Section 1:
Extended object tracking

Multi-Object Tracking

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Extended object tracking – motivation

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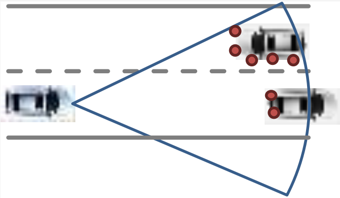
EXTENDED OBJECT TRACKING – DEFINITION

Extended object tracking

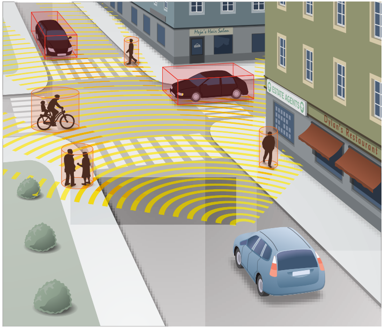
- Tracking objects that may generate multiple detections per time step.



- An extended object occupies more than one sensor resolution cell.
- Are cars extended?
  Depends on sensor and distance to sensor.

WHY EXTENDED OBJECT TRACKING?

- In autonomous applications, extended object tracking is useful for:
  - tracking vehicles using radar or lidar sensors,
  - mapping and localization using, e.g., radar sensors.
    Note: we are “tracking” stationary objects such as buildings, traffic signs and guard rails.



- In general, multiple detections from a single object may enable us to estimate the object’s **shape** and **orientation**.

Illustration by Per Thorneus, reproduced from Granström et al (2013), “Random set methods: Estimation of multiple extended objects”, *IEEE robotics & automation magazine* 21(2), 73-82.

WHAT'S NEW?

Bayesian filtering recursions – standard equations

Prediction: $$p(\mathbf{x}_k \middle| \mathbf{z}_{1:k-1}) = \int p(\mathbf{x}_k \middle| \mathbf{x}_{k-1}) p(\mathbf{x}_{k-1} \middle| \mathbf{z}_{1:k-1}) \, \delta\mathbf{x}_{k-1}$$

Update: $$p(\mathbf{x}_k \middle| \mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k \middle| \mathbf{x}_k) p(\mathbf{x}_k \middle| \mathbf{z}_{1:k-1})}{\int p(\mathbf{z}_k \middle| \mathbf{x}'_k) p(\mathbf{x}'_k \middle| \mathbf{z}_{1:k-1}) \, \delta\mathbf{x}'_k}$$

Differences:

- Measurement model $p(\mathbf{z}_k | \mathbf{x}_k)$ is different, since the single objects can generate multiple measurements.
- Data association hypothesis trees are different which requires new algorithms.
- The single object state x_k often contains shape information.

Single object measurement models

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MULTI-BERNOULLI RFSs

- **Objective:** model single object measurements, $\mathbf{o}_k \big| \{x_k\}$.
- Number of measurements? Spatial distribution?

Multi-Bernoulli (MB) models

- Given detailed knowledge about object: model $\mathbf{o}_k \big| \{x_k\}$ as a MB RFS.
- One Bernoulli for each “reflector point”.
- Both r^i and $p^i(o|x_k)$ depend on x_k.

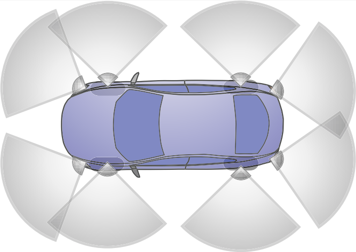


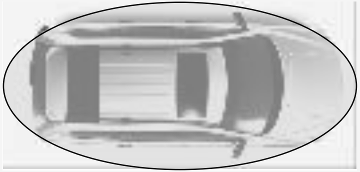
Image from Hammarstrand et al (2012), “Extended object tracking using a radar resolution model”, *IEEE TAES* 48(3).

THE POISSON POINT PROCESS AND ITS INTENSITY FUNCTION

- The PPP is arguably the **standard object measurement model** for extended object tracking.
- How can we **model the intensity function** $\lambda_o(o|x_k)$?

Random matrix model

- Approximate $\lambda_o(o|x_k)$ as a weighted Gaussian.
- The covariance matrix is part of x_k and can be estimated from data.



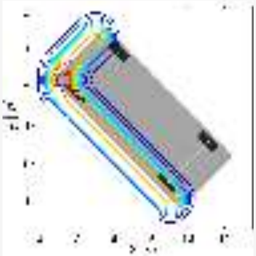
- **Note:** measurements are often described in a different coordinate system.

THE POISSON POINT PROCESS AND ITS INTENSITY FUNCTION

- The PPP is arguably the **standard object measurement model** for extended object tracking.
- How can we **model the intensity function** $\lambda_o(o|x_k)$?

Rectangular shapes

- Intensity is high along visible edges of object.
- Object dimensions may be part of x_k.
- Can be generalised to, e.g., 3D boxes.

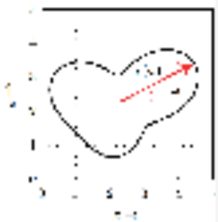


FLEXIBLE PARAMETRISATIONS OF THE INTENSITY FUNCTION

- A flexible parametrisation of $\lambda_o(o|x_k)$ may yield a more accurate model.

Star-convex contour

- Object contour is determined by $r(\phi)$.
- Here, $r(\phi)$ could be, e.g., a Fourier series.



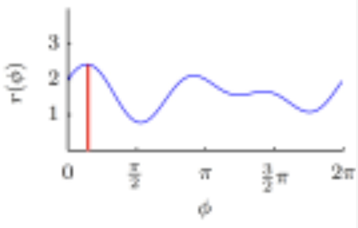


Image from Granström et al (2017), “Extended object tracking: Introduction, overview and applications”, *Journal of Advances in Information Fusion*.

Extended object tracking algorithms and conjugate priors

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EXTENDED OBJECT TRACKING ALGORITHMS

- Fairly rich literature on extended object tracking (EOT) algorithms, e.g.,
  - joint probabilistic data associations (JPDA),
  - particle filters,
  - probability hypothesis density (PHD) filter,
  - cardinalized probability hypothesis density (CPHD) filter,
  - probabilistic multi-hypothesis tracking (PMHT),
  - delta generalized labelled multi-Bernoulli (δ-GLMB) filters,
  - Poisson multi-Bernoulli mixture (PMBM) filters.

- **Key difference:** the family of distributions used to approximate $p(\mathbf{x}_k \middle| \mathbf{z}_{1:k})$ or $p(X_k \middle| Z_{1:k})$.

CONJUGATE PRIORS FOR EXTENDED OBJECT TRACKING (EOT)

PMBM conjugate prior

- The pdf $\mathcal{PMBM}_{k|k}(\mathbf{x}_k)$ is a conjugate prior to the standard models for EOT (Poisson birth, PPP object measurements):

Prediction:
$$\mathcal{PMBM}_{k|k-1}(\mathbf{x}_k) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})\mathcal{PMBM}_{k-1|k-1}(\mathbf{x}_{k-1})\,\delta\mathbf{x}_{k-1}$$

Update:
$$\mathcal{PMBM}_{k|k}(\mathbf{x}_k) = \frac{p(\mathbf{z}_k|\mathbf{x}_k)\mathcal{PMBM}_{k|k-1}(\mathbf{x}_k)}{\int p(\mathbf{z}_k|\mathbf{x}'_k)\mathcal{PMBM}_{k|k-1}(\mathbf{x}'_k)\,\delta\mathbf{x}'_k}.$$

- The MBM and delta-GLMB distributions are other EOT conjugate priors.
- Why study the PMBM conjugate prior?
  1) Describes the exact posterior: useful to understand EOT.
  2) PMBM filters arguably yield state of the art performance.

PMBM CONJUGATE PRIORS

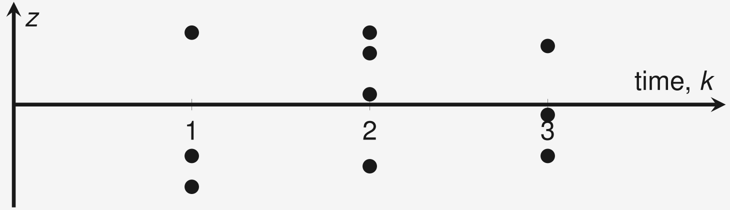
- According to $\mathcal{PMBM}_{k|k}(\mathbf{x}_k)$, $\mathbf{x}_k \big| \mathbf{z}_k$ is the union of two independent sets:
  - A **PPP**: set of objects that remain undetected.
  - A **multi-Bernoulli mixture RFS**: set of detected objects.
    One term in the mixture **for every global data association hypothesis**!

GLOBAL HYPOTHESES IN PMBM

- We obtain **one global hypothesis for every possible partition** of $\mathbf{z}_{1:k}$.
- A partition is a separation of $\mathbf{z}_{1:k}$ into disjoint subsets.

Visualizing global hypotheses, given $\mathbf{z}_{1:3}$

- Here, $\mathbf{z}_{1:3}$ contains ten measurements in total.
- In the MBM, every partition corresponds to a MB RFS and the subsets to B RFSs.



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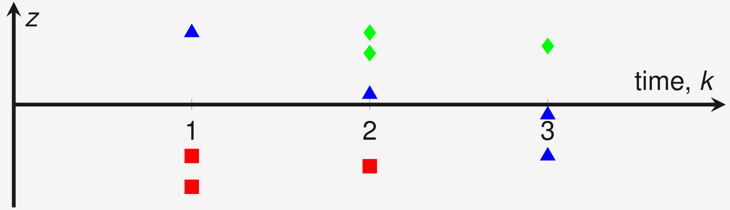

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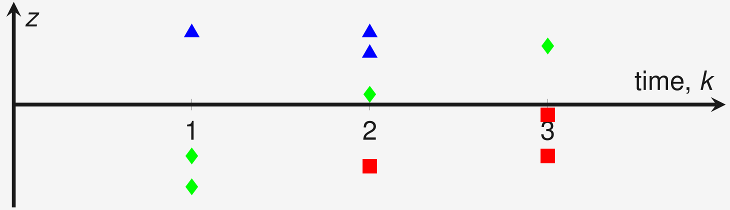

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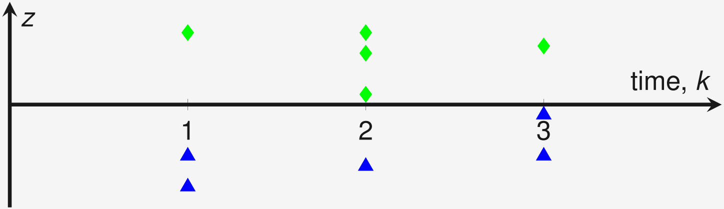

PMBM recursions for EOT

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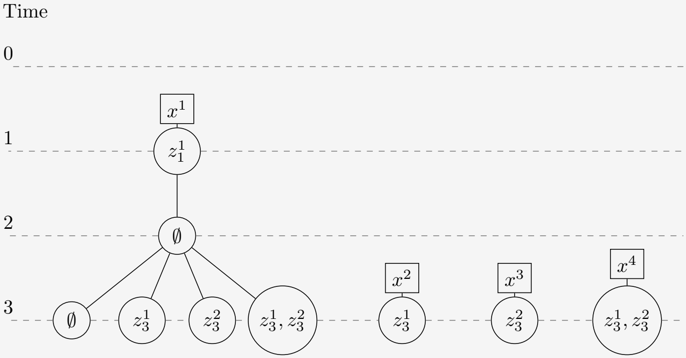
TRACK ORIENTED REPRESENTATION OF HYPOTHESES

- **Recursive algorithms:** useful to express global hypotheses as combinations of local hypotheses.

Local and global hypotheses, $\mathbf{z}_1 = \{z_1^1\}$, $\mathbf{z}_2 = \emptyset$, $\mathbf{z}_3 = \{z_3^1, z_3^2\}$



Look-up table:

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Local hypotheses indexed from left to right.

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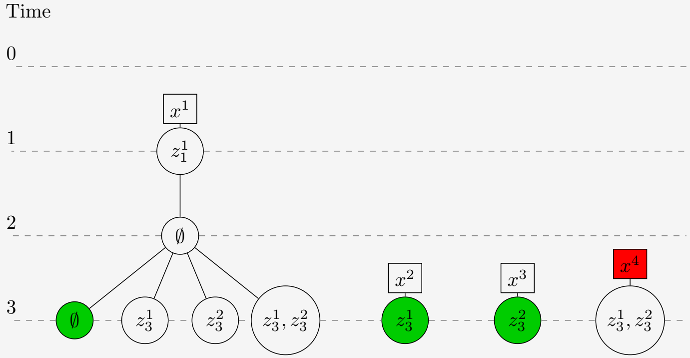

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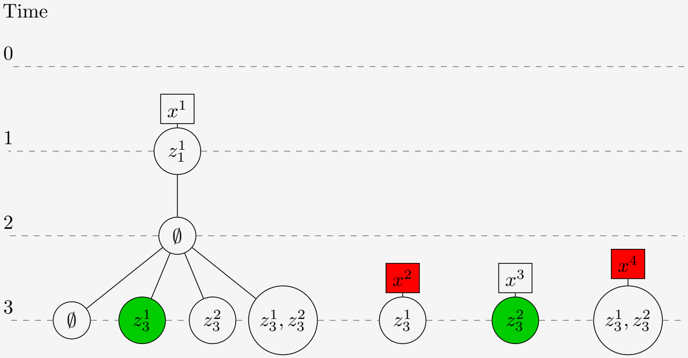

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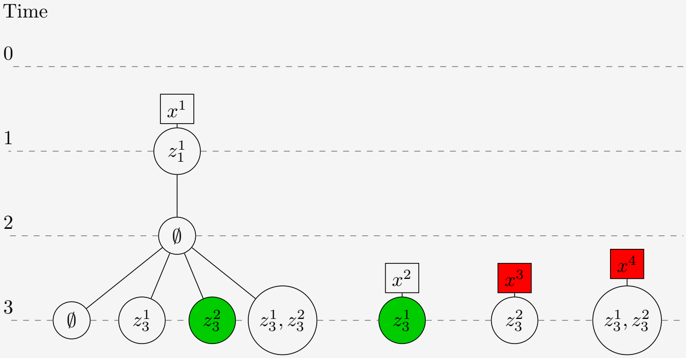

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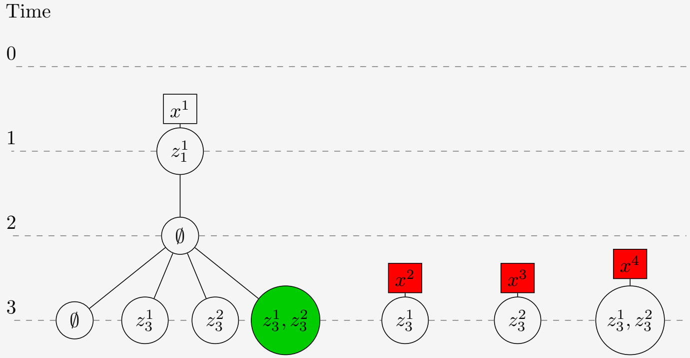

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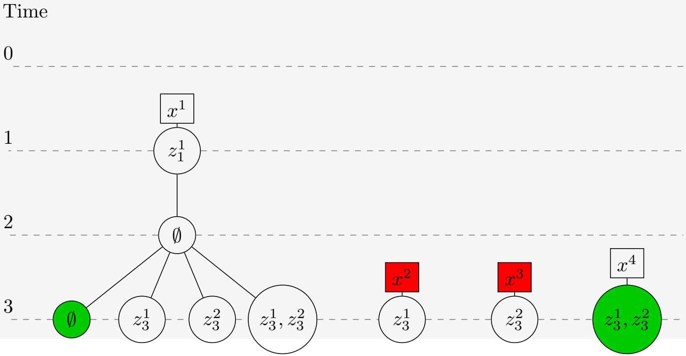

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PMBM – ALGORITHMIC OVERVIEW

- **Objective:** approximate $\mathcal{PMBM}_{k|k}(\mathbf{x}_k)$, given $\mathcal{PMBM}_{k-1|k-1}(\mathbf{x}_{k-1})$ and $\mathbf{z}_k$.
- **Prediction:** same equations as for point objects.
  Every component predicted independently. Existence prob. scaled by P^{S}.
  Add birth intensity to predicted Poisson intensity.
- **Update:**
  - **PPP**: scale intensity with probability that an object at x_k is undetected.
  - **MBM**:
    1. Identify global hypotheses with significant weights.
    2. Prune all other global hypotheses. Also prune local hypotheses that do not appear in global hyp. that we keep.
  - **Note:** assignments methods (auction, Murty's, ...) assume *at most one measurement per object* and cannot be used (directly) to find probably global hypotheses.

Pruning and clustering

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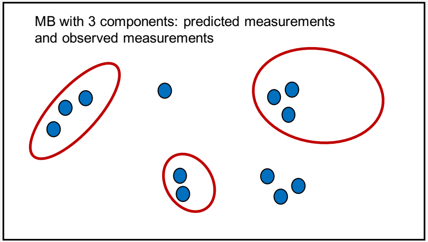
PROBLEM FORMULATION

Problem formulation

- Suppose $p(\mathbf{x}_k | \mathbf{z}_{1:k-1})$ is PMB.
- Given $\mathbf{z}_k$, find most probable association hypotheses.

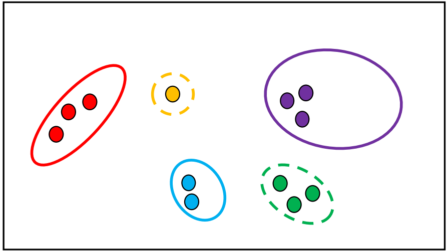


- If $p(\mathbf{x}_k | \mathbf{z}_{1:k-1}) = \mathcal{PMBM}_{k|k-1}(\mathbf{x}_k)$:
  1. Find the most probable hypotheses for every PMB in $\mathcal{PMBM}_{k|k-1}(\mathbf{x}_k)$.
  2. Store the most probable hypotheses found in step 1.

CLUSTERING AND ASSIGNMENT ALGORITHMS?

- Is it possible to make use of assignment algorithms?
  Yes! Using the following two-step procedure:

- Use **clustering:** to partition $\mathbf{z}_k$.
  Note: 1) we then associate clusters to potential objects, 2) two clusters cannot be associated to the same potential object.



- **Assignment:** we can now use Murty's algorithm to find the K best assignments.

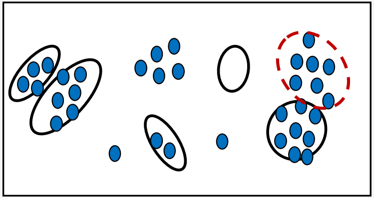
CHALLENGES WITH CLUSTERING

- **Clustering + Murty's**: fast and accurate if clustering works well.
- Sadly, clustering works poorly in many cases.
- **A simple fix:**
  1. Try several different clustering algorithms.
  2. Apply Murty's algorithms to all.

  Increases the chances of finding good clusters!

Challenging scenarios

- Clustering only considers measurements.
- Two predicted objects next to each other.
- Cluster from new object next to predicted object.



SAMPLING METHODS

Sampling methods to find likely associations

- Sample data associations according to their probabilities (weights).
- Standard approach: run a Markov chain that modifies the associations in each step.
  $\rightsquigarrow$ Finds associations with large weights!
- Yields very good performance.
- Difficult to analyse convergence.

Simulation example

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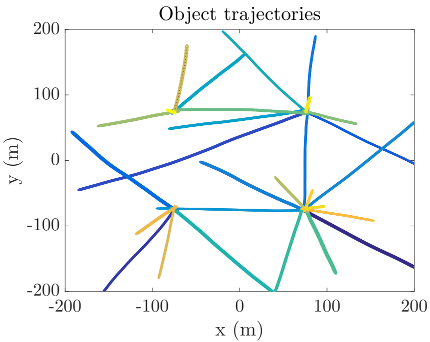
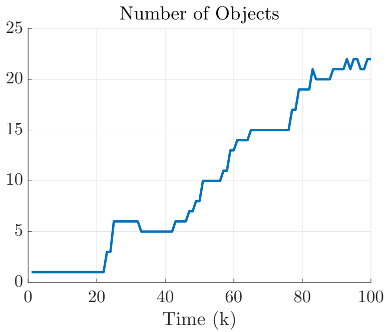
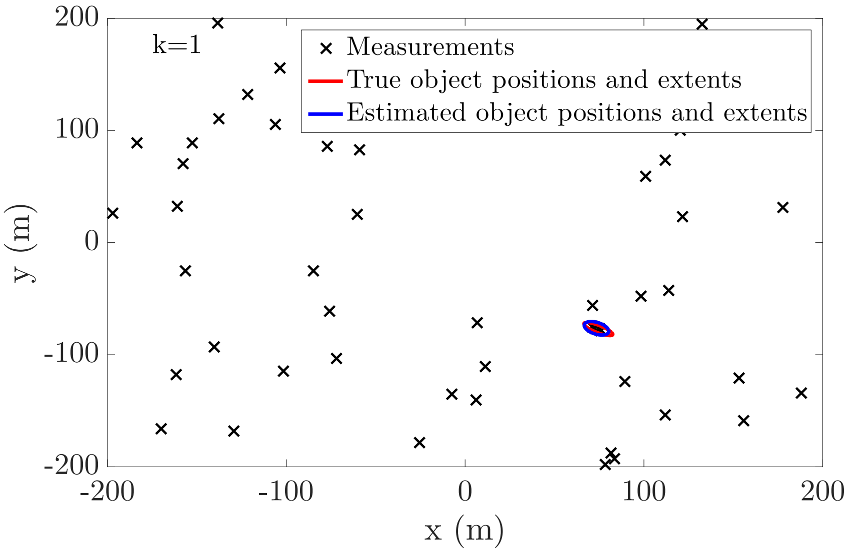
SIMULATION SCENARIO





- **Measurement model:**
  - Single object: PPP, random matrix, Poisson rate ≈ 8, $P^{\mathrm{D}} = 0.9$.
  - Clutter: Poisson rate 60.
- **Motion model:**
  - GGIW, constant velocity, $P^{\mathrm{S}} = 0.99$.
  - Birth intensity: peaks around $(\pm 75, \pm 75)$.

SIMULATION RESULTS



Lecture 6:
Outlook / Related topics
Version May 22, 2019

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Section 2:
Sets of trajectories

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Sets of trajectories – motivation

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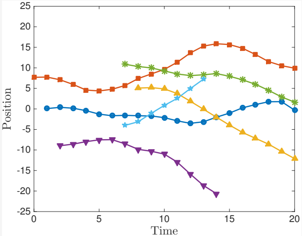
POSSIBLE PROBLEM FORMULATIONS (1)

The set of all trajectories

- Estimate trajectories of all objects present at some point in time ($\leq k$).
- **Example:** to collect ground truth data for self-driving vehicles. For instance, to train machine learning algorithms.
- **More examples:** to track human cells, sport athletes, etc.

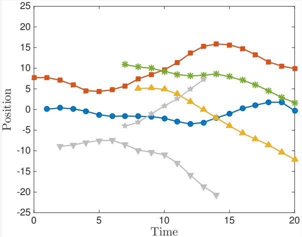


- We focus on recursive estimation algorithms, but batch solutions are also important.

POSSIBLE PROBLEM FORMULATIONS (2)

The set of current trajectories

- Estimate trajectories of all currently present objects.
- **Example:** for surveillance systems. For instance, of ships at sea, people at airports, etc.



- **Note:** from now on, we refer to recursively computing $p(\mathbf{x}_k | \mathbf{z}_{1:k})$, as multi-object filtering.
- In upcoming videos, **multi-object tracking** refers to recursively estimating the set of trajectories.

SETS OF TRAJECTORIES – BASIC IDEA

Basic idea

- We want to find the set of trajectories.
- Bayesian approach: compute posterior over quantities of interest.
- **Basic idea:** use the **set of trajectories as state variable**.

SINGLE TRAJECTORY: PARAMETRISATION

Single trajectory parametrisation

- We write single trajectories as

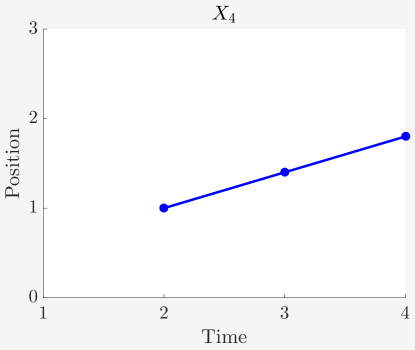
$$X_k = (\beta, \epsilon, x_{\beta:\epsilon})$$

where β is the time of birth, ϵ is the time of trajectory's most recent state and $x_{\beta:\epsilon}$ is the sequence of states.

- **Note:** $\epsilon = k$ means that trajectory is ongoing, whereas $\epsilon < k$ means that it ended at time ϵ.

Example

- The trajectory $X_4 = (2, 4, (1, 1.4, 1.8))$:



SETS OF TRAJECTORIES: PARAMETRISATION

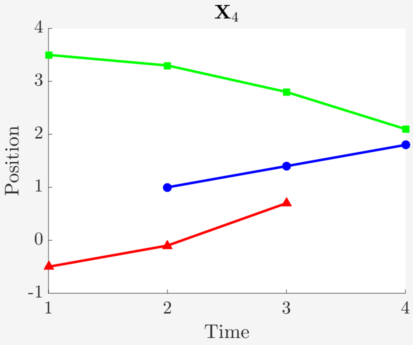
Sets of trajectories

- We use
$$\mathbf{X}_k = \left\{ X_k^1, \ldots, X_k^{N_k} \right\}$$
to denote the set of trajectories at time k.

- The number of trajectories N_k may be larger than the number of objects at time k, n_k.
- The set $\mathbf{X}_k$ is a random finite set.

Example

- A set $\mathbf{X}_4 = \{X_4^1, X_4^2, X_4^3\}$:



WHY USE $\mathbf{X}_K$ AS STATE VARIABLE?

- The set of trajectories is the quantity of interest.
- We now obtain the posterior of $\mathbf{X}_k$ using filtering.
- There is a one-to-one mapping between $\mathbf{X}_k$ and the physical reality:
  $\mathbf{X}_k$ is a minimal representation of the quantity of interest!
- Sets of trajectories can be used to develop performance metrics.
- Have been used to develop efficient and accurate tracking algorithms for both point objects and extended objects.

Sets of trajectories – basic concepts

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INTEGRATION

Single trajectory integral

- Suppose we consider trajectories up to time step t. The single trajectory integral is then defined as

$$\int f(X)\, \mathrm{d}X = \sum_{(\beta,\epsilon)\in \mathbf{i}_t} \int f\left((\beta, \epsilon, x_{\beta:\epsilon})\right) \mathrm{d}x_{\beta:\epsilon}$$

  where $\mathbf{i}_t = \{(\beta, \epsilon) : 1 \leq \beta \leq \epsilon \leq t\}$.

Set integrals

- The set integral is defined as

$$\int f(\mathbf{X})\, \delta\mathbf{X} = \sum_{i=0}^{\infty} \frac{1}{i!} \int f(\{X^1, \ldots, X^i\})\, \mathrm{d}X^{1:i}.$$

BAYESIAN FILTERING RECURSION FOR MOT

- Our variables
  - the set of trajectories, $\mathbf{X}_k$,
  - the set of measurement vectors, $\mathbf{z}_k$,

  are random finite sets (RFSs).

Bayesian filtering recursions

Prediction:
$$p(\mathbf{X}_k | \mathbf{z}_{1:k-1}) = \int p(\mathbf{X}_k | \mathbf{X}_{k-1}) p(\mathbf{X}_{k-1} | \mathbf{z}_{1:k-1}) \, \delta \mathbf{X}_{k-1}$$

Update:
$$p(\mathbf{X}_k | \mathbf{z}_{1:k}) = \frac{p(\mathbf{z}_k | \mathbf{X}_k) p(\mathbf{X}_k | \mathbf{z}_{1:k-1})}{\int p(\mathbf{z}_k | \mathbf{X}'_k) p(\mathbf{X}'_k | \mathbf{z}_{1:k-1}) \, \delta \mathbf{X}'_k}.$$

- Standard equations: Chapman-Kolmogorov for prediction and Bayes' rule for update.

MODELS

- Our **models** of the world **have not changed**.

Measurement model

- Measurements $\mathbf{z}_k$ only depend on $\mathbf{x}_k$

$$p(\mathbf{z}_k | \mathbf{X}_k) = p(\mathbf{z}_k | \mathbf{x}_k = \tau_k(\mathbf{X}_k)),$$

where τ_k extracts $\mathbf{x}_k$ from $\mathbf{X}_k$.

Motion model

- Objects appear, move and disappear. Trajectories at time k:
  - Appearing objects: $X_k = (k, k, x_k)$.
  - Surviving objects: $X_{k-1} = (\beta, k-1, x_{\beta:k-1}) \rightarrow X_k = (\beta, k, x_{\beta:k})$.
  - Disappearing objects, two cases:
    1. Tracking all objects that have been present: keep X_{k-1} in $\mathbf{X}_k, \mathbf{X}_{k+1}, \ldots$
    2. Tracking objects present at time k: remove X_{k-1} from set of trajectories.

COMMON RFSs: BERNOULLI

Bernoulli RFSs

- A Bernoulli RFS $\mathbf{X}$ has the multitrajectory pdf

$$p(\mathbf{X}) = \begin{cases} 1 - r & \text{if } \mathbf{X} = \emptyset \\ r\, p_X(X) & \text{if } \mathbf{X} = \{X\} \\ 0 & \text{if } |\mathbf{X}| > 1, \end{cases}$$

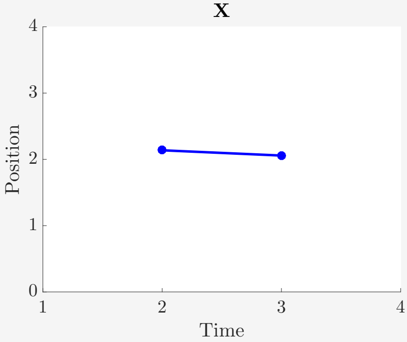
where $0 \le r \le 1$ and $p_X(X)$ is a single trajectory density.

- **Note:** r and $p_X(X)$ jointly determine existence probability at a specific time step t.

Sampling a Bernoulli RFS

- Suppose $r = 0.8$ and

$$p_X((\beta, \epsilon, x_{\beta:\epsilon})) = \delta_{\beta-2}\delta_{\epsilon-3} \times \\ \times \mathcal{N}(x_2; 2, 0.1)\,\mathcal{N}(x_3; 2.5, 0.3).$$



COMMON RFSs: PPPs

Poisson RFSs

- A Poisson RFS $\mathbf{X}$, with intensity function $\lambda(X)$, has the multitrajectory pdf

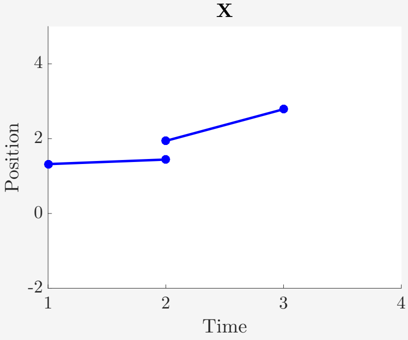
$$p(\mathbf{X}) = \exp(-\bar{\lambda}) \prod_{X \in \mathbf{X}} \lambda(X),$$

where $\bar{\lambda} = \int \lambda(X)\,\mathrm{d}X$ is the Poisson rate.

- **Note:** $|\mathbf{X}| \sim \mathrm{Po}(\bar{\lambda})$ and $\lambda(X)/\bar{\lambda}$ is the "spatial" distribution (including β and ϵ).

Sampling a Poisson RFS

- Suppose $\lambda((\beta, \epsilon, x_{\beta:\epsilon})) =$
  $\delta_{\beta-1}\delta_{\epsilon-2}\mathcal{N}(x_1; 1, 1)\mathcal{N}(x_2; 1, 1)+$
  $0.7\delta_{\beta-2}\delta_{\epsilon-3}\mathcal{N}(x_2; 2, 1)\mathcal{N}(x_3; 2, 1)$



PMBM trackers – part 1

Multi-Object Tracking

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SET OF TRAJECTORY CONJUGACY

PMBM conjugate prior

- The Poisson Multi-Bernoulli Mixture (PMBM) multitrajectory pdf $\mathcal{PMBM}_{k|k}(\mathbf{X}_k)$ is a conjugate prior to the standard models (point objects, Poisson birth):

$$\text{Prediction:} \quad \mathcal{PMBM}_{k|k-1}(\mathbf{X}_k) = \int p(\mathbf{X}_k | \mathbf{X}_{k-1}) \mathcal{PMBM}_{k-1|k-1}(\mathbf{X}_{k-1}) \, \delta\mathbf{X}_{k-1}$$

$$\text{Update:} \quad \mathcal{PMBM}_{k|k}(\mathbf{X}_k) = \frac{p(\mathbf{z}_k | \mathbf{X}_k) \mathcal{PMBM}_{k|k-1}(\mathbf{X}_k)}{\int p(\mathbf{z}_k | \mathbf{X}'_k) \mathcal{PMBM}_{k|k-1}(\mathbf{X}'_k) \, \delta\mathbf{X}'_k}.$$

- Hypotheses and weights are as in the PMBM filters (see Karl's videos).
- Conjugacy also holds for MBM birth and for extended objects.

PPP INTERPRETATION

The PPP component

- Models trajectories of the set of undetected objects.
- We parametrise the PPP using an intensity function, $\lambda^u_{k|k'}(X_k)$.

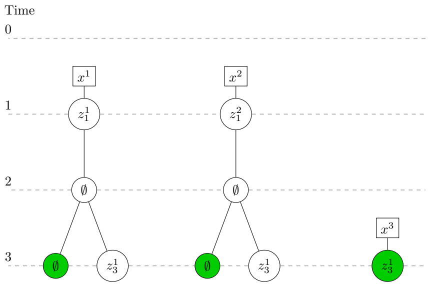
- The intensity $\lambda^u_{k|k'}(X_k)/\bar{\lambda}^u_{k|k'}$ describes a single trajectory distribution over $(\beta, \epsilon, x_{\beta:\epsilon})$.
- The distribution $\lambda^u_{k|k'}(X_k)/\bar{\lambda}^u_{k|k'}$ conveys information about where we may have undetected objects right now, but also when those objects might have entered the surveillance area.

MBM RFS INTERPRETATION

The MBM

- Models trajectories of objects that have been detected (in one of observed measurement sets).

- Why a multi-Bernoull mixture?
  - Mixture: multiple global hypotheses.
  - MB RFS: set of detected trajectories for a global hypothesis.



- **Note:** we describe the distribution of $\mathbf{X}_k$, not just $\mathbf{x}_k$.

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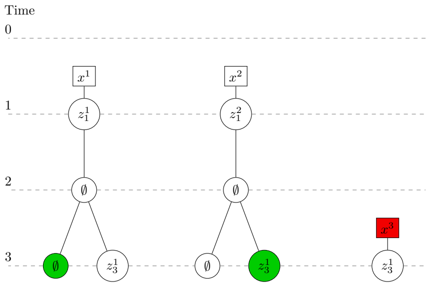

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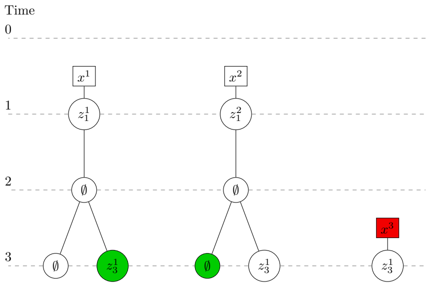

- **Note:** we describe the distribution of $\mathbf{X}_k$, not just $\mathbf{x}_k$.

PMBM trackers – part 2

Multi-Object Tracking

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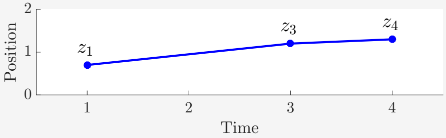
BERNOULLI RFS: INTERPRETATION

The Bernoulli RFS

- The Bernoulli RFSs model a single potential trajectory given a sequence of associations.

Example

- Suppose z_1, z_3 and z_4 are assoc. to a potential object.



- This would yield a Bernoulli RFS

$$p(\mathbf{X}_4^B) = \begin{cases} p_4^B(X) & \text{if } \mathbf{X}_4^B = \{X\} \\ 0 & \text{if } |\mathbf{X}_4| = 2, \end{cases}$$

$$p_4^B(X) = \delta_{\beta-1}\delta_{\epsilon-4}p_{4|4}(x_{\beta:\epsilon}|\beta,\epsilon)$$

- **Bottom line:** hypotheses describe the association history
$\Rightarrow$ enables us to compute trajectory distribution!

BERNOULLI RFS APPROXIMATIONS

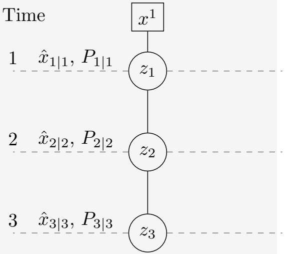
- In theory, complexity grows with length of trajectory.
- New measurement $\Rightarrow$ update all of $p(x_{\beta:\epsilon} | \beta, \epsilon = k)$.
- Kalman update is expensive when $\epsilon - \beta$ is large $\Rightarrow$ we need approximations!

Approximation example

$$\text{Exact:} \quad p(x_{1:3} | z_{1:3}) \propto p(x_{1:3} | z_{1:2}) p(z_3 | x_3)$$

$$\text{Approx.:} \quad p(x_{1:3} | z_{1:3}) \approx p(x_3 | z_{1:3}) p(x_2 | z_{1:2}) p(x_1 | z_1)$$

- The sequence $\hat{x}_{1|1}, \hat{x}_{2|2}, \ldots$ depends on the local hypothesis.
- $\Rightarrow$ same computational complexity as PMBM filtering.



MARGINALIZATION

- Consider a trajectory Bernoulli RFS, with parameters

$$\begin{cases} r_{k|k'} \\ p_{k|k'}(X) = \mathrm{Pr}_{k|k'}(\beta, \epsilon) p^x_{k|k'}(x_{\beta:\epsilon} | \beta, \epsilon). \end{cases}$$

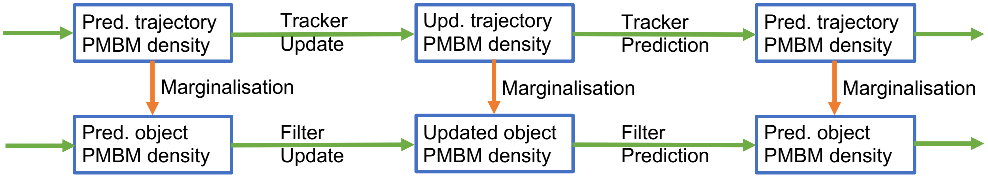
- Using marginalization, we can compute a Bernoulli RFS representing the set of objects at time k.
- This Bernoulli RFS has parameters

$$r^m_{k|k'} = r_{k|k'} \sum_{\beta=1}^{k} \mathrm{Pr}_{k|k'}(\beta, k)$$

$$p^m_{k|k'}(x_k) = \frac{r_{k|k'}}{r^m_{k|k'}} \sum_{\beta=1}^{k} \mathrm{Pr}_{k|k'}(\beta, k) \int p^x_{k|k'}(x_{\beta:\epsilon} | \beta, \epsilon) \, \mathrm{d}x_{\beta:k-1}.$$

PMBM TRACKERS VS PMBM FILTERS

- More interestingly, marginalizing $\mathcal{PMBM}_{k|k'}(\mathbf{X}_k)$ gives us $\mathcal{PMBM}_{k|k'}(\mathbf{x}_k)$.



- **Key insight:**
  - In PMBM filtering we marginalize out the history in every prediction step.
  - In PMBM trackers, we maintain knowledge about the past which enables us to estimate trajectories.

Simulation example 1 – missed detections

Multi-Object Tracking

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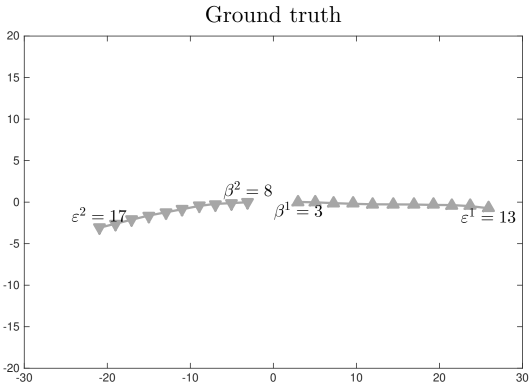
SCENARIO WITH MISSED DETECTIONS: SETUP

Two objects

- Twenty time steps
- Two objects appear at time step β^i, disappear at time step ε^i
- Left object misdetected at times $k = 11, 12, 13$



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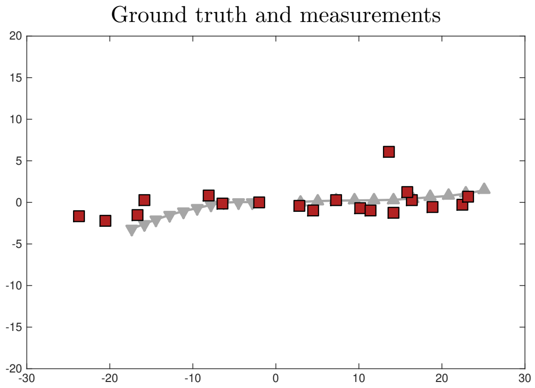
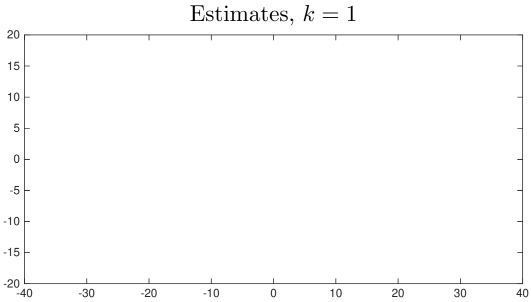


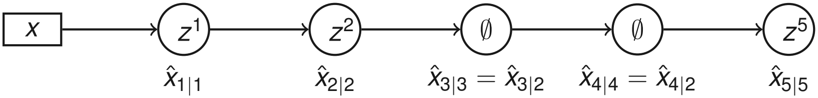
SCENARIO WITH MISSED DETECTIONS: NO SMOOTHING



- PMBM tracker
    - Gaussian state densities and a constant velocity model.
    - Trajectory estimates computed at time k. No smoothing.
    - Once object is detected again, we produce complete trajectory estimates.
    - Trajectory estimates are less accurate when object is not detected.

EXPLAINING THE TRACKING RESULTS

- Suppose there is only
  1. one potential object,
  2. one reasonable hypothesis at each time.



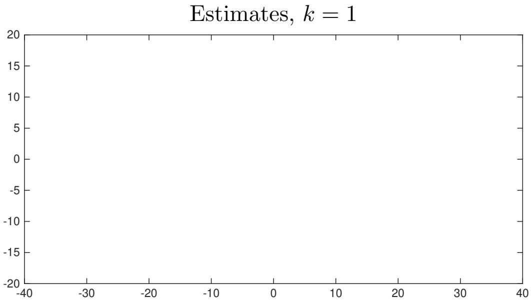
- At time 4, our best estimate may be $\hat{\mathbf{X}}_4 = \{(1, 2, (\hat{x}_{1|1}, \hat{x}_{2|2}))\}$.
- At time 5, our best estimate is that the trajectory continues until time 5:

$$\hat{\mathbf{X}}_5 = \{(1, 5, (\hat{x}_{1|1}, \hat{x}_{2|2}, \hat{x}_{3|3}, \hat{x}_{4|4}, \hat{x}_{5|5}))\}.$$

- Of course, an even better estimate would be:

$$\hat{\mathbf{X}}_5 = \{(1, 5, (\hat{x}_{1|5}, \hat{x}_{2|5}, \hat{x}_{3|5}, \hat{x}_{4|5}, \hat{x}_{5|5}))\}.$$

SCENARIO WITH MISSED DETECTIONS: SMOOTHING



- PMBM tracker
  - Smoothing: use measurements to improve earlier state estimates.
  - Final trajectory estimates are accurate also at times when we have missed detections.
  - Both versions of the PMBM tracker provide trajectory estimates without gaps.

Simulation example 2 – closely spaced objects

Multi-Object Tracking

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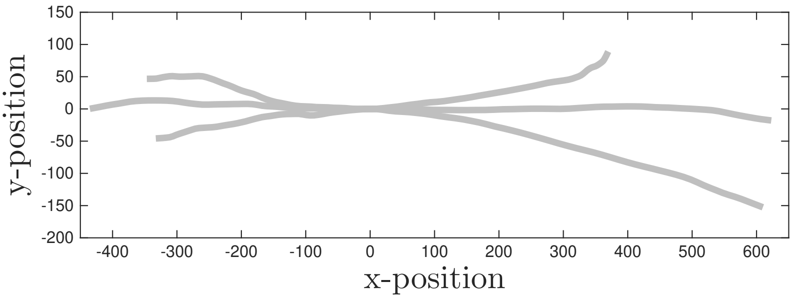
CLOSELY SPACED OBJECTS: SETUP

Three objects that become very close, and then separate

- Objects move left to right, are initially well-separated.
- Sequences last 100 time steps. After time 50, ambiguous which object goes where.



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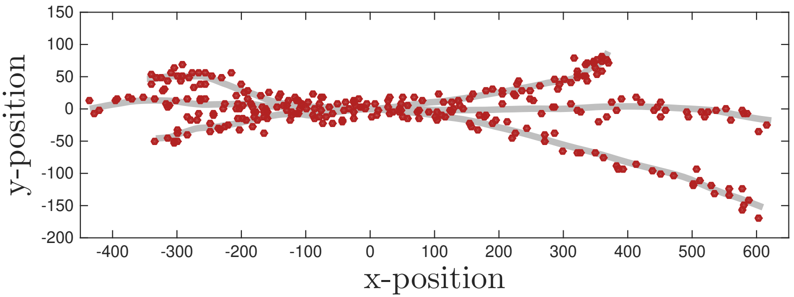
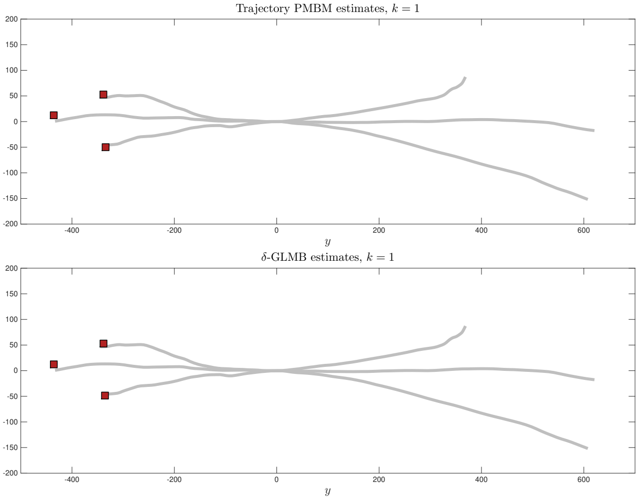


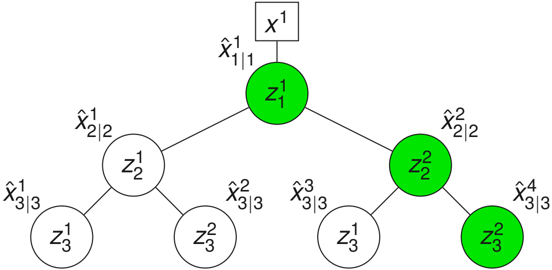
CLOSELY SPACED OBJECTS: TRACKING RESULTS



- PMBM tracker: 1) uses smoothing to improve trajectory estimates, 2) experiences track switching (at $k = 89$), 3) does not report trajectories with unrealistic switches.

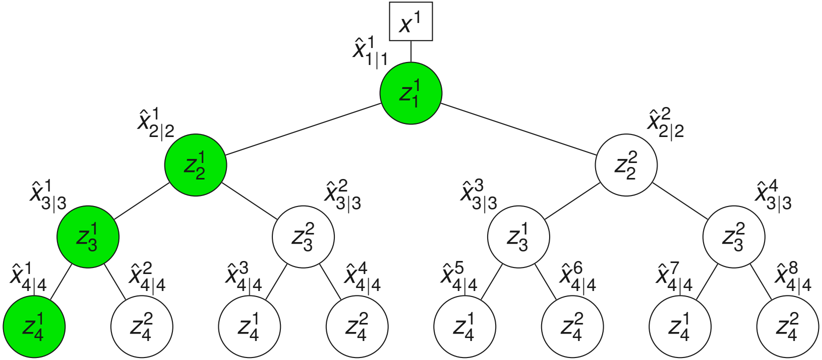
WHY NO TRACK SWITCHES?

- We select estimates from the most likely hypothesis, e.g., $\hat{\mathbf{X}}_3 = \{(1, 3, \hat{x}^1_{1|1}, \hat{x}^2_{2|2}, \hat{x}^4_{3|3})\}$.



WHY NO TRACK SWITCHES?

- We select estimates from the most likely hypothesis, e.g., $\hat{\mathbf{X}}_3 = \{(1, 3, \hat{x}^1_{1|1}, \hat{x}^2_{2|2}, \hat{x}^4_{3|3})\}$.
- Later estimates may be from a different branch, e.g., $\hat{\mathbf{X}}_4 = \{(1, 4, \hat{x}^1_{1|1}, \hat{x}^1_{2|2}, \hat{x}^1_{3|3}, \hat{x}^1_{4|4})\}$.
- **Note:** all estimates are from **the same branch**.



SETS OF TRAJECTORIES: CONCLUSIONS

- Sets of trajectories can leverage on the many advantages with PMBM filters, e.g., few hypotheses and initiate tracks based on measurements.
- We obtain trajectory estimates, essentially without increasing the computational complexity.
- The challenges that we observed with labels are resolved:
  every trajectory estimate in $\hat{\mathbf{X}}_k$ is from a single local hypothesis.
- Using smoothing, we can improve performance further.
- Sets of trajectories can be used for point objects, extended objects, track-before-detect, etc.

Section 3:
Deep learning

Multi-Object Tracking

Lennart Svensson

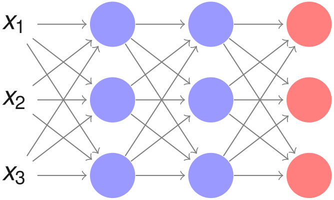
An introduction to deep learning

Multi-Object Tracking

Lennart Svensson

DEEP LEARNING VS ARTIFICIAL INTELLIGENCE

- The objective in **artificial intelligence** (A.I.) is to build “smart” machines.
- In **machine learning**, the idea is to give a machine access to data and ask it to learn for itself.
- **Deep learning** is a subfield of machine learning where we use **deep neural networks** to make decisions.



SPEECH RECOGNITION AND NATURAL LANGUAGE PROCESSING

- Deep learning has lead to much better **speech analysis**.

- It is also used by Google for **translations**.

Speech Recognition

Reduced word errors by more than 30%

Google Research Blog – August 2012, August 2015

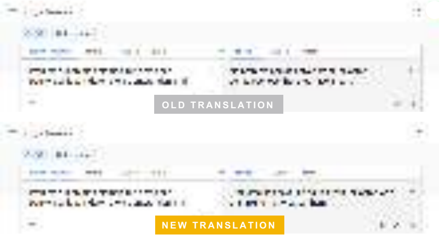


IMAGE CLASSIFICATION

- Top-5 ImageNet classification challenge:
  - 1000 categories,
  - 5 chances to guess,
  - $>$ 1 million images.

- First **deep learning** algorithm participated in 2012.

- Surpassed "human performance" in 2015.

OBJECT DETECTION AND REINFORCEMENT LEARNING

- Deep learning also yields very good **object detection** performance.

- In 2016, deep reinforcement learning won against the **world champion in Go**.

Deep neural networks

Multi-Object Tracking

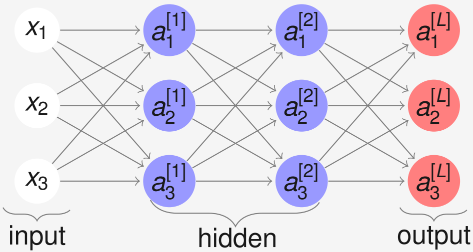
Lennart Svensson

FEEDFORWARD NEURAL NETWORKS

- A neural network is a function.
- Given enough neurons it can approximate (almost) any function.

Feedforward neural networks

- $L = \sharp$ layers, $L-1$ hidden layers
- $a_i^{[j]}$: value of neuron i in layer j
- We assume $x_i = a_i^{[0]}$.
- Number of neurons usually varies with depth.



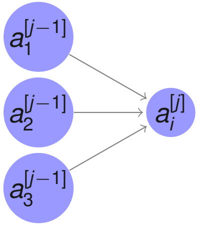
A SINGLE NEURON

- To evaluate a single neuron:

$$z_i^{[j]} = b_i^{[j]} + \sum_i w_i^{[j]} a_i^{[j-1]}$$

$$a_i^{[j]} = g^{[j]}(z_i^{[j]}).$$

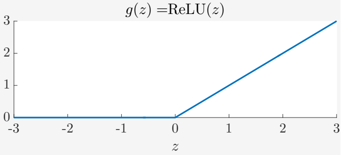
- Here $g^{[j]}(\cdot)$ is an **activation function**.
- $\left\{b_i^{[j]}, w_i^{[j]}\right\}_{i,j}$: learned from data.



Rectified linear units (ReLU)

- The rectified linear unit is a commonly used activation function:

$$g(z) = \text{ReLU}(z) = \max(0, z).$$



ACTIVATION FUNCTIONS – OUTPUT LAYER

- We use different output activation functions for different tasks.
- **Regression:** we often use $g(z) = z$ or $g(z) = \mathrm{ReLU}(z)$.

Softmax

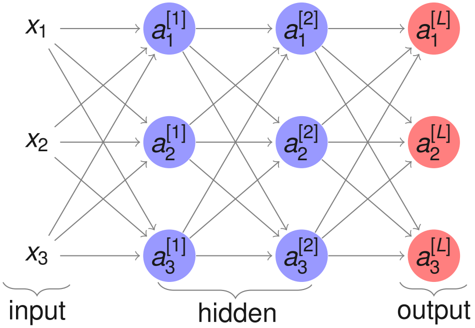
- For classification tasks, we often use a softmax activation

$$\tilde{a}_i^{[L]} = \exp(z_i^{[L]})$$
$$a_i^{[L]} = \frac{\tilde{a}_i^{[L]}}{\sum_s \tilde{a}_s^{[L]}}.$$

- **Note 1:** $a_i^{[L]}$ sum to one, and we use it to approximate $\Pr[Y = i \mid x]$.
- **Note 2:** $a_i^{[L]}$ depends on $z_1^{[L]}, z_2^{[L]}, \ldots$

GENERAL NEURAL NETWORKS

- Many neural networks can be viewed as feedforward networks.
- **Common adjustments:** remove edges and weight sharing.
- **Other techniques:** max-pooling, batch normalization, attention, etc.



Supervised learning

Multi-Object Tracking

Lennart Svensson

SUPERVISED LEARNING

- **Supervised learning:** learn to predict an output given an input.

Common approach:

1. Collect a dataset of inputs x (e.g., images) and labels y.
2. Train algorithm to predict y from x. That is, find θ such that
$$f(x;\theta) \approx y$$
(in some sense).
3. Evaluate algorithm on new data.

Image classification

In:

Out: dog.

- **More examples:**
  - Translation: x sentence in German, y sentence in English.
  - Object detections: x image, y object detections.

EMPIRICAL RISK

- Suppose we are given

$$\begin{aligned} \text{training data:} \quad & \left\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\right\} \\ \text{a loss function:} \quad & L(f, y). \end{aligned}$$

Empirical risk

- Normally, we then seek to minimize the **empirical risk**

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{m} L\left(f(x^{(i)}; \theta), y^{(i)} \right).$$

- We now learn how to solve the task by minimizing $J(\theta)$.
  $\Rightarrow$ Optimisation based learning!
- We use modified versions of gradient descent to minimize $J(\theta)$.

LOSS FUNCTIONS

Loss function for regression

- If $f(x;\theta) = \hat{y}(x;\theta)$, we can use, e.g.,

$$L(f, y) = \|f - y\|_2^2 = (f_1 - y_1)^2 + (f_2 - y_2)^2 + \cdots + (f_n - y_n)^2.$$

Note: the empirical risk then approximates the **mean squared error**.

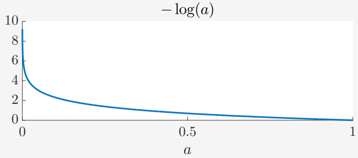
Cross-entropy and classification, $y \in \{1, 2, \dots, C\}$

- The **cross-entropy** is the standard loss:

$$L(a^{[L]}, y) = -\log a_y^{[L]}.$$

- We want to maximise

$$a_y^{[L]} = \Pr[Y = y \big| X = x, \theta].$$



An introduction to object detection

Multi-Object Tracking

Lennart Svensson

DEEP LEARNING AND MOT

- How can we use deep learning for multi-object tracking (MOT)?
  - Perform tracking directly on, e.g., lidar point clouds.
    No need for **deep learning**.
  - **Detect objects** using deep learning.
    Detections fed into a MOT alg. $\Rightarrow$ estimate set of objects/trajectories.
  - Compute **data association probabilities** using deep learning. Use object features to recognize an object at different times.
  - **Perform MOT** purely based on deep learning.
    Directly estimate object trajectories.

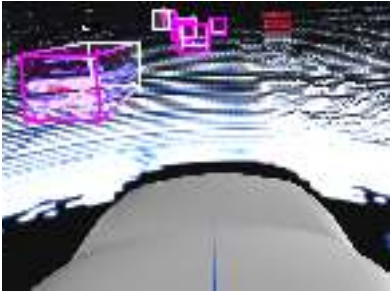


Image created using data from Geiger et al (2012), “Are we ready for Autonomous Driving? The KITTI Vision Benchmark Suite”, *CVPR*.

WHY OBJECT DETECTION?

– Why focus on using deep learning for **object detection**?

- Techniques are fairly “mature”: already yield good performance.
- Challenging to directly work with, e.g., raw images.
- Enables us to still leverage on motion models and MOT algorithms.
- Often manageable to obtain the data needed to train an object detection algorithm.

WHAT IS OBJECT DETECTION?

Object detection

- Determine the number of objects (of relevant classes).
- For each object, determine:

$$\begin{cases} \text{object class} \\ \text{object shape.} \end{cases}$$

- Object shape is often a bounding box in 2D or 3D.
- **Potential challenges:** many objects, partially occluded objects, objects of different sizes and distances, etc.

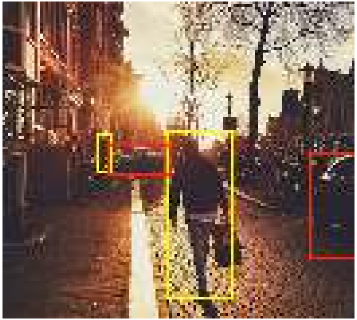
Single shot detectors – training

Multi-Object Tracking

Lennart Svensson

BASIC APPROACH

- Suppose we only have two classes: pedestrians and cars.
- **Key challenge:** number of objects is unknown $\Rightarrow$ variable number of outputs!



Suggested solution

- Provide a fixed number of bounding boxes+classifications.
- Classify bounded boxes as “object” or “not an object”.

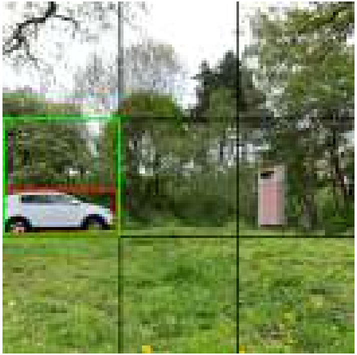
- By only considering “objects” we produce a variable number of boxes+classifications.

TRAINING

- Separate input into 3×3 cells.
- For each cell, we want to produce

$$y = \begin{bmatrix} p \\ c_1 \\ c_2 \\ x \\ y \\ w \\ h \end{bmatrix} = \begin{bmatrix} \text{0 if “no object”, 1 if “object”} \\ \text{1 if “pedestrian”, 0 “otherwise”} \\ \text{1 if “car”, 0 “otherwise”} \\ \text{x-position} \\ \text{y-position} \\ \text{width} \\ \text{height} \end{bmatrix}.$$

- During training, we try to make network output “similar” to these vectors.

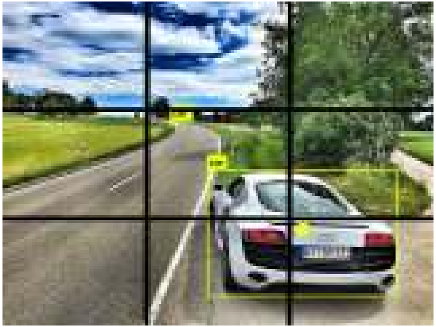


- For **green cell**:

$$y = \begin{bmatrix} 1 & 0 & 1 & x & y & w & h \end{bmatrix}^T$$

ANCHOR BOXES

- **Q:** Which cell is an object associated to?
  A: The cell that contains its center point.



ANCHOR BOXES

- **Q:** Which cell is an object associated to?
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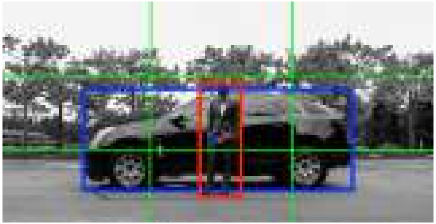


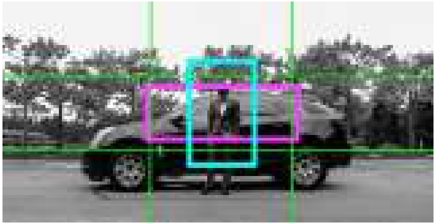
- **Q:** Can we handle multiple center points in one cell?

ANCHOR BOXES

- **Q:** Which cell is an object associated to?
  A: The cell that contains its center point.
  Note: we associate the object to an anchor box, in that cell, with similar shape (highest IoU).

- **Q:** Can we handle multiple center points in one cell?
  A: We have multiple **anchor boxes** in each cell.

- **Q:** What is an anchor box?
  A: An initial guess for a bounding box (fixed size). We output one y for every anchor box.



Single shot detectors – testing

Multi-Object Tracking

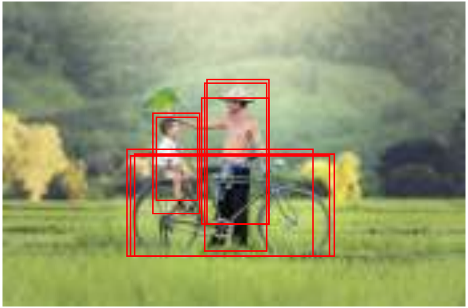
Lennart Svensson

TOO MANY BOUNDING BOXES

- We compute a vector y for every anchor box.
- We often then remove all bounding boxes:
$$p < 0.5.$$
- We are normally left with a few bounding boxes for each object.

NON-MAXIMUM SUPPRESSION

- 1. Remove all anchor boxes: $p < 0.5$.
- 2. Find anchor box, i, with largest p_i, and store i to set of approved anchor boxes.
- 3. Remove all anchor boxes:
  - the same most probable class,
  - the bounding box overlaps substantially with bounding box i.
- 4. Repeat.



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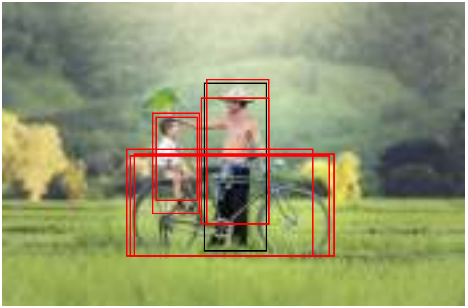

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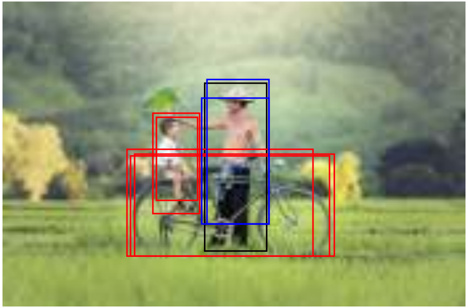

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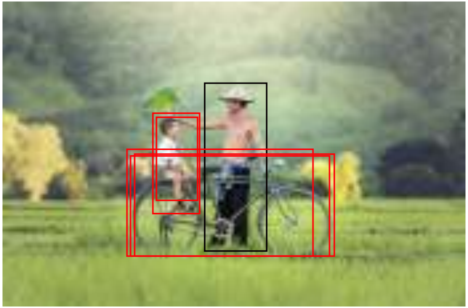

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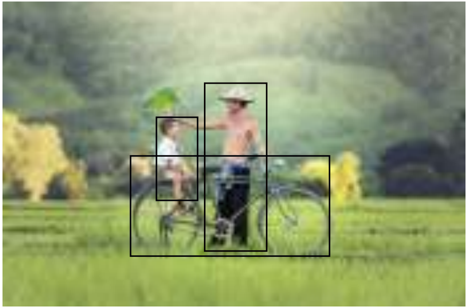

CONCLUSIONS

- We have studied **object detection** using deep neural networks.
- For every object in the image, we want to produce a classification and a bounding box.
- Object detection can also be used for lidar data, stereo images, etc, and combined MOT algorithms.