0.1 The Lie bracket

We can define the Lie bracket from the ring commutator.

We use the Lie bracket, rather than multiplication, as the operator over a field homomorphism.

$[A, B]$

This generates another element in the algebra.

This satisifies:

- Bilinearity: $[xA + yB, C] = x[A, C] + y[B, C]$
- Alternativity: $[A, A] = 0$
- Jacobi identity: $[A, [B, C]] + [C, [A, B]] + [B, [C, A]] = 0$
- Anticommutivity: $[A, B] = -[B, A]$

One option for the Lie bracket is the ring commutor. So that:

$[A, B] = AB - BA$