

Contents

0.1	Calculating eigenvalues and eigenvectors using the characteristic polynomial	1
0.2	Calculating eigenvalues	1
0.3	Calculating eigenvectors	2

0.1 Calculating eigenvalues and eigenvectors using the characteristic polynomial

The characteristic polynomial of a matrix is a polynomial whose roots are the eigenvalues of the matrix.

We know from the definition of eigenvalues and eigenvectors that:

$$Av = \lambda v$$

Note that

$$Av - \lambda v = 0$$

$$Av - \lambda Iv = 0$$

$$(A - \lambda I)v = 0$$

Trivially we see that $v = 0$ is a solution.

Otherwise matrix $A - \lambda I$ must be non-invertible. That is:

$$\text{Det}(A - \lambda I) = 0$$

0.2 Calculating eigenvalues

For example

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 2 - \lambda & 1 \\ 1 & 2 - \lambda \end{bmatrix}$$

$$\text{Det}(A - \lambda I) = (2 - \lambda)(2 - \lambda) - 1$$

When this is 0.

$$(2 - \lambda)(2 - \lambda) - 1 = 0$$

$$\lambda = 1, 3$$

0.3 Calculating eigenvectors

You can plug this into the original problem.

For example

$$Av = 3v$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 3 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

As vectors can be defined at any point on the line, we normalise $x_1 = 1$.

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3x_2 \end{bmatrix}$$

Here $x_2 = 1$ and so the eigenvector corresponding to eigenvalue 3 is:

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$