UNIVERSITY EXAMINATIONS UNIVERSITEITSEKSAMENS

COS1501/XOS1501

THEORETICAL COMPUTER SCIENCE

DURATION: 2 HOURS 50 marks

PRACTICE EXAMINATION PAPER – MULTIPLE CHOICE EXAM

EXAMINERS:

FIRST : MS HW DU PLESSIS

As from 2019, the format of the COS1501/XOS1501 exam paper will be an MCQ examination. The exam paper will have a similar format as this practice exam paper. Please note that no questions will be repeated in the exam.

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SECTION 1
SETS AND RELATIONS
(Questions 1 to 12) (12 marks)

Questions 1 to 8 relate to the following sets:

Suppose **U = {1, {c, 3}, 3, d, {d, e}, e}** is a universal set with the following subsets:

A = {{c, 3}, 3, {d, e}}, **B = {1, {c, 3}, d, e}** and **C = {1, 3, d, e}.**

Question 1

Which one of the following sets represents B ∪ C?

1. {1, c, 3, d, e}
2. {{c, 3}}
3. U – (B ∩ C)
4. {1, {c, 3}, 3, d, e}

Question 2

Which one of the following sets represents A ∩ C?

1. B – {{c, 3}}
2. {3}
3. {3, d, e}
4. (A ∪ C) – B

Question 3

Which one of the following sets represents (A ∪ C) + B?

1. {1, 3, {c, 3}, d, e}
2. {3, {d, e}}
3 { }
4. {3, d, e}

Question 4

Which one of the following sets represents U + A?

1. U
2. {1, d, e}
3. {3, {d, e}}
4. B

Question 5

Which one of the following sets represents (B + C)′ ?

1. {3, {c, 3}}
2. {1, d, e, {d, e}}
3. {{c, 3}, {d, e}}
4. {1, {c, 3}, 3, {d, e}}

ROUGH WORK

[TURN OVER]

Question 6

Which one of the following alternatives represents an element of $\mathcal{P}$(A)?

1. {c, 3}
2. {{c, 3}}
3. {{3}}
4. {{ }}

Question 7

Let T = {(1, 1), (1, d), ({c, 3}, 1), (d, d), (1, {c, 3}), (d, 1)} be a relation on the set B. Which one of the following statements is **false**?

1. T does not satisfy trichotomy.
2. T is not reflexive.
3. T is not transitive.
4. T is not symmetric.

Question 8

Which one of the following relations on set C is a strict partial order?

1. Q = {(1, 3), (1, d), (1, e), (d, e), (3, d)}
2. R = {(1, 3), (1, d), (1, e), (d, e), (3, d), (3, e), (d, 1)}
3. S = {(1, 1), (1, 3), (1, d), (1, e), (d, e), (3, d)}
4. T = {(1, 3), (1, d), (1, e), (d, e), (3, d), (3, e)}

ROUGH WORK

[TURN OVER]

[TURN OVER]

Questions 9 to 12 are based on set A = {1, 4, {4}, {{1}, 5}}

Question 9

Which one of the following statements provides a proper subset of A?

1. {{1, 4 ,{4}}}
2. {1, {4}}
3. {1, 4, {4}, {{1}, 5}}
4. {{1}, 5}

Question 10

Which one of the following is NOT a partition on A?

1. {{1, {4}}, {4}, {{{1}, 5}}}
2. {{1, 4, {{1}, 5}}, {4}}
3. {{{4}, {{1}, 5}}, {1, 4}}
4. {{1}, {4}, {{{1}, 5}, {4}}}

Question 11

Which one of the following relations is NOT a valid relation on A?

1. {(1, 4), ({4}, 1)}
2. {({1}, 5), ({4}, 4)}
3. {({4}, {{1}, 5}), (1, 1), ({4}, 1)}
4. {({4}, {4})}

Question 12

Which one of the following statements provides one or more elements of the set A?

1. {1, 4}
2. {{4}}
3. {1}, {{{1}, 5}}
4. {4}, {{1}, 5}

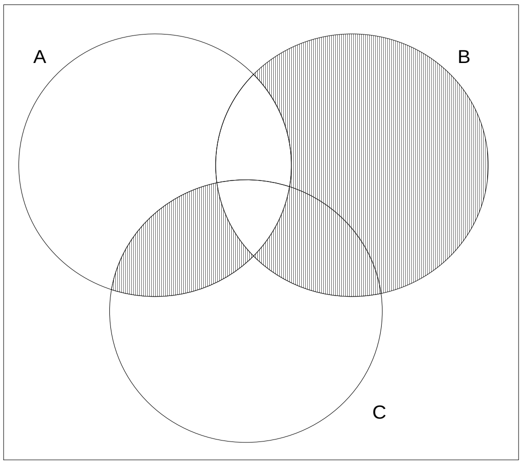
[TURN OVER]

ROUGH WORK

SECTION 2
SET THEORY
(Questions 13 to 17) (5 marks)

Question 13

Consider the following Venn diagram with A, B and C sets from the universal set U:



Which one of the following alternatives describes the set represented by the Venn diagram correctly? (***Hint***: Draw the Venn-diagrams for the alternatives on rough to find a match.)

1. (B – C) ∪ (A ∩ C)
2. [(A ∩ C) – B] ∪ (B – A)
3. [(A ∪ B) – C] + A
4. (B – A) ∪ (A ∩ B ∩ C)

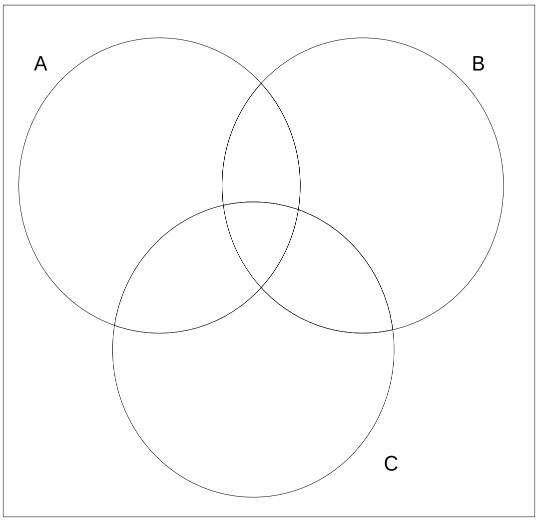
ROUGH WORK

[TURN OVER]

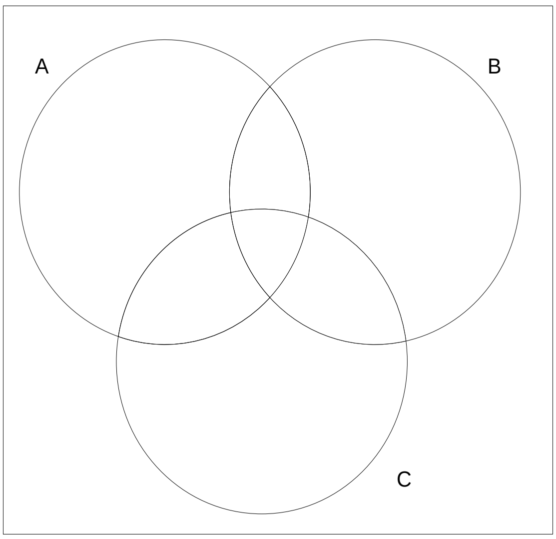
Question 14
Which one of the Venn diagrams in the alternatives below represents the set

$$\mathbf{(A - (B \cup C)) \cap (C + (A - B))}$$

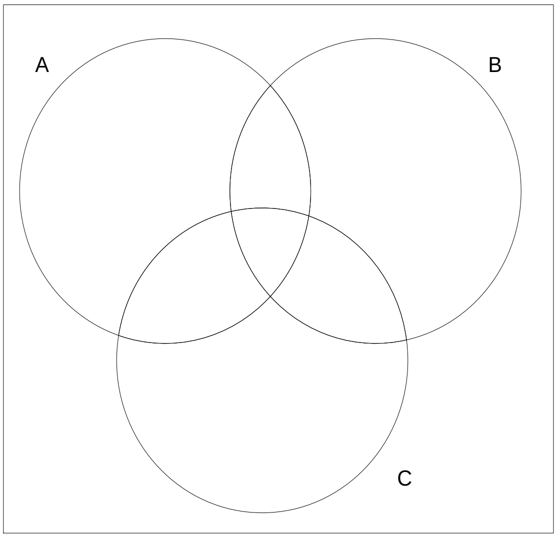
1.



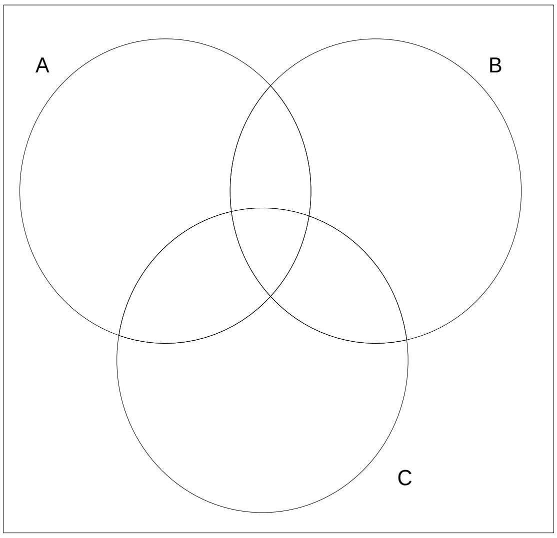
2.



3.



4.



Question 15

We want to prove that for all A, B, C $\subseteq$ U,

$$(A \cup C) - (C \cap B) = (A - C) \cup [(A - B) \cup (C - B)] \text{ is an identity.}$$

Consider the following incomplete proof:

$z \in (A \cup C) - (C \cap B)$

iff $(z \in A \text{ or } z \in C) \text{ and } (z \notin (C \cap B))$

iff $(z \in A \text{ or } z \in C) \text{ and } (z \notin C \text{ or } z \notin B)$

Step 4

iff $[(z \in A \text{ or } z \in C) \text{ and } (z \in C')] \text{ or } [(z \in A \text{ or } z \in C) \text{ and } (z \in B')]$

Step 6

iff $[(z \in A \text{ and } z \in C')] \text{ or } [(z \in A \text{ and } z \in B') \text{ or } (z \in C \text{ and } z \in B')]$

iff $[(z \in A - C)] \text{ or } [(z \in (A - B) \text{ or } (z \in C - B)]$

iff $z \in (A - C) \cup [(A - B) \cup (C - B)]$

Which one of the following alternatives contain the correct Step 4 and Step 6 to complete the proof correctly?

1. **Step 4**: iff $(z \in A \text{ or } z \in C) \text{ and } (z \in C' \text{ or } z \in B')$
   Step 6: iff $[(z \in A \text{ and } z \in C') \text{ or } (z \in C \text{ and } z \in C')] \text{ or}$
   $[(z \in A \text{ and } z \in B') \text{ or } (z \in C \text{ and } z \in B')]$
2. **Step 4**: iff $(z \in A \text{ or } z \in C) \text{ and } (z \in C' \text{ and } z \in B')$
   Step 6: iff $[(z \in A \text{ or } z \in C') \text{ and } (z \in C \text{ or } z \in C')] \text{ and}$
   $[(z \in A \text{ or } z \in B') \text{ and } (z \in C \text{ or } z \in B')]$
3. **Step 4**: iff $(z \in A \text{ or } z \in C) \text{ and } z \in (z \in C' \text{ and } z \in B')$
   Step 6: iff $[(z \in A \text{ and } z \in C') \text{ or } (z \in C \text{ and } z \in C')] \text{ or}$
   $[(z \in A \text{ and } z \in B') \text{ or } (z \in C \text{ and } z \in B')]$
4. **Step 4**: iff $(z \in A \text{ or } z \in C) \text{ and } (z \in C' \text{ or } z \in B')$
   Step 6: iff $[(z \in A \text{ or } z \in C') \text{ and } (z \in C \text{ or } z \in C')] \text{ and}$
   $[(z \in A \text{ or } z \in B') \text{ and } (z \in C \text{ or } z \in B')]$

ROUGH WORK

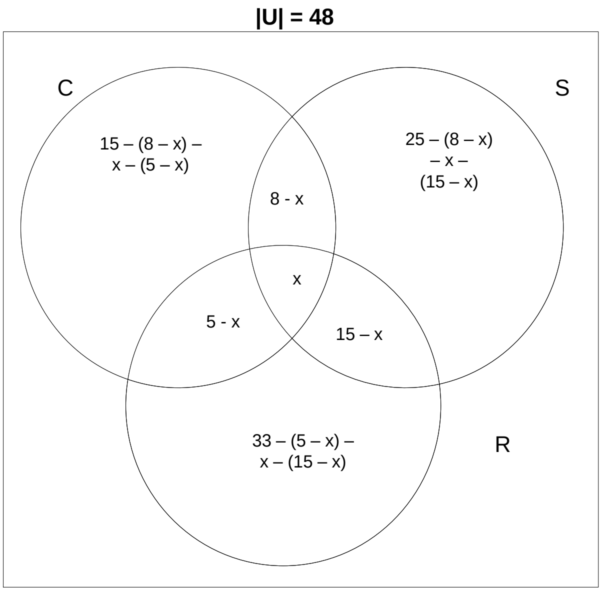
Question 16

Let U = {6, a, 8} and A, B and C be subsets of U. The set **(B – C)** = **(A ∩ (B ∩ C′))** is NOT an identity. Which one of the following alternatives contains sets A, B and C that can be used as counterexample to prove that the set **(B – C)** = **(A ∩ (B ∩ C′))** is not an identity.

1. A = {6}, B = {6}, C = {a, 8}
2. A = {6, a}, B = { }, C = {6, a}
3. A = {8}, B = {6, 8}, C = {a}
4. A = {a, 8}, B = { }, C = {a}

Question 17

The Venn diagram below represents the preference of ice cream flavors (C = Chocolate, R = Raspberry, S = Strawberry) in a group of 48 preschoolers.



Which one of the alternatives is true?

1. 3 preschoolers like all three of the ice cream flavors.
   5 preschoolers like chocolate ice cream only.
   15 preschoolers like both chocolate and strawberry ice cream, but not rasberry.

2. 3 preschoolers like all three of the ice cream flavors.
   5 preschoolers like chocolate ice cream only.
   5 preschoolers like both chocolate and strawberry ice cream, but not rasberry.

3. 3 preschoolers like all three of the ice cream flavors.
   15 preschoolers like chocolate ice cream only.
   5 preschoolers like chocolate and strawberry ice cream, but not rasberry.

4. 3 preschoolers like all three of the ice cream flavors.
   15 preschoolers like chocolate ice cream only.
   15 preschoolers like both chocolate and strawberry ice cream, but not rasberry.

ROUGH WORK

[TURN OVER]

SECTION 3
RELATIONS AND FUNCTIONS
(Questions 18 to 32) (15 marks)

Question 18
Let **C = {1, 2, 5, e}** and let **R = {(1, 1), (2, 5), (2, 2), (5, 1), (5, 2), (e, e)** be a relation on C. Which one of the following alternatives is needed to make R transitive?

1. Add the ordered pairs (5, 5) and (2, 1) to R.
2. Add the ordered pair (5, 5) to R.
3. Add the ordered pair (2, 1) to R.
4. Nothing needs to be added – R is already transitive.

Question 19
Let **A = {a, b, c, 3}.** Which one of the following relations on A satisfies trichotomy?

1. {(a, b), (c, a), (3, 3), (c, b), (b, 3), (3, c)}
2. {(3, b), (c, c), (b, b), (1, b), (2, b), (c, b)}
3. {(3, c), (c, a), (a, b), (a, 3), (c, b), (3, b)}
4. {(3, c), (c, b), (b, a), (3, a), (c, a), (b, c)}

Let **U = {1, {2}, {1, 2}, a, b}.** Let **A = {1, {2}, a}**, **B = {{2}, a, {1, 2}, b}** and **C = {{2}, 1, b}**.
Questions 20 to 23 are based on U, A, B and C.

Question 20
Which one of the following relations is functional from B to A?
1. {(b, {2}), (b, 1), (b, a)}
2. {(a, a), ({1, 2}, {2}), (a, 1)}
3. {({2}, 1)}
4. {({2}, {2}), (a, a), (b, {2}), ({1, 2}, {2}), (b, a)}

Question 21
Which one of the following relations is a function from C to U?
1. {({1, 2}, {2}), (1, b), (a, 1)}
2. {({2}, {2}), (1, 1), (b, b), (1, {2})}
3. {(b, {2}), (1, {1, 2}), ({2}, a)}
4. {({2}, {2}), (1, {2}), (b, {2}), ({2}, a), (b, {1, 2})}

Question 22

Which one of the following relations on C is NOT symmetric?

1. {(b, 1), (1, b), ({2}, {2})}
2. {(1, 1), (b, b)}
3. {({2}, 1), (b, {2}), (1, b), (b, b), ({2}, b), (1, {2})}
4. {({2}, b), (b, {2}), (b, b), ({2}, {2})}

Question 23

Which one of the following relations on A is symmetric and reflexive?

1. {(1, 1), ({2}, {2}), (a, 1), (1, a)}
2. {(a, a), (1, {2}), ({2}, 1), (1, 1), ({2}, a)}
3. {(a, a), (a, 1), (1, 1), ({2}, {2}), (1, a)}
4. {(1, a), (a, a), (1, 1)}

ROUGH WORK

[TURN OVER]

Let **A = {a, b, 1}, B = {b, 1, d}** and **C = {a, b, 1, d}.**
Answer Questions 24 and 25, based on these given sets:

Question 24

Which one of the following alternatives represents a surjective function from A to C?

1. {(a, a), (b, b), (1, 1)}
2. {(b, a), (b, b), (b, 1), (b, d)}
3. {(a, d), (b, 1), (1, a), (a, b)}
4. It is not possible to create a surjective function from A to C.

Question 25

Let F = {(b, b), (a, 1), (d, b), (1, d)} be a relation from C to B.
Which one of the following alternatives regarding F is TRUE?

1. F is an injective function from C to B.
2. F is a surjective function from C to B.
3. F is a bijective function from C to B.
4. F is neither an injective nor a surjective function from C to B.

Let **A = {1, 2, 3}** and **B = {2, 3, 4, 5}**. Consider the following two relations from **A** to **B**:

L = {(1, 4), (2, 2), (2, 3), (3, 2), (3, 5)} and
M = {(3, 3), (3, 2), (1, 3), (2, 4), (1, 5)}.

Question 26

Which one of the following alternatives represents L ○ M (ie M; L)?

1. {(3, 2), (3, 5), (3, 3), (1, 2), (1, 5)}
2. {(2, 4), (2, 3), (2, 2), (3, 4)}
3. {(2, 3), (2, 2), (2, 5), (3, 2), (3, 3)}
4. {(3, 2), (3, 4), (1, 3), (1, 2)}

ROUGH WORK

[TURN OVER]

Questions 27 to 32 is based on the following functions:
Let f and g be functions on ℤ defined by:

(x, y) ∈ g iff y = $2x^2 + 1$ and **(x, y) ∈ f iff y = -4x + 1**.

Question 27
Which one of the following statements regarding f and g is true?

1. Function f is bijective, but function g is not bijective.
2. Function f is surjective, but function g is not surjective.
3. Neither function f nor function g is injective.
4. Function f is injective, but function g is not injective.

Question 28
Which one of the following alternatives represents g ∘ f(x) (ie g(f(x))?

1. $-8x^2 - 3$
2. $8x^2 + 5$
3. $32x^2 - 16x + 3$
4. $32x^2 + 16x + 4$

Question 29
Which one of the following alternatives represents f ∘ f(x) (ie f(f(x))?

1. -16x - 3
2. 16x - 3
3. $16x^2 - 4x + 1$
4. $-16x^2 - 4x + 1$

Question 30
Which one of the following alternatives represents an ordered pair that does not belong to f?

1. (1, -3)
2. (-1, 5)
3. (-1, -3)
4. (3, -11)

Question 31
Which one of the following alternatives is FALSE regarding functions f and g?

1. Ordered pair (0, 1) is in both functions f and g.
2. Ordered pairs (2, 9) and (-2, 9) are both ordered pairs in function g.
3. Ordered pair (-2, 9) is in function f, but ordered pair (2, 9) is not in function f.
4. Ordered pair (-4, 17) is in function f, but not in function g.

Question 32
Which one of the following alternatives represents the range of g (ie ran(g))?

1. $\{y \mid \text{for some } y \in Z,\ y = 2x^2 + 1 \in Z\}$
2. $\{y \mid 2x^2 + 1 \in Z\}$
3. $\{y \mid \sqrt{\frac{y-1}{2}} \in Z\}$
4. Z

ROUGH WORK

[TURN OVER]

SECTION 4
OPERATIONS AND MATRICES
Questions 33 - 38

Question 33

Consider the following matrices:

Let A = $\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 3 & 2 & 1 & 3 \end{bmatrix}$ and C = $\begin{bmatrix} -1 & 0 \\ 0 & 1 \\ -3 & 3 \\ 2 & 1 \end{bmatrix}$

Which one of the following alternatives regarding operations on the given matrices is FALSE?

1. Performing the operation C • B will result in a 2 x 2 matrix.
2. The result of B • C is the matrix $\begin{bmatrix} -1 & 10 \\ 0 & 8 \end{bmatrix}$.
3. It is impossible to perform the operation A + B.
4. The result of A • B is the matrix $\begin{bmatrix} 8 & 6 & 6 & 12 \\ 6 & 5 & 7 & 12 \end{bmatrix}$.

ROUGH WORK

[TURN OVER]

ROUGH WORK

Question 34

Consider the following matrices:

A = $\begin{bmatrix} -4 & -5 \\ -3 & -1 \end{bmatrix}$ B = $\begin{bmatrix} -5 & 7 \\ -3 & 8 \end{bmatrix}$.

Which one of the following alternatives provides a matrix D such that D – A = 2B.

1. $\begin{bmatrix} -6 & 19 \\ -3 & 17 \end{bmatrix}$
2. $\begin{bmatrix} -14 & 9 \\ -9 & 15 \end{bmatrix}$
3. $\begin{bmatrix} -9 & -2 \\ -6 & 7 \end{bmatrix}$
4. $\begin{bmatrix} 6 & 9 \\ 3 & 15 \end{bmatrix}$

Question 35

What is the result of the operation $\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \bullet \begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 5 & 6 \end{bmatrix}$?

1. It is not possible to do the multiplication on these two matrices.
2. $\begin{bmatrix} 25 & 31 & 31 \end{bmatrix}$
3. $\begin{bmatrix} 21 & 7 \\ 14 & 28 \\ 35 & 42 \end{bmatrix}$
4. $\begin{bmatrix} 19 & 24 \end{bmatrix}$

Question 36

Consider the following binary operation * :

*	a	b	c
a	a	b	c
b	c	b	a
c	b	a	c

[TURN OVER]

Which one of the following statements regarding the binary operation * is TRUE?

1. **a** is the identity element of the binary operation *.
2. The binary operation * is commutative because **(b * c) = a** and **(c * b) = a**.
3. (c * a) * c ≠ c * (c * a) can be used as a counterexample to prove that the binary operation * is not associative.
4. [a * (b * (c * a))] = [b *((b * a) *(c * b))].

Question 37

Consider the incomplete binary operation ◊ below:

◊	a	b	c
a	b	a	
b			c
c			

Which one of the following tables represents the binary operation ◊ with the following properties:

(i) The operation ◊ is commutative.

(ii) The operation ◊ does not have an identity element.

1.

◊	a	b	c
a	b	a	a
b	a	b	c
c	a	c	b

2.

◊	a	b	c
a	b	a	b
b	a	c	c
c	c	c	a

3.

◊	a	b	c
a	b	a	b
b	a	c	c
c	b	c	a

4.

◊	a	b	c
a	b	a	a
b	a	b	c
c	a	b	c

Question 38

Consider the list notation for binary operation ☼ :

{((a, a), a), ((a, b), a), ((b, a), b), ((b, b), b)}

Which one of the following alternatives gives the correct table for the binary operation ☼ ?

1.

☼	a	b
a	b	a
b	b	a

2.

☼	a	b
a	a	a
b	b	b

3.

☼	a	b
a	b	a
b	a	b

4.

☼	a	b
a	a	b
b	b	a

ROUGH WORK

SECTION 5
TRUTH TABLES AND SYMBOLIC LOGIC
Questions 39 – 45 (7 marks)

Question 39
Consider the incomplete truth table below.

p	q	¬p	¬q	p ↔ (¬q ∨ ¬p)
T	T	F	F	
T	F	F	T	
F	T	T	F	
F	F	T	T	

Which one of the following alternatives provides the correct completed truth table?

1.

p	q	¬p	¬q	p ↔ (¬q ∨ ¬p)
T	T	F	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	F

2.

p	q	¬p	¬q	p ↔ (¬q ∨ ¬p)
T	T	F	F	F
T	F	F	T	T
F	T	T	F	F
F	F	T	T	F

3.

p	q	¬p	¬q	p ↔ (¬q ∨ ¬p)
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	F

4.

p	q	¬p	¬q	p ↔ (¬q ∨ ¬p)
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	F

Question 40

Which one of the statements in the following alternatives is equivalent to $p \vee q$?
(***Hint***: simplify the statement in each alternative using de Morgan's rules or a truth table to find the statement that is equivalent to $p \vee q$.)

1. $(\neg p \wedge \neg q) \rightarrow (\neg q \rightarrow p)$
2. $(p \wedge \neg q) \rightarrow (\neg q \rightarrow p)$
3. $(p \wedge \neg q) \rightarrow (p \rightarrow \neg q)$
4. $(p \wedge q) \rightarrow (\neg q \rightarrow p)$

Question 41

Which one of the statements in the following alternatives is equivalent to $p \rightarrow [\, q \vee (\neg p \vee q)]$?
(***Hint***: simplify the given statement using de Morgan's rules.)

1. $p \vee q$
2. $\neg p \vee q$
3. $\neg p \vee \neg q$
4. $p \vee \neg q$

ROUGH WORK

Question 42
Consider the following statement

$$[p \wedge (q \vee \neg r)] \leftrightarrow [(p \rightarrow \neg q) \vee \neg r]$$

and the incomplete truth table for the given statement below:

p	q	r	¬q	¬r	(q ∨ ¬r)	p ∧ (q ∨ ¬r)	↔	(p → ¬q)	(p → ¬q) ∨ ¬r
T	T	T	F	F	T	T			
T	T	F	F	T	T	T			
T	F	T	T	F				T	T
T	F	F	T	T	T	T	T	T	T
F	T	T	F	F	T	F			
F	T	F	F	T	T	F			
F	F	T	T	F				T	T
F	F	F	T	T	T	F		T	T

Which one of the following alternatives gives the correct completed truth table? The values that were completed are highlighted in each alternative.

1.

p	q	r	¬q	¬r	(q ∨ ¬r)	p ∧ (q ∨ ¬r)	↔	(p → ¬q)	(p → ¬q) ∨ ¬r
T	T	T	F	F	T	T	F	F	F
T	T	F	F	T	T	T	T	F	T
T	F	T	T	F	F	F	F	T	T
T	F	F	T	T	T	T	T	T	T
F	T	T	F	F	T	F	F	T	T
F	T	F	F	T	T	F	F	T	T
F	F	T	T	F	F	F	F	T	T
F	F	F	T	T	T	F	F	T	T

2.

p	q	r	¬q	¬r	(q ∨ ¬r)	p ∧ (q ∨ ¬r)	↔	(p → ¬q)	(p → ¬q) ∨ ¬r
T	T	T	F	F	T	T	T	F	F
T	T	F	F	T	T	T	T	F	T
T	F	T	T	F	F	T	T	T	T
T	F	F	T	T	T	T	T	T	T
F	T	T	F	F	T	F	T	T	T
F	T	F	F	T	T	F	T	T	T
F	F	T	T	F	F	F	F	T	T
F	F	F	T	T	T	F	F	T	T

3.

p	q	r	¬q	¬r	(q ∨ ¬r)	p ∧ (q ∨ ¬r)	↔	(p → ¬q)	(p → ¬q) ∨ ¬r
T	T	T	F	F	T	T	T	T	T
T	T	F	F	T	T	T	T	T	T
T	F	T	T	F	F	F	F	T	T
T	F	F	T	T	T	T	T	T	T
F	T	T	F	F	T	F	T	F	F
F	T	F	F	T	T	F	F	F	T
F	F	T	T	F	F	F	F	T	T
F	F	F	T	T	T	F	F	T	T

4.

p	q	r	¬q	¬r	(q ∨ ¬r)	p ∧ (q ∨ ¬r)	↔	(p → ¬q)	(p → ¬q) ∨ ¬r
T	T	T	F	F	T	T	F	F	F
T	T	F	F	T	T	T	F	F	F
T	F	T	T	F	F	F	T	T	F
T	F	F	T	T	T	T	T	T	T
F	T	T	F	F	T	F	T	T	F
F	T	F	F	T	T	F	F	T	T
F	F	T	T	F	F	F	T	T	F
F	F	F	T	T	T	F	F	T	T

Question 43

Consider the two statements below:

Statement 1: $\exists x \in \mathbb{Z}^+$, $[(3x - 5 > 0) \vee (2 - x^2 \geq 1)]$

Statement 2: $\exists x \in \mathbb{Z}$, $[(x^2 - 3 < 0) \wedge (3x - 4 \geq 0)]$

Which one of the following alternatives is true regarding statements 1 and 2?

1. Statement 1 is true and statement 2 is false.
2. Statement 1 is false and statement 2 is true.
3. Both statements 1 and 2 are false.
4. Both statements 1 and 2 are true.

Question 44

Consider the following statement:

$$\forall x \in \mathbb{Z}, [(3x - 5 > 0) \vee (1 + x^2 \leq 0)]$$

Which one of the following alternatives provides the correct simplification of the negation of the given statement such that the *not*-symbol (¬) does not occur to the left of any quantifier?

1. $\neg[\forall x \in \mathbb{Z}, [(3x - 5 > 0) \vee (1 + x^2 \leq 0)]]$

   $\equiv \forall x \in \mathbb{Z}, \neg[(3x - 5 > 0) \vee (1 + x^2 \leq 0)]$

   $\equiv \forall x \in \mathbb{Z}, [\neg(3x - 5 > 0) \wedge \neg(1 + x^2 \leq 0)]$

   $\equiv \forall x \in \mathbb{Z}, [(3x - 5 \leq 0) \wedge (1 + x^2 > 0)]$

2. $\neg[\forall x \in \mathbb{Z}, [(3x - 5 > 0) \vee (1 + x^2 \leq 0)]]$

   $\equiv \exists x \in \mathbb{Z}, \neg[(3x - 5 > 0) \vee (1 + x^2 \leq 0)]$

   $\equiv \exists x \in \mathbb{Z}, [\neg(3x - 5 > 0) \wedge \neg(1 + x^2 \leq 0)]$

   $\equiv \exists x \in \mathbb{Z}, [(3x - 5 < 0) \wedge (1 + x^2 \geq 0)]$

3. $\neg[\forall x \in \mathbb{Z}, [(3x - 5 > 0) \vee (1 + x^2 \leq 0)]]$

   $\equiv \exists x \in \mathbb{Z}, \neg[(3x - 5 > 0) \vee (1 + x^2 \leq 0)]$

   $\equiv \exists x \in \mathbb{Z}, [\neg(3x - 5 > 0) \wedge \neg(1 + x^2 \leq 0)]$

   $\equiv \exists x \in \mathbb{Z}, [(3x - 5 \leq 0) \wedge (1 + x^2 > 0)]$

4. $\neg[\forall x \in \mathbb{Z}, [(3x - 5 > 0) \vee (1 + x^2 \leq 0)]]$

   $\equiv \exists x \in \mathbb{Z}, \neg[(3x - 5 > 0) \wedge (1 + x^2 \leq 0)]$

   $\equiv \exists x \in \mathbb{Z}, [\neg(3x - 5 > 0) \vee \neg(1 + x^2 \leq 0)]$

   $\equiv \exists x \in \mathbb{Z}, [(3x - 5 \leq 0) \vee (1 + x^2 > 0)]$

ROUGH WORK

Question 45

Consider the following statement:

$$\exists x \in \mathbb{Z}^+, [(2x - 3 < 0) \wedge (x^2 + 1 \geq 10)]$$

Which one of the following statements about the given statement is TRUE?

1. $x = -1$ can be used as a counterexample to prove that the given statement is FALSE.
2. The negation of the given statement is $\exists x \in \mathbb{Z}^+, [(2x - 3 \geq 0) \vee (x^2 + 1 < 10)]$
3. The given statement is FALSE for all possible values of x.
4. The given statement is TRUE only for all positive values of x.

ROUGH WORK

[TURN OVER]

SECTION 6
MATHEMATICAL PROOFS
QUESTIONS 46 – 50 (5 marks)

Question 46

Consider the statement

If n is a multiple of 3, then $3n^2 + 6n + 9$ is even.

Which one of the following statements provides the **converse** of the given statement?

1. If n is a multiple of 3, then $3n^2 + 6n + 9$ is even.
2. If n is not a multiple of 3, then $3n^2 + 6n + 9$ is odd.
3. If $3n^2 + 6n + 9$ is even, then n is a multiple of 3.
4. If $3n^2 + 6n + 9$ is odd, then n is not a multiple of 3.

Question 47

Consider the statement

If n is even, then $4n^2 + 2n – 7$ is odd.

Which one of the following statements provides the **contrapositive** of the given statement?

1. If n is odd, then $4n^2 + 2n – 7$ is even.
2. If $4n^2 + 2n – 7$ is even, then n is odd.
3. If $4n^2 + 2n – 7$ is odd, then n is even.
4. If n is odd, then $4n^2 + 2n – 7$ is odd.

ROUGH WORK

Question 48

Which of the following alternatives provides a **direct** proof to show that for all n $\in \mathbb{Z}$,

if n + 1 is a multiple of 3, then n^2 + 3n + 5 is a multiple of 3.

1. Let n be a multiple of 3, then n = 3k, for some k $\in \mathbb{Z}$.
   ie $(3k)^2 + 3(3k) + 5$,
   ie $9k^2 + 9k + 5$, which can be written as $(9k^2 + 9k + 6) – 1$,
   ie $3(3k^2 + 3k + 2)$, which is a multiple of 3.

2. Assume that n is not a multiple of 3. We then have to prove that $n^2 + 3n + 5$ is also not a multiple of 3,
   Let n = 3k + 1, (because n = 3k is a multiple of 3),
   ie $(3k + 1)^2 + 3(3k + 1) + 5$,
   ie $9k^2 + 6k + 1 + 9k + 3 + 5$,
   ie $9k^2 + 15k + 9$,
   ie $3(3k^2 + 5k + 3)$, which is a multiple of 3.
   Our original assumption that n + 1 is not a multiple of 3 is therefore not true,
   ie we can deduce that the original statement is false.

3. Let n + 1 = 3, which is a multiple of 3, then n = 3 – 1 = 2,
   then $n^2 + 3n + 5$
   ie $(2)^2 + 3(2) + 5$,
   ie 18
   ie 3(6), which is a multiple of 3.

4. Let n + 1 be a multiple of 3, then n + 1 = 3k, for some k $\in \mathbb{Z}$, then n = 3k – 1,
   then $n^2 + 3n + 5$
   ie $(3k – 1)^2 + 3(3k – 1) + 5$,
   ie $9k^2 – 6k + 1 + 9k – 3 + 5$
   ie $9k^2 + 3k + 3$
   ie $3(3k^2 + k + 1)$, which is a multiple of 3.

ROUGH WORK

[TURN OVER]

Question 49

Consider the following statement, for all x $\in \mathbb{Z}$:

If $x^3 - 2x$ is odd, then x is odd.

Which one of the following alternatives contains the correct way to start a **contrapositive** proof to prove the statement?

1. Let x be odd, then x = 2k + 1 for some k $\in \mathbb{Z}$,
   ie $x^3 - 2x = (2k + 1)^3 – 2(2k +1)$,
   ie ………..

2. Let x be even, then x = 2k for some k $\in \mathbb{Z}$,
   ie $x^3 - 2x = (2k)^3 – 2(2k)$,
   ie ………..

3. Assume $x^3 - 2x$ is odd, then x can be odd or even. We will assume that x is even.
   Let x be even, then x = 2k for some k $\in \mathbb{Z}$,
   ie ………..

4. Let $x^3 - 2x$ be odd,
   We know that an odd number minus an even number is odd,
   ie x^3 must be odd, because odd * odd * odd is odd,
   ie let x = 2k + 1 for some k $\in \mathbb{Z}$,
   ie ………..

Question 50

Which one of the following values for x can be used in a counter-example to prove that the statement $\forall x \in \mathbb{Z}^+, -x^3 - 5x – 7 > 0$, is FALSE?

1. 1
2. -1
3. 0
4. -2

ROUGH WORK

[TURN OVER]

ROUGH WORK