Covariance derivatives

Rational quadratic

In[158]:=
```
RQ[s_, t_] := α^2 * (1 + (s - t)^2 / (2 * ν * ρ^2))^-ν
```

In[312]:=
```
RQ[s, t]
D[RQ[s, t], t] == RQ[s, t] * (2 * (s - t) * ν) / ((s - t)^2 + 2 * ν * ρ^2) // FullSimplify
D[RQ[s, t], {t, 2}] == RQ[s, t] * (2 * ν ((s - t)^2 (1 + 2 ν) - 2 * ν * ρ^2)) / ((s - t)^2 + 2 * ν * ρ^2)^2 //
 FullSimplify
D[D[RQ[s, t], t], s] == RQ[s, t] * (4 * ν^2 * ρ^2 - 2 * (s - t)^2 * ν * (1 + 2 * ν)) / ((s - t)^2 + 2 * ν * ρ^2)^2 //
 FullSimplify
D[D[RQ[s, t], {t, 2}], s] == RQ[s, t] *
   (4 * (s - t) * ν * (1 + ν) * (-(s - t)^2 * (1 + 2 * ν) + 6 * ν * ρ^2)) / ((s - t)^2 + 2 * ν * ρ^2)^3 // FullSimplify
D[D[RQ[s, t], {t, 2}], {s, 2}] == RQ[s, t] *
   ((4 * (s - t)^4 * ν * (1 + ν) * (3 + 8 * ν + 4 * ν^2)) / ((s - t)^2 + 2 * ν * ρ^2)^4 -
     (48 * (s - t)^2 * ν^2 * (1 + ν) * (3 + 2 * ν) * ρ^2) / ((s - t)^2 + 2 * ν * ρ^2)^4 +
     (48 * ν^3 * (1 + ν) * ρ^4) / ((s - t)^2 + 2 * ν * ρ^2)^4) // FullSimplify
```

Out[312]=
$$\alpha^2 \left(1 + \frac{(s-t)^2}{2\,\nu\,\rho^2}\right)^{-\nu}$$

Out[313]= True

Out[314]= True

Out[315]= True

Out[316]= True

Out[317]= True

Squared exponential

In[318]:=
```
SE[s_, t_] := α^2 * Exp[-(s - t)^2 / (2 * ρ^2)]
```

In[413]:=
```
SE[s, t]
D[SE[s, t], t] == SE[s, t] * (s - t)/ρ^2 // FullSimplify
D[SE[s, t], {t, 2}] == SE[s, t] * ((s - t)^2 - ρ^2)/ρ^4 // FullSimplify
D[D[SE[s, t], t], s] == SE[s, t] * (ρ^2 - (s - t)^2)/ρ^4 // FullSimplify
D[D[SE[s, t], {t, 2}], s] == SE[s, t] * (3 (s - t) ρ^2 - (s - t)^3)/ρ^6 // FullSimplify
D[D[SE[s, t], {t, 2}], {s, 2}] == SE[s, t] * ((s - t)^4 - 6 (s - t)^2 ρ^2 + 3 ρ^4)/ρ^8 // FullSimplify
```

Out[413]= $e^{-\frac{(s-t)^2}{2\rho^2}}\alpha^2$

Out[414]= True

Out[415]= True

Out[416]= True

Out[417]= True

Out[418]= True