

MAT1512 MAT112P

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CALCULUS A

Duration 2 Hours

100 Marks

EXAMINERS :
FIRST
SECOND

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This paper consists of 3 pages.

Instructions to candidates:

- The use of a pocket calculator is **NOT** permissible
- This paper consists of three pages Answer **ALL** questions
- Show **ALL** your workings

QUESTION 1

1.1 Determine the following limits (if it exists)

(a) $\lim_{x \rightarrow -3} \frac{x^2 - x - 12}{x + 3}$ (2)

(b) $\lim_{t \rightarrow 0} \frac{t \sin t}{1 - \cos t}$ (3)

(c) $\lim_{x \rightarrow \infty} \frac{\sqrt{2x^2 + 1}}{3x - 5}$ (3)

(d) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{|x - 2|}$ (3)

1.2 (a) Use the Squeeze Theorem to determine the limit: $\lim_{h \rightarrow 0} h \sin\left(\frac{1}{h}\right)$ (3)

(b) Now let

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Simplify $\frac{f(0+h) - f(0)}{h}$ and show from *first principles* that $f'(0)$ exists (3)

[17]

[TURN OVER]

QUESTION 2

Let

$$g(x) = \begin{cases} x^2 - 1 & \text{if } x < 3 \\ k & \text{if } x = 3 \\ 2x + 1 & \text{if } x > 3 \end{cases}$$

Show that, regardless of the value of $k \in \mathbb{R}$, g cannot be continuous at $x = 3$ **[4]****QUESTION 3**

3.1 Find the first derivatives of the following functions

$$(a) \ y = \sqrt{x} \left(3x^2 - \frac{2}{x^3} \right) \quad (3)$$

$$(b) \ y = 4^x \operatorname{cosec}(x^4). \quad (4)$$

$$(c) \ x^2 e^y = y^2 \ln x \quad (6)$$

$$(d) \ F(x) = \int_{x^2}^x \cos 2t \, dt \quad (5)$$

$$(3.2) \ \text{Use Logarithmic differentiation to find } \frac{dy}{dx} \text{ if } y = (\sin x)^{2x}. \quad (5)$$

(3.3) For the function

$$x^2 + y^3 - 2y = 3$$

find the equation of the normal line at the point (2, 1). (7)**[30]****QUESTION 4**

Determine the following integrals

$$(4.1) \ \int \tan^2 x + \sqrt{x} + \frac{2}{x} \, dx \quad (4)$$

$$(4.2) \ \int x 2^{x^2} \, dx \quad (4)$$

$$(4.3) \ \int \frac{x^3}{(x^2 + 1)^2} \, dx \quad (7)$$

$$(4.4) \ \int_0^1 \frac{4x + 8}{x^2 + 4x + 5} \, dx \quad (5)$$

[20]**[TURN OVER]**

QUESTION 5

Find the area enclosed by $f(x) = 1 - x^2$ and $g(x) = x^2 - 1$. Sketch the graphs of f and g on the same axes

[5]**QUESTION 6**

Make a direct substitution and change the integral limits to evaluate

$$\int_4^9 \frac{dx}{x - \sqrt{x}}$$

[6]**QUESTION 7**

Solve the following Initial Value Problem

$$\frac{dy}{dx} = \frac{9x^2 - \sin x}{\cos y + 5e^y}, \quad y(0) = \pi.$$

[6]**QUESTION 8**

A bacterial culture starts with 1000 bacteria and after 2 hours there are 2500 bacteria. Assuming that the culture grows at a rate proportional to its size, find the population after 6 hours

[6]**QUESTION 9**

If $z = \sin(x e^y)$ where

$$x = 3u^2 + uv \quad \text{and} \quad y = u^3 - \ln v,$$

use the chain rule for partial differentiation to find $\frac{\partial z}{\partial u}$

[6]**TOTAL: [100]**