

0.1 Cauchy-Schwarz inequality

This states that:

$$|\langle u, v \rangle|^2 \leq \langle u, u \rangle \langle v, v \rangle$$

Consider the vectors u and v . We construct a third vector $u - \lambda v$. We know the length of any vector is non-negative. $0 \leq \langle u - \lambda v, u - \lambda v \rangle$

$$0 \leq \langle u, u \rangle + \langle u, -\lambda v \rangle + \langle -\lambda v, u \rangle + \langle -\lambda v, -\lambda v \rangle$$

$$0 \leq \langle u, u \rangle - \bar{\lambda} \langle u, v \rangle - \lambda \langle v, u \rangle + \lambda \bar{\lambda} \langle v, v \rangle$$

We now look for a value of λ to simplify this equation.

$$\lambda = \frac{\langle u, v \rangle}{\langle v, v \rangle}$$

$$0 \leq \langle u, u \rangle - \frac{\langle v, u \rangle \langle u, v \rangle}{\langle v, v \rangle} - \frac{\langle u, v \rangle \langle v, u \rangle}{\langle v, v \rangle} + \frac{\langle u, v \rangle}{\langle v, v \rangle} \frac{\langle v, u \rangle}{\langle v, v \rangle} \langle v, v \rangle$$

$$0 \leq \langle u, u \rangle - \frac{|\langle u, v \rangle|^2}{\langle v, v \rangle}$$

$$|\langle u, v \rangle|^2 \geq \langle u, u \rangle \langle v, v \rangle$$