$3^{x+2} = 3^{2-2x} \Rightarrow 3x=0$

Problem 1 (5 marks). Solve for x : $\quad 9\cdot 3^x - 9^{1-x} = 0\,.$

$x=0$

Problem 2 (6 marks). Find the inverse $f^{-1}(x)$ of the function $f(x) = \ln(5+x) - 5$. $\quad D_f = R_{f^{-1}}: x>-5$

$x = \ln(5+y) - 5$

Determine the domain and the range of $f(x)$ **and** of $f^{-1}(x)$.

$\ln(y+5) = x+5 \Rightarrow y = e^{x+5} - 5 = f^{-1}(x) \qquad D_{f^{-1}} = R_f : (-\infty, +\infty)$

Problem 3 (6 marks). Find all **(a)** horizontal and **(b)** vertical asymptotes of the graph of

VA: $x=-3$

$$f(x) = \frac{(3x+6)\,|x-4|}{x^2+5x+6} = \frac{3\cancel{(x+2)}\,|x-4|}{\cancel{(x+2)}(x+3)}$$

HAs: $y = 3$ at $x \to +\infty$
$y = -3$ at $x \to -\infty$

Problem 4 (5 marks). Find the limit. If the limit does not exist explain why.

$$\lim_{x\to-\infty} \frac{\sqrt{1+8x^3+16x^6}}{2x^3-4x^2+1} = \lim_{x\to-\infty}\frac{-x^3\sqrt{1/x^6 + 8/x^3 + 16}}{x^3(2 - 4/x + 1/x^3)}$$

Ans: $\lim\limits_{x\to-\infty} = -2.$

Problem 5 (4+3 marks). Consider the piecewise function $f(x)$ with parameter a:

$$f(x) = \begin{cases} a + 2\sqrt{x} & \text{if} \quad x > 1 \\ 2 + x & \text{if} \quad x \le 1 \end{cases}$$

$a + 2 = 3 \Rightarrow a = 1$

$\frac{1}{\sqrt{x}}\Big|_1 = 1 = (2+x)'$

(A) Find the values of a that make $f(x)$ continuous everywhere.

(B) In that case, will the function $f(x)$ also be differentiable everywhere? Explain why yes or why not.

Yes. $f(x)$ is differentiable everywhere.

Problem 6. (4 marks) Find the derivative of the function:
(you have to show **at least one intermediate step** of your calculations)

$$f(x) = \frac{x^3 + \cancel{\sec x}}{\cancel{\sec x} + x^3} + e^3\,x^{3/2} \Rightarrow f'(x) = \frac{3}{2}e^3\sqrt{x}$$

Problem 7. (4 marks) Find the derivative of the function:
(you have to show **at least one intermediate step** of your calculations)

$$\left(\sqrt{x^2+1+\sqrt{x^2+1}}\,\right)' = \frac{1}{2\sqrt{x^2+1+\sqrt{x^2+1}}}\cdot\left(2x + \frac{1}{2\sqrt{x^2+1}}\cdot 2x\right)$$

Problem 8. (6 marks) Find the second derivative $f''(x)$ of the function

$f' = e^{bx} + bxe^{bx} + 2x$

$$f(x) = x^2\,e^{bx}(x^{-1} + e^{-bx}) = xe^{bx} + x^2$$

$f'' = 2be^{bx} + b^2xe^{bx} + 2$

where b is a parameter (a real number), and calculate its **exact** value $f''(0)$ at $x=0$.
(HINT: simplify the function before calculating $f''(x)$)

$f''(0) = 2b+2$

Problem 9 (4+3 marks) Given the function $f(x) = \dfrac{4}{x^2+1}$,

$$f' = \lim_{h\to0}\frac{\frac{4}{(x+h)^2+1} - \frac{4}{x^2+1}}{h} =$$

(a) Calculate $f'(x)$ using its definition as a limit of difference quotient.

$$= \lim_{h\to0}\frac{4(-2xh-h^2)}{h(x^2+1)((x+h)^2+1)}$$

(b) Write the equation of the tangent line to $y = f(x)$ at the point $(1,2)$.

$f'(1) = -2 \Rightarrow y_t = 2 - 2(x-1) = -2x+4.$

$$= \frac{-8x}{(x^2+1)^2}$$