0.1 Set union and intersection

We discuss functions. Just because we can write a function of sets which exist, does not mean the results of the functions exist. For that we need axioms discussed later.

0.1.1 Union function

We define a function on two sets, $a \vee b$, such that the result contains all elements from either sets.

$\forall a \forall x \forall y [a \in (x \vee y) \leftrightarrow (a \in x \vee a \in y)]$

This is commutative: $a \vee b = b \vee a$

This is associative: $(a \vee b) \vee c = a \vee (b \vee c)$

0.1.2 Intersection function

We define a function, $a \wedge b$, on two sets, such that the result contains all elements which are in both.

$\forall a \forall x \forall y [a \in (x \wedge y) \leftrightarrow (a \in x \wedge a \in y)]$

This is commutative: $a \wedge b = b \wedge a$

This is associative: $(a \wedge b) \wedge c = a \vee (b \wedge c)$

0.1.3 Distribution of union and intersection

Union is distributive over intersection: $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

Intersection is distributive over union: $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$