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0.1 Tensor product

We have spaces V and W over field F . If we have a linear operation which takes a vector from each space and returns a scalar from the underlying field, it is an element of the tensor product of the two spaces.

For example if we have two vectors:

$$v = e_i v^i$$

$$w = e_j w^j$$

A tensor product would take these and return a scalar.

There are three types of tensor products:

- Both are from the vector space
- $T_{ij} v^i w^j$
- $T_{ij} \in V \otimes W$
- Both are from the dual space
- $T^{ij} v_i w_j$
- $T_{ij} \in V^* \otimes W^*$
- One is from each space
- $T_i^j v^i w_j$
- $T_{ij} \in V \otimes W^*$

As a vector space, we can add together tensor products, and do scalar multiplication.

0.1.1 Homomorphisms

We can define homomorphisms in terms of tensor products.

$$Hom(V) = V \otimes V^*$$

$$T_j^i$$

We use the dual space for the second argument. This is because it ensures that changes to the bases do not affect the maps.

$$w^j = T_i^j v^i$$