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0.1 Differentiating exponents, logarithms and power functions

0.1.1 Differentiating the natural logarithm

$$\frac{\delta}{\delta x} \ln(x) = \lim_{\delta \rightarrow 0} \frac{\ln(x+\delta) - \ln(x)}{\delta}$$

$$\frac{\delta}{\delta x} \ln(x) = \lim_{\delta \rightarrow 0} \frac{\ln \frac{x+\delta}{x}}{\delta}$$

$$\frac{\delta}{\delta x} \ln(x) = \lim_{\delta \rightarrow 0} \frac{\ln(1 + \frac{\delta}{x})}{\delta}$$

$$\frac{\delta}{\delta x} \ln(x) = \frac{1}{x} \lim_{\delta \rightarrow 0} \frac{x}{\delta} \ln(1 + \frac{\delta}{x})$$

$$\frac{\delta}{\delta x} \ln(x) = \frac{1}{x} \ln(\lim_{\delta \rightarrow 0} (1 + \frac{\delta}{x})^{\frac{x}{\delta}})$$

$$\frac{\delta}{\delta x} \ln(x) = \frac{1}{x} \ln(e)$$

$$\frac{\delta}{\delta x} \ln(x) = \frac{1}{x}$$

0.1.2 Differentiating logarithms of other bases

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

$$\frac{\delta}{\delta x} \log_a(x) = \frac{\delta}{\delta x} \frac{\ln(x)}{\ln(a)}$$

$$\frac{\delta}{\delta x} \log_a(x) = \frac{1}{x \ln(a)}$$

0.1.3 Exponents

$$y = a^x$$

$$\ln(y) = x \ln(a)$$

$$\frac{\delta}{\delta x} \ln(y) = \frac{\delta}{\delta x} x \ln(a)$$

$$\frac{\delta}{\delta x} \ln(y) = \ln(a)$$

$$\frac{1}{y} \frac{\delta}{\delta x} y = \ln(a)$$

$$\frac{\delta}{\delta x} a^x = a^x \ln(a)$$

0.1.4 Power functions

$$y = x^n$$

$$\frac{\delta}{\delta x} y = \frac{\delta}{\delta x} x^n$$

$$\frac{\delta}{\delta x} y = \frac{\delta}{\delta x} e^{n \ln(x)}$$

$$\frac{\delta}{\delta x} y = \frac{n}{x} e^{n \ln(x)}$$

$$\frac{\delta}{\delta x} y = nx^{n-1}$$