

Game theory

There are many situations when one has to analyse problems where more than one party is pursuing conflicting objectives and the outcome of every action of one party depends on the action taken by the other. Such situations are called conflicting situations. If we construct a simplified formalized model of such situations, such formalized model of conflicting situations is called a game. A game can be defined as a set of rule for playing. A game may be considered as a clash of interests between two or more opponents. If two person's clash is involved, it is called Two-person game, ~~are~~ otherwise multiperson game.

Strategy: Strategy is one of the basic concepts of the game theory. A strategy is considered as a set of unambiguous rules determining the choice of every personal move of the player, depending on the situation that arises in the course of game.

Finite game: A game is said to be finite if either player has only a finite number of strategies. A finite game in which player A has m strategies, say A_1, A_2, \dots, A_m and player B has n strategies, say B_1, B_2, \dots, B_n is called $m \times n$ game.

Pay off matrix: The choice of strategies A_i by player A and B_j by player B determines the pay off (or gain) to player A and denoted by a_{ij} . Values of a_{ij} can be arranged in matrix form as shown in following table.

Pay off matrix for A

A \ B	B_1	B_2	\dots	B_m
A_1	a_{11}	a_{12}	\dots	a_{1m}
A_2	a_{21}	a_{22}	\dots	a_{2m}
\vdots	\vdots	\vdots	\vdots	\vdots
A_n	a_{n1}	a_{n2}	\dots	a_{nm}

This matrix of $m \times n$ game is denoted as $(a_{ij})_{m \times n}$ and called pay off (gain) matrix for player A. Every a_{ij} can either be positive (gain for A) or negative (loss for player A) or zero (no loss or no gain to A).

The pair strategy (A_i, B_j) when used rendering to a player the maximum possible gain (or minimum possible loss) is called the optimal strategy and corresponding value a_{ij} is called game value, denoted by α .

Characteristics of a game:

- ① Number of activities or course of actions:
In a game, no of activities may be finite or infinite. Based on this, a game may be called a finite or infinite game.
- ② Number of persons: If n persons are participating in a game, then it is called n person game. Here person refers to individual or group of participants.
- ③ Pay off: The payment off to a person as a result of the game is called pay off.
- ④ Courses of action available to a player:
In a game, the course of actions available to each player are known.

Two person Zero - Sum Games :

A game with only two players (say player A and player B) is called a two-person Zero-sum game, if the gain of one player, say A is equal to the loss of other player, say B, such that the sum of their net gain is zero. It is also known as rectangular game as the pay off matrix is rectangular.

Pay off matrix : Suppose the player A has m strategies A_1, A_2, \dots, A_m and player B has n activities B_1, B_2, \dots, B_n . Then a pay off matrix can be formed adopting the following rules:

1. The row designations of each matrix are strategies $A_1, A_2, \dots, A_i, \dots, A_m$ that are available to player A.
2. The column designations of each matrix are strategies $B_1, B_2, \dots, B_i, \dots, B_n$ that are available to player B.
3. The cell entry ' a_{ij} ' is the payment to player A in A's pay off matrix when A chooses the activity A_i and B chooses the activity B_j .

4. In a zero sum two-person game, the cell entry in B's pay-off matrix, will be the negative of the corresponding cell entry ' a_{ij} ' in the player A's pay off matrix, such that the sum of pay off matrices of both players is ultimately zero.

	B_1	B_2	...	B_j	...	B_n
A_1	a_{11}	a_{12}	...	a_{1j}	...	a_{1n}
A_2	a_{21}	a_{22}	...	a_{2j}	...	a_{2n}
...
A_i	a_{i1}	a_{i2}	...	a_{ij}	...	a_{in}
...
A_m	a_{m1}	a_{m2}	...	a_{mj}	...	a_{mn}

Representation of A's pay off matrix

	B_1	B_2	...	B_j	...	B_n
A_1	$-a_{11}$	$-a_{12}$...	$-a_{1j}$...	$-a_{1n}$
A_2	$-a_{21}$	$-a_{22}$...	$-a_{2j}$...	$-a_{2n}$
...
A_i	$-a_{i1}$	$-a_{i2}$...	$-a_{ij}$...	$-a_{in}$
...
A_m	$-a_{m1}$	$-a_{m2}$...	$-a_{mj}$...	$-a_{mn}$

Representation of B's pay off matrix

Strategies: When two players play a game, then they have different alternatives at their disposal to go ahead with the game. These alternatives are called strategies. A_1, A_2, \dots, A_m are strategies available to player A and B_1, B_2, \dots, B_n are available to player B.

Saddle point: A saddle point (A_r, B_s) of a pay off matrix is the position of an element with minimum value in its row and maximum value in its column.

If the pay off matrix (a_{ij}) is such that $\max_i (\min_j (a_{ij})) = \min_j (\max_i (a_{ij})) = \text{an ars}$,

then the matrix is said to have a saddle point at (A_r, B_s) .

A \ B	B ₁	B ₂	B ₃	α_i
A ₁	6	8	10	6
A ₂	4	12	2	2
B _j	6	12	10	-

Representation of saddle point

In this table, position $(1, 1)$ is the Saddle point with optimum pay off 6 to player A.

Optimal strategy: If the pay off matrix (a_{ij}) has a saddle point (r, s) , then A_r and B_s are called optimal strategy of player A and B resp.

In above table $r=s=1$, A_1 , B_1 are the optimal strategies for player A and B resp.

Pure Strategy: When only single alternative is used in the game, it is called pure strategy. The above table shows pure strategies. For optimum gain, both

players A and B use single courses of actions A_1 and B_1 , respectively.

Mixed Strategy: If several alternatives with different values are used to play the game, then it is called a ~~gen~~ game with mixed strategy. For example, the game problem with pay off matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ is a game with mixed strategies, as this game does not have a saddle point. When a saddle point does not exist, mixed strategies are used to find the value of the game.

Value of game: The optimum pay off to player A is called the value of game. In above table, ~~the~~ 6 is the game value with optimal strategies A_1 and B_1 . If any other of the players deviates from his/her optimal strategies, he/she will be in loss. Alternatively the pay off a_{rs} at the saddle in the position (A_r, B_s) is called the value of the game.