$$A = \left[\begin{array}{cc|cc|c} 1 & -2 & 0 & 1 & 3 \\ 2 & 3 & 5 & 7 & -2 \\ \hline 3 & 1 & 4 & 5 & 9 \\ 4 & 6 & -3 & 1 & 8 \end{array}\right]$$

$$\left[\begin{array}{cc|ccc} 1 & -2 & 0 & 1 & 3 \\ \hline 2 & 3 & 5 & 7 & -2 \\ 3 & 1 & 4 & 5 & 9 \\ \hline 4 & 6 & -3 & 1 & 8 \end{array}\right]$$

$$\left[\begin{array}{ccc|cc} 1 & -2 & 0 & 1 & 3 \\ 2 & 3 & 5 & 7 & -2 \\ \hline 3 & 1 & 4 & 5 & 9 \\ 4 & 6 & -3 & 1 & 8 \end{array}\right]$$

Definition 1.3.1 (Square block matrix)

Let M be a block matrix, the M is square block matrix if

1. M is square.
2. The block form a square matrix.
3. The diagonal blocks are also square matrices.

Example 1.3.2

$$A = \left[\begin{array}{cc|cc|c} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 1 & 3 \\ \hline 2 & 1 & 2 & 2 & 1 \\ \hline 3 & 2 & 1 & 4 & 5 \\ 5 & 2 & 3 & 1 & 4 \end{array}\right]$$

This is not a square block matrix

$$A = \left[\begin{array}{cc|cc|c} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 1 & 3 \\ \hline 2 & 1 & 2 & 2 & 1 \\ 3 & 2 & 1 & 4 & 5 \\ \hline 5 & 2 & 3 & 1 & 4 \end{array}\right]$$

This is a square block matrix

Definition 1.3.3 (Block diagonal matrix)

Let $M : [A_{ij}]$ be a square block matrix such that non-diagonal blocks are all zero matrices.
i.e. $A_{ij} = 0$ whenever $i \neq j$.
M is called block diagonal matrix, if $M = diag(A_{11}, A_{22}, A_{33}, \ldots, A_{rr})$ or $M = A_{11} \oplus A_{22} \oplus \ldots \oplus A_{rr}$.

$$M = \left[\begin{array}{c:c:c} & 0 & \\ \hdashline 0 & & 0 \\ \hdashline 0 & 0 & \end{array}\right]$$

Remark 1.3.4

Note : $M : diag\,(A_{11}, A_{22}, \ldots, A_{rr})$ *is invertible iff* A_{ii} *is invertible* $\forall i$. *Then* $M^{-1} = diag\left(A_{11}^{-1}, A_{22}^{-1}, \ldots, A_{rr}^{-1}\right)$

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Example 1.3.5

$$M = \left[\begin{array}{cc|cc} 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 3 \\ 0 & 0 & 2 & 0 \end{array}\right]$$

Example 1.3.6

Find Block matrix multiplication of

$$A = \begin{bmatrix} 1 & 3 & -2 & 3 & -3 \\ 2 & 4 & -2 & 2 & 1 \\ 0 & -2 & 1 & 1 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 3 \\ 2 & 2 \\ 1 & 2 \end{bmatrix}.$$

$$A = \left[\begin{array}{ccc:cc} 1 & 3 & -2 & 3 & -3 \\ 2 & 4 & -2 & 2 & 1 \\ \hdashline 0 & -2 & 1 & 1 & 1 \end{array}\right]_{3\times 5}, \; B = \left[\begin{array}{cc} 1 & 0 \\ 2 & 1 \\ 3 & 3 \\ \hdashline 2 & 2 \\ 1 & 2 \end{array}\right]_{5\times 2}$$

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}; A = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$AB = \begin{bmatrix} A_1B_1 & A_2B_2 \\ A_3B_1 & A_4B_2 \end{bmatrix}$$

$$A_1B_1 = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 4 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 4 & -2 \end{bmatrix}$$

$$A_2B_2 = \begin{bmatrix} 3 & -3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 5 & 6 \end{bmatrix}$$

$$A_3B_1 = \begin{bmatrix} 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \end{bmatrix}$$

$$A_4B_2 = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} A_1B_1 & A_2B_2 \\ A_3B_1 & A_4B_2 \end{bmatrix}$$

$$AB = \begin{bmatrix} \begin{bmatrix} 1 & -3 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 3 & 6 \end{bmatrix} \\ \begin{bmatrix} -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 4 \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 \\ 9 & 4 \\ 2 & 5 \end{bmatrix}$$

Example 1.4.1

Find the elementary matrices of $A = \begin{pmatrix} -3 & 2 & -1 \\ 6 & -6 & 7 \\ 3 & -4 & 4 \end{pmatrix}$.

$$A = \begin{pmatrix} -3 & 2 & -1 \\ 6 & -6 & 7 \\ 3 & -4 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} -3 & 2 & -1 \\ 0 & -2 & 5 \\ 3 & -4 & 4 \end{pmatrix} = M_1 A$$

$$M_1 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} -3 & 2 & -1 \\ 0 & -2 & 5 \\ 0 & -2 & 3 \end{pmatrix} = M_2 M_1 A$$

$$M_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} -3 & 2 & -1 \\ 0 & -2 & 5 \\ 0 & 0 & -2 \end{pmatrix} = M_3 M_2 M_1 A$$

$$M_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

$$U = M_3 M_2 M_1 A$$