$$
\begin{aligned}
\frac{\mathrm{d}S_a}{\mathrm{d}t} &= -\beta\frac{S_a}{N_a}\sum_{j=1}^{A} M_{a,j}\left(I_{N,j}+I_{H,j}+(1-\xi_j)\tilde{I}_{N,j}+(1-\xi_j)\tilde{I}_{H,j}\right) \\
&- \beta\frac{S_a}{N_a}\sum_{j=1}^{A} \tilde{M}_{a,j}\left(Q_{N,j}+Q_{H,j}+(1-\xi_j)\tilde{Q}_{N,j}+(1-\xi_j)\tilde{Q}_{H,j}\right) - \nu_a\mathbb{H}(N_V-D_V)S_a \qquad (1)\\
\frac{\mathrm{d}E_a}{\mathrm{d}t} &= \beta\frac{S_a}{N_a}\sum_{j=1}^{A} M_{a,j}\left(I_{N,j}+I_{H,j}+(1-\xi_j)\tilde{I}_{N,j}+(1-\xi_j)\tilde{I}_{H,j}\right) \\
&+ \beta\frac{S_a}{N_a}\sum_{j=1}^{A} \tilde{M}_{a,j}\left(Q_{N,j}+Q_{H,j}+(1-\xi_j)\tilde{Q}_{N,j}+(1-\xi_j)\tilde{Q}_{H,j}\right) - \sigma E_a - \nu_a\mathbb{H}(N_V-D_V)E_a \qquad (2)\\
\frac{\mathrm{d}F_a}{\mathrm{d}t} &= -\sigma F_a + \nu_a\mathbb{H}(N_V-D_V)E_a \qquad (3)\\
\frac{\mathrm{d}I_{N,a}}{\mathrm{d}t} &= (1-q_a)(1-h_a)\sigma\left(E_a+F_a\right) - (1-f)\gamma I_{N,a} - f\tau I_{N,a} \qquad (4)\\
\frac{\mathrm{d}Q_{N,a}}{\mathrm{d}t} &= q_a(1-h_a)\sigma\left(E_a+F_a\right) - \gamma Q_{N,a} + f\tau I_{N,a} \qquad (5)\\
\frac{\mathrm{d}I_{H,a}}{\mathrm{d}t} &= (1-q_a)h_a\sigma\left(E_a+F_a\right) - \delta I_{H,a} \qquad (6)\\
\frac{\mathrm{d}Q_{H,a}}{\mathrm{d}t} &= q_a h_a\sigma\left(E_a+F_a\right) - \delta Q_{H,a} \qquad (7)\\
\frac{\mathrm{d}V_a}{\mathrm{d}t} &= -\beta(1-\varepsilon_a)\frac{V_a}{N_a}\sum_{j=1}^{A} M_{a,j}\left(I_{N,j}+I_{H,j}+(1-\xi_j)\tilde{I}_{N,j}+(1-\xi_j)\tilde{I}_{H,j}\right) \\
&- \beta(1-\varepsilon_a)\frac{V_a}{N_a}\sum_{j=1}^{A} \tilde{M}_{a,j}\left(Q_{N,j}+Q_{H,j}+(1-\xi_j)\tilde{Q}_{N,j}+(1-\xi_j)\tilde{Q}_{H,j}\right) + \nu_a\mathbb{H}(N_V-D_V)S_a \qquad (8)\\
\frac{\mathrm{d}\tilde{E}_a}{\mathrm{d}t} &= \beta(1-\varepsilon_a)\frac{V_a}{N_a}\sum_{j=1}^{A} M_{a,j}\left(I_{N,j}+I_{H,j}+(1-\xi_j)\tilde{I}_{N,j}+(1-\xi_j)\tilde{I}_{H,j}\right) \\
&+ \beta(1-\varepsilon_a)\frac{V_a}{N_a}\sum_{j=1}^{A} \tilde{M}_{a,j}\left(Q_{N,j}+Q_{H,j}+(1-\xi_j)\tilde{Q}_{N,j}+(1-\xi_j)\tilde{Q}_{H,j}\right) - \sigma\tilde{E}_a \qquad (9)\\
\frac{\mathrm{d}\tilde{I}_{N,a}}{\mathrm{d}t} &= (1-\tilde{q}_a)(1-\tilde{h}_a)\sigma\tilde{E}_a - (1-\tilde{f})\gamma\tilde{I}_{N,a} - \tilde{f}\tau\tilde{I}_{N,a} \qquad (10)\\
\frac{\mathrm{d}\tilde{Q}_{N,a}}{\mathrm{d}t} &= \tilde{q}_a(1-\tilde{h}_a)\sigma\tilde{E}_a - \gamma\tilde{Q}_{N,a} + \tilde{f}\tau\tilde{I}_{N,a} \qquad (11)\\
\frac{\mathrm{d}\tilde{I}_{H,a}}{\mathrm{d}t} &= (1-\tilde{q}_a)\tilde{h}_a\sigma\tilde{E}_a - \delta\tilde{I}_{H,a} \qquad (12)\\
\frac{\mathrm{d}\tilde{Q}_{H,a}}{\mathrm{d}t} &= \tilde{q}_a\tilde{h}_a\sigma\tilde{E}_a - \delta\tilde{Q}_{H,a} \qquad (13)\\
\frac{\mathrm{d}H_a}{\mathrm{d}t} &= (1-c_a)\delta I_{H,a} + (1-c_a)\delta Q_{H,a} + (1-\tilde{c}_a)\delta\tilde{I}_{H,a} + (1-\tilde{c}_a)\delta\tilde{Q}_{H,a} - \left(m_H\mu_H + (1-m_H)\psi_H\right)H_a \qquad (14)\\
\frac{\mathrm{d}C_a}{\mathrm{d}t} &= c_a\delta I_{H,a} + c_a\delta Q_{H,a} + \tilde{c}_a\delta\tilde{I}_{H,a} + \tilde{c}_a\delta\tilde{Q}_{H,a} - \left(m_C\mu + (1-m_C)\psi_C\right)C_a \qquad (15)\\
\frac{\mathrm{d}N_a}{\mathrm{d}t} &= -m_C\mu_C C_a - m_H\mu_H H_a \qquad (16)\\
\frac{\mathrm{d}D_V}{\mathrm{d}t} &= \mathbb{H}(N_V-D_V)\sum_{j=1}^{A}\nu_j(S_j+E_j) \qquad (17)\\
\frac{\mathrm{d}W_V}{\mathrm{d}t} &= \mathbb{H}(N_V-D_V)\sum_{j=1}^{A}\nu_j E_j \qquad (18)
\end{aligned}
$$

Table 1: List of variables and their description

Variable	Description
S_a	Susceptibles in age class a
E_a	Cases in incubation period in age class a
F_a	Cases in incubation period and vaccinated after infection in age class a
$I_{N,a}$	Cases in symptomatic period that will not need hospitalization in age class a
$I_{H,a}$	Cases in symptomatic period that will need hospitalization in age class a
$Q_{N,a}$	Cases in symptomatic period and self-isolated that will not need hospitalization in age class a
$Q_{H,a}$	Cases in symptomatic period and self-isolated that will need hospitalization in age class a
V_a	Vaccinated and uninfected in age class a
$\tilde{E}_a$	Cases in incubation period in age class a who recieved vaccinatino prior to infection
$\tilde{I}_{N,a}$	Vaccinated cases in symptomatic period that will not need hospitalization in age class a
$\tilde{I}_{H,a}$	Vaccinated cases in symptomatic period that will need hospitalization in age class a
$\tilde{Q}_{N,a}$	Vaccinated cases in symptomatic period and self-isolated that will not need hospitalization in age class a
$\tilde{Q}_{H,a}$	Vaccinated cases in symptomatic period and self-isolated that will need hospitalization in age class a
H_a	Number of cases in the hospital for age class a
C_a	Number of cases in the ICU for age class a
N_a	Number of people in age class a
D_V	Number of vaccines distributed
W_V	Number of wasted vaccines distributed

Table 2: List of parameters and their description

Parameter	Description	Units
β	Transmission rate	per contact
$M_{a,c}$	Contact rate for person in age class a with a contact in age class c	contacts per day
$\tilde{M}_{a,c}$	Household contact rate for person in age class a with a contact in age class c	contacts per day
ν_a	Maximum vaccination rate for age class a	per day
$1/\sigma$	Average duration of the incubation period	days
$1/\tau$	Time from symptom onset to isolation of symtpmatic case	per contact
q	Proportion symptomatic cases being self-quaratined	
h	Proportion symptomatic cases requiring hospitalization or ICU	
c	Proportion for symptomatic cases requiring hospitalization that go to ICU	
f	Weight for symptomatic cases isolating after symptom onset	
$\tilde{q}$	Proportion vaccinated symptomatic cases being self-quaratined	
$\tilde{h}$	Proportion vaccinated symptomatic cases requiring hospitalization or ICU	
$\tilde{c}$	Proportion for vaccinated symptomatic cases requiring hospitalization that go to ICU	
$\tilde{f}$	Weight for vaccinated symptomatic cases isolating after symptom onset	
$1/\gamma$	Average time from symptom onset to recovery	days
$1/\delta$	Average time from symptom onset to hosptialization	days
$1\mu_C$	ICU admission to death	day
$1\mu_H$	Hospital admission to death	day
m_C	Weight for mortality in ICU for age class a	
m_H	Weight for mortality in hosptial for age class a	
$1/\psi_H$	Average time from hospitalization to recovery	days
$1/\psi_C$	Average time from ICU to recovery	days
ε_a	Vaccine efficacy from preventing infection in vaccinated individual	
ξ_a	Vaccine reduction in transmission of infected individuall	
ω	Proportion of deaths in ICU relative to ICU nd hospital	
N_V	Number of vaccines	
B_H	Number of hosptial beds	
B_C	Number of ICU beds	