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0.1 How many unique operators are there?

An arbitrary operator takes n inputs are returns T or F.

With 0 inputs there is one posible permutation. For every additional input the number of possible permutations doubles. Therefore there are 2^n possible permutations.

For the operator with one permutation there are two operators. For every additional permutation the number of operator doubles. Therefore there are $2^{(2^n)}$ possible operations.

With 0 inputs, we need 2 different operators to cover all outputs. For 1 input we need 4 and for 2 inputs we need 16.

0.2 We don't need 0-ary operators

There are two unique 0-ary operators. One always returns T and the other always returns F. These are already described.

0.3 We need one unary operator

For the operators with 1 input we have:

- one which always returns T
- one which always returns F
- one which always returns the same as the input
- one which returns the opposite of the input

It is this last one, negation, shown as $\neg$ and is of most interest.

0.4 We can use a subset of binary operators

The full list of binary operators are included below.

Of these, the first two are 0-ary operators, and so are not needed. The next four are unary operators, and so are not needed.

The non-implications can be rewritten using negation.

0.5 Brackets replace the need for n-ary operators

N-ary operators contain 3 or more inputs.

N-ary operators can be defined in terms of binary operators.

As an example if we want an operator to return positive if all inputs are true, we can use:

$(\theta \wedge \gamma) \wedge \beta$