

MAT1512

October/November 2017

CALCULUS A

Duration 2 Hours

100 Marks

EXAMINERS

FIRST

SECOND

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Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

This paper consists of 4 pages **ANSWER ALL QUESTIONS.**

ALL CALCULATIONS MUST BE SHOWN

Calculators may NOT be used

[TURN OVER]

QUESTION 1

(a) Determine the following limits (if they exist)

$$(i) \lim_{x \rightarrow -5} \frac{x^2 + x - 20}{3x + 15} \quad (3)$$

$$(ii) \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 4x} - 2x}{2x} \quad (3)$$

$$(iii) \lim_{x \rightarrow -3^-} \frac{|x + 3|}{x^2 - 9} \quad (3)$$

$$(iv) \lim_{t \rightarrow 4} \frac{1 + \sqrt{t}}{1 - t} \quad (3)$$

$$(v) \lim_{\theta \rightarrow 0} \frac{\sin(5\theta)}{\sin(2\theta)} \quad (3)$$

(b) Use Squeeze Theorem to determine

$$\lim_{x \rightarrow 0} x^3 \cos\left(\frac{10}{x}\right); (x \neq 0 \text{ and } x > 0) \quad (3)$$

(c) Let $h(x)$ be the function defined as:

$$h(x) = \begin{cases} 8 - x & \text{if } x < 0 \\ x & \text{if } 0 < x \leq 2 \\ \frac{1}{2}x^2 & \text{if } x > 2 \end{cases}$$

(i) Sketch the graph of $h(x)$ (3)(ii) Hence, determine if the function $h(x)$ is continuous at $x = 0$ and $x = 2$. (4)**[25]****QUESTION 2**

(a) Find the derivative of the following function(s) by using the appropriate rules of differentiation.

$$(i) y = \sqrt{x^2 \sin x} \quad (4)$$

$$(ii) h(x) = \frac{\cos \pi x}{1 - \cot x} \quad (4)$$

[TURN OVER]

$$(iii) \quad x^2 e^y = y^3 \ln x^2 \quad (4)$$

$$(iv) \quad k(\theta) = \sin(3\theta)^{\cos \theta^3} \quad (4)$$

$$(v) \quad F(x) = \int_{\sqrt{x}}^{-x^2} \sqrt{u^2 - 4} du \quad (4)$$

(b) Find the equation of the tangent and normal lines to the curve of

$$\pi \sin y + 2xy = 2\pi \text{ at the point } \left(1, \frac{\pi}{2}\right) \quad (5)$$

[25]

QUESTION 3

(a) Determine the following integrals, make a direct substitution and change of variable where necessary

$$(i) \quad \int \left(\frac{x^2 - 16}{x - 4} \right) dx \quad (3)$$

$$(ii) \quad \int 4x\sqrt{x^2 + 5} dx \quad (3)$$

$$(iii) \quad \int \frac{5dx}{(5 \ln x^2 + 3)x} \quad (3)$$

$$(iv) \quad \int_{\frac{3\pi}{2}}^{2\pi} 4 \sin^3 x \cos x dx \quad (5)$$

$$(v) \quad \int_0^1 \frac{3x^2}{(x^3 + 3)^2} dx \quad (5)$$

(b) Given that $f(x) = x^2 - 4$ and $g(x) = 2 - x$

Sketch the graphs of $f(x)$ and $g(x)$ on the same axes and evaluate the area of the region enclosed by both graphs

[25]

QUESTION 4

(a) Use the method of separation of variables to

(i) Solve the differential equation

$$(x^4 + 1) dy = 4x^3 (y - 1) dx \quad (5)$$

[TURN OVER]

(u) Solve the Initial Value Problem

$$\frac{dw}{dt} = t^2 w^2, \quad w(0) = a \quad (5)$$

(b) Assume a bacteria culture starts with 1000 bacteria and after 2 hours there are 2500 bacteria. The culture grows at a rate proportional to its size. Find the population after six hours. (5)

(c) Given that $T(x, y) = \cot(xy^2) + 3x^2y - 2y$

Determine

(i) T_x and T_y (3)

(u) Use your answers in (c)(i) above to write down $\frac{dy}{dx}$ (2)

(iii) Confirm your answer in c(ii) by finding $\frac{dy}{dx}$ using implicit differentiation when $T(x, y) = 0$. (5)

[25]

TOTAL: [100]