UNISA university of south africa

MAT1512   May/June 2012

CALCULUS A

Duration   2 Hours   100 Marks

EXAMINERS
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This paper consists of 3 pages

ANSWER ALL THE QUESTIONS.

Outline

Question 1   Limits and continuity

Question 2   Derivatives

Question 3   Intergrals

Question 4   Differential equations

QUESTION 1

(a) Determine the following limits (if they exists)

(i) $\lim\limits_{x \to -2} \dfrac{2x+4}{x^2-4}$ (3)

(ii) $\lim\limits_{x \to 2} \dfrac{2x+4}{x^2-4}$ (2)

(iii) $\lim\limits_{x \to 0} \dfrac{2x}{3-\sqrt{x+9}}$ (4)

(iv) $\lim\limits_{x \to \infty} \dfrac{2x^2}{x^2+3x-5}$ (3)

(b) Use the Squeeze Theorem to determine

$$\lim_{x \to -1} (x+1)^2 \sin\left(\frac{1}{x+1}\right)$$

(5)

(c) Let $f(x) = \begin{cases} 2x & \text{if } x \leq 0 \\ \sin x & \text{if } 0 < x \leq \pi \\ (x - \pi)^2 & \text{if } x > \pi \end{cases}$

Determine at which value(s) of x is $f(x)$ continuous   (6)

                                                                                                                    [23]

QUESTION 2

(a) Differentiate $f(x) = \dfrac{2}{2x - 1}$ from first principles   (5)

(b) Find the derivatives of the following functions by using the appropriate rules for differentiation

   (i) $f(x) = (x^3 + 1)\sqrt{x^2 + 1}$ (5)

   (ii) $f(x) = \dfrac{\sin x^3}{x^2 + 1}$ (5)

(c) Find the first, second and third derivatives of the function

$$y = 4x^2 - 12 + \frac{4}{x^2}$$

                                                                                                                     (4)

(d) Given $x^3 y^2 = -2xy - 3$, find

   (i) $\dfrac{dy}{dx}$ implicitly   (6)

   (ii) the equation of the normal line to the curve $x^3 y^2 = -2xy - 3$ at the point $(-1, -3)$.   (5)

                                                                                                                     [30]

QUESTION 3

(a) Use the appropriate substitution to evaluate the following integrals

   (i) $\displaystyle\int 4x \sec^2 x^2 \tan x^2 \, dx$ (6)

   (ii) $\displaystyle\int \frac{t}{(t^2 + 1)^3} \, dt$ (4)

   (iii) $\displaystyle\int 2 \sin 4x \, dx$ (4)

(b) Determine the exact values for the following integrals   (Use substitution if necessary)

   (i) $\displaystyle\int_0^2 x\sqrt{x^2 + 1}\, dx$ (5)

   (ii) $\displaystyle\int_1^4 \frac{x - 1}{\sqrt{x}}\, dx$ (3)

(c) Determine the area of the region enclosed by the curve $y = x^2 - 2$ and the line $y = x$ (8)

   Hint   Sketch the graphs on the same set of axis

[30]

QUESTION 4

(a) Solve the following Initial Value Problem

$$\frac{dy}{dx} = \frac{x - 1}{y}, \quad y(0) = -2$$

(5)

(b) Let $F(x, y) = y - \sin(xy)$

   (i) Find the first partial derivatives F_x and F_y (3)

   (ii) Using (i) above, find $\dfrac{dy}{dx}$ (3)

   (iii) Confirm your answer in part (ii) above by finding $\dfrac{dy}{dx}$ using implicit differentiation (6)

[17]
TOTAL: [100]