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0.1 Total differentiation

0.1.1 Scalar fields

A scalar field is a function on an underlying input which produces a real output.

Inputs are not limited to real numbers. In this section we consider functions on vector spaces.

0.1.2 Total differentiation

Consider a multivariate function.

$f(x)$.

We can define:

$$\Delta f(x, \Delta x) := f(x + \Delta x) - f(x)$$

$$\Delta f(x, \Delta x) = \sum_{i=1}^n f(x + \Delta x_i + \sum_{j=0}^{i-1} \Delta x_j) - f(x + \sum_{j=0}^{i-1} \Delta x_j)$$

$$\Delta f(x, \Delta x) = \sum_{i=1}^n \Delta x_i \frac{f(x + \Delta x_i + \sum_{j=0}^{i-1} \Delta x_j) - f(x + \sum_{j=0}^{i-1} \Delta x_j)}{\Delta x_i}$$

$$\frac{\Delta f}{\Delta x_k} = \sum_{i=1}^n \frac{\Delta x_i}{\Delta x_k} \frac{f(x + \Delta x_i + \sum_{j=0}^{i-1} \Delta x_j) - f(x + \sum_{j=0}^{i-1} \Delta x_j)}{\Delta x_i}$$

$$\lim_{\Delta x_k \rightarrow 0} \frac{\Delta f}{\Delta x_k} = \sum_{i=1}^n \lim_{\Delta x_k \rightarrow 0} \frac{\Delta x_i}{\Delta x_k} \frac{f(x + \Delta x_i + \sum_{j=0}^{i-1} \Delta x_j) - f(x + \sum_{j=0}^{i-1} \Delta x_j)}{\Delta x_i}$$

$$\frac{df}{dx_k} = \sum_{i=1}^n \frac{dx_i}{dx_k} \frac{\delta f}{\delta x_i}$$

0.1.3 Total differentiation of a univariate function

For a univariate function total differentiation is the same as partial differentiation.

$$\frac{df}{dx} = \frac{dx}{dx} \frac{\delta f}{\delta x}$$

$$\frac{df}{dx} = \frac{\delta f}{\delta x}$$