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## 0.1 Tensor contraction

We have a vector  $v \in V$  and  $w \in V^*$ .

$$\mathbf{v} = \sum_i v^i \mathbf{e}_i$$

$$\mathbf{w} = \sum_i w_i \mathbf{f}^i$$

$$\mathbf{w}\mathbf{v} = [\sum_i v^i \mathbf{e}_i][\sum_i w_i \mathbf{f}^i]$$

$$\mathbf{w}\mathbf{v} = \sum_i \sum_j [v^i \mathbf{e}_i][w_j \mathbf{f}^j]$$

$$\mathbf{w}\mathbf{v} = \sum_i \sum_j v^i w_j \mathbf{e}_i \mathbf{f}^j$$

We use the dual basis so:

$$\mathbf{w}\mathbf{v} = \sum_i \sum_j v^i w_j \mathbf{e}_i \mathbf{e}^j$$

$$\mathbf{w}\mathbf{v} = \sum_i \sum_j v^i w_j \delta_i^j$$

We can see that this value is unchanged when there is a change in basis.

What if these were both from  $V$ ?

$$\mathbf{v} = \sum_i v^i \mathbf{e}_i$$

$$\mathbf{w} = \sum_i w^i \mathbf{e}_i$$

$$\mathbf{w}\mathbf{v} = [\sum_i v^i \mathbf{e}_i][\sum_i w^i \mathbf{e}_i]$$

$$\mathbf{w}\mathbf{v} = \sum_i \sum_j v^i w^j \mathbf{e}_i \mathbf{e}_i$$

This term is dependent on the basis, and so we do not contract.

So if we have  $v_i w^i$ , we can contract, because the result (calculated from the components) does not depend on the basis.

But if we have  $v_i w_i$ , the result (calculated from the components) will change depending on the choice of basis.

We define a new object

$$c = \sum_i w^i v_i$$

This new term,  $c$ , does not depend on  $i$ , and so we have contracted the index.