0.1 Vector potential

Given a vector field $\mathbf{F}$ we may be able to identify another vector field A such that:

$\mathbf{F} = \nabla \times \mathbf{A}$

0.1.1 Existence

We know that the divergence of the curl for any vector field is 0, so this applies to A:

$\nabla.(\nabla \times \mathbf{A}) = 0$

Therefore:

$\nabla.\mathbf{F} = 0$

This means that if there is a vector potential of $\mathbf{F}$, then $\mathbf{F}$ has no divergence.

0.1.2 Non-uniqueness of vector potentials

Vector potentials are not unique.

If $\mathbf{A}$ is a vector potential of $\mathbf{F}$, then so is $\mathbf{A} + \nabla c$, where c is a scalar field and ∇c is its gradient.

0.1.3 Conservative vector fields

Not all vector fields have scalar potentials. Those that do are conservative.

For example if a vector field is the gradient of a scalar height function, then the height is a scalar potential.

If a vector field is the rotation of water, there will not be a scalar potential.