0.1 Differentiating exponents, logarithms and power functions

0.1.1 Differentiating the natural logarithm

$\frac{\delta}{\delta x}\ln(x) = \lim_{\delta\to 0}\frac{\ln(x+\delta)-ln(x)}{\delta}$

$\frac{\delta}{\delta x}\ln(x) = \lim_{\delta\to 0}\frac{\ln\frac{x+\delta}{x}}{\delta}$

$\frac{\delta}{\delta x}\ln(x) = \lim_{\delta\to 0}\frac{\ln(1+\frac{\delta}{x})}{\delta}$

$\frac{\delta}{\delta x}\ln(x) = \frac{1}{x}\lim_{\delta\to 0}\frac{x}{\delta}\ln(1+\frac{\delta}{x})$

$\frac{\delta}{\delta x}\ln(x) = \frac{1}{x}\ln(\lim_{\delta\to 0}(1+\frac{\delta}{x})^{\frac{x}{\delta}})$

$\frac{\delta}{\delta x}\ln(x) = \frac{1}{x}\ln(e)$

$\frac{\delta}{\delta x}\ln(x) = \frac{1}{x}$

0.1.2 Differentiating logarithms of other bases

$log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

$log_a(x) = \frac{\ln(x)}{\ln(a)}$

$\frac{\delta}{\delta x}\log_a(x) = \frac{\delta}{\delta x}\frac{\ln(x)}{ln(a)}$

$\frac{\delta}{\delta x}\log_a(x) = \frac{1}{xln(a)}$

0.1.3 Exponents

$y = a^x$

$\ln(y) = x\ln(a)$

$\frac{\delta}{\delta x}\ln(y) = \frac{\delta}{\delta x}x\ln(a)$

$\frac{\delta}{\delta x}\ln(y) = \ln(a)$

$\frac{1}{y}\frac{\delta}{\delta x}y = \ln(a)$

$\frac{\delta}{\delta x}a^x = a^x\ln(a)$

0.1.4 Power functions

$y = x^n$

$\frac{\delta}{\delta x}y = \frac{\delta}{\delta x}x^n$

$$\frac{\delta}{\delta x}y = \frac{\delta}{\delta x}e^{n\ln(x)}$$
$$\frac{\delta}{\delta x}y = \frac{n}{x}e^{n\ln(x)}$$
$$\frac{\delta}{\delta x}y = nx^{n-1}$$