

$$\frac{dS_a}{dt} = -\beta \frac{S_a}{N_a} \sum_{j=1}^A \left(M_{a,j}(I_j + (1 - \xi_j)\tilde{I}_j) + \tilde{M}_{a,j}(Q_j + (1 - \xi_j)\tilde{Q}_j) \right) - \nu_a \left(1 - \frac{D_V}{N_V} \right) S_a \quad (1)$$

$$\frac{dE_a}{dt} = \beta \frac{S_a}{N_a} \sum_{j=1}^A \left(M_{a,j}(I_j + (1 - \xi_j)\tilde{I}_j) + \tilde{M}_{a,j}(Q_j + (1 - \xi_j)\tilde{Q}_j) \right) - \sigma E_a - \nu_a \left(1 - \frac{D_V}{N_V} \right) E_a \quad (2)$$

$$\frac{dF_a}{dt} = -\sigma F_a + \nu_a \left(1 - \frac{D_V}{N_V} \right) E_a \quad (3)$$

$$\begin{aligned} \frac{dI_a}{dt} = & (1 - q_a)\sigma(E_a + F_a) - \left(1 - h_a \left(1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) - c_a \left(1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \right) \gamma \frac{\gamma}{\gamma + \tau} I_a - h_a \left(1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) \delta I_a \\ & - c_a \left(1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \theta I_a - \left(1 - h_a \left(1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) - c_a \left(1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \right) \tau \frac{\tau}{\gamma + \tau} I_a \\ & - \mu_a \left((1 - \omega) \frac{\sum_{j=1}^A H_j}{B_H} + \omega \frac{\sum_{j=1}^A C_j}{B_C} \right) I_a \end{aligned} \quad (4)$$

$$\begin{aligned} \frac{dQ_a}{dt} = & q_a\sigma(E_a + F_a) - h_a \left(1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) \delta Q_a - c_a \left(1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \theta Q_a \\ & - \left(1 - h_a \left(1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) - c_a \left(1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \right) \gamma Q_a - \mu_a \left((1 - \omega) \frac{\sum_{j=1}^A H_j}{B_H} + \omega \frac{\sum_{j=1}^A C_j}{B_C} \right) Q_a \\ & + \left(1 - h_a \left(1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) - c_a \left(1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \right) \tau \frac{\tau}{\gamma + \tau} I_a \end{aligned} \quad (5)$$

$$\frac{dH_a}{dt} = h_a \delta \left(1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) (I_a + Q_a) + \tilde{h}_a \left(1 - \frac{\sum_{i=1}^A H_a}{B_H} \right) \delta (\tilde{I} + \tilde{Q}_a) - (m_{a,H} \mu_H + (1 - m_{a,H}) \psi_H) H_a \quad (6)$$

$$\frac{dC_a}{dt} = c_a \left(1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \theta (I_a + Q_a) + \tilde{c}_a \left(1 - \frac{\sum_{i=1}^A C_a}{B_C} \right) \theta (\tilde{I} + \tilde{Q}_a) - (m_{a,C} \mu_C + (1 - m_{a,C}) \psi_C) C_a \quad (7)$$

$$\frac{dV_a}{dt} = -\beta(1-\varepsilon_a)\frac{V_a}{N_a}\sum_{j=1}^A\left(M_{a,j}(I_j+(1-\xi_j)\tilde{I}_j)+\tilde{M}_{a,j}(Q_j+(1-\xi_j)\tilde{Q}_j)\right)+\nu_a\left(1-\frac{D_V}{N_V}\right)S_a \quad (8)$$

$$\frac{d\tilde{E}_a}{dt} = \beta(1-\varepsilon_a)\frac{V_a}{N_a}\sum_{j=1}^A\left(M_{a,j}(I_j+(1-\xi_j)\tilde{I}_j)+\tilde{M}_{a,j}(Q_j+(1-\xi_j)\tilde{Q}_j)\right)-\sigma\tilde{E}_a \quad (9)$$

$$\begin{aligned} \frac{d\tilde{I}_a}{dt} &= (1-\tilde{q}_a)\sigma(\tilde{E}_a)-\left(1-\tilde{h}_a\left(1-\frac{\sum_{i=1}^AH_a}{B_H}\right)-\tilde{c}_a\left(1-\frac{\sum_{i=1}^AC_a}{B_C}\right)\right)\gamma\frac{\gamma}{\gamma+\tau}\tilde{I}_a-\tilde{h}_a\left(1-\frac{\sum_{i=1}^AH_a}{B_H}\right)\delta\tilde{I}_a \\ &- \tilde{c}_a\left(1-\frac{\sum_{i=1}^AC_a}{B_C}\right)\theta\tilde{I}_a-\left(1-\tilde{h}_a\left(1-\frac{\sum_{i=1}^AH_a}{B_H}\right)-\tilde{c}_a\left(1-\frac{\sum_{i=1}^AC_a}{B_C}\right)\right)\tau\frac{\tau}{\gamma+\tau}\tilde{I}_a \\ &- \tilde{\mu}_a\left((1-\omega)\frac{\sum_{j=1}^AH_j}{B_H}+\omega\frac{\sum_{j=1}^AC_j}{B_C}\right)\tilde{I}_a \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{d\tilde{Q}_a}{dt} &= \tilde{q}_a\sigma\tilde{E}_a+\left(1-\tilde{h}_a\left(1-\frac{\sum_{i=1}^AH_a}{B_H}\right)-\tilde{c}_a\left(1-\frac{\sum_{i=1}^AC_a}{B_C}\right)\right)\tau\frac{\tau}{\gamma+\tau}\tilde{I}_a \\ &- \left(1-\tilde{h}_a\left(1-\frac{\sum_{i=1}^AH_a}{B_H}\right)-\tilde{c}_a\left(1-\frac{\sum_{i=1}^AC_a}{B_C}\right)\right)\gamma\tilde{Q}_a-\tilde{h}_a\left(1-\frac{\sum_{i=1}^AH_a}{B_H}\right)\delta\tilde{Q}_a \\ &- \tilde{\mu}_a\left((1-\omega)\frac{\sum_{j=1}^AH_j}{B_H}+\omega\frac{\sum_{j=1}^AC_j}{B_C}\right)\tilde{Q}_a-\tilde{c}_a\left(1-\frac{\sum_{i=1}^AC_a}{B_C}\right)\theta\tilde{Q}_a \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{dN_a}{dt} &= -m_{a,C}\mu_C C_a - m_{a,H}\mu_H H_a - \mu_a\left((1-\omega)\frac{\sum_{j=1}^AH_j}{B_H}+\omega\frac{\sum_{j=1}^AC_j}{B_C}\right)(I_a+Q_a) \\ &- \tilde{\mu}_a\left((1-\omega)\frac{\sum_{j=1}^AH_j}{B_H}+\omega\frac{\sum_{j=1}^AC_j}{B_C}\right)(\tilde{I}_a+\tilde{Q}_a) \end{aligned} \quad (12)$$

$$\frac{dD_V}{dt} = \sum_{j=1}^A\left(\nu_j\left(1-\frac{D_V}{N_V}\right)(S_j+E_j)\right) \quad (13)$$

$$\frac{dW_V}{dt} = \sum_{j=1}^A\left(\nu_j\left(1-\frac{D_V}{N_V}\right)(E_j)\right) \quad (14)$$

$$(15)$$

Table 1: List of variables and their description

Variable	Description
S_a	Susceptibles in age class a
E_a	Cases in incubation period in age class a
F_a	Cases in incubation period and vaccinated after infection in age class a
I_a	Cases in symptomatic period in age class a
Q_a	Cases in symptomatic period in self-quarantine in age class a
H_a	Cases in symptomatic period in hospital in age class a
C_a	Cases in symptomatic period in ICU in age class a
V_a	Vaccinated and uninfected in age class a
\tilde{E}_a	Cases in incubation period and vaccinated before infection in age class a
\tilde{I}_a	Cases in symptomatic period and vaccinated before infection in age class a
\tilde{Q}_a	Cases in symptomatic period in self-quarantine and vaccinated before infection in age class a
N_a	Number of people in age class a
D_V	Number of vaccines distributed
W_V	Number of wasted vaccines distributed

Table 2: List of parameters and their description

Parameter	Description	Units
β_a	probability of transmission for age class a	per contact
$M_{a,c}$	Contact rate for person in age class a with a contact in age class c	contacts per day
$\tilde{M}_{a,c}$	Household contact rate for person in age class a with a contact in age class c	contacts per day
ν_a	Maximum vaccination rate for age class a	per day
$1/\sigma$	Average duration of the incubation period	days
τ	Early identification of symptomatic cases	per contact
q	Proportion symptomatic cases being self-quarantined	
h	Weight for symptomatic cases being hospitalized	
c	Weight for symptomatic cases going to ICU	
\tilde{q}	Proportion symptomatic cases being self-quarantined and vaccinated before infection	
\tilde{h}	Weight for symptomatic cases being hospitalized and vaccinated before infection	
\tilde{c}	Weight for symptomatic cases going to ICU and vaccinated before infection	
$1/\gamma$	Average time from symptom onset to recovery	days
$1/\delta$	Average time from symptom onset to hospitalization	days
$1/\theta$	Average time from symptom onset to ICU	days
$\tilde{\mu}_a$	Death rate in general vaccinated population when ICU/Hospital filling for age class a	per day
μ_a	Death rate in general population when ICU and Hospital filling for age class a	per day
$1/\mu_H$	Hospital admission to death	day
$1/\mu_C$	ICU admission to death	day
$m_{a,H}$	Weight for mortality in hospital for age class a	
$m_{a,C}$	Weight for mortality in ICU for age class a	
$1/\psi_H$	Average time from hospitalization to recovery	days
$1/\psi_C$	Average time from ICU to recovery	days
ε_a	Vaccine efficacy from preventing infection in vaccinated individual	
ξ_a	Vaccine reduction in transmission of infected individual	
ω	Proportion of deaths in ICU relative to ICU and hospital	
N_V	Number of vaccines	
B_H	Number of hospital beds	
B_C	Number of ICU beds	