0.1 Inverse matrices

An invertible matrix implies that if the matrix is multiplied by another matrix, the original matrix can be recovered.

That is, if we have matrix A, there exists matrix A^{-1} such that $AA^{-1} = I$.

Consider a linear map on a vector space.

$Ax = y$

If A is invertible we can have:

$A^{-1}Ax = A^{-1}y$

$x = A^{-1}y$

If we set $y = \mathbf{0}$ then:

$x = \mathbf{0}$

So if there is a non-zero vector x such that:

$Ax = \mathbf{0}$ then A is not invertible.

0.2 Left and right inverses

That is, for all matrices A, the left and right inverses of B, B_L^{-1} and B_R^{-1}, are defined such that:

$A(BB_R^{-1}) = A$

$A(B_L^{-1}B) = A$

Left and right inversions are equal

Note that if the left inverse exists then:

$B_L^{-1}B = I$

And if the right inverse exists:

$BB_R^{-1} = I$

Let's take the first:

$B_L^{-1}B = I$

$B_L^{-1}BB_L^{-1} = B_L^{-1}$

$B_L^{-1}BB_L^{-1} - B_L^{-1} = 0$

$B_L^{-1}(BB_L^{-1} - I) = 0$

0.3 Inversion of products

$(AB)(AB)^{-1} = I$

$A^{-1}AB(AB)^{-1} = A^{-1}$

$B^{-1}B(AB)^{-1} = B^{-1}A^{-1}$

$(AB)^{-1} = B^{-1}A^{-1}$

0.4 Inversion of a diagonal matrix

$DD^{-1} = I$

$D_{ii}D_{ii}^{-1} = 1$

$D_{ii}^{-1} = \frac{1}{D_{ii}}$