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0.1 Cauchy sequences

0.1.1 Cauchy sequence

A cauchy sequence is a sequence such that for an any arbitrarily small number ϵ, there is a point in the sequence where all possible pairs after this are even closer together.

$(\forall\epsilon > 0)(\exists N \in \mathbb{N} : \forall m, n \in \mathbb{N} > N)(|a_m - a_n| < \epsilon)$

This last term gives a distance between two entries. In addition to the number line, this could be used on vectors, where distances are defined.

As a example, $\frac{1}{n}$ is a cauchy sequence, $\sum_i \frac{1}{n}$ is not.

0.1.2 Completeness

Cauchy sequences can be defined on some given set. For example given all the numbers between 0 and 1 there are any number of different cauchy sequences converging at some point.

If it is possible to define a cauchy sequence on a set where the limit is not in the set, then the set is incomplete.

For example, the numbers between 0 and 1 but not including 0 and 1 are not complete. It is possible to define sequences which converge to these missing points.

More abstractly, you could have all vectors where $x^2 + y^2 < 1$. This is incomplete (or open) as sequences on these vectors can converge to limits not in the set.

Cauchy sequences are important when considering real numbers. We could define a sequence converging on $\sqrt{2}$, but as this number is not in the set, it is incomplete.