QUESTION 1

(a)(i)

$$\lim_{x\to -5}\frac{x^2+x-20}{3x+15}=$$

$$=\lim_{x\to -5}\frac{(x+5)(x-4)}{3(x+5)}$$

$$=\lim_{x\to -5}\frac{(x-4)}{3}$$

$$=\frac{(-5-4)}{3}$$

$$=-\frac{9}{3}$$

$$=-3$$

(a)(ii)

$$\lim_{x\to -\infty}\frac{\sqrt{x^2+4x}-2x}{2x}$$

$$=\lim_{x\to -\infty}\frac{\sqrt{x^2(1+4/x)}-2x}{2x}$$

$$\lim_{x\to -\infty}\frac{|x|\sqrt{1+4/x}-2x}{2x}$$

$$=\lim_{x\to -\infty}\frac{-x\sqrt{1+4/x}-2x}{2x}\quad (|x|=\ -x\ since\ x<0)$$

$$=\lim_{x\to -\infty}\frac{-x(\sqrt{1+4/x}+2)}{2x}$$

$$=\lim_{x\to -\infty}\frac{-(\sqrt{1+4/x}+2)}{2}$$

$$=\lim_{x\to -\infty}\frac{-\left(\sqrt{1+\frac{4}{-\infty}}+2\right)}{2}$$

$$=\frac{-\sqrt{1+0}+2}{2}=-\frac{3}{2}$$

(a)(iii)

$$\lim_{x\to -3^-}\frac{|x+3|}{(x^2-9)}$$

Rewrite $|x+3|$ as a piecewise function using definition of absolute values.

Thus $|x+3| = \begin{cases} x+3 \text{ if } x+3>0 \implies x>-3 \\ \\ -(x+3) \text{ if } -x-3\le 0 \implies x\le -3 \end{cases}$

$$\lim_{x\to -3^-}\frac{|x+3|}{(x^2-9)} \quad |x+3| = -(x+3) since\ x < -3$$

$$= \lim_{x\to -3^-}\frac{-(x+3)}{(x+3)(x-3)}$$

$$= \frac{-1}{-3-3}$$

$$= \frac{1}{6}$$

(a)(iv)

$$\lim_{t\to 4}\frac{1+\sqrt{t}}{1-t}$$

$$= \frac{1+\sqrt{4}}{1-4}$$

$$= \frac{3}{-3}$$

$$= -1$$

OR

$$\lim_{t\to 4}\frac{1+\sqrt{t}}{1-t} = \lim_{t\to 4}\left(\frac{1+\sqrt{t}}{1-t}.\frac{1-\sqrt{t}}{1-\sqrt{t}}\right) = \lim_{t\to 4}\left(\frac{1-t}{1-t1-\sqrt{t}}\right) = \lim_{t\to 4}\frac{1}{1-\sqrt{t}} = \frac{1}{1-\sqrt{4}} = -1$$

(a)(v)

$$\lim_{\theta\to 0}\frac{\sin(5\theta)}{\sin(2\theta)}$$

$$=\lim_{\theta\to 0}\left(\frac{\sin(5\theta)}{5\theta}.\frac{5\theta}{2\theta}.\frac{2\theta}{\sin(2\theta)}\right)$$

$$=\lim_{\theta\to 0}\left(\frac{\sin(5\theta)}{5\theta}.\frac{5}{2}.\frac{2\theta}{\sin(2\theta)}\right)$$

$$=1.\frac{5}{2}.1$$

$$=\frac{5}{2}$$

(b) $\lim_{x\to 0} x^3 cos\left(\frac{10}{x}\right)$

For all x it holds that: $-1\leq cosx\leq 1$ for all $x\neq 0$, so $-1\leq cos\left(\frac{10}{x}\right)\leq 1$

Multiply throughout by x^3:

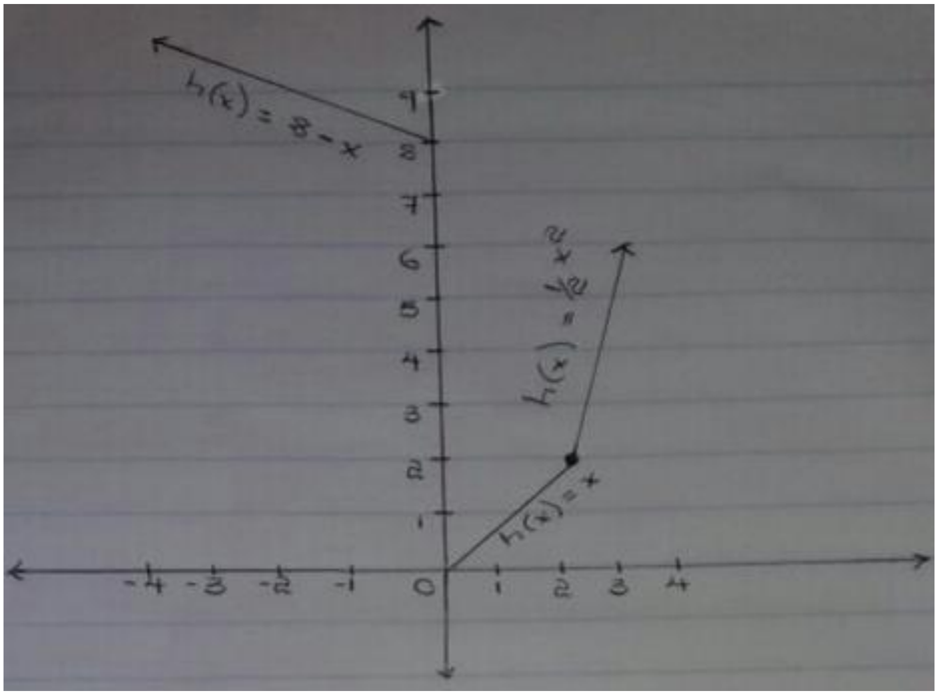
$$-x^3\leq x^3\, cos\left(\frac{10}{x}\right)\leq x^3$$

So, $\lim_{x\to 0} -x^3=-0^3=0$ and $\lim_{x\to 0} x^3=0^3=0$

Since $\lim_{x\to 0} -x^3=0=\lim_{x\to 0} x^3$

Therefore $\lim_{x\to 0} x^3 cos\left(\frac{10}{x}\right)=0$ by Squeeze Theorem.

(c)(i)



I will use a proper graphing tool for this one!!!

(c)(ii) $\lim_{x \to 0^-} 8 - x = 8 - 0 = 8$ and $\lim_{x \to 0^+} x = 0$, thus $\lim_{x \to 0} h(x)$ does not exist.

Hence $h(x)$ is discontinuous at $x = 0$.

$\lim_{x \to 2^-} x = 2$ and $\lim_{x \to 2^+} \frac{x^2}{2} = \frac{4}{2} = 2$, thus $\lim_{x \to 2} h(x)$ exists.

Hence $h(x)$ is discontinuous at $x = 2$.

QUESTION 2

(a)(i)

$$y = \sqrt{x^2 \ \sin x} \ = \sqrt{sinx}\sqrt{x^2} = (\ \text{sinx})^{\frac{1}{2}}(x^2)^{\frac{1}{2}}$$

$$\frac{d}{dx}\left((\ \text{sinx})^{\frac{1}{2}}(x^2)^{\frac{1}{2}}\right) = \frac{d}{dx}\left((\ \text{sinx})^{\frac{1}{2}}\right)(x^2)^{\frac{1}{2}} + \frac{d}{dx}\left((x^2)^{\frac{1}{2}}\right)(\ \text{sinx})^{\frac{1}{2}}$$

$$= \frac{1}{2}(\ \text{sinx})^{\frac{-1}{2}} \cos x \ (x^2)^{\frac{1}{2}} + \frac{1}{2}(x^2)^{\frac{-1}{2}}(2x)(\text{sinx})^{\frac{1}{2}}$$

$$= \frac{\cos x \ (x^2)^{\frac{1}{2}}}{2(\text{sinx})^{\frac{1}{2}}} + \frac{x(\text{sinx})^{\frac{1}{2}}}{(x^2)^{\frac{1}{2}}}$$

$$= \frac{\cos x \ (\csc x)^{\frac{1}{2}}(x^2)^{\frac{1}{2}}}{2} + \frac{x(\text{sinx})^{\frac{1}{2}}}{(x^2)^{\frac{1}{2}}}$$

(a)(ii)

$$h(x) = \frac{\cos(\ \pi x)}{1 - \cot(\ x)}$$

$$\frac{d}{dx}\left(\frac{\cos \pi x}{1 - \cot x}\right) = \frac{\frac{d}{dx}(\cos(\ \pi x))(1 - \cot(x)) - \frac{d}{dx}(1 - \cot(x))(\cos(\pi x))}{(1 - \cot(x))^2}$$

$$= \frac{(-\sin(\pi x).\pi)(1 - \cot x) - (\csc^2(x).(\cos(\pi x))}{(1 - \cot(x))^2}$$

$$= \frac{-\pi\sin(\pi x).(1 - \cot x) - \csc^2(x).\cos(\pi x)}{(1 - \cot(x))^2}$$

(a)(iii)

$$x^2e^y = y^3 \ln x^2$$

$$2xe^y + x^2e^y\frac{dy}{dx} = 3y^2 \ln x^2 \frac{dy}{dx} + \frac{2}{x}y^3$$

$$x^2e^y\frac{dy}{dx} - 3y^2 \ln x^2 \frac{dy}{dx} = \frac{2}{x}y^3 - 2xe^y$$

$$\frac{dy}{dx}(x^2e^y - 3y^2 \ln x^2) = \frac{2}{x}y^3 - 2xe^y$$

$$\frac{dy}{dx} = \frac{\frac{2}{x}y^3 - 2xe^y}{x^2e^y - 3y^2 \ln x^2}$$

(a)(iv)

$k(\theta) = \sin(3\theta)^{\cos(\theta)^3}$

$= \ln(k(\theta)) = \cos(\theta)^3 \ln(\sin(3\theta))$

$$= \frac{1}{k(\theta)}\frac{dk}{d\theta} = -3\theta^2 \sin(\theta)^3 \ln(\sin(3\theta)) + \frac{3\cos(3\theta)}{\sin(3\theta)}\cos(\theta)^3$$

$$= \frac{dk}{d\theta} = k(\theta)\left(-3\theta^2 \sin(\theta)^3 \ln(\sin(3\theta)) + \frac{3\cos(3\theta)}{\sin(3\theta)}\cos(\theta)^3\right)$$

(a)(v)

$$F(x)\int_{\sqrt{x}}^{-x^2} \sqrt{u^{2-4}}du$$

$$F(x) = \int_{\sqrt{x}}^{0} \sqrt{u^{2-4}}du + \int_{0}^{-x^2} \sqrt{u^{2-4}}du = -\int_{0}^{\sqrt{x}} \sqrt{u^{2-4}}du + +\int_{0}^{-x^2} \sqrt{u^{2-4}}du$$

$$F'(x) = -\sqrt{(\sqrt{x})^2 - 4}\,\frac{d}{dx}(\sqrt{x}) + \sqrt{(-x^2)^2 - 4}\,\frac{d}{dx}(-x^2)$$

$$= \frac{-\sqrt{(\sqrt{x})^2-4}}{2\sqrt{x}} - 2x\sqrt{(-x^2)^2 - 4}$$ I will revise this one.

(b)

$\pi \sin y + 2xy = 2\pi$

$$\pi \cos y \frac{dy}{dx} + 2y + 2x\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(\pi \cos y + 2x) = -2y$$

$$\frac{dy}{dx} = \frac{-2y}{\pi \cos y + 2x}$$

So the gradient at point $\left(1, \frac{\pi}{2}\right)$ is: $\frac{-2(\frac{\pi}{2})}{\pi\cos(\frac{\pi}{2})+2(1)} = -\frac{1}{2}$

∴ The equation of the tangent is: $y - \frac{\pi}{2} = -\frac{1}{2}(x-1)$

$$y = -\frac{1}{2}x - 1 + \frac{\pi}{2}$$

∴ The equation of the normal is: $y - \frac{\pi}{2} = 2(x-1)$

$$y = 2x - 2 + \frac{\pi}{2}$$