UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1512

October/November 2013

CALCULUS A

Duration 2 Hours

100 Marks

EXAMINERS
FIRST MRS SB MUGISHA
SECOND DR L LINDEBOOM

Closed book examination

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This paper consists of 3 pages

Answer all the questions There is a total of 100 marks. 100 marks will count as full marks

QUESTION 1

(a) Determine the following limits (if it exists)

(i) $\lim_{x \longrightarrow -2} \dfrac{x^3 + 2x^2 - 3}{5 - 3x}$ (3)

(ii) $\lim_{x \longrightarrow 1} \dfrac{1 - \sqrt{x}}{1 - x}$ (3)

(iii) $\lim_{x \longrightarrow 3^-} \dfrac{x^2 - 9}{|x - 3|}$ (3)

(iv) $\lim_{t \longrightarrow 0} \dfrac{t \tan t}{1 - \cos t}$ (3)

(v) $\lim_{x \longrightarrow \infty} \dfrac{x^2 + x}{5 - x}$ (3)

(b) (i) Use the Squeeze Theorem to determine the following limit $\lim_{k \longrightarrow 0} k^2 \cos\left(\dfrac{1}{\sin k}\right)$ (3)

(ii) Now let

$$T(x) = \begin{cases} x^2 \cos\left(\dfrac{1}{\sin x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Simplify $\dfrac{T(0 + k) - T(0)}{k}$ and show from *first principles* that $T'(0)$ exists (3)

[TURN OVER]

(c) Let

$$g(t) = \begin{cases} t^2 - 1 & \text{if } t < 3 \\ w & \text{if } t = 3 \\ 2t + 1 & \text{if } t > 3 \end{cases}$$

Show that g cannot be continuous at $t = 3$ for any value of $w \in \mathbb{R}$ (4)

[25]

QUESTION 2

(a) Find the first derivatives of the following functions

(i) $y = \sqrt{x \sin x}$ (4)

(ii) $y = \dfrac{\cos \pi x}{1 + \tan x}$ (4)

(iii) $y = 3^x \operatorname{cosec}(x^3)$ (4)

(iv) $x^3 e^y = y^2 \ln x$ (4)

(v) $F(x) = \int_{x^3}^{x} \sin 3t \, dt$ (4)

(b) Use Logarithmic differentiation to find $\dfrac{dy}{dx}$ if $y = (\cos x)^{2x}$ (5)

(c) For the function

$$x^2 + y^3 - 2y = 3$$

find the equation of the normal line at the point (2, 1) (5)

[30]

QUESTION 3

(a) Determine the following integrals

(i) $\int 5x\sqrt{4 + x} \, dx$ (4)

(ii) $\int \left(x - \frac{2}{x^2}\right)\left(x + \frac{2}{x^2}\right) dx$ (3)

(iii) $\int e^{\cos x} \sin x \; dx$ (4)

(iv) $\int x 2^{x^2} \, dx$ (4)

(b) Make a direct substitution and change the integral limits to evaluate the following

(i) $\int_0^1 \frac{5x}{(4+x^2)^2}\,dx$ (5)

(ii) $\int_{2\pi}^{3\pi} 3\cos^2 x \sin x \, dx$ (5)

(c) Find the area enclosed by $f(x) = 2 - x^2$ and $g(x) = -x$ Sketch the graphs of f and g on the same axes

(5)

[30]

QUESTION 4

(a) Solve the following initial value problem

$$\frac{dy}{dx} = \operatorname{cosec}^2 x \quad (e - 5y), \quad y(\frac{\pi}{2}) = 0 \tag{5}$$

(b) A bacterial culture starts with 2200 bacteria and after 3 hours there are 3700 bacteria Assuming that the culture grows at a rate proportional to its size, find the population after 6 hours (5)

(c) If $z = \sin(xe^y)$ where $x = 3u^2 + uv$ and $y = u^3 - \ln v$

Use the chain rule for partial differentiation to find $\frac{\partial z}{\partial u}$

(5)

[15]

TOTAL: [100]