1 Exterior algebra

1.1 The exterior (wedge) product

The exterior product of two vectors is:

$u \wedge v$

1.2 The exterior product is anticommutative

This is anticommutative (alternating).

$u \wedge v = -v \wedge u$

This implies that:

$u \wedge u = 0$

1.3 The exterior product is distributive

$(a + b) \wedge (c + d) = (a \wedge c) + (a \wedge d) + (b \wedge c) + (b \wedge d)$

1.4 Expanding the exterior product of two vectors

Consider 2-dimenional vector space with the following vectors:

$u = ae_1 + be_2$

$v = ce_1 + de_2$

The exterior product is:

$u \wedge v = (ae_1 + be_2) \wedge (ce_1 + de_2)$

$u \wedge v = (ae_1 \wedge ce_1) + (ae_1 \wedge de_2) + (be_2 \wedge ce_1) + (be_2 \wedge de_2)$

$u \wedge v = ac(e_1 \wedge e_1) + ad(e_1 \wedge e_2) + bc(e_2 \wedge e_1) + bd(e_2 \wedge e_2)$

$u \wedge v = ad(e_1 \wedge e_2) - bc(e_1 \wedge e_2)$

$u \wedge v = (ad - bc)(e_1 \wedge e_2)$

1.5 Exterior (Grassman) algebra

The exterior algebra is the algebra generated by the wedge product.

The term $u \wedge v$ can be interpreted as the area covered by the parallelogram generated by u and v.

As $a\mathbf{u} \wedge b\mathbf{v} = ab\mathbf{u} \wedge \mathbf{v}$, we can see that scaling the length of one of the vectors by a scalar, we also increase the exterior product by the same scalar.

1.6 Orientation

We can describe the exterior product of two vectors as $\mathbf{u} \wedge \mathbf{v}$ or $\mathbf{v} \wedge \mathbf{u}$.