0.1 Metric-preserving transformations and isometry groups

If we have a bilinear form we can write the form as:

$$u^T M v$$

After a transformation P to the vectors it is:

$$(Pu)^T M (Pv)$$

$$u^T P^T M P v$$

So the value of the metric will be unaffected if:

$$u^T P^T M P v = u^T M v$$

$$P^TMP = M$$

0.1.1 Equivalent metrics

Different metrics can produce the same group. For example multiplying the metric by a constant.

$$P^TMP = M$$