UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1503

May/June 2015

LINEAR ALGEBRA

Duration 2 Hours

100 Marks

EXAMINERS
FIRST DR L GODLOZA DR ZE MPONO

Closed book examination

This examination question paper remains the property of the University of South Africa and may not be removed from the examination venue

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This paper consists of 3 pages

Answer All Questions

QUESTION 1

(a) Solve the following system

$$\begin{cases} 3x_1 + 2x_2 + x_3 = 1 \\ \qquad x_2 - x_3 = 2 \\ \qquad\quad 2x_3 = 4 \end{cases}$$

(5)

(b) Find x, y, z such that

$$x\begin{bmatrix} 2 \\ 5 \end{bmatrix} + y\begin{bmatrix} 3 \\ -4 \end{bmatrix} + z\begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

(3)

(c) Let

$$A = \begin{bmatrix} 3 & 4 \\ 1 & 1 \\ 2 & 7 \end{bmatrix}$$

and verify that

(i) $5A = 3A + 2A$ (3)

(ii) $6A = 3(2A)$ (3)

(iii) $(A^T)^T = A$ (2)

[TURN OVER]

(d) Consider the following matrices

$$B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

Then compute B^2 and C^2 (4)

(e) Without using determinants, show that the following matrix

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

has no inverse (5)

[25]

QUESTION 2

(a) Let

$$E = \begin{bmatrix} 2 & 5 & 4 \\ 3 & 1 & 2 \\ 5 & 4 & 6 \end{bmatrix}$$

and find the determinant of E by the cofactor expansion

(i) along the first row (5)

(ii) along the second column (5)

(b) Given that

$$F = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$$

then

(i) show that $det(F) = det(F^T)$ (3)

(ii) find $det(3F)$ (3)

(iii) compare $det(3F)$ with $det(F)$ (2)

(c) Using Cramer's Rule, solve the following system of linear equations

$$\begin{cases} x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 2x_2 + x_3 = 6 \\ x_1 + 2x_2 + 3x_3 = 9 \end{cases}$$

(7)

[25]

[TURN OVER]

QUESTION 3

Consider the vectors $\underline{u} = (1, 1, 0)$ and $\underline{v} = (0\ 1, 1)$

(a) Determine $\cos(\theta)$ if θ is the angle between $\underline{u}$ and $\underline{v}$ (5)

(b) Calculate the area of the parallelogram determined by $\underline{u}$ and $\underline{v}$ (5)

(c) Find an equation of the plane **V** containing $\underline{u}$ and $\underline{v}$ (5)

(d) Determine the equation of the plane parallel to the plane **V** in (c) and passing through the point $(1, 1, 1)$ (5)

(e) Find an equation of the line perpendicular to the plane **V** in (c) and passing through the tip of $\underline{w}$, where $\underline{w}$ is the unit vector in the direction of $\underline{u}$ with the same initial point as $\underline{u}$ (5)

[25]

QUESTION 4

(a) Use de Moivre's Theorem to express $\cos 4\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$ (10)

(b) Determine the 4th roots of 16 in polar form (10)

(c) Let $w = 3 + 4i\ z = 5 - 2i$ Show that $\dfrac{w}{z} = \dfrac{7}{29} + \dfrac{26}{29}i$ (5)

[25]

TOTAL: 100 Marks

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