PART A

Control Charts for Variables

CONTROL CHARTS FOR VARIABLES

The first part of this lab course comprises Sessions 1 to 5 which will enable you to develop practical skills for constructing control charts for variables in MS Excel 2007. In Unit 1 of the course MSTE-001 (Industrial Statistics-I), you have learnt about the importance of control charts. You know that these are used in manufacturing processes to distinguish between the variation in a process resulting from chance causes and the variation resulting from assignable causes. These variations may arise due to low raw material quality, unskilled labour, machines, faulty equipment, etc. For example, when we purchase any product, we like to ensure that the product we buy is as per the specifications mentioned on it. The specifications may be in terms of length, diameter, thickness, strength, weight, etc., of the product:

- Weight of bottled juice and packet of chips,
- Wattage of bulbs, CFLs, etc.,
- Diameter and material of water pipes used in homes,
- Thickness of slabs or other metal parts used in construction,
- Diameter of fuse pins and ball bearing of fans,
- Length of aluminium foils, and so on.

Control charts help in detecting variations resulting from assignable causes. These can be eliminated and controlled through various ways. There are two types of control charts: (i) control charts for variables and (ii) control charts for attributes (see Fig. A.1).

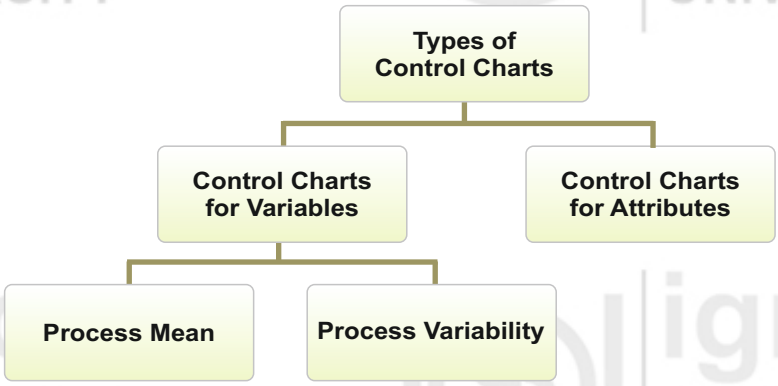


Fig. A.1

You have learnt in Unit 2 of MSTE-001 that the term **"variable"** means the quality characteristic which can be measured numerically, e.g., diameter of water pipes, length of aluminium foils, weight of cricket balls, etc. You know that the control charts based on numerical measurements of quality characteristics are called **control charts for variables**.

When we deal with a numerically measurable quality characteristic, it is necessary to control the central tendency as well as the dispersion (variability) of the quality characteristic for the process. For central tendency, we usually apply mean and for variability, we calculate range or standard deviation.

Therefore, there are different control charts for variables for controlling the mean and variability of the process. The following control charts for variables are used most frequently:

1. Control chart for process mean ($\bar{X}$-chart, read as X-bar chart)

2. Control chart for process variability (R-chart and S-chart)

To control the mean quality level or process average, we generally use the control chart for mean, i.e., $\bar{X}$-chart and to control the variability of the process, we use either control chart for range, i.e., R-chart or control chart for standard deviation, i.e., S-chart. You will learn how to construct each of these charts in Lab Sessions 1 to 5.

In Session 1, you will learn how to prepare the control chart for mean used to control variation in the average values of the quality characteristic when **process variability is known**. Sessions 2 and 3 deal with construction of control chart for mean when **process variability is unknown**. We shall consider the sample range or sample standard deviation, respectively, as estimators of process standard deviation (σ) to demonstrate control chart for mean using a specific problem.

In Session 4, we demonstrate the construction of R-chart to monitor the variation in the process variability using **sample range**. In Session 5, you will learn how to construct the S-chart to control the process variation using **sample standard deviation**.

All sessions of this part have been designed in such a way that after successfully completing them, you will be able to construct different variable control charts using MS Excel 2007 and interpret them. You will also be able to decide which control chart to use for any given data.

Control Chart for Mean

SESSION 1

Structure

1.1 Introduction

Objectives

1.2 Problem Description

1.3 Procedure for the Construction of $\bar{X}$-Chart

1.4 Steps Involved in the Construction of $\bar{X}$-Chart in Excel 2007

1.5 Interpretation

1.1 INTRODUCTION

In Unit 2 of MSTE-001 (Industrial Statistics-I), you have learnt that control charts for variables are used to control the measurable quality characteristics. We use the control chart for the process mean, i.e., $\bar{X}$-chart for controlling the mean of the quality characteristic or process mean. With the help of $\bar{X}$-chart, we monitor the variation in the mean of the samples that have been drawn from time to time from the process.

Prerequisite

- Lab Sessions 1, 3 and 6 of MSTL-001 (Basic Statistics Lab).
- Unit 2 of MSTE-001 (Industrial Statistics-I).

We can use control charts for mean in both cases: When process variability is known and when it is unknown (see Fig. 1.1). You have also learnt in Sec. 2.4 of Unit 2 that we estimate the unknown process variability with the help of sample range or standard deviation.

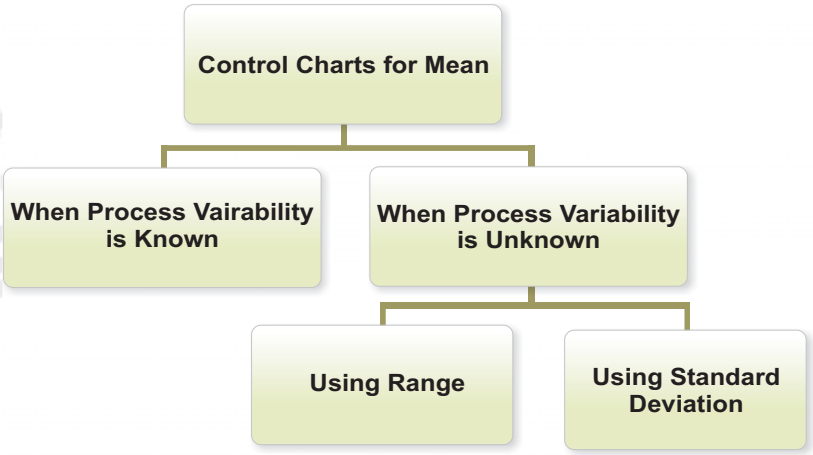


Fig. 1.1

In this lab session, you will learn how to prepare the control chart for mean in MS Excel 2007 **when process variability is known**. We shall illustrate it using a specific problem. You will learn how to prepare the control chart for mean for unknown process variability in Lab Sessions 2 and 3.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for control chart for mean;
- construct the control chart for mean; and
- interpret the control chart for mean.

1.2 PROBLEM DESCRIPTION

In this lab session, we consider the problem related to a manufacturing process for maintaining the quality of bottling procedure. A fruit juice manufacturing company uses automatic machines to fill 500 ml juice bottles. A quality control inspector at this company collected 25 samples of four observations of the juice bottles at four different times and measured the volume of each filled bottle. The data is given in Table 1.

Table 1: Volume of fruit juice in the filled bottles

Sample Number	Volume of Juice per Bottle (in ml)			
	Obs. 1	Obs. 2	Obs. 3	Obs. 4
1	497.32	500.62	498.68	497.82
2	504.76	500.00	498.32	500.32
3	499.24	497.18	498.12	498.68
4	499.26	496.32	498.88	497.82
5	502.32	503.62	504.56	503.12
6	502.12	500.32	501.38	500.94
7	499.34	498.32	497.32	497.62
8	499.38	498.12	500.62	498.12
9	501.26	502.38	500.68	501.38
10	498.60	497.62	499.25	498.56
11	502.44	500.00	501.32	499.38
12	501.26	502.32	500.76	502.68
13	497.32	498.50	497.18	499.38
14	499.56	498.00	498.76	501.12
15	502.24	500.32	503.12	501.25
16	501.76	500.50	502.68	501.12
17	500.65	497.82	494.06	496.25
18	501.12	501.26	500.44	502.76
19	501.00	500.50	501.56	501.76
20	497.50	498.82	499.76	497.82
21	503.44	500.62	500.00	501.26
22	499.38	498.38	497.56	498.56
23	501.56	499.56	498.00	499.82
24	498.32	497.32	499.56	498.62
25	499.50	501.12	502.50	500.38

The quality control inspector of the company needs to develop $\overline{X}$-chart to check whether the process of bottling is under control or out-of-control when process variability is known.

Therefore, the problem for this session is to construct the control chart for mean using the data given in Table 1. It is given that the standard deviation for the bottling operation is 2.5 ml.

1.3 PROCEDURE FOR THE CONSTRUCTION OF $\bar{X}$-CHART

You have learnt in Unit 2 of MSTE-001 that the $\bar{X}$-chart (average control chart) is used to monitor the changes in the average value of a quality characteristic. This helps us to detect whether the process is under control or out-of-control. The main steps involved in the construction of $\bar{X}$-chart for any process are as follows:

Step 1: We select k samples (subgroups) randomly from the process at different times, each of size n and measure the characteristic of interest.

Step 2: Suppose, the quality characteristic, say, X is measured from each unit of the sample and $X_{i1}, X_{i2}, ..., X_{in}$ are the measurements of the units of the ith sample of size n. Then we calculate the sample mean for each sample, say, $\bar{X}_1, \bar{X}_2, ..., \bar{X}_k$ where $\bar{X}_i$ is the sample mean for i^{th} sample given by

$$\bar{X}_i = \frac{1}{n}\sum_{j=1}^{n} X_{ij} \qquad ...(1)$$

Step 3: We find the mean of all sample means:

$$\bar{\bar{X}} = \frac{1}{k}\sum_{i=1}^{k} \bar{X}_i \qquad ...(2)$$

Step 4: To construct the $\bar{X}$-chart when process variability is known, we calculate the control limits given below:

✓ Centre line (CL) $= \bar{\bar{X}}$ ...(3)

✓ Upper control limit (UCL) $= \bar{X} + \frac{3\sigma}{\sqrt{n}} = \bar{X} + A\sigma$ ...(4)

✓ Lower control limit (LCL) $= \bar{X} - \frac{3\sigma}{\sqrt{n}} = \bar{X} - A\sigma$ ...(5)

where

n – number of observations in each sample,

k – total number of samples,

σ – process standard deviation, and

$A = 3/\sqrt{n}$ is a constant, which depends on the size of the sample.

It is tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 5: After setting the centre line and control limits, we construct the $\bar{X}$-chart by taking sample numbers on the X-axis and sample means on the Y-axis.

Step 6: Interpretation of the $\bar{X}$-chart.

For the given problem, we have $k = 25$, $n = 4$ and $\sigma = 2.5$ ml.

1.4 STEPS INVOLVED IN THE CONSTRUCTION OF $\bar{X}$-CHART IN EXCEL 2007

You have already learnt the manual computation of $\bar{X}$-chart in Unit 2 of MSTE-001. Here we describe the procedure of constructing $\bar{X}$-chart for controlling the process mean in MS Excel 2007. In order to calculate the $\bar{X}$-chart control limits and to plot the control chart for the given data, we follow the steps given below:

Step 1: We enter each one of the k samples given in Table 1, in the MS Excel 2007 spreadsheet row-wise. For this example, if we start from Row 3 in the Excel sheet, the entry will go up to Row 27. For the given data, the spreadsheet will look as shown in Fig. 1.2.

	A	B	C	D	E
1		Juice Volume (in ml)			
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4
3	1	497.32	500.62	498.68	497.82
4	2	504.76	500.00	498.32	500.32
5	3	499.24	497.18	498.12	498.68
6	4	499.26	496.32	498.88	497.82
7	5	502.32	503.62	504.56	503.12
8	6	502.12	500.32	501.38	500.94
9	7	499.34	498.32	497.32	497.62
10	8	499.38	498.12	500.62	498.12
11	9	501.26	502.38	500.68	501.38
12	10	498.60	497.62	499.25	498.56
13	11	502.44	500.00	501.32	499.38
14	12	501.26	502.32	500.76	502.68
15	13	497.32	498.50	497.18	499.38
16	14	499.56	498.00	498.76	501.12
17	15	502.24	500.32	503.12	501.25
18	16	501.76	500.50	502.68	501.12
19	17	500.65	497.82	494.06	496.25
20	18	501.12	501.26	500.44	502.76
21	19	501.00	500.50	501.56	501.76

X Bar Chart

Fig. 1.2: Partial screenshot of the spreadsheet for the given data.

Step 2: We calculate the mean of each sample, i.e.,

$$\bar{X}_i = \frac{\sum_{j=1}^{n} X_{ij}}{n}$$

For each sample of 4 observations of the volume of the filled bottle, we create a column for the sample mean. Here we are using Column F. We calculate the mean of 4 observations of the volume of bottle filled for the first sample using the inbuilt function of MS Excel. For calculating $\bar{X}$ for the first sample, we select Cell F3, click on the ***Formulas*** tab on the menu ribbon and select ***Insert Function***. The dialog box shown in Fig. 1.3 will appear.

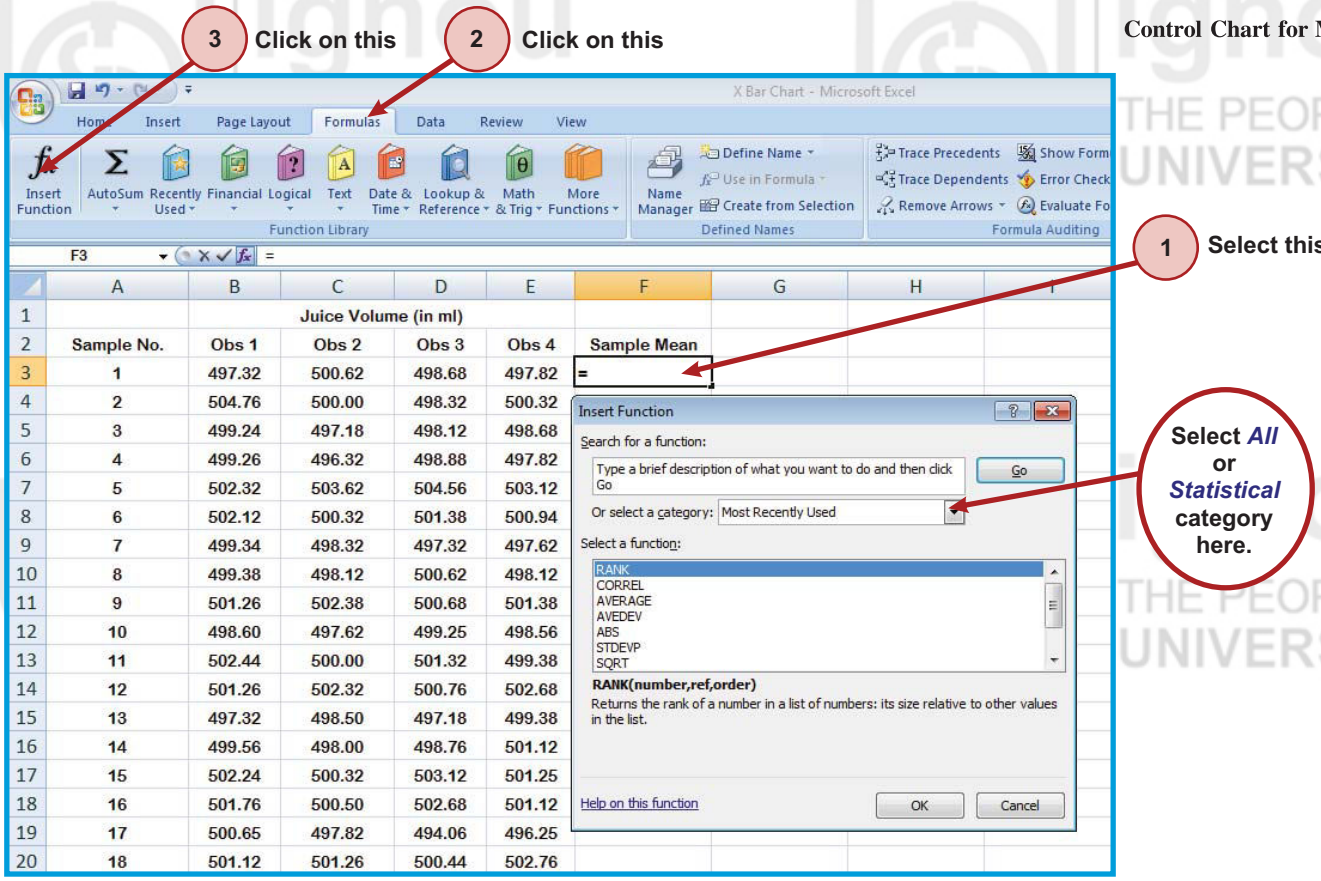


Fig. 1.3

We further select ***All*** or ***Statistical*** category as shown in Fig. 1.3. As a result, the following dialog box appears. We select ***Average*** function and click on ***OK*** (Fig. 1.4).

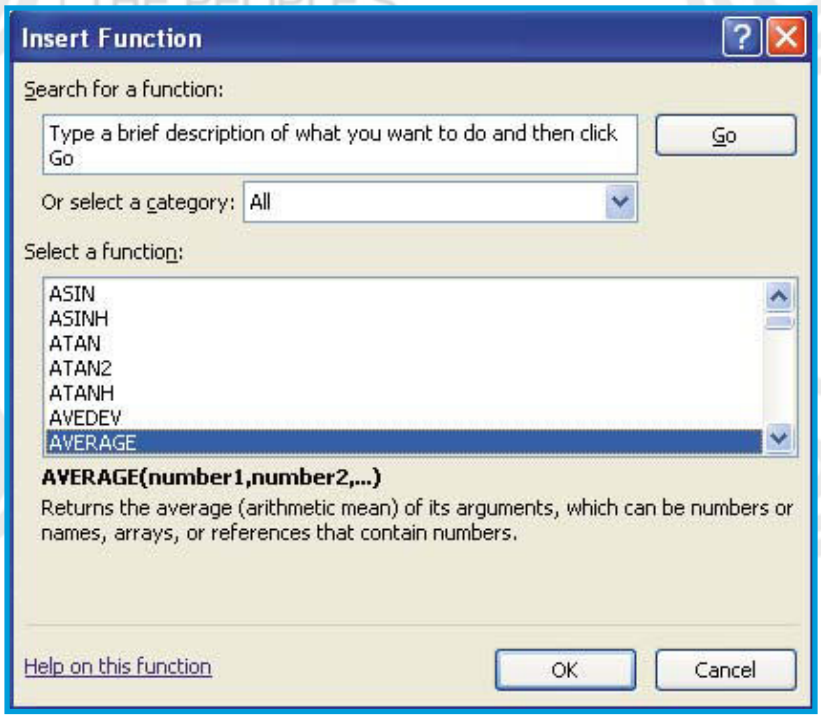


Fig. 1.4

Step 3: When we click on ***OK***, another dialog box appears as shown in Fig. 1.5. In this dialog box, data range can be taken by selecting observations of the sample given in Cells B3:E3. Then we click on ***OK*** or press the ***Enter*** key and get the average in Cell F3.

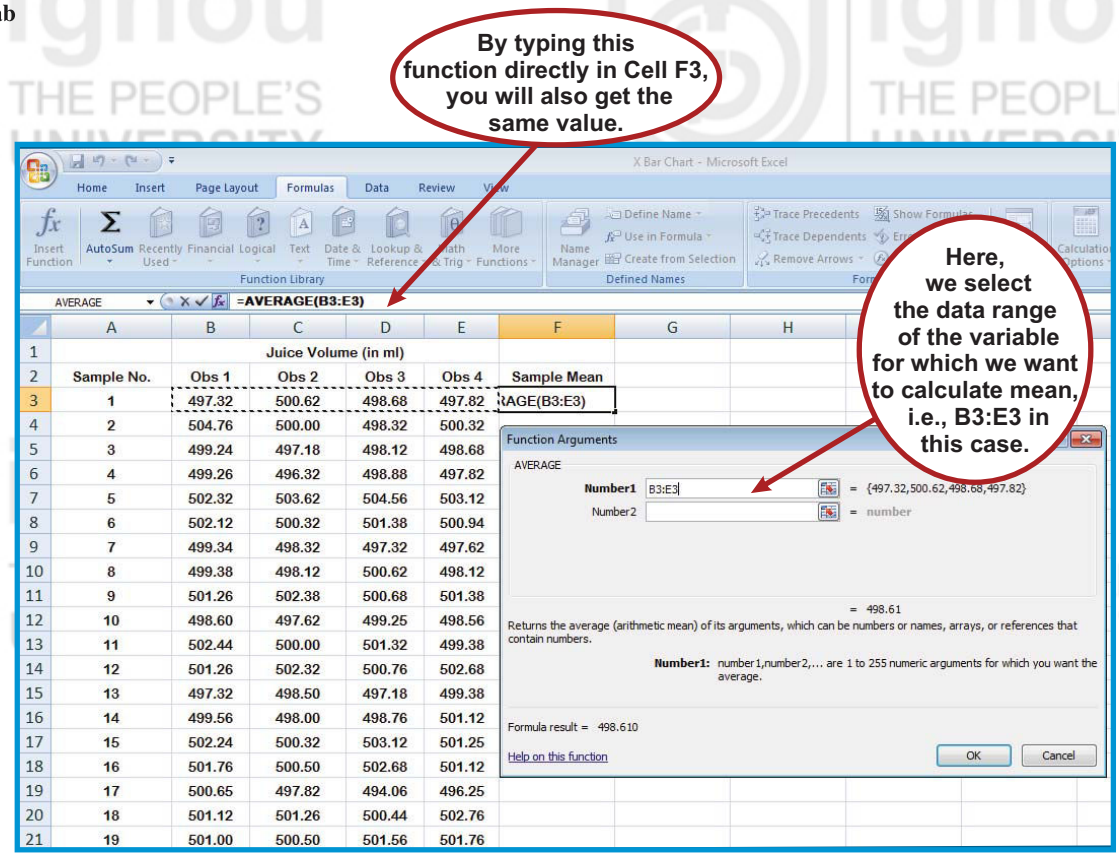


Fig. 1.5

Step 4: We put the cursor on the corner of Cell F3 and drag it down up to the last sample, i.e., Cell F27 as shown in Fig. 1.6.

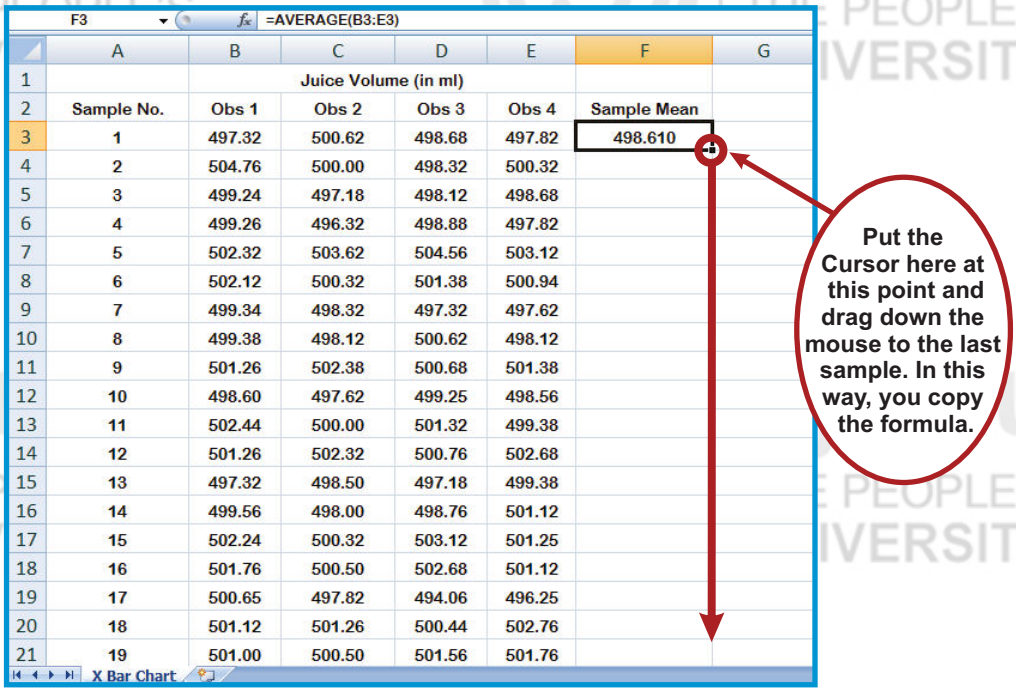


Fig. 1.6

Step 5: Step 4 will fill Column F up to the last sample, i.e., Cell F27. It will give the values of $\overline{X}$ for the remaining samples. The resulting spreadsheet is shown in Fig. 1.7.

	A	B	C	D	E	F
1			Juice Volume (in ml)			
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean
3	1	497.32	500.62	498.68	497.82	498.610
4	2	504.76	500.00	498.32	500.32	500.850
5	3	499.24	497.18	498.12	498.68	498.305
6	4	499.26	496.32	498.88	497.82	498.070
7	5	502.32	503.62	504.56	503.12	503.405
8	6	502.12	500.32	501.38	500.94	501.190
9	7	499.34	498.32	497.32	497.62	498.150
10	8	499.38	498.12	500.62	498.12	499.060
11	9	501.26	502.38	500.68	501.38	501.425
12	10	498.60	497.62	499.25	498.56	498.508
13	11	502.44	500.00	501.32	499.38	500.785
14	12	501.26	502.32	500.76	502.68	501.755
15	13	497.32	498.50	497.18	499.38	498.095
16	14	499.56	498.00	498.76	501.12	499.360
17	15	502.24	500.32	503.12	501.25	501.733
18	16	501.76	500.50	502.68	501.12	501.515
19	17	500.65	497.82	494.06	496.25	497.195
20	18	501.12	501.26	500.44	502.76	501.395
21	19	501.00	500.50	501.56	501.76	501.205

X Bar Chart

Fig. 1.7

Step 6: The sample means appear in Column F. To obtain the centre line and control limits for plotting the control chart, we first compute the grand average $\overline{\overline{X}}$ of the data, i.e., the average of 25 sample means (given in Cells F3 : F27). For this, we use the method explained in Step 3 in Cell F28 as shown in Fig. 1.8.

F28 f_x =AVERAGE(F3:F27)

	D	E	F	G
28		$\overline{\overline{X}}$ =	499.918	
29				

Fig. 1.8

Step 7: We type the values of k, n and σ in Cells F29, F30 and F31, respectively, as shown in Fig. 1.9.

	E	F	G
28	$\overline{\overline{X}}$ =	499.918	
29	k =	25	
30	n =	4	
31	σ =	2.50	ml

Fig. 1.9

Step 8: We use equations (3) to (5) given in Sec. 1.3 for computing both the control limits and the centre line. Here we shall use Columns G, H and I for putting the values of centre line, upper and lower control limits, respectively. We calculate these as follows:

i) The formula for centre line is (CL) = $\overline{\overline{X}}$ and $\overline{\overline{X}}$ is given in Cell F28 (see Fig. 1.9). So we type "=$F$28" in Cell G3 and press ***Enter*** to get the value of centre line as shown in Fig. 1.10a.

ii) To calculate the upper control limit, the formula is

$$UCL = \overline{\overline{X}} + 3\frac{\sigma}{\sqrt{n}}$$

The formula with dollar sign ($) is used for an absolute reference.

Since the values of $\overline{\overline{X}}$, n and σ are given in Cell F28, F30 and F31, respectively (see Fig. 1.9), we type "=$F$28+3*($F$31/(Sqrt ($F$30)))" in Cell H3 and press ***Enter***. Then we get the value of UCL in Cell H3 as shown in Fig. 1.10b.

iii) Similarly, we calculate the lower control limit $LCL = \overline{\overline{X}} - 3\frac{\sigma}{\sqrt{n}}$ by typing, "=$F$28–3*($F$31/(Sqrt ($F$30)))" in Cell I3 and pressing ***Enter***. We get the value of LCL as shown in Fig. 1.10c.

(a)

G3 | =$F$28

	G	H	I	J	K	L
1		Control Limits				
2	Centre Line	UCL	LCL			
3	499.918					
4						
5						

(b)

H3 | =$F$28+3*($F$31/(SQRT($F$30)))

	H	I	J	K	L	M
1	ontrol Limits					
2	UCL	LCL				
3	503.668					
4						
5						

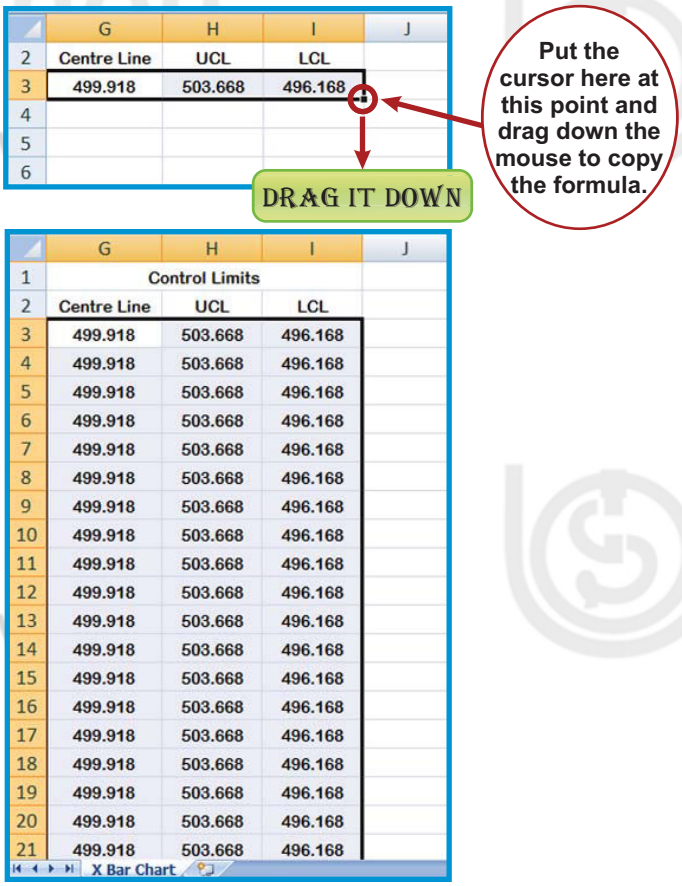
(c)

I3 | =$F$28-3*($F$31/(SQRT($F$30)))

	I	J	K	L	M	N
1						
2	LCL					
3	496.168					
4						
5						

Fig. 1.10

Step 9: So we have calculated the CL, UCL and LCL in Cells G3, H3 and I3, respectively. We now have to plot these limits on the chart. For this purpose, we first select Cells G3:I3 and drag them down up to Row 27 as shown in Fig. 1.11a. After this activity, the Excel sheet should look like Fig. 1.11b.



	G	H	I	J
2	Centre Line	UCL	LCL	
3	499.918	503.668	496.168	
4				
5				
6				

	G	H	I	J
1		Control Limits		
2	Centre Line	UCL	LCL	
3	499.918	503.668	496.168	
4	499.918	503.668	496.168	
5	499.918	503.668	496.168	
6	499.918	503.668	496.168	
7	499.918	503.668	496.168	
8	499.918	503.668	496.168	
9	499.918	503.668	496.168	
10	499.918	503.668	496.168	
11	499.918	503.668	496.168	
12	499.918	503.668	496.168	
13	499.918	503.668	496.168	
14	499.918	503.668	496.168	
15	499.918	503.668	496.168	
16	499.918	503.668	496.168	
17	499.918	503.668	496.168	
18	499.918	503.668	496.168	
19	499.918	503.668	496.168	
20	499.918	503.668	496.168	
21	499.918	503.668	496.168	

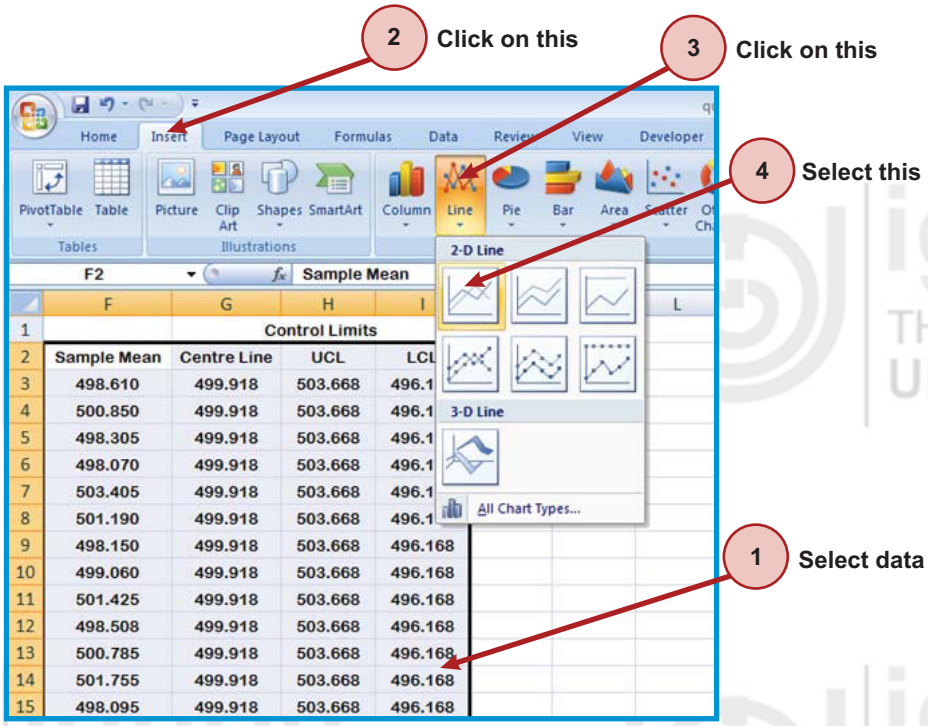
Fig. 1.11

To scroll through all available chart types and chart subtypes, click on

- ✓ Any chart type in ***Chart*** group dialog box under ***Insert*** tab and then click ***All Chart Types***,
- ✓ Click the arrows to the right down corner of the ***Chart*** group, and then click the ones that you want to use.

Step 10: To obtain the $\bar{X}$-chart, we refer to Fig. 1.12. It means that we

1. select the Cells F2:I27,
2. click on the ***Insert*** tab,
3. click on the ***Line*** option in the ***Charts*** group, and
4. select a chart subtype that we want to use.



	F	G	H	I
1			Control Limits	
2	Sample Mean	Centre Line	UCL	LCL
3	498.610	499.918	503.668	496.1
4	500.850	499.918	503.668	496.1
5	498.305	499.918	503.668	496.1
6	498.070	499.918	503.668	496.1
7	503.405	499.918	503.668	496.1
8	501.190	499.918	503.668	496.1
9	498.150	499.918	503.668	496.168
10	499.060	499.918	503.668	496.168
11	501.425	499.918	503.668	496.168
12	498.508	499.918	503.668	496.168
13	500.785	499.918	503.668	496.168
14	501.755	499.918	503.668	496.168
15	498.095	499.918	503.668	496.168

Fig. 1.12

Step 11: The values in Columns G, H and I provide the horizontal lines on the chart representing centre line, UCL and LCL, respectively. The values in Column F provide the averages for 25 samples on the chart. The resulting chart shown in Fig. 1.13 is called the $\bar{X}$-chart. Note that the sample means have been connected by straight line segments in Fig. 1.13.

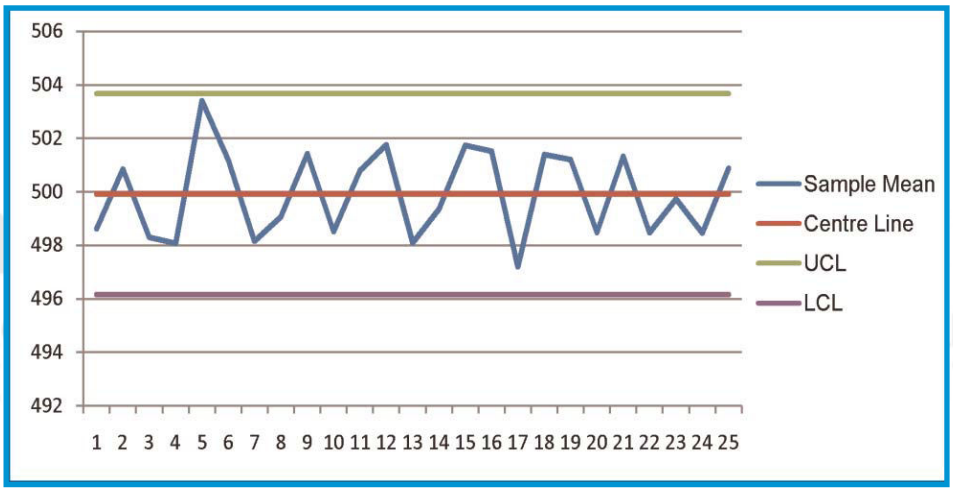


Fig. 1.13

Step 12: All that remains to be done is to format the chart the way you would like to present it. This is what you have already learnt in the Lab Session 3 of MSTL-001. For example, you may like to make the following changes:

- ✓ Eliminate the grid lines and the border around the chart.
- ✓ Change the UCL and LCL to dashed lines with no markers.
- ✓ Change the centre line to a bold solid line with no markers.
- ✓ Change the $\bar{X}$ data series to bold solid lines with large visible markers.
- ✓ Also change the fonts, axes, titles and background, etc. as desired.

We have formatted the chart as explained above. The resulting $\bar{X}$-chart is shown in Fig. 1.14.

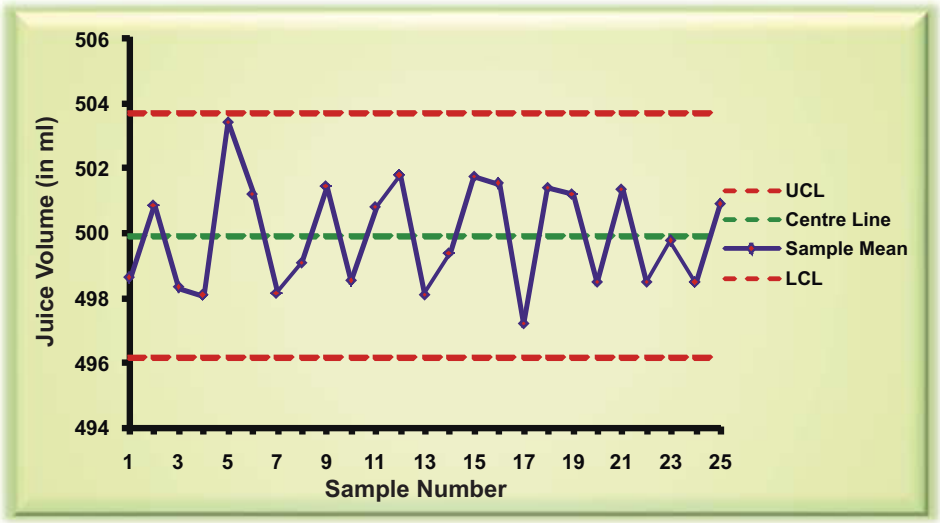


Fig. 1.14

1.5 INTERPRETATION

Now that you know how to make $\bar{X}$-chart in MS Excel, it is important for you to learn how to **interpret** it. If all points are within the control limits, we infer that the process is under statistical control. But, if any point goes outside the upper and lower control limits, the process is considered as out-of-control. In this kind of situation, the production process should be stopped and the causes of variations should be identified and eliminated.

The line segments in blue colour in Fig. 1.14 show the variability in the volume filled in the juice bottles. Since all these points lie between UCL and LCL, the process being monitored in the problem given in Sec. 1.2 is under statistical control (or stable). Here we may say that the process of bottling operation is free from assignable causes of variation and the variation is only under the influence of chance causes.

You should now apply this method to other problems for practice.

Activity

Construct the control charts for mean with the help of MS Excel 2007 and interpret the results for

A1) Example 3 given in Unit 2 of MSTE-001 if process standard deviation is 2.5.

A2) Exercise E6 given in Unit 2 of MSTE-001.

Match the results with the manual calculation done in Unit 2 of MSTE-001.

Continuous Assessment 1

Suppose a bulb manufacturing company wants to check whether the variation in the life of bulbs produced by a particular machine is due to chance causes or due to assignable causes. For this purpose, 12 samples, each of size 5, are selected and the life of each bulb is measured (in days). If the process variability is 0.25, develop the spreadsheet for the following data and construct a control chart for mean. Interpret it to infer whether the production is under control or not.

Table 2: Life of bulbs

Sample No.	Sample Observations				
1	42	65	75	78	87
2	42	45	68	72	90
3	19	24	80	81	81
4	36	54	89	77	84
5	42	51	57	59	78
6	51	74	75	78	132
7	60	60	72	95	138
8	18	20	27	42	60
9	15	30	39	62	84
10	69	109	113	118	153
11	64	90	93	109	112
12	61	78	94	109	136

Home Work: Do It Yourself

1) Follow the steps explained in Sec. 1.4 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 1" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute in your continuous assessment in the Laboratory.

Control Chart for Mean using Range

SESSION 2

Structure

2.1 Introduction

Objectives

2.2 Problem Description

2.3 Procedure for the Construction of $\bar{X}$-Chart

2.4 Steps Involved in the Construction of $\bar{X}$-Chart in Excel 2007

2.5 Revised $\bar{X}$-Chart

2.1 INTRODUCTION

In Lab Session 1, you have learnt how to prepare the control chart for mean in MS Excel 2007 when process variability is known. But the value of process variability is not known in practice. Therefore, we estimate it from the samples or subgroups taken when the process is thought to be in control. You have studied in Unit 2 of MSTE-001 (Industrial Statistics-I) that we express process variability in terms of standard deviation in quality control. We use sample range or sample standard deviation as an estimator of the process standard deviation (σ).

In this lab session, you will learn about the construction of control chart for mean in MS Excel 2007 **when process variability is unknown**. We shall consider the **sample range** as an estimator of process standard deviation (σ) to demonstrate control chart for mean using a specific problem. In the next lab session, you will learn how to use **sample standard deviation** for estimating the process standard deviation.

Prerequisite

- Lab Sessions 1, 3 and 6 of MSTL-001 (Basic Statistics Lab).
- Lab Session 1 of MSTE-002 (Industrial Statistics Lab).
- Unit 2 of MSTE-001 (Industrial Statistics-I).

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for control chart for mean using sample range to estimate the unknown process variability;
- construct the control chart for mean using sample range to estimate the unknown process variability;
- obtain the revised control limits for control charts for mean using sample range; and
- interpret the control chart.

2.2 PROBLEM DESCRIPTION

For this lab session, we consider the data given in Lab Session 1.

Suppose the quality control inspector of the juice manufacturing company needs to develop $\bar{X}$-chart to check whether the process of bottling is under control or out-of-control **when process variability is unknown**. Suppose he/she also needs to compute the revised control limits if the process is out-of-control.

Therefore, the problem for this session is to construct the control chart for mean using sample range to estimate the process variability.

2.3 PROCEDURE FOR THE CONSTRUCTION OF $\bar{X}$-CHART

In Sec. 2.4 of Unit 2 of MSTE-001, you have learnt various formulae and procedure of constructing the $\bar{X}$-chart when we estimate (σ) using sample range. Therefore, in this lab session, we will briefly revise the procedure of computing the centre line, upper and lower control limits using sample range to estimate the process standard deviation (σ). The main steps involved in the construction of $\bar{X}$-chart are as follows:

Steps 1-3: The first three steps are the same as explained in Sec. 1.3 of Lab Session 1.

Step 4: We calculate the sample range for each subgroup or sample, say, $R_1, R_2, \ldots, R_k$ where R_i is the sample range for i^{th} sample or subgroup given by

$$R_i = \max\left(X_{i1}, X_{i2}, \ldots, X_{ik}\right) - \min\left(X_{i1}, X_{i2}, \ldots, X_{ik}\right) \quad \ldots(1)$$

Step 5: We find the mean of all sample ranges given by

$$\bar{R} = \frac{1}{k}\left(R_1 + R_2 + \ldots + R_k\right) = \frac{1}{k}\sum_{i=1}^{k} R_i \quad \ldots(2)$$

Step 6: The control limits for $\bar{X}$-chart when σ is estimated by $\bar{R}/d_2$ are given by

- ✓ Centre line (CL) $= \bar{\bar{X}}$ ...(3)
- ✓ Upper control limit (UCL) $= \bar{\bar{X}} + A_2\bar{R}$ ...(4)
- ✓ Lower control limit (LCL) $= \bar{\bar{X}} - A_2\bar{R}$ ...(5)

where $A_2 = \dfrac{3}{d_2\sqrt{n}}$ is a constant and depends on the size of the sample. It is tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 7: Interpretation of the $\bar{X}$-chart.

2.4 STEPS INVOLVED IN THE CONSTRUCTION OF $\bar{X}$-CHART IN EXCEL 2007

We now apply the steps explained in Sec. 2.3 and construct the $\bar{X}$-chart for controlling process mean when process variability is estimated by the sample range for the given data. It is similar to the procedure we have explained in Lab Session 1. The main steps involved in this method are as follows:

Step 1: We calculate the sample mean for all 25 samples. To calculate sample means ($\bar{X}$), we repeat Steps 1-5 of Sec. 1.4 of Lab Session 1. To compute the grand sample mean ($\bar{\bar{X}}$) of the given 25 samples, we repeat Step 6 of Sec. 1.4. We get the output shown in Fig. 2.1.

	A	B	C	D	E	F
1		Juice Volume (in ml)				
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean
3	1	497.32	500.62	498.68	497.82	498.610
4	2	504.76	500.00	498.32	500.32	500.850
5	3	499.24	497.18	498.12	498.68	498.305
6	4	499.26	496.32	498.88	497.82	498.070
7	5	502.32	503.62	504.56	503.12	503.405
8	6	502.12	500.32	501.38	500.94	501.190
9	7	499.34	498.32	497.32	497.62	498.150
10	8	499.38	498.12	500.62	498.12	499.060
11	9	501.26	502.38	500.68	501.38	501.425
12	10	498.60	497.62	499.25	498.56	498.508
13	11	502.44	500.00	501.32	499.38	500.785
14	12	501.26	502.32	500.76	502.68	501.755
15	13	497.32	498.50	497.18	499.38	498.095
16	14	499.56	498.00	498.76	501.12	499.360
17	15	502.24	500.32	503.12	501.25	501.733
18	16	501.76	500.50	502.68	501.12	501.515
19	17	500.65	497.82	494.06	496.25	497.195
20	18	501.12	501.26	500.44	502.76	501.395
21	19	501.00	500.50	501.56	501.76	501.205
22	20	497.50	498.82	499.76	497.82	498.475
23	21	503.44	500.62	500.00	501.26	501.330
24	22	499.38	498.38	497.56	498.56	498.470
25	23	501.56	499.56	498.00	499.82	499.735

X Bar Chart

F28 =AVERAGE(F3:F27)

	A	B	C	D	E	F
25	23	501.56	499.56	498.00	499.82	499.735
26	24	498.32	497.32	499.56	498.62	498.455
27	25	499.50	501.12	502.50	500.38	500.875
28					Average	499.918
29						

Fig. 2.1: Partial screenshot of the spreadsheet for the given data.

Step 2: We calculate the sample range (R) for all 25 samples. In Column G, we determine the range for each sample. For this, we type "=Max(B3:E3)–Min(B3:E3)" in Cell G3 and then drag down Cell G3 up to Cell G27. We get the ranges for all 25 samples as shown in Fig. 2.2.

G3 =MAX(B3:E3)-MIN(B3:E3)

	A	B	C	D	E	F	G	H
1			Juice Volume (in ml)					
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean	Sample Range	
3	1	497.32	500.62	498.68	497.82	498.610	3.300	
4	2	504.76	500.00	498.32	500.32	500.850		
5	3	499.24	497.18	498.12	498.68	498.305		
6	4	499.26	496.32	498.88	497.82	498.070		
7	5	502.32	503.62	504.56	503.12	503.405		

DRAG IT DOWN

	A	B	C	D	E	F	G	H
1			Juice Volume (in ml)					
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean	Sample Range	
3	1	497.32	500.62	498.68	497.82	498.610	3.300	
4	2	504.76	500.00	498.32	500.32	500.850	6.440	
5	3	499.24	497.18	498.12	498.68	498.305	2.060	
6	4	499.26	496.32	498.88	497.82	498.070	2.940	
7	5	502.32	503.62	504.56	503.12	503.405	2.240	
8	6	502.12	500.32	501.38	500.94	501.190	1.800	
9	7	499.34	498.32	497.32	497.62	498.150	2.020	
10	8	499.38	498.12	500.62	498.12	499.060	2.500	
11	9	501.26	502.38	500.68	501.38	501.425	1.700	
12	10	498.60	497.62	499.25	498.56	498.508	1.630	
13	11	502.44	500.00	501.32	499.38	500.785	3.060	
14	12	501.26	502.32	500.76	502.68	501.755	1.920	
15	13	497.32	498.50	497.18	499.38	498.095	2.200	
16	14	499.56	498.00	498.76	501.12	499.360	3.120	
17	15	502.24	500.32	503.12	501.25	501.733	2.800	
18	16	501.76	500.50	502.68	501.12	501.515	2.180	
19	17	500.65	497.82	494.06	496.25	497.195	6.590	
20	18	501.12	501.26	500.44	502.76	501.395	2.320	
21	19	501.00	500.50	501.56	501.76	501.205	1.260	
22	20	497.50	498.82	499.76	497.82	498.475	2.260	
23	21	503.44	500.62	500.00	501.26	501.330	3.440	

X Bar Chart

Fig. 2.2

Step 3: To determine the average range ($\bar{R}$), we type "=Average(G3:G27)" in Cell G28 as shown in Fig. 2.3. We get the value of $\bar{R}$ (shown in the box, Fig. 2.3).

G28 =AVERAGE(G3:G27)

	A	B	C	D	E	F	G
25	23	501.56	499.56	498.00	499.82	499.735	3.560
26	24	498.32	497.32	499.56	498.62	498.455	2.240
27	25	499.50	501.12	502.50	500.38	500.875	3.000
28					Average	499.918	2.736

Fig. 2.3

Step 4: We type the values of k and n in Cells F30 and F31, respectively, as shown in Fig. 2.4. Note that we have also put the value of A_2 for n = 4 in Cell F32 from the Appendix given at the end of this lab course.

	C	D	E	F
30			k	25
31			n	4
32	A_2 Value from Table (for n = 4)			0.729

Fig. 2.4

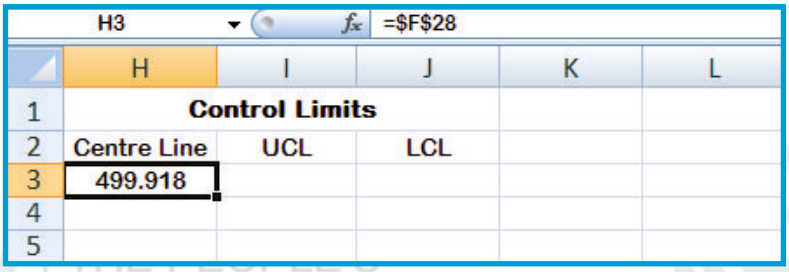
Step 5: We use the method for computing centre line and both control limits described in Step 8 of Sec. 1.4 in Lab Session 1. Here we shall use Columns H, I and J for putting the values of centre line, upper and lower control limits, respectively. We compute the centre line, upper and lower control limits as follows:

i) The formula for centre line is $(CL) = \overline{\overline{X}}$ and $\overline{\overline{X}}$ is given in Cell F28 (see Fig. 2.3). So we type "=$F$28" in Cell H3 as shown in Fig. 2.5a to obtain the centre line.

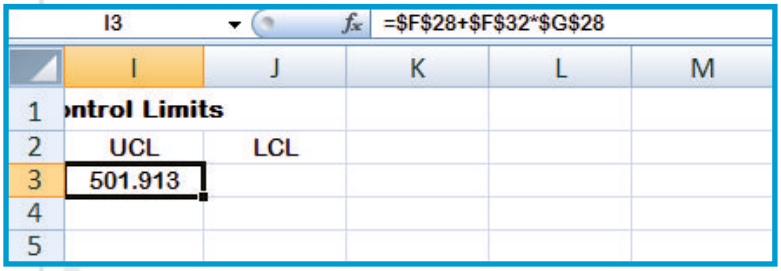
The formula with dollar sign ($) isused for an absolute references.

ii) To calculate upper control limit, the formula is $UCL = \overline{\overline{X}} + A_2\overline{R}$. Since the values of $\overline{\overline{X}}$, A_2 and $\overline{R}$ are given in Cells F28, F32 and G28, respectively (see Figs. 2.3 and 2.4), we type "=$F$28+$F$32*$G$28" in Cell I3 as shown in Fig. 2.5b.

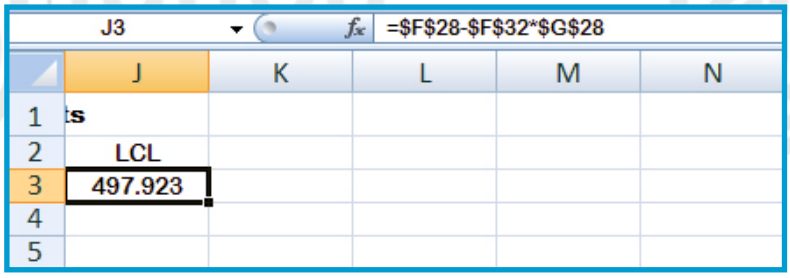
iii) Similarly, we calculate the lower control limit $LCL = \overline{\overline{X}} - A_2\overline{R}$ by typing "=$F$28-$F$32*$G$28" in Cell J3 as shown in Fig. 2.5c.



(a)



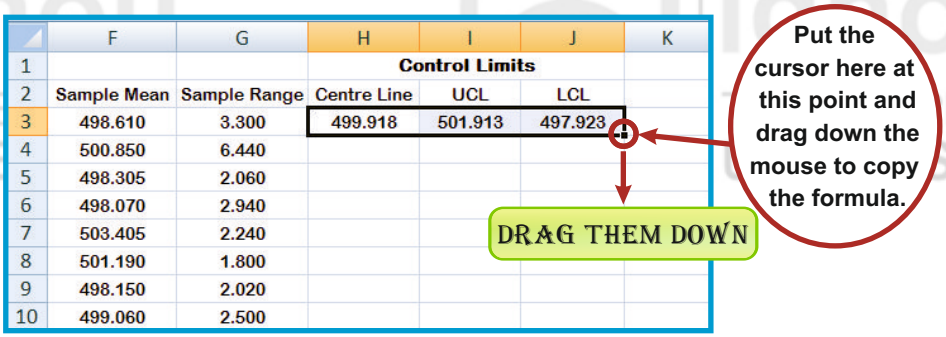
(b)



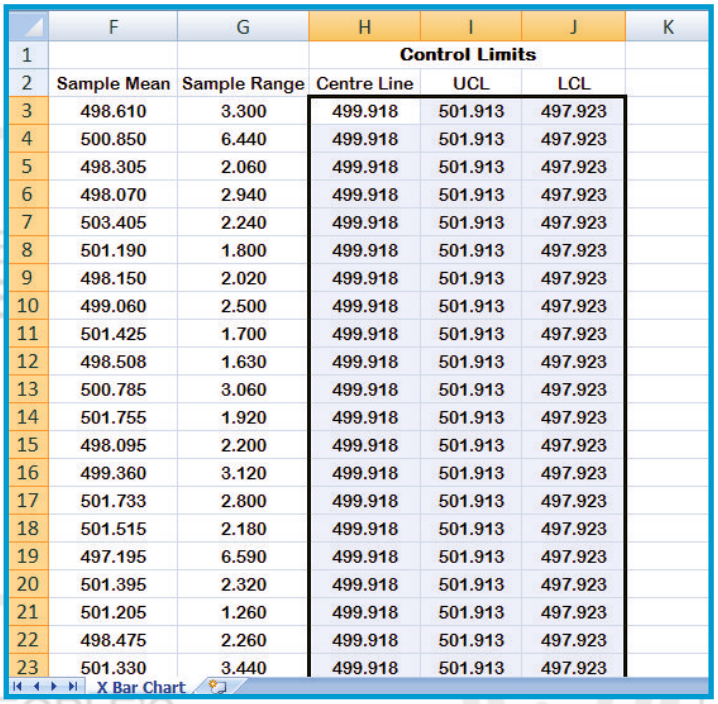
(c)

Fig. 2.5

Step 6: For plotting control limits on the chart, we first select Cells H3:J3 and drag Cells H3:J3 down up to Row 27. We get the output shown in Fig. 2.6.



	F	G	H	I	J	K
1			Control Limits			
2	Sample Mean	Sample Range	Centre Line	UCL	LCL	
3	498.610	3.300	499.918	501.913	497.923	
4	500.850	6.440				
5	498.305	2.060				
6	498.070	2.940				
7	503.405	2.240				
8	501.190	1.800				
9	498.150	2.020				
10	499.060	2.500				



	F	G	H	I	J	K
1			Control Limits			
2	Sample Mean	Sample Range	Centre Line	UCL	LCL	
3	498.610	3.300	499.918	501.913	497.923	
4	500.850	6.440	499.918	501.913	497.923	
5	498.305	2.060	499.918	501.913	497.923	
6	498.070	2.940	499.918	501.913	497.923	
7	503.405	2.240	499.918	501.913	497.923	
8	501.190	1.800	499.918	501.913	497.923	
9	498.150	2.020	499.918	501.913	497.923	
10	499.060	2.500	499.918	501.913	497.923	
11	501.425	1.700	499.918	501.913	497.923	
12	498.508	1.630	499.918	501.913	497.923	
13	500.785	3.060	499.918	501.913	497.923	
14	501.755	1.920	499.918	501.913	497.923	
15	498.095	2.200	499.918	501.913	497.923	
16	499.360	3.120	499.918	501.913	497.923	
17	501.733	2.800	499.918	501.913	497.923	
18	501.515	2.180	499.918	501.913	497.923	
19	497.195	6.590	499.918	501.913	497.923	
20	501.395	2.320	499.918	501.913	497.923	
21	501.205	1.260	499.918	501.913	497.923	
22	498.475	2.260	499.918	501.913	497.923	
23	501.330	3.440	499.918	501.913	497.923	

Fig. 2.6

Step 7: To obtain the $\bar{X}$-chart, we follow the procedure explained in Step 10 of Sec. 1.4, Lab Session 1 (see Fig. 2.7). It means that we

1. select Cells F2:F27 and H2:J27 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

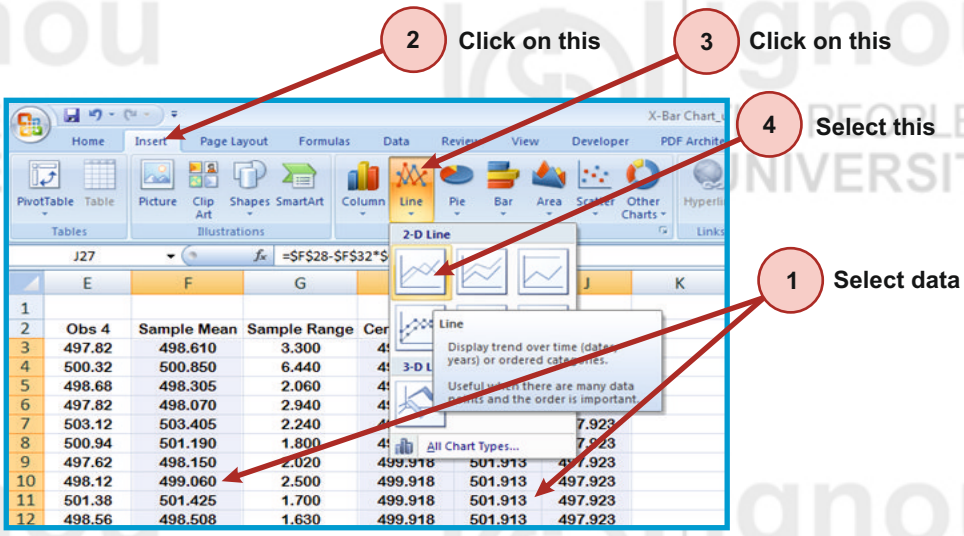


Fig. 2.7

Step 8: The values in Columns H, I and J provide the horizontal lines on the chart and represent the centre line, UCL and LCL, respectively. The values in Column F provide the averages for 25 samples on the chart. After we format the chart as explained in Step 12 of Sec. 1.4, we get the control chart shown in Fig. 2.8.

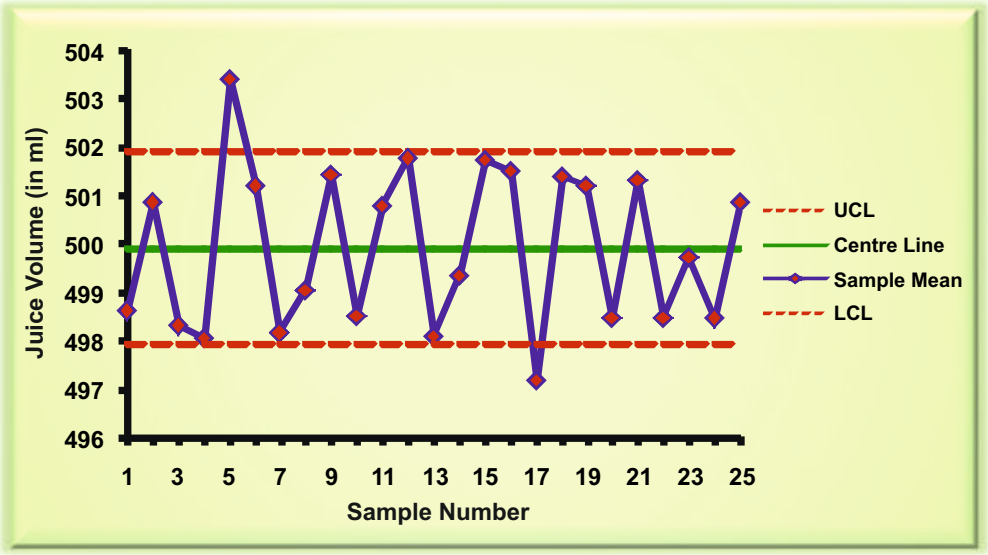


Fig. 2.8

Interpretation

Note from Fig. 2.8 that two points corresponding to Samples 5 and 17 lie outside the UCL and LCL, respectively. Therefore, this control chart indicates that the bottling process of juice manufacturing company **is not under statistical control**. Some **assignable causes** are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them.

Note: Let us compare the control limits obtained in Lab Session 1 and Lab Session 2. We have used a known value of process variability, i.e., 2.5 in Lab Session 1 and estimated the value of process variability using average sample range, i.e., 1.3288 in the Lab Session 2. We know that the control limits (UCL and LCL) depend on the process variability (σ). As σ increases, the gap between UCL and LCL also increases. Note that the estimated process variability (= 1.3288) in Lab Session 2 is smaller than the process variability (= 2.5) used in Lab Session 1. Therefore, the UCL and LCL in this case come closer to the centre line in comparison to the UCL and LCL in Lab Session 1. As a result, the 5th and 17th points lie beyond the control limits in Fig. 2.8. Recall that these points were close to the UCL and LCL, respectively, in Lab Session 1 (see Fig. 1.14).

The value of $d_2 = 2.059$ for $n = 4$ from the Appendix given at the end of this lab course.

$\therefore \hat{\sigma} = \bar{R}/d_2 = 1.3288$

2.5 REVISED $\bar{X}$-CHART

After eliminating the assignable causes from the process, we delete the out-of-control samples and calculate the revised centre line and control limits for the $\bar{X}$-chart using the remaining samples. These limits are known as the **revised control limits**.

The process of revising control limits will continue until the process comes under control.

The main steps for computing revised control limits are described below:

Step 1: For revised limits for $\bar{X}$-chart using sample range, we first calculate the new $\bar{\bar{X}}$ and new $\bar{R}$ using the following formulae:

$$\bar{\bar{X}}_{new} = \frac{\sum_{i=1}^{k} \bar{X}_i - \sum_{j=1}^{d} \bar{X}_j}{k-d} \qquad \ldots(6)$$

and $$\bar{R}_{new} = \frac{\sum_{i=1}^{k} R_i - \sum_{j=1}^{d} R_j}{k-d} \qquad \ldots(7)$$

where $\sum_{j=1}^{d} \bar{X}_j$ – sum of the averages of discarded samples,

d – number of discarded samples, and

$\sum_{j=1}^{d} R_j$ – sum of the ranges of discarded samples.

Step 2 : After determining the new $\bar{X}$ and new $\bar{R}$, we reconstruct the centre line and control limits of the chart by replacing $\bar{\bar{X}}$ by $\bar{\bar{X}}_{new}$ and $\bar{R}$ by $\bar{R}_{new}$ as given below:

- ✓ Centre line = $\bar{\bar{X}}_{new}$ ...(8)
- ✓ Upper control limit (UCL) = $\bar{\bar{X}}_{new} + A_2\bar{R}_{new}$ ...(9)
- ✓ Lower control limit (LCL) = $\bar{\bar{X}}_{new} - A_2\bar{R}_{new}$ ...(10)

Step 3: Interpretation of the $\bar{X}$-chart.

Generally, we control the process mean and process variability simultaneously. So we use both $\bar{X}$ and R-charts together.

Steps in Excel

The main steps for calculating the revised centre line and control limits for constructing the $\bar{X}$-chart in Excel 2007 using the remaining samples are as follows:

Step 1: We highlight the samples which lie outside the control limits, i.e., the 5th and 17th samples. We have shown them in light orange colour (Fig. 2.9).

	A	B	C	D	E	F	G	H	I	J
1			Juice Volume (in ml)					Control Limits		
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean	Sample Range	Centre Line	UCL	LCL
3	1	497.32	500.62	498.68	497.82	498.610	3.300	499.918	501.913	497.923
4	2	504.76	500.00	498.32	500.32	500.850	6.440	499.918	501.913	497.923
5	3	499.24	497.18	498.12	498.68	498.305	2.060	499.918	501.913	497.923
6	4	499.26	496.32	498.88	497.82	498.070	2.940	499.918	501.913	497.923
7	5	502.32	503.62	504.56	503.12	503.405	2.240	499.918	501.913	497.923
8	6	502.12	500.32	501.38	500.94	501.190	1.800	499.918	501.913	497.923
9	7	499.34	498.32	497.32	497.62	498.150	2.020	499.918	501.913	497.923
10	8	499.38	498.12	500.62	498.12	499.060	2.500	499.918	501.913	497.923
11	9	501.26	502.38	500.68	501.38	501.425	1.700	499.918	501.913	497.923
12	10	498.60	497.62	499.25	498.56	498.508	1.630	499.918	501.913	497.923
13	11	502.44	500.00	501.32	499.38	500.785	3.060	499.918	501.913	497.923
14	12	501.26	502.32	500.76	502.68	501.755	1.920	499.918	501.913	497.923
15	13	497.32	498.50	497.18	499.38	498.095	2.200	499.918	501.913	497.923
16	14	499.56	498.00	498.76	501.12	499.360	3.120	499.918	501.913	497.923
17	15	502.24	500.32	503.12	501.25	501.733	2.800	499.918	501.913	497.923
18	16	501.76	500.50	502.68	501.12	501.515	2.180	499.918	501.913	497.923
19	17	500.65	497.82	494.06	496.25	497.195	6.590	499.918	501.913	497.923

Fig. 2.9

Step 2: We put the value of d = 2 in Cell F33 in addition to the values of k, n and A_2 as shown in Fig. 2.10.

	C	D	E	F
30			k	25
31			n	4
32	A_2 Value from Table (for n = 4)			0.729
33			d	2

Fig. 2.10

Step 3: To calculate the value of $\bar{\bar{X}}_{new}$ given in equation (6), we type "=(Sum(F3:F27)-F7-F19)/(F30-F33)" in Cell F29 and then press ***Enter*** as shown in Fig. 2.11.

ROUND =(SUM(F3:F27)-F7-F19)/(F30-F33)

	D	E	F
29		Revised Average	=(SUM(F3:F27)-F7-F19)/(F30-F33)
30		k	25
31		n	4
32	A_2 Value from Table (for n=4)		0.729
33		d	2
34			

F29 =(SUM(F3:F27)-F7-F19)/(F30-F33)

	D	E	F	G
29		Revised Average	499.885	
30		k	25	

Fig. 2.11

Step 4: Similarly, for calculating $\bar{R}_{new}$, we use equation (7) and type "=(Sum(G3:G27)-G7-G19)/(F30-F33)" in Cell G29 as shown in Fig. 2.12. We get the result when we press ***Enter***.

AVERAGE =(SUM(G3:G27)-G7-G19)/(F30-F33)

	D	E	F	G	H
29		Revised Average	499.885	=(SUM(G3:G27)-G7-G19)/(F30-F33)	
30		k	25		
31		n	4		
32	A_2 Value from Table (for n = 4)		0.729		
33		d	2		
34					

G29 =(SUM(G3:G27)-G7-G19)/(F30-F33)

	E	F	G	H
29	Revised Average	499.885	2.590	
30	k	25		

Fig. 2.12

Step 5: We compute the revised centre line and control limits in the same way as explained in Step 5 in Sec. 2.4. Here we use Columns K, L and M for putting the values of the revised centre line, upper and lower control limits, respectively.

i) The formula for revised centre line is (CL) = $\bar{\bar{X}}_{new}$. Since the Value of $\bar{\bar{X}}_{new}$ is given in F29 (see Fig. 2.12), we type "=$F$29" in Cell K3 to obtain the centre line as shown in Fig. 2.13a.

ii) To calculate the upper control limit, we use equation (9). The values of $\bar{\bar{X}}_{new}$, A_2 and $\bar{R}_{new}$ are given in F29, F32 and G29, respectively (see Fig. 2.12). So we type "=$F$29+$F$32*$G$29" in Cell L3 as shown in Fig. 2.13b.

iii) Similarly, we calculate the lower control limit by typing "=$F$29-$F$32*$G$29" in Cell M3 as shown in Fig. 2.13c.

(a)

K3 =$F$29

	K	L	M	N	O
1	Revised Control Limits				
2	Centre Line	UCL	LCL		
3	499.885				
4					

(b)

L3 =$F$29+$F$32*$G$29

	L	M	N	O	P
1	d Control Limits				
2	UCL	LCL			
3	501.773				
4					

(c)

M3 =$F$29-$F$32*$G$29

	M	N	O	P	Q
1	Limits				
2	LCL				
3	497.997				
4					

Fig. 2.13

Step 6: For plotting the revised control limits on the chart using Excel, we select Cells K3:M3 and drag them down up to Row 27 as shown in Fig. 2.14.

	H	I	J	K	L	M	N
1	Control Limits			Revised Control Limits			
2	Centre Line	UCL	LCL	Centre Line	UCL	LCL	
3	499.918	501.913	497.923	499.885	501.773	497.997	
4	499.918	501.913	497.923				
5	499.918	501.913	497.923				
6	499.918	501.913	497.923				
7	499.918	501.913	497.923				
8	499.918	501.913	497.923				

DRAG THEM DOWN

	H	I	J	K	L	M	N
1	Control Limits			Revised Control Limits			
2	Centre Line	UCL	LCL	Centre Line	UCL	LCL	
3	499.918	501.913	497.923	499.885	501.773	497.997	
4	499.918	501.913	497.923	499.885	501.773	497.997	
5	499.918	501.913	497.923	499.885	501.773	497.997	
6	499.918	501.913	497.923	499.885	501.773	497.997	
7	499.918	501.913	497.923	499.885	501.773	497.997	
8	499.918	501.913	497.923	499.885	501.773	497.997	
9	499.918	501.913	497.923	499.885	501.773	497.997	
10	499.918	501.913	497.923	499.885	501.773	497.997	
11	499.918	501.913	497.923	499.885	501.773	497.997	
12	499.918	501.913	497.923	499.885	501.773	497.997	
13	499.918	501.913	497.923	499.885	501.773	497.997	
14	499.918	501.913	497.923	499.885	501.773	497.997	
15	499.918	501.913	497.923	499.885	501.773	497.997	

Fig. 2.14

Step 7: For plotting the revised $\bar{X}$-chart, we first select Cells F2:F6, F8:F18, F20:F27, K2:M6, K8:M18 and K20:M27 by holding *Ctrl* key as shown in Fig. 2.15. Then we follow Step 7 of Sec. 2.4.

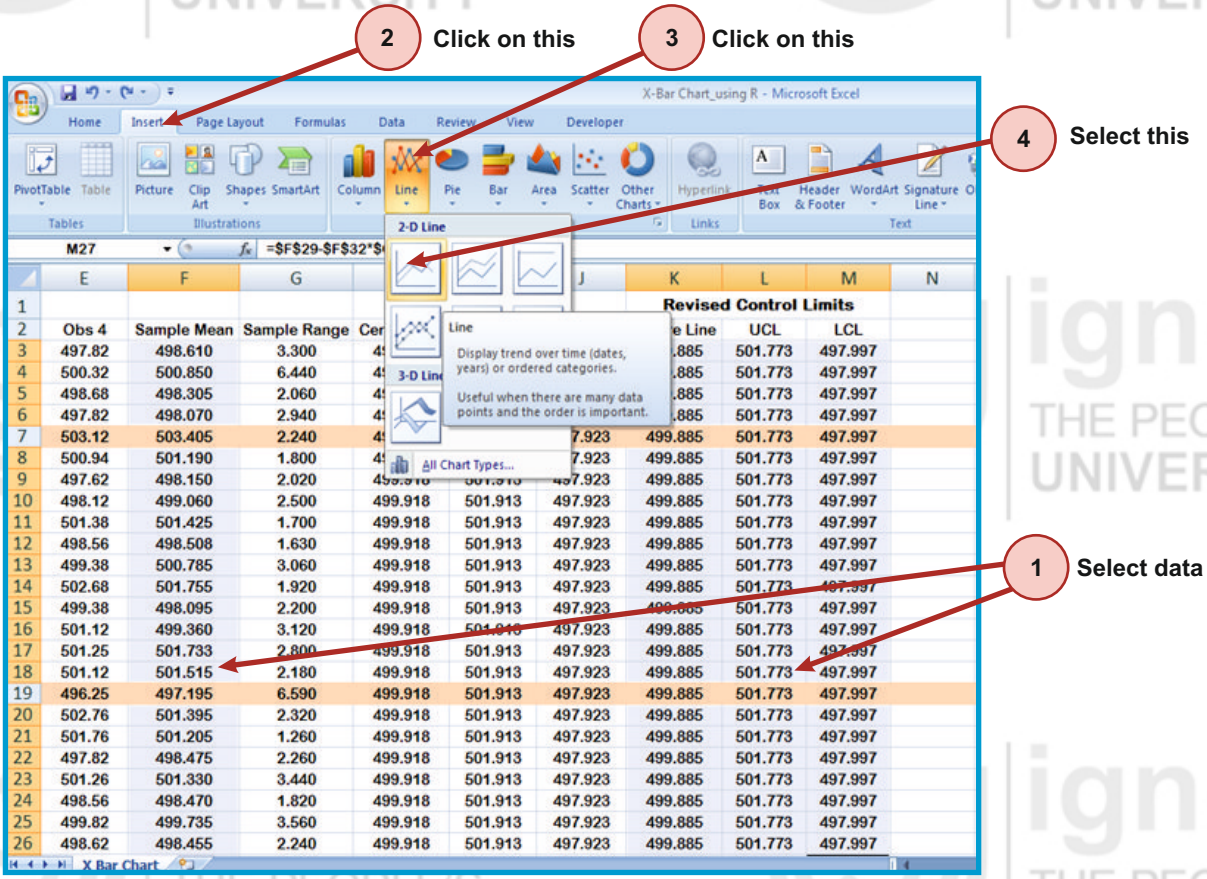


Fig. 2.15

Step 8: The values in Columns K, L and M provide the horizontal lines on the chart representing the revised centre line, UCL and LCL, respectively. The values in Column F provide the averages for 25 samples on the chart. The output is shown in Fig. 2.16.

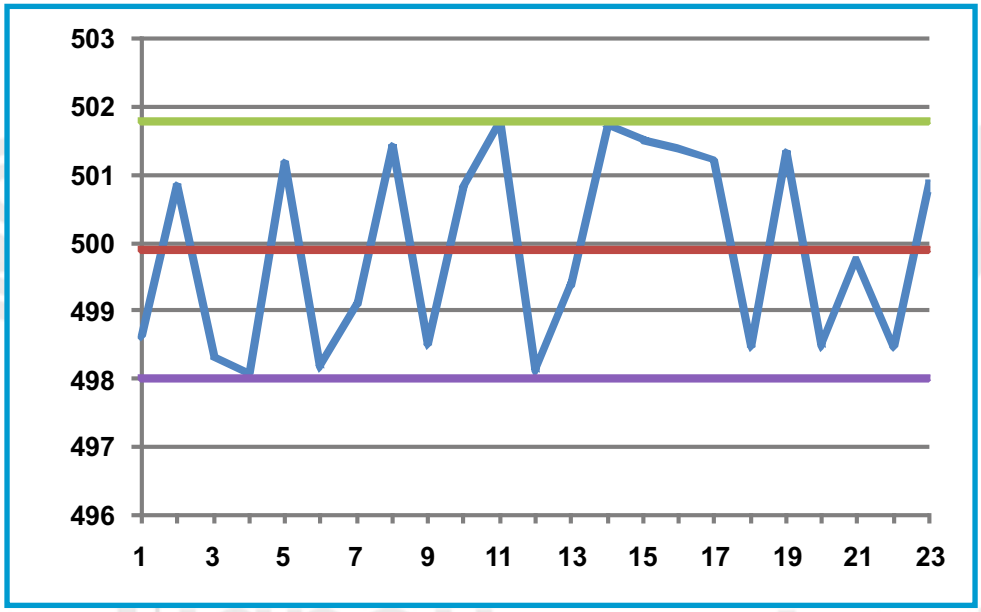


Fig. 2.16

Step 9: The horizontal axis shown in Fig. 2.16 also includes the 5th and 17th samples. To eliminate these points from the horizontal axis, we first click on the chart shown in Fig. 2.16. Then we

1. click on the ***Design*** tab under ***Chart Tools*** as shown in Fig. 2.17a,

2. choose ***Select Data*** option as shown in Fig. 2.17a. It opens a new dialog box as shown in Fig. 2.17b,

3. click on ***Edit*** under ***Horizontal (Category) Axis Labels*** as shown in Fig. 2.17b. It opens a new dialog box shown in Fig. 2.17c, and

4. select Cells A3:A6, A8:A18 and A20:A27, by holding the ***Ctrl*** key as shown in Fig. 2.17d and then press ***Enter***.

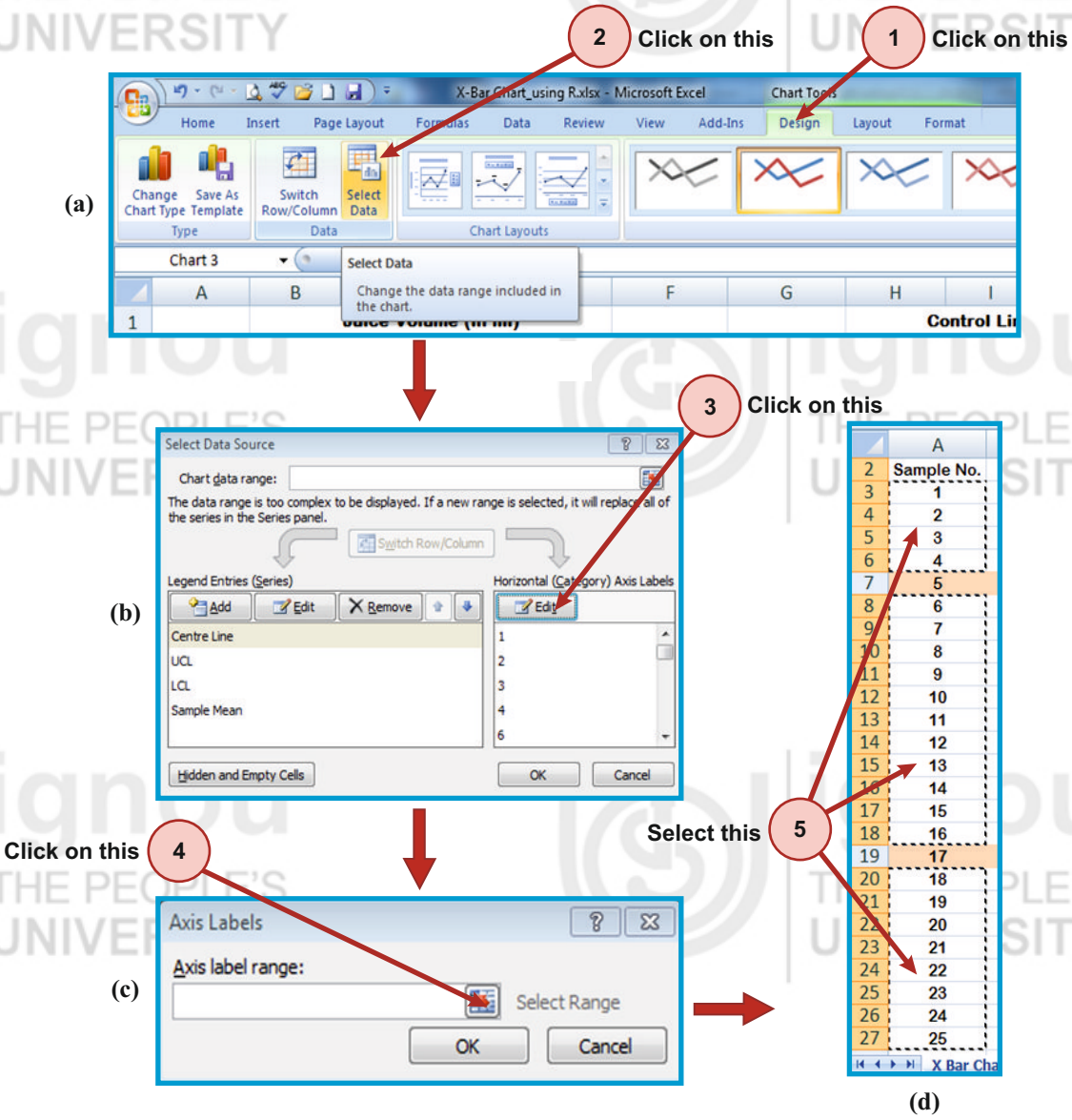


Fig. 2.17

Step 10: After we format as explained in Sec. 1.4, we get the control chart shown in Fig. 2.18. The resulting control chart is called the **revised** $\overline{X}$-chart.

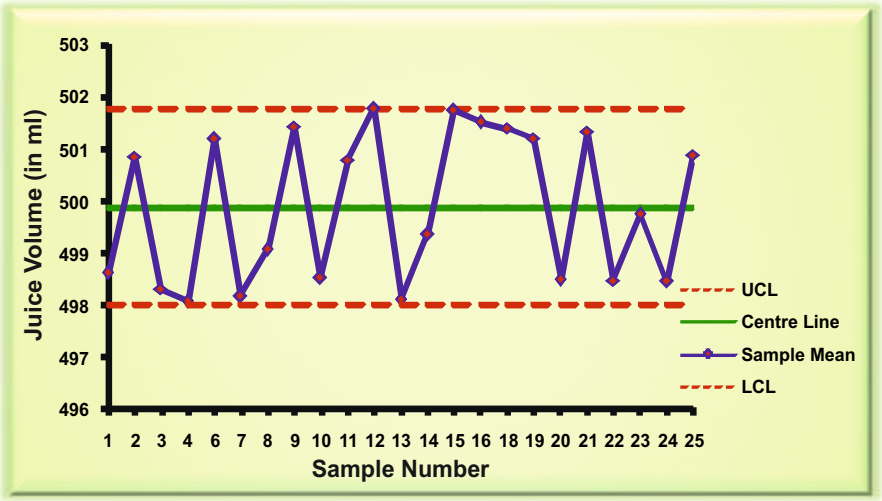


Fig. 2.18

Interpretation

The revised $\bar{X}$-chart shown in Fig. 2.18 indicates that all points lie within the control lines. We can conclude that the process is under statistical control with respect to mean.

Activity 1

You can also determine the revised control limits using another approach. For this purpose, follow the steps given below:

- Select Cells A1:E6, A8:E18 and A20:E27.
- Choose Cell A35 or any other cell and paste the values. You can also use a separate Excel sheet where you can paste these values.
- Repeat all steps given in Sec. 2.4.

It will give you the same results as you have obtained in Sec. 2.5.

You can now solve the following exercises to check whether you have learnt how to develop the control limits when process variability is unknown.

Activity 2

Construct the control charts for mean when process variability is unknown and estimated by sample range with the help of MS Excel 2007 and interpret the results for

A1) Example 3 given in Unit 2 of MSTE-001.

A2) Exercise E6 given in Unit 2 of MSTE-001.

Match the results with the manual calculation done in Unit 2 of MSTE-001.

Continuous Assessment 2

Consider the data given in Continuous Assessment 1 and develop $\overline{X}$ control limits to check whether the process of production of bulb is under control or out-of-control when process variability is unknown. If it is out-of-control, compute the revised control limits.

Home Work: Do It Yourself

1) Follow the steps explained in Sec. 2.4 and 2.5 to construct the control chart for the data of Table 1 in Lab Session 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 2" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute in your continuous assessment in the Laboratory.

Control Chart for Mean using Standard Deviation

SESSION 3

Structure

3.1 Introduction

Objectives

3.2 Problem Description

3.3 Procedure for the Construction of $\bar{X}$- Chart

3.4 Steps Involved in the Construction of $\bar{X}$- Chart in Excel 2007

3.5 Revised $\bar{X}$- Chart

3.6 Further Revision of $\bar{X}$- Chart

3.1 INTRODUCTION

In Lab Session 2, you have learnt how to construct the control chart for mean in MS Excel 2007 when process variability is unknown. We have used sample range to estimate the process standard deviation (σ) in Lab Session 2. You have studied in Unit 2 of MSTE-001 (Industrial Statistics-I) that sample standard deviation can also be used for estimating the process standard deviation.

In this lab session, you will learn about the construction of control chart for mean in MS Excel 2007 **when process variability is unknown**. We shall consider the **sample standard deviation** as an estimator of process standard deviation to develop the control chart for mean using a specific problem.

Prerequisite

- Lab Sessions 1, 3 and 6 of MSTL-001 (Basic Statistics Lab).
- Lab Sessions 1 and 2 of MSTL-002 (Industrial Statistics Lab).
- Unit 2 of MSTE-001 (Industrial Statistics-I).

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for control chart for mean using sample standard deviation to estimate the unknown process variability;
- construct the control chart for mean using sample standard deviation;
- obtain the revised control limits for control charts for mean using standard deviation; and
- interpret the control chart.

3.2 PROBLEM DESCRIPTION

Once again, we consider the data given in Lab Session 1. Suppose the quality control inspector of the juice manufacturing company wishes to develop $\bar{X}$-chart using sample standard deviation to estimate the process standard deviation. He/she also wants to check whether the process of bottling is under control or out-of-control. If the process is out-of-control, he/she would like to compute the revised control limits.

Therefore, the problem for this session is to construct the control chart for mean using sample standard deviation to estimate the process variability.

3.3 PROCEDURE FOR THE CONSTRUCTION OF $\bar{X}$-CHART

You have already learnt all the formulae for calculating control limits for control chart for mean when process variability is unknown and estimated by sample standard deviation in Sec. 2.4 of Unit 2 of MSTE-001.

The main steps involved in the construction of $\bar{X}$-chart using sample standard deviation to estimate the process standard deviation are as follows:

Steps 1-3: The first three steps are the same as explained in Sec. 1.3 of Lab Session 1.

Step 4: We calculate the sample standard deviation for each sample (subgroup), say, $S_1, S_2, \ldots, S_k$ where S_i is the sample standard deviation for i^{th} sample given by

$$S_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(X_{ij} - \bar{X}_i\right)^2} \qquad \ldots(1)$$

Step 5: We find the mean of all sample standard deviations given by

$$\bar{S} = \frac{1}{k}\left(S_1 + S_2 \ldots, S_k\right) = \frac{1}{k}\sum_{i=1}^{k} S_i \qquad \ldots(2)$$

Step 6: The control limits for $\bar{X}$-chart when σ is estimated by $\bar{S}/c_4$ are given by

- ✓ Centre line (CL) $= \bar{\bar{X}}$ ...(3)
- ✓ Upper control limit (UCL) $= \bar{\bar{X}} + A_3\bar{S}$ ...(4)
- ✓ Lower control limit (LCL) $= \bar{\bar{X}} - A_3\bar{S}$ ...(5)

where $A_3 = \dfrac{3}{\sqrt{n}\, c_4}$ is a constant and depends on the size of the sample. It is tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 7: Interpretation of the $\bar{X}$-chart.

3.4 STEPS INVOLVED IN THE CONSTRUCTION OF $\bar{X}$-CHART IN EXCEL 2007

In order to calculate the control limits of $\bar{X}$-chart and to plot the control chart when σ is estimated by $\bar{S}/c_4$ in Excel 2007 for the given data, we follow the steps given below:

Step 1: We repeat the Steps 1-5 of Sec. 1.4 of Lab Session 1 to compute the sample mean of the 25 samples given in Table 1 of Lab Session 1. To compute the grand sample mean ($\bar{\bar{X}}$) of these samples, we repeat Step 6 of Sec. 1.4. The output is given in Fig. 3.1.

	A	B	C	D	E	F
1			Juice Volume (in ml)			
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean
3	1	497.32	500.62	498.68	497.82	498.610
4	2	504.76	500.00	498.32	500.32	500.850
5	3	499.24	497.18	498.12	498.68	498.305
6	4	499.26	496.32	498.88	497.82	498.070
7	5	502.32	503.62	504.56	503.12	503.405
8	6	502.12	500.32	501.38	500.94	501.190
9	7	499.34	498.32	497.32	497.62	498.150
10	8	499.38	498.12	500.62	498.12	499.060
11	9	501.26	502.38	500.68	501.38	501.425
12	10	498.60	497.62	499.25	498.56	498.508
13	11	502.44	500.00	501.32	499.38	500.785
14	12	501.26	502.32	500.76	502.68	501.755
15	13	497.32	498.50	497.18	499.38	498.095
16	14	499.56	498.00	498.76	501.12	499.360
17	15	502.24	500.32	503.12	501.25	501.733
18	16	501.76	500.50	502.68	501.12	501.515
19	17	500.65	497.82	494.06	496.25	497.195
20	18	501.12	501.26	500.44	502.76	501.395
21	19	501.00	500.50	501.56	501.76	501.205
22	20	497.50	498.82	499.76	497.82	498.475
23	21	503.44	500.62	500.00	501.26	501.330
24	22	499.38	498.38	497.56	498.56	498.470
25	23	501.56	499.56	498.00	499.82	499.735

X Bar Chart

F28 f_x =AVERAGE(F3:F27)

	A	B	C	D	E	F
25	23	501.56	499.56	498.00	499.82	499.735
26	24	498.32	497.32	499.56	498.62	498.455
27	25	499.50	501.12	502.50	500.38	500.875
28					Average	499.918
29						

Fig. 3.1: Partial screenshot of the spreadsheet for the given data.

Step 2: We calculate standard deviation for each sample. For this, we type "=Stdev(B3:E3)" in Cell G3 as shown in Fig. 3.2 and click on the ***Enter*** key. We get the value of the standard deviation in the Cell G3.

AVERAGE =STDEV(B3:E3)

	A	B	C	D	E	F	G	H
1		Juice Volume (in ml)						
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean	SD	
3	1	497.32	500.62	498.68	497.82	498.610	=STDEV(B3:E3)	
4	2	504.76	500.00	498.32	500.32	500.850		

ENTER

	A	B	C	D	E	F	G	H
1		Juice Volume (in ml)						
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean	SD	
3	1	497.32	500.62	498.68	497.82	498.610	1.453	
4	2	504.76	500.00	498.32	500.32	500.850		
5	3	499.24	497.18	498.12	498.68	498.305		

Fig. 3.2

Step 3: We drag down Cell G3 up to Cell G27 to get the standard deviations for all 25 samples (see Fig. 3.3).

	A	B	C	D	E	F	G
1		Juice Volume (in ml)					
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean	SD
3	1	497.32	500.62	498.68	497.82	498.610	1.453
4	2	504.76	500.00	498.32	500.32	500.850	2.750
5	3	499.24	497.18	498.12	498.68	498.305	0.878
6	4	499.26	496.32	498.88	497.82	498.070	1.316
7	5	502.32	503.62	504.56	503.12	503.405	0.938
8	6	502.12	500.32	501.38	500.94	501.190	0.757
9	7	499.34	498.32	497.32	497.62	498.150	0.897
10	8	499.38	498.12	500.62	498.12	499.060	1.198
11	9	501.26	502.38	500.68	501.38	501.425	0.706
12	10	498.60	497.62	499.25	498.56	498.508	0.671
13	11	502.44	500.00	501.32	499.38	500.785	1.368
14	12	501.26	502.32	500.76	502.68	501.755	0.896
15	13	497.32	498.50	497.18	499.38	498.095	1.041
16	14	499.56	498.00	498.76	501.12	499.360	1.335
17	15	502.24	500.32	503.12	501.25	501.733	1.213
18	16	501.76	500.50	502.68	501.12	501.515	0.932
19	17	500.65	497.82	494.06	496.25	497.195	2.772
20	18	501.12	501.26	500.44	502.76	501.395	0.978
21	19	501.00	500.50	501.56	501.76	501.205	0.570
22	20	497.50	498.82	499.76	497.82	498.475	1.025
23	21	503.44	500.62	500.00	501.26	501.330	1.498
24	22	499.38	498.38	497.56	498.56	498.470	0.747
25	23	501.56	499.56	498.00	499.82	499.735	1.458

X Bar Chart

Fig. 3.3

Step 4: To obtain the average standard deviation ($\bar{S}$), we type "=Average(G3:G27)" in Cell G28 as shown in Fig. 3.4.

G28 =AVERAGE(G3:G27)

	A	B	C	D	E	F	G
25	23	501.56	499.56	498.00	499.82	499.735	1.458
26	24	498.32	497.32	499.56	498.62	498.455	0.923
27	25	499.50	501.12	502.50	500.38	500.875	1.270
28					Average	499.918	1.184
29							

Fig. 3.4

Step 5: We type the values of k and n in Cells F31 and F32, respectively, as shown in Fig. 3.5. Note that we have also typed the value of A_3 for n = 4 in Cell F33 from the Appendix given at the end of this lab course.

	C	D	E	F
31			k	25
32			n	4
33		A_3 Value from Table (for n=4)		1.628

Fig. 3.5

Step 6: We recall the method for computing centre line and both control limits described in Step 5, Sec. 2.4 of Lab Session 2. Here we use Columns H, I and J for putting the values of the centre line, upper and lower control limits, respectively. We compute the centre line, upper and lower control limits as follows:

i) The centre line is $CL = \overline{\overline{X}}$ and $\overline{\overline{X}}$ is given in Cell F28 (see Fig. 3.4). So in Excel, we type "=$F$28" in Cell H3 to find the centre line as shown in Fig. 3.6a.

ii) The upper control limit is $UCL = \overline{\overline{X}} + A_3\overline{S}$. The values of $\overline{\overline{X}}$, A_3 and $\overline{S}$ are given in Cells F28, F33 and G28, respectively (see Figs. 3.4 and 3.5). So we type "=$F$28+$F$33*$G$28" in Cell I3 as shown in Fig. 3.6b.

iii) Similarly, we calculate the lower control limit $LCL = \overline{\overline{X}} - A_3\overline{S}$ by typing "=$F$28-$F$33*$G$28" in Cell J3 as shown in Fig. 3.6c.

The formula with dollar sign ($) is used for an absolute reference.

H3 =$F$28

	H	I	J	K
1		Control Limits		
2	Centre Line	UCL	LCL	
3	499.918			
4				

(a)

I3 =$F$28+$F$33*$G$28

	I	J	K	L
1	Control Limits			
2	UCL	LCL		
3	501.845			
4				

(b)

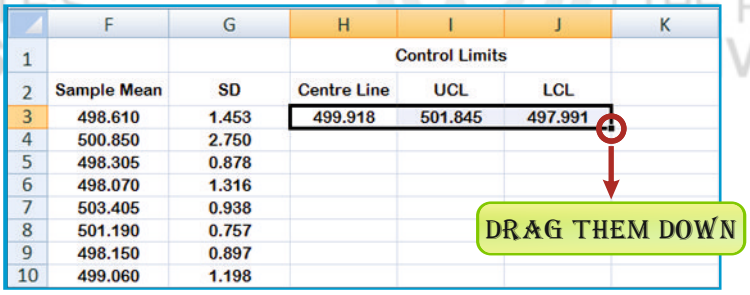
J3 =$F$28-$F$33*$G$28

	J	K	L	M
1				
2	LCL			
3	497.991			
4				

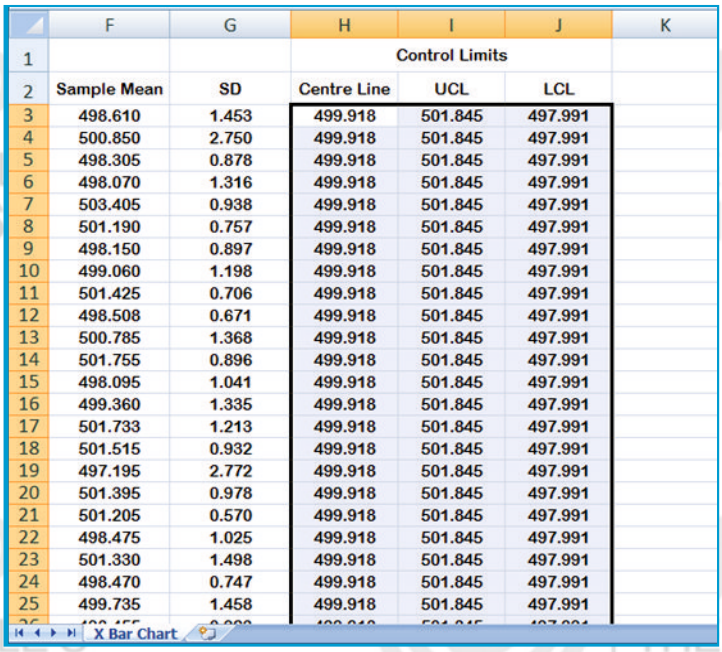
(c)

Fig. 3.6

Step 7: For plotting the control limits using Excel, we first select Cells H3:J3 and drag them down up to Row 27. We get the output shown in Fig. 3.7.



	F	G	H	I	J	K
1			Control Limits			
2	Sample Mean	SD	Centre Line	UCL	LCL	
3	498.610	1.453	499.918	501.845	497.991	
4	500.850	2.750				
5	498.305	0.878				
6	498.070	1.316				
7	503.405	0.938				
8	501.190	0.757				
9	498.150	0.897				
10	499.060	1.198				



	F	G	H	I	J	K
1			Control Limits			
2	Sample Mean	SD	Centre Line	UCL	LCL	
3	498.610	1.453	499.918	501.845	497.991	
4	500.850	2.750	499.918	501.845	497.991	
5	498.305	0.878	499.918	501.845	497.991	
6	498.070	1.316	499.918	501.845	497.991	
7	503.405	0.938	499.918	501.845	497.991	
8	501.190	0.757	499.918	501.845	497.991	
9	498.150	0.897	499.918	501.845	497.991	
10	499.060	1.198	499.918	501.845	497.991	
11	501.425	0.706	499.918	501.845	497.991	
12	498.508	0.671	499.918	501.845	497.991	
13	500.785	1.368	499.918	501.845	497.991	
14	501.755	0.896	499.918	501.845	497.991	
15	498.095	1.041	499.918	501.845	497.991	
16	499.360	1.335	499.918	501.845	497.991	
17	501.733	1.213	499.918	501.845	497.991	
18	501.515	0.932	499.918	501.845	497.991	
19	497.195	2.772	499.918	501.845	497.991	
20	501.395	0.978	499.918	501.845	497.991	
21	501.205	0.570	499.918	501.845	497.991	
22	498.475	1.025	499.918	501.845	497.991	
23	501.330	1.498	499.918	501.845	497.991	
24	498.470	0.747	499.918	501.845	497.991	
25	499.735	1.458	499.918	501.845	497.991	

Fig. 3.7

Step 8: To obtain the $\bar{X}$-chart, we follow the procedure explained in Step 10 of Sec. 1.4, Lab Session 1 (see Fig. 3.8). It means that we

1. select Cells F2:F27 and H2:J27 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

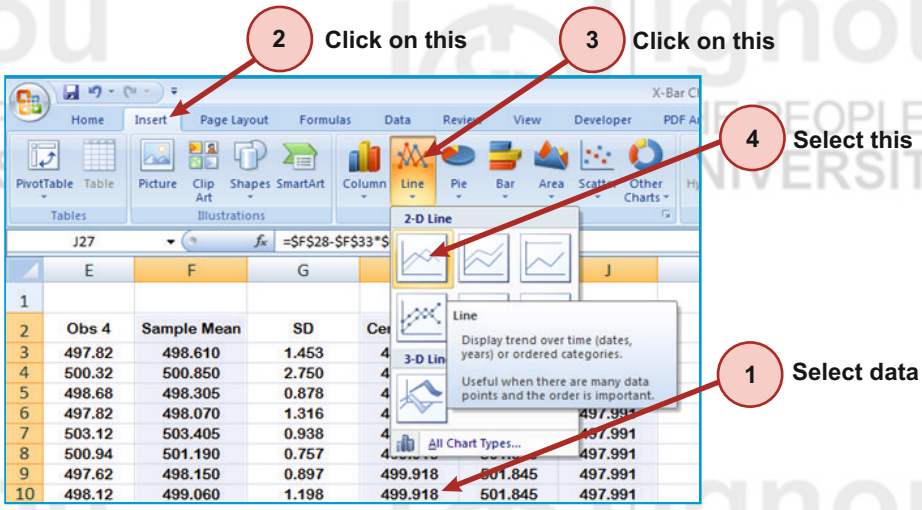


Fig. 3.8

Step 9: We find that the values in Columns H, I and J provide the horizontal lines on the chart representing the centre line, UCL and LCL, respectively. The values in Column F represent the averages for 25 samples We format the chart in the way explained in Sec. 1.4 of Lab Session 1 and obtain the control chart shown in Fig. 3.9.

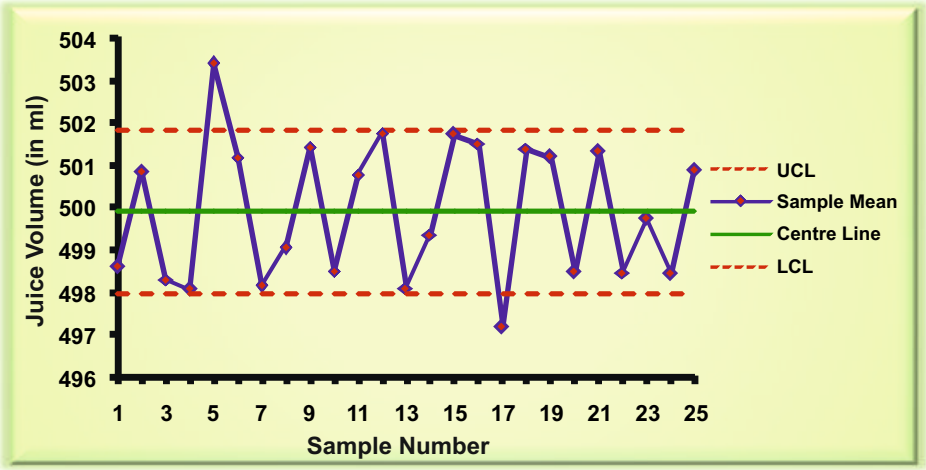


Fig. 3.9

Interpretation

Fig. 3.9 reveals that two points corresponding to Samples 5 and 17 lie outside the UCL and LCL, respectively. So the control chart indicates that the process is **not under statistical control**. Some **assignable causes** are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them.

3.5 REVISED $\bar{X}$- CHART

After eliminating the assignable causes from the process, we delete the out-of-control samples and calculate the revised centre line and control limits for the $\bar{X}$-chart using the remaining samples. These limits are known as the **revised control limits** which can be computed from the following equations:

Step 1: For revised limits for $\bar{X}$-chart using sample standard deviation, we first calculate the new $\bar{\bar{X}}$ and new $\bar{S}$ as follows:

$$\bar{\bar{X}}_{new} = \frac{\sum_{i=1}^{k} \bar{X}_i - \sum_{j=1}^{d} \bar{X}_j}{k-d} \qquad \ldots(6)$$

and $$\bar{S}_{new} = \frac{\sum_{i=1}^{k} S_i - \sum_{j=1}^{d} S_j}{k-d} \qquad \ldots(7)$$

where $\sum_{j=1}^{d} \bar{X}_j$ – sum of the averages of discarded samples,

d – number of discarded samples, and

$\sum_{j=1}^{d} S_j$ – sum of the standard deviations of discarded samples.

Step 2: After determining the new $\bar{X}$ and new $\bar{S}$, we reconstruct the centre line, upper and lower control limits by replacing $\bar{\bar{X}}$ by $\bar{\bar{X}}_{new}$ and $\bar{S}$ by $\bar{S}_{new}$ as follows:

- ✓ Centre line = $\bar{S}_{new}$ ...(8)
- ✓ Upper control limit (UCL) = $\bar{\bar{X}}_{new} + A_3\bar{S}_{new}$ ...(9)
- ✓ Lower control limit (LCL) = $\bar{\bar{X}}_{new} - A_3\bar{S}_{new}$ ...(10)

Step 3: Interpretation of the $\bar{X}$-chart.

Steps in Excel

The main steps for calculating the revised centre line and control limits for the $\bar{X}$-chart in Excel 2007 using the remaining samples are as follows:

Step 1: We highlight the samples outside the control limits, i.e., the 5th and 17th samples, with light orange colour as shown in Fig. 3.10.

	A	B	C	D	E	F	G	H	I	J
1		Juice Volume (in ml)						Control Limits		
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean	SD	Centre Line	UCL	LCL
3	1	497.32	500.62	498.68	497.82	498.610	1.453	499.918	501.845	497.991
4	2	504.76	500.00	498.32	500.32	500.850	2.750	499.918	501.845	497.991
5	3	499.24	497.18	498.12	498.68	498.305	0.878	499.918	501.845	497.991
6	4	499.26	496.32	498.88	497.82	498.070	1.316	499.918	501.845	497.991
7	5	502.32	503.62	504.56	503.12	503.405	0.938	499.918	501.845	497.991
8	6	502.12	500.32	501.38	500.94	501.190	0.757	499.918	501.845	497.991
9	7	499.34	498.32	497.32	497.62	498.150	0.897	499.918	501.845	497.991
10	8	499.38	498.12	500.62	498.12	499.060	1.198	499.918	501.845	497.991
11	9	501.26	502.38	500.68	501.38	501.425	0.706	499.918	501.845	497.991
12	10	498.60	497.62	499.25	498.56	498.508	0.671	499.918	501.845	497.991
13	11	502.44	500.00	501.32	499.38	500.785	1.368	499.918	501.845	497.991
14	12	501.26	502.32	500.76	502.68	501.755	0.896	499.918	501.845	497.991
15	13	497.32	498.50	497.18	499.38	498.095	1.041	499.918	501.845	497.991
16	14	499.56	498.00	498.76	501.12	499.360	1.335	499.918	501.845	497.991
17	15	502.24	500.32	503.12	501.25	501.733	1.213	499.918	501.845	497.991
18	16	501.76	500.50	502.68	501.12	501.515	0.932	499.918	501.845	497.991
19	17	500.65	497.82	494.06	496.25	497.195	2.772	499.918	501.845	497.991
20	18	501.12	501.26	500.44	502.76	501.395	0.978	499.918	501.845	497.991
21	19	501.00	500.50	501.56	501.76	501.205	0.570	499.918	501.845	497.991
22	20	497.50	498.82	499.76	497.82	498.475	1.025	499.918	501.845	497.991

Fig. 3.10

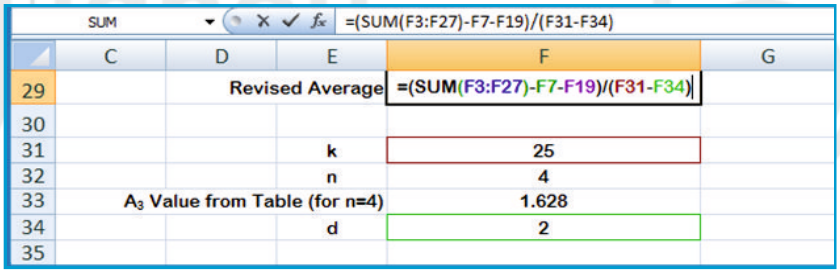
Step 2: We put the value of d = 2 in Cell F34 in addition to the values of k, n and A_3 as shown in Fig. 3.11.

The process of revising control limits will continue until the process comes under control.

	C	D	E	F
31			k	25
32			n	4
33		A_3 Value from Table (for n=4)		1.628
34			d	2

Fig. 3.11

Step 3: In Cell F29, we type "=(Sum(F3:F27)-F7-F19)/(F31-F34)" to get $\bar{\bar{X}}_{new}$ using equation (6) and then press ***Enter*** as shown in Fig. 3.12. We get the value of the revised average.



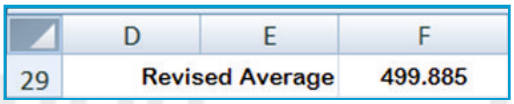
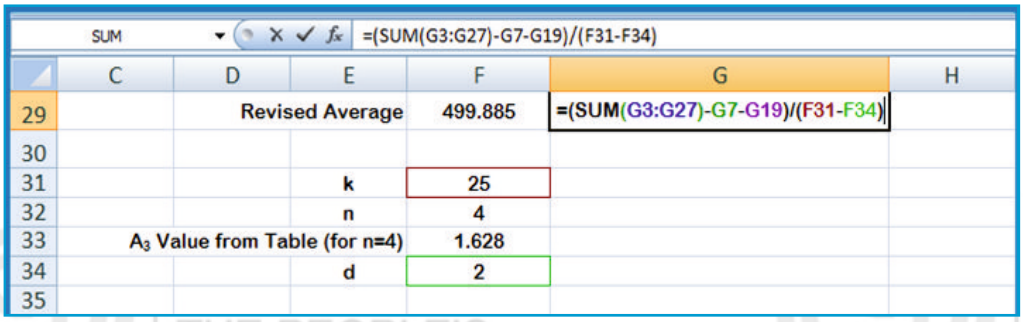


Fig. 3.12

Step 4: Similarly, for calculating $\bar{S}_{new}$, we use equation (7) and type "=(Sum(G3:G27)-G7-G19)/(F31-F34)" in Cell G29. To get the result, we press ***Enter*** as shown in Fig. 3.13.



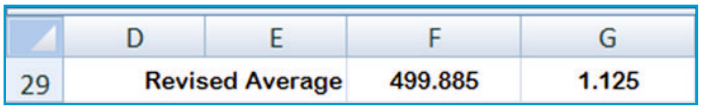


Fig. 3.13

Step 5: We follow Step 5 of Sec. 2.5 and use Columns K, L and M for putting the values of revised centre line, upper and lower control limits, respectively. We compute the centre line, upper and lower control limits as follows:

i) The revised centre line is given by (CL) = $\bar{\bar{X}}_{new}$ and $\bar{\bar{X}}_{new}$ is given in F29 (see Fig. 3.13). So in Excel, we type "=$F$29" in Cell K3 to find the centre line as shown in Fig. 3.14a.

ii) The upper control limit is given by $UCL = \bar{\bar{X}}_{new} + A_3\bar{S}_{new}$ and $\bar{\bar{X}}$, A_3 and $\bar{S}_{new}$ are given in Cells F29, F33 and G29, respectively (see Fig. 3.13). So we type "=$F$29+$F$33*$G$29" in Cell L3 as shown in Fig. 3.14b.

iii) Similarly, we calculate the lower control limit $LCL = \bar{\bar{X}}_{new} - A_3\bar{S}_{new}$ by typing "=$F$29-$F$33*$G$29" in Cell M3 as shown in Fig. 3.14c.

The formula with dollar sign ($) is used for an absolute reference.

(a)

K3 =$F$29

	K	L	M	N
1	Revised Control Limits			
2	Centre Line	UCL	LCL	
3	499.885			
4				

(b)

L3 =$F$29+$F$33*$G$29

	L	M	N	O
1	sed Control Limits			
2	UCL	LCL		
3	501.717			
4				

(c)

M3 =$F$29-$F$33*$G$29

	M	N	O	P
1	imits			
2	LCL			
3	498.053			
4				

Fig. 3.14

Step 6: For plotting control limits on the chart using Excel, we first select Cells K3:M3 and drag them down up to Row 27 as shown in Fig. 3.15.

	H	I	J	K	L	M	N
1	Control Limits			Revised Control Limits			
2	Centre Line	UCL	LCL	Centre Line	UCL	LCL	
3	499.918	501.845	497.991	499.885	501.717	498.053	
4	499.918	501.845	497.991				
5	499.918	501.845	497.991				
6	499.918	501.845	497.991				
7	499.918	501.845	497.991				
8	499.918	501.845	497.991				

DRAG THEM DOWN

	H	I	J	K	L	M	N
1	Control Limits			Revised Control Limits			
2	Centre Line	UCL	LCL	Centre Line	UCL	LCL	
3	499.918	501.845	497.991	499.885	501.717	498.053	
4	499.918	501.845	497.991	499.885	501.717	498.053	
5	499.918	501.845	497.991	499.885	501.717	498.053	
6	499.918	501.845	497.991	499.885	501.717	498.053	
7	499.918	501.845	497.991	499.885	501.717	498.053	
8	499.918	501.845	497.991	499.885	501.717	498.053	
9	499.918	501.845	497.991	499.885	501.717	498.053	
10	499.918	501.845	497.991	499.885	501.717	498.053	
11	499.918	501.845	497.991	499.885	501.717	498.053	
12	499.918	501.845	497.991	499.885	501.717	498.053	
13	499.918	501.845	497.991	499.885	501.717	498.053	
14	499.918	501.845	497.991	499.885	501.717	498.053	
15	499.918	501.845	497.991	499.885	501.717	498.053	
16	499.918	501.845	497.991	499.885	501.717	498.053	
17	499.918	501.845	497.991	499.885	501.717	498.053	
18	499.918	501.845	497.991	499.885	501.717	498.053	
19	499.918	501.845	497.991	499.885	501.717	498.053	
20	499.918	501.845	497.991	499.885	501.717	498.053	
21	499.918	501.845	497.991	499.885	501.717	498.053	
22	499.918	501.845	497.991	499.885	501.717	498.053	
23	499.918	501.845	497.991	499.885	501.717	498.053	
24	499.918	501.845	497.991	499.885	501.717	498.053	
25	499.918	501.845	497.991	499.885	501.717	498.053	

X Bar Chart

Fig. 3.15

Step 7: For plotting the $\bar{X}$-chart in Excel 2007, we refer to Fig. 3.16. It means that we

1. select Cells F2:F6, F8:F18, F20:F27, K2:M6, K8:M18 and K20:M27 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

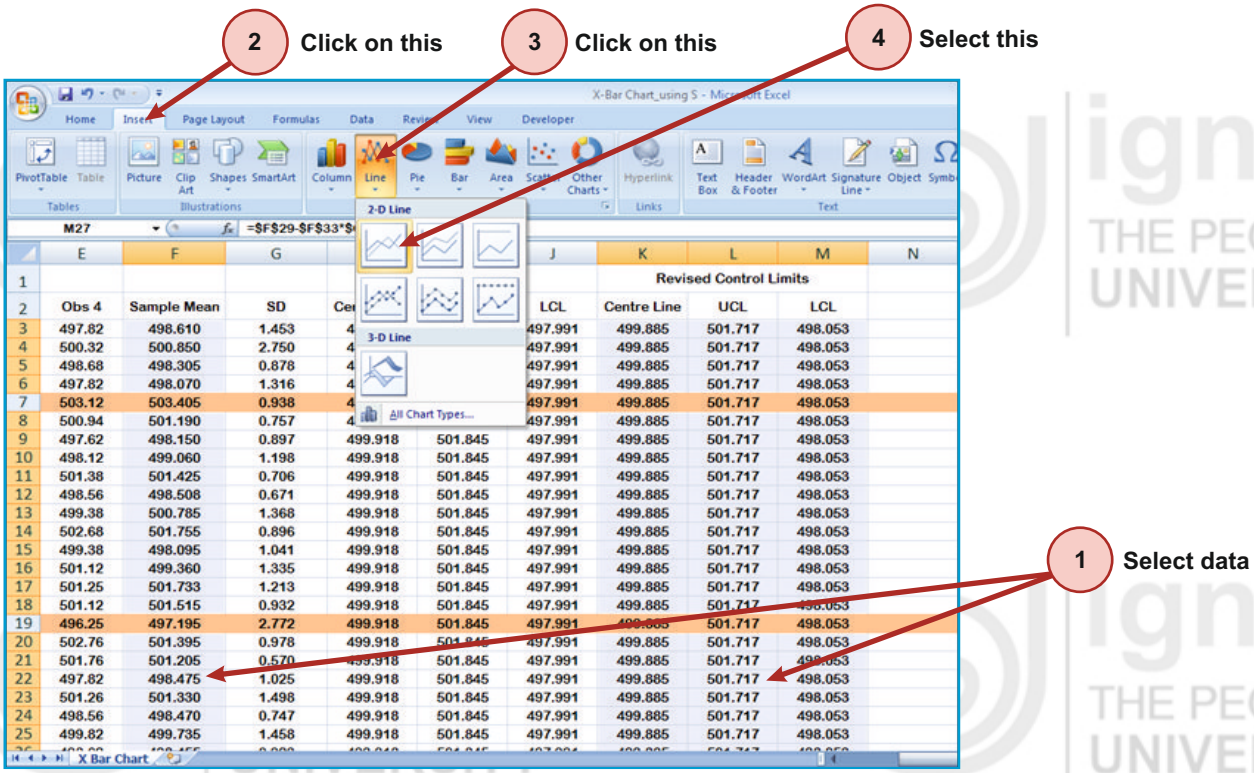


Fig. 3.16

Step 8: We find that the values in Columns K, L and M provide the horizontal lines on the chart representing the revised centre line, UCL and LCL, respectively.

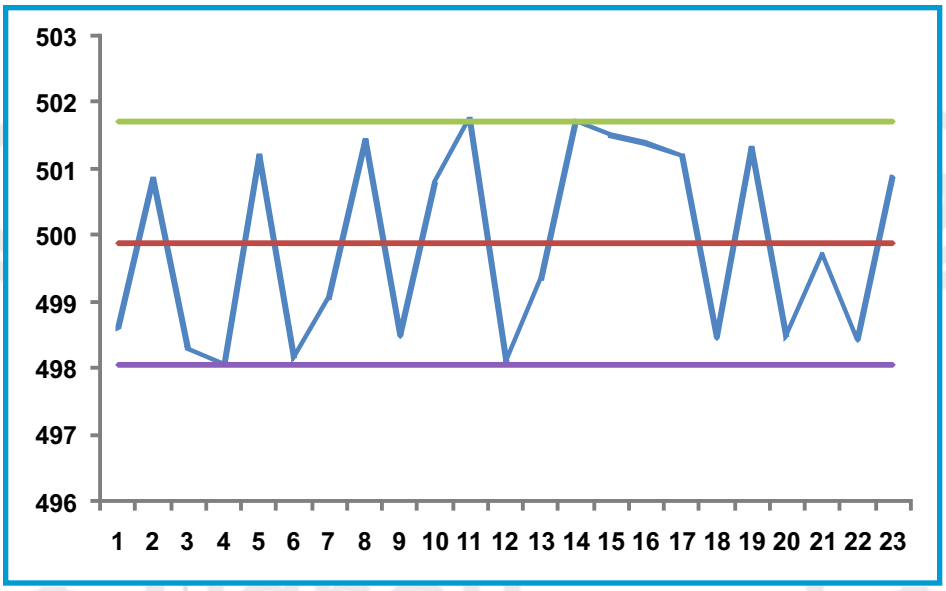


Fig. 3.17

Step 9: We note that the horizontal axis shown in Fig. 3.17 includes the 5th and 17th sample points. We need to eliminate these points from the horizontal axis. For this, we follow Steps 9 and 10 of Sec. 2.5 of Lab Session 2 as shown in Fig. 3.18.

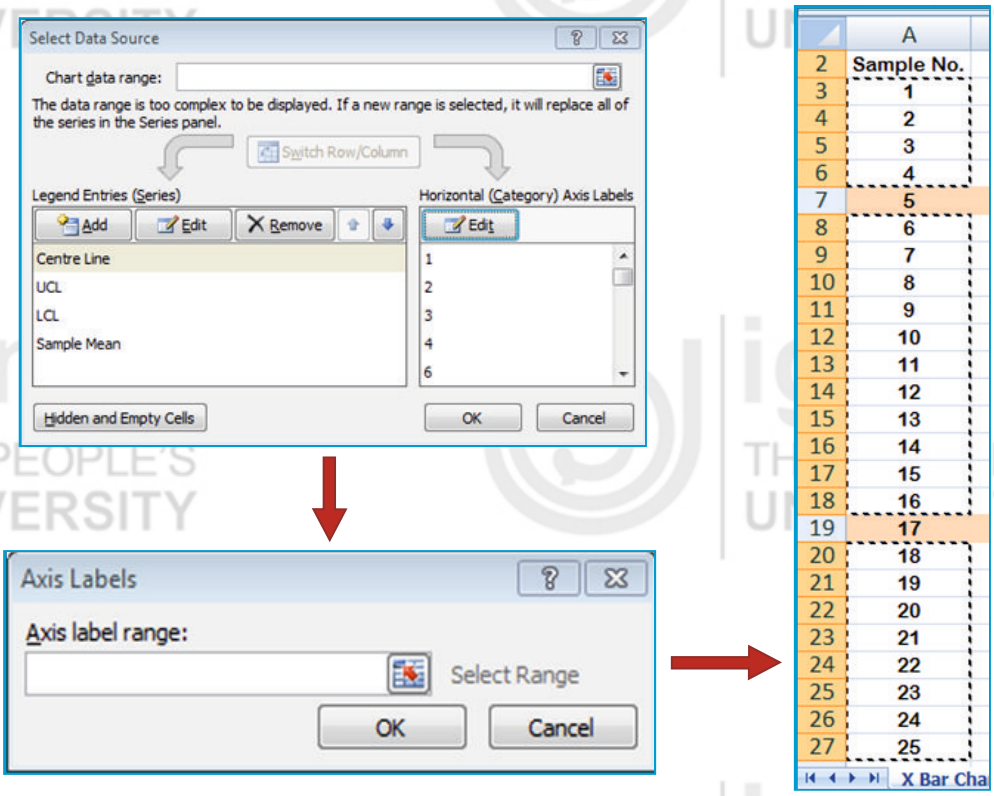


Fig. 3.18

Step 10: We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting control chart is shown in Fig. 3.19.

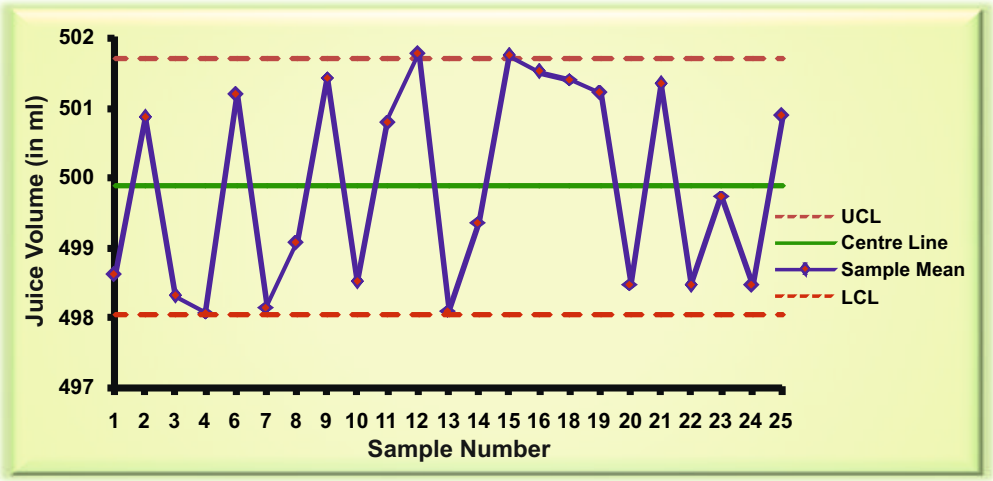


Fig. 3.19

Interpretation

Note from Fig. 3.19 that two points corresponding to Samples 12 and 15 lie outside the UCL and LCL, respectively, in the revised control chart. This control chart indicates that the process is **still not under statistical control**. Some assignable causes are still present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. After that, we delete the out-of-control points. In the next section, you will learn how to calculate the revised centre line and control limits again for the $\overline{X}$-chart for the remaining samples.

Activity 1

You can also determine the revised control limits using another approach. For this purpose, follow the steps given below:

- ✓ Select Cells A1:E6, A8:E18, and A20:E27.
- ✓ Choose Cell A35 or any other cell and paste the values. You can also use a separate Excel sheet where you can paste these values.
- ✓ Repeat all steps given in Sec. 3.4.

It will give you the same results as you have obtained in Sec. 3.5.

3.6 FURTHER REVISION OF $\bar{X}$-CHART

We can obtain the revised centre line and control limits for the $\bar{X}$-chart for the remaining samples as follows:

Step 1: We highlight the samples lying outside the revised control limits obtained in Sec. 3.5, i.e., the 12th and 15th samples, with purple colour as shown in Fig. 3.20.

	A	B	C	D	E	F	G	H	I	J
1		Juice Volume (in ml)						Control Limits		
2	Sample No.	Obs 1	Obs 2	Obs 3	Obs 4	Sample Mean	SD	Centre Line	UCL	LCL
3	1	497.32	500.62	498.68	497.82	498.610	1.453	499.918	501.845	497.991
4	2	504.76	500.00	498.32	500.32	500.850	2.750	499.918	501.845	497.991
5	3	499.24	497.18	498.12	498.68	498.305	0.878	499.918	501.845	497.991
6	4	499.26	496.32	498.88	497.82	498.070	1.316	499.918	501.845	497.991
7	5	502.32	503.62	504.56	503.12	503.405	0.938	499.918	501.845	497.991
8	6	502.12	500.32	501.38	500.94	501.190	0.757	499.918	501.845	497.991
9	7	499.34	498.32	497.32	497.62	498.150	0.897	499.918	501.845	497.991
10	8	499.38	498.12	500.62	498.12	499.060	1.198	499.918	501.845	497.991
11	9	501.26	502.38	500.68	501.38	501.425	0.706	499.918	501.845	497.991
12	10	498.60	497.62	499.25	498.56	498.508	0.671	499.918	501.845	497.991
13	11	502.44	500.00	501.32	499.38	500.785	1.368	499.918	501.845	497.991
14	12	501.26	502.32	500.76	502.68	501.755	0.896	499.918	501.845	497.991
15	13	497.32	498.50	497.18	499.38	498.095	1.041	499.918	501.845	497.991
16	14	499.56	498.00	498.76	501.12	499.360	1.335	499.918	501.845	497.991
17	15	502.24	500.32	503.12	501.25	501.733	1.213	499.918	501.845	497.991
18	16	501.76	500.50	502.68	501.12	501.515	0.932	499.918	501.845	497.991
19	17	500.65	497.82	494.06	496.25	497.195	2.772	499.918	501.845	497.991
20	18	501.12	501.26	500.44	502.76	501.395	0.978	499.918	501.845	497.991
21	19	501.00	500.50	501.56	501.76	501.205	0.570	499.918	501.845	497.991
22	20	497.50	498.82	499.76	497.82	498.475	1.025	499.918	501.845	497.991
23	21	503.44	500.62	500.00	501.26	501.330	1.498	499.918	501.845	497.991
24	22	499.38	498.38	497.56	498.56	498.470	0.747	499.918	501.845	497.991
25	23	501.56	499.56	498.00	499.82	499.735	1.458	499.918	501.845	497.991

X Bar Chart

Fig. 3.20

Step 2: Two points were outside the control limits as shown in Fig. 3.9 and two points are outside the revised control limits again as shown in Fig. 3.19. Thus, we need to eliminate a total of 4 samples. So we put the value of d′ (= 2 + 2 = 4) in Cell F35 as shown in Fig. 3.21.

	C	D	E	F
31			k	25
32			n	4
33		A_3 Value from Table (for n=4)		1.628
34			d	2
35			d'	4

Fig. 3.21

Step 3: We repeat Step 3 of Sec. 3.5 and compute the revised average $\bar{\bar{X}}'_{new}$ for this data as shown in Fig. 3.22.

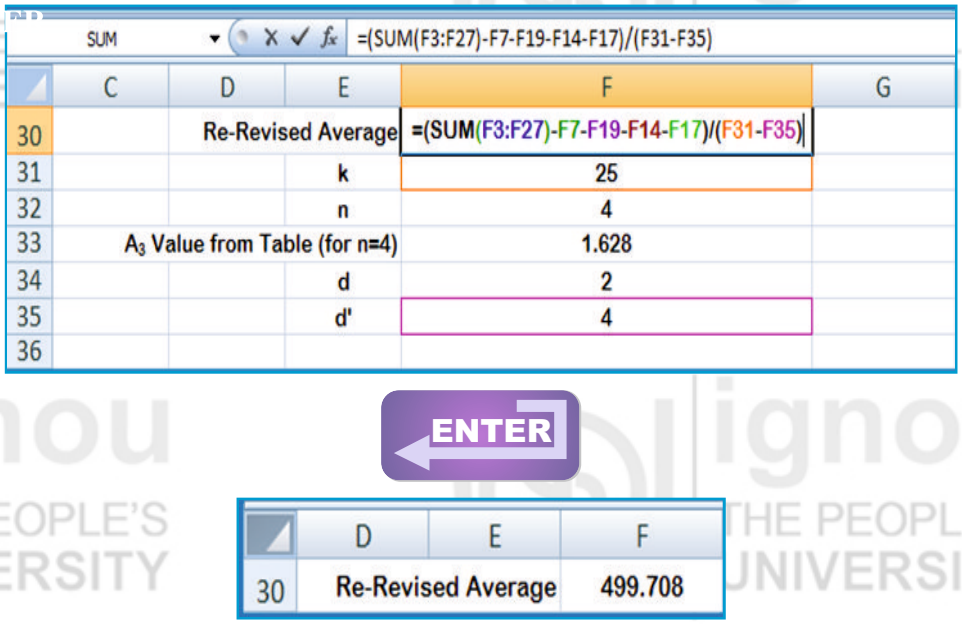


Fig. 3.22

Step 4: We repeat Step 4 of Sec. 3.5 and compute the revised average sample standard deviation $\bar{S}'_{new}$ for this data (see Fig. 3.23).

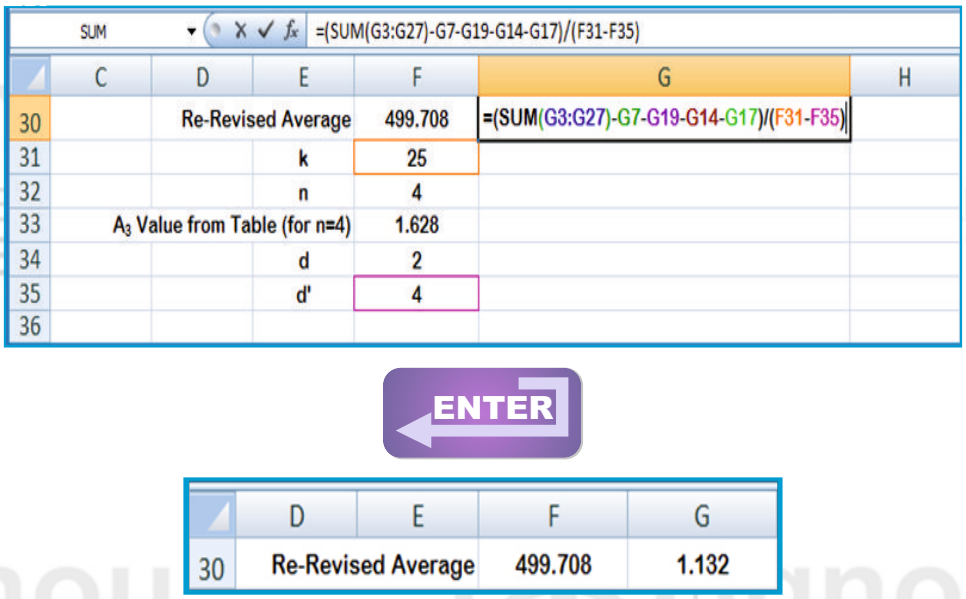


Fig. 3.23

Step 5: We follow Step 5 of Sec. 3.5 and compute the revised centre, upper and lower control limits, respectively:

i) The revised centre line is $(CL) = \bar{\bar{X}}'_{new}$. So we type "=$F$30" in Cell N3 to find the centre line as shown in Fig. 3.24a.

ii) We calculate the upper control limit $UCL = \bar{\bar{X}}'_{new} + A_3\bar{S}'_{new}$ by typing "=$F$30+$F$33*$G$30" in Cell O3 as shown in Fig. 3.24b.

iii) Similarly, we calculate the lower control limit $LCL = \bar{\bar{X}}'_{new} - A_3\bar{S}'_{new}$ by typing "=$F$30-$F$33*$G$30" in Cell P3 as shown in Fig. 3.24c.

N3 =$F$30

	N	O	P	Q
1	Re-Revised Control Limits			
2	Centre Line	UCL	LCL	
3	499.708			
4				

(a)

O3 =$F$30+$F$33*$G$30

	O	P	Q	R
1	vised Control Limits			
2	UCL	LCL		
3	501.551			
4				

(b)

P3 =$F$30-$F$33*$G$30

	P	Q	R	S
1	Limits			
2	LCL			
3	497.865			
4				

(c)

Fig. 3.24

Step 6: We follow Steps 6 and 7 of Sec. 3.5 and calculate the centre line, upper and lower control limits in Cells N3, O3 and P3, respectively, for the first sample. Then we select Cells N3:P3 and drag them up to Row 27. The output is shown in Fig. 3.25.

	K	L	M	N	O	P	Q
1	Revised Control Limits			Re-Revised Control Limits			
2	Centre Line	UCL	LCL	Centre Line	UCL	LCL	
3	499.885	501.717	498.053	499.708	501.551	497.865	
4	499.885	501.717	498.053	499.708	501.551	497.865	
5	499.885	501.717	498.053	499.708	501.551	497.865	
6	499.885	501.717	498.053	499.708	501.551	497.865	
7	499.885	501.717	498.053	499.708	501.551	497.865	
8	499.885	501.717	498.053	499.708	501.551	497.865	
9	499.885	501.717	498.053	499.708	501.551	497.865	
10	499.885	501.717	498.053	499.708	501.551	497.865	
11	499.885	501.717	498.053	499.708	501.551	497.865	
12	499.885	501.717	498.053	499.708	501.551	497.865	
13	499.885	501.717	498.053	499.708	501.551	497.865	
14	499.885	501.717	498.053	499.708	501.551	497.865	
15	499.885	501.717	498.053	499.708	501.551	497.865	
16	499.885	501.717	498.053	499.708	501.551	497.865	
17	499.885	501.717	498.053	499.708	501.551	497.865	
18	499.885	501.717	498.053	499.708	501.551	497.865	
19	499.885	501.717	498.053	499.708	501.551	497.865	
20	499.885	501.717	498.053	499.708	501.551	497.865	
21	499.885	501.717	498.053	499.708	501.551	497.865	

Fig. 3.25

Step 7: For plotting the $\bar{X}$-chart in Excel 2007, we refer to Fig. 3.26. It means that we

1. select Cells F2:F6, F8:F13, F15:F16, F18, F20:F27, N2:P6, N8:P13, N15:P16, N18:P18 and N20:P27 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

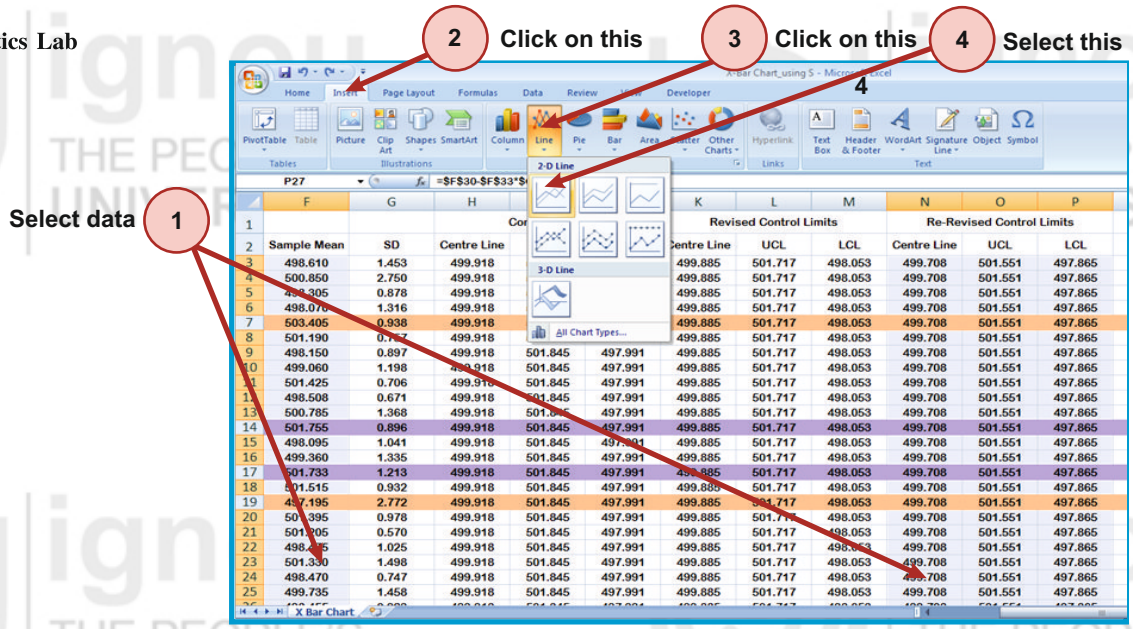


Fig. 3.26

Step 7: We can change the horizontal axis label as explained in Step 9 of Sec. 2.5. We select Cells A2:A6, A8:A13, A15:A16, A18 and A20:A27 by holding *Ctrl* key as shown in Fig. 3.27.

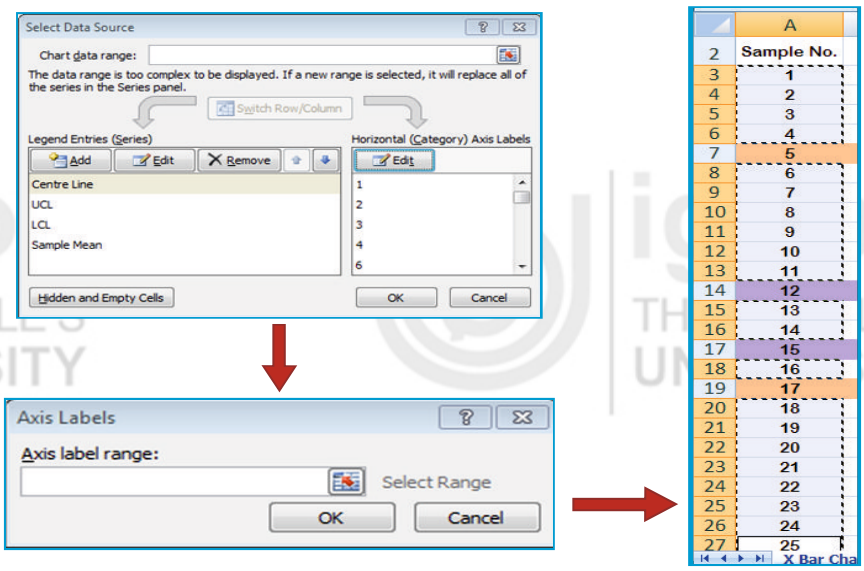


Fig. 3.27

Step 8: We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting control chart is shown in Fig. 3.28.

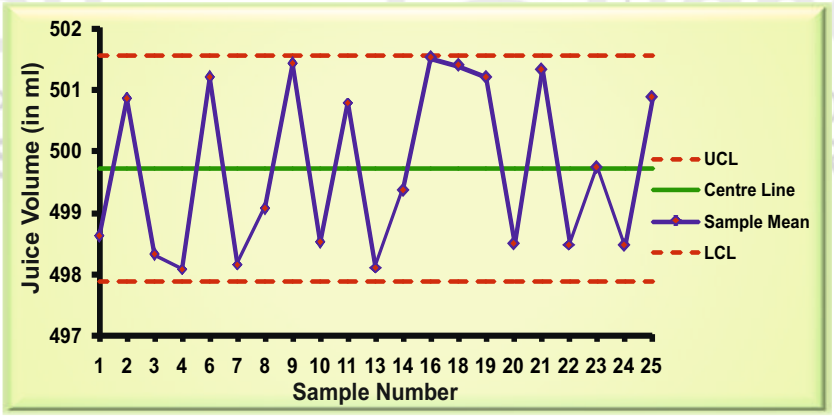


Fig. 3.28

Interpretation

The $\overline{X}$-chart revised further is shown in Fig. 3.28. It indicates that all points

lie within the control limits. So the process is under statistical control now with respect to mean.

Activity 2

You can also determine the revised control limits using another approach. For this purpose, follow the steps given below:

- ✓ Select Cells A1:E6, A8:E13, A15:E16, A18:E18 and A20:E27.
- ✓ Choose Cell A35 or any cell and paste the values. You can also use a separate Excel sheet where you can paste these values.
- ✓ Repeat all the steps given in Sec. 3.4.

It will give you the same results as you have obtained in Sec. 3.6.

You should now apply the method on other problems for practice.

Activity 3

Construct the control charts for mean when process variability is unknown and estimated by sample standard deviation with the help of MS Excel 2007 and interpret the results for

A1) Example 3 given in Unit 2 of MSTE-001.

A2) Exercise E6 given in Unit 2 of MSTE-001.

Match the results with the manual calculation done in Unit 2 of MSTE-001.

Continuous Assessment 3

Consider the data given in Continuous Assessment 1 and develop $\bar{X}$-chart to check whether the process of production of bulbs is under control or out-of-control when process variability is unknown. If it is out-of-control, compute the revised control limits. Use sample standard deviation to estimate the process standard deviation.

Home Work: Do It Yourself

1) Follow the steps explained in Sec. 3.4, 3.5 and 3.6 to construct the control chart for the data of Table 1 in Lab Session 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.
2) Develop the spreadsheet for the exercise "Continuous Assessment 3" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.
3) **Do not forget** to keep the screenshots in your record book as these will contribute in your continuous assessment in the Laboratory.

Control Chart for Range

SESSION 4

Structure

4.1 Introduction

Objectives

4.2 Problem Description

4.3 Procedure for the Construction of R-Chart

4.4 Steps Involved in the Construction of R-Chart in Excel 2007

4.5 Revised R-Chart

4.6 Revised $\bar{X}$-Chart

4.1 INTRODUCTION

Prerequisite

- Lab Sessions 1, 3 and 6 of MSTL-001 (Basic Statistics Lab).
- Lab Session 2 of MSTL-002 (Industrial Statistics Lab).
- Unit 2 of MSTE-001 (Industrial Statistics-I).

In Lab Sessions 1 to 3, you have learnt about the $\bar{X}$-chart which is used to control the process mean. We have discussed in Sec. 2.5 of Unit 2 of MSTE-001(Industrial Statistics-I) that process variability also affects the product quality. When variation in the production process is high, that is, the produced items have a wider range of values of the quality characteristics, the quality of the product is assumed to be poor. So we must reduce the process variability to improve the quality of the product. We use control chart for range (R-chart) and control chart for standard deviation (S-chart) to **control the process variability**.

R-chart is more widely used in quality control than the S-chart because in quality control, we generally use samples of sizes less than 10. We know that for $n \leq 10$, range and standard deviation reflect almost the same information about the variability. Range is easier to calculate and interpret as compared to standard deviation.

The underlying logic and basic form of R-chart are similar to the $\bar{X}$-chart. The primary difference between $\bar{X}$ and R-chart is that instead of plotting the sample means and monitoring their variation, we plot the **sample range** and monitor the **variation of sample ranges**. In this lab session, you will learn how to **construct R-chart** in MS Excel 2007. We shall explain the construction of **S-chart** in the next lab session.

Objectives

After performing the activities of this session, you should be able to:

- prepare spreadsheet in MS Excel 2007;
- determine the control limits for control chart for range;
- construct the control chart for range;
- obtain the revised control limits for control charts for range and mean; and
- interpret the control charts.

4.2 PROBLEM DESCRIPTION

Suppose a bulb manufacturing company wants to check whether the variation in the life of bulbs produced by a particular machine is due to chance causes or assignable causes. For this purpose, a quality control inspector at this company selects 35 samples, each of size 5, and measures the life of each bulb (in hours).

Table 1: Life of bulbs

Sample Number	Life of Bulbs (in hours)				
	Obs. 1	Obs. 2	Obs. 3	Obs. 4	Obs. 5
1	1152	650	1090	708	1065
2	688	540	1074	598	1049
3	876	1342	1187	1214	1162
4	725	1070	847	1128	822
5	900	846	1254	904	1229
6	1080	790	871	848	846
7	952	1087	1126	1145	1101
8	741	986	578	1044	837
9	1005	1244	1140	1302	1115
10	1494	1005	614	1063	575
11	1200	870	859	928	834
12	745	964	1050	1022	1025
13	926	1172	1200	1230	1175
14	781	876	981	934	956
15	1120	1350	750	1408	725
16	880	975	768	1033	743
17	1138	636	1076	694	1051
18	674	526	1060	584	1035
19	862	1328	1173	1200	1148
20	711	1056	833	1114	808
21	886	832	1240	890	1215
22	1066	776	857	834	832
23	938	1073	1112	1131	1087
24	727	972	564	1030	823
25	991	1230	1126	1288	1101
26	1227	991	600	1049	561
27	1186	856	845	914	820
28	731	950	1036	1008	1011
29	912	1158	1186	1216	1161
30	1124	862	967	920	942
31	1106	1336	578	1490	711
32	866	961	754	1019	729
33	1124	622	1062	680	1037
34	660	658	1046	784	1021
35	848	1314	1159	1186	1134

The quality control inspector of the company has to construct a control chart for range and mean to infer whether the production variability is under control or not. If it is out-of-control, he/she computes the revised control limits.

Therefore, the problem for this session is to construct the control charts for range and mean for the data given in Table 1.

4.3 PROCEDURE FOR THE CONSTRUCTION OF R-CHART

You have already learnt all the formulae for calculating control limits for R-chart and the method to draw R-chart manually in Sec. 2.5 of Unit 2 of MSTE-001. So here we focus on the main steps involved in the construction of R-chart for the data given in Table 1:

When the value of σ is known

Step 1: The control limits for R-chart for known σ are given by

- ✓ Centre line (CL) $= d_2\sigma$ ...(1)
- ✓ Upper control limit (UCL) $= D_2\sigma$ ...(2)
- ✓ Lower control limit (LCL) $= D_1\sigma$ ...(3)

where d_2, D_1 and D_2 are constants and depend on the size of the sample. These constants have been tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 2: To construct the R-chart, we take sample number on the X-axis and sample range R on the Y-axis and plot the value of sample range for each sample against the sample number. The consecutive sample points are joined by line segments.

Step 3: Interpretation of the R-chart.

When the value of σ is unknown

In practice, the value of σ is not known. Therefore, it is estimated from the samples which are taken when the process is assumed to be under control. In R-chart, σ is estimated by sample range (R).

Step 1-2: The first two steps are the same as Steps 4 and 5 of Sec. 2.3 of Lab Session 2.

Step 3: The control limits for R-chart when σ is estimated by $\bar{R}/d_2$ are given by

- ✓ Centre line (CL) $= \bar{R}$ ...(4)
- ✓ Upper control limit (UCL) $= D_4\bar{R}$ ...(5)
- ✓ Lower control limit (LCL) $= D_3\bar{R}$ ...(6)

where D_3 and D_4 are constants and depend on the size of the sample. These constants have been tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 4: Interpretation of the R-chart.

4.4 STEPS INVOLVED IN THE CONSTRUCTION OF R-CHART IN EXCEL 2007

In order to calculate the control limits of R-chart and to plot the control chart in Excel 2007 for the given data, we follow the steps given below:

Generally, in order to judge whether the process is in control, we use $\bar{X}$ and R-charts together. We first control the process variability and so we analyse the R-chart prior to the $\bar{X}$ - chart.

Step 1: We enter the given data in Excel Sheet and calculate range for the first sample by typing "=Max(B3:F3)-Min(B3:F3)" in Cell G3. After that, we drag down Cell G3 up to Cell G37 as shown in Fig. 4.1.

G3 =MAX(B3:F3)-MIN(B3:F3)

	A	B	C	D	E	F	G
1							
2	Sample No.	Life of Bulbs (in hours)					Range
3	1	1152.00	650.00	1090.00	708.00	1065.00	502.000
4	2	688.00	540.00	1074.00	598.00	1049.00	
5	3	876.00	1342.00	1187.00	1214.00	1162.00	
6	4	725.00	1070.00	847.00	1128.00	822.00	
7	5	900.00	846.00	1254.00	904.00	1229.00	
8	6	1080.00	790.00	871.00	848.00	846.00	
9	7	952.00	1087.00	1126.00	1145.00	1101.00	
10	8	741.00	986.00	578.00	1044.00	837.00	
11	9	1005.00	1244.00	1140.00	1302.00	1115.00	
12	10	1494.00	1005.00	614.00	1063.00	575.00	

DRAG IT DOWN

	A	B	C	D	E	F	G
1							
2	Sample No.	Life of Bulbs (in hours)					Range
3	1	1152	650	1090	708	1065	502.000
4	2	688	540	1074	598	1049	534.000
5	3	876	1342	1187	1214	1162	466.000
6	4	725	1070	847	1128	822	403.000
7	5	900	846	1254	904	1229	408.000
8	6	1080	790	871	848	846	290.000
9	7	952	1087	1126	1145	1101	193.000
10	8	741	986	578	1044	837	466.000
11	9	1005	1244	1140	1302	1115	297.000
12	10	1494	1005	614	1063	575	919.000
13	11	1200	870	859	928	834	366.000

Fig. 4.1

Step 2: In Cell G38, we compute the average range ($\bar{R}$) by typing "=Average(G3:G37)" and click on ***Enter*** key as shown in Fig. 4.2.

ROUND =AVERAGE(G3:G37)

	F	G	H
38	Average	=AVERAGE(G3:G37)	
39			

ENTER

G38 =AVERAGE(G3:G37)

	F	G	H	I
38	Average	418.457		
39				

Fig. 4.2

Step 3: Next we type the values of k and n in Cells G40 and G41, respectively. We also have to type the values of D_3 and D_4 for n = 5 in Cell G42 and G43, respectively, as shown in Fig. 4.3. We take them from the Appendix given at the end of this lab course.

	D	E	F	G
40			k	35
41			n	5
42	D_3 Value from Table (for n=5)			0
43	D_4 Value from Table (for n=5)			2.114

Fig. 4.3

Step 4: We recall the method for computing the centre line and both control limits described in Lab Sessions 1-3. Here we use Columns H, I and J for putting the values of centre line, upper and lower control limits, respectively. We calculate these values as follows:

The formula with dollar sign ($) is used for an absolute reference.

i) The centre line is given in equation (4) and $\bar{R}$ is given in Cell G38 (see Fig. 4.2). So we type "=$G$38" in Cell H3 to find the centre line as shown in Fig. 4.4a.

ii) We use equation (5) for the upper control limit. The values of D_4 and $\bar{R}$ are given in Cells G43 and G38, respectively (see Figs. 4.2 and 4.3). We type "=$G$43*$G$38" in Cell I3 as shown in Fig. 4.4b.

iii) Similarly, we calculate the lower control limit from equation (6) and type "=$G$43*$G$38" in Cell J3 as shown in Fig. 4.4c.

(a)

H3 | fx =$G$38

	H	I	J	K
1	Control Limits			
2	Centre Line	UCL	LCL	
3	418.457			
4				

(b)

I3 | fx =$G$43*$G$38

	I	J	K	L
1	ontrol Limits			
2	UCL	LCL		
3	884.618			
4				

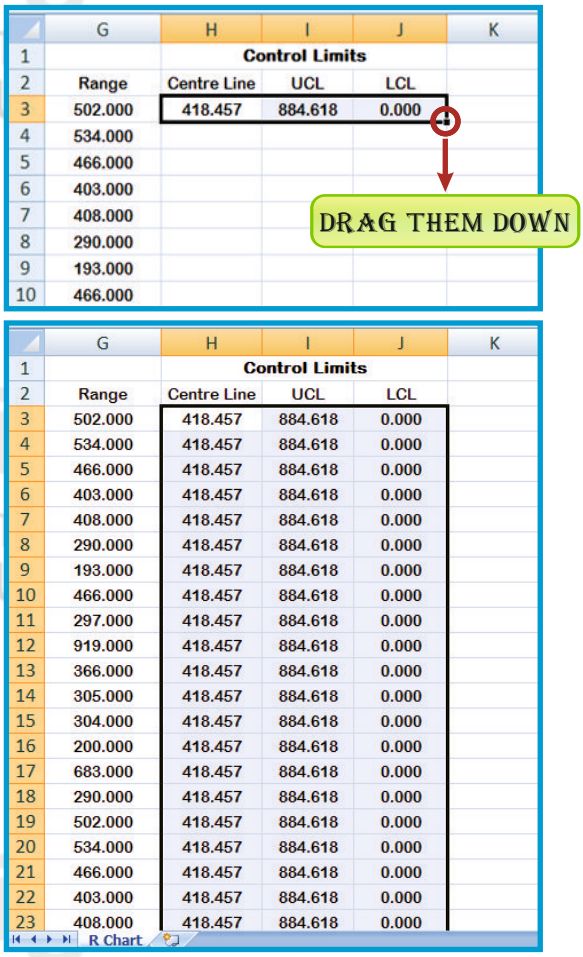
(c)

J3 | fx =$G$42*$G$38

	J	K	L	M
1	ts			
2	LCL			
3	0.000			
4				

Fig. 4.4

Step 5: For plotting control limits using Excel, we first select Cells H3:J3 and drag them down up to Row 37 as shown in Fig. 4.5.



	G	H	I	J	K
1		Control Limits			
2	Range	Centre Line	UCL	LCL	
3	502.000	418.457	884.618	0.000	
4	534.000				
5	466.000				
6	403.000				
7	408.000				
8	290.000				
9	193.000				
10	466.000				

	G	H	I	J	K
1		Control Limits			
2	Range	Centre Line	UCL	LCL	
3	502.000	418.457	884.618	0.000	
4	534.000	418.457	884.618	0.000	
5	466.000	418.457	884.618	0.000	
6	403.000	418.457	884.618	0.000	
7	408.000	418.457	884.618	0.000	
8	290.000	418.457	884.618	0.000	
9	193.000	418.457	884.618	0.000	
10	466.000	418.457	884.618	0.000	
11	297.000	418.457	884.618	0.000	
12	919.000	418.457	884.618	0.000	
13	366.000	418.457	884.618	0.000	
14	305.000	418.457	884.618	0.000	
15	304.000	418.457	884.618	0.000	
16	200.000	418.457	884.618	0.000	
17	683.000	418.457	884.618	0.000	
18	290.000	418.457	884.618	0.000	
19	502.000	418.457	884.618	0.000	
20	534.000	418.457	884.618	0.000	
21	466.000	418.457	884.618	0.000	
22	403.000	418.457	884.618	0.000	
23	408.000	418.457	884.618	0.000	

Fig. 4.5

Step 6: For plotting the R-chart in Excel 2007, we refer to Fig. 4.6. It means that we

1. select Cells G2:J37,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

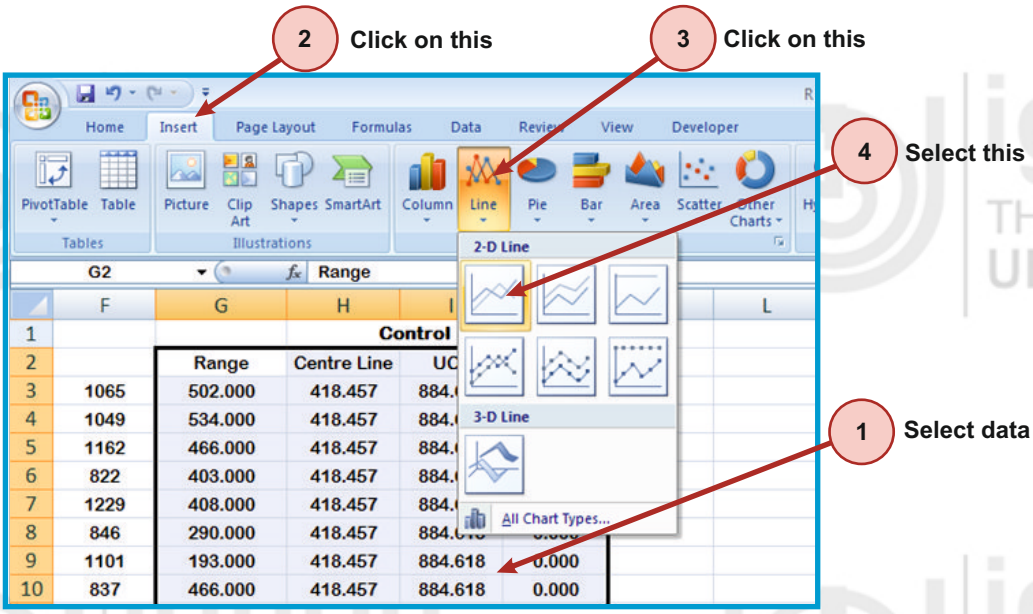


Fig. 4.6

Step 7: We find that Columns H, I and J will provide horizontal lines on the chart representing the centre line, UCL and LCL, respectively. Column G will provide the ranges for 35 samples on the chart. We format the chart as explained in Sec. 1.4 of Lab Session 1. Thus, we obtain the R-chart shown in Fig. 4.7.

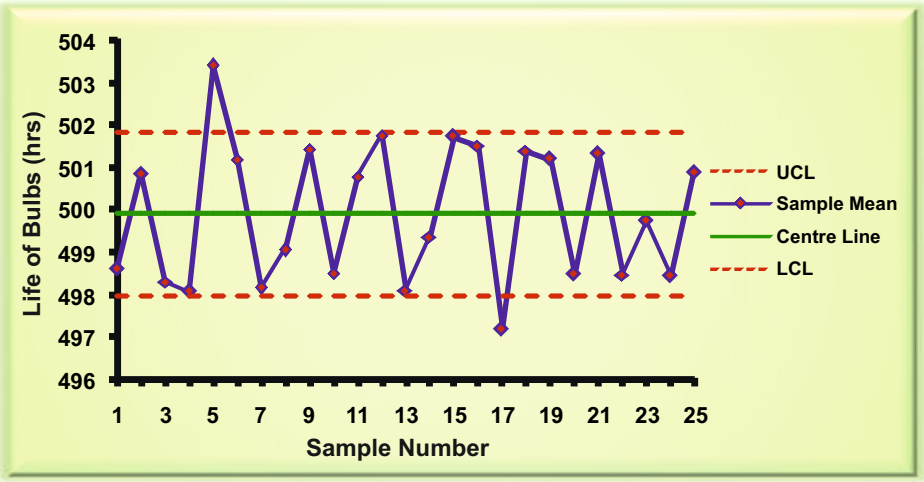


Fig. 4.7

Interpretation

Note from Fig. 4.7 that two points corresponding to Samples 10 and 31 lie outside the UCL. So the control chart indicates that the process is **not under statistical control**. Some **assignable causes** are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them.

4.5 REVISED R-CHART

After eliminating the assignable causes from the process, we revise the control chart of Fig.4.7 by deleting out-of-control samples. For this purpose, we obtain the revised control limits for the R-chart using only the remaining samples as explained below:

Step 1: For revised limits for R-chart, we first calculate new $\bar{R}$ as follows:

$$\bar{R}_{new} = \frac{\sum_{i=1}^{k} R_i - \sum_{j=1}^{d} R_j}{k-d} \quad \cdots(7)$$

where $\sum_{j=1}^{d} R_j$ – sum of the ranges of discarded samples, and

d – number of discarded samples.

Step 2: After finding the new $\bar{R}$, we reconstruct the centre line and control limits by replacing $\bar{R}$ by $\bar{R}_{new}$ as follows:

- ✓ Centre line (CL) $= \bar{R}_{new}$ …(8)
- ✓ Upper control limit (UCL) $= D_4\bar{R}_{new}$ …(9)
- ✓ Lower control limit (LCL) $= D_3\bar{R}_{new}$ …(10)

Step 3: Interpretation of the R-chart.

Steps in Excel

The main steps for calculating the revised centre line and control limits for the R-chart in Excel 2007 for the remaining samples are as follows:

Step 1: We highlight the samples outside the control limits, i.e., the 10th and 31st samples, with light orange colour (Fig. 4.8).

	A	B	C	D	E	F	G	H	I	J
1								Control Limits		
2	Sample No.	Life of Bulbs (in hours)					Range	Centre Line	UCL	LCL
3	1	1152	650	1090	708	1065	502.000	418.457	884.618	0.000
4	2	688	540	1074	598	1049	534.000	418.457	884.618	0.000
5	3	876	1342	1187	1214	1162	466.000	418.457	884.618	0.000
6	4	725	1070	847	1128	822	403.000	418.457	884.618	0.000
7	5	900	846	1254	904	1229	408.000	418.457	884.618	0.000
8	6	1080	790	871	848	846	290.000	418.457	884.618	0.000
9	7	952	1087	1126	1145	1101	193.000	418.457	884.618	0.000
10	8	741	986	578	1044	837	466.000	418.457	884.618	0.000
11	9	1005	1244	1140	1302	1115	297.000	418.457	884.618	0.000
12	10	1494	1005	614	1063	575	919.000	418.457	884.618	0.000
13	11	1200	870	859	928	834	366.000	418.457	884.618	0.000
14	12	745	964	1050	1022	1025	305.000	418.457	884.618	0.000
15	13	926	1172	1200	1230	1175	304.000	418.457	884.618	0.000
16	14	781	876	981	934	956	200.000	418.457	884.618	0.000
17	15	1120	1350	750	1408	725	683.000	418.457	884.618	0.000
18	16	880	975	768	1033	743	290.000	418.457	884.618	0.000
19	17	1138	636	1076	694	1051	502.000	418.457	884.618	0.000
20	18	674	526	1060	584	1035	534.000	418.457	884.618	0.000
21	19	862	1328	1173	1200	1148	466.000	418.457	884.618	0.000

	A	B	C	D	E	F	G	H	I	J
31	29	912.00	1158.00	1186.00	1216.00	1161.00	304.000	418.457	884.618	0.000
32	30	1124.00	862.00	967.00	920.00	942.00	262.000	418.457	884.618	0.000
33	31	1106.00	1336.00	578.00	1490.00	711.00	912.000	418.457	884.618	0.000
34	32	866.00	961.00	754.00	1019.00	729.00	290.000	418.457	884.618	0.000
35	33	1124.00	622.00	1062.00	680.00	1037.00	502.000	418.457	884.618	0.000
36	34	660.00	658.00	1046.00	784.00	1021.00	388.000	418.457	884.618	0.000
37	35	848.00	1314.00	1159.00	1186.00	1134.00	466.000	418.457	884.618	0.000

Fig. 4.8

Step 2: We have two samples outside the UCL, so we put d = 2 in Cell G44 as shown in Fig. 4.9.

	D	E	F	G
40			k	35
41			n	5
42	D_3 Value from Table (for n=5)			0
43	D_4 Value from Table (for n=5)			2.114
44			d	2

Fig. 4.9

Step 3: For calculating the value of $\bar{R}_{new}$ given in equation (7), we type "=(Sum(N3:N37)-N12-N33)/(G40-G44)" in Cell G39 and then press ***Enter***. We get the value of revised $\bar{R}$ i.e., $\bar{R}_{new}$ as shown in Fig. 4.10.

ROUND =(SUM(G3:G37)-G12-G33)/(G40-G44)

	E	F	G
39		Revised Average	=(SUM(G3:G37)-G12-G33)/(G40-G44)
40		k	35
41		n	5
42	lue from Table (for n=5)		0
43	lue from Table (for n=5)		2.114
44		d	2

ENTER

	E	F	G
39		Revised Average	388.333

Fig. 4.10

Step 4: We now calculate the revised centre line, upper and lower control limits as discussed in Step 4 of Sec. 4.3. Here we use Columns K, L and M for putting the values of the revised centre line, upper and lower control limits, respectively.

The formula with dollar sign ($) is used for an absolute reference.

i) The revised centre line is given by equation (8). The value of $\bar{R}_{new}$ is given in Cell G39 (see Fig. 4.10). So we type "=$G$39" in Cell K3 to find the centre line as shown in Fig. 4.11a.

ii) The upper control limit is given by equation (9). The values of D_4 and $\bar{R}_{new}$ are given in Cells G43 and G39, respectively (see Fig. 4.10). So we type "=$G$43*$G$39" in Cell L3 as shown in Fig. 4.11b.

iii) Similarly, we calculate the lower control limit given by equation (10) by typing "=$G$42*$G$39" in Cell M3 as shown in Fig. 4.11c.

(a)

K3 =$G$39

	K	L	M	N
1	Revised Control Limits			
2	Centre Line	UCL	LCL	
3	388.333			
4				

(b)

L3 =$G$43*$G$39

	L	M	N	O
1	d Control Limits			
2	UCL	LCL		
3	820.937			
4				

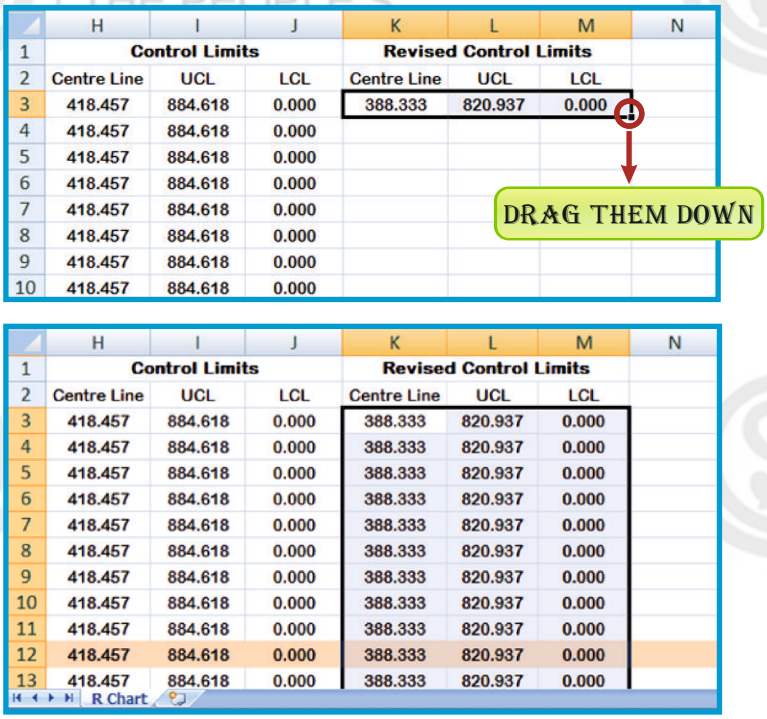
(c)

M3 =$G$42*$G$39

	M	N	O	P
1	Limits			
2	LCL			
3	0.000			
4				

Fig. 4.11

Step 5: For plotting control limits on the chart using Excel, we first select Cells K3:M3 and drag them down up to Row 37 as shown in Fig. 4.12.



	H	I	J	K	L	M	N
1	Control Limits			Revised Control Limits			
2	Centre Line	UCL	LCL	Centre Line	UCL	LCL	
3	418.457	884.618	0.000	388.333	820.937	0.000	
4	418.457	884.618	0.000				
5	418.457	884.618	0.000				
6	418.457	884.618	0.000				
7	418.457	884.618	0.000				
8	418.457	884.618	0.000				
9	418.457	884.618	0.000				
10	418.457	884.618	0.000				

	H	I	J	K	L	M	N
1	Control Limits			Revised Control Limits			
2	Centre Line	UCL	LCL	Centre Line	UCL	LCL	
3	418.457	884.618	0.000	388.333	820.937	0.000	
4	418.457	884.618	0.000	388.333	820.937	0.000	
5	418.457	884.618	0.000	388.333	820.937	0.000	
6	418.457	884.618	0.000	388.333	820.937	0.000	
7	418.457	884.618	0.000	388.333	820.937	0.000	
8	418.457	884.618	0.000	388.333	820.937	0.000	
9	418.457	884.618	0.000	388.333	820.937	0.000	
10	418.457	884.618	0.000	388.333	820.937	0.000	
11	418.457	884.618	0.000	388.333	820.937	0.000	
12	418.457	884.618	0.000	388.333	820.937	0.000	
13	418.457	884.618	0.000	388.333	820.937	0.000	

Fig. 4.12

Step 5: For plotting the R-chart in Excel 2007, we refer to Fig. 4.13. It means that we

1. select Cells G2:G11, G13:G32, G34:G37, K2:M11, K13:M32 and K34:M37 by holding *Ctrl* key,
2. click on the *Insert* tab,
3. select the *Line* option, and
4. choose the chart subtype.

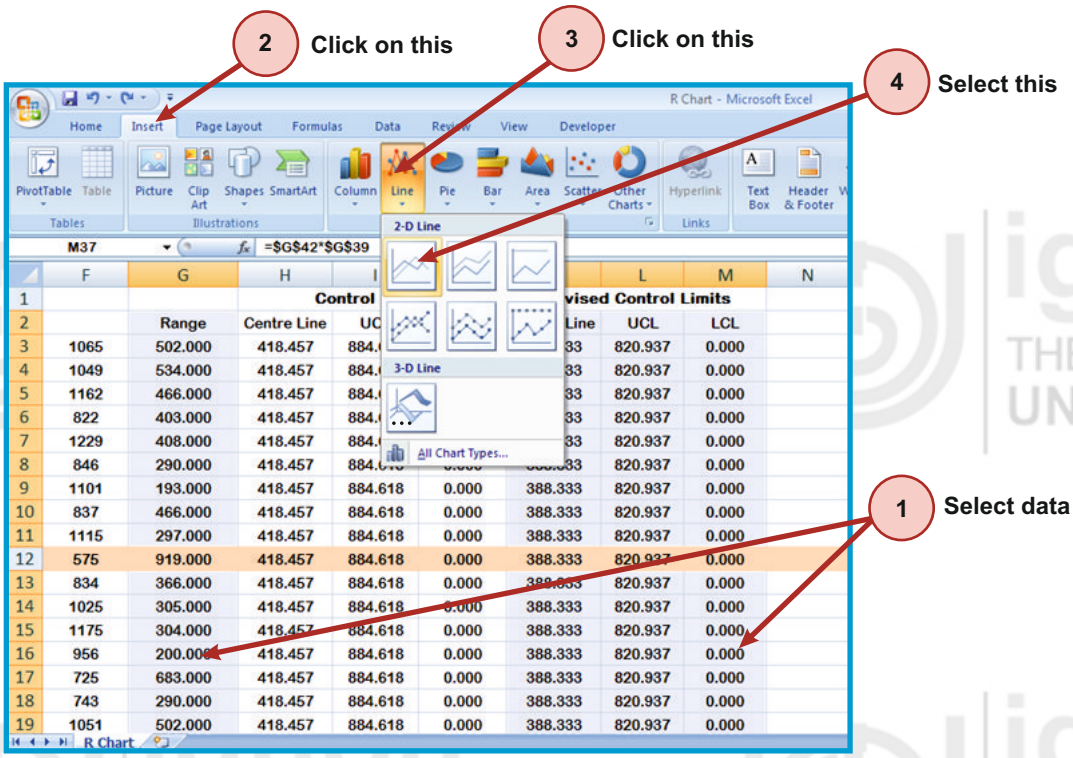


Fig. 4.13

Step 7: We format and change the horizontal axis as discussed in Steps 9 and 10 of Sec. 2.5 of Lab Session 2. The resulting R-chart is shown in Fig. 4.14.

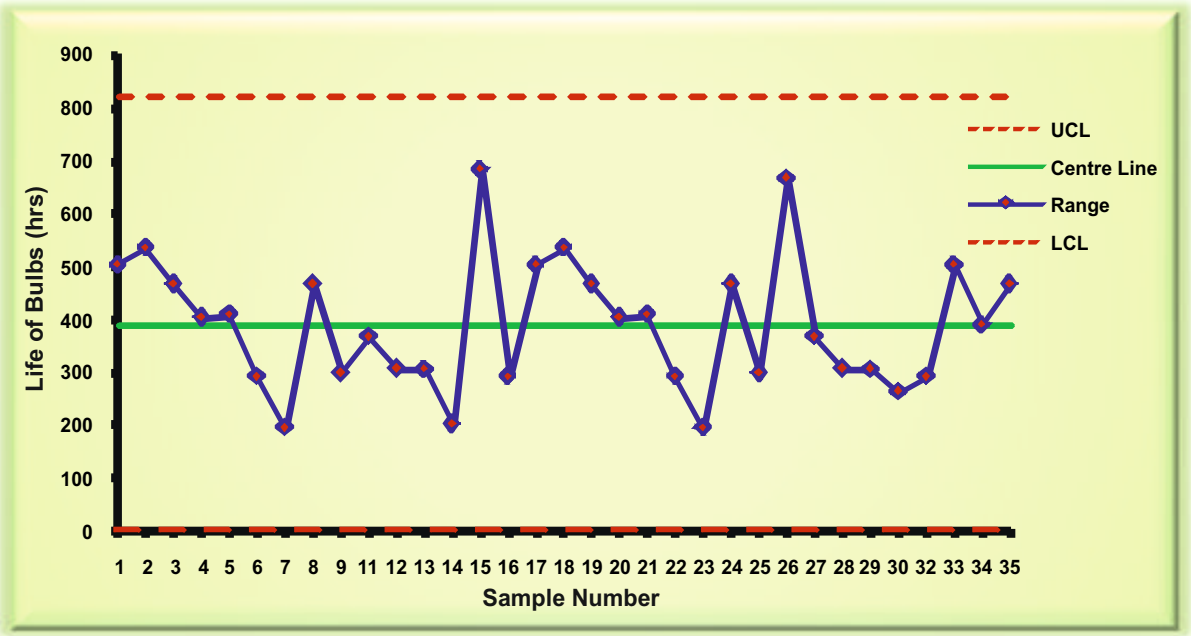


Fig. 4.14

If one or more points lie outside the revised control limits, we calculate the revised control limits of R-chart again. This continues until the process comes under control.

Interpretation

The revised R-chart shown in Fig. 4.14 indicates that all points lie within the control limits. So the process is under statistical control with respect to process variability.

Activity 1

You can also determine the revised control limits using another approach. For this purpose, follow the steps given below:

- ✓ Select Cells A2:F11, A13:F32 and A34:F37.
- ✓ Choose Cell A46 or any other cell and paste the values. You can also use a separate Excel sheet where you can paste these values.
- ✓ Repeat all the steps given in Sec. 4.4.

It will give you the same results as you have obtained in Sec. 4.5.

So far you have learnt how to control the process variability using the R-chart. We shall now study the process mean. For this, we calculate the control limits of the $\overline{X}$-chart using the remaining samples.

4.6 REVISED $\bar{X}$-CHART

We have already controlled the process variability using R-chart in Sec. 4.5 after eliminating the assignable causes present in the process. We now have to control the process mean. Here we use $\bar{R}_{new}$ to estimate the process variability. We calculate $\bar{\bar{X}}_{new}$ by deleting the out-of-control points. We compute the revised control limits for the $\bar{X}$-chart as explained in Sec. 2.5 of Lab Session 2.

Step 1: For the revised limits for $\bar{X}$-chart, we calculate new $\bar{\bar{X}}$ and new $\bar{R}$ as follows:

$$\bar{\bar{X}}_{new} = \frac{\sum_{i=1}^{k} \bar{X}_i - \sum_{j=1}^{d} \bar{X}_j}{k-d} \quad ...(11)$$

$$\text{and} \quad \bar{R}_{new} = \frac{\sum_{i=1}^{k} R_i - \sum_{j=1}^{d} R_j}{k-d} \quad ...(12)$$

where $\sum_{j=1}^{d} \bar{X}_j$ – sum of the averages of discarded samples,

d – number of discarded samples, and

$\sum_{j=1}^{d} R_j$ – sum of the ranges of discarded samples.

Step 2: Then we reconstruct the centre line and control limits of the chart by replacing $\bar{\bar{X}}$ by $\bar{\bar{X}}_{new}$ and $\bar{R}$ by $\bar{R}_{new}$ as follows:

- ✓ Centre line = $\bar{\bar{X}}_{new}$ ...(13)
- ✓ Upper control limit (UCL) = $\bar{\bar{X}}_{new} + A_2\bar{R}_{new}$ ...(14)
- ✓ Lower control limit (UCL) = $\bar{\bar{X}}_{new} - A_2\bar{R}_{new}$ ...(15)

Step 3: Interpretation of the $\bar{X}$-chart.

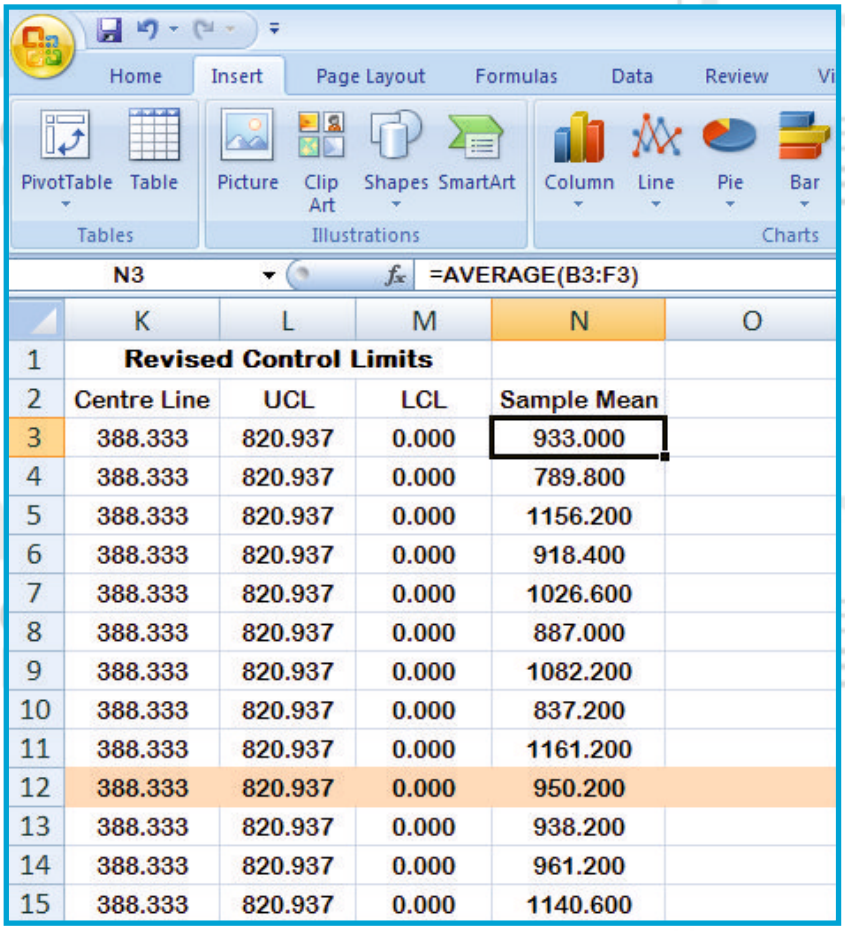
If R-chart indicates that the process variability is out-of-control, we first control the process variability by investigating the assignable causes and take corrective action to eliminate them. Then we analyse $\bar{X}$- chart. because when R-chart is brought to control, many assignable causes for $\bar{X}$-chart are also eliminated.

If R-chart indicates that the process variability is under control but the $\bar{X}$-chart indicates that the average of the process is out-of-control, we continue to use the same R-chart and revise the centre line and control limits of the $\bar{X}$-chart by eliminating these points.

Steps in Excel

The main steps for calculating the revised centre line and control limits for the $\bar{X}$-chart in Excel 2007 using the remaining samples are as follows:

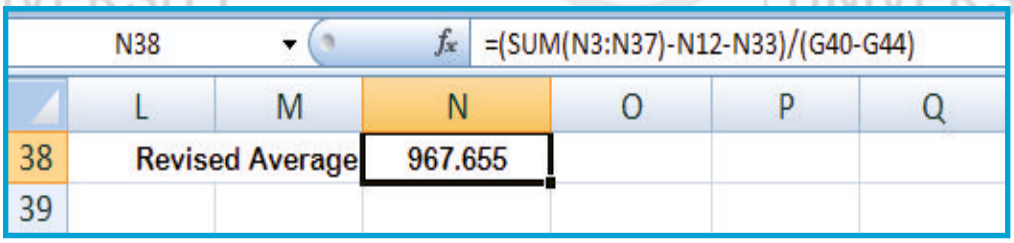
Step 1: We compute the sample means for all given samples in Cells N3:N37 as explained in Steps 1-5 of Sec. 1.4 of Lab Session 1. The output is shown in Fig. 4.15.



	K	L	M	N	O
1	Revised Control Limits				
2	Centre Line	UCL	LCL	Sample Mean	
3	388.333	820.937	0.000	933.000	
4	388.333	820.937	0.000	789.800	
5	388.333	820.937	0.000	1156.200	
6	388.333	820.937	0.000	918.400	
7	388.333	820.937	0.000	1026.600	
8	388.333	820.937	0.000	887.000	
9	388.333	820.937	0.000	1082.200	
10	388.333	820.937	0.000	837.200	
11	388.333	820.937	0.000	1161.200	
12	388.333	820.937	0.000	950.200	
13	388.333	820.937	0.000	938.200	
14	388.333	820.937	0.000	961.200	
15	388.333	820.937	0.000	1140.600	

Fig. 4.15

Step 2: We calculate $\bar{\bar{X}}_{new}$ by typing "= (Sum(N3:N37)-N12-N33)/(G40-G44)" in Cell N38 (Fig. 4.16).



	L	M	N	O	P	Q
38	Revised Average		967.655			
39						

Fig. 4.16

Step 3: We already have the value of d = 2 in Cell G44 (see Fig. 4.9). We now type the value of A_2 for n = 5 in Cell G45 (Fig. 4.17).

	D	E	F	G
44			d	2
45	A_2 Value from Table (for n=5)			0.577

Fig. 4.17

Step 4: We compute the centre line, UCL and LCL in Columns O, P and Q, respectively, as explained in Sec. 2.5 of Lab Session 2 (Fig. 4.18).

	O	P	Q
1	Control Limits		
2	Centre Line	UCL	LCL
3	967.655	1191.723	743.586
4	967.655	1191.723	743.586
5	967.655	1191.723	743.586
6	967.655	1191.723	743.586
7	967.655	1191.723	743.586
8	967.655	1191.723	743.586
9	967.655	1191.723	743.586
10	967.655	1191.723	743.586
11	967.655	1191.723	743.586
12	967.655	1191.723	743.586
13	967.655	1191.723	743.586
14	967.655	1191.723	743.586
15	967.655	1191.723	743.586
16	967.655	1191.723	743.586
17	967.655	1191.723	743.586
18	967.655	1191.723	743.586
19	967.655	1191.723	743.586
20	967.655	1191.723	743.586
21	967.655	1191.723	743.586
22	967.655	1191.723	743.586
23	967.655	1191.723	743.586

R Chart

Fig. 4.18

Step 5: To plot the $\bar{X}$-chart, we

1. select Cells N2:Q11, N13:Q32 and N34:Q37 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting control chart is shown in Fig. 4.19.

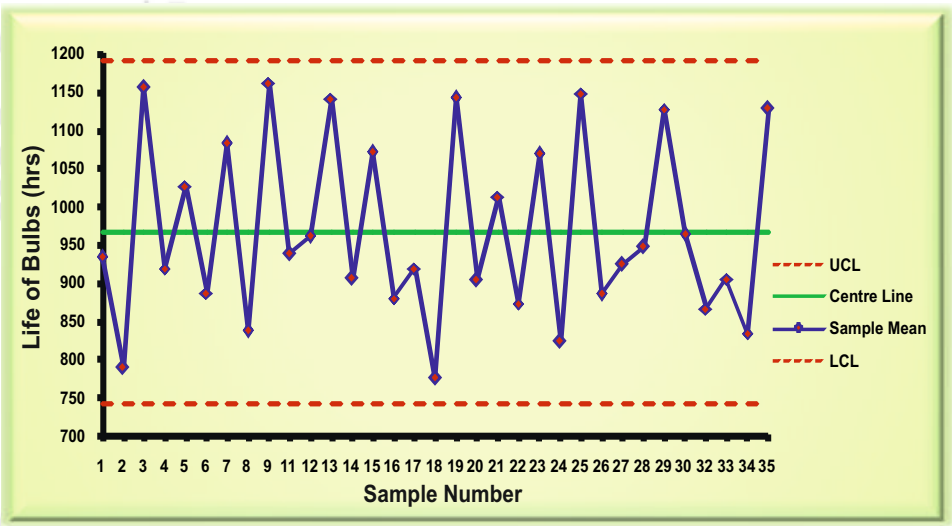


Fig. 4.19

Interpretation

Note from Fig. 4.19 that no point lies outside the control limits of the revised $\overline{X}$-chart. This indicates that the process mean is under control. Hence, we may conclude that the process is under statistical control with respect to the process variability and process mean.

Now, you can try the following exercises to revise the R-chart and $\overline{X}$-chart which are used to control the process variability and mean, respectively.

Activity 2

Construct the control charts for range with the help of MS Excel 2007 and interpret the results for

A1) Example 5 given in Unit 2 of MSTE-001.

A2) Exercise E10 given in Unit 2 of MSTE-001.

Match the results with the manual calculation done in Unit 2 of MSTE-001

Continuous Assessment 4

Consider a problem related to a process which maintains the quality of bottling procedure. Suppose a fruit juice manufacturing company uses automatic machines to fill 500 ml juice bottles. A quality control inspector at the company collects 25 samples each of four observations at different times and measures the volume of each filled bottle. The data is given in Table 2.

Table 2: Volume of fruit juice in the filled bottles

Sample	Volume of Juice per bottle (in ml)			Number
	Obs. 1	Obs. 2	Obs. 3	Obs. 4
1	497.32	500.62	498.68	497.82
2	504.76	500.00	498.32	500.32
3	499.24	497.18	498.12	498.68
4	499.26	496.32	498.88	497.82
5	498.32	500.62	499.56	500.12
6	499.12	500.32	499.38	500.94
7	499.34	498.32	497.32	497.62
8	499.38	498.12	500.62	498.12
9	499.26	498.38	500.68	500.38
10	498.60	497.62	499.25	498.56
11	499.44	500.00	501.32	499.38

Sample	Volume of Juice per bottle (in ml)			Number
	Obs. 1	Obs. 2	Obs. 3	Obs. 4
12	498.26	500.32	500.76	499.68
13	497.32	498.50	497.18	499.38
14	499.56	498.00	498.76	501.12
15	500.24	500.32	499.12	499.25
16	500.76	500.50	499.68	498.12
17	500.65	497.82	494.06	496.25
18	499.12	500.26	500.44	498.76
19	499.50	500.50	499.56	500.76
20	497.50	498.82	499.76	497.82
21	499.44	500.62	500.00	501.26
22	499.38	498.38	497.56	498.56
23	501.56	499.56	498.00	499.82
24	498.32	497.32	499.56	498.62
25	499.50	500.12	498.50	500.38

Develop the $\overline{X}$ and R-charts to check whether the process of bottling is under control or out-of-control. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Secs. 4.4, 4.5 and 4.6 to construct the control charts for the data of Table 1. Use a different format for the control charts. Take their screenshots and keep them in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 4" as explained in this lab session. Take screenshots of the final spreadsheet and the charts.

3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.

Control Chart for Standard Deviation

SESSION 5

Structure

5.1 Introduction

Objectives

5.2 Problem Description

5.3 Procedure for the Construction of S-Chart

5.4 Steps Involved in the Construction of S-Chart in Excel 2007

5.5 Revised S-Chart

5.6 Revised $\bar{X}$-Chart

5.1 INTRODUCTION

Prerequisite

- Lab Sessions 1, 3 and 7 of MSTL-001 (Basic Statistics Lab).
- Lab Sessions 3 and 4 of MSTL-002 (Industrial Statistics Lab).
- Unit 2 of MSTE-001 (Industrial Statistics-I).

You have learnt how to construct control charts for process mean, i.e., $\bar{X}$-chart in Lab Sessions 1 to 3. We have also described one control chart for process variability, i.e., R-chart in Lab Session 4. R-chart is easy to calculate and explain. But for large sample sizes (usually greater than 10), it does not give the correct picture of the process variability since range depends only on maximum and minimum values of the data.

In such situations, we prefer to use standard deviation as discussed in Sec. 2.6 of Unit 2 of MSTE-001 (Industrial Statistics-I). In this Lab Session, you will learn how to construct the S-chart using MS Excel 2007.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for control chart for standard deviation;
- construct the control chart for standard deviation;
- obtain the revised control limits for control charts for standard deviation and mean; and
- interpret the control charts.

5.2 PROBLEM DESCRIPTION

Suppose a bulb manufacturing company wants to check whether the variation in the life of bulbs produced by a particular machine is due to chance causes or due to assignable causes. For this purpose, a quality control inspector at this company selects 35 samples each of size 12 and measures the life of each bulb (in hours).

Table 1: Life of bulbs

Sample Number	Life of Bulbs (in hours)						
	1	2	3	4	5	6	7
Obs. 1	1152	854	876	725	900	1080	952
Obs. 2	834	822	987	1150	926	870	1167
Obs. 3	1170	954	1267	927	1334	951	924
Obs. 4	788	875	1067	1208	984	928	1154
Obs. 5	1145	1129	1242	902	1309	926	1181
Obs. 6	724	712	985	1040	816	760	1057
Obs. 7	1060	1044	1157	817	1224	841	1096
Obs. 8	678	765	957	1098	874	818	1044
Obs. 9	754	742	1015	1070	846	790	1087
Obs. 10	1090	1074	1187	847	1254	871	1126
Obs. 11	708	795	987	1128	904	848	1074
Obs. 12	1065	1049	1162	822	1229	846	1101

Table 1: Life of bulbs (continued…)

Sample Number	Life of Bulbs (in hours)						
	8	9	10	11	12	13	14
Obs. 1	741	1241	1194	1200	745	926	781
Obs. 2	1066	1124	1085	950	1044	1015	956
Obs. 3	658	987	694	939	1130	1210	1061
Obs. 4	1124	1158	1143	1008	1102	1124	1014
Obs. 5	917	1067	955	914	1105	987	1036
Obs. 6	956	1034	975	840	934	1142	846
Obs. 7	548	1110	875	829	1020	1024	951
Obs. 8	1014	957	1033	898	992	1068	904
Obs. 9	986	1064	1005	870	964	1172	876
Obs. 10	578	1140	614	859	1050	1054	981
Obs. 11	1044	1078	1063	928	1022	1098	934
Obs. 12	837	987	575	834	1025	1175	956

Table 1: Life of bulbs (continued…)

Sample Number	Life of Bulbs (in hours)						
	15	16	17	18	19	20	21
Obs. 1	1120	880	1138	674	862	711	886
Obs. 2	1430	1055	716	906	1127	1136	912
Obs. 3	830	848	1156	1140	987	913	1320
Obs. 4	1488	1113	774	664	1137	1194	970
Obs. 5	805	823	1131	1115	894	888	1295
Obs. 6	1320	945	857	696	1004	1026	802
Obs. 7	720	738	1046	1030	1143	803	1210
Obs. 8	1378	1003	664	754	1027	1084	860
Obs. 9	1350	975	636	826	1034	1056	832
Obs. 10	750	768	1076	1060	1173	833	1240
Obs. 11	1408	1033	694	584	1057	1114	890
Obs. 12	725	743	1051	1035	1148	808	1215

Table 1: Life of bulbs (continued…)

Sample Number	Life of Bulbs (in hours)						
	22	23	24	25	26	27	28
Obs. 1	1066	938	727	991	1227	1186	731
Obs. 2	856	1153	1052	1067	1071	936	1030
Obs. 3	937	1192	644	1206	878	925	1116
Obs. 4	914	1047	1110	1048	1129	994	1088
Obs. 5	912	1167	903	1181	758	900	1091
Obs. 6	746	1043	942	957	961	826	920
Obs. 7	827	1082	534	1096	768	815	1006
Obs. 8	804	937	1000	938	1019	884	978
Obs. 9	776	1073	972	987	991	856	950
Obs. 10	857	1112	564	1126	798	845	1036
Obs. 11	834	967	1030	968	1049	914	1008
Obs. 12	832	1087	823	1101	678	820	1011

Table 1: Life of bulbs (continued…)

Sample Number	Life of Bulbs (in hours)						
	29	30	31	32	33	34	35
Obs. 1	912	1124	1106	866	1124	660	848
Obs. 2	1238	942	1416	1041	1067	738	974
Obs. 3	967	1047	658	834	1142	1126	1239
Obs. 4	1064	1000	1570	1099	760	864	937
Obs. 5	1117	1022	791	809	1117	1101	1214
Obs. 6	1128	832	1306	931	957	628	957
Obs. 7	1156	937	548	724	1032	1016	1129
Obs. 8	954	890	1460	989	650	754	827
Obs. 9	1158	862	1336	961	987	658	1314
Obs. 10	1186	967	578	754	1062	1046	1159
Obs. 11	984	920	1490	1019	680	784	857
Obs. 12	1037	942	711	729	1037	1021	1134

The quality control inspector of this company needs to construct a control chart for standard deviation as well as mean to infer whether the production is under control or not. If it is out-of-control, he/she also computes the revised control limits.

Therefore, the problem for this session is to construct the control charts for standard deviation and mean for the data given in Table 1.

5.3 PROCEDURE FOR THE CONSTRUCTION OF S-CHART

You have already learnt how to calculate control limits for S-chart in Sec. 2.7 of Unit 2 of MSTE-001. The main steps involved in the construction of S-chart for the data given in Table 1 are as follows:

When the value of σ is known

Step 1: The control limits for S-chart for known σ are given by

- ✓ Centre line (CL) $= c_4\sigma$ ...(1)
- ✓ Upper control limit (UCL) $= B_6\sigma$ ...(2)
- ✓ Lower control limit (LCL) $= B_5\sigma$ ...(3)

where c_4, B_5 and B_6 are constants and depend on the size of the sample. These constants have been tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 2: Interpretation of the S-chart.

When the value of σ is unknown

In practice, the value of σ is not known. Therefore, it is estimated from the samples, which are taken when the process is assumed to be under control. In S-chart, σ is estimated by sample standard deviation (S).

Step 1-2: The first two steps are the same as Steps 4 and 5 of Sec. 3.3 of Lab Session 3.

Step 3: The control limits for S-chart when σ is estimated by $\bar{S}/c_4$ are given by

- ✓ Centre line (CL) $= \bar{S}$ ...(4)
- ✓ Upper control limit (UCL) $= B_4\bar{S}$ ...(5)
- ✓ Lower control limit (LCL) $= B_3\bar{S}$ ...(6)

where B_3 and B_4 are constants and depend on the size of the sample. These constants have been tabulated for various sample sizes in the Appendix given at the end of this lab course.

Step 4: Interpretation of the S-chart.

5.4 STEPS INVOLVED IN THE CONSTRUCTION OF S-CHART IN EXCEL 2007

In order to calculate the control limits of S-chart and to plot the control chart in Excel 2007 for the given data, we follow the steps given below:

Step 1: We enter the given data in Excel sheet as shown in Fig. 5.1.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1							Life of Bulbs (in hours)						
2	Sample No.	Obs. 1	Obs. 2	Obs. 3	Obs. 4	Obs. 5	Obs. 6	Obs. 7	Obs. 8	Obs. 9	Obs. 10	Obs. 11	Obs. 12
3	1	1152	834	1170	788	1145	724	1060	678	754	1090	708	1065
4	2	854	822	954	875	1129	712	1044	765	742	1074	795	1049
5	3	876	987	1267	1067	1242	985	1157	957	1015	1187	987	1162
6	4	725	1150	927	1208	902	1040	817	1098	1070	847	1128	822
7	5	900	926	1334	984	1309	816	1224	874	846	1254	904	1229
8	6	1080	870	951	928	926	760	841	818	790	871	848	846
9	7	952	1167	924	1154	1181	1057	1096	1044	1087	1126	1074	1101
10	8	741	1066	658	1124	917	956	548	1014	986	578	1044	837
11	9	1241	1124	987	1158	1067	1034	1110	957	1064	1140	1078	987
12	10	1194	1085	694	1143	955	975	875	1033	1005	614	1063	575
13	11	1200	950	939	1008	914	840	829	898	870	859	928	834
14	12	745	1044	1130	1102	1105	934	1020	992	964	1050	1022	1025
15	13	926	1015	1210	1124	987	1142	1024	1068	1172	1054	1098	1175
16	14	781	956	1061	1014	1036	846	951	904	876	981	934	956
17	15	1120	1430	830	1488	805	1320	720	1378	1350	750	1408	725
18	16	880	1055	848	1113	823	945	738	1003	975	768	1033	743
19	17	1138	716	1156	774	1131	857	1046	664	636	1076	694	1051
20	18	674	906	1140	664	1115	696	1030	754	826	1060	584	1035
21	19	862	1127	987	1137	894	1004	1143	1027	1034	1173	1057	1148
22	20	711	1136	913	1194	888	1026	803	1084	1056	833	1114	808
23	21	886	912	1320	970	1295	802	1210	860	832	1240	890	1215

S Chart

Fig. 5.1: Partial screenshot of the spreadsheet for the given data.

Step 2: We calculate the standard deviation of the first sample by typing "=Stdev(B3:M3)" in Cell N3. We then drag down Cell N3 up to Cell N37 as shown in Fig. 5.2.

N3 =STDEV(B3:M3)

	K	L	M	N	O
1					
2	Obs. 10	Obs. 11	Obs. 12	SD	
3	1090	708	1065	197.578	
4	1074	795	1049		
5	1187	987	1162		
6	847	1128	822		
7	1254	904	1229		

DRAG IT DOWN

	K	L	M	N	O
1					
2	Obs. 10	Obs. 11	Obs. 12	SD	
3	1090	708	1065	197.578	
4	1074	795	1049	143.633	
5	1187	987	1162	125.175	
6	847	1128	822	157.103	
7	1254	904	1229	200.582	
8	871	848	846	85.071	
9	1126	1074	1101	79.021	
10	578	1044	837	197.382	
11	1140	1078	987	81.686	
12	614	1063	575	204.506	
13	859	928	834	102.787	
14	1050	1022	1025	101.597	
15	1054	1098	1175	85.778	
16	981	934	956	80.437	
17	750	1408	725	317.527	
18	768	1033	743	128.421	
19	1076	694	1051	206.361	
20	1060	584	1035	198.013	
21	1173	1057	1148	101.567	

Fig. 5.2

Step 3: In Cell N38, we find the average of standard deviations obtained from 35 samples as shown in Fig. 5.3.

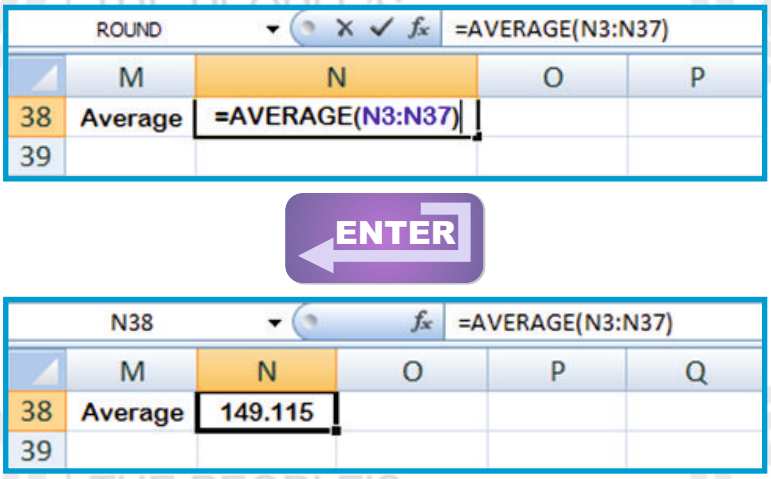


Fig. 5.3

Step 4: We type the values of k and n in Cells N40 and N41, respectively. We also type the values of B_3 and B_4 for n = 12 in Cells N42 and N43, respectively, from the Appendix given at the end of this lab course (Fig. 5.4).

	L	M	N
40		k	35
41		n	12
42	B_3 Value from Table		0.354
43	B_4 Value from Table		1.646

Fig. 5.4

Step 5: We now use the method for computing the centre line and both control limits described in Sec. 5.3. Here we use Columns O, P and Q for putting the values of the centre line, upper and lower control limits, respectively. We compute the centre line, upper and lower control limits as follows:

i) From equation (4), the centre line is CL = $\bar{S}$ and $\bar{S}$ is given in Cell N38 (see Fig. 5.3). So we type "=$N$38" in Cell O3 to obtain the centre line as shown in Fig. 5.5a.

The formula with dollar sign ($) is used for an absolute reference.

ii) From equation (5), the upper control limit is $UCL = B_4\bar{S}$. The values of B_4 and $\bar{S}$ are given in Cells N43 and N38, respectively (see Figs. 5.4 and 5.3). So we type "=$N$43*$N$38" in Cell P3 as shown in Fig. 5.5b.

iii) Similarly, we calculate the lower control limit $LCL = B_3\bar{S}$ (see equation (6)). The values of B_3 and $\bar{S}$ are given in Cells N42 and N38, respectively (see Figs. 5.4 and 5.3). So we type "=$N$42*$N$38" in Cell Q3 as shown in Fig. 5.5c.

(a)

O3 =$N$38

	O	P	Q	R
1	Control Limits			
2	Centre Line	UCL	LCL	
3	149.115			
4				

(b)

P3 =$N$43*$N$38

	P	Q	R	S
1	ntrol Limits			
2	UCL	LCL		
3	245.443			
4				

(c)

Q3 =$N$42*$N$38

	Q	R	S	T
1	ts			
2	LCL			
3	52.787			
4				

Fig. 5.5

Step 6: For plotting control limits on chart using Excel 2007, we first select Cells O3:Q3 and drag them down up to Row 37 as shown in Fig. 5.6.

	N	O	P	Q	R
1		Control Limits			
2	Standard Deviation	Centre Line	UCL	LCL	
3	197.578	149.115	245.443	52.787	
4	143.633				
5	125.175				
6	157.103				
7	200.582				
8	85.071				
9	79.021				
10	197.382				

DRAG THEM DOWN

	N	O	P	Q	R
1		Control Limits			
2	Standard Deviation	Centre Line	UCL	LCL	
3	197.578	149.115	245.443	52.787	
4	143.633	149.115	245.443	52.787	
5	125.175	149.115	245.443	52.787	
6	157.103	149.115	245.443	52.787	
7	200.582	149.115	245.443	52.787	
8	85.071	149.115	245.443	52.787	
9	79.021	149.115	245.443	52.787	
10	197.382	149.115	245.443	52.787	
11	81.686	149.115	245.443	52.787	
12	204.506	149.115	245.443	52.787	
13	102.787	149.115	245.443	52.787	
14	101.597	149.115	245.443	52.787	
15	85.778	149.115	245.443	52.787	
16	80.437	149.115	245.443	52.787	
17	317.527	149.115	245.443	52.787	
18	128.421	149.115	245.443	52.787	
19	206.361	149.115	245.443	52.787	
20	198.013	149.115	245.443	52.787	
21	101.567	149.115	245.443	52.787	
22	157.103	149.115	245.443	52.787	
23	200.582	149.115	245.443	52.787	

S Chart

Fig. 5.6

Step 7: To obtain the S-chart in Excel 2007, we refer to Fig. 5.7. It means that we

1. select Cells N2:Q37,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

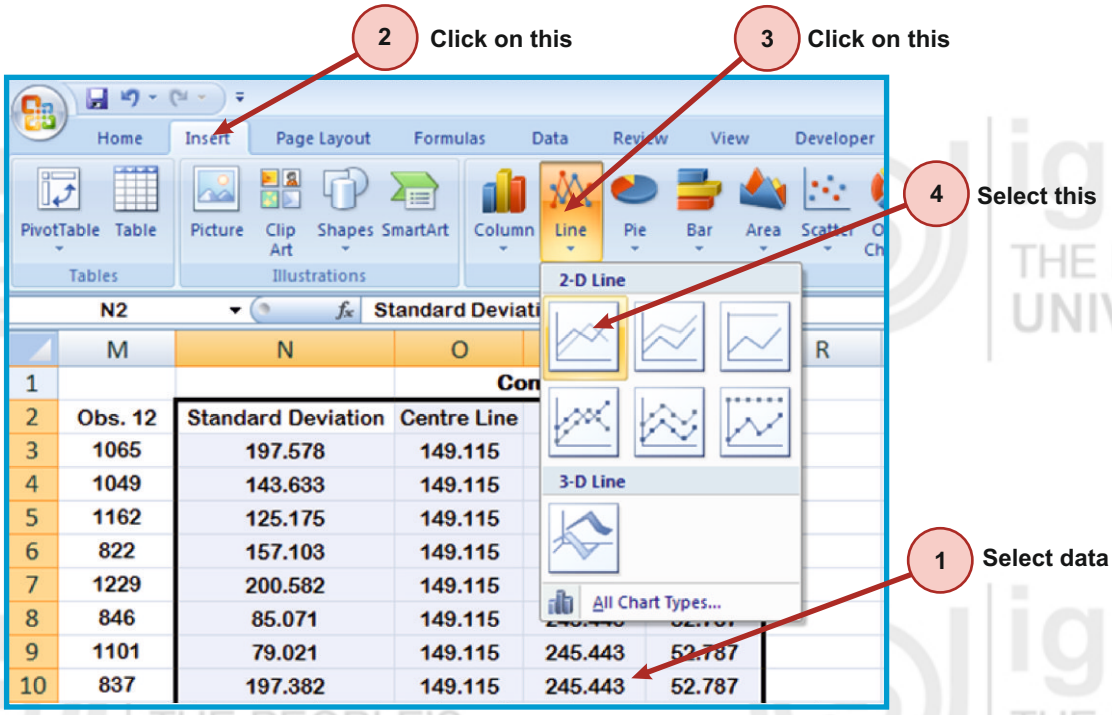


Fig. 5.7

Step 8: Columns O, P and Q provide horizontal lines on the chart representing the centre line, UCL and LCL, respectively. The values in Column N provide the sample standard deviations for 35 samples on the chart. We format the chart as explained in Sec.1.4 of Lab Session 1 and obtain the S-chart shown in Fig. 5.8.

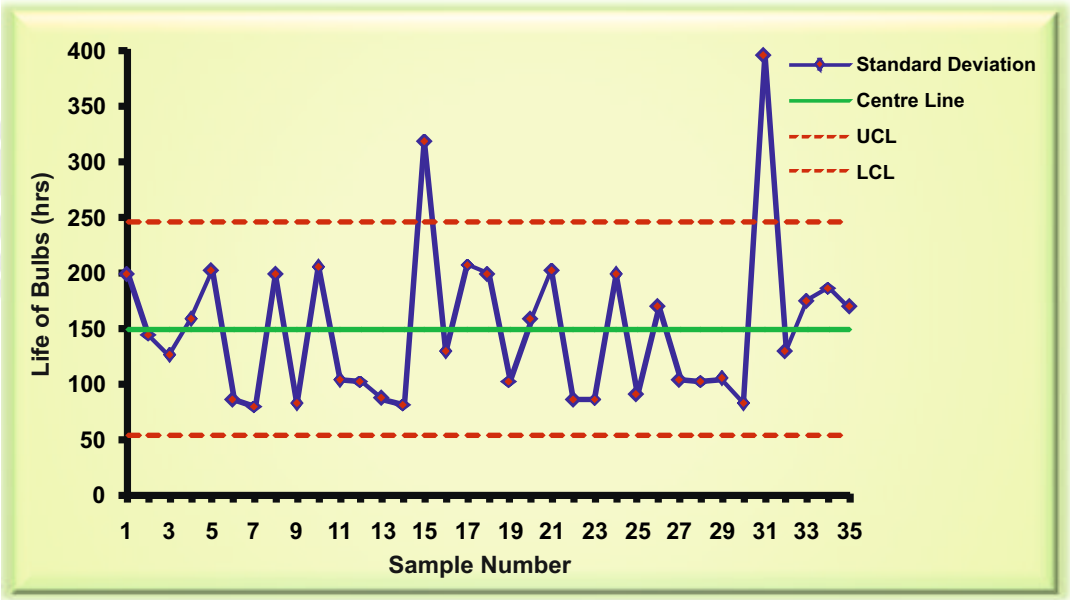


Fig. 5.8

Interpretation

We note from Fig. 5.8 that two points corresponding to Samples 15 and 31 lie outside the UCL. Therefore, the control chart indicates that the process is **not under statistical control**. Some **assignable causes** are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them.

5.5 REVISED S-CHART

After eliminating the assignable causes, we revise the S-chart given in Fig. 5.8. For this purpose, we delete the out-of-control samples and compute revised control limits for S-chart using only the remaining samples as described below:

Step 1: For revised limits for S-chart, we first calculate new $\bar{S}$ as follows:

$$\bar{S}_{new} = \frac{\sum_{i=1}^{k} S_i - \sum_{j=1}^{d} S_j}{k-d} \qquad \ldots(7)$$

where d – Number of discarded samples

$\sum_{j=1}^{d} S_j$ – Sum of the ranges of discarded samples

Step 2: After finding the new $\bar{S}$, we reconstruct the centre line and control limits the chart by replacing $\bar{S}$ by $\bar{S}_{new}$ as given below:

- ✓ Centre Line (CL) = $\bar{S}_{new}$ …(8)
- ✓ Upper control limit (UCL) = $B_4\bar{S}_{new}$ …(9)
- ✓ Lower control limit (LCL) = $B_3\bar{S}_{new}$ …(10)

Step 3: Interpretation of the S-chart.

Steps in Excel

The steps for obtaining the revised centre line and control limits for the S-chart in Excel 2007 using the remaining samples are as follows:

Step 1: We highlight the samples which lie outside the control limits, i.e., the 15th and 31st samples, with light orange colour as shown in Fig. 5.9.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1		Life of Bulbs (in hours)													Control Limits		
2	Sample No.	Obs. 1	Obs. 2	Obs. 3	Obs. 4	Obs. 5	Obs. 6	Obs. 7	Obs. 8	Obs. 9	Obs. 10	Obs. 11	Obs. 12	Standard Deviation	Centre Line	UCL	LCL
3	1	1152	834	1170	788	1145	724	1060	678	754	1090	708	1065	197.578	149.115	245.443	52.787
4	2	854	822	954	875	1129	712	1044	765	742	1074	795	1049	143.633	149.115	245.443	52.787
5	3	876	987	1267	1067	1242	985	1157	957	1015	1187	987	1162	125.175	149.115	245.443	52.787
6	4	725	1150	927	1208	902	1040	817	1098	1070	847	1128	822	157.103	149.115	245.443	52.787
7	5	900	926	1334	984	1309	816	1224	874	846	1254	904	1229	200.582	149.115	245.443	52.787
8	6	1080	870	951	928	926	760	841	818	790	871	848	846	85.071	149.115	245.443	52.787
9	7	952	1167	924	1154	1181	1057	1096	1044	1087	1126	1074	1101	79.021	149.115	245.443	52.787
10	8	741	1066	658	1124	917	956	548	1014	986	578	1044	837	197.382	149.115	245.443	52.787
11	9	1241	1124	987	1158	1067	1034	1110	957	1064	1140	1078	987	81.686	149.115	245.443	52.787
12	10	1194	1085	694	1143	955	975	875	1033	1005	614	1063	575	204.506	149.115	245.443	52.787
13	11	1200	950	939	1008	914	840	829	898	870	859	928	834	102.787	149.115	245.443	52.787
14	12	745	1044	1130	1102	1105	934	1020	992	964	1050	1022	1025	101.597	149.115	245.443	52.787
15	13	926	1015	1210	1124	987	1142	1024	1068	1172	1054	1098	1175	85.778	149.115	245.443	52.787
16	14	781	956	1061	1014	1036	846	951	904	876	981	934	956	80.437	149.115	245.443	52.787
17	15	1120	1430	830	1488	805	1320	720	1378	1350	750	1408	725	317.527	149.115	245.443	52.787
18	16	880	1055	848	1113	823	945	738	1003	975	768	1033	743	128.421	149.115	245.443	52.787
19	17	1138	716	1156	774	1131	857	1046	664	636	1076	694	1051	206.361	149.115	245.443	52.787

⋮ ⋮

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
32	30	1124	942	1047	1000	1022	832	937	890	862	967	920	942	81.754	149.115	245.443	52.787
33	31	1106	1416	658	1570	791	1306	548	1460	1336	578	1490	711	394.919	149.115	245.443	52.787
34	32	866	1041	834	1099	809	931	724	989	961	754	1019	729	128.421	149.115	245.443	52.787
35	33	1124	1067	1142	760	1117	957	1032	650	987	1062	680	1037	173.761	149.115	245.443	52.787
36	34	660	738	1126	864	1101	628	1016	754	658	1046	784	1021	185.803	149.115	245.443	52.787
37	35	848	974	1239	937	1214	957	1129	827	1314	1159	857	1134	168.538	149.115	245.443	52.787

Fig. 5.9

Step 2: We have two samples outside the UCL. So we put d = 2 in Cell N44 as shown in Fig. 5.10.

	L	M	N
40		k	35
41		n	12
42	B_3 Value from Table		0.354
43	B_4 Value from Table		1.646
44		d	2
45	A_3 Value from Table		0.886

Fig. 5.10

Step 3: In Cell N39, we use the formula "=(Sum(N3:N37)-N17-N33)/(N40-N44)" and press ***Enter*** to calculate the revised $\bar{S}$, i.e., $\bar{S}_{new}$ as shown in Fig. 5.11.

AVERAGE =(SUM(N3:N37)-N17-N33)/(N40-N44)

	L	M	N
39	Revised Average		=(SUM(N3:N37)-N17-N33)/(N40-N44)
40		k	35
41		n	12
42	B_3 Value from Table		0.354
43	B_4 Value from Table		1.646
44		d	2
45			

	L	M	N
39	Revised Average		136.563

Fig. 5.11

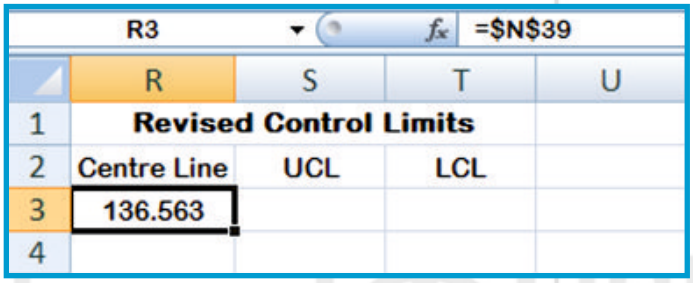
Step 4: As explained in Step 5 of Sec. 5.4 of Lab Session 5, we use Columns R, S and T for typing the values of revised centre line, upper and lower control limits, respectively.

i) From equation (8), the revised centre line is (CL) = $\bar{S}_{new}$ and $\bar{S}_{new}$ is given in N39 (see Fig. 5.11). So we type "=$N$39" in Cell R3 to find the centre line as shown in Fig. 5.12a.

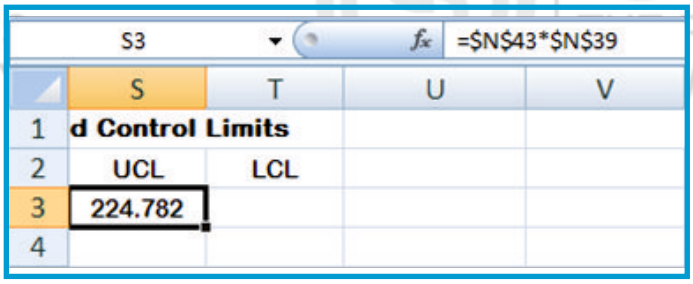
ii) From equation (9), the upper control limit is $UCL = B_4\bar{S}_{new}$. The values of B_4 and $\bar{S}_{new}$ are given in Cells N43 and N39, respectively (see Figs. 5.10 and 5.11). So we type "=$N$43*$N$39" in Cell S3 as shown in Fig. 5.12b.

The formula with dollar sign ($) is used for an absolute reference.

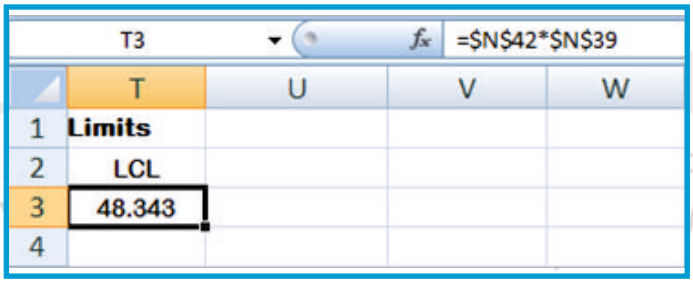
iii) Similarly, we calculate the lower control limit $LCL = B_3\bar{S}_{new}$ (see equation 10) by typing "=$N$42*$N$39" in Cell T3 as shown in Fig. 5.12c.



(a)



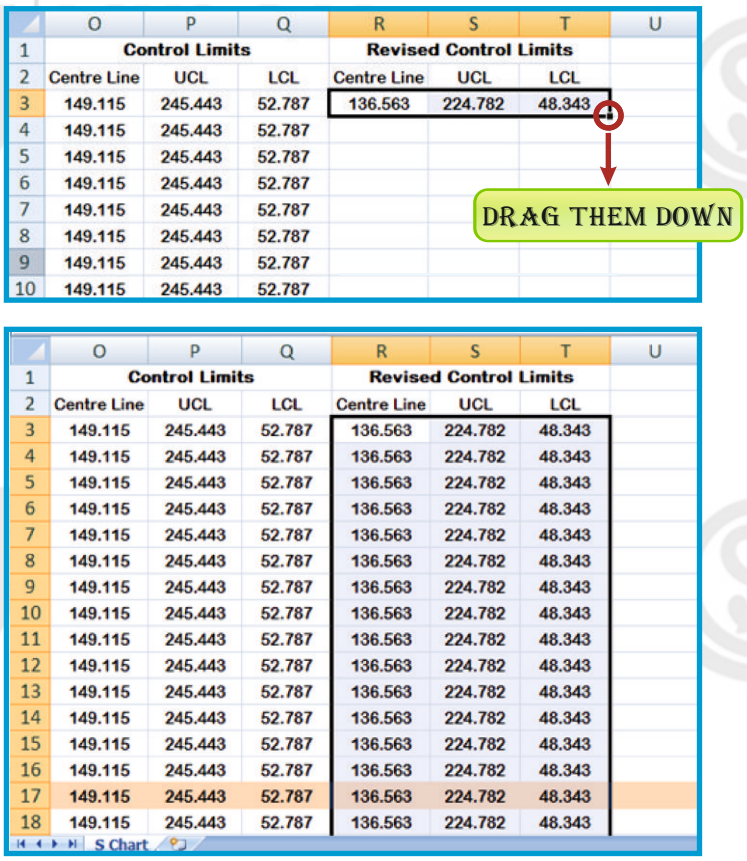
(b)



(c)

Fig. 5.12

Step 5: For plotting control limits on chart using Excel 2007, we select Cells R3:T3 and drag them down up to Row 37.



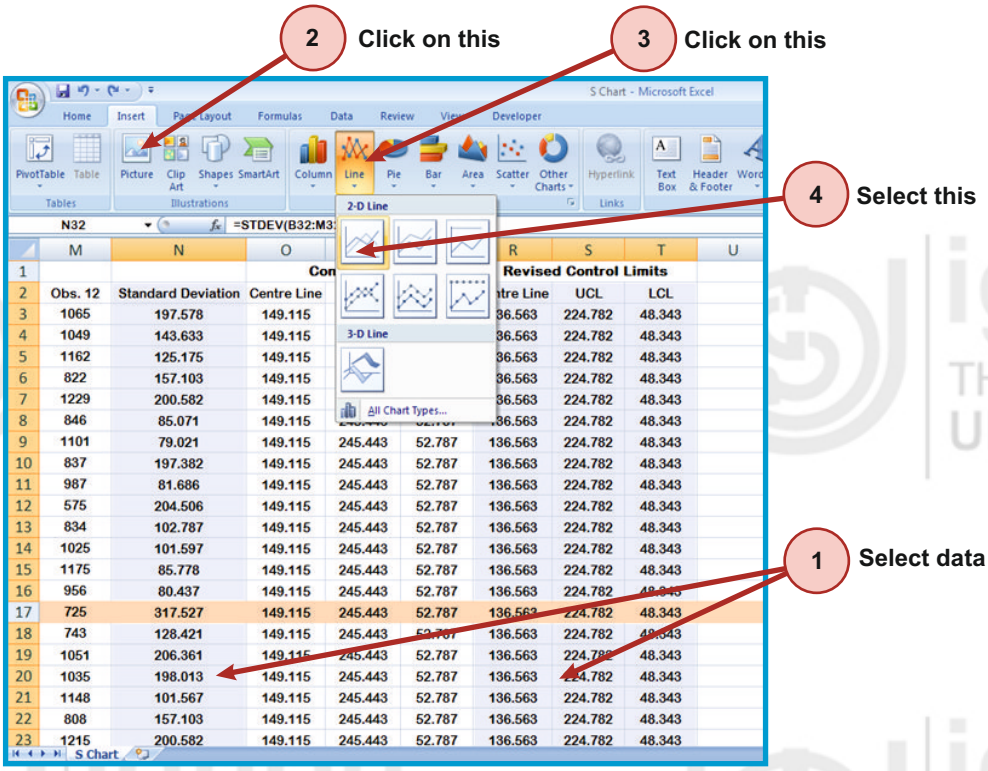
	O	P	Q	R	S	T	U
1	Control Limits			Revised Control Limits			
2	Centre Line	UCL	LCL	Centre Line	UCL	LCL	
3	149.115	245.443	52.787	136.563	224.782	48.343	
4	149.115	245.443	52.787				
5	149.115	245.443	52.787				
6	149.115	245.443	52.787				
7	149.115	245.443	52.787				
8	149.115	245.443	52.787				
9	149.115	245.443	52.787				
10	149.115	245.443	52.787				

	O	P	Q	R	S	T	U
1	Control Limits			Revised Control Limits			
2	Centre Line	UCL	LCL	Centre Line	UCL	LCL	
3	149.115	245.443	52.787	136.563	224.782	48.343	
4	149.115	245.443	52.787	136.563	224.782	48.343	
5	149.115	245.443	52.787	136.563	224.782	48.343	
6	149.115	245.443	52.787	136.563	224.782	48.343	
7	149.115	245.443	52.787	136.563	224.782	48.343	
8	149.115	245.443	52.787	136.563	224.782	48.343	
9	149.115	245.443	52.787	136.563	224.782	48.343	
10	149.115	245.443	52.787	136.563	224.782	48.343	
11	149.115	245.443	52.787	136.563	224.782	48.343	
12	149.115	245.443	52.787	136.563	224.782	48.343	
13	149.115	245.443	52.787	136.563	224.782	48.343	
14	149.115	245.443	52.787	136.563	224.782	48.343	
15	149.115	245.443	52.787	136.563	224.782	48.343	
16	149.115	245.443	52.787	136.563	224.782	48.343	
17	149.115	245.443	52.787	136.563	224.782	48.343	
18	149.115	245.443	52.787	136.563	224.782	48.343	

Fig. 5.13

Step 6: To obtain the S-chart, we refer to Fig. 5.14. It means that we

1. select Cells N2:N16, N18:N32, N34:N37, R2:T11, R18:T32 and R34:T37 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype



	M	N	O			R	S	T	U
1			Con			Revised Control Limits			
2	Obs. 12	Standard Deviation	Centre Line			tre Line	UCL	LCL	
3	1065	197.578	149.115			36.563	224.782	48.343	
4	1049	143.633	149.115			36.563	224.782	48.343	
5	1162	125.175	149.115			36.563	224.782	48.343	
6	822	157.103	149.115			36.563	224.782	48.343	
7	1229	200.582	149.115			36.563	224.782	48.343	
8	846	85.071	149.115			36.563	224.782	48.343	
9	1101	79.021	149.115	245.443	52.787	136.563	224.782	48.343	
10	837	197.382	149.115	245.443	52.787	136.563	224.782	48.343	
11	987	81.686	149.115	245.443	52.787	136.563	224.782	48.343	
12	575	204.506	149.115	245.443	52.787	136.563	224.782	48.343	
13	834	102.787	149.115	245.443	52.787	136.563	224.782	48.343	
14	1025	101.597	149.115	245.443	52.787	136.563	224.782	48.343	
15	1175	85.778	149.115	245.443	52.787	136.563	224.782	48.343	
16	956	80.437	149.115	245.443	52.787	136.563	224.782	48.343	
17	725	317.527	149.115	245.443	52.787	136.563	224.782	48.343	
18	743	128.421	149.115	245.443	52.787	136.563	224.782	48.343	
19	1051	206.361	149.115	245.443	52.787	136.563	224.782	48.343	
20	1035	198.013	149.115	245.443	52.787	136.563	224.782	48.343	
21	1148	101.567	149.115	245.443	52.787	136.563	224.782	48.343	
22	808	157.103	149.115	245.443	52.787	136.563	224.782	48.343	
23	1215	200.582	149.115	245.443	52.787	136.563	224.782	48.343	

Fig. 5.14

Step 7: We format the chart and change the horizontal axis as discussed in Steps 9 and 10 of Sec. 2.5 of Lab Session 2. The resulting S-chart is shown in Fig. 5.15.

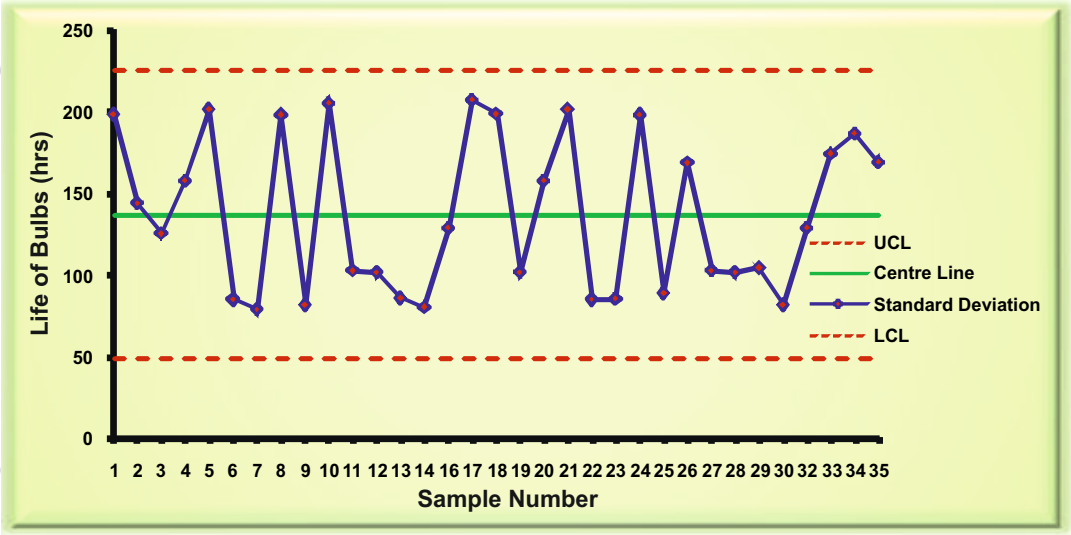


Fig. 5.15

If one or more points lie outside the revised control limits, we calculate the revised control limits of S-chart again. This process is continued until the process is brought under control.

Interpretation

This S-chart indicates that all points lie within the control limits. So the process variability is under statistical control.

After controlling the process variability, we now study the process mean. For this, we calculate the control limits of $\bar{X}$-chart using the remaining samples.

Before studying the next section, you should perform Activity 1.

Activity 1

You can also determine the revised control limits using another approach. For this purpose, follow the steps given below:

- ✓ Select Cells A2:M16, A18:M32, A34:M37.
- ✓ Choose Cell A48 or any other cell and paste the values. You can also use a separate Excel sheet where you can paste these values.
- ✓ Repeat all the steps given in Sec. 5.4.

It will give you the same results as you have obtained in Sec. 5.5.

5.6 REVISED $\bar{X}$-CHART

Recall from Unit 2 of MSTE-001 that we control the process mean and process variability simultaneously. So we use both $\bar{X}$ and S-charts together. We have already revised the S-chart in Sec. 5.5 to control the process variability. Now, we use $\bar{S}_{new}$ to estimate the process variability and calculate $\bar{\bar{X}}_{new}$ by deleting the out-of-control samples. We calculate the revised control

limits for the $\bar{X}$-chart in the same manner as discussed in Sec. 3.5 of the Lab Session 3.

We compute the revised centre line and control limits for the $\bar{X}$-chart in Excel 2007 for the remaining samples as follows:

Step 1: We determine the sample means for all samples in Cells U3:U37 as explained in Steps 1-5 of Sec. 1.4 of Lab Session 1. The output is shown in Fig. 5.16.

U3 =AVERAGE(B3:M3)

	M	N	O	P	Q	R	S	T	U	V
1				Control Limits		Revised Control Limits				
2	Obs. 12	Standard Deviation	Centre Line	UCL	LCL	Centre Line	UCL	LCL	Sample Mean	
3	1065	197.578	149.115	245.443	52.787	136.563	224.782	48.343	930.667	
4	1049	143.633	149.115	245.443	52.787	136.563	224.782	48.343	901.250	
5	1162	125.175	149.115	245.443	52.787	136.563	224.782	48.343	1074.083	
6	822	157.103	149.115	245.443	52.787	136.563	224.782	48.343	977.833	
7	1229	200.582	149.115	245.443	52.787	136.563	224.782	48.343	1050.000	
8	846	85.071	149.115	245.443	52.787	136.563	224.782	48.343	877.417	
9	1101	79.021	149.115	245.443	52.787	136.563	224.782	48.343	1080.250	
10	837	197.382	149.115	245.443	52.787	136.563	224.782	48.343	872.417	
11	987	81.686	149.115	245.443	52.787	136.563	224.782	48.343	1078.917	
12	575	204.506	149.115	245.443	52.787	136.563	224.782	48.343	934.250	
13	834	102.787	149.115	245.443	52.787	136.563	224.782	48.343	922.417	
14	1025	101.597	149.115	245.443	52.787	136.563	224.782	48.343	1011.083	
15	1175	85.778	149.115	245.443	52.787	136.563	224.782	48.343	1082.917	
16	956	80.437	149.115	245.443	52.787	136.563	224.782	48.343	941.333	
17	725	317.527	149.115	245.443	52.787	136.563	224.782	48.343	1110.333	
18	743	128.421	149.115	245.443	52.787	136.563	224.782	48.343	910.333	
19	1051	206.361	149.115	245.443	52.787	136.563	224.782	48.343	911.583	
20	1035	198.013	149.115	245.443	52.787	136.563	224.782	48.343	873.667	
21	1148	101.567	149.115	245.443	52.787	136.563	224.782	48.343	1049.417	
22	808	157.103	149.115	245.443	52.787	136.563	224.782	48.343	963.833	
23	1215	200.582	149.115	245.443	52.787	136.563	224.782	48.343	1036.000	

S Chart

Fig. 5.16

Step 2: We calculate $\bar{\bar{X}}_{new}$ by typing "=(Sum(U3:U37)-U12-U33)/(U40-U44)" in Cell U38 as shown in Fig. 5.17.

U38 =(SUM(U3:U37)-U17-U33)/(N40-N44)

	S	T	U	V	W	X
38	Revised Average		969.235			
39						

Fig. 5.17

Step 3: We already have d = 2 in Cell N44 (see Fig. 5.10). We now type the value of A_3 for n=12 in Cell N45 as shown in Fig. 5.18.

	K	L	M	N	O
44			d	2	
45		A_3 Value from Table		0.886	
46					

Fig. 5.18

Step 4: We compute the centre line, UCL and LCL shown in Fig. 5.19 in Columns V, W and X, respectively, as explained in Sec. 3.5 of Lab Session 3.

	U	V	W	X
1		Control Limits		
2	Sample Mean	Centre Line	UCL	LCL
3	930.667	969.235	1090.229	848.240
4	901.250	969.235	1090.229	848.240
5	1074.083	969.235	1090.229	848.240
6	977.833	969.235	1090.229	848.240
7	1050.000	969.235	1090.229	848.240
8	877.417	969.235	1090.229	848.240
9	1080.250	969.235	1090.229	848.240
10	872.417	969.235	1090.229	848.240
11	1078.917	969.235	1090.229	848.240
12	934.250	969.235	1090.229	848.240
13	922.417	969.235	1090.229	848.240
14	1011.083	969.235	1090.229	848.240
15	1082.917	969.235	1090.229	848.240
16	941.333	969.235	1090.229	848.240
17	1110.333	969.235	1090.229	848.240
18	910.333	969.235	1090.229	848.240
19	911.583	969.235	1090.229	848.240
20	873.667	969.235	1090.229	848.240
21	1049.417	969.235	1090.229	848.240
22	963.833	969.235	1090.229	848.240
23	1036.000	969.235	1090.229	848.240

S Chart

Fig. 5.19

Step 5: To plot the $\bar{X}$-chart, we

1. select Cells U2:X16, U18:X32 and U34:X37 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1 and obtain the revised $\bar{X}$-chart shown in Fig. 5.20.

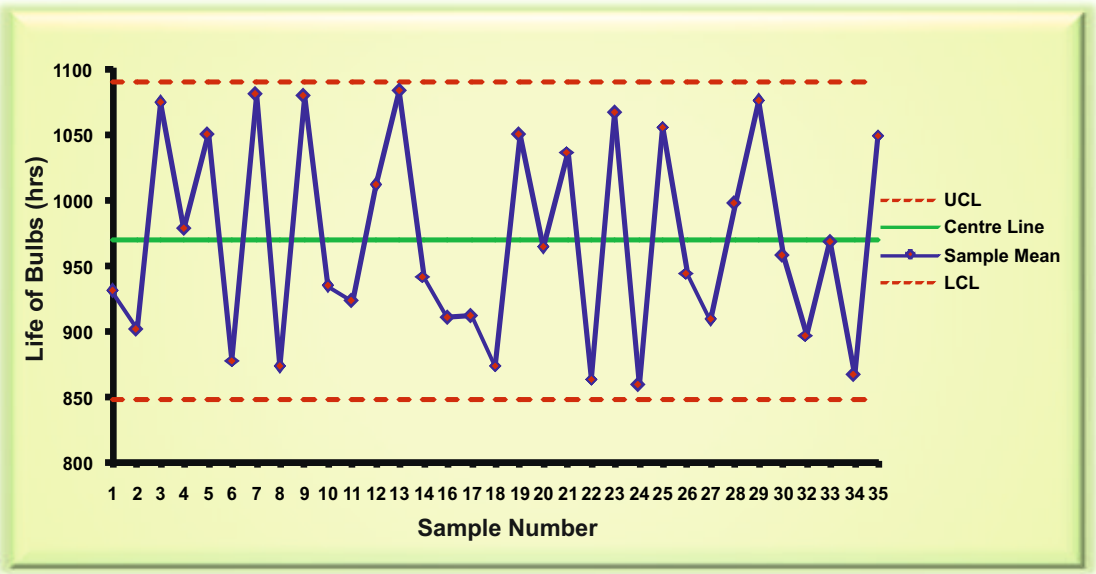


Fig. 5.20

Interpretation

Fig. 5.20 reveals that no point lies outside the control limits of the revised $\bar{X}$-chart and the process mean is under control. Hence, we may conclude that the process is under statistical control with respect to both process variability and process mean.

Now, you can check your understanding of constructing the S and $\bar{X}$-chart by solving the following exercises.

Activity 2

Construct a control charts for standard deviation with the help of MS Excel 2007 and interpret the results for

A1) Example 6 given in Unit 2 of MSTE-001.

A2) Exercise E12 given in Unit 2 of MSTE-001.

Match the results with the manual calculation done in Unit 2 of MSTE-001.

Continuous Assessment 5

Consider a problem related to a process which maintains the quality of bottling procedure. Suppose a fruit juice manufacturing company uses automatic machines to fill 500 ml juice bottles. A quality control inspector at this juice manufacturing company collects 25 samples of 10 observations of the juice bottles at different times and measures the volume of each filled bottle. The data is shown in Table 2.

Table 2: Volume of filled fruit juice in the bottles

Sample Number	Volume of Juice per bottle (in ml)									
	Obs. 1	Obs. 2	Obs. 3	Obs. 4	Obs. 5	Obs. 6	Obs. 7	Obs. 8	Obs. 9	Obs.10
1	497.32	500.62	498.68	497.82	497.32	498.68	497.32	500.62	498.68	497.82
2	504.76	500.00	498.32	500.32	504.76	498.32	504.76	500.00	498.32	500.32
3	499.24	497.18	498.12	498.68	499.24	498.12	499.24	497.18	498.12	499.68
4	499.26	496.32	498.88	497.82	499.26	498.88	499.26	499.32	498.88	498.82
5	498.32	500.62	499.56	500.12	498.32	499.56	498.32	500.62	499.56	500.12
6	499.12	500.32	499.38	500.94	499.12	499.38	499.12	500.32	499.38	500.94
7	499.34	498.32	497.32	497.62	499.34	497.32	499.34	498.32	499.32	498.62
8	499.38	498.12	500.62	498.12	499.38	500.62	499.38	498.12	500.62	498.12
9	499.26	498.38	500.68	500.38	499.26	500.68	499.26	498.38	500.68	500.38
10	498.60	497.62	499.25	498.56	498.60	499.25	498.60	497.62	499.25	498.56
11	499.44	500.00	501.32	499.38	499.44	501.32	499.44	500.00	499.32	499.38
12	498.26	500.32	500.76	499.68	498.26	500.76	498.26	500.32	500.76	499.68
13	497.32	498.50	497.18	499.38	499.32	498.18	498.32	498.50	499.18	499.38

Sample Number	Volume of Juice per bottle (in ml)									
	Obs. 1	Obs. 2	Obs. 3	Obs. 4	Obs. 5	Obs. 6	Obs. 7	Obs. 8	Obs. 9	Obs.10
14	499.56	498.00	498.76	501.12	499.56	498.76	499.56	498.00	498.76	501.12
15	500.24	500.32	499.12	499.25	500.24	499.12	500.24	500.32	499.12	499.25
16	500.76	500.50	499.68	498.12	500.76	499.68	500.76	500.50	499.68	498.12
17	500.65	497.82	494.06	496.25	500.65	494.06	500.65	497.82	494.06	496.25
18	499.12	500.26	500.44	498.76	499.12	500.44	499.12	500.26	500.44	498.76
19	499.50	500.50	499.56	500.76	499.50	499.56	499.50	500.50	499.56	500.76
20	497.50	498.82	499.76	497.82	497.50	499.76	497.50	498.82	499.76	497.82
21	499.44	500.62	500.00	501.26	499.44	500.00	499.44	499.62	500.00	500.26
22	499.38	498.38	497.56	498.56	499.38	497.56	499.38	498.38	497.56	498.56
23	501.56	499.56	498.00	499.82	501.56	498.00	501.56	499.56	498.00	499.82
24	498.32	497.32	499.56	498.62	498.32	499.56	498.32	497.32	499.56	498.62
25	499.50	500.12	498.50	500.38	499.50	498.50	499.50	500.12	498.50	500.38

Develop control limits for S-chart as well as $\bar{X}$-chart to check whether the process of bottling is under control or out-of-control. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Secs. 5.4, 5.5 and 5.6 to construct the control charts for the data of Table 1. Use a different format for the control charts. Take their screenshots and keep them in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 5" as explained in this lab session. Take screenshots of the final spreadsheet and the charts.

3) **Do not forget** to keep the screenshots in your record book as these will contribute in your continuous assessment in the Laboratory.