

# Algorithm for Gaussian elimination with scaled partial pivoting

## **I1: Find scaling factors**

Do for  $i = 1, \dots, n$

$$s_i = \max_{1 \leq j \leq n} |a_{ij}|.$$

## **I2: Check whether a non-zero $s_i$ can be found**

If  $s_i = 0$  for some  $i$  no unique solution exists

## **Gaussian elimination**

Do for  $i = 1, \dots, n - 1$  (Use row 1 to  $n - 1$  as pivoting row)

### **G1: Find pivot element**

Find the smallest integer  $p$  ( $i \leq p \leq n$ ) so that  $\frac{|a_{pi}|}{s_p} = \max_{i \leq j \leq n} \frac{|a_{ji}|}{s_j}$

If  $a_{pi} = 0$  a unique solution does not exist.

Else if  $p \neq i$  interchange row  $i$  and  $p$ :  $E_p \leftrightarrow E_i$ .

### **G2: Perform the Gaussian elimination with the non-zero pivot.**

Do for  $j = i + 1, \dots, n$  (On all rows below the pivot element)

$$m_{ji} = \frac{a_{ji}}{a_{ii}}$$

Do for  $k = i + 1, \dots, n$  (For all columns on the right of the pivot element)

$$a_{jk} = a_{jk} - m_{ji}a_{ik}$$

End Do (k-loop)

$$b_j = b_j - m_{ji}b_i$$

End Do (j-loop)

End Do (i-loop)

## **Backward substitution**

### **B1: Check whether backward substitution is possible.**

If  $a_{nn} = 0$  no unique solution exists

### **B2: Perform backward substitution.**

Do for  $i = n, \dots, 1$  (From the last row back to the first)

Do for  $j = i + 1, \dots, n$  (Subtract contributions from known values  $x_j$ )

$$b_i = b_i - a_{ij}x_j$$

End Do (j-loop)

$$x_i = \frac{b_i}{a_{ii}}$$

End Do (i-loop)

Remarks:

- Gaussian elimination with scaled partial pivoting is more robust than Gaussian elimination with partial pivoting.
- Gaussian elimination with scaled partial pivoting requires a little bit more work than Gaussian elimination with partial pivoting.