

000  
001  
002  
003  
004  
005  
006  
007  
008  
009  
010  
011  
012  
013  
014  
015  
016  
017  
018  
019  
020  
021  
022  
023  
024  
025  
026  
027  
028  
029  
030  
031  
032  
033  
034  
035  
036  
037  
038  
039  
040  
041  
042  
043  
044  
045  
046  
047  
048  
049  
050  
051  
052  
053

---

# Optimal independence tests for Bayesian Networks

---

Anonymous Author(s)  
Affiliation  
Address  
email

## Abstract

## 1 Introduction

Learning Bayesian networks for general distributions is intractable task Chickering [1996]. However, in practise for distributions appearing in real data, still we are able to recover Bayesian network structure. This implies that real data distribution has some special properties, which simplify process of structure recovery. This process is almost always based on computation local statistics, and then reasoning about global structure Jaakkola et al. [2010]; Tsamardinos et al. [2006]. Local statistics describe complexity (e.g. number of parameters in case of BIC), and level of independence between nodes (e.g. mutual information tests, conditional independence tests). We focus in this work on improvement of independence tests by learning it for CPDs present in data.

We consider parameterized family of independence tests. Before inferring about structure of Bayesian network, we tune classifier predict independence or dependence on CPDs present in data. This way, we obtain highly sensitive independence test, which fires on dependence or independence phenomenas present in our data.

## 2 Related work

There has been extensive research in area of scoring functions for Bayesian networks. This step is critical to recover no-complete graph structure. Without any scoring function, log-likelihood term would force optimization to choose fully connected graph. There have been proposed few regularizations (scoring functions) to address this problem. One most widely used is Bayesian Information Criterion (BIC) Schwarz et al. [1978]. Variety of such scoring functions calculate dependency between nodes conditioned on potential parents De Campos [2006]. Usual measures of dependency are based on mutual information, conditional independence test, or are fully Bayesian. Fully Bayesian methods assume probability distribution over CPDs of independent variables, and dependent variables.

We should discuss:

- LL (Log-likelihood) (1912-22)
- MDL/BIC (Minimum description length/Bayesian Information Criterion) (1978)
- AIC (Akaike Information Criterion) (1974)
- NML (Normalized Minimum Likelihood) (2008)
- MIT (Mutual Information Tests) (2006)

Existing independence tests:

- Pearson's  $\chi$ -squared. The problem is the null hypothesis is independence, but independence is what we're trying to show.

Margaritis [2003]

### 3 Independence testing

Manifold of Independence

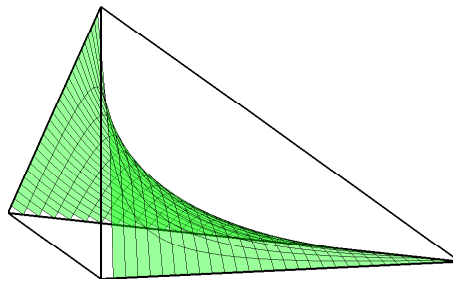


Figure 1: A

#### 3.1 Discrete variables formulation

#### 3.2 Continuous variables formulation

### 4 Experiments

#### 4.1 Classification of Synthetic CPDs

#### 4.2 Classification of CPDs from Gene Expression

#### 4.3 Synthetic Bayesian networks

#### 4.4 Gene expression data

### 5 Discussion

### References

- D. M. Chickering. Learning bayesian networks is np-complete. In *Learning from data*, pages 121–130. Springer, 1996.
- L. M. De Campos. A scoring function for learning bayesian networks based on mutual information and conditional independence tests. *The Journal of Machine Learning Research*, 7:2149–2187, 2006.
- T. Jaakkola, D. Sontag, A. Globerson, and M. Meila. Learning bayesian network structure using lp relaxations. In *International Conference on Artificial Intelligence and Statistics*, pages 358–365, 2010.
- D. Margaritis. *Learning Bayesian network model structure from data*. PhD thesis, University of Pittsburgh, 2003.
- G. Schwarz et al. Estimating the dimension of a model. *The annals of statistics*, 6(2):461–464, 1978.
- I. Tsamardinos, L. E. Brown, and C. F. Aliferis. The max-min hill-climbing bayesian network structure learning algorithm. *Machine learning*, 65(1):31–78, 2006.