0.1 Hessian matrix

We can take a function and make a matrix of its second order partial derivatives. This is the Hessian matrix, and it describes the local curvature of the function.

If the function f has n parameters, the Hessian matrix is $n \times n$, and is defined as:

$H_{ij} = \frac{\delta^2 f}{\delta x_i \delta x_j}$

If the function is convex, then the Hessian matrix is positive semi-definite for all points, and vice versa.

If the function is concave, then the Hessian matrix is negative semi-definite for all points, and vice versa.

We can diagnose critical points by evaluating the Hessian matrix at those points.

If it is positive definite, it is a local minimum. If it is negative definite it is a local maximum. If there are both positive and negative eigenvalues it is a saddle point.