UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1503

October/November 2014

LINEAR ALGEBRA

Duration 2 Hours

100 Marks

EXAMINERS :
FIRST DR L GODLOZA DR ZE MPONO

Closed book examination

This paper consists of 3 pages

Answer All Questions

QUESTION 1

(a) Consider the following system of equations

$$\begin{cases} x_1 - 3x_2 = 2 \\ \quad\quad 2x_2 = 6 \end{cases}$$

(i) write down the coefficient matrix of the above system (1)

(ii) using back substitution, solve the above system (3)

(b) Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \; B = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

and determine as to whether or not

(i) $A(BC) = (AB)C$ (5)

(ii) $A(B + C) = AB + AC$ (5)

(c) Let D, E be nonsingular matrices and show that $(DE)^{-1} = E^{-1}D^{-1}$ (5)

(d) Compute the inverse of the following matrix

$$F = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

and verify that the matrix you have computed indeed is the required inverse (6)

[25]

[TURN OVER]

QUESTION 2

(a) Given that

$$G = \begin{bmatrix} 2 & 5 & 4 \\ 3 & 1 & 2 \\ 5 & 4 & 6 \end{bmatrix}$$

compute $\det(G)$ (5)

(b) Find all values of λ for which

$$\det\begin{pmatrix} 2-\lambda & 4 \\ 3 & 3-\lambda \end{pmatrix} = 0$$ (5)

(c) Let

$$H = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

and find H^T and relate it to H (3)

(d) Let I, J be 3×3 matrices such that $\det(I) = 4$ and $\det(J) = 5$ Then find

(i) $\det(IJ)$ (2)

(ii) $\det(3I)$ (2)

(iii) $\det(2IJ)$ (2)

(iv) $\det(I^{-1}J)$ (2)

(e) Show that the coefficient matrix of the following system of equations is nonsingular

$$\begin{cases} x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 2x_2 + x_3 = 6 \\ x_1 + 2x_2 + 3x_3 = 9 \end{cases}$$

(4)

[25]

QUESTION 3

(a) Let $\underline{u} = (1\ 2, -2)$ and $\underline{v} = (3, 0\ 1)$ be vectors in $\mathbb{R}^3$

(i) Calculate $\underline{u} \times \underline{v}$ (3)

(ii) Determine the area of the parallelogram bounded by $\underline{u}$ and $\underline{v}$ (3)

[TURN OVER]

(iii) Verify that $\underline{u} \times \underline{v}$ is perpendicular to $\underline{v}$ (4)

(iv) Determine $Proj_{\underline{u}}\underline{v}$ (3)

(v) Determine the cosine of the angle between $\underline{u}$ and $\underline{v}$ (3)

(b) Consider the points $P(3, -1, 4)$ $Q(6\ 0, 2)$ and $R(5, 1, 1)$

(i) Find the point S in $\mathbb{R}^3$ whose first component is -1 and such that $\overrightarrow{PQ}$ is parallel to $\overrightarrow{RS}$ (4)

(ii) Determine the equation of the plane passing through R and perpendicular to the line passing through P and Q (5)

[25]

QUESTION 4

(a) Use de Moivre's Theorem to express $\sin 4\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$ (10)

(b) Determine the cube roots of i in polar form (10)

(c) Let $z = x + iy$ be any complex number Prove that if $z^3 = 1$, then $x^3 - 3xy^2 = 1$ and $3x^2y - y^3 = 0$

(5)

[25]

TOTAL: 100 Marks