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0.1 Cosets and normal subgroups

A coset is defined between a group and a subgroup of the group.

For a group G , and its subgroup H :

- The left coset is $\{gH\}$
- The right coset is $\{Hg\}$

For $\forall g \in G$.

For abelian groups, the left and right cosets are the same.

The left and right cosets can also be the same, even if the group G is not abelian.

0.1.1 Normal subgroups

If the left and right cosets are the same then H is a normal subgroup.

0.1.2 Cosets divide a group.

Consider two left cosets, aH and bH , with a common element.

This means that $ah_i = bh_j$.

We can use this to get:

$$a = bh_jh_i^{-1}$$

$$b = ah_ih_j^{-1}$$

We know that:

$$ah \in aH$$

$$bh \in bH$$

So:

$$bh_jh_i^{-1}h \in aH$$

$$ah_ih_j^{-1}h \in bH$$

And so:

$$bH \subset aH$$

$$aH \subset bH$$

Therefore:

$$aH = bH$$

0.1.3 Example 1

Consider the group $\{-1, 1\}, \times$

For the subgroup $\{1\}, \times$, the left coset is $\{gH\} = \{1, -1\}$.

The right coset is the same.

0.1.4 Example 2

Consider the group of integers and addition: $(\mathbb{Z}, +)$

For subgroup $(m\mathbb{Z}, +)$, the left and right cosets are the same because the group is abelian.

The coset of the subgroup is the subgroup multiplied by each element in G .

This is $m\mathbb{Z}$, $m\mathbb{Z} + 1$, $m\mathbb{Z} + 2$ and so on.

Once we reach $m\mathbb{Z} + m$ this has looped, and is already a coset, so we only need the sets upto $m\mathbb{Z} + m - 1$.