MAT1512

May/June 2017

CALCULUS A

Duration : 2 Hours   100 Marks

EXAMINERS :
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Closed book examination.

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This paper consists of 4 pages. **ANSWER ALL QUESTIONS.**
ALL CALCULATIONS MUST BE SHOWN.

Calculators may NOT be used.

QUESTION 1

(a) Determine the following limits (if it exists)

(i) $\lim\limits_{x \to -3} \dfrac{(x^2 - 9)}{2x + x^2 - 3}$ (3)

(ii) $\lim\limits_{x \to -\infty} \dfrac{2x + x^3 + 1}{2x^2 - x + 4}$ (3)

(iii) $\lim\limits_{t \to 1} \dfrac{1 - t}{1 - \sqrt{t}}$ (3)

(iv) $\lim\limits_{t \to 0} \dfrac{\tan 8t}{\tan 5t}$ (3)

(v) $\lim\limits_{x \to -3^+} \dfrac{x^2 - 9}{|x + 3|}$ (4)

(b) Use Sandwich theorem to determine the following limit

$$\lim\limits_{k \to 0} k^2 \sin\left(\frac{1}{\cos k}\right)$$ (3)

(c) Let $g(s) = \begin{cases} 2s + 1 & \text{if } s > 3 \\ w & \text{if } s = 3 \\ s^2 - 1 & \text{if } s < 3 \end{cases}$

   (i) Sketch the graph of g. (3)

   (ii) Show that $g(s)$ cannot be continous at $s = 3$ for any value of $w \in \mathbf{R}$. (3)

[25]

QUESTION 2

(a) Find the first derivative of the following function(s) by using the appropriate rules of differentiation.

(i) $y = \sqrt{x \cos x}$ (4)

(ii) $y = \dfrac{\sin \pi x}{\tan x + 1}$ (4)

(iii) $y^2 \ln x = x^3 e^y$ (4)

[TURN OVER]

(iv) $g\left(\theta\right) = \cos\left(5\theta\right)^{\sin\theta^2}$ (4)

(v) $F\left(x\right) = \int_{-x}^{\sqrt{x}} \sqrt{v^2 + 2}\, dv$ (4)

(b) Given that $h\left(x\right) = \dfrac{x}{\sqrt{x^2 + 1}}$

Determine the equation of the tangent and normal line to the curve of $h\left(x\right)$ at point where $x = 1$ (5)

[25]

QUESTION 3

(a) Determine the following integrals, make a direct substitution and change of variable where necessary.

(i) $\displaystyle\int \left(\dfrac{x^2 - 9}{x + 3}\right) dx$ (3)

(ii) $\displaystyle\int 5x\sqrt{4 + x}\, dx$ (3)

(iii) $\displaystyle\int \dfrac{3}{x\left(3\ln x + 1\right)}\, dx$ (3)

(iv) $\displaystyle\int_{2\pi}^{3\pi} 3\sin x \cos^2 x\, dx$ (5)

(v) $\displaystyle\int_{0}^{1} \dfrac{5x}{\left(4 + x^2\right)^2}\, dx$ (5)

(b) Let $g\left(x\right) = 2 - x^2$ and $h\left(x\right) = -x$.

Sketch the graph of g and h in the same axes and find the area of the region enclosed by the curves $g\left(x\right) = 2 - x^2$ and $h\left(x\right) = -x$. (6)

[25]

QUESTION 4

(a) Use the method of separation of variables to

(i) Solve the differential equation
$$\frac{dy}{dx} = \frac{\cos\left(x\right)\left(1 + y^2\right)}{y\sin\left(x\right)}$$
(5)

[TURN OVER]

(ii) Solve the following initial value problem

$$xy\frac{dy}{dx} = \ln x; \quad y(1) = 2 \tag{5}$$

(iii) A bactrial culture contains 100 cells at a certain point in time. Sixty minutes later, there are 450 cells. Assuming exponential growth, determine the number of cells present at time t. (5)

(b) If $f(x,y) = \dfrac{x}{x^2 + y^2}$

Determine

(i) f_x and f_y (3)

(ii) Use your answers in b(i) above to write down $\dfrac{dy}{dx}$. (2)

(iii) Confirm your answer in b(ii) by finding $\dfrac{dy}{dx}$ using implicit differentiation. (5)

[25]

TOTAL: [100]

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