$$\frac{\mathrm{d}S_a}{\mathrm{d}t} = -\beta\frac{S_a}{N_a}\sum_{j=1}^{A}\left(M_{a,j}(I_j+(1-\xi_j)\tilde{I}_j)+\tilde{M}_{a,j}(Q_j+(1-\xi_j)\tilde{Q}_j)\right)-\nu_a\left(1-\frac{D_V}{N_V}\right)S_a \tag{1}$$

$$\frac{\mathrm{d}E_a}{\mathrm{d}t} = \beta\frac{S_a}{N_a}\sum_{j=1}^{A}\left(M_{a,j}(I_j+(1-\xi_j)\tilde{I}_j)+\tilde{M}_{a,j}(Q_j+(1-\xi_j)\tilde{Q}_j)\right)-\sigma E_a-\nu_a\left(1-\frac{D_V}{N_V}\right)E_a \tag{2}$$

$$\frac{\mathrm{d}F_a}{\mathrm{d}t} = -\sigma F_a+\nu_a\left(1-\frac{D_V}{N_V}\right)E_a \tag{3}$$

$$\begin{aligned}\frac{\mathrm{d}I_a}{\mathrm{d}t} &= (1-q_a)\sigma(E_a+F_a)-\left(1-h_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)-c_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\right)\gamma\frac{\gamma}{\gamma+\tau}I_a-h_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)\delta I_a\\ &- c_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\theta I_a-\left(1-h_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)-c_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\right)\tau\frac{\tau}{\gamma+\tau}I_a \qquad (4)\\ &- \mu_a\left((1-\omega)\frac{\sum_{j=1}^{A}H_j}{B_H}+\omega\frac{\sum_{j=1}^{A}C_j}{B_C}\right)I_a\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}Q_a}{\mathrm{d}t} &= q_a\sigma(E_a+F_a)-h_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)\delta Q_a-c_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\theta Q_a\\ &- \left(1-h_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)-c_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\right)\gamma Q_a-\mu_a\left((1-\omega)\frac{\sum_{j=1}^{A}H_j}{B_H}+\omega\frac{\sum_{j=1}^{A}C_j}{B_C}\right)Q_a \qquad (5)\\ &+ \left(1-h_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)-c_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\right)\tau\frac{\tau}{\gamma+\tau}I_a\end{aligned}$$

$$\frac{\mathrm{d}H_a}{\mathrm{d}t} = h_a\delta\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)(I_a+Q_a)-(m_{a,H}\mu_H+(1-m_{a,H})\psi_H)H_a \tag{6}$$

$$\frac{\mathrm{d}C_a}{\mathrm{d}t} = c_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\theta(I_a+Q_a)-(m_{a,C}\mu_C+(1-m_{a,C})\psi_C)C_a \tag{7}$$

$$\frac{\mathrm{d}V_a}{\mathrm{d}t} = -\beta(1-\varepsilon_a)\frac{V_a}{N_a}\sum_{j=1}^{A}\left(M_{a,j}(I_j+\tilde{I}_j)+\tilde{M}_{a,j}(Q_j+\tilde{Q}_j)\right)+\nu_a\left(1-\frac{D_V}{N_V}\right)S_a \tag{8}$$

$$\frac{\mathrm{d}\tilde{E}_a}{\mathrm{d}t} = \beta(1-\varepsilon_a)\frac{V_a}{N_a}\sum_{j=1}^{A}\left(M_{a,j}(I_j+\tilde{I}_j)+\tilde{M}_{a,j}(Q_j+\tilde{Q}_j)\right)-\sigma\tilde{E}_a \tag{9}$$

$$\begin{aligned}\frac{\mathrm{d}\tilde{I}_a}{\mathrm{d}t} &= (1-\tilde{q}_a)\sigma(\tilde{E}_a)-\left(1-\tilde{h}_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)-\tilde{c}_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\right)\gamma\frac{\gamma}{\gamma+\tau}\tilde{I}_a-\tilde{h}_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)\delta\tilde{I}_a\\ &- \tilde{c}_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\theta\tilde{I}_a-\left(1-\tilde{h}_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)-\tilde{c}_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\right)\gamma\frac{\tau}{\gamma+\tau}\tilde{I}_a \qquad (10)\\ &- \tilde{\mu}_a\left((1-\omega)\frac{\sum_{j=1}^{A}H_j}{B_H}+\omega\frac{\sum_{j=1}^{A}C_j}{B_C}\right)\tilde{I}_a\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}\tilde{Q}_a}{\mathrm{d}t} &= \tilde{q}_a\sigma\tilde{E}_a+\left(1-\tilde{h}_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)-\tilde{c}_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\right)\gamma\frac{\tau}{\gamma+\tau}\tilde{I}_a\\ &- \left(1-\tilde{h}_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)-\tilde{c}_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\right)\gamma\tilde{Q}_a-\tilde{h}_a\left(1-\frac{\sum_{i=1}^{A}H_a}{B_H}\right)\delta\tilde{Q}_a \qquad (11)\\ &- \tilde{\mu}_a\left((1-\omega)\frac{\sum_{j=1}^{A}H_j}{B_H}+\omega\frac{\sum_{j=1}^{A}C_j}{B_C}\right)\tilde{Q}_a-\tilde{c}_a\left(1-\frac{\sum_{i=1}^{A}C_a}{B_C}\right)\theta\tilde{Q}_a\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}N_a}{\mathrm{d}t} &= -m_{a,C}\mu_C C_a-m_{a,H}\mu_H H_a-\mu_a\left((1-\omega)\frac{\sum_{j=1}^{A}H_j}{B_H}+\omega\frac{\sum_{j=1}^{A}C_j}{B_C}\right)(I_a+Q_a)\\ &- \tilde{\mu}_a\left((1-\omega)\frac{\sum_{j=1}^{A}H_j}{B_H}+\omega\frac{\sum_{j=1}^{A}C_j}{B_C}\right)(\tilde{I}_a+\tilde{Q}_a) \qquad (12)\end{aligned}$$

$$\frac{\mathrm{d}D_V}{\mathrm{d}t} = \sum_{j=1}^{A}\left(\nu_j\left(1-\frac{D_V}{N_V}\right)(S_j+E_j)\right) \tag{13}$$

$$\frac{\mathrm{d}W_V}{\mathrm{d}t} = \sum_{j=1}^{A}\left(\nu_j\left(1-\frac{D_V}{N_V}\right)(E_j)\right) \tag{14}$$

$$\tag{15}$$

Table 1: List of variables and their description

Variable	Description
S_a	Susceptibles in age class a
E_a	Cases in incubation period in age class a
F_a	Cases in incubation period and vaccinated after infection in age class a
I_a	Cases in symptomatic period in age class a
Q_a	Cases in symptomatic period in self-quaratine in age class a
H_a	Cases in symptomatic period in hosptial in age class a
C_a	Cases in symptomatic period in ICU in age class a
V_a	Vaccinated and uninfected in age class a
$\tilde{E}_a$	Cases in incubation period and vaccinated before infection in age class a
$\tilde{I}_a$	Cases in symptomatic period and vaccinated before infection in age class a
$\tilde{Q}_a$	Cases in symptomatic period in self-quaratine and vaccinated before infection in age class a
N_a	Number of people in age class a
D_V	Number of vaccines distributed
W_V	Number of wasted vaccines distributed

Table 2: List of parameters and their description

Parameter	Description	Units
β_a	probability of transmission for age class a	per contact
$M_{a,c}$	Contact rate for person in age class a with a contact in age class c	contacts per day
$\tilde{M}_{a,c}$	Household contact rate for person in age class a with a contact in age class c	contacts per day
ν_a	Maximum vaccination rate for age class a	per day
$1/\sigma$	Average duration of the incubation period	days
τ	Early identification of symptatic cases	per contact
q	Proportion symptomatic cases being self-quaratined	
h	Weight for symptomatic cases being hospitalized	
c	Weight for symptomatic cases going to ICU	
$\tilde{q}$	Proportion symptomatic cases being self-quaratined and vaccinated before infection	
$\tilde{h}$	Weight for symptomatic cases being hospitalized and vaccinated before infection	
$\tilde{c}$	Weight for symptomatic cases going to ICU and vaccinated before infection	
$1/\gamma$	Average time from symptom onset to recovery	days
$1/\delta$	Average time from symptom onset to hosptialization	days
$1/\theta$	Average time from symptom onset to ICU	days
$\tilde{\mu}_a$	Death rate in general vaccinated population when ICU/Hosptial filling for age class a	per day
μ_a	Death rate in general population when ICU and Hosptial filling for age class a	per day
$1\mu_H$	Hosptial admission to death	day
$1\mu_C$	ICU admission to death	day
$m_{a,H}$	Weight for mortality in hospital fpr age class a	
$m_{a,C}$	Weight for mortality in ICU for age class a	
$1/\psi_H$	Average time from hospitalization to recovery	days
$1/\psi_C$	Average time from ICU to recovery	days
ε_a	Vaccine efficacy from preventing infection in vaccinated individual	
ξ_a	Vaccine reduction in transmission of infected individuall	
ω	Proportion of deaths in ICU relative to ICU nd hospital	
N_V	Number of vaccines	
B_H	Number of hosptial beds	
B_C	Number of ICU beds	