

## 0.1 Boolean algebra

### 0.1.1 Boolean algebra in propositional logic

We previously discussed properties of normal form, and the results from these properties.

If another structure shares these properties then they will also share the results.

### 0.1.2 Sets satisfy the definitions of a boolean algebra

If a mathematical structure has the following properties, it shares the results from normal form, and is a boolean algebra.

- Both binary operators are commutative -  $A \wedge B = B \wedge A$  and  $A \vee B = B \vee A$
- Both binary operators are associative -  $(A \wedge B) \wedge C = A \wedge (B \wedge C)$  and  $(A \vee B) \vee C = A \vee (B \vee C)$
- Complementments -  $A \wedge \neg A = \emptyset$  and  $A \vee \neg A = U$
- Absorption -  $A \wedge (A \vee B) = A$  and  $A \vee (A \wedge B) = A$
- Identity -  $A \wedge U = A$  and  $A \vee \emptyset = A$
- Distributivity -  $A \wedge (B \vee C) = (A \wedge B) \vee (A \wedge C)$  and  $A \vee (B \wedge C) = (A \vee B) \wedge (A \vee C)$

These hold for sets, and so boolean algebra holds for sets.