UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1512

October/November 2014

CALCULUS A

Duration 2 Hours

100 Marks

EXAMINERS :
FIRST MR EF DOUNGMO GOUFO MRS SB MUGISHA
SECOND . DR L LINDEBOOM

Closed book examination.

This paper consists of 3 pages **ANSWER ALL QUESTIONS.**
ALL CALCULATIONS MUST BE SHOWN

QUESTION 1

(a) Given that $g(y) = \begin{cases} 3-y^2 & \text{if} \quad y<1 \\ 1+y & \text{if} \quad y>1 \end{cases}$

find $\lim_{y\to 1} g(y)$ (3)

(b) Determine the following limits (if they exist)

(i) $\lim_{x\longrightarrow -1} \dfrac{x^2-2x-3}{x^2-1}$ (3)

(ii) $\lim_{x\longrightarrow 0} \dfrac{3}{x} - \dfrac{3}{|x|}$ (3)

(iii) $\lim_{h\longrightarrow \infty} \dfrac{h^3-8h^2+1}{3h^4+7h^3-h^2}$ (3)

(iv) $\lim_{x\longrightarrow 1} \dfrac{x-1}{\sqrt{x^2+8}-3}$ (3)

(c) Use the Squeeze Theorem to determine

$$\lim_{x\longrightarrow \infty} \frac{2-\cos x^3}{2+x^2} \tag{5}$$

(d) Let $Y(t) = \begin{cases} t-2 & \text{if} \quad t\geq 2 \\ -(t-2) & \text{if} \quad t<2 \end{cases}$

(i) Draw the graph of $Y(t)$ (2)

[TURN OVER]

(ii) Determine $\lim_{t \longrightarrow 2} Y(t)$ (2)

(iii) Is $Y(t)$ continuous at $t = 2$? Give a reason for your answer (1)

[25]

QUESTION 2

(a) By the first principles of differentiation, find the derivative of $f(x) = 2x - 2$ at $x = 2$ (5)

(b) Find the derivatives of the following functions by using the appropriate rules for differentiation

(i) $y = \frac{4}{x}\left(x^2 - \frac{3}{4x^2}\right)$ (4)

(ii) $F(x) = \int_{3x}^{x^2} \sqrt{u^2 + 1}\ du$ (5)

(c) Given $x^2y^2 - 2x = 4 - 4y$, find:

(i) $\frac{dy}{dx}$ implicitly (6)

(ii) the equations of the tangent line and normal line to the curve $x^2y^2 - 2x = 4 - 4y$ at the point $(2, 1)$ (5)

[25]

QUESTION 3

(a) Use the appropriate substitution to evaluate the following integrals

(i) $\int x\sqrt{x+2}\ dx$ (5)

(ii) $\int \sin^3 x \cos^2 x\ dx$ (5)

(iii) $\int_0^2 \frac{e^x}{e^{2x}+1} dx$ (7)

(b) Let $f(x) = -x(x-2)$ and $g(x) = x^2$

Determine the area of the region enclosed by the curves f and g (8)

[25]

QUESTION 4

(a) Solve the following Initial Value Problem

$$\frac{dy}{dx} = \frac{\cos^2 y}{4x-3},\ y(1) = \frac{\pi}{4} \tag{8}$$

(b) Let $F(x,y) = x^2y - xy^2 + x^2 + y^2$

(i) Find the first partial derivatives F_x and F_y (6)

(ii) Using b(i) above, find $\frac{dy}{dx}$ (5)

(iii) If $F(x,y) = 0$, confirm your answer in part (b) (ii) above by finding $\frac{dy}{dx}$ using implicit differentiation (6)

[25]

TOTAL: [100]