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1 Vector spaces

1.1 Vector spaces

1.1.1 Vector spaces

A vector space is a group with additional structure.

The operation for each element is shown as addition. So we can say:

$$\forall u, v \in V [u + v \in V]$$

To this we add scalars, from a field F . We write this as multiplication.

$$\forall f \in F \forall v \in V [fv \in V]$$

1.1.1.1 Subspace

A subspace is a subset of V which still acts as a vector space. In practice, this means fewer dimensions.

1.1.2 Span

1.1.2.1 Span function

We can take a subset S of V . We can then make linear combinations of these elements.

This is called the linear span - $span(S)$.

1.1.3 Linear dependence

A collection of vectors in a vector space are linearly dependent if there exist values for α (other than all being 0) such that:

$$\sum_i \alpha_i v_i = 0.$$

If no such values for α exist we say the vectors are linearly independent.

1.1.4 Basis vectors

1.1.4.1 Basis

We can write vectors as combinations of other vectors.

$$v = \sum_i \alpha_i v_i$$

A subset which spans the vector space, and which is also linearly independent, is a basis of the vector space.

For an arbitrary vector of size n , we cannot use less than n elementary vectors. We could use more, but these would be redundant.

If we use n elementary vectors, there is a unique solution of weights of elementary vectors.

If we use more than n elementary vectors, there will be linear dependence, and so there will not be a unique solution.

1.1.5 Dimension function

For a basis S , the dimension of the vector space is $|S|$.

$$\dim(V) = |S|$$

$$S \subset V$$

1.1.5.1 Finite and infinite vector spaces

If $\dim(V)$ is finite, then we say the vector space is finite.

Otherwise, we say the vector space is infinite.

1.1.6 Points, lines and planes

$(1, 0)$ is point, $(x, 2x + 1)$ is a line $(1, x, y)$ is a plane

1.1.7 Parallel lines and planes

1.1.7.1 Parallel lines

If we have two lines:

1.1.7.2 Parallel planes