UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1512

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Calculus A

Duration 2 Hours

100 Marks

EXAMINERS

FIRST MRS SB MUGISHA

SECOND DR ZI ALI

Closed book examination

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This paper consists of 4 pages

ANSWER ALL QUESTIONS ALL CALCULATIONS MUST BE SHOWN

Calculators may NOT be used

QUESTION 1

(a) Determine the following limits (if they exist)

(i) $\displaystyle\lim_{x \to -3^-} \frac{(x^2-9)}{|x-3|}$ (3)

(ii) $\displaystyle\lim_{y \to 0} \frac{\sin 5y}{\sin 8y}$ (3)

(iii) $\displaystyle\lim_{t \to 4} \frac{1-\sqrt{t}}{1-t}$ (3)

(iv) $\displaystyle\lim_{x \to -\infty} \frac{x^3+2x^2-x+10}{2x^2-x+3}$ (3)

(v) $\displaystyle\lim_{x \to -2} \frac{x+2}{x^2-x-6}$ (3)

(b) Use the Squeeze Theorem to evaluate

$$\lim_{k \to \infty} \frac{2-\sin\left(e^k\right)}{\sqrt{k^2+3}} \qquad (5)$$

(c) Given that

$$f(x) = \begin{cases} 2x+3 & \text{if} \quad x < 0 \\ x^2 & \text{if} \quad 0 \le x \le 2 \\ 3x-2 & \text{if} \quad x > 2 \end{cases}$$

(i) Find $\displaystyle\lim_{x \to 2} f(x)$ Justify if it is continuous (3)

(ii) Sketch the graph of $f(x)$ (2)

[25]

QUESTION 2

(a) Differentiate $g(t) = 3t^3 + 2t - 1$ using the first principles of differentiation (5)

(b) Use appropriate rules of differentiation to differentiate the following functions

(i) $f(x) = (x^5 - 3x^{-2} + 3)(x^{\frac{5}{2}} - 4x)$ (3)

(ii) $g(\theta) = \sin(5\theta)^{\cos\theta^2}$ (4)

(iii) $\int_1^{\sqrt{x}} \frac{3u^3}{u^4+1}du$ (4)

(iv) $y = \frac{\cos(\pi x)}{\cot x + 1}$ (4)

(c) Given that $\sin(x + y) = 2x$, find the equation of the tangent line at the point $(0, \pi)$ (5)

[25]

QUESTION 3

(a) Determine the following integrals making a direct substitution and change of limit where necessary

(i) $\int x\sqrt{x^2+3}\,dx$ (3)

(ii) $\int \frac{2\sin x}{\cos x\,(1 + 2\ln\cos x)}\,dx$ (3)

(iii) $\int \frac{x^2-4}{x+2}\,dx$ (3)

(iv) $\int_0^{\frac{\pi}{3}} \left(1 + \cos^3 x\right)\sin x\,dx$ (5)

(v) $\int_0^1 \frac{u^3}{u^4+2}\,du$ (5)

[TURN OVER]

(b) Determine the area enclosed by the graph of f and g, where

$$f(s) = \begin{cases} 2 - s & \text{if} \quad s < 0 \\ s + 2 & \text{if} \quad s \geq 0 \end{cases}$$

and

$$g(s) = s^2$$

(6)

[25]

QUESTION 4

(a) Solve the initial value problem

$$\frac{dx}{dt} = \frac{3t^2 + \sec^2 t}{3x^2} \qquad x(0) = 5 \tag{6}$$

(b) If $z = \cos(xy) + y\cos x$ where $x = u^2 + v^2$ and $y = uv$, use the Chain Rule for partial derivatives to find $\dfrac{\partial z}{\partial u}$ (5)

(c) Let $F(x, y) = 2y - 3xy + \cot(xy^2)$

(i) Find the partial derivatives F_x and F_y (4)

(ii) Using c(i) above find $\dfrac{dy}{dx}$ (4)

(iii) If $F(x, y) = 0$ confirm your answer in c(ii) above by finding $\dfrac{dy}{dx}$ using implicit differentiation (6)

[25]

Total: [100]