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0.1 σ -algebra

0.1.1 Review of algebra on a set

An algebra, Σ , on set s is a set of subsets of s such that:

- Closed under intersection: If a and b are in Σ then $a \wedge b$ must also be in Σ
- $\forall ab[(a \in \Sigma \wedge b \in \Sigma) \rightarrow (a \wedge b \in \Sigma)]$
- Closed under union: If a and b are in Σ then $a \vee b$ must also be in Σ .
- $\forall ab[(a \in \Sigma \wedge b \in \Sigma) \rightarrow (a \vee b \in \Sigma)]$

If both of these are true, then the following is also true:

- Closed under complement: If a is in Σ then $s \setminus a$ must also be in Σ

We also require that the null set (and therefore the original set, null's complement) is part of the algebra.

0.1.2 σ -algebra

A σ -algebra is an algebra with an additional condition:

All countable unions of sets in A are also in A .

This adds a constraint. Consider the real numbers with an algebra of all finite sets.

This contains all finite subsets, and their complements. It does not contain \mathbb{N} .

However a σ -algebra requires all countable unions to be including, and the natural numbers are a countable union.

The power set is a σ -algebra. All other σ -algebras are subsets of the power set.