# Focused Transformer: Contrastive Training for Context Scaling

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#### Abstract

Large language models have an exceptional capability to incorporate new information in a contextual manner. However, the full potential of such an approach is often restrained due to a limitation in the effective context length. One solution to this issue is to endow an attention layer with access to an external memory, which comprises of (key, value) pairs. Yet, as the number of documents increases, the proportion of relevant keys to irrelevant ones decreases, leading the model to focus more on the irrelevant keys. We identify a significant challenge, dubbed the distraction issue, where keys linked to different semantic values might overlap, making them hard to distinguish. To tackle this problem, we introduce the Focused Transformer (FoT), a technique that employs a training process inspired by contrastive learning. This novel approach enhances the structure of the (key, value) space, enabling an extension of the context length. Our method allows for fine-tuning pre-existing, large-scale models to lengthen their effective context. This is demonstrated by our fine-tuning of 3B and 7B OpenLLaMA checkpoints. The resulting models, which we name LONGLLAMA<sup>2</sup>, exhibit advancements in tasks requiring a long context. We further illustrate that our LONGLLAMA models adeptly manage a 256k context length for passkey retrieval.

## 1 Introduction

Language models have served as a catalyst for substantial advancements in several areas, including natural language processing [Radford et al., 2019, Brown et al., 2020], code generation [Chen et al., 2021, Li et al., 2022], quantitative reasoning [Lewkowycz et al., 2022] and theorem proving [Polu and Sutskever, 2020, Jiang et al., 2022, Mikuła et al., 2023]. One of the central challenges with language models is the effective incorporation of extensive new knowledge. The common practice of fine-tuning the model is not only resource-intensive and complex to manage, but it also does not always clearly indicate how to incorporate new knowledge. For example, fine-tuning on a text such as "Alice in Wonderland" does not equip the model to answer questions about the story itself, but rather it trains the model to predict the next token or complete masked sentences. A promising alternative – integrating the new knowledge within the context – doesn't require training but is considerably

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<sup>&</sup>lt;sup>2</sup>We release the checkpoints and the source code of LONGLLAMA , see also our colab.

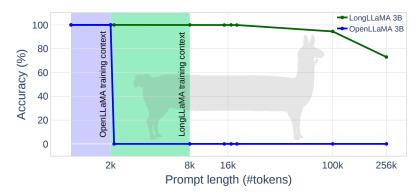


Figure 1: Accuracy of LONGLLAMA 3B on passkey retrieval compared to the original OpenLLaMA model. Our method extrapolates beyond the training length, achieving 94.5% accuracy at a context length of 100k and 73% at 256k tokens, while the baseline is unable to handle context longer than its training length (2k).

restricted by the model's effective context length. For this method to work with large knowledge databases, the model needs to manage a context length extending to millions of tokens.

In this research, we highlight one of the primary obstacles in augmenting the context length: as the number of documents increases, the ratio of pertinent to irrelevant tokens diminishes. The standard training procedure frequently results in overlaps between keys connected with irrelevant values and those related to relevant ones, exacerbating the model's task of differentiating between them. We term this challenge the *distraction issue*.

We propose the *Focused Transformer* (FoT), an innovative technique developed explicitly to address this issue. The Focused Transformer permits a subset of attention layers to access an external memory of (key, value) pairs through the k-nearest neighbors (kNN) algorithm, akin to the method used in [Wu et al., 2022]. This mechanism effectively extends the total context length. The distinctive aspect of the Focused Transformer is its training procedure, drawing from contrastive learning. This method addresses the distraction issue and facilitates larger memory capacities. Specifically, during the training phase, we deliberately expose the memory attention layers to both relevant and irrelevant keys (like negative samples from unrelated documents). This strategy incentives the model to differentiate keys connected with semantically diverse values, thereby enhancing their structure.

We introduce and make available LONGLLAMAs ( ), fine-tuned OpenLLaMA models with FoT, demonstrating that our method does not require long context during training and can be applied to existing models. Notably, LONGLLAMAs show significant improvements on tasks necessitating long-context modeling. In particular, they can manage a 256k context length on the passkey retrieval task [Mohtashami and Jaggi, 2023].

Our research contributions are the following:

- 1. We pinpoint the distraction issue as a significant challenge and a primary obstacle to scaling up the context length in Transformer models, particularly in multi-document scenarios.
- **2.** We develop the Focused Transformer (FoT), designed to alleviate the distraction issue. FoT includes a unique training objective that improves the (key, value) structure, enabling the use of extensive external memory and k-nearest neighbors lookup to scale the context length.
- 3. Our method is simple to implement, and it provides the benefit of augmenting existing models with memory without modifying their architecture, facilitated by cost-effective fine-tuning. We demonstrate this on the 3B and 7B OpenLLaMA checkpoints. The resulting models, named LongLLaMAs, display enhancements on tasks that benefit from increasing the number of few-shot demonstrations in the extended context, such as TREC [Li and Roth, 2002, Hovy et al., 2001] and WebQS [Berant et al., 2013]. We also prove that for passkey retrieval Mohtashami and Jaggi [2023], our LongLLaMA models successfully handle a 256k context length.

**4.** We further scrutinize FoT's capabilities across various datasets and model sizes. We show that a FoT trained with a total context of 512 tokens can extrapolate to 16 million tokens in a benchmark dictionary lookup task. We also assess FoT on long-context language modeling tasks such as books (PG-19), mathematics (arXiv), code (GitHub), and formal proofs (Isabelle), where it exhibits improvements in perplexity over baselines.

## 2 Related work

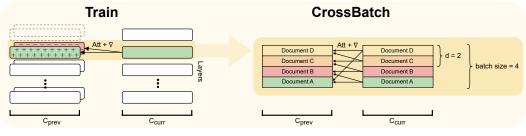
Long-context transformer architectures A multitude of approaches have been developed to increase the context length of transformers, mostly focusing on alleviating the quadratic complexity of the attention computation. For instance, Transformer-XL [Dai et al., 2019] caches the previous context and enables the linear extension of context with the number of layers. Longformer [Beltagy et al., 2020] employs an attention mechanism that allows tokens to attend to distant tokens sparsely, reducing the computational complexity. BigBird [Zaheer et al., 2020], LongT5 [Guo et al., 2021], and [Dao et al., 2022] also use sparse attention to handle long sequences. Hourglass [Nawrot et al., 2021] downsamples activations in intermediate layers to reduce computation and enable longer contexts. COLT5 [Ainslie et al., 2023] proposes conditional computation to save memory and enable larger contexts. Memorizing Transformer [Wu et al., 2022] uses kNN lookup to pick up the most relevant tokens, which might also be seen as a way to reduce the computational complexity of attention. Our work adheres to this approach and aims to train a key space that handles longer attention context length (e.g., by mitigating the distraction issue) and, thus, has better long-context capabilities.

Fine-tuning LLMs for longer context Prior works such as RETRO [Borgeaud et al., 2022] (RETROfitting) and Memorizing Transformer [Wu et al., 2022] have demonstrated a promising path for fine-tuning existing LMs to add new capabilities without the need to retrain the entire model. More recently, a number of works have explored fine-tuning LLaMA to extend its context length. Landmark attention [Mohtashami and Jaggi, 2023] proposes a compression scheme of LLM's context into landmarks, increasing the context length of LLaMA-7B to 32K. Position Interpolation (PI, [Chen et al., 2023] and [kaiokendev, 2023]) introduces a modification to the rotary positional encoding scheme that enables fine-tuning for 32K context. In contrast to this work, our method does not rely on positional encodings, following the findings from [Haviv et al., 2022]. Removing positional encoding in memory allows us to extrapolate to 256k tokens, although the model was only trained on sequences up to 8K, yielding theoretically unbounded context length.

Contrastive learning Contrastive learning aims to learn good representations by comparing positive and negative examples. CLIP [Radford et al., 2021] and SimCLR [Chen et al., 2020] are two popular contrastive learning methods that have achieved state-of-the-art performance in the image domain. During contrastive pre-training, negative examples are kept in the same batch to learn to distinguish them from positive examples. Scaling the batch size in contrastive learning has been demonstrated to enhance the quality of representations, as shown in [Gao et al., 2021b]. It has been suggested [Gao et al., 2019] that the embedding space in language modeling suffers from degeneracy, where embeddings are tightly packed in a narrow cone, making it difficult to distinguish between them. TRIME [Zhong et al., 2022] proposes a training approach designed for training LMs with memory augmentation, which uses in-batch negatives to improve the quality of representations. The main difference between this and our approach is that we incorporate negatives into the memory attention layer instead of interpolating in the output layer.

#### **3 FoT: Focused Transformer**

Our method, the Focused Transformer (FoT), is a simple plug-and-play extension of transformer models and can be used both to train new models or fine-tune existing, possibly large, models with longer context. To this end, FoT uses *memory attention layers* and the *crossbatch* training procedure. Memory attention layers enable the model to retrieve information from the external memory at inference time, effectively extending the context. The crossbatch training procedure biases the model to learn (*key*, *value*) representations, which are easy to use by a memory attention layer. See Figure 2 for an overview of the FoT architecture and Appendix F for pseudocode.



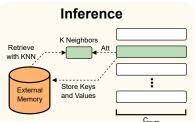


Figure 2: The Focused Transformer overview. During inference, a *memory attention layer* (green) uses external memory of (key, value) pairs via kNN lookup, which effectively extends its context length. This layer is trained using *crossbatch*. Namely, the tokens from the current context  $C_{curr}$  attend in a differentiable way  $(Att + \nabla)$  to the previous context  $C_{prev}$  of the same document and, importantly, d-1 contexts of other documents. The latter serve as 'negative' examples intended to better shape the (key, value) space.

## 3.1 Memory attention layers

Memory attention layers  $\mathcal L$  are endowed with access to an external memory database during inference. Namely, each query in  $\ell \in \mathcal L$  attends to preceding keys from the local context and the top k most matching keys from memory. The memory keys are ranked by the inner product with the query and retrieved using the kNN search algorithm. We use the exact kNN search implemented in FAISS [Johnson et al., 2017]. The memory is populated incrementally with (key, value) pairs processed by  $\ell$  beforehand. Our memory attention layer design is closely related to [Wu et al., 2022], we follow most of its design choices, except for the gating, which we replace with a simpler mechanism, which turns out to be more effective in our applications. See details in Section 5.6.3 and Appendix B.2. We remove positional encodings in memory layers in all our models except LONGLLAMAs. This allows LONGLLAMA checkpoints to be a drop-in replacement for LLaMA checkpoints.

#### 3.2 Crossbatch training procedure

Our training procedure is a novel way of training (or fine-tuning) transformer-based architectures in order to improve the structure of the (key, value) space. The main motivation is to shape this space so that a memory attention layer  $\ell \in \mathcal{L}$  can easily focus on relevant information. The key idea, inspired by contrastive learning, is to expose  $\ell$  to (key, value) pairs from the current and previous local context of the given document (positives) and d-1 contexts from unrelated documents (negatives). Importantly, this is done in a differentiable way.

To achieve this, we use a data pipeline in which each element of the batch corresponds to a different document. We embed the previous  $(C_{\text{prev}})$  and the current  $(C_{\text{curr}})$  local context for each of the processed documents. The overview of our procedure can be found in Figure 2. Specifically for each document  $\delta$  in  $C_{\text{curr}}$  we create a set  $\{p_i^{\delta}\}_{i=\{1,\dots,d\}}$  consisting of the (key, value) pairs from the previous local context of  $\delta$  (positives), along with pairs from d-1 other contexts coming from  $C_{\text{prev}}$  (negatives). We also experiment with varying the number of previous contexts and negatives for different batch elements.

We do not use external memory during training. This has two important consequences. One, the operation is fully differentiable, and thus, we improve all the (key, value) pairs in  $p^{\delta}$ . Two, the procedure is easy to implement; it does not require any additional loss (i.e., uses the standard transformer training objective) and is done on the level of the data loading pipeline and a minor self-attention change. The only new hyperparameter is d, which prescribes the ratio of positive to negative samples. Typically, we find it beneficial to start with small  $d \leq 8$  (otherwise, the model tends to ignore the previous local context) and later switch to bigger values, say  $d \geq 64$ . Appendix B.3 provides more details about the method.

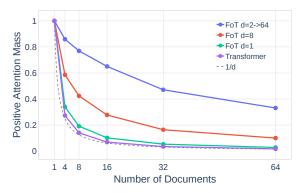


Figure 3: Distraction issue. We compare FoT trained with different values of parameter d to the standard Transformer baseline. During the evaluation, both models see the previous local context and some contexts from other documents in the chosen layer (as in crossbatch training procedure). For a document  $\delta$  we measure the distribution of attention mass on  $p^{\delta}$ . Scale x: the number of contexts from documents that the model can see. Scale y: avg attention mass to the previous local context of the current document.

#### 3.3 The distraction issue

In this section, we conceptualize what we call the distraction issue and hypothesize it is one of the key problems in using large memory databases. Namely, during the standard training, the model is not incentivized to distinguish the keys from different documents. We measure that the attention mass is evenly spread on the related and unrelated documents; see Figure 3. More precisely, for a document  $\delta$ , let  $w_{ij}$  be the softmax weights related to  $p_{ij}^{\delta}$  constructed as described in Section 3.2.

We define the positive attention mass as  $r_d := \sum_j w_{1j}/\sum_{i=1}^d \sum_j w_{ij}$ . We observe that  $r_d \approx 1/d$ , which can be interpreted as the fact that the attention is equally distracted by the positive (coming from the current document at i=1) and negative keys. This is an undesirable property since when scaling the memory, the attention becomes increasingly distracted. We show that the crossbatch mostly alleviates the distraction issue, resulting in a *focused* attention. More information can be found in Appendix B.4. In Section 5.4, we also show that the distraction issue has a harmful effect on metrics like perplexity.

## 4 LONGLLAMA : extending LLaMA's context length with FoT

One of the promises of our work is that FoT can be used to fine-tune already existing large models to extend their context length. In this section, we show that this is indeed the case. We use OpenLLaMA-3B and OpenLLaMA-7B models trained for 1T tokens as starting points and fine-tune them with FoT. We show that the resulting models, which we call LONGLLAMAs, are capable of extrapolating beyond their training context length (even up to 256K) and retain the performance on short-context tasks. We release the inference code on GitHub: https://github.com/CStanKonrad/long\_llama and the LONGLLAMA-3B checkpoint on Hugging Face: https://huggingface.co/syzymon/long\_llama\_3b. We note that our checkpoint is backward compatible, i.e. can be used with any existing LLaMA inference code (both in Hugging Face and other implementations), albeit without long-context capabilities.

## 4.1 Experimental setup

The architecture of the models is the same as OpenLLaMAs, see Geng and Liu [2023] and Appendix A.1. We use  $\mathcal{L} = \{6, 12, 18\}$  (resp.  $\mathcal{L} = \{8, 16, 24\}$ ) as the memory layers for 3B (resp. 7B) LONGLLAMA model. We fine-tune the models on 10B (resp. 3B) tokens using FoT, 8k context length and our dataset mixture based on RedPajama [TogetherComputer, 2023], see Appendix A.3.

There are three minor differences from the standard FoT procedure. First, we retain the positional encodings in the local context of the memory layers (this is not necessary for FoT, but makes our checkpoints fully compatible with any existing LLaMA inference codebase). To be more precise, queries and keys from the local context (up to 2K tokens) receive the standard LLaMA rotary positional encoding, whereas memory keys are encoded as if they had position 0 in the local context window. Second, we use dense attention instead of the kNN retrieval, as we found only marginal

performance differences, and it is simpler to implement. Third, we modify the crossbatch training procedure to have more fine-grained control over the number of additional contexts and the ratio of positive to negative samples. All these differences are detailed in Appendix A.2.

#### 4.2 Context length extrapolation on the passkey retrieval task

fective context length of LONGLLAMA, namely the We use passkey retrieval introduced in [Mohtashami and Jaggi, 2023], a synthetic task designed to measure this property. In this task, the model has to retrieve a passkey placed randomly in a long prompt. Results are shown in

We first measure the ef- There is an important info hidden inside a lot of irrelevant text. Find it and memorize them. I will quiz you about the important information there. effectively attend each other. green. The sky is blue. The sun is yellow. Here we go. There and back again.> The pass key is <PASS KEY>. Remember it. <PASS KEY> is the pass kev. <suffix filler> What is the pass key? The pass key is

> Prompt Format used in the passkey retrieval task, copied from [Mohtashami and Jaggi, 2023].

Figure 1 - importantly, our 3B model is capable of solving this task much beyond its training context length 8K, achieving 94.5% accuracy for prompts of length 100k and 73% for 256k.

#### 4.3 Improving few-shot learning accuracy with longer context

We measure long-context capabilities of these models on two downstream tasks, TREC question classification [Li and Roth, 2002, Hovy et al., 2001] and WebQS question answering [Berant et al., 2013]. We follow the experimental setup of [Hao et al., 2022]. Namely, we few-shot prompt the models with as many demonstration examples as possible up to the given context length. We do not use structured prompting like in [Hao et al., 2022] - instead, we directly provide all demonstrations in context.

We observe significant accuracy gains from longer contexts on TREC and some improvements on WebQS (see Table 1). The TREC dataset consists of 50 classes. A model is tasked to predict the class label given in-context examples. Only 100 examples fit the standard context length (2K); it is not unusual that no class example is present for a given question, making the task impossible. Increasing the context length and the number of examples mitigates this risk. Moreover, having more demonstrations of the given class is also likely to be beneficial.

Table 1: Few-shot in-context learning performance of LONGLLAMA; accuracy on TREC and WebQS. We see significant gains from the additional context on the TREC dataset. To calculate the results, we average over 20 trials for sampling in-context demonstrations from the train set; the resulting confidence intervals for TREC and WebQS are smaller than 1% and 0.1%, respectively.

Dataset	TREC		Wel	oQS
Context	LONGLLAMA 3B	LongLLaMA 7B	LONGLLAMA 3B	LongLLaMA 7B
2K	67.0	63.2	21.2	25.5
4K	71.6	72.7	21.4	26.4
6K	72.9	74.9	22.2	27.2
8K	73.3	75.9	22.4	27.7

#### 4.4 Comparison to standard long-context fine-tuning

In this section, we compare FoT to standard long-context fine-tuning, showing that it already achieves better performance for the context length used for fine-tuning and, importantly, that it can extrapolate beyond this context length, which is not the case for the baseline.

For comparisons, we fine-tune two models, one trained with FoT and another one (baseline) with standard fine-tuning (done similarly to [MosaicML, 2023, Nijkamp et al., 2023]). In both cases, we use 3B models fine-tuned on 1B tokens using the 4K context length. We evaluate both models on a number of few-shot downstream tasks in the setting described in Section 4.3.

In most cases, see Table 2, we observe accuracy improvements when more few-shot demonstrations are provided in the extended context (from 2K used by OpenLLaMA to 4K used in our fine-tuning). On TREC, the gains from additional context are significant for both models, while on WebQS, the standard fine-tuning baseline does not provide any improvement from extended context. Notably, the model fine-tuned with FoT enjoys further accuracy gains when evaluated with context lengths beyond its training length (6K and 8K). This shows extrapolation capabilities of FoT, which are not present in the baseline (see e.g. Figure 1).

Table 2: Few-shot in-context learning performance comparison between standard fine-tuning on 4K context (baseline) and FoT fine-tuning on the same context length for 1B tokens. On TREC, FoT is able to utilize additional examples beyond its training context length to achieve higher accuracy at 8K context length, which is not possible for the baseline since its context is bounded to 4K.

Dataset	T	TREC		ebQS
Context	baseline	FoT (ours)	baseline	FoT (ours)
$\overline{2K}$	52.8	55.6	20.7	20.8
4K	57.2	60.9	18.7	21.0
6K	_	61.7	_	21.2
8K	_	62.5	_	20.7

#### 4.5 Performance on short-context tasks

Fine-tuning for longer contexts could hurt performance on the original context length (2K), as the training data distribution changes. We show that this is not the case for the LONGLLAMA models by evaluating them using the LM Evaluation Harness library [Gao et al., 2021a]. On most tasks, the performance is kept intact; see Appendix A.4 for details, and Table 3 for the average scores. This also confirms that LONGLLAMAs could be used as a drop-in replacement of LLaMA models as they are compatible with the original LLaMA inference code.

Table 3: Comparsion with OpenLLaMA models on the original context length of 2K. We provide the average score calculated on Language Model Evaluation Harness [Gao et al., 2021a]. Detailed results can be found in Table 7 in Appendix A.4.

lm-eval	OpenLLaMA 3B	LONGLLAMA 3B	OpenLLaMA 7B	LongLLaMA 7B
Avg score	0.53	0.53	0.55	0.55

## 5 Analysis of FoT

In this section, we perform extensive experiments on smaller models to analyze and further validate our approach. In particular, we answer the following questions: (1) How does FoT perform when scaling the context length at inference time? (2) Can FoT be used to extend the context length of an existing, pre-trained model? (3) How effectively can it handle distractions, and how does this capability translate to enhanced performance in long-context language modeling tasks? Moreover, we provide ablation studies of our method and additional analysis.

#### 5.1 Experimental setup

**Architecture** For experiments described in this section we use decoder-only Transformer [Vaswani et al., 2017] models with 12 layers and 184M parameters (unless stated otherwise, e.g., in fine-tuning experiments in Section 5.3 we scale to 1.2B). Following Wu et al. [2022]; we pick  $\ell=8$  as the memory attention layer. We tune k=128, the number of top keys retrieved by kNN. In most experiments, we start training with a small crossbatch dimension  $d \le 8$  and switch to  $d \ge 64$  after some training. For more details about the architecture and hyperparameters, see Appendix B and Appendix C.

**Evaluation** We distinguish two evaluation settings: single-document (abbreviated to single-doc) and multi-document (abbreviated to multi-doc). The single-doc setting is typically used for evaluating

models that process long contexts. Here, we clear the memory for each new document, ensuring that only the current document is available in the context. The multi-doc setting retains memory across multiple documents without resets. This scenario is akin to a long-context retrieval task where the model must focus on tokens from useful documents without getting distracted by irrelevant ones.

**Datasets** We evaluate on the following long-context language modeling datasets: PG-19 (English books), arXiv (mathematical papers), GitHub (code), and Isabelle (formal proofs). PG-19 [Rae et al., 2019] is a large dataset of English-language books published prior to 1919, sourced from the Project Gutenberg archive. This dataset is a well-established benchmark for evaluating long-context language models [Sun et al., 2021]. The arXiv dataset contains LaTeX source of papers labeled as "Mathematics" that were obtained by downloading articles through the arXiv Bulk Data Access. The token count per paper in this dataset is comparable to that of a book in PG19. For details on the remaining datasets, refer to Appendix G.

## 5.2 Scaling of the context length to 16M

We use a synthetic dictionary lookup task to check whether the model trained with our method can utilize a large memory to extend its context length. In this task, the model is first provided with  $k_i : v_i$  mappings and then asked what value is associated with a particular key. We train models using documents of length 512. The first half of each document defines keys and values associated with them, whereas the second consists of questions about values associated with keys defined in the first half. Details about the task can be found in Appendix D. For this task, we use smaller 37M parameter models.

For FoT, we use a local context of 256, thus the model needs to use the memory attention layer to answer the questions correctly. We start with d=1 and increase to d=128 as soon as the model is able to reach 98% training accuracy. During the inference, we use k=32 (the number of keys retrieved by kNN). Figure 4 shows that FoT, after 5k steps of training, can effectively utilize memory consisting of 16M tokens achieving accuracy above 92%.

As a baseline, we use a standard transformer model trained with the context length of 512. In evaluation, we test different local context lengths, which quickly leads to very poor results.



Figure 4: Accuracy vs number of dictionary tokens in a dictionary look-up task. The task format is as follows:  $\langle k \rangle k_1 \langle v \rangle v_1 \langle k \rangle k_2 \langle v \rangle v_2 ... \langle k \rangle k_n \langle v \rangle v_n \langle q \rangle k_i \langle v \rangle v_i ...$ , where a dictionary is provided, followed by queries on randomly selected keys. Accuracy is determined by measuring the predicted values  $v_i$  after  $\langle q \rangle$  tokens. Models were trained on examples containing 512 tokens and evaluated with an extended context length. FoT demonstrates high accuracy even when the memory size is large. The baseline transformer fails already for 16K tokens. Error bars represent the minimum and maximum on 10 seeds.

#### 5.3 FoT fine-tuning and context length extrapolation

FOT is a minimal modification to the standard transformer architecture; therefore, it is possible to fine-tune existing models to endow them with a longer context length via the memory attention layer, as we already demonstrated in Section 4. In this section, we deepen this analysis (on a smaller model) by studying perplexity improvements on various datasets.

As a base model, we use a standard transformer model pre-trained for 100k steps with context of 1K tokens using the standard objective and fine-tune with the FoT objective (i.e. crossbatch). The data

used for both fine-tuning and pre-training is the C4 dataset Raffel et al. [2019a] (we omit documents shorter than 2K tokens). The fine-tuning phase takes 10k steps. We use the crossbatch dimension d=128 and local context of 1K tokens (context is 2K during training). We evaluate models in a zero-shot way on 4 language modeling datasets, which require long context: arXiv, PG-19, GitHub and Isabelle, see Section 5.1 and Appendix C for details.

Table 4: Perplexity for different context lengths after fine-tuning a standard transformer model. The model is fine-tuned using the FoT objective (i.e., crossbatch) on C4 and evaluated zero-shot varying the context size. Transformer-XL [Dai et al., 2019] and Memorizing Transformer [Wu et al., 2022] fine-tuned in the same setting are used as baselines.

Method	<b>Context Length</b>	GitHub	Isabelle	arXiv	PG-19
	2K	6.72	5.63	8.17	23.74
FoT	4K	5.88	4.93	7.44	23.25
FO1	16K	5.43	4.51	6.94	22.85
	64K	5.32	4.44	6.81	22.65
Transformer-XL	2K	6.85	5.76	8.21	23.57
	2K	8.10	7.34	9.39	24.03
Mamarizina Transformar	4K	7.55	6.93	8.95	23.62
Memorizing Transformer	16K	7.27	6.66	8.66	23.32
	64K	7.26	6.64	8.60	23.24

In Table 4, we observe that FoT enjoys steady perplexity gains up to 64K tokens, although it was fine-tuned only with the 2K total differentiable context length. We compare the model perplexity to the following baselines: Memorizing Transformer (MT) [Wu et al., 2022] fine-tuned with the local context of 1K and memory size of 16K, and Transformer-XL [Dai et al., 2019] fine-tuned with both local context and window length of 1K. To ensure a fair comparison, all three models are fine-tuned from the same base checkpoint. When evaluated with a context of 2K, our method achieves results on par with the Transformer-XL baseline, which has access to the previous context in all layers, unlike MT and FoT. Compared to the MT baseline, we achieve better scaling when evaluated with 64K context length and significantly better perplexity values. Unlike MT, our method does not require training on long sequences, which is reflected by the lower perplexities of FoT when evaluated in the zero-shot setting. For more details, see Appendix E.

#### 5.4 Handling distractions in language modeling tasks

In this section, we measure how handling distractions in the multi-document setting helps in language modeling.

We pick the PG-19 dataset [Rae et al., 2019] and measure the perplexity of the next token prediction (language modeling task) when varying the size of multi-doc memory (in this case consisting of books). Intuitively, the memory tokens corresponding to the current book might be beneficial (which is also confirmed in [Wu et al., 2022]), while the ones from the other books are unlikely to be useful and thus are distractions.

We observe, see Figure 5, that higher values of the crossbatch dimension d lead to better perplexity. This aligns with the observations in Section 3.3, indicating that by mitigating the distraction issue, we experience benefits in language modeling.

Moreover, all versions of FoT are able to utilize memory and achieve much better perplexity than the standard Transformer (no memory). Unsurprisingly, perplexity increases with memory size, but we stress that this happens gracefully. In the standard variant of FoT (bold line), the perplexity increases only by 0.18 when scaling to > 500k tokens. Importantly, the perplexity of FoT is close to this of Memorizing Transformer with the single-doc memory, which we treat as a soft lower bound since it is not exposed to distractions from unrelated books.

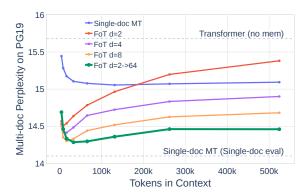


Figure 5: Perplexity in multi-doc setting. FoT was trained with local context of size 512 and different d. FoT 2->64 was initially trained with d=2 and then switched to d=64. Single-doc MT was trained with a memory size of 16K. As we increase the memory size, the number of distractions (irrelevant keys) increases, making the task harder. Single-doc MT evaluated in single-doc setting is a soft lower bound since it is not exposed to distractions.

## 5.5 Context length extrapolation in single-doc

The primary motivation behind FoT is to improve the multi-doc setting performance by handling distractions. Interestingly, our method also helps to extrapolate to longer contexts, even when evaluated in the single-doc setting.

To study this, we perform FoT fine-tuning (as in Section 5.3) and evaluate the perplexity of the resulting model on the PG-19 dataset with different context lengths in the zero-shot fashion. To deepen the analysis, we introduce an additional parameter w (the number of previous contexts used in cross batch training procedure). We provide results for w=1 (the standard setting for FoT, that corresponds to the total differentiable context being  $2 \cdot 1024$ ) and w=2 (corresponding to the total differentiable context  $3 \cdot 1024$ ).

We observe, see Figure 6, improvements when context grows, even far beyond the training context length, which reaffirms the hypothesis that FoT helps with extrapolation to longer contexts. Moreover, d=2 is significantly better than d=1. When comparing d=1 and w=2 to d=2 and w=1, we observe that the former is slightly better. This is natural, as the former has longer training context.

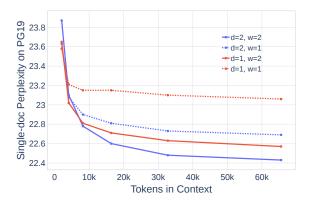
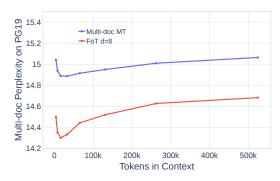


Figure 6: Zero-shot performance on PG19 of FoT pretrained on C4. Model fine-tuned with the crossbatch dimension d=2 outperforms the one with d=1. Using the double (w=2) training context of 2048 is beneficial.

#### 5.6 Ablations and design choices

In this section, we focus on two key properties of crossbatch training procedure: differentiability and the inclusion of negatives. We also discuss the relation to Memorizing Transformer in terms



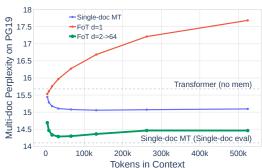


Figure 7: Perplexity on PG-19 in the multi-doc setting. Both FOT and Multi-doc MT were trained with local context of size 512. During training, Multi-doc MT utilized memory of size 4096 shared across 8 documents. Differentiability of keys and values results in better perplexity.

Figure 8: Importance of negatives in the multi-document setting. We compare FoT trained with d=1 to the one that started with d=2 and later switched to d=64. For additional comparison, we show the performance of MT trained with the memory of size 16K.

of the training protocol and memory integration. We refer to Appendix B.5 for a detailed technical description of differences between FoT and Memorizing Transformer.

#### 5.6.1 Impact of differentiable keys and values

We compare FoT to Memorizing Transformer, which uses a non-differentiable memory of keys and values during training. In the multi-doc experiment presented in Figure 7, both MT and FoT are trained with local context of 512. We observe that FoT is significantly better when the context is expanded during inference, which confirms that differentiable keys and values are beneficial.

We also check whether differentiable keys and values can improve the performance in the

Table 5: Perplexity on PG-19 in the single-doc setting for various local context lengths during training. In these experiments, we used the same context length both during training and evaluation.

<b>Context Length</b>	FoT d=1	MT
512	14.18	14.68
1024	14.17	14.46
2048	14.11	14.43

single-doc setting. For this, we compare FoT with d=1 to MT with memory consisting of the previous local context. Table 5 confirms that differentiable keys and values can also help in this scenario.

## 5.6.2 Importance of negatives

We reaffirm the importance of negatives in a multi-document setting. In previous experiments in Figure 3, we already observed that increasing the number of negatives (i.e., increasing d) results in more attention mass being dedicated to relevant tokens. In Figure 8, we additionally show that the lack of negatives in training (d=1) results in a significant deterioration in model perplexity when the context length grows. This confirms that both using negatives and differentiability are important for FoT to work well.

#### **5.6.3** Relation to Memorizing Transformer

Memorizing Transformer Wu et al. [2022] is closely related to our method. The two key differences are 1) the training protocol and 2) how the memory is integrated into the model. In this section, we provide additional insights into these differences.

**Training protocol** In the previous sections, we have discussed the benefits of the cross-batch training, namely using the contrastive-inspired objective and backpropagating through the previous context. A potential advantage of the MT approach is that it is exposed to the whole memory during training (instead of just the previous context). We performed a proof-of-concept experiment combining the two approaches to explore this further.

Namely, we trained the model for 499k steps using crossbatch and fine-tuned it with the MT objective for 1k steps. Interestingly, we observed a significant improvement compared to the MT training with the same step budget, see Figure 9. We believe there is further room to explore various training protocols combining the best of both worlds.

Memory integration FoT uses a simple memory integration approach where the (key, value) pairs retrieved by kNN lookup are treated the same way as the local context. In contrast, MT uses a gating mechanism, a weighted average of the memory, and local values; see details in Appendix B.2. We evaluated both approaches and found no difference in performance between these two memory integration methods. How-

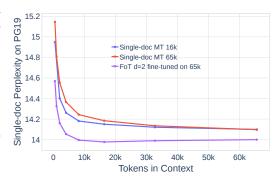


Figure 9: Single-doc eval of FoT finetuned for 1k steps on non differentiable memory. It achieves lower perplexity than MT, which has access to this memory for the whole training (500k steps).

ever, we decided to use our approach because it does not require any architectural changes (and thus makes fine-tuning existing models easy). For these reasons, we recommend using it. We speculate that the reason why the gating is not needed in FoT is another benefit of the fact that the crossbatch training backpropagates through the (key, value) pairs from the previous context  $C_{prev}$  in contrast to MT that cannot backpropagate there and needs to rely on local context when computing gradients for keys and values. Another reason might be the fact that  $C_{prev}$  is embedded for each batch, and thus staleness (see [Wu et al., 2022, Section 3.2]) is avoided.

#### 6 Limitations and future work

Our research opens a few avenues for future work. We list them as well as challenges and limitations.

Scaling up memory This is by far the most important future research direction. The challenges start from purely engineering, storing more than  $16M\ (key,value)$  pairs will require a distributed multi-node system. In our experiments, we use the exact kNN search, which is not scalable to large memory. Using approximate kNN search will require a lot of engineering effort, as well as careful evaluation of the impact of the approximation on the model performance.

Scaling up crossbatch We observed that increasing d is beneficial. In our experiments, we used d=64 or d=128, which is the maximum value that fits into the memory of a single TPUv3/TPUv2 machine, see also Appendix H. In future work, we want to further increase d as well as test on devices with bigger memory or utilize multi-node training.

**Exploring contrastive learning** The FoT training is inspired by rather basic contrastive learning (CL) techniques. We show that this improves the key structure so that the distraction issue is mitigated. We expect that other CL methods could be beneficial, for example, hard negative mining to utilize a larger memory during training (see [Lindgren et al., 2021]). We leave this for future work.

**Combining with other methods** Developing long-context methods is an active research field, see Section 2. We believe that some of these methods could be combined with FoT, resulting in mutually beneficial interactions.

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## A LONGLLAMA

#### A.1 Architecture

OpenLLaMA [Geng and Liu, 2023] is an open-source reproduction of LLaMA [Touvron et al., 2023]. It uses a decoder-only architecture with rotary positional embeddings, and a few changes including pre-normalization with RMSNorm [Zhang and Sennrich, 2019], and SiLU activation [Elfwing et al., 2017]. A SentencePiece [Kudo and Richardson, 2018] tokenizer with 32k vocabulary size is used.

## A.2 Extending context length with FoT

**Positional encodings** To achieve backward compatibility with the original LLaMA, we retain positional encodings in the local context. The tokens outside the local context are assigned the same position as the first token in the local context.

**Dense attention to longer context** To make the implementation simpler and less dependent on external software, we resign from using kNN lookup and perform attention over the whole memory. We have found only marginal performance differences between those two approaches to memory attention.

Crossbatch details For the 3B LongLLaMA model, we set  $\mathcal{L}=\{6,12,18\}$  as the memory layers. We vary the number of additional contexts  $d\in\{0,2,3\}$  across elements of the batch by dividing batch entries into four segments of equal size. Elements from the first segment only see local context (d=0). Elements from the second segment see two additional contexts (d=2), one from the same document (positive) and one from a different one (negative). Elements from the third segment see three additional contexts, two positives, and one negative. The last segment consists of elements exposed to three additional contexts coming from the same document. We abbreviate this setup as  $\frac{1}{4}(0,0), \frac{1}{4}(1,1), \frac{1}{4}(2,1), \frac{1}{4}(3,0)$ .

For the 7B LONGLLAMA model, we set  $\mathcal{L} = \{8, 16, 24\}$  as the memory layers. Here we divide batch entries into four segments and use the following setup:  $\frac{1}{4}(0,0), \frac{1}{4}(1,2), \frac{1}{4}(2,5), \frac{1}{4}(3,4)$ .

**Hyperparameters** We follow the choices of OpenLLaMA with respect to most of the hyperparameters, including using the same optimizer. During fine-tuning, we use a batch size of 256K tokens and constant learning rate of 2e-5, which is lower than the learning rate at the end of OpenLLaMA training (3e-5 after 1T tokens), and weight decay of 0.01.

## A.3 LLaMA fine-tuning dataset

We use a mixture based on RedPajama [TogetherComputer, 2023] and The Stack [Kocetkov et al., 2022] with the following proportions of each subset:

Table 6: Proportions of RedPajama subsets for the LONGLLAMA fine-tuning mixture. For python subset, data from The Stack is used (see text for details). We only train on documents with length being at least Min. doc. length, if specified (otherwise we train on all documents from that subset). The horizontal line separates long-context and short-context subsets.

Subset	Sampling proportion (%)	Min. doc. length
arxiv	25	-
python	25	4096
book	10	-
common_crawl	29	-
c4	5	1024
github	2	2048
stackexchange	2	1024
wikipedia	2	1024

All subsets apart from python are taken directly from RedPajama. For the python subset, we gather Python source code from The Stack and, to obtain long documents for training, concatenate files that

are in the same subdirectory in random order, using a similar procedure as for the GitHub dataset in Section G. Additionally, we filter out short documents for some subsets of the original RedPajama, namely shorter than the Min. doc. length column indicates.

## A.4 Language Model Evaluation Harness

To ensure that the performance of LONGLLAMAs has not degraded in short context scenarios, we evaluate our models on the Language Model Evaluation Harness benchmark [Gao et al., 2021a]. Table 7 compares our results with OpenLLaMA [Geng and Liu, 2023]. Similarly to the authors of OpenLLaMA, we omit CB and WSC tasks.

Table 7: Model comparison across different tasks/metrics on Language Model Evaluation Harness. The LONGLLAMA models were evaluated without context extension (i.e. as standard OpenLLaMA models). Results indicate that LONGLLAMAs maintain good performance in short context scenarios.

Task/Metric	OpenLLaMA 3B	LongLLaMA 3B	OpenLLaMA 7B	LongLLaMA 7B
anli_r1/acc	0.33	0.32	0.33	0.35
anli_r2/acc	0.32	0.33	0.36	0.37
anli_r3/acc	0.35	0.35	0.38	0.36
arc_challenge/acc	0.34	0.34	0.37	0.37
arc_challenge/acc_norm	0.37	0.37	0.38	0.38
arc_easy/acc	0.69	0.68	0.72	0.70
arc_easy/acc_norm	0.65	0.63	0.68	0.66
boolq/acc	0.68	0.68	0.71	0.71
hellaswag/acc	0.49	0.48	0.53	0.52
hellaswag/acc_norm	0.67	0.65	0.72	0.71
openbookqa/acc	0.27	0.28	0.30	0.30
openbookqa/acc_norm	0.40	0.38	0.40	0.41
piqa/acc	0.75	0.73	0.76	0.75
piqa/acc_norm	0.76	0.75	0.77	0.76
record/em	0.88	0.87	0.89	0.89
record/f1	0.89	0.87	0.90	0.90
rte/acc	0.58	0.60	0.60	0.59
truthfulqa_mc/mc1	0.22	0.24	0.23	0.24
truthfulqa_mc/mc2	0.35	0.38	0.35	0.35
wic/acc	0.48	0.50	0.51	0.50
winogrande/acc	0.62	0.60	0.67	0.67
Average score	0.53	0.53	0.55	0.55

#### **B** Architecture

#### **B.1** Transformer models

We use the transformer architecture introduced in [Vaswani et al., 2017] with a few standard changes. First, we use only the decoder without the encoder part. Secondly, we perform layer normalization before the input of both the attention and feed-forward modules. Additionally, we use Rotary Position Embedding [Su et al., 2021], normalize keys and queries [Henry et al., 2020], and introduce a learnable temperature parameter for each attention head.

The hyperparameters for each model size can be found in Appendix C. For training the models on PG-19, we use the standard T5 tokenizer with 32k vocabulary [Raffel et al., 2019b]. The larger models in Section 5.3 are trained with a custom SentencePiece tokenizer [Kudo and Richardson, 2018] with 64k vocabulary size.

#### **B.2** Memory attention layer

Memory attention layer  $\ell$  is one of the transformer layers, which has access to the external memory M. The memory stores (key, value) pairs. For each query q in  $\ell$ , we retrieve the k most matching entries from M and use them to compute the attention value. More precisely, we use the kNN algorithm to pick  $M_{top} := \{(key_1, value_1), \ldots, (key_k, value_k)\} \subset M$  such that  $\{\langle q, key_i \rangle\}_{i=1,\ldots,k}$  are the top k inner products in M. These are merged with the part of the local context before q denoted as  $C_{< q}$  and used to compute the attention value using the standard Transformer formula:

$$v := \sum_{(key,v)\in M_{top}\cup C_{\leq q}} s(key) \cdot v, \tag{1}$$

where s(key) is the softmax score for key. This softmax is calculated as follows:

$$softmax \left( \left[ \frac{\langle q, key \rangle}{\tau} \right]_{key \in M_{top} \cup C_{\leq q}} \right),$$

where  $\tau$  is a temperature parameter. In this approach, we do not distinguish between the local context and the memory.

Another way of integrating  $M_{top}$  is via gating. In this approach, we separately compute the attention value  $v_M$  for  $M_{top}$  and for the local context  $v_C$  (using the standard Transformer formula). Then we use a gating mechanism to combine them:

$$v := v_M \cdot g + v_C \cdot (1 - g), \quad g = \sigma(b_q),$$

where  $\sigma$  is the sigmoid function and  $b_g$  is a trainable bias. The gating approach was proposed in [Wu et al., 2022], see formula [Wu et al., 2022, (2)].

We found our approach, i.e. using (1), to be equally effective, see Figure 10. At the same time, (1) is simpler and does not require additional parameters. Thus, we use it in our experiments.

For kNN lookup, we use the exact kNN search implemented in FAISS [Johnson et al., 2017]. The memory attention layer does not use positional encodings. The memory is populated incrementally with (key, value) pairs processed by  $\ell$  beforehand. In the single-doc setting, the memory is erased after each document.

## **B.3** Crossbatch training procedure

In FoT we choose a subset  $\mathcal{L}$  of the attention layers for later augmentation with the memory of (key, value) pairs. Let  $\ell$  an attention layer from  $\mathcal{L}$ . During the training we expose this layer to a mixture of (key, value) pairs from the current local context,  $C_{\text{curr}}$ , and the previous local context and d-1 contexts from other documents,  $C_{\text{prev}}$ ; see also Figure 2 for an illustration. We achieve this by modifying the input pipeline so that each batch index corresponds to a different document (the batch index occupied by each document is fixed from the moment we load the document till we finish processing it).

More specifically, we embed the previous and the current local context for each document in the batch. Then we use  $C_{\rm prev}$  as a source of the previous local context for a given document and d-1 contexts from other documents. For each element of the batch, the choices of those d additional contexts are fixed. We disable positional encoding in  $\ell$ , as we envision it to handle global information.

To be more precise, for each document  $\delta$  within the batch and query q from the layer  $\ell$  we create the set  $p^{\delta}$  consisting of (key, value) pairs from the previous local context of document  $\delta$  along with pairs from d-1 contexts gathered from  $C_{\text{prev}}$ . The attention value for q is given by

$$v := \sum_{(key,v) \in p^{\delta} \cup C_{\text{curr}}^{\delta, < q}} s(key) \cdot v, \tag{2}$$

where  $C_{\text{curr}}^{\delta, \leq q}$  consists of (key, value) pairs that preceded q in its local context and s(key) is the softmax score for key. We use softmax with learnable temperature  $\tau$ :

$$softmax\left(\left[\frac{\langle q, key\rangle}{\tau}\right]_{key \in p^{\delta} \cup C_{\mathrm{curr}}^{\delta, < q}}\right).$$

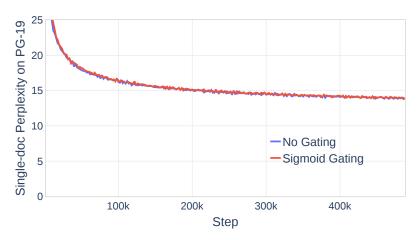


Figure 10: Perplexity (on the test set) during training on PG-19. We train two Memorizing Transformer models [Wu et al., 2022], one with original gating and one without (i.e. memory attention shared with local attention as described in (1)). We use the single-doc setting with 16k memory.

Note that the only difference between (1) and (2) is the source of the additional (key, value) pairs:  $p^{\delta}$ . This, in particular, implies that all the operations with respect to the previous context are differentiable.

The number of different documents is equal to  $b_S$  (the batch size, i.e. each document has a separate index in the batch). Assume that document  $\delta$  has index i. We include into  $p^{\delta}$  all tokens from  $C_{prev}$  with the batch indices in  $\{i, (i+1) \mod b_s, \ldots, (i+d-1) \mod b_s\}$ .

#### **B.4** Qualitative analysis

Table 8 provides a brief qualitative analysis of FoT. It shows that the model can handle distractions and retrieve the parts of the character name from the multi-document memory in the PG-19 task dataset and appropriate definitions from the large dictionary (dictionary lookup task).

Table 8: Example of elements retrieved by kNN search along with their scores on PG-19 and dictionary lookup task. The first column shows the token associated with a particular query (bolded) along with fragments of its context. The second column shows tokens associated with the keys retrieved by kNN for this query. The kNN score shows what fraction of the attention mass dedicated to retrieved keys corresponds to a particular key. The Focus score is calculated by taking the key along with 32 preceding and 32 following keys, calculating attention weights for them, and checking what fraction of attention the retrieved key gets. In the PG-19 setting the model was equipped with the memory of size 4096 spanning across parts of 8 documents. In the dictionary lookup setting the model was provided with memory of size 16M.

Text	kNN Results		Focus Score
	PG-19		
	Then if we're here with the closed carriage at ten-! [They go together into the library. DARBEY. [To SHE <b>BA</b> .]	0.69	0.99
S HE BA takes the gold-rimmed pince-nez which hangs upon THE DEAN'S waistcoat and places it before his eyes	Oh! [He sinks on to the settee with a vacant stare, his arms hanging helplessly. DAR-BEY. [To SHE <b>BA</b> ] There–now his career is a burden to him!	0.23	0.92
	Papa! SHE <b>BA</b> . Papsey! [THE DEAN rouses himself, discovers his children and removes his hat.	0.025	0.92
	Then if we're here with the closed carriage at ten-! [They go together into the library. DARBE Y . [To SHEBA.]	0.71	0.99
THE DEAN gives DAR BE Y a severe look, and with an important cough walks into the Library. The men and the girls speak in undertones.	Oh! [He sinks on to the settee with a vacant stare, his arms hanging helplessly. DARBE Y. [To SHEBA.] There—now his career is a burden to him!	0.19	0.99
	Oh, Salome! Papa! Papa! TARVER. The Dean? DARBE Y. The Dean!	0.08	0.99
Diction	onary Lookup Task		
<q> 14 42 23 38 <v> 40 41 05 56</v></q>	<k> 14 42 23 38 <v> 40 41 05 56</v></k>	0.96	0.99
<q> 30 55 07 23 <v> 36 17 26 63</v></q>	<k> 30 55 07 23 <v> 36 17 26 63</v></k>	0.88	1.0
<q> 10 41 26 39 <v> 48 38 11 24</v></q>	<k> 10 41 26 39 <v> 48 38 11 24</v></k>	0.87	0.99

## **B.5** Memorizing Transformer

The Focused Transformer shares many similarities with the Memorizing Transformer [Wu et al., 2022]. In this section, we summarize the differences between those two models.

**Training** The key difference between these two methods lies in the training procedure. Our method uses *crossbatch*, see Section B.3, which, in a nutshell, is the standard transformer training objective, but we additionally attend to tokens from the previous context window, both from the same and

different documents, see Appendix B.3 for details. The Memorizing Transformer trains on tokens retrieved from the same document (it was envisioned for single-doc).

This has a few important consequences:

- FoT does not use memory during training, while MT does. This may result in faster training; moreover, FoT always uses the most up-to-date values, while MT uses the values from memory, which may be outdated.
- FoT is differentiable through all (key, value) pairs, while MT does not differentiate through the retrieved tokens. We argue that this is key for joint training of well-structured key, value, and query embeddings and, consequently, good model performance.
- FoT does not require long documents in the training set, while MT does in order to capture long dependencies in memory. This is practically important, as many popular datasets consist of short documents.

We speculate that there may be benefits in blending these two approaches. One can, for example, argue that MT is better at providing 'hard' negatives for the model. We provide a proof-of-concept experiment in Section 5.6.3, otherwise leave this for future work.

**Inference** Both models use a very similar memory attention layer. The difference is how the retrieved (key, value) pairs are integrated. FoT treats the retrieved information in the same way as the local context. MT uses a gating mechanism. Details are provided in Section B.2.

## **C** Hyperparameters

Table 9 shows hyperparameters used in our experiments. We used context length 512 unless stated otherwise. In Section 5.2, Section 5.4, Section 5.6, we use the total batch size of 32K tokens. In Section 5.3 and Section 5.5, the total batch size is 128K tokens.

Table 9: Hyperparameters for different model sizes. The batch size is given in number of tokens. For the 37M model local context length was 256 for FoT and 512 for the baseline.

Hyperparameter	Value			
		Common		
#Layers		12		
Index of memory attention layer $(l)$		8		
Optimizer		AdaFa	ctor	
Learning rate schedule	Ir	verse Squ	are Root	
Warmup steps	1000			
$eta_1$	0.9			
	Model-specific			
#Params	37M	184M	600M	1.2B
Max learning rate	0.02	0.01	0.005	0.005
Min learning rate	0.01	0.0005	0.0005	0.0005
Embedding dim	512	1024	2048	2048
Head dim	64	128	128	128
#Heads	8	8	16	32
FeedForward dim	2048	4096	8192	16384
Local context length	256/512	512	2048	2048
Batch size	32K/64K	32K	256K	256K
#Number of training steps	5k	500k	150k	200k

For the experiments described in Section 5.4 and Section 5.6 we performed the following hyperparameter sweeps:

- Learning rate:  $\{1e-2, 5e-3, 3e-3, 1e-3\}$ , chosen: 1e-2,
- Batch size:  $\{8K, 16K, 32K\}$ , chosen: 32K.

For the dictionary lookup task (Section 5.2) we checked the following hyperparameters:

- Learning rate:  $\{4e-2, 2e-2, 1e-2, 5e-3, 3e-3\}$ , chosen: 2e-2,
- Number of dictionary tokens in training step:  $\{16K, 32K\}$ , chosen: 32K. Note that during the training number of document tokens dedicated to the dictionary is the same as the number of tokens dedicated to questions.

For most of the other hyperparameter choices, we followed [Wu et al., 2022], to provide a fair comparison.

## C.1 Schedule of d

In Sections 5.4, 5.5 and 5.6 for models with  $d \in \{1, 2, 4, 8\}$  we used constant schedule, and for models with d = 64 we trained with d = 2 for 450k steps and switched to d = 64 for the final 50k

steps. In Section 5.2 we trained with d=1 until model reached 98% accuracy and then we swtiched to d=128. For the 184M model in Section 5.3, we randomly sampled d from  $\{2,128\}$  in each training step, and the 600M and 1.2B models just used d=2.

## D Dictionary lookup task

We propose a dictionary lookup task to test whether the model trained using our crossbatch method can utilize a large memory database. Documents in this task are split into two parts. The first part defines keys and their associated values using the records of the format:

$$\langle k \rangle, k_1, k_2, k_3, k_4, \langle v \rangle, v_1, v_2, v_3, v_4,$$

where <k> is a special token that denotes the beginning of the defining sequence,

The second part consists of queries about the values associated with the previously defined keys. The queries are in the following format:

$$", k_1, k_2, k_3, k_4, , v_1, v_2, v_3, v_4,"$$

where  $\leq q >$  is a special token that denotes the beginning of the query. We mask the loss so that for such a question, only  $v_1, v_2, v_3, v_4$  are included. We use a vocabulary of 64 tokens, keys and values are described using 4 tokens.

During training, we use documents comprising 512 tokens. The first half of each document consists of definitions, whereas the second one consists of questions. In evaluation, we use longer documents but make only the last 256 tokens correspond to questions. That is, as the context gets bigger (the token axis on Figure 4), the number of definitions increases, but the number of queries remains the same.

## E FoT fine-tuning

For comparison in Table 4, our 184M model is pre-trained for 100k steps with a total batch size of 128 (128K tokens per step, with 1024 local context). Then we fine-tune both FoT and baselines for additional 10k steps with the same batch size. When fine-tuning FoT, we randomly sample d from  $\{2,128\}$  in each training step to prevent the model from overfitting to a large additional context length during training.

#### F Code

In Listing 1, we show the FoTs attention code (i.e., the code for the memory attention layers and crossbatch training), see Section 3, Appendix B.2, Appendix B.3. We note that the changes to the code are small; they are localized to the memory layer (the other layers follow the standard transformer protocol) and do not require any new trainable parameters.

Listing 1: Memory attention: Let  $\ell$  be a memory attention layer. During the training, we make  $\ell$  attend to the (key, value) pairs from the local context, previous local context, and d-1 contexts coming from other documents. During the inference, queries from  $\ell$  attend to the local context and k nearest neighbors retrieved from memory. For simplicity, we provide the code for one head and assume that the crossbatch dimension d is equal to the batch size.

```
def mem_attn_layer(Ql, Kl, Vl, Cl, Km, Vm, Kp, Vp, attn_scf, mode):
  """Attention mechanism for crossbatch and memory attention
  Args:
   Q1, K1, V1: tensors of shape [batch, ctx_len, dim]
                with queries, keys and values from the local context
                tensors of shape [batch, ctx_len, k, dim]
   Km. Vm:
                with k most matching memory keys for
                each query from Q1 along with associated values
   Kp, Vp:
                tensors of shape [batch, ctx_len, dim] with
                keys and values from the previous context
    attn_scf : a scale factor used before softmax
                either training or inference
   mode:
 Returns:
   y: a vector with shape [batch, ctx_len, dim]
 # attention to the local context
 local_attention = jnp.einsum("bqd,bkd ->bqk", Q1, K1)
  local_attention *= attn_scf
 local_attention = apply_causal_mask(local_attention)
  if mode == "train":
    # In train mode, we additionally use previous context
    \# and batch-1 contexts from other documents.
   prev_attention = jnp.einsum("bqd,ckd ->bqck", Q1, Kp)
    shape = prev_attention.shape
    additional_attention = prev_attention.reshape(shape[:-2] + (-1,))
  elif mode == "inference":
    # In the inference mode, we additionally use nearest neighbors
    # retrieved from memory. We retrieve k (key, value)
   # pairs for each query.
   memory_attention = jnp.einsum("bqd,bqnd -> bqn", Q1, Km)
   additional_attention = memory_attention
  else:
    raise Exception("Only train and inference modes are supported")
  additional_attention *= attn_scf
  # We merge the raw attention scores and calculate the softmax
  combined_attention = jnp.concatenate([local_attention,
                                         additional_attention],
                                        axis = -1)
 combined_weights = jax.nn.softmax(combined_attention, axis=-1)
  ctx_len = Ql.shape[1]
  local_weights = combined_weights[..., :ctx_len]
  additional_weights = combined_weights[..., ctx_len:]
 y = jnp.einsum("bqk,bkd -> bqd", local_weights, V1)
  if mode == "train":
   prev_weights = additional_weights
    shape = prev_weights.shape
   prev_weights = prev_weights.reshape(shape[:-1] + (-1, ctx_len))
   y += jnp.einsum("bqck,ckd -> bqd", prev_weights, Vp)
  else:
   memory_weights = additional_weights
   y += jnp.einsum("bqn,bqnd -> bqd", memory_weights, Vm)
  return y
```

## **G** Datasets

Section 5.1 outlines essential details concerning the PG-19 and arXiv datasets employed in this study. Now, we will present details about the remaining datasets:

**GitHub** We obtained a large corpus of permissively licensed Github repositories using BigQuery. By filtering for specific file extensions (C, C++, Java, Python, Go, and TypeScript), we captured individual source code files that are often short but have numerous dependencies and cross-references within the repository. To preserve the structure while shuffling the files and subdirectories in a random order, we concatenated all the files within each repository, treating subdirectories as a unit, similarly to Wu et al. [2022].

**Isabelle** The Isabelle corpus comprises formal mathematical proofs in the form of theories written in a formal language. We combined theories from The Archive of Formal Proofs (from October 2021) <sup>3</sup> and the Isabelle standard library to create a corpus of theories licensed as open source. Each theory focuses on topics like foundational logic, advanced analysis, algebra, or cryptography and consists of multiple files containing proofs. Similar to the GitHub corpus, the files within each theory are concatenated into a single document. However, unlike the Github corpus, we arrange the files based on their import dependencies, ensuring that later files can utilize sub-theorems proven in earlier files.

#### H Hardware and technical details

We used TPU virtual machines from the Google Cloud Platform (GCP). Each TPU virtual machine has 8 TPUv2 / TPUv3 cores totaling 64GB / 128GB of device memory, 96 CPU cores, and over 300GB of RAM. In larger-scale experiments (Section 5.3) we used machines with 32 TPUv3 cores. For training the LongllamA checkpoints, a TPUv3-128 pod provided by the TPU Research Cloud was used, which we gratefully acknowledge.

#### I Randomness

To evaluate the significance of our results, we conducted multiple runs for selected experiments in our study. In Figure 4, we calculate error bars, showing the minimum and maximum value over 10 runs of the same experiment. For the arXiv baseline experiment in Appendix J, we performed three runs with different random seeds and calculated their standard deviation, which is equal to 0.002 perplexity. However, due to resource constraints, we were unable to conduct multiple runs for all experiments. Our preliminary findings indicate that the observed variance was minimal compared to the impact observed from other factors under investigation.

For the calculation of test perplexities, we used 1M tokens.

## J Additional experimental results

This section presents additional empirical results, providing a detailed comparison of FoT with the Memorizing Transformer [Wu et al., 2022] baseline. Both models are trained for the same number of 500k steps with local context of 2K and evaluated on the arXiv dataset in the single-document setup, following [Wu et al., 2022]. In particular, we study how models trained with a given context length perform when evaluated with different context lengths. These experiments differ from those in Section 5.3, as the models were both trained and evaluated on the same dataset (arXiv), unlike the C4 training and zero-shot evaluation done in Section 5.3.

The MT baseline in Table 10 with a memory length of 2K struggles to utilize additional context beyond 32K tokens effectively. The model trained with 8K memory performs significantly better when evaluated with longer contexts, showing further perplexity gains at 64K tokens. We observe diminishing returns when scaling up the training memory length to 16K tokens and beyond.

Using the same setup, we study the performance of FoT while varying d and w configurations, similarly to Section 5.5, see Table 11. Parameter values w = 1 and w = 2 correspond to additional

<sup>&</sup>lt;sup>3</sup>https://www.isa-afp.org

context lengths of 2K and 4K, respectively. In an apples-to-apples comparison to MT with 2K additional context length, FoT outperforms the MT baseline, which shows the importance of trainable keys and values (see also Section 5.6.1). Moreover, we confirm the findings from Section 5.5 that d=2 works significantly better than d=1 in all settings. Our best configuration achieves 2.148 perplexity with 4K additional context during training, compared to 2.164 of MT with 16K additional context.

Table 10: Memorizing Transformer: arXiv perplexity values for different training memory lengths and evaluation context sizes

<b>Eval Context</b>	Training Memory Length				
	2k	8k	16k	32k	
4K	2.309	2.334	2.348	2.365	
8K	2.242	2.244	2.252	2.265	
16K	2.215	2.206	2.206	2.215	
32K	2.199	2.178	2.177	2.181	
64K	2.195	2.169	2.166	2.168	
128K	2.195	2.168	2.164	2.166	

Table 11: FoT: arXiv perplexity values for different parameter combinations and evaluation context sizes

<b>Evaluation Context</b>	Parameter Combinations (w, d), Training Memory Leng			
	(1, 1), 2048	(1, 2), 2048	(2, 1), 4096	(2, 2), 4096
4K	2.292	2.299	2.305	2.309
8K	2.229	2.224	2.214	2.217
16K	2.206	2.194	2.178	2.178
32K	2.192	2.176	2.159	2.156
64K	2.187	2.171	2.152	2.149
128K	2.187	2.171	2.152	2.148