## Contents

## 0.1 Density of the rationals

## 0.1.1 Rationals are dense in rationals

For any pair of rationals, there is another rational between them:

 $a = \frac{p}{a}$ 

 $b = \frac{m}{n}$ 

Where b > a.

We define a new rational:

 $c = \frac{a+b}{2}$ 

 $c = \frac{pn + qm}{2qn}$ 

This is a rational number.

We can write:

 $a = \frac{2pn}{2qn}$ 

 $b = \frac{2qm}{2qn}$ 

As b > a we know 2qm > 2pn

So: a < c < b