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0.1 Axiom of infinity

The axiom of infinity states that:

$$\exists I(\emptyset \in I \wedge \forall x \in I((x \vee \{x\}) \in I))$$

There exists a set, called the infinite set. This contains the empty set, and for all elements in I the set also contains the successor to it.

0.1.1 Sequential function

Let's define the sequential function:

$$s(n) := \{n \vee \{n\}\}$$

We can now rewrite the axiom of infinity as:

$$\exists \mathbb{N}(\emptyset \in \mathbb{N} \wedge \forall x \in \mathbb{N}(s(x) \in \mathbb{N}))$$

0.1.2 Zero

This set contains the null set: $\emptyset \in \mathbb{N}$.

Zero is defined as the empty set.

$$0 := \{\}$$

0.1.3 Natural numbers

For all elements in the infinite set, there also exists another element in the infinite set: $\forall x \in \mathbb{N}((x \vee \{x\}) \in \mathbb{N})$.

We then define all sequential numbers as the set of all preceding numbers. So:

$$1 := \{0\} = \{\{\}\}$$

$$2 := \{0, 1\} = \{\{\}, \{\{\}\}\}$$

$$3 := \{0, 1, 2\} = \{\{\}, \{\{\}\}, \{\{\}, \{\{\}\}\}\}$$

0.1.4 Existence of natural numbers

Does each natural number exist? We know the infinite set exists, and we also know the axiom schema of specification:

Point is: For each set, all finite subsets exist. PROVE ELSEWHERE

0.1.5 From infinite set to natural set

We don't know I is limited to natural numbers. Could contain urelements etc.

0.1.6 More

Infinite set axiom written using N. should be I

I could be superset of N, for example set of all natural numbers, and also the set containing the set containing 2.

Can extract N using axiom of specification

We need a way to define the set of natural numbers:

$$\forall n(n \in \mathbb{N} \leftrightarrow ([n = \emptyset \vee \exists k(n = k \vee \{k\})] \wedge))$$

If we can define N, we can show it exists from specification

$$\forall x \exists s [P(x) \leftrightarrow (x \in s)]$$

$$\forall n \exists s [n \in N \leftrightarrow (n \in s)]$$