

$$\frac{dS_a}{dt} = -p_a S_a \sum_{j=1}^A \left( M_{a,j} I_j + \tilde{M}_{a,j} Q_j \right) - \nu_a \left( 1 - \frac{D_V}{N_V} \right) S_a \quad (1)$$

$$\frac{dE_a}{dt} = p_a S_a \sum_{j=1}^A \left( M_{a,j} I_j + \tilde{M}_{a,j} Q_j \right) - \sigma E_a - \tau E_a \sum_{j=1}^A M_{a,j} (Q_j + H_j + C_j) - \nu_a \left( 1 - \frac{D_V}{N_V} \right) E_a \quad (2)$$

$$\frac{dV_a}{dt} = -p_a (1 - \varepsilon_a) V_a \sum_{j=1}^A \left( M_{a,j} I_j + \tilde{M}_{a,j} Q_j \right) + \nu_a \left( 1 - \frac{D_V}{N_V} \right) S_a \quad (3)$$

$$\frac{dF_a}{dt} = p_a (1 - \varepsilon_a) V_a \sum_{j=1}^A \left( M_{a,j} I_j + \tilde{M}_{a,j} Q_j \right) - \sigma F_a - \tau F_a \sum_{j=1}^A M_{a,j} (Q_j + H_j + C_j) + \nu_a \left( 1 - \frac{D_V}{N_V} \right) E_a \quad (4)$$

$$\begin{aligned} \frac{dI_a}{dt} &= (1 - q) \sigma (E_a + F_a) - \left( 1 - h \left( 1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) - c \left( 1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \right) \gamma I_a - h \left( 1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) \delta I_a \\ &- c \left( 1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \theta I_a - \tau I_a \sum_{j=1}^A M_{a,j} (Q_j + H_j + C_j) - \mu \frac{\sum_{j=1}^A (H_j + C_j)}{B_H + B_C} I_a \end{aligned} \quad (5)$$

$$\frac{dO_a}{dt} = \tau (F_a + E_a) \sum_{j=1}^A M_{a,j} (Q_j + H_j + C_j) - \sigma O_a \quad (6)$$

$$\begin{aligned} \frac{dQ_a}{dt} &= q \sigma (E_a + F_a) + \sigma O_a + \tau I_a \sum_{j=1}^A M_{a,j} (Q_j + H_j + C_j) - h \left( 1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) \delta Q_a - c \left( 1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \theta Q_a \\ &- \left( 1 - h \left( 1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) - c \left( 1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \right) \gamma Q_a - \mu \frac{\sum_{j=1}^A (H_j + C_j)}{B_H + B_C} Q_a \end{aligned} \quad (7)$$

$$\frac{dH_a}{dt} = h \delta \left( 1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) (I_a + Q_a) - (m_H \mu_H + (1 - m_H) \psi_H) H_a \quad (8)$$

$$\frac{dC_a}{dt} = c \left( 1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \theta (I_a + Q_a) - (m_C \mu_C + (1 - m_C) \psi_C) C_a \quad (9)$$

$$\frac{dD_V}{dt} = \sum_{j=1}^A \left( \nu_a \left( 1 - \frac{D_V}{N_V} \right) (S_a + E_a) \right) \quad (10)$$