0.1 Continous functions

A function is continuous if:

$\lim_{x\to c} f(x) = f(c)$

For example a function $\frac{1}{x}$ is not continuous as the limit towards 0 is negative infinity. A function like $y = x$ is continous.

More strictly, for any $\epsilon > 0$ there exists

$\delta > 0$

$c - \delta < x < c + \delta$

Such that

$f(c) - \epsilon < f(x) < f(c) + \epsilon$

This means that our function is continuous at our limit c, if for any tiny range around $f(c)$, that is $f(c) - \epsilon$ and $f(c) + \epsilon$, there is a range around c, that is $c - \delta$ and $c + \delta$ such that all the value of $f(x)$ at all of these points is within the other range.

0.1.1 Limits

Why can't we use rationals for analysis?

If discontinous at not rational number, it can still be continous for all rationals.

Eg f(x)=-1 unless x^2>2, where f(x)=1

Continous for all rationals, because rationals dense in reals.

But can't be differentiated.