## 1 Undirected graphs

### 1.1 Vertices and edges

A graph is a set of vertices V, a set of edges E which are subset of pairs from V. undirected so each edge is a set

### 1.1.1 Degree of a vertex

The degree of a vertex is the number of edges connections to it.

## 1.2 Order and size of graphs

The order of a graph is the number of vertices, |V|.

The size of a graph is the number of edges, |E|.

## 1.3 Subgraphs

We can take a subset of vertices, and all edges which only depend on these vertices. This is an induced subgraph.

### 1.3.1 Induced subgraph

## 1.4 Loops, multiple edges and simple graphs

### 1.4.1 Loops

A loop is an edge where both the vertices are the same.

### 1.4.2 Multiple edges

If there are two edges with the same pair of indices, there are multiple edges.

#### 1.4.3 Simple graphs

No loops or multiple edges.

# 2 Directed graphs

## 2.1 Direct acyclic graphs

# 3 Weighted graphs

## 3.1 Edge-weighted graph

An edge-weighted graphs has weights for each edge.

## 3.2 Vertex-weighted graph

A vertex-weighted graph has weights for each vertex.

## 4 Graph representation

## 4.1 Adjacency matrix

We can represent a finite graph as a square matrix.  $m_{ij}$  is the number of edges connecting vertex i to vertex j.

### 4.2 Incidence matrix

An incidence matrix has  $m_{ij}$  representing the number of connections from vertex i to edge j.

### 4.3 Degree matrix

A degree matrix is a diagonal matrix. Each diagonal contains the degree of the a vertex.

### 4.4 Laplacian matrix

The Laplacian matrix L is formed using the degree matrix D and the adjacency matrix A. L = D - A.

- 5 Representing manifolds
- 5.1 Nearest-neighbour graph
- 5.2 Triangular mesh