

COS1501

Notes on Sets and Venn diagrams

Before reading this document, please make sure that you have worked through the study guide lessons 1 to 4.

Below we give an example of a universal set U with 3 subsets A , B and C , and show how the sets are represented using a Venn diagram:

Let $U = \{1, 2, 3, 4, \dots, 16\}$, that is U includes the numbers 1 to 16 as its elements.

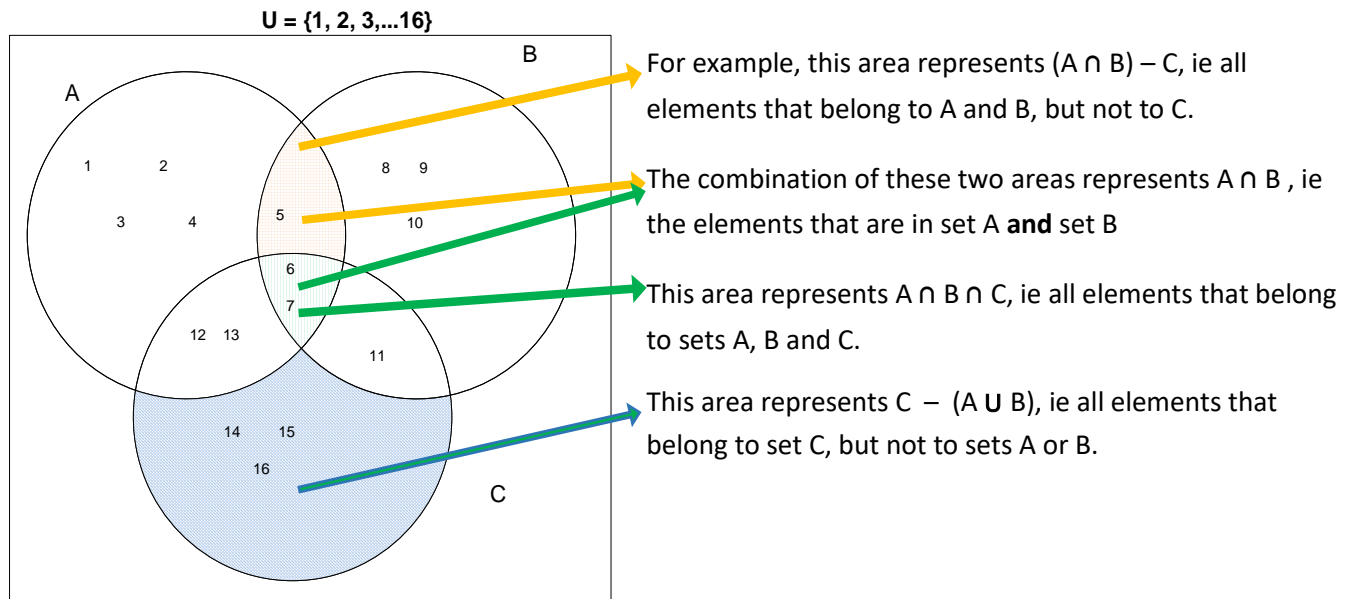
Let $A = \{1, 2, 3, 5, 6, 7, 12, 13\}$, $B = \{5, 6, 7, 8, 9, 10, 11\}$ and $C = \{6, 7, 11, 12, 13, 14, 15, 16\}$

Let us rewrite the above sets to make it clearer to see which elements are in more than one set:

$U = \{$	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12,	13,	14,	15,	16	$\}$
$A = \{$	1,	2,	3,		5,	6,	7,					12,	13				$\}$
$B = \{$					5,	6,	7,	8,	9,	10,	11						$\}$
$C = \{$						6,	7,				11,	12,	13,	14,	15,	16	$\}$

From the table it is easy to see, for example, that elements 6 and 7 are in sets A , B **and** C , ie $A \cap B \cap C = \{6, 7\}$, or that element 5 is in set A and B , but **not** in set C , ie $(A \cap B) - C = \{5\}$.

We can represent sets A, B and C as a Venn diagram as well:



Venn diagrams can also be used to solve mathematical problems. The study guide discusses a few examples. We give more examples below. It is very important to read the information given for a problem, carefully, otherwise we will not represent the information given correctly in the Venn diagram. Notice the difference in the wording of the different examples. Note that notes in red will not normally appear in the question – it is highlighted here to make you aware that the question should be read and understood carefully:

Example 1:

A school offers tennis, cricket and soccer as extramural activities. We know the following about the 34 pupils in Mr Smith's class:

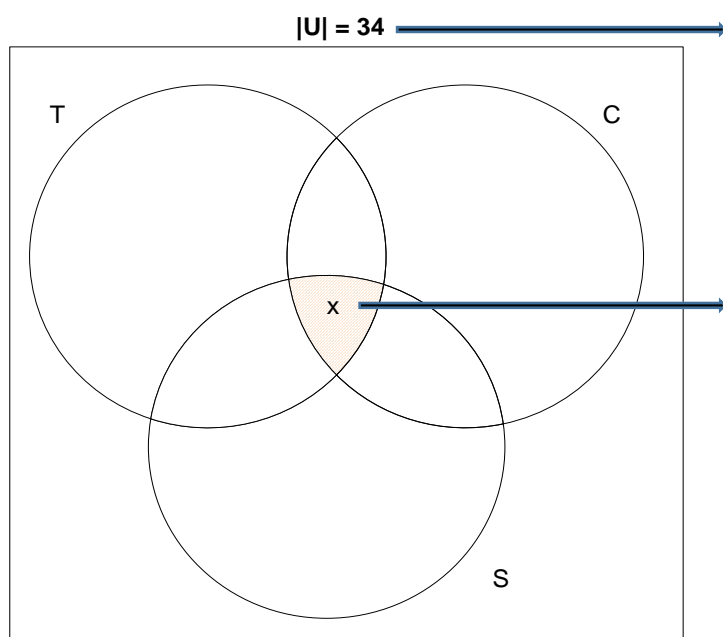
- 20 pupils play tennis;
 - 19 pupils play cricket;
 - 12 pupils play soccer.
- (pupils do not necessarily play one sport only)

Furthermore,

- 4 pupils play **tennis and cricket only**;
 - 5 pupils play **tennis and soccer only**;
 - 2 pupils play **cricket and soccer only**;
- (can you see that these groups of pupils each only play two sports?. This is important to note because it will influence the way the Venn diagram is completed)

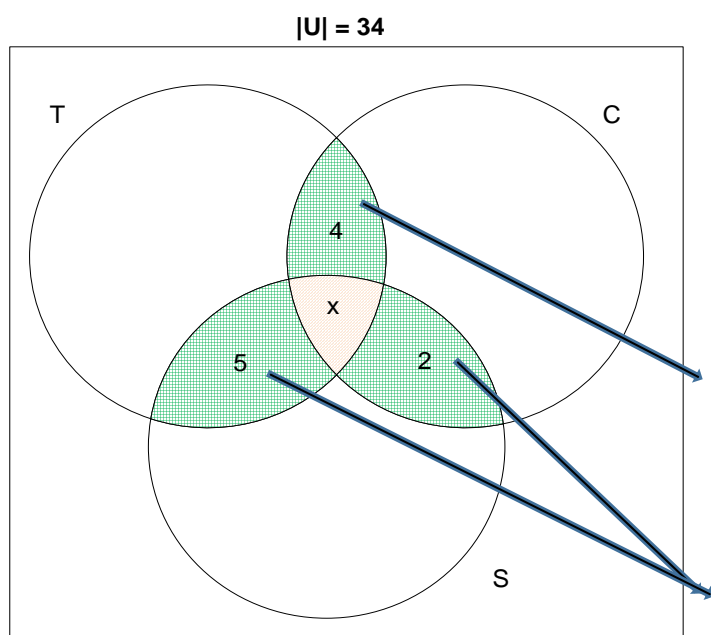
There will normally be one piece of information missing, that you have to find, using the given information. We call this piece of information the 'unknown' information. We normally use a variable like x or y to represent this unknown information.

We will complete the Venn diagram with the information that we have. We always start completing the Venn diagram from the middle outwards, ie we start with the intersection between the three sets. T represents tennis, C represents cricket and S represents soccer.



U represents all the pupils in the class. It has been given that there are 34 pupils in the class. $|U|$ indicates the number of elements (pupils) in the universal set U.

Step 1: We complete $T \cap C \cap S$. The question did not tell us how many pupils play all three sports. So this is the unknown that we have to solve. We call it x.



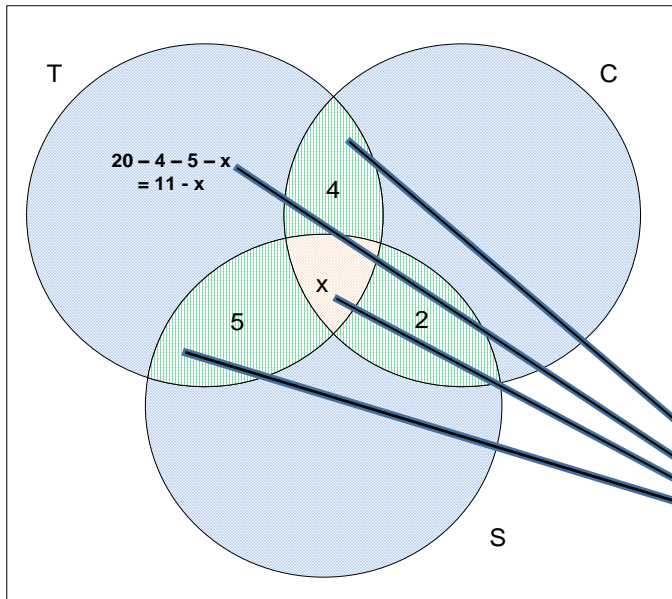
Step 2: We complete the green areas. It is very important here that you read the given information carefully. This area represents the pupils that play exactly 2 of the sports. We use the following given information to complete these areas:

- 4 pupils play **tennis and cricket only**;
- 5 pupils play **tennis and soccer only**;
- 2 pupils play **cricket and soccer only**;

This area represents the number of pupils that play tennis and cricket, but not soccer, in other words, the pupils that play tennis and cricket **only**, which is 4 pupils according to the given information. We therefore fill in 4 in this area. The word **only** here is key. Compare this to the wording in example 2.

Similarly these two areas can be filled in.

$$|U| = 34$$



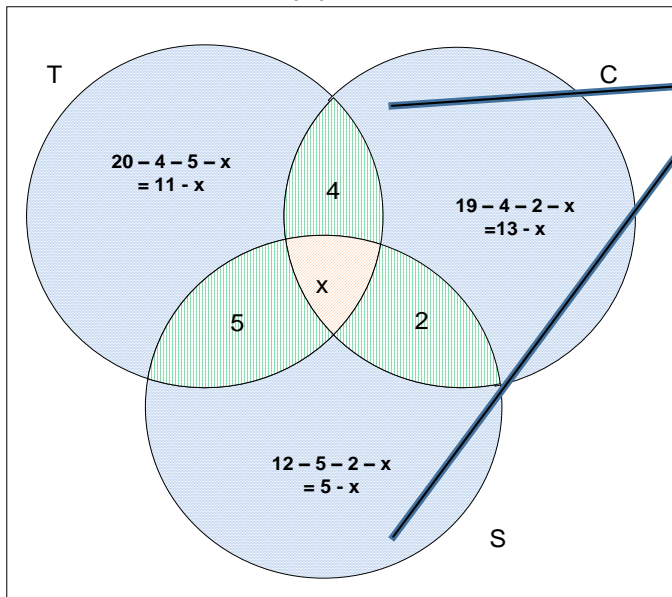
Step 3: We complete the blue areas. It is very important here that you read the given information carefully. This area represents the pupils that play exactly 1 of the sports. We use the following given information to complete these areas:

- 20 pupils play tennis;
 - 19 pupils play cricket;
 - 12 pupils play soccer.
- (pupils do not necessarily play one sport only)

The information tells us that 20 pupils play tennis, but that it does not mean that they play tennis only. These 20 pupils play at least 1 sport, which is tennis.

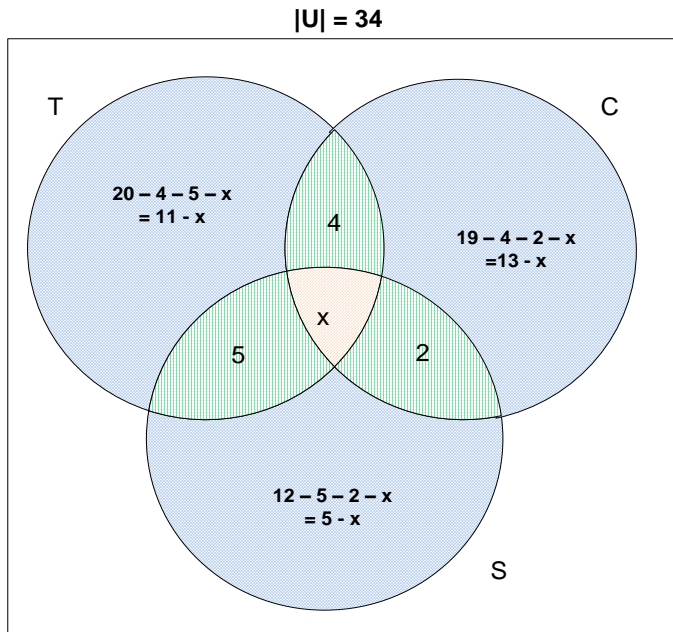
The 20 pupils are represented by all 4 these areas. In order to determine the number of pupils that should be represented in the blue area only, we need to deduct the number of pupils in the other three areas, which is 4, 5 and x .

$$|U| = 34$$



Step 4: In the same way these two areas are completed using the given information:

- 20 pupils play tennis;
 - 19 pupils play cricket;
 - 12 pupils play soccer.
- (pupils do not necessarily play one sport only)



Step 5: Now we have completed the whole Venn diagram and we can calculate the value of x. If we add the values in all the sections together, it must be equal to 34 as is given:

$$11 - x + 4 + 5 + x + 2 + 13 - x + 5 - x = 34$$

$$\text{ie } 40 - 2x = 34$$

$$\text{ie } 40 - 34 = 2x$$

$$\text{ie } x = 3.$$

Example 2:

A school offers tennis, cricket and soccer as extramural activities. We know the following about the 50 pupils in Mr Smith's class:

- 22 pupils play tennis;
- 25 pupils play cricket;
- 19 pupils play soccer.
(pupils do not necessarily play one sport only)

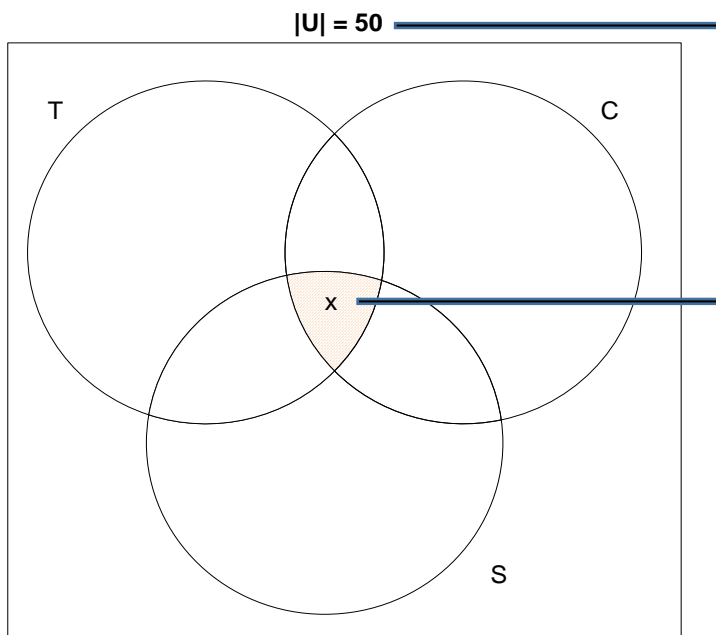
Furthermore,

- 7 pupils play **tennis and cricket**;
- 5 pupils play **tennis and soccer**;
- 6 pupils play **cricket and soccer**;
- (pupils do not necessarily play two sports only)

(can you see that these groups of pupils each play **at least** two sports?. This is important to note because it will influence the way the Venn diagram is completed. Compare this with example 1)

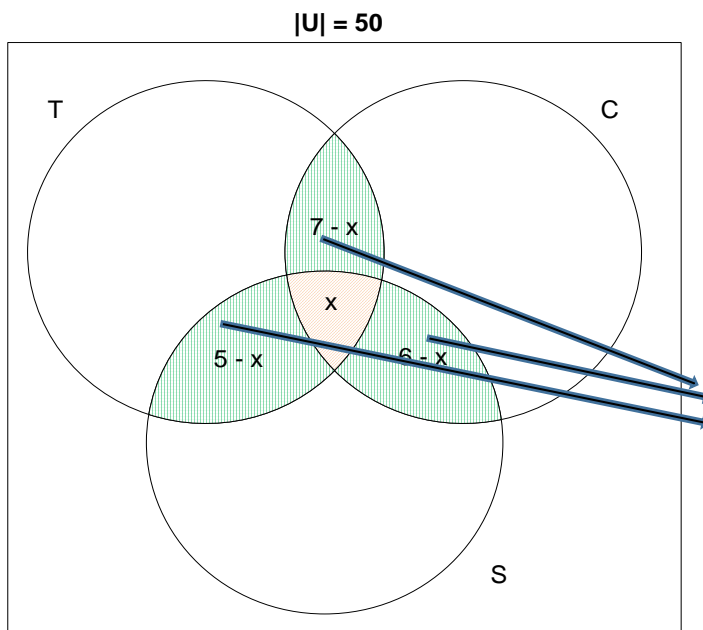
There will normally be one piece of information missing, that you have to find, using the given information. We call this piece of information the 'unknown' information. We normally use a variable like x or y to represent this unknown information.

We will complete the Venn diagram with the information that we have. We always start completing the Venn diagram from the middle outwards, ie we start with the intersection between the three sets. T represents tennis, C represents cricket and S represents soccer.



U represents all the pupils in the class. It has been given that there are 50 pupils in the class. $|U|$ indicates the number of elements (pupils) in the universal set U.

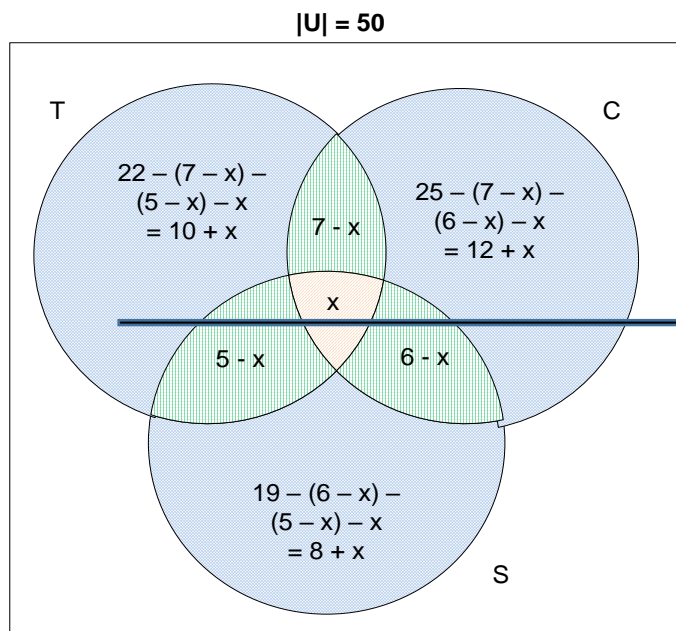
Step 1: We complete $T \cap C \cap S$. The question did not tell us how many pupils play all three sports. So this is the unknown that we have to solve. We call it x.



Step 2: We complete the green areas. It is very important here that you read the given information carefully. This area represents the pupils that play at least 2 of the sports. We use the following given information to complete these areas:

- 7 pupils play **tennis and cricket**;
- 5 pupils play **tennis and soccer**;
- 6 pupils play **cricket and soccer**;
- (pupils do not necessarily play two sports only)

Can you see that in this case, x, the number of pupils that play all 3 sports, are included in the 7, 5 and 6 respectively, that is why we have to deduct x from the values in the green areas.

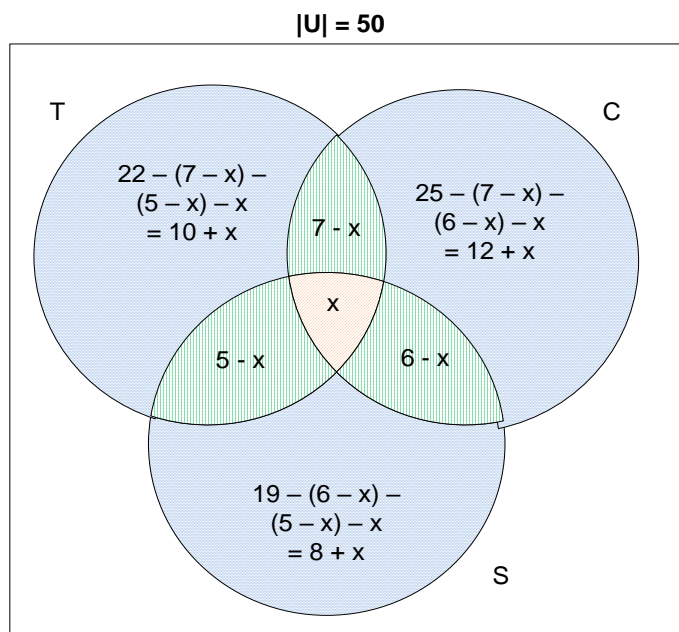


Step 3: Now we complete the blue areas, using the given information:

- 22 pupils play tennis;
- 15 pupils play cricket;
- 19 pupils play soccer.
(pupils do not necessarily play one sport only)

It is given that 22 pupils play tennis. Note that they do not necessarily play tennis only. The 22 represents the whole T set. This means that to get only the blue area, we need to subtract all the values in the green areas and the orange area that are part of T.

Similarly the other two blue areas can be determined.



Step 4: Now we have completed the whole Venn diagram and we can calculate the value of x . If we add the values in all the sections together, it must be equal to 50 as is given:

$$10 + x + 7 - x + x + 5 - x + 6 - x + 12 + x + 8 + x$$

$$\text{ie } 48 + x = 50$$

$$\text{ie } 50 - 48 = x$$

$$\text{ie } x = 2.$$