

## 0.1 Inequality constraints

### 0.1.1 Lagrangians with inequality constraints

We can add constraints to an optimisation problem. These constraints can be equality constraints or inequality constraints. We can write constrained optimisation problem as:

Minimise  $f(x)$  subject to

$$g_i(x) \leq 0 \text{ for } i = 1, \dots, m$$

$$h_i(x) = 0 \text{ for } i = 1, \dots, p$$

We write the Lagrangian as:

$$\mathcal{L}(x, \lambda, \nu) = f(x) + \sum_{i=1}^m \lambda_i g_i(x) + \sum_{i=1}^p \nu_i h_i(x)$$

If we try and solve this like a standard Lagrangian, then all of the inequality constraints will instead be equality constraints.

### 0.1.2 Affinity of the Lagrangian

The Lagrangian function is affine with respect to  $\lambda$  and  $\nu$ .

$$\mathcal{L}(x, \lambda, \nu) = f(x) + \sum_{i=1}^m \lambda_i g_i(x) + \sum_{i=1}^p \nu_i h_i(x)$$

$$\mathcal{L}_{\lambda_i}(x, \lambda, \nu) = g_i(x)$$

$$\mathcal{L}_{\nu_i}(x, \lambda, \nu) = h_i(x)$$

As the partial differential is constant, the partial differential is an affine function.