

## 0.1 Rational numbers

### 0.1.1 Defining rational numbers

We previously defined integers in terms of natural numbers. Similarly we can define rational numbers in terms of integers.

$$\forall ab \in \mathbb{I}(\neg(b = 0) \rightarrow \exists c(b.c = a))$$

A rational is an ordered pair of integers.

$$\{\{a\}, \{a, b\}\}$$

So that:

$$\{\{a\}, \{a, b\}\} = \frac{a}{b}$$

### 0.1.2 Converting integers to rational numbers

Integers can be shown as rational numbers using:

$$(i, 1)$$

Integers can then be turned into rational numbers:

$$\mathbb{Q} = \frac{a}{1}$$

$$a = \frac{a_1}{a_2}$$

$$b = \frac{b_1}{b_2}$$

$$c = \frac{c_1}{c_2}$$

### 0.1.3 Equivalence classes of rationals

There are an infinite number of ways to write any rational number, as with integers.  $\frac{1}{2}$  can be written as  $\frac{1}{2}$ ,  $\frac{-2}{-4}$  etc.

The class of these terms form an equivalence class.

We can show these are equal:

$$\frac{a}{b} = \{\{a\}, \{a, b\}\}$$

$$\frac{ca}{cb} = \{\{a\}, \{a, b\}\}$$

$$\frac{ca}{cb} = \{\{ca\}, \{ca, cb\}\}$$

$$\{\{a\}, \{a, b\}\} = \{\{ca\}, \{ca, cb\}\}$$