

0.1 The partial differential operator

0.1.1 Differential

When we change the value of an input to a function, we also change the output. We can examine these changes.

Consider the value of a function $f(x)$ at points x_1 and x_2 .

$$y_1 = f(x_1)$$

$$y_2 = f(x_2)$$

$$y_2 - y_1 = f(x_2) - f(x_1)$$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Let's define x_2 in terms of its distance from x_1 :

$$x_2 = x_1 + \epsilon$$

$$\frac{y_2 - y_1}{\epsilon} = \frac{f(x_1 + \epsilon) - f(x_1)}{\epsilon}$$

We define the differential of a function as:

$$\frac{\delta y}{\delta x} = \lim_{\epsilon \rightarrow 0^+} \frac{f(x + \epsilon) - f(x)}{\epsilon}$$

If this is defined, then we say the function is differentiable at that point.

0.1.2 Differential operator