Homework 1

24-677 Special Topics: Linear Control Systems

Prof. D. Zhao

Due: Sept 4, 2019, 8:30 am. Submit within deadline.

- Your online version and its timestamp will be used for assessment. The paper version, will only be used for backup purposes.
- We will use Gradescope (https://www.gradescope.com/) to grade. The link is on the panel of CANVAS. If you are confused about the tool, post your questions on Piazza.
- At the start of every exercise you will see topic(s) on what the given question is about and what will you be learning.
- Submit the pdf of your written work to Canvas and Gradescope.
- You will need to upload 1. A python script and 2. The programming assignment script, with this homework.
- The python script for submitting your theory homework answers will be given to you. The script is named **"hw1_answer_template.py"**. And you have to submit it with your answers. After filling out your answers, rename it to **"hw1_answer.py"**. Refer to the further instructions given in the script for submission format.
- For the programming assignment, copy and paste, or export your answer script into a python file named **"hw1.py"**. Follow the further instructions given in the jupyter notebook named **"PHW1_qs.ipynb"** for information on submission format of your answers.
- In total there will be **"4 files"** to submit in Gradescope and 1 file in canvas. Out of the 4, 3 to be submitted in the Gradescope assignment named "Programming Assignment 1". And the last one will be the pdf/images of your written work. Submit to Gradescope assignment named "Homework 1". The same pdf is to be submitted in Canvas.
- We advise you to start with the assignment early. All the submissions are to be done before the respective deadlines of each assignment. For information about the late days and scale of your Final Grade, refer to the Syllabus in Canvas.

Exercise 1. *Types of Systems*

A system has an input$u(t)$ and an output $y(t)$, which are related by the information provided below. Classify each system as a linear or non-linear and time invariant or time-varying.

1. $y(t) = 0$ for all t
2. $y(t) = u^3(t)$
3. $y(t) = u(3t)$
4. $y(t) = e^{-t}u(t-T)$
5. $y(t) = u(t-1)$
6. $y(t) = \int_{-\infty}^{t} u(\tau)d\tau$
7. $y(t) = \int_{-\infty}^{t} u^2(\tau)d\tau$
8. $y(t) = \begin{cases} 0 & t \leq 0 \\ u(t) & t > 0 \end{cases}$

Exercise 2. *Fields*

Is this given set a field?

1. Is $\mathbb{N}^*$ a field ?
2. Is $\mathbb{Q}$ a field ?(*Find out what the notations means!*)
3. Set of all polynomials
4. Set of rational functions with real coefficients.

Exercise 3. *Subspace*

The set W of $n \times n$ matrices with real entries is known to be a linear vector space. Determine which of the following set are subspaces of W :

1. The set of $n \times n$ skew-symmetric matrices?
2. The set of $n \times n$ diagonal matrices?
3. The set of $n \times n$ upper-diagonal matrices?
4. The set of $n \times n$ singular matrices?

Exercise 4. *Linearization*

Perform linearization on the given differential equation

$$\ddot{y} + (1 + y)\dot{y} - 2y + 0.5y^3 = 0$$

Exercise 5. *State space representations*

1. What is the dimension (or shape) of the matrix B and D in the given state space representation:

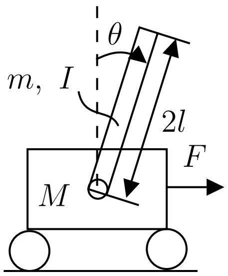
$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix} u(t)$$

$$\begin{bmatrix} y_1 & y_2 \end{bmatrix}^T = [B] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [D]u(t)$$

State how may inputs, states and outputs are present in this system.

Exercise 6. *Linearization of system*

Consider the below Inverted pendulum problem,



The system dynamics is illustrated as the below equations:

$$\begin{cases} (m+M)\ddot{x} + ml\ddot{\theta} - ml\dot{\theta}^2 sin\theta = F \\ (I + ml^2)\ddot{\theta} + ml\ddot{x}\cos\theta - mgl\sin\theta = \emptyset \end{cases}$$

Linearize the state equations about the equilibrium. Derive the A and B matrices by substituting the operating equilibrium point. Here m is mass of the rod, M mass of cart, l is length of the rod, I is moment of inertia of the rod, θ is the swing angle of the rod, g is gravitational acceleration, x is distance along cart, F is Force applied as in the diagram.