UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

MAT1512

October/November 2016

CALCULUS A

Duration 2 Hours

100 Marks

EXAMINERS

FIRST	MRS SB MUGISHA	
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Closed book examination

This paper consists of 4 pages **ANSWER ALL QUESTIONS.**
ALL CALCULATIONS MUST BE SHOWN

Calculators may NOT be used

QUESTION 1

(a) Determine the following limits (if they exist)

(i) $\lim\limits_{t \to -5} \dfrac{x^2 + x - 20}{3(x+5)}$ (3)

(ii) $\lim\limits_{t \to 0} \dfrac{\sin 5t}{t^2 + 4t}$ (3)

(iii) $\lim\limits_{x \to -\infty} \dfrac{3 - |x|}{2|x| + 1}$ (3)

(iv) $\lim\limits_{x \to 0} \dfrac{2x}{3 - \sqrt{x+9}}$ (3)

(v) $\lim\limits_{t \to +\infty} \dfrac{2x + x^2 + 1}{1 - x + 2x^2}$ (3)

(b) (i) Use the Squeeze Theorem to show that

$$\lim_{x \to \infty} \frac{\sin(e^x)}{x} = 0 \qquad (3)$$

(ii) Hence, evaluate

$$\lim_{x \to \infty} \frac{\sin(e^x)}{\sqrt{x^2 + 2}} \qquad (3)$$

(c) Let the function f be defined as

$$f(x) = \begin{cases} 4a & \text{if} \quad x \leq -2 \\ 3x^2 & \text{if} \quad -2 < x \leq 1 \\ x + b & \text{if} \quad x > 1 \end{cases}$$

Determine the values of the constants a and b so that f is continuous at $x = -2$ and $x = 1$ (4)

[25]

QUESTION 2

(a) By the first principle of differentiation, find the derivative of $f(x) = \dfrac{2}{2x - 1}$ at $x = 1$ (5)

(b) Find the derivative of the following functions by using the appropriate rules of differentiation

(i) $y = \frac{1}{\sqrt{x}}\left(x^2 - \frac{2}{x}\right)$ (3)

(ii) $g(x) = (\cos 5x)^{\sin x^2}$ (3)

(iii) $h(x) = \dfrac{\sin x}{1 + \cos x}$ (3)

(iv) $F(x) = \displaystyle\int_{\sqrt{x}}^{x} t\sqrt{t^2 + 1}\,dt$ (3)

(c) Given $2xy + \pi \sin y = 2\pi x$, find

(i) $\dfrac{dy}{dx}$ by using implicit differentiation (3)

(ii) the equation of the tangent and normal lines to the curve $2xy + \pi \sin y = 2\pi x$ at the point $\left(1, \frac{\pi}{2}\right)$ (5)

[25]

QUESTION 3

(a) Use the appropriate substitution to evaluate the following integrals

(i) $\displaystyle\int \frac{t}{(t^2 + 1)^3}\,dt$ (3)

(ii) $\displaystyle\int \frac{4^x - 5^x}{7^x}\,dx$ (3)

(iii) $\displaystyle\int 2\sin(4x)\,dx$ (3)

(b) Determine the exact values for the following integrals Use substitution if neccesary

(i) $\displaystyle\int_0^2 x\sqrt{x^2 + 1}\,dx$ (4)

(ii) $\displaystyle\int_e^{e^3} \frac{\ln(\ln t)}{t \ln t}\,dt$ (4)

(c) Determine the area of the region enclosed by the curves $f(x) = -x^2$ and $g(x) = x^2 - 2x$

(Hint Sketch the graphs of f and g on the same axes) (8)

[25]

QUESTION 4

(a) Solve the following initial value problem

$$\frac{dy}{dx} = x^3 \cos\left(x^4 + 2\right), \; y(0) = 1 \qquad (5)$$

(b) A bactrial culture starts with 2200 bacteria and after 3 hours there are 3700 bacteria Assuming that the culture grows at a rate proportional to its size, find the population after 6 hours (5)

(c) If $z = \sin(xy) + x \sin y$, where $x = u^2 + v^2$ and $y = uv$
Use Chain Rule for partial differentiation to find $\dfrac{\partial t}{\partial u}$ (5)

(d) Let $F(x, y) = x^4 - 3x^2y^3 + 5y$

(i) find the partial derivates F_x and F_y (2)

(ii) Use d(i) above find $\dfrac{dy}{dx}$ (2)

(iii) Confirm your answer in d(ii) by finding $\dfrac{dy}{dx}$ using implicit differentiation (6)

[25]

TOTAL: [100]