Tutorial letter 101/0/2022

LINEAR ALGEBRA

MAT1503

Year module

Department of Mathematical Sciences

IMPORTANT INFORMATION:

Please activate your *my*Unisa and myLife e-mail account and make sure that you have regular access to the *my*Unisa module website MAT1503-22-Y1, as well as your group website.

Note: This is an online module. It is therefore, only available on *my*Unisa.

CONTENTS

1 INTRODUCTION

Welcome to the MAT1503 module. We trust that you will find it both interesting and rewarding.

This tutorial letter contains important information about the scheme of work, resources and assignments for this module as well as exam admission. We urge you to read it carefully before working through the study material, preparing the assignment(s), preparing for the examination and addressing questions to your lecturers.

In this tutorial letter, you will find the assignments as well as instructions on the preparation and submission of the assignments. This tutorial letter also provides all the information you need with regard to the prescribed study material and other resources. Please study this information carefully and make sure that you obtain the prescribed material as soon as possible.

You will access all files online, a number of tutorial letters for example, solutions to assignments, during the year. These tutorial letters will be uploaded on myUnisa, under **Additional Resources** and **Lessons** tools on myUnisa platform. A tutorial letter is our way of communicating with you about teaching, learning and assessment.

Right from the start we would like to point out that **you must read all the tutorial letters** you access from the module site **immediately and carefully**, as they always contain important and, sometimes urgent information.

Because this is a fully online module, you will need to use *my*Unisa to study and complete the learning activities for this course. Please visit the website for MAT1503 on *my*Unisa frequently. The website for your module is MAT1503-21-Y1.

1.1 Getting started

Owing to the nature of this module, you can read about the module and find your study material online. Go to the website at https://my.unisa.ac.za and log in using your student number and password. Click on ”myModules” at the top of the web page and then on ”Sites” in the top right corner. In the new window, click on the grey Star icon next to the modules you want displayed on your navigation bar. Close the window in the top right corner. Then select the option ”Reload to see your updated favorite sites”. Now go to your navigation bar and click on the module you want to open.

We wish you every success with your studies!

2 OVERVIEW of MAT1503

2.1 Purpose

This module will be useful to students interested in developing the basic skills in linear algebra which can be applied in the natural sciences and social sciences. Students who have completed this module successfully will have an understanding of the basic ideas of linear algebra and be able to apply the basic techniques for handling systems of linear equations, matrices, determinants and vectors.

2.2 Outcomes

The broad outcomes for this module are to:

2.2.1 solve systems of linear equations.

2.2.2 perform basic matrix operations.

2.2.3 evaluate determinants and use them to solve certain systems of linear equations and to find inverses of invertible matrices.

2.2.4 perform various operations in 2-space, 3-space and n-space and to find equations for lines and planes in 3-space.

2.2.5 express complex numbers in Polar form, solve polynomial equations of a complex variable.

2.2.6 extract n*th* roots of any complex number where $n \in \mathbb{N}$.

2.2.7 express relationships between trigonometric functions using complex numbers.

3 CURRICULUM TRANSFORMATION

Unisa has implemented a transformation charter based on five pillars and eight dimensions. In response to this charter, we have placed curriculum transformation high on the teaching and learning agenda. Curriculum transformation includes the following pillars: student-centred scholarship, the pedagogical renewal of teaching and assessment practices, the scholarship of teaching and learning, and the infusion of African epistemologies and philosophies. These pillars and their principles will be integrated at both programme and module levels as a phased-in approach. You will notice a marked change in the teaching and learning strategy implemented by Unisa, together with how the content is conceptualised in your modules. We encourage you to embrace these changes during your studies at Unisa in a responsive way within the framework of transformation.

4 LECTURER(S) AND CONTACT DETAILS

4.1 Lecturer(s)

The primary lecturer for this module is:

Dr Ali
Department: Mathematical Sciences
Telephone: 011 670 9163
E-mail: alizi@unisa.ac.za

A notice will be posted on *myUnisa* if there are any changes and/or an additional lecturer is appointed to this module.

Please do not hesitate to consult your lecturer whenever you experience difficulties with your studies. You may contact your lecturer by phone or through correspondence or by making a personal visit to his/her office. **Please arrange an appointment in advance (by telephone or by e-mail) to ensure that your lecturer will be available when you arrive.** Please come to these appointments well prepared with specific questions that indicate your own efforts to have understood the basic concepts involved. If these difficulties concern exercises which you are unable to solve, you must send us your attempts so that we can see where you are going wrong.

If you should experience any problems with the exercises in the study guide or prescribed book, your lecturer will gladly help you with them, provided that you send in your bonafide attempts. **When sending in any queries or problems, please do so separately from your assignments and address them directly to your lecturer.**

4.2 Department

You can contact the Department of Mathematical Sciences as follows:

Department of Mathematical Sciences
Departmental Secretary: 011 670 9147 (RSA) +27 11 670 9147 (International)
e-mails: mathsciences@unisa.ac.za or swanem@unisa.ac.za

4.3 University

To contact the University, follow the instructions on the Contact us page on the Unisa website. Remember to have your student number available whenever you contact the University.

Whenever you contact a lecturer via e-mail, please include your student number in the subject line to enable the lecturer to help you more effectively.

5 RESOURCES

5.1 Joining myUnisa

The myUnisa learning management system is the University's online campus which will help you communicate with your lecturers, other students, and the administrative departments within Unisa. To claim your myUnisa account, please follow the steps below:

1. Visit the myUnisa website at https://my.unisa.ac.za/portal
2. Click on the "**Claim Unisa login**" link on the top of the screen under the orange user ID box.
3. A new screen will load, prompting you to **enter your student number**. Please enter your student number and click "**continue**".
4. Enter your surname, your full name, your date of birth and, finally, your South African ID number (for South African citizens) OR your passport number (for foreign students). Then click "continue". **Remember to enter either an ID number or a passport number, NOT both**.

5. Please read through the guidelines and click all the check boxes to acknowledge that you have read all the information provided. Once you are done, click the "**Acknowledge**" button to redirect you to the final page in the process.

6. The final page will display your myLife e-mail address, and your **myLife AND myUnisa password**. This password will also be sent to the cellphone number displayed on the page for safekeeping.

7. Please note that it can take up to 24 hours for your myLife e-mail account to be created

Remember, the password provided is your myUnisa **AND** myLife password.

5.2 Prescribed book(s)

The prescribed book for this module is

Title:	**Elementary Linear Algebra with Supplemental Applications**
Author:	Anton, Howard and Rorres, Chris
Publishers:	**WILEY**
Edition:	**Eleventh Edition**
Year:	2015
ISBN:	978-1-118-67745-2

You are welcome to use the newest edition below as a prescribed book:

Title:	**Elementary Linear Algebra, Applications Version**
Author:	Anton, Howard & Torres
Edition:	**12th Edition**, EMEA Edition (published 2020)
Year:	2020
Print Book ISB:	N: 978-1-119-66614-1
eBook ISBN:	978-1-119-67080-3

You are also welcome to download and use the 11th edition below as a prescribed book from the Library website:

Title:	**Elementary Linear Algebra, Applications Version**
Author:	Anton, Howard & Torres
Edition:	**11th Edition**, EMEA Edition (published 2020)
Year:	2020
Print Book ISBN:	978-1-118-43441-3

Please buy the textbook as soon as possible since you have to study from it directly – you cannot do this module without the prescribed textbook.
Please refer to the list of official booksellers and their addresses in the *Study @ Unisa* brochure. Prescribed books can be obtained from the University's official booksellers. If you have difficulty in locating your book(s) at these booksellers, please contact the Prescribed Book Section at Tel: 012 429-4152 or e-mail vospresc@unisa.ac.za.

5.3 Recommended book(s)

The following is a publication that you may consult in order to broaden your knowledge of MAT1503. A **limited** number of copies is available in the Library.

- Kolman, Bernard & Hill, David R.: *Introductory Linear Algebra; An Applied First Course* (8^{th} edition or earlier), Prentice Hall, 2005.

Recommended books can be requested online, via the Library catalog.
The following books are also available at the Unisa Library. However, there is a limited number of copies of these books.

- Ayres, Frank: *Schaum's Outline of Theory and Problems of Matrices*, McGraw-Hill, New York, 1974.
- Cullen, Charles G.: *Matrices and Linear Transformations*, Addison-Wesley, Reading, MASS., 1972.
- Johnson, Lee W.: *Introduction to Linear Algebra* (2^{nd} or earlier editions), Addison-Wesley, Reading, MASS., 1989.
- Knopp, Paul J.: *Linear Algebra, an Introduction*, Hamilton Publishing Co., Santa Barbara, CALIF., 1974.
- Lipschutz, Seymour: *Schaum's Outline of Theory and Problems of Linear Algebra,* McGraw-Hill, New York, 1968.
- Nering, Evar D.: *Elementary Linear Algebra*, W.B. Saunders Publishing Co., Philadelphia, 1974.
- Nicholson, W.K.: *Linear Algebra with Applications* (3^{rd} edition), PWS Publishing Company, Boston.
- Grossman, Stanley I.: *Elementary Linear Algebra* (any edition), Wadsworth Publishing Co.,Belmont, CA., 1991.
- Anton, Howard and Rorres, Chris:*Elementary Linear Algebra; Applications Version,*(10th edition, 2011), John Wiley & Sons, Inc

- **NOTE:** Do not feel that you **should** study from these books, simply because we have provided you with this list. Sometimes, however, if one really gets bogged down on a particular section or part of the work, a different presentation might just be what is needed to get going again.

5.4 Electronic reserves (e-reserves)

E-reserves can be downloaded from the Library catalogue. More information is available at: https://libguides.unisa.ac.za/request/request

5.5 Library services and resources

The Unisa Library offers a range of information services and resources:

- For a general Library overview, go to
  https://www.unisa.ac.za/sites/corporate/default/Library/About-the-Library

  Library @ a glance

- For detailed Library information, go to
  https://www.unisa.ac.za/sites/corporate/default/Library
- For research support and services (e.g. personal librarians and literature search services) go to

  http://www.unisa.ac.za/sites/corporate/default/Library/Library-services/Research-support

- The Library has created numerous **Library guides** to assist you:

  http://libguides.unisa.ac.za

Recommended guides:

- Request and recommended books and access e-reserve material:
  http://libguides.unisa.ac.za/request
- Requesting and finding library material: Postgraduate services:
  http://libguides.unisa.ac.za/request/postgrad
- Finding and using library resources and tools (Research Support):
  https://libguides.unisa.ac.za/research-support
- Frequently asked questions about the Library:
  http://libguides.unisa.ac.za/ask
- Services to students living with disabilities:
  http://libguides.unisa.ac.za/disability
- A-Z databases:
  https://libguides.unisa.ac.za/az.php
- Subject-specific guides:
  https://libguides.unisa.ac.za/?b=s
- Information on fines & payments:
  https://libguides.unisa.ac.za/request/fines

- Assistance with **technical problems** accessing the Unisa Library or resources: https://libguides.unisa.ac.za/techsupport

You may also send an e-mail to Lib-help@unisa.ac.za (insert your student number in the subject line please).

General library enquiries can be directed to Library-enquiries@unisa.ac.za

6 STUDENT SUPPORT SERVICES

The *Study @ Unisa* website is available on myUnisa: www.unisa.ac.za/brochures/studies

This website has all the tips and information you need to succeed at Unisa.

6.1 First-Year Experience Programme @ Unisa

For many students, the transition from school education to tertiary education is beset with anxiety. This is also true for first-time students to Unisa. Unisa is a dedicated open distance and e-learning institution. Unlike face-to-face/contact institutions, Unisa is somewhat different. It is a mega university and all our programmes are offered through a blended learning mode or fully online learning mode. It is for this reason that we thought it necessary to offer first-time students additional/extended support so that you can seamlessly navigate the Unisa teaching and learning journey with little difficulty and few barriers. In this regard we offer a specialised student support programme to students entering Unisa for the first time. We refer to this programme as Unisa's First-Year Experience (FYE) Programme. The FYE is designed to provide you with prompt and helpful information about services that the institution offers and how you can access information. The following FYE programmes are currently offered:

- FYE website: All the guides and resources you need to navigate through your first year at Unisa can be accessed using the following link: www.unisa.ac.za/FYE

- FYE e-mails: You will receive regular e-mails to help you stay focused and motivated.

- FYE broadcasts: You will receive e-mails with links to broadcasts on various topics related to your first-year studies (e.g., videos on how to submit assignments online).

- FYE mailbox: For assistance with queries related to your first year of study, send an e-mail to fye@unisa.ac.za

7 HOW TO STUDY ONLINE?

7.1 What does it mean to study fully online?

Studying fully online modules differs completely from studying some of your other modules at Unisa.

- **All your study material and learning activities for online modules are designed to be delivered online on myUnisa.**
- **All your assignments must be submitted online**. This means that you will do all your activities and submit all your assignments on myUnisa. In other words, you may **NOT** post your assignments to Unisa using the South African Post Office.
- **All communication between you and the University happens online.** Lecturers will communicate with you via e-mail and SMS, and use the **Announcements**, the **Discussion Forums** and the **Questions and Answers** tools. You can also use all of these platforms to ask questions and contact your lecturers.

7.2 myUnisa tools

The main tool that we will use is the **Lessons tool**. This tool will provide the content of and the assessments for your module. At times you will be directed to join discussions with fellow students and complete activities and assessments before you can continue with the module.

It is very important that you log in to myUnisa regularly. We recommend that you log in at least once a week to do the following:

- **Check for new announcements**. You can also set your myLife e-mail account so that you receive the announcement e-mails on your cellphone.
- **Do the Discussion Forum activities**. When you do the activities for each learning unit, we want you to share your answers with the other students in your group. You can read the instructions and even prepare your answers offline, but you will need to go online to post your messages.
- **Do other online activities**. For some of the learning unit activities you might need to post something on the **Blog tool**, take a quiz or complete a survey under the **Self-Assessment** tool. Do not skip these activities because they will help you complete the assignments and the activities for the module.

We hope that by giving you extra ways to study the material and practise all the activities, this will help you succeed in the online module. To get the most out of the online module, you **MUST** go online regularly to complete the activities and assignments on time.

8 ASSESSMENT

8.1 Assessment plan

Please note that this module has a total of **SIX** assignments consisting of FOUR written assignments (02-05) and two of which are multiple–choice assignments (Assignment 01 and 06).
The questions for the assignments are given at the end of this tutorial letter(see Addendum). For each assignment there is a **FIXED CLOSING DATE**; the date by which the assignment **must reach** the university. Solutions for each assignment as Tutorial Letter 202, ..., 205. will be uploaded on *myUnisa* under Additional Resources few days after the closing date.
Late assignments will be marked, but will be awarded 0%.

Written assignment
Not all the questions in the written assignment will be marked and you will also not be informed beforehand which questions will be marked. The reason for this is that Mathematics is learnt by "doing Mathematics", and it is therefore extremely important to do as many problems as possible. You can self assess the questions that are not marked by comparing your solutions with the solutions in the tutorial letter under Additional Resources.

Note that Assignment 01 is the compulsory assignment for admission to the examination and must reach (submit online) us by the due date.

The assignments have a combined 20% contribution towards the final mark.

The Written assignments can only be submitted online electronically through myUnisa.

Feedback to Assignments	Tutorial Letters
02	202
03	203
04	204
05	205

Please note that there is no feedback to assignment 06.

The assessments together with the contributions of assignments to the year mark are as follows;

Assignment	Format	Weight (%)	Due date
01	online	3	20th April 2022
02	Written	3	20th May 2022
03	Written	3	20th June 2022
04	Written	3	20th July 2022
05	Written	3	20th August 2022
06	Online	5	20th September 2022
Total			20

*Because this is an online module, the assignments are not provided in this tutorial letter. Instead, the assignments are provided online as they become due. You will see them when you go online.

The study plan below shows the content to be covered during specific periods of the year in terms of the broad concepts or topics, the study guide units and the prescribed book chapters. Your studies will be largely guided by the tutorial discussions and learning activities, and the assignments, which are all based on the same study plan. You should therefore participate as much as possible in the tutorial discussions and complete assignments and the learning or self-assessment activities linked to each topic in order to do well in the assignments, and for you to be well prepared for the final examination.

Month	**Activities**
January-April	Read Tutorial Letter 101 (this letter). Read pp iii to xii of the Study Guide and the sections of HC (the prescribed book) t which these pages refer. Make sure you have all your study material as well as other items such as assignment covers. Study Chapter 1 of HC as well as Unit 1 of the Study Guide. Prepare for Assignment 1.
April	Submit Assignment 1. Study Chapter 1 of HC as well as Unit 1 of the Study Guide. Prepare for Assignment 2.
May	Submit Assignment 2. Study Chapter 1 & 2 of HC as well as Units 1 & 2 of the Study Guide. Prepare for Assignment 3.
June	Submit Assignment 3. Study Chapter 2 & 3 of HC as well as Units 2 & 3 of the Study Guide. Prepare for Assignments 4 & 5.
July	Submit Assignments 4 & 5. Study Chapters 3 & 4 of HC as well as Units 3 & 4 of the Study Guide. Prepare for Assignment 5.
August	Submit Assignment 5. Study Chapters 3, 4 & 10 of HC as well as Units 3, 4 & 5 of the Study Guide. Prepare for Assignments 6.
September	Submit Assignments 6. Study all the Chapters and revise your work. Prepare for the exam.
August	Prepare for the exam. Work through the solutions of Assignments 3 to 6 and learn from your mistakes.
September-October October-November	Study for the exam. Write the exam.
December	ENJOY YOUR HOLIDAY!

8.2 Year mark and final examination/other options

The year mark and the examination mark for this module will be divided as follows:

Type of assessment	Contribution to the final mark
Formative	20
Summative	80
Final mark	100

Please note that the 20% contribution by the assignments makes it extremely important that you do all the assignments and score high marks, otherwise it is impossible for you to pass the module. This also means that if you do all the assignments well, there is less risk of you failing the module. The final examination is a 2-hours exam that will be conducted online, according to the examination calendar, which you can access on the Unisa website.

You only submit your assignments electronically via *myUnisa*. Assignments may **not** be submitted by fax or e–mail nor by post.

To submit an assignment via myUnisa:

- Go to *myUnisa*.
- Log in with your student number and password.
- Select the module.
- Click on ”Assignments” in the menu on the left–hand side of the screen.
- Click on the assignment number you wish to submit.
- Follow the instructions.

8.3 The examination

If you are registered then you will write the examination in October/November 2022 and the supplementary examination will be written in January/February 2023.

During the course of the year, the Examination Section will provide you with information regarding the examination in general, examination websites, examination dates and examination times.

Please note:

- The exam is a two hour examination.
- The use of a pocket calculator is not permitted during examination. You are **NOT** allowed to use a calculator during the exam.

The examination questions will be similar to the questions asked in the study guide and in the assignments.

9 CONCLUSION

Do not hesitate to contact us by e-mail if you are experiencing problems with the content of this tutorial letter or with any academic aspect of the module.

We wish you a fascinating and satisfying journey through the learning material, and trust that you will complete the module successfully.

Enjoy the journey!

Dr ZI Ali – lecturer for MAT1503
Department of Mathematical Sciences

ADDENDUM A: Additional Notes and Curriculum Transformation

Note that it is crucial to understand the content of this module in order to be able to properly do your assignments on your own and solve related problems. For this purpose, video notes will be uploaded under Additional Resources of the main module. In order to fully understand the concepts of this module and benefit from this course, you can request a MS Teams meeting for live discussions with the lecturer where recordings and video notes can be made available to your fellow students as well.

Note that the lecturers for this module will provide live discussions on Teams about the concepts in order to facilitate the understanding for this module.

Note that Feedback on Assignments will be discussed live on MS Teams with the lecturers for this module. The live discussion will be interactive between the lecturers and the students. We might also use Zoom for the discussion from time to time.

ADDENDUM B: Assignments

The multiple-choice assignments will be marked by computer. Hence the closing date is fixed and no extension of time can be granted.

Before you attempt enter your answers, please study in detail the relevant chapter of the publication My Studies @ Unisa.

Note that your assignment will not be returned to you. Please keep a record of your answers so that you can compare them with the worked out solutions.

In each of the following questions, four, five or six possible answers are given. In each case, mark/select the number of the answer that you think is correct.

For each correct answer you obtain 2 marks and for each incorrect one you may lose 1 mark. The multiple-choice Assignment 01 counts out of 40 marks.

COMPULSORY ASSIGNMENT FOR THE EXAM

ASSIGNMENT 01
Due date: Wednesday, 20 April 2022
Total Marks: 21

ONLY FOR YEAR MODULE

This assignment covers **chapter 1 of the prescribed book as well as the study guide**, **its specifically based on Study Units 1.1 & 1.2**

IMPORTANT

- This is a multiple choice assignment. **ALL** the questions must be answered. The best way to submit the assignment is online, using *my*Unisa. Before answering this assignment, consult the publication *Study @ Unisa* on how to use and complete a mark reading sheet.
- Keep your rough work so that you can compare your solutions with those uploaded on *my*Unisa.
- 1 marks will be awarded for every correct answer.
- 1 mark will be deducted for each incorrect answer.
- This assignment might cover more chapters than initially mentioned units.
- DO NOT USE A CALCULATOR.

Question 1: **1 Mark**

Which of the following is a linear equation in x; y and z?

1. $-x^{-1} + e^{-\sqrt{2}}y = 3z$, where $e = 2.71828\ldots.$
2. $2\pi \ln(e^{-\frac{1}{z}}) - 2y + z = \ln(3) - x$.
3. $\sqrt{y^2} + 4y - 2z = 7x$.
4. $y + 4y - 2z = 7x^{-2}$.

Question 2: **1 Mark**

Which of the following is a nonlinear equation in x; y and z?

1. $-2y - 4y - z = 0$.
2. $\pi \ln e^{2z} - \frac{1}{2}y - 2z = \ln(e^3) - x$.
3. $x + 4y - 2z = 0$.
4. $3x + xy = 3z$, where $e = 2.71828\ldots.$

In the following three questions, draw a table of logical operation in order to boil down statements into digestible operations through the corresponding logical formulas.

Question 3: **1 Mark**

Which of the following is the solution of the equation below?

$$0x + 0y = 0.$$

1. $(0, 0, 0)$.
2. $(1, 0, 0)$.
3. No such solution exists.
4. Infinitely many solution or $(-1, 2, 1)$.

Question 4: **1 Mark**

Consider the system of equations represented by the augmented matrix below.

$$\left[\begin{array}{rrr|r} 2 & 2 & 2 & 10 \\ 2 & 0 & -1 & 2 \end{array}\right]$$

Which of the following is not a solution to this system?

1. $(-1, 2, 4)$ or $(1, 1, 0)$.
2. $(1, 0, 0)$.
3. No such solution exists.
4. Infinitely many solution or $(-1, 2, 1)$.

Question 5: **1 Mark**

Determine which of the following is the solution set of the linear equations below.

$$\begin{aligned} 3x - y + z &= 2 \\ 2x - z &= 2 \end{aligned}$$

1. $\{(x, y, z) : x = \frac{1}{3}(t - s + 2), y = t, z = s$ with $s, t \in \mathbb{R}\}$
2. $\{(x, y, z) : x = \frac{1}{3}(t + s + 2), y = t, z = s$ with $s, t \in \mathbb{R}\}$
3. $\{(x, y, z) : x = \frac{1}{3}(t - s - 2), y = t, z = s$ with $s, t \in \mathbb{R}\}$
4. $\{(x, y, z) : x = -\frac{1}{3}(2 + t - s), y = t, z = s$ with $s, t \in \mathbb{R}\}$
6. $\{(x, y, z) : x = t, y = 5t + 4, z = 2t - 2$ with $t \in \mathbb{R}\}$

Question 6: **1 Mark**

Consider the system obtained from the augmented matrix below.

$$\left[\begin{array}{cc|c} a & b & c \\ d & e & f \end{array}\right]$$

Choose the correct statement(s):

1. The system has no solution if $ae = bd$.
2. The system has exactly one solution whenever $ae \neq db$.
3. The system is inconsistent for $af \neq ab$.
4. The system has infinitely many solution if $\frac{b}{e} = \frac{c}{f}$.
5. The system has no solution if $ae = bf$ or impossible.

Question 7: **1 Mark**

Consider given the system obtained from the augmented matrix below.

$$\left[\begin{array}{cc|c} a & b & c \\ d & e & f \end{array}\right]$$

Choose the correct option:

1. The system has infinitely many solution if $\frac{a}{c} = \frac{b}{d} = \frac{e}{f}$.
2. The system has exactly one solution whenever $af \neq db$.
3. The system is inconsistent for $af \neq ab$.
4. The system has no solution if $ae = bd$.
5. The system has no solution if $ae = bf$.

Question 8: **1 Mark**

Solve for X from the matrix equation below. Here I is the identity matrix and $\det(A) \neq 0$.

$$A^2X + I = AB$$

Choose the correct option:

1. X is the identity matrix.
2. $X = A^{-1}B - I$.
3. $X = A^{-1}B + A$.
4. $X = A^{-1}B - A$.
5. $X = AB + B$.

Question 9: **1 Mark**

Solve for X from the matrix equation below. Here I is the identity matrix and $\det(B) \neq 0$ and $\det(A) \neq 0$.

$$BXA + A = B$$

Choose the correct option:

1. No such matrix X.
2. $X = A^{-1}B - A$.
3. $X = A^{-1} - B + A$.
4. $X = A^{-1} + B - A$.
5. $X = -A^{-1} + B^{-1}$.
6. $X = -(A^{-1} + B^{-1})$.

Question 10: **1 Mark**

Solve for X from the matrix equation below. Here I is the identity matrix and $\det(B) \neq 0$ and $\det(A) \neq 0$.

$$B(X - I)A + B = A$$

Choose the correct option:

1. No such matrix X.
2. $X = A^{-1}B - A$.
3. $X = A^{-1} - B + A$.
4. $X = A^{-1} + B - A$.
5. $X = -A^{-1} + B^{-1} + I$.
6. $X = -(A^{-1} + B^{-1})$.

Question 11: **1 Mark**

Consider the following linear system:

$$\begin{cases} 2x - 3y = -1 \\ 2x - 3y = 1 \end{cases}$$

.

1. $x = 0$ and $y = 0$ satisfy the system.
2. The system has exactly one solution.
3. The system is inconsistent.
4. The system has infinitely many solution.

Question 12: **1 Mark**

Consider the system of linear equations given by

$$\begin{cases} x - y + 3z = 1 \\ 2x - y + z = -1 \\ x - 3y - z = 2 \end{cases}$$

Which of the following is the the augmented matrix for the above given system?

1. $$\left[\begin{array}{rrr|r} 1 & -1 & 3 & 1 \\ 2 & -1 & 1 & -1 \\ 1 & 3 & -1 & 2 \end{array}\right]$$
2. $$\left[\begin{array}{rrr|r} 1 & -1 & 3 & 1 \\ 2 & -1 & 1 & -1 \\ 1 & 3 & -1 & 1 \end{array}\right]$$
3. $$\left[\begin{array}{rrr|r} 1 & -1 & 3 & 1 \\ 2 & -1 & 1 & -1 \\ 1 & 3 & -1 & 3 \end{array}\right]$$
4. $$\left[\begin{array}{rrr|r} 1 & -1 & 3 & 1 \\ 2 & -1 & 1 & -1 \\ 1 & 3 & -1 & 0 \end{array}\right]$$

5.

$$\left[\begin{array}{ccc|c} 1 & -1 & 3 & 1 \\ 2 & -1 & 1 & -1 \\ 1 & -3 & -1 & 2 \end{array}\right]$$

Question 13: **1 Mark**

Assume that we are given a coefficient matrix A of the system of linear equations $AX = b$, where $b = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$,

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 5 & 6 \\ 1 & 0 & 0 \end{bmatrix}$

Solve for the variable x_2 and x_3 in the above system.

1. The system has infinitely many solution.
2. The system has exactly one solution.
3. The system is inconsistent
4. $x_2 = 0$ and $x_3 = 1$ satisfy the system.
5. $x_2 = 1 = x_3$.

Question 14: **1 Mark**

Use the Gauss-Jordan process to determine for which value (s) of λ will the following system have no solutions?

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & (\lambda^2 - 14) & \lambda + 2 \end{array}\right]$$

1. $\lambda = 4$.
2. $\lambda = 8$.
3. $\lambda = -2$.
4. $\lambda = -4$.

Question 15: **1 Mark**

What conditions must the constants α, β, and γ satisfy for the following system

$$\begin{cases} x + 2y + 3z = \alpha \\ 2x + 5y + 3z = \beta \\ 2x + 7y - 3z = \alpha + \gamma \end{cases}$$

to be consistent?

1. $\alpha = \beta$ and any value for γ.
2. $\alpha = -\gamma$ and any value for β.
3. The value for x is not unique and $\alpha = \gamma - \beta$.
4. The value for y is unique and $\alpha = \gamma - \beta$.
5. The values for x, y and z are unique and $\gamma = -5\alpha + 3\beta$.

Question 16: **1 Mark**

Let X, Y and Z be three matrices of the same size. Which of the following statement is correct?

1. $YX + XY = 2YX$.
2. $XY = YX$.
3. If $XY = YZ$ then $X = Z$.
4. If X is invertible and $X(Y - I)X = Z$ then Y does not exist.
5. If $Z = Z^{-1}$ where $Y + Z = X$ and Y is the identity matrix, then $X - \frac{1}{2}X^2 = 0$.

Question 17: **1 Mark**

Suppose that A and B are 4×4 matrices such that B is non-singular. Which of the following statement(s) is/are correct?

1. $\det(-3A^T) = 3^4 \det(A)$ and $\det(A) = \det(BAB^{-1})$.
2. $\det(3B^{-1}) = 3^4 \det(B)$ and $\det(A) = \det(BAB^{-1})$.
3. $\det(2A^T) = -16 \det(A)$ and $\det(A) = \det(BAB^{-1})$.
4. $\det(2A^T) = 2^3 \det(A)$ and $\det(A) = \det(BAB^{-1})$.

Question 18: **1 Mark**

Which of the following matrix (or given expression) results is a non-singular matrix

1. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & \sin\alpha & \frac{1}{2}\sin 2\alpha \\ 0 & -1 & -\cos\alpha \end{bmatrix}$.

2. $A \vee B$ where $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 0 \\ -2 & -4 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 0 \\ -2 & -4 & 2 \end{bmatrix}$.
   .

3. $\begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & 0 \\ 0 & -1 & 0 \end{bmatrix}$.

4. $\begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & 4 \\ 0 & 1 & -1 \end{bmatrix}$.

Question 19: **1 Mark**

Consider the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 2 & 5 \\ 0 & 3 & 4 & 10 \\ 0 & 0 & -3 & -3 \end{array}\right]$$

Solve the corresponding system and interpret the solution geometrically.
Choose which of the following is the correct option:

1. $(x, y, z) = (-1, 2, -2)$ is the unique solution and the geometric interpretation is: three planes intersecting in a point.

2. $(x, y, z) = (-1, 2, 3)$ is the unique solution and the geometric interpretation is: three planes intersecting in a line.

3. $(x, y, z) = (0, 0, 0)$ is the unique solution and the geometric interpretation is: three planes intersecting in a point.

4. The system has no solution and the geometric interpretation is: three planes intersecting in the line $r = (-1, 2, 2) + t(1, 1, 0)$.

5. $(x, y, z) = (-1, 2, 1)$ is the unique solution and the geometric interpretation is: three planes intersecting in a point.

Question 20: **1 Mark**

Consider the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & -4 & 0 \\ 0 & 0 & -3 & 15 \end{array}\right]$$

Solve the corresponding system and interpret the solution geometrically.
Choose which of the following is the correct option:

1. $(x, y, z) = (20, 0, -5) + t(-2, 0, -5)$ for $t \in \mathbb{R}$. Two planes intersecting in a point.
2. $(x, y, z) = 5(1, 0, -1) + t(-2, 1, -5)$ for $t \in \mathbb{R}$. Two planes intersecting in a line.
3. $(x, y, z) = (0, 0, 0)$ is the unique solution. Three planes intersecting in a Line.
4. $(x, y, z) = -5(4, 0, 1) - t(2, -1, 0)$ for $t \in \mathbb{R}$. Two planes intersecting in a line.
5. $(x, y, z) = (-1, 2, 2)$ is the unique solution. Three planes intersecting in a point.

Question 21: **1 Mark**

Use Gauss elimination to solve the system of linear equations obtained from the augmented matrix below.

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & 1 & 3 & 3 & 7 \\ -2 & -2 & 1 & 1 & 1 \end{array}\right]$$

Choose which of the following is the correct option:

1. There are infinitely many solutions given by the line $r(t) = (0, 0, -1) + t(-2, 0, -5)$ for $t \in \mathbb{R}$.
2. There are infinitely many solutions given by the line: $r(t) = (1, 0, -1) + t(-2, 1, -5)$ for $t \in \mathbb{R}$.
3. The unique solution is $(x, y, z) = (1, 3, 4)$.
4. $(x, y, z) = (2, 1, -3)$ is the only solution.
5. Impossible, no solution for this system.

ASSIGNMENT 02
Due date: Tuesday, 10 May 2022
Total Marks: 102

ONLY FOR YEAR MODULE

This assignment covers **chapter 1 of the prescribed book as well as the study guide**, it is specifically based on Study Units 1.3, 1.4 & 1.5

Question 1: **3 Marks**

Suppose that A, B, C, and D are matrices with the following sizes:

$$\overset{A}{(5\times 2)},\quad \overset{B}{(4\times 2)},\quad \overset{C}{(4\times 5)},\quad \overset{D}{(4\times 5)}$$

Determine in each in each of the following case whether a product is defined. If it is so, then give the size of the resulting matrix. [6]

(i) BD,

(ii) $AC - B$,

(iii) $DC + A$.

Question 2: **3 Marks**

Solve for x, y, z, and t in the matrix equation below.

$$\begin{bmatrix} 3x & y-x \\ t+\frac{1}{2}z & t-z \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ \frac{7}{2} & 3 \end{bmatrix}.$$

Question 3: **3 Marks**

Let $P(x) = x^2 - x - 6$. Compute $P(A)$ for $A = \begin{bmatrix} 3 & -1 \\ 0 & -2 \end{bmatrix}$

Question 4: **6 Marks**

Consider the matrices

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 4 & -6 & 3 \\ -2 & 1 & 8 \end{bmatrix}, B = \begin{bmatrix} -5 & 1 & 1 \\ 0 & 3 & 0 \\ 7 & 6 & 2 \end{bmatrix}, C = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -1 \\ 3 & -5 & -7 \end{bmatrix},$$

Verify the following expressions (where possible and give reasons)

(i) $A + (B - C) = (A + B) - C$ and $A(BC) = (AB)C$.

(ii) $(a + b)C = aC + bC$ and $a(B + C) = aB + aC$, where $a = -2,\ b = 3$.

(iii) $(-A^T)^T = -A$ and $(A + B)^T = A^T + B^T$.

Question 5: **3 Marks**

Assume that A and B are matrices of the same size. Determine an expression for A if $2A - B = 5(A + 2B)$.

Question 6: **10 Marks**

(6.1) Find $\det(C)$ if (1)

$$C = \begin{bmatrix} \lambda & \lambda + 1 \\ \lambda & \lambda - 1 \end{bmatrix}$$

(6.2) Use the cofactor expansion to determine (3)

$$\begin{vmatrix} 2 & 0 & 0 & 0 \\ 3 & 1 & 2 & 0 \\ 2 & -5 & 0 & 4 \\ 1 & 3 & 0 & 3 \end{vmatrix}$$

(6.3) Consider the matrix

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

(a) Compute A^{-1} (3)

(b) Find $\det(A^{-1})$ (2)

(c) Deduce a relation (if it exists) between $\det(A)$ and $\det(A^{-1})$ (1)

Question 7: **7 Marks**

Let

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ -2 & 1 & 3 \end{bmatrix}, \text{ and } B = \begin{bmatrix} 8 & -3 & -5 \\ 0 & 1 & 2 \\ 4 & -7 & 6 \end{bmatrix}.$$

Compute A^{-1}, $(B^T)^{-1}$ and $B^{-1}A^{-1}$. What do you observe about

(7.1) $(A^{-1})^{-1}$ in relation to A. (2)

(7.2) $((B^T)^{-1})^T$ in relation to B^{-1}. (2)

(7.3) $(AB)^{-1}$ in relation to $B^{-1}A^{-1}$. (3)

Question 8: **5 Marks**

Show that if A is a matrix with a row of zeros (or a column of zeros), then A cannot have an inverse

Question 9: **4 Marks**

Show that if A is an $n \times n$ matrix, then AA^T and $A + A^T$ are symmetric.

Question 10: **6 Marks**

Consider the given matrix

$$B = \begin{bmatrix} 2 & 2 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

Find $\det B$ and use it to determine whether or not B is invertible, and if so, find B^{-1}. (*Hint*: Use the matrix equation $BX = I$)

Question 11: **3 Marks**

Consider the following augmented matrix

$$\begin{bmatrix} 1 & -1 & 2 & 1 \\ 3 & -1 & 5 & -2 \\ -4 & 2 & x^2 - 8 & x + 2 \end{bmatrix}.$$

Determine the values of x for which the system has

(i) no solution,

(ii) exactly one solution,

(iii) infinitely many solutions.

Question 12: **3 Marks**

Assume that T is an $n \times n$ matrix with a row of zeros. Prove that T is a singular matrix.

Question 13: **4 Marks**

Prove that if A is a square matrix then AA^T and $A + A^T$ are symmetric.

Question 14: **2 Marks**

Determine whether or not the following matrices are in row echelon form or not?

(14.1) (1)

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & 2 \\ 0 & 0 & 5 \end{bmatrix}.$$

(14.2) (1)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}.$$

Question 15: **2 Marks**

Determine whether or not the following matrices are in reduced row echelon form or not?

(15.1) (1)

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 2 \end{bmatrix}.$$

(15.2) (1)

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Question 16: **4 Marks**

Consider the matrices

$$A = \begin{bmatrix} -2 & 7 & 1 \\ 3 & 4 & 1 \\ 8 & 1 & 5 \end{bmatrix}, B = \begin{bmatrix} 8 & 1 & 5 \\ 3 & 4 & 1 \\ -2 & 7 & 1 \end{bmatrix}, C = \begin{bmatrix} -2 & 7 & 1 \\ 3 & 4 & 1 \\ 2 & -7 & 3 \end{bmatrix}.$$

Find elementary matrices E_0, E_1, E_2 and E_3 such that

(16.1) $E_0 A = B$, (1)

(16.2) $E_1 B = A$, (1)

(16.3) $E_2 A = C$, (1)

(16.4) $E_3 C = A$. (1)

Question 17: **16 Marks**

(17.1) Find the values of x, y and z such the matrix below is skew symmetric. (3)

$$\begin{bmatrix} 0 & x & 3 \\ 2 & y & -1 \\ z & 1 & 0 \end{bmatrix}.$$

(17.2) Give an example of a symmetric and a skew symmetric 3 by 3 matrix. (2)

(17.3) Prove that A^2 is symmetric whenever A is skew symmetric. (4)

(17.4) Determine an expression for $\det(A)$ in terms of $\det(A^T)$ if A is a square skewsymmetric. (3)

(17.5) Assume that A is an odd order skew symmetric matrix. Prove that $\det(\cdot)$ is an old function in this case. (2)

(17.6) Use **(7.5)** to find the value for $\det(A)$. (2)

Question 18: **6 Marks**

Compute all the minors and cofactors of

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}.$$

Question 19: **4 Marks**

Use Cramer's rule to solve for y without solving for x, z and w in the system

$$\begin{cases} 2w + x + y + z = 3 \\ -8w - 7x - 3y + 5z = -3 \\ w + 4x + y + z = 6 \\ w + 3x + 7y - z = 1 \end{cases}$$

Question 20: **4 Marks**

Find an expression for a square matrix A satisfying $A^2 = I_n$, where I_n is the $n \times n$ identity matrix. Give 3 examples for the case $n = 3$.

Question 21: **4 Marks**

Give an example of 2×2 matrix with non-zero entries that has no inverse.

ASSIGNMENT 03
Due date: Sunday, 29 May 2022
Total Marks: 75

ONLY FOR YEAR MODULE

This assignment covers **chapters 1 & 2 of the prescribed book as well as the study guide**, it is specifically based on Study Units 1.5, 1.7 & 2.1

Question 1: **5 Marks**

Use the fact that

$$\begin{vmatrix} x & y & 1 \\ a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \end{vmatrix} = 0$$

to determine the equation of the line passing through the distinct points (a_1, b_1) and (a_2, b_2), where $|\cdot|$ stands for $\det(\cdot)$, the determinant.

Question 2: **2 Marks**

Evaluate $\det(-A)$ and $\det(-A^T)$. Compare $\det(-A)$ and $\det(-A^T)$ for:

(2.1) (2)

$$A = \begin{bmatrix} 3 & 1 & -2 \\ -5 & 3 & -6 \\ -1 & 0 & -4 \end{bmatrix}.$$

Question 3: **4 Marks**

Without calculating the determinant, inspect the following:

(3.1) (2)

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 \end{vmatrix}$$

(3.2) (2)

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{1}{4} & 0 \end{vmatrix}$$

Question 4: **2 Marks**

Use the reduced row echelon form to determine

(4.1) (2)

$$\begin{vmatrix} 2 & 4 & 6 \\ 0 & 0 & 2 \\ 2 & -1 & 5 \end{vmatrix}.$$

Question 5: **2 Marks**

Find $\det(-2A)$ and compare it to $\det(A)$ for

(5.1) (2)

$$A = \begin{bmatrix} -2 & 1 & 3 \\ 1 & 4 & 5 \\ 2 & 3 & 1 \end{bmatrix}$$

Question 6: **4 Marks**

Use Cramer's rule to solve for x, y and z

$$\begin{cases} 2x + y - 3z = 0 \\ 4x + 5y + z = 4 \\ x + y - 4z = -1 \end{cases}$$

Question 7: **7 Marks**

(7.1) (3)

Compute the product AB for

$$A = \begin{bmatrix} 0 & 4 & 0 \\ 2 & 3 & 1 \\ 3 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 1 & 5 \\ 2 & 3 & -1 \end{bmatrix}$$

(7.2) Use your answer in **(7.1)** to evaluate $\det(AB)$ and compare it to $\det(A)\det(B)$. (2)

(7.3) Determine whether or not if $\det(A - B)$ is related to $\det(A)$ - $\det(B)$. (2)

Question 8: **2 Marks**

Assume that A is a 3 by 3 matrix such that $\det(A) = 7$. Let B be a matrix obtained from A using the following elementary row operations:

$$\begin{aligned} R_3 + 2R_1 &\to R_3, \\ \frac{9}{4}R_1 &\to R_1, \\ -\frac{2}{3}R_2 &\to R_2 \\ R_2 &\leftrightarrow R_3. \end{aligned}$$

Find the determinant of B obtained from the resulting operations, *i.e.*, $\det(B)$.

Question 9: **2 Marks**

Determine for which value (s) of k will the matrix below be non-singular.

(9.1) (2)

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 3 & 1 & 3 \\ 1 & 3 & k \end{bmatrix}$$

Question 10: **3 Marks**

Let $\vec{u} =< 3, -1, -2 >$, $\vec{v} =< -1, 0, 2 >$ and $\vec{w} =< -6, 1, 4 >$. Compute the expressions below.

(10.1) $\vec{v} + 3\vec{w}$. (1)

(10.2) $3\vec{u} - 2\vec{v}$ (1)

(10.3) $-(\vec{v} + 3\vec{u})$. (1)

Question 11: **6 Marks**

(11.1) Find the area of the triangle with the given vertices $A(1, 3)$, $B(-3, 5)$, and C with $C = 2A$. (3)

(11.2) Use **(3.1)** to find the coordinates of the point D such that the quadrilateral $ABCD$ is a parallelogram. (3)

Question 12: **5 Marks**

Let $P = (1, 2, 3)$ and $Q = (-5, -1, 12)$.

(12.1) Find the midpoint of the line segment connecting P and Q. (1)

(12.2) Find the point on the line segment connecting P and Q that is $\frac{1}{4}$ of the way from P to Q. (2)

(12.3) Let $P = (-1, 5, 2)$. If the point $(0, -2, 3)$ is the midpoint of the line segment connecting P and Q, what is the point Q? (2)

Question 13: **5 Marks**

Consider the vectors $\vec{u} =< -2, 2, -3 >$, $\vec{v} =< -1, 3, -4 >$, $\vec{w} =< 2, -6, 2 >$ and the points $A(2, 6, -1)$ and $B(-3, -5, 7)$. Evaluate

(13.1) The distance between the two points. (1)

(13.2) $||2\vec{u} - 3\vec{v} + \frac{1}{2}\vec{w}||$. (1)

(13.3) The unit vector in the direction of $\vec{w}$. (1)

(13.4) Suppose $\vec{u}$; $\vec{v}$ and $\vec{w}$ are vectors in 3D, where $\vec{u} = (u_1, u_2, u_3)$; $\vec{v} = (v_1, v_2, v_3)$ and $\vec{w} = (w_1, w_2, w_3)$. (2)

Express $(\vec{u} \times \vec{v}) \cdot \vec{w}$ as a determinant.

Question 14: **4 Marks**

We assume given a plane $\mathcal{U}$ passing by the tip of the vectors $\vec{u} =< -1, 1, 2 >$, $\vec{v} =< 2, -1, 0 >$ and $\vec{w} =< 1, 1, 3 >$.

(14.1) Find the dot products $\vec{u} \cdot \vec{v}$ and $\vec{w} \cdot \vec{v}$ (1)

(14.2) Determine whether or not there is a vector $\vec{n}$ that is perpendicular to $\mathcal{U}$. If yes, then find the vector $\vec{n}$. Otherwise explain why such a vector does not exist? (3)

Question 15: **17 Marks**

Compute and find a relation between the expressions

(15.1) $\vec{u} \cdot (\vec{v} \times \vec{w})$, $\vec{w} \cdot (\vec{u} \times \vec{v})$, and $\vec{v} \cdot (\vec{w} \times \vec{u})$ (3)

Find an expression for

(15.2) $\vec{u} \cdot (\vec{v} \times \vec{w})$ and $\vec{u} \times (\vec{v} \times \vec{w})$ (3)

(15.3) Find the side lengths and angles of the triangle with vertices the tips the vectors in **Question 14** above. (2)

(15.4) Use $\vec{u}$ and $\vec{v}$ from **Question 14** to find the area of the parallelogram formed by $\vec{u}$ and $\vec{v}$. (2)

(15.5) Use $\vec{u}$, $\vec{v}$ and $\vec{w}$ from **Question 14** to find the volume of the parallelepiped with edges determined by the three vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$. (3)

Question 16: **1 Mark**

Knowing the fact that the cross product of two vectors $\vec{u} x \vec{v}$ is orthogonal to both vectors $\vec{u}$ and $\vec{v}$, find a case where this is not applicable.

Question 17: **4 Marks**

Assume that $\mathcal{U}$ is a plane. Find out whether or not the following vectors lie in $\mathcal{U}$:

(17.1) $\vec{u} =< 3.8, 1 >$, $\vec{v} =< -4, 1, 1 >$ and $\vec{w} = -\vec{v}$ (2)

(17.2) $\vec{u} =< 3.8, 1 >$, $\vec{v} =< -4, 1, 1 >$ and $\vec{w} = \vec{u} - \vec{v}$ (2)

ASSIGNMENT 04
Due date: Monday, 20 June 2022
Total Marks: 96

ONLY FOR YEAR MODULE

This assignment covers **chapter 2 & 3 of the prescribed book as well as the study guide**, it is based on Study Units 2.1, 2.2, 2.3 & 3.1

Question 1: **4 Marks**

Determine whether $\vec{u}$ and $\vec{v}$ are orthogonal vectors, make an acute or obtuse angle:

(1.1) $\vec{u} =< 1, 3, -2 >,\ \vec{v} =< -5, 3, 2 >$. (2)

(1.2) $\vec{u} =< 1, -2, 4 >,\ \vec{v} =< 5, 3, 7 >$. (2)

Question 2: **2 Marks**

Determine $\text{proj}_{\vec{a}}\vec{u}$ the orthogonal projection of $\vec{u}$ and $\vec{a}$ and deduce $||\text{proj}_{\vec{a}}\vec{u}||$ for

(2.1) $\vec{u} =< -2, 1, -3 >,\ \vec{a} =< -2, 1, 2 >$. (2)

Question 3: **8 Marks**

(3.1) Find an expression for $\frac{1}{2}||\vec{u} + \vec{v}||^2 + \frac{1}{2}||\vec{u} - \vec{v}||^2$ in terms of $||\vec{u}||^2 + ||\vec{v}||^2$. (4)

(3.2) Find an expression for $||\vec{u} + \vec{v}||^2 - ||\vec{u} - \vec{v}||^2$ in terms of $\vec{u} \cdot \vec{v}$ (3)

(3.3) Use the result of **(3.2)** to deduce an expression for $||\vec{u} + \vec{v}||^2$ whenever $\vec{u}$ and $\vec{v}$ are orthogonal to each other. (1)

Question 4: **13 Marks**

(4.1) Consider the point $A = (-1, 0, 1)$, $B = (0, -2, 3)$, and $C = (-4, 4, 1)$ to be vertices of a triangle Δ. Evaluate all side lengths of Δ. (2)

(4.2) Let Δ be the triangle with vertices the points $P = (3, 1, -1)$, $Q = (2, 0, 3)$ and $R = (1, 1, 1)$. Determine whether Δ is a right angle triangle. If it is not, explain with reason, why? (4)

(4.3) Let $\vec{u} =< 0, 1, 1 >$, $\vec{v} =< 2, 2, 0 >$ and $\vec{w} =< -1, 1, 0 >$ be three vectors in standard form.

(a) Determine which two vectors form a right angle triangle? (1)

(b) Find $\theta := \widehat{\vec{u}\vec{w}}$, the angel between the given two vectors. (2)

(4.4) Let $x < 0$. Find the vector $\vec{n} =< x, y, z >$ that is orthogonal to all three vectors $\vec{u} =< 1, 1, -2 >$, $\vec{v} =< -1, 2, 0 >$ and $\vec{w} =< -1, 0, 1 >$. (2)

(4.5) Find a unit vector that is orthogonal to both $\vec{u} =< 0, -1, -1 >$ and $\vec{v} =< 1, 0, -1 >$. (2)

Question 5: **4 Marks**

Assume that a vector $\vec{a}$ of length $||\vec{a}|| = 3$ units. In addition, $\vec{a}$ points in a direction that is 135° counter-clockwise from the positive x-axis, and a vector $\vec{b}$ in th xy-plane has a length $||\vec{b}|| = \frac{1}{3}$ and points in the positive y-direction.

(5.1) Find $\vec{a} \cdot \vec{b}$. (2)

(5.2) Calculate the distance between the point $(-1, \sqrt{3})$ and the line $2x - 2y - 5 = 0$. (2)

Question 6: **3 Marks**

Let $\vec{u} =< -2, 1, -1$, $\vec{v} =< -3, 2, -1 >$ and $\vec{w} =< 1, 3, 5 >$. Compute:

(6.1) $\vec{u} \times (\vec{v} \times \vec{w})$ and $(\vec{u} \times \vec{w}) \times \vec{v}$. (3)

Question 7: **12 Marks**

(7.1) Find a point-normal form of the equation of the plane passing through $P = (1, 2, -3)$ and having $\vec{n} =< 2, -1, 2 >$ as a normal. (4)

(7.2) Determine in each case whether the given planes are parallel or perpendicular:

(a) $x + y + 3z + 10 = 0$ and $x + 2y - z = 1$, (2)

(b) $3x - 2y + z - 6 = 0$ and $4x + 2y - 4z = 0$, (2)

(c) $3x + y + z - 1 = 0$ and $-x + 2y + z + 3 = 0$, (2)

(d) $x - 3y + z + 1 = 0$ and $3x - 4y + z - 1 = 0$. (2)

Question 8: **8 Marks**

(8.1) Determine whether the given line and the given plane are parallel:

(a) $x = 1 + t, y = -1 - t, z = -2t$ and $x + 2y + 3z - 9 = 0$, (4)

(b) $< 0, 1, 2 > +t < 3, 2, -1 >$ and $4x - y + 2z + 1 = 0$. (4)

Question 9: **13 Marks**

(9.1) Find parametric equations of the line that passes through the point $P = (2, 0, -1)$ and is parallel to the vector $\vec{n} =< 2, 1, 3 >$. (4)

(9.2) Find parametric equations of the line that passes through the points $A = (1, 2, -3)$ and $B = (7, 2, -4)$. (5)

(9.3) Find parametric equations for the line of intersection of the planes $-5x + y - 2z = 3$ and $2x - 3y + 5z = -7$. (4)

Question 10: **11 Marks**

(10.1) Find the line of intersection between the lines: $< 3, -1, 2 > + t < 1, 1, -1 >$ and $< -8, 2, 0 > + t < -3, 2, -7 >$. (3)

(10.2) Show that the lines $x + 1 = 3t$, $y = 1$, $z + 5 = 2t$ for $t \in \mathbb{R}$ and $x + 2 = s$, $y - 3 = -5s$, $z + 4 = -2s$ for $t \in \mathbb{R}$ intersect, and find the point of intersection. (5)

(10.3) Find the point of intersection between the planes: $-5x + y - 2z = 3$ and $2x - 3y + 5z = -7$. (3)

Question 11: **7 Marks**

Let L be the line given by $< 3, -1, 2 > + t < 1, 1, -1 >$, for $t \in \mathbb{R}$.

(11.1) Show that the above line L lies on the plane $-2x + 3y - 4z + 1 = 0$. (3)

(11.2) Find an equation for the plane through the point $P = (3, -2, 4)$ that is perpendicular to the line $< -8, 2, 0 > + t < -3, 2, -7 >$. (4)

Question 12: **11 Marks**

(12.1) Find the vector form of the equation of the plane that passes through the point $P_0 = (1, -2, 3)$ and has normal vector $\vec{n} =< 3, 1, -1 >$. (5)

(12.2) Find an equation for the plane that contains the line $x = -1 + 3t$, $y = 5 + 3t$, $z = 2 + t$ and is parallel to the line of intersection of the planes $x - 2(y - 1) + 3z = -1$ and $y - 2x - 1 = 0$. (6)

ASSIGNMENT 05
Due date: Sunday, 10 July 2022
Total Marks: 98

ONLY FOR YEAR MODULE

This assignment cover **chapter 3 of the prescribed book as well as the study guide**, it is specifically based on Study Units 3.1, 3.2, & 3.3 of Chapter 3 of HC

Question 1: **12 Marks**

(1.1) Let $\mathcal{U}$ and $\mathcal{V}$ be the planes given by: (2)

$$\mathcal{U} : \lambda x + 5y - 2\lambda z - 3 = 0,$$
$$\mathcal{V} : -\lambda x + y + 2z + 1 = 0.$$

Determine for which value(s) of λ the planes $\mathcal{U}$ and $\mathcal{V}$ are:

(a) orthogonal, (2)

(b) Parallel. (2)

(1.2) Find an equation for the plane that passes through the origin $(0, 0, 0)$ and is parallel to the plane $-x + 3y - 2z = 6$. (3)

(1.3) Find the distance between the point $(-1, -2, 0)$ and the plane $3x - y + 4z = -2$. (3)

Question 2: **10 Marks**

(2.1) Find the components of a unit vector satisfying $\vec{v} \cdot < 3, -1 >= 0$. (3)

(2.2) Show that there are infinitely many vectors in $\mathbb{R}^3$ with Euclidean norm 1 whose Euclidean inner product with $< -1, 3, -5 >$ is zero. (4)

(2.3) Determine all values of k so that $\vec{u} =< -3, 2k, -k >$ is orthogonal to $\vec{v} =< 2, \frac{5}{2}, -k >$. (3)

Question 3: **7 Marks**

(3.1)

(a) Find a and b such that $-3ai - (-1 - i)b = 3a - 2bi$. (1)

(b) Let $z_1 = 12 + 5i$ and $z_2 = (3 - 2i)(2 + \lambda i)$. Find λ without resorting to division such that $z_2 = z_1$. (1)

(3.2) Let $z = -2 + 3i$ and $z' = 5 - 4i$. Determine the complex numbers

(a) $z^2 - zz'$ (1)

(b) $\frac{1}{2}(z + \overline{z})^2$ (2)

(c) $\frac{1}{2}[z - \overline{z}] + [(-1 - z')]^2$. (2)

Question 4: **23 Marks**

(4.1) Determine the complex numbers i^{2666} and i^{145}. (2)

(4.2) Let $z_1 = \frac{-i}{-1+i}$, $z_2 = \frac{1+i}{1-i}$ and $z_3 = \frac{1}{10}\left[2(i-1)i + (-i+\sqrt{3})^3 + (1-i)\overline{(1-i)}\right]$. (6)
Express $\overline{\frac{z_1 z_3}{z_2}}$, $\frac{z_1 z_2}{z_3}$, and $\frac{\overline{z_1}}{z_3 z_2}$ in both polar and standard forms.

(4.3) Additional Exercises for practice:
Express $z_1 = -i$, $z_2 = -1 - i\sqrt{3}$, and $z_3 = -\sqrt{3} + i$ in polar form and use your results to find $\frac{z_3^4}{z_1^2 z_2^{-1}}$.

Find the roots of the polynomials below.

(a) $P(z) = z^2 + a$ for $a > 0$

(b) $P(z) = z^3 - z^2 + z - 1$.

(4.4)

(a) Find the roots of $z^3 - 1$ (4)

(b) Find in standard forms, the cube roots of $8 - 8i$ (3)

(c) Let $w = 1 + i$. Solve for the complex number z from the equation $z^4 = w^3$. (4)

(4.5) Find the value(s) for λ so that $\alpha = i$ is a root of $P(z) = z^2 + \lambda z - 6$. (4)

Question 5: **4 Marks**

Find the roots of the equation:

(5.1) $z^4 + 16 = 0$ and $z^3 - 27 = 0$ (4)

(5.2) Additional Exercises for practice are given below.

Find the roots of

(a) $z^8 - 16i = 0$

(b) $z^8 + 16i = 0$.

Question 6: **3 Marks**

Determine for which value (s) of λ the real part of $z = \frac{1+\lambda i}{1-\lambda i}$ equals zero.

Question 7: **16 Marks**

Use De Moivre's Theorem to

(7.1) Determine the 6th roots of $w = -729i$ (3)

(7.2) express $\cos(5\theta)$ and $\sin(4\theta)$ in terms of powers of $\cos\theta$ and $\sin\theta$ (6)

(7.3) expand $\cos^4\theta$ in terms of multiple powers of z based on θ (4)

(7.4) express $cos^3\theta\sin^4\theta$ in terms of multiple angles. (3)

Question 8: **5 Marks**

(8.1) Let $z = \frac{z_1}{z_2}$ where $z_1 = \tan\theta + i$ and $z_2 = \overline{z}_1$. Find an expression for z^n with $n \in \mathbb{N}$. (2)

(8.2) Let $z = \cos\theta - i(1 + \sin\theta)$. Determine (3)

$$\left| \overline{\frac{\overline{2z+i}}{-1-iz}} \right|$$

Question 9: **5 Marks**

Given that $z = \cos\theta + i\sin\theta$ and $\overline{u - iv} = (1 + z)(1 - i^2z^2)$. Show that

$$v = u\tan\left(\frac{3\theta}{2}\right)$$
$$r = 4^2\cos^2\left(\frac{\theta}{2}\right),$$

where r is the modulus of the complex number$u + -iv$.

Question 10: **13 Marks**

Let $z = \cos\theta + i\sin\theta$.

(10.1) Use de Moivre's theorem to find expressions for z^n and $\frac{1}{z^n}$ for all $n \in \mathbb{N}$. (2)

(10.2) Determine the expressions for $\cos(n\theta)$ and $\sin(n\theta)$. (2)

(10.3) Determine expressions for $\cos^n\theta$ and $\sin^n\theta$. (2)

(10.4) Use your answer from **(10.3)** to express $\cos^4\theta$ and $\sin^3\theta$ in terms of multiple angles. (4)

(10.5) Eliminate θ from the equations (3)

$$4x = \cos(3\theta) + 3\cos\theta$$
$$4y = 3\sin\theta - s\in(3\theta).$$

ASSIGNMENT 06
Due date: Saturday, 30 July 2022

ONLY FOR YEAR MODULE

This is and MCQ assignment and it covers **all the study material**

- This is an MCQ assignment that will be generated from an old exam paper.
- This MCQ assignment will be made available as soon as Assignment 5 closes.
- DO NOT USE A CALCULATOR.