1 Introduction

1.1 Rings

Consider an abelian group $(S, +)$.

A ring takes this and adds a multiplicative function which satisfies the distributive property.

Groups have an identity element for their function. Rings must have identity elements for both their functions.

The multiplicative function does not have to be a bijection. For example the set of integers, addition and multiplication form a ring.

1.2 Rngs

A rng is a ring without the multiplicative identity (hense no 'i').

2 Commutation

2.1 Commutative rings

The multiplication operation commutes.

2.2 Commutator

$[a, b] = ab - ba$

2.3 The Jacobi identity

3 Examples of rings

3.1 Zero (trivial) ring

The trivial ring is a ring with just one element.

0 with addition and multiplication work.

3.2 Integer rings

The integers with addition and multiplication form a ring.

3.3 Integer mod n rings

The integers mod n with addition and multiplication form a group.

3.3.1 Examples

The integers $\{1, 2, 3\}$ form a ring.

4 Properties of rings

4.1 Characteristic of a ring

The characteristic of a ring is the number of times the multiplicative identity must be added to get the additive identity.

If this never happens, the characteristic is 0.

4.1.1 Example

The integer mod 2 ring, the characteristic is 2.

5 Division

5.1 Division rings

A divison ring is a ring where every non-zero element has a multiplicative inverse.

5.1.1 Example

The rational numbers are a divison ring.

5.1.2 Relationship between divison rings and fields

Fields (not yet introduced) are different from division rings only in that multiplication for a field must be commutative.

5.2 Units

A unit is an element of a ring which has a multiplicative inverse.

5.2.1 Examples

The ring of integers with addition and multiplication, only -1 and 1 are units, as both have multiplicative inverses in the ring.

6 Subrings

6.1 Subrings

A subring is a subset of the ring, where the addition and multiplication operations on the subring result in elements also in the subring.

6.1.1 Example

The even numbers are a subring of the integers.

6.2 Ideals

An ideal is a subring where the multiplication of any element of the ideal with any element of the ring is also in the ideal.

6.2.1 Examples

Even numbers are an ideal of the integers.

Odd numbers are not an ideal. For example 1 is in the ideal, but multiplied by 2 gives 2, which is not in the ideal.