MACHINE DESIGN

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STATISTICAL METHODS AND FEA REVEAL MORE ABOUT YOUR PRODUCT

Combining statistical methods with finite elements delivers an analysis that provides a more realistic look at product performance.

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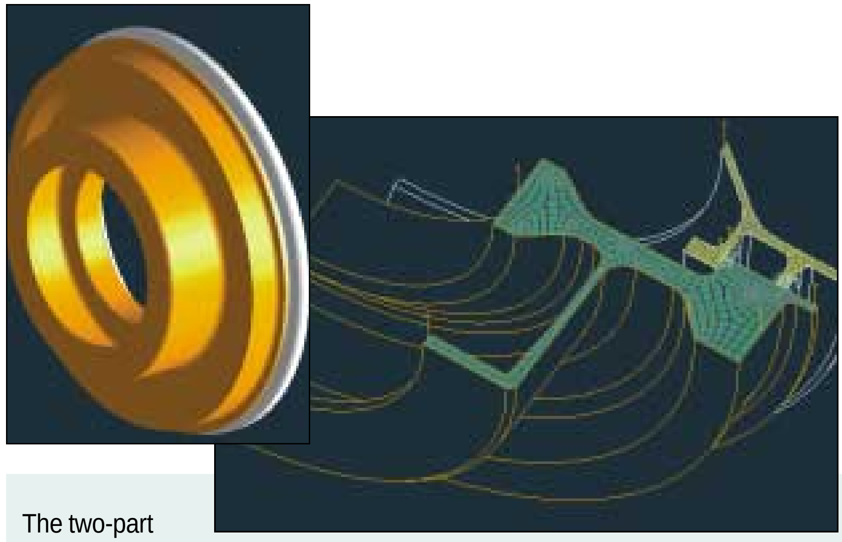
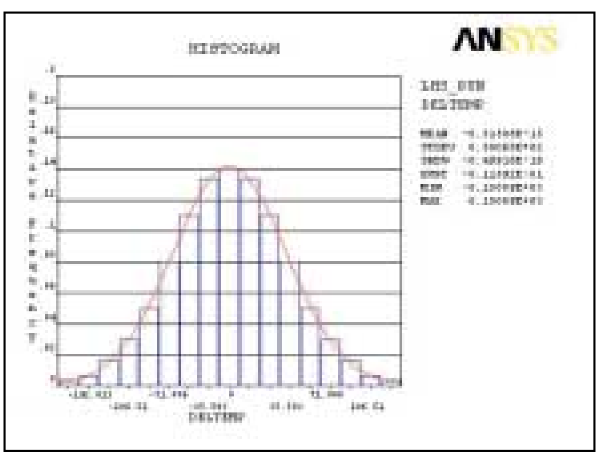


The two-part engine disk has to function at high speeds and temperatures. But how is the function of the disk affected by natural variations that occur in loads, material properties, and boundary conditions? The software called the Probabilistic Design System from Ansys marries finite-element analysis and statistics to answer such questions by considering a range of loads and boundary conditions at one time.

Digital simulations give the automobile industry a way to test preliminary designs without smashing lots of expensive and perfectly good cars. For example, finite-element analysis helps assess the behavior of a car and its passengers during an impact. A crash analysis could assume that the car hits a brick wall head on at 50 mph. The analysis set up can let one end of the wall line up with the center line of the car so the barrier blocks the exactly half the vehicle path.

While the insights gained from such an analysis are invaluable, it is also clear that results are realistic only if the assumptions are realistic. In real life the assumptions are unlikely and a crash can happen under significantly different conditions. In any real-life accident, the impact speed and angle as well as the barrier-vehicle path overlap will probably be different. This raises the question of what would happen to the passengers under a range of other speed and impact angles. It sounds like a daunting task because these other conditions can include numerous combinations of things that can possibly happen. Repeating a complex test hundreds of times to cover all variations to materials and boundary conditions gets expensive even for an auto company. Nevertheless, the multitude of things that can possibly happen represents what designers and analysts strive to grasp and what the quality of



Engineers have measured temperatures near 800°F at the engine seal. The Gaussian curve describes a range of temperature differences (deltas) that the engine seal also experiences. So the analysis includes values as much as 140° below and above 800°F.

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their work ultimately gets benchmarked with — the real world.

In real life, loads and material properties, such as yield strength, are more completely described by a spread of values with the average having the most likelihood of occurring. Slightly lower or higher values occur as well and could alter the product's behavior significantly. To illustrate the phenomenon, take 10 test specimens of any one material and measure their tensile strength. As you can imagine, results will show a distribution with an average close to published figures. Test 100 specimen and you get a better picture of the scatter.

Analysts dealing with computer models easily forget what is common knowledge for designers dealing with manufacturing drawings — that the geometry of a product can only be made within certain manufacturing tolerances. The difference between reality and a computer model make boundary conditions uncertain.

FEA, MEET STATISTICS

More recent analytical technology overcomes the drawbacks of the rigid approach described by combining statistical methods with finite-element analyses to account for variations that naturally occur with most input conditions. The software, called the Ansys Probabilistic Design System (PDS), delivers a clearer picture of a product's performance, robustness, and reliability under real-life conditions.

The idea of probabilistic analysis was initially developed in the nuclear industry. Its high reliability requirements make it mandatory to consider natural material variations and possible event scenarios. The technique has quickly proven useful solving problems for petroleum and aerospace industries. Another application has been the spate of recent micromechanical devices in which members can be 10 to 100 microns thick but variations to dimensions reach 10 to 20%. In general, the probabilistic-analysis technique is useful where issues such as safety, reliability, and robust quality are important.

The software takes uncertainties into account in a way that more closely describes how the design will function in the real world. The range of scatter on the output comes with probabilistic sensitivities, indicators of the most significant or influential characteristics.

As expected, model preparation time is longer than in a classical analy-

PUTTING THE PROBABILISTIC METHOD TO WORK

To better illustrate the inputs and outputs possible with the new software technology, it's useful to work through a simple example. The problem deals with a two-part turbine disk, which includes the disk itself and a cover plate. Both parts axially slide into each other with an interference fit, which also radially positions the cover plate. Given the manufacturing variability of the disk's geometry, the material properties and boundary conditions, what effect might these variations have on the proper function of the disk?

The analysis consists of a thermal study where metal temperatures are evaluated based on the thermal boundary conditions. A structural analysis considers where the displacements and stresses in the disk are calculated based on the structural loads of the disk (blade loads, rotational speed).

Initial inputs would include those in the Material properties box. The statistics of the random response parameters were computed using the derived results.

A proper function of the disk requires examining several output characteristics, such as:

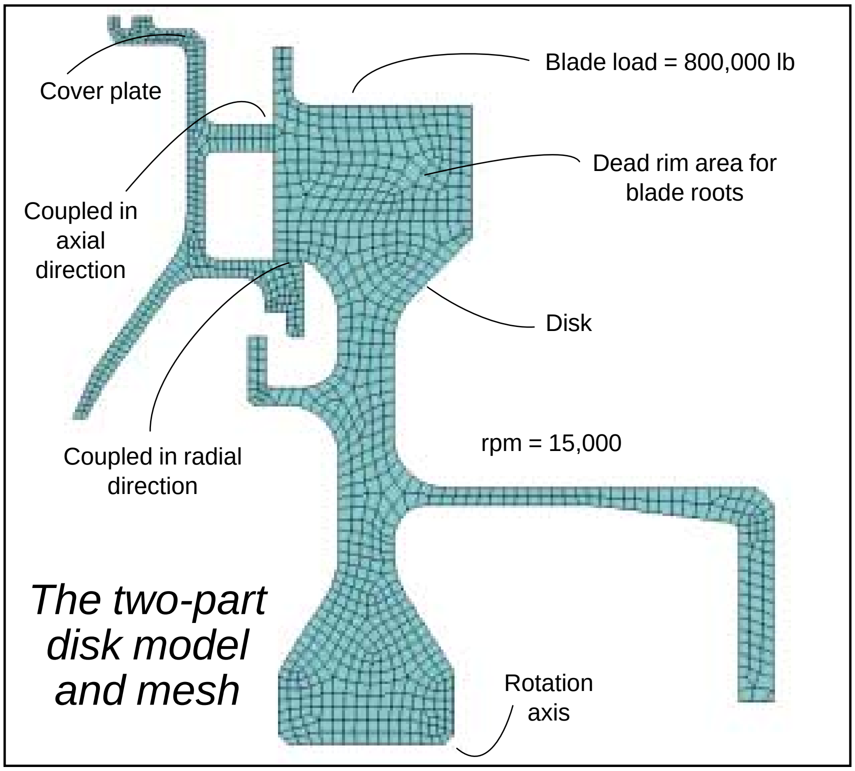
- The disk shall remain intact. The maximum equivalent (Von Mises) stress in the disk, which occurs at the center bore, is used as an output parameter. This parameter has been named SeqMxDsk. It also makes sense to require that this stress remain below a certain admissible limit. Otherwise, the disk might burst.
- The performance of the turbine stage is influenced by the radial clearance gap and the outer casing. This, in turn, is influenced by the radial displacement of the disk at the outer rim. This radial displacement has been stored in the output parameter named UxMaxDsk.
- A seal strip fits into a groove in the left upper part of the cover plate. The seal prevents hot gas from flowing into cavities between the mounted disks. The performance of the seal is influenced by the radial displacement of the groove. This radial displacement has been stored in the output parameter named UxMaxCvp.

Other output parameters could be added. The probabilistic design software has no limitations in that regard.

For this example, 300 Monte Carlo simulations have been used to analyze the problem from a probabilistic point of view. In each of the 300 simulations the values of the input parameter have been varied according to their statistical distributions as specified in *Statistical distributions of the input quantities* and the effect of these variations on the output parameters have been recorded. An overview of the results is listed in table *Statistics for output parameters*. Additional probabilistic results are illustrated. For the sake of brevity, we have focused on the radial displacement in the cover plate as the output parameter.

Material properties

	COVER PLATE	DISK
Modulus of elasticity	2.800E+07	3.000E+07
Density	7.764E-04	7.435E-04
Poisson's ratio	0.300	0.300
Thermal expansion coefficient	6.00E-06	5.00E-06



The two-part disk model and mesh

Statistical distributions of the input quantities

NAME	DISTRIBUTION TYPE	PARAMETER 1	PARAMETER 2
Delgap	Uniform	Min = -1.000E-02	Max = 1.000E-02
Deltemp	Gaussian	Mean = 0	Std. dev. = 50.0
Young1	Gaussian	Mean = 2.800E+07	Std. dev. = 1.400E+06
Young2	Gaussian	Mean = 3.000E+07	Std. dev. = 1.500E+06
Factload	Gaussian	Mean = 1.00	Std. dev. = 4.00E-02

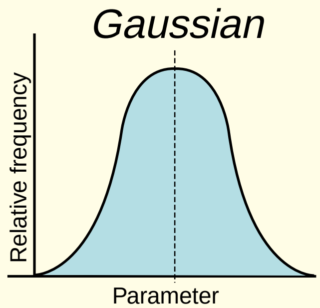
The abbreviated table of input characteristics shows input parameters associated with variability and how the variability is described by a distribution function. Delgap is the delta for the width of the interference gap between the two parts. It's described as a uniform distribution. Deltemp is a delta value on temperatures that have been used as boundary conditions for the thermal analysis. Young1 and 2 are Young's moduli for the two parts. And Factload is a factor on the centrifugal load of the blades carried by the disk. This load has a natural variation because the material density of the blades is subjected to scatter.

sis because rather than a single load or material selection, the characteristics are described as a range of values.

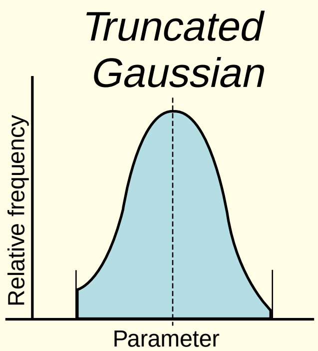
The scatter might follow a Gaussian distribution, the traditional bell-shaped curve. This plot places the relative frequency occurrence of the property values on the *Y* axis and the property values on the *X* axis. Most values usually occur around the peak of the curve indicating the greater likelihood of that value occurring. Values less likely to occur fall farther from the average. The Gaussian bell curve describes many material properties. It also works for geometry properties. The curve usually appears in well understood and stable mass-production processes such as die casting or stamping. Newer production methods, such as hydroforming, are less well understood, and might have a wider bell curve to describe its variability. The process could also have a different type of distribution.

The probabilistic FEA software allows describing the variability of the input data by several different statistical distributions. Others include:

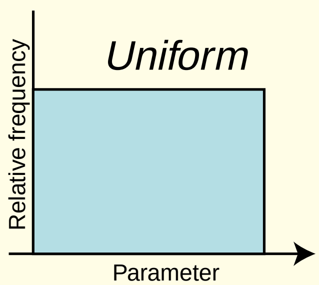
- A truncated Gaussian curve. As the name suggests, the extreme ends of the bell curve are cut off. This is typically used if the manufacturing process generates values for product properties following a Gaussian distribution, but those values falling outside a certain tolerance or acceptance band are sorted out by quality assurance.
- A uniform distribution is described by a straight horizontal line. Any value is as likely to occur as any other. It's appropriate when the only available information about the spread and distribution is a minimum and maximum value providing a lower and upper bound.
- A beta distribution is a more refined uniform version. It could be used to describe the variations in a large precise hole milled into a plate. A machinist, for example, might start by drilling a small hole and mill it further increasing its radius. After working a bit, he might stop and measure the diameter, mill a bit more, and measure again. Eventually, the diameter will fall within the required tolerance band. However, a final diameter exceeding the upper tolerance bound will lead to a part that eventually must be discarded. Having this in mind, the machinist will avoid the upper limits. More final diameters ends up in the lower half of the tolerance band. The beta distribution is generally suitable for describing such a biased or asymmetric variability.
- A triangular distribution provides a simple way to describe the variability for cases where, for example, the user only has a best guess value and assumes that certain departures from this value are possible. The best guess in this case would be the most likely value. Minimum and maximum values describe deviations from the peak and



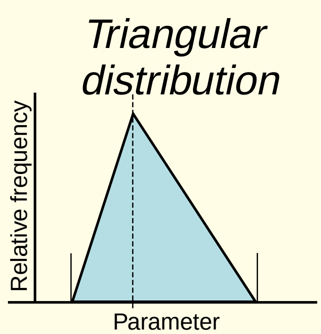
The Gaussian curve describes a symmetric distribution of values that occur most often about the average and less so away from it.



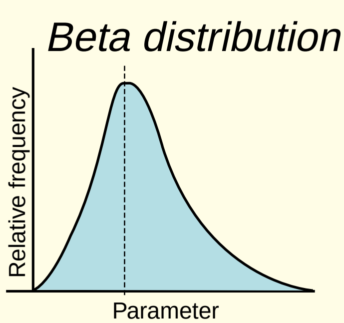
The truncated Gaussian distribution describes a range of values similar to the Gaussian distribution but with not-to-exceed limits.



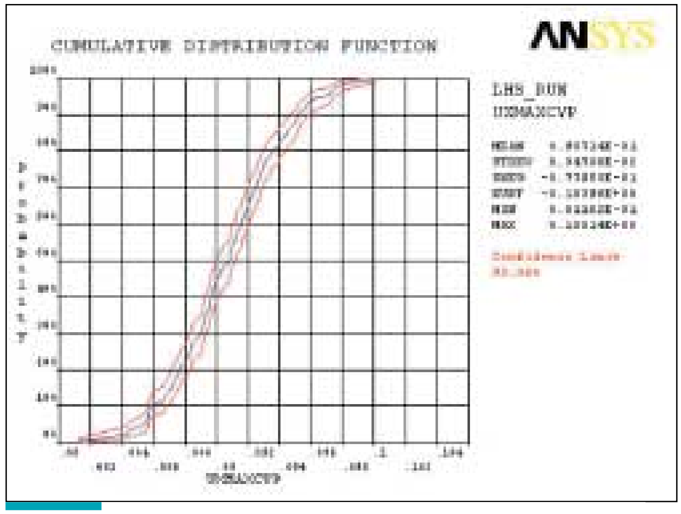
In a Uniform distribution all values are equally likely.



The triangular distribution has a best guess value as most likely. It marks the location of the peak.



The Beta distribution can be asymmetric to account for certain manufacturing characteristics.



A cumulative-distribution function puts probability relationships for quantities of interest into clearer focus. For example, if radial displacement drops below a certain limit, for example 0.090, then the clearance between the seal strip and the cover plate is getting too large, and hot gas is no longer properly sealed off. The curve shows a displacement of 0.090 units or less occurs with a probability of 55%. An engineering decision must accept or change the design, or the acceptable limit for radial displacement.

define the base of the triangle. This distribution is typically used for boundary conditions that are calculated from other software packages. For example, the temperature of the gas surrounding a turbine blade could be calculated using a CFD code, which would then be the most likely value.

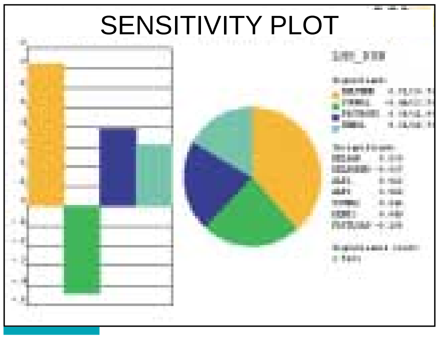
- Other more specialized inputs are described as Gamma, Weibull, and Exponential curves.

Material suppliers can usually provide a mean, upper, and lower bound for the distribution of material values in their literature. Most manuals provide a lower bound to the material figures which introduces a safety factor into calculations. When a supplier publishes a material value and a lower bound, the lower bound is usually a statistical value. The 99% exceedence limit provides an example. This means that in 99% of all cases the real material property value will exceed the specified limit. Sometimes material suppliers provide data for a mean value and standard deviation. In either case, two figures are enough to describe a distribution function having two distribution parameters such as the Gaussian.

RUNNING THE SOFTWARE

The general sequence of operation remains similar to traditional FE analyses: Users define a model, apply loads and boundary conditions, run the analysis, and interpret the output. The meshed models appear as they would in any FE analysis. No adjustments are needed. Also the same are ways of analyzing a product using a static analysis, eigen frequency analysis, thermal analysis, and so on.

However, model preparation takes more time than would a classical analysis. Solution times are also longer, but they are not affected by the distributions chosen.



The sensitivity chart shows that Deltemp most influences the radial displacement in the cover plate at the sealing groove. The controllable parameters under the heading Significant would be altered should a more robust design be needed. If the goal is to manufacturer the disk less expensively, then engineers would focus on the parameters listed under the Insignificant heading. For example, manufacturing tolerances on Delgap could be loosened (if other factors allow) because it has no influence on the radial displacement, or UxMaxCvp.

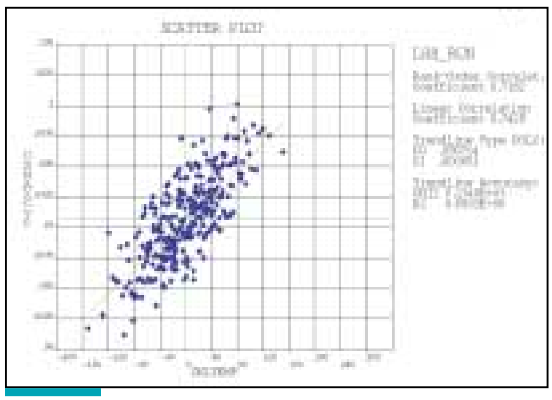
After model preparation, users must decide on a method to solve the probabilistic model. The available methods differ in how they treat or sample input values. For example, the Monte Carlo method picks input values randomly from the distributions specified by the user. Monte Carlo simulations are well known and easy to understand because they mimic the behavior and sampling process of nature. Another sampling method is called the Response Surface.

Because solution times will be longer, the software includes parallel-processing features to keep potentially enormous run times within reason. The software can distribute computation tasks to idle machines on the same network. For example, a master or controller computer would send the model and input variables to one idle computer with instructions to run analysis 1, while a second and a third computer might receive instructions to execute analysis run number 2 and 3. When the first of these machines finish it would receive instructions to run analysis number four and so on. The software can manage sending analysis jobs to as many as 255 computers. In the end, the master computer assembles results and produces output charts. This is all done automatically without user interaction.

Rather than a simple stress contour plot, the software provides a more complete look at a product that helps better identify the weak points and directs redesign work towards increased reliability and robustness, when necessary. The visualization and explanation of the probabilistic results include:

- Histogram plots to illustrate the relative frequency that output values might occur. A higher histogram curve indicates a more robust design than one with a wider base.
- Probability plots directly provide a way to judge the reliability of a product or its function.
- Sensitivity charts (pie and bar charts) show which input variables most influence the behavior of a product, and which have little influence. They illustrate where product-improvement efforts can be made most efficiently — by focusing on the important parameters. They also show how the manufacturing process can be made less expensive — by relaxing requirements on insignificant parameters.
- Scatter plots illustrate, for example, trends between inputs and outputs and provide quantitative information for how much an input value might be adjusted to achieve a particular desired output.
- And a report can automatically record all findings of the analysis with a descriptive text. The report in HTML format can be posted to a Web site (Internet or intranet) to share with colleagues and supervisors.

To illustrate the software's features, an accompanying box shows how the technology was applied to a problem relating to the new design for a two-part turbine disk for a jet engine. Materials were selected but temperatures are uncertain. The analysis is to help zero in on possible values for each. ■



Scatter plots provide indications of relations between characteristics. After examining the sensitivity charts of UxMaxCvp the user could, for example, request a scatter plot of UxMaxCvp as a function of the most important input parameter, Deltemp in this case. The diagonal line provides a best linear fit. An increase in Deltemp generally indicates an increase in radial displacement at the sealing groove. If Deltemp were an input parameter that could be controlled or adjusted, then the diagonal line shows in which direction and how far it should be adjusted for a certain required average level for the output parameter.

Statistics of the output parameters

NAME	MEAN	STD DEVIATION	MIN	MAX
SeqMxDsk	2.2899E+05	6703	-2.0969E+05	2.4523E+05
UxMaxRim	8.1102E-02	3.3956E-03	6.9585E-02	9.218E-02
UxMaxCvp	9.0714E-02	3.4700E-03	8.1182E-02	0.1001

The statistics of the output parameters show the spread of these variables.

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