Contents

0.1 Cauchy-Schwarz inequality

This states that:

$|\langle u, v\rangle|^2 \leq \langle u, u\rangle \dot{\langle} v, v\rangle$

Consider the vectors u and v. We construct a third vector $u - \lambda v$. We know the length of any vector is non-negative. $0 \leq \langle u - \lambda v, u - \lambda v\rangle$

$0 \leq \langle u, u\rangle + \langle u, -\lambda v\rangle + \langle -\lambda v, u\rangle + \langle -\lambda v, -\lambda v\rangle$

$0 \leq \langle u, u\rangle - \bar{\lambda}\langle u, v\rangle - \lambda\langle v, u\rangle + \lambda\bar{\lambda}\langle v, v\rangle$

We now look for a value of λ to simplify this equation.

$\lambda = \dfrac{\langle u, v\rangle}{\langle v, v\rangle}$

$0 \leq \langle u, u\rangle - \dfrac{\langle v, u\rangle\langle u, v\rangle}{\langle v, v\rangle} - \dfrac{\langle u, v\rangle\langle v, u\rangle}{\langle v, v\rangle} + \dfrac{\langle u, v\rangle}{\langle v, v\rangle}\dfrac{\langle v, u\rangle}{\langle v, v\rangle}\langle v, v\rangle$

$0 \leq \langle u, u\rangle - \dfrac{|\langle u, v\rangle|^2}{\langle v, v\rangle}$

$|\langle u, v\rangle|^2 \geq \langle u, u\rangle\langle v, v\rangle$