PART D

Time Series Modelling

TIME SERIES MODELLING

In Part C of this lab course, you have learnt about different types of regression models such as simple and multiple linear regressions when the predictors are quantitative as well as qualitative. We have also discussed different methods of selecting variables that truly contribute to the model.

The fourth part of this lab course comprises Sessions 15 to 17, which enable you to develop practical skills of predicting future values with the help of MS Excel 2007 using some elementary methods of time series data analysis. These projections can then be incorporated into the decision making process. For example, we can forecast production, product demand, sales revenues, inventory, etc. In time series techniques, we project the future values of a variable under consideration based on the past and present observations of that variable. In Block 4 of MSTE-002 (Industrial Statistics-II), we have explained the concept of time series that are usually combinations of four components, i.e., trend (T), cyclic (C), seasonal (S) and irregular (I).

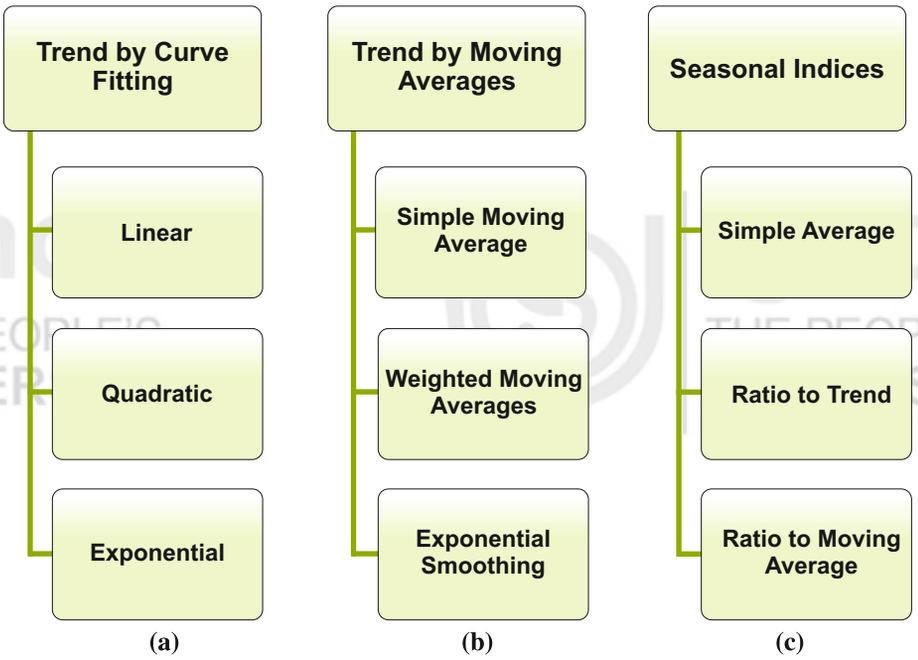


Fig. D.1

In **Session 15**, we use Excel 2007 to fit different models (such as linear, quadratic and exponential) that describe time series using the method of curve fitting. We then use them for forecasting the values of the variable (under consideration) of the given time series (Fig. D.1a).

In **Session 16**, you will learn how to eliminate seasonal and irregular fluctuation from the given time series data. You will learn three methods, namely, simple moving average, weighted moving average and exponential smoothing methods (Fig. D.1.b). You will also learn how to estimate the trend by smoothing (or filtering out) the effect of seasonal and irregular fluctuations from the given time series.

Sessions 17 deals with seasonal indices. You will learn three methods of estimating them: the simple average method, ratio to trend method and ratio to moving average method (Fig. D.1c). In this session, we also

deseasonalise the data by dividing the given time series data by the corresponding seasonal indices. Then we find the trend values after eliminating the effect of seasonal variation from the deseasonalised data.

All exercises of this part have been designed in such a way that after successfully completing them, you will be acquainted with the forecasting techniques for the given time series using MS Excel 2007. You will also be able to compute trend values and seasonal indices, and learn smoothing (or filtering) the seasonal and irregular effects and deseasonalising the data.

You should study Block 4 of MSTE-002 entitled Industrial Statistics-II thoroughly before performing the activities in the lab sessions of Part D as all concepts and formulae used there will be applied here.

Estimation of Trend by Curve Fitting

SESSION 15

Structure

15.1 Introduction

Objectives

15.2 Problem Description

15.3 Fitting of the Linear Trend

15.4 Fitting of the Quadratic Trend

15.5 Fitting of the Exponential Trend

15.1 INTRODUCTION

So far, you have learnt about different control charts for variables as well as attributes in the lab sessions of Part A and Part B, respectively. You have also learnt how to fit and analyse regression models and apply different variable selection methods in the lab sessions of Part C. In Unit 13 of MSTE-002 (Industrial Statistics-II), we have explained different components of the time series along with the estimation of trend using the methods of curve fitting. We have explained three methods of curve fitting: (i) linear, (ii) quadratic and (iii) exponential to fit an equation of the trend in Unit 13 of MSTE-002.

In this lab session, you will learn how to measure trend using all three methods of curve fitting. In the next lab session, we shall explain the methods of smoothing and filtering the time series data.

Prerequisite

- Lab Session 12 of MSTL-002 (Industrial Statistics Lab).
- Lab Session 5 of MSTL-001 (Basic Statistics Lab).
- Unit 13 of MSTE-002 (Industrial Statistics-II).

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- apply the method of curve fitting to fit linear, quadratic and exponential trends;
- identify the appropriate trend curve; and
- project the trend for a given period of time.

15.2 PROBLEM DESCRIPTION

We state three problems to illustrate the three trends:

1) Suppose the number of cars (in thousands) on a particular road were recorded for 48 months from 2007 to 2010 as a part of a hypothetical study of a car manufacturing company in a city. The data are given in Table 1.

Table 1: Number of cars on road

Month	Number of Cars (in thousands)	Month	Number of Cars (in thousands)	Month	Number of Cars (in thousands)
1	321	17	563	33	757
2	296	18	498	34	834
3	375	19	556	35	806
4	402	20	621	36	819
5	331	21	557	37	758
6	387	22	564	38	836
7	442	23	564	39	886
8	378	24	668	40	874
9	435	25	602	41	837
10	498	26	612	42	847
11	425	27	705	43	834
12	447	28	738	44	924
13	521	29	725	45	967
14	435	30	654	46	934
15	465	31	775	47	875
16	568	32	684	48	956

2) Suppose the owner of a store in Delhi wants to study the pattern of sales in his store and predict future sales. He records the monthly sales (in thousands) of the store for 48 months from 2010 to 2013. The data are given in Table 2.

Table 2: Monthly sales of a store

Month	Monthly Sales (in thousands)	Month	Monthly Sales (in thousands)	Month	Monthly Sales (in thousands)
1	589	17	495	33	473
2	754	18	369	34	750
3	517	19	521	35	575
4	487	20	354	36	734
5	690	21	345	37	795
6	606	22	478	38	669
7	478	23	387	39	845
8	425	24	494	40	645
9	504	25	523	41	715
10	458	26	398	42	918
11	523	27	578	43	804
12	584	28	587	44	994
13	423	29	467	45	989
14	548	30	624	46	897
15	365	31	645	47	1121
16	434	32	624	48	1087

3) The growth of a company in terms of the quarterly revenue generated (in crores) from 2002 to 2013 is given in Table 3.

Table 3 : Quarterly revenue of a production company

Year	Quarter	Revenue (in crores)	Year	Quarter	Revenue (in crores)
2002	1	25	2008	1	895
	2	180		2	2527
	3	864		3	1763
	4	683		4	1847
2003	1	244	2009	1	2630
	2	400		2	3224
	3	240		3	3076
	4	29		4	2699
2004	1	73	2010	1	4225
	2	135		2	4566
	3	884		3	4128
	4	384		4	4122
2005	1	827	2011	1	4900
	2	833		2	6526
	3	818		3	5949
	4	301		4	7022
2006	1	136	2012	1	8371
	2	1506		2	8906
	3	125		3	9325
	4	308		4	10450
2007	1	1531	2013	1	11891
	2	1709		2	13688
	3	1269		3	14068
	4	1294		4	16263

- Represent the data given in Tables 1, 2 and 3 graphically and fit the appropriate trend.
- Fit the appropriate trend using the method of curve fitting by matrix approach.
- Project the number of cars on the road in February 2012 for the data given in Table 1.
- Project the sales of May 2014 for the data given in Table 2.
- Project the revenue for the last quarter of the year 2014 from the data given in Table 3.

15.3 FITTING OF THE LINEAR TREND

You have studied about estimation of the linear trend by the method of curve fitting in Unit 13 of MSTE-002. We formulate the steps involved in this method as follows:

Step 1: Let x_t be the time period and y_t, the corresponding variable or trend values being studied. When the trend shows linear pattern, its equation can be obtained as

$$y_t = b_0 + b_1 x_t \qquad ...(1)$$

where b_0 and b_1 are unknown constants.

Step 2: The normal equations for estimating b_0 and b_1 using the method of least squares are given by

$$\sum y_t = nb_0 + b_1 \sum x_t \qquad ...(2)$$

$$\sum x_t y_t = b_0 \sum x_t + b_1 \sum x_t^2 \qquad ...(3)$$

The values of $\sum y_t$, $\sum x_t$ $\sum x_t y_t$ and $\sum x_t^2$ are obtained from the given data.

Step 3: To solve the normal equations in Excel 2007, we represent equations (2) and (3) in matrix notation as

$$\mathbf{C} = \mathbf{AB} \qquad ...(4)$$

where

$$\mathbf{C} = \begin{bmatrix} \sum y_t \\ \sum x_t y_t \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} n & \sum x_t \\ \sum x_t & \sum x_t^2 \end{bmatrix}, \text{ and}$$

$$\mathbf{B} = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}$$

Step 4: The optimum solution of b_0 and b_1, i.e., $\hat{b}_0$ and $\hat{b}_1$ can be obtained by solving the following equation:

$$\mathbf{B} = \mathbf{A}^{-1}\mathbf{C} \qquad ...(5)$$

Step 5: The desired trend line using the optimum values $\hat{b}_0$ and $\hat{b}_1$ is given by

$$y_t = \hat{b}_0 + \hat{b}_1 x_t \qquad ...(6)$$

Steps in Excel

We now describe the procedure for estimating the trend with the help of Problem 1. We first examine the data by plotting the line chart and appropriate trendline. For this, we follow the steps given below and fit the trend line using Excel 2007:

Step 1: We enter the data given in Table 1 in the Excel spreadsheet as shown in Fig. 15.1.

	A	B	C
1	Year	Month (x_t)	Number of Cars (y_t)
2	2007	1	321
3		2	296
4		3	375
5		4	402
6		5	331
7		6	387
8		7	442
9		8	378
10		9	435
11		10	498
12		11	425
13		12	447
14	2008	13	521
15		14	435
16		15	465
17		16	568
18		17	563
19		18	498
20		19	556

Linear

Fig. 15.1

Step 2: To plot the line chart for the given time series data, we

- ✓ select Cells C2:C49,
- ✓ select the ***Insert*** tab from the ***Charts*** group, and
- ✓ click on the ***Line*** chart as explained in Lab Session 5 of MSTL-001.

Step 3: We format the chart by

- ✓ eliminating the grid lines and border around the chart,
- ✓ changing the data series to bold solid lines with visible markers, and
- ✓ changing the marker option, size and colour.

We can also change the fonts, axes, titles, background, etc., as desired.

The resulting chart is shown in Fig. 15.2.

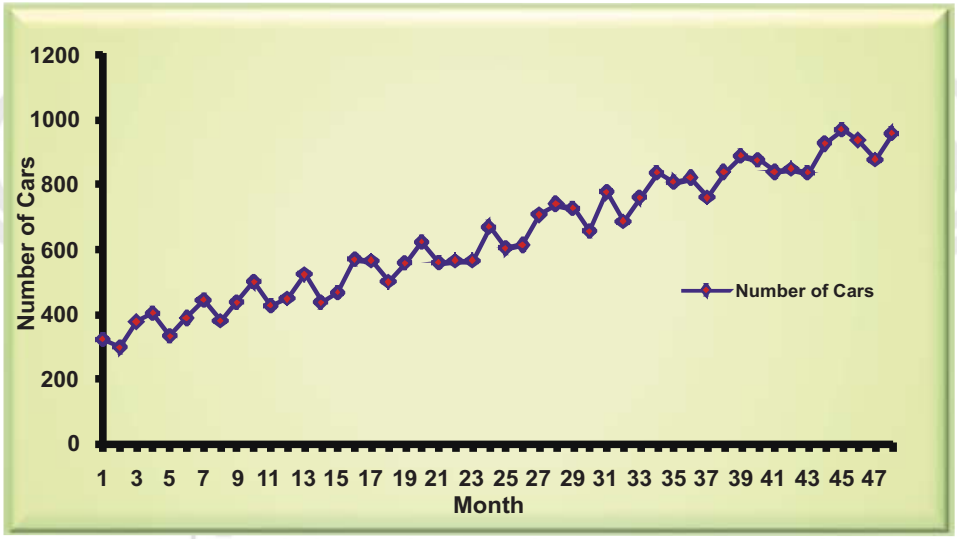


Fig. 15.2

Insert Tab→Charts Group→Line Chart

Step 4: To guess the appropriate trendline which will fit the given car data, we first click on the chart and select the ***Layout*** menu from ***Chart Tools*** as shown in Fig. 15.3.

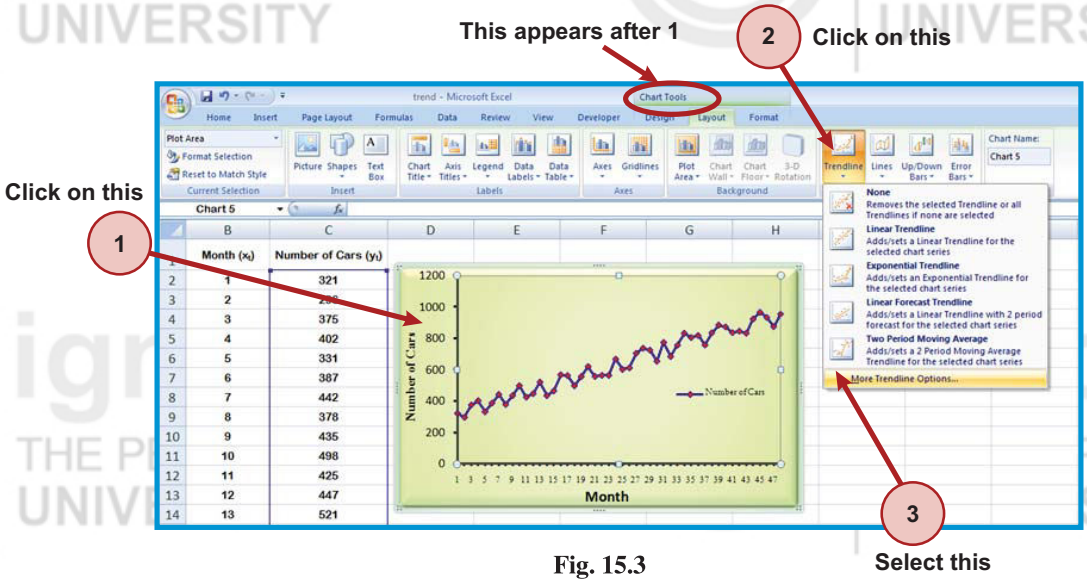


Fig. 15.3

Step 5: Then we select ***More Trendline Options***. A new dialog box gets opened as shown in Fig. 15.4.

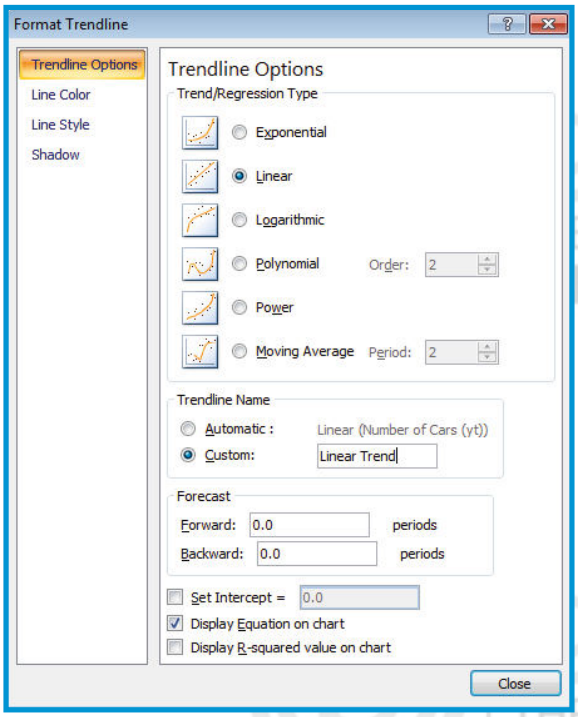


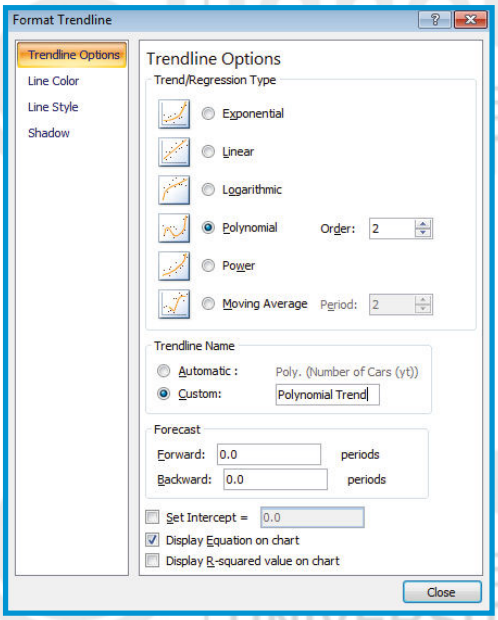
Fig. 15.4

Step 6: In the dialog box shown in Fig. 15.4, we can see different options such as **Exponential, Linear, Logarithmic, Polynomial, Power** and **Moving Averages**.

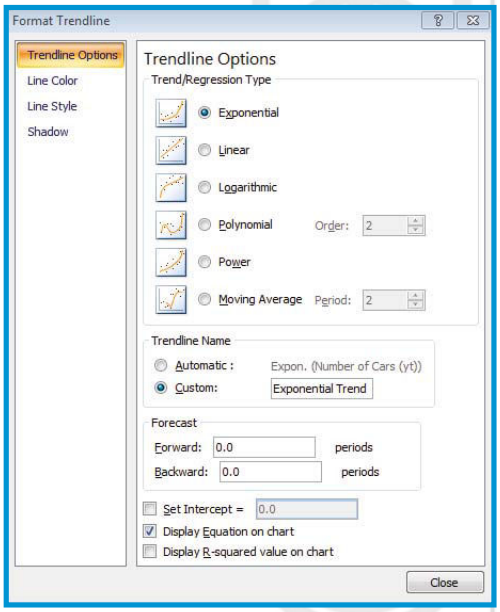
In Unit 13 of MSTE-002, we have discussed trends that are linear, polynomial of order 2 and exponential. So here we shall focus only on these three types. We fit these three trends together on the chart shown in Fig. 15.2 and then choose the best fitted trend.

We now fit the three trends one at a time.

- ✓ To fit linear trend, we tick on ***Linear***, ***Display Equation on chart*** and ***Custom***. Here we type "**Linear Trend**" as ***Trendline Name*** as shown in Fig. 15.4 and ***Close*** the box.
- ✓ To fit quadratic trend, we repeat Steps 4 and 5. We tick on ***Polynomial*** (select ***order 2***), ***Display Equation on chart*** and ***Custom***. Here we type "**Polynomial Trend**" as ***Trendline Name*** as shown in Fig. 15.5a and ***Close*** the box.
- ✓ To fit exponential trend, we repeat Steps 4 and 5. We now tick on ***Exponential***, ***Display Equation on chart*** and ***Custom***. Here we type "**Exponential Trend**" as ***Trendline Name*** as shown in Fig. 15.5b and ***Close*** the box.



(a)



(b)

Fig. 15.5

Step 7: From the ***Line Colour*** and ***Line Style*** specified on the left in the dialog boxes shown in Figs. 15.4, 15.5a and 15.5b, we can set different line formats. We have changed the trendlines to dashed lines of different colours. When we close the dialog box, we obtain the resulting chart with the three fits of the trend as shown in Fig. 15.6.

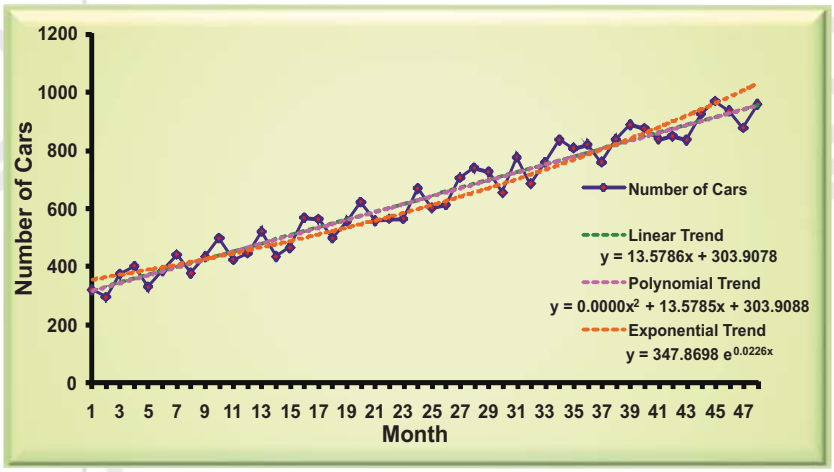


Fig. 15.6

Notice from Fig. 15.6 that the exponential curve is not fitting well and the fits of the linear and quadratic trends are approximately the same. The equation of the quadratic trend shown in Fig.15.6 is $y_t = 0.000\,x_t^2 + 13.578\,x_t + 303.909$, which is approximately the same as the equation of linear trend, i.e., $y_t = 13.579x_t + 303.908$. Note that the quadratic term (x_t^2) contributes approximately zero (0) to the equation. So we need not consider the quadratic trend. We choose the linear trend as it fits the given data well.

So far, we have obtained a linear trend for the data given in Table 1. We now explain how to fit the trend line using the method of least squares by the matrix approach in Excel 2007. You have already learnt this approach in Lab Session 12 of MSTL-002.

Step 8: As discussed in Sec. 15.3, we formulate the normal equations to find the equation of linear trend for which we need the values of $\sum x_t, \sum y_t, \sum y_t x_t$ and $\sum x_t^2$. The values of y_t and x_t are already given in Columns B and C, respectively (see Fig. 15.1). In Columns D and E, we compute the values of x_t^2 and $y_t x_t$, respectively.

In Cells D2 and E2, we type "=B2*B2" and "=C2*B2", respectively, as shown in Fig. 15.7.

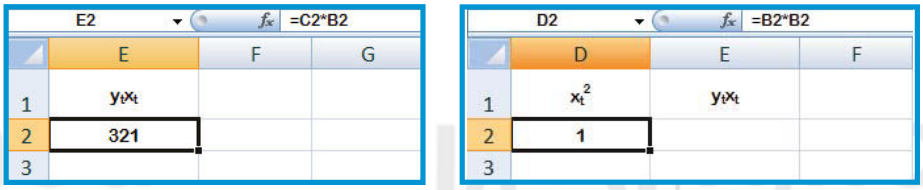
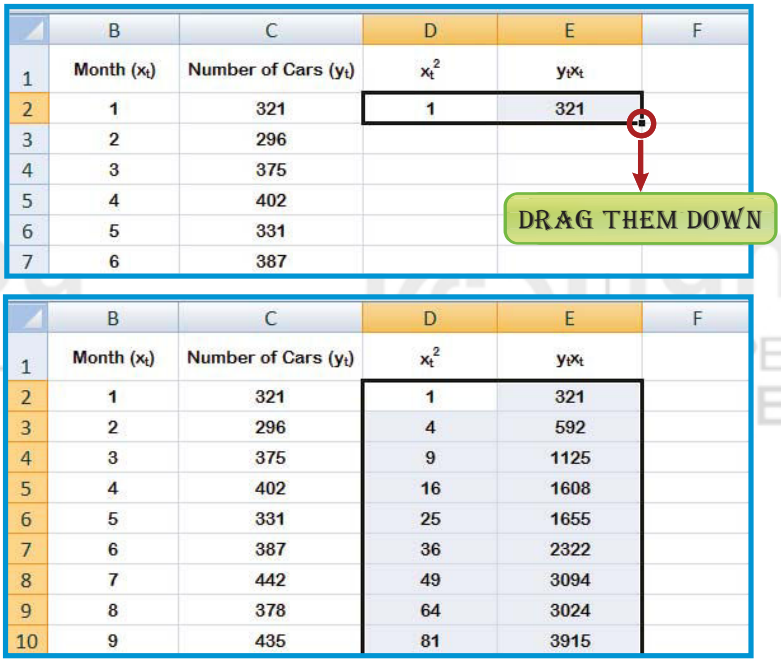


Fig. 15.7

Step 9: We select Cells D2:E2 and drag them down up to Row 49 to compute the values of x_t^2 and $y_t x_t$ for the remaining months as shown in Fig. 15.8.



	B	C	D	E	F
1	Month (x_t)	Number of Cars (y_t)	x_t^2	$y_t x_t$	
2	1	321	1	321	
3	2	296			
4	3	375			
5	4	402			
6	5	331			
7	6	387			

	B	C	D	E	F
1	Month (x_t)	Number of Cars (y_t)	x_t^2	$y_t x_t$	
2	1	321	1	321	
3	2	296	4	592	
4	3	375	9	1125	
5	4	402	16	1608	
6	5	331	25	1655	
7	6	387	36	2322	
8	7	442	49	3094	
9	8	378	64	3024	
10	9	435	81	3915	

Fig. 15.8

Step 10: We type "=Sum(B2:B49)" in Cell B50 to get the value of $\sum y_t$ as shown in Fig. 15.9.

B50 | =SUM(B2:B49)

	A	B	C
48		47	875
49		48	956
50	Sum	1176	
51			
52			

Fig. 15.9

Step 11: By dragging Cell B50 right up to Cell E50, we determine the values of $\sum x_t$, $\sum x_t^2$ and $\sum y_t x_t$, in Cells C50, D50 and E50, respectively, as shown in Fig. 15.10.

	A	B	C	D	E	F
48		47	875	2209	41125	
49		48	956	2304	45888	
50	Sum	1176	30556	38024	873708	
51						
52						

Fig. 15.10

Step 12: We substitute the values of summations in equation (5) given in Sec. 15.3 to determine the optimum values $\hat{b}_0$ and $\hat{b}_1$ as follows:

$$\mathbf{B} = \mathbf{A}^{-1}\mathbf{C}$$

i.e., $$\begin{pmatrix} \hat{b}_0 \\ \hat{b}_1 \end{pmatrix} = \begin{pmatrix} 48 & 1176 \\ 1176 & 38024 \end{pmatrix}^{-1} \begin{pmatrix} 30556 \\ 873708 \end{pmatrix}$$

To find the solution of $\hat{b}_0$ and $\hat{b}_1$ in Excel, we type the matrix **C** in Cells A52: A53 and matrix **A** in Cells C52:D53 as shown in Fig. 15.11.

	A	B	C	D
51				
52	(30556		(48	1176)
53	873708)		1176	38024)
54				

Fig. 15.11

Step 13: To find $\mathbf{A}^{-1}$, we first select the cells of the same size as the original matrix **A**, i.e., (2×2) where we want to put this A^{-1} matrix. Here we have selected Cells C56:D57. We then type "=Minverse(C52:D53)" and press ***Ctrl+Shift+Enter*** instead of simply ***Enter*** as shown in Fig. 15.12.

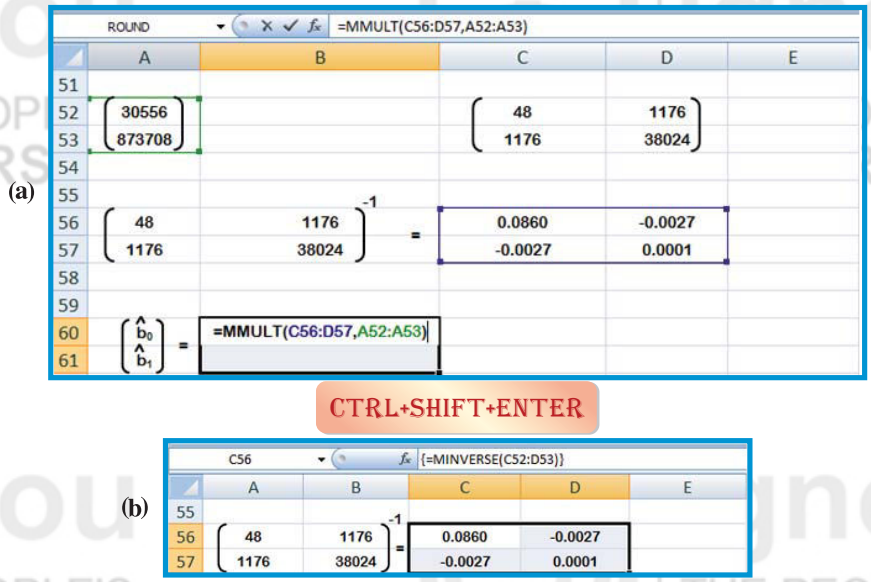


Fig. 15.12

Step 14: To find **B**, we select the cells of the same size as the resulting matrix, i.e., (2×1) where we want to put this **B** matrix. Here we have selected the Cells B60:B61. We then type "=Mmult(C56:D57, A52:A53)" and press ***Ctrl+Shift+Enter*** instead of simply ***Enter*** as shown in Fig. 15.13.

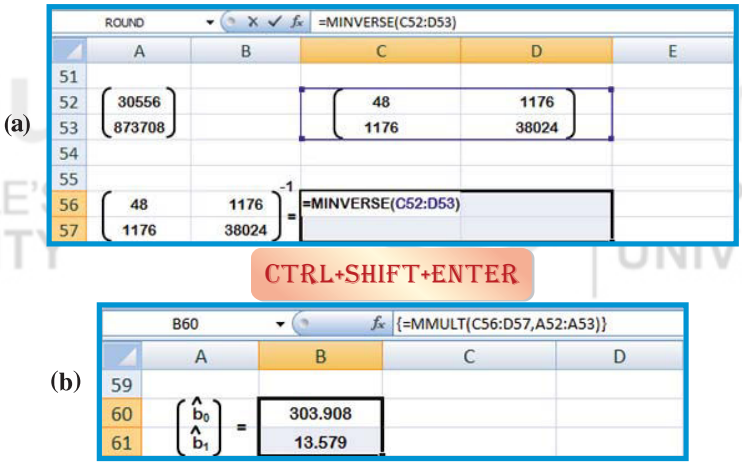


Fig. 15.13

Notice from Fig. 15.13 that the values of $\hat{b}_0 = 13.579$ and $\hat{b}_1 = 303.908$ are given in Cells B60 and B61, respectively. Hence, the equation of linear trend fitted to the given data is $y_t = 13.579 + 303.908x_t$ which is the same as obtained in Step 6.

Step 15: We type "=$B$60+$B$61*B2" in Cell F2 to obtain the trend value of the first month as shown in Fig. 15.14.

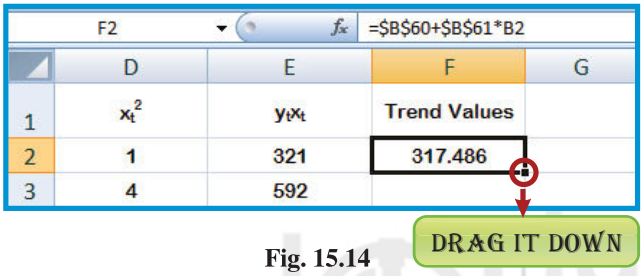


Fig. 15.14

Step 16: To determine the trend values for the remaining months, we drag down Cell F2 up to Row 49 as shown in Fig.15.15.

	x_t^2	$y_t x_t$	Trend Values
2	1	321	317.486
3	4	592	331.065
4	9	1125	344.644
5	16	1608	358.222
6	25	1655	371.801
7	36	2322	385.379
8	49	3094	398.958
9	64	3024	412.537
10	81	3915	426.115
11	100	4980	439.694
12	121	4675	453.272
13	144	5364	466.851
14	169	6773	480.430
15	196	6090	494.008
16	225	6975	507.587
17	256	9088	521.165
18	289	9571	534.744
19	324	8964	548.322
20	361	10564	561.901

Fig. 15.15

Step 17: To compute the trend projection for February 2012, we first locate the number at which the month of February 2012 is placed in the sequence of data given in Table 1. Note that January 2007 is placed at number 1 and December 2010 is placed at number 48. In this sequence, February 2012 will be placed at number 62 in the table. Now, the trend projection for February 2012 can be computed by typing "=$B$60+$B$61*62" in any Cell. Here we use Cell A64 as shown in Fig. 15.16.

A64 | =$B$60+$B$61*62

	A	B	C
63	Trend projection for February 2012 is		
64	1145.781		
65			

Fig. 15.16

Fig. 15.16 reveals that the number of cars on road in February 2012 is approximately 1146 (Cell A64).

You may now like to do the following activity.

Activity 1

Consider $x_t' = x_t - \bar{x}_t$ to simplify the calculations, where $\bar{x}_t$ is the mean of all given time periods x_t as discussed in Sec. 13.6 of MSTE-002. In Problem 1, x_t represents the month. Repeat Steps 8 to 17 taking x_t' instead of x_t. Then match your results with the result obtained in Step 17 and state the underlying reason.

15.4 FITTING OF THE QUADRATIC TREND

Sometimes trend is not linear and shows some curvature. We may check whether the data fits a second degree polynomial. The formulae and procedure involved in quadratic fitting are given below:

Step 1: The equation of a second degree polynomial may be written as:

$$y_t = b_0 + b_1 x_t + b_2 x_t^2 \qquad ...(7)$$

where b_0, b_1 and b_2 are constants.

Step 2: The normal equations for estimating b_0, b_1 and b_2 using the method of least squares are given by

$$\sum y_t = nb_0 + b_1 \sum x_t + b_2 \sum x_t^2 \qquad ...(8)$$

$$\sum x_t y_t = b_0 \sum x_t + b_1 \sum x_t^2 + b_2 \sum x_t^3 \qquad ...(9)$$

$$\sum x_t^2 y_t = b_0 \sum x_t^2 + b_1 \sum x_t^3 + b_2 \sum x_t^4 \qquad ...(10)$$

The values of $\sum y_t, \sum x_t \sum x_t y_t, \sum x_t^2 y_t, \sum x_t^2, \sum x_t^3$ and $\sum x_t^4$ are obtained from the given data.

Step 3: It is the same as explained in Sec. 15.3. We represent equations (8), (9) and (10) in matrix notation as

$$\mathbf{C} = \mathbf{A}\ \mathbf{B} \qquad ...(11)$$

where

$$\mathbf{C} = \begin{pmatrix} \sum y_t \\ \sum x_t y_t \\ \sum x_t^2 y_t \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} \text{ and}$$

$$\mathbf{A} = \begin{pmatrix} n & \sum x_t & \sum x_t^2 \\ \sum x_t & \sum x_t^2 & \sum x_t^3 \\ \sum x_t^2 & \sum x_t^3 & \sum x_t^4 \end{pmatrix}$$

Step 4: The optimum values of b_0 and b_1 and b_2, i.e., $\hat{b}_0$, $\hat{b}_1$ and $\hat{b}_2$ can be obtained by solving the following equation

$$\mathbf{B} = \mathbf{A}^{-1}\ \mathbf{C} \qquad ...(12)$$

Step 5: The desired quadratic trend using the optimum values $\hat{b}_0$, $\hat{b}_1$ and $\hat{b}_2$ is given by

$$y_t = \hat{b}_0 + \hat{b}_1 x_t + \hat{b}_2 x_t^2 \qquad ...(13)$$

Steps in Excel

Step 1: We enter the data given in Problem 2 in Sec. 15.2 of this lab session in Excel Sheet as shown in Fig. 15.17.

	A	B	C
1	Year	Month (x_t)	Monthly Sales (y_t)
2	2010	1	589
3		2	754
4		3	517
5		4	487
6		5	690
7		6	606
8		7	478
9		8	425
10		9	504
11		10	458
12		11	523
13		12	584
14	2011	13	423
15		14	548
16		15	365
17		16	434
18		17	495
19		18	369
20		19	521

Quadratic

Fig. 15.17

Step 2: As explained in Steps 2-7 of Sec. 15.3 under the heading 'Steps in Excel', we have plotted the data shown in Fig. 15.17 and also the three trends, i.e., linear, quadratic and exponential. The resulting plot is shown in Fig. 15.18.

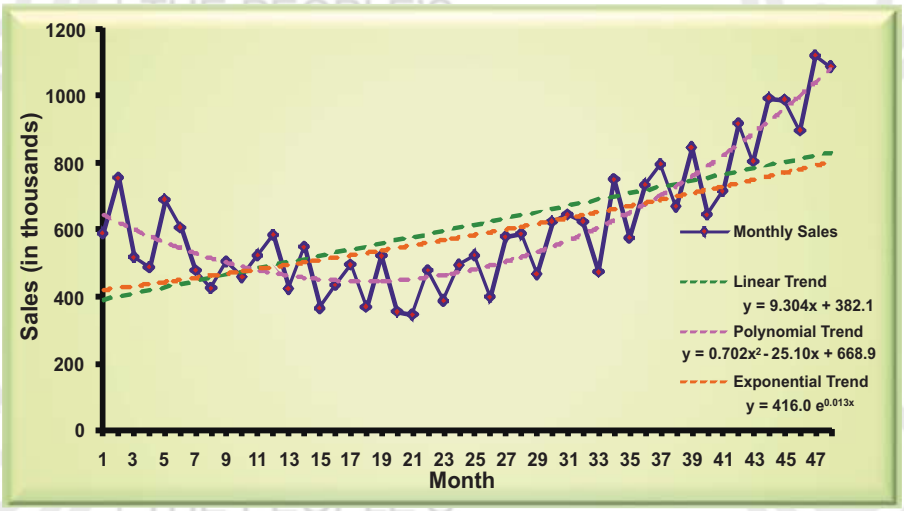


Fig. 15.18

From Fig. 15.18, we observe that the quadratic trend fits well in comparison with the linear and exponential trends. The equation of the quadratic trend is $y_t = 0.702x_t^2 - 25.108x_t + 668.908$.

We now explain how to fit the quadratic trend using the method of least squares in Excel 2007 in the steps given below:

Step 3: As explained in Sec. 15.3, we formulate the normal equations to find the optimum values $\hat{b}_0$, $\hat{b}_1$ and $\hat{b}_2$ using the method of least

squares. For this, we need the values of $\sum x_t, \sum y_t, \sum y_t x_t$, $\sum y_t x_t^2, \sum x_t^2, \sum x_t^3$ and $\sum x_t^4$. The values of x_t and y_t are already given in Columns B and C, respectively (Fig. 15.17). We determine the values of x_t^2, x_t^3, x_t^4, $y_t x_t$ and $y_t x_t^2$ in Columns D, E, F, G and H, respectively. These are shown in Fig. 15.19.

	D	E	F	G	H
1	x_t^2	x_t^3	x_t^4	$y_t x_t$	$y_t x_t^2$
2	1	1	1	589	589
3	4	8	16	1508	3016
4	9	27	81	1551	4653
5	16	64	256	1948	7792
6	25	125	625	3450	17250
7	36	216	1296	3636	21816
8	49	343	2401	3346	23422
9	64	512	4096	3400	27200
10	81	729	6561	4536	40824
11	100	1000	10000	4580	45800
12	121	1331	14641	5753	63283
13	144	1728	20736	7008	84096
14	169	2197	28561	5499	71487
15	196	2744	38416	7672	107408
16	225	3375	50625	5475	82125
17	256	4096	65536	6944	111104
18	289	4913	83521	8415	143055
19	324	5832	104976	6642	119556
20	361	6859	130321	9899	188081

Quadratic

Fig. 15.19

Step 4: We obtain the values of $\sum x_t, \sum y_t, \sum x_t^2, \sum x_t^3, \sum x_t^4$, $\sum y_t x_t$ and $\sum y_t x_t^2$ in Cells B50:H50, respectively, as explained in Steps 10 and 11 of Sec. 15.3 (see Fig. 15.20).

	A	B	C	D	E	F	G	H
50	Sum	1176	29285	38024	1382976	53651864	803199	28390715

Fig. 15.20

Step 5: As explained in Step 12 given in Sec. 15.3, we can find the solutions of $\hat{b}_0, \hat{b}_1$ and $\hat{b}_2$ easily by substituting the values of the summations in equation (12) as

$$\mathbf{B} = \mathbf{A}^{-1}\,\mathbf{C}$$

or
$$\begin{pmatrix} \hat{b}_0 \\ \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} 48 & 1176 & 38024 \\ 1176 & 38024 & 1382976 \\ 38024 & 1382976 & 53651864 \end{pmatrix}^{-1} \begin{pmatrix} 29285 \\ 803199 \\ 28390715 \end{pmatrix}$$

We type the matrix **C** in Cells A52:A54 and matrix **A** in Cells D52:F54 as shown in Fig. 15.21.

	A	B	C	D	E	F
51						
52	29285			48	1176	38024
53	803199			1176	38024	1382976
54	28390715			38024	1382976	53651864
55						

Fig. 15.21

Step 6: We repeat Steps 13 and 14 of Sec. 15.3 under the heading 'Steps in Excel' to compute the optimum values of b_0, b_1 and b_2 shown in Cells B60:B62 of Fig. 15.22.

	A	B	C		D	E	F
56							
57	48	1176	38024	$^{-1}$	0.20404	-0.01682	0.00029
58	1176	38024	1382976	=	-0.01682	0.00181	-0.00003
59	38024	1382976	53651864		0.00029	-0.00003	0.00000
60							
61	$\hat{b}_0$	668.9079					
62	$\hat{b}_1$ =	-25.1079					
63	$\hat{b}_2$	0.7023					
64							

Fig. 15.22

From Fig. 15.22, $\hat{b}_0 = 668.90\ 8$, $\hat{b}_1 = -\ 25.108$ and $\hat{b}_2 = 0.7023$.

Hence, the equation of the quadratic trend fitted to the data given in Table 2 is $y_t = 0.702x_t^2 - 25.108x_t + 668.908$.

It is the same as obtained in Step 2.

Step 7: We now type "=$B$61+$B$62*B2+$B$63*B2*B2" in Cell I2 to get the trend value of the first month. Then we drag down the Cell I2 up to Row 49 to find the trend values for all the given months as shown in Fig. 15.23.

I2 =$B$61+$B$62*B2+$B$63*B2*B2

	F	G	H	I	J
1	x_t^4	y_tx_t	$y_tx_t^2$	Trend Values	
2	1	589	589	644.502	
3	16	1508	3016		
4	81	1551	4653		
5	256	1948	7792		

DRAG IT DOWN

	F	G	H	I	J
1	x_t^4	y_tx_t	$y_tx_t^2$	Trend Values	
2	1	589	589	644.502	
3	16	1508	3016	621.501	
4	81	1551	4653	599.905	
5	256	1948	7792	579.713	
6	625	3450	17250	560.926	
7	1296	3636	21816	543.543	
8	2401	3346	23422	527.565	
9	4096	3400	27200	512.992	
10	6561	4536	40824	499.823	
11	10000	4580	45800	488.059	
12	14641	5753	63283	477.699	
13	20736	7008	84096	468.744	
14	28561	5499	71487	461.194	
15	38416	7672	107408	455.048	
16	50625	5475	82125	450.307	

Fig. 15.23

Step 8: To forecast the trend value (sales) for May 2014, we note that the month of May 2014 is placed at number 53 in the sequence of months given in Table 2. Now, the trend projection for May 2014 can be computed by typing "=$B$60+$B$61*53+$B$62*53*53" in any cell. Here we use Cell A65 as shown in Fig. 15.24.

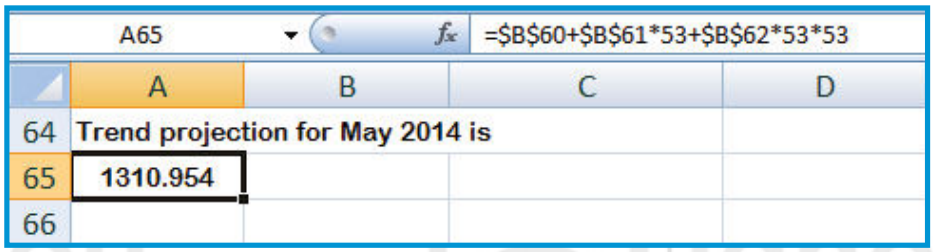


Fig. 15.24

So far, you have learnt the linear and quadratic curve fitting methods for trend. We now explain the exponential trend.

15.5 FITTING OF THE EXPONENTIAL TREND

In many situations, linear or quadratic trends do not give a good fit for the data. In this section, we explain how to use Excel 2007 to fit an exponential trend to the given data. We use the exponential function when the rate of change of a quantity is proportional to the initial amount of the quantity. We explain the procedure for fitting the exponential trend in the steps given below:

Step 1: The exponential model can be represented as

$$y_t = b_0 e^{b_1 x_t} \qquad ...(14)$$

Step 2: We transform the exponential model to a linear trend model by taking natural logarithm of equation (14):

$$\log_e(y_t) = \log_e(b_0) + x_t\, b_1 \qquad ...(15)$$

Step 3: We can represent the transformed model as a linear trend model given by

$$y'_t = b'_0 + b_1 x_t \qquad ...(16)$$

where $y'_t = \log_e(y_t)$ and $b'_0 = \log_e(b_0)$

Step 4: We determine the optimum values of $\hat{b}_0$ and b_1, i.e., $\hat{b}'_0$ and $\hat{b}_1$ as explained in Steps 3 and 4 of Sec. 15.3 using the matrix approach.

Step 5: We compute the value of b_0 as:

$$\hat{b}_0 = e^{\hat{b}'_0} \qquad ...(17)$$

Step 6: The desired exponential trend using the estimated values of $\hat{b}_0$ and $\hat{b}_1$ is given by

$$y_t = \hat{b}_0 e^{\hat{b}_1 x_t} \qquad ...(18)$$

Steps in Excel

Step 1: We enter the data given in Table 3 in Sec. 15.2 of this lab session in Excel Sheet as shown in Fig. 15.25.

	A	B	C
1	Year	Quarter (x_t)	Revenue (y_t)
2	2002	1	25
3		2	180
4		3	864
5		4	683
6	2003	5	244
7		6	400
8		7	240
9		8	29
10	2004	9	73
11		10	135
12		11	884
13		12	384
14	2005	13	827
15		14	833
16		15	818
17		16	301
18	2006	17	136
19		18	1506
20		19	125

Exponential

Fig. 15.25

Step 2: As explained in Steps 2-7 of Sec. 15.3 under the heading 'Steps in Excel', we plot the three trends, i.e., linear, quadratic and exponential along with the data of quarterly revenue shown in Fig. 15.25. The resulting chart is shown in Fig. 15.26.

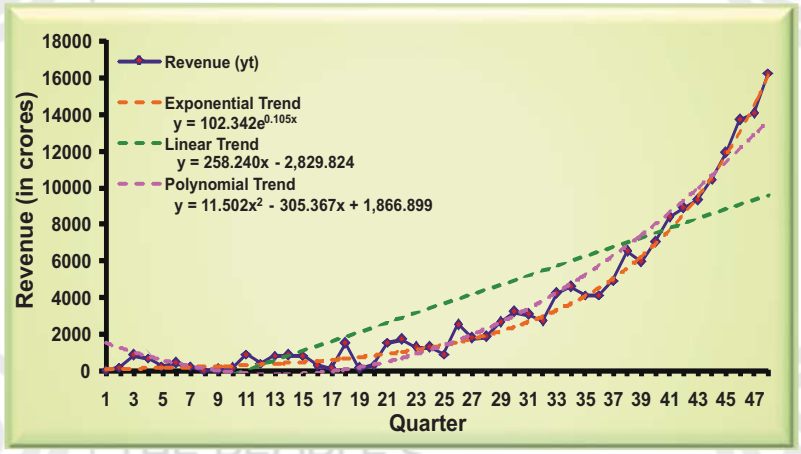


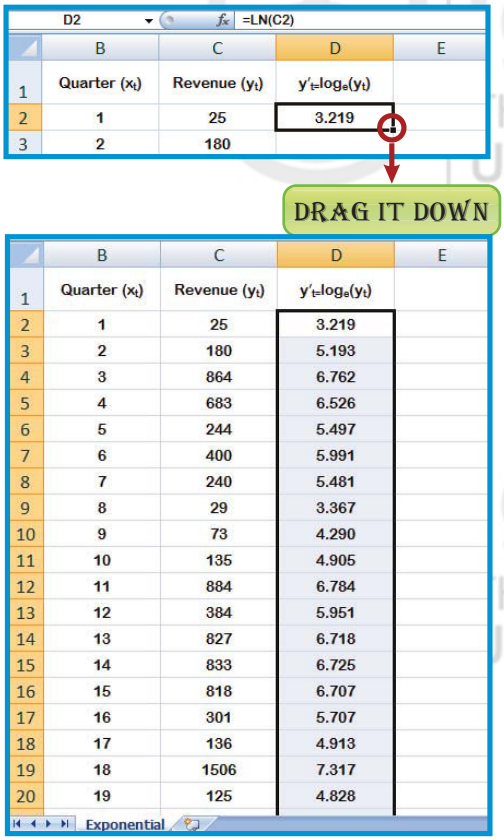
Fig. 15.26

From Fig. 15.26, it is observed that the exponential trend fits well in comparison with the linear and quadratic. The equation of exponential trend is given as

$$y_t = 102.342\, e^{0.105\, x_t}$$

We now explain how to fit exponential trend in Excel 2007 using the method of least squares in the following steps:

Step 3: We transform y_t, i.e., $Y_t = \log_e(y_t)$ by typing "=Ln(C2)" in Cell D2 and drag it down up to Row 49 as shown in Fig. 15.27.



	B	C	D	E
1	Quarter (x_t)	Revenue (y_t)	$y'_t=\log_e(y_t)$	
2	1	25	3.219	
3	2	180	5.193	
4	3	864	6.762	
5	4	683	6.526	
6	5	244	5.497	
7	6	400	5.991	
8	7	240	5.481	
9	8	29	3.367	
10	9	73	4.290	
11	10	135	4.905	
12	11	884	6.784	
13	12	384	5.951	
14	13	827	6.718	
15	14	833	6.725	
16	15	818	6.707	
17	16	301	5.707	
18	17	136	4.913	
19	18	1506	7.317	
20	19	125	4.828	

Fig. 15.27

Step 4: We formulate the normal equations to determine the optimum values $\hat{b}'_0$ and $\hat{b}_1$. For this we need the values of $\sum y'_t, \sum x_t \sum x_t y'_t$ and $\sum x_t^2$. The values of y'_t and x_t are already given in Columns D and B, respectively (Fig. 15.27). In Columns E and F, we compute the values of x_t^2 and $y'_t x_t$, respectively, as shown in Fig. 15.28.

	B	C	D	E	F
1	Quarter (x_t)	Revenue (y_t)	$y'_t=\log_e(y_t)$	x_t^2	$y'_t x_t$
2	1	25	3.219	1	3.219
3	2	180	5.193	4	10.386
4	3	864	6.762	9	20.285
5	4	683	6.526	16	26.106
6	5	244	5.497	25	27.486
7	6	400	5.991	36	35.949
8	7	240	5.481	49	38.364
9	8	29	3.367	64	26.938
10	9	73	4.290	81	38.614
11	10	135	4.905	100	49.053
12	11	884	6.784	121	74.629
13	12	384	5.951	144	71.408
14	13	827	6.718	169	87.331
15	14	833	6.725	196	94.150
16	15	818	6.707	225	100.603
17	16	301	5.707	256	91.314
18	17	136	4.913	289	83.515
19	18	1506	7.317	324	131.710
20	19	125	4.828	361	91.738

Exponential

Fig. 15.28

Step 5: We compute the values of $\sum x_t$, $\sum y'_t$, $\sum y'_t x_t$ and $\sum x_t^2$ in Cells B50, D50, E50 and F50, respectively, as explained in Steps 10 and 11 of Sec. 15.3 (Fig. 15.29).

	A	B	C	D	E	F
50	Sum	1176	167859	346.145	38024	9451.779
51						

Fig. 15.29

Step 6: We now follow Steps 12-14 given in Sec. 15.3 under the heading 'Steps in Excel' to obtain the solutions of $\hat{b}'_0$ and $\hat{b}_1$.

The computation of $\hat{b}'_0$ and $\hat{b}_1$ is shown in Fig. 15.30 in terms of matrices.

	A	B	C	D
51				
52	(346.145		(48	1176.000)
53	9451.779)		1176	38024.000)
54				
55				
56	(48	1176)$^{-1}$ =	0.0860	-0.003
57	1176	38024)	-0.0027	0.000
58				
59				
60	($\hat{b}'_0$) =	4.628		
61	$\hat{b}_1$)	0.105		
62				

Fig. 15.30

Step 7: We calculate the value of $\hat{b}_0$ by typing "=Exp(B60)" in Cell D60 as shown in Fig. 15.31.

D60 | f_x =EXP(B60)

	A	B	C	D	E
59					
60	($\hat{b}'_0$) =	4.628	$\hat{b}_0$ =	102.342	
61	$\hat{b}_1$)	0.105	$\hat{b}_1$ =	0.105	
62					

Fig. 15.31

From Fig. 15.31, $\hat{b}_0 = 102.342$ and $\hat{b}_1 = 0.105$. Hence, the equation of the exponential trend fitted to the given data is $y_t = 102.342\, e^{0.105\, x_t}$, which is the same as obtained in Step 2.

Step 8: We now type "=$D$60*Exp($D$61*B2)" in Cell G2 to get the trend value of the first month and then drag down the Cell G2 up to Row 49 to find the trend values for all given months as shown in Fig. 15.32.

G2 =$D$60*EXP($D$61*B2)

	E	F	G	H
1	x_t^2	$y'_t x_t$	Trend Values	
2	1	3.219	113.721	
3	4	10.386		

DRAG IT DOWN

	E	F	G	H
1	x_t^2	$y'_t x_t$	Trend Values	
2	1	3.219	113.721	
3	4	10.386	126.365	
4	9	20.285	140.416	
5	16	26.106	156.028	
6	25	27.486	173.377	
7	36	35.949	192.654	
8	49	38.364	214.075	
9	64	26.938	237.878	
10	81	38.614	264.327	
11	100	49.053	293.718	
12	121	74.629	326.376	
13	144	71.408	362.665	
14	169	87.331	402.989	
15	196	94.150	447.797	
16	225	100.603	497.587	
17	256	91.314	552.913	
18	289	83.515	614.390	
19	324	131.710	682.703	
20	361	91.738	758.612	

Exponential

Fig. 15.32

Step 9: To forecast the trend value (revenue) for the last quarter of 2014, we note that the last quarter of 2014 is placed at number 52 in the sequence given in Table 3. Now, the trend projection for the last quarter of 2014 can be computed by typing "=$D$60*Exp($D$61*52)" in any cell. Here we use Cell A64 as shown in Fig. 15.33.

A64 =$D$60*EXP($D$61*52)

	A	B	C	D
63	Trend projection for the last quarter of 2014 is			
64	24604.570			
65				

Fig. 15.33

You can now solve the following exercises to check whether you have learnt how to fit the appropriate trend to the given data and forecast the trend value for a specified period of time.

Activity 2

Fit an appropriate trend with the help of MS Excel 2007 and interpret the results for

A1) Examples 6, 7 and 8 given in Unit 13 of MSTE-002.

A2) Exercise E4 given in Unit 13 of MSTE-002.

Match the results with the calculation done in Unit 13 of MSTE-002.

Continuous Assessment 15

Consider the following situations:

1. Data of annual rainfall for the period of 1956–2011 were collected to study the rainfall patterns in a state. Data of rainfall for 56 years are given in Table 4.

Table 4: Annual rainfall data

Year	Annual Rainfall (in mm)	Year	Annual Rainfall (in mm)	Year	Annual Rainfall (in mm)
1	187.2	20	188.4	39	228.3
2	207.9	21	226.3	40	204.4
3	171.5	22	178.7	41	195.6
4	217.7	23	228.8	42	221.8
5	199.5	24	182.6	43	206.0
6	185.7	25	230.7	44	235.9
7	261.2	26	186.3	45	218.5
8	201.5	27	229.2	46	168.0
9	173.1	28	230.0	47	183.6
10	192.7	29	220.1	48	213.0
11	157.3	30	189.4	49	180.8
12	244.4	31	188.4	50	208.4
13	182.2	32	179.1	51	240.6
14	241.5	33	199.2	52	208.2
15	232.4	34	228.8	53	187.2
16	182.4	35	190.0	54	234.7
17	182.7	36	251.1	55	206.4
18	149.6	37	207.6	56	227.8
19	191.6	38	211.9		

2. Data of the annual yield of a particular crop for the period of 1966-2013 were collected to study the annual cropping pattern of a state and are given in Table 5.

Table 5: Yearly yield data

Year	Yield (in hectares)	Year	Yield (in hectares)	Year	Yield (in hectares)
1966	133	1982	95	1998	341
1967	72	1983	143	1999	237
1968	91	1984	113	2000	290
1969	59	1985	151	2001	323
1970	90	1986	144	2002	397
1971	84	1987	210	2003	334
1972	115	1988	132	2004	422
1973	112	1989	171	2005	322
1974	96	1990	169	2006	357
1975	61	1991	220	2007	459
1976	168	1992	157	2008	402
1977	92	1993	219	2009	497
1978	97	1994	200	2010	494
1979	74	1995	218	2011	448
1980	105	1996	287	2012	560
1981	136	1997	203	2013	543

3. Suppose a manufacturing company produces spare parts for two wheelers. Data for monthly production of the company for 44 months are given in Table 6.

Table 6: Monthly production of spare parts

Month	Production (in thousands)	Month	Production (in thousands)
1	81	23	587
2	133	24	615
3	80	25	876
4	9	26	1074
5	24	27	1025
6	45	28	899
7	294	29	1408
8	128	30	1522
9	275	31	1376
10	277	32	1374
11	272	33	1633
12	100	34	2175
13	45	35	1983
14	502	36	2340
15	41	37	2790
16	102	38	2968
17	510	39	3108
18	569	40	3483
19	423	41	3963
20	431	42	4875
21	298	43	5039
22	842	44	5967

- Represent the data given in Tables 4, 5 and 6 graphically and fit the appropriate trend.
- Fit the appropriate trend using the method of curve fitting by matrix approach.
- Project the annual rainfall in 2014 for the data given in Table 4.
- Project the yield in year 2015 for the data given in Table 5.
- Project the production of spare parts for the 50th month from the data given in Table 6.

Home Work: Do It Yourself

1) Follow the steps explained in Sec. 15.3 to 15.5 to fit an appropriate trend for the data of Tables 1, 2 and 3. Use a different format for plotting the given data along with the fitted trends and take the screenshots.
2) Develop the spreadsheets for the exercises given in "Continuous Assessment 15" as explained in this lab session. Take screenshots of the final spreadsheets and the charts.
3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.

Smoothing or Filtering the Time Series

SESSION 16

Structure

16.1 Introduction

Objectives

16.2 Problem Description

16.3 Simple Moving Average

16.4 Weighted (unequal) Moving Average

16.5 Exponential Smoothing Method

16.1 INTRODUCTION

We have explained in Unit 13 of MSTE-002 (Industrial Statistics-II) that the time series data are usually a combination of four components, i.e., trend (T), cyclic (C), seasonal (S) and irregular (I) effects. Suppose the data have been recorded on a monthly basis and reflect an annual seasonal effect. This means that after every twelve months, the data behave in a similar way. This is called a **period** of the seasonal effect.

Prerequisite

- Lab Session 15 of MSTL-002 (Industrial Statistics Lab).
- Unit 13 of MSTE-002 (Industrial Statistics-II).

For estimation of trend, it is very important to filter the effects of seasonal and irregular fluctuations from the time series so that the trend and cyclic effects can be estimated easily. In Unit 13 of MSTE-002, we have explained the simple moving average, weighted moving average and exponential smoothing methods for the estimation of trend values by smoothing or filtering the effects of the seasonal and irregular fluctuations from the time series data.

In Lab Session 15, you have learnt how to estimate trend values using the curve fitting method. You have applied three types of curve fitting: linear, quadratic and exponential. In this lab session, you will learn about the techniques of smoothing, i.e., simple and weighted moving average methods. You will also learn the exponential smoothing technique.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- apply the simple moving average, weighted moving averages and exponential smoothing method for filtering the seasonal and irregular fluctuations; and
- plot and interpret the data along with the moving averages or exponentially smoothed values.

16.2 PROBLEM DESCRIPTION

Suppose the owner of an ice-cream parlour situated in a locality of a metro city wants to analyse its monthly sales. The data of the total quantity of ice-cream sold every month for 5 years from 2008 to 2012 are recorded in Table 1.

Table 1: Monthly sales of ice-cream

Month	Quantity (in litre)	Month	Quantity (in litre)	Month	Quantity (in litre)
1	288	21	767	41	1210
2	316	22	738	42	1237
3	414	23	689	43	1248
4	540	24	596	44	1217
5	558	25	540	45	1215
6	586	26	630	46	1134
7	558	27	879	47	1024
8	548	28	990	48	898
9	540	29	1032	49	834
10	565	30	1020	50	934
11	465	31	1005	51	1312
12	396	32	1017	52	1398
13	396	33	967	53	1475
14	450	34	924	54	1446
15	648	35	848	55	1456
16	764	36	774	56	1411
17	824	37	694	57	1379
18	802	38	765	58	1358
19	812	39	1134	59	1189
20	776	40	1224	60	1064

For the data given in Table 1,

- Compute the 3, 6, 9 and 12 monthly moving averages. Plot the results along with the quantity of ice-cream sold.
- Compute the weighted moving average for the period of 3 months for the weights 0.2, 0.3 and 0.5. Plot the results on the chart.
- Compute the weighted moving average for the period of 9 months for the weights 0.05, 0.06, 0.08, 0.1, 0.11. 0.13, 0.14, 0.16, and 0.17. Plot the results on the chart.
- Use the smoothing coefficients of 0.05, 0.1, 0.25 and 0.5 to compute the exponentially smoothed series. Plot the results on the chart.

16.3 SIMPLE MOVING AVERAGE

You have studied about the estimation of trend values using the moving averages method in Unit 13 of MSTE-002. Here we mention the main steps involved in the method for both odd and even periods.

In this method, we find the simple moving averages of the time series data over m periods and these are called **m-period moving averages**. This method will also filter the effect of seasonal (S) and irregular (I) components. When the effects of S and I are removed, the data has only the trend (T) and cyclic (C) effects. The simple moving averages for odd and even periods can be computed as follows:

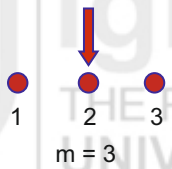
When m is odd

In some situations, the data may show seasonal effect over an **odd period** of time, e.g., 7 days, or 9 months, etc. It means that after every 7 days or 9 months, data behave in a similar manner. For odd periods, the method consists of the following steps:

Step 1: We calculate the average of the first m values of the time series and place it against the middle value, i.e., $\frac{(m+1)}{2}$.

For example, if m = 3, we compute the average of the first three values and place it against the second value.

Step 2: We discard the first value and include the next value. Then we take the average of m values again. For example, if m = 3, we discard the first value to determine the average of the second, third and fourth values and place it against the third value.



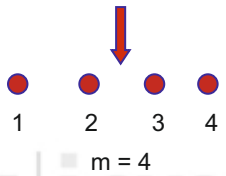
Step 3: We repeat this process until all data are exhausted. These steps provide us a new time series of m-period moving averages.

When m is even

If the period is even, we compute the centred simple moving averages. Sometimes, there may be a seasonal effect over an **even period** of time, e.g., 4 days or 12 months etc. This means that after every 4 days or 12 months, data behave in a similar way. The simple moving averages for even periods need to be centred. So it is also known as centred moving averages. If we take a centred moving average with m = 4 then it will filter (or eliminate) the effect of the season of a quarter. This method consists of the following steps:

Step 1: We calculate the average of the first m values of time series and place it against the middle value, i.e., $\frac{m}{2}$.

For example, if m = 4, we compute the average of the first four values and place it against the middle of the second and third values.



Step 2: We discard the first value and include the next value and then take the average of m values again. For example, if m = 4, we discard the first value, find the average of the second, third, fourth and fifth values and place it against the middle of third and fourth values.

Step 3: We repeat this process until all the data are exhausted. These steps provide us new time series of m-period moving averages.

Step 4: Since the period (m) is even, we determine the centred moving average. To determine the first centred moving average, we compute the average of the first two moving averages and place it against the middle value, i.e., $\left(\frac{m}{2}+1\right)$.

For example, if m = 4, we calculate the average of the first two moving averages and place it against the middle of the first and second moving averages, i.e., against the third period.

Step 5: We repeat this process until all data are exhausted. These steps give a new time series of m-period centred moving averages.

Steps in Excel

In this lab session, we compute 3, 6, 9 and 12 monthly moving averages in which 3 and 9 monthly moving averages are based on odd periods of time and the remaining two are based on even periods of time. The steps involved in the computation of 3, 6, 9 and 12 monthly moving averages are given below for each period:

3 Monthly Moving Averages

Step 1: We enter the given data in an Excel Sheet and name it "**Moving Averages**" as depicted in Fig. 16.1.

	A	B	C
1	Year	Month	Quantity (in litre)
2	2008	1	288
3		2	316
4		3	414
5		4	540
6		5	558
7		6	586
8		7	558
9		8	548
10		9	540
11		10	565
12		11	465
13		12	396
14	2009	13	396
15		14	450
16		15	648
17		16	764
18		17	824
19		18	802
20		19	812

Moving Averages

Fig. 16.1

Step 2: The first 3 monthly moving average is the average of first 3 values which will be placed at the middle, i.e., against the 2nd month. So we choose Cell D3 to calculate the first 3 monthly moving average. In Cell D3, we type "=Average(C2:C4)" as shown in Fig. 16.2.

D3 =AVERAGE(C2:C4)

	A	B	C	D	E
1	Year	Month	Quantity (in litre)	3 Monthly Moving Averages	
2	2008	1	288		
3		2	316	339.333	
4		3	414		
5		4	540		
6		5	558		

DRAG IT DOWN

Fig. 16.2

Step 3: We drag down Cell D3 up to Cell D60 as shown in Fig. 16.3, since the last moving average corresponds to the 59th month.

	A	B	C	D	E
1	Year	Month	Quantity (in litre)	3 Monthly Moving Averages	
2	2008	1	288		
3		2	316	339.333	
4		3	414	423.333	
5		4	540	504.000	
6		5	558	561.333	
7		6	586	567.333	
8		7	558	564.000	
9		8	548	548.667	
10		9	540	551.000	
11		10	565	523.333	
12		11	465	475.333	
13		12	396	419.000	
14	2009	13	396	414.000	
15		14	450	498.000	
16		15	648	620.667	
17		16	764	745.333	
18		17	824	796.667	
19		18	802	812.667	
20		19	812	796.667	

Moving Averages

Fig. 16.3

If we consider 3 monthly moving averages, we need to leave the first cell corresponding to the first month for calculating the first moving average and we should also leave the last cell corresponding to the last month for calculating the last moving average.

6 Monthly Moving Averages

Step 4: As discussed in Sec. 13.7 of Unit 13 in MSTE-002, we proceed as follows for finding the moving average for even period of 6 months:

i) We find the average of the first 6 months, which is placed at 3.5th month and then compute the average of the next 6 months after discarding the first month, i.e., 2nd to 7th months, which is placed at 4.5th month. The process continues until all data are used.

ii) In the end, we compute the average of the first two averages obtained in (i), which corresponds to (3.5 + 4.5)/2 = 4th month.

In Excel, we can type "=(Average (C2:C7)+Average (C3:C8))/2" in Cell E5 to find the first 6 monthly centred moving average which corresponds to the 4th month as shown in Fig. 16.4.

E5 =(AVERAGE(C2:C7)+AVERAGE(C3:C8))/2

	B	C	D	E	F
1	Month	Quantity (in litre)	3 Monthly Moving Averages	6 Monthly Moving Averages (Centred)	
2	1	288			
3	2	316	339.333		
4	3	414	423.333		
5	4	540	504.000	472.833	
6	5	558	561.333		
7	6	586	567.333		
8	7	558	564.000		

DRAG IT DOWN

Fig. 16.4

Step 5: We now drag down Cell E5 up to Cell E58 (Fig. 16.5.), as the last moving average corresponds to the 57th month.

	B	C	D	E	F
1	Month	Quantity (in litre)	3 Monthly Moving Averages	6 Monthly Moving Averages (Centred)	
2	1	288			
3	2	316	339.333		
4	3	414	423.333		
5	4	540	504.000	472.833	
6	5	558	561.333	514.667	
7	6	586	567.333	544.500	
8	7	558	564.000	557.083	
9	8	548	548.667	551.417	
10	9	540	551.000	527.833	
11	10	565	523.333	498.500	
12	11	465	475.333	476.833	
13	12	396	419.000	477.667	
14	13	396	414.000	503.250	
15	14	450	498.000	549.750	
16	15	648	620.667	613.500	
17	16	764	745.333	682.000	
18	17	824	796.667	743.833	
19	18	802	812.667	780.917	
20	19	812	796.667	788.667	

Moving Averages

Fig. 16.5

9 Monthly Moving Averages

Step 6: Using the procedure explained in Steps 1-3, we compute the first 9 monthly moving average in Cell F6 by typing "=Average (C2:C10)" and dragging down Cell F6 up to Cell F57 as shown in Fig.16.6.

F6 =AVERAGE(C2:C10)

	B	C	D	E	F	G
1	Month	Quantity (in litre)	3 Monthly Moving Averages	6 Monthly Moving Averages (Centred)	9 Monthly Moving Averages	
2	1	288				
3	2	316	339.333			
4	3	414	423.333			
5	4	540	504.000	472.833		
6	5	558	561.333	514.667	483.111	
7	6	586	567.333	544.500	513.889	
8	7	558	564.000	557.083	530.444	
9	8	548	548.667	551.417	528.444	
10	9	540	551.000	527.833	512.444	
11	10	565	523.333	498.500	500.444	
12	11	465	475.333	476.833	507.333	
13	12	396	419.000	477.667	530.222	
14	13	396	414.000	503.250	560.889	
15	14	450	498.000	549.750	590.000	
16	15	648	620.667	613.500	617.444	
17	16	764	745.333	682.000	652.000	
18	17	824	796.667	743.833	693.222	
19	18	802	812.667	780.917	731.222	
20	19	812	796.667	788.667	757.778	

Moving Averages

Fig. 16.6

12 Monthly Moving Averages

Step 7: We compute the first 12 monthly average by typing "=(Average(C2:C13)+Average(C3:C14))/2" in Cell G8 and dragging it down up to Cell G55 as shown in Fig. 16.7.

G8 =(AVERAGE(C2:C13)+AVERAGE(C3:C14))/2

	B	C	D	E	F	G	H
1	Month	Quantity (in litre)	3 Monthly Moving Averages	6 Monthly Moving Averages (Centred)	9 Monthly Moving Averages	12 Monthly Moving Averages (Centred)	
2	1	288					
3	2	316	339.333				
4	3	414	423.333				
5	4	540	504.000	472.833			
6	5	558	561.333	514.667	483.111		
7	6	586	567.333	544.500	513.889		
8	7	558	564.000	557.083	530.444	485.667	
9	8	548	548.667	551.417	528.444	495.750	
10	9	540	551.000	527.833	512.444	511.083	
11	10	565	523.333	498.500	500.444	530.167	
12	11	465	475.333	476.833	507.333	550.583	
13	12	396	419.000	477.667	530.222	570.667	
14	13	396	414.000	503.250	560.889	590.250	
15	14	450	498.000	549.750	590.000	610.333	
16	15	648	620.667	613.500	617.444	629.292	
17	16	764	745.333	682.000	652.000	645.958	
18	17	824	796.667	743.833	693.222	662.500	
19	18	802	812.667	780.917	731.222	680.167	
20	19	812	796.667	788.667	757.778	694.500	

Moving Averages

Fig. 16.7

Step 8: We plot and format the graph of the 3, 6, 9 and 12 monthly moving averages computed in Columns D, E, F and G, respectively, and the data given in Column C against the corresponding months as explained in Sec. 15.3 of Lab Session 15. The resulting chart is shown in Fig. 16.8.

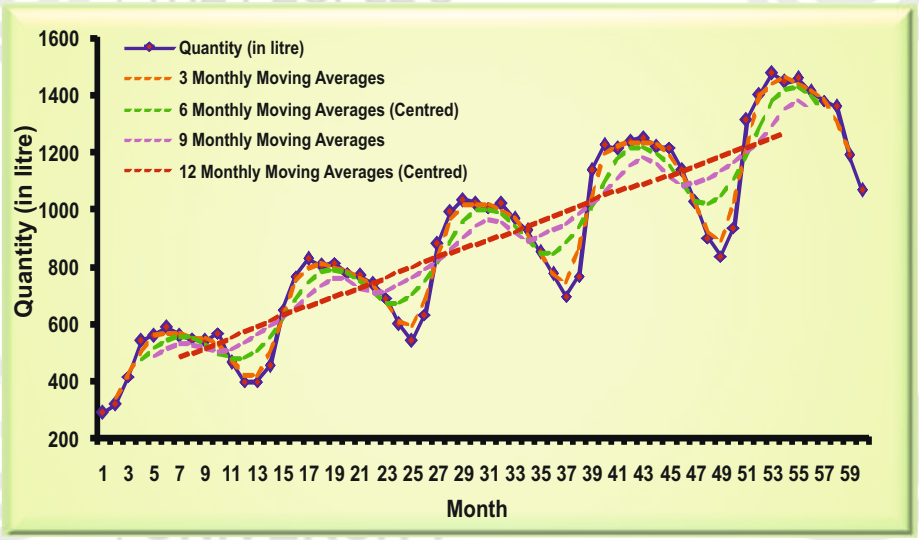


Fig. 16.8

From Fig. 16.8, we can notice the difference among the moving averages of the different periods along with the given data. Note that the fluctuations (crest and trough) are minimum when we consider 12 monthly moving average. The given data contained seasonal or irregular variation and we have eliminated these variations using the moving average method. Fig. 16.8 reveals that the smoothest curve is obtained for the 12 monthly moving average. So we can say that the period of the season is 12 months in the data given here. We can also study the long term trend after smoothing the data.

16.4 WEIGHTED (UNEQUAL) MOVING AVERAGE

As you have learnt in Sec. 13.5 of Unit 13 of MSTE-002, the simple moving average method described in Sec. 16.3 does not lie close to the latest values in the data. So we use weighted (unequal) moving averages method. In this method, we assign unequal weights to all values. The formulae and procedure involved in this method are given below:

Step 1: If w_i denotes the weight of the ith observation, the moving average value y_t is given by

$$y_t = \sum_{i=-q}^{q} w_i x_{t+i} \qquad \ldots(1)$$

such that $w_i \geq 0, \ \sum_{i=-q}^{q} w_i = 1$

where x_t is the original series.

Step 2: The procedure for getting the smooth trend values by the weighted moving average method is the same as that for the simple moving average in spite of weights. We calculate here the weighted average of the first m values given in time series as per equation (1).

Step 3: We now discard the first value and include the next one and take the average of the m values.

Step 4: We repeat this process until all values are exhausted. These steps produce a new time series of m period weighted moving averages.

Simple moving average becomes a particular case of weighted moving average which is obtained by taking m = 2q+1 and

$$w_i = \frac{1}{(2q+1)}.$$

In Sec. 16.3, we have considered m =3.

$\Rightarrow$ m = 2q+1 = 3

$\Rightarrow$ q = 1

$\Rightarrow$ w_i = 1/(2q+1) = 1/3.

Steps in Excel

In this lab session, we continue with the data given in Sec. 16.2 to help you learn how to compute the weighted moving averages of 3 and 9 months periods in Excel 2007. The steps are given below:

3 Monthly Weighted Moving Averages

Step 1: We enter the data given in Table 1 in Excel sheet and name it "**Weighted Moving Averages**" as shown in Fig. 16.9.

	A	B	C
1			
2	Year	Month	Quantity (in litre)
3	2008	1	288
4		2	316
5		3	414
6		4	540
7		5	558
8		6	586
9		7	558
10		8	548
11		9	540
12		10	565
13		11	465
14		12	396
15	2009	13	396
16		14	450
17		15	648
18		16	764
19		17	824
20		18	802

Exponential Smoothing

Fig. 16.9

Step 2: Since it is given that $w_1 = 0.2$, $w_2 = 0.3$ and $w_3 = 0.5$, we calculate the first weighted moving average of the first 3 values by typing "=0.2*C2+0.3*C3+0.5*C4" in Cell D3 as shown in Fig. 16.10.

D3 | fx =0.2*C2+0.3*C3+0.5*C4

	A	B	C	D	E
1	Year	Month	Quantity (in litre)	3 Monthly Weighted Moving Averages	
2	2008	1	288		
3		2	316	359.400	
4		3	414		
5		4	540		
6		5	558		

DRAG IT DOWN

Fig. 16.10

Step 3: We drag down Cell D3 up to Cell D60 to get the remaining values of 3 monthly weighted moving averages as shown in Fig. 16.11.

	A	B	C	D	E
1	Year	Month	Quantity (in litre)	3 Monthly Weighted Moving Averages	
2	2008	1	288		
3		2	316	359.400	
4		3	414	457.400	
5		4	540	523.800	
6		5	558	568.400	
7		6	586	566.400	
8		7	558	558.600	
9		8	548	546.000	
10		9	540	554.100	
11		10	565	510.000	
12		11	465	450.500	
13		12	396	409.800	
14	2009	13	396	423.000	
15		14	450	538.200	
16		15	648	666.400	
17		16	764	770.800	
18		17	824	801.000	
19		18	802	811.400	
20		19	812	792.000	

Weighted Moving Averages

Fig. 16.11

Step 4: For 9 monthly moving averages, we are given $w_1 = 0.05$, $w_2 = 0.06$, $w_3 = 0.08$, $w_4 = 0.1$, $w_5 = 0.11$, $w_6 = 0.13$ $w_7 = 0.14$, $w_8 = 0.16$ and $w_9 = 0.17$. We now type "=0.05*C2+0.06*C3+0.08*C4+0.1*C5+0.11*C6+0.13*C7+0.14*C8+0.16*C9 +0.17*C10" in Cell E6 to get the first 9 monthly weighted moving average and drag it down up to Cell E57 to get the remaining moving averages as shown in Fig. 16.12.

E6 =0.05*C2+0.06*C3+0.08*C4+0.1*C5+0.11*C6+0.13*C7+0.14*C8+0.16*C9+0.17*C10

	C	D	E	F	G
1	Quantity (in litre)	3 Monthly Weighted Moving Averages	9 Monthly Weighted Moving Averages		
2	288				
3	316	359.400			
4	414	457.400			
5	540	523.800			
6	558	568.400	515.64		
7	586	566.400			
8	558	558.600			
9	548	546.000			

DRAG IT DOWN

	C	D	E	F	G
1	Quantity (in litre)	3 Monthly Weighted Moving Averages	9 Monthly Weighted Moving Averages		
2	288				
3	316	359.400			
4	414	457.400			
5	540	523.800			
6	558	568.400	515.64		
7	586	566.400	535.81		
8	558	558.600	534.01		
9	548	546.000	514.46		
10	540	554.100	491.13		
11	565	510.000	478.52		
12	465	450.500	500.71		
13	396	409.800	543.1		
14	396	423.000	592.8		
15	450	538.200	636.31		
16	648	666.400	676.01		
17	764	770.800	709.64		
18	824	801.000	738.17		
19	802	811.400	755.5		
20	812	792.000	757.15		

Weighted Moving Averages

Fig. 16.12

Step 5: In Columns F and G, we compute the 3 and 9 monthly simple moving averages shown in Fig. 16.13 following Steps 2, 3 and 6 under the heading 'Steps in Excel' of Sec.16.3.

	F	G
1	3 Monthly Moving Averages	9 Monthly Moving Averages
2		
3	339.333	
4	423.333	
5	504.000	
6	561.333	483.111
7	567.333	513.889
8	564.000	530.444
9	548.667	528.444
10	551.000	512.444
11	523.333	500.444
12	475.333	507.333
13	419.000	530.222
14	414.000	560.889
15	498.000	590.000
16	620.667	617.444
17	745.333	652.000
18	796.667	693.222
19	812.667	731.222
20	796.667	757.778

Weighted Moving Averages

Fig. 16.13

Step 6: We now plot the simple and weighted moving averages along with the original data as explained in Step 8 of Sec. 16.3. The resulting graphical representation is shown in Fig. 16.14.

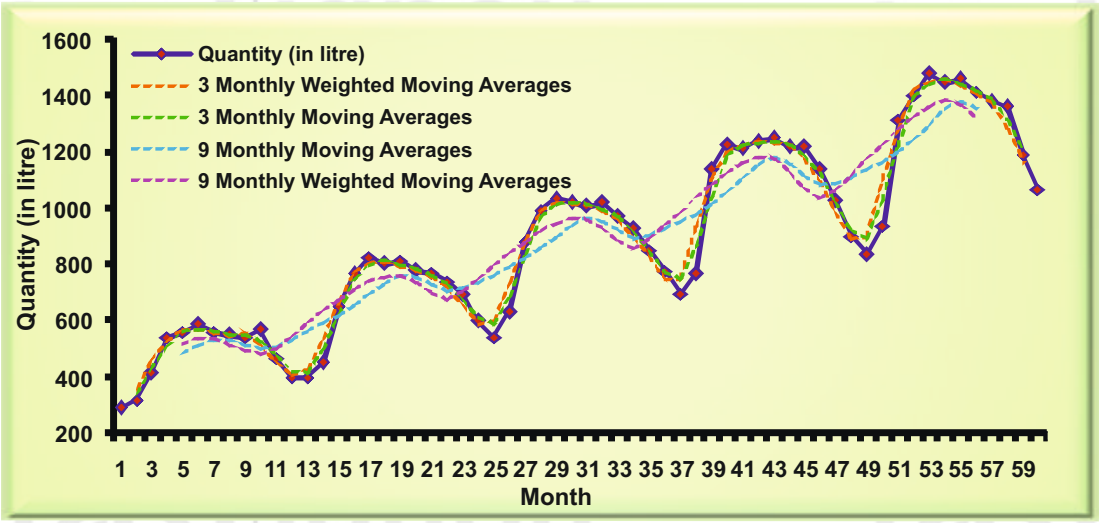


Fig. 16.14

Note from Fig. 16.14 that weighted moving averages (3 and 9 monthly) show the upward trend faster than the simple moving averages.

16.5 EXPONENTIAL SMOOTHING METHOD

This smoothing method is very popular. You have already learnt all formulae and procedure in Unit 13 of MSTE-002. We explain the main steps involved in this method to compute the exponentially smoothed time series as follows:

In this method, we assign weights in such a way that the latest observation gets the maximum weight and then the weights decrease exponentially. We select a single weight w, which is called the exponential smoothing constant.

Step 1: If $y_1, y_2, \ldots, y_t$ is a given time series, the smoothed value at time t is given by

$$y'_t = \sum_{j=0}^{t-2} w(1-w)^j y_{t-j} + (1-w)^{t-1} y_1 \qquad \ldots(2)$$

where $0<w<1$ and $\sum w_i = 1$. We choose the value of 'w' according to the requirement.

Step 2: We assume the first given value as the first smoothed value, i.e.,

$$y'_1 = y_1 \qquad \ldots(3)$$

Step 3: The second smoothed value is computed using the first smoothed value and the second value of the given time series from the following equation:

$$y'_2 = w\, y_2 + (1-w)\, y'_1 \qquad \ldots(4)$$

Step 4: The third smoothed value is computed using the second smoothed value and the third value of the given time series as follows:

$$y'_3 = w\, y_3 + (1-w)\, y'_2 \qquad \ldots(5)$$

Step 5: In the same way, the tth smoothed value can be computed as follows:

$$y'_t = w\, y_t + (1-w)\, y'_{t-1} \qquad ...(6)$$

Step 6: If the forecast for y_t is y'_t, we calculate the forecast error as

$$e_t = y_t - y'_t \qquad ...(7)$$

Steps in Excel

We consider the data given in Sec. 16.2 to filter the seasonal and irregular fluctuations using the exponential smoothing method. The following steps will make you familiar with the exponential smoothing method for different values of the weight (w):

Step 1: We enter the data given in Table 1 in an Excel sheet and name it "**Exponential Smoothing**" as shown in Fig. 16.15.

	A	B	C
1			
2	Year	Month	Quantity (in litre)
3	2008	1	288
4		2	316
5		3	414
6		4	540
7		5	558
8		6	586
9		7	558
10		8	548
11		9	540
12		10	565
13		11	465
14		12	396
15	2009	13	396
16		14	450
17		15	648
18		16	764
19		17	824
20		18	802

Exponential Smoothing

Fig. 16.15

Step 2: We calculate the exponentially smoothed values for w = 0.05. The first exponentially smoothed value will be the same as the original first value, i.e., $y'_1 = y_1$. So in Cell D3, we type "=C3" as shown in Fig. 16.16.

D3 | fx | =C3

	A	B	C	D	E
1				Exponentially Smoothed Values (y_t')	
2	Year	Month	Quantity (in litre)	w = 0.05	
3	2008	1	288	288	
4		2	316		
5		3	414		
6		4	540		
7		5	558		

Fig. 16.16

Step 3: In Cell D4, we type "$=(0.05*C4)+((1-0.05)*D3)$" to get the second smoothed value as shown in Fig. 16.17.

D4 =(0.05*$C4)+((1-0.05)*D3)

	A	B	C	D	E
1				Exponentially Smoothed Values (y_t')	
2	Year	Month	Quantity (in litre)	w = 0.05	
3	2008	1	288	288	
4		2	316	289.400	
5		3	414		
6		4	540		
7		5	558		
8		6	586		
9		7	558		
10		8	548		
11		9	540		

DRAG IT DOWN

Fig. 16.17

Step 4: We drag down Cell D4 up to Cell D62 to determine the remaining exponentially smoothed values for w = 0.05 as shown in Fig. 16.18.

	A	B	C	D	E
1				Exponentially Smoothed Values (y_t')	
2	Year	Month	Quantity (in litre)	w = 0.05	
3	2008	1	288	288	
4		2	316	289.400	
5		3	414	295.630	
6		4	540	307.849	
7		5	558	320.356	
8		6	586	333.638	
9		7	558	344.856	
10		8	548	355.014	
11		9	540	364.263	
12		10	565	374.300	
13		11	465	378.835	
14		12	396	379.693	
15	2009	13	396	380.508	
16		14	450	383.983	
17		15	648	397.184	
18		16	764	415.525	
19		17	824	435.948	
20		18	802	454.251	
21		19	812	472.138	
22		20	776	487.331	
23		21	767	501.315	
24		22	738	513.149	
25		23	689	521.942	

Exponential Smoothing

Fig. 16.18

The choice of w is very important. If we want to eliminate seasonal and irregular fluctuations, we choose a small value (close to zero) of w.

But if our aim is to forecast, we choose a large value of w (close to 1).

Step 5: We follow the procedure explained in Steps 2 to 4 and compute the exponentially smoothed values for other values of weight (w), i.e., 0.1, 0.25 and 0.5 as shown in Fig. 16.19.

The difference between original data given in Column C and exponentially smoothed values given in column D, E, F and G give the forecast errors for the weights 0.05, 0.1, 0.25 and 0.5, respectively.

	B	C	D	E	F	G
1			Exponentially Smoothed Values (y_t')			
2	Month	Quantity (in litre)	w = 0.05	w = 0.1	w = 0.25	w = 0.5
3	1	288	288	288	288	288
4	2	316	289.400	290.800	295.000	302.000
5	3	414	295.630	303.120	324.750	358.000
6	4	540	307.849	326.808	378.563	449.000
7	5	558	320.356	349.927	423.422	503.500
8	6	586	333.638	373.534	464.066	544.750
9	7	558	344.856	391.981	487.550	551.375
10	8	548	355.014	407.583	502.662	549.688
11	9	540	364.263	420.825	511.997	544.844
12	10	565	374.300	435.242	525.248	554.922
13	11	465	378.835	438.218	510.186	509.961
14	12	396	379.693	433.996	481.639	452.980
15	13	396	380.508	430.197	460.229	424.490
16	14	450	383.983	432.177	457.672	437.245
17	15	648	397.184	453.759	505.254	542.623
18	16	764	415.525	484.783	569.941	653.311
19	17	824	435.948	518.705	633.455	738.656
20	18	802	454.251	547.034	675.592	770.328

Exponential Smoothing

Fig. 16.19

Step 6: To compare the original series and the exponentially smoothed values obtained for different values of w given in Cells C3:G62, we plot them as shown in Fig. 16.20.

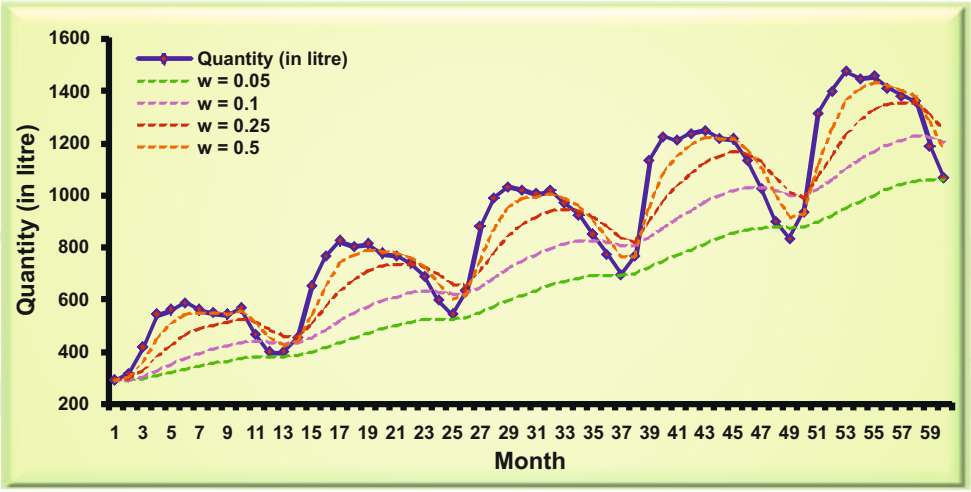


Fig. 16.20

Note that we get a smooth curve which changes slowly for small values of w and as we increase the value of w, the exponentially smoothed values come closer to the original values (Fig. 16.20).

You can now solve the following exercises to check whether you have learnt how to filter the seasonal and irregular fluctuations from the given data.

Activity

Filter the seasonal and irregular fluctuation with the help of MS Excel 2007 and interpret the results for

A1) Examples 1, 2 and 3 given in Unit 13 of MSTE-002.

A2) Exercises E1, E2 and E3 given in Unit 13 of MSTE-002.

Match the results with the calculation done in Unit 13 of MSTE-002.

Continuous Assessment 16

Suppose a company wants to start a sugar manufacturing unit. We know that the production of sugar depends on the yield of sugarcane. Hypothetical data for annual sugarcane yield for a state for 50 years from 1963 to 2012 are given in Table 2.

Table 2: Annual yield of sugarcane

Year	Yield (in hectares)	Year	Yield (in hectares)	Year	Yield (in hectares)
1	224	18	1060	35	1357
2	252	19	1046	36	1332
3	350	20	1073	37	1386
4	514	21	1084	38	1584
5	505	22	870	39	1763
6	533	23	823	40	1684
7	357	24	1123	41	1782
8	332	25	1234	42	1748
9	386	26	1311	43	1663
10	584	27	1243	44	1701
11	763	28	1292	45	1970
12	684	29	1170	46	2044
13	738	30	1134	47	1983
14	748	31	1350	48	2015
15	663	32	1514	49	1924
16	701	33	1505	50	1873

For the data given in Table 2,

- Compute the 3, 4 and 7 yearly moving averages. Plot the results along with the yield of sugarcane given in Table 2.
- Compute the weighted moving averages for a period of 3 years for the weights 0.2, 0.3 and 0.5. Plot the results on the chart.
- Compute the weighted moving averages for a period of 7 years for the weights 0.07, 0.09, 0.12. 0.14, 0.17, 0.19, and 0.22. Plot the results on the chart.
- Use the smoothing coefficients of 0.25, 0.1, 0.5 and 0.6 to compute the exponentially smoothed series. Plot the results on the chart.

Home Work: Do It Yourself

1) Follow the steps explained in Sec. 16.3 to 16.5 to filter the time series data given in Table 1. Use a different format for plotting the given data along with the smoothed time series values. Take the screenshots and keep them in your record book.
2) Develop the spreadsheets for the exercise given in "Continuous Assessment 16" as explained in this lab session. Take screenshots of the final spreadsheets and the charts.
3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.

Seasonal Component Analysis

SESSION 17

Structure

17.1 Introduction
Objectives
17.2 Problem Description
17.3 Simple Average Method
17.4 Ratio to Trend Method
17.5 Ratio to Moving Average Method

17.1 INTRODUCTION

Prerequisite

- Lab Sessions 15 and 16 of MSTL-002 (Industrial Statistics Lab).
- Unit 14 of MSTE-002 (Industrial Statistics-II).

In Lab Session 15, you have learnt the method for estimating the trend using the curve fitting method. You have also learnt the methods for smoothing or filtering the time series, i.e., the simple moving average, weighted moving average and exponential smoothing methods in Lab Session 16.

When time series data reflect seasonal variation, we estimate the seasonal component by removing the irregular component. In this lab session, you will learn three methods of estimating seasonal indices, i.e., the simple average method, ratio to trend method and ratio to moving average method.

If the effect of seasonal variation is not removed from the time series data containing the trend effect, the trend estimates are affected by the seasonal effects. Therefore, we have to deseasonalise the data by dividing the given time series data by the corresponding seasonal indices. Once the data is free from the seasonal effect, we estimate the trend components. For estimation of the trend components from the deseasonalised data, we apply the method of least squares as explained in Lab Session 15.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- obtain the seasonal indices using simple average, ratio to trend and moving average methods;
- determine the deseasonalised values; and
- compare the original and deseasonalised values graphically.

17.2 PROBLEM DESCRIPTION

For this lab session, we consider the data given in Lab Session 16 and apply the simple average, ratio to trend and moving average methods to

- compute the seasonal indices for 12 months,
- obtain the deseasonalised values and estimate the trend line by the method of least squares, and
- plot the given data and the deseasonalised values.

17.3 SIMPLE AVERAGE METHOD

The simple average method is the simplest method of measuring seasonality. This method is used to estimate the seasonal effect from the given time series data. It is based on the assumption that the given data do not contain any trend and cyclic components. This method consists of elimination of the irregular component by averaging the monthly (or quarterly, weekly or yearly) values over the years. It consists of the following steps:

The assumptions of no trend and cyclic components may or may not be true since most economic or business time series data exhibit trend.

Step 1: We arrange the data by months, quarters or weeks, etc., if monthly, quarterly or weekly data are given. For example, if we have monthly data, we arrange the data according to the months in rows and years in columns.

Step 2: After arranging the time series data, we calculate the averages (monthly, quarterly or weekly). For example, if we have monthly data, we calculate the average for each month for one year, i.e., $\bar{y}_i$ for $(i = 1, 2, ..., 12)$.

Step 3: We calculate the grand average of these averages. For example, if we have monthly data for one year, the grand average is

$$\bar{\bar{y}} = \frac{\bar{y}_1 + \bar{y}_2 + ... + \bar{y}_{12}}{12} \qquad ...(1)$$

Step 4: We express each average as the percentage of the grand average $\bar{\bar{y}}$. These percentages are known as **seasonal indices**. For example, if we have monthly data, the seasonal index for the ith month is given by

$$S_i = \frac{\bar{y}_i}{\bar{\bar{y}}} \times 100, \qquad \text{for } i = 1, 2, ..., 12. \qquad ...(2)$$

Step 5: To adjust seasonality of the data, we can also obtain the deseasonalised values by dividing the original (given) values with its corresponding adjusted seasonal indices which eliminate the seasonal effects, i.e.,

$$\text{Deseasonalised value} = \frac{\text{original value}}{\text{adjusted seasonal values}} \times 100 \qquad ...(3)$$

After we obtain the deseasonalised data, we can apply trend analysis as explained in Lab Session 15.

Steps in Excel

In the data given in Lab Session 16, we have monthly sales of the ice-cream parlour for 5 years. After going through the following steps, you will be able to apply the simple average method in Excel 2007:

Step 1: We enter the given data in an Excel sheet as shown in Fig.17.1.

	A	B	C	D	E	F
1		Quantity (in litre)				
2	Year / Month	2008	2009	2010	2011	2012
3	1	288	396	540	694	834
4	2	316	450	630	765	934
5	3	414	648	879	1134	1312
6	4	540	764	990	1224	1398
7	5	558	824	1032	1210	1475
8	6	586	802	1020	1237	1446
9	7	558	812	1005	1248	1456
10	8	548	776	1017	1217	1411
11	9	540	767	967	1215	1379
12	10	565	738	924	1134	1358
13	11	465	689	848	1024	1189
14	12	396	596	774	898	1064
15						

Fig. 17.1

Step 2: We type "=Average(B3:F3)" in Cell G3 to calculate the monthly average sales for the month of January. We drag down Cell G3 up to Cell G14 to get the monthly averages for all 12 months as shown in Fig. 17.2.

G3 =AVERAGE(B3:F3)

	A	B	C	D	E	F	G	H
1		Quantity (in litre)						
2	Year / Month	2008	2009	2010	2011	2012	Average	
3	1	288	396	540	694	834	550.400	
4	2	316	450	630	765	934		
5	3	414	648	879	1134	1312		
6	4	540	764	990	1224	1398		

DRAG IT DOWN

	A	B	C	D	E	F	G	H
1		Quantity (in litre)						
2	Year / Month	2008	2009	2010	2011	2012	Average	
3	1	288	396	540	694	834	550.400	
4	2	316	450	630	765	934	619.000	
5	3	414	648	879	1134	1312	877.400	
6	4	540	764	990	1224	1398	983.200	
7	5	558	824	1032	1210	1475	1019.800	
8	6	586	802	1020	1237	1446	1018.200	
9	7	558	812	1005	1248	1456	1015.800	
10	8	548	776	1017	1217	1411	993.800	
11	9	540	767	967	1215	1379	973.600	
12	10	565	738	924	1134	1358	943.800	
13	11	465	689	848	1024	1189	843.000	
14	12	396	596	774	898	1064	745.600	
15								

Fig. 17.2

Step 3: We compute the grand average of the monthly averages given in Cells G3:G14 by typing "=Average(G3:G14)" in Cell G15 as shown in Fig. 17.3.

G15 =AVERAGE(G3:G14)

	F	G	H
15	Average	881.967	
16			

Fig. 17.3

Step 4: To determine the seasonal indices, we type "=(G3/$G$15)*100" in Cell H3 and then drag it down up to Cell H14 as shown in Fig. 17.4. These seasonal indices are known as adjusted seasonal indices.

H3 =(G3/$G$15)*100

	G	H	I
2	Average	Seasonal Index	
3	550.400	62.406	
4	619.000		

DRAG IT DOWN

	G	H	I
2	Average	Seasonal Index	
3	550.400	62.406	
4	619.000	70.184	
5	877.400	99.482	
6	983.200	111.478	
7	1019.800	115.628	
8	1018.200	115.447	
9	1015.800	115.174	
10	993.800	112.680	
11	973.600	110.390	
12	943.800	107.011	
13	843.000	95.582	
14	745.600	84.538	

Fig. 17.4

Step 5: We can now verify that the average of the seasonal indices will be 100 as shown in Fig. 17.5.

H15 =AVERAGE(H3:H14)

	F	G	H	I
15	Average	881.967	100.000	
16				

Fig. 17.5

Step 6: We type (or ***Copy*** and ***Paste***) the given data in Cells A17:B77 as shown in Fig. 17.6.

	A	B
17	Month	Quantity (in litre)
18	1	288
19	2	316
20	3	414
21	4	540
22	5	558
23	6	586
24	7	558
25	8	548
26	9	540
27	10	565
28	11	465
29	12	396
30	13	396
31	14	450
32	15	648
33	16	764
34	17	824
35	18	802
36	19	812

Simple Averages

Fig. 17.6

Step 7: We now copy these adjusted seasonal indices from H3:H14 and paste these values in Cells C18, C30, C42, C54 and C66 using ***Paste Values*** option as shown in Fig. 17.7.

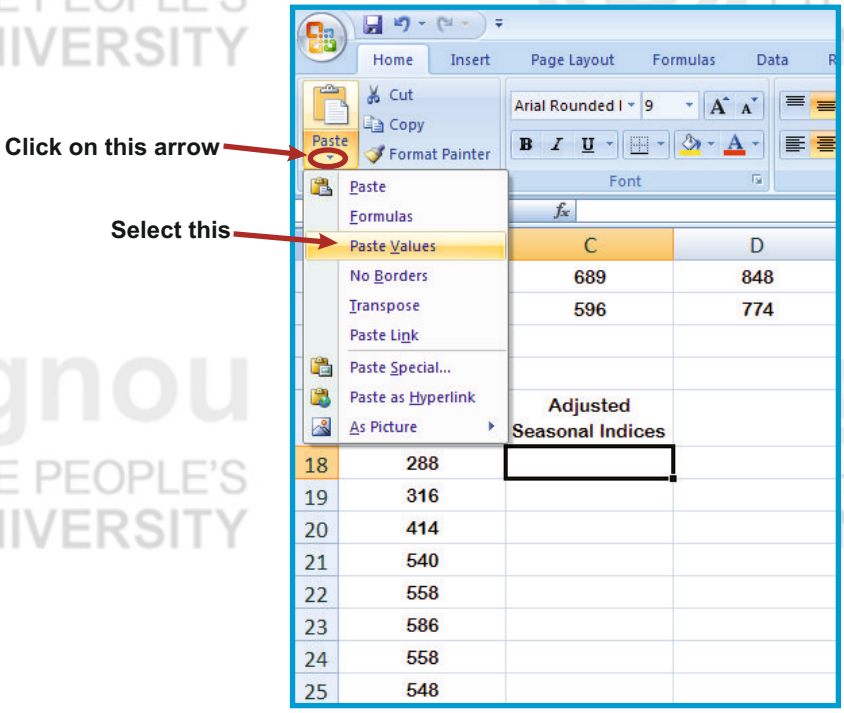


Fig. 17.7

The values given in Cells C18:C77 are the adjusted seasonal indices corresponding to the given data (Fig. 17.8).

	A	B	C
17	Month	Quantity (in litre)	Adjusted Seasonal Indices
18	1	288	62.406
19	2	316	70.184
20	3	414	99.482
21	4	540	111.478
22	5	558	115.628
23	6	586	115.447
24	7	558	115.174
25	8	548	112.680
26	9	540	110.390
27	10	565	107.011
28	11	465	95.582
29	12	396	84.538
30	13	396	62.406
31	14	450	70.184
32	15	648	99.482
33	16	764	111.478
34	17	824	115.628
35	18	802	115.447
36	19	812	115.174

Simple Averages

Fig. 17.8

Step 8: We also compute deseasonalised values for the given data by typing "=(B18/C18)*100" in Cell D18 and drag it down up to Cell D77 as shown in Fig. 17.9. Once the data are deseasonalised, the seasonal influences are removed from the data.

D18 =B18/C18*100

	B	C	D	E
17	Quantity (in litre)	Adjusted Seasonal Indices	Deseasonalised Values	
18	288	62.406	461.494	
19	316	70.184		

DRAG IT DOWN

	B	C	D	E
17	Quantity (in litre)	Adjusted Seasonal Indices	Deseasonalised Values	
18	288	62.406	461.494	
19	316	70.184	450.245	
20	414	99.482	416.155	
21	540	111.478	484.400	
22	558	115.628	482.582	
23	586	115.447	507.594	
24	558	115.174	484.483	
25	548	112.680	486.333	
26	540	110.390	489.176	
27	565	107.011	527.984	
28	465	95.582	486.494	
29	396	84.538	468.427	
30	396	62.406	634.555	
31	450	70.184	641.171	
32	648	99.482	651.373	
33	764	111.478	685.336	
34	824	115.628	712.630	
35	802	115.447	694.694	
36	812	115.174	705.018	

Simple Averages

Fig. 17.9

Step 9: We plot the given data and the deseasonalised values in Excel 2007 as explained in Sec. 15.3 of Lab Session 15. The resulting graph is shown in Fig. 17.10.

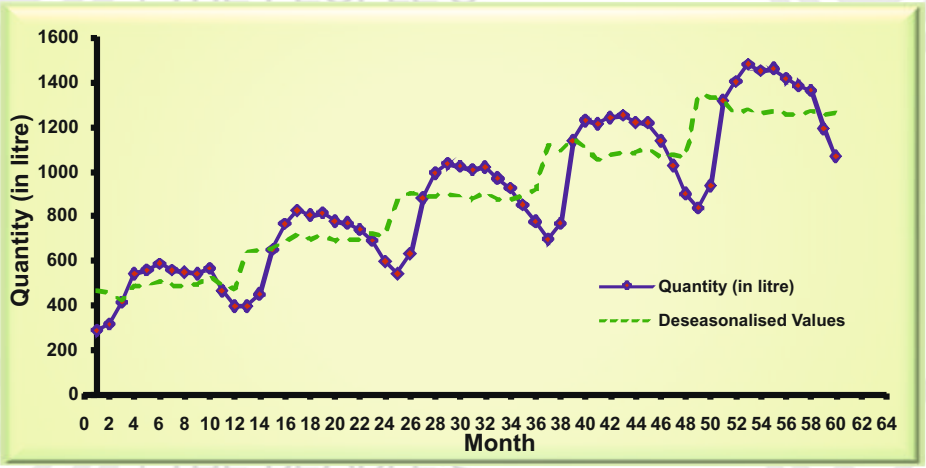


Fig. 17.10

Fig. 17.10 reveals that the deseasonalised values obtained after eliminating the seasonal effects have less crests and troughs in comparison with the original data and show an upward linear trend.

17.4 RATIO TO TREND METHOD

This method provides seasonal indices free from trend and is an improved version of the simple average method as it assumes that the seasonal variation for a given period is a constant fraction of trend. The measurement of the seasonal indices by this method consists of the following steps:

Step 1: We compute the yearly averages for the given characteristic or variable.

Step 2: We establish a trend by fitting a suitable curve using the method of least squares and taking the yearly averages as the dependent variable and year as the independent variable. Then we obtain the trend values for each period.

Step 3: On the basis of these yearly trend values, we determine the trend values for each season (e.g., month) by computing the monthly increment from the slope of the fitted trend curve.

Step 4: To express the original time series values as percentages of the trend values, we divide each original value by the corresponding trend value and multiply it by 100. The seasonal indices so obtained are free from the trend and known as the ratio to trend or trend eliminated values.

Step 5: To obtain the seasonal indices, we determine the average (mean or median) of the ratio to trend values for each season.

Step 6: We compute the grand average of all the seasonal indices obtained in Step 5.

Step 7: If the seasonal periods are quarters, the sum of the seasonal indices in case of multiplicative model should be 400. If seasonal periods are months, it should be 1200 and the average of these should be 100. But as often the sum (or average) of all seasonal indices is not exactly what it should be, we divide each seasonal index by the grand average obtained in Step 6 and multiply it by 100. It will make the average equal to 100.

Step 8: To adjust seasonality of the data, we can also obtain the deseasonalised values by dividing the original (given) values with the corresponding adjusted seasonal indices which eliminate the seasonal effects, i.e.,

$$\text{Deseasonalised value} = \frac{\text{original value}}{\text{adjusted seasonal value}} \times 100 \qquad ...(4)$$

After we obtain the deseasonalised data, we can apply trend analysis as explained in Lab Session 15.

Steps in Excel

Step 1: For applying the ratio to trend method, we arrange (or type) the data in the same way as explained in Step 1 of Sec. 17.3 (see Fig. 17.11).

	A	B	C	D	E	F
1		Quantity (in litre)				
2	Year / Month	2008	2009	2010	2011	2012
3	1	288	396	540	694	834
4	2	316	450	630	765	934
5	3	414	648	879	1134	1312
6	4	540	764	990	1224	1398
7	5	558	824	1032	1210	1475
8	6	586	802	1020	1237	1446
9	7	558	812	1005	1248	1456
10	8	548	776	1017	1217	1411

Fig. 17.11

Step 2: We type "=Average(B3: B14)" in Cell B15 to compute the yearly average of 2008 and drag it right up to Cell F15 to obtain the remaining yearly averages as shown in Fig. 17.12.

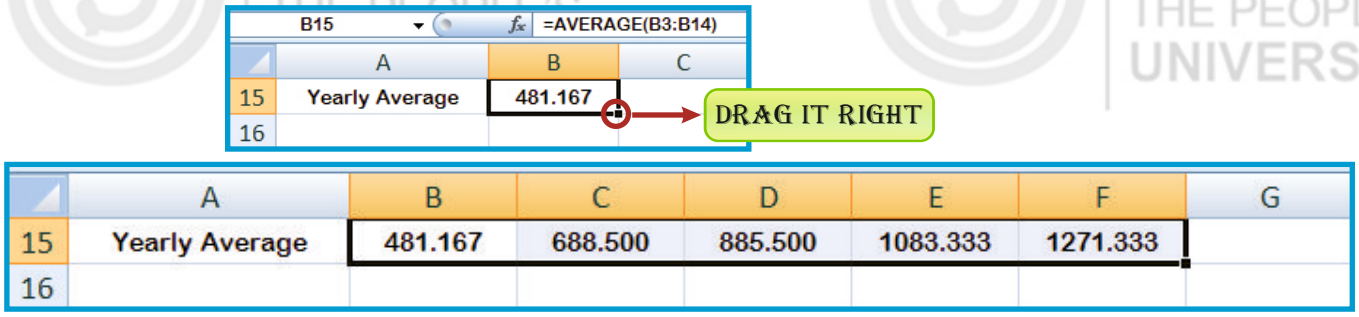


Fig. 17.12

Step 3: We now fit an appropriate trend line for the yearly average quantity of the ice-cream sold in different years. We follow the method explained in Lab Session 15 to fit a trend line. We plot the yearly averages on the graph and confirm that the linear trend line fits well (see Fig. 17.13).

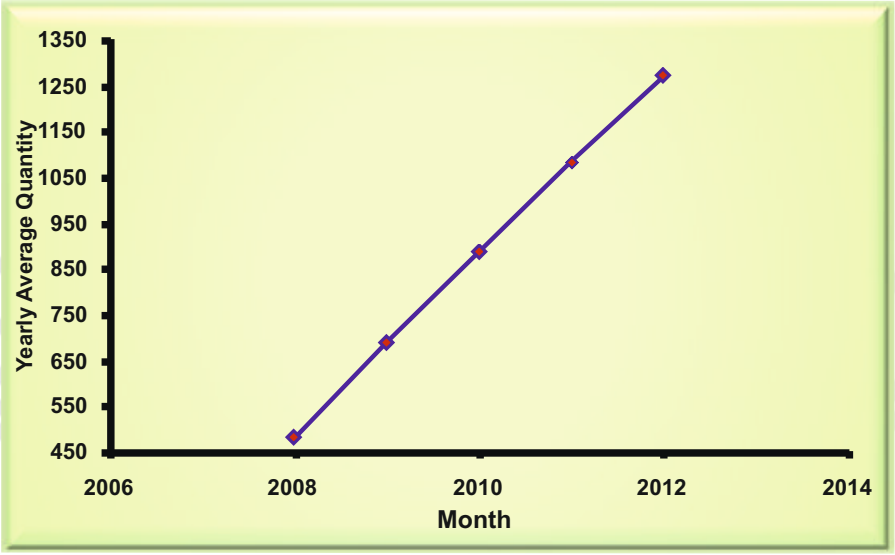


Fig. 17.13

Let the equation of the trend line be

$$\overline{y}_t = a_t + b_t x_t$$

where a_t is the intercept and b_t, the slope.

Here we are directly finding the values of intercept and slope by typing "=Intercept(B15:F15,B2:F2)" and "=Slope(B15:F 15, B2:F2)" in Cells B17 and B18, respectively, as shown in Fig. 17.14.

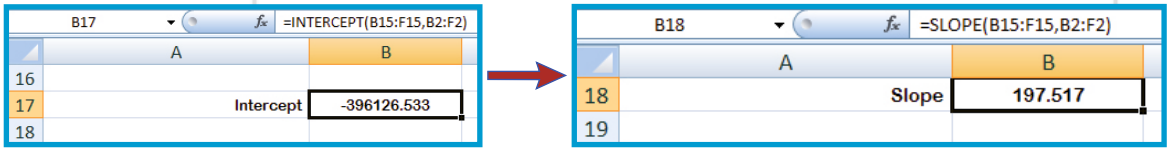


Fig. 17.14

Step 4: We type "=$B$17+$B$18*B2" in Cell B16 to obtain the trend value corresponding to the year 2008 and drag it right up to Cell F16 to determine the remaining trend values (see Fig. 17.15).

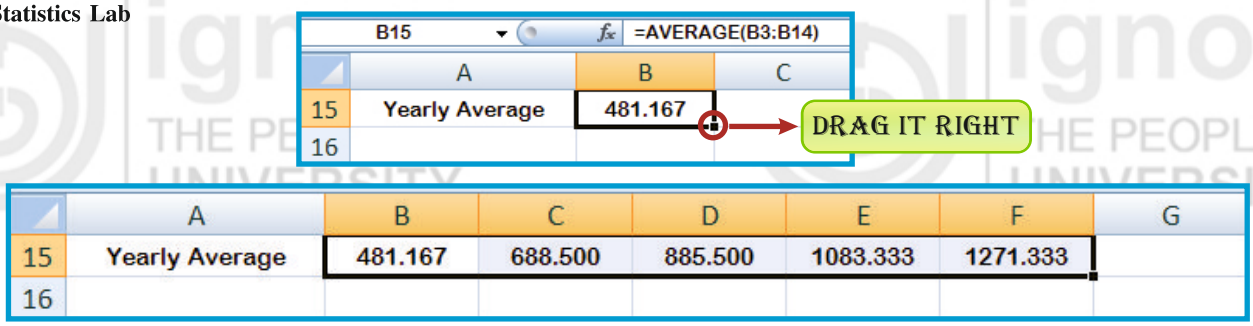


Fig. 17.15

Step 5: We obtained the equation of the linear trend for the yearly data in Step 3. The slope gives the yearly increment between the trend values. We can determine the monthly increment (as we have monthly data) between the trend value in Cell B20 by dividing the yearly increment by 12, i.e., by typing "=B19/12" in Cell B20 as shown in Fig. 17.16a. Then we compute the half monthly increment by typing "=B20/2" in Cell B21 as shown in Fig. 17.16b.

If we have quarterly data, we shall find quarterly increment by dividing yearly increment by 4 since we have 4 quarters in a year.

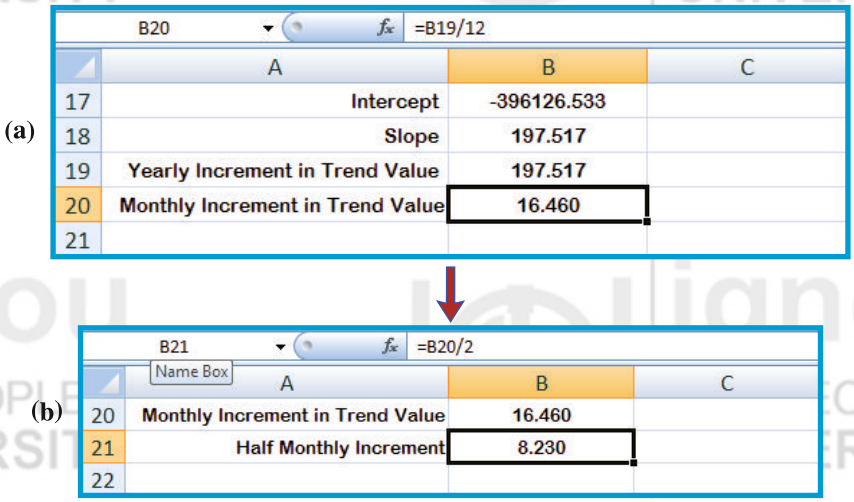


Fig. 17.16

Step 6: For finding trend values corresponding to each month, we consider Cells A23:F36 to compute the trend values as shown in Fig. 17.17.

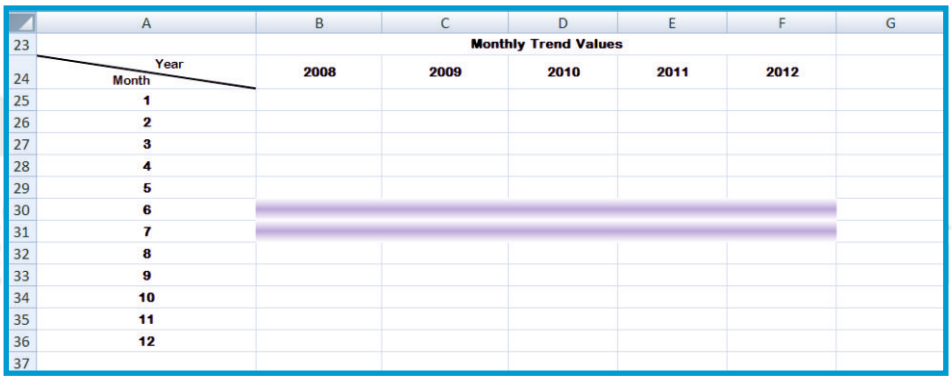


Fig. 17.17

In Cell B16, we have the trend value for the year 2008, i.e., 486.933 (Fig. 17.15). It corresponds to the middle of the months, i.e., between the 6th and 7th months. So we highlight Cells B30:F31 corresponding to the 6th and 7th months with purple colour as shown in Fig. 17.17.

Step 7: To obtain the trend values for the year 2008 corresponding to all months, we first need to compute the trend value of the 6th month of the year 2008.

To get the trend value for the 6th month, we type "=B$16-$B$21" in Cell B30 (Fig. 17.18a). We also type "=B$16+$B$21" in Cell B31 to get the trend values for the 7th month (see Fig. 17.18b).

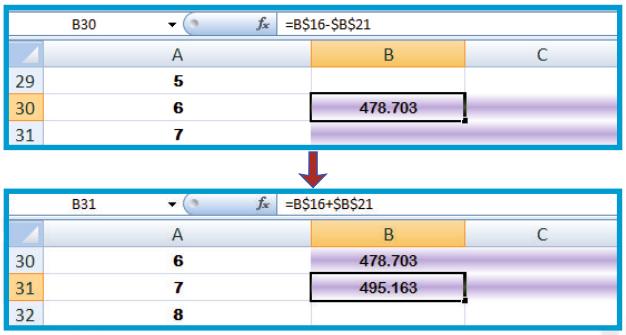


Fig. 17.18

Step 8: To obtain the trend value of the 5th month, we type "=B30-$B$20" in Cell B29 (Fig. 17.19a) and drag it up, up to Cell B25 to get the trend values of the 1st, 2nd, 3rd and 4th months (see Fig. 17.19b).

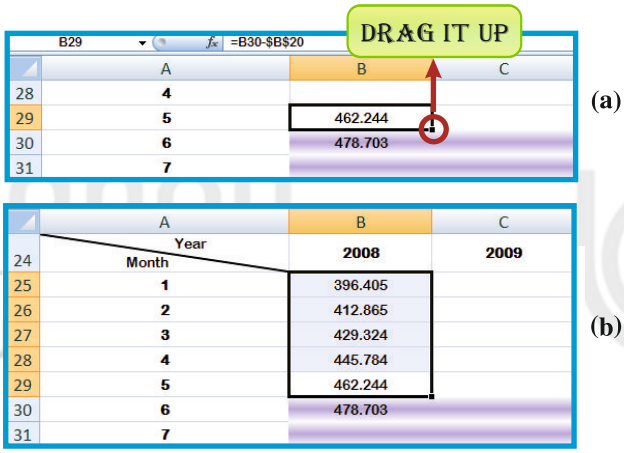


Fig. 17.19

Step 9: To obtain the trend value of the 8th month, we type "=B31+$B$20" in Cell B32 and drag it down up to Cell B36 to get the trend values of the 9th, 10th, 11th and 12th months (Fig. 17.20).

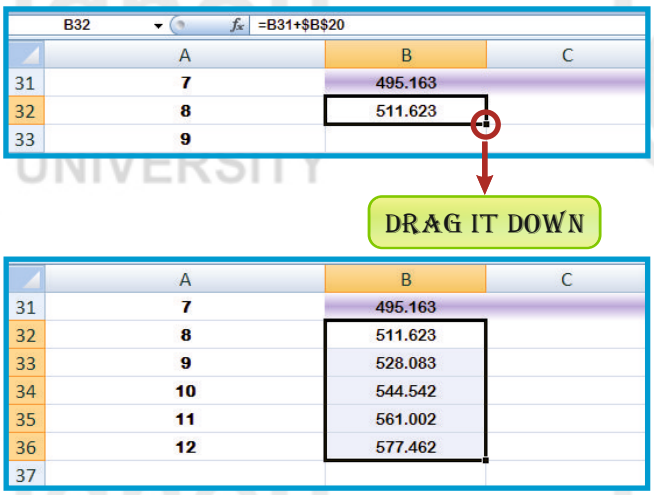
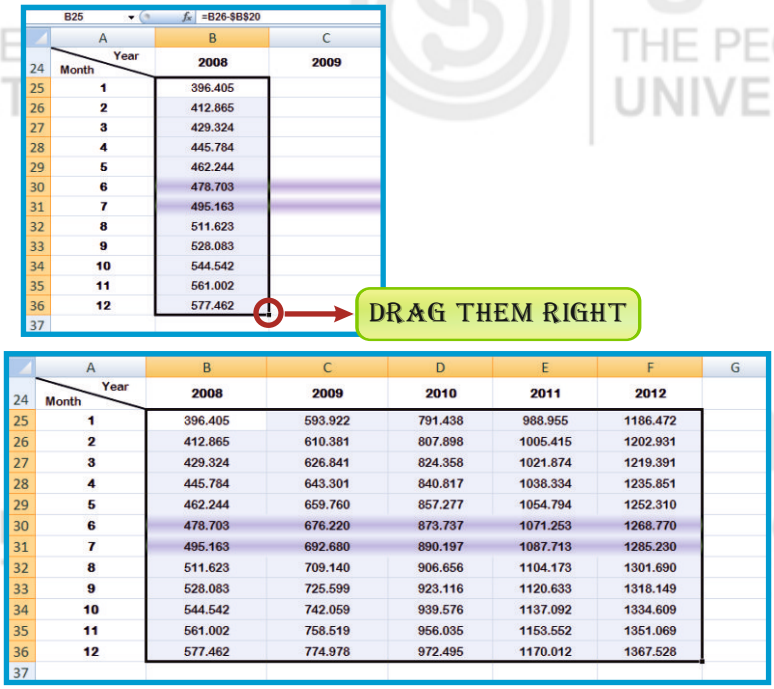


Fig. 17.20

The seasonal variations usually correspond to short term fluctuations. So before forecasting the future values, we should remove the seasonal component.

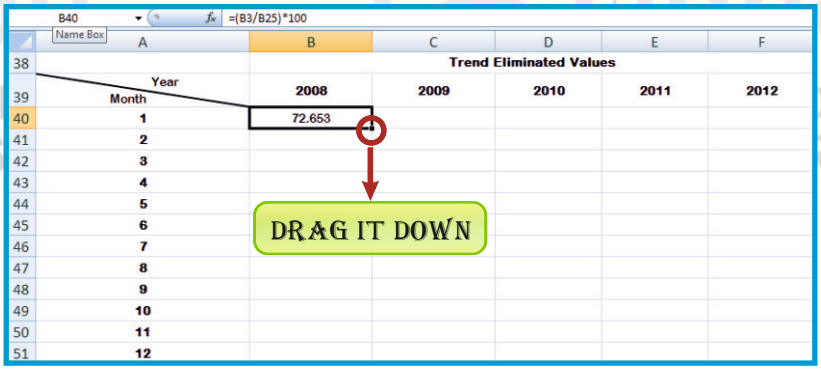
Step 10: We select Cells B25:B36 and drag them right up to Column F to obtain the month-wise trend values for all the years as shown in Fig. 17.21.



Month \ Year	2008	2009	2010	2011	2012
1	396.405	593.922	791.438	988.955	1186.472
2	412.865	610.381	807.898	1005.415	1202.931
3	429.324	626.841	824.358	1021.874	1219.391
4	445.784	643.301	840.817	1038.334	1235.851
5	462.244	659.760	857.277	1054.794	1252.310
6	478.703	676.220	873.737	1071.253	1268.770
7	495.163	692.680	890.197	1087.713	1285.230
8	511.623	709.140	906.656	1104.173	1301.690
9	528.083	725.599	923.116	1120.633	1318.149
10	544.542	742.059	939.576	1137.092	1334.609
11	561.002	758.519	956.035	1153.552	1351.069
12	577.462	774.978	972.495	1170.012	1367.528

Fig. 17.21

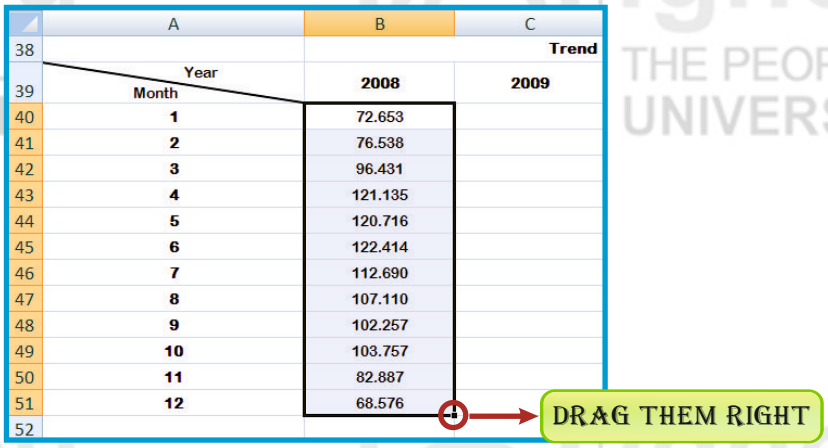
Step 11: We compute the trend eliminated values corresponding to the given original values. For this purpose, we type "=(B3/B25)*100" in Cell B40 (see Fig. 17.22).



Month \ Year	2008	2009	2010	2011	2012
1	72.653				
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Fig. 17.22

Step 12: We drag Cell B40 down up to Cell B51 (Fig. 17.23) to compute the trend eliminated values for all months of the year 2008.



Month \ Year	2008	2009
1	72.653	
2	76.538	
3	96.431	
4	121.135	
5	120.716	
6	122.414	
7	112.690	
8	107.110	
9	102.257	
10	103.757	
11	82.887	
12	68.576	

Fig. 17.23

Step 13: We select Cells B40:B51 and drag them right up to Column F (Fig. 17.24) to obtain the month-wise trend eliminated values for each given year.

	A	B	C	D	E	F
38		Trend Eliminated Values				
39	Month \ Year	2008	2009	2010	2011	2012
40	1	72.653	66.675	68.230	70.175	70.292
41	2	76.538	73.724	77.980	76.088	77.644
42	3	96.431	103.376	106.628	110.973	107.595
43	4	121.135	118.763	117.743	117.881	113.120
44	5	120.716	124.894	120.381	114.714	117.782
45	6	122.414	118.600	116.740	115.472	113.969
46	7	112.690	117.226	112.896	114.736	113.287
47	8	107.110	109.428	112.170	110.218	108.398
48	9	102.257	105.706	104.754	108.421	104.616
49	10	103.757	99.453	98.342	99.728	101.753
50	11	82.887	90.835	88.700	88.769	88.004
51	12	68.576	76.905	79.589	76.751	77.805

Fig. 17.24

Step 14: To compute the seasonal indices, we type "=Average(B40:F40)" in Cell G40 and drag it down up to Cell G51 as shown in Fig. 17.25.

G40 =AVERAGE(B40:F40)

	F	G	H
39	2012	Seasonal Index	
40	70.292	69.605	
41	77.644		
42	107.595		

DRAG IT DOWN

	F	G	H
39	2012	Seasonal Index	
40	70.292	69.605	
41	77.644	76.395	
42	107.595	105.000	
43	113.120	117.728	
44	117.782	119.697	
45	113.969	117.439	
46	113.287	114.167	
47	108.398	109.465	
48	104.616	105.151	
49	101.753	100.607	
50	88.004	87.839	
51	77.805	75.925	
52			

Fig. 17.25

Step 15: We calculate the average of the seasonal indices obtained in Step 14 by typing "=Average(G40:G51)" in Cell G52 as shown in Fig. 17.26.

G52 =AVERAGE(G40:G51)

	F	G	H
52	Average	99.918	
53			

Fig. 17.26

Step 16: The average given in Cell G52 (Fig. 17.26) is slightly different from 100, so we adjust the seasonal indices by typing "=(G40/$G$52)*100" in Cell H40. We drag Cell H40 down up to Cell H51 as shown in Fig. 17.27.

H40 | fx =G40/$G$52*100

	G	H	I
39	Seasonal Index	Adjusted Seasonal Index	
40	69.605	69.662	
41	76.395		

DRAG IT DOWN

	G	H	I
39	Seasonal Index	Adjusted Seasonal Index	
40	69.605	69.662	
41	76.395	76.457	
42	105.000	105.086	
43	117.728	117.825	
44	119.697	119.795	
45	117.439	117.535	
46	114.167	114.261	
47	109.465	109.555	
48	105.151	105.237	
49	100.607	100.689	
50	87.839	87.911	
51	75.925	75.987	
52	99.918		

Fig. 17.27

Step 17: We can verify these averages by computing the grand average in Cell H52. It should be 100 as shown in Fig. 17.28.

I78 | fx =AVERAGE(I66:I77)

	G	H	I	J
77		76.607	76.697	
78	Average	99.883	100	
79				
80				

Fig. 17.28

Step 18: We type the given data in Cells A54:B114. Then we copy the adjusted seasonal indices given in Cells H40:H51 and paste these values in Cells C55, C67, C79, C91 and C103 using ***Paste Values*** option as explained in Step 7 of Sec. 17.3. The values given in Cells C55:C114 are the adjusted seasonal indices corresponding to the given data (Fig. 17.29).

	A	B	C
54	Month	Quantity (in litre)	Adjusted Seasonal Indices
55	1	288	69.662
56	2	316	76.457
57	3	414	105.086
58	4	540	117.825
59	5	558	119.795
60	6	586	117.535
61	7	558	114.261
62	8	548	109.555
63	9	540	105.237
64	10	565	100.689
65	11	465	87.911
66	12	396	75.987
67	13	396	69.662
68	14	450	76.457
69	15	648	105.086
70	16	764	117.825
71	17	824	119.795
72	18	802	117.535
73	19	812	114.261

Ratio to Trend

Fig. 17.29

Step 19: We can also determine the deseasonalised values for the given data by typing "=(B55/C55)*100" in Cell D55 and dragging it down up to Cell D114 as shown in Fig. 17.30. By doing so, we may remove the seasonal influences on the data.

D55 =B55/C55*100

	B	C	D	E
54	Quantity (in litre)	Adjusted Seasonal Indices	Deseasonalised Values	
55	288	69.662	413.424	
56	316	76.457		

DRAG IT DOWN

	B	C	D	E
54	Quantity (in litre)	Adjusted Seasonal Indices	Deseasonalised Values	
55	288	69.662	413.424	
56	316	76.457	413.302	
57	414	105.086	393.962	
58	540	117.825	458.308	
59	558	119.795	465.794	
60	586	117.535	498.574	
61	558	114.261	488.358	
62	548	109.555	500.208	
63	540	105.237	513.129	
64	565	100.689	561.135	
65	465	87.911	528.944	
66	396	75.987	521.139	
67	396	69.662	568.458	
68	450	76.457	588.563	
69	648	105.086	616.636	
70	764	117.825	648.421	
71	824	119.795	687.840	
72	802	117.535	682.349	
73	812	114.261	710.657	

Ratio to Trend

Fig. 17.30

Step 20: We plot the given time series data and obtain deseasonalised values in Excel 2007 as explained in Sec. 15.3 of Lab Session 15. The resulting graph is shown in Fig. 17.31.

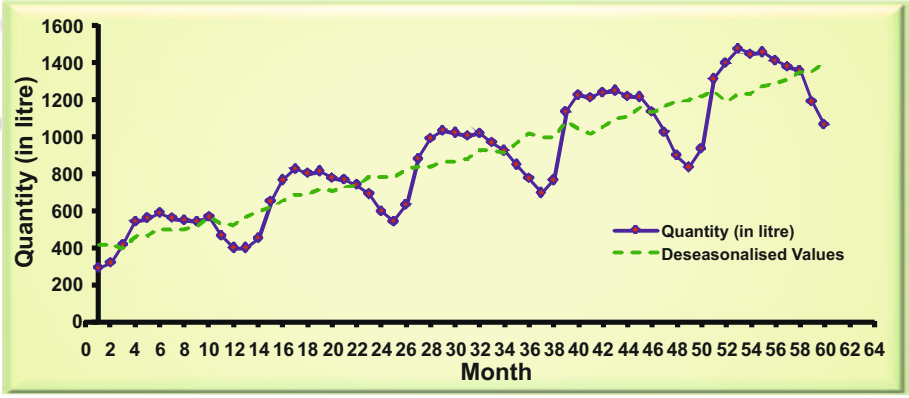


Fig. 17.31

If we have seasonal effect in our data, we should deseasonalise it before going for trend analysis or forecasting.

Fig. 17.31 reveals that the curve for deseasonalised values obtained after eliminating the seasonal effects is smooth (having fewer crests and troughs) in comparison with the original data and shows an upward linear trend.

17.5 RATIO TO MOVING AVERAGE METHOD

The ratio to moving average method is a better method than the methods discussed in Secs. 17.3 and 17.4 because of its accuracy. You have already studied the ratio to moving average method in Unit 13 of MSTE-002.

So here we briefly list the formulae and procedure in the steps given below:

Step 1: We eliminate the seasonal and irregular variations by computing moving averages as explained in Lab Session 16. It gives us estimates of the combined effect of trend and cyclic fluctuation, i.e., the time series obtained by the moving average method as

$$y_{MA} = TC \qquad ...(5)$$

Step 2: We divide the original (given) time series data by its corresponding moving average value. In this way, we obtain only seasonal and irregular components and eliminate the trend and cyclic fluctuation. These values are known as **Seasonal Relatives** and are given by

$$y_{SR} = \frac{TSCI}{y_{MA}} \times 100 = \frac{TSCI}{TC} \times 100$$

$$= SI \times 100 \qquad ...(6)$$

Step 3: We now determine the seasonal indices for all the given periods by calculating median of the seasonal relatives for each month.

Step 4: We compute the grand average of all the seasonal indices obtained in Step 3.

Step 5: If the seasonal periods are quarters, the sum of the seasonal indices in case of multiplicative model should be 400. If seasonal periods are months, it should be 1200 and the average of these should be 100. But as often the sum (or average) of all seasonal indices is not exactly what it should be, we divide each seasonal index by the grand average obtained in Step 4 and multiply it by 100. It will make the average equal to 100.

Step 6: To adjust seasonality of the data, we can also obtain the deseasonalised values by dividing the original (given) values with its corresponding adjusted seasonal indices, which eliminate the seasonal effects, i.e.,

$$\text{Deseasonalised value} = \frac{\text{original value}}{\text{adjusted seasonal value}} \times 100$$

$$= \frac{\text{TSCI}}{\text{S}} \times 100 = \text{TCI} \times 100 \qquad \text{...(7)}$$

After we obtain the deseasonalised data, we can apply trend analysis as explained in Lab Session 15.

Steps in Excel

In Secs. 17.3 and 17.4, you have learnt the simple average method and ratio to trend methods for calculating the seasonal indices using Excel 2007. In this section, you will learn how to use ratio to moving average method in Excel 2007 in the steps given below:

Step 1: To apply the ratio to moving average method, we consider the same data as used in Secs. 17.3 and 17.4. We enter the data in Excel sheet named "**Ratio to Moving Average**" as shown in Fig. 17.32.

	A	B	C
1	Year	Month	Quantity (in litre)
2	2008	1	288
3		2	316
4		3	414
5		4	540
6		5	558
7		6	586
8		7	558
9		8	548
10		9	540
11		10	565
12		11	465
13		12	396
14	2009	13	396
15		14	450
16		15	648
17		16	764
18		17	824
19		18	802
20		19	812

Ratio to Moving Average

Fig. 17.32

Step 2: We have already seen from Fig. 16.8 of Lab Session 16 that the peaks are minimum or smoother for 12 months. We compute the 12 monthly centred moving averages in Column D as obtained in Sec.16.3 of Lab Session 16 (Fig. 17.33).

	A	B	C	D
1	Year	Month	Quantity (in litre)	12 Monthly Centred Moving Averages
2	2008	1	288	
3		2	316	
4		3	414	
5		4	540	
6		5	558	
7		6	586	
8		7	558	485.667
9		8	548	495.750
10		9	540	511.083
11		10	565	530.167
12		11	465	550.583
13		12	396	570.667
14	2009	13	396	590.250
15		14	450	610.333
16		15	648	629.292
17		16	764	645.958
18		17	824	662.500
19		18	802	680.167
20		19	812	694.500

Ratio to Moving Average

Fig. 17.33

Step 3: We determine the seasonal relatives by typing "=(C8/D8)*100" in Cell E8 and drag Cell E8 down up to Cell E55. The output is shown in Fig. 17.34.

E8 =(C8/D8)*100

	C	D	E	F
1	Quantity (in litre)	12 Monthly Centred Moving Averages	Seasonal Relatives	
2	288			
3	316			
4	414			
5	540			
6	558			
7	586			
8	558	485.667	114.894	
9	548	495.750		
10	540	511.083		

DRAG IT DOWN

	C	D	E	F
7	586			
8	558	485.667	114.894	
9	548	495.750	110.540	
10	540	511.083	105.658	
11	565	530.167	106.570	
12	465	550.583	84.456	
13	396	570.667	69.393	
14	396	590.250	67.090	
15	450	610.333	73.730	
16	648	629.292	102.973	
17	764	645.958	118.274	
18	824	662.500	124.377	
19	802	680.167	117.912	
20	812	694.500	116.919	
21	776	708.000	109.605	
22	767	725.125	105.775	
23	738	744.167	99.171	
24	689	762.250	90.390	
25	596	780.000	76.410	
26	540	797.125	67.743	
27	630	815.208	77.281	

Ratio to Moving Average

Fig. 17.34

Step 4: We now arrange the month-wise seasonal relatives obtained in Cells E8:E55 for all the years as shown in Fig. 17.35.

	B	C	D	E	F	G
64		Seasonal Relatives				
65	Month \ Year	2008	2009	2010	2011	2012
66	1		67.090	67.743	69.863	70.286
67	2		73.730	77.281	75.605	77.618
68	3		102.973	105.448	110.044	107.695
69	4		118.274	116.516	116.618	113.252
70	5		124.377	119.439	113.544	117.941
71	6		117.912	116.162	114.732	114.361
72	7	114.894	116.919	112.679	114.583	
73	8	110.540	109.605	112.505	110.431	
74	9	105.658	105.775	105.085	108.822	
75	10	106.570	99.171	98.237	100.250	
76	11	84.456	90.390	88.541	89.085	
77	12	69.393	76.410	79.449	76.804	
78						

Fig. 17.35

Step 5: In Cell H66, we determine the seasonal index by computing the median of all seasonal relatives corresponding to the month of January by typing "=Median(C66:G66)". We drag Cell H66 down up to Cell H77 to compute the seasonal indices for the remaining months as shown in Fig. 17.36.

H66 f_x =MEDIAN(C66:G66)

	G	H	I
65	2012	Seasonal Indices	
66	70.286	68.803	
67	77.618		
68	107.695		

DRAG IT DOWN

	G	H	I
65	2012	Seasonal Indices	
66	70.286	68.803	
67	77.618	76.443	
68	107.695	106.572	
69	113.252	116.567	
70	117.941	118.690	
71	114.361	115.447	
72		114.738	
73		110.485	
74		105.716	
75		99.711	
76		88.813	
77		76.607	
78			

Fig. 17.36

Step 6: We determine the average of all seasonal indices computed in Cells H66:H77 by typing "=Average(H66:H77)" in Cell H78 as shown in Fig. 17.37. This is different from 100. So we need to adjust these seasonal indices so that the average turns out to be 100.

H78 =AVERAGE(H66:H77)

	G	H	I
78	Average	99.883	
79			

Fig. 17.37

Step 7: To compute the adjusted seasonal index for the month of January, we type "=(H66/$H$78)*100" in Cell I66 and drag it down up to Cell I77 to compute the adjusted seasonal indices for all months as shown in Fig. 17.38.

I66 =(H66/$H$78)*100

	H	I	J
65	Seasonal Indices	Adjusted Seasonal Indices	
66	68.803	68.884	
67	76.443		

DRAG IT DOWN

	H	I	J
65	Seasonal Indices	Adjusted Seasonal Indices	
66	68.803	68.884	
67	76.443	76.533	
68	106.572	106.697	
69	116.567	116.704	
70	118.690	118.829	
71	115.447	115.582	
72	114.738	114.873	
73	110.485	110.615	
74	105.716	105.840	
75	99.711	99.828	
76	88.813	88.917	
77	76.607	76.697	
78	99.883		

Fig. 17.38

Step 8: We can verify the average of the adjusted seasonal indices in Cell I78. It should be 100 as shown in Fig. 17.39.

I78 =AVERAGE(I66:I77)

	G	H	I	J
78	Average	99.883	100	
79				

Fig. 17.39

Step 9: We now copy these adjusted seasonal indices from Cells I66:I77 and paste these values in Cells F2, F14, F26, F38 and F50 using ***Paste Values*** option as explained in Step 7 of Sec. 17.3. The values given in Cells F2:F61 are the adjusted seasonal indices corresponding to the given data (Fig. 17.40).

	E	F
1	Seasonal Relatives	Adjusted Seasonal Indices
2		68.884
3		76.533
4		106.697
5		116.704
6		118.829
7		115.582
8	114.894	114.873
9	110.540	110.615
10	105.658	105.840
11	106.570	99.828
12	84.456	88.917
13	69.393	76.697
14	67.090	68.884
15	73.730	76.533
16	102.973	106.697
17	118.274	116.704
18	124.377	118.829
19	117.912	115.582
20	116.919	114.873

Ratio to Moving Average

Fig. 17.40

Step 10: We can also obtain the deseasonalised values for the given data by typing "=(C2/F2)*100" in Cell G2 and dragging it down up to Cell G61 as shown in Fig. 17.41 to eliminate the seasonal influences from the data.

G2 =C2/F2*100

	F	G	H
1	Adjusted Seasonal Indices	Deseasonalised Values	
2	68.884	418.095	
3	76.533		
4	106.697		
5	116.704		

DRAG IT DOWN

	F	G	H
1	Adjusted Seasonal Indices	Deseasonalised Values	
2	68.884	418.095	
3	76.533	412.895	
4	106.697	388.015	
5	116.704	462.710	
6	118.829	469.582	
7	115.582	506.997	
8	114.873	485.754	
9	110.615	495.411	
10	105.840	510.202	
11	99.828	565.974	
12	88.917	522.957	
13	76.697	516.317	
14	68.884	574.880	
15	76.533	587.983	
16	106.697	607.327	
17	116.704	654.649	
18	118.829	693.433	
19	115.582	693.877	
20	114.873	706.868	

Fig. 17.41

Step 11: We plot the given data and the deseasonalised values in Excel 2007 as explained in Sec. 15.3 of Lab Session 15. The resulting chart is shown in Fig. 17.42.

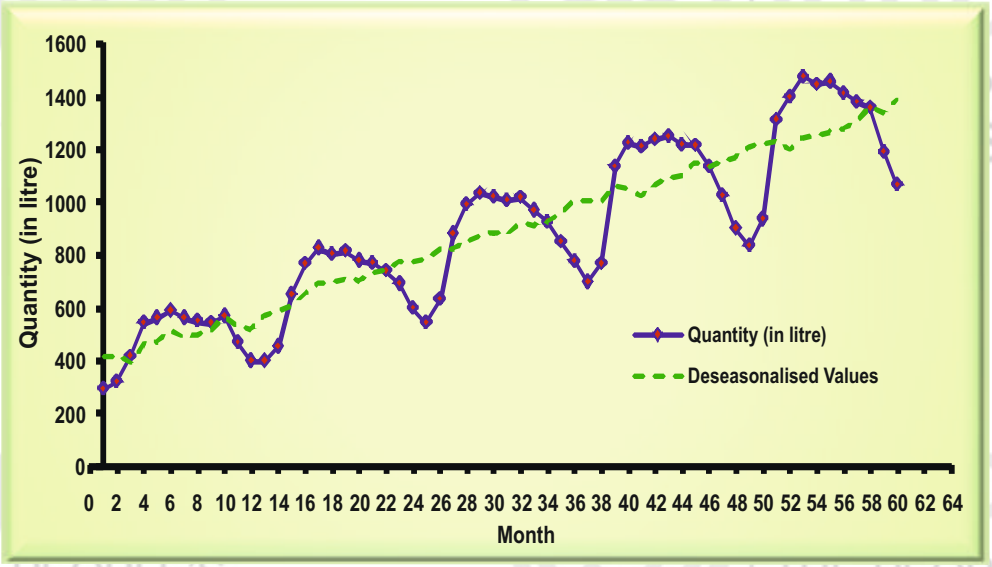


Fig. 17.42

Fig. 17.42 reveals that the deseasonalised data are smooth (having fewer crests and troughs) in comparison with the original data and show an upward trend.

You can now solve the following exercises to check whether you have learnt how to apply the seasonal analysis on the given data and deseasonalise the data.

Activity

Compute the seasonal indices with the help of MS Excel 2007 and interpret the results for

A1) Examples 1, 2 and 3 given in Unit 14 of MSTE-002.

A2) Exercises E1, E2, E3 and E4 given in Unit 14 of MSTE-002.

Match the results with the calculations done in Unit 14 of MSTE-002.

Continuous Assessment 17

Consider the data given in Continuous Assessment 16 of Lab Session 16 and use the simple average, ratio to trend and moving average methods to

- compute the seasonal indices for 7 years,
- obtain deseasonalised values and estimate the trend line by the method of least squares, and
- plot the given data and the deseasonalised values.

Home Work : Do It Yourself

1) Follow the steps explained in Secs. 17.3 to 17.5 to compute the seasonal indices and deseasonalised values for the data given in Table 1 of Lab Session 16. Use a different format for plotting the given data along with the deseasonalised values. Take the screenshots and keep them in your record book.

2) Develop the spreadsheets for the exercise given in "Continuous Assessment 17" as explained in this lab session. Take screenshots of the final spreadsheets and the charts.

3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.

APPENDIX

Table-I: Constants/Factors for Variable Control Charts

Sample Size (n)	c_4	d_2	d_3	A	A_2	A_3	B_3	B_4	D_1	D_2	D_3	D_4
2	0.798	1.128	0.853	2.121	1.880	2.659	0	3.267	0	3.686	0	3.267
3	0.886	1.693	0.888	1.732	1.023	1.954	0	2.568	0	4.358	0	2.574
4	0.921	2.059	0.880	1.500	0.729	1.628	0	2.266	0	4.698	0	2.282
5	0.940	2.326	0.864	1.342	0.577	1.427	0	2.089	0	4.918	0	2.114
6	0.952	2.534	0.848	1.225	0.483	1.287	0.030	1.970	0	5.078	0	2.004
7	0.959	2.704	0.833	1.134	0.419	1.182	0.118	1.882	0.204	5.204	0.076	1.924
8	0.965	2.847	0.820	1.061	0.373	1.099	0.185	1.815	0.388	5.306	0.136	1.864
9	0.969	2.970	0.808	1.000	0.337	1.032	0.239	1.761	0.547	5.393	0.184	1.816
10	0.973	3.078	0.797	0.949	0.308	0.975	0.284	1.716	0.687	5.469	0.223	1.777
11	0.975	3.173	0.787	0.905	0.285	0.927	0.321	1.679	0.811	5.535	0.256	1.744
12	0.978	3.258	0.778	0.866	0.266	0.886	0.354	1.646	0.922	5.594	0.283	1.717
13	0.979	3.336	0.770	0.832	0.249	0.850	0.382	1.618	1.025	5.647	0.307	1.693
14	0.981	3.407	0.763	0.802	0.235	0.817	0.406	1.594	1.118	5.696	0.328	1.672
15	0.982	3.472	0.756	0.775	0.223	0.789	0.428	1.572	1.203	5.741	0.347	1.653
16	0.984	3.532	0.750	0.750	0.212	0.763	0.448	1.552	1.282	5.782	0.363	1.637
17	0.985	3.588	0.744	0.728	0.203	0.739	0.466	1.534	1.356	5.820	0.378	1.622
18	0.985	3.640	0.739	0.707	0.194	0.718	0.482	1.518	1.424	5.856	0.391	1.608
19	0.986	3.689	0.734	0.688	0.187	0.698	0.497	1.503	1.487	5.891	0.403	1.597
20	0.987	3.735	0.729	0.671	0.180	0.680	0.510	1.490	1.549	5.921	0.415	1.585

Dear Learner,

While performing the Lab Sessions of MSTL-002, you may have found certain portions of the sessions difficult to perform. We wish to know your difficulties and suggestions in order to improve the course. Therefore, we request you to fill and send us the following feedback form, which pertains to this course. Just fill in the space provided or tick the most relevant option. If you find that the space provided is insufficient, kindly use a separate sheet.

Please mail to:

The Course Coordinator (MSTL-002)
Room No. 106
School of Sciences,
Block D, New Academic Complex,
IGNOU, Maidan Garhi
New Delhi – 110068

FEEDBACK FORM

1. Name: ……. 2. Male/Female: …… 3. Age: ……

4. Enrolment No: ……. 5. Email Address: ……

6. Mobile No: …….. 6. Employed/Unemployed: …….

7. Regional Centre Code and Name: ……

8. Study Centre Code and Name: ……

9. How many hours did you spend for performing all lab sessions?

Part No.	**A**					**B**					**C**				**D**		
Exercise No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
No. of Hours																	

10. How do you find the relevance of lab sessions for theory courses?

    i) Excellent ii) Good iii) Adequate iv) Poor v) Very poor

11. Did you enjoy practicing with MS Excel 2007? Yes/No

12. Do you have a personal computer? Yes/No

13. Where do you access the internet?

    i) At home ii) At internet cafe iii) At work place iv) Other ……

14) Put a tick mark on what you think is correct, based on your experience with the course from amongst the options given below:

Items	Excellent	Good	Adequate	Poor	Very Poor	Give Specific Examples, if Poor or Very Poor
Quality of Presentation						
Language and Style						
Screenshots Used						
Conceptual Clarity						
Activities						
Continuous Assessment Exercises						

15) **Specific Feedback:** If you have found some sessions of the course to be difficult and not clear, please mention the page numbers and what is not clear about each lab session. Use additional pages, if need be.

Lab Session and Page Number	Explanation is not Clear	Language is Difficult	Steps are Difficult	Screenshots are not Clear	Activity and Continuous Assessment are Difficult

16) Your overall comments and suggestions on the course (Please write the comments on the course after studying it).

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