0.1 Hermitian (self-adjoint) matrices

A matrix where $M = M^*$

For matrices over the real numbers, these are the same as symmetric matrices.

0.1.1 Sesqulinear forms on Hermitian matrices

$\phi(u, v) = u^* M v$

$(u^* M v)^* = v^* M^* u = v^* M u$

$\phi(u, v) = \overline{\phi(v, u)}$

0.1.2 The forms on the same vector are always real

$(v^* M v)^* = v^* M^* v = v^* M v$

So we have:

$(v^* M v)^* = v^* M v$

Which is only satisfied for reals.

0.1.3 If A and B are Hermitian

If A and B are Hermitian, AB is Hermitian if and only if AB commutes.

$(AB)^* = B^* A^* = BA$

If it commutes then

$(AB)^* = AB$

0.1.4 Real eigenvalues

Hermitian matrices have real eigenvalues.

$Hv = \lambda v$

$v^* H v = \lambda v^* v$

$v^* H v = \lambda$

0.1.5 Skew-Hermitian matrices

These are also known as anti-Hermitian matrices.

$M^* = -M$

0.1.6 If eigenvalues are different, eigenvectors are orthogonal