23_likelihood_function_and_dummy_coding

May 13, 2021

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The code so far:

[1]:
```
# uncomment the lines below to install the correct version of pymc3 and␣
↪dependencies
# !pip3 install --upgrade 'arviz==0.11.1'
# !pip3 install --upgrade 'pymc3==3.9.3'
```

[2]:
```
import numpy as np

%matplotlib inline
import matplotlib.pyplot as plt
plt.style.use('seaborn')
import seaborn as sns

import pandas as pd
import pymc3 as pm
```

[3]:
```
url = 'https://github.com/abrsvn/pyactr-book/blob/master/data/every_each.csv?
↪raw=true'
every_each = pd.read_csv(url)
every_each["quant"] = every_each["quant"].astype('category')
every_each.shape
```

[3]:
```
(347, 2)
```

[4]:
```
every_each.head(n=3)
```

[4]:

	logRTresid	quant
0	0.056128	each
1	0.241384	each
2	0.056128	every

[5]:
```
every_each.iloc[[0, 8, 18, 31], :]
```

[5]:

	logRTresid	quant
0	0.056128	each
8	0.869077	every
18	-0.073706	every
31	-0.187536	each

[6]:
```
np.min(every_each["logRTresid"]), np.max(every_each["logRTresid"])
```

[6]: (-0.678407840683957, 1.19278354190761)

[7]:
```
every_each_model = pm.Model()

with every_each_model:
    normal_density = pm.Normal('normal_density', mu=0, sd=10)
```

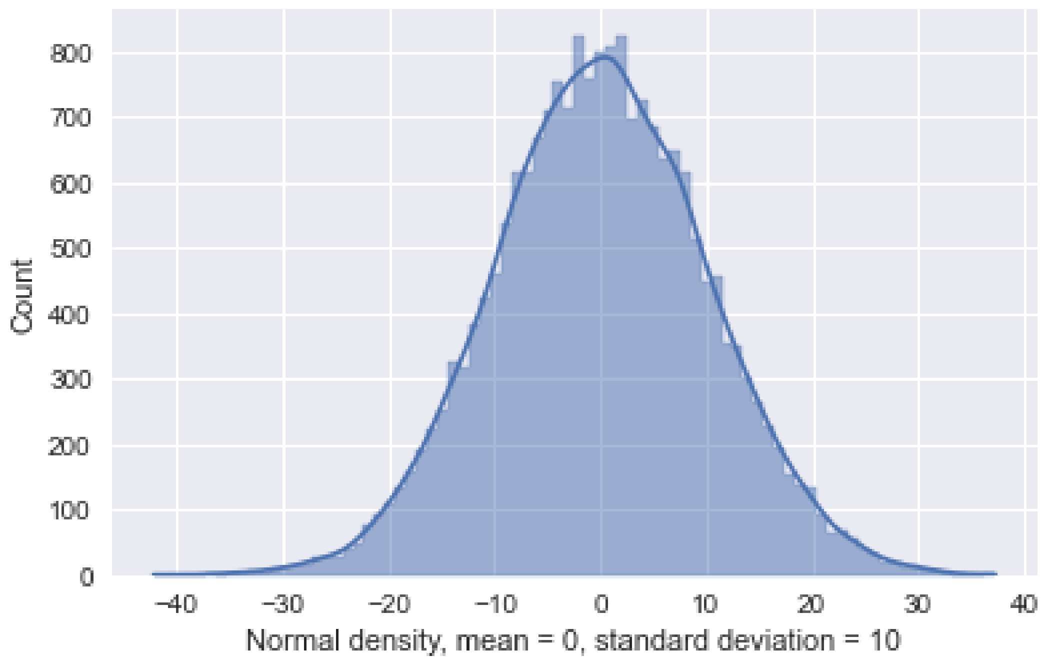
[8]:
```
from pymc3.backends import Text
from pymc3.backends.text import load

with every_each_model:
    db = Text('./data/normal_trace')
    trace = pm.sample(draws=5000, tune=500, cores=4, trace=db)
```

```
Auto-assigning NUTS sampler...
Initializing NUTS using jitter+adapt_diag...
Multiprocess sampling (4 chains in 4 jobs)
NUTS: [normal_density]
```

```
<IPython.core.display.HTML object>
```

```
Sampling 4 chains for 500 tune and 5_000 draw iterations (2_000 + 20_000 draws
total) took 2 seconds.
```

[9]:
```
fig, ax = plt.subplots(ncols=1, nrows=1)
fig.set_size_inches(5.5, 3.5)
sns.histplot(trace['normal_density'], element='step',
             kde=True, ax=ax)
ax.set_xlabel('Normal density, mean = 0, standard deviation = 10')
plt.tight_layout(pad=0.5, w_pad=0.2, h_pad=0.7)
```



0.1 Our model for generating the data (the likelihood)

Now that we specified our priors, we can go ahead and specify the model for how (we think) nature generated the data.

- we need to mathematically specify how RT is a function of quantifier

Recall that we have about 170 observations for each quantifier:

[10]:
```
every_each["quant"].value_counts()
```

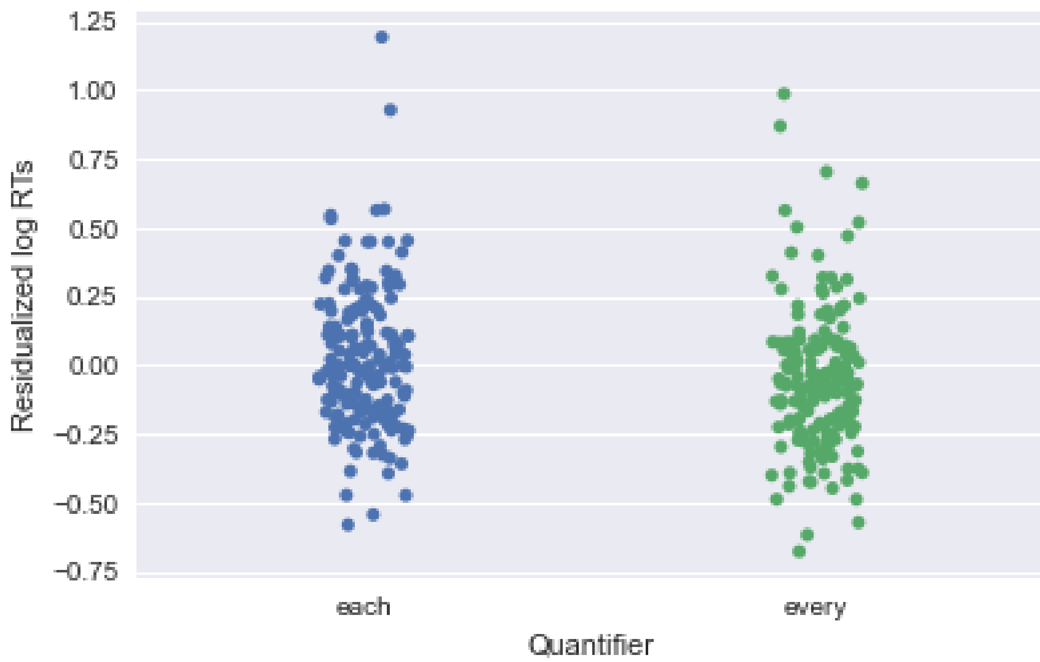
[10]:
```
each     174
every    173
Name: quant, dtype: int64
```

What we conjecture as our model for the data, a.k.a. our *likelihood* function, is that we have two mean RTs for the two quantifiers *every* and *each*.

- for each of the two quantifiers, the RTs we observed are imperfect reflections of the mean RT for that quantifier
  - they are somewhere around the mean for that quantifier
- specifically, the observed RTs for a quantifier are composed of the mean RT for that quantifier + some error
  - the error is due to our imperfect measurement, natural variability in the data source (e.g., a participant was faster pressing the space bar on one occasion than another) etc.

Plotting the RTs by quantifier will make this clearer:

[11]:
```
fig, ax = plt.subplots(ncols=1, nrows=1)
fig.set_size_inches(5.5, 3.5)
g = sns.stripplot(x="quant", y="logRTresid", data=every_each,
                  jitter=True)
g.set_xlabel("Quantifier")
g.set_ylabel("Residualized log RTs")
plt.tight_layout(pad=0.5, w_pad=0.2, h_pad=0.7)
```



The plot shos that:

- the 170+ observations collected for *each* are centered somewhere around 0.05 ms
- the 170+ observations collected for *every* are centered a little lower, around −0.05 ms

The observations are jittered (`jitter=True` on line 4 above):

- they are not plotted on a straight line, so that we can distinguish overlapping points in the plot

Our likelihood function is as follows:

- the observations for *each* are generated from the mean RT for *each* (which is, say, around 0.05) plus some error / noise around that mean RT
  - the noise is pretty substantial, with observed RTs spread between about −0.75 and 0.75
- similarly, the observations for *every* are generated from the mean RT for *every* (which seems to be around −0.05 ms) plus some error / noise around that mean RT
  - once again, the noise is substantial, spreading the observed values mostly between −0.75 and 0.75.

Our job right now is to write this story up in a single formula that will describe how the 347 RTs depend on quantifier.

Furthermore, recall that we are interested in the *difference* between the two quantifiers:

- we want to estimate it so that we can determine whether this difference is likely different from 0, i.e., whether the mean RT for *each* is different from the mean RT for *every*, as Tunstall's differentiation condition would predict

To this end, we will estimate two quantities:

- the mean RT for *every*: RT_{every}
- the mean difference in RT between *each* and *every*: $RT_{each-every}$

With these two quantities in hand, we can obtain the mean RT for *each* by summing them:

- $RT_{each} = RT_{every} + RT_{each-every}$

We will now use a simple reformulation of the *quant* variable (called 'dummy coding' of the categorical predictor variable `every_each["quant"]`) to be able to write a *single* formula describing how all 347 RTs are a function of the quantifier they are associated with.

- we'll rewrite the *quant* variable as taking either a value of 0 or a value of 1, depending on whether the RTs are associated with *every* or *each*
- we then multiply this rewritten / dummy-coded *quant* variable with the $RTs_{each-every}$ difference

Formula for RT as a function of quantifier:

$$\boxed{RT = RT_{every} + quant \cdot RT_{each-every} + noise}$$

- if RT is associated *every*, our dummy-coding for *quant* says that $quant = 0$
  - therefore, the RT is generated from the mean RT for *every* plus some noise
  - $RT = RT_{every} + 0 \cdot RT_{each-every} + noise = RT_{every} + noise$
- if RT is associated with *each*, our dummy-coding for *quant* says that $quant = 1$
  - therefore, the RT is generated from the mean RT for *each* plus some noise
  - $RT = RT_{every} + 1 \cdot RT_{each-every} + noise = RT_{each} + noise$

The code for the dummy coding of the *quant* variable is a one-liner (line 1 below)

- this takes advantage of the vectorial nature of both data and operations in `numpy` / `pandas`
- the resulting `"dummy_quant"` variable recodes *each* as 1 and *every* as 0, as expected

[12]:
```
every_each["dummy_quant"] = (every_each["quant"]=="each").astype("int")

every_each.head(n=6)
```

[12]:

	logRTresid	quant	dummy_quant
0	0.056128	each	1
1	0.241384	each	1
2	0.056128	every	0
3	0.037743	each	1
4	-0.208206	every	0
5	-0.113990	every	0

We can now use the variable `every_each["dummy_quant"]` and the model, a.k.a. likelihood function above, to generate synthetic datasets.

- below, we set our mean RT for *every* to -0.05 and our mean difference in RT to 0.1 (lines 1-2)
- this will result in a mean RT of 0.05 for *each*
- for convenience, we extract the dummy-coded `dummy_quant` variable and store it separately (line 3)
- we then assemble the means for the 347 synthetic observations we want to generate:
  - line 5 directly implements the likelihood function
- we can then look at the first 15 means thus assembled

[13]:
```
mean_every = -0.05
mean_difference = 0.1
quant = np.array(every_each["dummy_quant"])

synthetic_RT_means = mean_every + quant * mean_difference
synthetic_RT_means[:15]
```

[13]:
```
array([ 0.05,  0.05, -0.05,  0.05, -0.05, -0.05,  0.05,  0.05, -0.05,
        0.05, -0.05,  0.05,  0.05, -0.05, -0.05])
```

Note how the means match the quantifier:

- the first two are mean RTs associated with *each*, since the first two observations in our original data set are associated with *each*
  - their mean RT is therefore 0.05
- the third observation is associated with *every* since the third observation in our original data set was associated with *every*
  - its mean RT is therefore -0.05
- and so on

[14]:
```
every_each.head(n=15)
```

[14]:

	logRTresid	quant	dummy_quant
0	0.056128	each	1
1	0.241384	each	1
2	0.056128	every	0
3	0.037743	each	1
4	-0.208206	every	0
5	-0.113990	every	0
6	-0.041183	each	1
7	0.019087	each	1
8	0.869077	every	0
9	0.040079	each	1
10	0.090530	every	0
11	-0.019101	each	1
12	0.181284	each	1
13	-0.489288	every	0
14	-0.042091	every	0

The likelihood function has one final component: the noise.

- RTs from a specific quantifier are only imperfect, noisy reflections of the mean RT for that quantifier
- the noise comes from variations in the measuring equipment (keyboard etc.), or variations in the way the participants press the space bar at different times, or any other factor that we are not controlling for
- we generate noisy observations by drawing random numbers from a normal distribution: we use the `numpy` function `random.normal` for this purpose (line 2 below)
- the mean of the normal distribution is the mean RT for one quantifier or the other, and the standard deviation is set to 0.25, which generates noise of about $+/-0.75$
- the resulting RTs are randomly generated real numbers

[15]:
```
sigma = 0.25
synthetic_RTs = np.random.normal(synthetic_RT_means, sigma)
synthetic_RTs.round(2)[:25]
```

[15]:
```
array([ 0.21,  0.3 , -0.43, -0.18,  0.09,  0.39, -0.28,  0.04,  0.26,
        0.08,  0.1 ,  0.29,  0.05,  0.06, -0.26,  0.04,  0.32, -0.15,
        0.41,  0.03,  0.5 ,  0.07, -0.35,  0.1 ,  0.15])
```

We can compare these synthetic RTs to the actual RTs:

- we extract and store them in an independent variable `RTs` (line 2 below)
- we see that the range of variation in the synthetic data is pretty similar to the actual data

[16]:
```
# compare to the actual RTs in our dataset
RTs = np.array(every_each["logRTresid"])
RTs.round(2)[:25]
```

[16]:
```
array([ 0.06,  0.24,  0.06,  0.04, -0.21, -0.11, -0.04,  0.02,  0.87,
        0.04,  0.09, -0.02,  0.18, -0.49, -0.04,  0.17, -0.28, -0.16,
       -0.07, -0.18, -0.13, -0.27,  0.14, -0.34,  0.08])
```

Finally, if we want to synthesize more RT datasets that are similar to our actual dataset, we can simply do another set of draws from a normal distribution:

- centered at -0.05 or 0.05 (depending on the quantifier)
- with a standard deviation of 0.25

[17]:
```
# repeat to generate a different sample of synthetic RTs
synthetic_RTs = np.random.normal(synthetic_RT_means, sigma)
synthetic_RTs.round(2)[:25]
```

[17]:
```
array([-0.35,  0.31, -0.17, -0.16, -0.27,  0.35,  0.1 ,  0.42, -0.  ,
        0.07, -0.46,  0.26, -0.28,  0.29,  0.03,  0.09, -0.5 ,  0.15,
        0.04, -0.38, -0.16, -0.05,  0.03,  0.25, -0.4 ])
```

[ ]:
```

```