Tutorial letter 014/0/2021

Descriptive Statistics and Probability

STA1501

Year module

Department of Statistics

Assignment 4 Questions

ASSIGNMENT 04
Fixed closing date: 7 September 2021

Please study Chapters 9 and 10 and corresponding parts of the study guide before answering the following questions.

QUESTION 1

Which of the following statements are true and which are false? Justify your answers!

(a) The population variance is always larger than the standard error of the mean.

(b) The sample mean of a random variable X has normal distribution if X is normal, and is approximately normal even if X does not have the normal distribution, as long as the sample size is large enough.

(c) The sample proportion in a random sample has a binomial distribution, but can often be approximated by the normal distribution.

(d) The standard error of the proportion will increase when the sample size gets larger.

(e) A point estimator is better than an interval estimator because it is more precise.

(f) The sample mean is an unbiased estimator of the population mean but it is not always consistent, if the population variance is very large.

(g) Assume that we are estimating the population mean with confidence level $1-\alpha$ when the population standard deviation is known. Then $UCL - LCL = 2 * B$ where LCL is the lower confidence limit, UCL is the upper confidence limit and B is the limit on the error of estimation.

(h) Assume that we are estimating the population mean with confidence level $1-\alpha$ when the population standard deviation is known. Then $\overline{x} \in [LCL, UCL]$ holds with probability $1-\alpha$.

(i) Assume that we are estimating the population mean with confidence level $1-\alpha$ when the population standard deviation is known. For a given sample, if the confidence level is increased then the error of estimation will decrease.

(j) Assume that we are estimating the population mean with confidence level $1-\alpha$ when the population standard deviation is known. To make the confidence interval for a population mean interval estimate more narrow, we can either reduce the required confidence level or increase the sample size.

[20]

QUESTION 2

Assume that the height of South African adult males has the normal distribution, with an average of 166 cm and a standard deviation of 9 cm. Let $\overline{X}$ be the sample mean of the heights of a sample of 20 men.

(a) Describe the distribution of $\overline{X}$, giving the name of the distribution, its expected value and variance. (5)

(b) What is the probability that the sample mean is between 156 cm and 176 cm? (7)

(c) Find the first and third quartiles of the distribution of the sample mean. (8)

[20]

QUESTION 3

Assume that 20.5% of all cars in South Africa are silver in colour. Let $\widehat{p}$ be the sample proportion of silver cars in a random sample of 1000 cars.

(a) Find the mean and standard deviation of $\widehat{p}$. (6)

(b) What is the probability that the sample proportion $\widehat{p}$ is more than 25%? (6)

(c) What is the probability that the sample proportion $\widehat{p}$ is within 1% of the true population proportion? (8)

[20]

QUESTION 4

A random sample of 100 bags of potatoes found that the sample mean $\overline{X}$ for the weight of the bags was 10.2 kilograms. Assume that the population variance of the weights is known to be 2.5 kg.

(a) Compute an unbiased point estimate for the population mean μ. (3)

(b) Find the standard error of the mean. (3)

(c) Find the 90% and 99% confidence intervals for the population mean, μ. (10)

(d) Explain what the 99% confidence interval tells us about the population mean, μ. (4)

[20]

QUESTION 5

A new manufacturing method is supposed to increase the average life span of electronic components, while the variance of the life span is expected to stay the same. Using the previous manufacturing method, the average life span was 112.5 hours with a variance 12 hours. The manufacturer wishes to establish the new average life span by measuring the life spans of a sample of components manufactured using the new method.

(a) What sample size should be used, if the manufacturer wishes to establish the new average life span to within 1 hours, with 90% level of confidence? (8)

(b) How will the required sample size change, if the manufacturer wishes to establish the new average life span to within 1 hours, with 95% level of confidence? (6)

(c) How will the required sample size change, if the manufacturer wishes to establish the new average life span to within 1/2 hours, with 90% level of confidence? (6)

[20]

Total marks: [100]