

MAT1512

May/June 2011

CALCULUS A

Duration : 2 Hours

100 Marks

EXAMINERS :

FIRST :

SECOND :

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Instructions to candidates:

- The use of a pocket calculator is **NOT** permissible.
- This paper consists of three pages. Answer **ALL** questions
- Show **ALL** your workings.

QUESTION 1

1.1 Determine the following limits (if it exists)

$$(a) \lim_{x \rightarrow 2} f(x) \text{ if } f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ 3x - 2 & \text{if } x > 2 \end{cases} \quad (4)$$

$$(b) \lim_{x \rightarrow -3} \frac{x^2 - 9}{2x + x^2 - 3} \quad (4)$$

$$(c) \lim_{x \rightarrow -\infty} \frac{2x + x^2 + 1}{2x^2 - x + 1} \quad (4)$$

$$(d) \lim_{x \rightarrow 0^-} \left(\frac{2}{x} - \frac{2}{|x|} \right) \quad (4)$$

$$1.2 (a) \text{ Use the Squeeze Theorem to show that } \lim_{x \rightarrow \infty} \frac{\sin(e^x)}{x} = 0. \quad (5)$$

$$(b) \text{ Hence, evaluate } \lim_{x \rightarrow \infty} \frac{\sin e^x}{\sqrt{x^2 + 2}}. \quad (3)$$

[24]**[TURN OVER]**

QUESTION 2

Let $f(x) = |x - 1|$

(a) Show that f is continuous at $x = 1$ by using the definition of continuity. (6)

(b) Use the definition of the derivative to show that $f'(1)$ does not exist (4)

[10]

QUESTION 3

(3.1) Find the first derivatives of the following functions using the appropriate rules for differentiation. Simplify your answer.

(a) $y = \frac{1}{\sqrt{x}} \left(x^2 - \frac{2}{x} \right)$. (3)

(b) $y = \frac{\sin x}{1 + \cos x}$. (6)

(c) $y = e^{2x} \cos 4x$. (4)

(d) $F(x) = \int_{\sqrt{x}}^x \sqrt{t^2 + 1} dt$. (5)

(3.2) For the function $x^3 y^4 - 5 = x^3 - x^2 + y$ find the equation of the tangent line at the point $(2, -1)$. (7)

[25]

QUESTION 4

Determine the following integrals

(a) $\int \frac{4}{x(4 \ln x + 1)} dx$ (3)

(b) $\int x^2 \sqrt{x^3 + 2} dx$ (3)

(c) $\int \cos^2 x \sin^3 x dx$ (4)

[TURN OVER]

$$(d) \int_0^1 \frac{e^{2x}}{1 + e^{2x}} dx \quad (5)$$

[15]

QUESTION 5

Let $f(x) = -x^2$ and $g(x) = x^2 - 2x$

(a) Sketch the graphs of f and g on the same axes (4)

(b) Find the area of the region determined by the intersection of the curves of f and g . (6)

[10]

QUESTION 6

Solve the following Initial Value Problem

$$xy \frac{dy}{dx} = \ln x, \quad y(1) = 2$$

[6]

QUESTION 7

Let $F(x, y) = x^4 - 3x^2y^3 + 5y$.

(a) Find the first partial derivatives F_x and F_y . (2)

(b) Let $F(x, y) = 0$

(i) Use your answers in (a) above to write down $\frac{dy}{dx}$. (2)

(ii) Confirm your answer in (i) by finding $\frac{dy}{dx}$ using implicit differentiation (6)

[10]

TOTAL: [100]