1 HLLC overview

The HLLC solver puts the contact wave back into the HLL solver.

1. Get wave speeds S_L, $S_\star$, S_R.
2. Construct $\mathbf{U}_L^\star$ and $\mathbf{U}_R^\star$.
3. Calculate $\mathbf{F}_*^{hllc}$.

2 Wave speeds

Wave speeds are obtained from approximate simple Riemann solvers depending on the left-right states. These solvers are the primitive variable RS (PVRS), the two-rarefaction RS (TRRS), and the two-shock RS (TSRS). If the pressure jump at the interface is less than a user specified ratio (typically, $p_{max}/p_{pmin} < 2$) then the flow is smooth and the PVRS is used to estimate $p_\star$ and $u_\star$. If the pressure jump is larger than this ratio, there is likely either a shock or a rarefaction present. If the interface pressure, $p_\star$, given from the PVRS is less than p_{min}, then the rarefaction solver, TRRS, is used, else the shock solver, TSRS, is used.

The estimates for the three approximate solvers for the pressure and velocity are,

$$p_{pvrs} = \frac{1}{2}(p_L + p_R) - \frac{1}{2}(u_R - u_L)C \tag{1}$$

$$u_{pvrs} = \frac{1}{2}(u_L + u_R) - \frac{1}{2}\frac{p_R - p_L}{C} \tag{2}$$

$$C = \frac{\rho_L + \rho_R}{2}\frac{a_L + a_R}{2} \tag{3}$$

$$p_{trrs} = \left[\frac{a_L + a_R - \frac{\gamma-1}{2}(u_R - u_L)}{a_L/p_L^z + a_r/p_R^z}\right]^z \tag{4}$$

$$u_{trrs} = \frac{P_{LR}u_L/a_L + u_R/a_R + \frac{2(P_{LR}-1)}{\gamma-1}}{P_{LR}/a_L + 1/a_R} \tag{5}$$

$$z = \frac{\gamma - 1}{2\gamma} \qquad P_{LR} = \left(\frac{p_L}{p_R}\right)^z \tag{6}$$

$$p_{tsrs} = \frac{g_L(p_0)p_L + g_R(p_0)p_R - (u_R - u_L)}{g_L(p_0) + g_r(p_0)} \tag{7}$$

$$u_{tsrs} = \frac{1}{2}(u_L + u_R) + \frac{1}{2}\left[(p_{tsrs} - p_R)g_R(p_0) - (p_{tsrs} - p_L)g_L(p_0)\right] \tag{8}$$

$$g_K(p) = \sqrt{\frac{A_K}{p + B_K}} \qquad p_0 = max(0, p_{pvrs}) \tag{9}$$

$$A_K = \frac{2}{\rho_K(\gamma+1)} \qquad B_K = \left(\frac{\gamma-1}{\gamma+1}\right) p_K \tag{10}$$

The estimates for the interface pressure and velocity are then,

$$p_\star, u_\star = \begin{cases} p_{pvrs}, u_{pvrs} & \frac{p_{max}}{p_{min}} < 2 \\ p_{trrs}, u_{trrs} & \frac{p_{max}}{p_{min}} > 2 \,\text{and}\, p_{pvrs} < p_{max} \\ p_{tsrs}, u_{tsrs} & \frac{p_{max}}{p_{min}} > 2 \,\text{and}\, p_{pvrs} > p_{max} \end{cases} \tag{11}$$

Now that we have $p_\star$ and u_star we can calculate the minimum, maxmimum and intermediate wave speeds as,

$$S_L = u_L - a_L q_L \tag{12}$$

$$S_L = u_R + a_R q_R \tag{13}$$

$$S_\star = \frac{p_R - p_L + \rho_L u_L(S_L - u_L) - \rho_R u_R(S_R - u_R)}{\rho_L(S_L - u_L) - \rho_R(S_R - u_R)} \tag{14}$$

$$q_K = \begin{cases} 1 & p_\star \leq p_K \\ \sqrt{1 + \frac{\gamma+1}{2\gamma}\left(\frac{p_\star}{p_K} - 1\right)} & p_\star > p_K \end{cases} \tag{15}$$

3 Star region

Now that we have the wave speeds the conservative left and right states in the starred region are,

$$\mathbf{U}_K^\star = \rho_K \left(\frac{S_K - u_K}{S_K - S_\star}\right) \begin{bmatrix} 1 \\ S_\star \\ v_K \\ w_K \\ \frac{E_K}{\rho_K} + (S_\star - u_K)\left[S_\star + \frac{p_K}{\rho_K(S_K - u_K)}\right] \end{bmatrix} \tag{16}$$

Additionally, any passive scalar is advected in the same way as the tangential velocities, i.e

$$(\rho q)_\star^K = \rho_K \left(\frac{S_K - u_K}{S_K - S_\star}\right) q_k \tag{17}$$

4 Final flux

Finally, the HLLC flux is,

$$\mathbf{F}_{i+1/2}^{hllc} = \begin{cases} \mathbf{F}_L & 0 \leq S_L \\ \mathbf{F}_L + S_L(\mathbf{U}_\star^L - \mathbf{U}_L) & S_L \leq 0 \leq S_\star \\ \mathbf{F}_R + S_R(\mathbf{U}_\star^R - \mathbf{U}_R) & S_\star \leq 0 \leq S_R \\ \mathbf{F}_R & 0 \geq S_R \end{cases} \tag{18}$$