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1 Metric space

1.1 Linear metrics

1.1.1 Metrics

We defined a norm as:

$||v|| = v^T M v$

A metric is the distance between two vectors.

$d(u, v) = ||u - v|| = (u - v)^T M(u - v)$

1.1.1.1 Metric space

A set with a metric is a metric space.

1.1.2 Inducing a topology

Metric spaces can be used to induce a topology.

1.1.3 Translation symmetry

The distance between two vectors is:

$(v-w)^T M(v-w)$

So what operations can we do now?

As before, we can do the transformations which preserve $u^T Mv$, such as the orthogonal group.

But we can also do other translations

(v-w)TM(v-w)

v^TMv+w^TMw-v^TMw-w^TMv

so symmetry is now O(3,1) and affine translations

1.1.3.1 Translation matrix

[[1,x][0, 1]] moves vector by x

1.2 Specific groups

1.2.1 The affine group

1.2.2 The Euclidian group

1.2.3 The Galilean group

1.2.4 The Poincaré group

1.3 Non-linear norms

1.3.1 L_p norms (p-norms)

1.3.1.1 L^P norm

This generalises the Euclidian norm.

$||x||_p = (\sum_{i=1}^n |x|_i^p)^{1/p}$

This can defined for different values of p. Note that the absolute value of each element in the vector is used.

Note also that:

$||x||_2$

Is the Euclidian norm.

1.3.1.2 Taxicab norm

This is the L^1 norm. That is:

$||x||_1 = \sum_{i=1}^n |x|_i$

1.3.1.3 Angles

1.3.1.4 Cauchy-Schwarz

1.4 To linear forms

1.4.1 Norms

We can use norms to denote the "length" of a single vector.

$||v|| = \sqrt{\langle v, v\rangle}$

$||v|| = \sqrt{v^*Mv}$

1.4.1.1 Euclidian norm

If $M = I$ we have the Euclidian norm.

$||v|| = \sqrt{v^*v}$

If we are using the real field this is:

$||v|| = \sqrt{\sum_{i=1}^n v_i^2}$

1.4.1.2 Pythagoras' theorem

If $n = 2$ we have in the real field we have:

$||v|| = \sqrt{v_1^2 + v_2^2}$

We call the two inputs x and y, and the length z.

$z = \sqrt{x^2 + y^2}$

$z^2 = x^2 + y^2$

1.4.2 Angles

1.4.2.1 Recap: Cauchy-Schwarz inequality

This states that:

$|\langle u, v\rangle|^2 \leq \langle u, u\rangle \dot{\langle} v, v\rangle$

Or:

$\langle v, u\rangle\langle u, v\rangle \leq \langle u, u\rangle\dot{\langle} v, v\rangle$

1.4.2.2 Introduction

$\langle v, u\rangle\langle u, v\rangle \leq \langle u, u\rangle\dot{\langle} v, v\rangle$

$\frac{\langle v, u\rangle\langle u, v\rangle}{||u||.||v||} \leq ||u||.||v||$

$\frac{||u||.||v||}{\langle v, u\rangle} \geq \frac{\langle u, v\rangle}{||u||.||v||}$

$\cos(\theta) = \frac{\langle u, v\rangle}{||u||.||v||}$