

## 0.1 Cardinality of the rationals

### 0.1.1 Cardinality of rational numbers

We can see rational numbers as cartesian products of integers. That is:

$$\mathbb{Q} = \mathbb{Z} \cdot \mathbb{Z}$$

We can order the rational numbers like so:

$$\{\frac{1}{1}, \frac{2}{1}, \frac{1}{2}, \frac{1}{3}, \frac{2}{2}, \frac{3}{1} \dots\}$$

These can be mapped from natural numbers, so there is a bijective function.

So:

$$|\mathbb{Q}| = |\mathbb{Z} \cdot \mathbb{Z}| = |\mathbb{N}| = \aleph_0$$

$$\text{As: } |\mathbb{Z} \cdot \mathbb{Z}| = |\mathbb{Z}|^2$$

$$|\mathbb{N}|^n = \aleph_0$$