0.1 Morphism

Morphisms are functions which preserve the relationships between members of a set, and specified functions.

That is, if:

$a \odot b = c$

Then $f(x)$ is morphism if:

$f(a) \odot f(b) = f(a \odot b)$

Here we discuss morphisms in the context of groups, but we can define morphisms for sets with more than one function, for example with addition and multiplication.

Morphisms are also known as homomorphisms.

The following are morphisms of the additive group of integers.

Where we refer to c, $c \neq 0 \in \mathbb{I}$.

- $f(x) = 0$
- $f(x) = x$
- $f(x) = cx$
- Converting natural numbers to integers

The following are not morphisms

- $f(x) = x + 1$

0.1.1 Isomorphism

An isomorphism is a morphism which has an inverse.

This means the function is bijective.

The following are isomorphisms:

- $f(x) = x$
- $f(x) = cx$
- Converting natural numbers to integers

The following are not isomorphisms

- $f(x) = 0$
- $f(x) = x + 1$

0.1.2 Endomorphism

An endomorphism is one where the domain and codomain are the same.

The following are endomorphisms:

- $f(x) = 0$
- $f(x) = x$
- $f(x) = cx$

The following are not endomorphisms

- Converting natural numbers to integers
- $f(x) = x + 1$

0.1.3 Automorphism

An endomorphism which is also an isomorphism

The following are automorphisms:

- $f(x) = x$
- $f(x) = cx$

The following are not automorphisms

- $f(x) = 0$
- $f(x) = x + 1$
- Converting natural numbers to integers

0.1.4 Monomorphism

A morphism which is injective. That is:

$f(a) = f(b) \to a = b$

The following are monomorphisms:

- $f(x) = x$
- $f(x) = cx$
- Converting natural numbers to integers

The following are not monomorphisms:

- $f(x) = 0$
- $f(x) = x + 1$