

0.1 Singular value decomposition

The singular value decomposition of $m \times n$ matrix M is:

$$M = U\Sigma V^*$$

Where:

- U is a unitary matrix ($m \times m$)
- Σ is a diagonal matrix with non-negative real numbers ($m \times n$)
- V is a unitary matrix ($n \times n$)

Σ is unique. U and V are not.

0.1.1 Properties

$$M^*M = U\Sigma^2U^*$$

$$(M^*M)^{-1} = V\Sigma^{-2}V^*$$

0.1.2 Calculating the SVD

The SVD is generally calculated iteratively.