0.1 Taylor series

$f(x)$ can be estimated at point c by identifying its repeated differentials at point c.

The coefficients of an infinate number of polynomials at point c allow this.

$f(x) = \sum_{i=0}^{\infty} a_i(x-c)^i$

$f'(x) = \sum_{i=1}^{\infty} a_i(x-c)^{i-1}i$

$f''(x) = \sum_{i=2}^{\infty} a_i(x-c)^{i-2}i(i-1)$

$f^j(x) = \sum_{i=j}^{\infty} a_i(x-c)^{i-j}\frac{i!}{(i-j)!}$

For $x = c$ only the first term in the series is non-zero.

$f^j(c) = \sum_{i=j}^{\infty} a_i(c-c)^{i-j}\frac{i!}{(i-j)!}$

$f^j(c) = a_i j!$

So:

$a_j = \frac{f^j(c)}{j!}$

So:

$f(x) = \sum_{i=0}^{\infty}(x-c)^i\frac{f^i(c)}{i!}$