

3.10 Stochastic volatility models for financial time series

Model description Stochastic volatility models are used in mathematical finance to describe the evolution of asset returns, which typically exhibit changing variances over time. As an illustration we use a time series of daily pound/dollar exchange rates $\{z_t\}$ from the period 01/10/81 to 28/6/85, previously analyzed by Harvey, Ruiz & Shephard (1994). The series of interest are the daily mean-corrected returns $\{y_t\}$, given by the transformation

$$y_t = \log z_t - \log z_{t-1} - n^{-1} \sum_{i=1}^n (\log z_t - \log z_{t-1}).$$

The stochastic volatility model allows the variance of y_t to vary smoothly with time. This is achieved by assuming that $y_t \sim N(\mu, \sigma_t^2)$, where $\sigma_t^2 = \exp(\mu_x + x_t)$. The smoothly varying component x_t follows the autoregression

$$x_t = \beta x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2).$$

The vector of hyper-parameters for this model is thus $(\beta, \sigma, \mu, \mu_x)$.