

0.1 Incompleteness of the real numbers

0.1.1 The square root of 2 is not a rational number

Let's prove there are numbers which are not rational. Consider $\sqrt{2}$ and let's show that it being rational leads to a contradiction.

$$\sqrt{2} = \frac{x}{y}$$

$$2 = \frac{x^2}{y^2}$$

$$2y^2 = x^2$$

So we know that x^2 is even, and can be shown as $x = 2n$.

$$2y^2 = (2n)^2$$

$$y^2 = 2n^2$$

So y is even. But if both x and y are even, then the fraction was not reduced.

This presents a contraction so the original statement must have been false.

So we know there isn't a rational solution to $\sqrt{2}$.