Contents

0.1 Raising and lowering indices

We showed that the inner product between two vectors with the same basis can be written as:

$\langle v, w \rangle = \langle \sum_i e_i v^i, \sum_j f_j w^j \rangle$

$\langle v, w \rangle = v^i \overline{w^j} \langle e_i, e_j \rangle$

Defining the metric as:

$g_{ij} := \langle e_i, e_j \rangle$

$\langle v, w \rangle = v^i \overline{w^j} g_{ij}$

0.1.1 Metric inverse

We can use this to define the inverse of the metric.

$g^{ij} := (g_{ij})^{-1}$

We can use this to raise and lower vectors.

$v_i := v^j g_{ij}$

0.1.2 Raising and lowering indices of tensors

If we have tensor:

T_{ij}

We can define:

$T_i^k = T_{ij} g^{jk}$

$T^{il} = T_{ij} g^{jk} g^{kl}$

0.1.3 Tensor contraction

If we have:

$T_{ij} x^j$

We can contract it to:

$T_{ij}x^j = v_i$

Similarly we can have:

$T^{ij}x_j = v^i$