0.1 Cantors theorem

The cardinality of the powerset is strictly greater than the cardinality of the underlying set.

That is, $|P(s)| < |s|$.

This applies to finite sets and infinite sets. In particular, this means that the powerset of the natural numbers is bigger than the natural numbers.

0.1.1 Proof

If one set is at least as big as another, then then is a surjection from that set to the other.

That is, if we can prove that there is no surjection from a set to its powerset, then we have proved the theorem.

We consider $f(s)$. If there is a surjection, then for every subset of s there should be a mapping from s to that subset.

We take set s and have the powerset of this, $P(s)$.

Consider the set:

$A = \{x \in s | x \notin f(x)\}$

That is, the set of all elements of s which do not map to the surjection.