## 0.1 The chain rule, the product rule and the quotient rule

### 0.1.1 Chain rule

$f(x) = f(g(x))$

$\frac{\delta f}{\delta x} = \lim_{\epsilon \to 0^+} \frac{f(g(x+\epsilon)) - f(g(x))}{\epsilon}$

$\frac{\delta f}{\delta x} = \lim_{\epsilon \to 0^+} \frac{g(x+\epsilon) - g(x)}{g(x+\epsilon) - g(x)} \frac{f(g(x+\epsilon)) - f(g(x))}{\epsilon}$

$\frac{\delta f}{\delta x} = \lim_{\epsilon \to 0^+} \frac{g(x+\epsilon) - g(x)}{\epsilon} \frac{f(g(x+\epsilon)) - f(g(x))}{g(x+\epsilon) - g(x)}$

$\frac{\delta f}{\delta x} = \lim_{\epsilon \to 0^+} [\frac{g(x+\epsilon) - g(x)}{\epsilon}] \lim_{\epsilon \to 0^+} [\frac{f(g(x+\epsilon)) - f(g(x))}{g(x+\epsilon) - g(x)}]$

$\frac{\delta f}{\delta x} = \frac{\delta g}{\delta x} \frac{\delta f}{\delta g}$

### 0.1.2 Product rule

$y = f(x)g(x)$

$\frac{\delta y}{\delta x} = \lim_{\epsilon \to 0^+} \frac{f(x+\epsilon)g(x+\epsilon) - f(x)g(x)}{\epsilon}$

$\frac{\delta y}{\delta x} = \lim_{\epsilon \to 0^+} \frac{f(x+\epsilon)g(x+\epsilon) - f(x)g(x+\epsilon) + f(x)g(x+\epsilon) - f(x)g(x)}{\epsilon}$

$\frac{\delta y}{\delta x} = \lim_{\epsilon \to 0^+} \frac{f(x+\epsilon)g(x+\epsilon) - f(x)g(x+\epsilon)}{\epsilon} + \lim_{\epsilon \to 0^+} \frac{f(x)g(x+\epsilon) - f(x)g(x)}{\epsilon}$

$\frac{\delta y}{\delta x} = \lim_{\epsilon \to 0^+} g(x+\epsilon) \frac{f(x+\epsilon) - f(x)}{\epsilon} + \lim_{\epsilon \to 0^+} f(x) \frac{g(x+\epsilon) - g(x)}{\epsilon}$

$\frac{\delta y}{\delta x} = g(x) \frac{\delta f}{\delta x} + f(x) \frac{\delta g}{\delta x}$

### 0.1.3 Quotient rule

$y = \frac{f(x)}{g(x)}$

$\frac{\delta}{\delta x} y = \frac{\delta}{\delta x} \frac{f(x)}{g(x)}$

$\frac{\delta}{\delta x} y = \frac{\delta}{\delta x} f(x) \frac{1}{g(x)}$

$\frac{\delta}{\delta x} y = \frac{\delta f}{\delta x} \frac{1}{g(x)} - \frac{\delta g}{\delta x} \frac{f(x)}{g(x)^2}$

$\frac{\delta}{\delta x} y = \frac{\frac{\delta f}{\delta x} g(x) - \frac{\delta g}{\delta x} f(x)}{g(x)^2}$