0.1 Single equality constraint

0.1.1 Constrained optimisation

Rather than maximise $f(x)$, we want to maximise $f(x)$ subject to $g(x) = 0$.

We write this, the Lagrangian, as:

$\mathcal{L}(x, \lambda) = f(x) - \sum_k^m \lambda_k [g_k(x) - c_k]$

We examine the stationary points for both vector x and λ. By including the latter we ensure that these points are consistent with the constraints.

0.1.2 Solving the Langrangian with one constraint

Our function is:

$\mathcal{L}(x, \lambda) = f(x) - \lambda[g(x) - c]$

The first-order conditions are:

$\mathcal{L}_\lambda = -[g(x) - c]$

$\mathcal{L}_{x_i} = \frac{\delta f}{\delta x_i} - \lambda \frac{\delta g}{\delta x_i}$

The solution is stationary so:

$\mathcal{L}_{x_i} = \frac{\delta f}{\delta x_i} - \lambda \frac{\delta g}{\delta x_i} = 0$

$\lambda \frac{\delta g}{\delta x_i} = \frac{\delta f}{\delta x_i}$

$\lambda = \frac{\frac{\delta f}{\delta x_i}}{\frac{\delta g}{\delta x_i}}$

Finally, we can use the following in practical applications.

$\frac{\frac{\delta f}{\delta x_i}}{\frac{\delta g}{\delta x_i}} = \frac{\frac{\delta f}{\delta x_j}}{\frac{\delta g}{\delta x_j}}$