

0.1 Pauli matrices

Pauli matrices are 2×2 matrices which are unitary and hermitian.

That is, $P^* = P^{-1}$.

And $P^* = P$.

0.1.1 The Pauli matrices

The matrices are:

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The identity matrix is often considered alongside these as:

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

0.1.2 Pauli matrices are their own inverse

$$\sigma_i^2 = \sigma_i \sigma_i$$

$$\sigma_i^2 = \sigma_i \sigma_i^*$$

$$\sigma_i^2 = \sigma_i \sigma_i^{-1}$$

$$\sigma_i^2 = I$$

0.1.3 Determinants and trace of Pauli matrices

$$\det \sigma_i = -1$$

$$\text{Tr}(\sigma_i) = 0$$

As the sum of eigenvalues is the trace, and the product is the determinant, the eigenvalues are 1 and -1 .