

0.1 Elementary row operations

Some operations to a matrix can be reversed to arrive at the original matrix. Trivially, multiplying by the identity matrix is reversible.

Similarly, some operations are not reversible. Such as multiplying by the empty matrix.

All matrix operations which can be reversed are combinations of 3 elementary row operations. These are: Swapping rows

$$T_{12} = \begin{bmatrix} 0 & 1 & \dots & 0 \\ 1 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

Multiplying rows by a vector

$$D_2(m) = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & m & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

Adding rows to other rows

$$L_{12}(m) = \begin{bmatrix} 1 & 0 & \dots & 0 \\ m & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$