0.1 Tan

The $\tan(\theta)$ function is defined as:

$\tan(\theta) := \frac{\sin(\theta)}{\cos(\theta)}$

0.1.1 Behaviour around 0

$\sin(0) = 0$

$\cos(0) = 1$

$\tan(0) := \frac{\sin(0)}{\cos(0)}$

$\tan(0) = \frac{0}{1}$

$\tan(0) = 0$

0.1.2 Behaviour around $\cos(\theta) = 0$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

So $\tan(\theta)$ is undefined where $\cos(\theta) = 0$.

This happens where:

$\theta = \frac{\tau}{4} + \frac{1}{2}n\tau$

$\theta = \frac{1}{4}\tau(1 + 2n)$

Where $n \in \mathbb{Z}$.

0.1.3 Derivatives

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$

$\frac{\delta}{\delta\theta}\tan(\theta) = \frac{\delta}{\delta\theta}\frac{\sin(\theta)}{\cos(\theta)}$

$\frac{\delta}{\delta\theta}\tan(\theta) = \frac{\cos(\theta)}{\cos(\theta)} + \frac{\sin^2(\theta)}{\cos^n(\theta)}$

$\frac{\delta}{\delta\theta}\tan(\theta) = 1 + \tan^2(\theta)$

Note this is always positive. This means:

$\lim_{\cos(\theta)\to 0^+} = -\infty$

$\lim_{\cos(\theta)\to 0^-} = \infty$