

0.1 Defining groups

0.1.1 Magma

A magma, or groupoid, is a set with a single binary operation.

These can be defined as an ordered pair (s, \odot) where s is the set, and \odot is the binary operation.

If a and b are in s , then $a \odot b$ is also in s .

The following are magmas:

Natural numbers and addition $n \times n$ matrices with determinants other than 0
Natural numbers above 0 and addition Integers and addition Rational numbers
and division $\{-1, 1\}$ and multiplication

The following are not magmas:

- Natural numbers up to 10 and addition

0.1.2 Semigroup

A semigroup is a magma whose binary operation is associative.

The following are semigroups:

Natural numbers and addition $n \times n$ matrices with determinants other than 0
Natural numbers above 0 and addition Integers and addition

The following are not semigroups:

- $\{-1, 1\}$ and multiplication
- Rational numbers and division
- Natural numbers up to 10 and addition

0.1.3 Monoid

A monoid is a semigroup with an identity element

The following are monoids:

- Natural numbers and addition
- $n \times n$ matrices with determinants other than 0
- Integers and addition
- $\{-1, 1\}$ and multiplication

The following are not monoids:

- Natural numbers above 0 and addition
- Rational numbers and division
- Natural numbers up to 10 and addition

0.1.4 Group

A group is a monoid where there is an inverse operation for the binary operation.

The following are groups:

Integers and addition $n \times n$ matrices with determinants other than 0 $\{-1, 1\}$ and multiplication

The following are not groups:

Natural numbers above 0 and addition Rational numbers and division Natural numbers and addition Natural numbers up to 10 and addition