0.1 Total differentiation

0.1.1 Scalar fields

A scalar field is a function on an underlying input which produces a real output.

Inputs are not limited to real numbers. In this section we consider functions on vector spaces.

0.1.2 Total differentiation

Consider a multivariate function.

$f(x)$.

We can define:

$\Delta f(x, \Delta x) := f(x + \Delta x) - f(x)$

$\Delta f(x, \Delta x) = \sum_{i=1}^n f(x + \Delta x_i + \sum_{j=0}^{i-1} \Delta x_j) - f(x + \sum_{j=0}^{i-1} \Delta x_j)$

$\Delta f(x, \Delta x) = \sum_{i=1}^n \Delta x_i \frac{f(x+\Delta x_i+\sum_{j=0}^{i-1} \Delta x_j)-f(x+\sum_{j=0}^{i-1} \Delta x_j)}{\Delta x_i}$

$\frac{\Delta f}{\Delta x_k} = \sum_{i=1}^n \frac{\Delta x_i}{\Delta x_k} \frac{f(x+\Delta x_i+\sum_{j=0}^{i-1} \Delta x_j)-f(x+\sum_{j=0}^{i-1} \Delta x_j)}{\Delta x_i}$

$\lim_{\Delta x_k \to 0} \frac{\Delta f}{\Delta x_k} = \sum_{i=1}^n \lim_{\Delta x_k \to 0} \frac{\Delta x_i}{\Delta x_k} \frac{f(x+\Delta x_i+\sum_{j=0}^{i-1} \Delta x_j)-f(x+\sum_{j=0}^{i-1} \Delta x_j)}{\Delta x_i}$

$\frac{df}{dx_k} = \sum_{i=1}^n \frac{dx_i}{dx_k} \frac{\delta f}{\delta x_i}$

0.1.3 Total differentiation of a univariate function

For a univariate function total differentiation is the same as partial differentiation.

$\frac{df}{dx} = \frac{dx}{dx} \frac{\delta f}{\delta x}$

$\frac{df}{dx} = \frac{\delta f}{\delta x}$