

0.1 Existence of an infinite number of prime numbers

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If there are a finite number of primes, we can call the set of primes P .

We identify a new natural number a by taking the product of existing primes and adding 1.

$$a = 1 + \prod_{p \in P} p$$

From the fundamental theorem of arithmetic we know all numbers are primes or the products of primes.

If a is not a prime then it can be divided by one of the existing primes to form number n :

$$\prod_{p_j}^n p_i + 1 = n$$

$$\frac{p_j \prod_{i \neq j}^n p_i + 1}{p_j} = n$$

$$\prod_{i \neq j}^n p_i + \frac{1}{p_j} = n$$

As this is not a whole number, n must prime.

We can do this process for any finite number of primes, so there are an infinite number.