0.1 Lie algebra

Lie groups have symmetries. We can consider only the infintesimal symmetries.

For example the unit circle has many symmetries, but we can consider only those which rotate infintesimally.

0.1.1 Example

Take a continous group, such as $U(1)$. Its Lie algebra is all matrices such that their exponential is in the Lie group.

$\mathfrak{u}(1) = \{X \in \mathbb{C}^{1\times 1} | e^{tX} \in U(1) \forall t \in \mathbb{R}\}$

This is satisfied by the matrices where $M = -M^*$. Note that this means the diagonals are all 0.

0.1.2 Scale of specific Lie algebra matrices doesn't matter

Because of t.

0.1.3 Commutation of Lie group algebra

Consider two members of the Lie algebra: A and B. The commutator is:

A.

The corresponding Lie group member is:

$e^{t(A+B)} = e^{tA}e^{tB}$

While the Lie group multiplication may not commute, the corresponding addition of the Lie algebra does.