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A GENERALIZED GOODNESS-OF-FUNCTIONAL FORM TEST FOR BINARY AND FRACTIONAL REGRESSION MODELS*

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This paper proposes a new conditional mean test to assess the validity of binary and fractional parametric regression models. The new test checks the joint significance of two simple functions of the fitted index and is based on a very flexible parametric generalization of the postulated model. A Monte Carlo study reveals a promising behaviour for the new test, which compares favourably with that of the well-known RESET test as well as with tests where the alternative model is non-parametric.

1 Introduction

Parametric regression models for binary and fractional data are widely used in applied work. Because these models rely on the correct specification of the conditional expectation of the dependent variable given a set of explanatory variables, several conditional mean tests have been proposed in the literature. Most of those tests are based on the construction of general parametric models that incorporate the postulated model as a particular case. Examples are goodness-of-link (GOL) tests, which are very popular in the statistics literature (see *inter alia* Prentice, 1976; Pregibon, 1980; Aranda-Ordaz, 1981; Whitemore, 1983; Stukel, 1988; Czado, 1994; Koenker and Yoon, 2009), and the RESET test, which is more common in the econometrics literature. Recently, general tests that assess parametric specifications against non-parametric alternatives have also been proposed; see *inter alia* Zheng (1996) and Whang (2000).

While proposed originally for binary models, GOL tests may also be easily extended to deal with fractional responses, as noted by Ramalho *et al.* (2011). However, each GOL test is valid for testing the functional form of

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particular binary and fractional regression models, instead of *any* possible specification for those models. Therefore, not surprisingly, practitioners seemingly prefer to assess the specification of their models using the RESET test, which can be applied to *all* binary and fractional single index regression models as a simple significance test for some omitted variables. Nevertheless, RESET tests have also an important drawback: their size and power properties in finite samples may vary substantially according to the number of powers of the fitted index included in the test regression; see the recent comprehensive simulation study of Ramalho and Ramalho (2012) for binary models. On the other hand, non-parametric tests for parametric models, while sensitive to any type of departure from the null hypothesis, are typically much less powerful than parametric tests in cases where the alternative hypothesis underlying the latter approximates well the true model. Moreover, given its higher complexity, non-parametric tests are still rarely used in empirical work.

In this paper, we propose a new conditional mean test for parametric binary and fractional regression models. The derivation of the test closely follows the philosophy of GOL tests, but uses a much more general parametric model under the alternative hypothesis, which nests any plausible parametric specification for the conditional mean of binary and fractional responses. Therefore, the new test can be seen as a generalized GOL test and, in fact, encompasses the pioneering GOL test proposed by Prentice (1976) for binary logit models as a particular case. Moreover, the application of the new test is as simple as that of the RESET, the main difference being that instead of an arbitrary number of powers of the fitted index, the new test checks the significance of two simple functions of the fitted index.

The new test proposed in this paper includes also as particular cases the two 'goodness-of-functional form' (GOFF) tests proposed by Ramalho *et al.* (2011). In a simulation study, these authors found that GOFF tests often display a better power performance than RESET and GOL tests, namely in cases where the misspecification induces some type of asymmetry relative to the postulated model. However, for other types of misspecification, the power of the GOFF tests may be very low. The new test, which we designate by generalized GOFF (GGOFF) test, should circumvent this limitation of GOFF tests because, in contrast to the latter, it is based on a generalized model that is flexible enough to incorporate misspecifications that do not impose asymmetry.

The paper is organized as follows. Section 2 discusses the generalized model from which the GGOFF test is derived. Section 3 details the application of the new test. Section 4 uses Monte Carlo methods to compare the finite sample behaviour of the GGOFF test with that of RESET, GOFF and non-parametric tests. Section 5 illustrates the use of conditional mean tests in empirical work. Finally, Section 6 presents some concluding remarks.

Table 1
Alternative Non-linear Conditional Mean Specifications for Binary and Fractional Response Variables

Model	$G(x\theta)$	$g(x\theta)$
Cauchit	$\frac{1}{2}+\frac{1}{\pi}\arctan(x\theta)$	$\frac{1}{\pi}\frac{1}{(x\theta)^2+1}$
Complementary loglog	$1-e^{-e^{x\theta}}$	$e^{x\theta}[1-G(x\theta)]$
Logit	$\frac{e^{x\theta}}{1+e^{x\theta}}$	$G(x\theta)[1-G(x\theta)]$
Loglog	$e^{-e^{-x\theta}}$	$e^{-x\theta}G(x\theta)$
Probit	$\Phi(x\theta)$	$\phi(x\theta)$

2 Generalized Models for Binary and Fractional Regression Models

Consider a random sample of $i = 1, \ldots, N$ individuals. Let y be a binary or a fractional outcome, respectively, defined as $y \in \{0, 1\}$ or $y \in [0, 1]$, and x a vector of k exogenous variables. The conditional expectation of y given x is defined as

$$\mu \equiv E(y|x;\theta) = G(x\theta) \tag{1}$$

where θ is the vector of parameters of interest and $0 \le G(\cdot) \le 1$. In parametric models for both binary and fractional responses, the most usual choices for $G(\cdot)$ are the logit and probit specifications but there are many other alternatives such as the loglog, complementary loglog (cloglog) and cauchit models. Table 1 presents the specifications for $G(\cdot)$ under each of those models and also the corresponding expressions for the derivatives $g = \nabla_{x\theta}G(x\theta)$, which are necessary to implement the test proposed in Section 3. Alternatively, $E(y|x;\theta)$ could be defined simply as a single index model, with $G(\cdot)$ remaining unspecified; see, for example, Horowitz (2009). However, many practitioners still prefer using parametric models, given that semiparametric estimators are often not particularly simple to implement and interpret. Moreover, to the best of our knowledge, no applications of semiparametric estimators for fractional responses have so far been proposed.

With the aim of testing the adequacy of a particular specification for $G(\cdot)$, several alternative generalized models have been proposed in the econometrics literature on binary models. For example, Poirier (1980) and Smith (1989) introduce asymmetry in the logit model using the Burr II distribution, Bera *et al.* (1984) test the suitability of probit models by nesting the standard normal distribution assumed in the probit specification within the Pearson family of distributions and Koenker and Yoon (2009) suggested a generalized version of the cauchit functional form. On the other hand, in the statistics literature it has been much more common to work with generalizations of the so-called 'link' function $h(\cdot)$, which relates the linear predictor $x\theta$ to the conditional expected value μ, i.e. $h(\mu) = x\theta$. In this framework, generalizations of the binary logit model were proposed by Pregibon (1980), Aranda-Ordaz (1981) and Czado (1994), while Prentice (1976) and Stukel (1988) suggested models that encompass also the binary probit, loglog and complementary loglog models. Naturally, to each particular link function $h(\mu)$ corresponds a different functional form $G(x\theta)$ and, therefore, such generalizations may also be used as the basis for testing the specification adopted for model (1). In particular, as shown by Ramalho *et al.* (2011), first-order Taylor series approximations of the generalizations proposed for $E(y|x;\theta)$, via $h(\mu)$, may be written as

$$E(y|x;\theta,\alpha,\phi) = G[x\theta + \nabla_\alpha h(\mu;\alpha)\phi] \tag{2}$$

where $h(\mu;\alpha)$ is a generalized link function indexed by some vector of parameters α that includes the hypothesized link function as a special case for some specific values of α. All the various GOL tests proposed in the literature are thus tests for H_0: $\phi = 0$, differing only on the specification adopted for $h(\mu;\alpha)$. An unattractive feature of these tests is that none of the proposed $h(\mu;\alpha)$ functions is sufficiently general to include as particular case any possible choice for $h(\mu)$ and, hence, $G(x\theta)$.

While all previous generalizations, and the corresponding GOL tests, are only valid for particular specifications of $G(x\theta)$, the RESET test is based on an approximation of the true model that is valid for any $G(\cdot)$ function. Indeed, using standard approximation results for polynomials, it can be shown that any index model of the form $E(y|x;\theta) = H(x\theta)$ can be approximated by

$$E(y|x;\theta,\phi) = G\left[x\theta + \sum_{j=1}^{J}\phi_j(x\theta)^{j+1}\right] \tag{3}$$

for J large enough. Therefore, testing model (1) is equivalent to test H_0: $\phi = 0$, where ϕ is a J-dimensional vector, in the augmented model (3). In a recent paper, Ramalho and Ramalho (2012) found that, in the binary framework, the best RESET variants are clearly those that consider $J \leq 2$.

Similarly, the two generalized models recently proposed by Ramalho *et al.* (2011), which are defined by

$$E(y|x;\theta,\alpha) = G(x\theta)^\alpha \tag{4}$$

and

$$E(y|x;\theta,\alpha) = 1 - [1 - G(x\theta)]^\alpha \tag{5}$$

$\alpha > 0$, are also applicable to any binary and fractional regression model. Note that both (4) and (5) induce (complementary forms of) asymmetry in the postulated functional form $G(x\theta)$ and keep the property of producing values only on the interval [0, 1]. In a simulation study concerning fractional regression models, Ramalho *et al*. (2011) found that the overall performance of Lagrange Multiplier (LM) tests for the null hypothesis H_0: $\alpha = 1$ in both (4) and (5), respectively, designated as GOFF1 and GOFF2, was often superior to that of RESET and GOL tests. However, the study also revealed that both GOFF1 and GOFF2 have a strong drawback: they are both insensitive to misspecifications that do not induce asymmetry in $G(x\theta)$.

The GGOFF test proposed in the present paper circumvents this problem by using a more general model, which is a mixture of (4) and (5), and allows not only for a wider variety of asymmetric forms but also for many different symmetric shapes:

$$E(y|x;\theta,\lambda,\alpha_1,\alpha_2) = \lambda G(x\theta)^{\alpha_1} + (1-\lambda)\left\{1-[1-G(x\theta)]^{\alpha_2}\right\} \tag{6}$$

where $0 < \lambda < 1$ and α_1, $\alpha_2 > 0$ such that $0 < \mu < 1$. For $\lambda = 1$ and $\lambda = 0$, expression (6) would reduce to (4) and (5) respectively. On the other hand, as (6) reduces to $G(x\theta)$ when $\alpha_1 = \alpha_2 = 1$, we may test whether $G(x\theta)$ is the correct specification of μ by testing for H_0: $\alpha = 1$, where $\alpha = (\alpha_1, \alpha_2)$.

The consequences of introducing additional parameters in the conditional mean of y as in (6) are illustrated in Fig. 1 for several combinations of α_1, α_2 and λ, where a probit specification is assumed for $G(x\theta)$. The first two rows of Fig. 1 illustrate cases where $\alpha_1 = \alpha_2 = \bar{\alpha}$, while the third and fourth consider distinct values for those parameters. The graphs where $\lambda = 0$ or $\lambda = 1$ represent limiting cases of (6).

In general, different combinations of the parameters produce shifts of the original curve in different directions and of different magnitudes. For example, for $\alpha_1 = \alpha_2 = \bar{\alpha}$, it is clear that for $\bar{\alpha} > 1$ ($0 < \bar{\alpha} < 1$), the probit curve is shifted to the left (right) for small values of λ and to the right (left) for large values of λ, while for $\lambda = 0.5$ symmetric curves, with more dispersion than the original, are obtained. Moreover, in the cases where α_1 (α_2) is fixed and different values are considered for α_2 (α_1), the shifted curves become more similar as λ is increased (reduced); compare the third and fourth rows of Fig. 1, column by column. In fact, the shifted curves would coincide in the extreme case where $\lambda = 1$ ($\lambda = 0$), because model (6) would be only a function of α_1 (α_2). For this reason, the graph in the first row for $\lambda = 1$ ($\lambda = 0$) represents model 4 (5), which yields the GOFF1 (GOFF2) test, since it actually represents curves yielded by the values chosen for α_1 (α_2), irrespective of the values adopted for α_2 (α_1). Clearly, a test based on the generalized model (6) is potentially sensitive to a wider set of model misspecifications than GOFF1 or GOFF2.

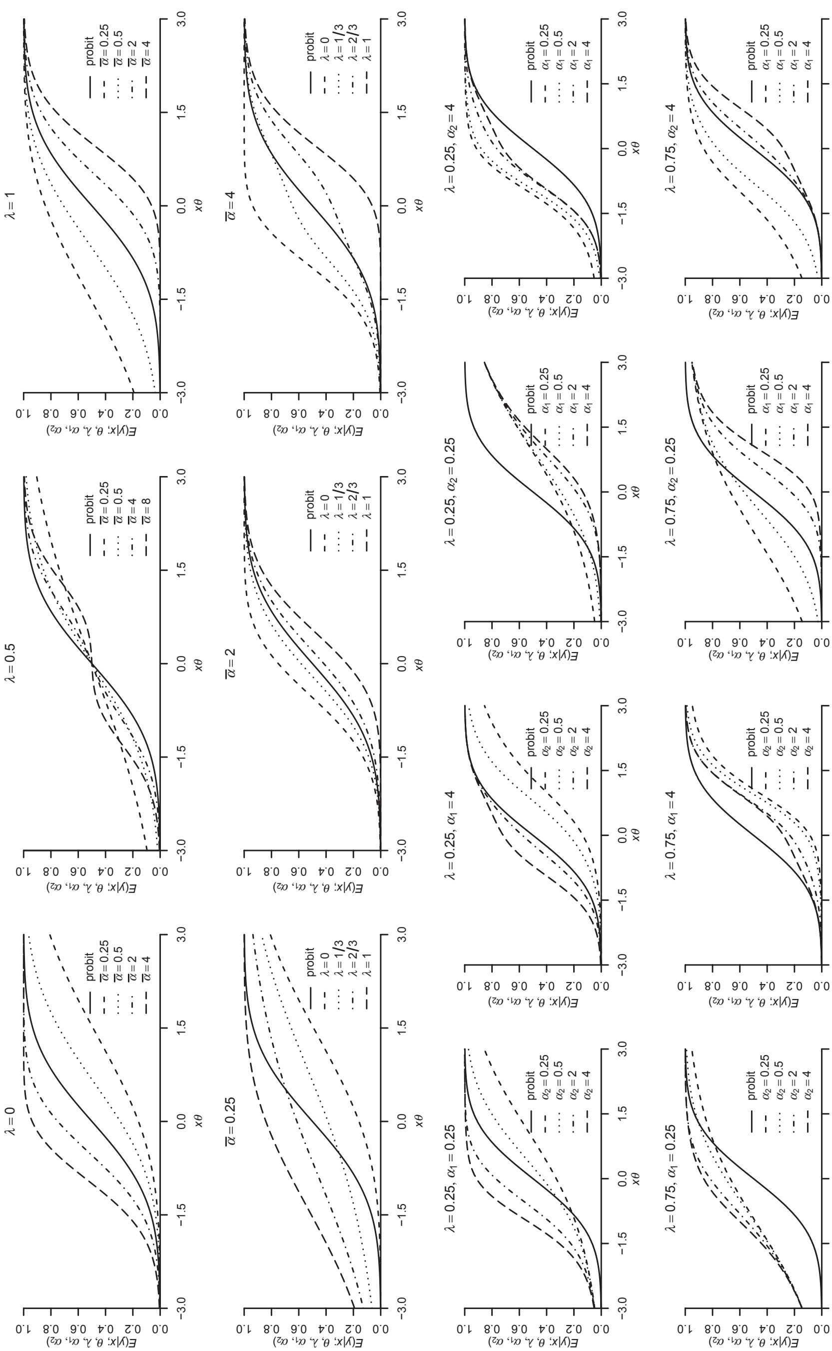

FIG. 1. Generalized Regression Models (Base Model: Probit)

Table 2
Summary of Alternative Models Used in the Tests and Respective H_0

Test	*Model*	H_0: G(xθ)
GOL	$G[x\theta + \nabla_{\alpha}h(\mu;\, \alpha)\phi]$	$\phi = 0$
RESET	$G\left[x\theta + \sum_{j=1}^{J} \phi_j (x\theta)^{j+1}\right]$	$\phi = 0$
GOFF1	$G(x\theta)^{\alpha}$	$\alpha = 1$
GOFF2	$1 - [1 - G(x\theta)]^{\alpha}$	$\alpha = 1$
GGOFF	$\lambda G(x\theta)^{\alpha_1} + (1-\lambda)\{1-[1-G(x\theta)]^{\alpha_2}\}$	$\alpha = 1$
Variable omission	$G(x\theta + z\phi)$	$\phi = 0$

3 The GGOFF Test

In the previous section, we presented five alternative types of generalized models that, for specific values of the additional parameters introduced, reduce to $G(x\theta)$; see Table 2 for a summary of those models and corresponding null hypotheses. Naturally, each of those models gives rise to a distinct statistic for testing H_0: $E(y|x) = G(x\theta)$. However, as discussed below, all tests may be implemented as simple LM tests for the omission of a set of J artificial regressors z from $G(x\theta + z\phi)$, where the null hypothesis is written as H_0: $\phi = 0$. Besides providing an integrated approach to all tests, the use of LM statistics in this framework has another very attractive feature: the generalized model underlying each test does not need to be estimated. Due to the computational issues that often arise from the estimation of generalized models with additional parameters (see *inter alia* Taylor, 1988), this feature of LM statistics is particularly relevant in this context.

Following Davidson and MacKinnon (1984), in maximum likelihood-based binary regression models, LM statistics for the significance of z can be straightforwardly computed as LM = ESS, where ESS is the explained sum of squares of the auxiliary regression

$$\tilde{u} = \tilde{g}x^{*}\delta + error \tag{7}$$

$\hat{u} = y - \hat{G}$, $\tilde{u} = \hat{u}\hat{\omega}$, $\tilde{g} = \hat{g}\hat{\omega}$, $\hat{\omega} = \left[\hat{G}\left(1-\hat{G}\right)\right]^{-0.5}$, $\hat{}$ indicates evaluation under H_0, $x^{*} = (x', z')$ and δ is a vector of parameters. For fractional regression models, which are usually estimated by quasi-maximum likelihood methods, it is in general preferable to compute heteroskedasticity-robust LM statistics. These statistics may be calculated also as LM = ESS, but based on the artificial regression

$$1 = \tilde{u}\tilde{r}\delta + error \tag{8}$$

where $\tilde{r}$ is the vector of residuals $\tilde{r}_j$, $j = 1, \ldots, J$, that results from regressing separately each element $\tilde{g}z_j$ on the entire vector $\tilde{g}x$; see Papke and Wooldridge (1996) for details. In both cases, the limiting distribution of the test is a chi-square distribution with J degrees of freedom.

From (2) and (3), see also Table 2, it follows immediately that $z = \nabla_\alpha h(\mu; \alpha)$ for GOL tests and $z = \left[\left(x\hat{\theta} \right)^2, \ldots, \left(x\hat{\theta} \right)^{J+1} \right]$ for RESET tests. For the GGOFF test, it is not so evident how the vector z may be defined, since the generalized model (6) is not written in the form $G(x\theta + z\phi)$. However, as shown by Wooldridge (2002, pp. 463–464), in models that may be written as $\mu = F[G(x\theta), \gamma]$ and reduce to $G(x\theta)$ for some particular value of the vector γ, testing H_0: $E(y|x; \theta) = G(x\theta)$ is equivalent to test for the omission of $z = \nabla_\gamma \hat{\mu} \hat{g}^{-1}$ from $G(x\theta + z\phi)$. In this framework, $F[\cdot]$ is given by the right-hand side of (6), $\gamma \equiv \alpha$, μ reduces to $G(x\theta)$ for $\gamma = (1, 1)$ and λ is a parameter that is not identified under the null hypothesis. Let $z = (z_1, z_2) = (\nabla_{\alpha_1} \hat{\mu} \hat{g}^{-1}, \nabla_{\alpha_2} \hat{\mu} \hat{g}^{-1})$. Then, $z_1 = \lambda \hat{G} \ln(\hat{G}) \hat{g}^{-1}$ and $z_2 = -(1 - \lambda)(1 - \hat{G}) \ln(1 - \hat{G}) \hat{g}^{-1}$, where λ appears in those expressions as an irrelevant multiplicative constant that can be dropped.[1] Thus, the two test variables are the following:

$$z_1 = \hat{G} \ln\left(\hat{G}\right) \hat{g}^{-1} \tag{9}$$

$$z_2 = \left(1 - \hat{G}\right) \ln\left(1 - \hat{G}\right) \hat{g}^{-1} \tag{10}$$

Note that (9) and (10) are the test variables used separately by the GOFF1 and GOFF2 tests respectively. When used for testing the loglog (cloglog) models, it is straightforward to demonstrate (see Table 1) that the test variable $z_1 = 1$ ($z_2 = 1$) and, therefore, must be dropped from the vector z, implying that for those models the GGOFF test coincides with the GOFF2 (GOFF1) test.

An interesting feature of the GGOFF test is that (9) and (10) coincide with the test variables resulting from the two-parameter model that underlies the pioneer GOL test proposed by Prentice (1976) for binary logit models; see Stukel (1988). Therefore, the GGOFF test can be seen as a generalization of Prentice's (1976) GOL test, because, unlike this test, it can be applied to any possible specification for $G(\cdot)$.

4 Monte Carlo Analysis

In this section we analyse the finite sample performance of the GGOFF test through a Monte Carlo simulation study for both binary and fractional regression models. In the former case, we follow the experimental design of Santos Silva (2001), assuming a linear index model with two covariates, $x\theta = \theta_0 + \theta_1 x_1 + \theta_2 x_2$, where x_1 and x_2 are generated, respectively, as a standard normal and a Bernoulli variate with mean 2/3. We set $\theta_2 = 1$ and consider several values for θ_0 and θ_1 in order to control the percentage of zeros and

[1]See, for example, Verbeek (2008, p. 190) for an analogous case where constant proportionality factors are eliminated from test variables of a heteroskedasticity test implemented as a variable omission test.

ones of y and the contribution of x_1 for the variance of the response index. For fractional responses, as in Ramalho *et al.* (2011), we consider $x\theta = \theta_0 + \theta_1 x_1$ and generate y according to a beta variate with mean $G(x\theta)$ and variance $G(x\theta)[1 - G(x\theta)]/(\phi + 1)$. We set $\theta_1 = 0.5$ and consider several values for the shape parameter ϕ, which produces different degrees of variability of y, and several values for θ_0, which influence the (a)symmetry of the data.

In all experiments, data are generated according to a symmetric (probit) and an asymmetric (loglog) specification for the conditional mean. In both cases, three different null models are considered: the true model, to evaluate empirical size; and a symmetric (cauchit) and an asymmetric (cloglog) model, to evaluate power. The performance of the GGOFF test is compared with that of: GOFF1 and GOFF2; the two most common RESET versions, which use one (RESET1) or two (RESET2) fitted powers; and two versions of Whang's (2000) non-parametric test, which generalize the Kolmogorov–Smirnov (KS) and Cramer–von Mises (CM) tests to the regression framework. While the parametric tests are implemented as LM tests based on both asymptotic and bootstrap critical values, for the KS and CM tests we use the bootstrap procedures described in Whang (2000), or some adaptation of them.[2] For the LM tests, we report only results based on asymptotic critical values, since, as shown next, the actual and nominal sizes are not significantly different in many cases.[3] All experiments are based on 10,000 replications and sample sizes of $N = \{100, 200, 400, 1000\}$.

Figure 2 (binary case) and Fig. 3 (fractional case) display the percentage of rejections of the null hypothesis for a nominal size of 5 per cent when this hypothesis is indeed true. For binary regression models, we consider $\theta_0 = \{-2, -1.5, \ldots, 2\}$ and $\theta_1 = \{0.5, 0.75, \ldots, 2.5\}$ and report results for $N = \{200, 1000\}$. For the fractional case, we consider $\phi = \{5, 10, \ldots 45\}$ and $\theta_0 = \{-0.5, -0.375, \ldots, 0.5\}$ and report results for $N = \{200, 400\}$. The horizontal lines in Figs 2 and 3 represent the limits of a 95 per cent confidence interval for the nominal size of 5 per cent.

For $N = 200$, the parametric tests are often undersized in the experiments with binary data, while the non-parametric tests are often slightly oversized with both types of data, especially the KS test. However, as the sample size increases, those problems disappear in most cases.[4] Overall, the GGOFF test seems to be as reliable as the other tests, displaying actual sizes that are not significantly different from the nominal ones, at the 5 per cent level, in most of the experiments considered in the second rows of Figs 2 and 3. For instance, for those cases, the empirical sizes of the GGOFF test range from

[2]In particular, in the binary case we use a parametric bootstrap. The number of bootstrap repetitions is 200.

[3]Full results are available upon request.

[4]In the case of parametric tests, the same happens when bootstrapped versions are considered, although in some cases it has occurred an overcorrection, with the tests becoming slightly oversized.

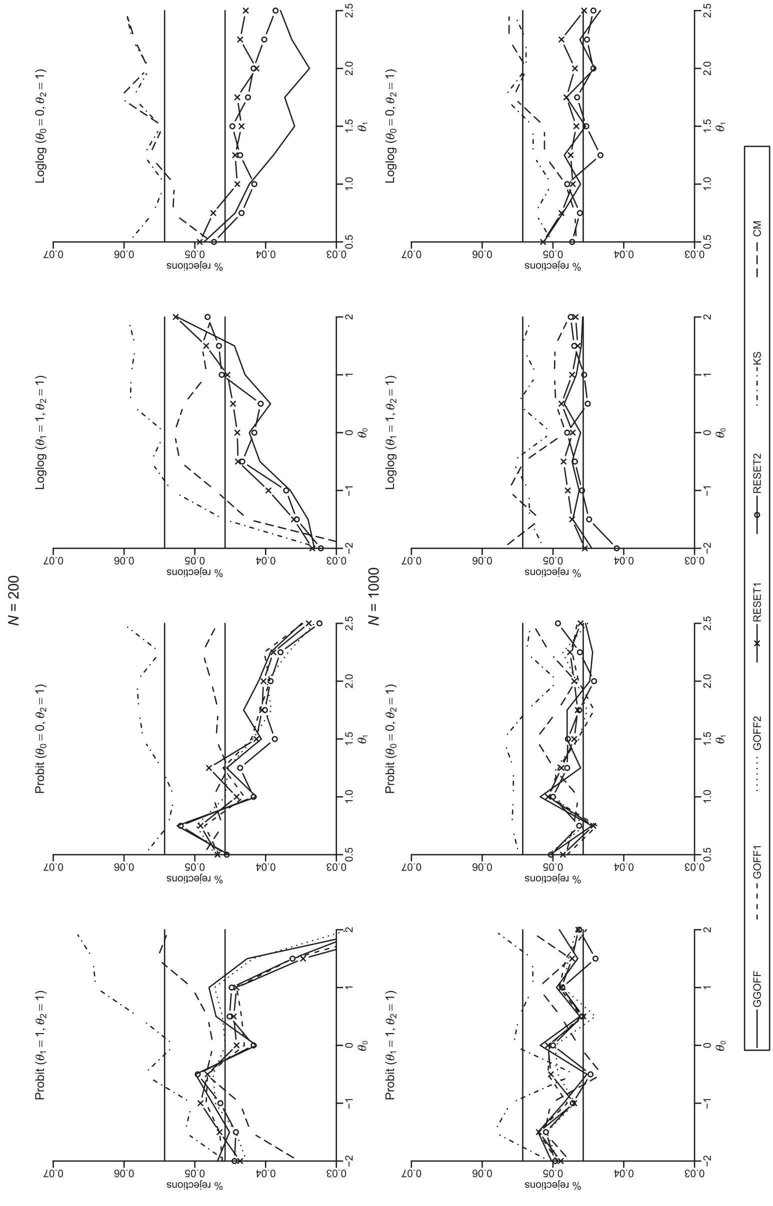

FIG. 2. Binary Regression Models—Empirical Size

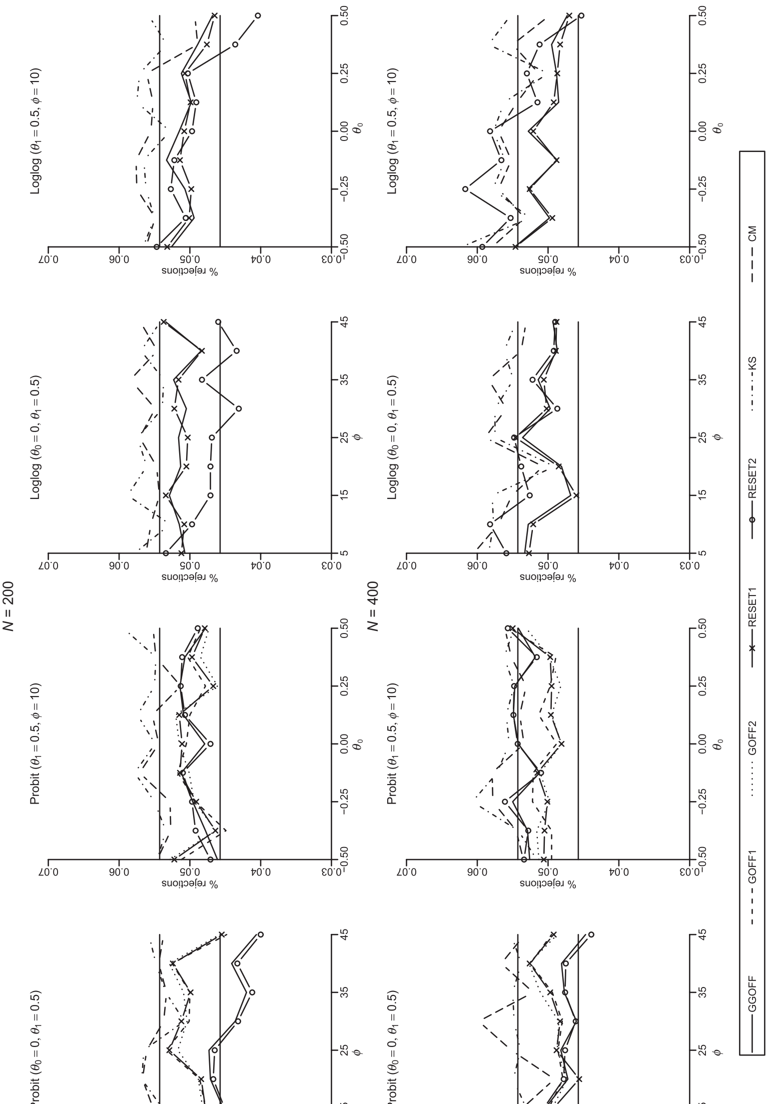

Fig. 3. Fractional Regression Models—Empirical Size

0.043 to 0.052 (binary models) and from 0.045 to 0.057 (fractional models), while those of the non-parametric tests range are in the range of 0.047–0.058 (KS) and 0.043–0.057 (CM), in the binary case, and 0.049–0.062 (KS) and 0.049–0.061 (CM), in the fractional case.

The same experimental designs are the basis for Figs 4–7, except that now the hypothesized and true models have no longer the same specification. Figures 4 and 5 consider the case of binary responses. Clearly, the GGOFF test displays a very promising behaviour in all experiments. When the null hypothesis is the cloglog model, the best performers are undoubtedly the GGOFF and RESET1 tests. In the experiments where the cauchit is the null model, either the GGOFF test is much more powerful than any of the other tests ($\theta_0 = 0$, changing θ_1) or the test that most consistently delivers, in relative terms, higher powers ($\theta_1 = 1$, changing θ_0). Regarding the latter case, note for example how RESET1 is the best power performer for some values of θ_0 and the worst for others, while GGOFF displays a much more uniform behaviour.

In the case of fractional responses, see Figs 6 and 7, the power performance of GGOFF is again very encouraging, being one of the most powerful tests in all experiments. In contrast, notice how the RESET1, GOFF1 and GOFF2 tests have no power at all when the distribution of y ($\theta_0 = 0$), the null model (cauchit) and the true model (probit) are symmetric. The non-parametric tests are reasonably powerful, particularly the CM test, but they are outperformed by the GGOFF test in most cases.

5 Empirical Application: Firms' Capital Structure Decisions

In this section, we illustrate the usefulness of the proposed test in an empirical application concerning firms' capital structure decisions. The main focus of many capital structure empirical studies lies in the investigation of the main determinants of the ratio of long-term debt to long-term capital assets (defined as the sum of long-term debt and equity) using regression techniques. By definition, this leverage ratio is bounded by 0 and 1, so fractional regression models have recently started to be used in this analysis. On the other hand, many firms do not use long-term debt in the financing of their activities, so binary regression models are also commonly used in this area to study the factors that influence the probability of a firm using debt. A model that considers both issues is the two-part fractional regression model proposed by Ramalho and Silva (2009), which first explains the decision on using debt or not (using a binary regression model) and then, conditional on this decision, the decision on the relative amount of debt to issue (using a fractional regression model).

Clearly, the two-part fractional regression model for financial leverage decisions provides a particularly interesting example for our purposes, since, on the one hand, the GGOFF test is applicable precisely to binary and

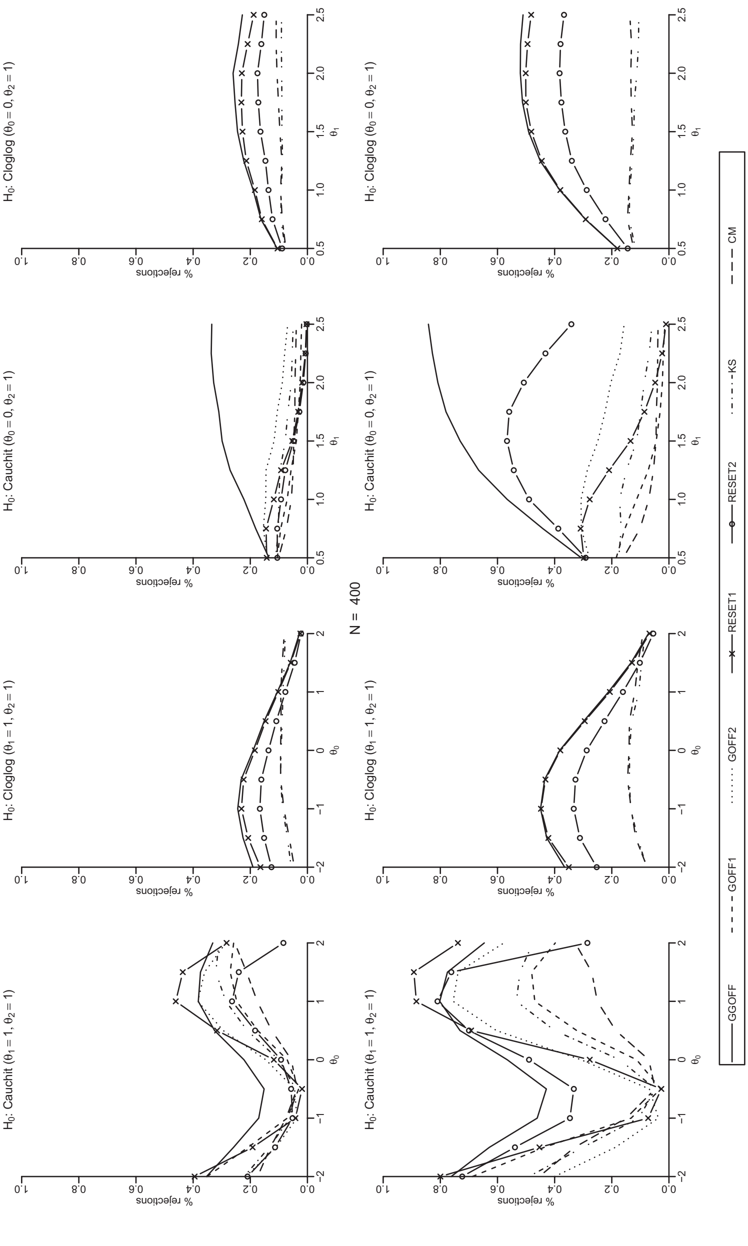

Fig. 4. Binary Regression Models—Misspecification of the Link Function (True Model: Probit)

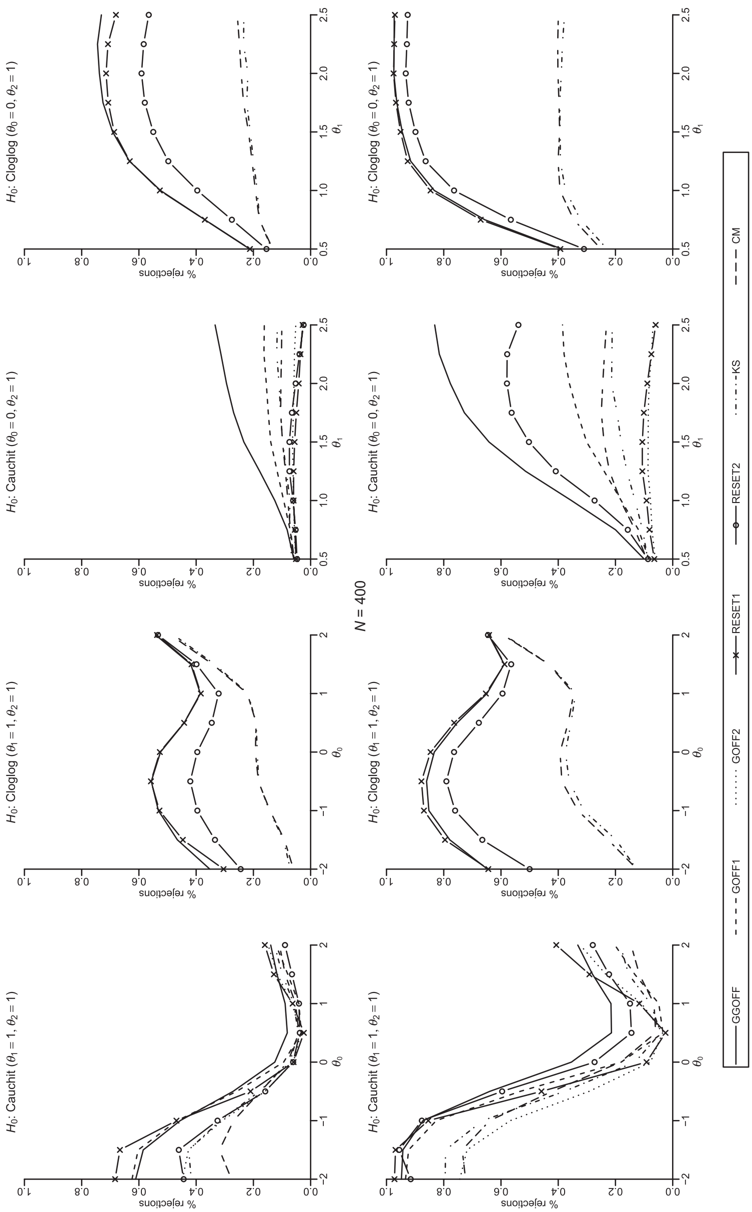

FIG. 5. Binary Regression Models—Misspecification of the Link Function (True Model: Loglog)

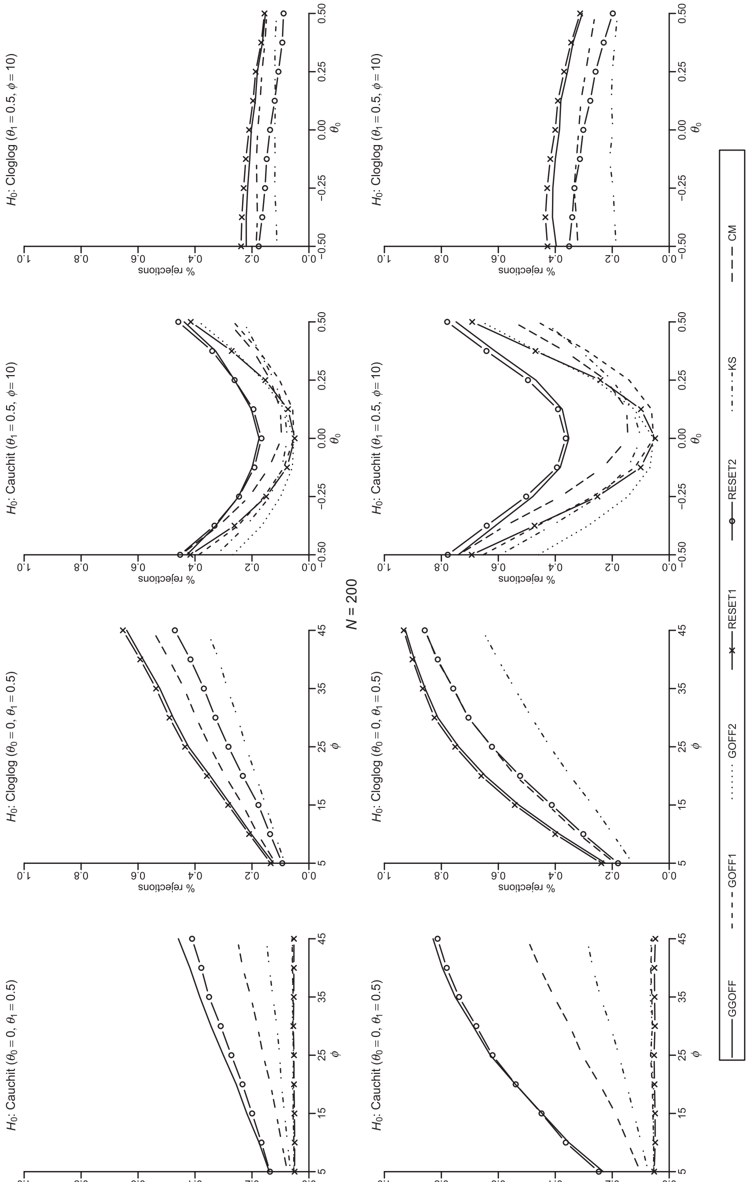

FIG. 6. Fractional Regression Models—Misspecification of the Link Function (True Model: Probit)

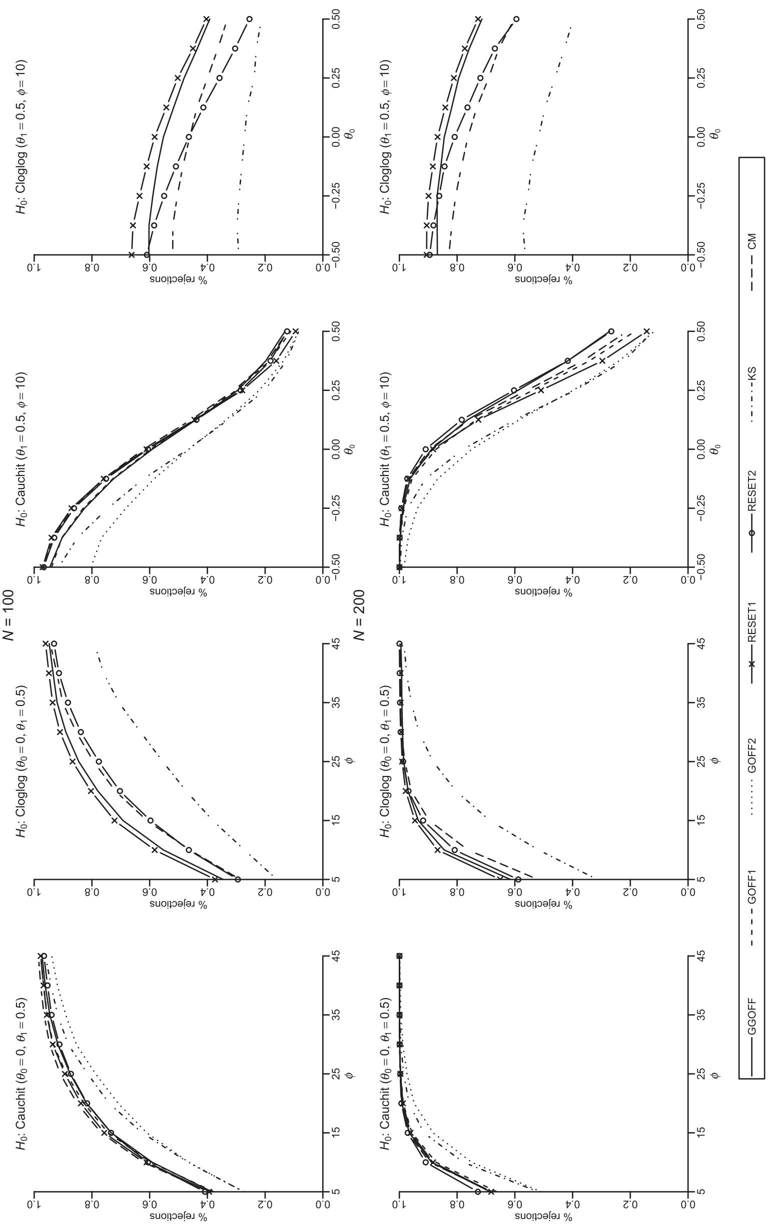

FIG. 7. Fractional Regression Models—Misspecification of the Link Function (True Model: Loglog)

fractional regression models and, on the other hand, economic theory does not suggest a specific functional form for each model component. Next, we use a subset of the data analysed in Ramalho and Silva (2009) to illustrate the application of the GGOFF and other parametric tests for binary and fractional regression models.[5] In particular, we consider only the subset of 3397 non-financial micro and small Portuguese firms.

We estimate five alternative specifications for each part of the model: cauchit, logit, probit, loglog and cloglog. In all cases, the same explanatory variables as those employed by Ramalho and Silva (2009) are contemplated: *Non-debt tax shields* (*NDTS*), measured by the ratio between depreciation and earnings before interest, taxes and depreciation; *Tangibility*, the proportion of tangible assets and inventories in total assets; *Size*, the natural logarithm of sales; *Profitability*, the ratio between earnings before interest and taxes and total assets; *Growth*, the yearly percentage change in total assets; *Age*, the number of years since the foundation of the firm; *Liquidity*, the sum of cash and marketable securities, divided by current assets; and four activity sector dummies: *Manufacturing*; *Construction*; *Trade* (wholesale and retail); and *Transport and Communication*.

Table 3 reports the estimation outcomes for the two-part fractional regression model. While for the fractional component of the model no specification test is able to reject any of the alternative functional forms considered, at the 5 per cent level, for the binary case the GGOFF test suggests that the loglog model is the only admissible specification. In fact, in this example, using the GGOFF test has the advantage of allowing to choose only one specification for the binary component of the two-part model, since the other tests are able to reject only the cloglog and cauchit (all tests) and logit (the GOFF2 test) specifications. While in this particular example the practical advantage of being able to select only one model seems to be relatively limited (the only relevant difference is that in the selected loglog model the variable *Construction* is statistically relevant and in three of the other models is not), in other applications more important differences across alternative models may arise. Moreover, if one is interested in the magnitude of partial effects, then sizable differences across models may appear, as Table 4 illustrates. For example, the average partial effects of the explanatory variables *Tangibility*, *Liquidity*, *Manufacturing* and *Construction* in the selected loglog binary model are at least 9.1 per cent higher or smaller than those produced by all the other models.

6 Concluding Remarks

In this paper we propose a new conditional mean test for binary and fractional regression models. In a Monte Carlo simulation study, we find that the

[5]We do not use the non-parametric tests due to the difficulty in dealing with the large number of explanatory variables that we consider in this application.

TABLE 3
TWO-PART FRACTIONAL REGRESSION MODELS FOR CAPITAL STRUCTURE DECISIONS

	Part I: Binary model					Part II: Fractional regression model				
	Cauchit	*Logit*	*Probit*	*Loglog*	*Cloglog*	*Cauchit*	*Logit*	*Probit*	*Loglog*	*Cloglog*
NDTS	–0.148	–0.066	–0.031	–0.023	–0.059	0.031	0.037	0.023	0.027	0.026
	(0.105)	(0.048)	(0.023)	(0.017)	(0.044)	(0.025)	(0.029)	(0.018)	(0.023)	(0.019)
Tangibility	1.297**	0.794**	0.421**	0.328**	0.717**	–0.276	–0.323	–0.200	–0.205	–0.258
	(0.301)	(0.225)	(0.126)	(0.107)	(0.196)	(0.177)	(0.190)	(0.117)	(0.123)	(0.147)
Size	0.642**	0.508**	0.287**	0.245**	0.432**	–0.102**	–0.118**	–0.072**	–0.078**	–0.087**
	(0.063)	(0.043)	(0.023)	(0.020)	(0.036)	(0.038)	(0.040)	(0.024)	(0.025)	(0.031)
Profitability	–5.089**	–3.737**	–2.052**	–1.710**	–3.226**	–1.801**	–2.207**	–1.376**	–1.520**	–1.622**
	(1.400)	(0.809)	(0.428)	(0.342)	(0.725)	(0.576)	(0.698)	(0.429)	(0.446)	(0.544)
Growth	0.001	0.001	0.000	0.000	0.001	0.003*	0.004*	0.002*	0.002*	0.003*
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)
Age	0.009	0.007	0.004	0.004	0.006	–0.008*	–0.009**	–0.006**	–0.006**	–0.007**
	(0.004)	(0.004)	(0.002)	(0.002)	(0.003)	(0.004)	(0.003)	(0.002)	(0.002)	(0.003)
Liquidity	–2.911**	–1.446**	–0.746**	–0.563**	–1.333**	–0.275	–0.305	–0.186	–0.165	–0.270
	(0.554)	(0.283)	(0.149)	(0.117)	(0.258)	(0.241)	(0.249)	(0.152)	(0.155)	(0.200)
Manufacturing	–0.478*	–0.544**	–0.327**	–0.312**	–0.425**	0.011	0.010	0.006	0.011	0.003
	(0.192)	(0.167)	(0.097)	(0.089)	(0.138)	(0.128)	(0.138)	(0.085)	(0.087)	(0.110)
Construction	–0.268	–0.384	–0.241*	–0.244*	–0.281	0.638**	0.783**	0.488**	0.566**	0.557**
	(0.242)	(0.200)	(0.115)	(0.102)	(0.169)	(0.142)	(0.171)	(0.105)	(0.117)	(0.128)
Trade	–1.642**	–1.423**	–0.811**	–0.699**	–1.178**	–0.055	–0.098	–0.064	–0.072	–0.080
	(0.504)	(0.339)	(0.187)	(0.158)	(0.298)	(0.339)	(0.338)	(0.206)	(0.200)	(0.284)
Communication	–1.378**	–1.120**	–0.636**	–0.556**	–0.939**	–0.226	–0.223	–0.134	–0.122	–0.193
	(0.371)	(0.246)	(0.136)	(0.116)	(0.214)	(0.215)	(0.208)	(0.126)	(0.125)	(0.171)
Constant	–9.622**	–7.694**	–4.368**	–3.447**	–6.902**	1.286*	1.459**	0.898**	1.320	0.727
	(0.948)	(0.618)	(0.338)	(0.283)	(0.535)	(0.516)	(0.567)	(0.349)	(0.368)	(0.433)
Number of observations	3397	3397	3397	3397	3397	616	616	616	616	616
RESET1	0.000**	0.202	0.767	0.175	0.020*	0.108	0.134	0.139	0.192	0.090
RESET2	0.000**	0.122	0.265	0.175	0.006**	0.070	0.118	0.129	0.195	0.082
GOFF1	0.000**	0.167	0.603	—	0.005**	0.099	0.121	0.156	—	0.103
GOFF2	0.000**	0.049*	0.825	0.159	—	0.166	0.166	0.126	0.177	—
GGOFF	0.000**	0.005**	0.042*	0.159	0.005**	0.061	0.116	0.128	0.177	0.103

Notes: Below the coefficients we report standard errors in parentheses; for the test statistics we report p values; ** and * denote coefficients or test statistics which are significant at 1 per cent or 5 per cent respectively.

TABLE 4
AVERAGE PARTIAL EFFECTS FOR THE BINARY COMPONENT OF TWO-PART FRACTIONAL REGRESSION MODELS

	Cauchit	*Logit*	*Probit*	*Loglog*	*Cloglog*
NDTS	–0.015	–0.009	–0.007	–0.006	–0.009
	(0.010)	(0.006)	(0.005)	(0.005)	(0.007)
Tangibility	0.128**	0.106**	0.099**	0.088**	0.111**
	(0.029)	(0.030)	(0.030)	(0.029)	(0.030)
Size	0.063**	0.068**	0.068**	0.066**	0.067**
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Profitability	–0.501**	–0.500**	–0.484**	–0.460**	–0.497**
	(0.136)	(0.108)	(0.100)	(0.091)	(0.112)
Growth	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Age	0.001*	0.001	0.001	0.001	0.001
	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)
Liquidity	–0.286**	–0.194**	–0.176**	–0.151**	–0.205**
	(0.054)	(0.038)	(0.035)	(0.031)	(0.040)
Manufacturing	–0.047*	–0.073**	–0.077**	–0.084**	–0.065**
	(0.019)	(0.022)	(0.023)	(0.024)	(0.021)
Construction	–0.026	–0.051	–0.057*	–0.066*	–0.043
	(0.024)	(0.027)	(0.027)	(0.027)	(0.026)
Trade	–0.162**	–0.191**	–0.191**	–0.188**	–0.182**
	(0.049)	(0.045)	(0.044)	(0.042)	(0.046)
Communication	–0.136**	–0.150**	–0.150**	–0.150**	–0.145**
	(0.036)	(0.033)	(0.032)	(0.031)	(0.033)

Notes: Below the partial effects we report standard errors in parentheses; ** and * denote partial effects that are significant at 1 per cent or 5 per cent respectively.

suggested GGOFF test is potentially sensitive to a wider set of model misspecifications than the most common functional form parametric tests, since, unlike them, its performance is clearly stable across all experiments. We also find that the GGOFF test seems to be more powerful than non-parametric tests when applied to traditional regression models for binary or fractional responses. Therefore, at least for the type of models simulated in the paper, the GGOFF test seems to be an attractive statistical tool for detecting functional form misspecifications.

Despite the complexity of the generalized model from which the GGOFF test is derived, its implementation as an LM statistic is straightforward in applied work.[6] Relative to RESET tests, the GGOFF test has also the clear advantage of not requiring the choice of the number of test variables, arbitrarily made in practice. Moreover, also in contrast to RESET tests, in case of rejection of the null hypothesis, the generalized model underlying the GGOFF test may be used as a modelling device. However, note that, in general, as in many other generalized models with additional parameters, estimates for all parameters of the model may not be easily computed.

[6]A canned Stata command that allows automatic computation of the GGOFF test is available at http://evunix.uevora.pt/~jsr/FRM.htm.

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