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0.1 Vector potential

Given a vector field \mathbf{F} we may be able to identify another vector field \mathbf{A} such that:

$$\mathbf{F} = \nabla \times \mathbf{A}$$

0.1.1 Existence

We know that the divergence of the curl for any vector field is 0, so this applies to \mathbf{A} :

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$

Therefore:

$$\nabla \cdot \mathbf{F} = 0$$

This means that if there is a vector potential of \mathbf{F} , then \mathbf{F} has no divergence.

0.1.2 Non-uniqueness of vector potentials

Vector potentials are not unique.

If \mathbf{A} is a vector potential of \mathbf{F} , then so is $\mathbf{A} + \nabla c$, where c is a scalar field and ∇c is its gradient.

0.1.3 Conservative vector fields

Not all vector fields have scalar potentials. Those that do are conservative.

For example if a vector field is the gradient of a scalar height function, then the height is a scalar potential.

If a vector field is the rotation of water, there will not be a scalar potential.