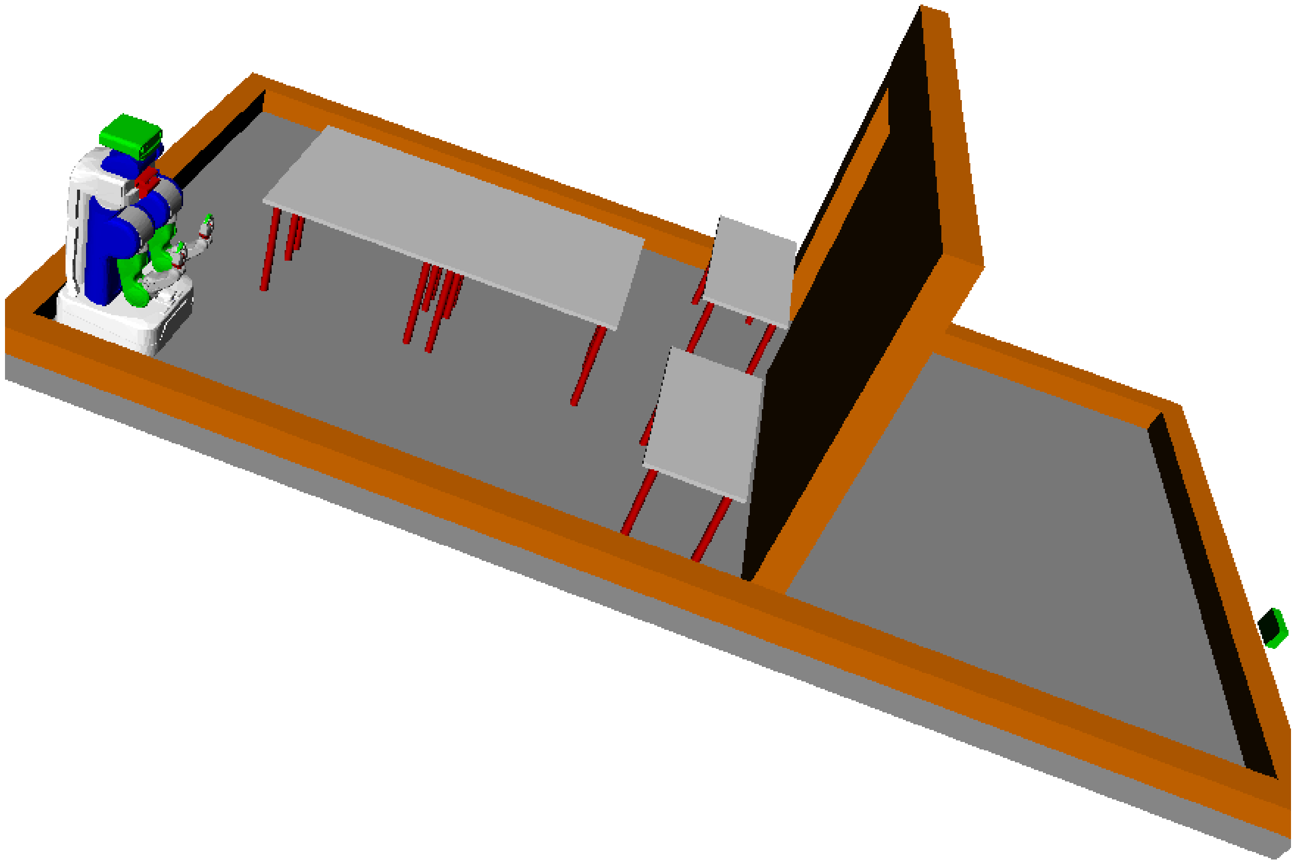
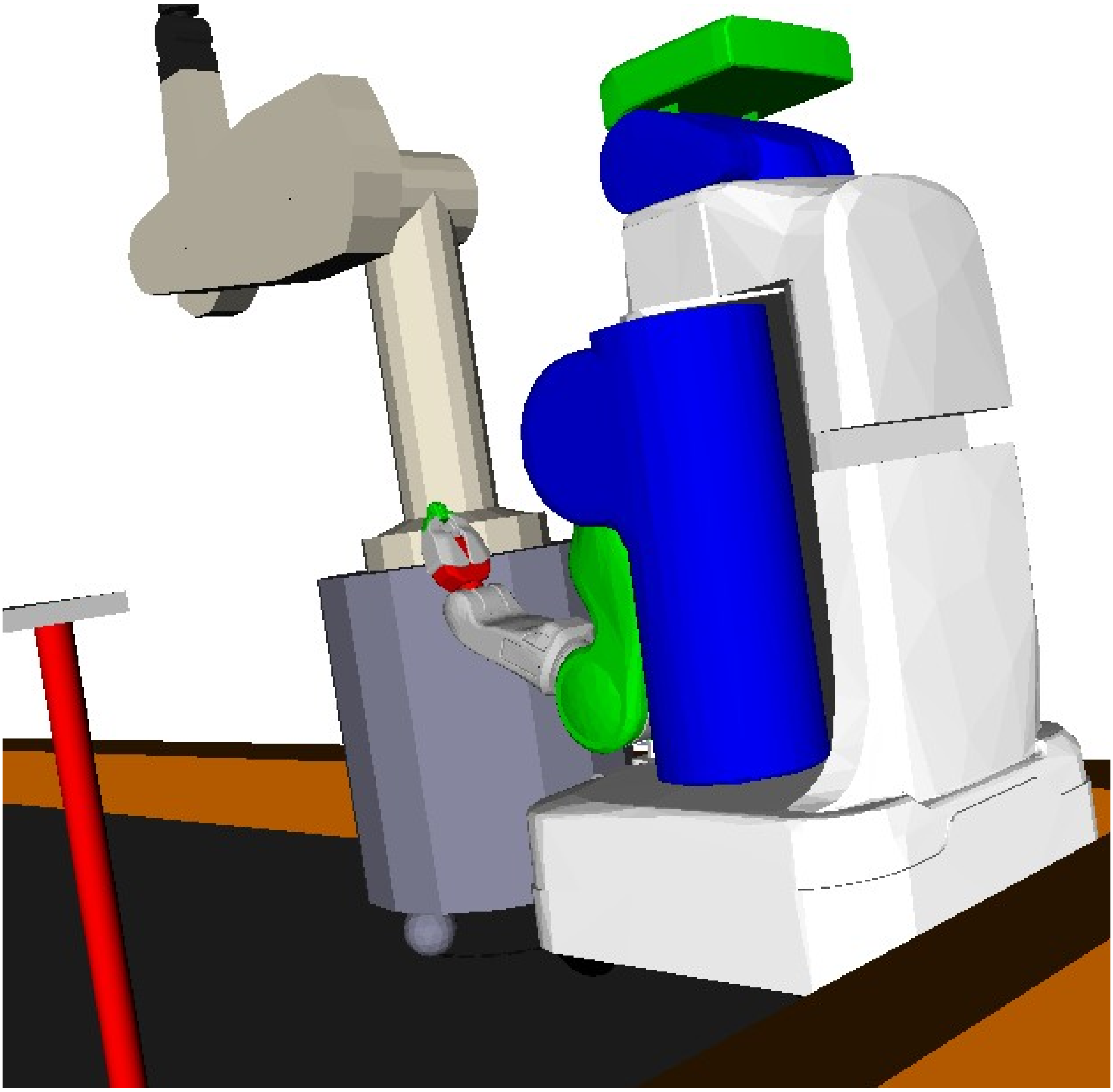
Aditya Gupta
RBE 550
HW1

1)

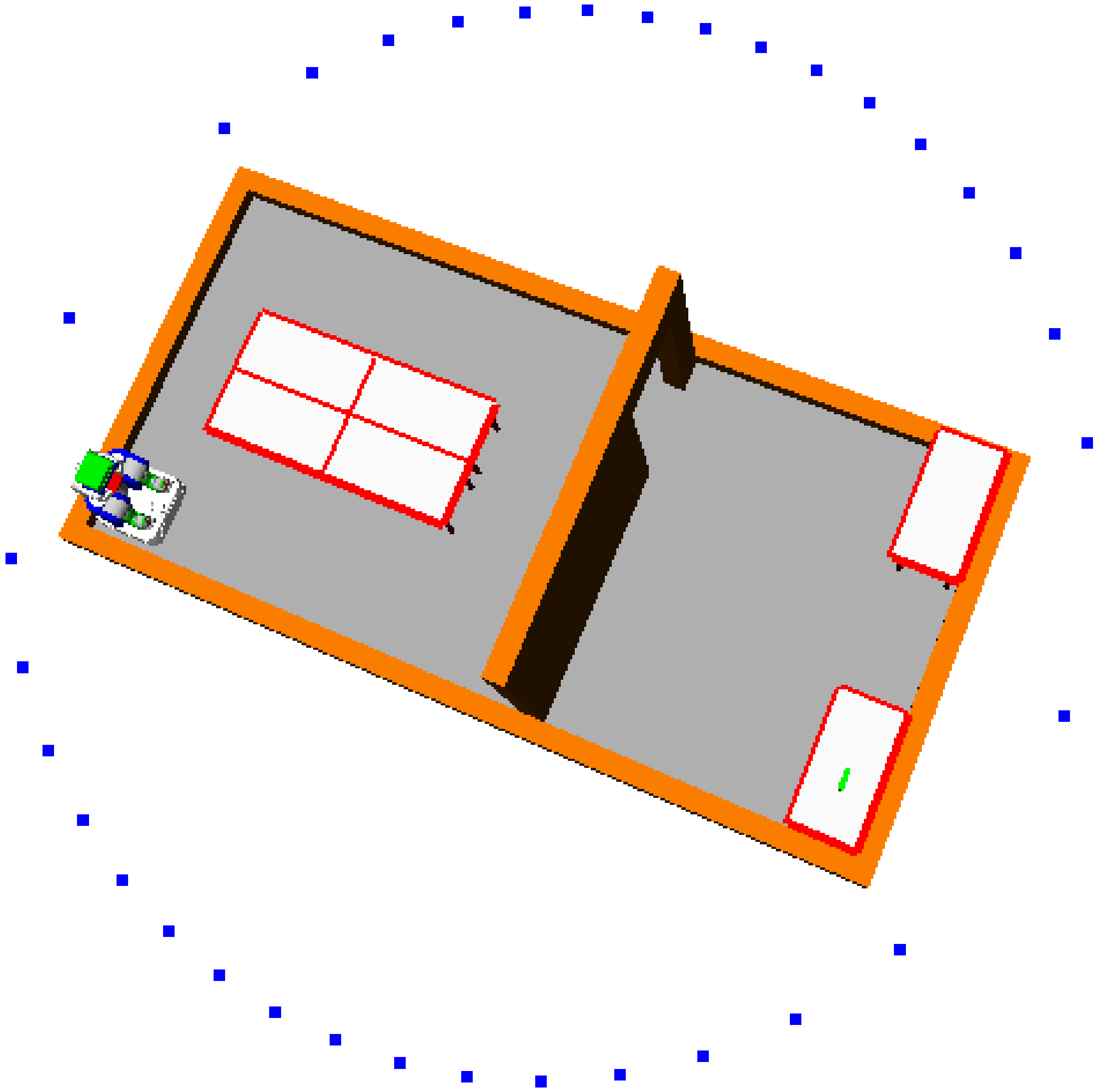


)

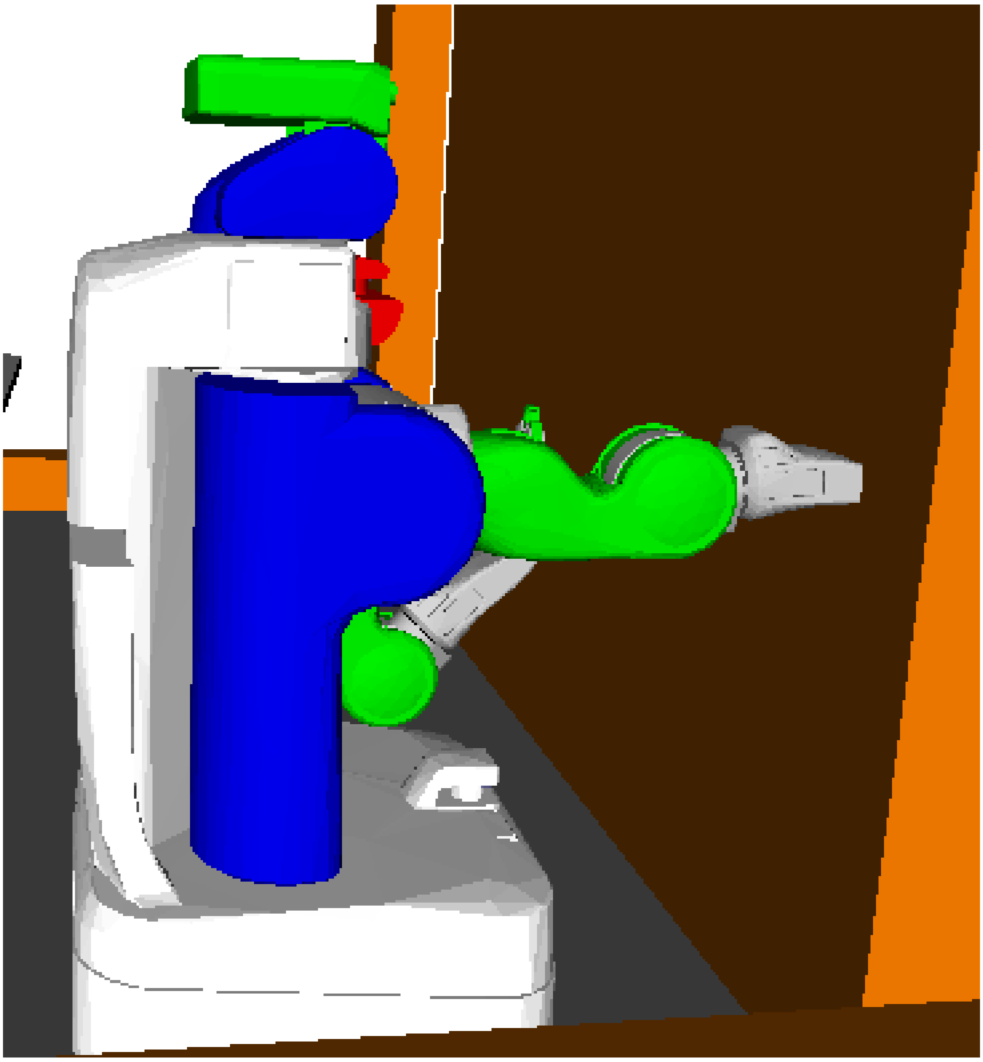
2



3)



4)



We will keep “waitrobot()” outside “with env:” because if kept inside, the position of robot does not change. This happens beacuse it will aways make itslef true if environment is true.

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Q1 Generally a rigid body in C-Space having properties of translation and rotation will have $R^3 \times RP^3$ configration.

However, given the condition, there will be a contraint of Degree of Freedom (DOF), so
$$C\text{space} = R^3 \times RP^2$$
$$\text{Dimension} = 5$$

Q2 Translation + Rotation in 2D space hence a the following C-space configration : $R^2 S^1$
Since top and botton of screen are further identified R^2 becomes Torus T^2
Hence, now the Cspace of the spacecraft becomes $T^2 S^1$.

Q3 a) we will obtain another Mobius Bend.

b) No, because holes doesn't have homeomorphic neibhourhood.

(A.) Unit Quaternion h_1 : $\frac{\pi}{2}$ rotation around $\left[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right]$

$$h = \cos\frac{\theta}{2} + (u_x i + u_y j + u_z k)\sin\frac{\theta}{2}$$

{ Standard form of quaternion
θ = rotation of $\angle\theta$ about axis unit vector $\vec{u} = (u_x, u_y, u_z)$. }

$$\therefore h_1 = \cos\left(\frac{\pi}{4}\right) + \left[\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right] \sin\left[\frac{\pi}{\sqrt{3}}\right]$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$$

(B.) $\theta = \pi$ $\quad u = [0, 1, 0]$

$$\therefore h_2 = \cos\left(\frac{\pi}{2}\right) + \left[0.\hat{i} + 1.\hat{j} + 0.\hat{k}\right]\sin\left(\frac{\pi}{2}\right)$$

$$\boxed{h_2 = \hat{j}}$$

(c) Combination of quaternion from h_1 and h_2 in terms of quaternion.

$$h_3 = h_2 h_1$$

$$= (\hat{j})\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\left(\hat{i} + \hat{j} + \hat{k}\right)\right)$$

$$= -\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$$

We know,

$$\theta = 2\cos^{-1}(h_w)$$

$$U_x = \frac{h_x}{\sqrt{1-h_x^2}} \quad U_y = \frac{h_y}{\sqrt{1-h_y^2}} \quad U_z = \frac{h_z}{\sqrt{1-h_z^2}}$$

$$\theta = 2\cos^{-1}(1/\sqrt{6}) = 1.27\pi = 3.983 \text{ radians}$$

$$U_x = 1/\sqrt{5}$$

$$U_y = \sqrt{3/5}$$

$$U_z = -1/\sqrt{5}$$

$$\therefore \theta = 1.27\pi \qquad \text{Axis is} \equiv \left[\frac{1}{\sqrt{5}}, \sqrt{\frac{3}{5}}, -\sqrt{\frac{1}{5}}\right]$$

(4) For single rigid body in 3D space with translation and rotational properties
$R^3 \times PR^3$.

For 6 <u>un attached</u> bodies:

$(R^3 \times RP^3)(R^3 \times RP^3) \ldots$ 6 times each for 1 body

$= R^{18} \times RP^3 \times RP^3 \times RP^3 \times RP^3 \times RP^3 \times RP^3$

Dimension will be $\underline{\underline{36}}$