

## 0.1 Binary functions

### 0.1.1 Properties of binary functions

Binary functions can be written as:

$$f(a, b) = a \oplus b$$

A function is commutative if:

$$x \oplus y = y \oplus x$$

A function is associative if:

$$(x \oplus y) \oplus z = x \oplus (y \oplus z)$$

A function  $\otimes$  is left distributive over  $\oplus$  if:

$$x \otimes (y \oplus z) = (x \otimes y) \oplus (x \otimes z)$$

Alternatively, function  $\otimes$  is right distributive over  $\oplus$  if:  $(x \oplus y) \otimes z = (x \otimes z) \oplus (y \otimes z)$

A function is distributive over another function if it both left and right distributive over it.