0.1 Density of the rationals

0.1.1 Rationals are dense in rationals

For any pair of rationals, there is another rational between them:

$a = \frac{p}{q}$

$b = \frac{m}{n}$

Where $b > a$.

We define a new rational:

$c = \frac{a+b}{2}$

$c = \frac{pn+qm}{2qn}$

This is a rational number.

We can write:

$a = \frac{2pn}{2qn}$

$b = \frac{2qm}{2qn}$

As $b > a$ we know $2qm > 2pn$

So: $a < c < b$