Contents

0.1 Diagonalisable matrices and eigendecomposition

If matrix M is diagonalisable if there exists matrix P and diagonal matrix A such that:

$M = P^{-1}AP$

0.1.1 Diagonalisiable matrices and powers

If these exist then we can more easily work out matrix powers.

$M^n = (P^{-1}AP)^n = P^{-1}A^nP$

A^n is easy to calculate, as each entry in the diagonal taken to the power of n.

0.1.2 Defective matrices

Defective matrices are those which cannot be diagonalised.

Non-singular matries can be defective or not defective, for example the identiy matrix.

Singular matrices can also be defective or not defective, for example the empty matrix.

0.1.3 Eigen-decomposition

Consider an eigenvector v and eigenvalue λ of matrix M.

We known that $Mv = \lambda v$.

If M is full rank then we can generalise for all eigenvectors and eigenvalues:

$MQ = Q\Lambda$

Where Q is the eigenvectors as columns, and Λ is a diagonal matrix with the corresponding eigenvalues. We can then show that:

$M = Q\Lambda Q^{-1}$

This is only possible to calculate if the matrix of eigenvectors is non-singular. Otherwise the matrix is defective.

If there are linearly dependent eigenvectors then we cannot use eigen-decomposition.

0.2 Using the eigen-decomposition to invert a matrix

This can be used to invert M.

We know that:

$M^{-1} = (Q\Lambda Q^{-1})^{-1}$

$M^{-1} = Q^{-1}\Lambda^{-1}Q$

We know Λ can be easily inverted by taking the reciprocal of each diagonal element. We already know both Q and its inverse from the decomposition.

If any eigenvalues are 0 then Λ cannot be inverted. These are singular matrices.