1 Vector spaces

1.1 Vector spaces

A vector space is a group with additional structure.

The operation for each element is shown as addition. So we can say:

$\forall u, v \in V[u + v \in V]$

To this we add scalars, from a field F. We write this as multiplication.

$\forall f \in F \forall v \in V[fv \in V]$

1.1.1 Subspace

A subspace is a subset of V which still acts as a vector space. In practice, this means fewer dimensions.

1.2 Span

1.2.1 Span function

We can take a subset S of V. We can then make linear combinations of these elements.

This is called the linear span - $span(S)$.

1.3 Linear dependence

A collection of vectors in a vector space are linearly dependent if there exist values for α (other than all being 0) such that:

$\sum_i \alpha_i v_i = 0$.

If no such values for α exist we say the vectors are linearly independent.

1.4 Basis vectors

1.4.1 Basis

We can write vectors as combinations of other vectors.

$v = \sum_i \alpha_i v_i$

A subset which spans the vector space, and which is also linearly independent, is a basis of the vector space.

For an arbitrary vector of size n, we cannot use less than n elementary vectors. We could use more, but these would be redundant.

If we use n elementary vectors, there is a unique solution of weights of elementary vectors.

If we use more than n elementary vectors, there will be linear dependence, and so there will not be a unique solution.

1.5 Dimension function

For a basis S, the the dimension of the vector space is $|S|$.

$\dim(V) = |S|$

$S \subset V$

1.5.1 Finite and infinite vector spaces

If $\dim(V)$ is finite, then we say the vector space is finite.

Otherwise, we say the vector space is infinite.

1.6 Points, lines and planes

$(1, 0)$ is point, $(x, 2x + 1)$ is a line $(1, x, y)$ is a plane

1.7 Parallel lines and planes

1.7.1 Parallel lines

If we have two lines:

1.7.2 Parallel planes