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0.1 Single equality constraint

0.1.1 Constrained optimisation

Rather than maximise $f(x)$, we want to maximise $f(x)$ subject to $g(x) = 0$.

We write this, the Lagrangian, as:

$$\mathcal{L}(x, \lambda) = f(x) - \sum_k^m \lambda_k [g_k(x) - c_k]$$

We examine the stationary points for both vector x and λ . By including the latter we ensure that these points are consistent with the constraints.

0.1.2 Solving the Langrangian with one constraint

Our function is:

$$\mathcal{L}(x, \lambda) = f(x) - \lambda[g(x) - c]$$

The first-order conditions are:

$$\mathcal{L}_\lambda = -[g(x) - c]$$

$$\mathcal{L}_{x_i} = \frac{\delta f}{\delta x_i} - \lambda \frac{\delta g}{\delta x_i}$$

The solution is stationary so:

$$\mathcal{L}_{x_i} = \frac{\delta f}{\delta x_i} - \lambda \frac{\delta g}{\delta x_i} = 0$$

$$\lambda \frac{\delta g}{\delta x_i} = \frac{\delta f}{\delta x_i}$$

$$\lambda = \frac{\frac{\delta f}{\delta x_i}}{\frac{\delta g}{\delta x_i}}$$

Finally, we can use the following in practical applications.

$$\frac{\frac{\delta f}{\delta x_i}}{\frac{\delta g}{\delta x_i}} = \frac{\frac{\delta f}{\delta x_j}}{\frac{\delta g}{\delta x_j}}$$