0.1 Summation of natural numbers

0.1.1 Goal

Let's prove that:

$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$

0.1.2 Proof by induction

We use the inference rules Modus Ponens, which says that if X is true, and $X \rightarrow Y$ is true, then Y is true.

0.1.3 True for $n = 0$

We know this is true for $n = 0$:

$0 = \frac{0(0+1)}{2}$

$0 = 0$

0.1.4 If it's true for n, it's true for $n + 1$

We can also prove that if it true for n, it is true for $n + 1$.

$\sum_{i=0}^{n+1} i = \frac{(n+1)(n+2)}{2}$

$(n + 1) + \sum_{i=0}^{n} i = \frac{n^2+3n+2}{2}$

If it is true for n, then:

$(n + 1) + \frac{n(n+1)}{2} = \frac{n^2+3n+2}{2}$

$\frac{n^2+3n+2}{2} = \frac{n^2+3n+2}{2}$

$1 = 1$

0.1.5 Result

So we know that it is true for $n = 0$, and if it is true for n, then it is true for $n + 1$. As a result it is true for all natural numbers.