Masaryk University Faculty of Informatics



ECG Arrhythmia Detection and Classification

BACHELOR'S THESIS

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This is where a copy of the official signed thesis assignment and a copy of the Statement of an Author is located in the printed version of the document.

Declaration

Hereby I declare that this paper is my original authorial work, which I have worked out on my own. All sources, references, and literature used or excerpted during elaboration of this work are properly cited and listed in complete reference to the due source.

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Acknowledgements

Abstract

In the thesis we try to design a machine learning (ML) model for classification of heart arrhythmia from a 1-lead ECG signal recorded by telemonitoring devices. A part of the thesis is devoted to comparison of multiple ML methods on three datasets, two of which are publicly available.

Both the classical ML methods based on domain knowledge features and neural networks were tried in the comparison.

The output of the thesis is a public repository with the comparison of the methods mentioned above on the publicly available arrhythmia datasets. The resulting model is running in test mode as a part of a bigger solution on the telemonitoring server and is going to replace the existing solution over the coming months.

Keywords

machine learning, electrocardiography, telemedicine, signal processing, neural networks

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Introduction

Abnormalities of the heart rhythm (arrhythmia) are among the most common health problems in the population. They range in severity from benign palpitations to total loss of cardiac function and death [1]. One of the methods to diagnose heart arrhythmia is the use of electrocardiogram (ECG). However, a trained human expert is needed for precise diagnosis of a heart condition; some arrhythmia require long-term monitoring. When using a portable device for long-term cardiac monitoring, the resulting signal is too long to be thoroughly analysed by a human effectively. With automated arrhythmia classification, the expert only has to look at parts of the signal suggested by the model.

The model has to be able to classify six various heart rate abnormalities, the normal rhythm and noisy electrocardiograms.

In chapter 1, there is a brief explanation of the physiological background of ECG and description of heart arrhythmia we will classify. Chapter 2 contains an introduction to supervised learning lightly focused on learning from one-dimensional signals. Chapter 3 is a summary of software tools and libraries used (scikit-learn, PyTorch, ONNX Runtime, and more). Chapter 4 includes the description of datasets we used to train the models. The practical part of the thesis starts from chapter 5, where we discuss various machine learning models we tried to solve the problem along with their comparison on the ECG datasets. Chapter 6 is concerned with the training of the models. Chapter 7 is about the evaluation of the models and model export. In the last chapter 8, we will talk about further improvements and maintenance of the model.

We show the final model performance in the comparison table in chapter 4.

The full source code for the experiment is available under the MIT licence. For more information, see Appendix A.

1 Down the Electrocardiography Hole

1.1 Electrocardiogram

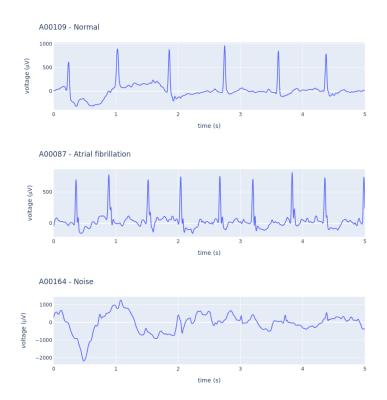


Figure 1.1: Electrocardiograms of normal rhythm, atrial fibrillation and a noisy rhythm.

Electrocardiogram (ECG) is a recording of the electrical activity of the heart. Electrodes on the skin detect small changes of voltage which are caused by heart muscle depolarization and the following repolarization. Two electrodes can measure the electric potential between them. This pair of electrodes is called an ECG lead. Every lead provides a different viewpoint of a heart's electrical activity. The most clinically used lead formation is 12-lead from ten electrodes. This work focuses on 1-lead (two electrodes) ECG often used in telemonitoring devices.

1.1.1 Telemonitoring



Figure 1.2: Holter monitor, a type of ECG telemonitoring device [2].

Telemonitoring is defined as "the use of information technology to monitor patients at a distance" [3]. In the context of ECG monitoring, a telemonitoring device is usually a small, wearable device such as a Holter monitor (Figure 1.2). Telemonitoring system can help decrease hospital admissions in cardiac patients [4].

1.1.2 Specific ECG elements

In order to identify heart arrhythmias, we have to be able to recognize the characteristic patterns in ECG. The essential elements of an ECG include P wave, QRS complex and T wave [1]. All these waves are shown together in Figure 1.3.

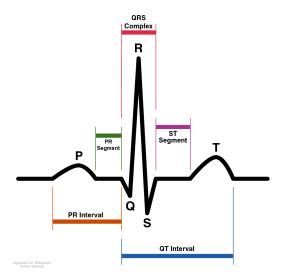


Figure 1.3: ECG of a single beat in normal sinus rhythm [5].

The P wave represents the electrical depolarization and the resulting muscle contraction of both atria, the upper chambers of the heart. The QRS complex represents the depolarization of the ventricles, lower chambers of the heart. The horizontal ST segment follows, and the cardiac cycle ends with a broad T wave, which represents the ventricular repolarization [6].

RR interval

RR interval is the time elapsed between two consecutive R waves which can be directly used to calculate the average heart rate HR:

$$HR = \frac{60}{RR \text{ interval in seconds'}}$$
 (1.1)

where the RR interval in seconds is averaged over several consecutive R peaks.

Several relevant machine learning features can be extracted from the RR intervals. Heart rate variability methods are concerned precisely with the problem of analysing inter-beat intervals. We discuss these methods more thoroughly in section 2.3.5.

1.2 Atrial fibrillation

Atrial fibrillation (AF) is an arrhythmia usually detected from ECG by irregular heartbeats (RR interval length is unstable) and an absence of a P wave preceding the QRS complex. AF is the most common cardiac rhythm disorder and is dangerous for the patient – it can lead to decreased quality of life, stroke or heart failure [7]. An example of atrial fibrillation electrocardiogram is shown in Figure 1.1. In the figure, we can see the characteristic irregular RR intervals. Atrial fibrillation increases the risk of heart failure, stroke, coronary heart disease and death [8].

1.3 Aims of the thesis

The thesis aims to design various machine learning models on multiple ECG arrhythmia databases. The resulting experiment is designated with respect to reproducibility and extensibility of both the datasets and models.

2 Feature Processing

Two different approaches to feature extraction are being compared in this work. One is the approach of crafting hand-engineered features from the signal by using domain knowledge. The second one is the use of neural networks to be used as automated feature extractors trained on the complete signal.

2.1 Signal processing

An electrocardiogram is an example of a discrete one-dimensional signal. Thus, the natural way to work with ECGs is to use methods of digital signal processing. A brief explanation of some signal processing key concepts follows. A more detailed introduction into signal processing is out of this thesis' scope, and there is already a great deal of quality material on the subject available online or in signal processing textbooks, e.g., *Discrete-time signal processing* [9].

Sampling

Heart electrical activity is a continuous analogue signal. To get a digital representation of the underlying analogue signal, we need to sample and quantize it. The sampling rate of a signal is the number of samples per time interval. The standard unit of sampling rate is Hz (1 Hz = 1 sample per second). ECG sampling rate usually falls within the range of 100-1000 Hz, but there does not exist a single sampling rate used ubiquitously.

Filtering

A discrete-time signal can be mapped from the real space into the Fourier (frequency) space by the Fourier transform. That means we can think about the signal not as a function of time, but a function of frequency [10].

A band-pass filter has two cut-off frequencies. The signals under the lower cut-off frequency and over the higher cut-off frequency are attenuated. Filter design is the study of designing signal processing filters that satisfy the requirements. More information about filter design can be found in *Discrete-time signal processing* [11].

2.2 Preprocessing

We used resampling to reduce the dimensionality of the signal and band-pass filtering to filter out frequency spectra which contain little information about heart rhythm.

2.2.1 Resampling

We can reduce the dimensionality of the signal without losing too much of its quality by resampling – changing its sampling rate. Most of the heartbeat information is in the lower frequencies under 50 Hz, and thus we can still see the electrocardiogram clearly when using relatively small sampling rate.

As we can see in Figure 2.1, the second recording resampled to 100 Hz still retains the quality of the signal, and the different segments of ECG are easily detectable. A sampling rate of 10 Hz is too low; parts of the recording are inevitably lost.

For feature extraction, we did not do any resampling. Some features, especially the frequency-domain ones, benefit from larger sampling rate. A sampling rate of 250 Hz or greater is ideal. For time-domain analysis, a frequency of 100 Hz is acceptable [12]. Thus, we resampled all recordings for neural network-based methods to 100 Hz using a polyphase filter. We used the resample_poly function with default parameters from SciPy, a Python library [13].



Figure 2.1: Recording CINC2017/A00002 sampled at original frequency, 100 Hz and 10 Hz.

2.2.2 Band-pass filtering

As most of the relevant information of the signal is only in a narrow frequency spectrum, we can suppress the frequencies out of the spectrum in the signal. For this, we can use a digital band-pass filter which removes both the very low $(0-0.5~{\rm Hz})$ and high $(>40~{\rm Hz})$ frequency noise.

In the cases of datasets where there is no information about filtering, we preprocess the signals using a band-pass filter in the frequency range of 0–40 Hz. We do zero-phase filtering using a fifth-order Butterworth finite impulse response filter. We use the SciPy Python library for filter design.

2.2.3 Standardization

Models with hand-engineered features

For the methods where the features are extracted from the signal using domain knowledge, we standardize each feature individually by removing the mean of the training set and dividing by the standard deviation of the feature.

Models with direct signal input

We use a sample-wise standardization when working with direct signal input (distance-based methods, neural networks). For a signal $X = X_1 X_2 X_3 ... X_n$, the standardized version Z is defined as:

$$Z = \frac{X - \bar{X}}{\sigma(X)},\tag{2.1}$$

where \bar{X} is the mean and $\sigma(X)$ is the standard deviation of the signal. The sample-wise standardization helps us to get all samples to the same range and baseline of sample values, as every patient can have its characteristic voltage range. When there is a large amount of high-amplitude noise in the recording, the sample standard deviation can be extreme, and the resulting heart waves can become hard to read after standardization. We do not deal with this problem, and it could theoretically be a problem which affects performance; thus, it is promising to explore the phenomenon more in the future.

2.3 Feature extraction

Electrocardiogram signal is high-dimensional (thousands of samples), and it can be helpful to extract features of the signal. We use generic signal statistics (mean, range, standard deviation, skew, kurtosis, extremes, percentiles). We also add domain-level features based on the length of the signal segments. For most of these methods, we need to extract and label the R peaks. Robust R peak detection in noisy electrocardiograms is not trivial, and it is still an open research problem [14].

2.3.1 R peak detection

For R peak detection, the Python physiological signal library NeuroKit2 (Section 3.2) was used. It supports a variety of R peak detection algorithms. We chose the NeuroKit custom method as it can extract R peaks even from some noisy signals (see Figure 2.2). In comparison, the standard Pan–Tompkins algorithm [15] does not detect any real R peaks in the noisy signal in the figure.

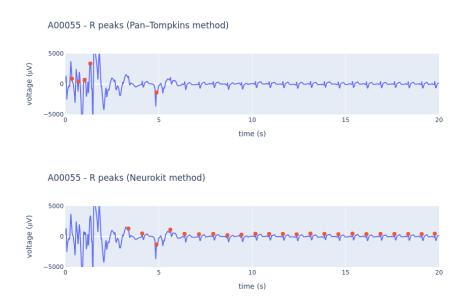


Figure 2.2: R peak detection in a noisy signal. Recordings from CINC2017.

2.3.2 R peak features

With R peak annotations, we can extract potentially useful features for arrhythmia detection. These include RR interval statistics, the corresponding heart rate, and heart rate variability features (Section 2.3.5).

2.3.3 QRS correlation features

We also create features by correlating the area near the QRS peaks. Missing P waves, a symptom of atrial fibrillation, can be numerically detected by low correlation of the area before the QRS complex. These features have already been successfully tried for the detection of atrial fibrillation [16].

2.3.4 PRT segments

Neurokit2 also has an algorithm for segmenting the electrocardiogram into all the ECG segments referenced in Figure 1.3. We can extract

intervals between the segments. Some arrhythmia are even defined by prolonged intervals of these segments. For example, the first degree atrioventricular block (I-AVB) is, by definition, a heart rhythm with a PR interval longer than 0.2 seconds [17]. The segmentation is a problem of at least the same complexity as R peak detection, so we cannot expect the detected PRT segments will be accurate all the time.

2.3.5 Heart rate variability

Heart rate variability (HRV) is the variation of the times between heartbeats (RR intervals). Heart rate variability features can be divided into three categories – time domain, frequency domain and non-linear features [18]. We use the Neurokit2 library, which is self-proclaimed to be "the most comprehensive software for computing HRV indices" [19]. As of NeuroKit2 version 0.0.40, the library can compute 19 time-domain, 11 frequency-domain and nine non-linear features. The inputs for all HRV methods are the positions of R peaks.

List of hand-engineered features

Here we present the summarized list of hand-engineered features used for training and inference. For a complete list of all features, see Appendix B.

Table 2.1: Features extracted from the signal.

Туре	Count
Signal statistics	12
RR interval features	11
QRS correlation features	11
PRT interval features	55
HRV features	24
Total	113

3 Software Tools

The whole implementation was written in Python with the support of several libraries, mainly the machine learning and signal processing ones. A brief description of the tools used follows.

3.1 SciPy

The SciPy library is a scientific computation library with a heavy emphasis on time and memory-efficient numerical routines. It consists of about 600,000 lines of code in 16 subpackages and is being actively developed since 2001 [20].

In this work, the signal subpackage is used for signal processing and the stats subpackage for various statistic features computation.

3.2 NeuroKit2

NeuroKit2 is a work-in-progress successor to the library NeuroKit.py. The library provides access to advanced biological signal processing functions. The supported biosignals comprise ECG (electrocardiography signals), EDA (electrodermal activity), PPG (photoplethysmogram), EMG (electromyography signals) and EEG (electroencephalography signals) [21].

We use the ECG processing subpart of the package, namely the QRS detector (Section 2.3.1) and the PQRST segmentation routine (Section 2.3.4). We also compute the HRV features (Section 2.3.5) using this package.

3.3 Scikit-learn

Scikit-learn is a library which encompasses a range of machine learning algorithms for supervised and unsupervised classification problems. It includes functions for data preprocessing, model selection, dimensionality reduction, classification and more [22].

We use scikit-learn for the classical ML pipeline: preprocessing, training and evaluating models. The random forest and logistic regression models are also implemented by this library.

3.4 PyTorch

PyTorch is a high-performance object-oriented deep learning library. A dynamical computational graph is the core structure for neural network layers. It integrates acceleration libraries for both CPU (central processing unit) and GPU (graphics processing unit). An automatic differentiation framework is also included [23].

We use PyTorch for training neural network models. PyTorch also supports export of trained models to the ONNX format, which is useful for inference.

3.5 ONNX

Open Neural Network Exchange (ONNX) is an open-source format for trained ML models. It supports inference on both PC and mobile devices using the ONNX Runtime. The ONNX Runtime also supports multiple inference accelerators, one of which is also a GPU acceleration [24].

ONNX bindings exist for a great variety of programming languages: Python, C#, C++, C, Java, Ruby and JavaScript. We use the ONNX models and the ONNX Runtime C# package for easy model replacement/updates.

3.6 Other libraries

We use Pandas [25] for data analysis, progress bars from tqdm [26], NumPy [27] for its efficient vectorized computation model, tslearn [28] for the *k*-NN DTW classifier (Section 5.1.1), XGBoost [29] for the gradient boosted tree model (Section 5.2.2) and Plotly [30] for generating the plots in this thesis.

4 Datasets

We use three datasets in total for evaluating the models. Several arrhythmia open datasets exist with various lengths of recordings, the number of classes and number of ECG leads. From these, we have chosen The PhysioNet Computing in Cardiology Challenge 2017 and The China Physiological Signal Challenge 2018 dataset. For an interested reader, comprehensive lists of arrhythmia datasets is a part of the PTB-XL publication [31]. We also used a private dataset which is larger than any open one we have found.

Our practical implementation is extensible and adjusted for new datasets to be added to the comparison.

4.1 The PhysioNet Computing in Cardiology Challenge 2017

AF Classification from a short single lead ECG recording was the theme of The PhysioNet Computing in Cardiology Challenge (CINC) 2017 [32]. It is of our interest since the samples provided resemble our use case – single lead data from a telemonitoring device. The 8528 recordings ranging from 9 to 61 seconds are sampled at 300 Hz and are already preprocessed by a band-pass filter with a bandwidth of 0.5-40 Hz. There are only three classes included in the challenge original scoring metric – the noise class *X* was added subsequently. Five teams shared the first place in the competition. The winners mainly used ensemble methods: XGBoost + RNN [33], random forests [34, 35] and custom ensemble algorithms [36, 37].

Table 4.1: CINC 2017 labels

Label	Condition	Count
N	Sinus Rhythm	5154
A	Atrial Fibrillation	771
0	Other Rhythm	2557
X	Noisy	46
Total		8528

4.2 The China Physiological Signal Challenge 2018

A set of 12-lead ECGs was made public for The China Physiological Signal Challenge (CPSC) 2018 [38]. There is a total of 9831 recordings from 9458 different patients. Out of those, 6877 is available in the form of a training set. The recordings are sampled at 500 Hz. The length of the recording ranges from six to 60 seconds. There is a total of nine distinct classes. There is a total of 680 multi-label recordings (the patient had more than one heart condition); we do not use them as some of our models are not fit for multi-label classification. We used only the first lead I (channel) as it is similar to a single-lead ECG. Moreover, the winners of the challenge achieved only slightly worse performance when using a single lead for classification [39].

Table 4.2: CPSC 2018 labels

Label	Condition	Count
Normal	Sinus Rhythm	918
AF	Atrial Fibrillation	1098
I-AVB	Atrioventricular Block	704
LBBB	Left Bundle Branch Block	207
RBBB	Right Bundle Branch Block	1695
PAC	Premature Atrial Contraction	556
PVC	Premature Ventricular Contraction	672
STD	ST Segment Depression	825
STE	ST Segment Elevation	202
Total		6877

4.3 Private dataset

This dataset was acquired by telemonitoring thousands of patients throughout several years. The sampling frequency is 200 Hz; the length of every recording is 30 seconds. In the table below, we can see the distribution of class labels in the data. This dataset is the one that has

been created for the task of training the previous model for arrhythmia classification.

Table 4.3: Private dataset labels

Label	Condition	Count
AF	Atrial Fibrillation	9966
AVB	Atrioventricular Block	9980
NK	Noisy	5312
PAC	Premature Atrial Contraction	9988
PVC	Premature Ventricular Contraction	9996
SR	Sinus Rhythm	8247
SVT	Supraventricular Tachycardia	9961
VT	Ventricular Tachycardia	10000
Total		73450

4.4 Other datasets

There exist more publicly available electrocardiogram datasets. Although we did not use them in the practical part, we wanted to give an overview of some of them and the reason why we do not compare the machine learning models on them in this work.

4.4.1 MIT-BIH

The MIT-BIH (Massachusetts Institute of Technology - Beth Israel Hospital) Arrhythmia Database [40] has been the standard for analysis of arrhythmia classification methods. The dataset is small – it contains 48 recordings from 47 patients. There is a high number of papers investigating this dataset. We get about 12,100 results on Google Scholar if we search for "MIT-BIH Arrhythmia Database" related articles [41]. We decided not to use the dataset because of its small size and over saturation of articles analysing it.

4.4.2 PTB

The PTB (Physikalisch-Technische Bundesanstalt) Diagnostic ECG Database [42] is another ECG database that is widely cited (3,150 results on Google Scholar). It contains 549 records from 290 patients. The only class with a significant number of recordings is Myocardial infarction, with 148 recordings. As we do not try to detect myocardial infarction in this thesis, it does not make much sense to include it in the benchmarks.

4.4.3 PTB-XL

This dataset [31] contains 21,837 clinical 12-lead ECG short recordings from 18885 patients and is the largest public ECG dataset as of July 2020. As it was published in April 2020, there was not enough time to do a detailed analysis of it. One comparison study focused on deep learning networks on the PTB-XL dataset is already available [43]. Further analysis of machine learning methods for classification of arrhythmia in the dataset is a promising future direction to explore.

5 Models

This chapter lists all the models that were tested on the selected arrhythmia datasets. The selection of models was influenced by the winning entries of CINC 2017 and CPSC 2018. Therefore, we chose models from the family of ensemble methods (random forest – Section 5.2.1, XGBoost – Section 5.2.2) based on hand-engineered features and neural network models trained on direct signal (CNN – Section 5.3.1, RNN – Section 5.3.2). As baseline classifiers, we used *k*-nearest neighbour classifier with DTW distance metric (Section 5.1.1) and a logistic regression (Section 5.1.2) model based on the hand-engineered features.

The descriptions of the machine learning models are not complete or detailed. For a more thorough review, we recommend the textbooks *Hands-On Machine Learning with Scikit-Learn & TensorFlow* [44] and *Deep Learning* [45].

5.1 Baselines

These baselines are simple models which can give us an idea of how are the other models better than a trivial classifier.

5.1.1 k-nearest neighbors with dynamic time warping

k-nearest neighbors (*k*-NN) is one of the simplest supervised learning algorithms. The main idea is that new examples are of the class of their nearest neighbours. The important parameter is *k*, which means how many neighbours influence the classification. The metric used when comparing the examples is also very important. The commonly used metrics include, but are not limited to, Minkowski distance, correlation, cosine similarity or dynamic time warping [46].

As for the implementation, we used the KNeighborsTimeSeries-Classifier with its default dtw metric from the tslearn Python library.

Dynamic time warping

The problem of classifying arrhythmia from ECG is a time series classification problem. Time series classification is a mature research field,

and there is plenty of algorithms trying to solve the problem [47]. However, most of the algorithms are evaluated on small datasets (hundreds of records) from The UCR (University of California, Riverside) Time Series Archive [48]. Dynamic time warping (DTW) is considered to be the standard for measuring time-series similarity. It was shown that a 1-NN classifier with DTW distance is a powerful time-series classification method [47].

5.1.2 Logistic regression

Logistic regression is one of the simpler supervised learning algorithms. For each sample \mathbf{x} and class k, a score $s_k(\mathbf{x})$ is computed:

$$s_k(\mathbf{x}) = \mathbf{x}^\mathsf{T} \theta^{(k)},\tag{5.1}$$

where $\theta^{(k)}$ is the parameter vector of the model for class k and the k^{th} row of the full parameter matrix Θ . Then the predicted class is:

$$\hat{y} = \operatorname{argmax}_k s_k(\mathbf{x}). \tag{5.2}$$

The model is usually trained by minimizing the cross entropy cost function *J*:

$$J(\Theta) = -\frac{1}{m} \sum_{i=1}^{m} \sum_{k=1}^{K} y_k^{(i)} \log(\hat{p}_k^{(i)}), \tag{5.3}$$

where $y_k^{(i)}$ is an indicator variable in the case of mutually exclusive classes, m is the total number of samples, K is the total number of classes and $\hat{p}_k^{(i)}$ is the model estimated probability of sample i belonging to class k calculated using the softmax function from the score $s_k^{(i)}$ [49].

We use the multinomial logistic regression class LogisticRegression from the scikit-learn package.

5.2 Models with hand-engineered features

These models take as the input the features from Section 2.3. Random forest and XGBoost classifiers are examples of ensemble machine learning models using the bagging and boosting ensemble techniques, respectively. Ensemble tree models were chosen because of ease of feature selection – they are not as sensitive to the quality of features.

Bagging

Bagging (bootstrap aggregating) is the technique of training the same type of model using random subsets of the training set. The sampling of random subsets is done with replacement. By collective voting of the classifiers trained on all subsets, the resulting ensemble model can gain robustness and be better than any single base classifier [50].

5.2.1 Random forest

Random forest is an ensemble model of decision tree classifiers using the bagging technique. A set of decision trees is trained on different random subsets on the data. Moreover, the nodes in the decision trees are not split by the best feature globally, but the best feature selected from a random set of features [50].

Boosting

Boosting is the technique of training multiple weak classifiers sequentially. Every classifier is trying to correct the mistakes that the previous models made. By sequentially improving the results of previous models, the final ensemble classifier is robust machine learning model. One of the boosting algorithms is Gradient Boosting [51]. The underlying weak classifier is usually a decision tree.

5.2.2 XGBoost

XGBoost is an optimized library using the gradient boosting algorithm for training ensemble classifiers. XGBoost stands for Extreme Gradient Boosting [29].

We use the XGBoostClassifier which the Python package provides as a scikit-learn compatible classifier.

5.2.3 Multilayer perceptron

5.3 Models with direct signal input

These methods take as the input the band-filtered, standardized electrocardiogram signal.

5.3.1 Convolutional networks

TODO

5.3.2 Recurrent networks

6 Training

6.1 Train-test split

We split all the datasets into train and test partitions which are same for all the models. The models are trained on the train partition, and the final evaluation is done on the test partition, which the model has not seen so that we can evaluate the generalization power of the model. For neural network-based models, we further separate a hold-out validation set out of the training set to evaluate learning performance during training.

The split is 90-10 % of data for train and test partitions, respectively. When using the validation partition, the final split is 80-10-10 % of train, validation and test partitions.

6.1.1 Cross-validation

Cross-validation is an alternative way of evaluating the generalization performance of a model. The K-fold cross-validation splits a dataset into K partitions (folds), then trains the model K times training on K-1 of the folds and evaluating on the last one. The K scores can be averaged to get the validation score.

For all models except CNN and RNN models, we use 5-fold cross-validation as the validation score. For CNN and RNN, we use the hold-out validation set score.

6.2 Classical models

TODO

6.3 Neural network models

6.4 Model checkpointing

7 Evaluation

7.1 Metrics

We need to evaluate the performance different models on the same data. For a binary classification problem with two classes, say *Pos* and *Neg*, a confusion matrix has two rows and two columns. True positives are the examples labelled as *Pos* and classified as *Pos*, true negatives are *Neg* classified as *Neg*, false positives are *Neg* classified as *Pos*, and false negatives are *Pos* classified as *Neg*.

Table 7.1: A binary classification task confusion matrix.

True Positives (TP)	True Negatives (TN)
False Positives (FP)	False Negatives (FN)

A confusion matrix for a multi-class problem has shape $N \times N$, where N is the number of classes. A single score instead of a full matrix is preferable for comparing the classifiers' performance. We calculate the F_1 score for all classifiers and use it for a rough comparison.

7.1.1 F1 score

The F_1 score is the harmonic mean of precision (positive predictive value) and recall (sensitivity) [52]:

$$F_1 = 2 \times \frac{\text{precision} \times \text{recall}}{\text{precision} + \text{recall}'}$$
 (7.1)

where

precision =
$$\frac{TP}{TP + FP}$$
, and recall = $\frac{TP}{TP + FN}$. (7.2)

The above definition is valid for binary classification. With multiclass classification, we do macro averaging. That means, we take a separate F_1 score for each class versus all the others as a binary problem; then, we get the resulting macro F_1 score as the mean of all the class scores. F_1 score is a simple way to compare classifiers. It favours classifiers with similar precision and recall; this is not always what we want when our task is classifying heart pathologies.

We use F_1 score for simplicity, but some types of arrhythmia are more dangerous than the others. For example, ventricular tachycardia is life-threatening as it can lead to deadly ventricular fibrillation. Therefore, a metric that takes into account the severity of different classes could prove useful in future work.

7.2 Results

TODO discussion of results.

Table 7.3: The final F_1 macro averaged scores on the test partitions of used datasets.

Model	CINC2017	CPSC2018	Private
LogisticRegression	TBA	TBA	TBA
KNNTimeSeriesClassifier	TBA	TBA	TBA
RandomForestClassifier	TBA	TBA	TBA
XGBClassifier	TBA	TBA	TBA
MLPClassifier	TBA	TBA	TBA
CNNClassifier	TBA	TBA	TBA
RNNClassifier	TBA	TBA	TBA

7.3 Inference and deployment

The models are automatically converted to the Open Neural Network Exchange format (Section 3.5). Exported models can be loaded in any application that supports the ONNX Runtime. We use the models in a .NET Core cross-platform analyser application for processing of electrocardiograms which come to the server every hour. The full description of the analyser is not in the scope of this thesis.

8 Conclusion, Future Works

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A Attachments

The source code for all experiments is available as a .zip file in the archive of the thesis at https://is.muni.cz/th/ybs7e/. It is also publicly available on GitHub at https://github.com/adamivora/ecg_arrhythmia_classification. A README.md file is included with the instructions to run the experiments.

The archive version contains the extracted feature Pandas data frames and trained models, so only the download of datasets is needed to run the comparison. The GitHub version does not contain any data and the full run through the pipeline can take two or more hours.

B Complete list of hand-engineered features

The table presents the full list of the features used to train models with hand-engineered features, along with descriptions of the features. The total number of numeric columns used for training is 113 – 24 HRV features and 89 segment and signal features.

The features of type [float] are actually a list of 11 statistical features listed in Table B.3. The HRV feature descriptions are based on the Neurokit2 documentation [19].

Name	Description	Туре		
Segment and signa	Segment and signal features:			
N_QRS	Number of QRS complexes detected.	integer		
P_Stats	Statistics of the P segment length.	[float]		
R_Stats	Statistics of the R segment length.	[float]		
T_Stats	Statistics of the T segment length.	[float]		
PR_Stats	Statistics of the PR intervals of the same beat.	[float]		
RR_Stats	Statistics of the consecutive RR intervals.	[float]		
RT_Stats	Statistics of the RT intervals of the same beat.	[float]		
QRS_Stats	Consecutive beat correlation statistics.	[float]		
Signal_Stats	Statistics of the band-pass preprocessed signal.	[float]		

Time-domain HRV features:			
HRV_RMSSD	Root mean square of successive differences between RR intervals.	float	
HRV_MeanNN	Mean of the RR intervals.	float	
HRV_SDNN	Standard deviation of RR intervals.	float	

HRV_SDSD	Standard deviation of successive differences between RR intervals.	float
HRV_CVNN	Calculated from the features HRV_SDNN / HRV_MeanNN.	float
HRV_CVSD	HRV_RMSSD / HRV_MeanNN.	float
HRV_MedianNN	Median of the absolute values of successive differences between RR intervals.	float
HRV_MadNN	Median absolute deviation of the RR intervals.	float
HRV_MCVNN	HRV_MadNN / HRV_MedianNN.	float
HRV_TINN	An approximation of the RR interval distribution.	float
HRV_HTI	The number of RR intervals divided by the height of histogram of RR intervals.	float
HRV_pNN20	Number of pairs of successive RR intervals that differ by more than 20 ms divided by total number of RR intervals.	float
HRV_pNN50	Number of pairs of successive RR intervals that differ by more than 50 ms divided by total number of RR intervals.	float

Frequency-domain HRV features:			
HRV_HF	SPD of the high frequency band.	float	
HRV_VHF	SPD of the very high frequency band.	float	
HRV_HFn	Normalized high frequency (HRV_HF / total power).	float	
HRV_LnHF	Log transformation of HRV_HF.	float	

Non-linear HRV features:		
HRV_SD1	Beat-to-beat variability.	float

B. Complete list of hand-engineered features

HRV_SD2	Index of long-term RR interval fluctuations.	float
HRV_SD2SD1	The ratio of HRV_SD2 / HRV_SD1.	float
HRV_CSI	Cardiac Sympathetic Index.	float
HRV_CSI_Modified	Modified Cardiac Sympathetic Index.	float
HRV_CVI	Cardial Vagal Index.	float
HRV_SampEn	Sample entropy.	float

Table B.3: List of statistics used.

Name	Description
min	The minimum.
max	The maximum.
mean	The arithmetic mean.
median	The median (50 th percentile).
perc25	The 25 th percentile (first quartile).
perc75	The 75 th percentile (third quartile).
perc99	The 99 th percentile.
range	The range $(max - min)$.
std	The standard deviation.
skew	The sample skewness (symmetry of the distribution).
kurtosis	The fourth standardized moment (tailedness of the distribution).