Homework 5

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24-677 Special Topics: Linear Control Systems

Due: Oct 8, 2019, 08:30 am. Submit within deadline.

- At the beginning of each question you will find the key words for the knowledge that the exercise will help you to practice.
- Right after each week's recitation, we have a half-hour homework Q&A session. I request you to work on the assignment early and bring your questions to this session to take advantage of this support.
- You can also post your questions on the Piazza. We will try our best to give feedback within 24 hours during the workday and within 48 hours during the weekend. We will keep answering questions until 12 hours before the deadline.
- You need to upload your homework to Gradescope (https://www.gradescope.com/) to be graded. The link is on the panel of CANVAS. If you are not familiar about the tool, post your questions on Piazza or ask during the office hours/homework Q&A sessions. We will use the online submission time as the timestamp.
- We designed an Autograder to provide you instantaneous feedback for most of the questions. Submit **hw5_theory.py**, **hw5_script.py**, **hw5_output.npy** under "Programming Assignment 5" and a your derivations in *.pdf* format to "Homework 5". We will manually check all of the answers marked as wrong by Autograder to make sure you get the points you deserve.

Exercise 1. *DT Dynamics*

Consider the following system, Given $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ and the input $u(k) = 1,\ k = 0, 1, \cdots \infty$. Find $y(5)$.

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0 & 1 \\ -0.5 & -1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k) \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) \end{aligned}$$

Exercise 2. *Discretizing, DT Dynamics*

Discretize the system below using a sample time of $T = 1s$.

$$\dot{x}(t) = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) + 2u(t)$$

Exercise 3. *CT Dynamics*

Given the following system, find y(5).

$$\dot{x(t)} = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 2 & 3 \end{bmatrix} x(t)$$

Assume $x(0) = 0$ and u is a unit step input.

Exercise 4. *Linear Discrete-Time State Equations*

Solve for $x(3)$ where $x = [x_1\ x_2\ x_3]^T$.

$$x_1(k+1) = \frac{1}{2}x_1(k) - \frac{1}{2}x_2(k) + x_3(k)$$

$$x_2(k+1) = \frac{1}{2}x_2(k) + 2x_3(k)$$

$$x_3(k+1) = \frac{1}{2}x_3(k)$$

$$x(0) = \left[\begin{array}{ccc} 2 & 4 & 6 \end{array}\right]^T$$

When put in matrix form $x(k+1) = Ax(k)$, the system matrix is $A = \left[\begin{array}{ccc} \frac{1}{2} & -\frac{1}{2} & 1 \\ 0 & \frac{1}{2} & 2 \\ 0 & 0 & \frac{1}{2} \end{array}\right]$

Exercise 5. *State transition matrix*

Find the state transition matrix of

$$\dot{x} = \begin{bmatrix} -\sin t & 0 \\ 0 & -\cos t \end{bmatrix} x$$

(*Autograder will not be used for this question. It shall be graded manually.*)

Exercise 6. *CT Dynamics*

An oscillation can be generated by

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x$$

Show that the solution is

$$x(t) = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix} x(0)$$

(*Autograder will not be used for this question. It shall be graded manually.*)