An example of a Karnaugh map for a function with 4 variables, created from a truth table for the given function F.

Please work through Appendix E in the textbook and the relative material in Tut102 before watching the slide show.

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

Assume that the truth table represents function F, ie function F is *true* where there appear a 1 in column F, and *false* otherwise.

Each horizontal value underneath A, B, C and D represents a binary value.

A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

Assume that the truth table represents function F, ie function F is *true* where there appear a 1 in column F, and *false* otherwise.

Each horizontal value underneath A, B, C and D represents a binary value.

When converted to decimal, this number represents the minterm number:

0010 = $(2^3 \times 0) + (2^2 \times 0) + (2^1 \times 1) + (2^0 \times 0) = 2$, representing $\mathbf{m_2}$ (or minterm 2)

1000= $(2^3 \times 1) + (2^2 \times 0) + (2^1 \times 0) + (2^0 \times 0) = 8$, representing $\mathbf{m_8}$ (or minterm 8)

1100= $(2^3 \times 1) + (2^2 \times 1) + (2^1 \times 0) + (2^0 \times 0) = 8 + 4 = 12$, representing $\mathbf{m_{12}}$ (or minterm 12)

A	B	C	D	F	minterm number
0	0	0	0	0	m_0
0	0	0	1	0	m_1
0	0	1	0	1	m_2
0	0	1	1	1	m_3
0	1	0	0	0	m_4
0	1	0	1	0	m_5
0	1	1	0	0	m_6
0	1	1	1	0	m_7
1	0	0	0	1	m_8
1	0	0	1	1	m_9
1	0	1	0	0	m_{10}
1	0	1	1	0	m_{11}
1	1	0	0	1	m_{12}
1	1	0	1	1	m_{13}
1	1	1	0	0	m_{14}
1	1	1	1	0	m_{15}

As in the previous slide, calculate the minterm for each line to make sure you understand how we get it.

Now, when A = 1, the complement of A (A’) is 0, and similarly, when A = 0, A’ = 1.

We can rewrite each of the binary values in terms of A, B, C, D, A’, B’, C’ and D’ as well, for example:

0011 is similar to **A’B’CD**

1000 is similar to **AB’C’D’**

1101 is similar to **ABC’D**, etc.
We add all these terms in the next slide:

A	B	C	D	F	term	minterm number
0	0	0	0	0	A'B'C'D'	m_0
0	0	0	1	0	A'B'C'D	m_1
0	0	1	0	1	A'B'CD'	m_2
0	0	1	1	1	A'B'CD	m_3
0	1	0	0	0	A'BC'D'	m_4
0	1	0	1	0	A'BC'D	m_5
0	1	1	0	0	A'BCD'	m_6
0	1	1	1	0	A'BCD	m_7
1	0	0	0	1	AB'C'D'	m_8
1	0	0	1	1	AB'C'D	m_9
1	0	1	0	0	AB'CD'	m_{10}
1	0	1	1	0	AB'CD	m_{11}
1	1	0	0	1	ABC'D'	m_{12}
1	1	0	1	1	ABC'D	m_{13}
1	1	1	0	0	ABCD'	m_{14}
1	1	1	1	0	ABCD	m_{15}

0011 is similar to **A'B'CD**

1000 is similar to **AB'C'D'**

1101 is similar to **ABC'D**, etc.

A	B	C	D	F	term	minterm number
0	0	0	0	0	A’B’C’D’	m_0
0	0	0	1	0	A’B’C’D	m_1
0	0	1	0	1	A’B’CD’	m_2
0	0	1	1	1	A’B’CD	m_3
0	1	0	0	0	A’BC’D’	m_4
0	1	0	1	0	A’BC’D	m_5
0	1	1	0	0	A’BCD’	m_6
0	1	1	1	0	A’BCD	m_7
1	0	0	0	1	AB’C’D’	m_8
1	0	0	1	1	AB’C’D	m_9
1	0	1	0	0	AB’CD’	m_{10}
1	0	1	1	0	AB’CD	m_{11}
1	1	0	0	1	ABC’D’	m_{12}
1	1	0	1	1	ABC’D	m_{13}
1	1	1	0	0	ABCD’	m_{14}
1	1	1	1	0	ABCD	m_{15}

Now we can write down function F in terms of the terms that we have just created. Remember F is true wherever F is 1:

F = A’B’CD’ + A’B’CD + AB’C’D’ + AB’C’D + ABC’D’ + ABC’D
= m_2 + m_3 + m_8 + m_9 + m_{12} + m_{13}

F needs to be simplified. We can do it in two ways:
(i) By simplifying it algebraically using the Boolean rules as described on pp. 41-42 of tutorial letter 102;
(ii) By drawing a Karnaugh graph as explained in Tutorial letter 102 and Appendix E of the textbook.

(i) We look for term pairs where only one variable differs in the 2 terms, and then use the Boolean rules to get rid of the specific variable:

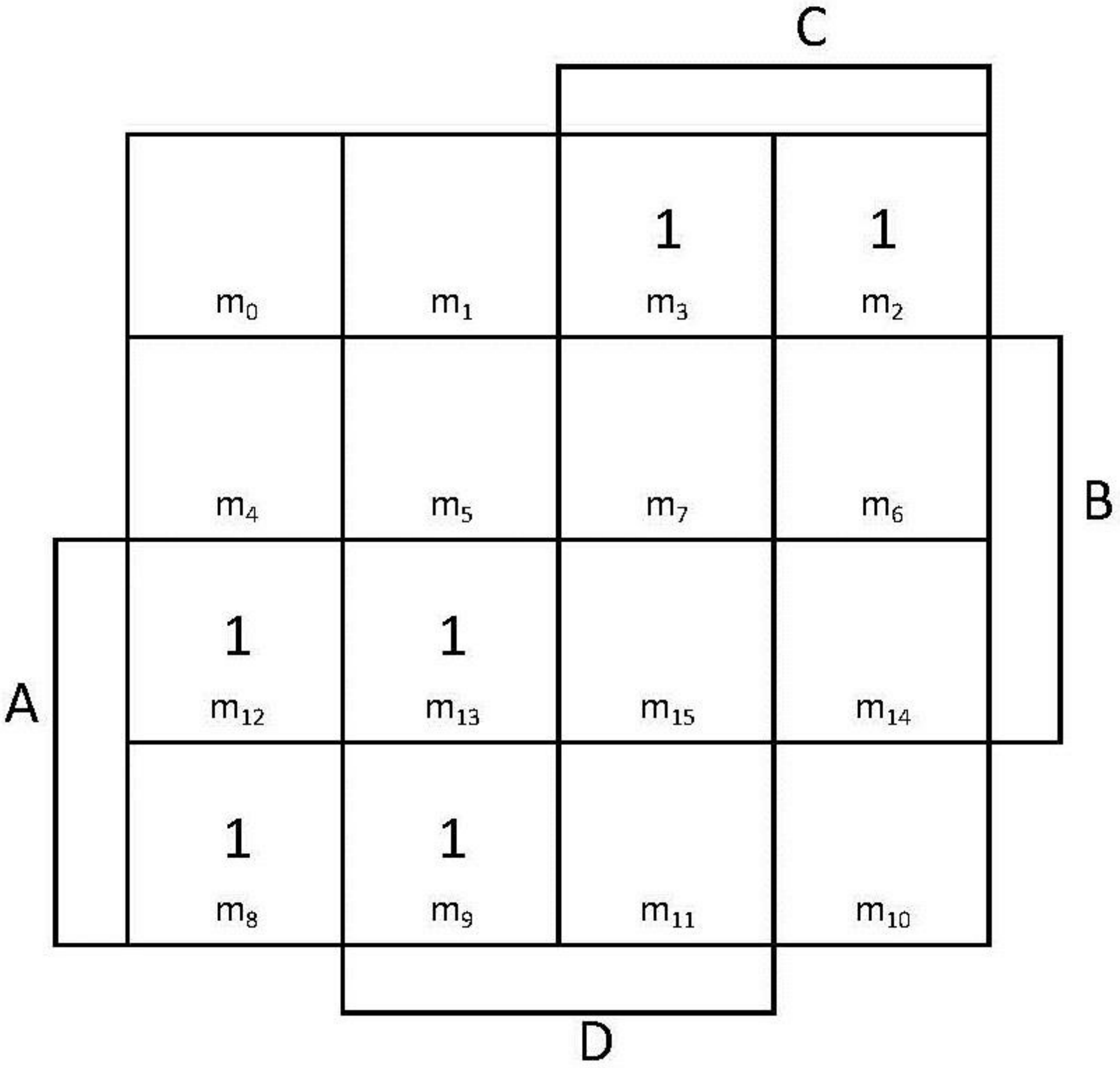
F = A'B'CD' + A'B'CD + AB'C'D' + AB'C'D + ABC'D' + ABC'D

= A'B'C(D' + D)	+ AB'C'(D' + D)	+ ABC'(D' + D)	Distributive rule (b), Tut 102, p.41
= A'B'C(1)	+ AB'C'(1)	+ ABC'(1)	Complement rule (a), Tut 102, p.41
= A'B'C	+ AB'C'	+ ABC'	Identity rule (b), Tut 102, p.41

= A'B'C	+ AC'(B' + B)	Distributive rule (b), Tut 102, p.41
= A'B'C	+ AC' (1)	Complement rule (a), Tut 102, p.41
= A'B'C	+ AC'	Identity rule (b), Tut 102, p.41

This is the simplest form for function F.

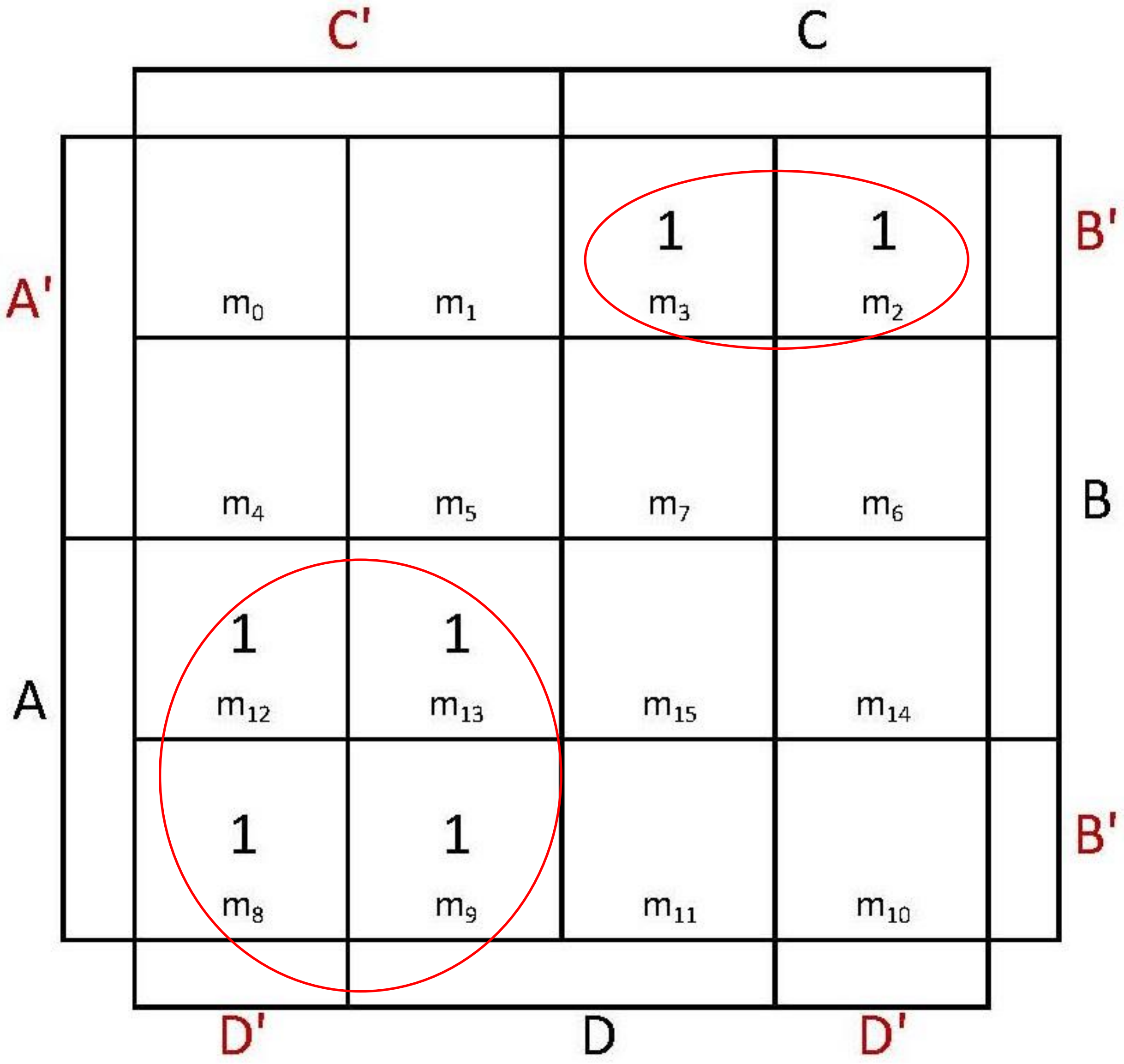
Next we will draw a Karnaugh map representing F.



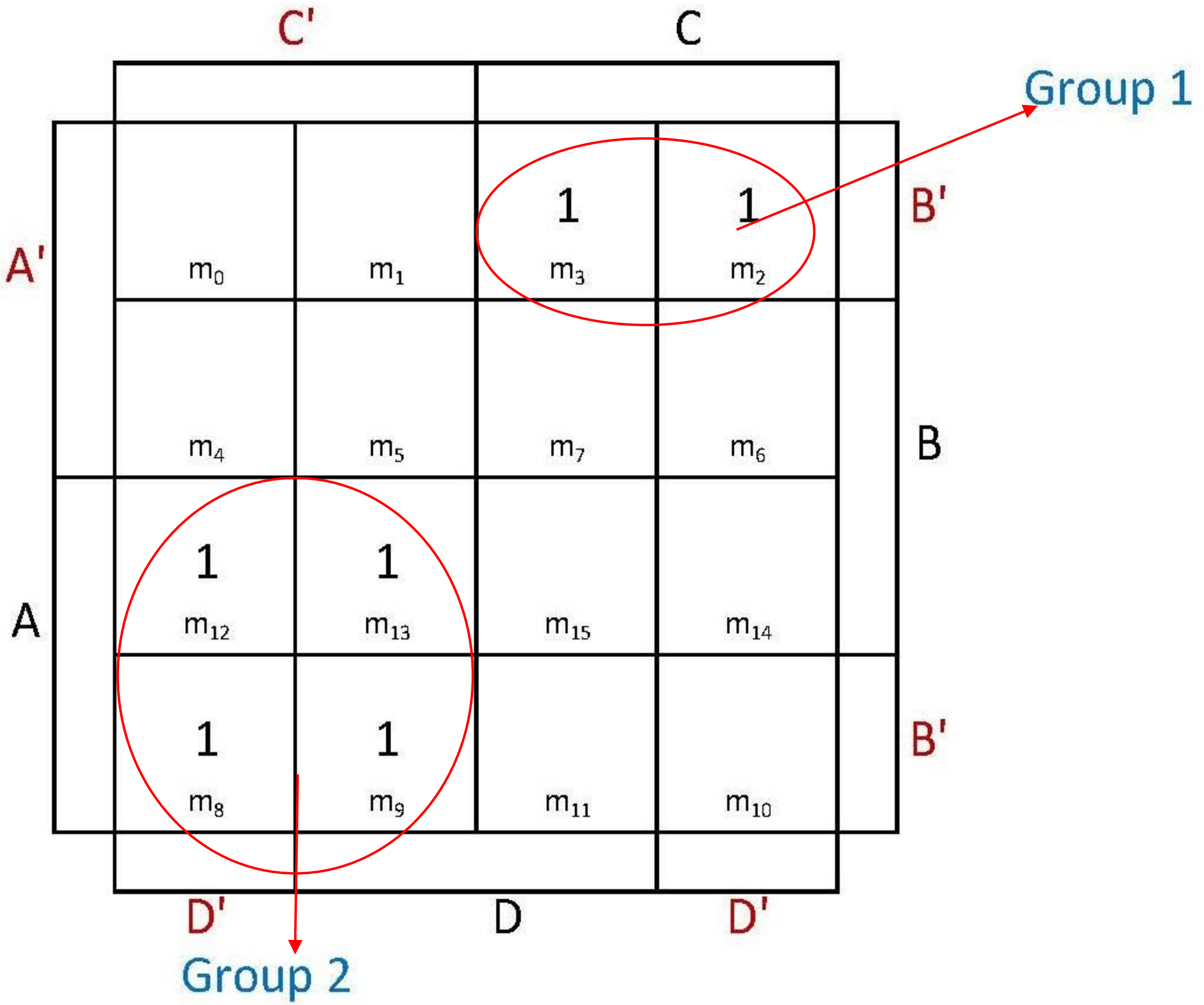
- We have determined that function F is:

$F = m_2 + m_3 + m_8 + m_9 + m_{12} + m_{13}$

- Firstly note that columns 3 and 4 get swapped around, and so does rows 3 and 4. It did not have an effect for this example, but could have a major effect on the outcome if you do not swap them around. Note carefully where the minterms are placed. This is also explained in Appendix E.
- We now fill in a 1 (ie where function F is true) at the applicable minterm according to the definition of F above
- Next we form groups of 1's of 1, 2, 4 or 8, trying to make the groups as big as possible. This is an easy example – we only have two groups:



- We also indicate where on the graph A’, B’, C’ and D’ is represented, so that it makes reading statements from the graph easier.
- We will now name the groups, and then determine the statement/expression for each group.



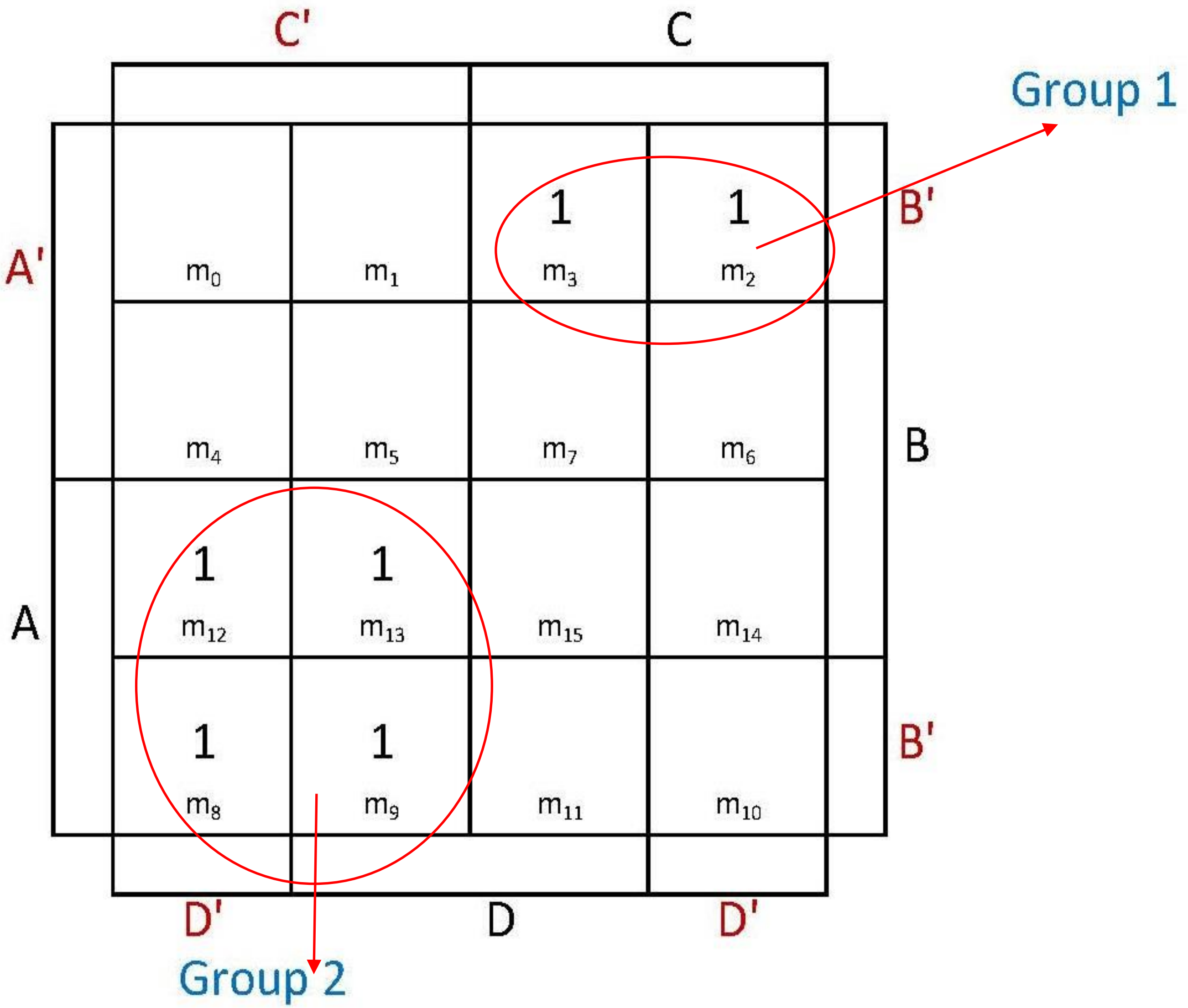
A Karnaugh graph example

Let us look at Group 1 first:

In the same way that we removed variables for which the variable and the complement appears in a group in the algebraic method, we do not include a variable in a group if a variable and its complement appears in the group.

We look at each variable for Group 1:

- Both m_2 and m_3 fall within the scope of A', but not in the scope of A. So A' must be part of the expression for Group 1;
- Similarly, both m_2 and m_3 fall within the scope of B', but not in the scope of B. So B' must be part of the expression for Group 1;
- Similarly, both m_2 and m_3 fall within the scope of C, but not in the scope of C'. So C must be part of the expression for Group 1;
- But m_2 falls within the scope of D' and m_3 falls within the scope of D, So we have both D and D', which we then discard from this group;
- This means that only A', B' and C is part of the expression. Group 1 is therefore represented by A'B'C.



Now we look at Group 2:

We look at each variable for Group 2:

- m_{12} , m_{13} , m_8 and m_9 all fall within the scope of A, but not within the scope of A'. So A must be part of the expression for Group 2;
- m_{12} and m_{13} fall within the scope of B and m_8 and m_9 fall within the scope of B'. So we have both B and B', which we then discard from this group;
- m_{12} , m_{13} , m_8 and m_9 all fall within the scope of C', but not within the scope of C. So C' must be part of the expression for Group 2;
- m_{12} and m_8 fall within the scope of D' and m_{13} and m_9 fall within the scope of D. So we have both D and D', which we then discard from this group;
- This means that only A and C' are part of the expression. Group 2 is therefore represented by AC'.
- Therefore function F, represented by Groups 1 and 2, is A'B'C + AC'.

We started of the example by giving a truth table for F. What if all that was given was that F = A'B'C + AC'?

By completing the terms algebraically, we can get to all the terms needed to generate the truth table:
(This is the opposite of what we did in slide 7).

$$
\begin{aligned}
F &= A'B'C + AC' \\
&= A'B'C(D + D') + A(B + B')C'(D + D') \\
&= A'B'CD + A'B'CD' + ABC'D + AB'C'D + ABC'D' + AB'C'D' \\
&= m_3 + m_2 + m_{13} + m_9 + m_{12} + m_8
\end{aligned}
$$

From this the truth table can be created, or the Karnaugh map can directly be created.

So you can see that one can always start somewhere, and work your way to an answer, depending on what was given in the question.

It is important that you work through Appendix E, as well as the examples in Tut 102.