Quality-Based Design with Probabilistic Methods

Accounting for scatter of input parameters improves analysis results.

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"It needs to be recognized that a single answer to a set of inputs is of limited value without an understanding of the variation." In the last issue of *ANSYS Solutions* magazine, G. Deleonardo, a Sr. Engineer at the Engineering Mechanics Laboratory of General Electric, provided this provocative but correct statement. He stressed the significant difference between finite element (FE) models and reality: in real-life all input parameters are subjected to scatter. The geometry of components can only be manufactured within certain tolerances. To strive for perfection is neither physically possible nor financially acceptable. Lab tests clearly reveal that material properties also are subjected to scatter. The same holds true for boundary conditions and loads. With the development of the ANSYS Probabilistic Design System (PDS), ANSYS, Inc., has acknowledged how important the variations of input parameters are. ANSYS/PDS will be implemented in ANSYS 5.7. In general, probabilistic methods can be used to answer the following questions:

1. How large is the scatter of the parameters describing the behavior of the component?
2. What is the probability that a performance criterion of the component is no longer met?
3. What are the input parameters that need to be addressed in order to achieve a reliable design and improved quality?

The answers automatically lead to measures that can be implemented as part of the quality control in the manufacturing process. The remainder of this article outlines a probabilistic analysis performed on a micro-electro-mechanical system (MEMS) that illustrates how ANSYS/PDS can be used to understand variations and derive measures for quality improvement and cost reduction.

The Analysis Model

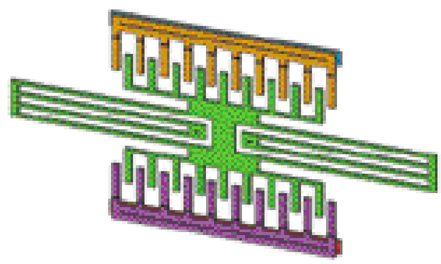

Figure 1: Micro-electro-mechanical linear resonator

The model is a micromachined lateral resonator consisting of two comb drives and a folded-spring mass assembly (Figure 1).

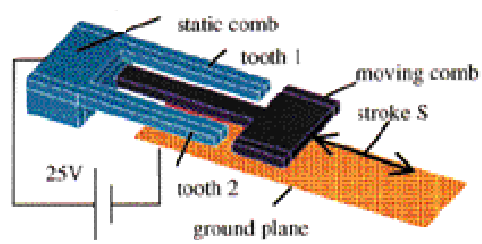

Figure 2: Illustration of the tooth model

The model used for the analysis considers one tooth of the moving comb and the two neighboring static comb teeth, plus the ground plane electrode (Figure 2). For the probabilistic analysis, the effects of the manufacturing tolerances on the electrostatic force on the moving comb tooth are investigated. A full electrostatic finite element analysis (FEA) using ANSYS 5.6 is performed, and the electrostatic force is calculated. The FE model is constructed by extruding a large air volume around a tooth model and subtracting the tooth model from the air volume with Boolean operations. The resulting electrostatic "domain" is meshed (Figure 3). The electrostatic far-field behavior is modeled by "infinite" elements at the exterior of the "domain."

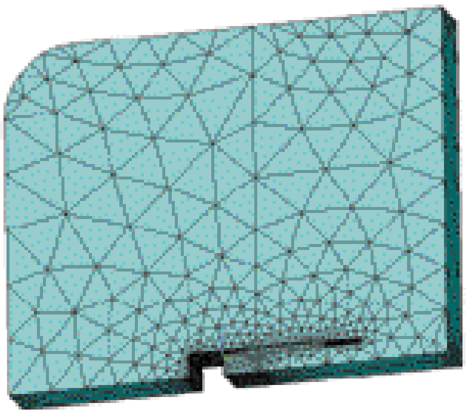

Figure 3: FE mesh – only electrostatic "domain" shown

As illustrated in Figure 2, the potential difference of 25V is applied across the ground plane and the static comb. Equipotential contours of the voltage are shown in Figure 4.

The electrostatic force, *K*, is the derivative of the electrostatic energy, *W*, with respect to the stroke, *S*. Since the energy is a very smooth function of the stroke, a simple forward finite-differencing scheme given by Equation 1 has been applied:

$$F = \frac{W(S+\Delta S) - W(S)}{\Delta S}$$

The electrostatic energy, *W*,can be extracted from the FE results.

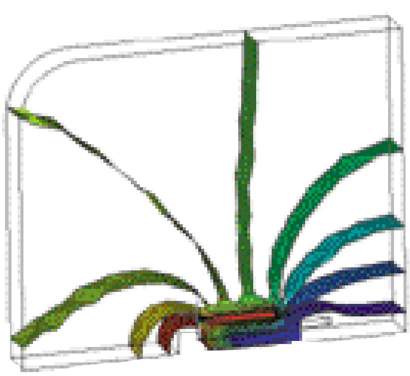

Figure 4: Equipotential contours of voltage

The Probabilistic Model

ANSYS/PDS allows for the definition of random variables (and random fields) based on various statistical distribution functions. Due to the lack of data, a normal distribution was assumed for all 14 input variables. All tolerances are defined as a deviation from the nominal geometry, i.e., with a mean value of 0.0. The 14 random input variables and their standard deviations (in µm) are listed in Table 1.

Name	Description	St. Dev.
MCTHDL	Moving comb tooth delta length	0.125
MCTHDW	Moving comb tooth delta width	0.125
MCSPDW	Moving comb spine delta width	0.125
MCTHDY	Moving comb tooth delta y-position	0.125
SCTH1DL	Static comb tooth 1 delta length	0.125
SCTH1DW	Static comb tooth 1 delta width	0.125
SCTH1DY	Static comb tooth 1 delta y-position	0.125
SCTH2DL	Static comb tooth 2 delta length	0.125
SCTH2DW	Static comb tooth 2 delta width	0.125
SCTH2DY	Static comb tooth 2 delta y-position	0.125
SCSPDW	Static comb spine delta width	0.125
GPDX	Ground plane delta length	0.125
POLY_DT	Polysilicon delta thickness	0.01
Z_GAPDZ	Polysilicon above nitride delta height	0.01

Table 1: Uncertain input variables in the tooth model

There are various probabilistic methods available in the literature, and several of them have been implemented in ANSYS/PDS. For this example, the Latin Hypercube Sampling techniques have been used.

To demonstrate the use of probabilistic methods to guide the design process of a MEMS device to achieve a more reliable and robust design, 270 Monte Carlo simulations were run at a stroke of 6 µm. The resulting

Statistics	Force
Mean value	1.5792e-2
Standard deviation	6.6248e-4
Sample minimum	1.4062e-2
Sample maximum	1.7902e-2

Table 2: Statistics of the electrostatic force

statistics of the electrostatic force are given in Table 2.

In response to question 1 listed in the introduction, the histogram of the electrostatic force is shown in Figure 5. It illustrates the scatter induced in the output parameter due to the scatter of the input variables.

Products are typically designed such that certain design criteria, based on the output parameters, are fulfilled. Here, a good tooth design is one, where the force, *F*, remains within a small range. Hence, the failure probability of the device is given by the probability that the force, *F*, falls outside that range. To answer question 2 concerning the failure probability, the ANSYS/PDS provides cumulative distribution curves (Figure 6).

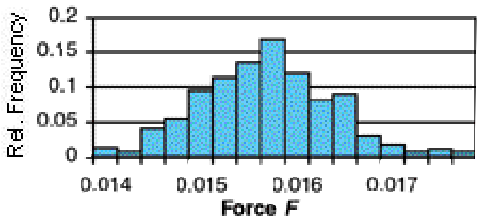

Figure 5: Histogram of the electrostatic force

The black center line is the probability, *P*, that the force remains lower than a certain limit value, *F-limit*. The complement, *1.0-P*, is the probability that the force, *F*, exceeds this limit. The upper and lower curves in Figure 6 quantify the accuracy of the probability results.

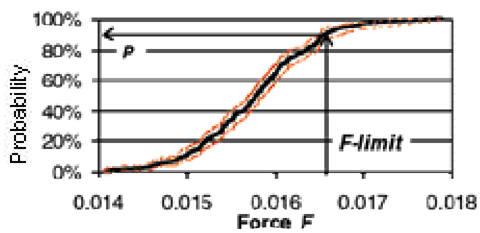

Figure 6: Probability curve of the electrostatic force

If the reliability of the device is not sufficient, then question 3 from the introduction must be answered, i.e., which input variables should be addressed to improve the quality? The answer can be derived from probabilistic sensitivity diagrams and scatter plots. To display the sensitivities, ANSYS/PDS sorts the input parameters into two groups, namely those having a significant influence on the output parameters and those being insignificant.

Figure 7 shows the sensitivities of only the significant input variables. There are two important conclusions that can be derived from sensitivity diagrams.

First, if the design is not sufficient, then the most important input variables must be modified or better controlled. There is no point in focusing on input variables of little or no importance. Here, the force is sensitive to only three input variables. This is a reduction of the problem complexity from 14 input variables down to 3.

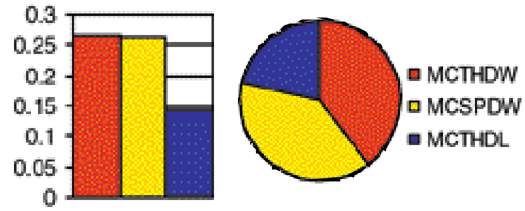

Figure 7: Sensitivities of the electrostatic force, F

This complexity reduction ensures that necessary design changes are identified in the most efficient way.

Second, if the design is satisfactory, there is usually the need to reduce the manufacturing costs without sacrificing reliability or quality. In this case, the manufacturing tolerances of the insignificant or the less important input parameters can be relaxed or expensive quality assurance measures for them can be reduced.

If, for example, the electrostatic force should be modified, then the question remains how this could be done. A scatter plot of the electrostatic force as a function of the parameter MCTHDW can be used to answer this question (Figure 8). The trendline describes the amount of scatter in the force due to the scatter of MCTHDW. The trendline can be used to estimate to what extent the scatter of the force might be reduced if the scatter of MCTHDW is reduced.

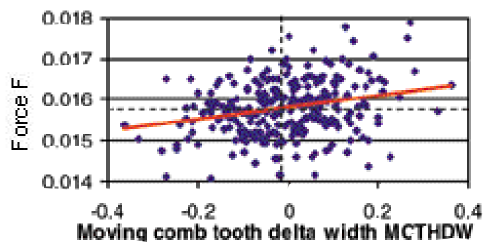

Figure 8: Electrostatic force, F, as a function of MCTHDW

Conclusions

"A probabilistic design approach cannot be realized with a single analysis," as G. Deleonardo points out in the abovementioned article. However, the above example illustrates that the additional computational effort is well rewarded. A probabilistic analysis provides a wealth of information that otherwise cannot be discovered. A probabilistic analysis helps the users to better understand the behavior of a product under real-life conditions and to efficiently derive measures for quality improvement and cost reductions in an automated way. It helps the users to get closer to what their analysis work ultimately gets benchmarked against – real life.