0.1 Length of a curve

We have a curve from a to b in $\mathbf{R}^n$.

$f : [a, b] \to \mathbf{R}^n$

We divide this into n segments.

The ith cut is at:

$t_i = a + \frac{i}{n}(b - a)$

So the first cut is at:

$t_0 = a$

$t_n = b$

The distance between two sequential cuts is:

$||f(t_i) - f(t_{i-1}||$

The sum of all these differences is:

$L = \sum_{i=1}^n ||f(t_i) - f(t_{i-1})||$

The limit is:

$L = \lim_{n\to\infty} \sum_{i=1}^n ||f(t_i) - f(t_{i-1})||$

0.1.1 Method 1

$L = \lim_{n\to\infty} \sum_{i=1}^n ||f(t_i) - f(t_{i-1})||$

$L = \lim_{n\to\infty} \sum_{i=1}^n ||\frac{f(t_i) - f(t_{i-1})}{\Delta t}||\Delta t$

$L = \lim_{n\to\infty} \sum_{i=1}^n ||f'(t)||\Delta t$

$L = \int_a^b ||f'(t)||dt$

0.1.2 Method 2

$L = \lim_{n\to\infty} \sum_{i=1}^n ||f(t_i) - f(t_{i-1})||$

$L = \lim_{n\to\infty} \sum_{i=1}^n \sqrt{(f(t_i) - f(t_{i-1}))^* M(f(t_i) - f(t_{i-1}))}$

$L = \int_a^b \sqrt{(dt)^T M(dt)}$