LIBRATION POINT ORBIT RENDEZVOUS USING LINEARIZED RELATIVE MOTION DYNAMICS AND NONLINEAR DIFFERENTIAL CORRECTION

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This paper presents a technique for computing a rendezvous trajectory with a target satellite in a libration point orbit. The chaser satellite completes the rendezvous by executing a series of impulsive maneuvers to travel between waypoints approaching the target satellite. Linearized equations of relative motion of the chaser with respect to the target in the circular restricted three body problem are used to compute the required magnitude and direction of the maneuvers; these results are then refined using differential correction with the nonlinear equations of motion. The performance of this technique is discussed and several rendezvous strategies are evaluated.

INTRODUCTION

Libration point orbit rendezvous is a critical component for many possible satellite mission architectures. If a large satellite is launched in components and assembled in orbit, the individual components will need to rendezvous and dock prior to assembly. If a valuable space asset such as a telescope requires an on-orbit repair, the satellite servicing mission will need to rendezvous with the object. A libration point orbit is a potentially useful place to build a space station, and rendezvous capabilities will be important during the construction of the station as well as every crew and cargo mission to visit the station.

The history of satellites being deployed to libration point orbits dates back to 1978, when ISEE-3 was launched to the Sun-Earth L1 point. ACE, WIND, SOHO, and DSCOVR are currently orbiting the same point, with the LISA Pathfinder satellite set to join them in 2015. Several spacecraft have been deployed to the Sun-Earth L2 point, and the ARTEMIS mission explored L1 and L2 in the Earth-Moon system. Several additional libration point orbiting missions are currently planned, including the James Webb Space Telescope. So far, all libration point missions have consisted of a single satellite that operates independently of any other satellite. There has been research on deploying a formation of satellites to fly together around libration points, such as the Terrestrial Planet Finder mission.[1] However, little research has been conducted regarding rendezvous with libration point orbiters.

Satellite rendezvous in low-Earth orbit is well-studied, due to many years of experience operating the International Space Station and other applications. The Hill's/Clohessy-Wiltshire (HCW) equations are frequently used to closely approximate the relative motion of a chaser vehicle with respect to a target vehicle in a circular orbit.[2] These equations of relative motion are used to compute the

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approximate ΔV (instantaneous change in velocity) to travel between waypoints defining an approach trajectory. However, for libration point orbits, the dynamical environment is quite different and the same equations of motion are not applicable.

Luquette has developed linearized equations of relative motion for formation flying in libration point orbits.[3] Lian et al. have used these linearized dynamics to compute impulses for a chaser satellite to travel between waypoints in order to approach a target orbiting a libration point.[4] This paper discusses the results of applying this technique for a set of test cases, and presents an additional step in which the shooting method of differential correction is used to refine the computed ΔV for use with the nonlinear equations of motion. It is demonstrated that without the differential correction step, applying the ΔV computed using the linear equations of relative motion can lead to significant position errors when modeling the maneuver in a simulation using the nonlinear dynamics.

DYNAMICS

Circular Restricted Three-Body Dynamics

The circular restricted three body problem (CRTBP) deals with two larger objects orbiting each other in a circular orbit and a third object of infinitesimal mass. Examples include the Trojan asteroids orbiting in the Sun-Jupiter system, or man-made satellites in the Earth-Moon system. When dealing with the Sun-Earth system, the second body is often modeled by treating the Earth and Moon as a single object at the Earth-Moon barycenter; this is called the Sun-Earth/Moon system.

The nonlinear equations of motion for a satellite in the CRTBP are:

$$
\begin{aligned}
\ddot{x} &= x + 2\dot{y} + \frac{(1-\mu)(-\mu-x)}{r_1^3} + \frac{\mu(1-\mu-x)}{r_2^3} \\
\ddot{y} &= y - 2\dot{x} - \frac{(1-\mu)y}{r_1^3} - \frac{\mu y}{r_2^3} \\
\ddot{z} &= \frac{-(1-\mu)z}{r_1^3} - \frac{\mu z}{r_2^3}
\end{aligned}
\tag{1}
$$

where μ is the mass ratio of the primary bodies:

$$
\mu = \frac{M_2}{M_1 + M_2}
\tag{2}
$$

and r_1 and r_2 are the distances from the larger and smaller bodies to the target satellite:

$$
\begin{aligned}
r_1 = \|\mathbf{x} - \mathbf{X_1}\| &= \sqrt{(x - \mathbf{X_{1x}})^2 + y^2 + z^2} \\
r_2 = \|\mathbf{x} - \mathbf{X_2}\| &= \sqrt{(x - \mathbf{X_{2x}})^2 + y^2 + z^2}
\end{aligned}
\tag{3}
$$

where $\mathbf{X_1}$ and $\mathbf{X_2}$ are the positions of the larger and smaller bodies along the X-axis of the rotating CRTBP frame:

$$\mathbf{X_1} = \begin{bmatrix} -\mu \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{X_2} = \begin{bmatrix} 1-\mu \\ 0 \\ 0 \end{bmatrix}$$

(4)

When working in canonical units in the CRTBP, one distance unit is defined by the distance between the primary bodies, one mass unit is defined by the combined mass of the primary bodies, and one time unit is defined by the period of the primary bodies divided by 2π. Finally, the gravitational constant G is also defined as one. Note that as a result, the rotation rate of the system ω also evaluates to one. A summary of the canonical units in the CRTBP is provided in Equation (5).

$$
\begin{aligned}
1\,\text{DU} &= r_{12} = 1 \\
1\,\text{MU} &= M_1 + M_2 = 1 \\
1\,\text{TU} &= \frac{P}{2\pi} = 1 \\
\text{G} &= 1 \\
\omega &= \sqrt{GM/r^3} = 1
\end{aligned}
$$

(5)

Linearized Relative Motion Dynamics

Luquette has derived linearized equations of relative motion of a chaser satellite with respect to a target satellite orbiting in the restricted three body problem (RTBP).[3] The RTBP is distinct from the CRTBP discussed above in that the RTBP provides the option to model the primary bodies in an elliptic orbit instead of a circular orbit around each other; the rotation rate of the system is therefore a function of time. Luquette provides the equations of relative motion in two reference frames: the inertial frame as well as the rotating (RTBP) frame. These equations of motion are valid anywhere in the RTBP system; they are not assumed to be near any specific libration point. They are also useful for analyzing relative motion around any type of libration point orbit, such as Lyapunov, halo, or Lissajous orbits.

In this work, the equations of motion as given in the rotating reference frame are used. These equations are functions of the offset state of the chaser vehicle with respect to the target vehicle in the rotating frame, the position of the target vehicle in the rotating frame with respect to the origin, the mass ratio of the primary bodies, and the rotation rate of the rotating frame.

The linearized equations of relative motion of a chaser satellite with respect to a target satellite, given in the RTBP reference frame, are:

$$\dot{\boldsymbol{\xi}}_R = \mathbf{A}_R(t)\boldsymbol{\xi}_R \tag{6}$$

where $\boldsymbol{\xi}_R$ is the offset state of the chaser vehicle with respect to the target vehicle in the rotating frame:

$$\boldsymbol{\xi}_R = \begin{bmatrix} \mathbf{x}_c \\ \dot{\mathbf{x}}_c \end{bmatrix}_R \tag{7}$$

3

and $\mathbf{A}_R(t)$ is the 6×6 linearized relative motion dynamics matrix in the rotating frame:

$$\mathbf{A}_R(t) = \begin{bmatrix} \mathbf{0} & \mathbf{I_3} \\ \mathbf{\Xi}_R(t) & -2[\boldsymbol{\omega}\times] \end{bmatrix} \tag{8}$$

where

$$\mathbf{\Xi}_R(t) = -(c_1 + c_2)\mathbf{I_3} + 3c_1\hat{\mathbf{r}}_1(t)\hat{\mathbf{r}}_1(t)^T + 3c_2\hat{\mathbf{r}}_2(t)\hat{\mathbf{r}}_2(t)^T + [\dot{\boldsymbol{\omega}}\times] - [\boldsymbol{\omega}\times][\boldsymbol{\omega}\times] \tag{9}$$

and

$$\begin{aligned} c_1 &= \frac{1 - \mu}{r_1^3} \\ c_2 &= \frac{\mu}{r_2^3} \end{aligned} \tag{10}$$

Note that $\boldsymbol{\omega}$ is the rotation rate of the rotating RTBP frame with respect to the inertial frame:

$$\boldsymbol{\omega} = \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} \tag{11}$$

The notation $[\boldsymbol{\omega}\times]$ refers to the cross-product matrix of $\boldsymbol{\omega}$:

$$[\boldsymbol{\omega}\times] = \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{12}$$

Also note that if we assume that the rotation rate of the RTBP frame is constant (that is, assume we can use the CRTBP where the massive bodies are in circular orbits around each other), then the $\dot{\boldsymbol{\omega}}$ term cancels to zero.

These equations of motion do not include any model of solar radiation pressure (SRP) and assume that neither satellite is applying any propulsive thrusting force to perturb or change their orbits.

In order to integrate a chaser satellite's trajectory using the equations of relative motion given in Equation (6), the integration state vector must contain the absolute state of the target satellite in the RTBP frame (with respect to the origin of the RTBP frame) and the relative state of the chaser satellite in the RTBP frame (with the origin located at the target satellite). If using the CRTBP assumption, then the target satellite's state over time can be integrated using the classical CRTBP equations of motion as given in Equation (1), which must be done concurrently with the integration of the relative motion of the chaser so that the time-dependent linearized relative motion dynamics matrix $\mathbf{A}_R(t)$ given in Equation (8) can be computed. The CRTBP model is employed in the test cases presented in this work.

TRAVELING BETWEEN WAYPOINTS WITH IMPULSIVE ΔV'S

The concept of waypoints can be used to divide a rendezvous approach trajectory into a series of shorter segments. The starting and ending waypoints of a segment are defined with respect to the location of a target satellite. When considering rendezvous with a satellite in a circular orbit, the HCW equations supply a linearized dynamics model for a chaser satellite with respect to a target satellite. The inverse of the linear dynamics matrix is used to compute the velocity required to travel from one waypoint to the next within a specified amount of time. This approach is applied in the CRTBP using the linear dynamics matrix presented in Equation (8) by following the procedure below.

Using the Linearized Relative Motion Dynamics Matrix to Compute ΔV

The linearized relative motion dynamics matrix given in Equation (8) can be used to numerically accumulate a State Transition Matrix (STM), $\mathbf{\Phi}$, of the chaser satellite with respect to the target satellite over time:

$$\dot{\mathbf{\Phi}} = \mathbf{A}_R(t)\mathbf{\Phi} \tag{13}$$

The initial "state vector" for the STM passed to the integration process should be the 6×6 identity matrix, $\mathbf{I_6}$. As above, the STM must be integrated concurrently with the target satellite's state because of the time-dependence in $\mathbf{A}_R(t)$. When integrated from the initial state at time t_i to a future time t_{i+1}, this accumulated State Transition Matrix represents the relative position and velocity of the chaser at time t_{i+1} with respect to its relative position and velocity at time t_i. More explicitly:

$$\begin{bmatrix} \mathbf{r}_{i+1} \\ \mathbf{v}_{i+1} \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{11} & \mathbf{\Phi}_{12} \\ \mathbf{\Phi}_{21} & \mathbf{\Phi}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{r}_i \\ \mathbf{v}_i \end{bmatrix} \tag{14}$$

Analogous to the use of the HCW equations in the two-body problem, this STM can be used to compute the required velocity $\mathbf{v}_i^+$ for a chaser satellite in a CRTBP system to travel from waypoint $\mathbf{r}_i$ to waypoint $\mathbf{r}_{i+1}$ in time $\Delta t = t_{i+1} - t_i$. This approach was employed by Lian et al. using the RTBP relative motion dynamics derived by Luquette (see Reference 4).

$$\mathbf{v}_i^+ = \mathbf{\Phi}_{12}^{-1}(\mathbf{r}_{i+1} - \mathbf{\Phi}_{11}\mathbf{r}_i) \tag{15}$$

The instantaneous change in velocity (ΔV) for a maneuver at waypoint $\mathbf{r}_i$ is then simply the difference between the required (post-maneuver) velocity and the velocity that the chaser satellite had before the maneuver:

$$\Delta\mathbf{v}_i = \mathbf{v}_i^+ - \mathbf{v}_i^- \tag{16}$$

Shooting Method with Nonlinear Dynamics

The approach described above for computing ΔV is based on the linearized equations of relative motion developed by Luquette. Of course, the true dynamical environment in the CRTBP is nonlinear, as seen in Equation (1). The linear-based approximation of ΔV can be corrected for use

with this nonlinear propagation model using the shooting method of differential correction. The linear-based approximated velocity is used as an initial guess for this iterative process. Following the procedure below, each of the three components of the velocity vector is varied in order to achieve convergence in the CRTBP on the desired three-dimensional waypoint $\mathbf{w}_{\text{desired}}$ within some specified tolerance.

Step 1: Using the initial guess for the chaser relative velocity $\mathbf{v}_i^+$ from Equation (15), propagate both the target and chaser satellite from time t_i to t_{i+1} using the nonlinear CRTBP equations of motion given in Equation (1) and compute the nominally achieved waypoint $\mathbf{w}$.

Step 2: Add to the x-component of the chaser's velocity a pre-chosen scalar value, called the perturbation. Propagate both satellites from time t_i to t_{i+1} a second time using this perturbed velocity and compute the achieved waypoint $\mathbf{w}'$.

Step 3: Compute the difference between $\mathbf{w}$ and $\mathbf{w}'$, $\frac{d\mathbf{w}}{dv_x}$.

Step 4: Reset the x-component of the velocity to its original value, and repeat steps 2 and 3 when perturbing the y-component of the velocity.

Step 5: Reset the y-component of the velocity to its original value, and repeat steps 2 and 3 when perturbing the z-component of the velocity.

Step 6: Gather the results into a partial derivatives matrix, $\mathbf{M}$:

$$\mathbf{M} = \left[\frac{d\mathbf{w}}{d\mathbf{v}} \right] = \begin{bmatrix} \frac{dw_x}{dv_x} & \frac{dw_y}{dv_x} & \frac{dw_z}{dv_x} \\ \frac{dw_x}{dv_y} & \frac{dw_y}{dv_y} & \frac{dw_z}{dv_y} \\ \frac{dw_x}{dv_z} & \frac{dw_y}{dv_z} & \frac{dw_z}{dv_z} \end{bmatrix} \tag{17}$$

Step 7: Compute an updated estimate of the velocity required to travel to the desired location $\mathbf{w}_{\text{desired}}$:

$$\mathbf{v}_i^+ = \mathbf{v}_i^+ + [\mathbf{M}]^{-1}(\mathbf{w}_{\text{desired}} - \mathbf{w}) \tag{18}$$

Step 8: Repeat steps 1 through 7 until all three components of $\mathbf{w}$ converge to $\mathbf{w}_{\text{desired}}$ within some tolerance.

Definition of Waypoint Reference Frames

In this work, local RIC (Radial, In-track, Cross-track) and VNB (Velocity, Normal, Binormal) reference frames are defined with respect to the target satellite's orbit around its libration point. These local reference frames are used to define the locations of the rendezvous trajectory waypoints. Each of these frames will rotate once for each full orbit of the target around the libration point, with the origin of both frames located at the target satellite's position.

The two libration points of most interest for this work are L_1 and L_2. The position of L_1 in the CRTBP frame is found by computing the real root l_1 of this polynomial in x:

$$(1 - \mu)(p^3)(p^2 - 3p + 3) - \mu(p^2 + p + 1)(1 - p)^3 = 0 \tag{19}$$

where:

$$p = 1 - \mu - x \tag{20}$$

The coordinates of L_1 are then $[l_1, 0, 0]$ in the CRTBP frame. Likewise, the position of L_2 in the CRTBP frame is found by computing the real root l_2 of this polynomial in x:

$$(1 - \mu)(p^3)(p^2 + 3p + 3) - \mu(p^2 + p + 1)(1 - p)(p + 1)^2 = 0 \tag{21}$$

where:

$$p = \mu - 1 + x \tag{22}$$

The coordinates of L_2 are then $[l_2, 0, 0]$ in the CRTBP frame.

The local RIC and VNB reference frames are then defined with the origin of each of these frames located at the target satellite's position. The RIC frame as has its first primary axis the vector pointing from the libration point radially out to the target satellite in the CRTBP frame. For example, in the case of a target satellite orbiting L_1, the fundamental $\hat{\mathbf{R}}$, $\hat{\mathbf{I}}$, and $\hat{\mathbf{C}}$ axes of the RIC frame are defined as:

$$
\begin{aligned}
\hat{\mathbf{R}} &= \frac{\mathbf{x} - \mathbf{L}_1}{\|\mathbf{x} - \mathbf{L}_1\|} \\
\hat{\mathbf{C}} &= \frac{\mathbf{R} \times \dot{\mathbf{x}}}{\|\mathbf{R} \times \dot{\mathbf{x}}\|} \\
\hat{\mathbf{I}} &= \frac{\mathbf{C} \times \mathbf{R}}{\|\mathbf{C} \times \mathbf{R}\|}
\end{aligned}
\tag{23}
$$

where $\mathbf{x}$ and $\dot{\mathbf{x}}$ are the target satellite's position and velocity vectors in the CRTBP frame.

The VNB frame has as its first primary axis the target satellite's velocity vector in the CRTBP frame. In the case of a target satellite orbiting L_1, the fundamental $\hat{\mathbf{V}}$, $\hat{\mathbf{N}}$, and $\hat{\mathbf{B}}$ axes of the VNB frame are defined as:

$$
\begin{aligned}
\hat{\mathbf{V}} &= \frac{\dot{\mathbf{x}}}{\|\dot{\mathbf{x}}\|} \\
\hat{\mathbf{N}} &= \frac{\mathbf{R} \times \mathbf{V}}{\|\mathbf{R} \times \mathbf{V}\|} \\
\hat{\mathbf{B}} &= \frac{\mathbf{V} \times \mathbf{N}}{\|\mathbf{V} \times \mathbf{N}\|}
\end{aligned}
\tag{24}
$$

RESULTS

Performance of Linear ΔV Estimate and Shooting Method

To begin evaluating the performance of the techniques presented above, consider the following scenario. The target satellite is in a planar Lyapunov orbit around the Earth-Moon L1 point as shown in Figure 1. The target satellite's initial conditions are presented in Table 1 in canonical units. For

this test, the rendezvous starts with the target satellite at the location indicated by the asterisk on the CRTBP X-axis, that is, crossing the XZ-plane.

Table 1. Initial Conditions of the Target Satellite in the CRTBP Frame

Parameter	Value
X (DU)	0.862307159058101
Z (DU)	0.0
$\dot{Y}$ (DU/TU)	-0.187079489569182
Period (TU)	2.79101343456226
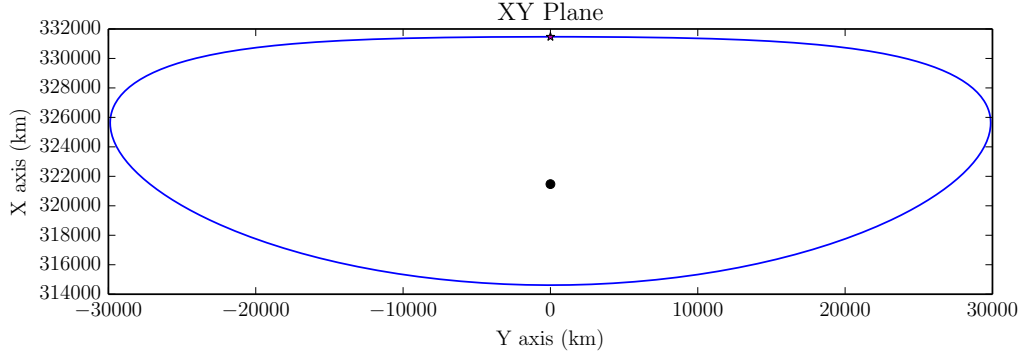
Figure 1. Target Satellite Orbit in Earth-Moon L1 CRTBP Frame

To convert between canonical and dimensional units, the relevant properties of the Earth-Moon CRTBP system are presented in Table 2.

Table 2. Earth-Moon CRTBP Parameters

| Parameter	Value
Earth mass (kg)	5.97219e24
Moon mass (kg)	7.34767309e22
Mass ratio μ	0.012277471
Combined mass (kg), or 1 MU	6.045667e24
r_{12} (km), or 1 DU	384400.0
Time Constant (s), or 1 TU	375201.9
Period of Moon around Earth (s)	2πTU

Three waypoints are defined for the chaser satellite to travel between on its rendezvous with the target, with a final waypoint located at the target satellite itself. The waypoint locations are presented in Table 3 in the RIC reference frame. The waypoints are converted into the CRTBP frame for the propagation. Note that these waypoints represent an approach along the **I** axis of the RIC frame.

Figure 2 shows the result of propagating the target and chaser satellite through a rendezvous using these waypoints. The integration is performed in the CRTBP frame, and the results are converted to the RIC frame for visualization, with the target satellite at the origin and the chaser satellite offset with respect to the target shown in kilometers. The green curve shows the path of the chaser satellite

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Table 3. Waypoints in RIC Frame

Waypoint	Time (days)	R (km)	I (km)	C (km)
1	0.00	0.0	15.0	0.0
2	0.36	0.0	5.0	0.0
3	0.97	0.0	1.0	0.0
4	1.59	0.0	0.0	0.0

when propagated using the linear relative motion dynamics as given in Equation (6), with the application of impulsive ΔV's as computed using Equation (15). The model employed to compute the impulsive maneuvers matches the linear propagation model, and so the chaser satellite travels precisely to each waypoint. The red curve shows the result when the chaser satellite is propagated using the nonlinear CRTBP dynamics as shown in Equation (1) when the nominal ΔVs (as computed using the linear model) are still applied. It is easily seen that the nominally planned maneuvers do not bring the chaser exactly to the desired waypoints when propagating with the nonlinear model. In this test, the trajectory is then reset to start at the correct waypoint in order to visualize the trajectory for the next segment. Finally, the blue curve shows the result when the chaser satellite is propagated using the nonlinear CRTBP dynamics and the ΔVs have been corrected (or "targeted") through the iterative differential correction procedure described above. The path does not precisely match the original green curve due to the different dynamical models in use; however, each waypoint is achieved successfully.
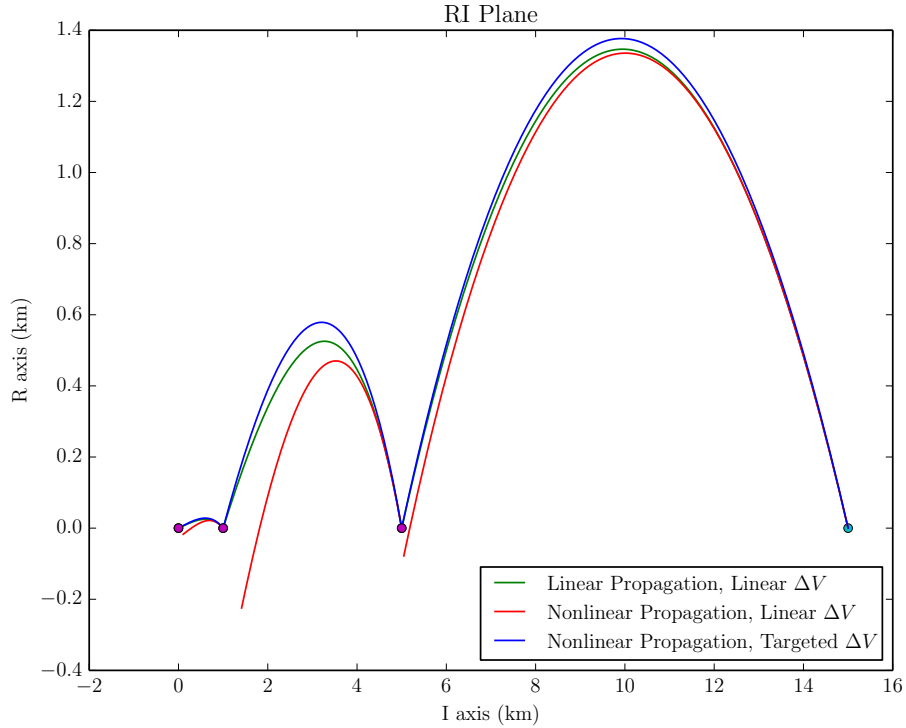
Figure 2. Relative Motion of a Chaser Satellite with respect to a Target

The results for this case are also summarized in Table 4. To compute the first ΔV, the chaser satellite is assumed to start precisely at waypoint 1 with its pre-maneuver velocity at waypoint 1 equal to the target satellite's initial velocity in the CRTBP frame. The final maneuver at waypoint 4 is computed to equalize the chaser's velocity with the target satellite's velocity for a completed rendezvous.

We can note that, while the difference in ΔV magnitude is quite small between the linear computation and the differentially corrected computation, on the order of millimeters/second, the angular difference between the two ΔV vectors is more significant, reaching almost 6° at waypoint 3. These differences result in achieved position errors on the order of hundreds of meters at each waypoint when the linear approximation is applied in the nonlinear propagation model; the errors are reduced to the order of centimeters when the differential correction procedure is applied.

Regarding the tuning parameters of the differential correction procedure, the perturbation value for the velocity components was set to 1e-5 DU/TU (or speed units, SU), which equals about 1.0 centimeters/second. The tolerance on the achieved waypoints was set to 1e-9 DU, or about 0.38 meters.

Table 4. ΔV and Position Error Summary at each Waypoint

Waypoint	Linear ΔV	Corrected ΔV	ΔV Angle Difference	$\|\Delta V\|$ Difference	Position Error, Linear ΔV	Position Error, Corrected ΔV
	(m/s)	(m/s)	(deg)	(m/s)	(m)	(m)
1	0.346	0.345	0.466	-0.001	N/A	N/A
2	0.293	0.295	2.609	0.002	91.394	0.011
3	0.064	0.059	5.890	-0.005	470.653	0.063
4	0.019	0.018	0.445	-0.001	107.663	0.056
Total	0.722	0.717	9.410	0.008	669.709	0.131

Rendezvous Initial Clock Angle

This technique can be used to evaluate the relative ΔV cost of performing a rendezvous at different points along a target spacecraft's orbit. The following orbital plots present twelve test cases that each begin with the target satellite at a different "clock angle" in its orbit; each test case is separated by one twelfth of an orbit in time, with the twelve starting positions indicated by the asterisks shown in Figure 3.

Each of these twelve rendezvous sequences are computed using the same waypoint locations along the **I** axis of the RIC frame as shown in Table 3. However, due to the differing local curvature of the target orbit at each of the twelve starting locations, the relative motion patterns in the RIC frame for each rendezvous are different, as shown in Figure 4.

When viewed using the CRTBP reference frame's rotating axes as in Figure 5, it is more intuitively seen that each of these in-track rendezvous sequences approach the target satellite from a different direction in the CRTBP frame, due to the rotation of the RIC frame with the target satellite's orbit.
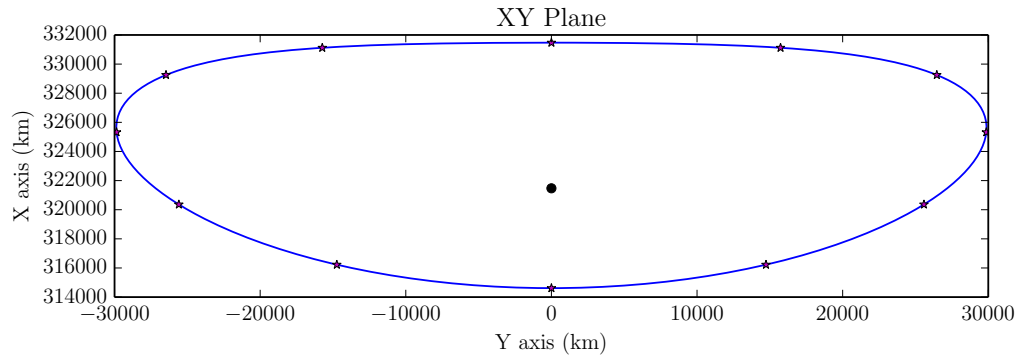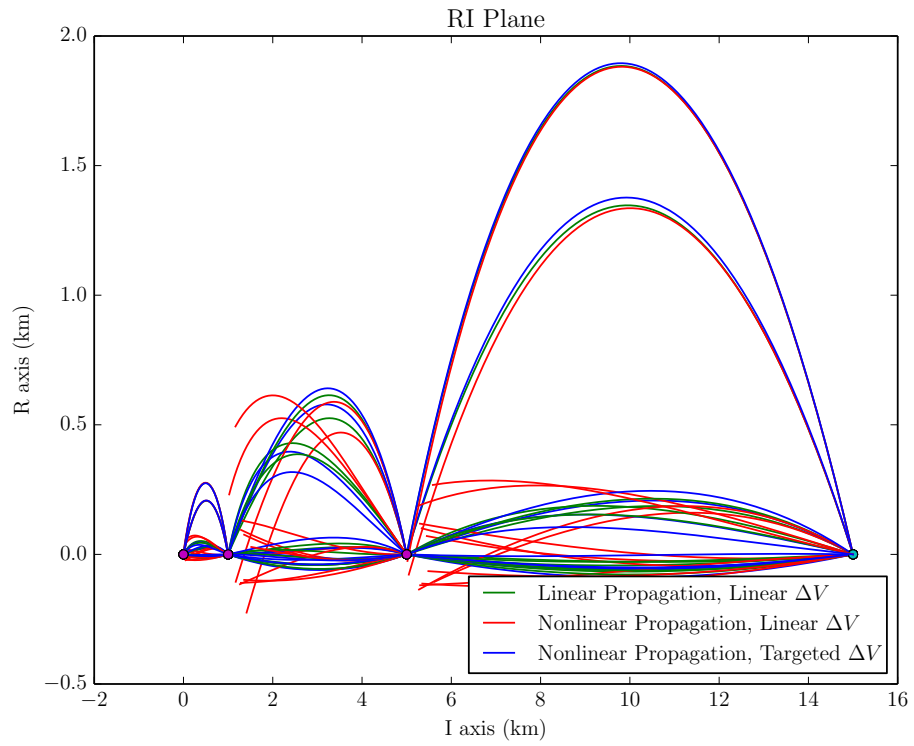
Figure 3. Twelve Target Satellite Initial Conditions

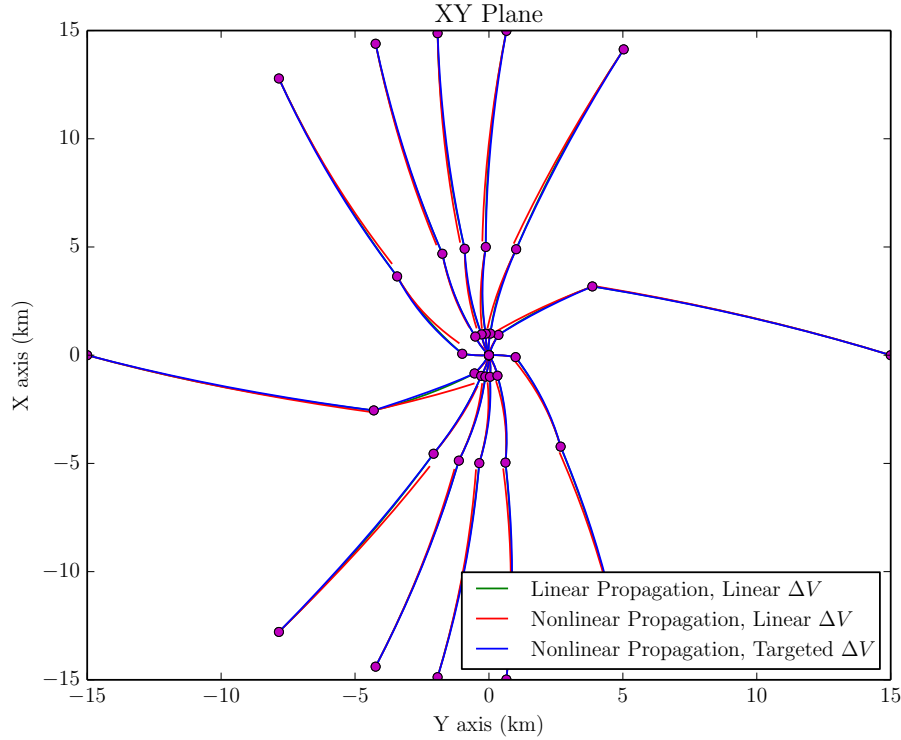**Figure 4. Rendezvous using 12 Different Initial Clock Angles**

Figure 5. Rendezvous using 12 Different Initial Clock Angles; CRTBP Axes

The same rendezvous was then computed for 360 test cases with the initial conditions spaced evenly in time around the same libration point orbit, in order to examine the trends in total ΔV to complete the rendezvous as a function of the initial clock angle. The total ΔV is shown in Figure 6 as a function of the initial clock angle; the magnitude of both the linear approximated ΔV and the nonlinear corrected ΔV are shown. From this we can see that the total ΔV cost does not change dramatically when the initial clock angle is varied; however the peak costs are at initial clock angles of 0° and 180°. This trend is apparent in the linear approximation of the ΔV as well as in the corrected or "targeted" results; this means that the linear approximation may safely be used in this case to provide order-of-magnitude indications of the relative costs of the different rendezvous sequences.

Although the differences in the magnitude of the ΔV vector are not large when comparing the linear approximation to the nonlinear differential correction, the direction of the ΔV vector is sometimes quite different. The angle between these two ΔV vectors, summed up over each impulse of the rendezvous, is shown in Figure 7.

The total rendezvous position error, summed up over waypoints 2 through 4, is shown in Figure 8 as a function of the initial clock angle. The position error when the linear approximated ΔV is applied in the nonlinear model is on the order of 1 km, while the order of magnitude of the position error after the differential correction has been applied is 10 cm, which was about the tolerance level of the correction algorithm and could even be improved with more differential correction iterations.
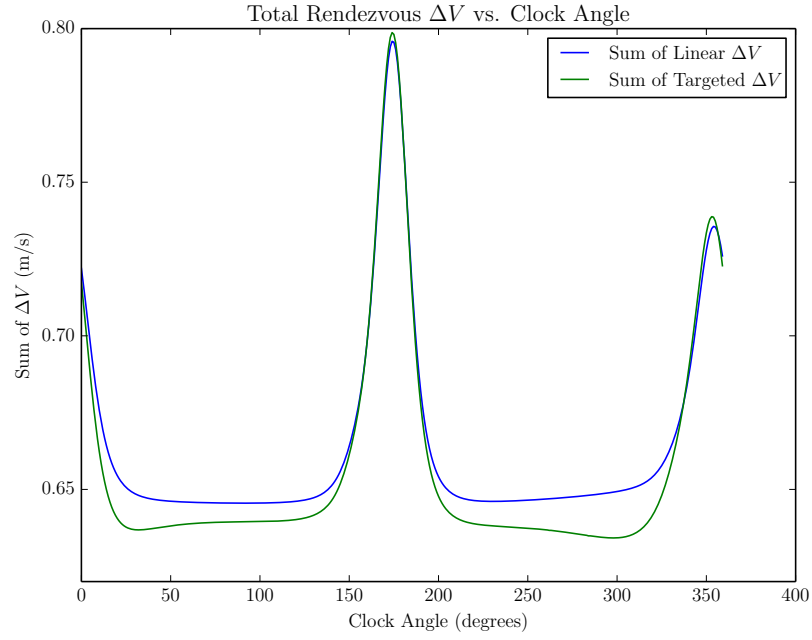
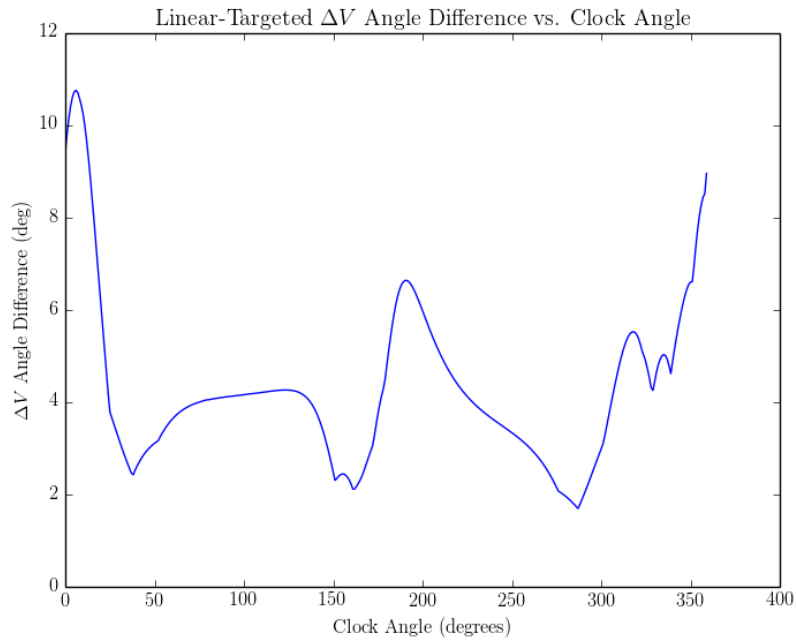**Figure 6. Total Rendezvous ΔV for Different Initial Clock Angles**

Figure 7. Total Rendezvous ΔV Angle Difference for Different Initial Clock Angles
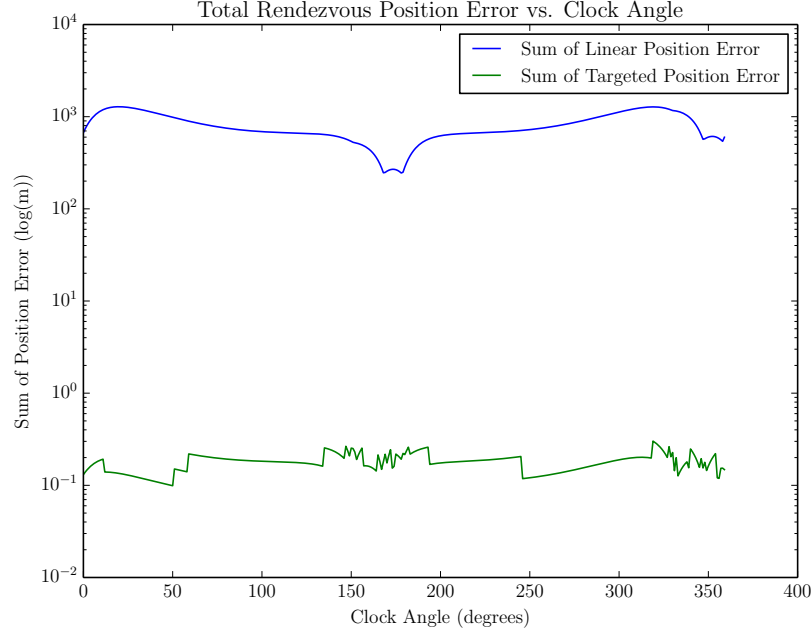
Figure 8. Total Rendezvous Position Error for Different Initial Clock Angles

CONCLUSIONS

This procedure and the types of metrics presented in the plots discussed above can be used to evaluate other rendezvous approach strategies, such as approaching the target satellite from a different direction, or with different waypoint spacing and time intervals, or in a different target orbit.

Going forward, there is much that can be done to further develop this procedure. The procedure should be tested against many more rendezvous scenarios, to identify when the procedure works well and when it may fail (in particular, it will be useful to characterize when the linear approximation is not good enough to compute a useful initial ΔV approximation). The nonlinear differential correction step of the procedure can be expanded to model not only the CRTBP dynamics, but the full planetary ephemeris model. These enhancements to the procedure will allow analysis to determine whether the linear approximation is generally accurate enough to establish the ΔV budget for a rendezvous sequence, or if the nonlinear differential correction step is necessary even early in the mission design process.

This procedure can be used to search for trends in rendezvous ΔV cost as a function of the total time of flight, approach direction, and the target orbit size and shape. The safety of a given rendezvous configuration can also be studied: for example, if a maneuver at one of the rendezvous waypoints is missed for some reason, is the configuration of the two satellites passively safe? As with low-Earth orbit rendezvous, this type of safety concern will be important to consider when designing rendezvous sequences.

ACKNOWLEDGMENT

REFERENCES

[1] C. Beichman, G. Gómez, M. Lo, J. Masdemont, and L. Romans, "Searching for life with the Terrestrial Planet Finder: Lagrange point options for a formation flying interferometer," *Advances in Space Research*, Vol. 34, No. 3, 2004, pp. 637–644.

[2] W. H. Clohessy and R. S. Wiltshire, "Terminal guidance system for satellite rendezvous," *Journal of the Aerospace Sciences*, 1960.

[3] R. J. Luquette and R. M. Sanner, "Linear state-space representation of the dynamics of relative motion, based on restricted three body dynamics," *AIAA Guidance, Navigation, and Control Conference and Exhibit, AIAA*, Vol. 4783, 2004.

[4] Y. Lian, L. Liu, Y. Meng, G. Tang, and K. Chen, "Constant thrust glideslope guidance algorithm for rendezvous in multi-body realm," *Recent Advances in Space Technologies (RAST), 2011 5th International Conference on*, IEEE, 2011, pp. 232–236.