

## 0.1 Bilinear maps

A bilinear map (or function) is a map from two inputs to an output which preserves addition and scalar multiplication. This is in contrast to a linear map, which only has one input.

In addition, the function is linear in both arguments.

That is if function  $f$  is bilinear then:

$$X = aM + bN$$

$$Y = cO + dP$$

$$f(X, Y) = f(aM + bN, cO + dP)$$

$$f(X, Y) = f(aM, cO + dP) + f(bN, cO + dP)$$

$$f(X, Y) = f(aM, cO) + f(aM, dP) + f(bN, cO) + f(bN, dP)$$

$$f(X, Y) = acf(M, O) + adf(M, P) + bcf(N, O) + bdf(N, P)$$

Note that:

$$f(X, Y) = f(X + 0, Y)$$

$$f(X, Y) = f(X, Y) + f(0, Y)$$

$$(0, Y) = 0$$

That is, if any input is 0 in an additive sense, the value of the map must be zero.