

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	203	All
Examination	Date	Duration
Midterm Test	30 October, 2022	1 h 30 min
Special Instructions:	Only approved calculators are allowed Show your work for full marks	

1. (12 marks): (a) Solve for x (find the *exact* values, do not approximate):

$$8 = 2^{5x} 4^{x^2}$$

- (b) Let $f(x) = \sqrt{9 - 3x}$ and $g(x) = 3 - x^2$. Find the composite functions $f \circ g$ and $g \circ f$ and determine their domains.

- (c) Let $f(x) = 5^{-x} + 1$. Find the inverse function f^{-1} and the domain and range of f^{-1} .

2. (8 marks) Find the limit or explain why the limit does not exist:

(a) $\lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$

(b) $\lim_{x \rightarrow -3} \frac{2x^2 - 18}{|x + 3|}$

3. (6 marks) Find (a) all horizontal and (b) all vertical asymptotes of the graph

$$y = \frac{\sqrt{4x^6 + 2x^2 + 1}}{x^3 + x^2 - 6x}$$

4. (4 marks) Consider the following piecewise-defined function:

$$f(x) = \begin{cases} ax + 2b & \text{if } x \leq 0 \\ x^2 + 3a - b & \text{if } 0 < x \leq 2 \\ 3x - 5 & \text{if } x > 2 \end{cases}$$

For what values of a and b is the function continuous at every x ? Explain.

(continued on the other side)

5. (12 marks) Find the derivatives of the following functions (you have to show at least **one intermediate step** of your calculations):

(a) $f(x) = x\sqrt{x}(x + x^{-3/2})$

(b) $f(x) = \frac{xe^x}{(x + e^{-x})} + e^2$

(c) $f(x) = \cos(x)\sec^2(x) + 2\tan(x)$

(d) $f(x) = \sin[x \cos(x) + x^2 + \cos^2(x)]$

6. (8 marks) Given the function $f(x) = \sqrt{3x + 7}$,

(a) Calculate $f'(x)$ using its definition as a limit of difference quotient.

(b) Check that your calculation is correct using standard differentiation rules.

(c) Write equation of the tangent line to the curve $y = f(x)$ at the point $(3, 4)$.

Bonus Question (3 marks). Find a formula for the n -th derivative $f^{(n)}(x)$ if $f(x) = \frac{1}{1-x}$ and n is any positive integer.