0.1 Linear first-order Ordinary Differential Equations

0.1.1 Linear ODEs

For some we can write:

$\frac{dy}{dt} = f(t, y)$

$\frac{dy}{dt} = q(t) - p(t)y$

This can be solved by multiplying by an unknown function $\mu(t)$:

$\frac{dy}{dt} + p(t)y = q(t)$

$\mu(t)[\frac{dy}{dt} + p(t)y] = \mu(t)q(t)$

We can then set $\mu(t) = e^{\int p(t)dt}$. This means that $\frac{d\mu}{dt} = p(t)u(t)$

$\frac{d}{dt}[\mu(t)y] = \mu(t)q(t)$

$\mu(t)y = \int \mu(t)q(t)dt + C$

In some cases, this can then be solved.

0.1.2 Example

$\frac{\delta y}{\delta x} = cy$

$y = Ae^{c(y+a)}$

$\frac{\delta^2 y}{\delta x^2} = cy$

$y = Ae^{\sqrt{c}(y+a)}$