

## **0.1 The infinite cyclic group ( $Z$ )**

### **0.1.1 The additive group of integers**

### **0.1.2 Generating cyclic groups**

We can generate a group with a single element, it is a cyclic group.

For example, we can define a group  $G = \langle 1 \rangle$  which gives us the additive group of integers.

### **0.1.3 Infinite cyclic groups are isomorphic to the additive group of integers**

More generally, any infinite cyclic group is isomorphic to the additive group of integers.

Consider the multiplicative group of  $\langle i \rangle$ .

This contains  $\{1, -1, i, -i\}$ .

This is also isomorphic to the natural number and modulo addition group above.

We can define finite cyclic groups of size  $n$  using the generating element  $z^{\frac{1}{n}}$ . This is isomorphic to the general cyclic group  $C_n$ , and to  $Z/nZ$ .

### **0.1.4 Abelian cyclic groups**

Cyclic groups are abelian.