why an iron-core coil will have more inductance than an air-core coil of the same number of turns.
13. What is the meaning of coefficient of coupling between two coils? When is this value equal to unity and equal to zero?
14. What are Faraday's laws of electromagnetic induction?
15. What is the Lenz's law? Give an example.
16. What is the magnitude of force experienced by a current-carrying conductor placed in a magnetic field?
17. How do you determine the direction of force developed in a current-carrying conductor placed in a magnetic field?
18. What are the factors on which inductance of a coil depends?
19. Why does the inductance of a coil increase if the core has a magnetic material instead of air?
20. Derive the following expression for self-inductance of a coil

$$L = \frac{\mu N^2 A}{\ell} \text{ Henry.}$$

21. You have to make an inductance of high value. How will you proceed?
22. What is Fleming's Right-Hand Rule? Where is it used?
23. What rule do you apply to determine the direction of force on a current-carrying conductor placed in magnetic field?
24. What is the magnitude of force on a current-carrying conductor placed in a magnetic field?
25. Show that the energy stored in a magnetic field produced by an inductor is $\frac{1}{2} LI^2$.
26. Distinguish between self-inductance and mutual inductance.
27. Explain why inductance of a coil increases if an iron piece forms its core instead of air or any non-magnetic material.
28. Establish the relation, $M = K\sqrt{L_1 L_2}$ for two adjacent coils linking flux.
29. On what factors does the reluctance of a magnetic material depend?

32. When capacitors are connected in parallel, their equivalent capacitance is increased. Explain why?
33. Explain why capacitors are called energy storage devices.
34. What is the meaning of relative permittivity or dielectric constant? What is it's unit?
35. Write three formulae of electrical power.
36. Prove that 1 kWh is equal to 3.6×10^{6} Joules.
37. The most important property of a capacitor is its ability to block steady dc voltage while passing ac signals, explain.
38. Define the Farad unit of capacitance.
39. How is energy stored in a capacitor? On what factors does it depend?
40. What are the physical factors that affect the capacitance of a capacitor?
41. Two coils of $N_1 = 50$ and $N_2 = 500$ turns, respectively, are wound side by side on an iron ring of cross-sectional area of 50 cm^2 and mean length of 120 cm. Calculate the mutual inductance between the coils, self inductance of the coils, and the coefficient of coupling assuming permeability of iron as 1000.

    [Ans 0.13 H, 0.013 H, 1.3 H, 1.0]

42. Two coils of $N_1 = 1500$ and $N_2 = 200$ turns are wound on a common magnetic circuit of reluctance 25×10^{4} AT/Wb. Calculate the mutual inductance between the coils.

    [Ans 1.2 H]

43. Two coils have a mutual inductance of 400 mH. Calculate the EMF induced in one coil when current in the second coil varies at a rate of 6000 Amperes per second.

    [Ans 2.4 V]

44. Two similar coils have a coupling coefficient of 0.4. When the coils are connected in series cumulatively, the total inductance becomes equal to 140 mH. Calculate the self-inductance of each coil.

    [Ans 50 mH]

45. Two coils when connected in series cumulatatively show to have a total inductance of 2.4 H and when connected in series but differentially show a total inductance of 0.4 H. The inductance of one coil when isolated is calculated as equal to 0.8 H. Calculate (a) the mutual inductance and (b) the coefficient of coupling between the coils.

    [Ans M = 0.5 H, 0.75]

46. Calculate the inductance of a coil having 100 turns wound

[Ans L = 50.24 mH]

47. A conductor of length 25 cm is placed in a uniform magnetic field of strength 0.5 Wb/m^2. Calculate the EMF induced in the conductor when it is moved at the rate of 10 m/sec (a) parallel to the magnetic field, (b) perpendicular to the magnetic field.

[Ans (a) 0 V; (b) 1.25 V]

Multiple Choice Questions

1. The number of electrons per Coulomb is equal to
    1. 1.602×10^{-19}
    2. 6.28×10^{18}
    3. 1.602×10^{18}
    4. 6.28×10^{-19}.
2. In insulators the outermost orbit of their atoms is filled with
    1. 4 electrons
    2. 8 electrons
    3. 1 electron
    4. 18 electrons.
3. In the atoms of semiconducting materials like silicon and germanium the outermost orbit has
    1. 1 electron
    2. 2 electrons
    3. 8 electrons
    4. 4 electrons.
4. Which of the following expressions is incorrect?
    1. Current, $I = \frac{q}{t}$
    2. Charge = current × time
    3. $R = \rho \frac{A}{l}$
    4. Volt = joules per Coulomb.
5. Which is the following expressions does not represent power?
    1. $I^2 R$
    2. $\frac{V^2}{R}$
    3. VI
    4. $\frac{V^2}{I}$.
6. Which of the following is not the unit of power?
    1. Joules/second
    2. Watt-hour
    3. KW
    4. Volt-Ampere
7. A conductor of length ℓ and diameter d has resistance of R ohms. If the diameter is reduced to one-third and length increased by three times, the resistance of the conductor will be
    1. 3 R
    2. 6 R

1. $e = -N\frac{d\phi}{dt}$
2. $L = -N\frac{d\phi}{dt}$
3. $L = \frac{\mu N^2 A}{\ell}$.
4. $e = L\frac{di}{dt}$.

9. Which of the following expressions is incorrect?
    1. $C = \frac{\varepsilon d}{A}$
    2. $C = \frac{Q}{V}$
    3. $Q = \int idt$
    4. $C = \frac{\varepsilon A}{d}$.
10. Inductance of an air-core coil will increase if the core is made of
    1. copper
    2. aluminium
    3. iron
    4. porcelain.
11. Which of the following statements is not true?
    1. Inductance of a coil will increase by four times if the number of terms eq is doubled
    2. inductance of a coil will increase if the area of cross section of the coil, i.e., the flux path is increased
    3. inductance of a coil will increased if the length of flux path is increased
    4. inductance of a coil will increase if the core is made up of material having higher permeability.
12. The direction of the induced EMF in the coil sides of a coil rotating in a magnetic field can be determined by applying
    1. Fleming's left-hand rule
    2. Right-hand-grip rule
    3. Fleming's-left-hand rule
    4. Cork screw rule.
13. Which of the following is not the unit of energy?
    1. kWh
    2. Joules/second
    3. Watt-hour
    4. Joules.
14. Self-inductance of two magnetically coupled coils are 8 H and 2 H, respectively. What coefficient of coupling will make their mutual inductance equal to 4 H?
    1. K = 0.5
    2. K = 0.25
    3. 0.1
    4. 1.0.
15. Which of the following eq. is incorrect with respect of increase in resistance with increase in temperature of a conducting material?
    1. $R_t = R_0(1 + \alpha t)$

4. $R_2 = R_1[1-\alpha_1(t_2-t_1)]$.

Answers to Multiple Choice Questions

1. (b)
2. (b)
3. (d)
4. (c)
5. (d)
6. (b)
7. (d)
8. (b)
9. (a)
10. (c)
11. (c)
12. (c)
13. (b)
14. (d)
15. (d)

PREV
Preface

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