

## Contents

|       |                           |   |
|-------|---------------------------|---|
| 0.1   | Defining groups . . . . . | 1 |
| 0.1.1 | Magma . . . . .           | 1 |
| 0.1.2 | Semigroup . . . . .       | 1 |
| 0.1.3 | Monoid . . . . .          | 2 |
| 0.1.4 | Group . . . . .           | 2 |

### 0.1 Defining groups

#### 0.1.1 Magma

A magma, or groupoid, is a set with a single binary operation.

These can be defined as an ordered pair  $(s, \odot)$  where  $s$  is the set, and  $\odot$  is the binary operation.

If  $a$  and  $b$  are in  $s$ , then  $a \odot b$  is also in  $s$ .

The following are magmas:

Natural numbers and addition  $n \times n$  matrices with determinants other than 0  
Natural numbers above 0 and addition Integers and addition Rational numbers  
and division  $\{-1, 1\}$  and multiplication

The following are not magmas:

- Natural numbers up to 10 and addition

#### 0.1.2 Semigroup

A semigroup is a magma whose binary operation is associative.

The following are semigroups:

Natural numbers and addition  $n \times n$  matrices with determinants other than 0  
Natural numbers above 0 and addition Integers and addition

The following are not semigroups:

- $\{-1, 1\}$  and multiplication
- Rational numbers and division
- Natural numbers up to 10 and addition

### 0.1.3 Monoid

A monoid is a semigroup with an identity element

The following are monoids:

- Natural numbers and addition
- $n \times n$  matrices with determinants other than 0
- Integers and addition
- $\{-1, 1\}$  and multiplication

The following are not monoids:

- Natural numbers above 0 and addition
- Rational numbers and division
- Natural numbers up to 10 and addition

### 0.1.4 Group

A group is a monoid where there is an inverse operation for the binary operation.

The following are groups:

Integers and addition  $n \times n$  matrices with determinants other than 0  $\{-1, 1\}$  and multiplication

The following are not groups:

Natural numbers above 0 and addition Rational numbers and division Natural numbers and addition Natural numbers up to 10 and addition