

CS202 ASSIGNMENT

SUDOKU IN SAT

SUDOKU GENERATION CONSTRAINTS

- ▶ For each number (1-9) each cell has atmost 1 number.
- ▶ For each row all number occur at least one.
- ▶ For each column all number occur at least one.
- ▶ For each box number occur at least one.
- ▶ For each diagonal number occur at least one.
- ▶ Point 1 and 2 will suffice that every cell has exactly one number.
- ▶ Fixed values constraint.

ENCODING

- ▶ Expressed each number corresponding to a cell in the base-9 notation.
- ▶ $9^2 (\text{row}) + 9 (\text{column}) + (\text{number in cell})$
- ▶ Hence having total of 729 variables.

HOW IS MY SUDOKU ALWAYS DIFFERENT?

- ▶ Uses initial -rnd-seed function of minisat and seed itself is rand()
- ▶ Seed value set to time(NULL) to ensure it changes every time.

MINIMAL SUDOKU GENERATION

- ▶ Generate a sudoku using sudoku solver.
- ▶ Follow the algorithm in next slide.
- ▶ How to check if a sudoku has more than 1 solution?
- ▶ Negate the fixed values, if there exist another solution-SAT else SAT.

AS A COMPUTER SCIENCE STUDENT OUR DUTY IS TO GIVE ROCK SOLID PROOF FOR CORRECTNESS.

Let S be a set of nodes that give a unique solution. The two lemma are stated without proof:

- If S is a set of nodes that give a unique solution. Then any S' such that $S \subset S'$ also gives a unique solution.
- If S is a set of nodes that gives more than one solution. Then any S' such that $S' \subset S$ also gives more than one solution.

We keep adding numbers randomly until we get a set S which gives unique solution.

Let $S = \{e_1, e_2, \dots, e_n\}$

We pick an e_i .

- If $S - e_i$ gives a unique solution remove e_i as there must be $S' \subset S - e_i$ which gives unique solution. (Lemma 1)
- If $S - e_i$ does not gives a unique solution **do not** remove e_i as then there will be **no** $S' \subset S - e_i$ which gives unique solution. (Lemma 2)

By following above algorithm we reach a solution which is minimal, as removing any element now will result in a non-unique solution.

Link: <http://home.iitk.ac.in/~abhyuday/minimalsudoku.html>

IMPROVISATIONS

- ▶ Ref Slide 2, halved the time by removing additional constraints.
- ▶ Instead of removing numbers in `generator.cpp`, we added starting from an empty sudoku, almost reducing the time quadruple times*.
- ▶ *Elements in minimal sudoku lie in range 17-22 (It can't be less than that). So, removing 60 values take almost quadruple time.

THANK YOU!