$$\frac{\mathrm{d}S_a}{\mathrm{d}t} = -p_a S_a \sum_{j=1}^{A} M_{a,j} \left((I_j + (1-\varepsilon_q)Q_j) + (1-\xi_j)(\tilde{I}_j + (1-\varepsilon_q)\tilde{Q}_j) \right) - \nu_a \left(1 - \frac{D_V}{N_V} \right) S_a \tag{1}$$

$$\begin{aligned} \frac{\mathrm{d}E_a}{\mathrm{d}t} &= p_a S_a \sum_{j=1}^{A} M_{a,j} \left((I_j + (1-\varepsilon_q)Q_j) + (1-\xi_j)(\tilde{I}_j + (1-\varepsilon_q)\tilde{Q}_j) \right) - \sigma E_a \\ &- \tau E_a \sum_{j=1}^{A} M_{a,j} \left(Q_j + \tilde{Q}_j + H_j + \tilde{H}_j + C_j + \tilde{C}_j \right) E_a \end{aligned} \tag{2}$$

$$\frac{\mathrm{d}F_a}{\mathrm{d}t} = -\sigma F_a - \tau F_a \sum_{j=1}^{A} M_{a,j} \left(Q_j + \tilde{Q}_j + H_j + \tilde{H}_j + C_j + \tilde{C}_j \right) + \nu_a \left(1 - \frac{D_V}{N_V} \right) \tag{3}$$

$$\frac{\mathrm{d}I_a}{\mathrm{d}t} = \sigma(E_a + F_a) - (1-q-h)\gamma I_a - q\omega I_a - h\delta I_a - \tau I_a \sum_{j=1}^{A} M_{a,j} \left(Q_j + \tilde{Q}_j + H_j + \tilde{H}_j + C_j + \tilde{C}_j \right) \tag{4}$$

$$\frac{\mathrm{d}O_a}{\mathrm{d}t} = \tau(F_a + E_a) \sum_{j=1}^{A} M_{a,j} \left(Q_j + \tilde{Q}_j + H_j + \tilde{H}_j + C_j + \tilde{C}_j \right) - \sigma O_a \tag{5}$$

$$\begin{aligned} \frac{\mathrm{d}Q_a}{\mathrm{d}t} &= q\omega I_a + \sigma O_a - (1-h_q)\rho Q_a - h_q \zeta \left(1 - \frac{\sum_{i=1}^{A}(H_a + \tilde{H}_a)}{B_H} \right) Q_a \\ &+ \tau I_a \sum_{j=1}^{A} M_{a,j} \left(Q_j + \tilde{Q}_j + H_j + \tilde{H}_j + C_j + \tilde{C}_j \right) \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{\mathrm{d}H_a}{\mathrm{d}t} &= h\delta \left(1 - \frac{\sum_{i=1}^{A}(H_a + \tilde{H}_a)}{B_H} \right) I_a + h_q \zeta \left(1 - \frac{\sum_{i=1}^{A}(H_a + \tilde{H}_a)}{B_H} \right) Q_a \\ &- c\theta \left(1 - \frac{\sum_{i=1}^{A}(C_a + \tilde{C}_a)}{B_C} \right) H_a - (1-c)\left(m_H \mu_H + (1-m_H)\psi_H \right) H_a \end{aligned} \tag{7}$$

$$\frac{\mathrm{d}C_a}{\mathrm{d}t} = c \left(1 - \frac{\sum_{i=1}^{A}(C_a + \tilde{C}_a)}{B_C} \right) \theta H_a - m_C \mu_C C_a - (1-m_C)\psi_C C_a \tag{8}$$

$$\frac{\mathrm{d}V_a}{\mathrm{d}t} = -p_a(1-\varepsilon_a)V_a \sum_{j=1}^{A} M_{a,j}\left((I_j + (1-\varepsilon_q)Q_j) + (1-\xi_j)(\tilde{I}_j + (1-\varepsilon_q)\tilde{Q}_j)\right) - \nu_a\left(1 - \frac{D_V}{N_V}\right)S_a \quad (9)$$

$$\frac{\mathrm{d}\tilde{E}_a}{\mathrm{d}t} = p_a(1-\varepsilon_a)V_a \sum_{j=1}^{A} M_{a,j}\left((I_j + (1-\varepsilon_q)Q_j) + (1-\xi_j)(\tilde{I}_j + (1-\varepsilon_q)\tilde{Q}_j)\right) - \sigma\tilde{E}_a$$

$$- \tau\tilde{E}_a \sum_{j=1}^{A} M_{a,j}\left(Q_j + \tilde{Q}_j + H_j + \tilde{H}_j + C_j + \tilde{C}_j\right) \quad (10)$$

$$\frac{\mathrm{d}\tilde{I}_a}{\mathrm{d}t} = \sigma(\tilde{E}_a) - (1-\tilde{q}-\tilde{h})\gamma\tilde{I}_a - \tilde{q}\omega I_a - \tilde{h}\delta\tilde{I}_a - \tau\tilde{I}_a \sum_{j=1}^{A} M_{a,j}\left(Q_j + \tilde{Q}_j + H_j + \tilde{H}_j + C_j + \tilde{C}_j\right) \quad (11)$$

$$\frac{\mathrm{d}\tilde{O}_a}{\mathrm{d}t} = \tau(\tilde{E}_a) \sum_{j=1}^{A} M_{a,j}\left(Q_j + \tilde{Q}_j + H_j + \tilde{H}_j + C_j + \tilde{C}_j\right) - \sigma\tilde{O}_a \quad (12)$$

$$\frac{\mathrm{d}\tilde{Q}_a}{\mathrm{d}t} = \tilde{q}\omega\tilde{I}_a + \sigma\tilde{O}_a - (1-\tilde{h}_q)\rho\tilde{Q}_a - \tilde{h}_q\zeta\left(1 - \frac{\sum_{i=1}^{A}(H_a + \tilde{H}_a)}{B_H}\right)\tilde{Q}_a$$

$$+ \tau\tilde{I}_a \sum_{j=1}^{A} M_{a,j}\left(Q_j + \tilde{Q}_j + H_j + \tilde{H}_j + C_j + \tilde{C}_j\right) \quad (13)$$

$$\frac{\mathrm{d}\tilde{H}_a}{\mathrm{d}t} = \tilde{h}\delta\left(1 - \frac{\sum_{i=1}^{A}(H_a + \tilde{H}_a)}{B_H}\right)\tilde{I}_a + \tilde{h}_q\zeta\left(1 - \frac{\sum_{i=1}^{A}(H_a + \tilde{H}_a)}{B_H}\right)\tilde{Q}_a$$

$$- \tilde{c}\theta\left(1 - \frac{\sum_{i=1}^{A}(C_a + \tilde{C}_a)}{B_C}\right)\tilde{H}_a - (1-\tilde{c})\left(\tilde{m}_H\mu_H + (1-\tilde{m}_H)\psi_H\right)\tilde{H}_a \quad (14)$$

$$\frac{\mathrm{d}\tilde{C}_a}{\mathrm{d}t} = \tilde{c}\left(1 - \frac{\sum_{i=1}^{A}(C_a + \tilde{C}_a)}{B_C}\right)\theta\tilde{H}_a - \tilde{m}_C\mu_C\tilde{C}_a - (1-\tilde{m}_C)\psi_C\tilde{C}_a \quad (15)$$