# Cavity mechanics and vibrations under Lorentz forces excitation

Danilo Barni

INFN Milan-LASA

Tel.+39-02-2392544

Fax +39-02-2392543

E-mail: Danilo.Barni@mi.infn.it

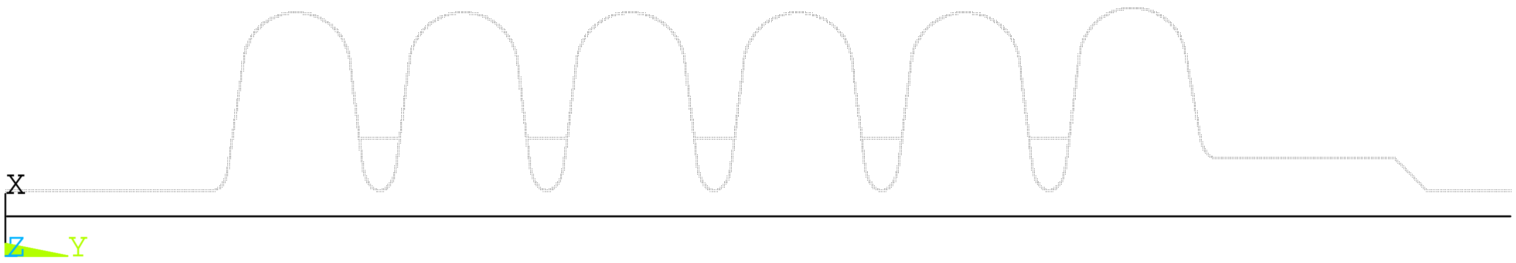
# Overview of the analysis presented here

- The behavior of the real system (cavity + environment) cannot be precisely determined until all details are fixed
- A “bare” multicell cavity is taken as a reference and excited near its fundamental resonance applying a pulsed Lorentz pressure load, to determine the induced RF phase shift on the “Qloaded” curve

# Reference Parameters for calculation

- Cavity is the SNS Beta 061 (627 in BuildCavity Database)
  - Stiffening radius: 80mm
  - Niobium Thickness: 3.8mm
  - Young Modulus: 103 GPa
  - Poisson Ratio: 0.3

- Operating conditions:
  - $E_{acc}$= 10 MV/m
  - $Q_{ext}$= $1\cdot10^6$
  - Macro bunch length= 1ms
  - Repetition rate= 60Hz (but I will force resonance excitation of the modes!)



# Lorentz Forces and Microphonic Vibrations: Some Considerations

The system unders study is ONLY the “bare” cavity:

- No ancillaries components
  - No external mass added
  - No external position constrain
- No external load: cavity length is fixed
- Cavity has not been pre-stressed
  - No tuning compression
  - No residual stress influence

The results do not have absolute value but they represent a good exercise to understand the problem

# Cavity Eigenfrequencies

A longitudinally axi-symmetric geometry has been studied.

Modal shapes have been computed for the unstiffened cavity

| Hz |
|---|
| 81.9 |
| 161.8 |
| 237.4 |
| 304.5 |
| 356.9 |
| 381.9 |
| 680.2 |
| 717.0 |
| 761.5 |
| 804.7 |
| 837.9 |

Two possible kinds of modal shape:

"Bellow" shape:

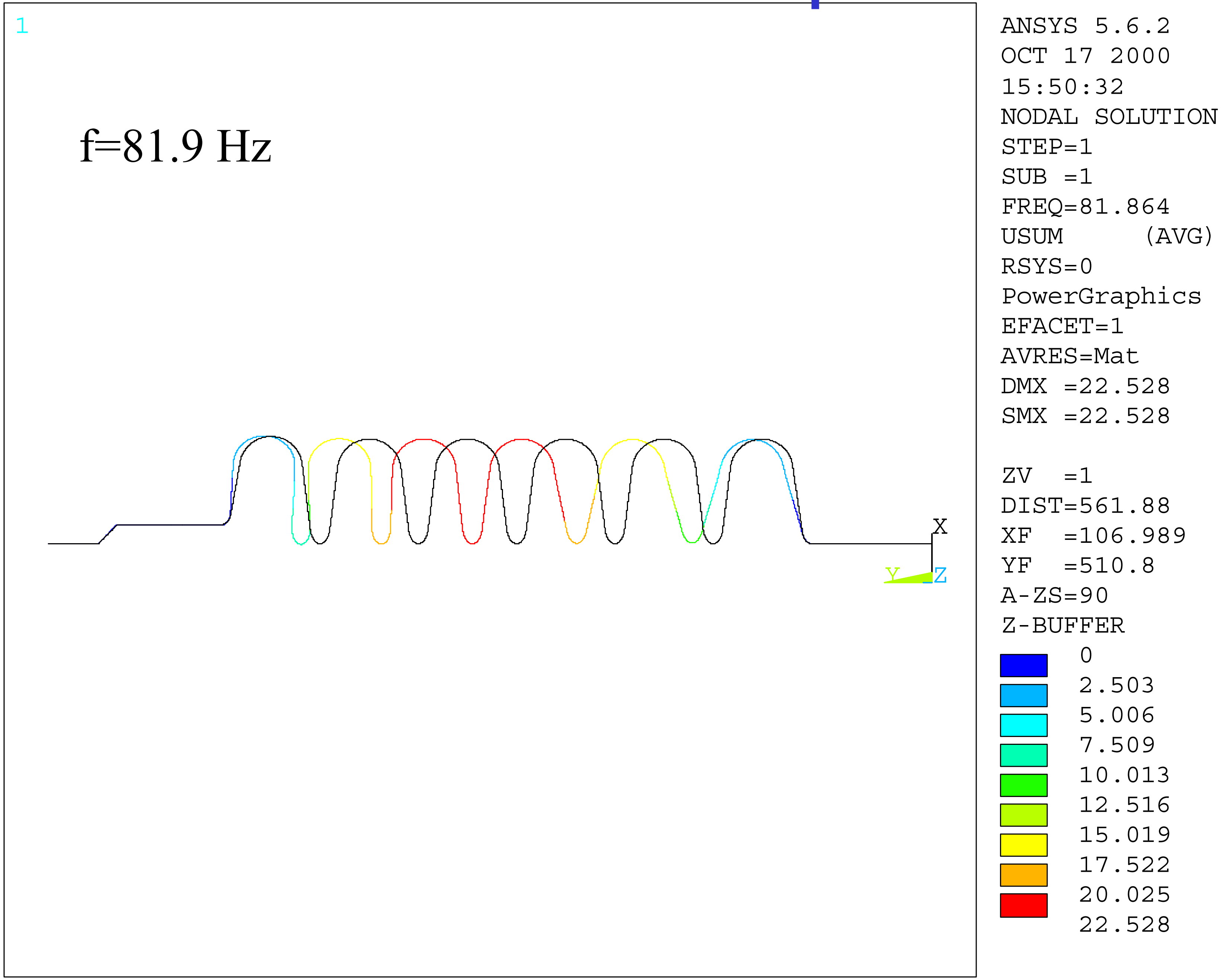
- Lower frequencies
- Produce "semirigid" displacement of cells

"Cell" shape

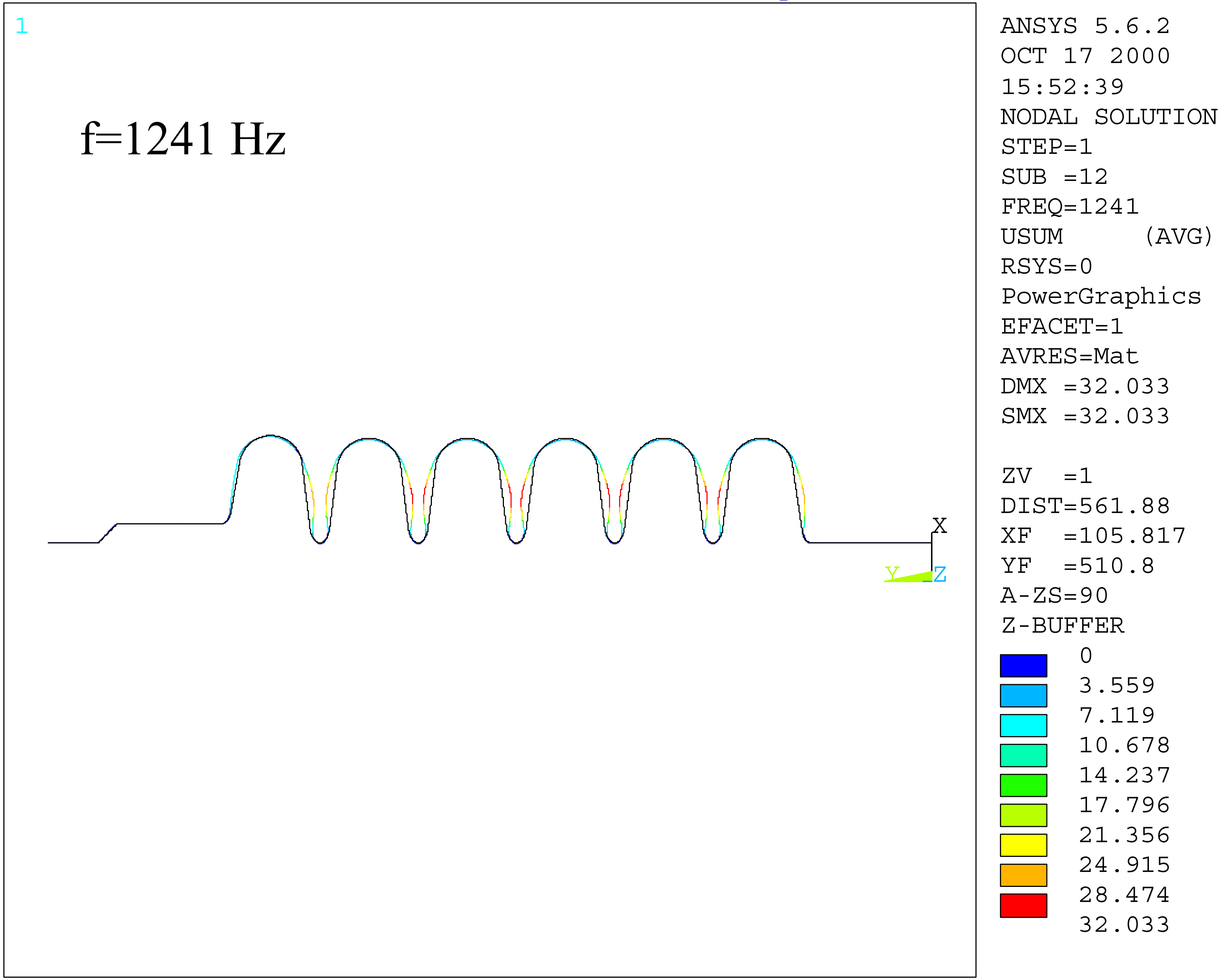
- Higher frequencies (>1 kHz)
- Cell are "independent"

Static Deformation is "similar" to both but...

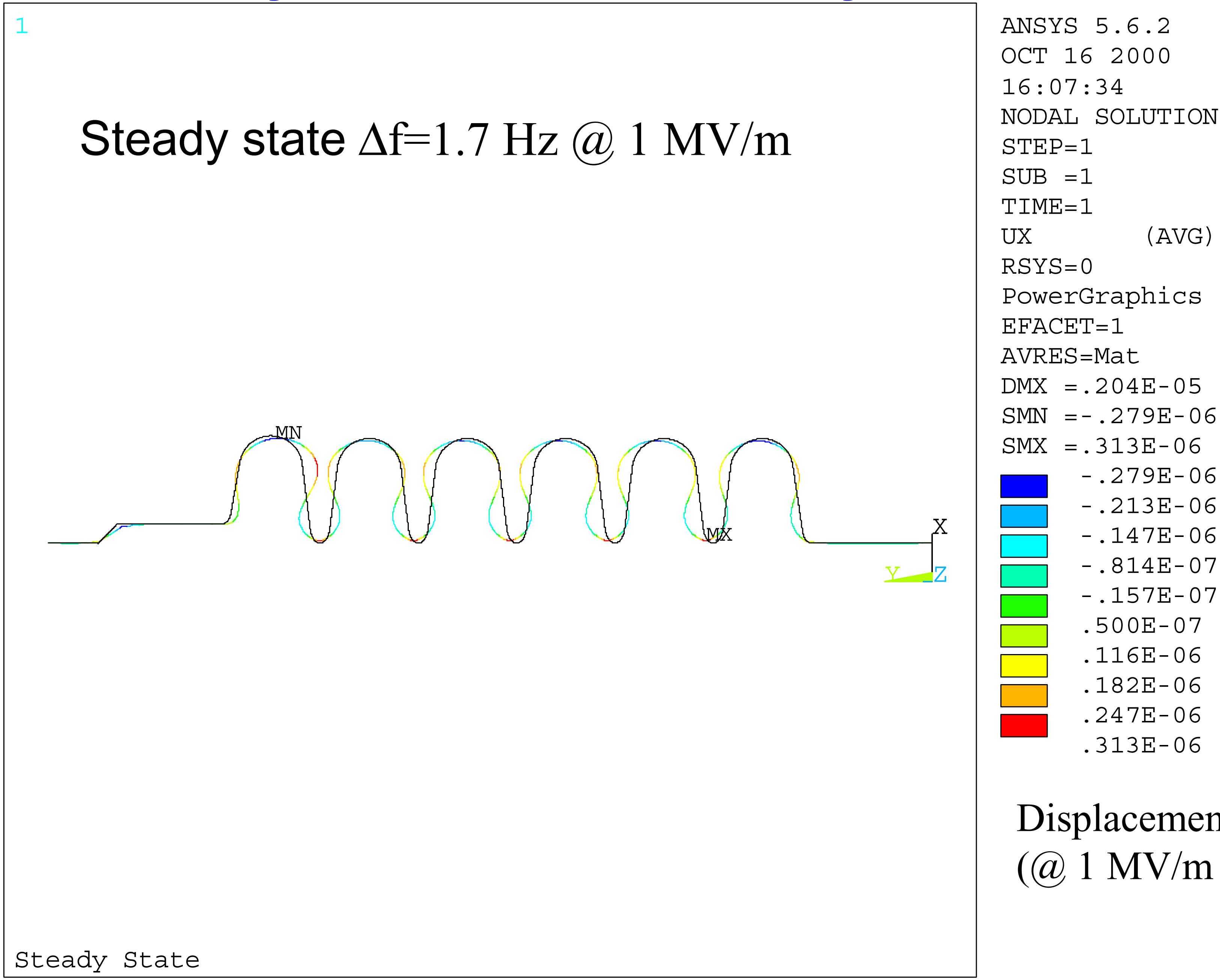
# Bellow shape



# Cell shape


1
f=1241 Hz
X
Y
Z
ANSYS 5.6.2
OCT 17 2000
15:52:39
NODAL SOLUTION
STEP=1
SUB =12
FREQ=1241
USUM (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =32.033
SMX =32.033
ZV =1
DIST=561.88
XF =105.817
YF =510.8
A-ZS=90
Z-BUFFER
0
3.559
7.119
10.678
14.237
17.796
21.356
24.915
28.474
32.033

# Steady State deformation by Lorentz Forces



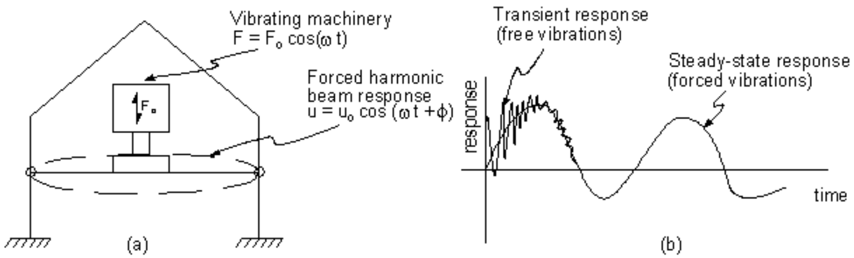

Displacements (mm)
(@ 1 MV/m Eacc)

# Deterministic Harmonic Analysis

Only the shape information can be extracted from modal analysis.

The load is the Lorentz pressure distribution (calculated from Cavity SFO file)

Suppose that load is varying harmonically at different frequencies and forget the transitory effects



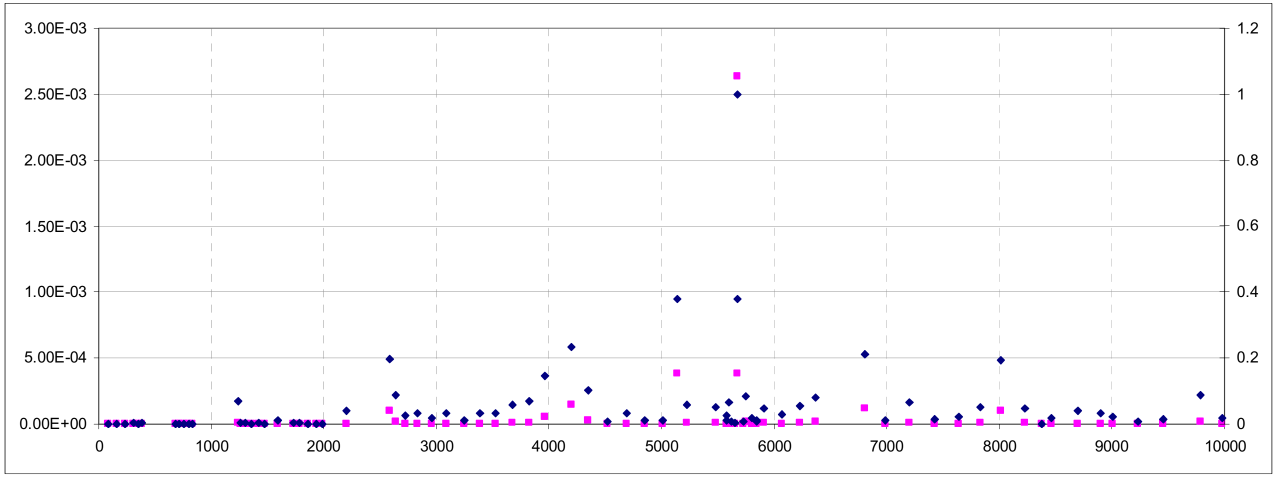

Solution can be expressed by superimposition of natural modes.

$$Y(x,\Omega) = \sum \frac{f_n}{\left(\omega_n^2 - \Omega^2\right) + i \cdot \left(2 \cdot \omega_n \cdot \Omega \cdot \xi_n\right)} \cdot \phi_n(x)$$

$f_n$ is the participation factor and gives information on the “compatibility” between a mode shape and the load .

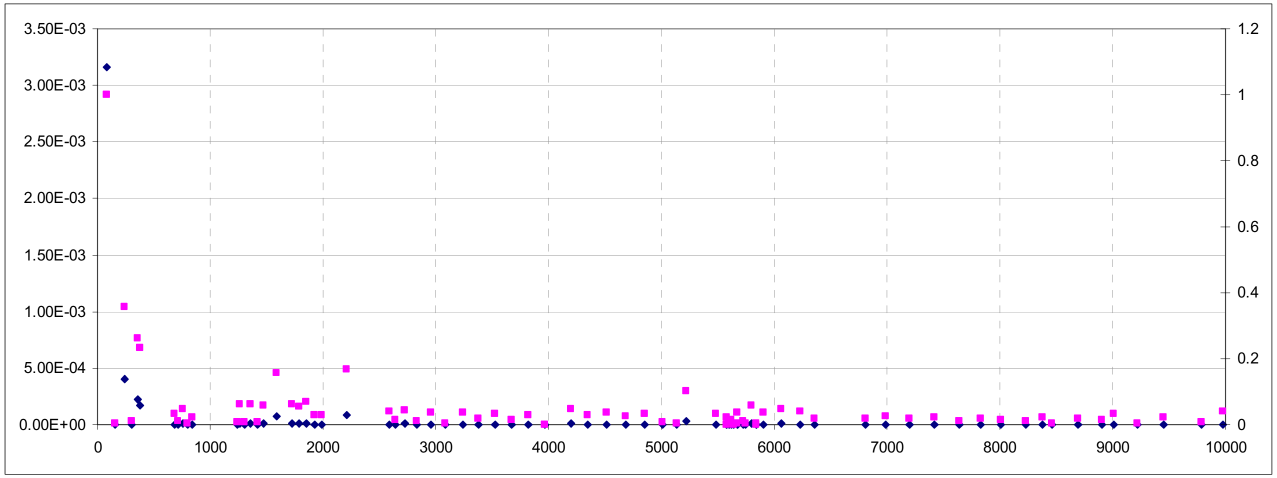
# Partecipation Factor Effects

R direction




Formally the participation factor is the dot product between the modal shape and the load profile. Modal shapes are also normalized to a proper unity vector so also normalization has to be kept into account

Z direction




Low frequency modes are well coupled to transversal components while higher frequencies are coupled to radial components

# Modal analysis completed

| Hz | Rcoupling | Zcoupling |
|---|---|---|
| 81.9 | 0.0001 | 1.0000 |
| 161.8 | 0.0010 | 0.0021 |
| 237.4 | 0.0001 | 0.3554 |
| 304.5 | 0.0019 | 0.0127 |
| 356.9 | 0.0010 | 0.2630 |
| 381.9 | 0.0029 | 0.2344 |
| 680.2 | 0.0002 | 0.0336 |
| 717.0 | 0.0014 | 0.0108 |
| 761.5 | 0.0002 | 0.0480 |
| 804.7 | 0.0013 | 0.0051 |
| 837.9 | 0.0000 | 0.0206 |
| | (...) | |
| 5619.4 | 0.0075 | 0.0145 |
| 5647.3 | 0.0048 | 0.0011 |
| 5670.2 | 0.3805 | 0.0357 |
| 5672.9 | 1.0000 | 0.0034 |
| 5731.0 | 0.0085 | 0.0112 |
| 5745.3 | 0.0842 | 0.0033 |
| 5803.3 | 0.0188 | 0.0590 |
| 5840.2 | 0.0063 | 0.0042 |
| 5844.1 | 0.0116 | 0.0002 |

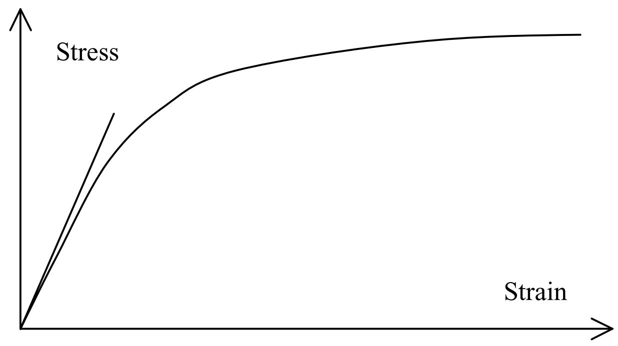
Once the load shape is defined the mode table can be completed
(i.e. loads can be projected on each mode shape)

Each mode is characterized by its influence in the response of the system to a specified load

# Damping

We consider only material inner damping:

- No damping coming from supports
- No damping from ancillary components




Material damping is almost due to plasticity effects during the oscillation.

Material damping depends on the amplitude of the oscillation.

Analytical energy consideration and measurements pointed out that the energy lost during an oscillation with respect to the elastic energy can be generally expressed by a simplified relation that is a reasonable approximation for all metals

$$\psi = \frac{\varepsilon}{\varepsilon_{pl}} \cdot 8 \rightarrow Q = {}^{1}\!/_{\psi}$$

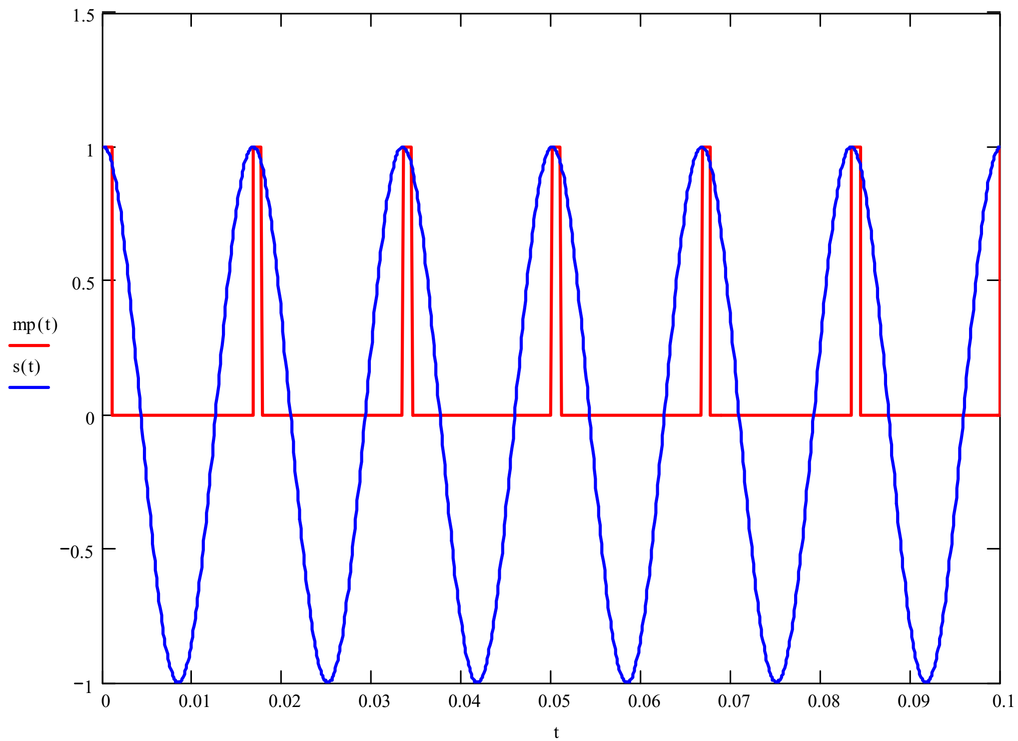
As reference I take Q=50, that corresponds to 0.25 % of plasticity effects.

# ‘Real’ pulse description

Lorentz excitation is not harmonic, but the correspondence can be analytically corrected by means of Laplace formalism

$$Out(s) := \sum_{n=0}^{\infty} \frac{\left(1 - e^{-s \cdot \lambda}\right)}{s} \cdot e^{\frac{-n \cdot s}{f}} \cdot H(s)$$

$$End(s) := \lim_{s \to 0} s \cdot Out(s)$$

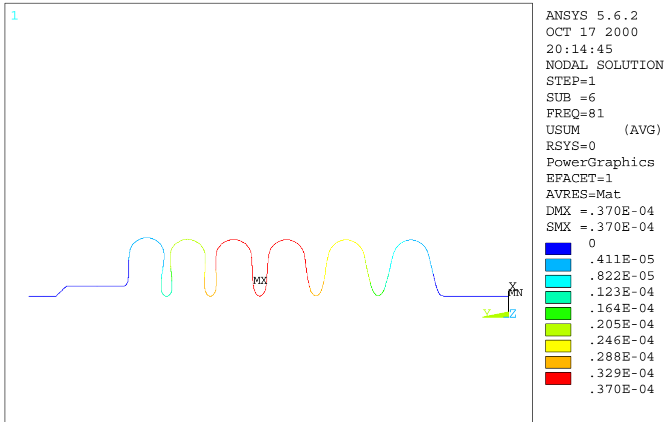


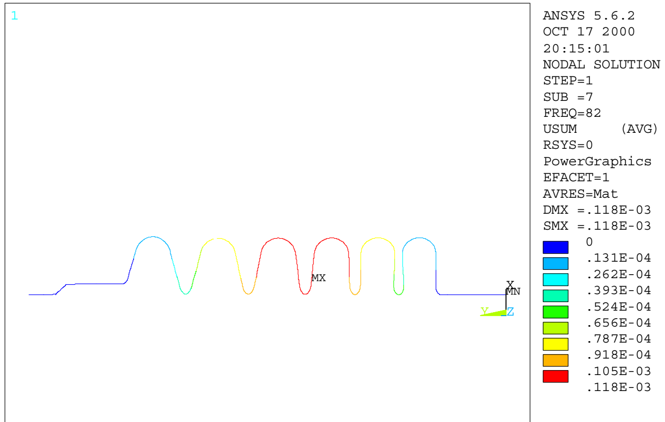

The results of the harmonic analysis give the H(s) function

λ is the pulse length (1 ms)

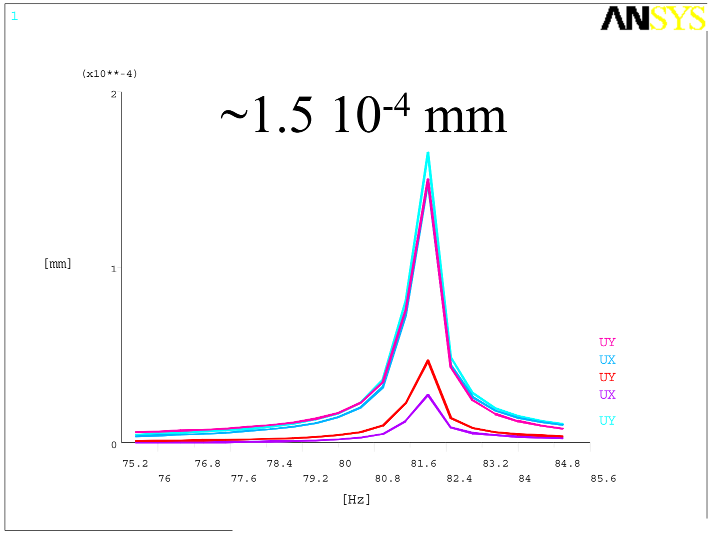
$$Amplitude_{Norm} = Amplitude \cdot (\lambda \cdot f)$$

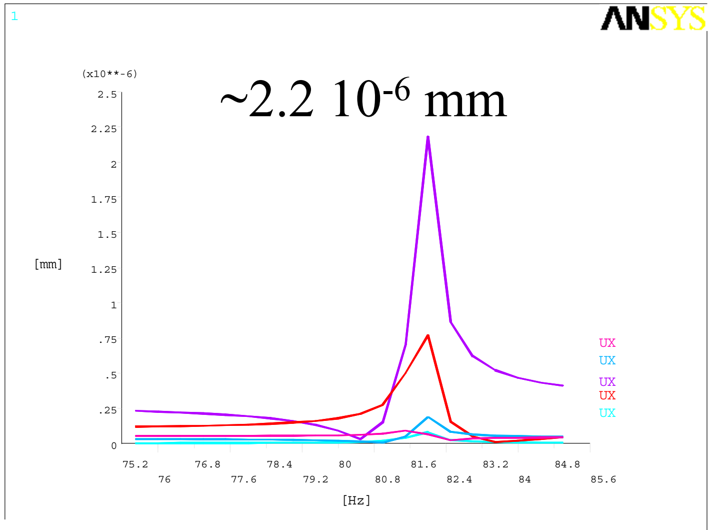
# Unstiffened Cavity Analysis




Phase change over the resonance



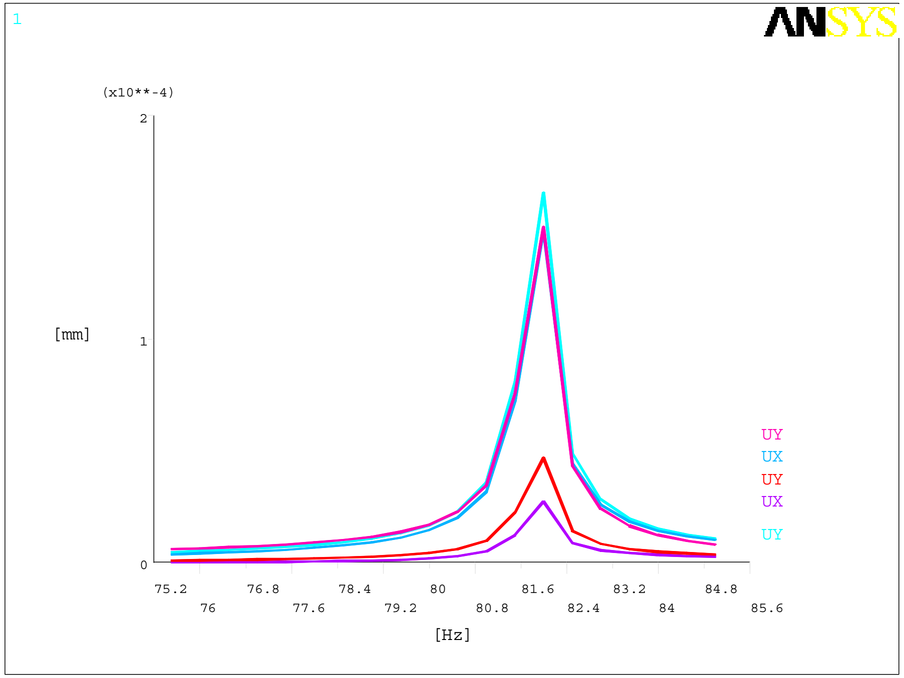

Modulus of displacement in Z amd R direction

# Unstiffened Cavity Analysis

The real and immaginary part of oscillation displacement **near the resonance** are extracted and used to calculate the frequency shift by SLATER theorem

Using the $Q_{loaded}$ curve: the frequency shift is moved to RF phase shift

$$\tan(\phi) = \frac{\omega \cdot \omega_0}{Q \cdot \left(\omega_0^2 - \omega^2\right)} \longrightarrow \tan(\phi) = \frac{F}{2 \cdot Q \cdot \Delta F}$$




$$\Delta\phi = 2.9° @ 81\text{Hz}$$

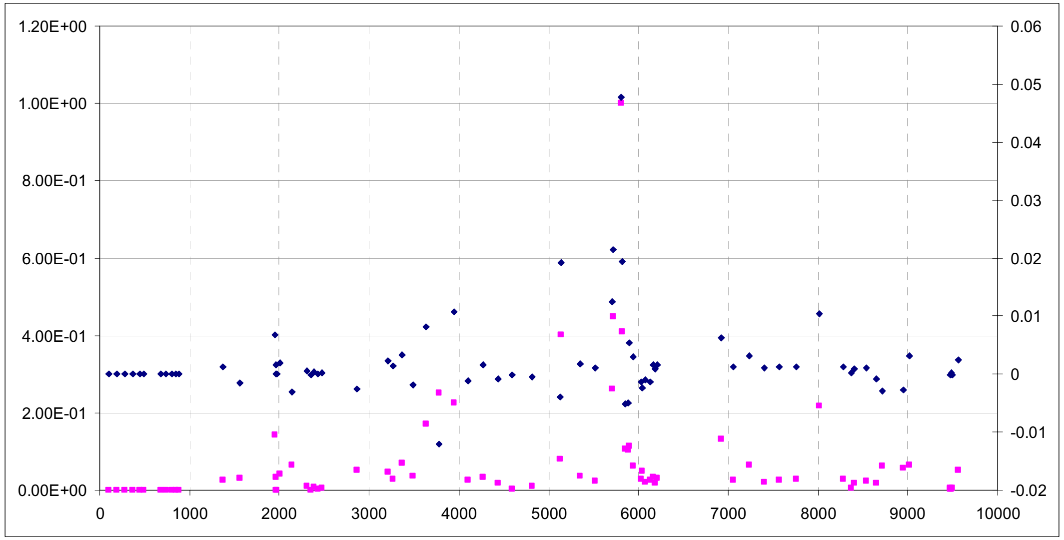
$$\Delta\phi = 6.1° @ 81.9\text{Hz}$$

$$\Delta\phi = 0.5° @ 83\text{Hz}$$

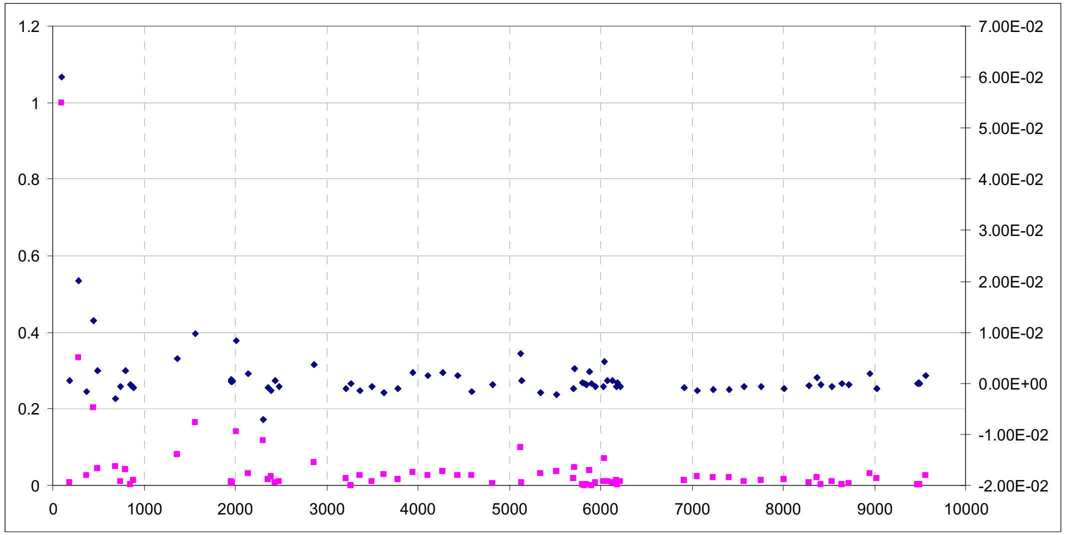
**Also at the exact resonance the Lorentz load is able to trasfer a very small amount of energy into the mode!**

# Stiffened Cavity Analysis

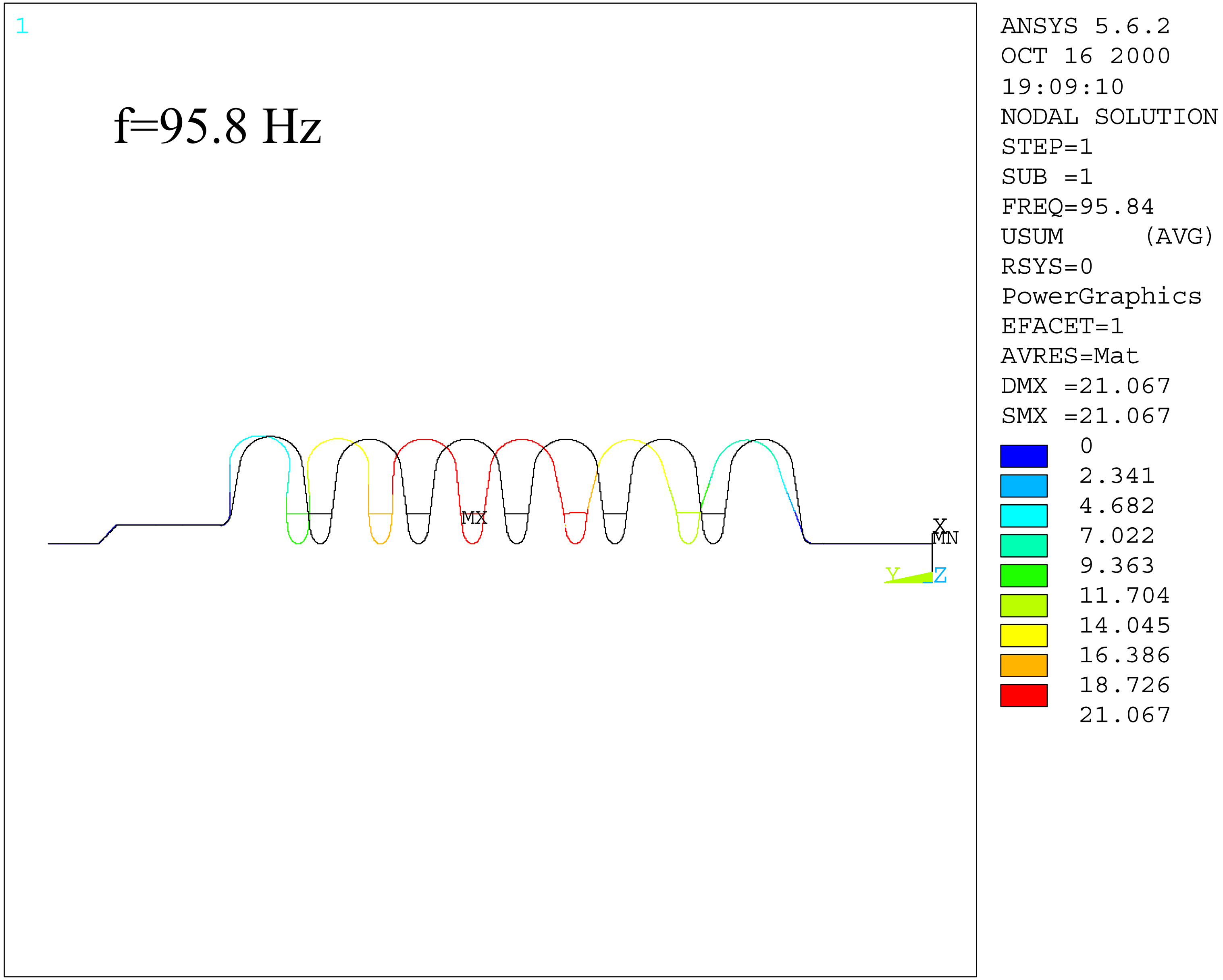
| Hz | Rcoupling | Zcoupling |
|---|---|---|
| 95.8 | 0.0001 | 1.0000 |
| 190.7 | 0.0003 | 0.0088 |
| 282.9 | 0.0004 | 0.3334 |
| 369.6 | 0.0004 | 0.0263 |
| 444.4 | 0.0007 | 0.2038 |
| 491.7 | 0.0003 | 0.0431 |
| 685.4 | 0.0004 | 0.0496 |
| 740.6 | 0.0004 | 0.0104 |
| 798.9 | 0.0005 | 0.0425 |
| 849.3 | 0.0006 | 0.0028 |
| 883.7 | 0.0002 | 0.0141 |
| | (...) | |
| 5946.4 | 0.0239 | 0.0374 |
| 6029.8 | 0.2615 | 0.0176 |
| 6041.5 | 0.4489 | 0.0473 |
| 6080.6 | 1.0000 | 0.0015 |
| 6132.0 | 0.4084 | 0.0013 |
| 6170.3 | 0.1064 | 0.0035 |
| 6189.1 | 0.1039 | 0.0395 |
| 6214.6 | 0.1130 | 0.0012 |
| 6922.9 | 0.0610 | 0.0090 |



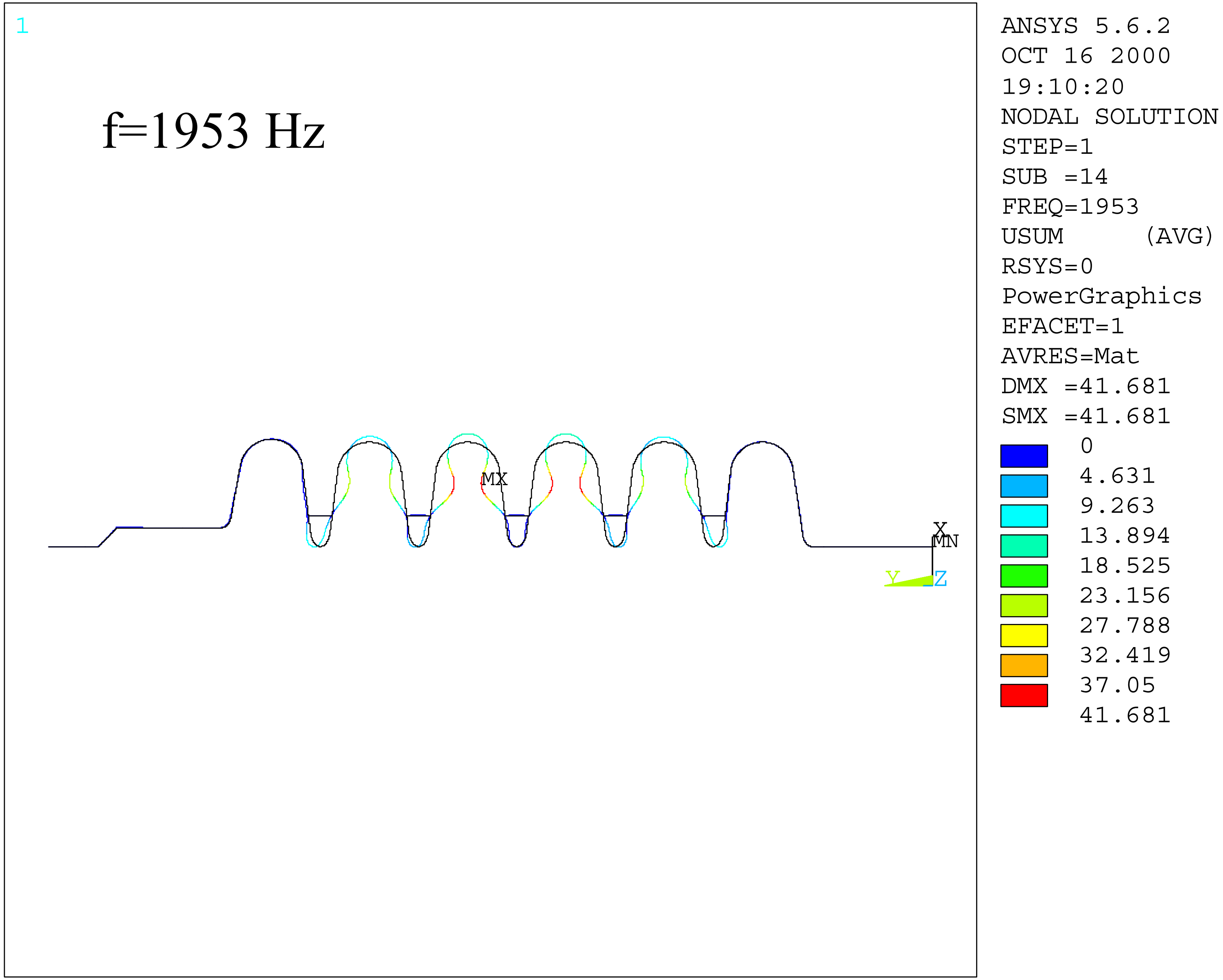

Same kind of modes and same peculiarities



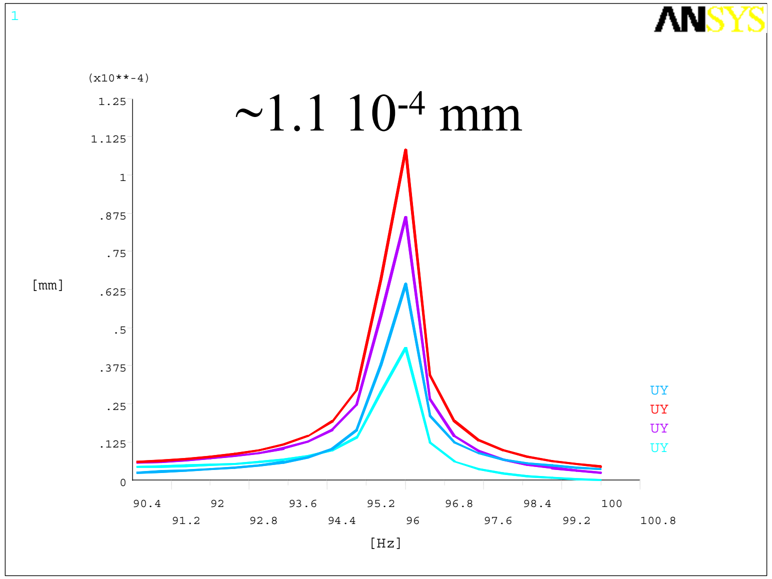
# Bellow mode shape

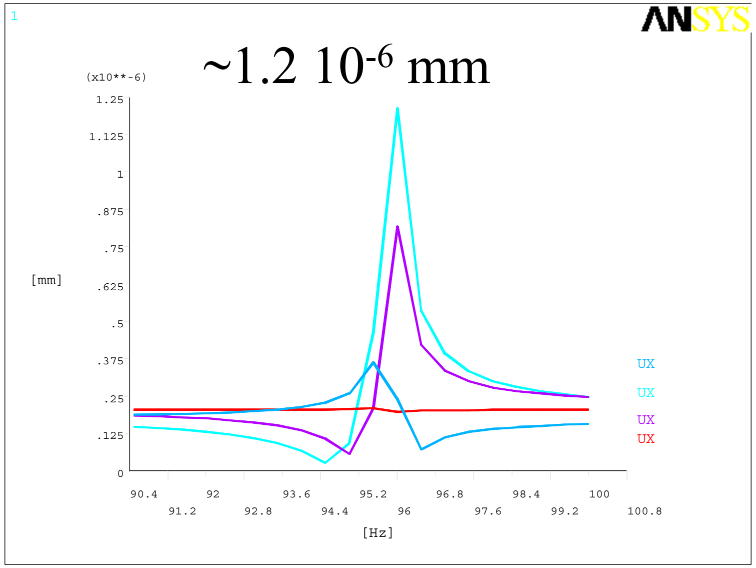


# Cell Shape


1
f=1953 Hz
MX
MN
X
Y
Z
ANSYS 5.6.2
OCT 16 2000
19:10:20
NODAL SOLUTION
STEP=1
SUB =14
FREQ=1953
USUM (AVG)
RSYS=0
PowerGraphics
EFACET=1
AVRES=Mat
DMX =41.681
SMX =41.681
0
4.631
9.263
13.894
18.525
23.156
27.788
32.419
37.05
41.681

# Stiffened Cavity Analysis



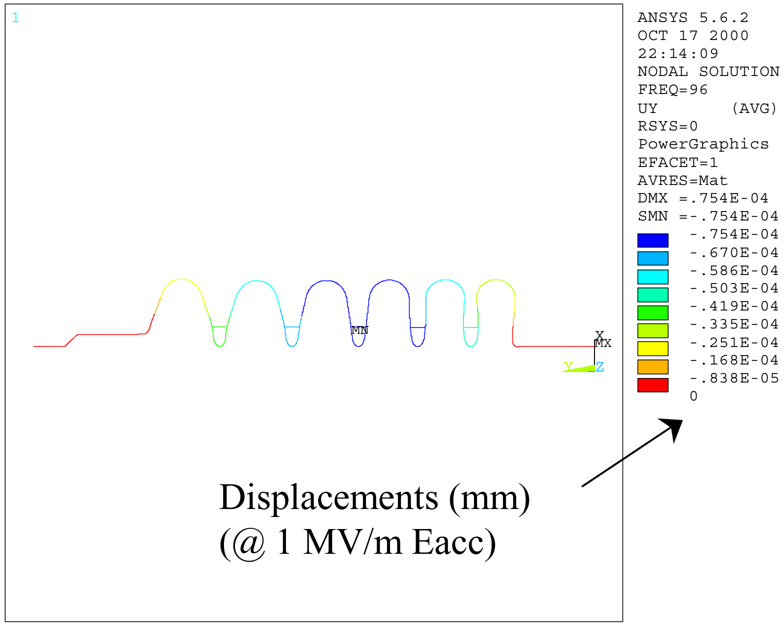




Deformations and frequency shifts are lower by ~ 25%

$$\Delta\phi = 0.8° @ 95Hz$$

$$\Delta\phi = 4.5° @ 95.8Hz$$

$$\Delta\phi = 1.5° @ 97Hz$$




Displacements (mm)
(@ 1 MV/m Eacc)

# Conclusion and plans

- Results are preliminary and must be checked.
  - It is, however, a pessimistic scenario, since modes are always excited near the resonance condition, independently from the 60 Hz “clock”
  - Only material damping
- Lorentz contribute **in full resonance condition** looks “small” with respect to static Lorentz contribution and:
  - Real system damping has to be considered
  - Transversal modes have not been checked but due to the axi-simmetry of the Lorentz load they should be weakly coupled and also Slater contributions should be self compensating
  - Transient effects could be significant during the pulsing and have to be studied (overelongation)
- Different harmonic loads could be checked:
  - Uniform pressure
  - Concentrated loads on the supports
- Prestress effects must be considered in the calculation for the real system
  - Change the frequency and the damping