

0.1 The exterior (wedge) product

The exterior product of two vectors is:

$$u \wedge v$$

0.2 The exterior product is anticommutative

This is anticommutative (alternating).

$$u \wedge v = -v \wedge u$$

This implies that:

$$u \wedge u = 0$$

0.3 The exterior product is distributive

$$(a + b) \wedge (c + d) = (a \wedge c) + (a \wedge d) + (b \wedge c) + (b \wedge d)$$

0.4 Expanding the exterior product of two vectors

Consider 2-dimensional vector space with the following vectors:

$$u = ae_1 + be_2$$

$$v = ce_1 + de_2$$

The exterior product is:

$$u \wedge v = (ae_1 + be_2) \wedge (ce_1 + de_2)$$

$$u \wedge v = (ae_1 \wedge ce_1) + (ae_1 \wedge de_2) + (be_2 \wedge ce_1) + (be_2 \wedge de_2)$$

$$u \wedge v = ac(e_1 \wedge e_1) + ad(e_1 \wedge e_2) + bc(e_2 \wedge e_1) + bd(e_2 \wedge e_2)$$

$$u \wedge v = ad(e_1 \wedge e_2) - bc(e_1 \wedge e_2)$$

$$u \wedge v = (ad - bc)(e_1 \wedge e_2)$$