Homework 3

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24-677 Special Topics: Linear Control Systems

Due: Sept 17, 2019, 08:30 am. Submit within deadline.

- Your online version and its timestamp will be used for assessment. The paper version, will only be used for backup purposes.
- We will use Gradescope (https://www.gradescope.com/) to grade. The link is on the panel of CANVAS. If you are confused about the tool, post your questions on Piazza.
- At the start of every exercise you will see topic(s) on what the given question is about and what will you be learning.
- The Gradescope Autograder python script for submitting your theory homework answers will be given to you. The script is named **"hw3_theory.py"** and you have to submit it with your answers. Carefully refer to the further instructions given in the script for submission format.
- For the programming assignment, copy and paste, or export your answer script into a python file named **"hw3_script.py"**. Follow the further instructions given in the jupyter notebook named **"PHW3_qs.ipynb"** for information on submission format of your answers.
- Submit **hw3_theory.py**, **hw3_script.py**, **hw3_output.npy** to Gradescope under "Programming Assignment 3" and **pdf/images** to "Homework 3".
- We advise you to start with the assignment early. For information about the late days refer to the Syllabus in Canvas.
- Any extraneous changes made to the Gradescope Autograder script which may result in error in compilation of the Autograder will result in point deduction.

Exercise 1. *Gram-schmidt method*

Find the orthonormal basis using Gram-Schmidt orthogonalization for the following vectors in $\mathbb{R}^3$:

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 8 \\ 1 \\ -6 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Assume v_1 as your first vector.

Exercise 2. *Orthogonal complement*

Find the orthogonal complement of the subspace of $\mathbb{R}^3$ spanned by the two vectors

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 5/3 \end{bmatrix} \text{ and } v_2 = \begin{bmatrix} 0 \\ 1 \\ -1/3 \end{bmatrix}$$

Exercise 3. *Orthogonal projection*

$$4x_1 + 10x_2 - 4x_3 = -8$$

1. Find the minimum distance between the given plane and the origin.
2. Find coordinates of the point on the plane closest to the origin.

Exercise 4. *Analyzing solution of equations*

Consider the linear algebraic equation: Ax = y

$$\begin{bmatrix} 2 & -1 \\ -3 & 3 \\ -1 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

It has three equations and two unknowns.

1. Does a solution $\mathbf{x}$ exist ?

2. Is the solution unique?

3. Does the solution exist if $\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T$.

Exercise 5. *Solution of equations and norm.*

1. Find the general solution of

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

2. Find the solution that has the smallest Euclidean norm.

Exercise 6. *Rank and Nullity*

Find the rank and nullity of the following matrices:

$$\mathbf{A_1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{A_2} = \begin{bmatrix} 4 & 1 & -1 \\ 3 & 2 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad \mathbf{A_3} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Exercise 7. *Linear operators*

Determine which of the following operations on vectors are linear operators:

1. $Ax = x + \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, where $x \in \mathbb{R}^3$

2. $Ax = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, where $x \in \mathbb{R}^3$

3. $Ax = \int_{-\infty}^{\infty} f(\sigma)x(t-\sigma)d\sigma$, where $f(\sigma)$ is a continuous function, and x is in the vector space of continuous functions.

4. $$Ax = \begin{bmatrix} x_1 + 2x_2 + 3x_3 \\ x_1 + x_2 + x_3 \\ x_1 \end{bmatrix}, \text{ where } x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$