

0.1 Concave and convex functions

0.1.1 Convex functions

A convex function is one where:

$$\forall x_1, x_2 \in \mathbb{R} \forall t \in [0, 1] [f(tx_1 + (1-t)x_2) \leq tf(x_1) + (1-t)f(x_2)]$$

That is, for any two points of a function, a line between the two points is above the curve.

A function is strictly convex if the line between two points is strictly above the curve:

$$\forall x_1, x_2 \in \mathbb{R} \forall t \in (0, 1) [f(tx_1 + (1-t)x_2) < tf(x_1) + (1-t)f(x_2)]$$

An example is $y = x^2$.

0.1.2 Concave functions

A concave function is an upside down convex function. The line between two points is below the curve.

$$\forall x_1, x_2 \in \mathbb{R} \forall t \in [0, 1] [f(tx_1 + (1-t)x_2) \geq tf(x_1) + (1-t)f(x_2)]$$

A function is strictly concave if the line between two points is strictly below the curve:

$$\forall x_1, x_2 \in \mathbb{R} \forall t \in (0, 1) [f(tx_1 + (1-t)x_2) > tf(x_1) + (1-t)f(x_2)]$$

An example is $y = -x^2$.

0.1.3 Affine functions

If a function is both concave and convex, then the line between two points must be the function itself. This means the function is an affine function.

$$y = cx$$