1 Defining measure spaces

1.1 Measure space

In a metric space, the structure was defining a value for each two elements of the set.

In a measure space, the structure defines a value of subsets of the set.

A measure space includes the set X, subsets of the set, Σ, and a function μ which maps from Σ to $\mathbb{R}$.

1.1.1 Sigma algebra

Requirement for Σ.

1.2 Axioms for measures

1.2.1 Measures are non-negative

$\forall E \in \Sigma : \mu(E) \geq 0$

1.2.2 The measure for the null set is 0.

$\mu() = 0$

1.2.3 Disjoint sets are additive

$\mu(\vee_{k=1}^{\infty} E_k) = \sum \mu(E_k)$

Where all elements E_k are disjoint. That is, they have no elements in common.

2 Examples of measure spaces

2.1 The counting measure

$\mu(E)$

This provides the number of elements in E.

2.2 The probability measure

This is discussed in more detail in Statistics.