Finite Element Based Reduced Order Modeling of Micro Electro Mechanical Systems (MEMS)

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ABSTRACT

The paper demonstrates reduced order modeling (ROM) of the strongly coupled system level dynamic analysis of micro electro mechanical systems (MEMS) by lumped finite elements (FE) and substructuring implemented in the ANSYS/Multiphysics program.

Index terms - Micro Electromechanical Systems (MEMS), Finite elements (FE), Coupled analysis, Reduced order modeling (ROM)

1 INTRODUCTION

With the rapid advancement of micromachining technologies there has been an ever-increasing need for simulation tools to perform virtual prototyping of Micro Electro Mechanical Systems (MEMS). Complicating the simulation issue is the need to perform sophisticated multiphysics analyses of the physical components as well as a broad system level simulation using electronic design automation (EDA) tools.

The ANSYS finite element (FE) software product line is suited for performing the myriad of physics simulations required for MEMS [1]. Release 5.6 of the ANSYS/Multiphysics program represents a major milestone in providing simulation tools to accurately characterize and simulate static and dynamic performance of electrostatically actuated MEMS devices [2,3].

Full dynamic FE simulation of coupled electrostatic-structural problems is prohibitively expensive. The need exists to create simplifications and provide accurate, high-fidelity time-harmonic and time-domain solutions in a fast and efficient manner. The simplification process and simulation thereof is often referred to as Reduced Order Modeling (ROM). ROM takes on two forms in ANSYS, one consisting of lumped finite elements (spring, mass, damper, transducers, capacitor, inductor, resistor, etc.) and substructuring of large linear systems when reduction to simple lumped elements is not feasible or is cumbersome. ANSYS offers simultaneous analysis with distributed finite elements, lumped circuit elements and substructures. The ultimate goal will be to extract accurate reduced models for use in system simulators running under VHDL-AMS or other languages.

2 LUMPED FINITE ELEMENTS

2.1 Classical Circuit Elements

Circuit elements are frequently used to describe the response of a distributed system in terms of lumped equivalents. The ANSYS program contains many zero dimensional lumped elements spanning multiple physics. The focus here is on electro-mechanical coupling via lumped elements.

On the mechanical side, linear and nonlinear lumped elements are available to model discrete spring, mass, and dampers. The elements work on the principle of “force” as the through variable, and “displacement” as the across variable. Typical elements used in the simulations in this paper include the ANSYS elements MASS21 for mass, and COMB14 for springs and dampers.

For electrical characterization, a lumped finite element has been developed to simulate linear electrical systems (CIRCU124). The element works on the principle of “current” as the through variable and “voltage” as the across variable. Options for this element include modeling a resistor, capacitor, inductor, current and voltage sources, and controlled sources.

2.2 Electro Mechanical Transducer

The coupling between electrical and mechanical lumped elements was achieved through the development of an electro-mechanical transducer element (TRANS126) introduced in ANSYS 5.6 [2,4]. Coupling between electrostatic forces and mechanical forces can be characterized by mapping the capacitance as a function of the motion of the device. The transducer element converts electrostatic energy to mechanical energy and visa versa as well as store electrostatic energy, thus completely modeling the coupled system. TRANS126 takes on the form of a lumped element with voltage and structural DOFs as across variables and current and force as through variables. Input for the element consists of a capacitance-stroke relationship that can be derived from electrostatic field solutions. The element can characterize up to three independent translation degrees of freedom at any point to simulate 3-D coupling. Thus, the electrostatic mesh is removed from the problem domain and replaced by a set of TRANS126 elements hooked to the mechanical and electrical model providing a ROM of a coupled electrostatic-structural system (Fig. 1 and 2).

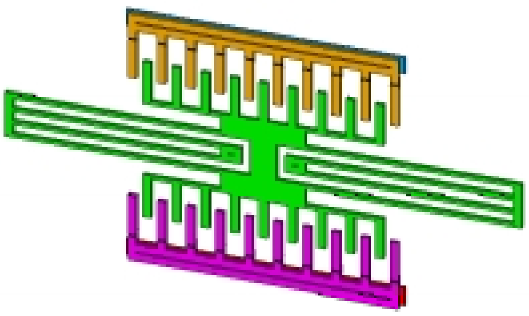

Figure 1: Linear comb drive resonator model

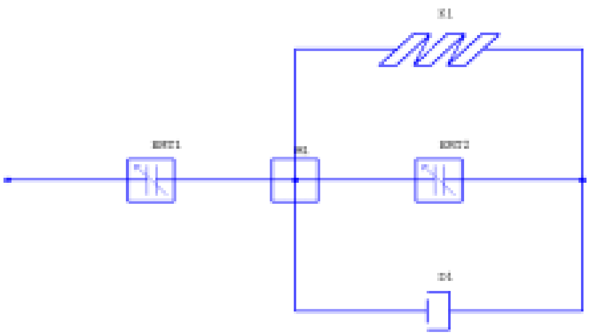

Figure 2: ROM using lumped elements

The through and across variables are related by:

$$i = \frac{d}{dt}\left[C(x)u\right] = uv\frac{dC}{dx} + Cw = i(u,\ x,\ w,\ v) \qquad (1)$$

$$f = \frac{d}{dx}\left[\frac{C(x)}{2}u^2\right] = f(u,x,w,v) \qquad (2)$$

where C(x) is the capacitance and w is the voltage rate. The first term in (1) is the motion induced current; the second is the current due to voltage change with fixed capacitor plates. (2) can be easily obtained from virtual work principle.

For small changes the through variables are:

$$i = i_0 + K_{uu}du + K_{ux}dx + D_{uu}dw + D_{ux}dv \qquad (3)$$

$$f = f_0 + K_{xu}du + K_{xx}dx + D_{xu}dw + D_{xx}dv \qquad (4)$$

where:

$$K_{uu} = \frac{di}{du}; K_{ux} = \frac{di}{dx}; K_{xu} = \frac{df}{du}; K_{xx} = \frac{df}{dx}$$

$$D_{uu} = \frac{di}{dw}; D_{ux} = \frac{di}{dv}; D_{xu} = \frac{df}{dw}; D_{xx} = \frac{df}{dv}$$

where i_0 and f_0 are the entries of the coupled system Newton-Raphson restoring force vector corresponding to a large signal nonlinear solutions vector u_0 and x_0; K_{uu}, K_{ux}, K_{xu} and K_{xx} as well as D_{uu}, D_{ux}, D_{xu} and D_{xx} are the entries of the tangent coupled system stiffness and damping matrices, respectively.

2.3 Lumped and Finite Elements Together

The lumped elements have through and across variables corresponding to degrees of freedom and reaction forces of standard finite elements. The circuit equations are formulated by variational energy principles, thus the lumped elements remain compatible with distributed finite element regions. This enables nonlinear mechanical devices to be modeled with standard finite elements and connected to lumped elements where applicable. Fig. 3 illustrates lumped transducer elements (representing the drive and pick-up comb) attached to a distributed finite element model. The combined mechanical, electrical and transducer lumped elements provide a convenient tool for rapidly simulating static, time-harmonic, time-transient, and eigenfrequency simulations of strongly coupled systems. When the system is nonlinear, time-harmonic and eigenvalue analyses are performed for small changes around operation solutions. This is ofter referred to as small signal prestressed harmonic and eigenvalue analysis. Naturally a prestress analysis should be preceeded by a large signal static or harmonic simulation to obtain a bias solution.

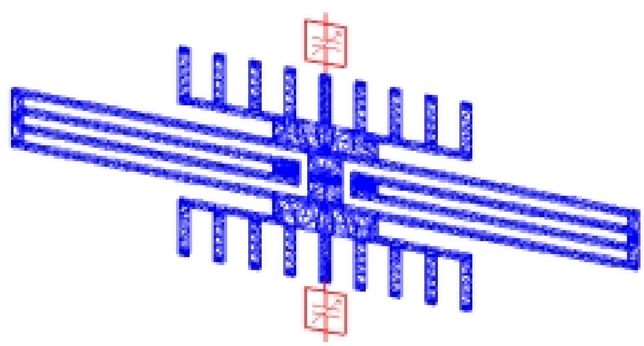

Figure 3: Finite element model with ROM transducers replacing stationary combo

3 SUBSTRUCTURING

Another useful technique for large finite element systems is a matrix reduction process that can reduce the system matrices to a smaller subset of unknowns. This allows for an efficient reduced order model, which accurately captures the three-dimensional response of a system. The method is applicable to any kind of simulation: large signal static and time-transient or small signal prestressed harmonic and eigenvalue analyses. The substructure can contain lumped elements, or work along side lumped elements.

Consider a matrix representation of the transient dynamic equations:

$$\mathbf{M}\,\ddot{\mathbf{u}} + \mathbf{D}\,\dot{\mathbf{u}} + \mathbf{K}\,\mathbf{u} = \mathbf{f} \qquad (5)$$

where **M**, **D**, and **K** represent the mass, damping, and stiffness matrices, **u** the degree of freedom vector, and **f** the forcing function; dot denotes time derivative. A reduced set of “master” degrees of freedom can be defined which include nodes that attach to lumped elements, other distributed components, and interior nodes sufficient to capture the dynamic response of the system. Given the master degrees of freedom, the above matrix equation is partitioned into master and “slave” degrees of freedom. The slave degrees of freedom are condensed out of the matrix equation. Matrix condensation for the stiffness matrix is exact, condensation for the damping and mass matrices is approximated as detailed by Guyan [5]. The number of master degrees of freedom required to accurately represent the linear system depends on the number of modes present in the dynamic response. For MEMS structures which excite only the lowest modes, only a small fraction of the total system degrees of freedom are required.

Substructuring techniques can greatly aid the user in accurately characterizing a large system in terms of only a few degrees of freedom. The method removes the burden of computing lumped element parameters such as stiffness, material damping, and effective mass, which can be cumbersome for general three-dimensional motion.

4 EXAMPLE PROBLEMS

A linear comb drive resonator shown in Figure 1 is modeled to compute the transfer function relating input voltage to output current I_o/V_i. Several simulations are made. In all cases, the comb drive is replaced with the TRANS126 transducer. A single transducer is used for the drive and pick-up combs. A three-dimensional finite element model of the comb structure is modeled over the stroke-range of the device to compute the capacitance as a function of stroke. This relationship forms the input parameters for the transducer element. Only uniaxial motion is considered at the connection point to the resonator.

The first simulation uses a complete lumped element approach as shown in Figure 3. The stiffness of the system is computed from static simulations where applied force and displacement are recorded to compute stiffness. The effective mass is determined from an eigenvalue analysis. The damping factor is preset to correspond to a Q-factor of 100.

The second simulation uses the substructuring technique to reduce a distributed finite element model of the resonator (mass and beam system structure shown in Figure 2) to a small subset of degrees of freedom. A ½ symmetry view of the substructure outline, transducer, and dampers are shown in Figure 4. The original resonator structure contained 18201 degrees of freedom. The substructure simulation was run using 100 master degrees of freedom. The master degrees of freedom are automatically selected by the program and consider full three-dimensional motion. The excitation force generated by a voltage applied to the transducer element is transmitted to the substructure at a single master node. In a similar fashion, the pick-up comb displacement and current is obtained from a second transducer element connected at another single master node connection point. The solution time for the harmonic sweep (60 frequencies) takes only seconds with the substructure model.

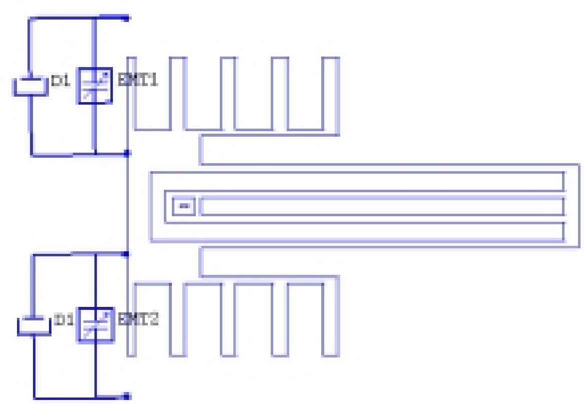

Figure 4: ROM using lumped elements and substructure

The system transfer function results are illustrated in Figure 5. Results are illustrated about the expected eigen frequency (93 kHz.). Note that both the lumped and substructure results track very closely. A slight perturbation in the results of the substructure run is noticed around 109 kHz. This frequency was confirmed to be at another resonance eigen frequency. We see the ability of the substructure to pick up the additional resonance effects which are completely lost in the fully lumped model.

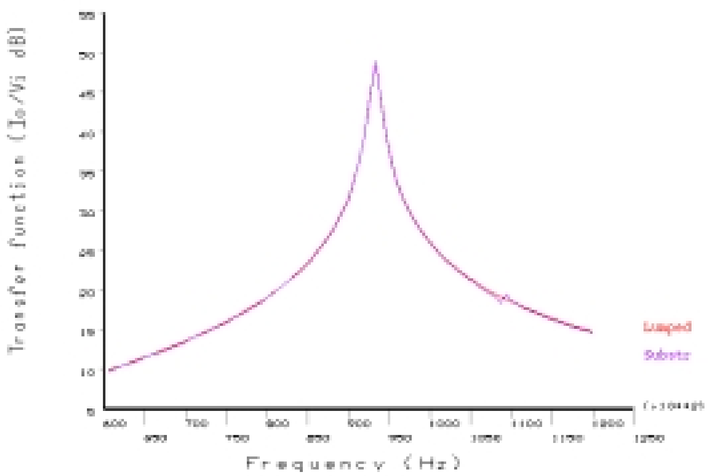

Figure 5: Frequency response
(output current/input voltage)

Nonlinear large-signal simulation can easily be performed using the combined mechanical, electrical, and transducer element. Figure 6 illustrates a parallel plate electrostatic transducer coupled to a mechanical resonator. The problem is fully described in [6]. Displacement of the mass is tracked over a series of pulse voltage excitations and is shown in Figure 7. Results are in good agreement with [6].

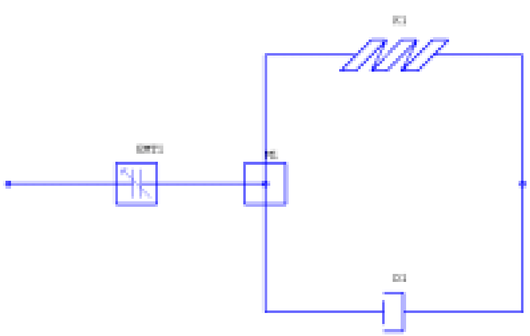

Figure 6: ROM of a parallel-plate electrostatic
transducer-resonator

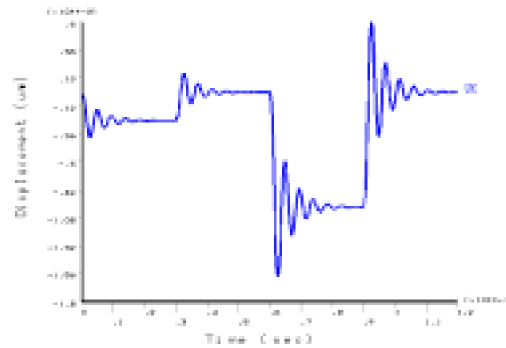

Figure 7: Large-signal nonlinear displacement response

4 CONCLUSIONS

Lumped finite elements have been developed for mechanical, electrical, and field coupling to enable rapid prototyping of fully coupled electro-mechanical systems. The lumped elements are compatible with standard finite element systems and thus can be linked to distributed finite element models. Large linear finite element systems can make use of substructuring techniques to reduce the total number of unknowns to a fraction of the original set, and yet capture the dynamic response of the distributed system. Substructures can be directly linked to lumped elements to effectively model complex electro-mechanical systems. Future work will center on deriving equivalent behavioral models directly from the reduced FE system for incorporation into EDA simulators to produce high fidelity models of physical systems components.

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