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0.1 Concave and convex functions

0.1.1 Convex functions

A convex function is one where:

$\forall x_1, x_2 \in \mathbb{R} \forall t \in [0,1][f(tx_1 + (1-t)x_2 \leq tf(x_1) + (1-t)f(x_2)]$

That is, for any two points of a function, a line between the two points is above the curve.

A function is strictly convex if the line between two points is strictly above the curve:

$\forall x_1, x_2 \in \mathbb{R} \forall t \in (0,1)[f(tx_1 + (1-t)x_2 < tf(x_1) + (1-t)f(x_2)]$

An example is $y = x^2$.

0.1.2 Concave functions

A concave function is an upside down convex function. The line between two points is below the curve.

$\forall x_1, x_2 \in \mathbb{R} \forall t \in [0,1][f(tx_1 + (1-t)x_2 \geq tf(x_1) + (1-t)f(x_2)]$

A function is strictly concave if the line between two points is strictly below the curve:

$\forall x_1, x_2 \in \mathbb{R} \forall t \in (0,1)[f(tx_1 + (1-t)x_2 > tf(x_1) + (1-t)f(x_2)]$

An example is $y = -x^2$.

0.1.3 Affine functions

If a function is both concave and convex, then the line between two points must be the function itself. This means the function is an affine function.

$y = cx$