

MAT1512

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CALCULUS A

Duration 2 Hours

100 Marks

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This paper consists of 3 pages

ANSWER ALL THE QUESTIONS.**Outline**

Question 1 Limits and continuity

Question 2 Derivatives

Question 3 Integrals

Question 4. Differential equations

QUESTION 1

(a) Determine the following limits (if they exist)

$$(i) \lim_{t \rightarrow 2} \frac{t^2 - 4}{t^3 - 8} \quad (3)$$

$$(ii) \lim_{x \rightarrow 2} \frac{\sin(x^2 - 4)}{x - 2} \quad (4)$$

$$(iii) \lim_{x \rightarrow 2} 2 \frac{\sqrt{x} - \sqrt{2}}{x - 2} \quad (4)$$

$$(iv) \lim_{x \rightarrow \infty} \frac{8x^2 - 9x + 1}{3x^2 + 9x - 3} \quad (3)$$

$$(b) \text{ If } 1 \leq f(x) \leq x^2 + 6x + 6 \text{ for all } x, \text{ find } \lim_{x \rightarrow -1} f(x) \quad (3)$$

[TURN OVER]

(c) Let $f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leq x < 3 \\ (3 - x)^2 & \text{if } x > 3 \end{cases}$

Determine at which value(s) of x is $f(x)$ discontinuous (6)

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QUESTION 2

(a) Differentiate $f(x) = x^2 - 2x - 15$ from first principles (5)

(b) Find the derivatives of the following functions by using the appropriate rules for differentiation

(i) $y = \sqrt{x} \cos \sqrt{x}$ (4)

(ii) $f(x) = \frac{2 \sin x}{\sin \pi x}$ (4)

(iii) $h(x) = -\int_{\sqrt{x}}^1 \frac{2z^2}{z^4 + 1} dz$ (4)

(c) Find the first and second derivatives of the function

$$y = \sqrt{2x + 1} \quad (3)$$

(d) Given $y \sin 2x = x \cos 2y$, find

(i) $\frac{dy}{dx}$ implicitly (5)

(ii) the equation of the normal line to the curve $y \sin 2x = x \cos 2y$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{4}\right)$ (5)

[30]

QUESTION 3

(a) Use the appropriate substitution to evaluate the following integrals

(i) $\int \frac{4 + 6x}{\sqrt{6 + 4x + 3x^2}} dx$ (3)

(ii) $\int \frac{x}{1 - x} dx$ (3)

[TURN OVER]

$$(iii) \int e^{\sin x} \cos x \, dx \quad (3)$$

(b) Determine the exact values for the following integrals (Use substitution if necessary)

$$(i) \int_{-4}^3 |2x - x^2| \, dx \quad (10)$$

$$(ii) \int_{-1}^0 \frac{2r}{\sqrt{1-r^2}} \, dr \quad (4)$$

(c) Determine the area of the region enclosed by the curve $y = x^2 - 2x$ and the line $y = x + 4$ (7)

Hint Sketch the graphs on the same set of axis

[30]

QUESTION 4

(a) Solve the following Initial Value Problem

$$\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}, \quad u(0) = -5 \quad (6)$$

(b) Let $G(x, y) = x^2y^4 - 3x^4y$

(i) Find the first partial derivatives G_x and G_y (3)

(ii) Using (i) above, find $\frac{dy}{dx}$ (3)

(iii) If $G(x, y) = 0$, confirm your answer in part (ii) above by finding $\frac{dy}{dx}$ using implicit differentiation (5)

[17]

TOTAL: [100]