In this document some FAQs are addressed. I hope these explanations help. You can post questions if there are more things you want to be explained.

Everything of the best with the exams!

Daphne Becker.

Some explanations about sets:

A set can be a set containing any elements - they need not be of the same kind. One can create a set as you wish:

Eg {Mary, Mother, Tom, Dog} - these could be the set of people living in a house, but they are *not of the same kind*.

Just accept that a set is defined in a certain way, i.e. a set has certain elements.

Other examples: {hat, rock, bird, computer, a, b, c}; {1,2,a,b,c, {c}}; T = {1, 2, 3, 4, 0 ,{1}}

One can put *any* elements in a set - there is no rule that says there must be some logical explanation why some element is in a set, so it is OK if {1} is in T. T does not have a specific logical meaning , it is just defined a set with certain elements.

Some comments (please refer to the definitions of these concepts in the study guide):

The empty set **{ }**: Remember, the empty set is not nothing. Think of the empty set as being an empty bag. It is something that could even be an element of a set. In a set the elements are separated by commas, so with a set {_,_,_,_} three commas separate 4 elements. These elements could be anything you which to put in the set, e.g. {table, 6, dog, empty bag}. The cardinality of this set is 4, and the cardinality of e.g. the set **{** table, 6, dog, { } **}** is also 4.

Subsets: Another thing to remember is how subsets are formed: you can throw away all, none or some of the elements of a set to form subsets:
From {table, 6, dog, empty bag}, throw away the elements “table” and “6” then the subset that is formed is {dog, empty bag}. From {table, 6, dog, { }} throw away “table”, “6” and “dog”, then the subset that is formed is {{ }}. Three members were thrown away so one member is left in the set.

Powersets: Some set A and its power set P(A) cannot contain the same element, except when A contains { } as a member (remember { } is always a member of a powerset), but if { } is a member of A then { } and also {{ }} are members of P(A).

EG if A = {{1}, 2}, then P(A) = {{ }, {{1}}, {2}, {{1}, 2}}. (Remember how power sets are formed: throw away none, some or all elements of a set.) { }, {{1}}, {2} and {{1}, 2} are all the subsets of A.

Let B={{ }, 1} then P(B) = {{ }, {{ }}, {1}, {{ }, 1}}.

Hints regarding Venn diagrams and proofs.

The basic principle when drawing Venn diagrams or doing formal proofs, is to apply the definitions for union, intersection, and so on. Refer to the defs. in the guide, they are not repeated here. If one knows and understands these definitions one should be able to follow the explanations provided here.
Take note that the <u>words</u> "**and**", "**or**", and so on, may only connect *statements*, e.g. $x \in (A + B)$ **or** $x \in C$, and <u>symbols</u> such as "$\cap$", "+", and so on, may only connect *sets*, e.g. $(A + B) \cup C$. (One may **not** write
$x \in (A + B)$ or C because "or" is now connecting two sets and not two statements.).
Also take note that a proof is invalid if there is no connective between statements. We usually use the connective "iff" between statements.

In the defs for union, intersection, and so on, *single* sets such as A or B are mentioned. However, **t**hese definitions can be applied when we have some *composite* set in some statement
(e.g. $x \in X \cap (\underline{Y \cap W})$, where $Y \cap W$ is a composite set). We will look at Venn diagrams and formal proof statements.

For **union**, the word "**or**" appears in the def., so some element can appear anywhere in some mentioned sets.
Refer to guide p. 60: Venn diagram for $(A + B) \cup C$: all the areas that are coloured in in A + B, combined with the coloured area C, are now coloured in in $(A + B) \cup C$ because elements can live in A + B or C.
Formal statement: $x \in (A + B) \cup C$ iff $x \in (A + B)$ **or** $x \in C$, i.e. x can belong to A + B or to C (or to both). (We use "or" in the inclusive sense.)

For **intersection**, the word "**and**" appears in the def., thus some element can only appear in **both** some mentioned sets.
Refer to guide p. 59: Venn diagram for $(X - Y) \cap (X - W)$: only the area that is coloured in in exactly the same areas in X – Y and in X – W are now coloured in in $(X - Y) \cap (X - W)$ because some element can only live in **both**
(X – Y) **and** (X – W).
Formal statement: $x \in (X - Y) \cap (X - W)$ iff $x \in (X - Y)$ **and** $x \in (X - W)$, i.e. x belongs to **both** X – Y **and** X – W.

For **difference**, some element can only live in one set but not in any other set mentioned.
Refer to guide p. 52: Venn diagram for $(A \cup B) - (A \cap B)$: the areas that are coloured in in $A \cup B$ but

not in

$A \cap B$, is coloured in in $(A \cup B) - (A \cap B)$. (Think of it as the coloured areas in $A \cup B$ take away the coloured area in $A \cap B$.) Some element can live in $(A \cup B)$ **but not** in $(A \cap B)$.

Formal statement: $x \in (A \cup B) - (A \cap B)$ iff $x \in (A \cup B)$ **and** $x \notin (A \cap B)$, i.e. x belongs to $A \cup B$ **but not to** $A \cap B$.

For **complement**, elements do not live in some mentioned set, thus elements can only live outside this set.

Refer to Activity 4-5 1.a: Venn diagram for $(X \cup Y)'$: the area outside $X \cup Y$ is coloured in because elements cannot belong to $X \cup Y$, i.e. some element can only live outside $X \cup Y$.

Formal statement: $x \in (X \cup Y)'$ iff $x \notin (X \cup Y)$, i.e. x does not belong to $X \cup Y$. (Refer to guide p. 57.)

For **symmetric difference**, the words "**or…but not both**" appears in the def., thus some element can appear anywhere in two mentioned sets but not in both.

Refer to guide p. 60: Venn diagram for $(A \cup C) + (B \cup C)$: combine all the areas that are coloured in in $A \cup C$ with those areas coloured in in $B \cup C$, but then leave out those areas coloured in in both these sets, because some element can live in $A \cup C$ or in $B \cup C$, but not in both. (Think of it as all the coloured areas in

$(A \cup C) \cup (B \cup C)$ take away the coloured areas in $(A \cup C) \cap (B \cup C)$.)

Remember, $(A \cup C) + (B \cup C) = [(A \cup C) \cup (B \cup C)] - [(A \cup C) \cap (B \cup C)]$ (Refer to Activity 4-4.)

Formal statement: $x \in (A \cup C) + (B \cup C)$ iff $x \in (A \cup C)$ or $(B \cup C)$, but not both,

i.e. x can belong to $A \cup C$ or to $B \cup C$ but not both). (We use "+" in the exclusive sense.)

Some explanations regarding Activity 6-6(b):

Remember, sets are indicated with curly brackets.

For example, one can get a power set *P*(A) of some set A (say A has n members), with the members of *P*(A) being all subsets of A, with the cardinality of P(A), i.e. $|P(A)| = 2^n$) Outside brackets indicate a set **{ }** and inside the set are the **elements** of the set, with different elements being separated by commas **{. , . , . }**. If there is only one element # in a set, we just have **{** # **}.**

A sub**set** is formed if you throw away none, one, some or all **elements** of a set.

For example: T = {a,b,c}; a, b and c are the **elements** of the set T.

Form subsets of T: Throw away all elements of T: $\{\} \subseteq T$, throw away no element of T: $T \subseteq T$, throw away one element (a) of T: $\{b,c\} \subseteq T$, etc.

In Activity 6-6(b) Given X = {ф, **{**ф**}**, **{**{ф}**}**} and R the relation ⊆ on X x X. In the ordered pairs of R the first co-ordinates are subsets of the second co-ordinates.

FAQ: why is (**{**ф**}**,**{**{ф}**}**) not an element of R?

Let's say that ф = { } is the empty set. We have
R = ⊆ = { (ф, ф), (ф, **{**ф**}**), (ф, **{**{ф}**}**), (**{**ф**}**, **{**ф**}**), (**{**{ф}**}**, **{**{ф}**}**) } – a set which elements are ordered pairs.

The **entries in the ordered pairs** of ⊆ are **sets** which are *elements* of X. The **sets** are indicated with black brackets **{ }** and the **members** of the sets are indicated in **red**. This means that **{ }** (=ф) is the empty set; **{**ф**}** is the set with element ф; **{**{ф}**}** is the set with element {ф}.

Now remember, by definition, A ⊆ B iff every element of A is an element of B. The only element of **{**ф**}** is ф and the only element of **{**{ф}**}** is {ф}. Clearly the element of **{**ф**}** is not an element of **{**{ф}**}**, thus **{**ф**}** is not a subset of **{**{ф}**}**, thus (**{**ф**}**,**{**{ф}**}**) ∉ R.

Some explanations about equivalence classes:

The definition says: [x] = {y| y ∈ A and x R y} This means that one must find **all** the y-values that can be paired with some **specific** x-value.

The x -value is put in square brackets: **[x]** All the y-values paired with a specific x, is put together in a set **{. , . . , .}**

Given X = {a,b,c}

R1 = { (a,a), (b,b), (c,c) } – equivalence class [a] = {a} (the x-value a is paired with the y-value a), also [b] = {b} and [c] = {c}.

R2 = { (a,a), (b,**b**), (c,c), (a,b), (b.**a**) } – equivalence class [a] = [b] = {a,b} and [c] = {c}

In R2: The x-value a is paired with a and b in two separate pairs. We get [a] = {a, b}
The x-value b is paired with **b** and **a** in two separate pairs. We get [b] = {**b**, **a**}
Clearly [a] = {a, b} = {**b**, **a**} = [b] and also [c] = {c}.

Refer to tut letter 102 Activity 6-9.3(c)
Let R be the relation on $\mathbb{Z}$ defined by
(x, y) ∈ R iff x – y = 4k.
We determine the equivalence classes of R:

[x] = {y| y ∈ $\mathbb{Z}$ and x R y}
R is defined by x – y = 4k, so [x] = {y| x – y = 4k, k ∈ $\mathbb{Z}$} (1)

Take a specific value of x (we usually start with x=0), then all the y-values (second coordinates) that go with this x-value must be found.
Thus [0] = {y: 0 - y = 4k, k ∈ $\mathbb{Z}$} in other words, x = 0 is substituted in (1), and then the y values can be determined from the resulting 0 - y = 4k, I.e. y = -4k = 4(-k) = 4t i.e. the y-values are all the multiples of 4.

Next we take x = 1 etc until we do not get any new equivalence class. Refer to tut letter 102 Activity 6-9.3(c)

Antisymmetry & Trichotomy

Antisymmetry: If (x, y) ∈ R, then the mirror image of the ordered pair, namely (x, y) may never appear in the relation.

Eg Let R be a relation on A: A = {1, 2, 3} and R = {(1, 2), (2, 3)}. It is clear that the mirror pairs (2,1) and (3,2) do not appear in R, so R is antisymmetric.

Trichotomy: for any x, y ∈ A, if x is not equal to y then (x, y) or (y, x) must be members of a relation T. This just means that if you pick **any** two different elements from A then they must appear in some ordered pair in T. Refer to R above: the elements in A, namely 1 and 3 are not equal, but they do not appear together in an ordered pair in R. Thus R is not trichotomous.
However, the relation T = {(1, 2), (1, 3), (2, 3)} is trichotomous - **all** elements in A that are different from each other are paired in ordered pairs in T. As long as all these pairs are in T, T is trichotomous.

An example of another trichotomous relation P on A:
P = {(2, 1), (1, 3), (2, 3), (1, 1), (3, 2)} Because the ordered pairs (2, 1), (1, 3) and (2, 3) are in P, one can safely say that P is trichotomous.

Proving properties of relations:

The definitions of reflexivity, symmetry and transitivity must be applied when proving that a relation is an equivalence relation.

Eg the def for symmetry says that for all x, y∈ A, if (x, y) ∈ R then (y,x) must be an element of R.

*In these type of proofs, you **assume** that the "if" part ((x, y) ∈ R) is true, then you **use this information to prove** that the "then" part ((y, x) ∈ R) is true.*

Please refer to tut letter 102 Activity 6-9.3(a):

For symmetry:

Start the proof: Suppose (x, y) ∈ R is true, i.e. x – y = 4k.
(Now **use** this information to show that (y, x) ∈ R. When will (y, x) ∈ R? It will be the case if y – x = 4t.)
Back to our solution:
we have that x – y = 4k, but this means that y - x = -4k (multiply with "-")
So y - x = 4(-k)
i.e. y - x = 4t
Thus (y, x)∈R
Therefore R is symmetric.

For Transitivity, **Assume** (x, y)∈R and (y,z)∈R, then **use** this information to prove that (x,z)∈R – see how this is proof method is applied in Activity 6-9.3(a).

You can watch the following videos:
Sets: http://www.youtube.com/watch?v=mfB13y4y0oA&feature=related

Relations and functions:
http://www.youtube.com/watch?v=4DQcTbN0eeY&list=LPYxU00Bvrj24&index=1&feature=plcp

Logic: http://www.youtube.com/watch?v=OLGVhszBlq4&feature=related

If you cannot open these links by clicking on them while you are on myUnisa, copy and then paste them to the window on the internet.

One student recommended the following sites:

Great website discussing many of the areas of discrete maths relevant to this course:
http://www.abstractmath.org/MM/MMIntro.htm

The site author also made his study notes available, free to use:
http://www.abstractmath.org/MM/dm.pdf