

Tutorial 2

$$1) A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

$$R_2 \rightarrow -3R_1 + R_2$$

$$R_3 \rightarrow (-1)R_1 + R_3$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -4 & -2 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 5 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix} = E_1 A$$

$$R_2 \rightarrow \frac{2}{3} R_3 + R_2$$

$$R_1 \rightarrow -\frac{1}{3} R_3 + R_1$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 2/3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 2/3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 \rightarrow \frac{1}{2} R_2 + R_1 :$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 2/3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & -1/2 & 1/3 \\ 0 & 1 & -2/3 \\ 0 & 0 & 1 \end{pmatrix}$$

2) $\alpha = (a_1, b_1, c_1) \in V_3(\mathbb{R})$

$\beta = (a_2, b_2, c_2) \in V_3(\mathbb{R})$

$T(\alpha) = T(\beta)$

$T(a_1, b_1, c_1) = T(a_2, b_2, c_2)$

$(3a_1, a_1 - b_1, 2a_1 + b_1 + c_1) = (3a_2, a_2 - b_2, 2a_2 + b_2 + c_2)$

$3a_1 = 3a_2$

$a_1 - b_1 = a_2 - b_2$

$a_2 = b_2$

$c_1 = c_2$

$(a_1, b_1, c_1) = (a_2, b_2, c_2)$

$\alpha = \beta$

$$\text{If } T(a, b, c) = (p, q, r)$$

$$\text{then } T^{-1}(p, q, r) = (a, b, c)$$

$$T(a, b, c) = (p, q, r)$$

$$(3a, a-b, 2a+b+c) = (p, q, r)$$

$$3a = p \quad a = p/3$$

$$a-b = q$$

$$\frac{p}{3} - b = q$$

$$\frac{p}{3} - q = b$$

$$2a + b + c = r$$

$$2\left(\frac{p}{3}\right) + \left(\frac{p}{3} - q\right) + c = r$$

$$c = r - p + q$$

$$T^{-1}(p, q, r) = (a, b, c)$$

$$= \left(\frac{p}{3}, \frac{p}{3} - q, r - p + q \right)$$