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24-677 HW2

Q1

(1) $\left\{ \begin{bmatrix} -1 \\ -9 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right\}$

$\quad\; c_1 \qquad c_2 \qquad c_3$

$$-c_1 + c_2 + 2c_3 = 0$$
$$-9c_1 + 3c_2 - 2c_3 = 0$$
$$0c_1 + 0c_2 + c_3 = 0$$

$\boxed{c_3 = 0}$

$$-c_1 + c_2 = 0 \quad -(1)$$
$$-9c_1 + 3c_2 = 0 \quad -(2)$$

from (1) $c_1 = c_2$ put in (2)

$$9c_1 = 3c_2 \Rightarrow 3c_1 = c_2$$

$3c_1 = c_1$

$2c_1 = 0$

$c_1 = 0 = c_2$

$\boxed{c_1 = c_2 = c_3 = 0}$

so linearly independent

② $\left\{ \begin{bmatrix} 2-i \\ -i \end{bmatrix}, \begin{bmatrix} 1+2i \\ -i \end{bmatrix}, \begin{bmatrix} -i \\ 3+4i \end{bmatrix} \right\}$

$\quad c_1 \qquad\qquad c_2 \qquad\qquad c_3$

$(2-i)c_1 + (1+2i)c_2 + (-i)c_3 = 0$

$(-i)c_1 + (-i)c_2 + (3+4i)c_3 = 0$

$(2c_1 + c_2) + i(-c_1 + 2c_2 - c_3) = 0 + 0i$

$(3c_3) + i(-c_1 - c_2 + 4c_3) = 0 + 0i$

$-c_1 + 2c_2 - 0 = 0$

$2c_1 + c_2 = 0$ — ①

$c_1 = 2c_2$ — ③

$\boxed{3c_3 = 0}$

$\boxed{c_3 = 0}$

① becomes $4c_2 + c_2 = 0 \Rightarrow \boxed{c_2 = 0}$

put c_2 in ③ $c_1 = 2c_2 = \boxed{0 = c_1}$

$-c_1 - c_2 + 4c_3 = 0$

$\boxed{0 + 0 + 0 = 0}$

linearly independent

③ $\{2s^2+2s-1,\ -2s^2+2s+1,\ s^2-s-5\}$
$\quad c_1 \qquad\qquad c_2 \qquad\qquad c_3.$

$$c_1(2s^2+2s-1) + c_2(-2s^2+2s+1)$$
$$+ c_3(s^2-s-5) = 0$$

$$\underline{2c_1s^2} + 2c_1s - c_1 - \underline{2c_2s^2} + 2c_2s + c_2$$
$$+ \underline{c_3s^2} - c_3s - 5c_3 = 0$$

$$(2c_1 - 2c_2 + c_3)s^2 + (2c_1 + 2c_2 - c_3)s$$
$$+ (-c_1 + c_2 - 5c_3) = 0$$

for this equation to be 0 (zero)

$$2c_1 - 2c_2 + c_3 = 0 \quad -①$$
$$2c_1 + 2c_2 - c_3 = 0 \quad -②$$
$$-c_1 + c_2 - 5c_3 = 0 \quad -③$$

from ②
$$c_3 = 2c_1 + 2c_2 \quad -④$$

put in ① & ③

① $2c_1 \cancel{-2c_2} + 2c_1 \cancel{+2c_2} = 0$
$$4c_1 = 0 \qquad \boxed{c_1 = 0}$$

③ $-c_1 + c_2 - 5(2c_1 + 2c_2) = 0$

$$-c_1 + c_2 - 10c_1 - 10c_2 = 0$$
$$-11c_1 - 9c_2 = 0$$
$$-9c_2 = 0 \qquad \boxed{c_2 = 0}$$

put c_1 & c_2 in (4)

$$c_3 = 2\times 0 + 2\times 0 \quad = 0$$
$$\boxed{c_3 = 0}$$
$$\boxed{c_1 = c_2 = c_3 = 0}$$

So linearly independent

(4) $x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ $x_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ $x_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$ $x_4 = \begin{bmatrix} 3 \\ 5 \\ 5 \\ 7 \end{bmatrix}$

$$X = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 2 & 5 \\ 1 & 3 & 1 & 5 \\ 1 & 4 & 2 & 7 \end{bmatrix} \end{array}$$

$\det(X) = 0$

A set of vector is linearly independent iff the matrix made of the vectors has a non-zero determinant.

So, linearly dependant

Q2 **Norms.**

$$x_1 = \begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$$

1 norm $= \|x_1\|_1 = |2| + |-3| + |5|$

$= 2 + 3 + 5 = 10$

2 norm $= \|x_1\|_2 = \sqrt{(|2|)^2 + (|-3|)^2 + (|5|)^2}$

$= \sqrt{4 + 9 + 25}$

$= \sqrt{38}$

$= 6.16$

∞ norm $= \|x_1\|_\infty$

$= \max |x_i|$

$= \max (2, 3, 5)$

$= 5$

$$x_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$1 \text{ norm} = \|x_2\|_1 = |1| + |1| + |-1|$$
$$= 3$$

$$2 \text{ norm} = \|x_2\|_2 = \sqrt{(|1|)^2 + (|1|)^2 + (|-1|)^2}$$
$$= \sqrt{1+1+1}$$
$$= \sqrt{3}$$
$$= 1.73$$

$$\infty \text{ norm} = \|x_2\|_\infty = \max |x_i|$$
$$= \max(1, 1, 1)$$
$$= 1$$

Q3 $B = \{b_1, b_2\}$ $C = \{c_1, c_2\}$

$$b_1 = 6c_1 - 2c_2$$
$$b_2 = 9c_1 - 4c_2 \quad \text{--- } 1$$

I

$$[b_i]_c = \begin{bmatrix} A^{b_i}_{c_1} c_1 \\ A^{b_i}_{c_2} c_2 \end{bmatrix}$$

$$\therefore [b_1]_c = A^{b_1}_{c_1} c_1 + A^{b_1}_{c_2} c_2$$
$$[b_2]_c = A^{b_2}_{c_1} c_1 + A^{b_2}_{c_2} c_2 \quad \text{--- } 2$$

compare 1 & 2

$$b_1 \begin{bmatrix} 6 \\ -2 \end{bmatrix} \quad b_2 = \begin{bmatrix} 9 \\ -4 \end{bmatrix}$$

$$A_{B \to C} = \begin{bmatrix} 6 & 9 \\ -2 & -4 \end{bmatrix}$$

(2) $x = -3b_1 + 2b_2$

$$x = \begin{bmatrix} -3 \\ 2 \end{bmatrix} = [x]_B$$

for B to C

$$[x]_C = [A_{B\to C}][x]_B$$

$$= \begin{bmatrix} 6 & 9 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} -18 + 18 \\ +6 \ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$$

Q4

$$\begin{bmatrix} -1 & 2 & 3 & 5 \\ 1 & 1 & 2 & 5 \\ -5 & 7 & 10 & 15 \\ 7 & -8 & -11 & -15 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$R_3 \to R_3 - \left(\frac{+5}{+1}\right) R_1$

$R_4 \to R_4 - (-7) R_1$

$14 - 8$

6

$$\begin{bmatrix} -1 & 2 & 3 & 5 \\ 0 & 3 & 5 & 10 \\ 0 & -3 & -5 & -10 \\ 0 & 6 & 10 & 20 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$R_4 \to R_4 - \left(\frac{\cancel{6}^{\,2}}{\cancel{3}}\right) R_2$

$$\begin{bmatrix} -1 & 2 & 3 & 5 \\ 0 & 3 & 5 & 10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

columns corresponding to the pivots is the basis of W.

Basis: $\left[\begin{bmatrix} -1 \\ 1 \\ -5 \\ 7 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 7 \\ -8 \end{bmatrix} \right]$

$$Z = \begin{bmatrix} 3 \\ -1 \\ 13 \\ -17 \end{bmatrix}$$

$$\alpha \begin{bmatrix} -1 \\ 1 \\ -5 \\ 7 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 1 \\ 7 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 13 \\ -17 \end{bmatrix}$$

$-\alpha + 2\beta = 3 \Rightarrow \alpha = 2\beta - 3$ — (1)

$\alpha + \beta = -1 \Rightarrow 2\beta - 3 + \beta = -1$

$3\beta - 3 = -1$

$3\beta = 2$

$\boxed{\beta = 2/3}$

put β in (1)

$\alpha = 2 \times \frac{2}{3} - 3$

$= \frac{4-9}{3} = \boxed{\frac{-5}{3}} = \alpha$

yes, it is a subspace

$\Rightarrow -5\alpha + 7\beta = 13$

$-5 \times \frac{-5}{3} + 7 \times \frac{2}{3} = \frac{25}{3} + \frac{14}{3} = \frac{\cancel{39}}{\cancel{3}}$ 13

$\Rightarrow 7\alpha - 8\beta = -17$

$7 \times \frac{-5}{3} - 8 \times \frac{2}{3} = \frac{-35-16}{3} = -\frac{\cancel{51}}{\cancel{3}}$ $\boxed{-17}$

$$u = \begin{bmatrix} 4 \\ 9 \\ 12 \\ -8 \end{bmatrix} \qquad \alpha \begin{bmatrix} -1 \\ 1 \\ -5 \\ 7 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 1 \\ 7 \\ -8 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 12 \\ -8 \end{bmatrix}$$

$\rightsquigarrow$ $-\alpha + 2\beta = 4 \Rightarrow \alpha = 2\beta - 4$ — (1)

$\rightsquigarrow$ $\alpha + \beta = 9$

$2\beta - 4 + \beta = 9$

$3\beta = 13$

$\boxed{\beta = \frac{13}{3}}$

$\alpha = 2 \times \frac{13}{3} - 4$

$= \frac{26 - 12}{3}$

$\boxed{\alpha = \frac{14}{3}}$

$\rightsquigarrow$ $-5\alpha + 7\beta = 12$

$-5 \times \frac{14}{3} + \frac{7 \times 13}{3} = \frac{-70 + 91}{3} = \frac{\cancel{21}}{\cancel{3}}$ 7 $\neq 12$

$\rightsquigarrow$ $7\alpha - 8\beta = -8$

$7 \times \frac{14}{3} - 8 \times \frac{13}{3} = \frac{98 - 104}{3} = \frac{-\cancel{6}}{\cancel{3}}$ -2 $\neq -8$

So $\alpha = \beta = 0$

is not a subspace.

$$v = \begin{bmatrix} -1 \\ -1 \\ -3 \\ 3 \end{bmatrix} \qquad \alpha \begin{bmatrix} -1 \\ 1 \\ -5 \\ 7 \end{bmatrix} + \beta \begin{bmatrix} 2 \\ 1 \\ 7 \\ -8 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -3 \\ 3 \end{bmatrix}$$

$\rightsquigarrow -\alpha + 2\beta = -1 \implies \alpha = 2\beta + 1$

$\rightsquigarrow \alpha + \beta = -1 \qquad \alpha = 2 \times \frac{-2}{3} + 1$

$2\beta + 1 + \beta = -1 \qquad = \frac{-4}{3} + 1$

$3\beta = -2 \qquad = \frac{-4+3}{3} = \boxed{\frac{-1}{3} = \alpha}$

$\boxed{\beta = \frac{-2}{3}}$

$\rightsquigarrow -5\alpha + 7\beta = -3$

$-5 \times \frac{-1}{3} + 7 \times \frac{-2}{3} = \frac{5}{3} - \frac{14}{3} = \frac{-9}{3} = \boxed{-3}$

$\rightsquigarrow 7\alpha - 8\beta = 3$

$7 \times \left(-\frac{1}{3}\right) - 8\left(-\frac{2}{3}\right) = \frac{-7}{3} + \frac{16}{3} = \frac{9}{3} = \boxed{3}$

Yes, it is a subspace

Q5

$$x_1 = \begin{bmatrix} 1 \\ 12 \\ 18 \end{bmatrix} \qquad x_2 = \begin{bmatrix} 25 \\ 37 \\ 11 \end{bmatrix}$$

$$\langle x_1, x_2 \rangle = x_1^T x_2$$

$$= \begin{bmatrix} 1 & 12 & 18 \end{bmatrix} \begin{bmatrix} 25 \\ 37 \\ 11 \end{bmatrix}$$

$$= 25 + 12 \times 37 + 198$$

$$= 25 + 444 + 198$$

$$= 667$$