Tutorial Letter 101/0/2022

Calculus A

MAT1512

Year Module

Name of Department

IMPORTANT INFORMATION

Please register on myUnisa, activate your myLife e-mail account and make sure that you have regular access to the myUnisa module website, MAT1512-2022-Y1, as well as your group website.

Note: This is a fully online module. It is, therefore, only available on myUnisa.

BARCODE

UNISA university of south africa

CONTENTS

1 INTRODUCTION

Dear Student

Welcome to the *MAT1512* module. We trust that you will find the mathematics studied in this module interesting and useful, and that you will enjoy doing it.

This tutorial letter contains important information about the scheme of work, resources and assignments for this module as well as exam admission. We urge you to read it carefully before working through the study material, preparing the assignment(s), preparing for the examination and addressing questions to your lecturers.

In this tutorial letter, you will find the assignments as well as instructions on the preparation and submission of the assignments. This tutorial letter also provides all the information you need with regard to the prescribed study material and other resources. Please study this information carefully and make sure that you obtain the prescribed material as soon as possible.

You will access all files online, a number of tutorial letters for example, solutions to assignments, during the semester/ year. These tutorial letters will be uploaded on *myUnisa*, under Additional Re-sources and Lessons tools on *myUnisa* platform. A tutorial letter is our way of communicating with you about teaching, learning and assessment.

Right from the start we would like to point out that you must read all the tutorial letters you access from the module site immediately and carefully, as they always contain important and, sometimes urgent information.

Because this is a fully online module, you will need to use myUnisa to study and complete the learning activities for this course. Please visit the website for MAT1512 on myUnisa frequently.
The website for your module is MAT1512-22-Y1.

1.1 Getting started

Owing to the nature of this module, you can read about the module and find your study material online. Go to the website at https://my.unisa.ac.za and log in using your student number and password. Click on "myModules" at the top of the webpage and then on "Sites" in the top right corner. In the new window, click on the grey Star icon next to the modules you want displayed on your navigator bar. Close the window in the right corner. The select the option "Reload to see your updated favorite sites". Now go to your navigation bar and click on the module you want to open.

We wish you every success with your studies!

2 OVERVIEW OF MAT1512

2.1 Purpose

This module will be able useful to students interested in developing the basic skills in Calculus which can be applied in the natural sciences and social sciences. Students who have completed this module successfully will have an understanding of the basic ideas of Calculus.

This module is useful to students interested in developing the basic skills in differential and integral calculus. Differential and integral calculus are essential for physical, life and economic sciences. Students credited with this module will have a firm understanding of the limit, continuity at a point, differentiation and integration, together with a background in the basic techniques and some applications of Calculus.

2.1.1 Learning Assumptions: The learning is based on the assumption that students are already competent in terms of the following outcomes or areas of learning and must:

- Have a Senior Certificate or equivalent qualification (as required) for further study.
- Have obtained an NQF/HEQF Level equivalent to 4 with the ability to:
- Be able to learn from predominantly written material in the language of tuition
- Take responsibility for their own progress and independently adjust to the learning environment
- Have basic computer skills like using a mouse, keyboard and windows features
- Demonstrate an understanding of the most current topics in mathematics including
  - Functions
  - The ability to algebraically manipulate real numbers and solve equations.
  - An ability to sketch graphs and find equations from these graphs.
  - Substantive knowledge about basic trigonometry
  - Knowledge about the following mathematical concepts: absolute values, partial fractions and inequalities.

Recognition of prior learning will take place in accordance with the institution's policy and guidelines. Recognition takes place, where prior learning corresponds to the required NQF-HEQF level and in terms of applied competencies relevant to the content and outcomes of the qualification, at the discretion of the department.

2.1.2 Range statement for the module: The techniques selected involve polynomial, rational, trigonometric, exponential and logarithmic functions and their composites. This introductory calculus module covers differentiation and integration of functions of one variable, with applications.

2.2 Outcomes

2.2.1 **Specific outcome 1:**

Demonstrate knowledge of the concept of a limit of a function and its application. Range:

The knowledge includes limits of one variable and an introduction to limits of two or more variables.

Assessment criteria

- A formal definition of the limit with the correct mathematical notation is given which embraces an understanding of the limit as the y-value of a function.
- A distinction between the limits of a function as x approaches $(\lim_{x\to a} f(x))$ and the value of the function at $x = a$ is made correctly.
- Laws governing limits are stated and used to determine and evaluate limits of sums, products, quotients and composition of functions.
- The limits of functions are evaluated graphically and numerically.
- The limit definition of continuity is used to determine whether a function is continuous or discontinuous at a point.
- The Squeeze Theorem is used to determine certain undefined limits.

2.2.2 **Specific outcome 2:**

Demonstrate an understanding of differentiation.

Assessment criteria

- The derivative is defined as an instantaneous rate of change of a function.
- The first principle of differentiation is presented using different expressions.
  Range: These different expressions include:

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \; ; \; f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x} \; ; \; f'(a) = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

- Alternate derivative notations are given. Range: These include:

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x)$$

- A distinction between continuity and differentiability of a function at point is made correctly.
- A representation of the first derivative as the slope of the tangent line at the point of tangency is given.

2.2.3 **Specific outcome 3**:

- Calculate derivatives.

Assessment criteria

- The derivative of a function is computed from the first principle of differentiation.
- The basic rules of differentiation such as the power rule, product and quotient rules are used to compute derivatives of different functions.
- Range: The functions are in the form: $[h(x) = f(x) \pm g(x)]$; $[h(x) = f(x).g(x)]$

$\left[h(x) = \dfrac{f(x)}{g(x)}\right]$. The chain rule is used, together with other rules of differentiation to find derivatives of composite functions.

2.2.4 **Specific outcome 4**:

– **Use derivatives to solve applied problems.**

Assessment criteria

- For the problem solving, the differentiation technique chosen is appropriate to the problem.
- Mathematical notations and language are used appropriately.
- The derivative is used to find equations of tangent and normal lines of different curves.
- Where appropriate, the Mean Value Theorem is applied.

2.2.5 **Specific outcome 5**:

– **Demonstrate understanding of basic integration and the Fundamental Theorem of Calculus**

Assessment criteria

- The definite integral is defined and interpreted using:
  - the concept of definite integral to obtain areas under the curve.
  - as the net change in a quantity from $x = a$ to $x = b$ if $f(x)$ is the rate of change of the quantity with respect to x.
- A function F is defined as an anti-derivative (indefinite integral) of the function f if the derivative $F' = f$ .Anti-differentiation (integration) is recognised as the inverse of the differentiation process.
- The Fundamental Theorem of Calculus for a function f on an interval $[a, b]$ as:

$$\int_a^b f(x)dx = F(b) - F(a) \text{ where } F(x) \text{ is such that } F'(x) = f(x)$$

is reproduced and used to:-

  - explain the way in which differentiation and integration are related.
  - evaluate given integrals.
- Integral notation is used appropriately.

2.2.6 **Specific outcome 6**:

– Use integrals of simple functions to solve applied problems
Range: Simple integrals are applied but not limited to problems involving the length of a curve, area between curves, velocity and acceleration.

Assessment criteria

- Substitution or term by term integration techniques are used appropriately.
- The anti-derivatives of basic algebraic and trigonometric functions are determined correctly.
- For the problem solving process:-
  - The estimations of the definite integrals of the functions are correct.
  - The solution is consistent with the problem.

2.2.7 **Specific outcome 7**

– **Analyse logarithmic and exponential functions.**

Assessment criteria

- The graphs of the functions $y = e^x$ and $y = \ln x$ are reproduced.
- The relationship between e^x and $\ln x$ as inverse differentiable functions is recognised and used as a device for simplifying calculations.
- Rules of differentiation and integration are applied to functions involving logarithmic and exponential functions.
- Logarithmic differentiation is used correctly.
- Exponentials and logarithmic models for solving applied problems are identified.

2.2.8 **Specific outcome 8**

- Solve exponential growth and decay problems using elementary differential equations.
  Range: The solutions are limited to first-order, separable, constant coefficient initial-value problems, with contextual situations involving exponential growth and decay. Assessment criteria
- The contextual situation (problem) is analysed and represented with a differential equation.
- A suitable method for determining the solution is chosen.
- Initial or boundary conditions are identified and used to determine the constant of integration.
- The differential equation is solved correctly.
- Partial derivatives are computed where necessary.
- Mathematical notation is used to communicate the results clearly.

3 CURRICULUM TRANSFORMATION

Unisa has implemented a transformation charter based on five pillars and eight dimensions. In response to this charter, we have placed curriculum transformation high on the teaching and learning agenda. Curriculum transformation includes the following pillars: student-centred scholarship, the pedagogical renewal of teaching and assessment practices, the scholarship of teaching and learning, and the infusion of African epistemologies and philosophies. These pillars and their principles will be integrated at both programme and module levels as a phased-in approach. You will notice a marked change in the teaching and learning strategy implemented by Unisa, together with how the content is conceptualised in your modules. We encourage you to embrace these changes during your studies at Unisa in a responsive way within the framework of transformation.

4 LECTURER(S) AND CONTACT DETAILS

4.1 Lecturer(s)

The primary lecturer for this module is Dr SB Mugisha:
Department: Mathematical Sciences
Telephone: 011 670 9154
E-mail: mugissb@unisa.ac.za

A notice will be posted on *myUnisa* if there are any changes and/or an additional lecturer is appointed to this module.

Please do not hesitate to consult your lecturer whenever you experience difficulties with your stud- ies. You may contact your lecturer by phone or through correspondence or by making a personal visit to his/her office. **Please arrange an appointment in advance (by telephone or by e-mail) to ensure that your lecturer will be available when you arrive.**

Please come to these appointments well prepared with specific questions that indicated your own efforts to have understood the basic concepts involved. If these difficulties concern exercises which you are unable to solve, you must send us your attempts so that we can see where you are going wrong.

If you should experience any problems with the exercises in the study guide, your lecturer will gladly help you with them, provided that you send in your bona fide attempts. **When sending in any queries or problems, please do so separately from your assignments and address them directly to your lecturer.**

4.2 Department

You can contact the Department of Mathematical Sciences as follows:

Department of Mathematical Sciences
Fax number: 011 670 9171 (RSA) +27 11 670 9171 (International)
Departmental Secretary: 011 670 9147 (RSA) +27 11 670 9147 (International) e-mails: mathsciences@unisa.ac.za or swanem@unisa.ac.za

4.3 University

To contact the University, follow the instructions on the Contact us page on the Unisa website. Remember to have your student number available whenever you contact the University.

Whenever you contact a lecturer via e-mail, please include your student number in the subject line to enable the lecturer to help you more effectively.

5 RESOURCES

5.1 Joining myUnisa

The myUnisa learning management system is the University's online campus which will help you communicate with your lecturers, other students and the administrative departments within Unisa. To claim your myUnisa account, please follow the steps below:

1. Visit the myUnisa website at https://my.unisa.ac.za/portal
2. Click on the **"Claim Unisa login"** link on the top of the screen under the orange user ID box.
3. A new screen will load, prompting you to **enter your student number**. Please enter your student number and click **"continue"**.

4. Enter your surname, your full name, your date of birth and, finally, your South African ID number (for South African citizens) OR your passport number (for foreign students). Then click "continue". **Remember to enter either an ID number or a passport number, NOT both.**
5. Please read through the guidelines and **click all the check boxes** to acknowledge that you have read all the information provided. Once you are done, click the **"Acknowledge"** button to redirect you to the final page in the process.
6. The final page will display your myLife e-mail address, and your **myLife AND myUnisa password**. This password will also be sent to the cellphone number displayed on the page for safekeeping.
7. Please note that it can take up to 24 hours for your myLife e-mail account to be created.

Remember, the password provided is your myUnisa **AND** myLife password.

5.2 Prescribed book(s)

James Stewart

Calculus, Metric Version 8E, Early Transcendentals, Cengage Learning,

ISBN 13: 978-1-305-27237-8.

Please buy the textbook as soon as possible since you have to study from it directly- you cannot do this module without the prescribed textbook.
Please refer to the list of official booksellers and their addresses in the *Study@Unisa* brochure. The prescribed book can be obtained from the University's official booksellers. If you have difficulty locating your book at these booksellers, please contact the Prescribed Books Section at (012) 429 4152 or e-mail vospresc@unisa.ac.za.

CHAPTERS AND SECTIONS TO BE COVERED IN THE PRESCRIBED TEXTBOOK FOR MAT1512 MODULE:
Chapter 1 (Functions and Models): Sections 1.1, 1.2, 1.3, 1.4 and 1.5 (for your own revision).
Chapter 2 (Limits and Derivatives): Sections 2.1, 2.2, 2.3, 2.5, 2.6, 2.7 and 2.8 with Section 2.4 to be ready only.
Chapter 3 (Differentiation Rules) Sections: 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9 and 3.10.
Chapter 4 (Applications of Differentiation): Sections 4.2 and 4.9. Sections: 4.3, 4.4 and 4.5 to be read only.
Chapter 5 (Integrals): Sections 5.1, 5.2, 5.3, 5.4 and 5.5.
Chapter 6 (Applications of Integration): Section 6.1 only.
Chapter 9 (Differential Equations): Sections 9.1 and 9.3 with Section 9.2 to be read only.
Chapter 14 (Partial Derivatives) Sections: 14.1, 14.2, 14.3 and 14.5.
APPENDIXES: A, B, C, D and E to be read for your own revision (high school work).
APPENDIX: F: Section 2.3: Limit laws (without proofs) and The Squeeze Theorem (with proof).
APPENDIX: F: Section 2.5 (Theorem on page A42 without proof), Theorem 8 on page A43 (without proof) and Section 3.3, Theorem on page A43 (without proof).
APPENDIX: G: All the Laws of Logarithms and Exponents without proofs.

5.3 Recommended book(s)

There are no recommended books for this module.

5.4 Electronic reserves (e-reserves)

E-reserves can be downloaded from the Library catalogue. More information is available at https://libguides.unisa.ac.za/request/request

Videos for MAT1512 made by your lecturer and put on You-Tube

We managed to put online (account Youtube) the video from your Lecturer Dr. Mugisha on Calculus A. As the video is too long we had to cut it into four parts. The videos are all about the module MAT1512. The videos cover the sections of this module which most student tend to have difficulties. The videos were made using an old prescribed textbook, but follow the videos with the new prescribed textbook by James Stewart.
The students can just click on the given list below or copy and paste them on their internet browser bar.
Video 1 - Limits
http://www.youtube.com/watch?v=GuRGhrt19tM&feature=youtu.be
video 2- Limits and continuity
http://www.youtube.com/watch?v=tEenlPFx6Mk&feature=youtu.be
Video 3-Calculus A-Differentiation
http://www.youtube.com/watch?v=Eyc7C54sPgA
Video 4-Calculus A-Integration
http://www.youtube.com/watch?v=sChEcFeuqT8

5.5 Library services and resources

The Unisa Library offers a range of information services and resources:

- For a general Library overview, go to
  https://www.unisa.ac.za/sites/corporate/default/Library/About-the-Library
  Library @ a glance
- For detailed Library information, go to
  https://www.unisa.ac.za/sites/corporate/default/Library
- For research support and services (eg personal librarians and literature search services) go to
  https://www.unisa.ac.za/sites/corporate/default/Library/Library-services/Research-support

The Library has created numerous **Library guides** to assist you: http://libguides.unisa.ac.za

Recommended guides:

- Request recommended books and access e-reserve material:
  https://libguides.unisa.ac.za/request
- Requesting and finding library material: Postgraduate services:
  https://libguides.unisa.ac.za/request/postgrad
- Finding and using library resources and tools (Research Support):

https://libguides.unisa.ac.za/research-support

- Frequently asked questions about the Library:
  https://libguides.unisa.ac.za/ask
- Services to students living with disabilities:
  https://libguides.unisa.ac.za/disability
- A-Z databases:
  https://libguides.unisa.ac.za/az.php
- Subject-specific guides:
  https://libguides.unisa.ac.za/?b=s
- Information on fines & payments:
  https://libguides.unisa.ac.za/request/fines

Assistance with **technical problems** accessing the Unisa Library or resources:
https://libguides.unisa.ac.za/techsupport

Lib-help@unisa.ac.za (insert your student number in the subject line please)

General library enquiries can be directed to Library-enquiries@unisa.ac.za

6 STUDENT SUPPORT SERVICES

The *Study @ Unisa* website is available on myUnisa: www.unisa.ac.za/brochures/studies

This website has all the tips and information you need to succeed at Unisa.

6.1 First-Year Experience Programme @ Unisa

For many students, the transition from school education to tertiary education is beset with anxiety. This is also true for first-time students to Unisa. Unisa is a dedicated open distance and e-learning institution. Unlike face-to-face/contact institutions, Unisa is somewhat different. It is a mega university and all our programmes are offered through a blended learning mode or fully online learning mode. It is for this reason that we thought it necessary to offer first-time students additional/extended support so that you can seamlessly navigate the Unisa teaching and learning journey with little difficulty and few barriers. In this regard we offer a specialised student support programme to students entering Unisa for the first time. We refer to this programme as Unisa's First-Year Experience (FYE) Programme. The FYE is designed to provide you with prompt and helpful information about services that the institution offers and how you can access information. The following FYE programmes are currently offered:

- FYE website: All the guides and resources you need to navigate through your first year at Unisa can be accessed using the following link: www.unisa.ac.za/FYE

- FYE e-mails: You will receive regular e-mails to help you stay focused and motivated.

- FYE broadcasts: You will receive e-mails with links to broadcasts on various topics related to your first-year studies (eg videos on how to submit assignments online).

- FYE mailbox: For assistance with queries related to your first year of study, send an e-mail to fye@unisa.ac.za

7. HOW TO STUDY ONLINE

7.1 What does it mean to study fully online?

Studying fully online modules differs completely from studying some of your other modules at Unisa.

- **All your study material and learning activities for online modules are designed to be delivered online on myUnisa.**
- **All your assignments must be submitted online.** This means that you will do all your activities and submit all your assignments on myUnisa. In other words, you may **NOT** post your assignments to Unisa using the South African Post Office.
- **All communication between you and the University happens online.** Lecturers will communicate with you via e-mail and SMS, and use the **Announcements,** the **Discussion Forums** and the **Questions and Answers** tools. You can also use all of these platforms to ask questions and contact your lecturers.

7.2 myUnisa tools

The main tool that we will use is the **Lessons tool**. This tool will provide the content of and the assessments for your module. At times you will be directed to join discussions with fellow students and complete activities and assessments before you can continue with the module.

It is very important that you log in to myUnisa regularly. We recommend that you log in at least once a week to do the following:

- **Check for new announcements.** You can also set your myLife e-mail account so that you receive the announcement e-mails on your cellphone.
- **Do the Discussion Forum activities.** When you do the activities for each learning unit, we want you to share your answers with the other students in your group. You can read the instructions and even prepare your answers offline, but you will need to go online to post your messages.
- **Do other online activities.** For some of the learning unit activities you might need to post something on the **Blog tool**, take a quiz or complete a survey under the **Self-Assessment** tool. Do not skip these activities because they will help you complete the assignments and the activities for the module.

We hope that by giving you extra ways to study the material and practise all the activities, this will help you succeed in the online module. To get the most out of the online module, you **MUST** go online regularly to complete the activities and assignments on time.

8. ASSESSMENT

Please note that this module has a total of SIX compulsory assignments which contribute
20%

to the final mark and one examination which contributes 80% to the final mark.

Please note that lecturers are not responsible for examination admission, and ALL inquiries about examination admission should be directed by e-mail to exams@unisa.ac.za

You will be admitted to the examination if and only if Assignment 01 reaches the University and is submitted/ uploaded by the stipulated due date.

Note that your marks for the assignments contribute 20% to your final mark (the remaining 80% is contributed by the examinations).

Please note that this module has a total of SIX compulsory assignments which contribute 20% to the final mark.

The questions for the assignments are available online on *myUnisa* platform. For each assignment there is a FIXED CLOSING DATE; the date by which the assignment must be uploaded on the system. Solutions for each assignment will be given as Tutorial Letters 201, 202, etc. will be uploaded on *myUnisa* under Additional Resources a few weeks after the closing date.
Late assignments will not be marked & will be awarded 0%.

**Written-
assignment:**
Not all the questions in the written assignments will be marked and you will also not be informed beforehand which questions will be marked. The reason for this is that Mathematics is learnt by “doing Mathematics”, and it is therefore extremely important to do as many problems as possible.

You can self-assess the questions that are not marked by comparing your solutions with the solutions that will be given to you.

Note that Assignment 01 is the compulsory assignment for admission to the examination and must be uploaded by the due date.

The written/ Online assignments can only be submitted online electronically through myUnisa.

Assignments	Feedback as Tutorial Letters
01	201
02, etc, …, 06	202, etc,…, 206.

Please note that Assignment 01 & Assignment 06 are Multiple-Choice Assignments and are marked by the system.

The Assignments for the Module are as follow:

Assignment Nr.	01	02	03
Unique Nr.			

Due date	08 April 2022	06 May 2022	03 June 2022

Assignment Nr.	04	05	06
Unique Nr.			
Due date	01 July 2022	29 July 2022	26 August 2022

Because this is an online module, the assignments contained in this Tutorial Letter are made available online. You will see the Assignments when you go online. We urge you to tune to the Website MAT1512-22-Y1 for more updated information.

8.1 Assessment plan

The table in the Tutorial Letter, which gives an Overview of the Module, indicates which sections in the textbook cover the syllabus of the module and have to be studied.

At the beginning of each assignment there is an indication of the sections in the textbook and study guide, which have to be studied properly before the assignment is attempted.

It is very important to study each section well at this stage. Make a good start by reading through the text, studying each and every example and doing the indicated exercises. Study the specific sections as if the assignment that follows, is a test of your knowledge and understanding of these sections.

The due dates of the assignments set the pace at which you should work through the content.

Month	**Activities**
January February	Read Tutorial Letter 101 (this letter). Read pp. vii to ix of the Study Guide and the sections of James Stewart Calculus to which these pages refer. Make sure you have all your study material. Study Chapters 1 & 2 of James Stewart Calculus as well as Units 1, 2 & 3 of the Study Guide. Prepare for Assignment 1.
March	Study Chapters 1 & 2 of James Stewart Calculus as well as Units 1, 2 & 3 of the Study Guide. Prepare for Assignment 1.
April	Submit Assignment 1. Study Chapters 2 & 3 of James Stewart Calculus as well as Unit 4 of the Study Guide. Prepare for Assignments 2.

May	Submit Assignments 2. Study Chapters 3 & 4 of James Stewart Calculus as well as Units 4 & 5 of the Study Guide. Prepare for Assignments 3.
June	Submit Assignment 3. Study Chapters 5, 6.1, 14.1 – 14.5 (14.4 read only) of James Stewart Calculus as well as Units 4 & 5 of the Study Guide. Prepare for Assignments 4 & 5.
July	Submit Assignments 4 & 5. Study Chapter 9 & ALL the above mentioned Chapters of James Stewart Calculus with Unit 6 in the Study Guide and Units: 1, 2, 3, 4 & 5 in the Study Guide. Prepare for Assignment 6.
August	Submit Assignments 6. Study all the Chapters and revise your work. Prepare for the exam.
September	Work through the solutions of Assignments 1 to 6 and learn from your mistakes. Prepare for the exam.
October November	Study for the exam. Write the exam.
December	ENJOY YOUR HOLIDAY!

Draw up your own study schedule and keep to it!

See the brochure*Study @ Unisa* for general time management and planning skills.

Assessment criteria

There are six written assignments and one examination.

Assessment plan

Please note that this module has a total of six compulsory assignments which contribute 20% to the final mark.

Assignments should be submitted electronically via *myUnisa*.

The assignments contribute to the year mark as follows:

Assignment Number	Type of assignment	Contribution to the final mark (%)
01	Multiple Choice	3
02	Written	3
03	Written	3
04	Written	3
05	Written	3
06	Multiple Choice	5

Total		20

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8.2 Year mark and final examination/other options

The year mark and the examination mark for this module will be divided as follows:

Type of assessment	Contribution to the final mark (%)
Formative	20
Summative	80
Final mark	100

Please note that the 20% contribution by the assignments makes it extremely important that you do all the assignments and score high marks, otherwise it is impossible for you to pass the module. This also means that if you do all the assignments well, there is less risk for you failing the module. The final examination is a 2-hours written exam that will be conducted online, according to the examination calendar, which you can access on the Unisa website

You only submit your assignments electronically via myUnisa. Assignments may not be submitted by fax or e-mail.

To submit an assignment via myUnisa:

- Go to *myUnisa*.
- Log in with your student number and password.
- Select the module.
- Click on “Assignments” in the menu on the left-hand side of the screen.
- Follow the instructions.

The Examination

You will write the examination in October/ November 2022 and the supplementary examination will be written in January/ February 2023.

The Examination Section will provide you with relevant information regarding the examination in general, examination dates and examination times.

Please note:

- The exam is a two-hour examination.
- The use of a pocket calculator is not permitted during the examination.

The examination questions will be similar to the questions asked in the study guide and in the assignments.

9. CONCLUSION

Do not hesitate to contact us by e-mail if you are experiencing problems with the content of this tutorial letter or with any academic aspect of the module.

Remember that there are no "short cuts" to studying and understanding mathematics. You need to be dedicated, work consistently and practice, practice and practice some more. Do not hesitate to contact us by e-mail if you are experiencing problems with the content of this tutorial letter or with any academic aspect of the module.

We wish you a fascinating and satisfying journey through the learning material, and trust that you will complete the module successfully.

Enjoy the journey!

Dr. SB. Mugisha – lecturer for MAT1512.

DEPARTMENT OF MATHEMTICAL SCIENCES

ASSIGNMENT 01
Fixed Closing Date: 08 April 2022
Total Marks: 100

1. Determine the following limits:

$$\lim_{x\to 2}\frac{\sqrt{6-x}-2}{\sqrt{3-x}-1} \quad \text{and} \quad \lim_{x\to -\infty}\frac{3-|x|}{2|x|+1}$$

Then the correct answers are:

(1) 2 and -2
(2) $\frac{1}{2}$ and $-\frac{1}{2}$
(3) *6* and *3*
(4) None of the above. (5)

2.
Determine the following limit:

$$\lim_{x\to -\frac{2}{5}}\frac{-5x^2-2x}{|2+5x|}$$

Then the correct answer is:

(1) 0
(2) Does not exist
(3) -5
(4) None of the above. (5)

3.
Determine the following limit:

$$\lim_{x\to \frac{\pi}{2}}\frac{\sin^2 x-1}{\sin x-1}$$

Then the correct answer is:

(1) $\frac{\pi}{2}$
(2) 2

(3) -1
(4) None of the above. (5)

4.
Determine the following limit:

$$\lim_{x\to -\infty}\frac{\left|25-x^2\right|}{x(x+5)}$$

Then the correct answer is:

(1) $-\infty$
(2) 1
(3) Does not exist
(4) None of the above. (5)

5. Determine the following limit:

$$\lim_{x\to 4}h(x)$$

where $h(x)=\begin{cases}\dfrac{2x^2-8}{3} & if \quad x<4\\ 2x & if \quad x\geq 4\end{cases}$

Then the correct answer is:

(1) 2
(2) $-\dfrac{8}{3}$
(3) 8
(4) None of the above. (5)

The following Questions from **Question 6**, below, to **Question 11**, below, refer to the function h below:

Let h be a function defined as:

$$h(x)=\begin{cases} 8-x & if \;\; x<0 \\ x & if \;\; 0<x\leq 2 \\ \dfrac{1}{2}x^2 & if \;\; x>2 \end{cases}$$

6. Determine the following limit:

$$\lim_{x\to 0^-} h(x)$$

Then the correct answer is:

(1) 0
(2) 8
(3) 2
(4) None of the above. (5)

7. Determine the following limit:

$$\lim_{x\to 0^+} h(x)$$

Then the correct answer is:

(1) 0
(2) 8
(3) 2
(4) None of the above. (5)

8. Determine the following limit:

$$\lim_{x\to 0} h(x)$$

Then the correct answer is:

(1) 0
(2) 8
(3) Does not exist.
(4) None of the above. (5)

9. Determine the following limit:

$$\lim_{x\to 2^-} h(x)$$

Then the correct answer is:

(1) 2
(2) 8
(3) 6
(4) None of the above. (5)

10. Determine the following limit:

$$\lim_{x \to 2^+} h(x)$$

Then the correct answer is:

(1) 2
(2) 8
(3) 6
(4) None of the above. (5)

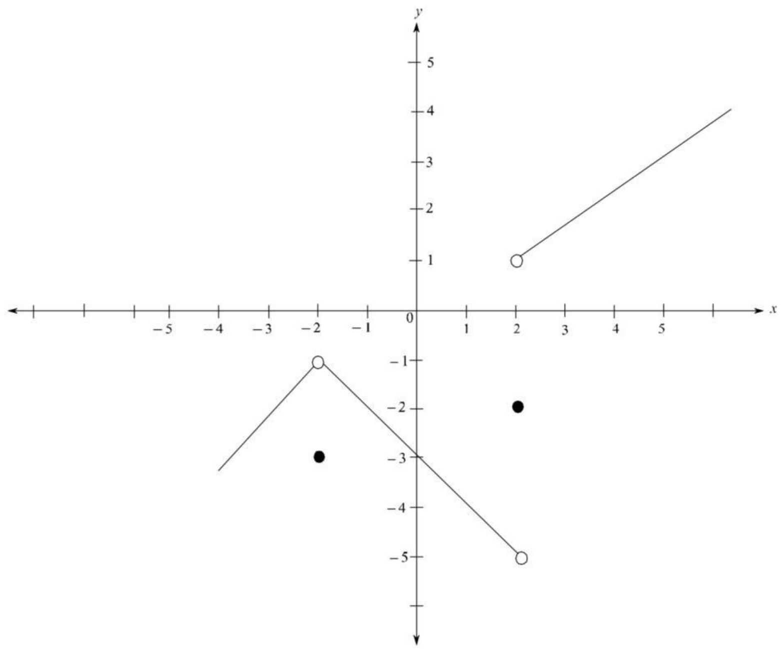
11. Determine if the function h is continuous at $x = 0$ and $x = 2$.

Then the correct answer is:

(1) Yes, the function h is continuous at both $x = 0$ and $x = 2$.
(2) No, the function h is NOT continuous at both $x = 0$ and $x = 2$.
(3) The function h is NOT continuous at $x = 0$ but is continuous at $x = 2$.
(4) None of the above. (5)

The following Questions from **Question 12**, below, up to and including **Question 18**, below, are about finding Limits from a graph.

Let the graph of the particular function $g(x)$ be represented as shown below (the graph is NOT drawn to scale):



Use the graph of g in the figure above to find the following values, if they exist:

12. $g(-2)$

Then the correct answer is:

(1) -1
(2) -3
(3) Undefined.
(4) None of the above. (5)

(4) None of the above. (5)

13. $\lim_{x \to -2^-} g(x)$

Then the correct answer is:

(1) -2
(2) -1
(3) Undefined.
(4) None of the above. (5)

14. $\lim_{x \to -2^+} g(x)$

Then the correct answer is:

(1) -2
(2) -3
(3) -1
(4) None of the above.
(5)

15. $\lim_{x \to -2} g(x)$

Then the correct answer is:

(1) -1
(2) -2
(3) -3
(4) None of the above.
(5)

16.
$\lim_{x \to -2^+} g(x)$

Then the correct answer is:

(1) -2
(2) -3
(3) -1
(4) None of the above. (5)

17.
$\lim_{x \to -2} g(x)$

Then the correct answer is:

(1) -1

(2) -2

(3) -3

(4) None of the above. (5)

18. Identify the discontinuities in the function $g(x)$ graphed above.

Then the correct answer is:

(1) $x = -2$, removable discontinuity and $x = 2$, Jump discontinuity.
(2) $x = -3$, essential discontinuity and $x = 2$, jump discontinuity.
(3) $x = -2$, Jump discontinuity and $x = 2$, removable discontinuity.
(4) None of the above. (5)

19. Use the squeeze Theorem to determine

$$\lim_{x \to \infty} \frac{3 - \sin(e^x)}{\sqrt{x^2 + 2}}$$

Then the correct answer is:

(1) 0

(2) $\frac{4}{\sqrt{2}}$

(3) $\frac{2}{\sqrt{2}}$

(4) None of the above. (5)

20. Let

$$f(x)=\begin{cases}(x^2+1)\dfrac{\sin(x^2-4)}{x^2-4} & if \quad x<2 \\ b & if \quad x=2 \\ a\cos(x-2) & if \quad x>2 \end{cases}$$

Determine the values of a and b that make the function $f(x)$ continuous at $x=2$.

Then the correct answers are:

(1) $a=2$ and $b=5$

(2) $a=5$ and $b=2$

(3) $a=5$ and $b=5$

(4) None of the above. (5)

GRAND TOTAL: [100]

ASSIGNMENT 02
Fixed Closing Date: 06 May 2022
Total Marks: 100

1. Use the first principles of differentiation to determine $f'(x)$ for the following functions:

(a) $f(x) = 3x^2 - 4x + 1$ (5)

(b) $f(x) = \dfrac{2x+1}{x+3}$ (5)

(c) $f(x) = \dfrac{4}{\sqrt{1-x}}$ (5)

2. By using the first principles of differentiation, find the following:

(a) $f(x) = \dfrac{1-x}{2+x}$ (3)

(b) $f'(-3)$ (2)

3. Use appropriate differentiation techniques to determine the first derivatives of the following functions (simply your answers as far as possible).

(a) $f(v) = \dfrac{3\sqrt{v} - 2ve^{v}}{v}$ (5)

(b) $c_0(t) = \left(\sqrt{5}\right)t + \dfrac{\sqrt{7}}{t}$ (5)

(c) $f(x) = \dfrac{x^2+1}{x^3-1}$ (5)

(d) $f(x) = \cos\sqrt{\sin(\tan \pi x)}$ (5)

(e) $f(x) = \dfrac{(\tan x) - 1}{\sec x}$ (5)

(f) $y = \dfrac{\cos x}{1 + \sin x}$ (5)

4. Determine the derivatives of the following functions:

(a) $f(x)=\ln(x+\ln x)$ (5)

(b) $g(x)=\sqrt{\dfrac{x-1}{x^4+1}}$ (5)

(c) $h(x)=\sqrt{x}e^{x^2-x}(x+1)^{\frac{2}{3}}$ (5)

(d) $y(x)=(\sin x)\ln x$ (5)

(e) $y=\left(\sqrt{x}\right)^{\sin x}$ (5)

5. Determine the derivatives of the following inverse trigonometric functions:

(a) $f(x)=\tan^{-1}\sqrt{x}$ (5)

(b) $y(x)=\ln\left(\dfrac{x^2\cot^{-1}x}{\sqrt{x-1}}\right)$ (5)

(c) $g(x)=\sin^{-1}(3x)+\cos^{-1}\left(\dfrac{x}{2}\right)$ (5)

(d) $h(x)=\tan^{-1}\left(x-\sqrt{x^2+1}\right)$ (5)

(e) $k(x)=\left(\sqrt{7x^3-5x^2+x}\right)\cot^{-1}\left(3-5x^2\right)-9\operatorname{cosec}^{-1}\left(2-3x^2\right)$ (5)

Total: [100]

ASSIGNMENT 03
Fixed Closing Date: 03 June 2022
Total Marks: 100

1. Use the method of implicit differentiation to determine the derivatives of the following functions:

(a) $x\sin y + y\sin x = 1$ (5)

(b) $\tan(x-y) = \frac{y}{1+x^2}$ (5)

(c) $\sqrt{x+y} = x^4 + y^4$ (5)

(d) $y + x\cos y = x^2 y$ (5)

(e) $2y + \cot(xy^2) = 3xy$ (5)

2. Find the number “c” that satisfy the Mean Value Theorem (M.V.T.) on the given intervals.

(a) $f(x) = e^{-x}$, $[0, 2]$ (5)

(b) $f(x) = \frac{x}{x+2}$, $[1, \pi]$ (5)

3. Determine the equation of the tangent and normal at the given points:

(a) $y + x\cos y = x^2 y$, $\left[1, \frac{\pi}{2}\right]$ (5)

(b) $h(x) = \frac{2}{\sqrt{x^2+1}}$, at $x = 1$. (5)

4. Find the derivative of $f(x) = \int_{-x}^{\sqrt{x}} \left(\sqrt{v^2+2}\right) dv$ (5)

5. Find the derivative of the following functions using the appropriate rules for differentiation.

Simplify your answer: $F(x) = \int_{2x}^{x^2} \sqrt{t^2+1}\,dt$ (5)

6. Find the derivatives of the following functions by using the appropriate rules of differentiation:

$y = \int_{1-3x}^{1} \frac{u^3}{1+u^2} du$ (5)

7. The equation of the ellipse is given as

$$\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1$$

Use implicit differentiation to determine the derivative of the equation of the ellipse given above. (5)

8. Determine the slope of the equation in Question 1., above, at $(x_0,\ y_0)$. (5)

9. Hence or otherwise find the equation of the tangent at $(x_0,\ y_0)$.
The equation referred to in Question 1, above. (5)

10. Let $x^2-xy+y^2=3$ be the equation of an ellipse. By implicit differentiation

Determine the equation of the normal of the equation given above at $(-1,1)$. (5)

11. Given that $\sin(x+y)=2x$, find the equation of the tangent line at the point $(0,\ \pi)$. (5)

12. Find the equation of the tangent and normal lines to the curve of:

$\pi\sin y+2xy=2\pi$ at the point $\left(1,\ \frac{\pi}{2}\right)$. (5)

13. Let $x^4+5y=3x^2y^3$. Find $\frac{dy}{dx}$ using implicit differentiation. (5)

14. For the equation $x^2+y^3-2y=3$

Find the equation of the normal line at the point (2, 1). (5)

Total:[100

ASSIGNMENT 04
Fixed Closing Date: 01 July 2022
Total Marks: 100

1. Determine the first order partial derivative of the following functions:

(a) $z = \ln(x + t^2)$ (5)

(b) $F(x, y) = \int_y^x \cos(e^t)\,dt$ (5)

(c) $f(x, y, z) = xy^2 e^{-xz}$ (5)

2. Clairaut's Theorem holds that $U_{xy} = U_{yx}$, show that the following equations obey Clairaut's Theorem.

(a) $U = \ln(x + 2y)$ (5)

(b) $U = e^{xy} \sin y$ (5)

3. Laplace's equations holds that $U_{xx} + U_{yy} = 0$, verify that the second derivative of the following equation are Laplace's equation:

(a) $U = \ln\sqrt{x^2 + y^2}$ (5)

(b) $U = x^2 - y^2$ (5)

4. Determine the following integrals:

(a) $\int\left(U^6 - 2U^5 + \frac{2}{7}\right)dU$ (5)

(b) $\int\left(\frac{1 + \sqrt{x} + x}{x}\right)dx$ (5)

(c) $\int_1^4\left(\frac{4 + 6u}{\sqrt{u}}\right)du$ (5)

(d) $\int_0^2 |2x - 1|\,dx$ (5)

5. Determine the following integrals:

(a) $\int_0^t \sin(3\pi t)\,dt$ (5)

(b) $\int_0^1 \frac{\sin 2x}{\cos^2 x}dx$ (5)

(c) $\int e^{\cos x} \sin x\,dx$ (5)

(d) $\int_0^4 \left|\sqrt{x} - 1\right|dx$ (5)

6. Use substitution method to determine the following integrals:

(a) $\int \frac{x^3}{1+x^4}dx$ (5)

(b) $\int \cos^4\theta \sin\theta d\theta$ (5)

(c) $\int_{2\pi}^{3\pi} 3\cos^2 x \sin x dx$ (5)

7. Let $F(x, y) = y - \sin(xy)$. Find the partial derivatives F_x and F_y. Then find $\frac{dy}{dx}$.

Confirm your answer above by finding $\frac{dy}{dx}$ using implicit differentiation. (5)

8. Let $F(x, y) = y\cos(x^2y^2) + y$, then find the first partial derivatives F_x and F_y.

Then using your answer, find $\frac{dy}{dx}$. Using implicit differentiation conform your answer. (3)

9. If $z = \sin(xe^y)$ where $x = 3u^2 + uv$ and $y = u^3 - \ln v$.

Use the chain rule for partial differentiation to find $\frac{\partial z}{\partial u}$. (2)

Total:[100]

ASSIGNMENT 05
Fixed Closing Date: 29 July 2022
Total Marks: 100

1. Show whether or not the following differential equations are separable:

1.1 $\frac{dy}{dx} = \frac{x+1}{y-1}$ (5)

1.2 $\frac{dy}{dx} = \frac{ye^{x+y}}{x^2+2}$ (5)

1.3 $\frac{dS}{dt} = t(\ln(S^{2t})) + 8t^2$ (5)

2. Solve the following differential equation by using separation of variables method:

2.1 $x\dfrac{dy}{dx}=4y$ (5)

2.2 $\dfrac{dp}{dt}=\dfrac{(1+p^2)\cos t}{p\sin t}$ (5)

3. Solve the following differential equations subject to the given initial conditions:

3.1 $\dfrac{dy}{d\theta}=y\sin\theta$, $y(\pi)=3$. (5)

3.2 $x^2\dfrac{dy}{dx}=y-xy$, $y(1)=1$. (5)

4. The population of a certain community is known to increase at a rate proportional to the number of people

present any time. If the population had doubled in 5 years, how long will it take to triple? (5)

5. Solve the initial value problem $\dfrac{dx}{dt}=\dfrac{3t^2+\sec^2 t}{3x^2}$, $x(0)=5$. (5)

6. Solve the following initial value problem $\dfrac{dy}{dx}=\operatorname{cosec}^2 x(e-5y)$, $y\left(\dfrac{\pi}{2}\right)=0$. (5)

7. A bacterial culture starts with 2200 bacteria and after 3 hours there are 3700 bacteria.

Assuming that the culture grows at a rate proportional to its size, find the population after 6 hours. (5)

8. Solve the following initial value problem: $\dfrac{dy}{dx}=\dfrac{\cos^2 y}{4x-3}$, $y(1)=\dfrac{\pi}{4}$. (5)

9. Solve the fooling Initial Value problem: $\dfrac{dy}{dx}=\dfrac{9x^2-\sin x}{\cos y+5e^y}$, $y(0)=\pi$. (5)

10. A bacterial culture starts with 1000 bacteria and after 2 hours there are 2500 bacteria. Assuming that the culture grows at a rate proportional to its size, find the population after 6 hours. (5)

11. Solve the following Initial value problem $\dfrac{dw}{dt}=\dfrac{2t+\sec^2 t}{2w}$, $w(0)=-5$. (5)

12. Determine the solution of $\dfrac{y'}{x}=\dfrac{1}{y^2-y}$ that passes through the point (1, 2). (5)

13. Solve the Initial Value Problem $\dfrac{dw}{dt}=t^2w^2$, $w(0)=a$. (5)

14. Use the method of separation of variables to find a general solution to the differential equation

$\dfrac{dy}{dx}=2xy+3y-4x-6$. (5)

15. Find the solution to the initial-value problem $6\frac{dy}{dx}=(2x+1)(y^2-2y-8)$ with $y(0)=-3$ using the method of separation of variables. (3)

16. Solve the equation $xdx+\sec x\sin ydy=0$. (2)

17. A bacterial culture contains 100 cells at a certain point in time. Sixty minutes later, there are 450 cells. Assuming exponential growth, determine the number of cells present at time t. (3)

18. Solve the following Initial Value Problem $xy\frac{dy}{dx}=\ln x$, $y(1)=2$. (2)

Total:[100]

ASSIGNMENT 06
Fixed Closing Date: 26 August 2022
Total Marks: 100

QUESTION 1

1. Determine the following limit (if it exists):

$$\lim_{x \to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1}$$

Then the correct answer is:

(1) 3
(2) 2
(3) $\frac{1}{2}$
(4) None of the above. **(4)**

2. Determine the following limit (if it exists):

$$\lim_{x \to -3^-} \frac{|x+3|}{x^2-9}$$

Then the correct answer is:

(1) $\frac{1}{3}$
(2) 3
(3) $\frac{1}{6}$
(4) None of the above. **(4)**

3. Determine the following limit (if it exists):

$$\lim_{x \to -\infty} \frac{\sqrt{x^2+4x}-2x}{2x}$$

Then the correct answer is:
(1) -1

(2) 2
(3) $-\frac{3}{2}$
(4) None of the above. **(4)**

4. Determine the following limit (if it exists):

$$\lim_{x \to 1} \frac{1-\sqrt{x}}{1-x}$$

Then the correct answer is:

(1) 1
(2) $\frac{1}{2}$
(3) -2
(4) None of the above. **(4)**

5. Determine the following limit (if it exists):

$$\lim_{x \to 0} \frac{2x}{3-\sqrt{x+9}}$$

Then the correct answer is:
(1) $\frac{2}{3}$
(2) -12
(3) 2
(4) None of the above. **(4)**

6. Determine the following limit (if it exists):

$$\lim_{t \to 0} \frac{\sin t - \tan 2t}{t}$$

Then the correct answer is:
(1) 0
(2) -1
(3) 2
(4) None of the above. **(4)**

7. Use the Squeeze Theorem to determine the following limit:

$$\lim_{k \to \infty} \frac{5k^2 - \cos 3k}{k^2 + 10}.$$

(1) - 3

(2) $\frac{5}{10}$

(3) 5

(4) None of the above. **(4)**

The following questions, **Question 8** and **Question 9** are based on the piecewise function below:

Let $G(x) = \begin{cases} -(x-2) & if \quad x < 2 \\ x-2 & if \quad x \geq 2 \end{cases}$

8. Draw the graph of $G(x)$ and determine $\lim_{x \to 2} G(x)$.

Then the correct answer is:

(1) 2
(2) -2
(3) 0
(4) None of the above. **(4)**

9. Is $G(x)$ continuous at $x = 2$? Give a reason for your answer.

Then the correct answer is:
(1) No, since $G(x)$ is a piecewise function.
(2) Yes, since $G(2) = 0 = \lim_{x \to 2} G(x)$.
(3) No, since $G(x)$ is v-shaped.
(4) None of the above. **(4)**

10. By the first principles of differentiation, find the derivative of $f(x) = 2x^2 + 3x + 4$ at $x = -2$.

Then the correct answer is:

(1) $f'(-2) = 5$
(2) $f'(-2) = -5$
(3) $f'(-2) = \pm 5$
(4) None of the above. **(4)**

11. Find the derivative of the following function by using the appropriate rules of differentiation:

$$f(x)=\frac{x^2+\sqrt{x}}{\sin x\cos x}$$

Then the correct answer is:

(1) $\frac{\sin x\cos x-\cos 2x}{\sin^2 x\cos^2 x}$

(2) $\frac{\sin x\cos x\left(2x+\frac{1}{2\sqrt{x}}\right)-\cos 2x\left(x^2+\sqrt{x}\right)}{\sin^2 x\cos^2 x}$

(3) $\sin x\cos x-\cos 2x$

(4) None of the above. **(4)**

12. Find the derivative of the following function by using the appropriate rules of differentiation:

$$g(x)=(\cos 5x)^{\sin\left(x^2\right)}$$

Then the correct answer is:

(1) $\cos^{\sin\left(x^2\right)}(5x)\left[2x\cos\left(x^2\right)\ln(\cos(5x))-5\tan(5x)\sin\left(x^2\right)\right]$

(2) $\sin\left(x^2\right)\ln(\cos(5x))$

(3) $-\sin\left(x^2\right)\ln(\cos(5x))$

(4) None of the above. **(4)**

13. Find the derivative of the following function by using the appropriate rules of differentiation:

$$F(z)=\int_z^{z^3}\sin 3z\,dz$$

Then the correct answer is:

(1) $3z^2\sin\left(3z^3\right)-\sin(3z)$

(2) $3z^2\cos\left(3z^3\right)-\cos(3z)$

(3) $3z^2\cos\left(3z^3\right)-\sin(3z)$

(4) None of the above. **(4)**

14. Given: $\sin(x+y)=2x$, find $\frac{dy}{dx}$ by using implicit differentiation. Then the correct answer is:

Then the correct answer is:

(1) $\frac{dy}{dx}=\frac{\cos(x+y)-2}{\cos(x+y)}$

(2) $\frac{dy}{dx}=\frac{2-\cos(x+y)}{\cos(x+y)}$

(3) $\frac{dy}{dx}=\frac{\sin(x+y)-2}{\sin(x+y)}$

(4) None of the above. **(4)**

15. The equations of the tangent line and normal line to the curve $\sin(x+y)=2x$ at the point $(0,\ \pi)$ are given by:

(1) $y=3x-\pi$ and $y=-\frac{1}{3}x-\pi$

(2) $y=-3x+\pi$ and $y=\frac{1}{3}x+\pi$

(3) $y=\pm 3x$ and $y=\pm 3x-\pi$

(4) None of the above. **(4)**

16. Determine the following integral:

$$\int\left(x-\frac{2}{x^2}\right)\left(x+\frac{2}{x^2}\right)dx$$

Then the correct answer:

(1) $3x^3+\frac{4}{x^3}+c$

(2) $\frac{1}{3}x^3+\frac{4}{3}\left(\frac{1}{x^3}\right)+c$

(3) $x^2-\frac{4}{x^4}$

(4) None of the above. **(4)**

17. Determine the following integral

$$\int e^{5x}\left(\frac{e^{2x}}{7}+\frac{3}{e^{3x}}\right)dx$$

Then the correct answer is:

(1) $\frac{1}{49}e^{7x}+\frac{3}{2}e^{2x}+c$

(2) $\frac{e^{7x}}{7}+3e^{2x}+c$

(3) $\frac{1}{49}e^{7x}+3$

(4) None of the above. **(4)**

18. Determine the following integral:

$$\int \frac{1}{\left(4-\left(\sqrt{3}\right)x\right)^3}dx$$

Then the correct answer:

(1) $\frac{1}{\left(4-\left(\sqrt{3}\right)x\right)^2}+c$

(2) $\frac{1}{2\sqrt{3}}\left[\frac{1}{\left(4-\left(\sqrt{3}\right)x\right)^2}\right]+c$

(3) $\frac{-1}{\left(4-\left(\sqrt{3}\right)x\right)^2}+c$

(4) None of the above. **(4)**

19. Determine the following integral:

$$\int_0^{\frac{\pi}{4}} (\tan x)^3 (\sec x)^3\, dx$$

Then the correct answer:

(1) $\frac{2\sqrt{2}-2}{15}+c$

(2) $\frac{2\left(\sqrt{2}+1\right)}{15}+c$

(3) $\frac{2-2\sqrt{2}}{15}+c$

(4) None of the above. **(4)**

20. Let $f(x)=x^2-2$ and $g(x)=-|x|$, then sketch the graphs of f and g on the same axes. Find the area enclosed by $f(x)=x^2-2$ and $g(x)=-|x|$.

Then the correct answer is:

(1) 2 squareunits.

(2) 7 square units.

(3) $\frac{7}{3}$ square units.

(4) None of the above. **(4)**

21. Solve the following Initial Value Problem:

$$\frac{dx}{dt} = \frac{3t^2 + \sec^2 t}{3x^2} \quad ; \quad x(0) = 5 .$$

Then the correct answer is:

(1) $x = \sqrt[3]{t^3 + \tan t - 125}$

(2) $x = \sqrt[3]{t^3 + \tan t + 125}$

(3) $x = \sqrt[3]{t^3 - \tan t + 125}$

(4) None of the above. **(4)**

22. Use the Chain Rule for partial derivatives to find $\frac{\partial z}{\partial u}$ if $z = \cos(xy) + y\cos x$, where

$x = u^2 + v^2$ and $y = uv$.

Then the correct answer is:

(1) $\frac{\partial z}{\partial u} = -2u^2 v\sin(u^3 v + v^3 u) - 2u^2 v\sin(u^2 + v^2) - (u^2 v + v^3)\sin(u^3 v + v^3 u) + v\cos(u^2 + v^2)$

(2) $\frac{\partial z}{\partial u} = 2u^2 v\sin(u^3 v + v^3 u) - 2u^2 v\sin(u^2 + v^2) - (u^2 v + v^3)\sin(u^3 v + v^3 u) + v\cos(u^2 + v^2)$

(3) $\frac{\partial z}{\partial u} = -2u^2 v\sin(u^3 v + v^3 u) + 2u^2 v\sin(u^2 + v^2) - (u^2 v + v^3)\sin(u^3 v + v^3 u) + v\cos(u^2 + v^2)$

(4) None of the above. **(4)**

23. Let $F(x, y) = 2y - 3xy + \cot(xy^2)$, find the first partial derivatives F_x and F_y. Then the correct answer is:

(1) $F_x = -3y - y^2 \csc^2(xy^2)$ and $F_y = 2 - 3x - 2xy\csc^2(xy^2)$

(2) $F_x = 3y + y^2 \csc^2(xy^2)$ and $F_y = -2 + 3x - 2xy\csc^2(xy^2)$

(3) $F_x = -3y + y^2 \csc^2(xy^2)$ and $F_y = 2 - 3x - 2xy\csc^2(xy^2)$

(4) None of the above.

(4)

24. Using Question 25. above, find $\frac{dy}{dx}$

Then the correct answer is:

(1) $\frac{dy}{dx} = \frac{y^2 \csc^2(xy^2) + 3y}{2 - 3x - 2xy\csc^2(xy^2)}$

(2) $\frac{dy}{dx} = \frac{-y^2 \csc^2(xy^2) - 3y}{2 + 3x + 2xy\csc^2(xy^2)}$

(3) $\frac{dy}{dx} = \frac{-y^2 \csc^2(xy^2) - 3y}{2 - 3x - 2xy\csc^2(xy^2)}$

(4) None of the above. **(4)**

25. If $F(x, y) = 0$, then find $\frac{dy}{dx}$ using implicit differentiation to confirm your answer in Question 26, above. Given that $F(x, y) = 2y - 3xy + \cot(xy^2)$.

Then the correct answer is:

(1) $\frac{dy}{dx} = \frac{y^2 \csc^2(xy^2) + 3y}{2 - 3x - 2xy\csc^2(xy^2)}$

(2) $\frac{dy}{dx} = \frac{-y^2 \csc^2(xy^2) + 3y}{2 - 3x - 2xy\csc^2(xy^2)}$

(3) $\frac{dy}{dx} = \frac{-y^2 \csc^2(xy^2) - 3y}{2 - 3x - 2xy\csc^2(xy^2)}$

(4) None of the above. **(4)**

TOTAL: [100]