Greibach Normal Form (GNF) & CNF to GNF conversion

Greibach Normal form (GNF)

⇒ A CFG is in Greibach Normal form if the productions are in the following forms:

$$A \rightarrow b$$
$$A \rightarrow bC_1C_2\cdots C_n$$

where $A, C_1, \ldots, C_n$ are Non-Terminals and $b \in T$

Steps to Convert a given CFG to GNF:

Step 1: Check if the given CFG has any Unit productions or NULL Productions and Remove if there are any.
[Using Unit and Null production Removal Techniques]

Step 2: Check whether the CFG is already in CNF and Convert it to CNF if it is not.
[Using CFG to CNF conversion technique]

Step 3: Change the names of the Non-Terminal Symbols into some A_i in ascending order of i.

CFG to GNF

Example:

$S \rightarrow CA \mid BB$

$B \rightarrow b \mid SB$

$C \rightarrow b$

$A \rightarrow a$

Replace: S with A_1, C with A_2, A with A_3, B with A_4 (Not Random ↓)

we set:

$A_1 \rightarrow A_2 A_3 \mid A_4 A_4$

$A_4 \rightarrow b \mid A_1 A_4$ *

$A_2 \rightarrow b$

$A_3 \rightarrow a$

~~Step 2~~:

Step 4: Alter the rules so that the Non-Terminals are in ascending order, such that, If the production is of the form $A_i \rightarrow A_j x$, then, $i < j$ and should never be $i \geq j$

* $A_4 \rightarrow b \mid A_1 A_4$

$A_4 \rightarrow b \mid A_2 A_3 A_4 \mid A_4 A_4 A_4$

$A_4 \rightarrow b \mid b A_3 A_4 \mid A_1 A_4 A_4$ [$i = j$]

$A_4 \rightarrow b \mid b A_3 A_4 \mid$ Left ~~Recursion~~ Recursion

[A_4 is calling itself]

CFG to GNF

Step 5: Remove Left Recursion.

⇒ Introduce a new variable to remove the Left Recursion

$A_4 \rightarrow b \mid bA_3A_4 \mid A_4A_4A_4$

$Z \rightarrow A_4A_4Z \mid A_4A_4$ ✓

$A_4 \rightarrow b \mid bA_3A_4 \mid bZ \mid bA_3A_4Z$ ✓

Now the grammar is:

$A_1 \rightarrow A_2A_3 \mid A_4A_4$ *

$A_4 \rightarrow b \mid bA_3A_4 \mid bZ \mid bA_3A_4Z$

$Z \rightarrow A_4A_4 \mid A_4A_4Z$ * *

$A_2 \rightarrow b$

$A_3 \rightarrow a$

GNF

$A_1 \rightarrow bA_3 \mid bA_4 \mid bA_3A_4A_4 \mid bZA_4 \mid bA_3A_4ZA_4$

$A_4 \rightarrow b \mid bA_3A_4 \mid bZ \mid bA_3A_4Z$

$Z \rightarrow bA_4 \mid bA_3A_4A_4 \mid bZA_4 \mid bA_3A_4ZA_4 \mid$
$bA_4Z \mid bA_3A_4A_4Z \mid bZA_4Z \mid bA_3A_4ZA_4Z$

$A_2 \rightarrow b$

$A_3 \rightarrow a$