Homework 8

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24-677 Special Topics: Linear Control Systems

Due: Oct 29, 2019, 08:30 am. Submit within deadline.

- At the beginning of each question you will find the key words for the knowledge that the exercise will help you to practice.
- Right after each week's recitation, we have a half-hour homework Q&A session. I request you to work on the assignment early and bring your questions to this session to take advantage of this support.
- You can also post your questions on the Piazza. We will try our best to give feedback within 24 hours during the workday and within 48 hours during the weekend. We will keep answering questions until 12 hours before the deadline.
- You need to upload your homework to Gradescope (https://www.gradescope.com/) to be graded. The link is on the panel of CANVAS. If you are not familiar about the tool, post your questions on Piazza or ask during the office hours/homework Q&A sessions. We will use the online submission time as the timestamp.
- Autograder would not be used for this homework **(No Autograder)**. Submit your solutions and derivations in *.pdf* format to "Homework8". We will manually check all the answers.
- **Note: we do not have programming questions in HW8.**
- **Remember to submit derivations of all questions in your pdf submission**

Exercise 1. *Lyapunov's direct method*

An LTI system is described by the equations

$$\dot{x} = \begin{bmatrix} a & 0 \\ 1 & -1 \end{bmatrix} x$$

Use Lyapunov's direct method to determine the range of variable a for which the system is asymptotically stable. Consider the Lyapunov function,

$$V = x_1^2 + x_2^2$$

Exercise 2. *Stability of Non-Linear Systems*

Consider the following system:

$$\dot{x_1} = x_2 - x_1 x_2^2$$
$$\dot{x_2} = -x_1^3$$

Is the system stable:

(a) Based on Lyapunov's Indirect method?
[Hint: is the approximated linearized system stable?]

(b) Based on Lyapunov's Direct method? Consider the Lyapunov function:

$$V(x_1, x_2) = x_1^4 + 2x_2^2$$

(c) Plot the Phase Portrait plot of the resultant system (linearized at the equilibrium point) in 2.a.
[Hint: Use python matplotlib-streamplot library]

(d) Generate a 3D plot showing the variation of $\dot{V}$ with respect to x_1 and x_2.
[Hint: Use Axes3D python library]. Submit the code along-with the plot in pdf.

Note: For (c) and (d), include the code along with the plot in the pdf to be submitted. No need to submit .py file.

Exercise 3. *BIBO Stability*

For each of the systems given below, determine whether it is BIBO stable.

(a)

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 1 & 0 \\ -0.5 & 0.5 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} 5 & 5 \end{bmatrix} x(k) \end{aligned}$$

(b)

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -7 & -2 & 6 \\ 2 & -3 & -2 \\ -2 & -2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix} u \\ y &= \begin{bmatrix} -1 & -1 & 2 \\ 1 & 1 & -1 \end{bmatrix} x \end{aligned}$$

Exercise 4. *Canonical forms*

Consider system given by:

$$\frac{U(s)}{Y(s)} = \frac{s+3}{s^2+3s+2}$$

Find out the controllable Canonical form state representation.

Exercise 5. *Realization matrix form of realizable MIMO system*

Find a state space realization for

$$\hat{G}_1(s) = \begin{bmatrix} \dfrac{1}{s} & \dfrac{s+3}{s+1} \\ \dfrac{1}{s+3} & \dfrac{s}{s+1} \end{bmatrix}$$

Exercise 6. *Minimum Realizations*

Are the two state equations

$$\dot{x} = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 2 & 2 \end{bmatrix} x$$

and

$$\dot{x} = \begin{bmatrix} 2 & 0 \\ -1 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$
$$y = \begin{bmatrix} 2 & 0 \end{bmatrix} x$$

equivalent i.e. have the same transfer function? Are they minimal realizations?

Additional Exercise (Optional)

"William L. Brogan. Modern Control Theory", Example 10.11 (Page 367) and Example 12.1 (page 426).