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Homework 6

16-720 A
May 5, 2020

Problem 1

Q 1.a

Let's assume, n as surface normal, v as viewing direction, l as light direction and θ as angle between the surface normal and the light direction.

We can define n-dot-l as the product between the surface normal and light direction. In the product, both the vectors are considered normalized. From comparison, we can see the n-dot-l product is same as cosine of angle between two vectors.

As we are dealing with Lambertian surface, these surfaces have a constant Bidirectional Reflectance Distribution Function (BRDF) of value $\frac{\rho_d}{\pi}$, where ρ_d is known as albedos.

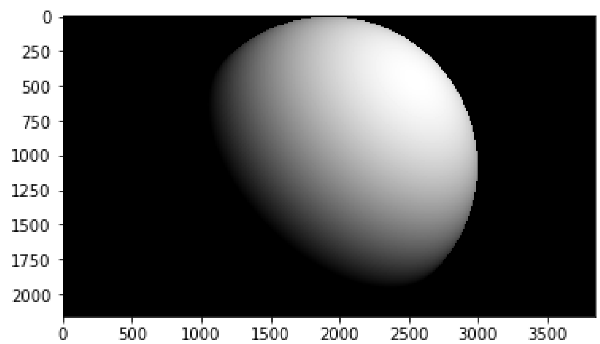
The dot product comes from Lambert's Cosine Law, using this we can write the final equation for surface radiance as:

$$L = \frac{\rho_d}{\pi} I cos(\theta_i)$$

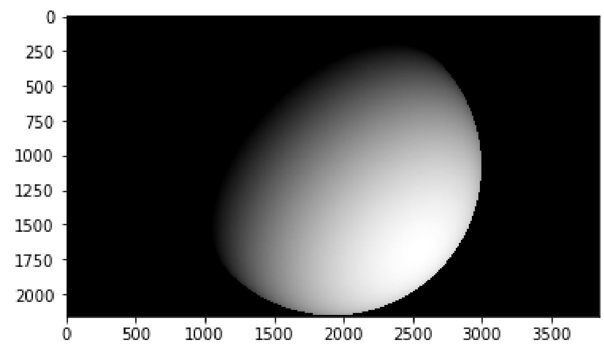
$$L = \frac{\rho_d}{\pi} I \hat{n}.\hat{s}$$

The viewing angle doesn't not matter because of the constant BRDF value of the surface. Due to constact BRDF, the surface appears equally bright in all the directions which makes surface brightness independent of the viewing angle i.e independent of v (viewing direction).

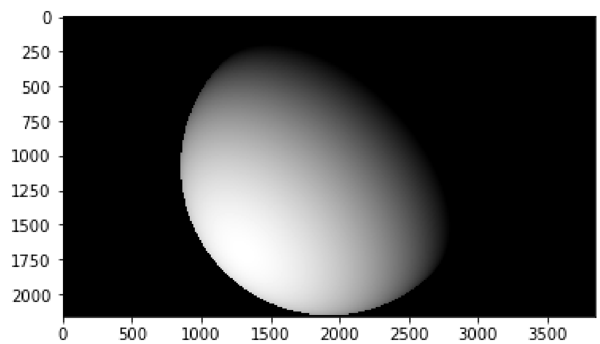
Q 1.b



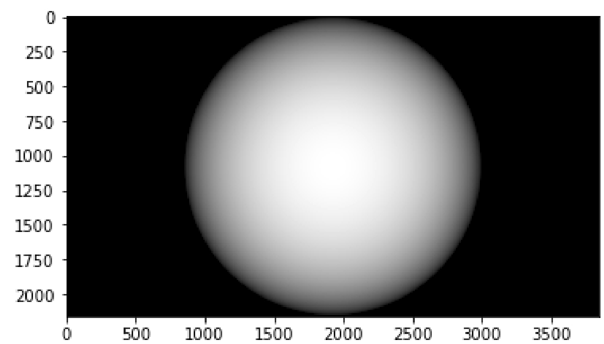
(a) L = (1,1,1)/$\sqrt{3}$



(b) L = (1,-1,1)/$\sqrt{3}$



(c) L = (-1,-1,1)/$\sqrt{3}$



(d) L = (0,0,1)

Figure 1: Output from renderNDotLSphere using different light direction and radius as 0.75cm

Q 1.d

In the equation:

$$I = L^T B$$

The dimension of L is 3×7 and B is $3 \times P$. Therefore, the dimension of I is $7 \times P$. We perform SVD on the matrix I, and get singular values of I as:

```
[79.36348099 13.16260676  9.22148403  2.41472899  1.61659626  1.26289066
  0.89368301]
```

From the singular values of I, we can say the **rank of I is 7**. No, the singular values do not agree with the rank-3 requirement. If more data than the required 3 and viewpoints that have independent light source directions, the system becomes an overdetermined system and results in higher rank of I.

Q 1.e

We have equation for I as:

$$I = L^T B$$

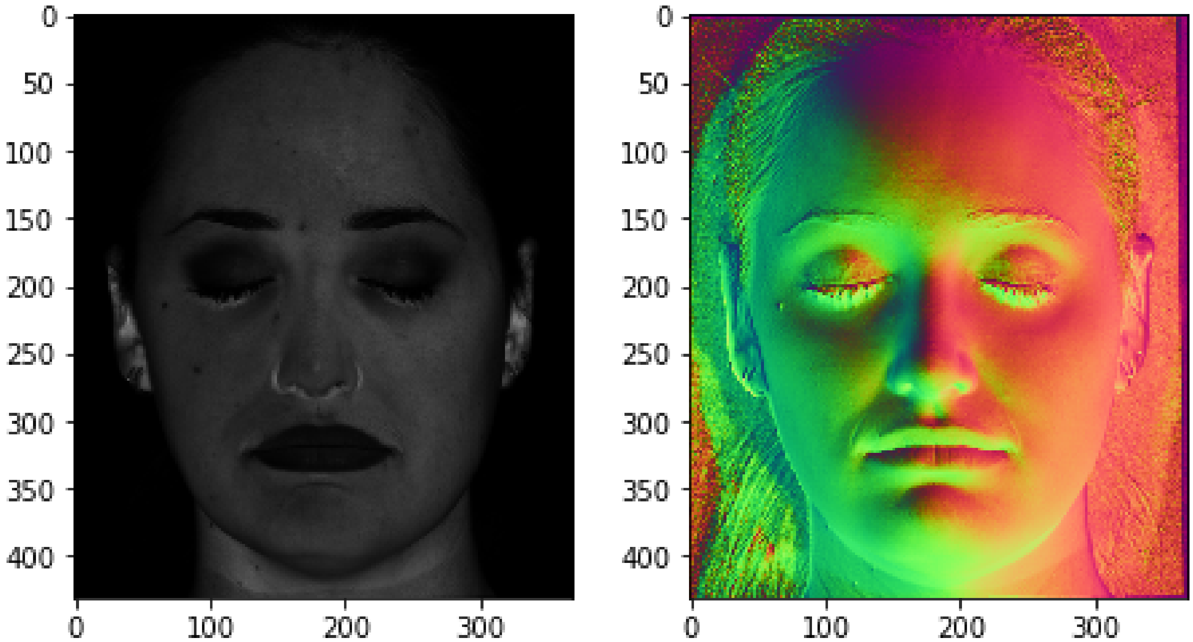
We can easily write the equation in the form of $Ax = y$, compare we get,

$$A = L^T$$

$$x = B$$

$$y = I$$

Q 1.f



(a) Albedos (b) Normals

Figure 2: Output from displayAlbedosNormals

From the albedos image, we can see that areas near the nose boundaries, ears and under eyes, we observe higher brightness. This higher brightness might be because of Global Inter-reflection, that occurs due to curved surfaces where light bounces through multiple points before coming out.

From the normals image, as explained before, there are some differences near the nose, ears, chin etc. and hence, the curvature of the face is different than the original image.

Q 1.g

From the question, we know that the surface can be represented as (x, y, f(x,y)), in which f(x,y) is z value and is depth of the surface. Also, the partial derivatives or tangents are described as:

$$f_x = \frac{\partial z}{\partial x} \qquad f_y = \frac{\partial z}{\partial y}$$

We know that the normal is perpendicular to all tangents i.e in the direction of $(-f_x, -f_y, 1)^T$. Therefore to get the direction of the normal, we can compute normal as:

$$normal = \frac{(-f_x, -f_y, 1)^T}{\sqrt{f_x^2 + f_y^2 + 1^2}}$$

According to the question, normal = (n_1, n_2, n_3), therefore, we can split the normals in (n_1, n_2, n_3) as:

$$n_1 = \frac{-f_x}{\sqrt{f_x^2 + f_y^2 + 1^2}}$$

$$n_2 = \frac{-f_y}{\sqrt{f_x^2 + f_y^2 + 1^2}}$$

$$n_3 = \frac{1}{\sqrt{f_x^2 + f_y^2 + 1^2}}$$

Using the above equations, we can find a relationship for f_x and f_y as:

$$f_x = \frac{-n_1}{n_3}$$

$$f_x = \frac{\partial z}{\partial x} = \frac{-n_1}{n_3}$$

$$f_y = \frac{-n_2}{n_3}$$

$$f_y = \frac{\partial z}{\partial y} = \frac{-n_2}{n_3}$$

Therefore, from above results, we can say that normal (n) is related to the partial derivatives of f at $f(x, y)$.

Q 1.h

We know,

$$g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$g_x(x_i, y_j) = g(x_{i+1}, y_j) - g(x_i, y_j)$$
$$g_y(x_i, y_j) = g(x_i, y_{j+1}) - g(x_i, y_j)$$

Using the above matrix and equations, we can get g_x and g_y as:

$$g_x = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad g_y = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{bmatrix}$$

The reconstructed matrix using the procedure one, Use g_x to construct the first row of g, then use g_y to construct the rest of g, we get:

$$g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

The reconstructed matrix using the procedure two, Use g_y to construct the first column of g, then use g_x to construct the rest of g, we get:

$$g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

We get exactly the same reconstruction matrices using both the procedures.

For non-integrability, let's assume g_x and g_y as:

$$g_x = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \qquad g_y = \begin{bmatrix} 4 & 4 & 4 & 4 \\ 4 & 0 & 4 & 4 \\ 0 & 4 & 4 & 4 \\ 0 & 0 & 4 & 4 \end{bmatrix}$$

The reconstructed matrix using the procedure one, Use new g_x to construct the first row of g, then use new g_y to construct the rest of g, we get:

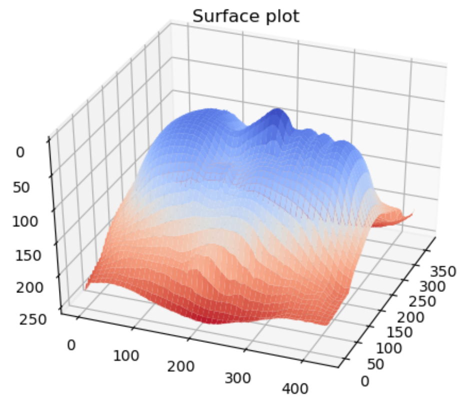
$$g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 2 & 7 & 8 \\ 5 & 6 & 11 & 12 \\ 5 & 6 & 15 & 16 \end{bmatrix}$$

The reconstructed matrix using the procedure two, Use new g_y to construct the first column of g, then use new g_x to construct the rest of g, we get:

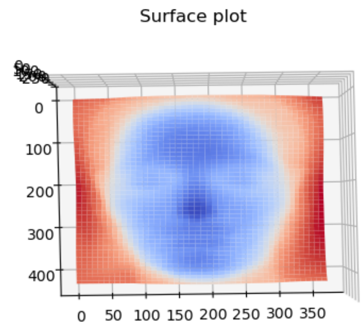
$$g = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 5 & 5 & 6 \\ 5 & 6 & 6 & 7 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$

From above reconstructed g matrices are different and this is because I have set the gradients are discontinuous whereas previous the gradients were continuous and results in same reconstruction.

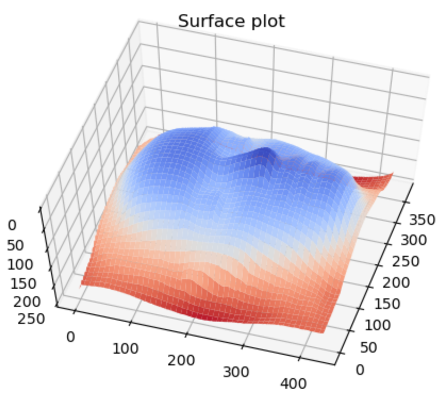
Q 1.i



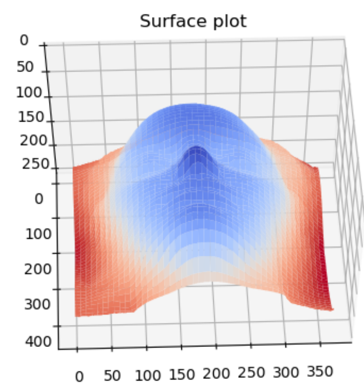
(a)



(b)



(c)



(d)

Figure 3: Plot of the surface using plotSurface with different views

Problem 2

Q 2.a

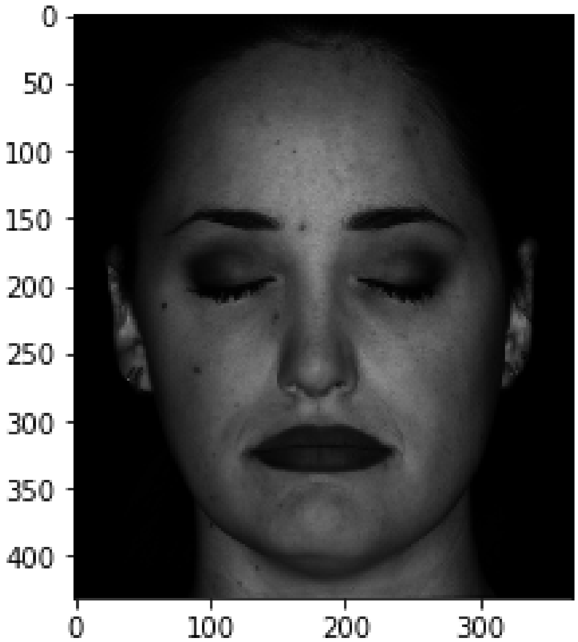
In the uncalibrated case, we need to factorize the I matrix to generate L and B matrix. Where, The dimension of L is 3×7, B is $3 \times P$ and I is $7 \times P$. As we know, the rank of I matrix is 7, However, we need rank of I matrix to be 3, we simply leave the top-3 singular values and zero the others.

To factorize, we perform SVD and split the matrices to generate L and B matrix. We follow the following steps to factorize:

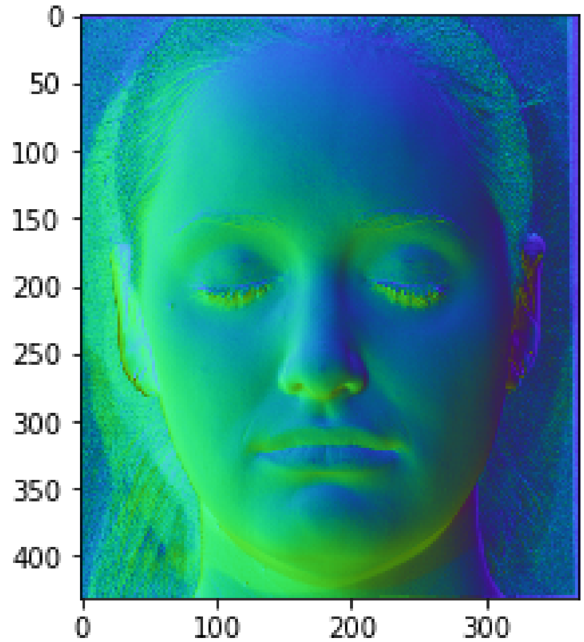
1. Decompose the I matrix using SVD. $I = USV^T$
2. Take first 3 columns of U as U_3.
3. Take first 3 columns of V as V_3.
4. Take 3×3 upper left block of S as S_3.
5. Split S_3 into $\sqrt{S_3}\sqrt{S_3}$.
6. Create L^T and B as:

$$L^T = U_3\sqrt{S_3}$$
$$B = \sqrt{S_3}V_3^T$$

Q 2.b



(a) Albedos



(b) Normals

Figure 4: Output from estimatePseudonormalsUncalibrated

Q 2.c

L_0 matrix is as:

```
[[-0.1418  0.1215 -0.069   0.067  -0.1627  0.      0.1478]
[-0.1804 -0.2026 -0.0345 -0.0402  0.122   0.1194  0.1209]
[-0.9267 -0.9717 -0.838  -0.9772 -0.979  -0.9648 -0.9713]]
```

$\hat{L}$ matrix is as:

```
[[-2.99267472 -3.86998525 -2.40803005 -3.74500806 -3.59135539 -3.38666635
  -3.3525448 ]
[ 0.94780484 -2.31708946  0.49911094 -0.62599426  2.32568155  0.46605103
  -0.79271078]
[ 1.87934697  1.01461664  0.42942606 -0.01730299 -0.31077291 -0.91273581
  -1.8830081 ]]
```

No, the estimated lighting directions ($\hat{L}$) estimated by the factorization above are not similar to the ground truth lighting directions given in L_0.

There are infinite solutions to have different L and B to attain the same I. Let's assume, If I have an invertible matrix P and multiply it with the relationship as $P^{-1}P$. As, $P^{-1}P = identity$ it won't change the I matrix. So,

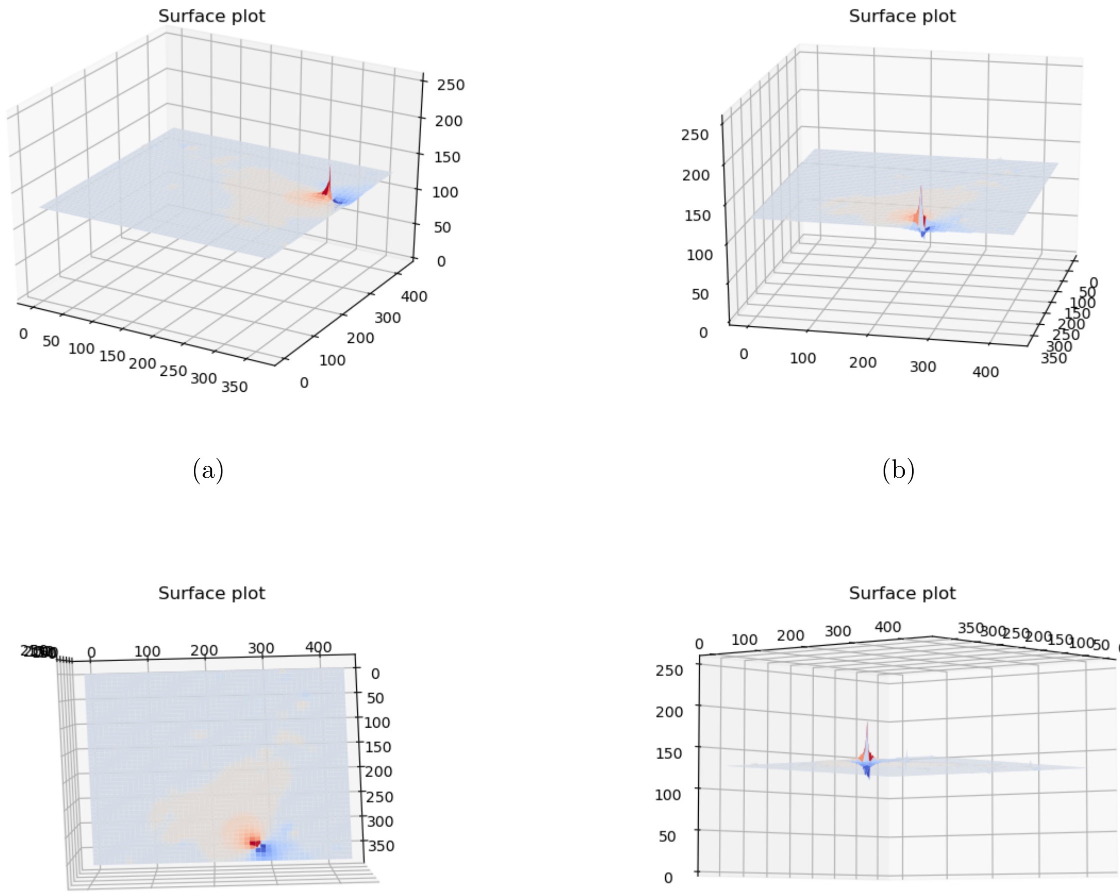
$$I = L^T P^{-1} P B$$

$$I = (L^T P^{-1})(PB)$$

$$I = (L^{'})^T B^{'}$$

Therefore, I can change infinite P matrices to generate different L and B, which keeping the rendered image same.

Q 2.d



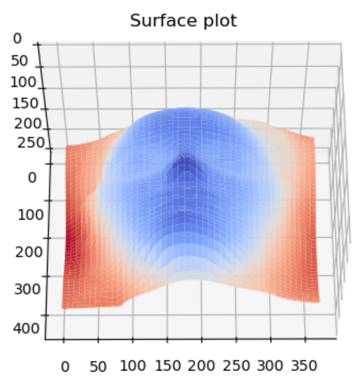
(a) (b)

(c) (d)

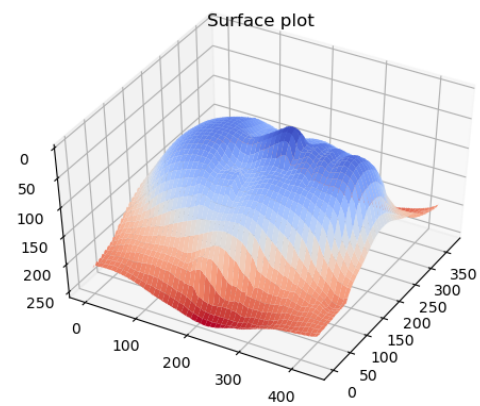
Figure 5: Plot of the surface reconstruction using Frankot-Chellappa algorithm implementation with different views

No, the surface does not look anywhere near a face.

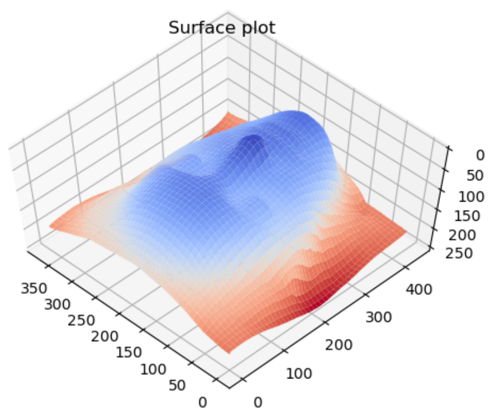
Q 2.e



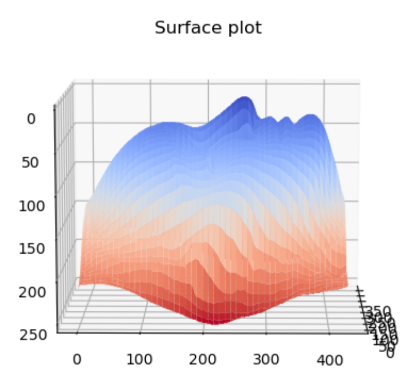
(a)



(b)



(c)

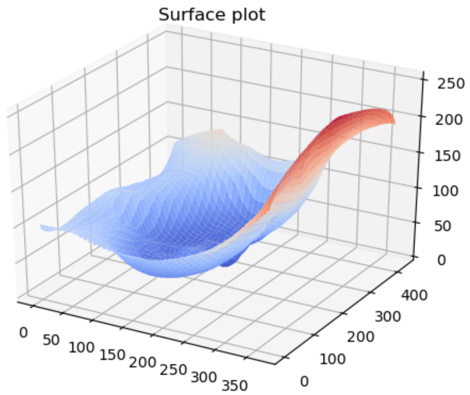


(d)

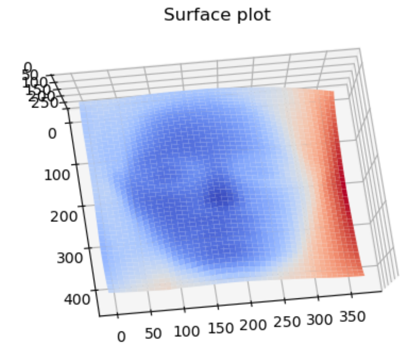
Figure 6: Plot of the surface reconstruction using enforceIntegrability and Frankot-Chellappa algorithm implementation with different views

The output after using enforceIntegrability and Frankot-Chellappa algorithm looks a lot like the reconstruction from the calibrated reconstruction as shown in figure 3. The reconstruction now looks like a face.

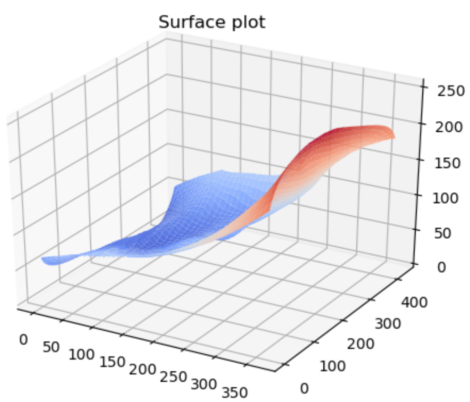
Q 2.f



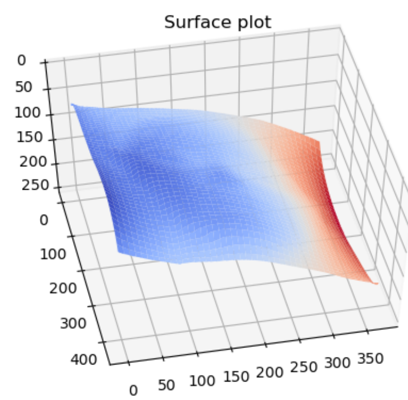
(a) mu = 3, v = 1, lambda = 1 view 1



(b) mu = 3, v = 1, lambda = 1 view 2



(c) mu = 8, v = 1, lambda = 1 view 1



(d) mu = 8, v = 1, lambda = 1 view 2

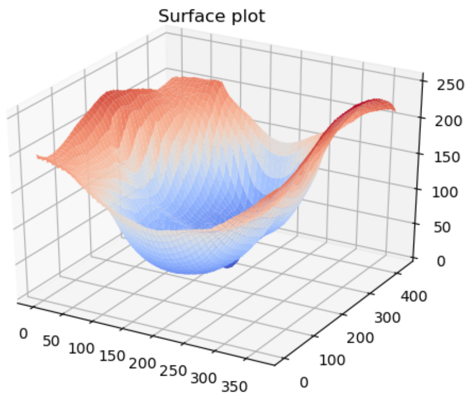
Figure 7: Plots for varying mu in bas-relief transformation

From the above example figure 7, we can see that if we increase the value for mu, the surface starts to become flat.

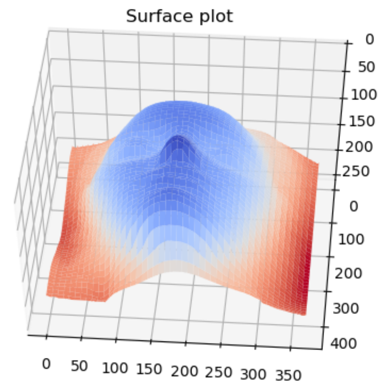
From the above example figure 8, we can see that if we increase the value for lambda, the surface starts to become more curvy.

From the above example figure 9, we can see that if we increase the value for v, the surface starts to tilt and the face features are lost.

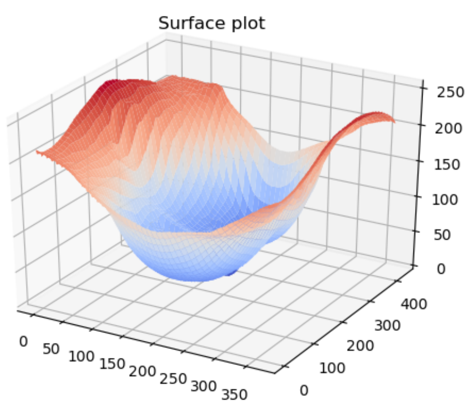
The bas-relief ambiguity is so named as when the relief is low and one single viewpoint is



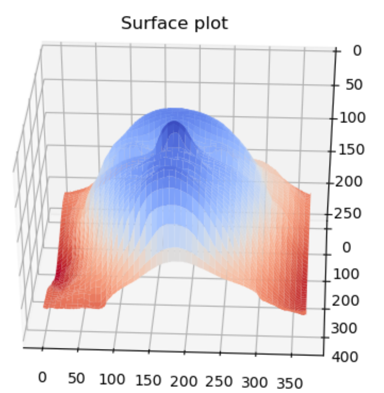
(a) mu = 2, v = 1, lambda = 3 view 1



(b) mu = 2, v = 1, lambda = 3 view 2



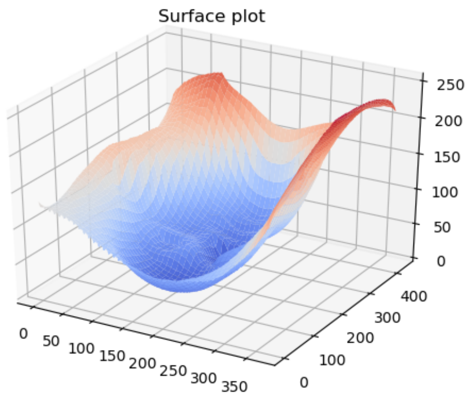
(c) mu = 2, v = 1, lambda = 8 view 1



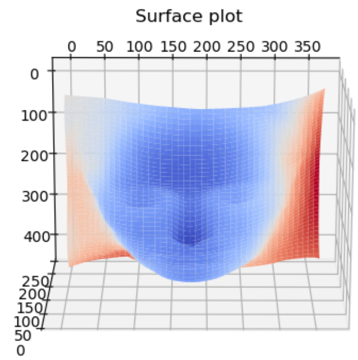
(d) mu = 2, v = 1, lambda = 8 view 2

Figure 8: Plots for varying lambda in bas-relief transformation

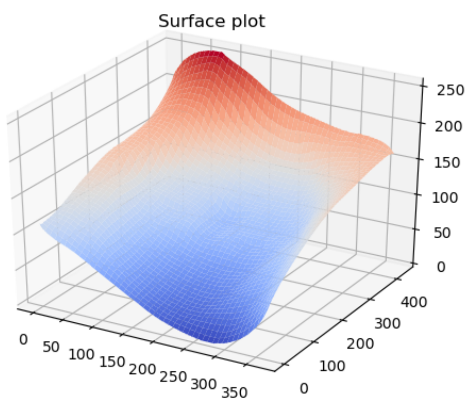
considered, there is an ambiguity in the recovery of the surface. No information in either the shadowing or shading can resolve this.



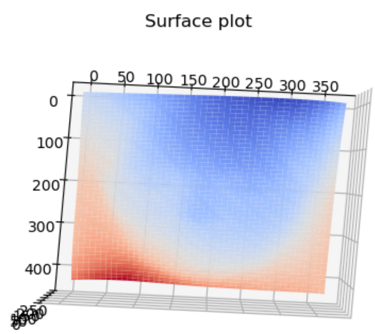
(a) mu = 2, v = 2, lambda = 1 view 1



(b) mu = 2, v = 2, lambda = 1 view 2



(c) mu = 2, v = 8, lambda = 1 view 1



(d) mu = 2, v = 8, lambda = 1 view 2

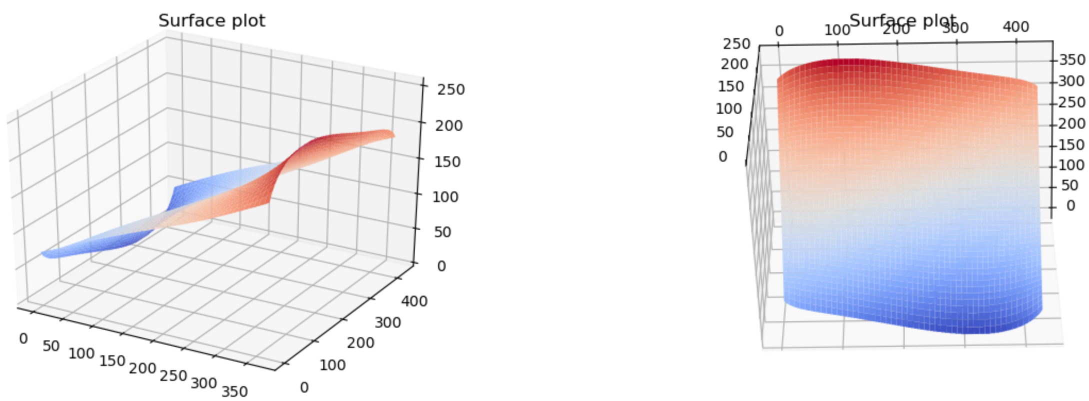
Figure 9: Plots for varying v in bas-relief transformations

Q 2.g

Based on the inference made in Q 2.f for all 3 parameters, we can find the best case for the flattest surface with hit and trial. For the flattest surface possible, I have used G matrix as:

```
[[1.e+00 0.e+00 0.e+00]
 [0.e+00 1.e+00 0.e+00]
 [8.e+00 1.e-03 1.e-02]]
```

The result using the above matrix G, is shown in the image below.



(a) View 1

(b) View 2

Figure 10: Flattest surface with mu = 8, v = 0.001 and lambda = 0.01

Q 2.h

No, Acquiring more pictures from more lighting directions won't help resolve the ambiguity. Knowledge about surface shape, surface albedo, light source direction, or light source intensity must be used to resolve this ambiguity.

Reference: https://doi.org/10.1023/A:1008154927611

Problem 3

Q 3.a

The homework was really interesting and had a lot of things to learn. But, was unclear and consumed way more time than it should have. There should be proper referencing and resources should be provided (to be specific atleast for q1b). Some hint should be provided in the q2e to enforce pseudonormals B instead of normals to get good results.

Otherwise, this homework was really interesting and gives an insight of the advanced computer vision based courses and help students decide on the future courses.