Midterm revision

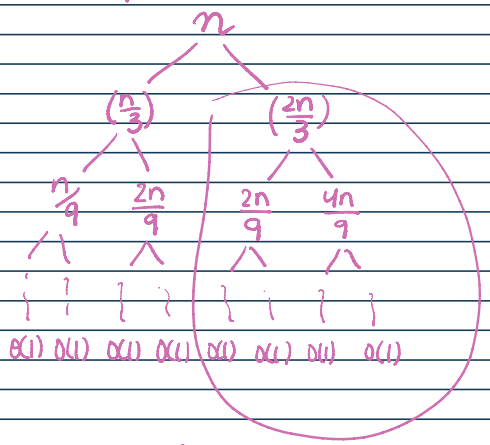
Thursday, November 3, 2022 12:37 PM

Use the recursion tree method to get an upperbound for the following

$T(n) = T(\frac{n}{3}) + T(\frac{2n}{3}) + \Theta(n)$

2 recursive parts

Solution



longer than the other branch

$(\frac{2}{3})^k n = 1$

$(\frac{2}{3})^h = \frac{1}{n}$

$n = (\frac{3}{2})^h$

$\log_{3/2}(n) = h$

$\sum_{i=0}^{\log_{3/2} n} n$ ← constant series

$= n(\log_{3/2}(n) + 1)$

$= O(n \log_{3/2}(n))$

maximum height will be from the $\frac{2n}{3}$ branch

Consider the following recurrence

$T(n) = T(n/2) + 5^{\lfloor (\log_5 n) \rfloor}$

$T(1) = \Theta(1)$

Solve using master theorem.

Cost of leaves $= n^{\log_2 1} = c$

Cost of root $= 5^{\lfloor \log_5 n \rfloor} =$

n is a power of 5 → Cost $= 5^{\log_5(n)} = n$

n is not a power of 5 → Cost $= 5^{(\log_5(n) + (\log_5(n) \% 1))} = 5^{\log_5(n)} \cdot 5^{-d}$

Note: any no. % 1 → gets decimal part

$0 < d < 1$

substitute by d to get lower & higher bound

$0.2 < 5^{-d} < 1$

root $= en$

where $0.2 < e < 1$

3rd Case ? → Prove: ① $en \geq n^{0+\varepsilon}$ $\varepsilon = 0.1$ ✓

② Regularity → $5^{\lfloor \log_5 \frac{n}{2} \rfloor} \leq C\, 5^{\lfloor \log_5 n \rfloor}$

$n = 2m$

$5^{\lfloor \log_5 m \rfloor} \leq C\, 5^{\lfloor \log_5 (2m) \rfloor}$

m is power of 5 ↙ ↘ m not power of 5

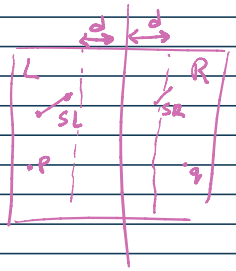
$5^{\log_5 m} \leq C 5^{\lfloor \log_5 (2) + \log_5 m \rfloor}$ (.5 omitted for flooring)

$5^{\log_5 m} \leqslant C\, 5^{\log_5 (m)}$

$1 \leq C$ → master theorem will not work

you are given an array of n points in a 2D plane.
Design a divide and conquer algorithm to find out the closest pair of points in the array.

* Split plane in 2 halves



$\min(S_L, S_R) = d$