

Simplification of Context-free Grammar

Simplification of Context Free Grammar

Reduction of CFG

In CFG, sometimes all the production rules and symbols are not needed for the derivation of strings. Besides this, there may also be some NULL productions and Unit productions. Elimination of these productions and symbols is called Simplification of CFG.

Simplification consists of the following steps:

- ① Reduction of CFG.
- ② Removal of unit productions.
- ③ Removal of NULL production.

Reduction of CFG

⇒ CFG are reduced in two phases:

Phase 1: Derivation of an equivalent grammar G_1' from the CFG G_1 , such that each variable derives some terminal string.

Derivation Procedure:

Step 1: Include all symbols w_1 , that derives some terminal and initialize $i=1$.

Step 2: Include symbols w_{i+1} , that derives w_i .

Step 3: Increment i and repeat Step 2, until $w_{i+1} = w_i$.

Step 4: Include all production rules that have w_i in it.

Phase 2: Derivation of an equivalent grammar G_1'' from the CFG, G_1' , such that each symbol appears in a sentential form.

Derivation Procedure:

Step 1: Include the Start Symbol in Y_1 and initialize $i=1$.

Step 2: Include ~~the~~ all symbols Y_{i+1} , that can be derived from Y_i and include all production rules that have been applied.

Step 3: Increment i and repeat Step 2, until $Y_{i+1} = Y_i$.

- Example -

\Rightarrow Find a reduced grammar equivalent ~~the~~ to the grammar G , having production rules

$$P: \underline{S \rightarrow AC \mid B, A \rightarrow a, C \rightarrow c \mid BC, E \rightarrow aA \mid e}$$

Phase 1

$$T = \{a, c, e\}$$

$$W_1 = \{A, C, E\}$$

$$W_2 = \{A, C, E, S\}$$

$$W_3 = \{A, C, E, S\}$$

$$\underline{G'} = \{(A, C, E, S), \{a, c, e\}, P, \{S\}\}$$

$$\underline{P}: S \rightarrow AC, A \rightarrow a, C \rightarrow c, E \rightarrow aA \mid e$$

Phase 2:

$$Y_1 = \{S\}$$

$$Y_2 = \{S, A, c\}$$

$$Y_3 = \{S, A, C, a, c\}$$

$$Y_4 = \{S, A, C, a, c\}$$

$$\underline{G''} = \{(A, c, S), \{a, c\}, P, \{S\}\}$$

$$\underline{P}: S \rightarrow AC, A \rightarrow c, C \rightarrow c. \quad \checkmark$$

Simplification of CFG

② Removal of unit productions:

Any production rule of the form $A \rightarrow B$ where $A, B \in$
Non-Terminals is called Unit Production.

Procedure for removal

Step 1: To remove $A \rightarrow B$, add production $A \rightarrow \underline{x}$ to the grammar rule whenever $B \rightarrow \underline{x}$ occurs in the grammar. [$x \in \text{Terminal}$, x can be NULL]

Step 2: Delete $A \rightarrow B$ from the grammar.

Step 3: Repeat from step 1 until all unit productions are removed.

- Example -

Remove Unit productions from the Grammar whose production rule is given by

$$P: \underline{S \rightarrow XY, X \rightarrow a, Y \rightarrow z|b, Z \rightarrow M, M \rightarrow N, N \rightarrow a}$$

~~Ans~~ Ans:

$$Y \rightarrow z, Z \rightarrow M, \underline{M \rightarrow N}$$

i> Because $N \rightarrow a$, we add $M \rightarrow a$

$$P: S \rightarrow XY, X \rightarrow a, \cancel{Y \rightarrow z|b}, Z \rightarrow M, \underline{M \rightarrow a}, \underline{N \rightarrow a}$$

ii> Because $M \rightarrow a$, we add $Z \rightarrow a$

$$P: S \rightarrow XY, X \rightarrow a, Y \rightarrow z|b, \underline{Z \rightarrow a}, M \rightarrow a, N \rightarrow a$$

iii> Because $Z \rightarrow a$, we add $Y \rightarrow a$

$$P: \underline{S \rightarrow XY, X \rightarrow a, \underline{Y \rightarrow a|b}, Z \rightarrow a, M \rightarrow a, N \rightarrow a}$$

⇒ Unreachable Symbols: Z, M, N .

↓
From Start symbol there is no way to reach those Symbols. [Remove it]

$$P: \underline{S \rightarrow XY, X \rightarrow a, Y \rightarrow a|b.}$$

Simplification of CFG

③ Removal of NULL productions.

In a CFG, a Non-Terminal Symbol 'A' is a nullable Variable if there is a production $A \rightarrow \epsilon$ or there is a derivation that starts at 'A' and leads to ϵ .

Procedure for Removal:

Step 1: To remove $A \rightarrow \epsilon$, look for all productions whose right side contains A.

Step 2: Replace each occurrence of 'A' in each of these productions with ϵ .

Step 3: Add the resultant production to the Grammar.

- Example -

Remove NULL productions from the following Grammar

P: $S \rightarrow ABAC$, $A \rightarrow aA/\epsilon$, $B \rightarrow bB/\epsilon$, $C \rightarrow c$.

NULL production: $A \rightarrow \epsilon$, $B \rightarrow \epsilon$

1) To eliminate $A \rightarrow \epsilon$

$\rightarrow S \rightarrow \underline{ABAC}$

$S \rightarrow \underline{ABC} \mid \underline{BAC} \mid BC$

$\rightarrow A \rightarrow aA$

$A \rightarrow a$

P: $S \rightarrow ABAC \mid ABC \mid BAC \mid BC$

$A \rightarrow aA/a$, $B \rightarrow bB/\epsilon$, $C \rightarrow c$

2) To eliminate $B \rightarrow \epsilon$

$\rightarrow B \rightarrow bB$

$B \rightarrow b$

$\rightarrow \underline{S \rightarrow AAC \mid AC \mid C}$

✓ P: $S \rightarrow ABAC \mid ABC \mid BAC \mid BC \mid AAC \mid AC \mid C$

$A \rightarrow aA/a$

$B \rightarrow bB/b$

$C \rightarrow c$