0.1 Ordered sets

0.1.1 Totally ordered sets

A totally ordered set is one where the relation is defined on all pairs:

$\forall a \forall b (a \leq b) \vee (b \leq a)$

Note that totality implies reflexivity.

0.1.2 Partially ordered sets (poset)

A partially ordered set, or poset, is one where the relation is defined between each element and itself.

$\forall a (a \leq a)$

That is, every element is related to itself.

These are also called posets.

0.1.3 Well-ordering

A well-ordering on a set is a total order on the set where the set contains a minimum number. For example the relation $\leq$ on the natural numbers is a well-ordering because 0 is the minimum.

The relation $\leq$ on the integers however is not a well-ordering, as there is no minimum number in the set.