UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1503

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LINEAR ALGEBRA

Duration · 2 Hours

100 Marks

EXAMINERS .
FIRST DR L GODLOZA DR ZE MPONO

Closed book examination.

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This paper consists of 4 pages

ANSWER ALL THE QUESTIONS.

THE USE OF A POCKET CALCULATOR IS NOT PERMITTED.

Question 1

(a) Consider the following system of linear equations

$$\begin{cases} x + 2y = 0 \\ \quad\quad y = 1 \end{cases}$$

(i) write down the augmented matrix of the above system (1)

(ii) solve for x in the above system (2)

(b) Reduce the following matrix

$$\begin{bmatrix} 1 & 1 & -1 & 4 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & -5 & 8 \end{bmatrix}$$

to row-echelon form (3)

(c) Let

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(i) let

$$B = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 3 \\ -1 & -2 & 5 \end{bmatrix}$$

show that $B = A^{-1}$

Hint find AB and BA (6)

(ii) Given the following system of linear equations

$$\begin{cases} x_1 - 2x_2 + 2x_3 = 3 \\ 2x_1 + x_2 + x_3 = 0 \\ x_1 \qquad + x_3 = -2 \end{cases}$$

let

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad Y = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$$

then observe that the above system can be written in matrix form as $AX = Y$, where A is the matrix given above Hence solve the system (5)

Hint Use A^{-1} given in c(i) above

(d) Let B, C, D be matrices such that C, D are both inverses of B Then show that $C = D$ (3)

(e) Let T be an $m \times n$ matrix Show that $T = -T$ if and only if T is the $m \times n$ zero matrix (5)

[25]

Question 2

(a) Let

$$E = \begin{bmatrix} 8 & 1 & 2 \\ 1 & 0 & 3 \\ 1 & 2 & -1 \end{bmatrix}, \quad F = \begin{bmatrix} 8 & 1 & 2 \\ 3 & 0 & 9 \\ 1 & 2 & -1 \end{bmatrix}$$

with all your calculations, show that $\det(F) = 3\det(E)$ and explain why this is the case (5)

(b) Let

$$G = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad H = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

(i) show that $\det(GH) = \det(G)\det(H)$ (5)

(ii) show that $\det(GH) = \det(HG)$ (3)

(c) Let J, K be invertible $n \times n$ matrices Then find $\det(J^{-1}K^{-1}JK)$ (3)

(d) Find all values of λ for which

$$\det\left(\begin{bmatrix} \lambda - 1 & -4 \\ 0 & \lambda - 4 \end{bmatrix}\right) = 0$$

(2)

(e) Using Cramer's Rule, solve the following system of linear equations

$$\begin{cases} -2x_1 + 3x_2 - x_3 = 1 \\ x_1 + 2x_2 - x_3 = 4 \\ -2x_1 - x_2 + x_3 = -3 \end{cases}$$

(7)

[25]

Question 3

(a) Let $\underline{u} = (2, -1, 3)$ and $\underline{a} = (4, -1, 2)$

- (i) Calculate $\underline{u}\ \ \underline{a}$ (2)
- (ii) Calculate $\underline{u} \times \underline{a}$ (6)
- (iii) Calculate $||\underline{u}||$ and $||\underline{a}||$ (4)
- (iv) Determine the vector component of $\underline{u}$ along $\underline{a}$ and the vector component of $\underline{u}$ orthogonal to $\underline{a}$ (6)
- (v) Find a vector perpendicular to both $\underline{u}$ and $\underline{a}$ (2)

(b)
- (i) Find the distance between the point $(1, -4, -3)$ and the plane $2x - 3y + 6z = -1$ (3)
- (ii) Determine the equation of the plane that passes through the points $P_1(-2, 1, 3)$, $P_2(-1, -1, 1)$, $P_3(3, 0, 2)$ (7)

[30]

Question 4

(a) Use de Moivre's Theorem to express $\cos(3\theta)$ in terms of powers of $\sin(\theta)$ and $\cos(\theta)$ (6)

(b) Determine the 4th roots of -16 (8)

(c) Use the polar forms of the complex numbers $z_1 = 1 + \sqrt{3}i$ and $z_2 = \sqrt{3} + i$ to compute

(i) $z_1 z_2$ (3)

(ii) $\dfrac{z_1}{z_2}$ (3)

[20]

TOTAL: [100]