UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1512

October/November 2015

CALCULUS A

Duration 2 Hours

100 Marks

EXAMINERS

FIRST MRS SB MUGISHA

SECOND DR B HUNTLEY DR L LINDEBOOM

Closed book examination

This paper consists of 3 pages **ANSWER ALL QUESTIONS.**

ALL CALCULATIONS MUST BE SHOWN

Calculators may NOT be used

QUESTION 1

(a) Determine the following limits (if they exist)

(i) $\lim\limits_{m\to 2}\dfrac{3m-5}{m^3-2m^2+3}$ (3)

(ii) $\lim\limits_{t\to 1}\dfrac{\sqrt{t}-1}{t-1}$ (3)

(iii) $\lim\limits_{r\to -2}\left(\dfrac{1}{r+2}+\dfrac{4}{r^2-4}\right)$ (3)

(iv) $\lim\limits_{t\to 0}\dfrac{1-\cos^2 t}{t^2}$ (3)

(v) $\lim\limits_{x\to -\infty}\dfrac{5x^3-x^2+2}{2x^3+x-3}$ (3)

(b) $\lim\limits_{x\to\infty}\dfrac{3x-\sin x}{4x+5}$ (5)

(c) Let $L(w)=\begin{cases}\frac{w+2}{\sqrt{w+2}} & \text{if} \quad w>-2\\ 2w+c & \text{if} \quad w\leq -2\end{cases}$

Find the value of 'c' which will make the function $L(w)$ continuous at $w=-2$ (5)

[25]

QUESTION 2

(a) By the first principles of differentiation find the derivative of $G(t)=(t+1)(t-2)$ at $t=3$ (5)

(b) Find the derivatives of the following functions by using the appropriate rules for differentiation

(i) $f(x)=\frac{x^2+x-2}{x}$ (3)

(ii) $f(r)=e^{e^{\cos\left(e^{3r}\right)}}$ (4)

(iii) $y=\int\limits_{\sqrt{x}}^{3}\cos t\,dt$ (4)

(c) Given $\cos(x+y)=2x$ determine the following

(i) $\dfrac{dy}{dx}$ by using implicit differentiation (4)

(ii) the equations of the tangent and normal line to the curve $\cos(x+y)=2x$ at the point $\left(0,\frac{\pi}{2}\right)$ (5)

[25]

[TURN OVER]

QUESTION 3

(a) Determine the following integrals

(i) $\int \left(\frac{x^4 - 2}{x^2} \right) dx$ (3)

(ii) $\int e^{7t} \left(\frac{e^{2t}}{5} + \frac{3}{e^{3t}} \right) dt$ (3)

(iii) $\int \frac{x^3}{x^4 - 5} \, dx$ (4)

(iv) $\int_0^{\frac{\pi}{4}} \sin 2\theta \sin \theta d\theta$ (5)

(b) Let $x = 2y$ and $4x = y^2$

(i) sketch the graphs of the above two curves on the same axes (4)

(ii) find the area of the region enclosed by the curves $x = 2y$ and $4x = y^2$ (6)

[25]

QUESTION 4

(a) Solve the following Initial Value Problem

$$\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}, \; u(0) = -5 \tag{7}$$

(b) Let $T(x\ y) = x^2y - y^3 + \ln x$

(i) find the first order partial derivatives T_x and T_y (6)

(ii) using b(i) above, find $\frac{dy}{dx}$ (6)

(iii) If $T(x, y) = 0$ then find $\frac{dy}{dx}$ using implicit differentiation to confirm your answer in part (b) (ii) above (6)

[25]

TOTAL: [100]