Online-Class 11-02-2021

Probability, Statistics and Reliability (MAT3003)

SLOT: B21 + B22 + B23

MODULE - 2

Topic: Random variables (RV): Discrete RV & Continuous RV

Reference: Probability and Statistics for Engineers & Scientists
by R.E.Walpole, Pearson (2012).

Brief Contents

- Review of Previous Class
- Concept of Random Variable (RV)
- Types of RV: DRV and CRV
- Distribution and Density Functions: CDF & PDF
- Worked Out Questions
- Practice Questions

Concept of Random Variables (RV)

- A numerically valued variable *x* will change depending on the particular outcome of the experiment being measured.
- For example, suppose we toss a die and measure *x*, the number observed on the upper face. The variable *x* can take on any of six values—1, 2, 3, 4, 5, 6—depending on the *random* outcome of the experiment. For this reason, we refer to the variable *x* as a **random variable (RV).**
- **Definition** **A random variable is a variable whose value is a numerical outcome of a random experiment.**

Definition A variable x is a **random variable** if the value that it assumes, corresponding to the outcome of an experiment, is a chance or random event.

- **Definition** A RV is a function from the sample space to real line.

$$X: S \to R$$

where X is RV, S is sample space, R is the real line. Thus, as described in *Walpole book*,

"A RV is a function that associates a real number with each element in the sample space"

- **Caution:**
  1. A RV is not a probability.
  2. RV can be defined in practically any way. Their values do not have to be positive or between 0 and 1 as with probabilities.
  3. RVs are typically named using capital letters such as X, Y, Z. Values of RVs are denoted with their respective lower-case letters. Thus, the expression

$$X = x$$

means that the RV X has the value x.

Reference: Probability and Statistics for Engineers & Scientists
by R.E.Walpole, Pearson (2012).

Examples of RV

Example 1. X : Number of defects on a *randomly selected* piece of furniture.

Example 2. X : GATE score for a *randomly selected* college applicant.

Example 3. X : Number of telephone calls received by a crisis intervention hotline during a *randomly selected* time period.

Example 1

- Two balls are drawn in succession without replacement from an urn containing 4 red balls and 3 black balls. The possible outcomes and the values y of the random variable Y, where Y is the number of red balls, are

Sample Space	y
RR	2
RB	1
BR	1
BB	0

Example 2

- Consider the simple condition in which components are arriving from the production line and they are stipulated to be defective or not defective. Define the random variable *X* by

$$X = \begin{cases} 1, & \text{if the component is defective,} \\ 0, & \text{if the component is not defective.} \end{cases}$$

- Clearly, the assignment of 1 or 0 is arbitrary though quite convenient. The random variable for which 0 and 1 are chosen to describe the two possible values is called a **Bernoulli random variable**.

Example 3

- Statisticians use **sampling plans** to either accept or reject batches or lots of material. Suppose one of these sampling plans involves sampling independently 10 items from a lot of 100 items in which 12 are defective.
- Let X be the random variable defined as the number of items found defective in the sample of 10. In this case, the random variable takes on the values $0, 1, 2, \ldots, 9, 10$.

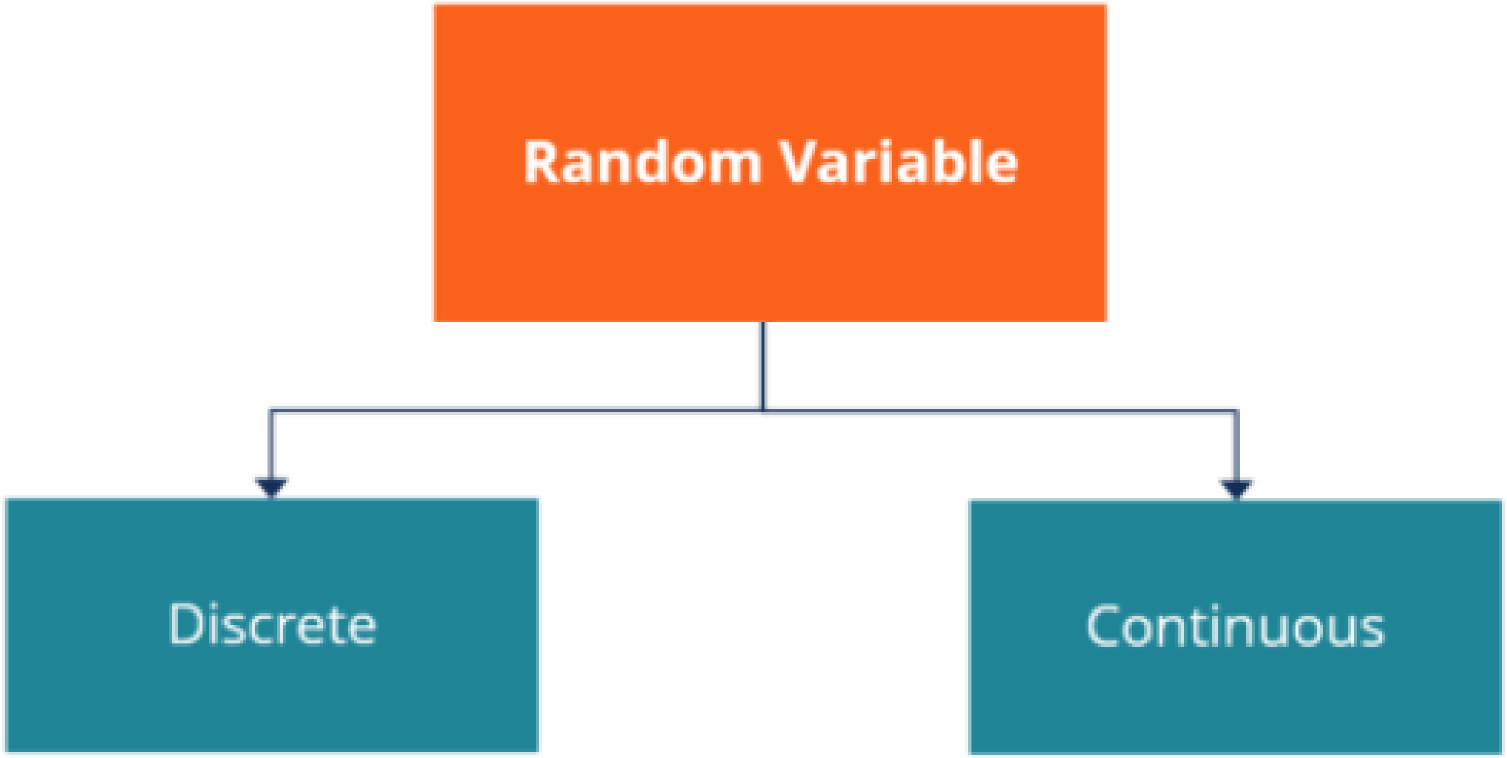
Example 4

- Suppose a sampling plan involves sampling items from a process until a defective is observed. The evaluation of the process will depend on how many consecutive items are observed. In that regard, let X be a random variable defined by the number of items observed before a defective is found. With N a non-defective and D a defective, sample spaces are:
- $S = \{D\}$ given $X = 1$,
- $S = \{ND\}$ given $X = 2$,
- $S = \{NND\}$ given $X = 3$, and so on.

Example 5

- Let X be the random variable defined by the waiting time, in hours, between successive speeders spotted by a radar unit. The random variable X takes on all values x for which $x \geq 0$.

Types of RVs



Discrete Random Variable (DRV)

- **Definition:** A random variable is DRV if its range is either finite or countable.

Reference: Probability and Statistics for Engineers & Scientists
by R.E.Walpole, Pearson (2012).

Examples of DRV

1) Number of courses taken by a student in the semester Winter 2021, is a DRV because it can take any value: 1, 2, 3, 4, or 5.
2) Number of heads obtained in three tosses of a coin.
3) Number of cars sold at a dealership during a given month.
4) Number of houses in a certain block.
5) Number of complaints received at the office of airline on a given day.
6) Number of customers who visit a bank during any given hour.
7) Number of children in a family.
8) Number of Facebook likes.
9) Number of votes in an election.

Continuous Random Variable (CRV)

- **Definition**: A random variable is called a CRV if it can take on values on a continuous scale.
- A CRV takes on all values in an interval of numbers.
- CRVs are usually measurements.
- Examples:

  -- Height.

  -- Weight.

  -- Time required to run a mile.

  -- Water temperature.

  -- Volts of electricity

  -- Wind speed.

  -- Average age of students in a class.

Reference: Probability and Statistics for Engineers & Scientists
by R.E.Walpole, Pearson (2012).

Random Variables

Example: Decide if the random variable X is discrete or continuous.

a.) The distance your car travels on a tank of gas

The distance your car travels is a continuous random variable because it is a measurement that cannot be counted. (All measurements are continuous random variables.)

b.) The number of students in a statistics class

The number of students is a discrete random variable because it can be counted.

Probability Distributions

There are two types:

1. **Probability Density Function (pdf) denoted by** $f(x)$

2. **Cumulative Distribution Function (cdf) denoted by** $F(x)$

Probability Density Function $f(x)$ for a DRV

<u>Definition:</u> The set of ordered pairs $(x, f(x))$ is a **probability density function (probability mass function**, or **probability distribution)** of the discrete random variable X if, for each possible outcome x,

1. $f(x) \geq 0$,
2. $\sum_x f(x) = 1$,
3. $P(X = x) = f(x)$.

Example 1

- A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives.

Solution

- Let *X* be a random variable whose values *x* are the possible numbers of defective computers purchased by the school. Then *x* can only take the numbers 0, 1, and 2.

Now,

$$f(0) = P(X = 0) = \frac{\binom{3}{0}\binom{17}{2}}{\binom{20}{2}} = \frac{68}{95}, \quad f(1) = P(X = 1) = \frac{\binom{3}{1}\binom{17}{1}}{\binom{20}{2}} = \frac{51}{190},$$

$$f(2) = P(X = 2) = \frac{\binom{3}{2}\binom{17}{0}}{\binom{20}{2}} = \frac{3}{190}.$$

Thus, the probability distribution of X is

x	0	1	2
$f(x)$	$\frac{68}{95}$	$\frac{51}{190}$	$\frac{3}{190}$

Cumulative Distribution Function $F(x)$ for a DRV

- Definition

The cumulative distribution function $F(x)$ of a discrete random variable X with probability distribution $f(x)$ is

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t), \quad \text{for } -\infty < x < \infty.$$

Practice Questions

1. Determine the value *c* so that each of the following functions can serve as a probability distribution of the discrete random variable *X*:

   (a) $f(x) = c(x^2 + 4)$, for $x = 0, 1, 2, 3$;

   (b) $f(x) = c\binom{2}{x}\binom{3}{3-x}$, for $x = 0, 1, 2$.

THANK YOU