Project 1
Robot Localization and Navigation

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Introduction

In this project, we were supposed to implement an extended Kalman filter for state estimation using Vicon and IMU data. We used simulated random noise for the system to mimic real world noise.

Part 1

Process Model

The state x is as follows.

- p is the position
- q is the orientation
- The third element is pdot, which is the linear velocity
- Bg is the gyroscope bias
- Ba is the accelerometer bias

$$x = \begin{bmatrix} p \\ q \\ \dot{p} \\ b_g \\ b_a \end{bmatrix}$$

The process model xdot is as follows.

$$\dot{x} = f(x, u, n) = \begin{bmatrix} \dot{p} \\ G(q)^{-1}R(q)(w_m - b_g - n_g) \\ g + R(q)(a_m - b_a - n_a) \\ n_{bg} \\ n_{ba} \end{bmatrix}$$

The orientation q is as follows. X, y, z are the roll, pitch and yaw angles.

$$q = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$G(q) = \begin{bmatrix} \cos(y)\cos(z) & -\sin(z) & 0 \\ \sin(z)\cos(y) & \cos(z) & 0 \\ -\sin(y) & 0 & 1 \end{bmatrix}$$

R is obtained using the euler angle representation of rotation matrices.

$$R(q) = R_z(z) \cdot R_y(y) \cdot R_x(x)$$

Measurement Model

The measurement for part 1 is the position (px, py, pz) and orientation (roll, pitch, yaw – x, y, z) from the Vicon system.

Z_t = C_t * x + W_t * v

Z_t = [p q] ^ T

v ~ N (0, R) – measurement noise

Final Equations

Prediction step:

- $\bar{\mu}_t = \mu_{t-1} + \delta t\, f(\mu_{t-1}, u_t, 0)$
- $\bar{\Sigma}_t = F_t\, \Sigma_{t-1}\, F_t^T + V_t\, Q_d\, V_t^T$

Assumptions:

- $\dot{x} = f(x, u, n)$
- $n \sim N(0, Q)$

Linearization:

- $A_t = \left.\frac{\partial f}{\partial x}\right|_{\mu_{t-1}, u_t, 0}$
- $U_t = \left.\frac{\partial f}{\partial n}\right|_{\mu_{t-1}, u_t, 0}$

Discretization:

- $F_t = I + \delta t\, A_t$
- $V_t = U_t$
- $Q_d = Q\, \delta t$

Update step:

- $\mu_t = \bar{\mu}_t + K_t\, (z_t - g(\bar{\mu}_t, 0))$
- $\Sigma_t = \bar{\Sigma}_t - K_t\, C_t\, \bar{\Sigma}_t$
- $K_t = \bar{\Sigma}_t\, C_t^T\, (C_t\, \bar{\Sigma}_t\, C_t^T + W_t\, R\, W_t^T)^{-1}$

Assumptions:

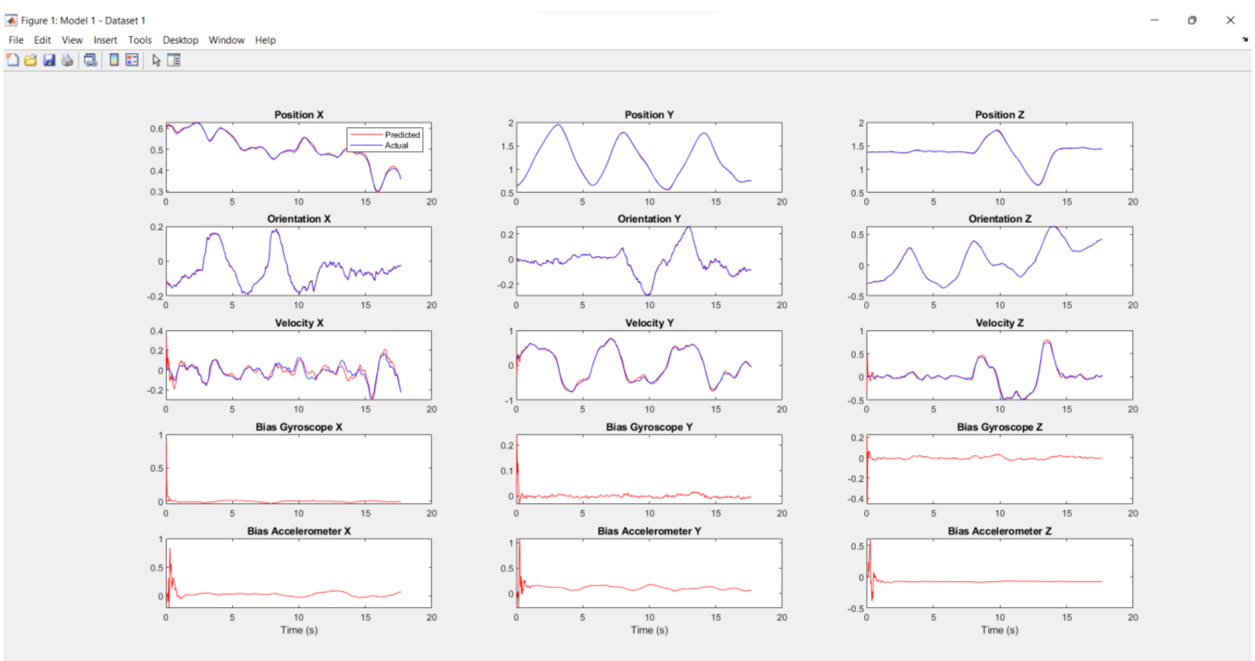
- $z = g(x, v)$
- $v \sim N(0, R)$

Linearization:

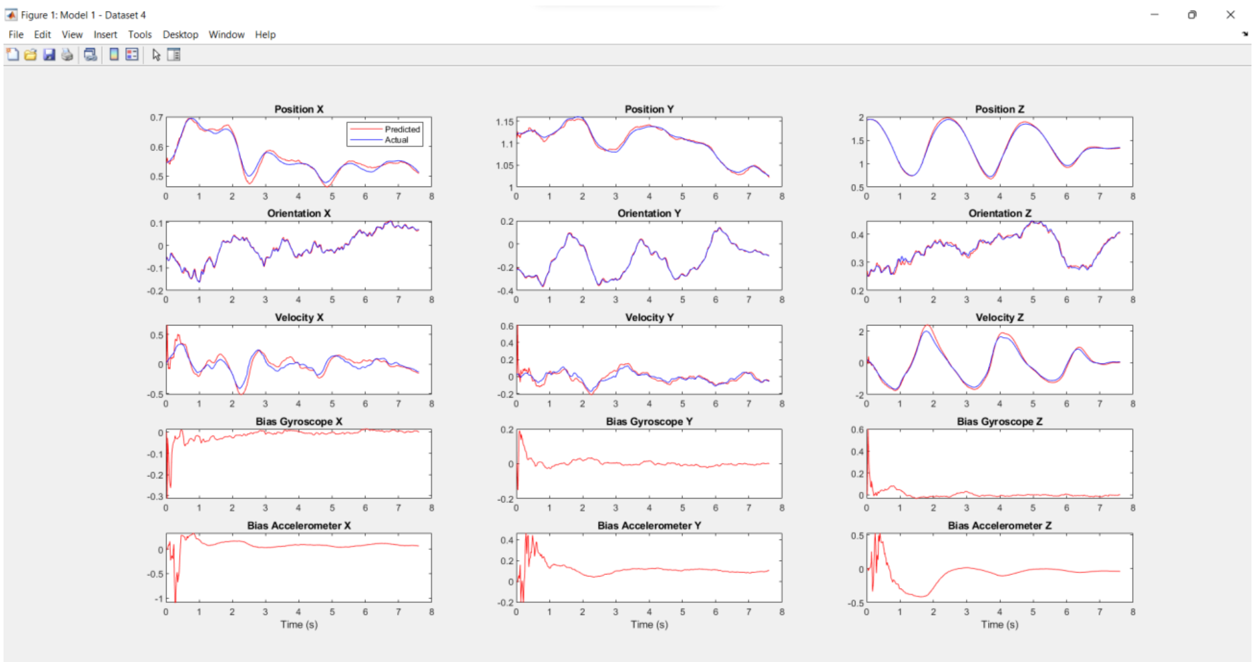
- $C_t = \left.\frac{\partial g}{\partial x}\right|_{\bar{\mu}_t, 0}$
- $W_t = \left.\frac{\partial g}{\partial v}\right|_{\bar{\mu}_t, 0}$

Q_d = 0.00015 * eye(12). It's a 12x12 matrix of covariances of process model noise n. n is 12x1 noise vector with mean zero and covariance Q_d.

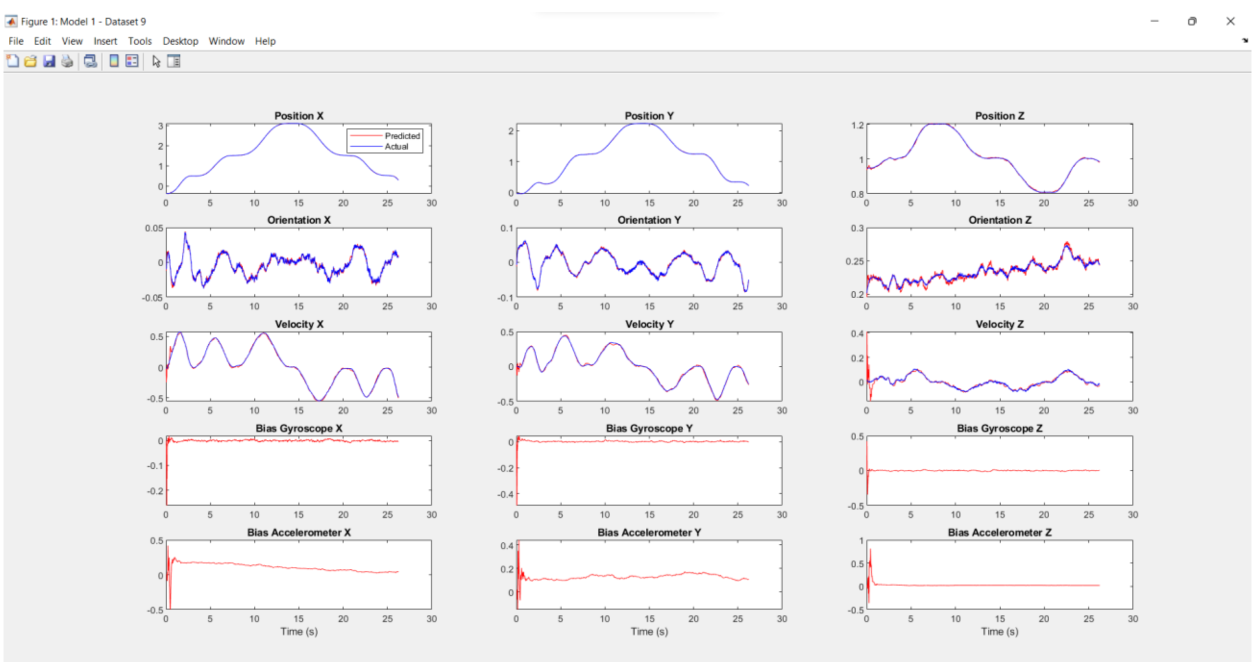
Results for Dataset 1



Results for Dataset 4



Results for Dataset 9



Part 2

Process Model

The process model is same as part 1.

Measurement Model

The measurement for part 2 is the linear velocity from the vicon.

Z_t = C_t * x + W_t * v

Z_t = [pdot] ^ T

v ~ N (0, R) – measurement noise

Final Equations

Prediction step:

- $\bar{\mu}_t = \mu_{t-1} + \delta t\, f(\mu_{t-1}, u_t, 0)$
- $\bar{\Sigma}_t = F_t\, \Sigma_{t-1}\, F_t^T + V_t\, Q_d\, V_t^T$

$$\left.\begin{array}{l} \dot{x} = f(x, u, n) \\ n \sim N(0, Q) \end{array}\right\} \text{Assumptions}$$

$$\left.\begin{array}{l} A_t = \left.\frac{\partial f}{\partial x}\right|_{\mu_{t-1}, u_t, 0} \\ U_t = \left.\frac{\partial f}{\partial n}\right|_{\mu_{t-1}, u_t, 0} \end{array}\right\} \text{Linearization}$$

$$\left.\begin{array}{l} F_t = I + \delta t\, A_t \\ V_t = U_t \\ Q_d = Q\, \delta t \end{array}\right\} \text{Discretization}$$

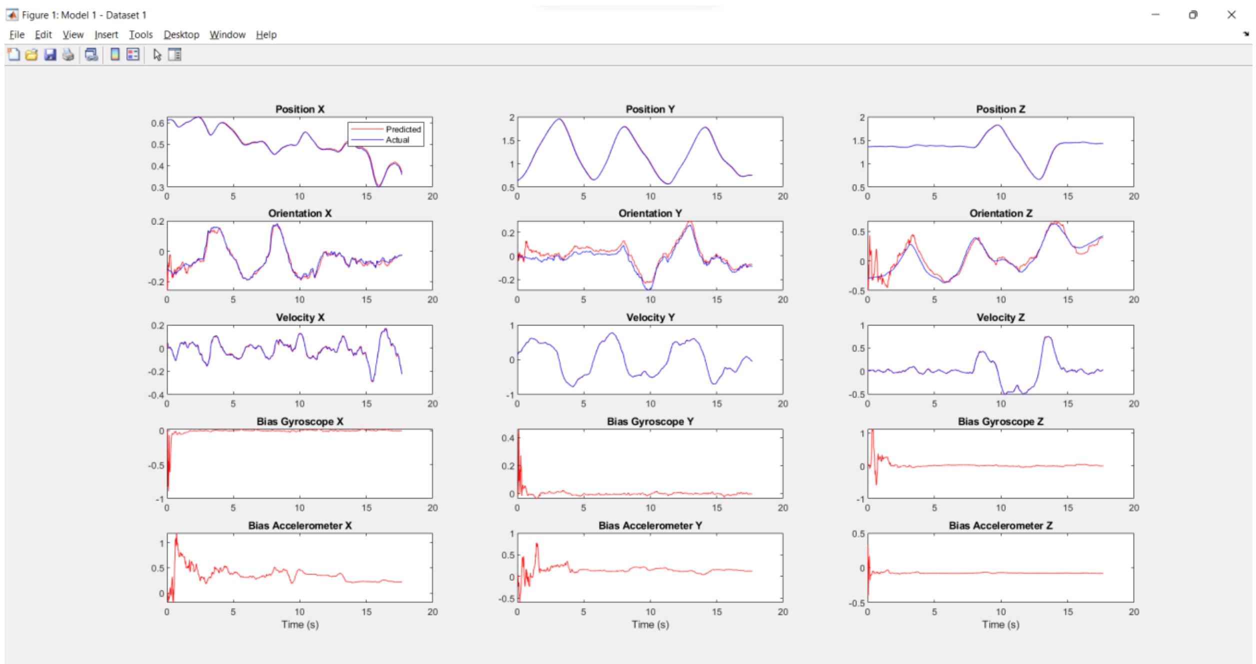
Update step:

- $\mu_t = \bar{\mu}_t + K_t\, (z_t - g(\bar{\mu}_t, 0))$
- $\Sigma_t = \bar{\Sigma}_t - K_t\, C_t\, \bar{\Sigma}_t$
- $K_t = \bar{\Sigma}_t\, C_t^T\, (C_t\, \bar{\Sigma}_t\, C_t^T + W_t\, R\, W_t^T)^{-1}$

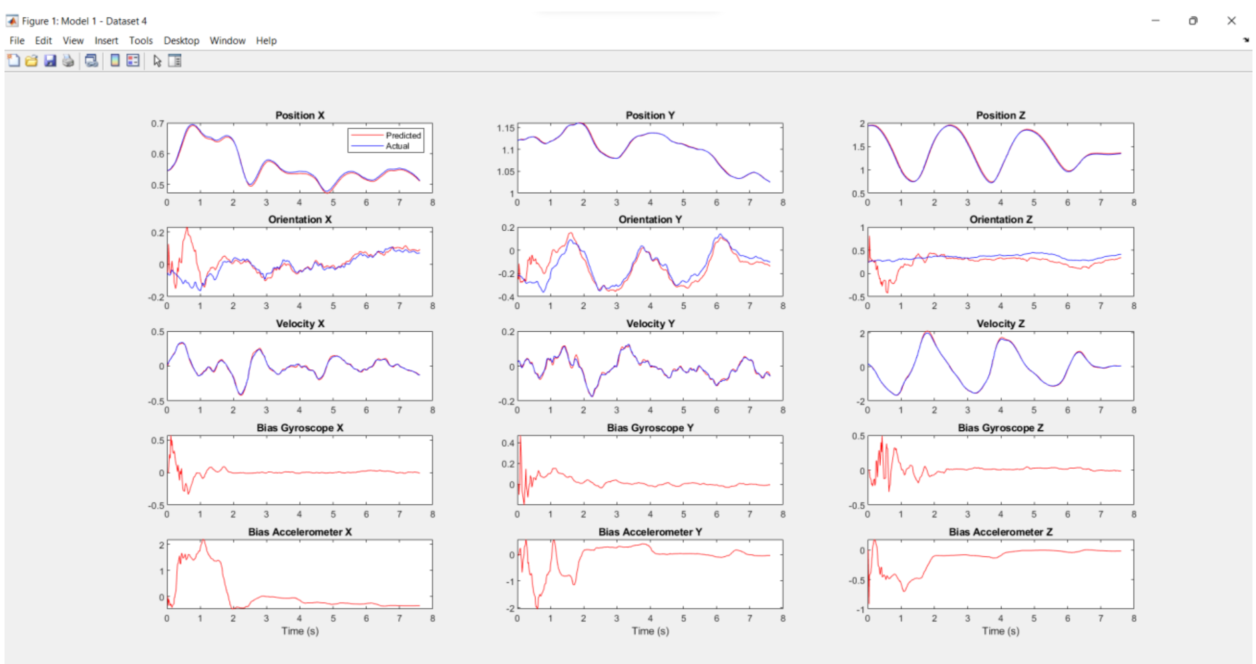
$$\left.\begin{array}{l} z = g(x, v) \\ v \sim N(0, R) \end{array}\right\} \text{Assumptions}$$

$$\left.\begin{array}{l} C_t = \left.\frac{\partial g}{\partial x}\right|_{\bar{\mu}_t, 0} \\ W_t = \left.\frac{\partial g}{\partial v}\right|_{\bar{\mu}_t, 0} \end{array}\right\} \text{Linearization}$$

Results for Dataset 1



Results for Dataset 4



Results for Dataset 9

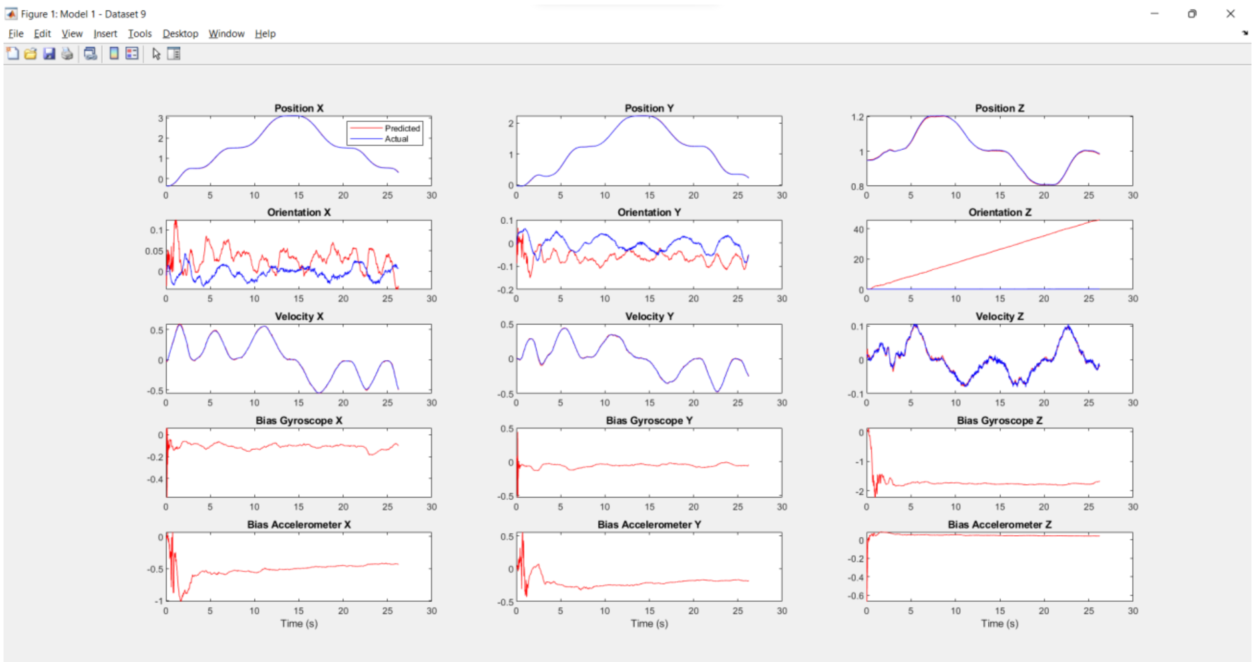


Figure 1: Model 1 - Dataset 9
File Edit View Insert Tools Desktop Window Help
Position X
Predicted
Actual
Position Y
Position Z
Orientation X
Orientation Y
Orientation Z
Velocity X
Velocity Y
Velocity Z
Bias Gyroscope X
Bias Gyroscope Y
Bias Gyroscope Z
Bias Accelerometer X
Bias Accelerometer Y
Bias Accelerometer Z
Time (s)
Time (s)
Time (s)