PART B

Control Charts for Attributes

CONTROL CHARTS FOR ATTRIBUTES

The second part of this lab course comprises Sessions 6 to 10 which will enable you to develop practical skills for constructing control charts for attributes in MS Excel 2007. In lab sessions of Part A, you have learnt how to construct control charts for variables. These charts are for quality characteristics that can be measured and expressed numerically. For example, diameter, length, weight, position, thickness, etc. You have constructed control charts for mean, range and standard deviation.

In Unit 3 of MSTE-001 (Industrial Statistics-I), you have learnt that a separate variable chart is required for each quality characteristic. However, if a manufacturer specifies more than one (say, 10) measurable quality characteristics for its product, it becomes impracticable and uneconomical to construct 10 control charts. In such situations, we use control charts for attributes, which provide overall information about the quality of the product. There are many situations in which numerical measurement is not possible, e.g., the number of failures in a production run, number of defects in a laptop, mobile, furniture, fabric, etc. In such situations, we classify the items into one of the two classes: conforming or non conforming, defective or non-defective, good or bad, etc.

In such cases, actual measurements are not taken. Instead, the number of **defectives** or **defects** are **counted**. The size of the defect and its location are not important. The products are inspected and either accepted or rejected.

You have also learnt in Units 3 and 4 of MSTE-001 that we construct control charts for attributes for controlling the defectives and defects. The following control charts for attributes are used most frequently in quality control:

1. Control charts for defectives (p-chart and np-chart)
2. Control chart for defects (c-chart and u-chart)

The control charts for defectives are based on the **binomial** distribution. The control charts for defects are based on the **Poisson** distribution. The control charts for attributes are shown in Fig. B.1.

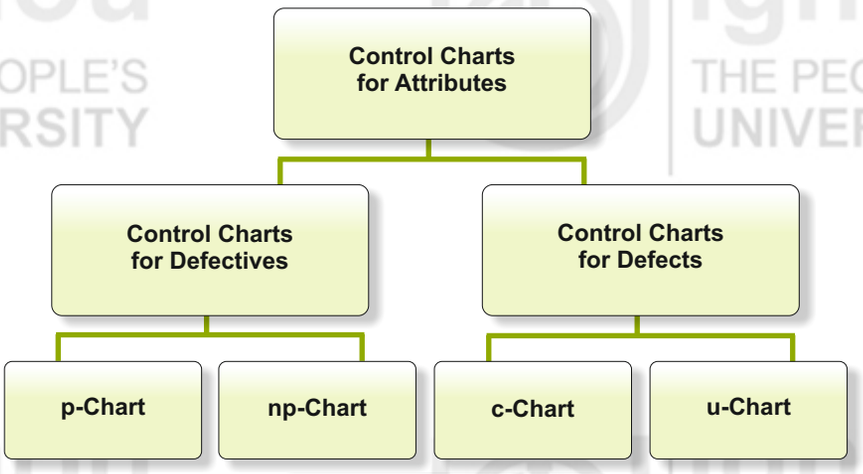


Fig. B.1

In Sessions 6 and 7, you will learn how to prepare the p-chart for **fraction defective** for both **constant** and **variable sample sizes**, respectively. It is applied to quality characteristics, which are not measurable and the items or units are classified as defective or non-defective. Session 8 deals with construction of the np-chart for the **number of defectives**.

In Session 9, you will learn how to construct the c-chart used to monitor and control the **number of defects**. In Session 10, we demonstrate the construction of u-chart to control the **number of defects per unit**.

All sessions of this part have been designed in such a way that after successfully completing them, you will be able to construct different control charts for attributes using MS Excel 2007. You will also be able to use suitable attribute control charts for any given data.

Before studying the lab sessions of Part B, we expect that you have studied Units 3 and 4 of the course MSTE-001 entitled Industrial Statistics-I thoroughly. All concepts and formulae used there will be applied here.

Control Chart for Fraction Defective

SESSION 6

Structure

6.1 Introduction
Objectives
6.2 Problem Description
6.3 Procedure for the Construction of p-Chart
6.4 Steps Involved in the Construction of p-Chart in Excel 2007
6.5 Revised p-Chart

6.1 INTRODUCTION

In Lab Sessions 1 to 5, you have learnt how to construct the $\bar{X}$-chart, R-chart and S chart. $\bar{X}$-chart is used for controlling the mean of the quality characteristic whereas R-chart and S-chart are used for monitoring and controlling the process variability. We have already explained in Sec. 3.3 of Unit 3 of MSTE-001 (Industrial Statistics-I) that the p-chart is applied to control the **fraction or proportion defective** in the process.

In this lab session, you will learn how to prepare the control chart for fraction defective, i.e., the p-chart in MS Excel 2007 using a specific problem. In the next lab session, we shall explain how to construct the p-chart for **variable sample size**.

Prerequisite

- Lab Sessions 1 and 3 of MSTL-001 (Basic Statistics Lab).
- Lab Sessions 1 to 5 of MSTL-002 (Industrial Statistics Lab).
- Unit 3 of MSTE-001 (Industrial Statistics-I).

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for control chart for fraction defective;
- construct the control chart for fraction defective; and
- interpret the control chart.

6.2 PROBLEM DESCRIPTION

Let us consider a company which manufactures cream and is facing the problem of leakage in tubes of cream. Suppose these tubes are packed in boxes of 50 tubes each.

A quality control inspector in this company selected a sample of 30 boxes having 50 tubes each. The numbers of leaky tubes per box are recorded in Table 1.

Table 1: Number of leaky tubes

Sampled Box	Total number of Tubes in the Box	Number of Leaky Tubes	Sampled Box	Total number of Tubes in the Box	Number of Leaky Tubes
1	50	6	16	50	4
2	50	5	17	50	3
3	50	4	18	50	6
4	50	7	19	50	5
5	50	2	20	50	4
6	50	3	21	50	1
7	50	4	22	50	4
8	50	2	23	50	2
9	50	1	24	50	2
10	50	6	25	50	4
11	50	6	26	50	8
12	50	3	27	50	6
13	50	4	28	50	2
14	50	2	29	50	2
15	50	3	30	50	7

The quality control inspector needs to construct a control chart to check whether the process is under statistical control or not and also compute the revised control limits, if necessary.

Therefore, the problem for this session is to construct the control chart for fraction defective, i.e., p-chart for the data given in Table 1.

6.3 PROCEDURE FOR THE CONSTRUCTION OF p-CHART

You have already studied the detailed procedure for the construction of p-chart in Sec. 3.3 of Unit 3 of MSTE-001. It is similar to the control charts for variables discussed in the lab sessions of Part A. The main steps involved in the construction of the p-chart are as follows:

Step 1: We select k samples (subgroups) randomly from the process at different times and each selected item or unit is classified as defective or non-defective with respect to certain defects.

Step 2: After that, we count the number of defective items in each sample. Suppose there are k samples of the same size n and $d_1, d_2, ..., d_k$ are the numbers of defectives in 1st, 2nd, …, kth sample of size n, respectively. Then for each sample, we calculate the fraction defective given by

$$\text{Fraction defective} = \frac{\text{Number of defectives}}{\text{Total number of items inspected}}$$

$$p_i = \frac{d_i}{n}; \quad (i = 1, 2, ..., k) \qquad ...(1)$$

Step 3: We find the average of all fraction defectives as follows:

$$\bar{p} = \frac{1}{k}(p_1 + p_2, \ldots, p_k) = \frac{1}{k}\sum_{i=1}^{k} p_i \qquad \ldots(2)$$

Step 4: We set up the control limits to check whether the process is under control or out-of-control. The control limits for the p-chart are:

✓ Centre line (CL) $= \bar{p}$ ...(3)

✓ Upper control limit (UCL) $= \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ ...(4)

✓ Lower control limit (LCL) $= \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ ...(5)

Note : If the value of LCL comes out to be negative, we should consider it as zero because negative fraction defective is not possible.

Step 5: Having set the centre line and control limits, we construct the p-chart by taking sample numbers on the X-axis and sample fraction defectives (p) on the Y-axis.

Step 6: Interpretation of the p-chart.

In practice, fraction defective (P) of the process is not known. So it is necessary to estimate the fraction defective of the process. We estimate it using the sample fractions. The best estimator of process fraction defective is the average fraction defective of the samples.

6.4 STEPS INVOLVED IN THE CONSTRUCTION OF p-CHART IN EXCEL 2007

We now describe the procedure for construction of the p-chart with the help of the problem given in Sec. 6.2. For plotting a p-chart in Excel 2007, we follow the steps given below:

Step 1: We enter the given data in MS Excel spreadsheet as shown in Fig. 6.1.

	A	B	C
1			
2	Sampled Box	Tubes in Box	Leaky Tubes
3	1	50	6
4	2	50	5
5	3	50	4
6	4	50	7
7	5	50	2
8	6	50	3
9	7	50	4
10	8	50	2
11	9	50	1
12	10	50	6
13	11	50	6
14	12	50	3
15	13	50	4
16	14	50	2

p Chart

Fig. 6.1: Partial screenshot of the spreadsheet for the given data.

Fraction defective is always less than or equal to unity and expressed as a decimal fraction.

Step 2: To calculate the fraction defective for each sample, we use equation (1). For example, the 1st sample contains 50 tubes and each tube is inspected for leakage. After the inspection, 6 tubes are found defective (leaky). The fraction defective for the 1st sample is obtained as:

$$\text{Fraction defective (p)} = \frac{\text{Number of leaky tubes}}{\text{Total number of tubes inspected}}$$

$$= \frac{6}{50} = 0.12$$

To calculate the fraction defective of the 1st sample in Excel 2007, we type "=C3/B3" in Cell D3 as shown in Fig. 6.2. We drag it down up to the last sample, i.e., Cell D32 to find the fraction defective for the remaining samples (Fig. 6.2).

D3 | fx =C3/B3

	A	B	C	D
1				
2	Sampled Box	Tubes in Box	Leaky Tubes	Fraction Defective (p)
3	1	50	6	0.12
4	2	50	5	0.1
5	3	50	4	0.08
6	4	50	7	0.14
7	5	50	2	0.04
8	6	50	3	0.06
9	7	50	4	0.08
10	8	50	2	0.04
11	9	50	1	0.02
12	10	50	6	0.12
13	11	50	6	0.12
14	12	50	3	0.06
15	13	50	4	0.08
16	14	50	2	0.04
17	15	50	3	0.06
18	16	50	4	0.08
19	17	50	3	0.06
20	18	50	6	0.12

p Chart

Fig. 6.2

Step 3: We now use equation (2) to compute average fraction defective $\bar{p}$, i.e., the average of 30 sample fraction defectives (given in Cells D3 to D32) as shown in Fig. 6.3.

D33 | fx =AVERAGE(D3:D32)

	A	B	C	D
29	27	50	6	0.12
30	28	50	2	0.04
31	29	50	2	0.04
32	30	50	7	0.14
33			Average	0.0787
34				

Fig. 6.3

Step 4: Here we also use Columns E, F and G for putting the values of the centre line, upper and lower control limits, respectively, as discussed in Lab Sessions 1 to 5. We can calculate these limits from equations (3) to (5) as follows:

i) The centre line is $CL = \bar{p}$. Since the value of $\bar{p}$ is given in Cell D33 (see Fig. 6.3), we type "=$D$33" in Cell E3 to find the centre line as shown in Fig. 6.4a.

ii) The upper control limit is $UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$. We type "= $D$33+3*Sqrt(($D$33*(1-$D$33))/(B3))" in Cell F3 (Fig. 6.4b).

iii) Similarly, we calculate the lower control limit

$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$ by typing

"=$D$33–3*Sqrt($D$33*(1-$D$33))/(B3))" in Cell G3 (Fig. 6.4c).

The formula with dollar sign ($) is used for an absolute reference.

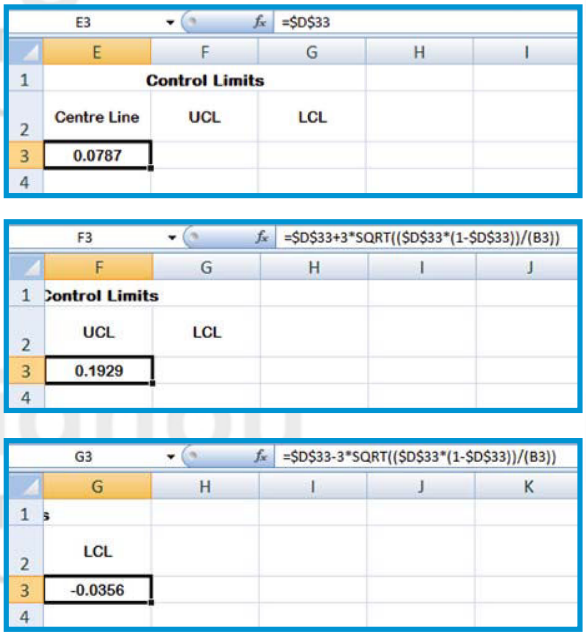


Fig. 6.4

Step 5: To plot these limits on the chart, we first select Cells E3:G3 and drag them down up to Row 32. After dragging, we get the result shown in Fig. 6.5.

	E	F	G
1		Control Limits	
2	Centre Line	UCL	LCL
3	0.0787	0.1929	-0.0356
4	0.0787	0.1929	-0.0356
5	0.0787	0.1929	-0.0356
6	0.0787	0.1929	-0.0356
7	0.0787	0.1929	-0.0356
8	0.0787	0.1929	-0.0356
9	0.0787	0.1929	-0.0356
10	0.0787	0.1929	-0.0356
11	0.0787	0.1929	-0.0356
12	0.0787	0.1929	-0.0356
13	0.0787	0.1929	-0.0356
14	0.0787	0.1929	-0.0356
15	0.0787	0.1929	-0.0356
16	0.0787	0.1929	-0.0356
17	0.0787	0.1929	-0.0356
18	0.0787	0.1929	-0.0356
19	0.0787	0.1929	-0.0356
20	0.0787	0.1929	-0.0356

Fig. 6.5

Step 6: We notice from Fig. 6.5 that the value of LCL is negative, which is not possible. We set it to the value 0 (zero) and denote the limit by LCL* as shown in Fig. 6.6 to plot the control chart.

If the value of lower control limit is negative, we set the lower control limit as zero because a negative fraction defective is not possible.

	E	F	G	H
1	Control Limits			
2	Centre Line	UCL	LCL	LCL*
3	0.0787	0.1929	-0.0356	0
4	0.0787	0.1929	-0.0356	0
5	0.0787	0.1929	-0.0356	0
6	0.0787	0.1929	-0.0356	0
7	0.0787	0.1929	-0.0356	0
8	0.0787	0.1929	-0.0356	0
9	0.0787	0.1929	-0.0356	0
10	0.0787	0.1929	-0.0356	0
11	0.0787	0.1929	-0.0356	0
12	0.0787	0.1929	-0.0356	0
13	0.0787	0.1929	-0.0356	0
14	0.0787	0.1929	-0.0356	0
15	0.0787	0.1929	-0.0356	0
16	0.0787	0.1929	-0.0356	0
17	0.0787	0.1929	-0.0356	0
18	0.0787	0.1929	-0.0356	0
19	0.0787	0.1929	-0.0356	0
20	0.0787	0.1929	-0.0356	0

p Chart

Fig. 6.6

Step 7: To obtain the p-chart in Excel 2007, we follow the procedure explained in Step 10 of Sec. 1.4, Lab Session 1. It means that we

1. select Cells D2:F32 and H2:H32 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1. Thus, we obtain the p-chart shown in Fig. 6.7.

If the p-chart continually indicates an increase in the value of the average proportion of non-conforming items, the management should investigate the reasons behind this increase rather than constantly revising the centre line, upper and lower control limits. The possible reasons behind the increase in the non-conforming items may be poor incoming quality from vendors, tightening of specification limits, etc.

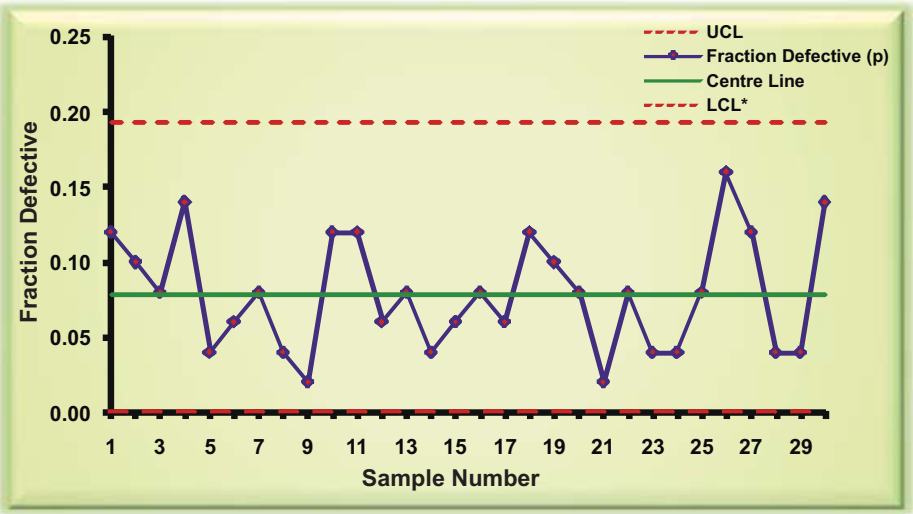


Fig. 6.7

Interpretation

Study Fig. 6.7. Since all points lie between the upper and lower control limits, we may conclude that the process of manufacturing cream tubes is under statistical control with respect to the fraction defective. So, there is no need to revise the control limits.

In the next section, we state the formulae and procedure of computing the revised control limits for the p-chart if you come across this situation.

6.5 REVISED p-CHART

If one or more points lie outside the upper or lower or both control limits, the control chart indicates that the process is not under statistical control. Some assignable causes may be present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. Once this is done, we remove out-of-control samples and calculate the revised centre line, upper and lower control limits for the p-chart using the remaining samples. These limits are known as revised control limits.

You have already studied various formulae and procedure for revising the p-chart in Sec. 3.3 of Unit 3 of MSTE-001. Here we are listing the steps to compute the revised p-chart:

Step 1: To calculate the revised control limits of p-chart, we first calculate the new $\bar{p}$ as follows:

$$\bar{p}_{new} = \frac{\sum_{i=1}^{k} p_i - \sum_{j=1}^{d} p_j}{k-d} \quad \text{or} \quad \bar{p}_{new} = \frac{\sum_{i=1}^{k} d_i - \sum_{j=1}^{d} d_j}{n(k-d)} \qquad \ldots (6)$$

where d – number of discarded samples,

$\sum_{j=1}^{d} p_j$ – sum of fraction defectives in the discarded samples, and

$\sum_{j=1}^{d} d_j$ – sum of number of defectives in the discarded samples.

Step 2: After finding $\bar{p}_{new}$, we reconstruct the centre line and control limits of the chart by replacing $\bar{p}$ by $\bar{p}_{new}$ as given below:

- ✓ Centre line (CL) $= \bar{p}_{new}$ ...(7)
- ✓ Upper control limit (UCL)

$$= \bar{p}_{new} + 3\sqrt{\frac{\bar{p}_{new}(1-\bar{p}_{new})}{n}} \qquad \ldots(8)$$

- ✓ Lower control limit (LCL)

$$= \bar{p}_{new} - 3\sqrt{\frac{\bar{p}_{new}(1-\bar{p}_{new})}{n}} \qquad \ldots (9)$$

Step 3: We plot the p-chart for the revised control limits until all points lie within UCL and LCL in the same way as discussed in the Lab Sessions 2 to 5.

Step 4: Interpretation of the p-chart.

You should now apply the methods discussed in Secs. 6.4 and 6.5 on other problems for practice.

Activity

Construct the control charts for fraction defective with the help of MS Excel 2007 and interpret the results for

A1) Examples 1, 2 and 3 given in Unit 3 of MSTE-001.

A2) Exercises E4 and E5 given in Unit 3 of MSTE-001.

Also match the results with the manual calculation done in Unit 3 of MSTE-001.

Continuous Assessment 6

To monitor the manufacturing of laptops, a quality controller randomly selected a fixed number of laptops from the production line, each day over a period of 25 days. The laptops were inspected for defectives and the number of defective laptops found each day was recorded. The data is given in Table 2.

Table 2: The number of defective laptops

Day	Number of Defective Laptops	Day	Number of Defective Laptops
1	3	14	5
2	4	15	2
3	4	16	4
4	11	17	3
5	4	18	6
6	2	19	2
7	4	20	4
8	4	21	3
9	5	22	1
10	4	23	5
11	6	24	3
12	6	25	4
13	6		

Construct the p-chart for fixed sample size and comment on the process. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Sec. 6.4 and 6.5 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 6" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.

Control Chart for Fraction Defective with Variable Sample Size

SESSION 7

Structure

7.1 Introduction
Objectives
7.2 Problem Description
7.3 Procedure for the Construction of p-Chart
7.4 Steps Involved in the Construction of p-Chart in Excel 2007
7.5 Revised p-Chart

7.1 INTRODUCTION

Prerequisite

- Lab Sessions 1 and 3 of MSTL-001 (Basic Statistics Lab).
- Lab Session 6 of MSTL-002 (Industrial Statistics Lab).
- Unit 3 of MSTE-001 (Industrial Statistics-I).

In Lab Session 6, you have learnt how to construct the control chart for fraction defective (p-chart) when we have constant sample size, i.e., we have inspected the same number of units in each sample and counted the number of defective items. But sometimes, we cannot take the same number of units in each sample. Generally, such a situation arises when the p-chart is used for 100% inspection of output that varies from day to day. The variation in the output per day can be due to breakdown of machines, different production requirements, etc. When the sample size is not uniform, we use the p-chart for variable sample size using two approaches as discussed in Sec. 3.4 of Unit 3 in MSTE-001 (Industrial Statistics-I). Since the control limits for the p-chart are functions of sample size (n_i), these vary with the sample size. We calculate the control limits separately for each sample. The control limits in such a situation are known as variable control limits.

In this lab session, you will learn how to construct the p-chart for variable sample size in MS Excel 2007. The procedure for drawing the p-chart for variable sample size is similar to the p-chart with constant sample size except for the control limits.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine control limits for the control chart for fraction defective with variable sample size;
- construct the control chart for fraction defective with variable sample size; and
- interpret the control chart for fraction defective.

7.2 PROBLEM DESCRIPTION

To monitor the manufacturing of laptops, a quality controller randomly selected laptops from the production line each day over a period of 25 days. The laptops were inspected for defectives and the number of defective laptops found each day is recorded in Table 1.

Table 1: Number of defective laptops

Day	Number of Laptops Inspected	Number of Defective Laptops	Day	Number of Laptops Inspected	Number of Defective Laptops
1	50	2	14	55	5
2	52	4	15	52	1
3	57	4	16	48	4
4	50	11	17	50	3
5	50	4	18	56	6
6	48	2	19	52	2
7	51	4	20	53	4
8	54	6	21	50	3
9	52	5	22	55	1
10	50	1	23	50	5
11	55	6	24	50	3
12	60	3	25	47	4
13	55	6			

The quality control inspector of this company needs to construct the control chart with the help of both approaches to check whether the process is under statistical control or not. He/she also computes the revised control limits, if necessary.

Therefore, the problem for this session is to construct the control chart for fraction defective, i.e., the p-chart with variable sample size for the data given in Table 1.

7.3 PROCEDURE FOR THE CONSTRUCTION OF p-CHART

You have already learnt all formulae for calculating control limits for the p-chart and the method to draw the p-chart manually in Unit 3 of MSTE-001. The steps involved in the computation of p-chart for variable sample size are given below:

First Approach

According to the first approach, we calculate the control limits for each sample as follows:

Step 1: If $d_1, d_2, ..., d_k$ represent the number of defective, items in 1st, 2nd, …, kth samples of size $n_1, n_2, ..., n_k$, respectively, the unknown value of fractions defective (P) can be estimated by

$$\bar{p} = \frac{d_1 + d_2 + ... + d_k}{n_1 + n_2 + ... + n_k} = \frac{\sum_{i=1}^{k} d_i}{\sum_{i=1}^{k} n_i} \quad ...(1)$$

Step 2: The centre line remains constant for each sample:

Centre line $(CL) = \bar{p}$ ...(2)

Step 3: The upper and lower control limits change due to variable sample size. The control limits for i^{th} sample are given by

✓ Upper control limit $(UCL_i) = \bar{p} + 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n_i}}$ ...(3)

✓ Lower control limit $(LCL_i) = \bar{p} - 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n_i}}$ ...(4)

Step 4: Interpretation of the p-chart.

Second Approach

According to the second approach, the control limits are calculated using the average sample size. This approach is used only when there is no large variation in the sample sizes and future sample sizes are not expected to differ significantly from the average sample size. We get constant control limits using this approach as we get with the constant sample size discussed in Lab Session 6.

Step 1: We calculate the average sample size as follows:

$$\bar{n} = \frac{n_1 + n_2 + \ldots + n_k}{k} = \frac{1}{k}\sum_{i=1}^{k} n_i \quad \ldots(5)$$

Step 2: The unkown value of fraction defective (P) can be estimated by

$$\bar{p} = \frac{\sum_{j=1}^{k} d_i}{k\bar{n}} = \frac{\sum_{i=1}^{k} d_i}{\sum_{i=1}^{k} n_i} \quad \ldots(6)$$

Step 3: The centre line, upper and lower control limits are given by

✓ Centre line $(CL) = \bar{p}$ ...(7)

✓ Upper control limit $(UCL) = \bar{p} + 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{\bar{n}}}$ ...(8)

✓ Lower control limit $(LCL) = \bar{p} - 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{\bar{n}}}$ ... (9)

Step 4: Interpretation of the p-chart.

7.4 STEPS INVOLVED IN THE CONSTRUCTION OF p-CHART IN EXCEL 2007

We now describe the procedure for the construction of p-chart for the problem given in Sec. 7.2. For plotting a p-chart in Excel 2007, we follow the steps given ahead.

Step 1: We enter the given data in MS Excel spreadsheet as shown in Fig. 7.1.

	A	B	C
1			
2	Sample No.	Laptops Inspected	Defective Laptops
3	1	50	2
4	2	52	4
5	3	57	4
6	4	50	11

p Chart

Fig. 7.1: Partial screenshot of the spreadsheet for given data.

Step 2: Following Step 2 of Lab Session 6, we compute the fraction defective for each sample as shown in Fig. 7.2.

D3 =C3/B3

	A	B	C	D
1				
2	Sample No.	Laptops Inspected	Defective Laptops	Fraction Defective (p)
3	1	50	2	0.040
4	2	52	4	0.077
5	3	57	4	0.070
6	4	50	11	0.220
7	5	50	4	0.080
8	6	48	2	0.042

Fig. 7.2

Step 3: We determine the total number of laptops inspected and total number of defective laptops in Cells B28 and C28, respectively, as shown in Fig. 7.3.

	A	B	C
28	Total	1302	99

Used **"=Sum(B3:B27)"** function Used **"=Sum(C3:C27)"** function

Fig. 7.3

Step 4: We now compute the average fraction defective, i.e., $\bar{p}$ by typing "=C28/B28" in Cell B29 and clicking on ***Enter*** as shown in Fig. 7.4.

ROUND =C28/B28

	A	B	C
28	Total	1302	99
29	$\bar{p}$	=C28/B28	

	A	B	C
28	Total	1302	99
29	$\bar{p}$	0.076	
30			

Fig. 7.4

The formula with dollar sign ($) is used for an absolute reference.

Step 5: We now determine the centre line, upper and lower control limits in Columns E, F and G, respectively, by typing

i) "=$B$29" in Cell E3 to find centre line (Fig. 7.5a).

ii) "=E3+3*Sqrt(($B$29*(1-$B$29))/(B3))" in Cell F3 (Fig. 7.5b).

iii) "=E3-3*Sqrt(($B$29*(1-$B$29))/(B3))" in Cell G3 (Fig. 7.5c).

(a)

E3 =$B$29

	E	F	G	H	I	J
1		Control Limits				
2	Centre Line	UCL	LCL			
3	0.076					
4						

(b)

F3 =E3+3*SQRT(($B$29*(1-$B$29))/(B3))

	F	G	H	I	J	K
1	ontrol Limits					
2	UCL	LCL				
3	0.188					
4						

(c)

G3 =E3-3*SQRT(($B$29*(1-$B$29))/(B3))

	G	H	I	J	K	L
1						
2	LCL					
3	-0.036					
4						

Fig. 7.5

Step 6: We select Cells E3:G3 and drag them down up to Row 27 to get the centre line, upper and lower control limits corresponding to each sample as shown in Fig. 7.6. Here we get different values for UCL and LCL because we have different sample sizes.

	E	F	G
1		Control Limits	
2	Centre Line	UCL	LCL
3	0.076	0.188	-0.036
4	0.076	0.186	-0.034
5	0.076	0.181	-0.029
6	0.076	0.188	-0.036
7	0.076	0.188	-0.036
8	0.076	0.191	-0.039
9	0.076	0.187	-0.035
10	0.076	0.184	-0.032
11	0.076	0.186	-0.034
12	0.076	0.188	-0.036
13	0.076	0.183	-0.031
14	0.076	0.179	-0.027
15	0.076	0.183	-0.031
16	0.076	0.183	-0.031
17	0.076	0.186	-0.034
18	0.076	0.191	-0.039
19	0.076	0.188	-0.036
20	0.076	0.182	-0.030
21	0.076	0.186	-0.034
22	0.076	0.185	-0.033

p Chart

Fig. 7.6

Step 7: We notice from Fig. 7.6 that all values of LCL are negative, which is not possible and not acceptable. Hence, we set these values as 0 (zero) and denote the limit by LCL* as shown in Fig. 7.7.

	E	F	G	H
1		Control Limits		
2	Centre Line	UCL	LCL	LCL*
3	0.076	0.188	-0.036	0
4	0.076	0.186	-0.034	0
5	0.076	0.181	-0.029	0
6	0.076	0.188	-0.036	0
7	0.076	0.188	-0.036	0
8	0.076	0.191	-0.039	0
9	0.076	0.187	-0.035	0
10	0.076	0.184	-0.032	0
11	0.076	0.186	-0.034	0
12	0.076	0.188	-0.036	0
13	0.076	0.183	-0.031	0
14	0.076	0.179	-0.027	0
15	0.076	0.183	-0.031	0
16	0.076	0.183	-0.031	0
17	0.076	0.186	-0.034	0
18	0.076	0.191	-0.039	0
19	0.076	0.188	-0.036	0
20	0.076	0.182	-0.030	0
21	0.076	0.186	-0.034	0
22	0.076	0.185	-0.033	0

p Chart

Fig. 7.7

If the value of lower control limits are negative, we set the lower control limit as zero because a negative fraction defective is not possible.

Step 8: To obtain the p-chart in Excel 2007, we follow the procedure explained in Step 10 of Sec.1.4, Lab Session 1. It means that we

1. select Cells D2:F27 and H2:H27 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1. Thus, we obtain the p-chart shown in Fig. 7.8.

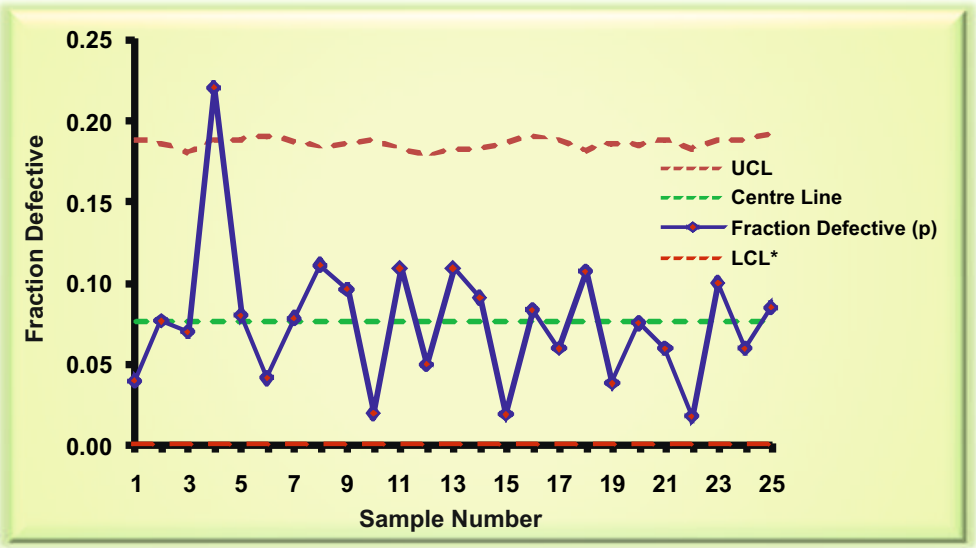


Fig. 7.8

Interpretation

The control chart for defective laptops (shown in Fig. 7.8) indicates that the process is not under statistical control. The point corresponding to Sample 4 is outside the upper control limit. Some assignable causes are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. Once this is done, we remove the out-of-control Sample 4 and calculate the revised centre line and control limits for the p-chart using the remaining samples. These limits are known as revised control limits and will be discussed in the next section.

We hope that you have grasped the method for plotting the p-chart for variable sample size. Before going to the next section, you should perform Activity 1.

Activity 1

- ✓ Compute the centre line and control limits in Excel 2007 using the second approach which you have learnt in Sec. 3.4 of MSTE-001.
- ✓ Plot the p-chart and compare the result with the output obtained above. If any point goes out of control, compute the revised control limits in the same manner as discussed in Sec. 7.5.

7.5 REVISED p-CHART

From the p-chart shown in Fig. 7.8, we observe that the point corresponding to Sample 4 lies outside the upper control limit. So we plot the p-chart for the revised control limits until all points come within UCL and LCL as discussed in the Lab Sessions 2 to 5. The main steps for computing the revised control limits are as follows:

First Approach

Step 1: To calculate the revised limits of p-chart, we first calculate new $\bar{p}$ as follows:

$$\bar{p}_{new} = \frac{\sum_{i=1}^{k} d_i - \sum_{j=1}^{d} d_j}{\sum_{i=1}^{k} n_i - \sum_{j=1}^{d} n_j} \qquad \ldots(10)$$

where d – number of discarded samples, and

$\sum_{j=1}^{d} d_j$ – sum of number of defectives in the discarded samples.

Step 2: After finding $\bar{p}_{new}$, we reconstruct the centre line and control limits of the chart by replacing $\bar{p}$ by $\bar{p}_{new}$ as follows:

✓ Centre line = $\bar{p}_{new}$ ...(11)

✓ Upper control limit (UCL_i) $= \bar{p}_{new} + 3\sqrt{\dfrac{\bar{p}_{new}(1-\bar{p}_{new})}{n_i}}$ ...(12)

✓ Lower control limit (LCL_i) $= \bar{p}_{new} - 3\sqrt{\dfrac{\bar{p}_{new}(1-\bar{p}_{new})}{n_i}}$ ...(13)

Step 3: Interpretation of the p-chart.

Second Approach

Step 1: To calculate the revised limits of p-chart, the value of new $\bar{p}$ will be the same as equation (10).

Step 2: After finding $\bar{p}_{new}$, we reconstruct the centre line and control limits of the chart by replacing $\bar{p}$ by $\bar{p}_{new}$ as follows:

✓ Centre line = $\bar{p}_{new}$ ...(14)

✓ Upper control limit (UCL) $= \bar{p}_{new} + 3\sqrt{\dfrac{\bar{p}_{new}(1-\bar{p}_{new})}{\bar{n}}}$ ...(15)

✓ Lower control limit (LCL) $= \bar{p}_{new} - 3\sqrt{\dfrac{\bar{p}_{new}(1-\bar{p}_{new})}{\bar{n}}}$ ...(16)

Step 3: Interpretation of the p-chart.

Steps in Excel

The revised centre line and control limits for the p-chart using the remaining samples are described below:

Step 1: We highlight the samples outside the control limits, i.e., the 4th sample, with light orange colour (Fig. 7.9).

	A	B	C	D	E	F	G	H
1					Control Limits			
2	Sample No.	Laptops Inspected	Defective Laptops	Fraction Defective (p)	Centre Line	UCL	LCL	LCL*
3	1	50	2	0.040	0.076	0.188	-0.036	0
4	2	52	4	0.077	0.076	0.186	-0.034	0
5	3	57	4	0.070	0.076	0.181	-0.029	0
6	4	50	11	0.220	0.076	0.188	-0.036	0
7	5	50	4	0.080	0.076	0.188	-0.036	0
8	6	48	2	0.042	0.076	0.191	-0.039	0
9	7	51	4	0.078	0.076	0.187	-0.035	0
10	8	54	6	0.111	0.076	0.184	-0.032	0
11	9	52	5	0.096	0.076	0.186	-0.034	0
12	10	50	1	0.020	0.076	0.188	-0.036	0

Fig. 7.9

Step 2: We calculate $\bar{p}_{new}$ by typing "=(C28-C6)/(B28-B6)" in Cell B30 as shown in Fig. 7.10.

B30 | =(C28-C6)/(B28-B6)

	A	B	C
28	Total	1302	99
29	$\bar{p}$	0.076	
30	$\bar{p}_{new}$	0.070	
31			

Fig. 7.10

Step 3: We determine the centre line and control limits by replacing $\bar{p}$ by $\bar{p}_{new}$ as shown in Fig. 7.11 and also replace the negative LCL by zero (0) and denote the limit by LCL* as discussed in Step 5 of Sec. 7.4.

	I	J	K	L
1	Revised Control Limits			
2	Centre Line	UCL	LCL	LCL*
3	0.070	0.179	-0.038	0
4	0.070	0.177	-0.036	0
5	0.070	0.172	-0.031	0
6	0.070	0.179	-0.038	0
7	0.070	0.179	-0.038	0
8	0.070	0.181	-0.040	0
9	0.070	0.178	-0.037	0
10	0.070	0.175	-0.034	0
11	0.070	0.177	-0.036	0
12	0.070	0.179	-0.038	0

p Chart

Fig. 7.11

Step 4: We now plot fraction defectives, revised centre line, UCL and LCL* as discussed in Step 8 of Sec. 7.4. We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting chart is shown in Fig. 7.12.

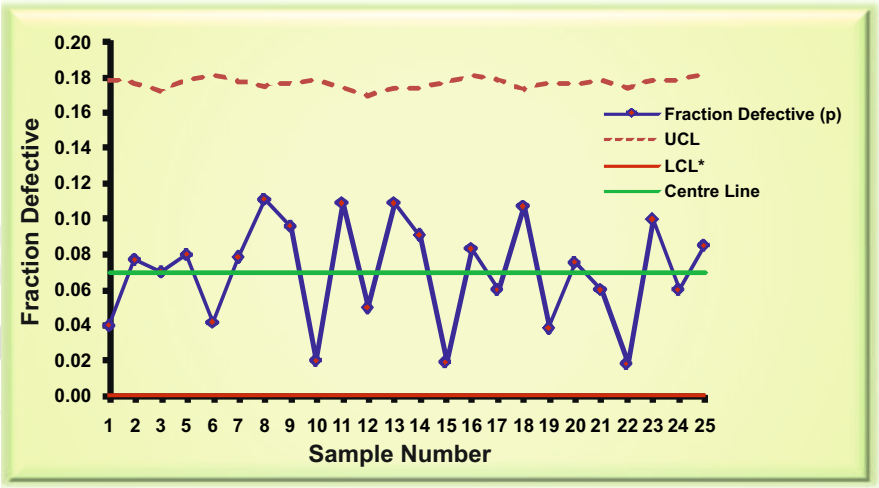


Fig. 7.12

Interpretation

The p-chart given in Fig. 7.12 reveals that no point lies outside the control limits. So we may conclude that the process is under statistical control with respect to fraction defectives.

You should now apply this method on other problems for practice.

Activity 2

Construct the control charts for fraction defective with the help of MS Excel 2007 and interpret the results for

A1) Example 4 given in Unit 3 of MSTE-001.

A2) Exercise E6 given in Unit 3 of MSTE-001.

Match the results with the manual calculation done in Unit 3 of MSTE-001.

Continuous Assessment 7

In the production of tyres, the output of given size was inspected every day prior to the tyres being given to finished goods stores. The number of defective tyres found everyday is summarised in the following table:

Table 2: Number of defective tyres

Sample No.	Number of Tyres Inspected	Number of Defective Tyres	Sample No.	Number of Tyres Inspected	Number of Defective Tyres
1	650	70	14	550	78
2	510	74	15	540	64
3	600	58	16	610	90
4	590	61	17	670	96
5	630	65	18	660	99
6	650	108	19	650	78
7	700	82	20	590	60
8	740	51	21	650	56
9	580	80	22	640	55
10	600	90	23	580	57
11	670	71	24	530	58
12	660	75	25	520	44
13	600	77			

Draw the p-chart using both approaches and comment about the state of the process. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Secs. 7.4 and 7.5 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.
2) Develop the spreadsheet for the exercise "Continuous Assessment 7" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.
3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.

Control Chart for Number of Defectives

SESSION 8

Structure

8.1 Introduction

Objectives

8.2 Problem Description

8.3 Procedure for the Construction of np-Chart

8.4 Steps Involved in the Construction of np-Chart in Excel 2007

8.5 Revised np-Chart

8.1 INTRODUCTION

Prerequisite

- Lab Sessions 1 and 3 of MSTL-001 (Basic Statistics Lab).
- Lab Session 6 of MSTL-002 (Industrial Statistics Lab).
- Unit 3 of MSTE-001 (Industrial Statistics-I).

In Lab Sessions 6 and 7, you have studied the p-chart for fixed and varying sample size, respectively. In p-chart, we calculated the fraction defective for each sample and plotted the fraction defective against the sample number.

In this lab session, we count the number of defective items in the samples instead of calculating the fraction defective. We plot them against the sample numbers. This chart is known as the np-chart. Using np-chart, we may better understand the actual number of defective items than the fraction of defective. The np-chart is used only for equal sample size because we cannot compare the number of defectives for variable sample size as discussed in Sec. 3.5 of Unit 3 of MSTE-001 (Industrial Statistics-I). For example, if we get one defective item in a sample of size 10 and one defective item in a sample of size 100, the first sample shows a much higher rate of defectives, i.e., 10% in comparison with the second sample.

The np-chart is also known as the d-chart.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for control chart for number of defectives;
- construct the control chart for number of defectives; and
- interpret the control chart for number of defectives.

8.2 PROBLEM DESCRIPTION

To monitor the manufacturing of mobile phones, a quality controller randomly inspects 45 mobile phones at the end of the production each day and notes the number of defective mobile phones. The number of defective mobile phones found each day is recorded in Table 1.

Table 1: Number of defective mobile phones

Sample No.	1	2	3	4	5	6	7	8	9	10
Number of Defective Phones	13	12	13	18	7	14	12	10	16	16
Sample No.	11	12	13	14	15	16	17	18	19	20
Number of Defective Phones	12	18	10	12	14	10	17	8	13	12
Sample No.	21	22	23	24	25	26	27	28	29	30
Number of Defective Phones	16	13	10	17	14	18	10	12	12	13

The quality control inspector of this company needs to construct the control chart for number of defective mobile phones to check whether the process is under statistical control or not.

Therefore, the problem for this session is to construct the control chart for number of defective mobile phones, i.e., the np-chart for the data given in Table 1 and also compute the revised control limits, if necessary.

8.3 PROCEDURE FOR THE CONSTRUCTION OF np-CHART

In Sec. 3.5 of Unit 3 of MSTE-001, you have grasped the various formulae and procedure for the construction of np-chart. Therefore, in this lab session, we briefly discuss the procedure of computing the centre line, upper and lower control limits. The main steps involved in the construction of the np-chart are as follows:

Step 1: We draw k samples of the same size n and $d_1, d_2, ..., d_k$ are the numbers of defectives in 1st, 2nd, …, kth sample, respectively. The average fraction defective of k samples is given by

$$\bar{p} = \frac{\text{Sum of defective items}}{\text{Total number of items inspected}} = \frac{1}{nk}\sum_{i=1}^{k} d_i \qquad \ldots(1)$$

Step 2: The centre line and control limits for the np-chart are given by

- ✓ Centre line (CL) $= n\bar{p}$ …(2)
- ✓ Upper control limit (UCL) $= n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$ …(3)
- ✓ Lower control limit (LCL) $= n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$ …(4)

Step 3: After setting the centre line and control limits, we construct the np-chart by taking sample number on the X-axis and number of defectives in the sample on the Y-axis.

Step 4: Interpretation of the np-chart.

8.4 STEPS INVOLVED IN THE CONSTRUCTION OF np-CHART IN EXCEL 2007

You have already learnt about the manual computation of np-chart in Sec. 3.5 of Unit 3 of MSTE-001. The steps for plotting the np-chart in Excel 2007 are given below:

Step 1: We enter the given data in MS Excel spreadsheet as shown in Fig. 8.1.

	A	B
1	Sample No.	Number of Defective Phones
2	1	13
3	2	12
4	3	13
5	4	18
6	5	7
7	6	14
8	7	12
9	8	10
10	9	16
11	10	16
12	11	12
13	12	18
14	13	10
15	14	12
16	15	14
17	16	10
18	17	17
19	18	8
20	19	13

np chart

Fig. 8.1: Partial screenshot of the spreadsheet for given data.

Step 2: We find the total number of defective mobile phones by typing "=Sum(B2:B31)" in Cell B2 and then clicking on ***Enter*** as shown in Fig. 8.2.

ROUND =SUM(B2:B31)

	A	B	C
32	Total	=SUM(B2:B31)	
33			

ENTER

	A	B
32	Total	392

Fig. 8.2

Step 3: After calculating the value of $\sum_{i=1}^{30} d_i$, we put the values of n and k in Cells B34 and B35, respectively (Fig. 8.3).

	A	B
32	Total	392
33		
34	n	45
35	k	30

Fig. 8.3

The values of $\sum_{i=1}^{30} d_i$, n and k are given in Cells B32, B34 and B35, respectively.

Step 4: We now use equation (1) and compute the average fraction defective $\bar{p}$ by typing "=B32/(B34*B35)" in Cell B33 and clicking on ***Enter*** as shown in Fig. 8.4.

ROUND | =B32/(B34*B35)

	A	B	C
32	Total	392	
33	$\bar{p}$	=B32/(B34*B35)	
34	n	45	
35	k	30	
36			

	A	B
32	Total	392
33	$\bar{p}$	0.290
34	n	45
35	k	30

Fig. 8.4

Step 5: We determine the values of centre line, upper and lower control limits in Columns C, D and E, respectively. We can calculate these limits from equations (2) to (4) by typing

i) "=$B$34*$B$33" in Cell C2 to find centre line as shown in Fig. 8.5a.

ii) "=$B$34*$B$33+3* Sqrt(($B$34*$B$33*(1-$B$33)))" in Cell D2 as shown in Fig. 8.5b.

iii) "=$B$34*$B$33-3* Sqrt(($B$34*$B$33*(1-$B$33)))" in Cell E2 as shown in Fig. 8.5c.

The formula with dollar sign ($) is used for an absolute reference.

(a)

C2 =$B$34*$B$33

	C	D	E	F	G
1	Centre Line	UCL	LCL		
2	13.067				
3					

(b)

D2 =$B$34*$B$33+3*SQRT(($B$34*$B$33*(1-$B$33)))

	D	E	F	G	H
1	UCL	LCL			
2	22.202				
3					

(c)

E2 =$B$34*$B$33-3*SQRT(($B$34*$B$33*(1-$B$33)))

	E	F	G	H	I
1	LCL				
2	3.931				
3					

Fig. 8.5

Step 6: We first select Cells C2:E2 and drag them down up to Row 31 to get the control limits corresponding to each sample as shown in Fig. 8.6.

	A	B	C	D	E
1	Sample No.	Number of Defective Phones	Centre Line	UCL	LCL
2	1	13	13.067	22.202	3.931
3	2	12	13.067	22.202	3.931
4	3	13	13.067	22.202	3.931
5	4	18	13.067	22.202	3.931
6	5	7	13.067	22.202	3.931
7	6	14	13.067	22.202	3.931
8	7	12	13.067	22.202	3.931
9	8	10	13.067	22.202	3.931
10	9	16	13.067	22.202	3.931
11	10	16	13.067	22.202	3.931
12	11	12	13.067	22.202	3.931
13	12	18	13.067	22.202	3.931
14	13	10	13.067	22.202	3.931
15	14	12	13.067	22.202	3.931
16	15	14	13.067	22.202	3.931
17	16	10	13.067	22.202	3.931
18	17	17	13.067	22.202	3.931
19	18	8	13.067	22.202	3.931
20	19	13	13.067	22.202	3.931

np chart

Fig. 8.6

Step 7: To plot the np-chart in Excel 2007, we follow the procedure explained in Step 10 of Sec. 1.4, Lab Session 1. It means that we

1. select Cells B1:E31,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting np-chart is shown in Fig. 8.7.

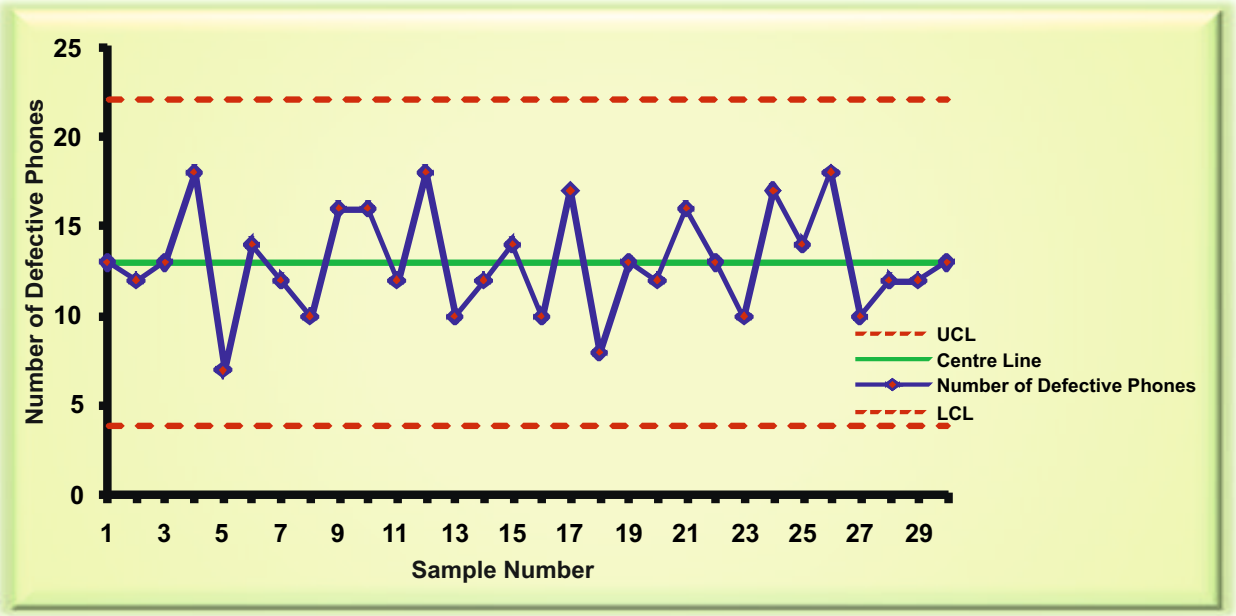


Fig. 8.7

Interpretation

The control chart for defective mobile phones (shown in Fig. 8.7) reveals that the process is under statistical control since no point lies outside the control limits of the np-chart. We may conclude that the process is under statistical control with respect to number of defective mobile phones.

In the next section, we are listing the formulae and procedure of computing the revised control limits for the np-chart if you come across this situation.

8.5 REVISED np-CHART

If one or more points lie outside the control limits (UCL or LCL or both), the control chart indicates that the process is not under statistical control. Some assignable causes are present in the process. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. Once this is done, we eliminate the out-of-control samples and calculate the revised centre line, upper and lower control limits for the np-chart using the remaining samples as follows:

Step 1: To get the revised limits for np-chart, we first calculate the new $\bar{p}$ as follows:

$$\bar{p}_{new} = \frac{\sum_{i=1}^{k} d_i - \sum_{j=1}^{d} d_j}{n(k-d)} \quad ...(5)$$

where d – number of discarded samples, and

$\sum_{j=1}^{d} d_j$ – sum of number of defectives in the discarded samples.

Step 2: After finding $\bar{p}_{new}$, we reconstruct the centre line, upper and lower control limits by replacing $\bar{p}$ by $\bar{p}_{new}$ as follows:

- ✓ Centre line (CL) $= n\bar{p}_{new}$ ...(6)
- ✓ Upper control limit (UCL)

$$= n\bar{p}_{new} + 3\sqrt{n\bar{p}_{new}(1-\bar{p}_{new})} \quad ...(7)$$

- ✓ Lower control limit (LCL)

$$= n\bar{p}_{new} - 3\sqrt{n\bar{p}_{new}(1-\bar{p}_{new})} \quad ..(8)$$

Step 3: As discussed in Lab Sessions 2 to 5, we plot the np-chart for the revised control limits until all points lie within UCL and LCL.

Step 4: Interpretation of the p-chart

You should do the following exercises to construct the np-chart.

Activity

Construct the control charts for the number of defectives with the help of MS Excel 2007 and interpret the results for

A1) Examples 5 and 6 given in Unit 3 of MSTE-001.

A2) Exercises E9 and E10 given in Unit 3 of MSTE-001.

Match the results with the manual calculation done in Unit 3 of MSTE-001.

Continuous Assessment 8

In the production of tyres, 650 tyres were inspected every day before sending them to finished goods stores. The number of defective tyres found everyday is summarised in Table 2.

Table 2: Number of defective tyres

Sample No.	Number of Defective Tyres	Sample No.	Number of Defective Tyres
1	70	14	78
2	74	15	64
3	58	16	90
4	61	17	87
5	65	18	91
6	108	19	78
7	82	20	60
8	56	21	56
9	80	22	55
10	90	23	57
11	71	24	78
12	75	25	68
13	77		

Draw the np-chart and comment about the state of the process. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Secs. 8.4 and 8.5 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 8" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.

Control Chart for Number of Defects

SESSION 9

Structure

9.1 Introduction

Objectives

9.2 Problem Description

9.3 Procedure for the Construction of c-Chart

9.4 Steps Involved in the Construction of c-Chart in Excel 2007

9.5 Revised c-Chart

9.1 INTRODUCTION

Prerequisite

- Lab Sessions 1 and 3 of MSTL-001 (Basic Statistics Lab).
- Lab Session 6 of MSTL-002 (Industrial Statistics Lab).
- Unit 4 of MSTE-001 (Industrial Statistics-I).

You have learnt in Lab Sessions 6 and 8 that the p-chart and np-chart are used to control the fraction of defectives and number of defectives, respectively, by considering only one criterion, i.e., whether the unit or item is defective or non-defective. But in many situations, it is economically advantageous to know the number of defects within the unit rather than knowing if a unit is defective or non-defective. For example, suppose a glass bottle has a small air bubble and another one has many small air bubbles in it. Even though both bottles are defective, the second bottle is of inferior quality as compared to the first because it has many more defects. Therefore, it is important to control the number of defects within the unit. In such situations, we use the c-chart and u-chart to monitor and control the number of defects.

You have studied in Sec. 4.2 of Unit 4 in MSTE-001 (Industrial Statistics-I) that we use the c-chart when the **sample size is 1 or constant**, i.e., we have inspected the same number of units in each sample, e.g., one TV, one aeroplane, twenty laptops, ten bottles, one bolt of cloth, and so on. The u-chart is used to **control the number of defects per unit**.

In this lab session, you will learn how to construct the c-chart to monitor and **control the number of defects within the unit** with the help of MS Excel 2007. In the next lab session, we shall discuss the application of u-chart when the sample size varies.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;

- determine the control limits for c-chart;
- construct the c-chart; and
- interpret the c-chart.

9.2 PROBLEM DESCRIPTION

Let us look at the problem faced by a textile company observing some defects in its woven fabric. As a part of overall quality improvement programme, the company decided to monitor the number of defects found in the woven fabric each day. The fabric produced each day is one metre wide and 60 metres in length. For this purpose, the total number of defects in the woven fabric each day were counted for 35 days. These are recorded in Table 1.

Table 1: Number of defects in fabric

Day	Number of Defects	Day	Number of Defects
1	7	19	8
2	3	20	4
3	1	21	2
4	3	22	5
5	6	23	12
6	2	24	1
7	4	25	5
8	5	26	3
9	7	27	6
10	3	28	5
11	12	29	7
12	2	30	2
13	4	31	4
14	8	32	5
15	3	33	4
16	7	34	3
17	6	35	6
18	3		

The quality control inspector of the company needs to construct a control chart to check whether the process is under statistical control or not and also computes the revised control limits, if necessary.

Therefore, the problem for this session is to construct the control chart for the number of defects, i.e., the c-chart for the data given in Table 1.

9.3 PROCEDURE FOR THE CONSTRUCTION OF C-CHART

You have already studied about the construction of c-chart in Unit 4 of MSTE-001 in depth. Here we briefly discuss how to compute control limits.

Generally, the average number of defects within a unit is not known. So we estimate it from the sample information as follows:

Step 1: We draw k samples of constant size and $c_1, c_2, ..., c_k$ are the numbers of defects the in the 1st, 2nd, …, kth sample, respectively. We estimate the average number of defects within a unit by the average number of defects in the sample which is calculated by

$$\bar{c} = \frac{\text{Total number of defects}}{\text{Total number of inspected samples}} = \frac{1}{k}\sum_{i=1}^{k} c_i \qquad ...(1)$$

Step 2: The control limits of c-chart are given by

- ✓ Centre Line $(CL) = \bar{c}$ …(2)
- ✓ Upper control limit $(UCL) = \bar{c} + 3\sqrt{\bar{c}}$ …(3)
- ✓ Lower control limit $(LCL) = \bar{c} - 3\sqrt{\bar{c}}$ …(4)

Step 3: We construct the c-chart by taking sample number on the X-axis and number of defects in the sample (c) on the Y-axis.

Step 4: Interpretation of the c-chart.

9.4 STEPS INVOLVED IN THE CONSTRUCTION OF c-CHART IN EXCEL 2007

We now describe the construction of c-chart in Excel 2007 with the help of the data given in Sec. 9.2. We follow the steps given below:

Step 1: We enter the given data in MS Excel spreadsheet as shown in Fig. 9.1.

	A	B
1		
2	Days	Number of Defects
3	1	7
4	2	3
5	3	1
6	4	3
7	5	6
8	6	2
9	7	4
10	8	5
11	9	7
12	10	3
13	11	12
14	12	2
15	13	4
16	14	8
17	15	3
18	16	7
19	17	6
20	18	3
21	19	8

c Chart

Fig. 9.1: Partial screenshot of the spreadsheet for the given data.

Step 2: We determine the total number of defects in the fabric by typing "=Sum(B3:B37)" in Cells B38 as represented in Fig. 9.2.

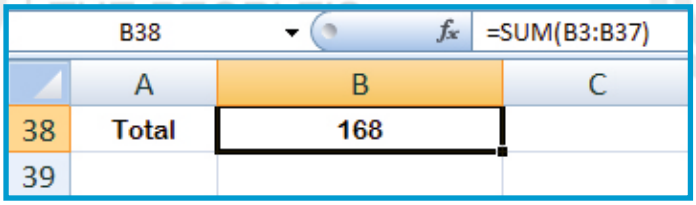


Fig. 9.2

Step 3: We type the values of n and k in Cells B41 and B42, respectively, as shown in Fig. 9.3.

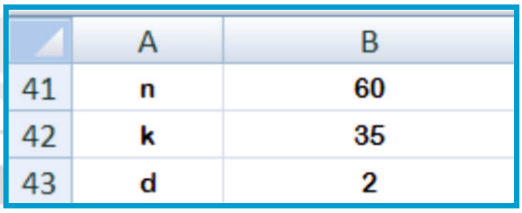
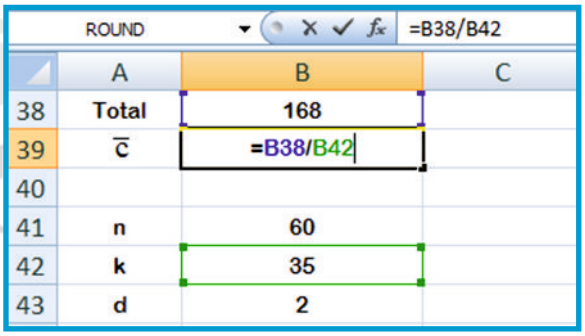


Fig. 9.3

Step 4: We use equation (1) to compute the average defects in fabric, i.e., $\bar{c}$ by typing "=B38/B42" in Cell B39 and then press ***Enter*** (Fig. 9.4).



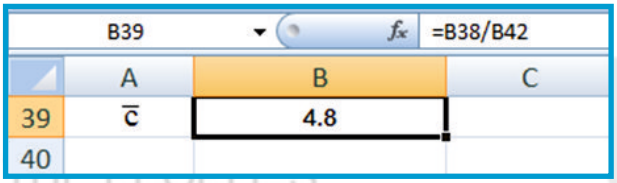


Fig. 9.4

Step 5: We compute the values of the centre line, upper and lower control limits using equations (2) to (4) in Columns C, D and E, respectively, by typing

i) "=$B$39" in Cell C3 to find the centre line as shown in Fig. 9.5a.

ii) "=C3+3*Sqrt(C3)" in Cell D3 as shown in Fig. 9.5b.

iii) "=C3-3*Sqrt(C3)" in Cell E3, as shown in Fig. 9.5c.

(a)

C3 =$B$39

	C	D	E	F
1		Control Limits		
2	Centre Line	UCL	LCL	
3	4.8			
4				

(b)

D3 =C3+3*SQRT(C3)

	D	E	F	G
1	Control Limits			
2	UCL	LCL		
3	11.373			
4				

(c)

E3 =C3-3*SQRT(C3)

	E	F	G	H
1				
2	LCL			
3	-1.773			
4				

Fig. 9.5

Step 6: We select Cells C3:E3 and drag them down up to Row 37 to get the control limits corresponding to each sample as shown in Fig. 9.6.

	B	C	D	E
1			Control Limits	
2	Number of Defects	Centre Line	UCL	LCL
3	7	4.8	11.373	-1.773
4	3	4.8	11.373	-1.773
5	1	4.8	11.373	-1.773
6	3	4.8	11.373	-1.773
7	6	4.8	11.373	-1.773
8	2	4.8	11.373	-1.773
9	4	4.8	11.373	-1.773
10	5	4.8	11.373	-1.773
11	7	4.8	11.373	-1.773
12	3	4.8	11.373	-1.773
13	12	4.8	11.373	-1.773
14	2	4.8	11.373	-1.773
15	4	4.8	11.373	-1.773
16	8	4.8	11.373	-1.773
17	3	4.8	11.373	-1.773
18	7	4.8	11.373	-1.773
19	6	4.8	11.373	-1.773
20	3	4.8	11.373	-1.773
21	8	4.8	11.373	-1.773

c Chart

Fig. 9.6

Step 7: Look at Fig. 9.6. The value of LCL is negative, which is not possible and, therefore, not acceptable. So we consider the value 0 (zero) instead of the negative value of LCL to plot the control chart as shown in Fig. 9.7.

=C2+3*Sqrt(C2)

=C2-3*Sqrt(C2)

→ 0

	B	C	D	E	F
1			Control Limits		
2	Number of Defects	Centre Line	UCL	LCL	LCL*
3	7	4.8	11.373	-1.773	0
4	3	4.8	11.373	-1.773	0
5	1	4.8	11.373	-1.773	0
6	3	4.8	11.373	-1.773	0
7	6	4.8	11.373	-1.773	0
8	2	4.8	11.373	-1.773	0
9	4	4.8	11.373	-1.773	0
10	5	4.8	11.373	-1.773	0
11	7	4.8	11.373	-1.773	0
12	3	4.8	11.373	-1.773	0
13	12	4.8	11.373	-1.773	0
14	2	4.8	11.373	-1.773	0
15	4	4.8	11.373	-1.773	0
16	8	4.8	11.373	-1.773	0
17	3	4.8	11.373	-1.773	0
18	7	4.8	11.373	-1.773	0
19	6	4.8	11.373	-1.773	0
20	3	4.8	11.373	-1.773	0
21	8	4.8	11.373	-1.773	0

c Chart

If the value of lower control limit is negative, we change the lower control limit to zero because a negative number of defects is not possible.

Fig. 9.7

Step 8: To plot the c-chart in Excel 2007, we follow the procedure explained in Step 10 of Sec. 1.4, Lab Session 1. It means that we

1. select Cells B3:D37 and F3:F37 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting c-chart is shown in Fig. 9.8.

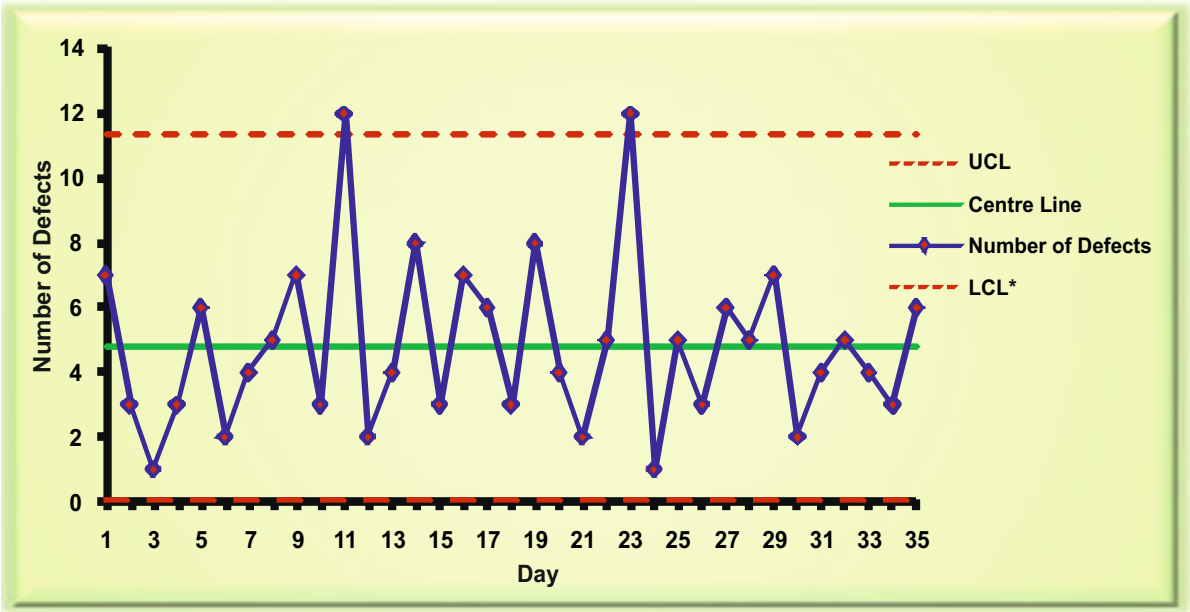


Fig. 9.8

Interpretation

The control chart for the number of defects in a fabric (shown in Fig. 9.8) indicates that the process is not under statistical control and some assignable causes are present in the process. The samples corresponding to the 11th and 23rd days are outside the upper control limit. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. Then we eliminate out-of-control Samples 11 and 23 and calculate the revised centre line, upper and lower control limits for the c-chart using the remaining samples. These limits are known as revised control limits and are discussed in Sec. 9.5.

9.5 REVISED c-CHART

We plot the c-chart for the revised control limits using the remaining samples in the same way as explained in Lab Session 7. We repeat the procedure until all points come within UCL and LCL. The main steps for computing the revised control limits are as follows:

Step 1: For revised control limits for c-chart, we first calculate new $\bar{c}$ as follows:

$$\bar{c}_{new} = \frac{\sum_{i=1}^{k} c_i - \sum_{j=1}^{d} c_j}{k - d} \qquad \ldots(5)$$

where d – number of discarded samples, and

$\sum_{j=1}^{d} c_j$ – Total number of defects within the discarded samples.

Step 2: After finding the new $\bar{c}$, we reconstruct the centre line and control limits for the c-chart by replacing $\bar{c}$ by $\bar{c}_{new}$ as given below:

- ✓ Centre line (CL) $= \bar{c}_{new}$ …(6)
- ✓ Upper control limit (UCL) $= \bar{c}_{new} + 3\sqrt{\bar{c}_{new}}$ …(7)
- ✓ Lower control limit (LCL) $= \bar{c}_{new} - 3\sqrt{\bar{c}_{new}}$ …(8)

Step 3: Interpretation of the c-chart.

Steps in Excel

The revised centre line and control limits for constructing the c-chart in Excel 2007 using the remaining samples are described below:

Step 1: From the c-chart given in Fig. 9.8, we observe that the samples corresponding to the 11th and 23rd days lie outside the upper control limit. We highlight these samples with light orange colour as shown in Fig. 9.9.

	A	B	C	D	E	F
1				Control Limits		
2	Days	Number of Defects	Centre Line	UCL	LCL	LCL*
3	1	7	4.8	11.373	-1.773	0
4	2	3	4.8	11.373	-1.773	0
5	3	1	4.8	11.373	-1.773	0
6	4	3	4.8	11.373	-1.773	0
7	5	6	4.8	11.373	-1.773	0
8	6	2	4.8	11.373	-1.773	0
9	7	4	4.8	11.373	-1.773	0
10	8	5	4.8	11.373	-1.773	0
11	9	7	4.8	11.373	-1.773	0
12	10	3	4.8	11.373	-1.773	0
13	11	12	4.8	11.373	-1.773	0
14	12	2	4.8	11.373	-1.773	0

	A	B	C	D	E	F
24	22	5	4.8	11.373	-1.773	0
25	23	12	4.8	11.373	-1.773	0
26	24	1	4.8	11.373	-1.773	0
27	25	5	4.8	11.373	-1.773	0

Fig. 9.9

Step 2: For revised limits of the c-chart, we calculate new $\bar{c}$ by typing "=(B38-B13-B25)/(B42-B43)" in Cell B40. We put the value d = 2 in Cell B43 as shown in Fig. 9.10.

B40 =(B38-B13-B25)/(B42-B43)

	A	B	C	D
38	Total	168		
39	$\bar{c}$	4.8		
40	$\bar{c}_{new}$	4.364		
41	n	60		
42	k	35		
43	d	2		

Fig. 9.10

Step 3: After finding the new $\bar{c}$, we reconstruct the centre line and control limits by replacing $\bar{c}$ by $\bar{c}_{new}$ and also replace the negative LCL by zero (0) and denote the limit by LCL* as shown in Fig. 9.11.

	G	H	I	J
1		Revised Control Limits		
2	Centre Line	UCL	LCL	LCL*
3	4.4	10.630	-1.903	0
4	4.4	10.630	-1.903	0
5	4.4	10.630	-1.903	0
6	4.4	10.630	-1.903	0
7	4.4	10.630	-1.903	0
8	4.4	10.630	-1.903	0
9	4.4	10.630	-1.903	0
10	4.4	10.630	-1.903	0
11	4.4	10.630	-1.903	0
12	4.4	10.630	-1.903	0
13	4.4	10.630	-1.903	0
14	4.4	10.630	-1.903	0
15	4.4	10.630	-1.903	0
16	4.4	10.630	-1.903	0
17	4.4	10.630	-1.903	0

Fig. 9.11

Step 4: Finally, we plot the number of defects, centre line, UCL and LCL* as discussed in Step 8 of Sec. 9.4. We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting chart is shown in Fig. 9.12.

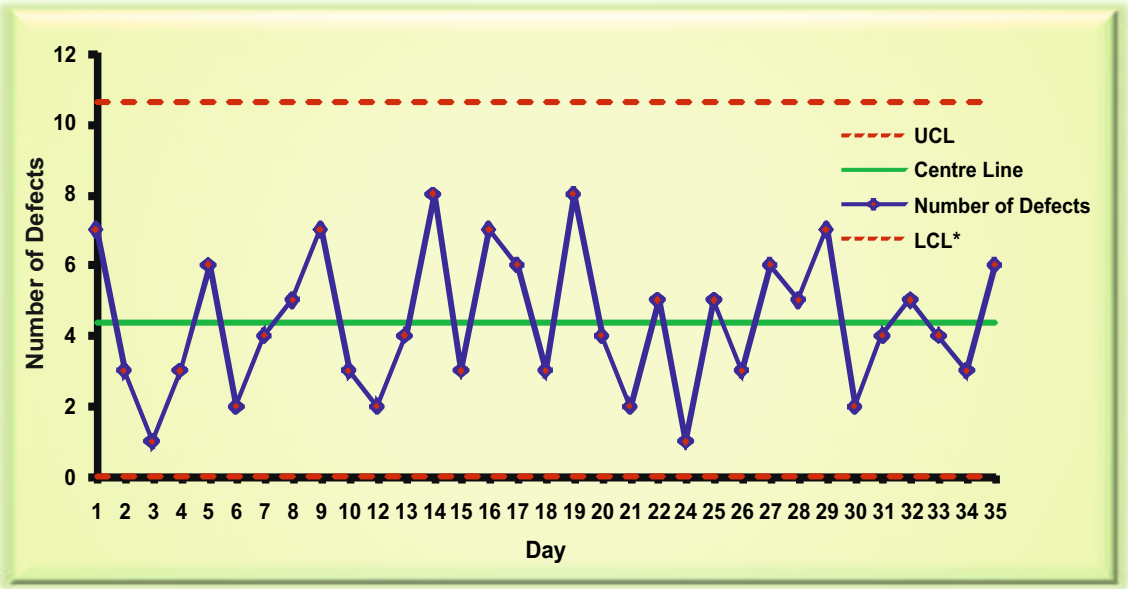


Fig. 9.12

Interpretation

From the revised c-chart given in Fig. 9.12, we observe that no point lies outside the control limits. This indicates that the process is under statistical control with respect to the number of defects.

The following activity will help you understand the c-chart more clearly.

Activity

Construct the control charts for the number of defects with the help of MS Excel 2007 and interpret the results for

A1) Examples 1, 2 and 3 given in Unit 4 of MSTE-001.

A2) Exercises E3 and E4 given in Unit 4 of MSTE-001.

Match the results with the manual calculation done in Unit 4 of MSTE-001.

Continuous Assessment 9

Suppose an almirah manufacturing company wants to judge the quality of the almirahs produced. The number of scratch marks on each almirah is recorded for this purpose. The data for 30 samples are given in Table 2.

Table 2: Number of scratch marks

Sample No.	1	2	3	4	5	6	7	8	9	10
Scratch Marks	7	3	1	3	6	2	4	5	7	3
Sample No.	11	12	13	14	15	16	17	18	19	20
Scratch Marks	6	3	14	7	2	5	9	4	7	3
Sample No.	21	22	23	24	25	26	27	28	29	30
Scratch Marks	2	7	6	8	4	10	5	4	6	7

Which chart will be used in this case? Construct the suitable control chart. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Secs. 9.4 and 9.5 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 9" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.

Control Chart for Number of Defects per Unit

SESSION 10

Structure

10.1 Introduction

Objectives

10.2 Problem Description

10.3 Procedure for the Construction of u-Chart

10.4 Steps Involved in the Construction of u-Chart in Excel 2007

10.5 Revised u-Chart

10.1 INTRODUCTION

Prerequisite

- Lab Sessions 1 and 3 of MSTL-001 (Basic Statistics Lab).
- Lab Session 7 of MSTL-002 (Industrial Statistics Lab).
- Unit 4 of MSTE-001 (Industrial Statistics-I).

In Lab Session 9, you have learnt how to apply the c-chart for monitoring and controlling the number of defects. You have also noted that the c-chart is applied only when the **sample size is 1 or constant**, i.e., the number of units or items is the same in each sample. When the sample size varies for some reason, we use the u-chart for controlling the number of defects. The u-chart can also be used when the **sample size is constant**.

You have learnt how to construct the u-chart in Unit 4 of MSTE-001 (Industrial Statistics-I). In this lab session, we explain how to construct the u-chart for monitoring and controlling the number of defects for **varying sample sizes** with the help of MS Excel 2007.

Objectives

After performing the activities of this session, you should be able to:

- prepare the spreadsheet in MS Excel 2007;
- determine the control limits for the u-chart;
- construct the u-chart; and
- interpret the u-chart.

10.2 PROBLEM DESCRIPTION

Let us consider the problem faced by a textile company, which observes defects in its woven fabric. As part of an overall quality improvement programme, the company decided to monitor the number of defects found in the woven fabric each day. The fabric produced each day is one metre

wide but varies in length. For this purpose, the length of woven fabric and the total number of defects inspected on each day were recorded for 35 days. The data is given in Table 1.

Table 1: Number of defects in fabric

Day	Length of Fabric (in metre)	Number of Defects	Day	Length of Fabric (in metre)	Number of Defects
1	50	7	19	35	8
2	40	3	20	50	4
3	45	1	21	33	2
4	35	3	22	43	5
5	45	6	23	38	12
6	30	2	24	38	1
7	40	4	25	48	5
8	35	5	26	43	3
9	35	7	27	43	6
10	45	3	28	33	5
11	40	12	29	48	7
12	40	2	30	53	2
13	30	4	31	48	4
14	45	8	32	38	5
15	50	3	33	43	4
16	45	7	34	38	3
17	35	6	35	53	6
18	40	3			

The quality control inspector of the company needs to construct a control chart to check whether the process is under statistical control or not and also compute the revised control limits, if necessary.

Therefore, the problem for this session is to construct the control chart for number of defects per unit, i.e., the u-chart for the data given in Table 1.

10.3 PROCEDURE FOR THE CONSTRUCTION OF u-CHART

You have learnt how to construct the u-chart in Unit 4 of MSTE-001. You know that the control limits are the function of sample size (n_i) in the u-chart. So the control limits vary with sample size. We calculate the control limits separately for each sample as explained in Lab Session 7. Here we list the formulae and procedure for computing control limits.

The u-chart plays the same role for the number of defects as the p-chart for the number of defectives with variable sample size discussed in Session 7. Therefore, we can use two approaches for constructing the u-chart in the same way as the p-chart with variable sample size.

First Approach

According to the first approach, we calculate the control limits for each sample separately as follows:

Step 1: If $c_1, c_2, ..., c_k$ represent the number of defects in the 1st, 2nd, …, kth samples of size $n_1, n_2, ..., n_k$, respectively, the number of defects per unit for each sample is given by

$$u_i = \frac{c_i}{n_i}; \quad (i = 1, 2, ..., k) \qquad ...(1)$$

Step 2: The average number of defects per unit is given by

$$\bar{u} = \frac{\sum_{I=1}^{k} c_i}{\sum_{I=1}^{k} n_i} \qquad ...(2)$$

Step 3: The centre line remains constant for each sample because it is not a function of sample size and is given as

$$\text{Centre line}(CL) = \bar{u} \qquad ...(3)$$

Step 4: The upper and lower control limits for each sample are:

- ✓ Upper control limit $(UCL_i) = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$ ...(4)
- ✓ Lower control limit $(LCL_i) = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}$ ...(5)

Step 5: Interpretation of the u-chart.

<u>Second Approach</u>

According to the second approach, the control limits are calculated using the average sample size. This approach is used only when the variations among the sample sizes are not large and future sample sizes are not expected to differ significantly from the average sample size. Using this approach, we get constant control limits just as in c-chart.

Step 1: The average sample size is given by

$$\bar{n} = \frac{n_1 + n_2 + ... + n_k}{k} = \frac{1}{k}\sum_{i=1}^{k} n_i \qquad ...(6)$$

Step 2: The average number of defects per unit are given by

$$\bar{u} = \frac{\sum_{i=1}^{n} c_i}{k\bar{n}} = \frac{\sum_{i=1}^{n} c_i}{\sum_{i=1}^{n} n_i} \qquad ...(7)$$

Step 3: The centre line, upper and lower control limits are:

- ✓ Centre line$(CL) = \bar{u}$ ...(8)
- ✓ Upper control limit $(UCL) = \bar{u} + 3\sqrt{\frac{\bar{u}}{\bar{n}}}$ ...(9)
- ✓ Lower control limit $(LCL) = \bar{u} - 3\sqrt{\frac{\bar{u}}{\bar{n}}}$ ...(10)

Step 4: Interpretation of the u-chart.

10.4 STEPS INVOLVED IN THE CONSTRUCTION OF u-CHART IN EXCEL 2007

We now give the steps for constructing the u-chart in Excel 2007 for the data given in Sec.10.2.

Step 1: We enter the given data in the MS Excel spreadsheet as shown in Fig.10.1.

	A	B	C
1			
2	Days	Length of Fabric	Number of Defects
3	1	50	7
4	2	40	3
5	3	45	1
6	4	35	3
7	5	45	6
8	6	30	2
9	7	40	4
10	8	35	5
11	9	35	7
12	10	45	3
13	11	40	12
14	12	40	2
15	13	30	4
16	14	45	8
17	15	50	3
18	16	45	7
19	17	35	6
20	18	40	3
21	19	35	8

u Chart

Fig. 10.1: Partial screenshot of the spreadsheet for the given data.

Step 2: We compute the number of defects per metre fabric for each sample in Column D. We type "=C3/B3" in Cell D3 and drag down Cell D3 up to Cell D37 to obtain the number of defects per metre fabric for all 35 samples as shown in Fig. 10.2.

D3 =C3/B3

	A	B	C	D
1				
2	Days	Length of Fabric	Number of Defects	u
3	1	50	7	0.140
4	2	40	3	
5	3	45	1	
6	4	35	3	
7	5	45	6	

DRAG IT DOWN

	A	B	C	D
1				
2	Days	Length of Fabric	Number of Defects	u
3	1	50	7	0.140
4	2	40	3	0.075
5	3	45	1	0.022
6	4	35	3	0.086
7	5	45	6	0.133
8	6	30	2	0.067
9	7	40	4	0.100
10	8	35	5	0.143
11	9	35	7	0.200
12	10	45	3	0.067
13	11	40	12	0.300

Fig. 10.2

Step 3: We calculate the total length of fabric and total number of defects by using "=Sum(B3:B37)" and "=Sum(C3:C37)" function in Cells B38 and C38, respectively. The output is shown in Fig. 10.3.

	A	B	C
38	Total	1450	168

Used **"=Sum(B3:B37)"** function Used **"=Sum(C3:C37)"** function

Fig. 10.3

Step 4: We determine the average number of defects per unit ($\bar{u}$) by typing "=C38/B38" in Cell B39 as shown in Fig. 10.4.

B39 | =C38/B38

	A	B	C
39	$\bar{u}$	0.116	
40			

Fig. 10.4

Step 5: We calculate the values of the centre line, upper and lower control limits from equations (3) to (5) in Columns E, F and G, respectively, by typing

The formula with dollar sign ($) is used for an absolute reference.

i) "=$B$39" in Cell E3 to find the centre line as shown in Fig. 10.5a.

ii) "=E3+3*Sqrt(E3/B3)" in Cell F3 to calculate the upper control limit as shown in Fig. 10.5b.

iii) "=E3-3*Sqrt(E3/B3)" in Cell G3 to compute the lower control limit as shown in Fig. 10.5c.

(a)

E3 | =$B$39

	E	F	G	H
1		Control Limits		
2	Centre Line	UCL	LCL	
3	0.116			
4				

(b)

F3 | =E3+3*SQRT(E3/B3)

	F	G	H	I
1	Control Limits			
2	UCL	LCL		
3	0.260			
4				

(c)

G3 | =E3-3*SQRT(E3/B3)

	G	H	I	J
1				
2	LCL			
3	-0.029			
4				

Fig. 10.5

Step 6: We select Cells E3:G3 and drag them down up to Row 37 to get the control limits corresponding to each sample. The output is shown in Fig. 10.6.

	E	F	G
1		Control Limits	
2	Centre Line	UCL	LCL
3	0.116	0.260	-0.029
4	0.116	0.277	-0.046
5	0.116	0.268	-0.036
6	0.116	0.288	-0.057
7	0.116	0.268	-0.036
8	0.116	0.302	-0.071
9	0.116	0.277	-0.046
10	0.116	0.288	-0.057
11	0.116	0.288	-0.057
12	0.116	0.268	-0.036
13	0.116	0.277	-0.046
14	0.116	0.277	-0.046
15	0.116	0.302	-0.071
16	0.116	0.268	-0.036
17	0.116	0.260	-0.029
18	0.116	0.268	-0.036
19	0.116	0.288	-0.057
20	0.116	0.277	-0.046
21	0.116	0.288	-0.057

u Chart

Fig. 10.6

If the values of lower control limits are negative, we set the lower control limits as zero because negative number of defects is not possible.

Step 7: The values of LCL are negative (Fig. 10.6), which is not possible and, therefore, not acceptable. We set it to the value 0 (zero) and denote the limit by LCL* as shown in Fig. 10.7 to plot the control chart.

	D	E	F	G	H
1		Control Limits			
2	u	Centre Line	UCL	LCL	LCL*
3	0.140	0.116	0.260	-0.029	0
4	0.075	0.116	0.277	-0.046	0
5	0.022	0.116	0.268	-0.036	0
6	0.086	0.116	0.288	-0.057	0
7	0.133	0.116	0.268	-0.036	0
8	0.067	0.116	0.302	-0.071	0
9	0.100	0.116	0.277	-0.046	0
10	0.143	0.116	0.288	-0.057	0
11	0.200	0.116	0.288	-0.057	0
12	0.067	0.116	0.268	-0.036	0
13	0.300	0.116	0.277	-0.046	0
14	0.050	0.116	0.277	-0.046	0
15	0.133	0.116	0.302	-0.071	0
16	0.178	0.116	0.268	-0.036	0
17	0.060	0.116	0.260	-0.029	0
18	0.156	0.116	0.268	-0.036	0
19	0.171	0.116	0.288	-0.057	0
20	0.075	0.116	0.277	-0.046	0
21	0.229	0.116	0.288	-0.057	0

u Chart

Fig. 10.7

Step 8: To plot the u-chart in Excel 2007, we follow the procedure explained in Step 10 of Sec. 1.4, Lab Session 1. It means that we

1. select Cells D3:F37 and H3:H37 by holding ***Ctrl*** key,
2. click on the ***Insert*** tab,
3. select the ***Line*** option, and
4. choose the chart subtype.

We format the chart as explained in Sec. 1.4 of Lab Session 1. The resulting u-chart is shown in Fig. 10.8.

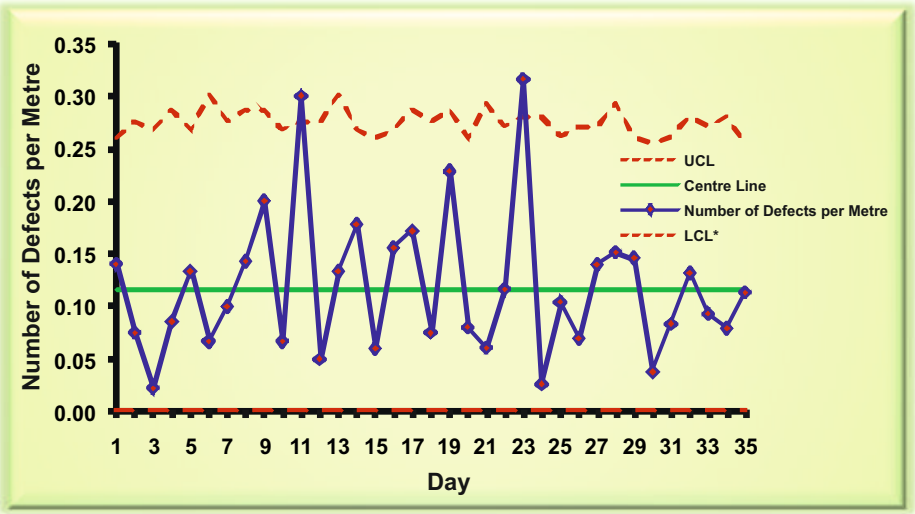


Fig. 10.8

Interpretation

The control chart for number of defects (shown in Fig. 10.8) indicates that the process is not under statistical control. Some assignable causes are present in the process. The samples corresponding to the 11th and 23rd days are outside the upper control limit. To bring the process under statistical control, it is necessary to investigate the assignable causes and take corrective action to eliminate them. We now remove the out-of-control samples for the 11th and 23rd days and calculate the revised centre line and control limits for the u-chart using the remaining samples. These limits are known as the revised control limits and are discussed in Sec.10.5. But before studying Sec. 10.5, you should practice how to construct the u-chart using second approach.

Activity 1

- ✓ Compute the centre line and control limits in Excel 2007 using the second approach which you have learnt in Sec. 4.4 of MSTE-001.
- ✓ Plot the u-chart and compare the result with the output obtained in Sec. 10.4. If any point goes out of control, compute the revised control limits in the same manner as discussed in Sec. 10.5.

10.5 REVISED u-CHART

We plot the u-chart for the revised control limits using the remaining samples. We repeat the procedure until all points come within UCL and LCL. The main steps for computing the revised control limits are as follows:

First Approach

Step 1: For the revised limits for u-chart, we calculate the new $\bar{u}$ as follows:

$$\bar{u}_{new} = \frac{\sum_{i=1}^{k} c_i - \sum_{j=1}^{d} c_j}{\sum_{i=1}^{k} n_i - \sum_{j=1}^{d} n_j} \quad \ldots(10)$$

where d – number of discarded samples, and

$\sum_{j=1}^{d} c_j$ – total number of defects within the discarded sample.

Step 2: Then we reconstruct the centre line and control limits for the u-chart by replacing $\bar{u}$ by $\bar{u}_{new}$ as follows:

✓ Centre line (CL) = $\bar{u}_{new}$ ...(11)

✓ Upper control limit (UCL_i) $= \bar{u}_{new} + 3\sqrt{\frac{\bar{u}_{new}}{n_i}}$ ...(12)

✓ Lower control limit (LCL_i) $= \bar{u}_{new} - 3\sqrt{\frac{\bar{u}_{new}}{n_i}}$ ...(13)

Step 3: Interpretation of the u-chart.

Second Approach

Step 1: For the revised limits for u-chart, the value of new $\bar{u}$ will be same as equation (10).

Step 2: Then we reconstruct the centre line and control limits for the u-chart by replacing $\bar{u}$ by $\bar{u}_{new}$ as follows:

✓ Centre line (CL) = $\bar{u}_{new}$ ...(14)

✓ Upper control limit (UCL) $= \bar{u}_{new} + 3\sqrt{\frac{\bar{u}_{new}}{\bar{n}}}$ ...(15)

✓ Lower control limit (LCL) $= \bar{u}_{new} - 3\sqrt{\frac{\bar{u}_{new}}{\bar{n}}}$ ...(16)

Step 3: Interpretation of the u-chart.

Steps in Excel

The revised centre line and control limits for the u-chart in Excel 2007 using the remaining samples are as follows:

Step 1: We observe from Fig. 10.8 that the samples corresponding to the 11th and 23rd days lie outside the upper control limit. So we highlight the samples which lie outside the control limits, i.e., the 11th and 23rd samples, with light orange colour as shown in Fig. 10.9.

	A	B	C	D	E	F	G	H
1					Control Limits			
2	Days	Length of Fabric	Number of Defects	u	Centre Line	UCL	LCL	LCL*
3	1	50	7	0.140	0.116	0.260	-0.029	0
4	2	40	3	0.075	0.116	0.277	-0.046	0
5	3	45	1	0.022	0.116	0.268	-0.036	0
6	4	35	3	0.086	0.116	0.288	-0.057	0
7	5	45	6	0.133	0.116	0.268	-0.036	0
8	6	30	2	0.067	0.116	0.302	-0.071	0
9	7	40	4	0.100	0.116	0.277	-0.046	0
10	8	35	5	0.143	0.116	0.288	-0.057	0
11	9	35	7	0.200	0.116	0.288	-0.057	0
12	10	45	3	0.067	0.116	0.268	-0.036	0
13	11	40	12	0.300	0.116	0.277	-0.046	0
14	12	40	2	0.050	0.116	0.277	-0.046	0
15	13	30	4	0.133	0.116	0.302	-0.071	0
16	14	45	8	0.178	0.116	0.268	-0.036	0
			⋮			⋮		
	A	B	C	D	E	F	G	H
23	21	33	2	0.061	0.116	0.294	-0.062	0
24	22	43	5	0.116	0.116	0.272	-0.040	0
25	23	38	12	0.316	0.116	0.282	-0.050	0
26	24	38	1	0.026	0.116	0.282	-0.050	0
27	25	48	5	0.104	0.116	0.263	-0.032	0

Fig. 10.9

Step 2: For the revised limits of u-chart, we calculate the new $\bar{u}$ by typing "=(C38-C13-C25)/(B38-B13-B25)" in Cell B40 as shown in Fig. 10.10. We also put d = 2 in Cell B41.

B40 f_x =(C38-C13-C25)/(B38-B13-B25)

	A	B	C	D
38	Total	1450	168	
39	$\bar{u}$	0.116		
40	$\bar{u}_{new}$	0.105		
41	d	2		

Fig. 10.10

Step 3: We reconstruct the centre line and control limits by replacing $\bar{u}$ by $\bar{u}_{new}$. We also replace the negative LCL by zero (0) and denote the limit by LCL* as shown in Fig. 10.11.

	I	J	K	L
1	Revised Control Limits			
2	Centre Line	UCL	LCL	LCL*
3	0.105	0.242	-0.032	0
4	0.105	0.259	-0.049	0
5	0.105	0.250	-0.040	0
6	0.105	0.269	-0.059	0
7	0.105	0.250	-0.040	0
8	0.105	0.282	-0.072	0
9	0.105	0.259	-0.049	0
10	0.105	0.269	-0.059	0
11	0.105	0.269	-0.059	0
12	0.105	0.250	-0.040	0
13	0.105	0.259	-0.049	0
14	0.105	0.259	-0.049	0

u Chart

Fig. 10.11

Step 4: Finally, we plot the defects per metre of the fabric, centre line, UCL and LCL* as discussed in Step 8 of Sec. 10.4. We also format the chart as explained in Sec.1.4 of Lab Session 1. The resulting chart is shown in Fig. 10.12.

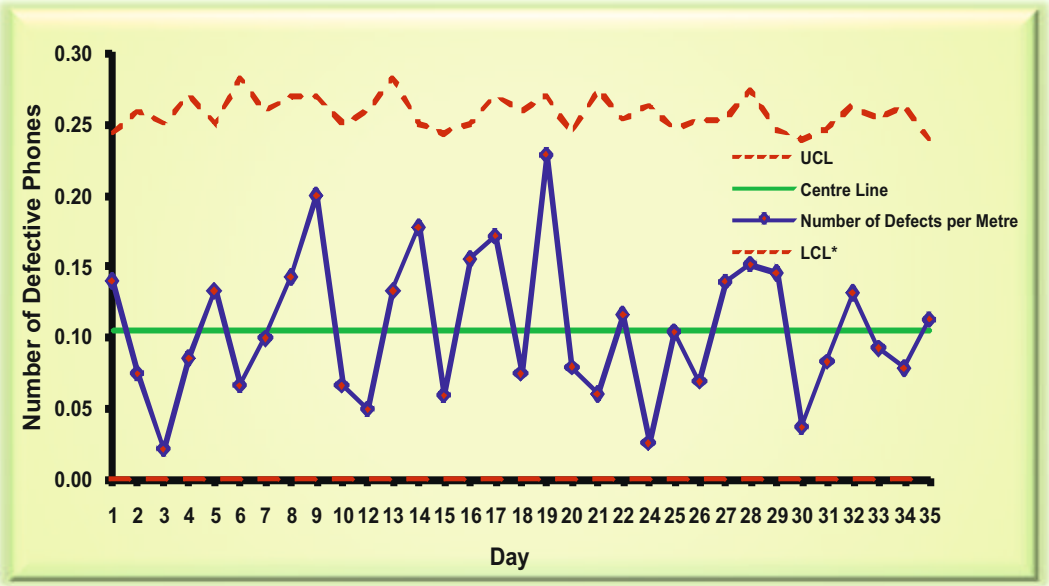


Fig. 10.12

Interpretation

The u-chart shown in Fig. 10.12 reveals that no point lies outside the control limits. So we may conclude that the process is under statistical control with respect to defects per metre fabric.

You should now apply the method on other problems for practice.

Activity 2

Construct the control charts for the number of defects per unit with the help of MS Excel 2007 and interpret the results for

A1) Example 4 given in Unit 4 of MSTE-001.

A2) Exercise E6 given in Unit 4 of MSTE-001.

Match the results with the manual calculation done in Unit 4 of MSTE-001.

Continuous Assessment 10

Suppose an almirah manufacturing company wants to judge the quality of almirah produced. The number of scratch marks on an almirah was recorded for this purpose. The data of 30 samples are given as follows:

Table 2: Number of scratch marks on almirah

Sample No.	1	2	3	4	5	6	7	8	9	10
Scratch Marks	7	3	1	3	6	2	4	5	7	3
Sample No.	11	12	13	14	15	16	17	18	19	20
Scratch Marks	6	3	14	7	2	5	9	4	7	3
Sample No.	21	22	23	24	25	26	27	28	29	30
Scratch Marks	2	7	6	8	4	10	5	4	6	7

Which chart will be used in this case? Construct the suitable control chart. Also plot the revised control chart, if necessary.

Home Work: Do It Yourself

1) Follow the steps explained in Secs. 10.4 and 10.5 to construct the control chart for the data of Table 1. Use a different format for the control chart. Take its screenshot and keep it in your record book.

2) Develop the spreadsheet for the exercise "Continuous Assessment 10" as explained in this lab session. Take screenshots of the final spreadsheet and the chart.

3) **Do not forget** to keep the screenshots in your record book as these will contribute to your continuous assessment in the Laboratory.