

MAT1503

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LINEAR ALGEBRA

Duration 2 Hours

100 Marks

 EXAMINERS .
 FIRST
 EXTERNAL

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Closed book examination.

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This paper consists of 3 pages

ANSWER ALL THE QUESTIONS

QUESTION 1

- (a) (i) Find the solution set of the following linear equation

$$7x - 5y = 3 \quad (2)$$

- (ii) Reduce the following matrix to row-echelon form

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \quad (3)$$

- (b) Show that if
- A
- is an
- $m \times n$
- matrix and
- $A(BA)$
- is defined, then
- B
- is an
- $n \times m$
- matrix

(5)

- (c) Show that the following matrices are inverses of each other

$$\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \text{ and } \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \quad (5)$$

- (d) Let

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 3 \\ 1 & -1 \\ -1 & 0 \end{bmatrix}$$

- (i) Check as to whether or not
- AB
- and
- BA
- are defined (explain fully)

(4)

- (ii) If defined, what will their sizes be?

(2)

- (iii) Compute
- AB
- and
- BA
- if defined

(4)

[25]**[TURN OVER]**

QUESTION 2

- (a) (i) Compute
- $\det(A)$
- , where
- A
- is given by

$$A = \begin{bmatrix} a+1 & a \\ a & a-1 \end{bmatrix} \quad (3)$$

- (ii) Let
- A, B, C
- be
- $n \times n$
- matrices such that
- $\det(A) = -1$
- ,
- $\det(B) = 2$
- and
- $\det(C) = 3$
- . Then evaluate
- $\det(B^2 C^{-1} A B^{-1} C^T)$
- (5)

- (b) Given that

$$A = \begin{bmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{bmatrix}$$

then find $\det(A)$ (5)

- (c) (i) If
- B
- is a square matrix such that
- $B^n = 0$
- where
- 0
- is a zero matrix, then show that
- B
- is not invertible (2)

- (ii) Find
- $\det(C)$
- if
- $C^2 = I$
- , where
- I
- is an identity matrix (2)

- (d) Solve by Cramer's rule

$$\begin{cases} 3x + 4y = 9 \\ 2x - y = -1 \end{cases} \quad (8)$$

[25]

QUESTION 3

- (a) Consider the vectors
- $\underline{u} = (1, 0, 2)$
- and
- $\underline{v} = (1, 1, 0)$

- (i) Determine the orthogonal projection
- $\text{proj}_{\underline{u}} \underline{v}$
- (4)

- (ii) Calculate the area of the parallelogram bounded by
- \underline{v}
- and
- \underline{u}
- (6)

- (iii) Determine the perimeter of the parallelogram bounded by
- \underline{v}
- and
- \underline{u}
- (5)

- (b) Find an equation of the plane containing the point
- $(1, 1, 1)$
- and perpendicular to the line passing through the points
- $(2, 1, 1)$
- and
- $(1, 1, 0)$
- (6)

- (c) Determine whether the point
- $(0, -4, 6)$
- lies on the plane whose parametric equations are
- $x = 2 - t, y = 2 - 3t, z = 4 + t$
- (4)

[25]

QUESTION 4

- (a) Use De Moivre's theorem to express
- $\sin 3\theta$
- in terms of powers of
- $\sin \theta$
- and
- $\cos \theta$
- (10)

- (b) Determine the cube roots of 8 in polar form (10)

[TURN OVER]

(c) Let a and b be real numbers such that $(a - ib)^2 = 4i$

(i) Prove that $a^2 - b^2 = 0$

(2)

(ii) Verify that $ab = 2$

(3)

[25]

TOTAL MARKS: [100]