Contents

0.1 Sequences

0.1.1 Definition

A sequence is an ordered list of terms.

These are commonly maps from natural numbers to real (or complex) numbers.

We can use $a_i = f(i)$ to denote this.

If $f(i)$ is defined on all $i \in \mathbb{N}$ then the sequence is infinite. Otherwise it is finite.

If a sequence is defined on $n \in \mathbb{N}$ and $n \neq 0$ then the sequence must be defined on $n - 1$.

For example $a_0, a_1, a_2, ...$ is a sequence, but $a_1, a_2, ...$ is not.

0.1.2 Monotone sequence

A monotone sequence is one where each element is succeeded ordinally by the next entry.

For example:

$< 1, 2, 3, 6, 7 >$ is monotone

$< 1, 2, 3, 3, 4 >$ is not monotone

An increasing sequence is one where:

$\forall m \in \mathbb{N} \forall n \in \mathbb{N}[m > n \leftrightarrow a_m \geq a_n]$

A strictly increasing sequence is one where:

$\forall m \in \mathbb{N} \forall n \in \mathbb{N}[m > n \leftrightarrow a_m > a_n]$

A decreasing sequence is one where:

$\forall m \in \mathbb{N} \forall n \in \mathbb{N}[m > n \leftrightarrow a_m \leq a_n]$

A strictly decreasing sequence is one where:

$\forall m \in \mathbb{N} \forall n \in \mathbb{N}[m > n \leftrightarrow a_m < a_n]$

All strictly decreasing sequences are decreasing, and all strictly increasing sequences are increasing.

A monotone sequence is one which is either increasing or decreasing.

0.1.3 Subsequences

A subsequence of a sequence is the original sequence with some elements of the original removed, not changing the order.

For example:

$< 1, 3, 5 >$ is a subsequence of $< 2, 1, 3, 4, 7, 5 >$

0.1.4 Bounded sequence

A function $f(x)$ on set X is bounded if:

$\exists M \in \mathbb{R} [\forall x \in X f(x) \leq M]$

A bounded sequence is a special case of a bounded function where:

$X = \mathbb{N}$

That is, a sequence is bounded by M iff:

$\forall n \in \mathbb{R} |f(a_n)| < leM$