Homework 4

Ding Zhao
24-677 Special Topics: Linear Control Systems

Due: Sept 25, 2019, 08:30 am. Submit within deadline.

- At the beginning of each question you will find the key words for the knowledge that the exercise will help you to practice.
- Right after each week's recitation, we have an about half-hour homework Q&A session. I request you to work on the assignment early and bring your questions to this session to take advantage of this support.
- You can also post your questions on the Piazza. We will try our best to give feedback within 24 hours during the workday and within 48 hours during the weekend. We will keep answering questions until 12 hours before the deadline.
- You need to upload your homework to Gradescope (https://www.gradescope.com/) to be graded. The link is on the panel of CANVAS. If you are not familiar about the tool, post your questions on Piazza or ask during the office hours/homework Q&A sessions. We will use the online submission time as the timestamp.
- We designed an Autograder to provide you an instantaneous feedback for most of the questions. The purpose is to help you know what you do not know and solve them during the office hours/recitations/homework Q&A sessions. Submit **hw4_theory.py** under "Programming Assignment 4" and a photocopy of your derivation in *.pdf* format to "Homework 4". We will manually check all of the answers marked as wrong by Autograder to make sure you get the points you deserve.
- **Note that we do not have programming questions in Homework 4**.

Exercise 1. *eigenvalues* ***(practice solving the eigenvalues manually instead of using Python, the same for the entire homework 4)***

For $\mathbf{A_1} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{A_2} = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

Find the eigenvalues of $\mathbf{A_1}$ and $\mathbf{A_2}$, such that the eigenvalues are in ascending order ($\lambda_1 \leq \lambda_2 \leq \lambda_3$)

Exercise 2. *Singular values.*

For $\mathbf{A_1} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ $\mathbf{A_2} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}$

1. Find the eigenvalues of $\mathbf{A_1}^T \mathbf{A_1}$.
2. Find the eigenvalues of $\mathbf{A_1} \mathbf{A_1}^T$.
3. Find the singular values of $\mathbf{A_1}$
4. Find the singular values of $\mathbf{A_2}$

Write eigenvalues in ascending order.

Exercise 3. *Characteristic polynomial*

$$\mathbf{A} = \begin{bmatrix} -\alpha_1 & -\alpha_2 & -\alpha_3 & -\alpha_4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

A matrix in this form is called a companion form matrix

1. Calculate the characteristic polynomial in terms of $\alpha_i, i = 1, 2, 3, 4$.

2. Derive the normalized eigenvector v of $\mathbf{A}$ in terms of its eigenvalues λ, s.t. $\|v\|_2 = 1$.

(As the solution is analytical, we will check it manually. No AutoGrader for this question.)

Exercise 4. *Jordan form, decomposition*

Find the Jordan-form of the following matrices.

$$\mathbf{A_1} = \begin{bmatrix} 1 & 4 & 8 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad \mathbf{A_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix}$$

$$\mathbf{A_3} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad \mathbf{A_4} = \begin{bmatrix} 0 & 4 & 3 \\ 0 & 20 & 16 \\ 0 & -25 & -20 \end{bmatrix}$$

(Write the Jordan form such that eigenvalues should be in ascending order of their absolute values. The absolute value of a complex number is defined as $|a + bi| = \sqrt{a^2 + b^2}$)

Exercise 5. *Function of matrices*

Given

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Find A^{10}, A^{103}, and e^{At} (when submitting to the Autograder, substitute t with 1).

Exercise 6. *Diagonalization*

Diagonlize the following matrix.

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Such that $\mathbf{A} = \mathbf{M}\boldsymbol{\Lambda}\mathbf{M}^{-1}$
(Write the diagonal matrix $\boldsymbol{\Lambda}$ such that its eigenalues are in ascending order of their absolute values.)