ASSIGNMENT 03
Fixed Closing Date: 03 June 2022
Total Marks: 100
UNIQUE ASSIGNMENT NUMBER: _____

1. Use the method of implicit differentiation to determine the derivatives of the following functions:

(a) $x\sin y + y\sin x = 1$ (5)

(b) $\tan(x-y) = \dfrac{y}{1+x^2}$ (5)

(c) $\sqrt{x+y} = x^4 + y^4$ (5)

(d) $y + x\cos y = x^2 y$ (5)

(e) $2y + \cot(xy^2) = 3xy$ (5)

2. Find the number "c" that satisfy the Mean Value Theorem (M.V.T.) on the given intervals.

(a) $f(x) = e^{-x}$, $[0, 2]$ (5)

(b) $f(x) = \dfrac{x}{x+2}$, $[1, \pi]$ (5)

3. Determine the equation of the tangent and normal at the given points:

(a) $y + x\cos y = x^2 y$, $\left[1, \dfrac{\pi}{2}\right]$ (5)

(b) $h(x) = \dfrac{2}{\sqrt{x^2+1}}$, at $x = 1$. (5)

4. Find the derivative of $f(x) = \int_{-x}^{\sqrt{x}} \left(\sqrt{v^2+2}\right) dv$ (5)

5. Find the derivative of the following functions using the appropriate rules for differentiation.

Simplify your answer: $F(x) = \int_{2x}^{x^2} \sqrt{t^2+1}\, dt$ (5)

6. Find the derivatives of the following functions by using the appropriate rules of differentiation:

$$y = \int_{1-3x}^{1} \frac{u^3}{1+u^2} du$$ (5)

7. The equation of the ellipse is given as

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

Use implicit differentiation to determine the derivative of the equation of the ellipse given above. (5)

8. Determine the slope of the equation in Question 1., above, at (x_0, y_0). (5)

9. Hence or otherwise find the equation of the tangent at (x_0, y_0).
The equation referred to in Question 1, above. (5)

10. Let $x^2 - xy + y^2 = 3$ be the equation of an ellipse. By implicit differentiation determine the equation of the normal of the equation given above at $(-1, 1)$. (5)

11. Given that $\sin(x+y) = 2x$, find the equation of the tangent line at the point $(0, \pi)$. (5)

12. Find the equation of the tangent and normal lines to the curve of:

$\pi \sin y + 2xy = 2\pi$ at the point $\left(1, \frac{\pi}{2}\right)$. (5)

13. Let $x^4 + 5y = 3x^2y^3$. Find $\frac{dy}{dx}$ using implicit differentiation. (5)

14. For the equation $x^2 + y^3 - 2y = 3$

Find the equation of the normal line at the point (2, 1). (5)

Total: [100]