

# Contents

0.1	Powers . . . . .	1
0.1.1	Exponents and logarithms . . . . .	1
0.1.2	Powers . . . . .	1
0.1.3	Axioms . . . . .	1
0.1.4	Example . . . . .	2
0.1.5	Exponential rules . . . . .	2
0.1.6	Powers of natural numbers . . . . .	2
0.1.7	Powers of integers . . . . .	2
0.1.8	Powers of rational numbers . . . . .	2

## 0.1 Powers

### 0.1.1 Exponents and logarithms

Previously we defined addition and multiplication in terms of successive use of the sucessor function. That is, the definition of addition was:

$$\forall a \in \mathbb{N}(a + 0 = a)$$

$$\forall ab \in \mathbb{N}(a + s(b) = s(a + b))$$

And similarly for multiplication:

$$\forall a \in \mathbb{N}(a.0 = 0)$$

$$\forall ab \in \mathbb{N}(a.s(b) = a.b + a)$$

Additional functions could also be defined, following the same pattern:

$$\forall a \in \mathbb{N}(a \oplus_n 0 = c)$$

$$\forall ab \in \mathbb{N}(a \oplus_n s(b) = (a \oplus_n b) \oplus_{n-1} a)$$

### 0.1.2 Powers

Exponents can also be defined:

### 0.1.3 Axioms

$$\forall a \in \mathbb{N}a^0 = 1$$

$$\forall ab \in \mathbb{N}a^{s(b)} = a^b.a$$

#### 0.1.4 Example

So  $2^2$  can be calculated like:

$$2^2 = 2^{s(1)}$$

$$2^{s(1)} = 2.2^1$$

$$2.2^1 = 2.2.2^0$$

$$2.2.2^0 = 2.2.1$$

$$2.2.1 = 4$$

Unlike addition and multiplication, exponentiation is not commutative. That is

$$a^b \neq b^a$$

#### 0.1.5 Exponential rules

$$a^b a^c = a^{b+c}$$

$$(a^b)^c = a^{bc}$$

$$(ab)^c = a^c b^c$$

#### 0.1.6 Powers of natural numbers

#### 0.1.7 Powers of integers

#### 0.1.8 Powers of rational numbers