

1 Expanding functions of natural numbers

1.1 Gamma function

The gamma function expands the factorial function to the real (and complex) numbers

We want:

$$f(1) = 1$$

$$f(x+1) = xf(x)$$

There are an infinite number of functions which fit this. The function could fluctuate between the natural numbers.

The function we use is:

$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx$$

1.2 Beta function

The beta function expand the binomial coefficient formula to the real (and complex) numbers.

We want to expand the binomial coefficient function.

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

We do this as:

$$B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

2 Other

2.1 Dirac delta

2.1.1 Kronecker delta

The function is: δ_{ij}

If $i = j$ this is 1. Otherwise it is 0.

We introduced this in linear algebra.

2.1.2 Dirac delta

The Dirac delta replaces the Kronecker delta for continuous functions.

That is, we want:

- $\delta(x \neq 0) = 0$
- $\delta(0) = +\infty$
- $\int_{-\infty}^{\infty} \delta(x) dx = 1$