UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1512

May/June 2014

CALCULUS A

Duration 2 Hours

100 Marks

EXAMINERS
FIRST MRS SB MUGISHA
SECOND DR L LINDEBOOM

Closed book examination

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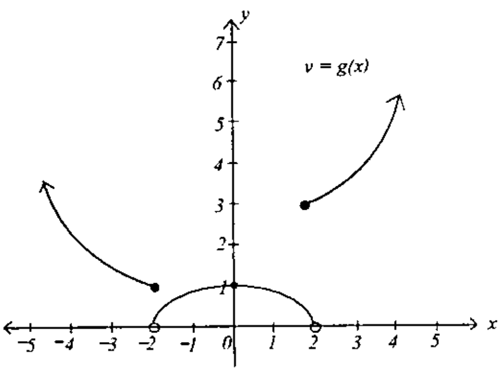
This paper consists of 3 pages **ANSWER ALL QUESTIONS**
ALL CALCULATIONS MUST BE SHOWN

QUESTION 1

(a) Let the graph of the function $g(x)$ be represented as shown below



Answer each of the following by using the graph of $g(x)$ above

(i) $\lim_{x\to-2} g(x) =$ ____________

(ii) $\lim_{x\to0} g(x) =$ ____________

(iii) $\lim_{x\to2} g(x) =$ ____________

(iv) $\lim_{x\to3} g(x) =$ ____________

(4)

[TURN OVER]

(b) Given that

$$G(t) = \begin{cases} \sin t & \text{if} \quad t < 0 \\ t^2 & \text{if} \quad 0 \leq t \leq 2 \\ 3t - 2 & \text{if} \quad t > 2 \end{cases}$$

find $\lim\limits_{t \longrightarrow 2} G(t)$ (3)

(c) Determine the following limits (if they exist)

(i) $\lim\limits_{x \longrightarrow -2} \dfrac{x+2}{x^2 - x - 6}$ (3)

(ii) $\lim\limits_{x \longrightarrow 3} \left[\dfrac{2x^2}{x-3} + \dfrac{6x}{3-x} \right]$ (4)

(iii) $\lim\limits_{t \longrightarrow 3^+} \dfrac{3-t}{|t-3|}$ (3)

(iv) $\lim\limits_{x \longrightarrow \infty} \dfrac{x^3 + 6x + 1}{2x^2 - 5x}$ (3)

(v) Use the Squeeze Theorem to determine $\lim\limits_{y \longrightarrow \infty} \dfrac{3 - \sin(e^y)}{\sqrt{y^2 + 2}}$ (5)

[25]

QUESTION 2

(a) Use the first principles of differentiation to find the derivative of $g(t) = 3t^3 + 2t - 1$ (5)

(b) Find the derivatives of the following functions by using the appropriate rules for differentiation

(i) $f(x) = \left(x^{\frac{3}{2}} - 4x \right) \left(x^4 - 3x^{-2} + 2 \right)$ (3)

(ii) $g(t) = \dfrac{6t - 2t^{-1}}{t^2 + \sqrt{t}}$ (4)

(c) Use the Fundamental Theorem of Calculus and find the derivative of $h(x) = \int\limits_{\sqrt{x}}^{1} \dfrac{2t^2}{t^4 + 1} dt$ (4)

(d) Find the first and second derivatives of the function $T(z) = \sqrt{z} + \sqrt[5]{z}$ (4)

(e) Given $\sin\left(x^2 y\right) = x - y^2$ find

(i) $\dfrac{dy}{dx}$ implicitly (5)

(ii) the equation of the normal line to the curve $\sin\left(x^2 y\right) = x - y^2$ at the point $\left(\dfrac{1}{2}, 0 \right)$ (3)

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QUESTION 3

(a) Use the appropriate substitution to evaluate the following integrals

(i) $\int x\sqrt{x^2+3}\ dx$ (4)

(ii) $\int \dfrac{2y}{\sqrt{1-y^2}}\,dy$ (4)

(iii) $\int e^{\sin\theta}\cos\theta\ d\theta$ (4)

(b) Determine the exact values for the following integrals (use substitution if necessary)

(i) $\int_0^1 \dfrac{w^3}{2+w^4}\ dw$ (4)

(ii) $\int_0^{\frac{\pi}{3}} \left(\cos^3 x+1\right)\sin x\ dx$ (5)

(c) Let $g(t)=t^2$

and

$$h(t)=\begin{cases} 2-t & \text{if} \quad t<0 \\ t+2 & \text{if} \quad t\geq 0 \end{cases}$$

Determine the area of the region enclosed by the curves g en h (6)

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QUESTION 4

(a) Solve the following Initial Value Problem

$$\frac{dz}{dt}=\frac{2t+\sec^2 t}{2z} \quad z(0)=-5$$ (6)

(b) Let $T(x,y)=\tan\left(xy^2\right)+3y-2xy$

(i) Find the first partial derivatives T_x and T_y (4)

(ii) Using (i) above, find $\dfrac{dy}{dx}$ (4)

(iii) If $T(x,y)=0$ confirm your answer in part (b) (ii) above by finding $\dfrac{dy}{dx}$ using implicit differentiation (6)

[20]

TOTAL [100]

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