0.1 Membership relation

Say we have a preterite $P(x)$ which is true for some values of x. Sets allow us to explore the properties of these values.

We may want to talk about a collection of terms for which $P(x)$ is true, which we call a set.

To do this we need to introduce new axioms, however first we can add (conservative) definitions to help us do this.

We introduce a new relation: membership. If element x is in set s then the following relation is true, otherwise it is false:

$x \in s$

Sets are also terms. In first-order logic they will be included in quantifiers. Indeed, in set theory, we aim to treat everything as sets.

If a term is not a member of another term, we can write this using the non-member relation as follows:

$\forall x \forall s[\neg(x \in s) \leftrightarrow x \notin s]$