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Homework 4

16-720 A
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Problem 1

Q 1.1

Consider the fundamental matrix F of 3x3 dimension as:

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

We know that $x^T F x = 0$, we can assume $x = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$. Therefore,

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{13} \\ f_{23} \\ f_{33} \end{bmatrix} = 0$$

$$f_{33} = 0$$

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Q 1.2

Pure translation in the direction of x-axis:

$$t = \begin{bmatrix} t_x \\ 0 \\ 0 \end{bmatrix}$$

As there is no rotation, rotation matrix can be considered as 3x3 identity matrix. So, we can combine the rotation and translation as $E = txR$:

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

We know,

$$p_r^T(tRp_l) = 0$$

We know, equations for epipolar lines are as:

$$l_1 = p_l^T E \tag{1}$$

$$l_1 = \begin{bmatrix} x_1 & y_1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

$$l_1 = t_x - t_x y_1$$

$$l_2 = p_r^T E^T \tag{2}$$

$$l_2 = \begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & t_x \\ 0 & -t_x & 0 \end{bmatrix}$$

$$l_2 = -t_x + t_x y_1$$

From equations l_1 and l_2m we can say that the epipole lines are parallel to x-axis.

Q 1.3

Let's assume two instances as:

$$p_1 = R_1 P + t_1 \tag{3}$$

$$p_2 = R_2 P + t_2 \tag{4}$$

Using equation (3), we find an equation for P as:

$$P = R_1^{-1}(p_1 - t_1)$$

Using the equation for P in equation (4), we get:

$$p_2 = R_2(R_1^{-1}(p_1 - t_1)) + t_2$$

$$p_2 = R_2 R_1^{-1} p_1 - R_2 R_1^{-1} t_1 + t_2$$

We can simplify the above equation as:

$$p_2 = R_{com} p_1 + t_{com}; \quad R_{com} = R_2 R_1^{-1}, t_{com} = -R_2 R_1^{-1} t_1 + t_2$$

As we know,

$$E = tR$$

$$E = (-R_2 R_1^{-1} t_1 + t_2)(R_2 R_1^{-1})$$

Equation for F can be write as:

$$F = K^{-T} E K^{-1}$$

$$F = K^{-T}(-R_2 R_1^{-1} t_1 + t_2)(R_2 R_1^{-1}) K^{-1}$$

$$F = K^{-T} t_{com} R_{com} K^{-1}$$

Q 1.4

Let 2D image points for an object, O can be denoted by x_1, x_2 and $x_1^{'}, x_2^{'}$ for $O^{'}$ (reflection of object O). Therefore, we have:

$$x_1^T F x_2 = 0$$

$$x_2^T F^T x_1 = 0$$

Adding the equations:

$$x_1^T F x_2 + x_2^T F^T x_1 = 0$$

We know, If T is a transition matrix, $M_2 = TM_1$. Also, $x_1 = KM_1P$ and $x_2 = KTM_1P$, we get:

$$K^T M_1^T P^T F K T M_1 P + K^T T^T M_1^T P^T F^T K M_1 P = 0$$

Simplify using,

$$K^T F K + K^T F^T K = 0$$

We get:

$$F + F^T = 0$$

$$F = -F^T$$

Therefore, from the above we can conclude F is skew-symmetric.

Problem 2

Q 2.1

The recovered **F** matrix is given below and visualization of epipolar lines is shown in figure (1).

```
[[ 9.80213865e-10 -1.32271663e-07  1.12586847e-03]
 [-5.72416248e-08  2.97011941e-09 -1.17899320e-05]
 [-1.08270296e-03  3.05098538e-05 -4.46974798e-03]]
```

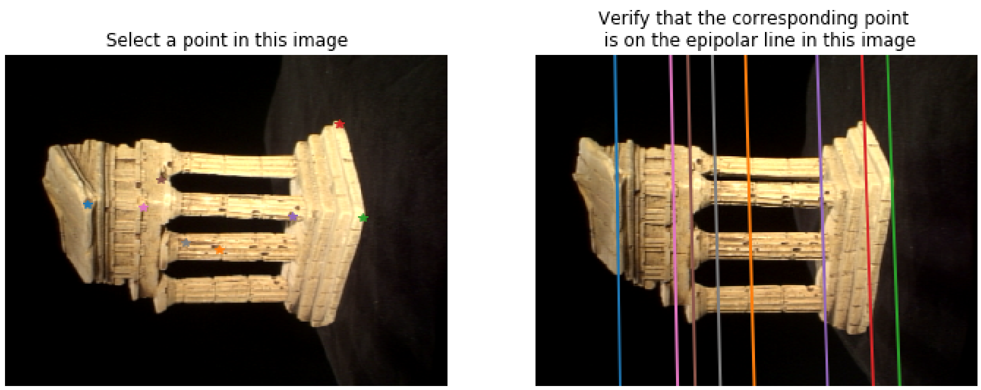


Figure 1: Visualizing epipolar lines using displayEpipolarF in helper.py

Problem 3

Q 3.1

The estimated **E** matrix using **F** from Q 2.1 is shown below,

```
[[ 2.26587821e-03 -3.06867395e-01  1.66257398e+00]
 [-1.32799331e-01  6.91553934e-03 -4.32775554e-02]
 [-1.66717617e+00 -1.33444257e-02 -6.72047195e-04]]
```

Q 3.2

Let C_{1i} and C_{2i} represent i^th row of C_1 and C_2 respectively. In this, each C_{1i} and C_{2i} is a row matrix of size 1x4. Also, assume $x1_i$ and $y1_i$, represent x and y coordinate of the i^th point from set of points 1 and similarly, $x2_i$ and $y2_i$ for points 2. Using the above notation, we can write A matrix as:

$$A = \begin{bmatrix} x1_iC_{13} - C_{11} \\ y1_iC_{13} - C_{12} \\ x2_iC_{23} - C_{21} \\ y2_iC_{23} - C_{22} \end{bmatrix}$$

Problem 4

Q 4.1

With window size set to 10, the visualization of epipolarMatchGUI is shown below.

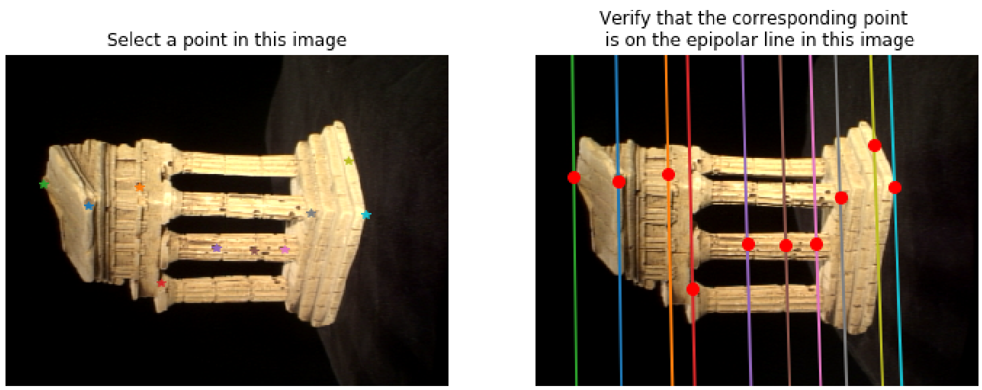


Figure 2: Visualization of epipolar correspondences

Q 4.2

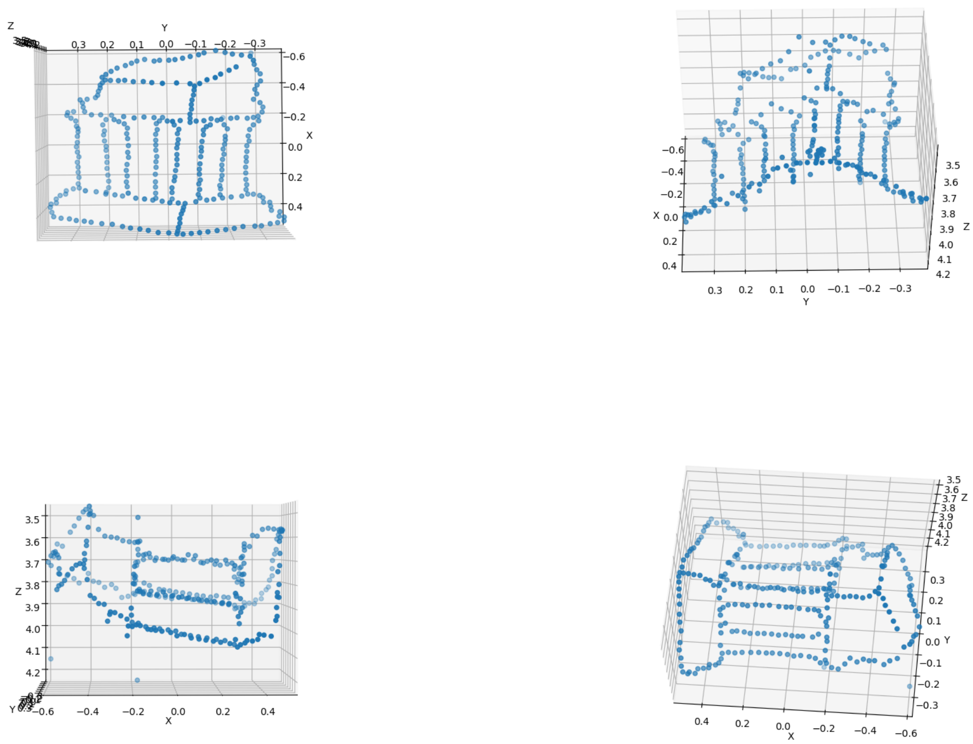


Figure 3: Point Cloud of the temple

Problem 5

Q 5.1

Results of epipolar correspondences without and with ransac are shown in figure x and y below.

Figure 4: Epipolar correspondences without ransac

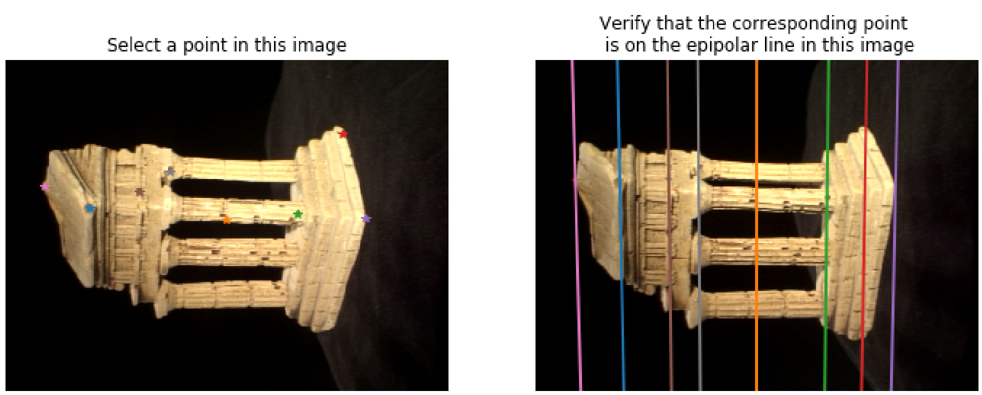


Figure 5: Epipolar correspondences with ransac

For finding fundamental matrix using Ransac, following approach was adopted. We know, the equation for fundamental matrix is:

$$x_2^T F x_1 = 0$$

A error value was calculated using the equation below to set a tolerance to determine if a point is inlier:

$$error = abs(x_{2i}^T F x_{1i})$$

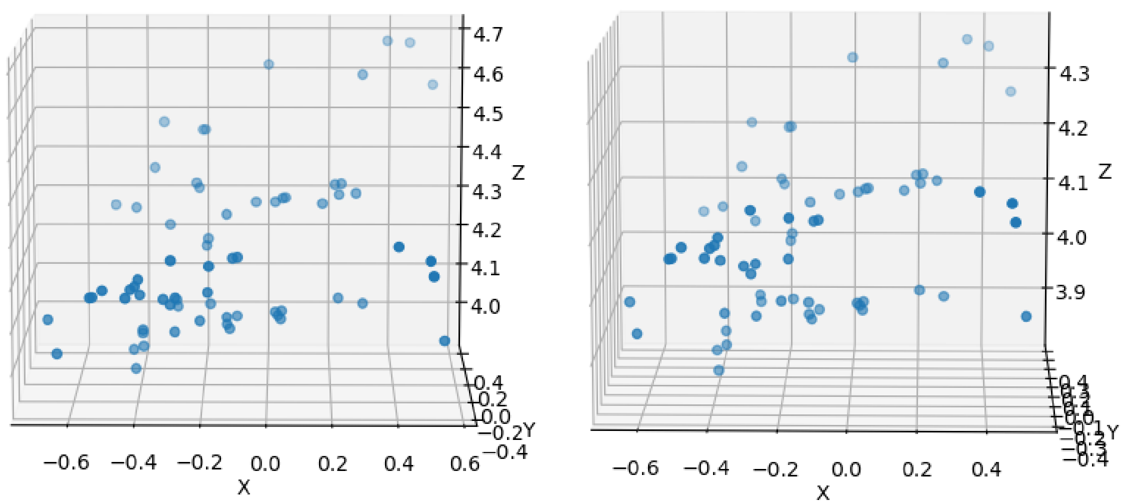
To tune the ransac parameters like tolerance and number of iterations. I observed that increasing the tolerance value allows more points to be inliers and inlier count drops with drop in tolerance. For the number of iterations, Higher the number of iterations, higher are the chances of finding optimal fundamental matrix. In our case, it is able to find a good F matrix in around 200 iterations.

I ran the ransac function with 1000 iterations and 0.5 tolerance and obtained the following fundamental matrix F.

```
[[ 1.55129727e-08, -1.33013862e-08, -1.15667262e-03]
[ 1.99834980e-07,  1.65742475e-09, -2.71655895e-05]
[ 1.10458826e-03,  1.09625130e-05,  4.67986435e-03]]
```

Q 5.3

The error for without adjustment using original P and M matrix was at 1352.62. After using bundle adjustment and optimizing extrinsic matrix for camera 2, the final error dropped to 1.8 which is clear way better than the original. The scatter plot for the point cloud without and with adjustment is shown in Figure 6.



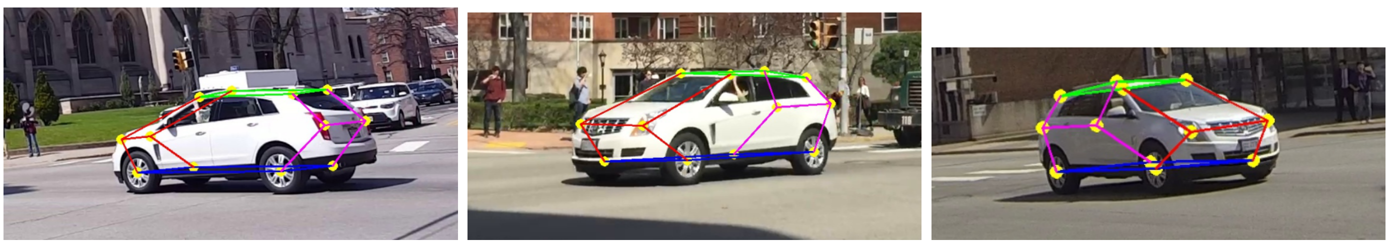
(a) Without bundle adjustment (b) Without bundle adjustment

Figure 6: without and with bundle adjustment point cloud

Problem 6

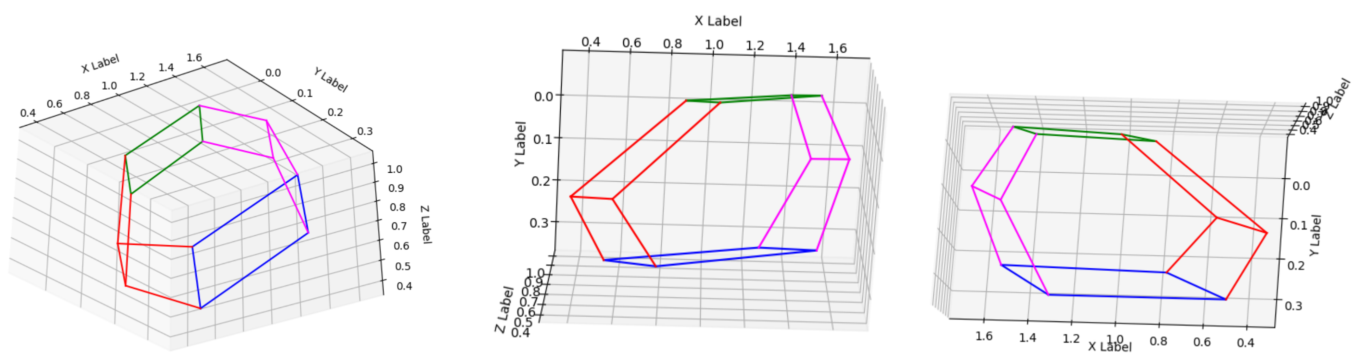
Q 6.1

In MultiviewReconstruction function, I have extracted the points with more than threshold confidence and triangulation is performed. In the existing triangulation, we had A matrix as 4x4 as now we have 3 view points, we need to augment the A matrix to handle 3rd view points. Two new rows are augumented in the A matrix and triangulation is performed. Also, lower the threshold for confidence more points contribute to reconstruction so lower value is preferred but setting it too low might also result in considering even the noisy points. If we set the threshold to be too high, we will remove most of the points and which will result in poor or no reconstruction. I'm able to get 3D reconstruction of the points and the results are shown below.



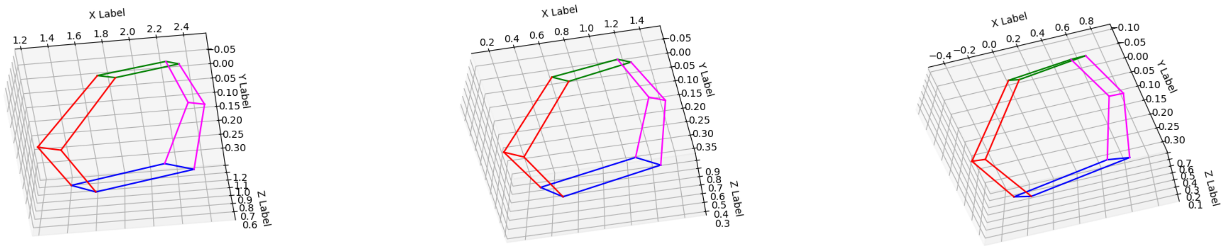
(a) cam 1 (b) cam 2 (c) cam 3

Figure 7: Time stamp 5 with different cam views with visualized points



(a) approx cam 1 (b) approx cam 2 (c) approx cam 3

Figure 8: Time stamp 5 Multiview reconstruction results



(a) Time stamp 1 (b) Time stamp 6 (c) Time stamp 9

Figure 9: Multiview reconstruction for different time stamps

Q 6.2

Results are shown below:

(a) Time stamp 0 (b) Time stamp 9

Figure 10: Views for cam2 for time stamp 0 and 9

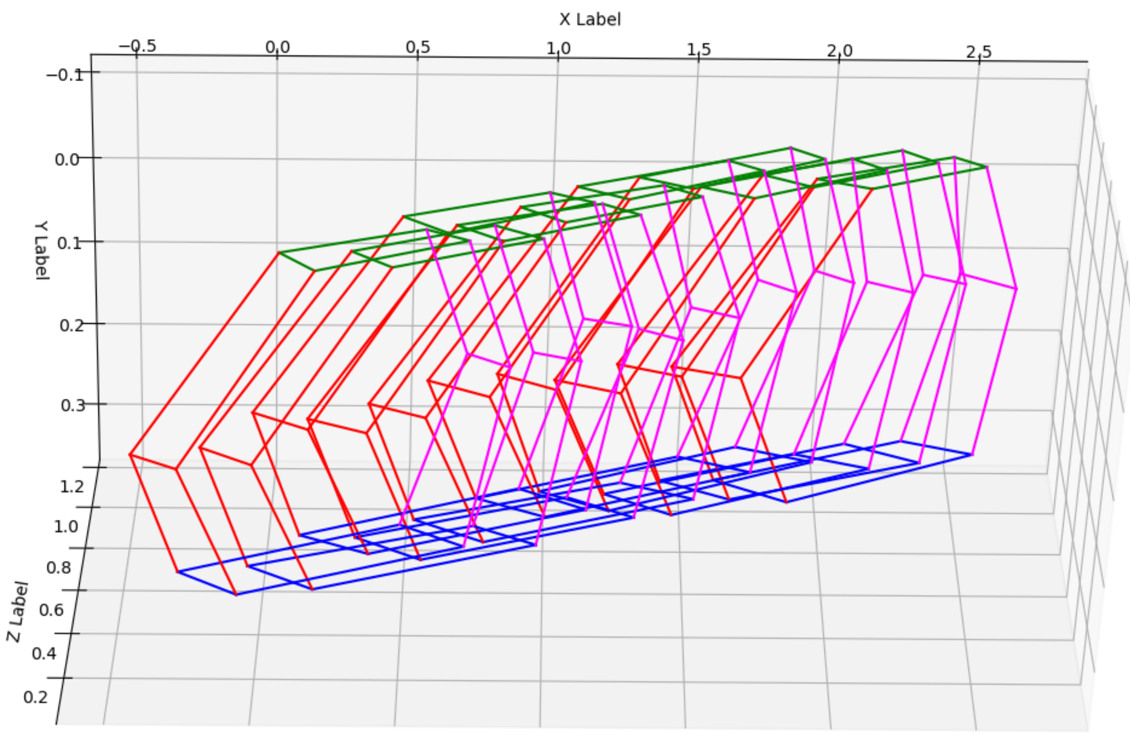


Figure 11: 3D reconstruction for all time stamps