ASSIGNMENT 04
Fixed Closing Date: 01 July 2022
Total Marks: 100
UNIQUE ASSIGNMENT NUMBER: ______

1. Determine the first order partial derivative of the following functions:

(a) $z = \ln\left(x + t^2\right)$ (5)

(b) $F\left(x,\ y\right) = \int_y^x \cos\left(e^t\right)dt$ (5)

(c) $f\left(x, y,\ z\right) = xy^2 e^{-xz}$ (5)

2. Clairaut's Theorem holds that $U_{xy} = U_{yx}$, show that the following equations obey Clairaut's Theorem.

(a) $U = \ln\left(x + 2y\right)$ (5)

(b) $U = e^{xy} \sin\, y$ (5)

3**.** Laplace's equations holds that $U_{xx} + U_{yy} = 0$, verify that the second derivative of the following equation are Laplace's equation:

(a) $U = \ln\sqrt{x^2 + y^2}$ (5)

(b) $U = x^2 - y^2$ (5)

4. Determine the following integrals:

(a) $\int\left(U^6 - 2U^5 + \frac{2}{7}\right)dU$ (5)

(b) $\int\left(\frac{1 + \sqrt{x} + x}{x}\right)dx$ (5)

(c) $\int_1^4\left(\frac{4 + 6u}{\sqrt{u}}\right)du$ (5)

(d) $\int_0^2 |2x-1|dx$ (5)

5. Determine the following integrals:

(a) $\int_0^t \sin(3\pi t)dt$ (5)

(b) $\int_0^1 \frac{\sin 2x}{\cos^2 x}dx$ (5)

(c) $\int e^{\cos x} \sin x dx$ (5)

(d) $\int_0^4 \left|\sqrt{x}-1\right|dx$ (5)

6. Use substitution method to determine the following integrals:

(a) $\int \frac{x^3}{1+x^4}dx$ (5)

(b) $\int \cos^4 \theta \sin \theta \, d\theta$ (5)

(c) $\int_{2\pi}^{3\pi} 3\cos^2 x \sin x dx$ (5)

7. Let $F(x, y) = y - \sin(xy)$. Find the partial derivatives F_x and F_y. Then find $\frac{dy}{dx}$.

Confirm your answer above by finding $\frac{dy}{dx}$ using implicit differentiation. (5)

8. Let $F(x, y) = y\cos(x^2y^2) + y$, then find the first partial derivatives F_x and F_y.

Then using your answer, find $\frac{dy}{dx}$. Using implicit differentiation conform your answer. (3)

9. If $z = \sin(xe^v)$ where $x = 3u^2 + uv$ and $y = u^3 - \ln v$.

Use the chain rule for partial differentiation to find $\frac{\partial z}{\partial u}$. (2)

Total: [100]