

0.1 Writing first-order logic

0.1.1 Existential quantifier

We introduce a shorthand for “at least one term satisfies a predicate”, that is:

$$P(x_0) \vee P(x_1) \vee P(x_2) \vee P(x_2) \vee P(x_3) \dots$$

The short hand is:

$$\exists x P(x)$$

0.1.2 universal quantifier

We introduce another shorthand, this time for:

$$P(x_0) \wedge P(x_1) \wedge P(x_2) \wedge P(x_2) \wedge P(x_3) \dots$$

The shorthand is

$$\forall x P(x)$$

0.1.3 Free and bound variables

A bound variable is one which is quantified in the formula. A free variable is one which is not. Consider:

$$\forall x P(x, y)$$

In this, x is bound while y is free.

Free variables can be interpreted differently, while bound variables cannot.

We can also bind a specific variable to a value. For example 0 can be defined to be bound.

0.1.4 Ground terms

A ground term does not contain any free variables. A ground formula is one which only includes ground terms.

$\forall x x$ is a ground term.

$\forall x P(x)$ is a ground formula.