

Contents

0.1	Ordering of the integers	1
0.1.1	Ordering integers	1

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Integers are an ordered pair of naturals.

$$\{\{x\}, \{x, y\}\}$$

For example -4 can be:

$$\{\{4\}, \{4, 8\}\}$$

$$\{\{0\}, \{0, 8\}\}$$

We extend the ordering to say:

$$\{\{x\}, \{x, y\}\} \leq \{\{s(x)\}, \{s(x), y\}\}$$

$$\{\{x\}, \{x, s(y)\}\} \leq \{\{x\}, \{x, y\}\}$$

So can we define this on an arbitrary pair:

$$\{\{a\}, \{a, b\}\} \leq \{\{c\}, \{c, d\}\}$$

We know that:

$$\{\{a\}, \{a, b\}\} = \{\{s(a)\}, \{s(a), s(b)\}\}$$

And either of:

$$\{\{a\}, \{a, b\}\} = \{\{0\}, \{0, A\}\}$$

$$\{\{a\}, \{a, b\}\} = \{\{B\}, \{B, 0\}\}$$

$$\{\{a\}, \{a, b\}\} = \{\{0\}, \{0, 0\}\}$$

As the latter is a case of either of the other 2, we consider only the first 2.

So we can define:

$$\{\{a\}, \{a, b\}\} \leq \{\{c\}, \{c, d\}\}$$

As any of:

$$1 : \{\{0\}, \{0, A\}\} \leq \{\{0\}, \{0, C\}\}$$

$$2 : \{\{0\}, \{0, A\}\} \leq \{\{D\}, \{D, 0\}\}$$

$$3 : \{\{B\}, \{B, 0\}\} \leq \{\{0\}, \{0, C\}\}$$

$$4 : \{\{B\}, \{B, 0\}\} \leq \{\{D\}, \{D, 0\}\}$$

Case 1:

$$\{\{0\}, \{0, A\}\} \leq \{\{0\}, \{0, C\}\}$$

Trivial, depends on relative size of A and C .

Case 2:

$$\{\{0\}, \{0, A\}\} \leq \{\{D\}, \{D, 0\}\}$$

We can see that:

$$\{\{D\}, \{D, A\}\} \leq \{\{D\}, \{D, 0\}\}$$

And therefore this holds.

Case 3:

$$\{\{B\}, \{B, 0\}\} \leq \{\{0\}, \{0, C\}\}$$

We can see that:

$$\{\{B\}, \{B, 0\}\} \leq \{\{B\}, \{B, C\}\}$$

And therefore this does not hold.

Case 4:

$$\{\{B\}, \{B, 0\}\} \leq \{\{D\}, \{D, 0\}\}$$

Trivial, like case 1.