UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1512

May/June 2015

CALCULUS A

Duration 2 Hours

100 Marks

EXAMINERS
FIRST MRS SB MUGISHA
SECOND DR L LINDEBOOM

Closed book examination

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This paper consists of 3 pages **ANSWER ALL QUESTIONS**
ALL CALCULATIONS MUST BE SHOWN

QUESTION 1

(a) Determine the following limits (if they exist)

(i) $\lim\limits_{t\longrightarrow -2} \dfrac{5-3t}{t^3+2t^2-3}$ (3)

(ii) $\lim\limits_{x\longrightarrow 1} \dfrac{1-\sqrt{x}}{1-x}$ (3)

(iii) $\lim\limits_{x\longrightarrow -4} \dfrac{x^2-16}{|x|-4}$ (3)

(iv) $\lim\limits_{t\longrightarrow 0} \dfrac{\sin t-\tan 2t}{t}$ (3)

(v) $\lim\limits_{x\longrightarrow \infty} \dfrac{x^2-ax-bx+ab}{a^2x^2+2abx+b^2}$ (3)

(b) Use the Squeeze Theorem to determine the following limit

$$\lim_{k\longrightarrow\infty} \frac{5k^2-\cos 3k}{k^2+10} \tag{5}$$

(c) Let $G(s)=\begin{cases} -(s-2) & \text{if } s<2 \\ s-2 & \text{if } s\geq 2 \end{cases}$

(i) Draw the graph of $G(s)$ (2)

(ii) Determine $\lim\limits_{s\longrightarrow 2} G(s)$ (2)

(iii) Is $G(s)$ continuous at $s=2$? Give a reason for your answer (1)

[25]

QUESTION 2

(a) By the first principles of differentiation, find the derivative of $f(x) = 2x^2 + 3x + 4$ at $x = -2$ (5)

(b) Find the derivatives of the following functions by using the appropriate rules for differentiation

(i) $f(x) = \dfrac{x^2 + \sqrt{x}}{\sin x \cos x}$ (3)

(ii) $f(x) = e^{e^{\sin(e^{2x})}}$ (3)

(iii) $F(s) = \displaystyle\int_s^{s^3} \sin 3u \, du$ (5)

(c) Given $\sin(x + y) = 2x$, find the following

(i) $\dfrac{dy}{dx}$ by using implicit differentiation (4)

(ii) the equations of the tangent line and normal line to the curve $\sin(x + y) = 2x$ at the point $(0, \pi)$ (5)

[25]

QUESTION 3

(a) Determine the following integrals

(i) $\int \left(x - \dfrac{2}{x^2}\right)\left(x + \dfrac{2}{x^2}\right) dx$ (3)

(ii) $\int e^{5x} \left(\dfrac{e^{2x}}{7} + \dfrac{3}{e^{3x}}\right) dx$ (3)

(iii) $\int \dfrac{1}{(4 - \sqrt{3x})^3} \, dx$ (4)

(iv) $\int_0^{\frac{\pi}{4}} \tan^3 x \sec^3 x \, dx$ (5)

(b) Let $f(x) = x^2 - 2$ and $g(x) = -|x|$, then

(i) sketch the graphs of f and g on the same axes (4)

(ii) find the area enclosed by $f(x) = x^2 - 2$ and $g(x) = -|x|$ (6)

[25]

[TURN OVER]

QUESTION 4

(a) Solve the following Initial Value Problem

$$\frac{dy}{dx} = \frac{\cos^2 y}{4x - 3},\ y(1) = \frac{\pi}{4} \tag{7}$$

(b) Let $F(x\ y) = y\cos(x^2y^2) + y$, then

(i) find the first partial derivatives F_x and F_y (6)

(ii) using b(i) above find $\frac{dy}{dx}$ (6)

(iii) If $F(x, y) = 0$ then find $\frac{dy}{dx}$ using implicit differentiation to confirm your answer in part (b) (ii) above (6)

[25]

TOTAL: [100]