## 0.1 Calculating eigenvalues and eigenvectors using the characteristic polynomial

The characteristic polynomial of a matrix is a polynomial whose roots are the eigenvalues of the matrix.

We know from the definition of eigenvalues and eigenvectors that:

$Av = \lambda v$

Note that

$Av - \lambda v = 0$

$Av - \lambda Iv = 0$

$(A - \lambda I)v = 0$

Trivially we see that $v = 0$ is a solution.

Otherwise matrix $A - \lambda I$ must be non-invertible. That is:

$Det(A - \lambda I) = 0$

## 0.2 Calculating eigenvalues

For example

$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$

$Det(A - \lambda I) = (2 - \lambda)(2 - \lambda) - 1$

When this is 0.

$(2 - \lambda)(2 - \lambda) - 1 = 0$

$\lambda = 1, 3$

## 0.3 Calculating eigenvectors

You can plug this into the original problem.

For example

$Av = 3v$

$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 3 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

As vectors can be defined at any point on the line, we normalise $x_1 = 1$.

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3x_2 \end{bmatrix}$$

Here $x_2 = 1$ and so the eigenvector corresponding to eigenvalue 3 is:

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$