0.1 Fundamental Theorem of Arithmetic

0.1.1 Statement

Each natural number is a prime or unique product of primes.

0.1.2 Proof: existance of each number as a product of primes

If n is prime, no more is needed.

If n is not prime, then $n = ab$, $a, b \in \mathbb{N}$.

If a and b are prime, this is complete. Otherwise we can iterate to find:

$n = \prod_{i=1} p_i$

0.1.3 Proof: this product of primes is unique

Consider two different series of primes for the same number:

$s = \prod_{i=1}^{n} p_i = \prod_{i=1}^{m} q_i$

We need to show that $n = m$ and $p = q$.

We know that p_i divides s. We also know that through Euclid's lemma that if a prime number divides a non-prime number, then it must also divide one of its components. As a result p_i must divide one of q.

But as all of q are prime then p_i=q_j.

We can repeat this process to to show that $p = q$ and therefore $n = m$.