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0.1 Complements and disjoint sets

0.1.1 Disjoint sets

Sets are disjoint if there is no overlap in their elements. Two sets s_i and s_j are mutually exclusive if:

$$s_i \wedge s_j = \emptyset$$

A collection of events s are all mutually exclusive if all pairs are mutually exclusive. That is:

$$\forall s_i \in s \forall s_j \in s [s_i \wedge s_j \neq \emptyset \rightarrow s_i = s_j]$$

0.1.2 Complement function

x^C is the complement. It is defined such that:

$$\forall x [x \wedge x^C = \emptyset]$$

For a set b , the complement with respect to a is all elements in a which are not in b .

$$\forall x \in a \forall y \in b [x \in (a \setminus b) \wedge y \in (a \setminus b)]$$

$$b \wedge (a \setminus b) = \emptyset$$

That is, b and $a \setminus b$ are disjoint.

0.1.3 Existence of the complement

For two sets a and b we can write $(a \setminus b)$. This is the set of elements of a which are not in b .

Consider the axiom of specification:

$$\forall x \forall a \exists s [(P(x) \wedge x \in a) \leftrightarrow (x \in s)]$$

We can also write

$$\forall x \forall a \forall b \exists s [(x \notin b \wedge x \in a) \leftrightarrow (x \in s)]$$

Which provides the complement, s .