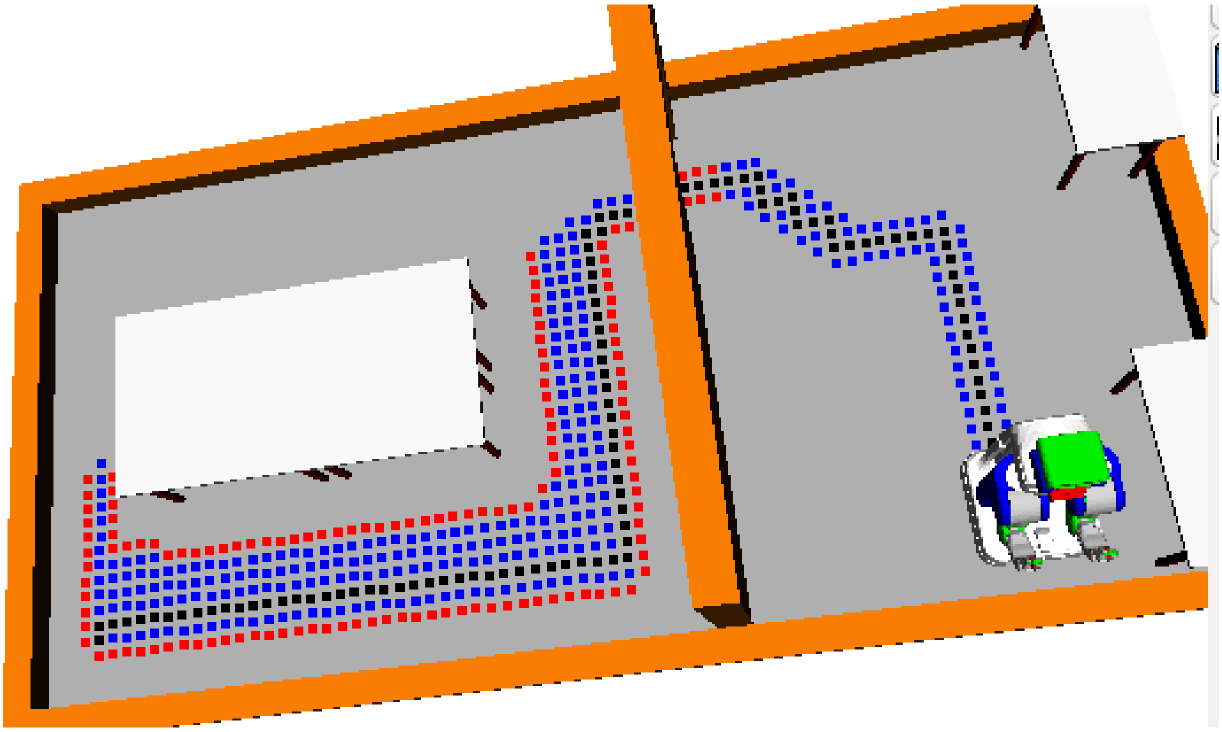
Aditya Gupta
RBE 550

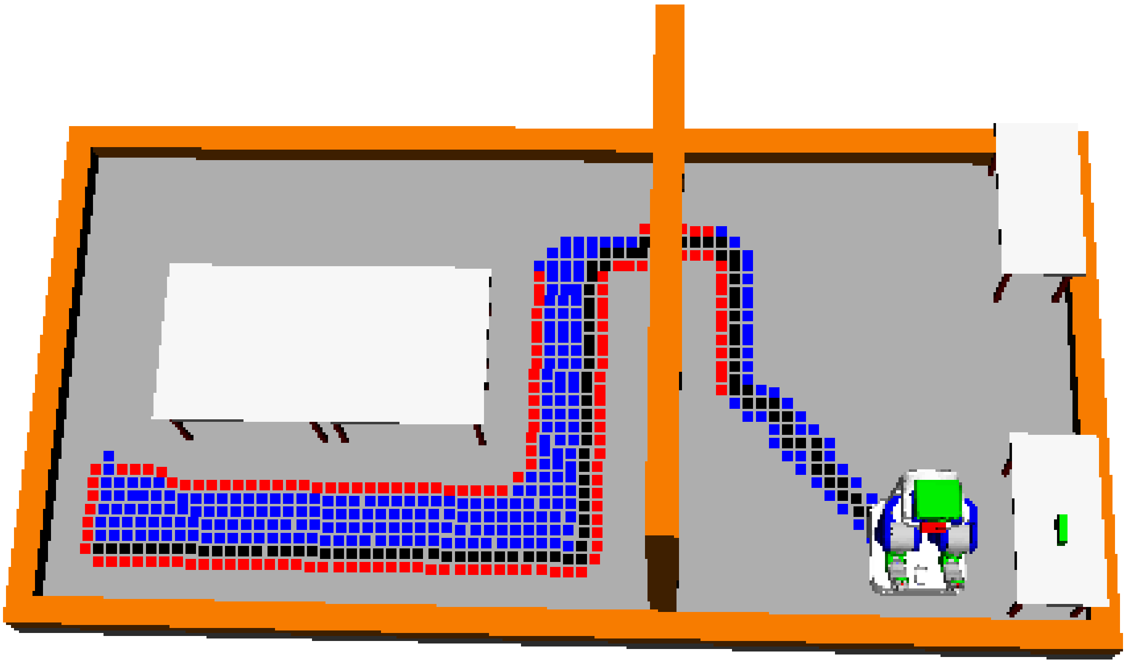
SoftwarePart.

(a) “4-connected” space, with the manhattan distance heuristic



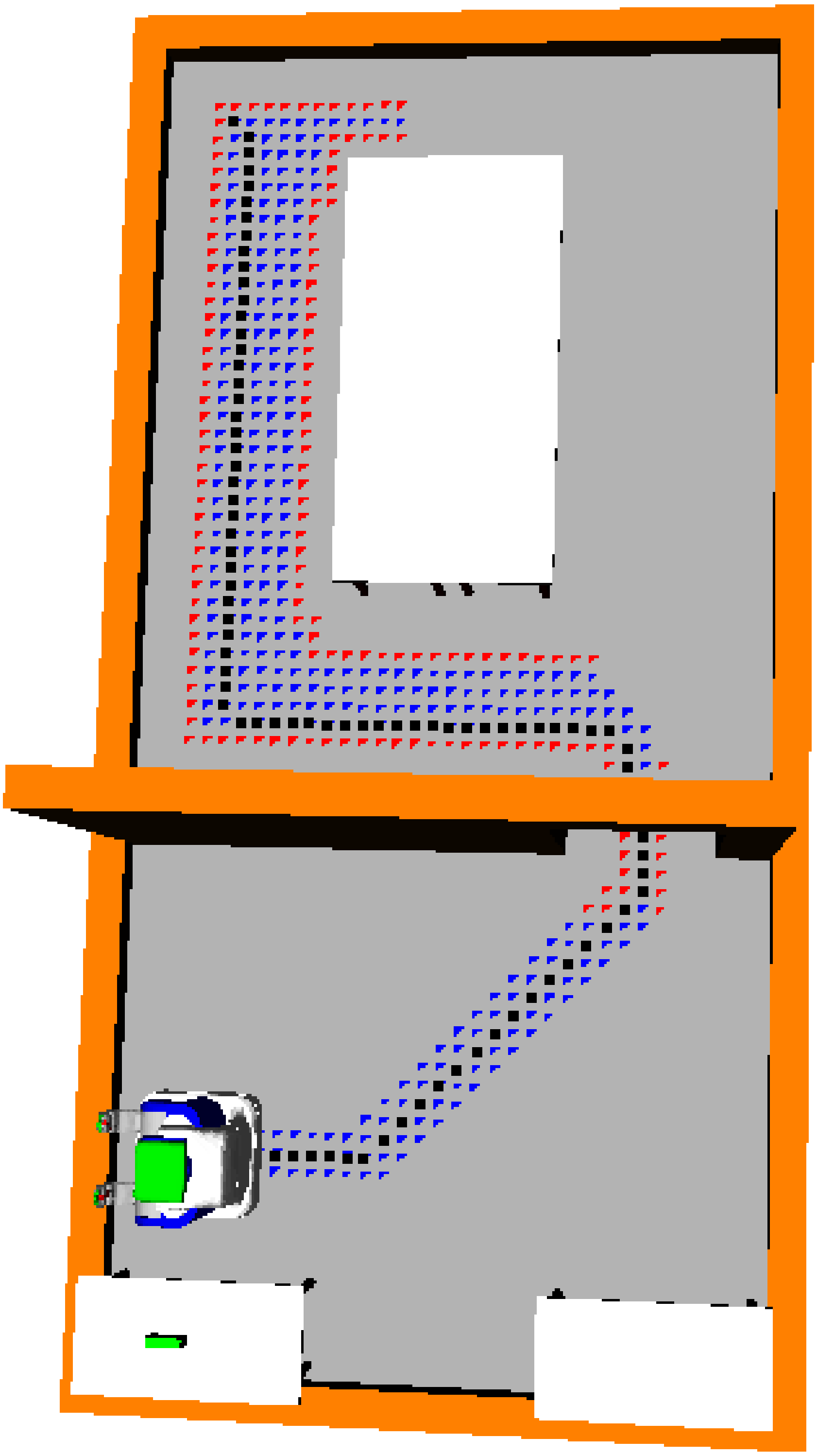
Nodes:828

(b) “4-connected” space, with the euclidian distance heuristic



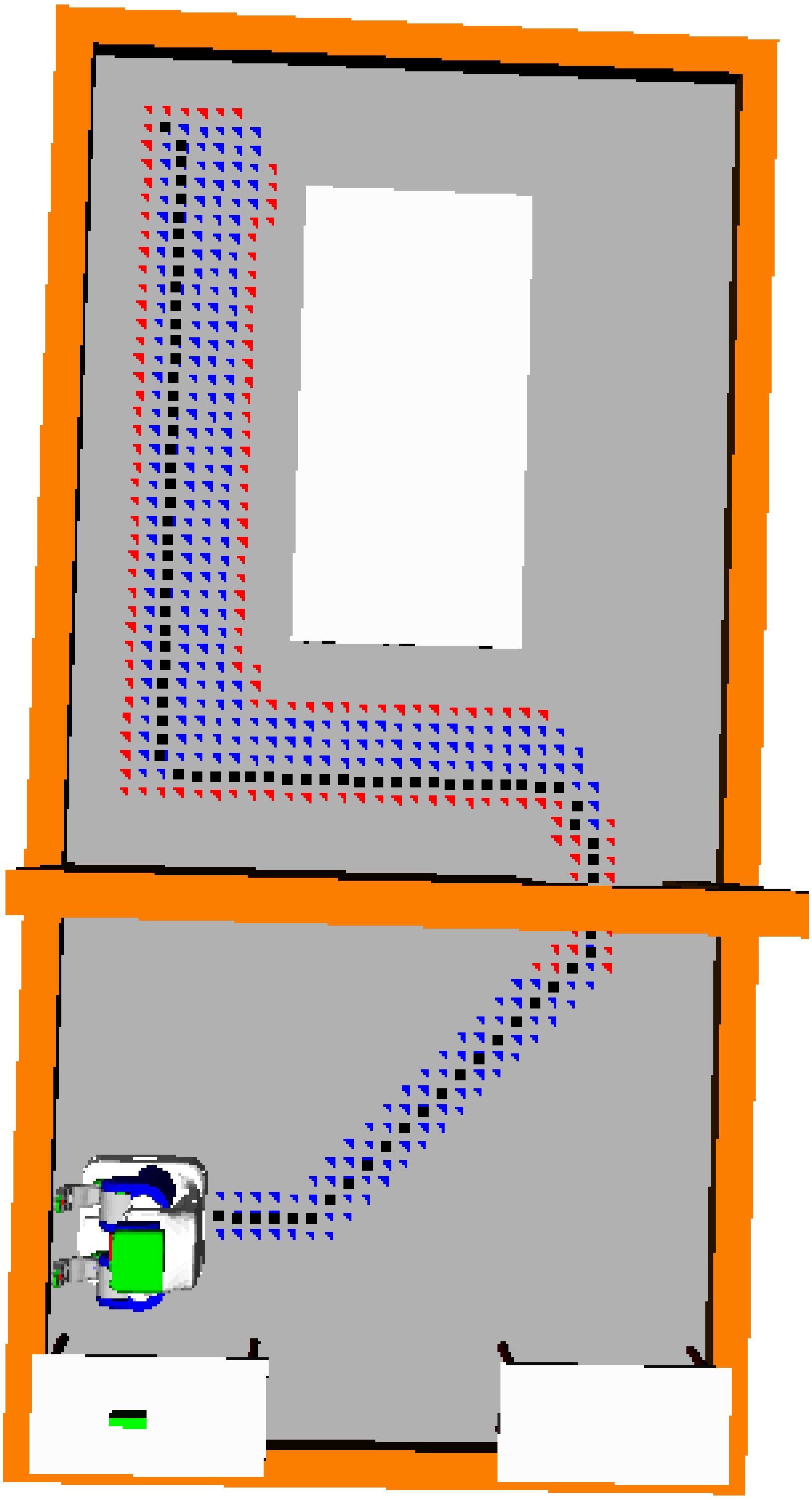
Nodes:741

(c) “8-connected” space, with the manhattan distance heuristic



Nodes: 1257

(d) “8-connected” space, with the euclidian distance heuristic



Nodes:1160

1.1) Manhatten is better as it has low running time.

1.2) Eucleadean is better as it searches less nodes.

1.2)It is not admissible in Manhatten for 8 conntected as it over-estimate the cost og goal. For all the rest configration it is admissible.

ADITYA GUPTA

HW2

RBE 550

(1) We are given,

$$R(\alpha, \beta, \gamma) = R(\alpha', \beta', \gamma')$$

s.t. at least $\alpha \neq \alpha'$, $\beta \neq \beta'$ or $\gamma = \gamma'$

For $R(\alpha, \beta, \gamma)$, Rotation Matrix is defined as

$$= \begin{bmatrix} \cos\alpha \cos\beta & \cos\gamma \sin\alpha \sin\beta - \cos\alpha \sin\gamma & \cos\alpha \cos\gamma \sin\beta + \sin\alpha \sin\gamma \\ \sin\alpha \cos\beta & \sin\alpha \sin\beta \sin\gamma + \cos\alpha \cos\gamma & \sin\beta \sin\gamma \cos\alpha - \cos\gamma \sin\alpha \\ -\sin\beta & \cos\beta \sin\gamma & \cos\alpha \cos\beta \end{bmatrix}$$

similarly for $R(\alpha', \beta', \gamma')$

we can define Rotation Matrix for angles $\equiv (\alpha', \beta', \gamma')$

On comparing both matrices, we can see

$$-\sin\beta = -\sin\beta'$$

$$\therefore \sin\beta = \sin\beta'$$

$$\Rightarrow \beta' = (2n+1)\pi - \beta$$

similarly,

$$\cos\alpha . \cos\beta = \cos\alpha' - \cos\beta'$$

$$\cos\beta . \sin\alpha = \cos\beta' . \sin\alpha'$$

$$\Rightarrow \cot\alpha = \cot\alpha'$$

$$\therefore \alpha' = \alpha - (2n+1)\pi$$

Similarly, for ~~axis~~ '$\gamma$'

$$\cos\beta \sin\gamma = \cos\beta' \sin\gamma'$$

$$\Rightarrow \cos\beta \sin\gamma = \cos(\pi-\beta)\sin\gamma'$$

$$\Rightarrow \cos\beta \sin\gamma = -\cos\beta \cdot \sin\gamma'$$

$$\Rightarrow \sin\gamma = -\sin\gamma'$$

$$\Rightarrow \gamma' = \gamma - (2n+1)\pi$$

(2) For 2D Rotation Matrix.

Let $R_1$ is 2D Matrix for $\theta_1$

" $R_2$ " " " " $\theta_2$

$$\therefore R_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}$$

$$R_1 R_2 = \begin{bmatrix} c_1c_2 - s_1s_2 & -c_1s_2 - s_1c_2 \\ s_1c_2 + c_1s_2 & -s_1s_2 + c_1c_2 \end{bmatrix}$$

$$= \begin{bmatrix} c(\theta_1+\theta_2) & -s(\theta_1+\theta_2) \\ s(\theta_1+\theta_2) & c(\theta_1+\theta_2) \end{bmatrix} \text{ (A)}$$

$$R_2R_1 = \begin{bmatrix} c_1c_2 - s_1s_2 & -c_2s_1 - c_1s_2 \\ c_1s_2 + s_1c_2 & -s_1s_2 + c_1c_2 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{bmatrix} \quad \text{— (B)}$$

$c\theta_1 = \cos\theta_1 \qquad c\theta_2 = \cos\theta_2$

$s\theta_1 = \sin\theta_1 \qquad s\theta_2 = \sin\theta_2$

We can see,

$A = B$

$\Rightarrow R_1R_2 = R_2R_1$

Hence 2D Rotation Matrix is commutative.

## For 3D Rotation:

Lets take 3D Rota. Matix about $x$ axis as $R_1$, $y$ axis as $R_2$

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_1 & -s_1 \\ 0 & s_1 & c_1 \end{bmatrix} \qquad R_2 = \begin{bmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{bmatrix}$$

$$R_1R_2 = \begin{bmatrix} c_2 & 0 & s_2 \\ s_1s_2 & c_1 & -s_1c_2 \\ -c_1s_2 & s_1 & c_1c_2 \end{bmatrix} \quad \text{— (1)}$$

$$R_2 R_1 = \begin{bmatrix} c_2 & s_1 s_2 & c_1 s_2 \\ 0 & c_1 & -s_1 \\ -s_2 & s_1 c_2 & c_1 c_2 \end{bmatrix} \quad (2)$$

As we can see

$$(1) \neq (2)$$

Hence $R_1 \neq R_2$

Hence not commutative in this case.

(3) Use Homogenous Transformation Matrix for

$$(\theta_1, \theta_2, \theta_3) = \left(\frac{\pi}{4}, \frac{\pi}{2}, \frac{\pi}{4}\right)$$

$T_0^a$ : Transform Matrix betn Origin and 'a'

$T_a^b$ : " " " 'a' and 'b'

$T_b^c$ : " " " 'b' and 'c'

$T_0^b$ : " " " 'b' and origin

$T_0^c$ : " " " 'c' and origin

A) $T_0^{\,a} = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 5\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 5\sin\theta_1 \\ 0 & 0 & 1 \end{bmatrix}$

Position of point a $\equiv (5\cos\theta_1 , 5\sin\theta_1)$

$T_0^{\,b} = T_0^{\,a} \times T_a^{\,b}$

$$= \begin{bmatrix} C\theta_1 & -S\theta_1 & 10\cos\theta_1 \\ S\theta_1 & C\theta_1 & 10\sin\theta_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 11C\theta_2 \\ S\theta_2 & C\theta_2 & 11S\theta_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{12} & -S_{12} & 11C_1C_2 - 11S_1S_2 + 10C_1 \\ S_{12} & C_{12} & 11S_1C_2 + 11C_1S_2 + 10S_1 \\ 0 & 0 & 1 \end{bmatrix}$$

Pos$^n$ of pt b $\equiv \left(11\cos(\theta_1+\theta_2) + 10C_1 ,\ 11S(\theta_1+\theta_2) + 10S_1\right)$

Assumption: $C_i = \cos(\theta_i)$ || $S_i = \sin(\theta_i)$ ||

We know $\theta_1 = \pi/4$ $\theta_2 = \pi/2$ $\theta_3 = -\pi/4$

substituting the values in

$T_0^c = T_0^a \, T_a^b \, T_b^c$ ; Note that $T_a^b$ will have differt component.

$$T_0^c = \begin{bmatrix} c_1 & -s_1 & 10c_1 \\ s_1 & c_1 & 10s_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 10c_2 \\ s_2 & c_2 & 10s_2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} c_3 & -s_3 & \sqrt{122}\, c(\theta_3 + \theta) \\ s_3 & c_3 & \sqrt{122}\, s(\theta_3 + \theta) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\theta = \tan^{-1}\left(\frac{1}{11}\right)$$

Notice that in $T_a^b$, $T_a^b(1,3)$ and $(2,3)$ have different coefficent representing length of link.

$$= \begin{bmatrix} c_{123\theta} & -s_{123\theta} & \sqrt{122}\, c_{123\theta} + 10c_{12} + 10c_1 \\ s_{123\theta} & c_{123\theta} & \sqrt{122}\, s_{123\theta} + 10s_{12} + 10s_1 \\ 0 & 0 & 1 \end{bmatrix}$$

Putting values of $\theta_1, \theta_2\, \theta_3$

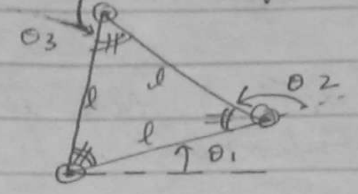
$a = (5/\sqrt{2}, 5/\sqrt{2})$

$b = (-1/\sqrt{2}, 21/\sqrt{2})$

$c = (-1, 11 + 20/\sqrt{2})$

3b.

Required Configration



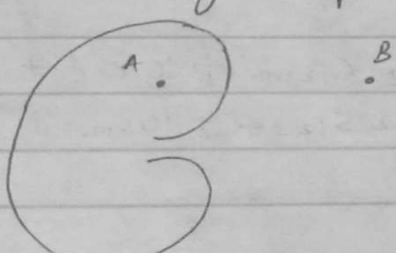

As all sides are having some length, the internal triangle will form equilateral $\Delta$.

$\therefore$ $\theta_2$ and $\theta_3$ will have $\frac{2\pi}{3}$ rad as this measurment.

and

$\theta_1$ can take any value with in its range.

(4) Bug Trap eg:-




A, B possible configration of Goal and start (interchangable)

Difficult for unidirectional planner to compute a path because, at some space the possibilities is reduced very much. and if step size is beyond that limit it might act as trap ≡ 'bug trap'. And ~~bug~~ robot may never find a solution, hence reffered as bug trap

5)  Approximate solutions are computaionally less expansive than exact solution, a paramter 'resolution parameter' is required for its computation. The basic idea is to approximate each path segment  by inserting intermediate vertices along long line segments. The vertices are added whenever a new sample, is added and a condition is ensured that no two consecutive vertices are ever further than 'resolution paramter' defined earlier.
Reducind the paramter will lead to better search but with added running time, one should decide upon number of dimensions, its not reccomoned to use it beyond n=20.
A good way to select 50 neighbors for a grid in R^10, can be implimented using KD-tree. Where, KD tree will select 50 nearest neibhour around the grid.

Reference: Planning Algorithms (Steven M LaValle)

6) Searching a high resolution grid can lead our algorithm to get trapped in local minima, in addition to un-recovorable state due to high dimnesionality of space. It would also search around local minima which would increase the runtime for the algorithm. High computational cost is another drawback of this technique. On the other hand if trees are grown in C-space, it is less likely to get trapped in local minima ultimately improving space and time complexity. But still bug trap is still a problem in this case.