0.1 Riemann integral

0.1.1 Riemann sums

Given a function $f(x)$ and an interval $[a, b]$, we can divide $[a, b]$ into n sections and calculate:

$\sum_{j=0}^{n(b-a)} f(a + \frac{j}{n})$

This is the Riemann sum.

0.1.2 Riemann integral

We take the limit of the Riemann sum as $n \to \infty$

$\int_a^b f(x)dx := \lim_{n\to\infty} \sum_{j=0}^{n(b-a)} f(a + \frac{j}{n})$

0.1.3 Linearity

$\int_a^b f(x) + g(x)dx = \lim_{n\to\infty} \sum_{j=0}^{n(b-a)} f(a + \frac{j}{n}) + g(a + \frac{j}{n})$

$\int_a^b f(x) + g(x)dx = \lim_{n\to\infty} \sum_{j=0}^{n(b-a)} f(a + \frac{j}{n}) + \lim_{n\to\infty} \sum_{j=0}^{n(b-a)} g(a + \frac{j}{n})$

$\int_a^b f(x) + g(x)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

0.1.4 Continuation

$\int_a^b f(x)dx + \int_b^c f(x)dx = \lim_{n\to\infty} \sum_{j=0}^{n(b-a)} f(a + \frac{j}{n}) + \lim_{n\to\infty} \sum_{j=0}^{n(c-b)} f(b + \frac{j}{n})$

$\int_a^b f(x)dx + \int_b^c f(x)dx = \lim_{n\to\infty}[\sum_{j=0}^{n(b-a)} f(a + \frac{j}{n}) + \sum_{j=0}^{n(c-b)} f(b + \frac{j}{n})]$

$\int_a^b f(x)dx + \int_b^c f(x)dx = \lim_{n\to\infty}[\sum_{j=0}^{n(b-a)} f(a + \frac{j}{n}) + \sum_{j=n(b-a)}^{n(c-b)+n(b-a)} f(b + \frac{j-n(b-a)}{n})]$

$\int_a^b f(x)dx + \int_b^c f(x)dx = \lim_{n\to\infty}[\sum_{j=0}^{n(b-a)} f(a + \frac{j}{n}) + \sum_{j=n(b-a)}^{n(c-a)} f(a + \frac{j}{n})]$

$\int_a^b f(x)dx + \int_b^c f(x)dx = \lim_{n\to\infty}[\sum_{j=0}^{n(c-a)} f(a + \frac{j}{n})$

$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$