UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1512

May/June 2016

CALCULUS A

Duration 2 Hours

100 Marks

EXAMINERS

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Closed book examination.

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This paper consists of 4 pages **ANSWER ALL QUESTIONS.**
ALL CALCULATIONS MUST BE SHOWN

Calculators may NOT be used

QUESTION 1

(a) Determine the following limits (if they exist)

(i) $\lim\limits_{t\to 2}\left(\dfrac{t^2-2}{t^3-3t+5}\right)^2$ (3)

(ii) $\lim\limits_{r\to 9}\dfrac{\sqrt{r}}{(r-9)^4}$ (3)

(iii) $\lim\limits_{x\to 1}\left(\dfrac{1}{x-1}+\dfrac{1}{x^2-3x+2}\right)$ (3)

(iv) $\lim\limits_{y\to 0}\dfrac{1-\cos^2 y}{y^2}$ (3)

(v) $\lim\limits_{t\to\infty}\dfrac{t-t\sqrt{t}}{2t^{\frac{3}{2}}+3t-5}$ (3)

(b) If $4x-9\leq f(x)\leq x^2-4x+7$ for $x\geq 0$, find $\lim\limits_{x\to 4} f(x)$ (4)

(c) Let $f(x)=\begin{cases} cx^2+2x & \text{if} \quad x<2 \\ x^3-cx & \text{if} \quad x\geq 2 \end{cases}$

Find the values of "c" which will make the function $f(x)$ continuous at $x=2$ (6)

[25]

QUESTION 2

(a) By the first principles of differentiation, find the following

(i) derivative of $F(x)=\dfrac{1-x}{2+x}$ (3)

(ii) $F'(-3)$ (2)

(b) Find the derivatives of the following functions by using the appropriate rules for differentiation

(i) $g(x)=x^2(1-2x)$ (3)

(ii) $f(v)=\dfrac{3\sqrt{v}-2ve^v}{v}$ (4)

(iii) $y = \int_{1-3x}^{1} \frac{u^3}{1+u^2} du$ (4)

(c) Given $y + x\cos y = x^2 y$, determine the following

(i) $\frac{dy}{dx}$ by using implicit differentiation (4)

(ii) the equations of the tangent line and normal line to the curve $y + x\cos y = x^2 y$ at the point $\left(1, \frac{\pi}{2}\right)$ (5)

[25]

QUESTION 3

(a) Determine the following integrals

(i) $\int \frac{4x^3 - 1}{x^4 - x} dx$ (3)

(ii) $\int_1^4 \frac{x^2 + 1}{\sqrt{x}} dx$ (3)

(iii) $\int \tan^2 \sec^4 x \, dx$ (4)

(iv) $\int \frac{6x}{x^2 - 1} dx$ **Hint** Use integration by partial fractions (5)

(b) Given that $f(x) = x^2 - 9$ and $g(x) = 3 - x$

(i) Sketch the graphs of $f(x)$ and $g(x)$ on the same axes (4)

(ii) Evaluate the area bounded by the graphs of $f(x)$ and $g(x)$ (6)

[25]

QUESTION 4

(a) Given that $f(x, y) = \frac{\sin(x+y)}{y} + x^2 \tan y$

(i) Determine f_x and f_y, hence find $\frac{dy}{dx}$ (6)

(ii) Show that, for the function $f(x, y) = \frac{\sin(x+y)}{y} + x^2 \tan y$, $f_{xy} = f_{yx}$ (6)

(iii) Let $f(x, y) = 0$, then use implicit differentiation technique to determine $\frac{dy}{dx}$ (6)

[TURN OVER]

(iv) Compare your answer in 4a(i) to 4a(iii) (2)

(b) Determine the solution of $\frac{y'}{x} = \frac{1}{y^2 - y}$ that passes through the point $(1, 2)$ (5)

[25]

TOTAL: [100]