

Contents

0.1	Fourier series	1
0.1.1	Fourier series	1
0.1.2	Combinations of wave functions	1
0.1.3	As a summation of series	1
0.1.4	Ordering by b	2
0.1.5	Adding waves with same frequency	2

0.1 Fourier series

0.1.1 Fourier series

Motivation: we have a function we want to display as another sort of function.
 More specifically, a function can be shown as a combination of sinusoidal waves.
 To frame this let's imagine a sound wave, with values $f(t)$ for all time values t .
 We can imagine this as a summation of sinusoidal functions. That is:

$$f(t) = \sum_{n=0}^{\infty} a_n \cos(n\omega_0 t)$$

We want to get another function $F(\xi)$ for all frequencies ξ .

0.1.2 Combinations of wave functions

We can add sinusoidal waves to get new waves.

For example

$$s_N(x) = 2 \sin(x + 3) + \sin(-4x) + \frac{1}{2} \cos(x)$$

0.1.3 As a summation of series

We can simplify arbitrary series using the following identities:

$$\cos(x) = \sin(x + \frac{\pi}{2})$$

$$\sin(-x) = -\sin(x)$$

So we have:

$$s(x) = 2 \sin(x + 3) - \sin(4x) + \frac{1}{2} \sin(x + \frac{\pi}{2})$$

We can put this into the following format:

$$s(x) = \sum_{i=1}^m a_i \sin(b_i x + c_i)$$

Where:

$$a = [2, -1, \frac{1}{2}]$$

$$b = [1, 4, 1]$$

$$c = [3, 0, \frac{\pi}{8}]$$

0.1.4 Ordering by b

We can move terms around to get:

$$s(x) = \sum_{i=1}^m a_i \sin(b_i x + c_i)$$

Where:

$$a = [2, \frac{1}{2}, -1]$$

$$b = [1, 1, 4]$$

$$c = [3, \frac{\pi}{8}, 0]$$

0.1.5 Adding waves with same frequency

We know that:

$$\sin(a + b) = \sin(a) \cos(b) + \sin(b) \cos(a)$$

So:

$$\sin(b_i x + c_i) = \sin(b_i x) \cos(c_i) + \sin(c_i) \cos(b_i x)$$

If 2 terms have the same value for b_i , then:

$$a_i \sin(b_i x + c_i) + a_j \sin(b_j x + c_j) = a_i \sin(b_i x + c_i) + a_j \sin(b_i x + c_j)$$

$$a_i \sin(b_i x + c_i) + a_j \sin(b_j x + c_j) = a_i \sin(b_i x) \cos(c_i) + a_i \sin(c_i) \cos(b_i x) + a_j \sin(b_i x) \cos(c_j) + a_j \sin(c_j) \cos(b_i x)$$

So we now get for:

$$s(x) = \sum_{i=1}^m a_i \sin(b_i x + c_i)$$

$$a = [, -1]$$

$$b = [, 4]$$

$$c = [, 0]$$