0.1 Addition of sine and cosine

0.1.1 Adding waves with same frequency

We know that:

$a\sin(bx+c) = a\sin(bx)\cos(c) + a\sin(c)\cos(bx)$

So:

$a\sin(bx+c) + d\sin(bx+e) = a\sin(bx)\cos(c) + a\sin(c)\cos(bx) + d\sin(bx)\cos(e) + d\sin(e)\cos(bx)$

We know that:

$\sin(\theta) = \frac{e^{i\theta}-e^{-i\theta}}{2i}$

So:

$a\sin(bx+c) + d\sin(bx+f) = a\frac{e^{i(bx+c)}-e^{-i(bx+c)}}{2i} + d\frac{e^{i(bx+f)}-e^{-i(bx+f)}}{2i}$

$a\sin(bx+c) + d\sin(bx+f) = \frac{a(e^{i(bx+c)}-e^{-i(bx+c)})+d(e^{i(bx+f)}-e^{-i(bx+f)})}{2i}$

$a\sin(bx+c) + d\sin(bx+f) = \frac{a(e^{ibx}e^{ic}-e^{-ibx}e^{-ic})+d(e^{ibx}e^{if}-e^{-ibx}e^{-if})}{2i}$

$a\sin(bx+c) + d\sin(bx+f) = \frac{(e^{ibx}(ae^{ic}+de^{if})-e^{-ibx}(ae^{-c}+d^{-if})}{2i}$

$a_i\sin(b_ix+c_i) + a_j\sin(b_jx+c_j) = a_i\sin(b_ix+c_i) + a_j\sin(b_ix+c_j)$

$a_i\sin(b_ix+c_i) + a_j\sin(b_jx+c_j) = a_i\sin(b_ix)\cos(c_i) + a_i\sin(c_i)\cos(b_ix) + a_j\sin(b_ix)\cos(c_j) + a_j\sin(c_j)\cos(b_ix)$