Behavioural modeling for heterogeneous systems based on FEM descriptions

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Abstract

The component design of heterogeneous systems and microsystems as sensor and actors is well supported by FEM programs, e.g. ANSYS. Interactions between different physical domains are partly taken into consideration by these programs. To simulate more complex systems using Spice-compatible circuit and system simulation programs, behavioural models of the components are required. The focus of the paper is to discuss two possibilities to derive such models from FEM descriptions. On the one hand, FEM ideas can be applied to derive analytically formulas for the description of the terminal behaviour with languages like MAST, HDL-A and VHDL-AMS. On the other hand, reduced system matrices from FEM programs can be used to generate behavioural models with fixed numerical values. Analytically and numerically derived models can be linked together. Some examples will be presented.

1 Introduction

The basis of modern Spice-compatible circuit and system simulation programs is the solution of systems of differential algebraic equations (DAEs). Originally, these programs were developed for the simulation of electronic circuits. Because of the well-known analogies between electrical and nonelectrical networks, these simulators can also be used to solve nonelectrical problems.

Modern description languages allow the integration of user-defined elements into these programs. In the time domain the subsystems are described by time-continuous quantities that can occur at their border and special „internal“ quantities. A very general form of the description of the terminal behaviour is given by

$$i(t) = f_1(v|t, s|t, p) \tag{1}$$

$$0 = f_2(v|t, s|t, p) \tag{2}$$

i(t) is a vector of dependent terminal quantities at time *t*. Without loss of generality it can be set to the actual values of through quantities into the terminals. $v|t$ and $s|t$ characterize the independent terminal quantities *v* and some state values *s*, resp. at time *t*. Without loss of generality *v* is the vector of across quantities measured between the terminals and the assigned reference node(s). *p* is a vector of parameters. In general, the characterization results in the usage of the values of *v* and *s* at the actual time *t* and the values of their derivatives [1, 2].

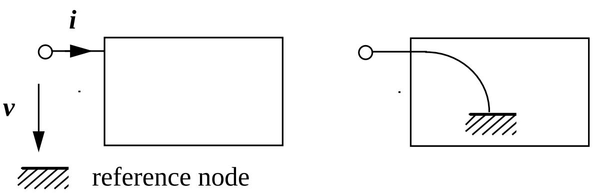


Fig. 1: Terminal quantities of a subsystem (i vector of through quantities, v vector of across quantities) and internal branch structure of a VHDL-AMS model based on (1) and (2)

The mathematical oriented descriptions (1) and (2) can be used as basis for the implementation of models in different behavioural description languages like MAST and HDL-A. VHDL-AMS models can also formally be derived from this form. In this case, the vector *s* is represented by free quantities. Branches are introduced that connect the terminals to the assigned reference node. Their through quantities establish the vector *i* and their across quantities the vector *v* (see Fig. 1).

In conclusion, that means that the step from (1) and (2) to a behavioural model is a formal one. The essential problem is to find the suitable description (1) and (2) for a given task.

2 Relations between FEM descriptions and Terminal Behaviour

One systematic way to get behavioural models is to derive functions *f* of subsystems similar to the way to establish finite elements for FEM solvers. These programs in principle solve a partial differential equation by minimization of an associated functional. For the beginning, only linear dynamic descriptions are taken into account.

The idea behind the Finite Element Method from an engineering point of view is to determine an functional of a system as the sum of the functionals of the subsystems. We use energy as such a functional. Then, the energy of a subsystem can be determined with the help of some signals at specified points at the border of the subsystem. These points are called nodes. The objective of the Finite Element Analysis is to determine the node signals so that the energy of the system is minimized. There is a close relation to the formulation of Ritz to minimize the original functional in a finite dimensional subspace [3].

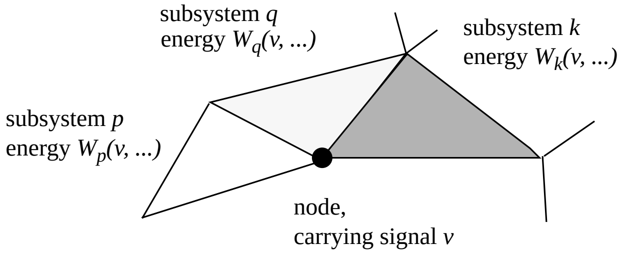


Fig. 2: Part of a system (principle)

Fig. 2 shows a detail of a system. The energies of the subsystems p, q and k depend on the signal v at the node. The energy W_S of the system (in Fig. 2) is given by

$$W_S = \ldots + W_p(v, \ldots) + W_q(v, \ldots) + W_k(v, \ldots) + \ldots \quad (3)$$

The energy W_S is a minimum if all the derivatives to all node signals are equal to zero. That means, the derivative of W_S to v has also to be zero. Because only W_p, W_q and W_k depend on v it follows

$$\frac{\partial W_S}{\partial v} = \frac{\partial W_p}{\partial v} + \frac{\partial W_q}{\partial v} + \frac{\partial W_k}{\partial v} = 0 \quad (4)$$

This equation can be interpreted as a generalized Kirchhoff's Current Law (KCL) at the node.

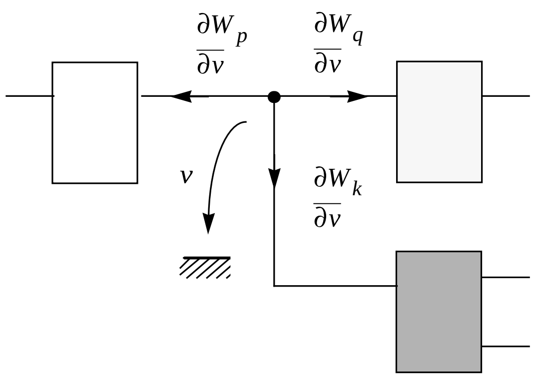


Fig. 3: KCL for finite element models (principle)

Thus, the same description used in FEM-programs can (in principle) be used in network analysis programs. The „FEM-network“ has to be composed of blocks which represent the subsystems. We use as flow quantities through the terminals the derivatives of the energy of the subsystem with respect to the across quantities at the terminals. The negative derivatives could also be used for this purpose. It is only important that the derivation is done for all later connected subsystems in the same way. The choice of the sign influences furthermore the direction of external „flow sources“ with respect to their direction.

The terminal description of a subsystem can now be derived in the following way. $W(v)$ is the „energy“ of a subsystem. The behaviour is expressed with the help of the potentials (across quantities) v at the subsystems nodes (terminals). The through quantities are in principle the derivatives of W with respect to v. That means, the mathematical description is given by

$$i(t) = \frac{\partial W}{\partial v} \quad (5)$$

In the case of an electrical terminal with the across voltage quantity v_{term} the associated relation (5) delivers the electrical charge as a flow quantity. If the current shall be used as terminal flow quantity of a model of a subsystem a free quantity s_{term} (equal to charge) has to be introduced. Then s_{term} can to be computed via (5). The terminal current is computed by derivation of s_{term} with respect to the time.

The discussed approach (5) is also used for finite elements in many FEM programs. A FEM program sets up the system matrices from this description and solves them. The steps to set up a system of DAE's from a netlist description are corresponding. The main advantage of the relation between FEM and circuit analysis programs is that well-known analytical descriptions of finite elements can be used to construct behavioural models. On the other hand, numerical values of system matrices established with a FEM program can be directly used for behavioural models. Similar ideas are used by Romanowicz [4] to model transducers (see also [5]).

3 Model Generation

3.1 Analytical Models

Main experiences applying this way were achieved in modeling the mechanical part of sensors. The general form of the description of a linear mechanical component is given by

$$F = D^T \cdot \overline{M} \cdot D \cdot \frac{d^2 v}{dt^2} + D^T \cdot \overline{D} \cdot D \cdot \frac{d}{dt} v + D^T \cdot \overline{K} \cdot D \cdot v \quad (6)$$

For a beam, F is a vector of through quantities measured in a fixed reference coordinate system. It consists of forces and torques. v is the vector of across quantities consisting of displacements and angles of rotation. D is a matrix to transform values from a description in a local coordinate system to a global coordinate system.

$\overline{M}$, $\overline{D}$ and $\overline{K}$ are mass, damping and stiffness matrices of a finite element as known from textbooks. This is the main advantage of this way. Known formulas from FEM can be used to describe a behavioural model. (6) is a special form of (1). So it is easy to establish a special model implementation. Geometrical parameters of elements can easily be changed because the element matrices depend on them analytically. Further information can be found in [6]. An example is given in section 4.1.

3.2 Numerical Models

Some FEM simulators like ANSYS allow to access the numerical values of the system matrices. Based on the element descriptions like (6), the description of the geometry of components and the connection of basic finite elements the simulator establishes these matrices.

These matrices can be used to describe the terminal behaviour of a subsystem in the following way:

1. Mark the geometry of the subsystem that has to be modelled inside of the FEM simulator.
2. Define the nodes that shall be used as terminals of the behavioural model.
3. Generate the system matrices for the defined component. During the generation some simulators carry out a reduction of the matrices [7]. The accuracy of the description of the dynamic behaviour of the defined component can be influenced by defining some additional internal nodes in step 2.
4. Generate a behavioural model in MAST, HDL-A or VHDL-AMS from these matrices. The signals at the internal nodes correspond to free quantities in the behavioural model.

In the linear dynamic case the description of the terminal behaviour of the subsystem has the form

$$\begin{bmatrix} i \\ 0 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \cdot \begin{bmatrix} \ddot{v} \\ \ddot{s} \end{bmatrix} + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \cdot \begin{bmatrix} \dot{v} \\ \dot{s} \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \cdot \begin{bmatrix} v \\ s \end{bmatrix} \quad (7).$$

4 Examples

Some examples shall demonstrate the power of the described approach. Because of limits concerning the range of this paper all details cannot be explained but we hope that the main ideas will be comprehensible.

4.1 Simple Beam

A standard example in textbooks (e. g. [8]) to describe the main ideas of FEM is a simple beam (see Fig. 5)

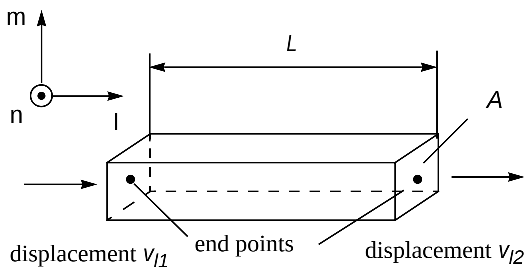


Fig. 5: Simple beam in local (*l, m, n*) coordinate system

It is also an example to demonstrate the procedure from section 2.1. Only static displacements and forces in the direction of the axis of the beam are taken into account. In a local coordinate system l, m, n (with the l-axis equal to the beam axis) there is the following relation between displacements and forces at the ends of the beam. The indices refer to the coordinate directions.

$$\begin{bmatrix} F_{l1} \\ F_{m1} \\ F_{n1} \\ F_{l2} \\ F_{m2} \\ F_{n2} \end{bmatrix} = \begin{bmatrix} \frac{E \cdot A}{L} & 0 & 0 & -\frac{E \cdot A}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{E \cdot A}{L} & 0 & 0 & \frac{E \cdot A}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} v_{l1} \\ v_{m1} \\ v_{n1} \\ v_{l2} \\ v_{m2} \\ v_{n2} \end{bmatrix} = \overline{K} \cdot \begin{bmatrix} v_{l1} \\ v_{m1} \\ v_{n1} \\ v_{l2} \\ v_{m2} \\ v_{n2} \end{bmatrix} \quad (8)$$

To model systems composed of different simple beams and other basic elements, the formulation of the terminal equations in a global coordinate system (x, y, z) is more convenient (see Fig. 6).

C is a matrix (submatrix of D) to transform values from a local (l, m, n)-system to a global (x, y, z)-system which is equal for all simple beams of a system. If (x_1, y_1, z_1) and (x_2, y_2, z_2) are the end points of a simple beam then the interesting matrix elements of C are

$$c_{11} = \frac{x_2 - x_1}{L}, \quad c_{12} = \frac{y_2 - y_1}{L} \quad \text{and} \quad c_{13} = \frac{z_2 - z_1}{L} \quad (9).$$

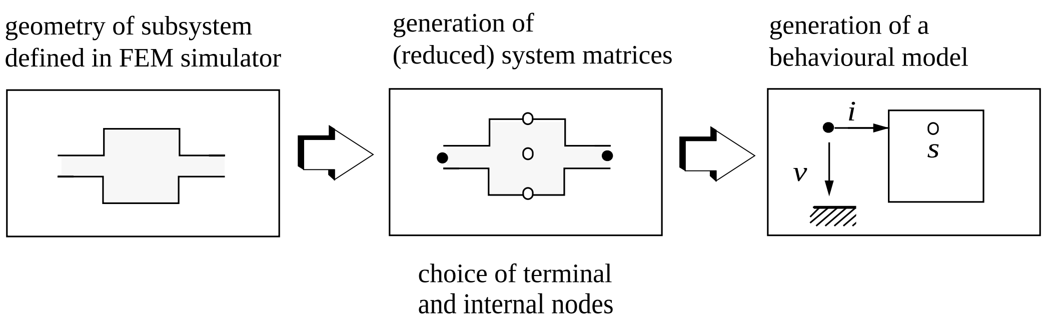
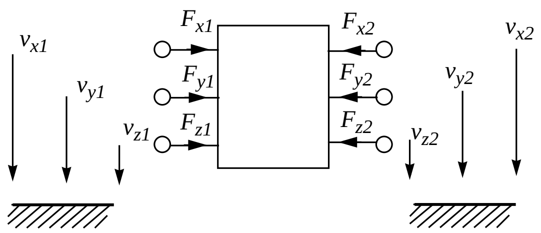


Fig. 4: Construction of behavioural models with fixed numerical values using FEM descriptions



$$\begin{bmatrix} F_{x1} \\ F_{y1} \\ F_{z1} \\ F_{x2} \\ F_{y2} \\ F_{z2} \end{bmatrix} = \begin{bmatrix} C^T & 0 \\ 0 & C^T \end{bmatrix} \cdot \bar{K} \cdot \begin{bmatrix} C & 0 \\ 0 & C \end{bmatrix} \cdot \begin{bmatrix} v_{x1} \\ v_{y1} \\ v_{z1} \\ v_{x2} \\ v_{y2} \\ v_{z2} \end{bmatrix}$$

Fig. 6: FEM-block for a simple beam in a global coordinate system

A HDL-A model (in accordance with Fig. 6) has the following form:

```
ENTITY stabelement IS
        GENERIC ( x1, y1, z1 : real;
              -- coordinates 1. end point
                 x2, y2, z2 : real;
              -- coordinates 2. end point
                 E : real;  -- elasticity module
                 B : real;  -- witdh
                 H : real); -- height
       PIN    ( p : mechanical2_vector (1 to 6));
              -- terminals
END ENTITY stabelement;

ARCHITECTURE v1 OF stabelement IS

 CONSTANT K    : real_matrix (1 to 6, 1 to 6);
              -- stiffness matrix divided by E*A/L
 CONSTANT C    : real_vector (1 to 3);
               -- transformation matrix
 VARIABLE L    : real;           -- length
 CONSTANT EAL  : real;           -- E*A/L
 VARIABLE k_ij : real;
 VARIABLE val  : analog;
 VARIABLE I, J : integer;

BEGIN
  RELATION
    PROCEDURAL FOR init =>

     E    := 2.0E8;    -- in kN/m^2
     B    := 5.0E-2;   -- in m
     H    := 1.0E-2;   -- in m

    L    := sqrt ((x2 - x1)*(x2 - x1) + (y2 - y1)*
             (y2 - y1) + (z2 - z1)*(z2 - z1));
     C(1) := (x2 - x1)/L;
     C(2) := (y2 - y1)/L;
     C(3) := (z2 - z1)/L;

     EAL  := (E*B*H)/L;  -- E*A/L
     FOR I IN 1 TO 3 LOOP
         FOR J IN 1 TO 3 LOOP
             k_ij       := C(I)*C(J);
             K(I,J)     := k_ij;
             K(I+3,J+3) := k_ij;
             K(I,J+3)   := -k_ij;
             K(I+3,J)   := -k_ij;
         END LOOP;
     END LOOP;

  PROCEDURAL FOR ac, dc, transient =>

     FOR I IN 1 TO 6 LOOP
         val := 0.0;
         FOR J IN 1 TO 6 LOOP
             val := val + K(I,J)*p(j).d;
         END LOOP;
         p(i).f %= EAL*val;
     END LOOP;

  END RELATION;
END ARCHITECTURE v1;
```

In a network consisting of behavioural models of simple beam elements external forces have to be taken into consideration as flow sources from the node where the force occurs to ground. Additional conditions concerning the displacements at special nodes are realized by across sources connecting the corresponding node and ground.

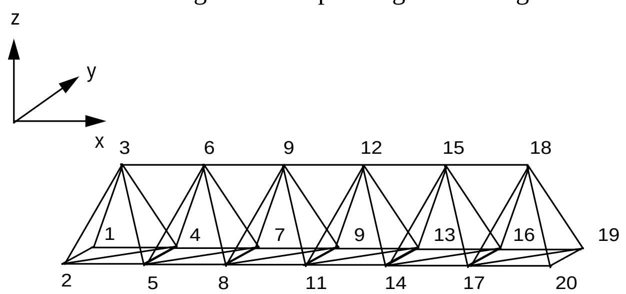


Fig. 7: Girders of a crane [8]

The following statements are part of an ELDO netlist to simulate the example from Fig. 7:

```
* Simple beam from 3 to 6
y1 stabelement(v1)   pin: nodex3 nodey3 nodez3
+                         nodex6 nodey6 nodez6
+   generic:  x1=1.0  y1=1.0 z1=2.0
+             x2=3.0  y2=1.0 z2=2.0
* Fixed points in y- and z-direction (at 1)
v1   nodey1   0    0.0
v2   nodez1   0    0.0

* external forces at 7 and 8

i1   nodez7   0    30.0
i2   nodez8   0    30.0
```

4.2 Electro-thermal coupling

The self-heating of an operational amplifier µA741 as a result of a jump of the difference input signal (400 µV) at T1 and T2 shall be investigated. The heat flow from T14 and T20 warms the chip with the opamp. Thus, the parameters of the circuit elements change. The temperature of T1 and T2 shall be determined.

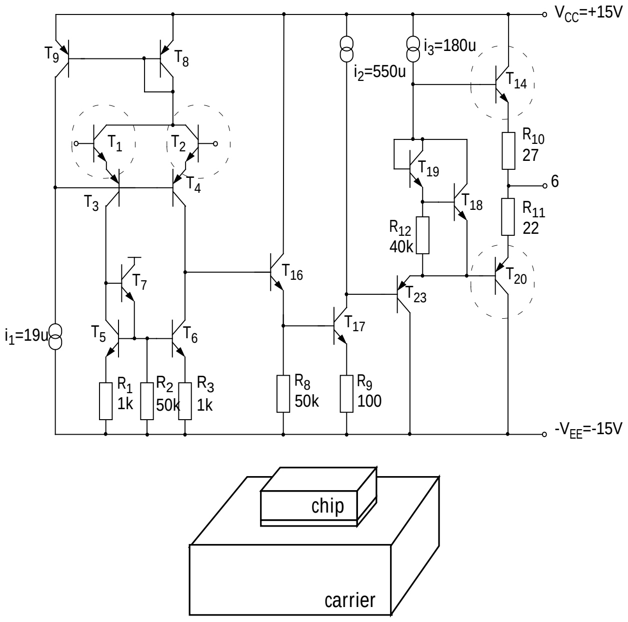


Fig. 8: Operational amplifier µA741 (see e. g. [9] and references) and chip structure

In [9] this task is solved by coupling the simulators Saber and ANSYS. We constructed now a MAST-model [10] for the chip from the ANSYS description as discussed in section 3.2 and simulated operational amplifier and thermal chip model together (Fig. 9). Heat flow is the through quantity of the thermal model and temperature its across quantity.

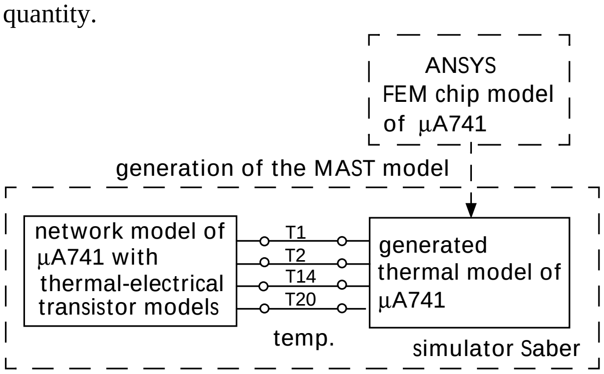


Fig. 9: Simulation of thermal electrical coupling with Saber

The simulation with Saber takes about some seconds, its preparation some minutes. In comparision the computation of the same results (with the same accuracy) using a coupling of Saber and ANSYS takes some hours.

4.3 Acceleration sensor

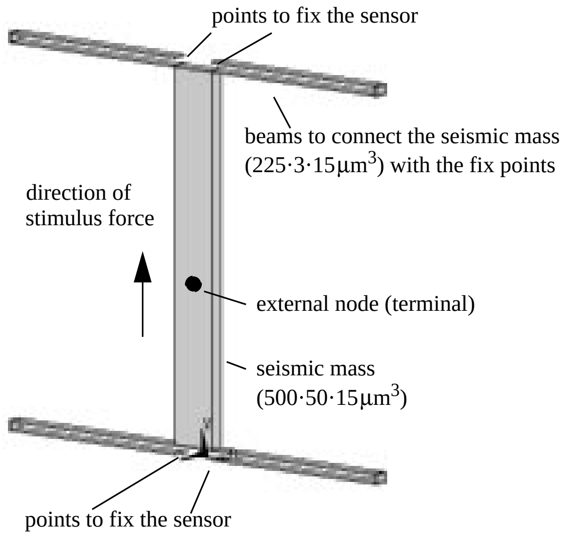


Fig. 10: Structure of an acceleration sensor

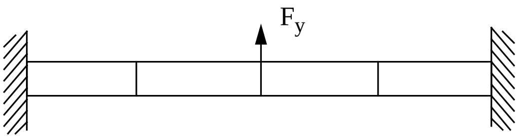
For the mechanical part of an acceleration sensor (Fig. 10, [11]) a VHDL-AMS model was generated in accordance with the principles of section 3.2. The behavioural model has one terminal and 40 internal nodes. The ANSYS beam elements take dynamic effects into consideration. Displacements and forces were taken into account. In the following, parts of the generated model are shown.

```
library DISCIPLINES;
use DISCIPLINES.KINEMATIC_SYSTEM.all;

ENTITY file41 IS
PORT (
TERMINAL t121_8609 : kinematic;
TERMINAL t122_8609 : kinematic;
TERMINAL t123_8609 : kinematic
);
END file41;

ARCHITECTURE model OF file41 IS

QUANTITY v_t121_8609 ACROSS i_t121_8609 THROUGH
                     t121_8609;
QUANTITY v_t122_8609 ACROSS i_t122_8609 THROUGH
                     t122_8609;
QUANTITY v_t123_8609 ACROSS i_t123_8609 THROUGH
                     t123_8609;

QUANTITY v_t1_635 : displacement;
QUANTITY v_t2_635 : displacement;
...

BEGIN
i_t121_8609 == 8.000003e-24*v_t1_635'DOT'DOT +
(-2.368012e-23)*v_t2_635'DOT'DOT + (-2.786784e-34
)*v_t3_635'DOT'DOT + 6.614110e-23*
v_t4_671'DOT'DOT + 5.316135e-23*v_t5_671'DOT'DOT
+ 1.088774e-33*v_t6_671'DOT'DOT ...
...
END ARCHITECTURE model;
```

Experiences with an equivalent MAST model show a good correspondence of the simulation with a circuit and a FEM simulator stimulating the seismic mass at its center. Resonance frequencies are detected at 15.941 kHz resp. 15.512 kHz, 112.21 kHz resp. 112.54 kHz and so on.

4.4 Mechanical systems with Geometric Nonlinearities

If in the transformation matrix from a local to a global coordinate system (see e. g. equation (8) in section 4.1) the coordinates of the end points change with the displacements of these points, we obtain models that consider geometric nonlinearities. That means in equation (6) the matrix D is a function of v.



length	$l=500\mu m$	density	$rho=2329 kg/m^3$
width	$b=15\mu m$	E-module	$E=1.302\cdot 10^{11} N/m^2$
height	$h=3\mu m$	G-module	$G=0.7962\cdot 10^{11} N/m^2$

Fig. 11: Test structure

Fig. 11 shows a test structure modelled with beam elements with geometric nonlinearities. The structure was stimulated with a sinussoidal force. Once the frequency f was increased from lower to higher values than it was decreased from higher to lower values. In Fig. 12 the transfer function shows the amplitude of the displacement v_y depending on f. The graph shows that there are two different steady states for some frequencies. Further results are discussed in [12].

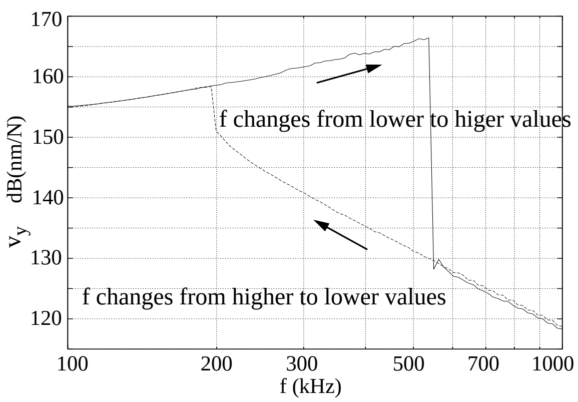


Fig. 12: Transfer function for $F_y=10^{-4}N\cdot \sin(2\pi ft)$

In this context it is evident that our approach can also be used as a basis for more complicated nonlinear models.

5 Conclusion

We have discussed an approach that allows

- the usage of analytical FEM formulas for the construction of behavioural models,
- to derive behavioural models with fixed numerical values for components from FEM descriptions,
- the implementation of models in different languages (MAST, HDL-A, VHDL-AMS).

The models can be used in a Spice-compatible circuit and system simulator. The approach is demonstrated with some examples. The models obtained in this approach can be used as basis for the consideration of more complicated effects (e.g. geometric nonlinearities).

References

[1] Casinovi, G.; Yang, J.-M.: *Multi-Level Simulation of Large Analog Systems Containing Behavioural Models*, IEEE Transactions on CAD 13 (1994) 11, pp. 1391-1399.

[2] Clauss, C.; Haase, J.; Schwarz, P.: *An Approach To Analogue Behavioural Modelling*, Proc. SIG-VHDL Spring ‘96 Working Conference, Dresden, May 1996, pp. 181-192.

[3] Braess, D.: *Finite Elemente*, Berlin/Heidelberg: Springer-Verlag, 1997.

[4] Romanowicz, B. F.: *Methodology for the Modeling and Simulation of Microsystems*, Boston/Dordrecht/London: Kluwer Academic Publishers, 1998.

[5] Senturia, S. D.: *CAD Challenges for Microsensors, Microactuators, and Microsystems*, Proceedings of the IEEE 86 (1998) 8, pp. 1611-1626.

[6] Neul, R.; Becker, U.; Lorenz, G.; Schwarz, P.; Haase, J.; Wünsche, S.: *A Modeling Approach to Include Mechanical Microsystem Components into the System Simulation*, Proc. Design, Automation and Test in Europe DATE 1998, Paris, 23.-26.2.98, pp. 510-517.

[7] Guyan, R. J.: *Reduction of Stiffness and Mass Matrices*, AIAA Journal 3 (1965) 2, 380.

[8] Schwarz, H. R.: *FORTRAN-Programme zur Methode der finiten Elemente*, Stuttgart: B. G. Teubner, 1991.

[9] Wünsche, S.; Clauß, C.; Schwarz, P.; Winkler, F.: *Electro-Thermal Circuit Simulation Using Simulator Coupling*, IEEE Transactions on VLSI 5 (1997) 3, 277-282.

[10] Mantooth, H. A.; Fiegenbaum, M. F.: *Modeling with an Analog Hardware Description Language*, Kluwer Academic Publisher, 1994.

[11] Haase, J.; Reitz, S.; Wünsche, S.; Schwarz, P.; Becker, U.; Lorenz, G.; Neul, R.: *Netzwerk- und Verhaltensmodellierung eines mikromechanischen Beschleunigungssensors*, Proc. 6. Workshop „Methoden und Werkzeuge zum Entwurf von Mikrosystemen“, Paderborn, December 1997, pp. 23-30.

[12] Reitz, S.; Haase, J.; Schwarz, P.: *Dynamisches Verhalten von mikromechanischen Systemen unter Berücksichtigung von Nichtlinearitäten*, Proc. 7. Workshop „Methoden und Werkzeuge zum Entwurf von Mikrosystemen“, Paderborn, January 1999, pp. 59-66.