0.1 Fourier series

0.1.1 Fourier series

Motivation: we have a function we want to display as another sort of function.

More specifically, a function can be shown as a combination of sinusoidal waves.

To frame this let's imagine a sound wave, with values $f(t)$ for all time values t. We can imagine this as a summation of sinusoidal functions. That is:

$f(t) = \sum_{n=0}^{\inf} a_n cos(nw_0 t)$

We want to get another function $F(\xi)$ for all frequencies ξ.

0.1.2 Combinations of wave functions

We can add sinusoidal waves to get new waves.

For example

$s_N(x) = 2\sin(x+3) + \sin(-4x) + \frac{1}{2}\cos(x)$

0.1.3 As a summation of series

We can simplify arbitrary series using the following identities:

$\cos(x) = \sin(x + \frac{\tau}{8})$

$\sin(-x) = -\sin(x)$

So we have:

$s(x) = 2\sin(x+3) - \sin(4x) + \frac{1}{2}\sin(x + \frac{\tau}{8})$

We can put this into the following format:

$s(x) = \sum_{i=1}^{m} a_i \sin(b_i x + c_i)$

Where:

$a = [2, -1, \frac{1}{2}]$

$b = [1, 4, 1]$

$c = [3, 0, \frac{\tau}{8}]$

0.1.4 Ordering by b

We can move terms around to get:

$s(x) = \sum_{i=1}^{m} a_i \sin(b_i x + c_i)$

Where:

$a = [2, \frac{1}{2}, -1]$

$b = [1, 1, 4]$

$c = [3, \frac{\tau}{8}, 0]$

0.1.5 Adding waves with same frequency

We know that:

$\sin(a + b) = \sin(a)\cos(b) + \sin(b)\cos(a)$

So:

$\sin(b_i x + c_i) = \sin(b_i x)\cos(c_i) + \sin(c_i)\cos(b_i x)$

If 2 terms have the same value for b_i, then:

$a_i \sin(b_i x + c_i) + a_j \sin(b_j x + c_j) = a_i \sin(b_i x + c_i) + a_j \sin(b_i x + c_j)$

$a_i \sin(b_i x + c_i) + a_j \sin(b_j x + c_j) = a_i \sin(b_i x)\cos(c_i) + a_i \sin(c_i)\cos(b_i x) + a_j \sin(b_i x)\cos(c_j) + a_j \sin(c_j)\cos(b_i x)$

So we now get for:

$s(x) = \sum_{i=1}^{m} a_i \sin(b_i x + c_i)$

$a = [, -1]$

$b = [, 4]$

$c = [, 0]$