UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1512

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CALCULUS A

Duration 2 Hours

100 Marks

EXAMINERS
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This paper consists of 3 pages

Instructions to candidates.

- The use of a pocket calculator is **NOT** permissible
- This paper consists of three pages Answer **ALL** questions
- Show **ALL** your workings

QUESTION 1

1 1 Determine the following limits (if it exists)

(a) $\lim_{x \to 1} f(x)$ if $f(x) = \begin{cases} x+1 & \text{if } x > 1 \\ -1 & \text{if } x = 1 \\ 3-x^2 & \text{if } x < 1 \end{cases}$ (4)

(b) $\lim_{x \to 2} \dfrac{x^3-8}{x^2-4}$ (4)

(c) $\lim_{x \to -\infty} (\sqrt{x^2+2x}+x)$ (4)

(d) $\lim_{x \to 0} \left(\dfrac{2}{x} - \dfrac{2}{|x|}\right)$ (4)

1 2 (a) Use the Squeeze Theorem to show that $\lim_{x \to \infty} \dfrac{\sin(x^2)}{x^3} = 0$ (5)

(b) Hence, evaluate $\lim_{x \to \infty} \dfrac{2x^3+\sin(x^2)}{1+x^3}$ (3)

[24]

[TURN OVER]

QUESTION 2

Let $f(x) = |x - 2|$

(*a*) Show that f is continuous at $x = 2$ by using the definition of continuity (5)

(*b*) Use the definition of the derivative to show that $f'(2)$ does not exist (6)

[11]

QUESTION 3

(3 1) Find the first derivatives of the following functions using the appropriate rules for differentiation Simply your answer

(*a*) $y = \frac{3}{x}\left(x^2 - \frac{2}{3x^2}\right)$ (4)

(*b*) $y = \frac{\cos x}{1 + \sin x}$ (6)

(*c*) $y = (\sqrt{x})^{\sin x}$ (6)

(*d*) $F(x) = \int_{2x}^{x^2} \sqrt{t^2 + 1}\, dt$ (6)

(3 2) For the function $x^2y^2 - 2x + 4y = 4$ find the equation of the tangent line at the point $x = 2$ (6)

[28]

QUESTION 4

Determine the following integrals

(*a*) $\int \frac{4}{x(4 \ln x + 1)}\, dx$ (3)

(*b*) $\int x\sqrt{4 + x}\, dx$ (3)

(*c*) $\int \cos^3 x \sin^2 x\, dx$ (4)

[TURN OVER]

(*d*) $\int_0^2 \frac{e^x}{1+e^{2x}}\,dx$ (5)

[15]

QUESTION 5

Let $f(x) = -x(x-2)$ and $g(x) = x^2$

(*a*) Sketch the graphs of f and g on the same axes (2)

(*b*) Find the area of the region determined by the intersection of the curves of f and g (6)

[8]

QUESTION 6

Solve the following Initial Value Problem

$$\frac{dy}{dx} = \frac{\cos^2 y}{4x-3}, \quad y(1) = \frac{\pi}{4}$$

[6]

QUESTION 7

Let $F(x, y) = x^2y - xy^2 + x^2 + y^2$

(*a*) Find the first partial derivatives F_x and F_y (2)

(*b*) Let $F(x, y) = 0$

(i) Use your answers in (*a*) above to write down $\frac{dy}{dx}$ (1)

(ii) Confirm your answer in (i) by finding $\frac{dy}{dx}$ using implicit differentiation (5)

[8]

TOTAL: [100]