0.1 Powers

0.1.1 Exponents and logarithms

Previously we defined addition and multiplication in terms of successive use of the sucessor function. That is, the definition of addition was:

$\forall a \in \mathbb{N}(a + 0 = a)$

$\forall ab \in \mathbb{N}(a + s(b) = s(a + b))$

And similarly for multiplication:

$\forall a \in \mathbb{N}(a.0 = 0)$

$\forall ab \in \mathbb{N}(a.s(b) = a.b + a)$

Additional functions could also be defined, following the same pattern:

$\forall a \in \mathbb{N}(a \oplus_n 0 = c)$

$\forall ab \in \mathbb{N}(a \oplus_n s(b) = (a \oplus_n b) \oplus_{n-1} a)$

0.1.2 Powers

Exponents can also be defined:

0.1.3 Axioms

$\forall a \in \mathbb{N} a^0 = 1$

$\forall ab \in \mathbb{N} a^{s(b)} = a^b.a$

0.1.4 Example

So 2^2 can be calculated like:

$2^2 = 2^{s(1)}$

$2^{s(1)} = 2.2^1$

$2.2^1 = 2.2.2^0$

$2.2.2^0 = 2.2.1$

$2.2.1 = 4$

Unlike addition and multiplication, exponention is not commutative. That is

$a^b \neq b^a$

0.1.5 Exponential rules

$a^b a^c = a^{b+c}$

$(a^b)^c = a^{bc}$

$(ab)^c = a^c b^c$

0.1.6 Powers of natural numbers

0.1.7 Powers of integers

0.1.8 Powers of rational numbers