0.1 Unitary groups $U(n, F)$

0.1.1 Metric preserving transformations for sesquilinear forms

For bilinear forms, the transformations which preserved metrics were:

$P^T = P^{-1}$

For sesquilinear they are different:

$u^* M v$

$(Pu)^* M (Pv)$

$u^* P^* M P v$

So we want the matrices where:

$P^* M P = M$

0.1.2 The unitary group

The unitary group is where $M = I$

$P^* P = I$

$P^* = P^{-1}$

We refer to these using U instead of P.

$U^* = U^{-1}$

0.1.3 Parameters of the unitary group

The unitary group depends on the dimension of the vector space, and the underlying field. So we can have:

- $U(n, R)$; and
- $U(n, C)$.

0.1.4 We generally refer only to the complex

For the $U(n, R)$ we have:

$U^* = U^{-1}$

$U^T = U^{-1}$

This is the condition for the orthogonal group, and so we would instead write $O(n)$.

As a result, $U(n)$ refers to $U(n, C)$.

0.1.5 $U(1)$: The circle group