# n - Parameter Logistic Models (nPL)

For a through understanding of MIRT and Item Response Theory (IRT), I want to describe each logistic model. I generalized to *nPL* for $n = \{1, 2, 3\}$ parameters.

# Parameters:

## Ability $(\theta)$:

- In IRT, the *ability* parameter is denoted as $\theta$. This refers to the latent trait or *characteristic being measured by the test*. This could be the acedemic proficiciency in a subject to a personality trait.
- (e.g.) In a math test, $\theta$ represents the student's mathematical ability. A higher $\theta$ indicates a higher level in math.

## Difficulty $(b)$:

- The *difficulty* parameter $b$, indicates how challenging the item is. Each item in the test has this difficulty level.
- For 1PL, this is crucial this parameter is crucial. A higher $b$ value means that the question is more difficult.
- (e.g.) In the math test, a question on basic addition would have a low $b$ value (easy), while a more complex calculus question has a higher $b$ value.

## Discrimination $(a)$:

- This *discrimination* parameter is introduced in the 2PL, denoted $a$, indicates how well the item can differentiate between the individuals of diffferent abilities.
- A higher $a$ value means that the item is better at distinguishing between those who have mastered the material and those who haven't.
- (e.g.) A well designed quesition that most students with a good understanding of algebra get right, but those who get it wrong, would have a high value. A quesiton that has both high and low ability tend to answer similarity (either correct/ incorrect) would have a low value.

## Guessing $(c)$:

- The *guessing* parameter is unique to the 3PL model. It represents the probability that a person with low ability could guess the correct answer.
- $c$ is particularily relavent in multiple-choice tests where random guesses could lead to correct answers.
- (e.g.) In the multiple-choice math test, with four options per question, a question where one could easily eliminate two wrong answers can have a higher $c$ value since the probability of guessing between two options is higher.

## One Parameter Logistic Model: *Rasch Model*

- Equation:

$$P(\theta) = \frac{1}{1 + e^{-a(\theta - b)}}$$

- $\theta$: Ability of the individual.
- $b$: Difficulty of the item.
- The 1PL, the only item that is being estimated is $b$. It's assumed that all items have the same discrimination power (a = 1), and there's no guessing (c = 0).
- This model posits the probability of a correct response is a logistic function of the difference between the person's ability and the item's difficulty.

## Two Parameter Logistic Model: *(2PL)*

- Adds the *discrimination* parameter.
- Equation:

$$P(\theta) = c + (1 - c)\frac{1}{1 + e^{-a(\theta - b)}}$$

- $\theta$: Ability of the individual.
- $a$: Discrimination of the item.
- $b$: Difficulty of the item.
- Both the difficulty $b$ and the discrimination $a$ of an item are estimated. The discrimination parameter indicates how well an item distinguishes between the individuals with different levels of ability.

- The higher the discrimination parameter means the item is better at differentiating between individuals with abilities above or below the item's difficulty level.

## Three Parameter Logistic Model: *(3PL)*

- Further extends the modelk by including a guessing parameter.
- Equation:

$$P(\theta) = c + (1 - c)\frac{1}{1 + e^{-a(\theta - b)}}$$

- $\theta$: Ability of the individual.
- $a$: Discrimination of the item.
- $b$: Difficulty of the item.
- $c$: Guessing parameter.
- In the 3PL model, it estimates the difficulty $b$, discrimination $a$, and guessing parameters $c$. $c$ accounts for the likelihood that a low-ability $\theta$ individual might guess the answer correctly.
- Model is often used in multiple choice tests where the possibility of guessing correctly is non-zero.

$$P(\theta) = c + (1 - c)\frac{1}{1 + e^{-a(\theta - b)}}$$