0.1 Partial fraction decomposition

We have: $\frac{1}{A.B}$

We want this in the form of:

$\frac{a}{A} + \frac{b}{B}$

First, lets define M as the mean of these two numbers, and define $\delta = M - B$. Then:

$\frac{1}{AB} = \frac{1}{(M+\delta)(M-\delta)} = \frac{a}{M+\delta} + \frac{b}{M-\delta}$

We can rearrange the latter two to find:

$1 = a(M - \delta) + b(M + \delta)$

Now we need to find values of a and b to choose.

Let's examine a.

$a = \frac{1-b(M+\delta)}{M-\delta}$

$a = -\frac{bM+b\delta-1}{M-\delta}$

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For this to divide neatly we need both the numerator to be a constant multiplier of the denominator. This means the ratio the multiplier for the left hand side of the denominator is equal to the right:

$\frac{bM}{M} = \frac{b\delta-1}{-\delta}$

$b = \frac{b\delta-1}{-\delta}$

$b = \frac{1}{2\delta}$

We can do the same for a.

$a = -\frac{1}{2\delta}$

We can plug these back into our original formula:

$\frac{1}{(M+\delta)(M-\delta)} = \frac{-\frac{1}{2\delta}}{M+\delta} + \frac{\frac{1}{2\delta}}{M-\delta}$

$\frac{1}{(M+\delta)(M-\delta)} = \frac{1}{2\delta}[\frac{1}{M-\delta} - \frac{1}{M+\delta}]$