0.1 Axiom of infinity

The axiom of infinity states that:

$\exists I(\varnothing \in I \wedge \forall x \in I((x \vee \{x\}) \in I))$

There exists a set, called the infinite set. This contains the empty set, and for all elements in I the set also contains the successor to it.

0.1.1 Sequential function

Let's define the sequential function:

$s(n) := \{n \vee \{n\}\}$

We can now rewrite the axiom of infinity as:

$\exists \mathbb{N}(\varnothing \in \mathbb{N} \wedge \forall x \in \mathbb{N}(s(x) \in \mathbb{N}))$

0.1.2 Zero

This set contains the null set: $\varnothing \in \mathbb{N}$.

Zero is defined as the empty set.

$0 := \{\}$

0.1.3 Natural numbers

For all elements in the infinite set, there also exists another element in the infinite set: $\forall x \in \mathbb{N}((x \vee \{x\}) \in \mathbb{N})$.

We then define all sequential numbers as the set of all preceding numbers. So:

$1 := \{0\} = \{\{\}\}$

$2 := \{0, 1\} = \{\{\}, \{\{\}\}\}$

$3 := \{0, 1, 2\} = \{\{\}, \{\{\}\}, \{\{\}, \{\{\}\}\}\}$

0.1.4 Existence of natural numbers

Does each natural number exist? We know the infinite set exists, and we also know the axiom schema of specification:

Point is: For each set, all finite subsets exist. PROVE ELSEWHERE

0.1.5 From infinite set to natural set

We don't know I is limited to natural numbers. Could contain urelements etc.

0.1.6 More

Infinite set axiom written using N. should be I

I could be superset of N, for example set of all natural numbers, and also the set containing the set containing 2.

Can extract N using axiom of specification

We need a way to define the set of natural numbers:

$\forall n(n \in \mathbb{N} \leftrightarrow ([n = \emptyset \vee \exists k(n = k \vee \{k\})] \wedge))$

If we can can define N, we can show it exists from specicication

$\forall x \exists s[P(x) \leftrightarrow (x \in s)]$

$\forall n \exists s[n \in N \leftrightarrow (n \in s)]$