0.1 Complements and disjoint sets

0.1.1 Disjoint sets

Sets are disjoint is there is no overlap in their elements. Two sets are s_i and s_j are mutually exclusive if:

$s_i \wedge s_j = \emptyset$

A collection of events s are all mutually exclusive if all pairs are mutually exclusive. That is:

$\forall s_i \in s \forall s_j \in s[s_i \wedge s_j \neq \emptyset \rightarrow s_i = s_j]$

0.1.2 Complement function

x^C is the completement. It is defined such that:

$\forall x[x \wedge x^C = \varnothing]$

For a set b, the complement with respect to a is all elements in a which are not in b.

$\forall x \in a \forall y \in b[x \in (a \setminus b) \wedge y \in (a \setminus b)]$

$b \wedge (a \setminus b) = \varnothing$

That is, b and $a \setminus b$ are disjoint.

0.1.3 Existence of the complement

For two sets a and b we can write $(a \setminus b)$. This is the set of elements of a which are not in b.

Consider the axiom of specification:

$\forall x \forall a \exists s[(P(x) \wedge x \in a) \leftrightarrow (x \in s)]$

We can also write

$\forall x \forall a \forall b \exists s[(x \notin b \wedge x \in a) \leftrightarrow (x \in s)]$

Which provides the complement, s.