Homework 2

24-677 Special Topics: Linear Control Systems

Prof. D. Zhao

Due: Sept 10, 2019, 08:30 am. Submit within deadline.

- Your online version and its timestamp will be used for assessment. The paper version, will only be used for backup purposes.
- We will use Gradescope (https://www.gradescope.com/) to grade. The link is on the panel of CANVAS. If you are confused about the tool, post your questions on Piazza.
- At the start of every exercise you will see topic(s) on what the given question is about and what will you be learning.
- The Gradescope Autograder python script for submitting your theory homework answers will be given to you. The script is named **"hw2_theory.py"** and you have to submit it with your answers. Carefully refer to the further instructions given in the script for submission format.
- For the programming assignment, copy and paste, or export your answer script into a python file named **"hw2_script.py"**. Follow the further instructions given in the jupyter notebook named **"PHW2_qs.ipynb"** for information on submission format of your answers.
- Submit **hw2_theory.py**, **hw2_script.py**, **hw2_output.npy** to Gradescope under "Programming Assignment 2" and **pdf/images** to "Homework 2".
- We advise you to start with the assignment early. All the submissions are to be done before the respective deadlines of each assignment. For information about the late days refer to the Syllabus in Canvas.
- Any extraneous changes made to the Gradescope Autograder script which may result in error in compilation of the Autograder will result in point deduction.

Exercise 1. *Linear dependence*

Consider the following set of vectors.
Determine whether they are Linearly dependent or independent.

1. $\left\{ \begin{bmatrix} -1 \\ -9 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right\}$ in the space of real 3 -tuples over the field of reals, $(\mathbb{R}^2, \mathbb{R})$.

2. $\left\{ \begin{bmatrix} 2-i \\ -i \end{bmatrix}, \begin{bmatrix} 1+2i \\ -i \end{bmatrix} \cdot \begin{bmatrix} -i \\ 3+4i \end{bmatrix} \right\}$ in the space of complex pairs over the field of reals,$(\mathbb{C}^2, \mathbb{R})$.

3. $\{2s^2 + 2s - 1, -2s^2 + 2s + 1, s^2 - s - 5\}$ in the space of polynomials over the field of reals.

4. $\{x_1, x_2, x_3, x_4\}$ such that: $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, x_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, x_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$ and $x_4 = \begin{bmatrix} 3 \\ 5 \\ 5 \\ 7 \end{bmatrix}$

Exercise 2. *Norms*

What are the 1-norm, 2-norm, and infinite-norm of the vectors

$$\mathbf{x_1} = \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix} \quad \mathbf{x_2} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

Exercise 3. *Change of basis*

Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be bases for a vector space V, and suppose

$$\mathbf{b}_1 = 6\mathbf{c}_1 - 2\mathbf{c}_2 \text{ and } \mathbf{b}_2 = 9\mathbf{c}_1 - 4\mathbf{c}_2$$

1. Find the change-of-basis matrix from $\mathcal{B}$ to $\mathcal{C}$.
2. Consider $\mathbf{x} = -3\mathbf{b}_1 + 2\mathbf{b}_2$. Using the change-of-basis matrix obtained in question 1, find the coordinates of x in basis $\mathcal{C}$.

Exercise 4. *Basis and span*

Determine the basis for $\mathbf{W} \subset \mathbb{R}^4$ spanned by four vectors

$$\{y_1, y_2, y_3, y_4\} = \left\{ \begin{bmatrix} -1 \\ 1 \\ -5 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 7 \\ -8 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 10 \\ -11 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 15 \\ -15 \end{bmatrix} \right\}$$

Is $z = \begin{bmatrix} 3 \\ -1 \\ 13 \\ -17 \end{bmatrix} \in \mathbf{W}$? is $u = \begin{bmatrix} 4 \\ 9 \\ 12 \\ -8 \end{bmatrix} \in \mathbf{W}$? is $v = \begin{bmatrix} -1 \\ -1 \\ -3 \\ 3 \end{bmatrix} \in \mathbf{W}$?

Exercise 5. *Inner product*

Find the inner product of the following given vectors $\mathbf{x}_1 = [1, 12, 18]^T$ and $\mathbf{x}_2 = [25, 37, 11]^T$.