Contents

1 Riemann manifolds

1.1 Introduction

1.1.1 Metric tensors

A metric tensor assigns a bilinear form to each point on the manifold.

We can then take two vectors in the tangent space and return a scalar.

1.1.2 Riemann manifolds and pseudo-Riemann manifolds

1.1.2.1 Riemann manifolds

Metric is positive definite.

1.1.2.2 Pseudo-Riemann manifolds

The metric isn't necessarily positive definite.

1.2 Connections on Riemann manifolds

1.2.1 Metric compatibility

If we have two vectors in the tangent space of a manifold with a metric tensor, we can get a scalar:

$v^i u^j g_{ij}$

1.2.1.1 Transported metric

If we transport two vectors along a connection, we have the metric at the new point.

1.2.1.2 Metric preserving connections

If the connection preserves the metric, then the connection is metric compatible.

1.2.2 Torsion tensor

1.2.3 The Levi-Civita connection

For any metric tensor there is only one connection which preserves the metric and is torsion free.

1.3 Sort

1.3.1 The circle as a topology

1.3.2 Cylinders

1.3.3 Embeddings and immersions

1.3.4 Conformal maps

1.3.5 Geodesics

How do we have straight line on a curve? eg going round equator, but not going via uk.

Take start direction and find tangent vectors. geodesic is where tangent vectors stay parallel.

1.3.6 Curvature tensor

1.3.7 Ricci curvature