0.1 σ-algebra

0.1.1 Review of algebra on a set

An algebra, Σ, on set s is a set of subsets of s such that:

- Closed under intersection: If a and b are in Σ then $a \wedge b$ must also be in Σ
- $\forall ab[(a \in \Sigma \wedge b \in \Sigma) \rightarrow (a \wedge b \in \Sigma)]$
- Closed under union: If a and b are in Σ then $a \vee b$ must also be in Σ.
- $\forall ab[(a \in \Sigma \wedge b \in \Sigma) \rightarrow (a \vee b \in \Sigma)]$

If both of these are true, then the following is also true:

- Closed under complement: If a is in Σ then $s \backslash a$ must also be in Σ

We also require that the null set (and therefore the original set, null's complement) is part of the algebra.

0.1.2 σ-algebra

A σ-algebra is an algebra with an additional condition:

All countable unions of sets in A are also in A.

This adds a constraint. Consider the real numbers with an algebra of all finite sets.

This contains all finite subsets, and their complements. It does not contain $\mathbb{N}$.

However a σ-algebra requires all countable unions to be including, and the natural numbers are a countable union.

The power set is a σ-algebra. All other σ-algebras are subsets of the power set.