0.1 Integers

0.1.1 Defining integers

To extend the number line to negative numbers, we define:

$\forall ab \in \mathbb{N} \exists c(a + c = b)$

For any pair of numbers there exists a terms which can be added to one to get the other.

For $1 + x = 3$ this is another natural number, however for $3 + x = 1$ there is no such number.

Integers are defined as the solutions for any pair of natural numbers.

There are an infinite number of ways to write any integer. -1 can be written as $0 - 1$, $1 - 2$ etc.

The class of these terms form an equivalence class.

0.1.2 Integers as ordered pairs

Integers can be defined as an ordered pair of natural numbers, where the integer is valued at: $a - b$.

For example -1 could be shown as:

$-1 = \{\{0\}, \{0, 1\}\}$

$-1 = \{\{5\}, \{5, 6\}\}$

$(a, b) = a - b$

0.1.3 Converting natural numbers to integers

Natural numbers can be shown as integers by using:

$(n, 0)$

Natural numbers can be converted to integers:

$\{\{a\}, \{a, 0\}\}$

0.1.4 Cardinality of integers