Q1

1) $y(t) = 0$ for all t

$y_1(t) = 0$

$y_2(t) = 0$

$u(t) = \alpha u_1(t) + \beta u_2(t)$

$y(t) = 0$

$= 0$

$y_1(t) + y_2(t) = y(t)$

linear

$u_a(t) = u(t-a)$

$y_a(t) = 0$

time invariant

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24-677 HW 1

② $y(t) = u^3(t)$

$y_1(t) = u_1^3(t)$

$y_2(t) = u_2^3(t)$

$u(t) = \alpha u_1(t) + \beta u_2(t)$

$y(t) = u^3(t)$

$y(t) = \left(\alpha u_1(t) + \beta u_2(t)\right)^3 \quad — ①$

$y_1(t) + y_2(t) = u_1^3(t) + u_2^3(t) — ②$

① ≠ ②   so non-linear

$u_a(t) = u(t-a)$

$y_a(t) = u_a^3(t)$

$y_a(t) = u^3(t-a)$

$= y(t-a)$ time invariant

(3) $y(t) = u(3t)$

$$y_1(t) = u_1(3t)$$
$$y_2(t) = u_2(3t)$$

$$u(t) = \alpha u_1(t) + \beta u_2(t)$$

$$y(t) = u(3t)$$
$$= \alpha u_1(3t) + \beta u_2(t)$$
$$y(t) = \alpha y_1(t) + \beta y_2(t)$$

linear

$$u_a(t) = u(t-a)$$

$$y_a(t) = u_a(3t)$$
$$= u(3t-a)$$
$$\neq y(t-a)$$

time variant

(4) $y(t) = e^{-t} u(t-T)$

$y_1(t) = e^{-t} u_1(t-T)$

$y_2(t) = e^{-t} u_2(t-T)$

$u(t) = \alpha u_1(t) + \beta u_2(t)$

$$y(t) = e^{-t} u(t-T)$$
$$= e^{-t}(\alpha u_1(t-T) + \beta u_2(t-T))$$

$$= e^{-t} \alpha u_1(t-T) + e^{-t} \beta u_2(t-T)$$

$$\boxed{y(t) = \alpha y_1(t) + \beta y_2(t)}$$

linear

$u_a(t) = u(t-a)$

$y(t) = e^{-t} u(t-T)$

$y(t) = e^{-t} u((t-a)-T)$
$\neq y(t-a)$

Time variant

(5) $y(t) = u(t-1)$

$y_1(t) = u_1(t-1)$

$y_2(t) = u_2(t-1)$

$u(t) = \alpha u_1(t) + \beta u_2(t)$

$y(t) = u(t-1)$

$= \alpha u_1(t-1) + \beta u_2(t-1)$

$\underline{\underline{y(t) = \alpha y_1(t) + \beta y_2(t)}}$

linear

$u_a(t) = u(t-a)$

$y_a(t) = u_a(t-1)$

$= u((t-a)-1)$

$y_a(t) = y(t-a)$

$\underline{\text{time invariant}}$

⑥ $y(t) = \int_{-\infty}^{t} u(T)\,dT$

$y_1(t) = \int_{-\infty}^{t} u_1(T)\,dT = u_1(t)$

$y_2(t) = \int_{-\infty}^{t} u_2(T)\,dT = u_2(t)$

$\alpha y_1(t) + \beta y_2(t) = \alpha u_1(t) + \beta u_2(t)$

$y_3(t) = \alpha y_1(t) + \beta y_2(t)$
$= \alpha u_1(t) + \beta u_2(t)$

so linear

$u_a(t) = u(t-a)$

$y_a(t) = u_a(t)$
$= u(t-a)$
$= y(t-a)$ so time invariant

(7) $y(t) = \int_{-\infty}^{t} u^2(t)\, dT$

$y_1(t) = \int_{-\infty}^{t} u_1^2(t)$

$y_2(t) = \int_{-\infty}^{t} u_2^2(t)$

$u_3(t) = \left(\alpha u_1(t) + \beta u_2(t)\right)^2$

$\alpha y_1(t) + \beta y_2(t) = \alpha u_1^2(t) + \beta u_2^2(t)$

not equal to
non-linear

$u_a(t) = u(t-a)$

$y_a(t) = u^2(t-a)$

$= y(t-a)$

to time
invariant

(8) $y(t) = \begin{cases} 0 & t \le 0 \\ u(t) & t > 0 \end{cases}$

time variant

$a(t)\, 0 + b(t)\, u(t)$

$a(t) = \begin{cases} 1 & t \le 0 \\ 0 & t > 0 \end{cases}$ $b(t) = \begin{cases} 0 & t \le 0 \\ 1 & t > 0 \end{cases}$

$\alpha\, a(t)\, 0 + \alpha\, b(t) u(t)$

$\beta\, a(t)\, 0 + \beta\, b(t)\, u(t)$

$\alpha\, b(t)\, u(t) + \beta\, b(t)\, u(t)$

linear

Q2.

① N^* a field?
↳ natural number (1, 2, 3 ....)
Does not follow A_4 (additive inverse)

② Q is a field
↳ Rational number

is a field, follows all the conditions.

③ Set of polynomials

Does not follow M_4

$x^{-1} \notin F$ $\frac{1}{x^2+x+3}$ is not a polynomial.

④ set of rational functions with real coefficients
is a field, follows all the conditions

Q3

1. $n \times n$ skew symmetric matrices

(a) yes is a sub set
(b) yes is a space

⇒ Is a subspace

2. $n \times n$ diagonal matrix

(a) yes is a sub set
(b) yes is a space

⇒ Is a subspace

3. $n \times n$ upper diagonal matrix

(a) yes is a sub set
(b) yes is a space

⇒ Is a subspace

(4) $n \times n$ singular matrix

(a) Yes is a subset

(b) No not a space

eg: $\begin{bmatrix} 0 & 0 \\ 0 & x \end{bmatrix} + \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix} = \begin{bmatrix} x & 0 \\ 0 & x \end{bmatrix}$

which is not a singular matrix

doesn't follow Ao

Q4 $\ddot{y} + (1+y)\dot{y} - 2y + 0.5y^3 = 0$

let

$x_1 = y \qquad \therefore \quad \dot{x}_1 = \dot{y} = x_2$

$x_2 = \dot{y} \qquad \dot{x}_2 = \ddot{y}$

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$

$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ 2x_1 - (1+x_1)x_2 - 0.5x_1^3 \end{bmatrix} = 0 \quad (\equiv m)$

$\therefore \quad \dot{x}_1 = 0 = \dot{y} = x_2$

$2x_1 - (1+x_1)0 - 0.5x_1^3 = 0$

$2x_1 - 0.5x_1^3 = 0$

$x_1(2 - 0.5x_1^2) = 0$

$2 - 0.5x_1^2 = 0$

$2 = \frac{1}{2}x_1^2$

$4 = x_1^2 \qquad x_1 = \pm 2$ let it be $\underline{2}$

$$\frac{df}{dx} = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} \\ \frac{df_2}{dx_1} & \frac{df_2}{dx_2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 2 - x_2 - \frac{3}{2} x_1^2 & -(1-x_1) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -4 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Q5

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} +$$

$$\underset{(3\times4)}{\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \end{bmatrix}} \underset{(4\times1)}{u(t)}$$

$$[y_1\ y_2]^T = \underset{(2\times3)}{[B]} \underset{3\times1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} + \underset{(2\times4)}{[D]} \underset{(4\times1)}{u(t)}$$

$\left\{\text{from } u(t)\right\}$ inputs = 4

$B = 2\times3$

$\left\{\text{from } \dot{x}\right\}$ states = 3

$D = 2\times4$

$\left\{\text{from } y\right\}$ outputs = 2

Q6

$$(m+M)\ddot{x} + ml\ddot{\theta} - ml\dot{\theta}^2 \sin\theta = f \quad -(1)$$

$$(I+ml^2)\ddot{\theta} + ml\ddot{x}\cos\theta - mgl\sin\theta = \phi \quad -(2)$$

$$\ddot{x} = \frac{f - ml\ddot{\theta} + ml\dot{\theta}^2 \sin\theta}{m+M} \quad -(3)$$

$$\ddot{\theta} = \frac{mgl\sin\theta - ml\ddot{x}\cos\theta}{(I+ml^2)} \quad -(4)$$

(3) in (2) & (4) in (1)

$$(I+ml^2)\ddot{\theta} + ml\cos\theta\left[\frac{f - ml\ddot{\theta} + ml\dot{\theta}^2 \sin\theta}{m+M}\right] - mgl\sin\theta = 0$$

$$(I+ml^2)\ddot{\theta} + \frac{ml\cos\theta f}{m+M} - \frac{m^2l^2\cos\theta\ddot{\theta}}{m+M} + \frac{m^2l^2\dot{\theta}^2\sin\theta\cos\theta}{m+M}$$

$$- mgl\sin\theta = 0$$

$$\ddot{\theta}\left\{(I+ml^2) - \frac{m^2l^2\cos\theta}{m+M}\right\} = mgl\sin\theta - \frac{ml\cos\theta f}{m+M} - \frac{m^2l^2\dot{\theta}^2\sin\theta\cos\theta}{m+M} \quad -(5)$$

4 in 1

$$(m+M)\ddot{x} + ml\left[\frac{mgl\sin\theta - ml\ddot{x}\cos\theta}{(I+ml^2)}\right] - ml\dot{\theta}^2\sin\theta = f$$

$$(m+M)\ddot{x} + \frac{m^2l^2g\sin\theta}{(I+ml^2)} - \frac{m^2l^2\ddot{x}\cos\theta}{(I+ml^2)} - ml\dot{\theta}^2\sin\theta = f$$

$$\ddot{x}\left\{(m+M) - \frac{m^2l^2\cos\theta}{(I+ml^2)}\right\} = f + ml\dot{\theta}^2\sin\theta - \frac{m^2l^2g\sin\theta}{(I+ml^2)}$$

$$\ddot{x} = \frac{f + ml\dot{\theta}^2\sin\theta - \frac{m^2l^2g\sin\theta}{(I+ml^2)}}{(m+M) - \frac{m^2l^2\cos\theta}{(I+ml^2)}} \quad - 6$$

$$\ddot{\theta} = \frac{mgl\sin\theta - \frac{ml\cos\theta\, f}{m+M} - \frac{m^2l^2\dot{\theta}^2\sin\theta\cos\theta}{m+M}}{(I+ml^2) - \frac{m^2l^2\cos\theta}{m+M}} \quad - 7$$

$$X = \begin{bmatrix} x \\ \dot{x} \\ \theta \\ \dot{\theta} \end{bmatrix} \qquad \dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta} \\ \ddot{\theta} \end{bmatrix}$$

$x_1 = x \qquad x_4 = \dot{\theta}$

$x_2 = \dot{x}$

$x_3 = \theta$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad \dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = 0$$

{for equillibrium}

$\dot{x}_1 = 0 = x_2$

$\dot{x}_3 = 0 = x_4$

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{\partial f_2}{\partial \theta} & \frac{\partial f_2}{\partial \dot{\theta}} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{\partial f_4}{\partial \theta} & \frac{\partial f_4}{\partial \theta} \end{bmatrix}$$

$\frac{df}{dx}$

(6) putting values $m, M, I, l = 1 \quad g = 10$

$$\frac{F \times 2 + \dot{\theta}^2 \sin\theta \times 2 - 10 \sin\theta}{2 \times 2 - \cos\theta}$$

$$\ddot{x} = \frac{2F + 2\dot{\theta}^2 \sin\theta - 10 \sin\theta}{4 - \cos\theta}$$

$$\frac{df_2}{d\theta} = \frac{\cancel{2\dot{\theta}^2}\cos(\theta) - 10\cos(\theta)}{4 - \cos(\theta)} -$$

$$\frac{\cancel{\sin(\theta)}\left(2\dot{\theta}^2\sin(\theta) - 10\sin(\theta) + 2f\right)}{(4-\cos(\theta))^2}$$

$\theta = 0$

$\dot{\theta} = x_4 = 0$

$$= \frac{-10\cos(0)}{4 - \cos(0)}$$

$$= \frac{-10}{4-1} = \boxed{\frac{-10}{3}}$$

$$\frac{df_2}{d\dot{\theta}} = \frac{4\,\cancel{\sin(\theta)}\,\dot{\theta}}{4-\cos(\theta)} = 0$$

(7) $m, M, l, I = 1 \quad g = 10$

$$= \frac{10\times 2\sin\theta - \cos\theta f - \dot{\theta}^2\sin\theta\cos\theta}{2\times 2 - \cos\theta}$$

$$f_4 = \frac{20\sin\theta - f\cos\theta - \dot{\theta}^2\sin\theta\cos\theta}{4-\cos\theta}$$

$$\frac{df_4}{d\theta} = \frac{\cancelto{0}{\dot{\theta}^2 \sin^2\theta} + \cancelto{0}{f\sin\theta} - \cancelto{0}{\dot{\theta}^2 \cos^2\theta} + 20\cos\theta}{4-\cos(\theta)} - \frac{\cancelto{0}{\sin(\theta)}\left(-\dot{\theta}^2\cos(\theta)\sin\theta + 20\sin\theta - f\cos\theta\right)}{(4-\cos(\theta))^2}$$

$$= \frac{20\cos\theta}{4-\cos\theta} = \frac{20}{3}$$

$$\frac{df_4}{d\dot{\theta}} = -\frac{2\cos(\theta)\cancelto{0}{\sin(\theta)}\cancelto{0}{\dot{\theta}}}{4-\cos(\theta)}$$

$$= 0$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{10}{3} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{20}{3} & 0 \end{bmatrix}$$

$$B = \frac{df}{du} = \begin{bmatrix} 0 \\ \frac{2}{4-\cos\theta} \\ 0 \\ -\frac{\cos\theta}{4-\cos\theta} \end{bmatrix} \approx \begin{bmatrix} 0 \\ \frac{2}{3} \\ 0 \\ -\frac{1}{3} \end{bmatrix}$$

used derivative-calculator.net for derivative of the terms.

Collabrated / Discussed with (Q 3,6)

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