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0.1 Defining e as a binomial

0.1.1 Lemma

$$f(n, i) = \frac{n!}{n^i(n-i)!}$$

$$f(n, i) = \frac{(n-i)! \prod_{j=n-i+1}^n j}{n^i(n-i)!}$$

$$f(n, i) = \frac{\prod_{j=n-i+1}^n j}{n^i}$$

$$f(n, i) = \frac{\prod_{j=1}^i (j+n-i)}{n^i}$$

$$f(n, i) = \prod_{j=1}^i \frac{j+n-i}{n}$$

$$f(n, i) = \prod_{j=1}^i \left(\frac{n}{n} + \frac{j-i}{n} \right)$$

$$f(n, i) = \prod_{j=1}^i \left(1 + \frac{j-i}{n} \right)$$

$$\lim_{n \rightarrow \infty} f(n, i) = \lim_{n \rightarrow \infty} \prod_{j=1}^i \left(1 + \frac{j-i}{n} \right)$$

$$\lim_{n \rightarrow \infty} f(n, i) = \prod_{j=1}^i 1$$

$$\lim_{n \rightarrow \infty} f(n, i) = 1$$

0.1.2 Defining e

We know that:

$$(a+b)^n = \sum_{i=0}^n a^i b^{n-i} \frac{n!}{i!(n-i)!}$$

Let's set $b = 1$

$$(a+1)^n = \sum_{i=0}^n a^i \frac{n!}{i!(n-i)!}$$

Let's set $a = \frac{1}{n}$

$$\left(1 + \frac{1}{n}\right)^n = \sum_{i=0}^n \frac{1}{n^i} \frac{n!}{i!(n-i)!}$$

$$\left(1 + \frac{1}{n}\right)^n = \sum_{i=0}^n \frac{1}{i!} \frac{n!}{n^i(n-i)!}$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \lim_{n \rightarrow \infty} \sum_{i=0}^n \frac{1}{i!} \frac{n!}{n^i(n-i)!}$$

From the lemma above:

$$\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = \sum_{i=0}^{\infty} \frac{1}{i!}$$

$$e = \sum_{i=0}^{\infty} \frac{1}{i!}$$

0.1.3 Defining e^x

$$e = \lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n$$

$$e^x = \lim_{n \rightarrow \infty} (1 + \frac{1}{n})^{nx}$$

$$e^x = \lim_{n \rightarrow \infty} \sum_{i=0}^{nx} \frac{1}{n^i} \frac{(nx)!}{i!(nx-i)!}$$

$$e^x = \lim_{n \rightarrow \infty} \sum_{i=0}^{nx} \frac{x^i}{i!} \frac{(nx)!}{(nx)^i (nx-i)!}$$

From the lemma:

$$e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$$