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0.1 Primal and dual problems

0.1.1 The primal problem

We already have this.

0.1.2 The dual problem

We can define the Lagrangian dual function:

$g(\lambda, \nu) = \inf_{x \in X} \mathcal{L}(x, \lambda, \nu)$

That is, we have a function which chooses the returns the value of the optimised Lagrangian, given the values of λ and ν.

This is an unconstrained function.

We can prove this function is concave (how?).

The infimum of a set of concave (and therefore also affine) functions is concave.

The supremum of a set of convex (and therefore also affine) functions is convex.

Given a function with inputs x, what values of x maximise the function?

We explore constrained and unconstrained optimisation. The former is where restrictions are placed on vector x, such as a budget constraint in economics.

0.1.3 The dual problem is concave

0.1.4 The duality gap

We refer to the optimal solution for the primary problem as p^*, and the optimal solution for the dual problem as d^*.

The duality gap is $p^* - d^*$.