UNISA university of south africa

MAT1503

May/June 2013

LINEAR ALGEBRA

Duration 2 Hours 100 Marks

EXAMINERS
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Closed book examination

This paper consists of 3 pages

QUESTION 1

(1.1) Describe the elementary row operations on a matrix (3)

(1 2) Verify that (4)

$$\begin{aligned} x &= 19t - 35 \\ y &= 25 - 13t \\ z &= t \end{aligned}$$

is a solution of

$$\begin{cases} 2x + 3y + z = 5 \\ 5x + 7y - 4z = 0 \end{cases}$$

(1.3) **(a)** Compute (2)

$$\begin{bmatrix} 3 & 2 & 1 \\ 5 & 1 & 0 \end{bmatrix} - 5 \begin{bmatrix} 3 & 0 & -2 \\ 1 & -1 & 2 \end{bmatrix}$$

(b) Find A in terms of B if $2A - B = 5(A + 2B)$ (3)

(1.4) **(a)** Given (5)

$$A = \begin{bmatrix} 3 & -1 \\ 0 & -2 \end{bmatrix}, \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

compute $A^2 - A - 6I_2$

(b) Given (4)

$$B = \begin{bmatrix} 6 & 9 \\ -4 & -6 \end{bmatrix}$$

Compute B^2 and say what you observe about B^2 in relation to B

(1 5) Let (4)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

and show that A and B are inverses of each other

[25]

QUESTION 2

(2 1) Find $\det(A)$ if

(a) (3)

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

(b) (3)

$$A = \begin{bmatrix} a+1 & a \\ a & a-1 \end{bmatrix}$$

(2 2) Using the cofactor expansion, find $\det(B)$, where (5)

$$B = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 5 & 1 & 2 & 0 \\ 2 & 6 & 0 & -1 \\ -6 & 3 & 1 & 0 \end{bmatrix}$$

(2 3) Let (5)

$$C = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

and show that $\det(C^{-1}) = \dfrac{1}{\det(C)}$

(2.4) Let (4)

$$D = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$$

find $\det(2D)$ and compare it to $\det(D)$

(2 5) Solve the following system by Cramer's rule (5)

$$\begin{cases} 2x + y = 1 \\ 3x + 7y = -2 \end{cases}$$

[25]

QUESTION 3

Consider the vectors

$$\underline{u} = \left(1, 0, \sqrt{3}\right) \quad \text{and} \quad = \left(1, \sqrt{3}, 0\right) \text{ in standard position}$$

(3 1) Determine the orthogonal projection $\text{proj}_{\,\underline{u}}\underline{v}$ (5)

[TURN OVER]

(3.2) Calculate the area of the parallelogram determined by $\underline{u}$ and $\underline{v}$ (5)

(3.3) Find an equation of the plane containing $\underline{u}$ and $\underline{v}$ (5)

(3.4) Determine the parametric equations of the plane in (3.3) (10)

[25]

QUESTION 4

(4.1) Use De Moivre's theorem to express $\cos 2\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$ (8)

(4.2) Determine the cube roots of -1 in the form $a + ib$ where $a, b \in \mathbb{R}$ (10)

(4.3) Use De Moivre's theorem to determine $(-1 + i)^{134}$ in the form $x + iy$ (7)

[25]

TOTAL MARKS: [100]