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1 Ordinary Differential Equations (ODEs)

1.1 Introduction

1.1.1 Order of differential equations

1.1.2 Implicit and explit differential equations

An ordinary differential equation is one with only one independent variable. For example:

$\frac{dy}{dx} = f(x)$

The order of a differential equation is the number of differentials of y included. For example one with the second derivative of y is of order 2.

Ordinary equations can can either implicit or explicit. An explicit function shows the highest order derivative as a function of other terms.

An implicit function is one which is not explicit.

A linear ODE is an explicit ODE where the derivative terms of y do not multiply together, that is, in the form:

$y^{(n)} = \sum_i a_i(x)y^{(i)} + r(x)$

1.1.2.1 First-order ODEs

We have an evolution:

$\frac{dy}{dt} = f(t, y)$

And a starting condition:

$y_0 = f(t_0)$

We now discuss various ways to solve these.

1.2 First-order Ordinary Differential Equations

1.2.1 Ordinary differential equations

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We now discuss various ways to solve these.

1.2.2 Linear first-order Ordinary Differential Equations

1.2.2.1 Linear ODEs

For some we can write:

$\frac{dy}{dt}=f(t,y)$

$\frac{dy}{dt}=q(t)-p(t)y$

This can be solved by multiplying by an unknown function $\mu(t)$:

$\frac{dy}{dt}+p(t)y=q(t)$

$\mu(t)[\frac{dy}{dt}+p(t)y]=\mu(t)q(t)$

We can then set $\mu(t) = e^{\int p(t)dt}$. This means that $\frac{d\mu}{dt} = p(t)u(t)$

$\frac{d}{dt}[\mu(t)y] = \mu(t)q(t)$

$\mu(t)y = \int \mu(t)q(t)dt + C$

In some cases, this can then be solved.

1.2.2.2 Example

$\frac{\delta y}{\delta x} = cy$

$y = Ae^{c(y+a)}$

$\frac{\delta^2 y}{\delta x^2} = cy$

$y = Ae^{\sqrt{c}(y+a)}$

1.2.3 Separable first-order Ordinary Differential Equations

For some we can write:

$\frac{dy}{dt} = f(t, y)$

$\frac{dy}{dt} = \frac{g(t)}{h(y)}$

We can then do the following:

$h(y)\frac{dy}{dt} = g(t)$

$\int h(y)\frac{dy}{dt}dt = \int g(t)dt + C$

$\int h(y)dy = \int g(t)dt + C$

In some cases, these functions can then be integrated and solved.

1.3 Second-order Ordinary Differential Equations

1.3.1 Linear second-order Ordinary Differential Equations

These are of the form

$\frac{d^2y}{dt^2} + p(t)\frac{dy}{dt} + q(t)y = g(t)$

There are two types. Homogenous equations are where $g(t) = 0$. Otherwise they are heterogenous.

We explore the case with constants:

$a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + cy = 0$