Formula Sheet – MATH 401 – Final Exam – Spring2022

1. **Sample mean:** $\overline{X} = \dfrac{\sum X}{n}$, & **Sample variance:** $s^2 = \dfrac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1}$.
2. **Binomial**: $P(X = k) = {}^nC_k \cdot p^k \cdot (1-p)^{n-k}$, $E(X) = np, V(X) = np(1-p)$.
3. **Geometric**: $P(X = k) = p \cdot (1-p)^{k-1}$, $E(X) = \dfrac{1}{p}$ & $V(X) = \dfrac{1-p}{p^2}$.
4. **Poisson**: $P(X_\lambda = k) = e^{-\lambda} \cdot \dfrac{\lambda^k}{k!}$, $E(X_\lambda) = \lambda = V(X_\lambda)$.
5. **Exponential**: $P(T \leq t) = F(t) = 1 - e^{-\lambda t}$, $E(T) = \dfrac{1}{\lambda}$ & $V(T) = \dfrac{1}{\lambda^2}$.
6. **Standard normal distribution**: $Z = \dfrac{X - \mu}{\sigma}$, is a N.D. with $\mu_Z = 0$ & $\sigma_Z = 1$.
7. **Sampling & Central L. T.**: "As the sample size **n** increases the shape; of the distribution of sample means from a population "normal or $n \geq 30$" with μ and σ; will approach a N.D. with $\mu_{\overline{X}} = \mu$ and $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}}$." And we use $z = \dfrac{\overline{X} - \mu}{\sigma} \cdot \sqrt{n}$.
8. **Binomial $\approx$ N.D.**: If a discrete binomial probability distribution satisfies both: $n.p \geq 5$ and $n.q \geq 5$, then it can be approximated by a continuous N.D. with mean $\mu = n.p$ and a standard deviation $\sigma = \sqrt{npq}$.
9. **Confidence interval for μ**: If σ is given, then
   - <u>2-sided</u>: $\overline{X} - z_{\frac{\alpha}{2}} \cdot \dfrac{\sigma}{\sqrt{n}} < \mu < \overline{X} + z_{\frac{\alpha}{2}} \cdot \dfrac{\sigma}{\sqrt{n}}$
   - <u>1-sided</u>: $\mu > \overline{X} - z_\alpha \cdot \dfrac{\sigma}{\sqrt{n}} \overset{=}{\leftrightsquigarrow}$ "<u>lower</u>" & $\mu < \overline{X} + z_\alpha \cdot \dfrac{\sigma}{\sqrt{n}} \overset{=}{\leftrightsquigarrow}$ "<u>upper</u>"
10. **Confidence interval for σ^2**:
    - <u>2-sided</u>: $\dfrac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}} < \sigma^2 < \dfrac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}}$
    - <u>1-sided</u>: $\sigma^2 > \dfrac{(n-1)s^2}{\chi^2_\alpha} \overset{=}{\leftrightsquigarrow}$ "<u>lower</u>" & $\sigma^2 < \dfrac{(n-1)s^2}{\chi^2_{1-\alpha}} \overset{=}{\leftrightsquigarrow}$ "<u>upper</u>"
11. **Hypothesis Testing**: <u>p-value</u> $= P(|Z| > |z_0|)$, where z_0 is the test value
    (i) For **2-tailed** test: p-value $= 2[1 - \Phi(|z_0|)]$
    (ii) For **left-tailed**: p-value $= \Phi(z_0)$. For **right-tailed**: p-value $= 1 - \Phi(z_0)$
12. The **test power** is "$1 - \beta$", where $\beta = \Phi(z_{\alpha/2} - \dfrac{\delta\sqrt{n}}{\sigma}) - \Phi(-z_{\alpha/2} - \dfrac{\delta\sqrt{n}}{\sigma})$ for **2-tailed** test. A sample size that guarantees this **power** is $\boxed{n \approx \dfrac{(z_\beta + z_{\alpha/2})^2\sigma^2}{\delta^2}}$.
    For 1-tailed: <u>if $\delta > 0$</u>, $\beta = \Phi(z_\alpha - \dfrac{\delta\sqrt{n}}{\sigma})$. <u>If $\delta < 0$</u>, $\beta = 1 - \Phi(-z_\alpha - \dfrac{\delta\sqrt{n}}{\sigma})$.