

## 0.1 Linear maps

### 0.1.1 Homomorphisms between vector spaces

Homomorphisms map between algebras, preserving the underlying structure.

A homomorphism between vector space  $V$  and vector space  $W$  can be described as:

$$\text{hom}(V, W)$$

Homomorphism between vector spaces must preserve the group-like structure of the vector space.

$$f(u + v) = f(u) + f(v)$$

The homomorphism must also preserve scalar multiplication.

$$f(\alpha v) = \alpha f(v)$$

A linear map (or function) is a map from one input to an output which preserves addition and scalar multiplication.

That is if function  $f$  is linear then:

$$f(aM + bN) = af(M) + bf(N)$$

### 0.1.2 Alternative names for homomorphisms

Vector spaces homomorphisms are also called linear maps or linear functions.