European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS 2000
Barcelona, 11-14 September 2000

ADVANCED ANALYSIS OF STRUCTURAL RELIABILITY USING COMMERCIAL FE-CODES

Christian Bucher*, Dagmar Hintze*, and Dirk Roos*

*Institute of Structural Mechanics, Bauhaus-University Weimar
Marienstraße 15, D-99423 Weimar, Germany
e-mail: Christian.Bucher@bauing.uni-weimar.de
Web page: http://www.uni-weimar.de/Bauing/ism/

Key words: Stochastic Analysis, Structural Reliability, Response Surface Method, Nonlinear Analysis, Monte Carlo Simulation, Directional Sampling

Abstract. *Recent trends in structural mechanics applications of Finite Element codes show an increasing demand for efficient stochastic analysis tools. While in principle virtually all stochastic problems can be solved by Monte-Carlo simulation, the computational cost may be prohibitively large. Therefore more effective approximations based on e.g. the Response Surface Method become quite popular. Alternatively, advanced Monte-Carlo methods such as importance sampling or adaptive sampling can be utilized. Depending on the class of problem to be analyzed, these methods may even be more efficient than the Response Surface Method.*

The paper focuses on two main issues, i.e.

- *The application of existing code for probabilistic analysis together with existing Finite Element Code for advanced Monte Carlo analysis*
- *Presentation of a local-global approximation strategy for the response surface method. This so-called "weighted radii" approximation appears to provide the flexibility and accuracy needed for the reliability analysis of complex structures.*

A numerical example from structural analysis under static loading conditions shows the applicability of these concepts. In particular, the stochastics package SLang *is utilized together with the FE-code ANSYS in the context of geometrically and materially nonlinear analysis.*

1 INTRODUCTION

Computational Mechanics traditionally relates mechanical modeling with numerical analysis tools. Considering the properties of the mechanical or structural model realistically it is necessary to take into account some uncertainty. This uncertainty can be conveniently described in terms of probability measures, such as distribution functions. It is a major goal of stochastic computational mechanics to relate the uncertainties of the input variables to the uncertainty of the structural performance. Based on their meaning in structural analysis, the sources of uncertainty may be classified in three categories, i.e. actions (e.g. loads, thermal stress), system data (geometry, boundary conditions, mass density), and structural resistance (yield limit, thickness).

Within many engineering fields, structural design must meet challenging demands of cost-effectiveness. This leads to light-weight, highly flexible, and consequently vulnerable structures. In order to assess potential risks associated with such a design, the structural analysis must take into account available information on the randomness of loads and structural properties. It means that the analysis should include reliability estimates in an appropriate manner. The stochastic models for these uncertain parameters can be one of the following:

- Random Variables (constant in time and space)
- Random Fields (correlated fluctuations in space)
- Random Processes (correlated fluctuations in time)

Within a computer implementation it becomes necessary to discretize random fields/processes in time and space. This immediately leads to a considerable number of random variables.

However, although the need for stochastic analysis is apparent, there is a lack of software which enables the application of reliability methods by engineers not specialized in the field. Currently, stochastic analysis software is being developed which may be applied successfully by practitioners. Within the scope of practical structural analysis, the concept of Probability Integrator (PI) with external Finite Element Analysis (FEA) seems to be most appropriate. In this case, the software can be considered as symbiosis of two functionally independent stand-alone applications. Alternatively, integrated software for Stochastic Structural Analysis (SSA) is currently being developed in several places.

Several examples for these classes are available. Some of them are mentioned in the following (this list is incomplete). In the first category (PI/FEA), I would like to mention as examples the packages ISPUD [2] PROBAN [11], RYFES [13]. In the category SSA, there are e.g. the packages NESSUS [12], CALREL [10] STRUREL/COMREL [9], SLang [15].

The aims of stochastic analysis include safety measures such as time-invariant failure probabilities or safety indices, response variability in terms of standard deviations or time-variant first passage probabilities. The available solution methods for these tasks are exact methods (numerical integration, Monte Carlo simulation - MCS), approximate (FORM/SORM, Response Surface Method - RSM) or other techniques (e.g. based on Markov vector theory in stochastic dynamics). In general, these solution methods determine the strategy and drive structural analysis through a Finite-Element package.

Although Monte Carlo methods are most versatile, intuitively clear, and well understood, the computational cost (the number of FE-runs required) in many cases is prohibitive. Thus approximations become important, which can be based e.g. on the RSM. For the feasibility of Response Surface approaches it is quite essential to reduce the number of variables to a tractable amount. This may require extensive sensitivity analysis in order to identify the relevant random variables. This is particularly important for random variables arising from discretization of random fields or processes. In this context, close coupling between the tools for stochastic and structural analyses is essential. Such coupling is provided by the so-called “Stochastic Finite Element Methods”.

Simplifications can be based on the following items

- Representation of the random phenomenon in terms of uncorrelated random variables. This is frequently called “Spectral Representation”. It aims at a reduction of the number of random variables required [8, 5].
- Sensitivity Analysis of the structural response with respect to the random variables [1]. Again, this aims at reducing the number of random variables needed.
- Concentrate random sampling in the region which contributes most to the total failure probability [3]. This is generally called “Importance Sampling”. It is important to note that most importance sampling strategies work best with a low number of random variables.
- Approximation of the structural response by a class of simple mathematical functions. This is the so-called “Response Surface Method” [4] Again, it is vital that the number of random variables be kept small.

As a very simplistic rule-of-the-thumb, Fig.1 gives the accuracy/speed ratio for some solution methods as mentioned above.

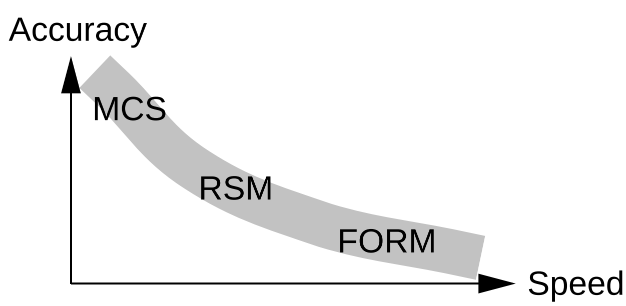

Figure 1: Performance of methods for stochastic analysis.

However, there are situations in which MCS can be comparably fast or FORM can be comparably accurate.

2 SOFTWARE CONCEPT

Software for stochastic structural analysis should encompass state-of-the-art technology in both the structural analysis as well stochastic analysis parts. This does not necessarily imply that one single program package must include everything. The following items, however, should at least be made accessible:

a) Nonlinear Finite Element modeling

b) Dynamic analysis and stability

c) Randomly spatially distributed loads and structural properties

d) FORM/SORM reliability measures

e) Response Surface methodology and Monte Carlo Simulation

Here the first two items fall into the traditional FEA domain and the last two are parts of most PI packages. The item c) lies somewhere in between, and is partly covered by codes in the area of Stochastic Finite Elements.

In many applications of stochastic structural analysis (such as e.g. structural reliability assessment), the PI-part controls the flow of execution, i.e. the structural analysis to be performed by FEA-part. This is of specific importance with adaptive strategies (i.e. if the next step of the PI depends on the answers from the FEA), requiring continuous two-way exchange of information between PI and FEA.

Following a long tradition in software development for structural engineering applications, it seems useful to formulate tasks in small, easily controllable steps. It should be possible to combine these steps into larger segments which can be executed repeatedly. Such a solution can be achieved by implementing a problem oriented module set in which the individual modules pertain to stochastic and structural analysis. A dedicated software package for stochastic structural analysis along this line has conceptually been presented several years ago by Bucher et al. [6] and substantiated subsequently by Bucher and Schuëller [7]. In order to meet the above mentioned requirements regarding interaction between PI and FEA, a software system comprising data management, user interactivity, finite element analysis, and reliability tools - SLang - has been developed [15]. The name is derived from the term **S**tructural **Lang**uage. In a way, SLang can be seen as a toolbox containing the basic software products for both PI and FEA (amongst others) which can interact smoothly and transparently. SLang integrates Finite Elements with stochastic modeling at a level which appears to be sufficient for a wide range of engineering problems. In addition, the recent developments in the area of component software allow further enhancements regarding inter-process communication and parallel processing which can be utilized advantageously for stochastic analysis [23].

3 RESPONSE SURFACE METHOD

3.1 General Remarks

The structural behavior near the structural failure state is most important in the reliability analysis. The structural design parameters, such as loadings, material parameters and geometry, are the set of basic random variables **X** which determine the probabilistic response of structural systems. The failure condition is defined by a deterministic limit state function

$$g(\mathbf{x}) = g(x_1, x_2, \ldots, x_n) \leq 0 \tag{1}$$

The failure probability of a structural system is given by

$$p_f = P[\mathbf{X} : g(\mathbf{X}) \leq 0] = \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} I(g(\mathbf{x}))\, f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{2}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function of the basic random variables and where $I(g(\mathbf{x}))$ is an indicator function with

$$I(g(\mathbf{x})) = \begin{cases} 0 & : \quad g(\mathbf{x}) > 0 \\ 1 & : \quad g(\mathbf{x}) \leq 0 \end{cases} \tag{3}$$

An unbiased estimator of p_f is its sample average

$$\bar{p}_f = \frac{1}{p} \sum_{k=1}^{p} I(g(\mathbf{x}_k)) \tag{4}$$

with p basic random variables realizations. Normally, $g(\mathbf{x})$ is described implicitly, e.g. through an algorithmic procedure within Finite Element analysis. Alternatively, the original limit state function can be replaced by a response surface function which has polynomial [4, 17, 20, 22] or polyhedral [18, 19, 23] form. One of the major advantages of the response surface method lies in its potential to selectively determine the number of structural analyses of the support points. This is especially helpful if some overall knowledge on the system behavior - particularly near to the failure region - is a priori available. By such means the computational effort can be substantially reduced. On the other hand, the global interpolation schemes widely used in the application of the response surface method (e.g. second order polynomials) can be quite misleading due to the lack of information in certain regions of the random variable space. It is therefore required to avoid such undesirable interpolation errors at reasonable computational effort. The polynomial approximations are not sufficiently flexible. They always need a certain amount of limit state check points in unimportant directions in order to avoid approximation problems. Moreover, the maximum number of limit state check points is limited too. The polyhedral response surface functions avoid some problems but need a rather high of check points for a closed safe domain. The following new approximation scheme will solve these difficulties.

3.2 Weighted radii response surfaces

Let the normalized limit state check points $P_i(\mathbf{x})$ be defined by

$$\mathbf{p}_i = \mathbf{l}_i - \mathbf{m} \tag{5}$$

with the limit state check point vector in cartesian coordinates $\mathbf{l}_i(\mathbf{x})$ and the mean value vectors $\mathbf{m}(\mathbf{x})$.

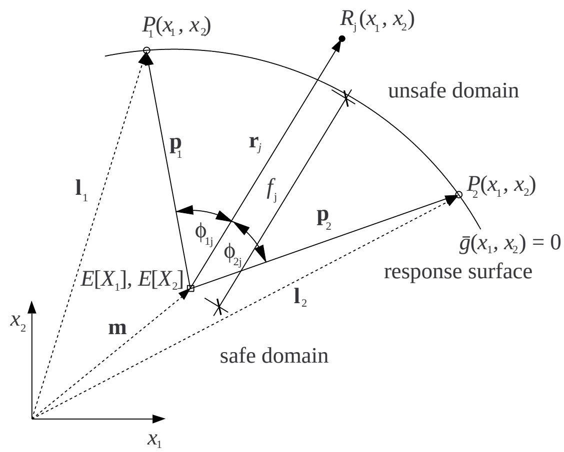

Figure 2: Limit state approximation using weighted radii.

The normalized sampling point $R_j(\mathbf{x})$ is calculated from a random sample $\mathbf{x}$ by subtracting the means according to $\mathbf{r}(\mathbf{x}) = \mathbf{x} - \mathbf{m}$. Thus the angle between the directions from the mean to sampling point j and any limit state check point i, respectively, is given by

$$\cos\phi_{ij} = \frac{\mathbf{p}_i^T \mathbf{r}_j}{\|\mathbf{p}_i\| \|\mathbf{r}_j\|} \qquad 0 \le \phi_{ij} \le \pi \tag{6}$$

Assume that the weights of any sampling point are given by

$$w_{ij} = \frac{1}{\phi_{ij}} \tag{7}$$

then the factors

$$f_j = \frac{\sum_i \|\mathbf{p}_i\| w_{ij}}{\sum_i w_{ij}} \tag{8}$$

as shown in Fig. 2, define the response surface function

$$\bar{\mathbf{g}}(\mathbf{x}) = f_j - \|\mathbf{r}(\mathbf{x})\| \tag{9}$$

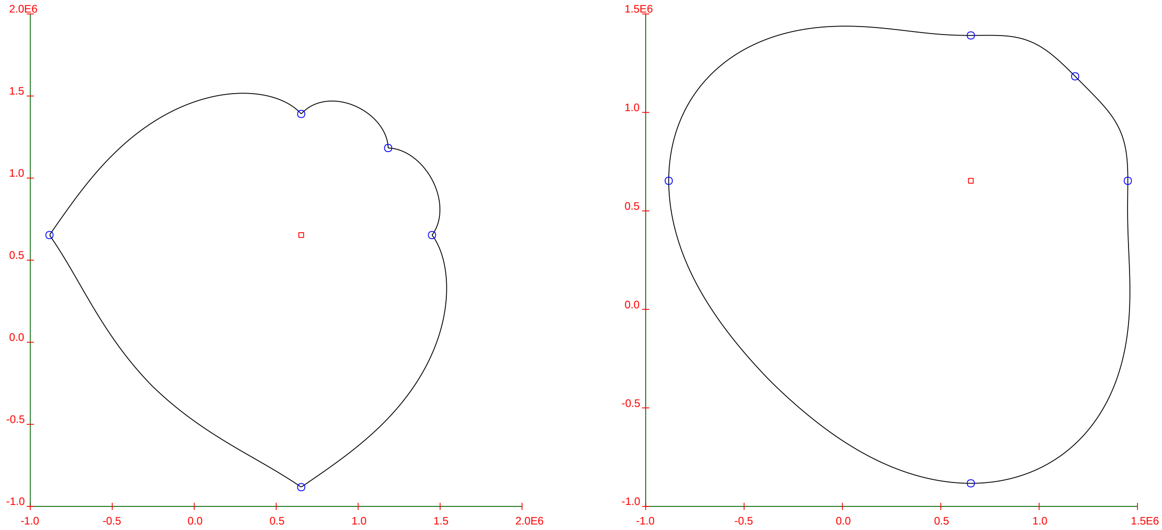

Figure 3: Response surface $\bar{\mathbf{g}}(\mathbf{x}) = 0$ using a) linear weights, b) nonlinear weights.

on the cartesian coordinates.

The simple assumption (7) generates a closed continuous response surface function. For an elimination of the numerical discontinuity in $\phi_{ij} = 0$ we introduce a small value ε in the equation (7)

$$w_{ij} = \frac{1}{\phi_{ij} + \varepsilon} \tag{10}$$

The response surface function $bar\mathbf{g}(\mathbf{x}) = 0$ in two dimensions based on 5 limit state check points using linear weights is shown in Fig. 3a. Nonlinear weights such as

$$w_{ij} = \left(\frac{1}{\phi_{ij} + \varepsilon}\right)^2 \tag{11}$$

can be introduced which will form a differentiable function in the supporting points, as shown in Fig. 3b. The weighted radii type response surface is of course convex in any case. It should be mentioned that application of this type of response surface should preferably be done in standard Gaussian space.

4 APPLICATION EXAMPLE

4.1 Parabolic Shell under Vertical and Horizontal Load

This example is intended to show the application of ANSYS as Finite Element Analyzer within the PI/FEA concept. Herein the mechanical system is a parabolic shell, as sketched in this Fig.4, subjected to horizontal (11 x H) and vertical loads (40 x V). The constitutive relation of the shell material is von Mises plasticity without hardening. The structure is modeled by 120 geometrically nonlinear shell elements (SHELL93). It is assumed that the vertical and

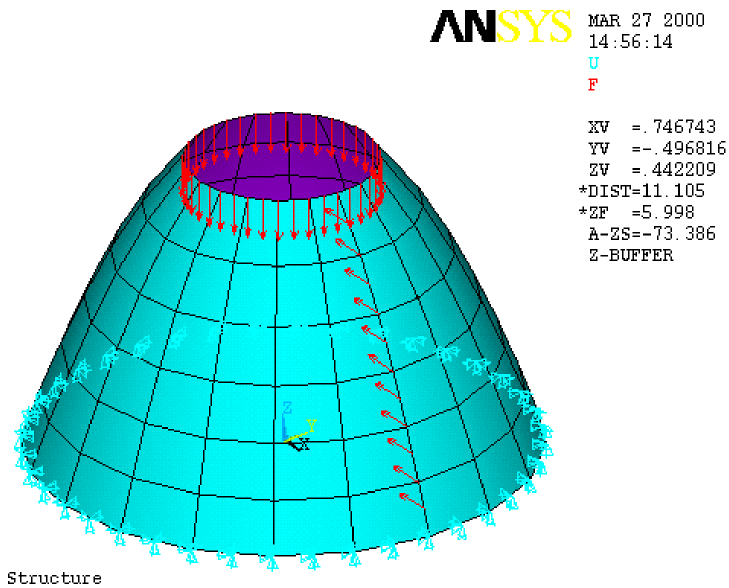

Figure 4: Structural model (parabolic shell) with loads

horizontal loads as well as the shell thickness are random variables with statistics given in Table 1.

Table 1: Statistical data for loads and thickness

	Mean	Std.Dev	Type
Horizontal Load	10 MN	1 MN	normal
Vertical Load	10 MN	1MN	normal
Thickness	0.32 m	0.01 m	normal

Limit state points were determined by means of incremental analysis. The failure in equilibrium iteration was used to determine collapse of the structure. The distribution of the von-Mises stress at collapse under equal horizontal and vertical load is shown in Fig.5. In obtaining these result, the loads were incremented proportionally up to collapse of the structure. Hence no path dependence was considered in the analysis.

A set of 540 failure points was simulated using Directional Sampling as shown in Fig. 6. These points were generated for a set of 10 discrete thicknesses (equally spaced from 0.30 m to 0.36 m, chosen according to the respective probability law, cf. Fig.8) which were held constant during the search for limit states. This is due to the fact that during an incremental collapse analysis within ANSYS it is not possible to change the structural system. The individual thicknesses can be clearly seen in Fig.6. A section through these points (for the mean thickness) is shown in Fig.7. Each thickness leads to a conditional failure probability which is weighted with the occurrence probability of the respective thickness (cf. Fig.8). For the statistical data as given in

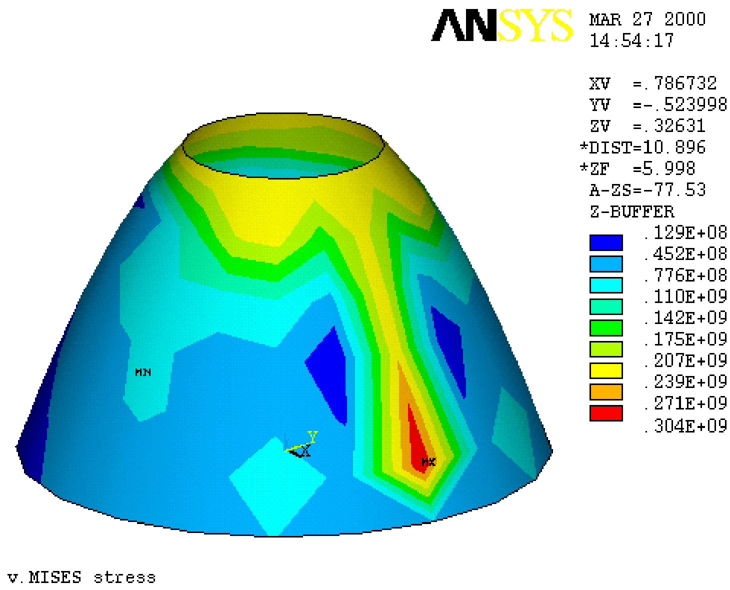

Figure 5: von-Mises stress at collapse under equal horizontal and vertical load

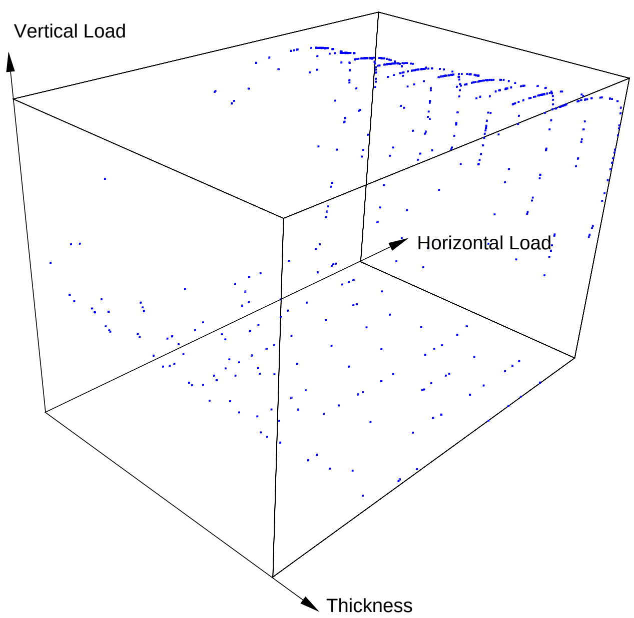

Figure 6: Points on limit state surface

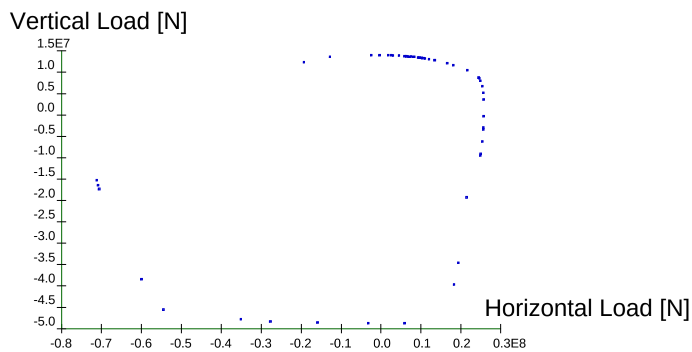

Figure 7: Points on limit state surface for mean thickness

Table 1 the failure probability is calculated as $p_f = 3.35 \cdot 10^{-5}$ with a statistical error (standard deviation) of $1.07 \cdot 10^{-6}$. This corresponds to a safety index $\beta = 3.976$. An analysis based on

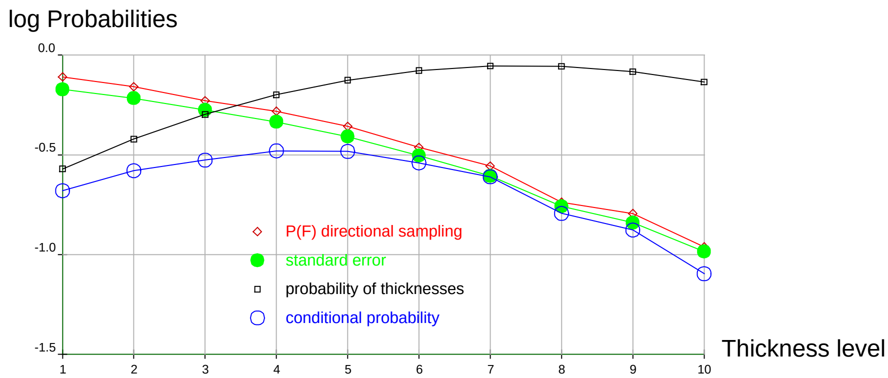

Figure 8: Conditional failure probabilities vs. thickness

the weighted radii response surface method was carried out with a resulting failure probability of $p_f = 7.82 \cdot 10^{-5}$. This corresponds to a safety index $\beta = 3.781$. The agreement with the exact result from directional sampling is quite good. The corresponding response surface is shown in Fig.9. In the same figure, an adapted response surface concentrating the samples in the most important region is indicated as well. It is quite important to note that re-analysis with different statistical data does *not* necessarily require repetition of the FE analysis. This makes the PI/FEA concept especially attractive for problems with deterministic system data but with several sets of (possibly only slightly different) random loading data to be considered.

4.2 Parallel Implementation

The following excerpts from the SLang -command file are intended to demonstrate the interaction between PI (SLang) and FEA (ANSYS). In a first step, the random directions required for the Response Surface strategy are simulated in a master process. These values are then distributed among different SLang slave processes which are launched in parallel. Each of these

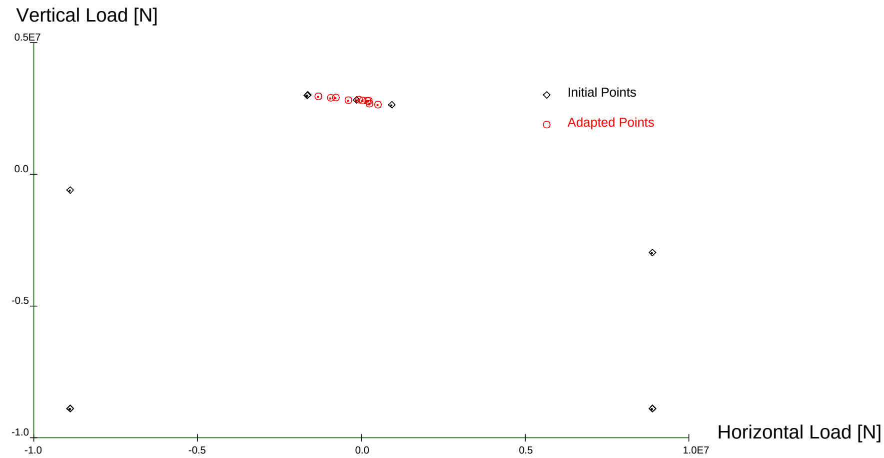

Figure 9: Adapted Response Surface

SLang processes then assembles an individual startup command for ANSYS and then in turn launches ANSYS. Upon return from ANSYS, the slave processes check in with the master process. Finally the master process evaluates the data collected by the slave processes and determines the strategy for the next run. The analysis was carried out on a SGI Origin 2000 with 20 processors.

The following listing shows how the startup command for ANSYS is assembled in a slave SLang process.

```
object create, , , theThread /
object read, , theThread -1, /
input file, replace text, "toThread.t", theThread /
GetThreadInfo theThread
input file, text, szThreadFile, PartDirections /
input file, text, szThreadFile, Thickness /
string append, replace,
      3 "echo;hostname;echo starting " szThread ";echo;", szTmp /
control system,, szTmp,/
control system,, "chmod a+x toThread.t", /
control gosub, , PrepareRanset, /
control gosub, , CalcPartOfSimulations, /
string append, replace, 2 "chmod a+x " szThreadFile, checkout /
control system,, checkout, /
control quit,,,/
```

The following code piece (SLang -subroutine `Search` assembles the command line and launches ANSYS by means of the command `control system, ...`.

```
#label SEARCH
string convert, replace real_string no_space, ansysfactors1, szthefactor1/
string convert, replace real_string no_space, ansysfactors2, szthefactor2/
string create, replace, 131, szTmp/
string read, , szTmp "ansys55 -p ansysrf -b -j ansj",/
```

```
string convert, replace int_string no_space, theThread, sztheThread/
string convert, replace int_string no_space, iSim, sziSim/
string append, replace, 2 szTmp sztheThread, szTmp /
string append, replace, 2 szTmp "_", szTmp /
string append, replace, 2 szTmp sziSim, szTmp /
string append, replace, 2 szTmp " -m1000 -db300 < schale.mac > ansc", szTmp /
string append, replace, 2 szTmp sztheThread, szTmp /
string append, replace, 2 szTmp "_", szTmp /
string append, replace, 2 szTmp sziSim, szTmp /
string append, replace, 2 szTmp " -sh ", szTmp /
string append, replace, 2 szTmp szthefactor1, szTmp /
string append, replace, 2 szTmp " -sv ", szTmp /
string append, replace, 2 szTmp szthefactor2, szTmp /
string append, replace, 2 szTmp " -dicke ", szTmp /
string convert, replace real_string no_space, Thickness, szThickness/
string append, replace, 2 szTmp szThickness, szTmp /
string append, replace, 2 szTmp " -aus \ ansj", szTmp /
string append, replace, 2 szTmp sztheThread, szTmp /
string append, replace, 2 szTmp "_", szTmp /
string append, replace, 2 szTmp sziSim, szTmp /

control system, soft_fail, szTmp, tmp /
control if, integer equal goto, tmp 1 AnsysFailure,/
control return,,,/

#label AnsysFailure
control message,, "Cannot start ANSYS",/
control quit,,,/
```

5 CONCLUDING REMARKS

The example, as presented above, indicates that traditional reliability issues, involving randomness of a small number of loading parameters only, can be handled satisfactorily by separate software packages for probability integration and finite element analysis. The proposed response surface method is suitable for computing the reliability of complex structures. The major advantage of this method is the flexibility for the approximation of highly nonlinear limit state functions. In addition, these response surfaces can be refined adaptively to consistently increase the accuracy of the estimated failure probability.

The suggested cooperation model between commercial Finite-Element-Software (ANSYS) and intelligent Probability Integrator (SLang) fully utilized available capacities for parallel processing, and thus can provide a substantial speedup at virtually no implementation cost.

ACKNOWLEDGEMENT

The results as presented here were obtained within the research project “Probabilistic numerical analysis based on component software” which is supported by the Deutsche Forschungsgemeinschaft under Contract No. Be-2160/3-1. This support is gratefully acknowledged.

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