0.1 Tensor product

We have spaces V and W over field F. If we have a linear operation which takes a vector from each space and returns a scalar from the underlying field, it is an element of the tensor product of the two spaces.

For example if we have two vectors:

$v = e_i v^i$

$w = e_j w^j$

A tensor product would take these and return a scalar.

There are three types of tensor products:

- Both are from the vector space
- $T_{ij} v^i w^j$
- $T_{ij} \in V \otimes W$
- Both are from the dual space
- $T^{ij} v_i w_j$
- $T_{ij} \in V^* \otimes W^*$
- One is from each space
- $T_i^j v^i w_j$
- $T_{ij} \in V \otimes W^*$

As a vector space, we can add together tensor products, and do scalar multiplication.

0.1.1 Homomorphisms

We can define homomorphisms in terms of tensor products.

$Hom(V) = V \otimes V^*$

T_j^i

We use the dual space for the second argument. This is because it ensures that changes to the bases do not affect the maps.

$w^j = T_i^j v^i$