0.1 Ordering of the integers

0.1.1 Ordering integers

Integers are an ordered pair of naturals.

$\{\{x\},\{x,y\}\}$

For example -4 can be:

$\{\{4\},\{4,8\}\}$

$\{\{0\},\{0,8\}\}$

We extend the ordering to say:

$\{\{x\},\{x,y\}\} \leq \{\{s(x)\},\{s(x),y\}\}$

$\{\{x\},\{x,s(y)\}\} \leq \{\{x\},\{x,y\}\}$

So can we define this on an arbitrary pair:

$\{\{a\},\{a,b\}\} \leq \{\{c\},\{c,d\}\}$

We know that:

$\{\{a\},\{a,b\}\} = \{\{s(a)\},\{s(a),s(b)\}\}$

And either of:

$\{\{a\},\{a,b\}\} = \{\{0\},\{0,A\}\}$

$\{\{a\},\{a,b\}\} = \{\{B\},\{B,0\}\}$

$\{\{a\},\{a,b\}\} = \{\{0\},\{0,0\}\}$

As the latter is a case of either of the other 2, we consider only the first 2.

So we can define:

$\{\{a\},\{a,b\}\} \leq \{\{c\},\{c,d\}\}$

As any of:

$1 : \{\{0\},\{0,A\}\} \leq \{\{0\},\{0,C\}\}$

$2 : \{\{0\},\{0,A\}\} \leq \{\{D\},\{D,0\}\}$

$3 : \{\{B\},\{B,0\}\} \leq \{\{0\},\{0,C\}\}$

$4 : \{\{B\},\{B,0\}\} \leq \{\{D\},\{D,0\}\}$

Case 1:

$\{\{0\},\{0,A\}\} \leq \{\{0\},\{0,C\}\}$

Trivial, depends on relative size of A and C.

Case 2:

$\{\{0\},\{0,A\}\} \leq \{\{D\},\{D,0\}\}$

We can see that:

$\{\{D\},\{D,A\}\} \leq \{\{D\},\{D,0\}\}$

And therefore this holds.

Case 3:

$\{\{B\},\{B,0\}\} \leq \{\{0\},\{0,C\}\}$

We can see that:

$\{\{B\},\{B,0\}\} \leq \{\{B\},\{B,C\}\}$

And therefore this does not hold.

Case 4:

$\{\{B\},\{B,0\}\} \leq \{\{D\},\{D,0\}\}$

Trivial, like case 1.