0.1 Einstein summation convention

A vector can be written as a sum of its components.

$v = \sum_i e_i v^i$

The Einstein summation convention is to remove the $\sum_i$ symbols where they are implicit.

For example we would instead write the vector as:

$v = e_i v^i$

0.1.1 Adding vectors

$v + w = (\sum_i e_i v^i) + (\sum_i f_i w^i)$

$v + w = \sum_i (e_i v^i + f_i w^i)$

$v + w = e_i v^i + f_i w^i$

If the bases are the same then:

$v + w = e_i (v^i + w^i)$

0.1.2 Scalar multiplication

$cv = c \sum_i e_i v^i$

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0.1.3 Matrix multiplication

$AB_{ik} = \sum_j A_{ij} B_{jk}$

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0.1.4 Inner products

$\langle v, w \rangle = \langle \sum_i e_i v^i, \sum_j f_j w^j \rangle$

$\langle v, w \rangle = \sum_i v^i \langle e_i, \sum_j f_j w^j \rangle$

$\langle v, w \rangle = \sum_i \sum_j v^i \overline{w^j} \langle e_i, f_j \rangle$

If the two bases are the same then:

$\langle v, w \rangle = \sum_i \sum_j v^i \overline{w^j} \langle e_i, e_j \rangle$

We can define the metric as:

$g_{ij} := \langle e_i, e_j \rangle$

$\langle v, w \rangle = v^i \overline{w^j} g_{ij}$