

0.1 Translation symmetry

The distance between two vectors is:

$$(v - w)^T M (v - w)$$

So what operations can we do now?

As before, we can do the transformations which preserve $u^T M v$, such as the orthogonal group.

But we can also do other translations

$$(v-w) \text{TM} (v-w)$$

$$v^{\wedge} \text{TM} v + w^{\wedge} \text{TM} w - v^{\wedge} \text{TM} w - w^{\wedge} \text{TM} v$$

so symmetry is now $O(3,1)$ and affine translations

0.1.1 Translation matrix

$\begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix}$ moves vector by x