0.1 Unconstrained envelope theorem

Consider a function which takes two parameters:

$f(x, \alpha$

We want to choose x to maximise f, given α.

$V(\alpha) = \sup_{x \in X} f(x, \alpha)$

There is a subset of X where $f(x, \alpha) = V(\alpha)$.

$X^*(\alpha) = \{x \in X | f(x, \alpha) = V(\alpha)\}$

This means that $V(\alpha) = f(x^*, \alpha)$ for $x^* \in X^*$.

Let's assume that there is only one x^*.

$V(\alpha) = f(x^*, \alpha)$

What happens to the value function as we relax α?

$V_{\alpha_i}(\alpha) = f_{\alpha_i}(x^*(\alpha), \alpha)$.

$V_{\alpha_i}(\alpha) = f_x \dfrac{\delta x^*}{\delta \alpha} + f_{\alpha_i}$.

We know that $f_x = 0$ from first order conditions. So:

$V_{\alpha_i}(\alpha) = f_{\alpha_i}$.

That is, at the optimum, as the constant is relaxed, we can treat the x^* as fixed, as the first-order movement is 0.