UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1512

May/June 2013

CALCULUS A

Duration 2 Hours

100 Marks

EXAMINERS :

FIRST MRS SB MUGISHA

SECOND PROF I NAIDOO

Closed book examination.

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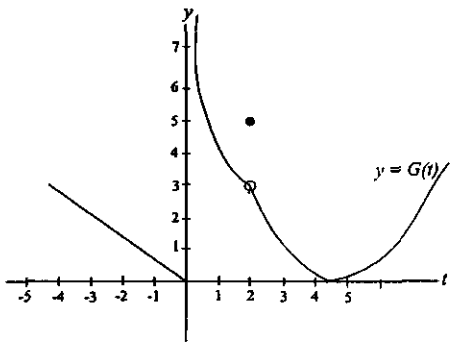
This paper consists of 3 pages.

ANSWER ALL THE QUESTIONS
ALL CALCULATIONS MUST BE SHOWN.

QUESTION 1

(a) Let the graph of the function $G(t)$ be represented as shown below



Answer each of the following by using the graph of $G(t)$ above

(i) $\lim_{t\to -3} G(t) =$ _____

(ii) $\lim_{t\to 2} G(t) =$ _____

(iii) $\lim_{t\to 0} G(t) =$ _____

(iv) $\lim_{t\to 4} G(t) =$ _____ (4)

(b) Given that

$$f(x)\begin{cases} 3x+1 & \text{if } x<2 \\ 7 & \text{if } x=2 \\ x^2+3 & \text{if } x>2 \end{cases},$$

find $\lim_{x\to 2} f(x)$. (3)

(c) Determine the following limits (if they exist)

(i) $\lim_{x\to 1} \dfrac{x^2-6x+5}{x-1}$ (3)

(ii) $\lim_{x\to 3} \left[\frac{2x^2}{x-3} + \frac{6x}{3-x}\right]$ (4)

(iii) $\lim_{x\to 3} \dfrac{\sqrt{x}-\sqrt{3}}{x-3}$ (3)

(iv) $\lim_{t\to -\infty} \dfrac{t^3+6t+1}{2t^3-5t}$ (3)

(v) Use the Squeeze Theorem to determine

$\lim_{t\to \infty} \dfrac{1-\cos x}{x}$ (5)

[25]

QUESTION 2

(a) Use the first principles of differentiation to find the derivative of $f(x) = x^3 - 2x^2 + 3$ (5)

(b) Find the derivatives of the following functions by using the appropriate rules for differentiation

(i) $f(x) = (x^2+x)(3x+1)$ (3)

(ii) $g(t) = \frac{1-t^{-1}}{2-t^{-2}}$ (4)

(c) Use the Fundamental Theorem of Calculus and find the derivative of $h(x) = -\int_{\sqrt{x}}^{1} \frac{2z^2}{z^4+1} dz$ (4)

(d) Find the first and second derivatives of the function $G(r) = \sqrt{r} + \sqrt[5]{r}$ (4)

(e) Given $\cos(xy) = x^2 - y$, find

(i) $\frac{dy}{dx}$ implicitly (5)

(ii) the equation of the normal line to the curve $\cos(xy) = x^2 - y$ at the point $(\frac{1}{4}, 0)$ (3)

[28]

QUESTION 3

(a) Use the appropriate substitution to evaluate the following integrals

(i) $\int x\sqrt{x^2+3}\, dx$ (3)

(ii) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ (3)

(iii) $\int e^{\cos x}\sin x\,dx$ (3)

(b) Determine the exact values for the following integrals (use substitution if necessary)

(i) $\int_{-1}^{0} \frac{2t}{\sqrt{1-t^2}}\,dt$ (4)

(ii) $\int_{0}^{4} |x^2 - 2x|\,dx$ (8)

(c) Let $g(t) = t^2$
and

$$h(t) = \begin{cases} 2-t & \text{if } t < 0 \\ t+2 & \text{if } t \geq 0 \end{cases},$$

Determine the area of the region enclosed by the curves g and h (6)

[27]

QUESTION 4

(a) Solve the following Initial Value Problem
$\frac{dw}{dt} = \frac{2t+sec^2t}{2w}$, $w(0) = -5$ (6)

(b) Let $T(x,y) = \tan(xy^2) + 3y - 2xy$

(i) Find the first partial derivatives T_x and T_y. (4)

(ii) Using (i) above, find $\frac{dy}{dx}$ (4)

(iii) If $T(x,y) = 0$, confirm your answer in part (b) above by finding $\frac{dy}{dx}$ using implicit differentiation (6)

[20]

TOTAL MARKS: [100]

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