0.1 Addition of natural numbers

0.1.1 Definition

Let's add another function: addition. Defined by:

$\forall a \in \mathbb{N}(a + 0 = a)$

$\forall ab \in \mathbb{N}(a + s(b) = s(a + b))$

That is, adding zero to a number doesn't change it, and $(a+b)+1 = a+(b+1)$.

0.1.2 Example

Let's use this to solve $1 + 2$:

$1 + 2 = 1 + s(1)$

$1 + s(1) = s(1 + 1)$

$s(1 + 1) = s(1 + s(0))$

$s(1 + s(0)) = s(s(1 + 0))$

$s(s(1 + 0)) = s(s(1))$

$s(s(1)) = s(2)$

$s(2) = 3$

$1 + 2 = 3$

All addition can be done iteratively like this.

0.1.3 Commutative property of addition

Addition is commutative:

$x + y = y + x$

0.1.4 Associative property of addition

Addition is associative:

$x + (y + z) = (x + y) + z$