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Space-Time Finite Element Approximation and Numerical Solution of Hereditary Linear Viscoelasticity Problems

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Vorwort

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Darüberhinaus bietet die Reihe ein Forum für die Berichterstattung über die zahlreichen Kooperationsprojekte des Instituts mit Partnern aus Industrie und Wirtschaft.

Berichterstattung heißt hier Dokumentation darüber, wie aktuelle Ergebnisse aus mathematischer Forschungs- und Entwicklungsarbeit in industrielle Anwendungen und Softwareprodukte transferiert werden, und wie umgekehrt Probleme der Praxis neue interessante mathematische Fragestellungen generieren.

Prof. Dr. Dieter Prätzel-Wolters
Institutsleiter

Kaiserslautern, im Juni 2001

Space-Time Finite Element Approximation and Numerical Solution of Hereditary Linear Viscoelasticity Problems

Julia Orlik [*], Arina Ostrovska

July 19, 2006

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Abstract

In this paper we suggest a fast numerical approach to treat problems of the hereditary linear viscoelasticity, which results in the system of elliptic partial differential equations in space variables , who's coefficients are Volterra integral operators of the second kind in time. We propose to approximate the relaxation kernels by the product of purely time- and space-dependent terms, which is achieved by their piecewise-polynomial space-interpolation. A priori error estimate was obtained and it was shown, that such approximation does not decrease the convergence order, when an interpolation polynomial is chosen of the same order as the shape functions for the spatial finite element approximation, while the computational effort is significantly reduced.

1 Introduction

Our task is to develop an approach for the numerical treatment of mathematical problems, which arise from considering the behavior of hereditary viscoelastic solids. These result in a system of elliptic partial differential equations in space variables, whose coefficients are Volterra integral operators of the second kind in time, which allow for weak-singular kernels, i.e.,

$$\frac{\partial}{\partial x}\left(a_0(x,t)\frac{\partial u(x,t)}{\partial x}\right)+\int_0^t \frac{\partial}{\partial x}\left(a(x,t,\tau)\frac{\partial u(x,\tau)}{\partial x}\right)d\tau = f(x,t). \tag{1}$$

Section 2 gives an overview about existing models for the viscoelasticity. In Sections 3 and 4 a general mathematical model of the boundary value problem of the inhomogeneous hereditary ageing viscoelasticity is given in classical and weak formulations. The solvability of such a Volterra integral equation in Sobolev spaces and the stability of the solution with respect to the right-hand side is recalled here in Theorem 4 and proven in [1], [4], [3].
Our basic idea for the numerical solution of such a problem, was to treat the space and time dependence of the solution separately, with Finite Elements technique in x and with spline collocation in t,τ. This can be done trivially, if the time and space dependence in the instantaneous elastic coefficients $a_0(x,t)$, relaxation kernels $a(x,t,\tau)$ and the external forces $f(x,t)$ can be separated in a straightforward manner. In Section 6 we suggest to approximate the integral kernels out-of-integral terms and right-hand side in space by polynomial interpolation, thus representing them by a product of purely time- and space-dependent terms, and reduce the performance of the spatial FE-analysis in such a way. This approach allows us to raduce calculation time. In Sec. 7 the errors, introduced by the approximation of kernels, out-of-integral coefficients and external loads as well as the total error due to the numerical treatment are estimated. It is shown, that choosing an interpolation polynomial of the same or even one order less compared to the shape functions in the finite element approximation of the solution, we do not decrease the convergence order. Therefore we suggest the analyst to use the kernel approximation method, even though it requires more effort in preliminary work.
For the software realization of our numerical method we have chosen ANSYS as the basic simulation tool due to its extensive modeling capabilities and convenient user interface. The operations, that are not standard for ANSYS, like,

for example, spline collocation in time or kernel approximation in space, were coded in separate procedures and integrated into the ANSYS environment as User Predefined Routines. A corresponding numerical example is considered in Sec. 8

2 Models of viscoelasticity

It has long been known that all visco-elastic materials, e.g., polymers, when subjected to mechanical loading, exhibit two distinct deformation mechanisms. There always is initially an elastic deformation obeying Hooke's law followed by some sort of viscous flow. For better understanding, visco-elastic models often are represented as combinations of springs and dash-pots in mechanical engineering. The stress in the spring(s) is proportional to the strain, while the stress in the dash-pot(s) is proportional to the strain-rate. All such models valu on a **differential form of the constitutive law**

$$\sum_{i=0}^{m} D_i \left(\frac{d}{dt}\right)^i \sigma(t) = \sum_{i=0}^{r} B_i \left(\frac{d}{dt}\right)^i e(t), \tag{2}$$

where σ and e denote stress and strain, respectively, m, r, D_i, B_i are some constants depending on the number of springs and dash-pots, their stiffness and their interconnection in the arrangement. These models can be subdivided into two main groups: Maxwell models and Voigt models. All arrangements of Maxwell elements lead to the constitutive equation (2) with $m \equiv r$. Alternatively, in the Voigt model, one has $m < r$. Note that such an equation (for the simple Voigt model $m = 0$, $r = 1$) implies the following: when the strain rate changes discontinuously, it results in the overall stress, which however can not be observed in the experiment. Moreover, instantaneously applied deformation will cause infinite stress in this situation, and therefore all these models only partially reflect a physical reality.
All Maxwell models can be rewritten in an equivalent integral form, where elastic coefficients (out-of-integral terms) are independent of time and the kernels are of the convolution type:

$$\sigma(t) = A_0 e(t) + \int_0^t A(t-\tau) e(\tau) d\tau. \tag{3}$$

This type of stress-strain relation is called **integral model without aging**.

Example 1 *Consider Kelvin's model*

$$\frac{d\sigma(t)}{dt} + D_0 \sigma(t) = B_1 \frac{de(t)}{dt} + B_0 e(t),$$

consisting of a spring and a dash-pot connected in parallel and then this arrangement is further connected to another spring in series. The equivalent integral form is (3) with $A_0 = B_1$, $A(s) = (B_0 - B_1 D_0) e^{-D_0 s}$.

All kernels $A(s)$ obtained from the differential models are represented by a sum of exponents.

The viscoelastic differential model of the form (2), for which $m < r$, can be rewritten in the equivalent integral form

$$\sigma(t) = A_0 e(t) + \frac{de(t)}{dt} + \int_0^t A(t-\tau)e(\tau)d\tau, \tag{4}$$

which can be rewritten via further integration by parts as **Volterra equation of the second kind**:

$$\sigma(t) = \frac{de(t)}{dt} + \int_0^t K(t-\tau)\frac{de(\tau)}{d\tau}d\tau. \tag{5}$$

For $m > r$, we obtain a **Volterra equation of the first kind**

$$\sigma(t) = \int_0^t A(t-\tau)e(\tau)d\tau \tag{6}$$

that can be transformed to an equivalent **Volterra equation of the second kind**:

$$\sigma(t) = (A(0) - A(t))\tilde{e}(t) + \int_0^t \frac{dA(t-\tau)}{d\tau}\tilde{e}(\tau)d\tau, \tag{7}$$

where

$$\tilde{e}(t) := \int_0^t e(\tau)d\tau.$$

From the mechanical literature, e.g. [9], [10], there are also known some models of the form (2), including not only integer, but also fractional orders of the time derivatives:

$$\sigma(t) = \kappa\frac{d^\alpha e(t)}{dt^\alpha}, \tag{8}$$

where $0 < \alpha < 1$. Note that for $\alpha = 0$, $\kappa = E$ this yields Hooke's law, for $\alpha = 1$, $\kappa = \mu$ - Newtonian viscous flow.

All such models, according to [10], [11], can also be transformed to an integral (10) of the convolutional type with weakly singular kernels, like Abel's type:

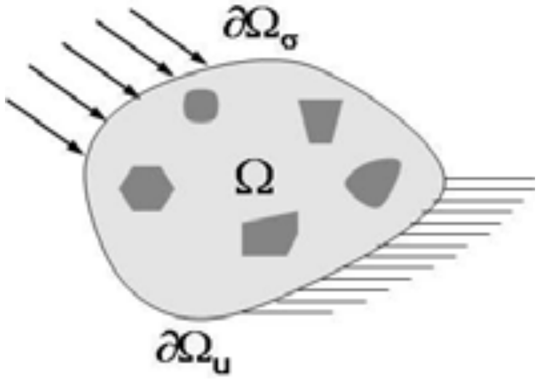
$$\sigma(t) = \frac{\kappa}{\Gamma(1-\alpha)}\int_{-\infty}^t \frac{de(\tau)}{(t-\tau)^\alpha}. \tag{9}$$

Some hereditary (viscoelastic) materials, e.g., concrete, are subject to aging that means that the material properties change in time, independently of the applied loading. Such materials are described by **Integral models with aging**, in which the out-of-integral term is dependent on time, and the kernels are of the non-convolution type, i.e.

$$\sigma(t) = A_0(t)e(t) + \int_0^t A(t,\tau)e(\tau)d\tau. \tag{10}$$

3 Definition of the problem.

We consider a linear viscoelastic and aging (of the non-convolutional integral type) body, which is subject to some external loading. We denote the volume occupied by the body by Ω, which is assumed to be a Lipschitz domain.
We are going to consider the equilibrium equations for such a solid. Note that a viscoelastic solid is still a solid and therefore its deformation is slow and we restrict ourselves to the quasi-static case description, i.e., classical for the solid mechanics statement of problem, without the inertial term. A summation from 1 to n over repeating indices is assumed in all the present work, unless the opposite is stated.



$$\frac{\partial}{\partial x_h}\Big(\Big(a_{ij\,0}^{hk}(x,t)+a_{ij}^{hk}(x)\star\Big)\frac{\partial u_j(x,t)}{\partial x_k}\Big)=-f_{i0}(x,t),\quad x\in\Omega \tag{11}$$
$$i,j,h,k=1,2,...,n$$

with boundary conditions:

$$u_i(x,t)=\psi_i(x,t),\qquad x\in\partial\Omega_u, \tag{12}$$

$$\Big(\Big(a_{ij\,0}^{hk}(x,t)+a_{ij}^{hk}(x)\star\Big)\frac{\partial u_j(x,t)}{\partial x_k}\Big)n_h(x)=\phi_i(x,t),\quad x\in\partial\Omega_\sigma, \tag{13}$$

$i=1,2,3$, $\forall x\in\Omega$ holding for any $t\in[0,T]$.

$$(a_{ij}^{hk}(x)\star e_k^j)(t):=\int_0^t a_{ij}^{hk}(x,t,\tau)\cdot e_k^j(x,\tau)d\tau \tag{14}$$

are Volterra integral operators with kernels $a_{ij}^{hk}(x,t,\tau)$; $a_{ij\,0}^{hk}(x,t)$ are instantaneous elastic coefficients (out-of-integral terms) and $\underline{a}_{ij}^{hk}(x):=a_{ij\,0}^{hk}(x,t)+a_{ij}^{hk}(x)\star$; f_{i0} are components of a vector of external forces; $\phi_i(x,t)$ are components of a vector of boundary traction on the part $\partial\Omega_\sigma$ of the external boundary; $\psi_i(x,t)$ are components of the displacement vector on the rest part $\partial\Omega_u$ of the boundary, n_h are components of the outer unit normal vector to the boundary of Ω. All functions are supposed to be continuous w.r.t. $t\in[0,T]$ and sufficiently smooth w.r.t. x in domain Ω (for performing a partial integration).

The whole viscoelastic operator tensor $(\underline{a}_{ij}^{hk}(x))_{n\times n}^{n\times n}$ is assumed to be symmetric at each point $x \in \Omega$:

$$\underline{a}_{ij}^{hk}(x) = \underline{a}_{ji}^{kh}(x) = \underline{a}_{hj}^{ik}(x) = \underline{a}_{ik}^{hj}(x). \tag{15}$$

The tensor $(a_{ij\ 0}^{hk}(x,t))_{n\times n}^{n\times n}$ is additionally positive-definite, with elements bounded at each point $x \in \Omega$

$$c_0 \eta_k^j \eta_k^j \leq a_{ij\ 0}^{hk}(x,t)\eta_h^i \eta_k^j \leq C_0 \eta_k^j \eta_k^j, \tag{16}$$

for all $\eta_k^j = \eta_j^k \in \mathbb{R}$ and $t \in [0,T]$, where the constants $0 < c_0 \leq C_0 < \infty$ are independent of x and t. For isotropic materials we get:

$$\underline{a}_{ij}^{hk} = \underline{\lambda}\delta_{hi}\delta_{kj} + \underline{\mu}\delta_{ij}\delta_{hk} + \underline{\mu}\delta_{ik}\delta_{hj}. \tag{17}$$

Example 2 *(i) Often, the kernels $a_{ij}^{hk}(x,t,\tau)$ are of the convolution type and are taken in the exponential form:*

$$a_{ij}^{hk}(x,t,\tau) = \begin{cases} \sum_{p=1}^{m} \alpha_{ij\ p}^{hk}(x) e^{-\beta_p(x)(t-\tau)}, & \textit{if } t \geq \tau \\ 0 & \textit{if } t < \tau; \end{cases} \tag{18}$$

where the $\beta_p(x)$ and the $\alpha_{ij\ p}^{hk}(x)$ are piecewise continuous functions for $x \in \Omega$, often β_p are just constants;
(ii) The $a_{ij}^{hk}(x,t,\tau)$ may also be kernels of the Abel type (typical example for the relaxation kernels of concrete and cement):

$$a_{ij}^{hk}(x,t,\tau) = \begin{cases} A_{ij\ 1}^{hk}(x,t,\tau)(t-\tau)^{-\alpha} + A_{ij\ 2}^{hk}(x,t,\tau)(\tau)^{-\beta} & \\ \qquad +A_{ij\ 3}^{hk}(x,t,\tau)t^{-\gamma}, & \textit{if } t \geq \tau \\ 0 & \textit{otherwise}; \end{cases} \tag{19}$$

with $0 \leq \alpha, \beta, \gamma < 1$. The $A_{ij\ p}^{hk}$, $p = 1,2,3$, are continuous in t and τ, and piece-wise continuous in $x \in \Omega$.

4 Weak problem formulation and main results

In this section we derive a weak problem formulation from the classical one, given by (11) - (13) in the previous section, by partial integration, and then assume general functional classes for its coefficients and right-hand sides.

In order to obtain the variational formulation, we multiply equation (11) by test functions $v_i(x) \in H_0^1(\Omega, \partial\Omega_u)$, $i = 1, ..., n$, where $H_0^1(\Omega, \partial\Omega_u) := \{v \in H^1(\Omega) : \ v(x) = 0,\ x \in \partial\Omega_u\}$, and integrate over the whole domain Ω. Integrating by parts and taking into account boundary condition (12), we obtain:

$$\int_\Omega -\underline{a}_{ij}^{hk} \frac{\partial u_j}{\partial x_k} \frac{\partial v_i}{\partial x_h} dx + \int_{\partial\Omega_\sigma} \left(\underline{a}_{ij}^{hk} \frac{\partial u_j}{\partial x_k} \right) v_i n_h ds = -\int_\Omega f_{i0} v_i dx. \tag{20}$$

If for equation (20) we take into consideration the boundary condition, we will obtain the following weak problem:

Find $u_j \in H^1(\Omega)$, $j = 1, ..., n$, satisfying (12) and equation:

$$\int_\Omega \underline{a}_{ij}^{hk} \frac{\partial u_j}{\partial x_k} \frac{\partial v_i}{\partial x_h} dx = l(v),$$

$$l(v) := \int_\Omega f_{i0} v_i dx + \int_{\partial\Omega_\sigma} \phi_i v_i ds \tag{21}$$

$\forall v_i \in H_0^1(\Omega, \partial\Omega_u)$, $i = 1, ..., n$.

Definition 3 (General weak formulation) *Consider the matrix of instantaneous elastic coefficients* $(a_{ij\,0}^{hk})_{n\times n} \in C([0,T]; \mathbf{L}^\infty(\Omega))$, *the relaxation operators* $(a_{ij}^{hk}\star)_{n\times n}$, *such that* $a_{ij}^{hk}(t,\tau) = 0$ $\forall \tau > t$, *and* $a_{ij}^{hk} \in C([0,T]; L^\infty(\Omega))$, *and* $f_0 := (f_{i0})_n \in C([0,T]; \mathbf{H^{-1}}(\Omega))$, *the boundary tractions* $\phi := (\phi_i)_n \in C([0,T]; \mathbf{H^{-1/2}}(\partial\Omega_\sigma))$ *and boundary displacements* $\psi := (\psi_i)_n \in C([0,T]; \mathbf{H^{1/2}}(\partial\Omega_u))$. *We define a weak solution of problem (11)-(13) as a vector-valued function* $u \in C([0,T]; \mathbf{H^1}(\Omega))$, *which can be represented in the form* $u = \hat{u} + \tilde{\psi}$, *where* $\tilde{\psi} \in C([0,T]; \mathbf{H^1}(\Omega))$ *satisfies* $(\tilde{\psi}|_{\partial\Omega_u}) = \psi$ *and* $\hat{u}_i \in C([0,T]; H_0^1(\Omega, \partial\Omega_u))$, $i = 1, ..., n$, *satisfies the integral identity*

$$[\underline{a}(\hat{u}, v)](t) := \int_\Omega \left[\underline{a}_{ij}^{hk} \frac{\partial \hat{u}_j}{\partial x_k}\right](t) \frac{\partial v_i}{\partial x_h} dx = \hat{l}(v)(t) \quad \forall t \in [0,T], \tag{22}$$

for any $v_i \in H_0^1(\Omega, \partial\Omega_u)$. *The right hand-side of (22) is, for all* $t \in [0,T]$, *a linear functional on the* $\mathbf{H_0^1}(\Omega, \partial\Omega_u)$

$$\hat{l}(v)(t) \;:=\; \int_\Omega \left(f_{i0}(t) v_i - \left[\underline{a}_{ij}^{hk} \frac{\partial \tilde{\psi}_j}{\partial x_k}\right](t) \frac{\partial v_i}{\partial x_h} \right) dx + \int_{\partial\Omega_\sigma} \phi_i(t) v_i ds. \tag{23}$$

The space of linear bounded functionals on $\mathbf{H_0^1}(\Omega, \partial\Omega_u)$ *is denoted by* $H^{-1}(\Omega)$.

We denote further

$$a_0(\hat{u}, v)(t) \;:=\; \int_\Omega a_{ij\,0}^{hk}(x,t) \frac{\partial \hat{u}_j(x,t)}{\partial x_k} \frac{\partial v_i(x)}{\partial x_h} dx \qquad \text{and}$$

$$a(\hat{u}, v)(t,\tau) \;:=\; \int_\Omega a_{ij}^{hk}(x,t,\tau) \frac{\partial \hat{u}_j(x,\tau)}{\partial x_k} \frac{\partial v_i(x)}{\partial x_h} dx. \tag{24}$$

Obviously, $[\underline{a}(\hat{u}, v)](t) = a_0(\hat{u}, v)(t) + \int_0^t a(\hat{u}, v)(t,\tau) d\tau$.
Note that $a_0(\hat{u}, v)$ and $a(\hat{u}, v)$ are bilinear forms on $\mathbf{H_0^1}(\Omega, \partial\Omega_u)$ for every t and almost every τ. We can rewrite the weak formulation (22) as follows

$$a_0(\hat{u}, v)(t) + \int_0^t a(\hat{u}, v)(t,\tau) d\tau = \hat{l}(v)(t), \quad t \in [0,T] \tag{25}$$

Now let us rewrite equation (25) in the operator form. For this purpose we introduce the following notations:

$$A_{0x}\hat{u} := a_0(\hat{u}, \cdot), \qquad A_x \hat{u} := a(\hat{u}, \cdot), \qquad F(t) := \hat{l}(v)(t). \tag{26}$$

$A_{0x}(t), A_x(t,\tau) : \mathbf{H_0^1}(\Omega, \partial\Omega_u) \to H^{-1}(\Omega)$ for all fixed t and almost all $\tau \in [0,T]$. Now we can represent equation 22 in the form

$$\underline{A}\hat{u} = F(t),$$

where $\underline{A}\cdot$ is an infinite dimensional integro-differential operator of the following form:

$$\underline{A} = A_{0x} \cdot + A_x \star$$

and the weak problem formulation (22) takes the form:

$$A_{0x}(t)\hat{u}(t) + [A_x \star \hat{u}]\,(t) = F(t). \tag{27}$$

Equation (27) provides the most general form of the time-space integro-differential dependencies of the considered problem. The following theorem is used as an auxillary result for showing the solvability of such equations.

Theorem 4 (Data stability.) *Let $\Omega \subset \mathbb{R}^n$ be a Lipschitz domain and $\partial\Omega_u \subseteq \partial\Omega$, let $A_{0x} \in C([0,T]; \mathcal{L}(\mathbf{H_0^1}(\Omega, \partial\Omega_u), H^{-1}))$, and let $A_{0x}(t)$ be boundedly-invertible uniformly in $[0,T]$, $A_x(t,\tau) = 0\ \forall\tau > t$, $A_x \in C\big([0,T]; L^1\big([0,T], \mathcal{L}(\mathbf{H_0^1}(\Omega, \partial\Omega_u), H^{-1})\big)\big)$, and $F \in C([0,T]; H^{-1})$.*
Then there exists a unique global solution u of the problem

$$A_{0x}(t)u(t) + [A_x \star u]\,(t) = F(t) \tag{28}$$

in $C([0,T]; \mathbf{H_0^1}(\Omega, \partial\Omega_u))$, which depends continuously on F, that is

$$\|u\|_{C([0,T];H^1(\Omega))} \le C_1 \|F\|_{C([0,T];H^{-1})} \tag{29}$$

where the constant C_1 is independent of F, and if $\|A_{0x}^{-1}(t)\|_{\mathcal{L}(H^{-1},H^1)} \le \frac{1}{c_0}$, then

$$C_1 \le \tilde{C}\Big(\frac{1}{c_0} \max_{i,j,h,k} \|a_{ij}^{hk}(t,\tau)\|_{C(([0,T],L^1([0,T],L^\infty(\Omega)))}\Big) \tag{30}$$

where $\tilde{C}$ is some real-valued function, independent of f.

The proof of this theorem can be found in [1],[4].

Lemma 5 *Let Ω be a Lipschitz domain in $\mathbb{R}^n$, the instantaneous elastic (out-of-integral) coefficients ${a_{ij}^{hk}}_0 \in C([0,T]; L^\infty(\Omega))$ satisfy the symmetry (15) and positivity condition (the first of (16)) with a constant c_0, and the relaxation kernels $a_{ij}^{hk} \in C(([0,T], L^1([0,T], L^\infty(\Omega)))$ also satisfy the symmetry condition (15). Then*
(i) A_{0x} belongs to $C([0,T]; \mathcal{L}(\mathbf{H_0^1}(\Omega, \partial\Omega_u),\ H^{-1}))$, and $A_{0x}(t)$ has an inverse operator $A_{0x}^{-1}(t)$ $\forall t \in [0,T]$. This inverse operator is uniformly bounded in $[0,T]$, that is, the following estimate

$$\|A_{0x}^{-1}(t)\|_{\mathcal{L}(H^{-1},H^1)} \le \frac{1}{c_0}, \tag{31}$$

holds for any $t \in [0,T]$, and c_0 is independent from t.
(ii) $A_x(t,\tau)$ satisfies the following estimate

$$\|A_x(t,\tau)\|_{\mathcal{L}(H^1(\Omega),H^{-1})} \le \max_{i,j,h,k} \|a_{ij}^{hk}(t,\tau)\|_{L^\infty(\Omega)}, \tag{32}$$

$\forall t$ and almost all $\tau \in [0,T]$. Furthermore, the condition $a_{ij}^{hk} \in C(([0,T], L^1([0,T], L^\infty(\Omega)))$ implies that $A_x \in C(([0,T], L^1([0,T], \mathcal{L}(\mathbf{H_0^1}(\Omega, \partial\Omega_u), H^{-1})$.

See [1],[4] for proof.

Lemma 6 *In both cases of Example 2, $a_{ij}^{hk} \in C(([0,T], L^1([0,T], L^\infty(\Omega)))$.*

See [1],[4] for proof.

Remark 7 *Consider the weak problem given by definition 3. The functional defined by (23) is a continuous functional, satisfying the following estimate*

$$\begin{aligned} \|\hat{l}(t)\|_{C([0,T],H^{-1})} &\leq C \left(||f_0||_{C([0,T],L^2(\Omega))} \right. &(33)\\ &+ \left. ||\phi||_{C([0,T],H^{-1/2}(\partial\Omega_\sigma))} + ||\psi||_{C([0,T],H^{1/2}(\partial\Omega_u))} \right), \end{aligned}$$

where C depends on Ω, $\partial\Omega_\sigma$, $\max_{i,j,h,k} \|a_{ij}^{hk}(t,\tau)\|_{C(([0,T],L^1([0,T],L^\infty(\Omega)))}$.

See [1],[4],[8] for proof.

5 Preliminaries w.r.t. approximation and interpolation

We consider the problem similar to the one considered in [12] with only difference that we allow for the non-convolution and weak singular relaxation kernels, like (19), i.e., our kernels must be only integrable and continuous after the integration and not necessary essentially bounded in the integrating time-variable as in [12]. We also use the fine property of the Volterra integral, that if split the full integration interval on many subintervals and start the solution from the first subinterval, we are able to solve the problem on each subinterval separately and get a recurrent formula containing solution on all previous subintervals in the right-hand side.

$$\begin{aligned} &\frac{\partial}{\partial x}\left(a_0(x,t)\frac{\partial u_n(x,t)}{\partial x}\right) + \int_{t_{n_1-1}}^{t} \frac{\partial}{\partial x}\left(a(x,t,\tau)\frac{\partial u_n(x,\tau)}{\partial x}\right) d\tau = \hat{f}(x,t,\{u_i\}_{i=1,\ldots,n_1-1}),\\ &\hat{f}(x,t,\{u_i\}_{i=1,\ldots,n_1-1}) := f(x,t) - \sum_{i=1}^{n_1-1}\int_{t_{i-1}}^{t_i} \frac{\partial}{\partial x}\left(a(x,t,\tau)\frac{\partial u_i(x,\tau)}{\partial x}\right) d\tau. \qquad (34) \end{aligned}$$

We use the same approach here as one demonstrated in [12], where the authors employ a discontinuous Galerkin approximation in time, i.e., they start with the approximation of the integral operators in time and go on with the recurrent suxessive spatial FE-discretization for each next discrete time step, allowing the space mashes to undergo arbitrary changes at each time level. Since our problem satisfies general assumptions of [12], with the only difference that function $\phi(t-\tau) \in L^\infty(J\times[0;\infty))$ from part (ii) of Assumption 2 should be in general replaced by $\phi(t,\tau) \in C(J, L^1(J))$, we can without loss of generality refer to the framework of this paper for proof of data stability for the discrete problem.
Our innovation is a suggestion to approximate the coefficients and the right-hand side w.r.t. the space variable by interpolation probably with the same shape functions that we use in the FE approximation of the solution to separate

space and time dependences.
First we divide the time interval by (n_1+1) points $0=t_0<t_1<...<t_{n_1}=T$ and for each subinterval $J_i=(t_{i-1},t_i)$, $(i=1,...,n_1)$ construct on domain $\bar{\Omega}$ a triangular/tetrahedral space mesh of N_{el} elements and denote the domain Ω with this mesch by Ω_i and each elament j of Ω_i by Ω_{ij}. We introduce the spatial finite element spaces as

$$S^r_{hi} \;:=\; \Big\{v\in (C[0,T],H^1(\Omega)) : v\Big|_{J_i\times\Omega_{ij}} \in C(J_i,(\mathbb{P}_r(\Omega_{ij})),\ \forall\Omega_{ij}\subset\Omega_i, j=1,...,N_{el},$$
$$h_i=\max_j(diam\,\Omega_{ij})\Big\}, \tag{35}$$

where $\mathbb{P}_r$ is the space of polynomials of degree at most r. We also denote $S^r_h|_{J_i} := S^r_{hi}$. On each J_i we perform the FE approximation of the solution function in space by:

$$u_{i\ h}(x,t) := P_r u = \sum_{j=1}^{n_{nodes}} N_j(x)U_i^{(j)}(t), \qquad i=1,...,n \tag{36}$$

on every finite element, where P_r is the map $P_r : C([0,T],(H^1(\Omega))^n) \to S^r_h$, $N_j(x)$ are shape functions and n_{nodes} denotes the number of nodes in the finite element.
In the next step we introduce the time poynomial space:

$$H^p_q := \Big\{v\in C[0,T] : v\Big|_{J_i} \in \mathbb{P}_p(J_i)\ \forall J_i\subset[0,T], i=1,...,n_1, q=\max_i|t_i-t_{i-1}|\Big\}, \tag{37}$$

and approximate $U_i^{(j)}(t)\in C[0,T]$ by $\pi_p U_i^{(j)}\in H^p_q$, $j=1,...,n_{nodes}$, in time:

$$U^{(j)}_{i\ q}(t) := \sum_{k=0}^{n_1} Poly^p_k(t)U_i^{(j,k)},\ \ i=1,...,n, \tag{38}$$

where $Poly^p_k$ are polynomials of the power p on intervals $[t_i,t_{i+1}]\subset[0,T]$,

$$\pi_p : C([0,T]) \to H^p_q.$$

Like in [12] we restrict us also on $p=0$ or $p=1$.
In this way we obtain the complete space-time approximation $\pi_p P_r u \in V^{r+p}_{hq}$ of the exact solution $u(x,t)$, where

$$V^{r+p}_{hq} := \Big\{v\in S^r_h : v\Big|_{J_i} \in \mathbb{P}_p(J_i), \forall J_i\subset[0,T],\ i=1,...,n_1\Big\}. \tag{39}$$

It is known from the standard theory of elliptic second order problems, that if Ω is convex, $\partial\Omega_u$ is non-empty and the coefficients are piece-wise smooth in the space variable then the solution of the problem (11)-(13) is regular in H^2. Furthermore, if Ω has a C^s-boundary, then the problem (11)-(13) is H^s-regular. Note only that this is difficult to reach for s higher than 2, since the boundary should be C^3-regular and some problems with a suitable triangulation at the boundaries may arise. Therefore the best choice for the FE-approximation are linear shape functions. Using further the stability estimate (29), the following error estimates, known for the elliptic second order problems, can be applied to the problem (11)-(13) for the quasi-uniform triangulation of Ω:

$$\|u - u_h\|_{H^1(\Omega)} \leq Ch^r \|u\|_{H^{r+1}(\Omega)} \stackrel{(29)}{\leq} Ch^r \|f\|_{L^2(\Omega)}, \quad u \in H^{r+1}(\Omega) \tag{40}$$

or

$$\|u - u_h\|_{L^2(\Omega)} \leq Ch^{r+1} \|f\|_{L^2(\Omega)}. \tag{41}$$

Here r is the degree of approximating polynomials or shape functions.
Note that our approximating functions in V_{hq}^{r+p} are continuous in space, but in general discontinuous at the time levels J_i, $i = 1, ..., n_1$. These discontinuities allow the space meshes to change with time. We choose the mesh on Ω_i here, like in [12], in such a way that it is a union of valid meshes on each of the subdomains in $\{\Omega_i\}$. Then we can refer to Theorem 6 from [12] for discrete data stability estimate

$$\|u_h\|_{L^\infty([0,t_i);H^1(\Omega))} \leq C_S \|F\|_{L^\infty([0,t_i);H^{-1})}, \ \forall i = 1, ..., n_1. \tag{42}$$

We should only replace in its proof the estimate of the history term by the following estimate

$$\sum_{m=1}^{i-1} \int_{t_{m-1}}^{t_m} \|a_{ij}^{hk}(t,\tau)\|_{L^\infty(\Omega)} \|u_h(\tau)\|_{H^1(\Omega)} d\tau \leq \max_{i,j,h,k} \|a_{ij}^{hk}\|_{C(([0,T],L^1([0,T],L^\infty(\Omega)))} \|u_h\|_{C([0,t_i);H^1(\Omega))} \tag{43}$$

Since u is continuous in $t \in [0,T]$ w.r.t. its H^1-norm in Ω, we can use estimate (40) to estimate the error of the spatial FE-discretization:

$$\|\theta_r\|_{L^\infty([0,T],H^1(\Omega))} = \|u - P_r u\|_{L^\infty([0,T],H^1(\Omega))} \leq Ch^r \|f\|_{C([0,T],L^2(\Omega))}. \tag{44}$$

For the spline collocation method, the following estimates are valid:

$$\|U - U_q\|_{L^\infty[0,T]} \leq C_1 q^{\hat{p}}, \tag{45}$$

where $\hat{p} = p + 1$, if the collocation parameters $\{c_j\}$ are chosen as equidistant points in [0,1]. We refere to [6] (Theorem 1.1) and [14] (Theorem 4.4.7) for proof.
The number of FEM calculations, needed to solve this equation strongly depends on the form of the out-of-integral coefficients, the integral kernels and the right-hand side of the equation. This means that the very first step that should be done is to analyze the space-time dependencies in these terms. In the following considerations we will represent the integral kernels in the form:

$$a_{ij}^{hk}(x,t,\tau) = \Big(\alpha(x,t,\tau)\beta(t,\tau)\Big)_{ij}^{hk}, \tag{46}$$

where $\alpha_{ij}^{hk}(x,t,\tau)$ are the well-behaved parts of the kernel, normally piecewise-smooth and bounded, and $\beta_{ij}^{hk}(t,\tau)$ are singular parts.
If it is possible to separate the space and time dependence in the non-singular part of the kernels, as well as in the out-of-integral coefficients and the right-hand side functions, we suggest to start the purely elastic finite element analysis of both left-hand side terms with their space-dependent coefficients parts and use their global stiffness matrix (matrices) as the constant coefficient matrix in

constructing the system of finite dimensional integral equations later on.
If such separation is not applicable, we are forced to discretize the time interval and carry out the series of FEM analyzes for large number of values of t_m and τ_l. The stiffness matrices, which we obtain, are used afterwards for approximating the coefficient matrices of the resulting system of integral equations.
It is obvious, that for the second, more complicated case, the time interval discretization points must be chosen very carefully in order to minimize the number of spatial analyzes on the one hand (note that even a single run of the FEM package on a non-trivial geometry can appear extremely time- and resource-consuming), and on the other hand, to capture all the non-smoothness properties of the kernel, like oscillations, jumps etc.
Now let us consider the case, when the "well-behaved" part of kernel $\alpha_{ij}^{hk}(x,t,\tau)$, the instantaneous elastic coefficients (out-of-integral terms) $a_{ij\,0}^{hk}(x,t)$, components of vectors of external forces f_{i0} and boundary traction $\phi_i(x,t)$, depend on the time and space variables so that this dependence can not be separated.
Let us recall the following error estimate for multidimensional interpolation from [15].

Theorem 8 *Let $\Omega \subset \mathbb{R}^n$, $f \in C^{r+1}(\Omega)$ and $S^r f(\cdot)$ be its unique interpolation by Lagrange polynomials of degree $\leq r$ taken on the triangular/tetrahedral discretization $(\Omega_j)_{j=1,\dots,N_{el}}$ of domain Ω, defined by (35). Let further $m = m(r) = \dim S^r$ be the number of interpolation points in Ω_j, $j = 1,\dots,N_{el}$. Then the following interpolation estimate*

$$\left\| f - S^r f \right\|_{C(\Omega)} \leq C M_{r+1} h^{r+1} \tag{47}$$

is valid, where $C = C(r,m,n)$ and

$$M_{r+1} = \sup_{x\in\Omega} ||D^{r+1} f(x)|| < +\infty,$$

where the notation $D^r f = \{\partial_{x_1}^{\alpha_1} \dots \partial_{x_n}^{\alpha_n} f\}$, $\alpha_1 + \dots + \alpha_n = r$ is used.
Furthermore, if $f \in H^{r+1}(\Omega)$,

$$\left\| f - S^r f \right\|_{H^m(\Omega)} \leq C ||D^{r+1} f||_{L^2(\Omega)} h^{r+1-m} \qquad \forall\ 0 \leq m \leq k+1. \tag{48}$$

Example 9 *Let $m = 4$ and the interpolation points $\{x_q\}_{q=1}^4$ be taken in vertices of the tetrahedral elements $\Omega_j \subset \mathbb{R}^3$. $S^1 f(x)|_{\Omega_j} := \sum_{q:x_q\in\Omega_j} f(x_q) P_q^1(x)$, $j = 1,\dots,N_{el}$, where $P_q^1(x)$ are linear shape functions of nodes $\{x_q\}_{q=1}^4$ of element Ω_j.*

In the sections below we suggest approaches to the numerical treatment of the equations with relaxation kernels with inseparable time and space dependencies.

6 Spatial approximation via interpolation and subsequent FE/collocation methods .

Let us rewrite (1) w.r.t. the discretization of the time interval $[0,T]$ and shift the memory term depending on the solutions at all previous time-steps and supposed to be known, while we perform calculations on the subinterval (t_{n_1-1}, t),

in to the right-hand side, as shown in (34). We start up with the discretization and solution of the problem from the first interval $J_1 = [0, t_2)$ and go on using the recurrent formula (34).
On each subiterval J_i, consider the non-singular part of the relaxation kernel $\alpha_{ij}^{hk}(x,t,\tau)$, $a_{ij\,0}^{hk}(x,t)$, f_{i0} and $\phi_i(x,t)$ containing inseparable space-time dependencies. For example, consider the case of exponentially-dependent kernels $\alpha_{ij}^{hk}(x,t,\tau) = e^{-\beta_{ij}^{hk}(x)(t-\tau)}$.
Suppose from now on that $\psi \equiv 0$, and functions α_{ij}^{hk}, $a_{ij\,0}^{hk}$, f_{i0} and ϕ_i are $m+1$ time differentiable in Ω. We suggest to approximate all these terms with respect to $x \in \Omega$ by continuous functions that are piecewise-polynomial on the finite elements Ω_p, $p = 1, ..., N_{el}$, i.e.

$$\alpha_{ij}^{hk}(x,t,\tau) \approx S^m \alpha_{ij}^{hk}(\cdot,t,\tau) := \sum_{p=1}^{N_{el}} \sum_{q:x_q\in\Omega_p} \alpha_{ij}^{hk}(x_q,t,\tau) P_q^m(x)\chi_p(x), \qquad x \in \Omega \tag{49}$$

where N_{el} is the number of elements in the model, x_q is the node of the p-th element, χ_p is a characteristic function on the elements, i.e.:

$$\chi_p(x) = \begin{cases} 1 & \text{if } x \text{ belongs to the } p\text{-th element} \\ 0 & \text{otherwise} \end{cases}$$

and $P_q^m(x)$ is a Lagrange polynomial of power m in the q-th node of the p-th element.
The same idea of the kernel approximation by interpolation is widely used in spatial boundary integral equations of the kind $\lambda u(x) + \int_\Gamma k(x,y)u(y)dy = f(x)$, $\forall x \in \Gamma \subset \mathbb{R}^n$, see, e.g., works of Hackbusch [14], [13].
By the space-time approximations (36) and (38) we reduce the infinite dimensional system of integral equations in Hilbert spaces to the sistem of linear algebraic equations and the viscoelastic problem with memory to the system of recurrent pure elastic problems. For the subsequent analysis we introduce the integrals

$$\Phi_{nd,\gamma}(x_q,t,[u_d]) := \begin{cases} \int_0^1 (\alpha(x_q,t_n,t_d+sq_d)\beta(t_n,t_d+sq_d))_\gamma u_d(t_d+sq_d)ds & \text{if } 0 \le d \le n-1; \\ \int_0^t (\alpha(x_q,t_n,t_n+sq_n)\beta(t_n,t_n+sq_n))_\gamma u_n(t_n+sq_n)ds & \text{if } d = n. \end{cases}, \; n = 0, ...n \tag{50}$$

Here γ stands for a generic $4-$index.

Using this notation and performing FE-approximation (36), we obtain entries of the global stiffness matrix

$$\Lambda_{nd,\;ij}^{(u,l)}(t,[U_d\,j]) := \sum_{\delta:x_\delta\in\Omega_p} q_d \Phi_{nd\;ij}^{hk}(x_q,t,[U_d\,j]) \int_{\Omega_p} P_\delta^m(x) \frac{\partial N_u(x)}{\partial x_k}\frac{\partial N_l(x)}{\partial x_h}dx, \qquad i,j = 1,...,n, \tag{51}$$

where u, l are global numbers of the nodes belonging to element p .
Now, let us apply approximation (49) to the instantaneous elastic coefficients $a_{ij\,0}^{hk}(x,t)$ and then apply the Finite Element approximation (36) to the out-of-integral term of operator equation (25). Thus we eliminate the space depen-

dence, and obtain elements of the out-of-integral stiffness matrix

$$
\begin{aligned}
\int_{\Omega_p} a_{ij\ 0}^{hk}(x,t)\frac{\partial N_u(x)}{\partial x_k}\frac{\partial N_l(x)}{\partial x_h}dx &\approx \sum_{q:x_q\in\Omega_p} a_{ij\ 0}^{hk}(x_q,t)\int_{\Omega_p} P_q^m(x)\frac{\partial N_u(x)}{\partial x_k}\frac{\partial N_l(x)}{\partial x_h}dx,; \qquad (52)\\
\Lambda_{ij\ 0}^{(u,l)}(t) &:= \sum_{q:x_q\in\Omega_p} a_{ij\ 0}^{hk}(x_q,t)\int_{\Omega_p} P_q^m(x)\frac{\partial N_u(x)}{\partial x_k}\frac{\partial N_l(x)}{\partial x_h}dx, \quad i,j=1,...,n.
\end{aligned}
$$

Finally, we calculate the elements entries of the global load vector. For this reason we refer to (23), keeping in mind that we let for simplicity $\psi(x,t)\equiv 0$.

$$
\begin{aligned}
&\int_{\Omega_p} f_{i0}(x,t)N_l(x)dx + \int_{\partial\Omega_\sigma\cap\Omega_p}\phi_i(s,t)N_l(s)ds \approx \qquad (53)\\
F_i^{(l)}(t) &:= \sum_{q:x_q\in\Omega_p} f_{i0}(x_q,t)\int_{\Omega_p} P_q^m(x)N_l(x)dx + \sum_{q:x_q\in\Omega_p\cap\partial\Omega_\sigma}\phi_i(x_q,t)\int_{\partial\Omega_\sigma\cap\Omega_p} P_q^m(s)N_l(s)ds.
\end{aligned}
$$

Collecting these element stiffness matrices and load vectors to the global ones by the standard assembly procedure, we can rewrite the variational statement of the problem in the form:

$$
\Lambda_{ij\ 0}(t)U_{n\ j}(t)+\Lambda_{nn,\ ij}[t,U_{n\ j}] = F_i(t)-\sum_{d=0}^{n-1}\Lambda_{nd,\ ij}[t_d,U_{d\ j}],\ n=0,...n_1 \qquad (54)
$$

Further, we look for the solution of (54) recurrent in H_q^p defined by (37) using (38), which can be done by standard collocation procedure.

Example 10 *Let $p=0$ in (38), i.e. we consider H_q^0. Then we can extract $U_{n\ j}$ from the integrals and redefine (50) and (51) as follows*

$$
\Phi_{nd,\gamma}(x_q,t) := \begin{cases} \int_0^1(\alpha(x_q,t_n,t_d+sq_d)\beta(t_n,t_d+sq_d))_\gamma ds & \text{if } 0\le d\le n-1;\\ \int_0^t(\alpha(x_q,t_n,t_n+sq_n)\beta(t_n,t_n+sq_n))_\gamma ds & \text{if } d=n.\end{cases},\ n=0,...n_1 \qquad (55)
$$

and

$$
\Lambda_{nd,\ ij}^{(u,l)}(t) := \sum_{\delta:x_\delta\in\Omega_p} q_d\Phi_{nd\ ij}^{hk}(x_q,t)\int_{\Omega_p} P_\delta^m(x)\frac{\partial N_u(x)}{\partial x_k}\frac{\partial N_l(x)}{\partial x_h}dx, \qquad i,j=1,...,n. \qquad (56)
$$

Now problem 54 obtain a form of the pure elastic one:

$$
(\Lambda_{ij\ 0}(t_n)+\Lambda_{nn,\ ij}(t))(t_n))U_{n\ j} = F_i(t)-\sum_{d=0}^{n-1}\Lambda_{nd,\ ij}(t_d)U_{d\ j},\ n=0,...n_1 \qquad (57)
$$

Remark 11 *Note that we can replace $P_q^m(x)$ in definitions (51), (53), (53) by $N_q(x)$, if the shape functions approximating the solution and the Lagrange shape functions $P_q^m(x)$, approximating the inhomogeneous coefficients and the external loading, all are of the same order.*

Now let us recall the collocation method for Volterra integral equations of the second kind from [5], slitely modifies for weak singular kernels.

Let $\{c_j\}$ with $0 \le c_1 < ... < c_{p+1} \le 1$, be a given set of collocation parameters, and define the sets X_n:

$$X_n := \{t_{n,j} := t_n + c_j q_n : j = 1, ..., p+1, \quad n = 0, ..., n_1 - 1\}$$

and

$$X(n_1) := \cup_{i=0}^{n_1-1} X_i$$

Setting

$$U_n(t_n + sq_n) = \sum_{l=1}^{p+1} L_l(s) Y_{n,l}, \tag{58}$$

with $Y_{n,l} := u_n(t_{n,l})$ and where the polynomials

$$L_l(s) := \prod_{k=1, k\neq l}^{p+1} \frac{s - c_k}{c_l - c_k}$$

represent the Lagrange canonical polynomials for the collocation parameters $\{c_j\}$, we see that (54) represents, for each $n = 0, ..., n_1 - 1$, a linear system in $\mathbb{R}^{p+1}$ for the vector $Y_n := (Y_{n,1}, ..., Y_{n,p+1})^T$.
In most applications the integrals (50) occurring in the collocation equation (54) cannot be evaluated analytically. Besides, the term $\beta(t, \tau)$ possesses a weak singularity in the end of every subinterval (see 19). That is why one is forced to resort to employing suitable quadrature formulae for kernel approximation. In the following we shall use (p+1)-point interpolatory quadrature formulae of the form:

$$\hat{\Phi}_{n,i}^{(j)}[u_i] := \sum_{l=1}^{p+1} w_{n,i,l}^{(j)} \alpha(t_{n,j}, t_{i,l}) u_i(t_{i,l}) \;\; if \; 0 \le i \le n-1; \tag{59}$$

For the approximation of $\hat{\Phi}_{n,n}^{(j)}[u_n]$ we shall employ either

$$\tilde{\Phi}_{n,n}^{(j)}[u_n] := \sum_{l=1}^{p+1} w_{n,n,l}^{(j)} \alpha(t_{n,j}, t_{n,l}) u_n(t_{n,l}) \tag{60}$$

or

$$\hat{\Phi}_{n,n}^{(j)}[u_n] := \sum_{l=1}^{p+1} w_{j,n,i,l}^{(j)} \alpha(t_{n,j}, t_n + c_j c_l q_n) u_n(t_n + c_j c_l q_n) \tag{61}$$

Here, the quadrature weights are given by

$$w_{n,i,l}^{(j)} := \int_0^1 L_l(s) \beta(t_{n,j}, t_i + sq_i) ds \tag{62}$$

and by

$$w_{j,n,i,l}^{(j)} := c_j w_{n,i,l}^{(j)}, \quad (j, l = 1, ..., p+1) \tag{63}$$

Note, that the same idea was presented in [18].

Example 12 *Let $\beta(t, s) \equiv 1$. Then we are in a framework of the standard collocation and*

$$w_{n,i,l}^{(j)} := \int_0^1 L_l(s) ds. \tag{64}$$

Example 13 *Let $\beta(t,s)=(t-s)^{-\alpha}$ and let the discretization by t_n, $n=\{0,n_1\}$ be equidistant with step size q. Then*

$$w_{n,i,l}^{(j)} := q^{-\alpha}\int_0^1 L_l(s)(n-i+c_j-s)^{-\alpha}ds$$

This integral could be analytically taken in the form of a hypergeometric series (see [16]). However it can be reduced to the finite expression through Euler's $\Gamma-$ or $\mathcal{B}-$ functions only for $n-i+c_j=1$. Therefore, we prefer to avoid the weak singularity by partial integration

$$\begin{aligned}
w_{n,i,l}^{(j)} \quad &:= q^{-\alpha}\int_0^1 L_l(s)(n-i+c_j-s)^{-\alpha}ds \qquad (65)\\
&= -q^{-\alpha}L_l(s)\frac{(n-i+c_j-s)^{1-\alpha}}{1-\alpha}\Big|_{s=0}^{s=1} + q^{-\alpha}\int_0^1 L_l'(s)\frac{(n-i+c_j-s)^{1-\alpha}}{1-\alpha}ds\\
&= \begin{cases} 0, & \text{if } l\neq 1 \text{ and } l\neq p+1\\ q^{-\alpha}L_1(0)\frac{(n-i+c_j)^{1-\alpha}}{1-\alpha}, & \text{if } l=1\\ -q^{-\alpha}L_{p+1}(1)\frac{(n-i+c_j-1)^{1-\alpha}}{1-\alpha}, & \text{if } l=p+1\end{cases} + q^{-\alpha}\int_0^1 L_l'(s)\frac{(n-i+c_j-s)^{1-\alpha}}{1-\alpha}ds
\end{aligned}$$

and then approximate the integral in the following way:

$$w_{n,i,l}^{(j)} \approx \begin{cases} 0, & \text{if } l\neq 1 \text{ and } l\neq p+1\\ q^{-\alpha}L_1(0)\frac{(n-i+c_j)^{1-\alpha}}{1-\alpha}, & \text{if } l=1\\ -q^{-\alpha}L_{p+1}(1)\frac{(n-i+c_j-1)^{1-\alpha}}{1-\alpha}, & \text{if } l=p+1\end{cases} + \frac{q^{-\alpha}(n-i+c_j-c_l)^{1-\alpha}}{1-\alpha}\int_0^1 L_l'(s)ds$$

The detailed recurrent formulas for the weights can be found in [17].

7 A priori error estimates

Let us first suppose that the instantaneous elastic coefficients $a_{ij\ 0}^{hk}$ as well as the external loads f_{i0} and ϕ_i are either space-independent functions or change homogeneously in space, i.e. they are represented as a product of purely space and time dependent functions respectively. Therefore, we apply the suggested in section 6 approximation in space by interpolation to the relaxation kernels only. Let

$$A_x u := \int_\Omega \Big(\alpha(x,t,\tau)\beta(t,\tau)\Big)_{ij}^{hk}\frac{\partial u_j(x,\tau)}{\partial x_k}\frac{\partial \cdot_i (x)}{\partial x_h}dx, \qquad (66)$$

$$A_{xS}u_S := \int_\Omega \Big(S^m\alpha(\cdot,t,\tau)\beta(t,\tau)\Big)_{ij}^{hk}\frac{\partial u_{Sj}(x,\tau)}{\partial x_k}\frac{\partial \cdot_i (x)}{\partial x_h}dx, \qquad (67)$$

where $S^m\alpha_{ij}^{hk}(x,t,\tau)$ is the approximation of the non-singular part of the kernels, defined by (49), and u_S denotes the solution of our problem (28) corresponding to this approximation. We introduce the error in the solution caused by the kernel approximation by interpolation as:

$$\epsilon := u - u_S. \qquad (68)$$

Lemma 14 *The error in the solution caused by kernel approximation (49) is of the same order as the error of kernel approximation itself, i.e.*

$$\|\epsilon(t)\|_{H^1(\Omega)} := \|u(t)-u_S(t)\|_{H^1(\Omega)} \leq Ch^{m+1} \qquad (69)$$

for any $t \in [0,T]$*, where*

$$C = \frac{\tilde{C}\Big(\frac{1}{c_0} a \star\Big)}{c_0^2} \|f\|_{C([0,T],L^2(\Omega))} \max_{i,j,h,k} \Big(\Big\| D^{m+1} \alpha_{ij}^{hk} \Big\|_{C([0,T]\times[0,T]\times\Omega)} \Big\| \beta_{ij}^{hk} \Big\|_{C([0,T],L^1([0,T]))} \Big) \tag{70}$$

Proof. To begin with, let us represent equation (28) in the form:

$$u(t) + \int_0^t A_{0x}^{-1}(t) A_x(t,\tau) u(\tau) d\tau = A_{0x}^{-1}(t) F(t) \tag{71}$$

Similarly, we can write for $u_S(t)$:

$$u_S(t) + \int_0^t A_{0x}^{-1}(t) A_{xS}(t,\tau) u_S(\tau) d\tau = A_{0x}^{-1}(t) F(t) \tag{72}$$

Then the error (68) can be estimated as follows:

$$\begin{aligned}
& \|u(t) - u_S(t)\|_{H^1(\Omega)} \\
= \quad & \Big\| A_{0x}^{-1}(t) \int_0^t \Big(A_{xS}(t,\tau) u_S(\tau) - A_x(t,\tau) u(\tau) \Big) d\tau \Big\|_{H^1(\Omega)} \\
\overset{(31)}{\leq} \quad & \frac{1}{c_0} \max\big\{ \|u\|_{C([0,T],H^1(\Omega))}, \|u_S\|_{C([0,T],H^1(\Omega))} \big\} \int_0^t \Big\| A_{xS}(t,\tau) - A_x(t,\tau) \Big\|_{\mathcal{L}(H^1(\Omega),H^{-1})} d\tau \\
\overset{(32)}{\leq} \quad & \frac{1}{c_0} \max\big\{ \|u\|_{C([0,T],H^1(\Omega))}, \|u_S\|_{C([0,T],H^1(\Omega))} \big\} \times \\
& \max_{i,j,k,h} \Big(\Big\| S^m \alpha_{ij}^{hk} - \alpha_{ij}^{hk} \Big\|_{C([0,T]\times[0,T],L^\infty(\Omega))} \Big\| \beta_{ij}^{hk} \Big\|_{C([0,T],L^1([0,T]))} \Big) \\
\overset{Theorem\,4}{\leq} \quad & \frac{1}{c_0^2} \tilde{C}\Big(\frac{1}{c_0} a \star \Big) \Big\| F \Big\|_{C([0,T],H^{-1}(\Omega))} \times \\
& \max_{i,j,k,h} \Big(\Big\| S^m \alpha_{ij}^{hk} - \alpha_{ij}^{hk} \Big\|_{C([0,T]\times[0,T],L^\infty(\Omega))} \Big\| \beta_{ij}^{hk} \Big\|_{C([0,T],L^1([0,T]))} \Big) \\
\overset{Theorem\,8}{\leq} \quad & \frac{1}{c_0^2} \tilde{C}\Big(\frac{1}{c_0} a \star \Big) \Big\| f \Big\|_{C([0,T],L^2(\Omega))} \times \\
& \max_{i,j,k,h} \Big(\Big\| D^{m+1} \alpha_{ij}^{hk} \Big\|_{C([0,T]\times[0,T]\times\Omega)} \Big\| \beta_{ij}^{hk} \Big\|_{C([0,T],L^1([0,T]))} \Big) h^{m+1}
\end{aligned}$$

■

Let now the instantaneous elastic coefficients ${a_{ij}^{hk}}_0$ and the external loads f_{i0} and ϕ_i also possess inseparable time-space dependencies. So, we apply the approximation S^m, defined by (49), to these terms too. We define, additionally to (66), (67),

$$A_{0xS}(t) u_S(t) := \int_\Omega S^m {a_{ij}^{hk}}_0(\cdot,t) \frac{\partial u_{Sj}(x,t)}{\partial x_k} \frac{\partial \cdot_i (x)}{\partial x_h} dx, \tag{73}$$

$$F_S(t) \quad := \quad \int_\Omega S^m f_{i0}(\cdot,t) u_{Si}(x,t) dx + \int_{\partial\Omega_\sigma} S^m \phi_i(\cdot,t) u_{Si}(s,t) ds, \tag{74}$$

where S^m is the approximation defined by (49), and u_S denotes the solution of (28) corresponding to this approximation.

Lemma 15 *The error in the solution of (28) caused by approximations* $S^m a_{ij\ 0}^{hk}$, $S^m \alpha_{ij}^{hk}$, $S^m f_{i0}$ *and* $S^m \phi_i$ *is again of the order* $O(h^{m+1})$, *i.e.,*

$$\|\epsilon(t)\|_{H^1(\Omega)} := \|u(t) - u_S(t)\|_{H^1(\Omega)} \leq Ch^{m+1} \tag{75}$$

for any $t \in [0, T]$.

Proof. Consider again equations (71) and

$$u_S(t) + \int_0^t A_{0xS}^{-1}(t) A_{xS}(t, \tau) u_S(\tau) d\tau = A_{0xS}^{-1}(t) F_S(t) \tag{76}$$

We estimate the error:

$$\begin{aligned}
& \|u(t) - u_S(t)\|_{H^1(\Omega)} \\
\leq \; & \max\left\{\|u\|_{C([0,T],H^1(\Omega))}, \|u_S\|_{C([0,T],H^1(\Omega))}\right\} \Big\| \int_0^t \Big(A_{0xS}^{-1}(t) A_{xS}(t,\tau) - {A_{0xS}}^{-1}(t) A_x(t,\tau)\Big) d\tau \\
+ \; & \int_0^t \Big(A_{0xS}^{-1}(t) A_x(t,\tau) - {A_{0x}}^{-1}(t) A_x(t,\tau)\Big) d\tau \Big\|_{\mathcal{L}(H^1(\Omega),H^1(\Omega))} \\
+ \; & \Big\| A_{0x}^{-1}(t) F(t) - A_{0x}^{-1}(t) F_S(t) + A_{0x}^{-1}(t) F_S(t) - A_{0xS}^{-1}(t) F_S(t) \Big\|_{H^1(\Omega)} \\
\overset{Sec.\,4}{\leq} \; & \frac{1}{c_0} \tilde{C}\Big(\frac{1}{c_0} a \star\Big) \max\left\{\Big\|F\Big\|_{C([0,T],H^{-1}(\Omega))}, \Big\|F_S\Big\|_{C([0,T],H^{-1}(\Omega))}\right\} \\
\times \; & \left[\frac{1}{c_0} \max_{i,j,k,h} \left(\Big\|\beta_{ij}^{hk}\Big\|_{C([0,T],L^1([0,T]))} \Big\|S^m \alpha_{ij}^{hk} - \alpha_{ij}^{hk}\Big\|_{C([0,T]\times[0,T],L^\infty(\Omega))}\right)\right. \\
+ \; & \left. \max_{i,j,k,h} \left(\Big\|\alpha_{ij}^{hk}\Big\|_{C([0,T]\times[0,T],L^\infty(\Omega))} \Big\|\beta_{ij}^{hk}\Big\|_{C([0,T],L^1([0,T]))}\right) \Big\|A_{0xS}^{-1} - A_{0x}^{-1}\Big\|_{C([0,T],\mathcal{L}(H^{-1},H^1(\Omega)))}\right] \\
+ \; & \frac{1}{c_0}\left(\Big\|f_0 - S^m f_0\Big\|_{C([0,T],L^2(\Omega))} + \Big\|\phi - S^m \phi\Big\|_{C([0,T],H^{-1/2}(\partial\Omega_\sigma))}\right) \\
+ \; & \Big\|F_S\Big\|_{C([0,T],H^{-1}(\Omega))} \Big\|A_{0xS}^{-1} - A_{0x}^{-1}\Big\|_{C([0,T],\mathcal{L}(H^{-1},H^1(\Omega)))}.
\end{aligned} \tag{77}$$

Let us estimate the term $\Big\|A_{0xS}^{-1} - A_{0x}^{-1}\Big\|_{C([0,T],\mathcal{L}(H^{-1},H^1(\Omega)))}$:

$$\begin{aligned}
& \Big\|A_{0xS}^{-1} - A_{0x}^{-1}\Big\|_{C([0,T],\mathcal{L}(H^{-1},H^1(\Omega)))} = \Big\|A_{0xS}^{-1}(A_{0x} - A_{0xS}) A_{0x}^{-1}\Big\|_{C([0,T],\mathcal{L}(H^{-1},H^1(\Omega)))} \\
\leq \; & \frac{1}{c_0^2}\Big\|A_{0x} - A_{0xS}\Big\|_{C([0,T],\mathcal{L}(H^1(\Omega),H^{-1}))} \leq \frac{1}{c_0^2} \max_{i,j,k,h} \Big\|a_{ij\ 0}^{hk} - S^m a_{ij\ 0}^{hk}\Big\|_{C([0,T],L^\infty(\Omega))}
\end{aligned} \tag{78}$$

Application of Theorem 8 to each term of (77) and to (78) completes the proof.

■

The following lemma gives the a priori estimate for the total solution error.

Lemma 16 (A priori error estimate.) *The error estimate satisfies:*

$$\Big\|u - \pi_p P_r u_S\Big\|_{L^\infty([0,T],H^1(\Omega)} \leq C_S h^{m+1} + Ch^r + C_2(|\Omega|) q^{\hat{p}} \tag{79}$$

Proof. Using triangle inequality, we can see that:

$$
\begin{aligned}
& \left\|u - \pi_p P_r u_S\right\|_{L^\infty([0,T],H^1(\Omega))} \\
\leq\ & \left\|u(t) - u_S(t)\right\|_{L^\infty([0,T],H^1(\Omega))} + \left\|u_S(t) - P_r u_S(t)\right\|_{L^\infty([0,T],H^1(\Omega))} \\
+\ & \left\|P_r u_S(t) - \pi_p P_r u_S(t)\right\|_{L^\infty([0,T],H^1(\Omega))}
\end{aligned}
$$

The estimates for the first term in the right-hand side are given by (69), (75) and for the second by (41) and (44) for L^2- and H^1-norms respectively. Proceeding with the second term, we get:

$$
\begin{aligned}
\|\rho_p\|_{L^\infty([0,T],B)} &= \|P_r u - P_r \pi_p u\|_{L^\infty([0,T],B)} = \\
&= \Big\|\sum_j N_j(x)\Big(U_j(t) - \sum_k Poly^p(t) U_{jk}\Big)\Big\|_{L^\infty([0,T],B)} \leq \\
&\leq \sup_j \Big\|\Big(U_j(t) - \sum_k Poly^p(t) U_{jk}\Big)\Big\|_{L^\infty([0,T])} \sum_j \Big\|N_j(x)\Big\|_B \leq \\
&\stackrel{(45)}{\leq} C_2(|\Omega|) q^{\hat{p}},
\end{aligned}
$$

where $B = \{H^1(\Omega), L^2(\Omega)\}$. ■

Thus, we can conclude that the error, introduced by spatial approximation of relaxation kernels, instantaneous elastic coefficients and external loads, does not increase the total error, if $m \geq r$. On the other hand, it significantly simplifies the calculation procedure, allowing us to significantly reduce the computations of the spatial finite element analysis.

8 Numerical example

Consider the homogeneous isotropic viscoelastic prismatic rod of length ℓ, as shown in Figure 1. It is subject to stretching under its own weight and under an external tension pressure p, homogeneously distributed over its lower end. The rod is rigidly fixed in the middle point A = (0,0,0) on its upper face. Besides, the zero displacement constraints in vertical direction are applied to the circle of diameter m, which completely belongs to the upper face of the rod.

Consider the system of equilibrium equations (11)-(13) under isotropy condition (17), and switch over to Poisson's ratio and Young's modulus through the relation:

$$\lambda = \frac{\nu\, E}{(1+\nu)\,(1-2\,\nu)}, \qquad \mu = \frac{E}{2(1+\nu)}. \tag{80}$$

We assume further that the Poisson's ratio ν is time-independent and that the Young's modulus $\underline{E}$ is taken in the form of the following Volterra integral operator

$$\underline{E} = E_0 + E\star; \qquad E\star = -E_0 \int_0^t k(t-\tau)\cdot d\tau. \tag{81}$$

Now, the system of equilibrium equations (11)-(13) can be rewritten as follows:

$$\frac{\underline{E}}{2(1+\nu)(1-2\nu)} \frac{\partial}{\partial x_i} div\mathbf{u}(x,t) + \frac{\underline{E}}{2(1+\nu)} \Delta u_i(x,t) = -f_i(x,t), \tag{82}$$

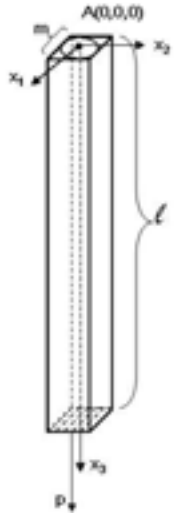


Figure 1: Test problem: isotropic viscoelastic rod.

with boundary conditions:

$$u_i\big|_A = 0, \quad i = 1,2,3, \tag{83}$$

$$u_3\big|_{\substack{x_3=0,\\ x_1^2+x_2^2=(\frac{m}{2})^2}} = 0, \tag{84}$$

$$\sigma_3\big|_{x_3=\ell} = pf_2(t), \tag{85}$$

$$\sigma_{ij}n_j\big|_{\text{on the rest of boundary}} = 0, \; i,j = 1,2,3, \tag{86}$$

holding for any $t \in [0,T]$. Here

$$\mathbf{f}^{\mathbf{T}}(x,t) = (f_i(x,t))_n = (0,0,\rho g f_2(t)), \tag{87}$$

where ρ is the material density, and we choose

$$k(s) = \begin{cases} & \frac{1}{\sqrt{s}}, \\ \text{or} & \frac{s^2}{2}e^{-s} \end{cases}, \tag{88}$$

$$f_2(t) = \begin{cases} & t+1-\left(2\sqrt{t}+\frac{4}{3}\sqrt{t^3}\right), \\ \text{or} & (t^2+t+1)e^{-t}. \end{cases} \tag{89}$$

Since the right-hand side of our system can be represented as a product of a purely space- and time-dependent function, and the kernel (88) of operator (81) is space-independent, the solution of the problem (82) - (86) will be of the form:

$$u_i(x,t) := u_{i1}(x)u_2(t). \tag{90}$$

We rewrite our system as:

$$\left(\frac{1}{2(1+\nu)(1-2\nu)}\frac{\partial}{\partial x_i}div\ u_1(x) + \frac{1}{2(1+\nu)}\Delta u_{i1}(x)\right)u_2(t) = -\left[(\underline{E})^{-1}f_i\right](t). \tag{91}$$

Then the pair of equations, consisting of the purely spatial one:

$$\frac{E_0}{2(1+\nu)(1-2\nu)}\frac{\partial}{\partial x_i} div\ \mathbf{u_1}(x) + \frac{E_0}{2(1+\nu)}\Delta u_{i1}(x) = -\left\{\begin{array}{l} 0,\ i=1,2; \\ \rho g,\ i=3 \end{array}\right. \tag{92}$$

and the temporal one:

$$u_2(t) = (t+1) - \left(2\sqrt{t} + \frac{4}{3}\sqrt{t^3}\right) + \int_0^t \frac{u_2(\tau)}{\sqrt{t-\tau}}\,d\tau \tag{93}$$

or

$$u_2(t) = (t^2+t+1)e^{-t} + \int_0^t \frac{(t-\tau)^2}{2}e^{\tau-t}u_2(\tau)d\tau \tag{94}$$

can be solved analytically to yield the desired solution:

$$u_1(x) \quad = \quad \frac{1}{E_0}\left(\begin{array}{c} -\rho g\nu x_1x_3 - \nu(\rho g\ell - p)x_1, \\ -\rho g\nu x_2x_3 - \nu(\rho g\ell - p)x_2, \\ \frac{\rho g x_3^2}{2} + (\rho g\ell - p)x_3 + \frac{\rho g\nu}{2}(x_1^2+x_2^2) - \frac{\rho g\nu m^2}{8} \end{array}\right) \tag{95}$$

for the spatial part of the solution, and

$$u_2(t) = 1 + t \tag{96}$$

or

$$u_2(t) = 1 + \frac{1}{3}\left(1 - e^{-\frac{3}{2}t}\left[\cos\left(\frac{\sqrt{3}}{2}t\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}}{2}t\right)\right]\right) \tag{97}$$

For a numerical simulation of the system's space-dependent part we used the ANSYS Finite Element package. Since the ANSYS element library does not contain elements supporting materials with weakly singular kernels, we combined ANSYS with the collocation method implemented in an own fortran code based on the algorithm from [6]. The latter algorithm was modified to allow for weak-singular kernel parts as well as a coefficient matrix in front of the out-of-integral term.

The FEM discretization of $\frac{\partial}{\partial x}\left(\alpha(x,t_m,\tau_l)\frac{\partial u}{\partial x}\right)$ w.r.t. the space variable is obtained as the global stiffness matrix of the problem with appropriate material properties, and the one for the right-hand side as a global load vector. For the extraction of the global stiffness matrix from ANSYS in a text format, we modified **rdsubs.F** code, taken from the ANSYS distribution medium and incorporated into original ANSYS as a User Predefined Routine (UPF). Thus, we reduced the infinite-dimensional system of the Volterra integral equations to a finite-dimensional one. Then, the obtained integral equation system was solved numerically with the spline collocation method described in section 6. Finally the numerical solution for the system of integral equations was obtained by iteratively solving a system of linear algebraic equations with help of the conjugate gradient method, as $u_{ij} = u_i(t_j)$, $j = 0, ..., n_1$, $i = 1, ..., 3N$, where n_1 is the number of discretization points for the time interval and N is the number of nodes in the finite element discretization of the body.

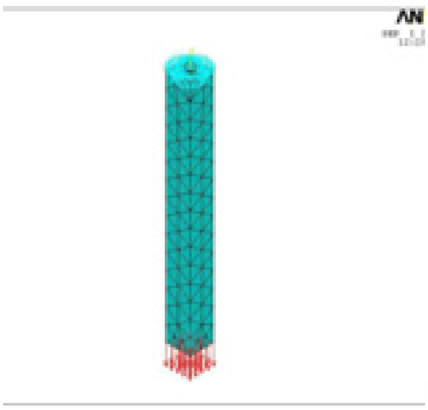


Figure 2: FE-mesh and boundary conditions.

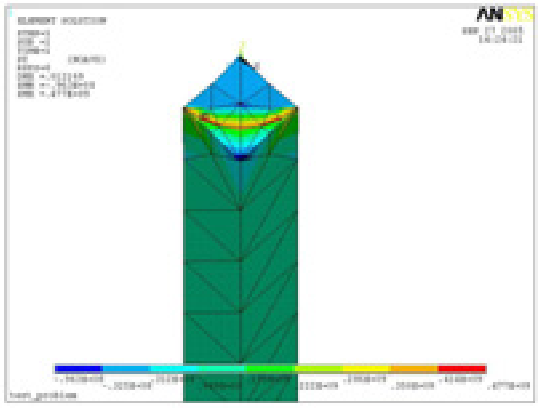


Figure 3: FE-mesh of the quarter of rod and σ_{22} at the time-step $t = 0$.

8.1 Convergence in time.

To test the temporal convergence of the collocation method, we solve both problems, (93) and (94), on the unit time interval with five collocation points on a unit subinterval and compare the analytical $u_2(t)$ and numerical $U_2(t)$ solutions in the arbitrary chosen time point $T = \frac{1}{3}$. The summary of numerical performance is presented in Table 1.

Time, Equation	$u_2(t)$	$U_2(t)$	$\lvert u_2(t) - U_2(t) \rvert$
T=1/3, (93)	0.1333371E+01	0.132760E+01	0.577E-02
T=1/3, (94)	0.13983189911	0.13983187675	0.224E-07

Table 1: Comparison of analytical ($u_2(t)$) and numerical ($U_2(t)$) solutions of equations (93) and (94).

8.2 Spatial and full convergence.

The following parameter set:
$E_0 = 8.5 \cdot 10^{10}$ Pa, $\nu = 0.16$, $\rho = 1.2770\ kg/m^3$, $p = -10 \cdot 10^7$ Pa, $m = 1$ cm, $l = 10$ cm
was used for a numerical example. To perform the numerical simulation with ANSYS, we took advantage of the symmetry of the modeled body and therefore considered only a quarter of it (see Fig. 3). The body has been discretized with 244 elements, 620 nodes. The summary of successive spatial and temporal numerical performances of ANSYS and the collocation fortran-routine is presented in Table 2 and 3.

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Time	$\|u_1(t)\|_{L_\infty(\Omega)}$	$\|U_1(t)\|_{L_\infty(\Omega)}$	$\|u_1(t) - U_1(t)\|_{L_\infty(\Omega)}$
T = 0	0.946305E-04	0.94118E-04	0.512456E-06

Table 2: Comparison of analytical ($u_1(t)$) and numerical ($U_1(t)$) solution.

Time	$\|u_3(t)\|_{L_\infty(\Omega)}$	$\|U_3(t)\|_{L_\infty(\Omega)}$	$\|u_3(t) - U_3(t)\|_{L_\infty(\Omega)}$
T = 0	0.11812E-01	0.12168E-01	0.35534E-03
T=1/3, (96)	0.157497E-01	0.161542E-01	0.54400936E-03
T=1/3, (97)	0.1651694E-02	0.1701474E-02	0.496881E-04

Table 3: Comparison of analytical ($u_3(t)$) and numerical ($U_3(t)$) solution.

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1. D. Hietel, K. Steiner, J. Struckmeier

A Finite - Volume Particle Method for Compressible Flows

We derive a new class of particle methods for conservation laws, which are based on numerical flux functions to model the interactions between moving particles. The derivation is similar to that of classical Finite-Volume methods; except that the fixed grid structure in the Finite-Volume method is substituted by so-called mass packets of particles. We give some numerical results on a shock wave solution for Burgers equation as well as the well-known one-dimensional shock tube problem.
(19 pages, 1998)

2. M. Feldmann, S. Seibold

Damage Diagnosis of Rotors: Application of Hilbert Transform and Multi-Hypothesis Testing

In this paper, a combined approach to damage diagnosis of rotors is proposed. The intention is to employ signal-based as well as model-based procedures for an improved detection of size and location of the damage. In a first step, Hilbert transform signal processing techniques allow for a computation of the signal envelope and the instantaneous frequency, so that various types of non-linearities due to a damage may be identified and classified based on measured response data. In a second step, a multi-hypothesis bank of Kalman Filters is employed for the detection of the size and location of the damage based on the information of the type of damage provided by the results of the Hilbert transform.
Keywords: Hilbert transform, damage diagnosis, Kalman filtering, non-linear dynamics
(23 pages, 1998)

3. Y. Ben-Haim, S. Seibold

Robust Reliability of Diagnostic Multi-Hypothesis Algorithms: Application to Rotating Machinery

Damage diagnosis based on a bank of Kalman filters, each one conditioned on a specific hypothesized system condition, is a well recognized and powerful diagnostic tool. This multi-hypothesis approach can be applied to a wide range of damage conditions. In this paper, we will focus on the diagnosis of cracks in rotating machinery. The question we address is: how to optimize the multi-hypothesis algorithm with respect to the uncertainty of the spatial form and location of cracks and their resulting dynamic effects. First, we formulate a measure of the reliability of the diagnostic algorithm, and then we discuss modifications of the diagnostic algorithm for the maximization of the reliability. The reliability of a diagnostic algorithm is measured by the amount of uncertainty consistent with no-failure of the diagnosis. Uncertainty is quantitatively represented with convex models.
Keywords: Robust reliability, convex models, Kalman filtering, multi-hypothesis diagnosis, rotating machinery, crack diagnosis
(24 pages, 1998)

4. F.-Th. Lentes, N. Siedow

Three-dimensional Radiative Heat Transfer in Glass Cooling Processes

For the numerical simulation of 3D radiative heat transfer in glasses and glass melts, practically applicable mathematical methods are needed to handle such problems optimal using workstation class computers. Since the exact solution would require super-computer capabilities we concentrate on approximate solutions with a high degree of accuracy. The following approaches are studied: 3D diffusion approximations and 3D ray-tracing methods.
(23 pages, 1998)

5. A. Klar, R. Wegener

A hierarchy of models for multilane vehicular traffic
Part I: Modeling

In the present paper multilane models for vehicular traffic are considered. A microscopic multilane model based on reaction thresholds is developed. Based on this model an Enskog like kinetic model is developed. In particular, care is taken to incorporate the correlations between the vehicles. From the kinetic model a fluid dynamic model is derived. The macroscopic coefficients are deduced from the underlying kinetic model. Numerical simulations are presented for all three levels of description in [10]. Moreover, a comparison of the results is given there.
(23 pages, 1998)

Part II: Numerical and stochastic investigations

In this paper the work presented in [6] is continued. The present paper contains detailed numerical investigations of the models developed there. A numerical method to treat the kinetic equations obtained in [6] are presented and results of the simulations are shown. Moreover, the stochastic correlation model used in [6] is described and investigated in more detail.
(17 pages, 1998)

6. A. Klar, N. Siedow

Boundary Layers and Domain Decomposition for Radiative Heat Transfer and Diffusion Equations: Applications to Glass Manufacturing Processes

In this paper domain decomposition methods for radiative transfer problems including conductive heat transfer are treated. The paper focuses on semi-transparent materials, like glass, and the associated conditions at the interface between the materials. Using asymptotic analysis we derive conditions for the coupling of the radiative transfer equations and a diffusion approximation. Several test cases are treated and a problem appearing in glass manufacturing processes is computed. The results clearly show the advantages of a domain decomposition approach. Accuracy equivalent to the solution of the global radiative transfer solution is achieved, whereas computation time is strongly reduced.
(24 pages, 1998)

7. I. Choquet

Heterogeneous catalysis modelling and numerical simulation in rarified gas flows
Part I: Coverage locally at equilibrium

A new approach is proposed to model and simulate numerically heterogeneous catalysis in rarefied gas flows. It is developed to satisfy all together the following points:
1) describe the gas phase at the microscopic scale, as required in rarefied flows,
2) describe the wall at the macroscopic scale, to avoid prohibitive computational costs and consider not only crystalline but also amorphous surfaces,
3) reproduce on average macroscopic laws correlated with experimental results and
4) derive analytic models in a systematic and exact way.
The problem is stated in the general framework of a non static flow in the vicinity of a catalytic and non porous surface (without aging). It is shown that the exact and systematic resolution method based on the Laplace transform, introduced previously by the author to model collisions in the gas phase, can be extended to the present problem. The proposed approach is applied to the modelling of the EleyRideal and LangmuirHinshelwood recombinations, assuming that the coverage is locally at equilibrium. The models are developed considering one atomic species and extended to the general case of several atomic species. Numerical calculations show that the models derived in this way reproduce with accuracy behaviors observed experimentally.
(24 pages, 1998)

8. J. Ohser, B. Steinbach, C. Lang

Efficient Texture Analysis of Binary Images

A new method of determining some characteristics of binary images is proposed based on a special linear filtering. This technique enables the estimation of the area fraction, the specific line length, and the specific integral of curvature. Furthermore, the specific length of the total projection is obtained, which gives detailed information about the texture of the image. The influence of lateral and directional resolution depending on the size of the applied filter mask is discussed in detail. The technique includes a method of increasing directional resolution for texture analysis while keeping lateral resolution as high as possible.
(17 pages, 1998)

9. J. Orlik

Homogenization for viscoelasticity of the integral type with aging and shrinkage

A multiphase composite with periodic distributed inclusions with a smooth boundary is considered in this contribution. The composite component materials are supposed to be linear viscoelastic and aging (of the nonconvolution integral type, for which the Laplace transform with respect to time is not effectively applicable) and are subjected to isotropic shrinkage. The free shrinkage deformation can be considered as a fictitious temperature deformation in the behavior law. The procedure presented in this paper proposes a way to determine average (effective homogenized) viscoelastic and shrinkage (temperature) composite properties and the homogenized stressfield from known properties of the components. This is done by the extension of the asymptotic homogenization technique known for pure elastic nonhomogeneous bodies to the nonhomogeneous thermoviscoelasticity of the integral nonconvolution type. Up to now, the homogenization theory has not covered viscoelasticity of the integral type. SanchezPalencia (1980), Francfort & Suquet (1987) (see [2], [9]) have considered homogenization for viscoelasticity of the differential form and only up to the first derivative order. The integralmodeled viscoelasticity is more general then the differential one and includes almost all known differential models. The homogenization procedure is based on the construction of an asymptotic solution with respect to a period of the composite structure. This reduces the original problem to some auxiliary boundary value problems of elasticity and viscoelasticity on the unit periodic cell, of the same type as the original non-homogeneous problem. The existence and uniqueness results for such problems were obtained for kernels satisfying some constrain conditions. This is done by the extension of the Volterra integral operator theory to the Volterra operators with respect to the time, whose 1 kernels are space linear operators for any fixed time variables. Some ideas of such approach were proposed in [11] and [12], where the Volterra operators with kernels depending additionally on parameter were considered. This manuscript delivers results of the same nature for the case of the spaceoperator kernels.
(20 pages, 1998)

10. J. Mohring

Helmholtz Resonators with Large Aperture

The lowest resonant frequency of a cavity resonator is usually approximated by the classical Helmholtz formula. However, if the opening is rather large and the front wall is narrow this formula is no longer valid. Here we present a correction which is of third order in the ratio of the diameters of aperture and cavity. In addition to the high accuracy it allows to estimate the damping due to radiation. The result is found by applying the method of matched asymptotic expansions. The correction contains form factors describing the shapes of opening and cavity. They are computed for a number of standard geometries. Results are compared with numerical computations.
(21 pages, 1998)

11. H. W. Hamacher, A. Schöbel

On Center Cycles in Grid Graphs

Finding "good" cycles in graphs is a problem of great interest in graph theory as well as in locational analysis. We show that the center and median problems are NP hard in general graphs. This result holds both for the variable cardinality case (i.e. all cycles of the graph are considered) and the fixed cardinality case (i.e. only cycles with a given cardinality p are feasible). Hence it is of interest to investigate special cases where the problem is solvable in polynomial time. In grid graphs, the variable cardinality case is, for instance, trivially solvable if the shape of the cycle can be chosen freely.
If the shape is fixed to be a rectangle one can analyze rectangles in grid graphs with, in sequence, fixed dimension, fixed cardinality, and variable cardinality. In all cases a complete characterization of the optimal cycles and closed form expressions of the optimal objective values are given, yielding polynomial time algorithms for all cases of center rectangle problems.
Finally, it is shown that center cycles can be chosen as rectangles for small cardinalities such that the center cycle problem in grid graphs is in these cases completely solved.
(15 pages, 1998)

12. H. W. Hamacher, K.-H. Küfer

Inverse radiation therapy planning - a multiple objective optimisation approach

For some decades radiation therapy has been proved successful in cancer treatment. It is the major task of clinical radiation treatment planning to realize on the one hand a high level dose of radiation in the cancer tissue in order to obtain maximum tumor control. On the other hand it is obvious that it is absolutely necessary to keep in the tissue outside the tumor, particularly in organs at risk, the unavoidable radiation as low as possible.
No doubt, these two objectives of treatment planning - high level dose in the tumor, low radiation outside the tumor - have a basically contradictory nature. Therefore, it is no surprise that inverse mathematical models with dose distribution bounds tend to be infeasible in most cases. Thus, there is need for approximations compromising between overdosing the organs at risk and underdosing the target volume.
Differing from the currently used time consuming iterative approach, which measures deviation from an ideal (non-achievable) treatment plan using recursively trial-and-error weights for the organs of interest, we go a new way trying to avoid a priori weight choices and consider the treatment planning problem as a multiple objective linear programming problem: with each organ of interest, target tissue as well as organs at risk, we associate an objective function measuring the maximal deviation from the prescribed doses.
We build up a data base of relatively few efficient solutions representing and approximating the variety of Pareto solutions of the multiple objective linear programming problem. This data base can be easily scanned by physicians looking for an adequate treatment plan with the aid of an appropriate online tool.
(14 pages, 1999)

13. C. Lang, J. Ohser, R. Hilfer

On the Analysis of Spatial Binary Images

This paper deals with the characterization of microscopically heterogeneous, but macroscopically homogeneous spatial structures. A new method is presented which is strictly based on integral-geometric formulae such as Crofton's intersection formulae and Hadwiger's recursive definition of the Euler number. The corresponding algorithms have clear advantages over other techniques. As an example of application we consider the analysis of spatial digital images produced by means of Computer Assisted Tomography.
(20 pages, 1999)

14. M. Junk

On the Construction of Discrete Equilibrium Distributions for Kinetic Schemes

A general approach to the construction of discrete equilibrium distributions is presented. Such distribution functions can be used to set up Kinetic Schemes as well as Lattice Boltzmann methods. The general principles are also applied to the construction of Chapman Enskog distributions which are used in Kinetic Schemes for compressible Navier-Stokes equations.
(24 pages, 1999)

15. M. Junk, S. V. Raghurame Rao

A new discrete velocity method for Navier-Stokes equations

The relation between the Lattice Boltzmann Method, which has recently become popular, and the Kinetic Schemes, which are routinely used in Computational Fluid Dynamics, is explored. A new discrete velocity model for the numerical solution of Navier-Stokes equations for incompressible fluid flow is presented by combining both the approaches. The new scheme can be interpreted as a pseudo-compressibility method and, for a particular choice of parameters, this interpretation carries over to the Lattice Boltzmann Method.
(20 pages, 1999)

16. H. Neunzert

Mathematics as a Key to Key Technologies

The main part of this paper will consist of examples, how mathematics really helps to solve industrial problems; these examples are taken from our Institute for Industrial Mathematics, from research in the Technomathematics group at my university, but also from ECMI groups and a company called TecMath, which originated 10 years ago from my university group and has already a very successful history.
(39 pages (4 PDF-Files), 1999)

17. J. Ohser, K. Sandau

Considerations about the Estimation of the Size Distribution in Wicksell's Corpuscle Problem

Wicksell's corpuscle problem deals with the estimation of the size distribution of a population of particles, all having the same shape, using a lower dimensional sampling probe. This problem was originary formulated for particle systems occurring in life sciences but its solution is of actual and increasing interest in materials science. From a mathematical point of view, Wicksell's problem is an inverse problem where the interesting size distribution is the unknown part of a Volterra equation. The problem is often regarded ill-posed, because the structure of the integrand implies unstable numerical solutions. The accuracy of the numerical solutions is considered here using the condition number, which allows to compare different numerical methods with different (equidistant) class sizes and which indicates, as one result, that a finite section thickness of the probe reduces the numerical problems. Furthermore, the relative error of estimation is computed which can be split into two parts. One part consists of the relative discretization error that increases for increasing class size, and the second part is related to the relative statistical error which increases with decreasing class size. For both parts, upper bounds can be given and the sum of them indicates an optimal class width depending on some specific constants.
(18 pages, 1999)

18. E. Carrizosa, H. W. Hamacher, R. Klein, S. Nickel

Solving nonconvex planar location problems by finite dominating sets

It is well-known that some of the classical location problems with polyhedral gauges can be solved in polynomial time by finding a finite dominating set, i. e. a finite set of candidates guaranteed to contain at least one optimal location.
In this paper it is first established that this result holds for a much larger class of problems than currently considered in the literature. The model for which this result can be proven includes, for instance, location problems with attraction and repulsion, and location-allocation problems.
Next, it is shown that the approximation of general gauges by polyhedral ones in the objective function of our general model can be analyzed with regard to the subsequent error in the optimal objective value. For the approximation problem two different approaches are described, the sandwich procedure and the greedy algorithm. Both of these approaches lead - for fixed epsilon - to polynomial approximation algorithms with accuracy epsilon for solving the general model considered in this paper.
Keywords: Continuous Location, Polyhedral Gauges, Finite Dominating Sets, Approximation, Sandwich Algorithm, Greedy Algorithm
(19 pages, 2000)

19. A. Becker

A Review on Image Distortion Measures

Within this paper we review image distortion measures. A distortion measure is a criterion that assigns a "quality number" to an image. We distinguish between mathematical distortion measures and those distortion measures in-cooperating a priori knowledge about the imaging devices (e. g. satellite images), image processing algorithms or the human physiology. We will consider representative examples of different kinds of distortion measures and are going to discuss them.
Keywords: Distortion measure, human visual system
(26 pages, 2000)

20. H. W. Hamacher, M. Labbé, S. Nickel, T. Sonneborn

Polyhedral Properties of the Uncapacitated Multiple Allocation Hub Location Problem

We examine the feasibility polyhedron of the uncapacitated hub location problem (UHL) with multiple allocation, which has applications in the fields of air passenger and cargo transportation, telecommunication and postal delivery services. In particular we determine the dimension and derive some classes of facets of this polyhedron. We develop some general rules about lifting facets from the uncapacitated facility location (UFL) for UHL and projecting facets from UHL to UFL. By applying these rules we get a new class of facets for UHL which dominates the inequalities in the original formulation. Thus we get a new formulation of UHL whose constraints are all facet–defining. We show its superior computational performance by benchmarking it on a well known data set.
Keywords: integer programming, hub location, facility location, valid inequalities, facets, branch and cut
(21 pages, 2000)

21. H. W. Hamacher, A. Schöbel

Design of Zone Tariff Systems in Public Transportation

Given a public transportation system represented by its stops and direct connections between stops, we consider two problems dealing with the prices for the customers: The fare problem in which subsets of stops are already aggregated to zones and "good" tariffs have to be found in the existing zone system. Closed form solutions for the fare problem are presented for three objective functions. In the zone problem the design of the zones is part of the problem. This problem is NP hard and we therefore propose three heuristics which prove to be very successful in the redesign of one of Germany's transportation systems.
(30 pages, 2001)

22. D. Hietel, M. Junk, R. Keck, D. Teleaga

The Finite-Volume-Particle Method for Conservation Laws

In the Finite-Volume-Particle Method (FVPM), the weak formulation of a hyperbolic conservation law is discretized by restricting it to a discrete set of test functions. In contrast to the usual Finite-Volume approach, the test functions are not taken as characteristic functions of the control volumes in a spatial grid, but are chosen from a partition of unity with smooth and overlapping partition functions (the particles), which can even move along pre- scribed velocity fields. The information exchange between particles is based on standard numerical flux functions. Geometrical information, similar to the surface area of the cell faces in the Finite-Volume Method and the corresponding normal directions are given as integral quantities of the partition functions. After a brief derivation of the Finite-Volume-Particle Method, this work focuses on the role of the geometric coefficients in the scheme.
(16 pages, 2001)

23. T. Bender, H. Hennes, J. Kalcsics, M. T. Melo, S. Nickel

Location Software and Interface with GIS and Supply Chain Management

The objective of this paper is to bridge the gap between location theory and practice. To meet this objective focus is given to the development of software capable of addressing the different needs of a wide group of users. There is a very active community on location theory encompassing many research fields such as operations research, computer science, mathematics, engineering, geography, economics and marketing. As a result, people working on facility location problems have a very diverse background and also different needs regarding the software to solve these problems. For those interested in non-commercial applications (e. g. students and researchers), the library of location algorithms (LoLA can be of considerable assistance. LoLA contains a collection of efficient algorithms for solving planar, network and discrete facility location problems. In this paper, a detailed description of the functionality of LoLA is presented. In the fields of geography and marketing, for instance, solving facility location problems requires using large amounts of demographic data. Hence, members of these groups (e. g. urban planners and sales managers) often work with geographical information too s. To address the specific needs of these users, LoLA was inked to a geographical information system (GIS) and the details of the combined functionality are described in the paper. Finally, there is a wide group of practitioners who need to solve large problems and require special purpose software with a good data interface. Many of such users can be found, for example, in the area of supply chain management (SCM). Logistics activities involved in strategic SCM include, among others, facility location planning. In this paper, the development of a commercial location software tool is also described. The too is embedded in the Advanced Planner and Optimizer SCM software developed by SAP AG, Walldorf, Germany. The paper ends with some conclusions and an outlook to future activities.
Keywords: facility location, software development, geographical information systems, supply chain management
(48 pages, 2001)

24. H. W. Hamacher, S. A. Tjandra

Mathematical Modelling of Evacuation Problems: A State of Art

This paper details models and algorithms which can be applied to evacuation problems. While it concentrates on building evacuation many of the results are applicable also to regional evacuation. All models consider the time as main parameter, where the travel time between components of the building is part of the input and the overall evacuation time is the output. The paper distinguishes between macroscopic and microscopic evacuation models both of which are able to capture the evacuees' movement over time.
Macroscopic models are mainly used to produce good lower bounds for the evacuation time and do not consider any individual behavior during the emergency situation. These bounds can be used to analyze existing buildings or help in the design phase of planning a building. Macroscopic approaches which are based on dynamic network flow models (minimum cost dynamic flow, maximum dynamic flow, universal maximum flow, quickest path and quickest flow) are described. A special feature of the presented approach is the fact, that travel times of evacuees are not restricted to be constant, but may be density dependent. Using multicriteria optimization priority regions and blockage due to fire or smoke may be considered. It is shown how the modelling can be done using time parameter either as discrete or continuous parameter.
Microscopic models are able to model the individual evacuee's characteristics and the interaction among evacuees which influence their movement. Due to the corresponding huge amount of data one uses simulation approaches. Some probabilistic laws for individual evacuee's movement are presented. Moreover ideas to model the evacuee's movement using cellular automata (CA) and resulting software are presented.
In this paper we will focus on macroscopic models and only summarize some of the results of the microscopic approach. While most of the results are applicable to general evacuation situations, we concentrate on building evacuation.
(44 pages, 2001)

25. J. Kuhnert, S. Tiwari

Grid free method for solving the Poisson equation

A Grid free method for solving the Poisson equation is presented. This is an iterative method. The method is based on the weighted least squares approximation in which the Poisson equation is enforced to be satisfied in every iterations. The boundary conditions can also be enforced in the iteration process. This is a local approximation procedure. The Dirichlet, Neumann and mixed boundary value problems on a unit square are presented and the analytical solutions are compared with the exact solutions. Both solutions matched perfectly.
Keywords: Poisson equation, Least squares method, Grid free method
(19 pages, 2001)

26. T. Götz, H. Rave, D. Reinel-Bitzer, K. Steiner, H. Tiemeier

Simulation of the fiber spinning process

To simulate the influence of process parameters to the melt spinning process a fiber model is used and coupled with CFD calculations of the quench air flow. In the fiber model energy, momentum and mass balance are solved for the polymer mass flow. To calculate the quench air the Lattice Boltzmann method is used. Simulations and experiments for different process parameters and hole configurations are compared and show a good agreement.
Keywords: Melt spinning, fiber model, Lattice Boltzmann, CFD
(19 pages, 2001)

27. A. Zemitis

On interaction of a liquid film with an obstacle

In this paper mathematical models for liquid films generated by impinging jets are discussed. Attention is stressed to the interaction of the liquid film with some obstacle. S. G. Taylor [Proc. R. Soc. London Ser. A 253, 313 (1959)] found that the liquid film generated by impinging jets is very sensitive to properties of the wire which was used as an obstacle. The aim of this presentation is to propose a modification of the Taylor's model, which allows to simulate the film shape in cases, when the angle between jets is different from 180°. Numerical results obtained by discussed models give two different shapes of the liquid film similar as in Taylors experiments. These two shapes depend on the regime: either droplets are produced close to the obstacle or not. The difference between two regimes becomes larger if the angle between jets decreases. Existence of such two regimes can be very essential for some applications of impinging jets, if the generated liquid film can have a contact with obstacles.
Keywords: impinging jets, liquid film, models, numerical solution, shape
(22 pages, 2001)

28. I. Ginzburg, K. Steiner

Free surface lattice-Boltzmann method to model the filling of expanding cavities by Bingham Fluids

The filling process of viscoplastic metal alloys and plastics in expanding cavities is modelled using the lattice Boltzmann method in two and three dimensions. These models combine the regularized Bingham model for viscoplastic with a free-interface algorithm. The latter is based on a modified immiscible lattice Boltzmann model in which one species is the fluid and the other one is considered as vacuum. The boundary conditions at the curved liquid-vacuum interface are met without any geometrical front reconstruction from a first-order Chapman-Enskog expansion. The numerical results obtained with these models are found in good agreement with available theoretical and numerical analysis.
Keywords: Generalized LBE, free-surface phenomena, interface boundary conditions, filling processes, Bingham viscoplastic model, regularized models
(22 pages, 2001)

29. H. Neunzert

»Denn nichts ist für den Menschen als Menschen etwas wert, was er nicht mit Leidenschaft tun kann«
Vortrag anlässlich der Verleihung des Akademiepreises des Landes Rheinland-Pfalz am 21.11.2001

Was macht einen guten Hochschullehrer aus? Auf diese Frage gibt es sicher viele verschiedene, fachbezogene Antworten, aber auch ein paar allgemeine Gesichtspunkte: es bedarf der »Leidenschaft« für die Forschung (Max Weber), aus der dann auch die Begeisterung für die Lehre erwächst. Forschung und Lehre gehören zusammen, um die Wissenschaft als lebendiges Tun vermitteln zu können. Der Vortrag gibt Beispiele dafür, wie in angewandter Mathematik Forschungsaufgaben aus praktischen Alltagsproblemstellungen erwachsen, die in die Lehre auf verschiedenen Stufen (Gymnasium bis Graduiertenkolleg) einfließen; er leitet damit auch zu einem aktuellen Forschungsgebiet, der Mehrskalenanalyse mit ihren vielfältigen Anwendungen in Bildverarbeitung, Materialentwicklung und Strömungsmechanik über, was aber nur kurz gestreift wird. Mathematik erscheint hier als eine moderne Schlüsseltechnologie, die aber auch enge Beziehungen zu den Geistes- und Sozialwissenschaften hat.
Keywords: Lehre, Forschung, angewandte Mathematik, Mehrskalenanalyse, Strömungsmechanik
(18 pages, 2001)

30. J. Kuhnert, S. Tiwari

Finite pointset method based on the projection method for simulations of the incompressible Navier-Stokes equations

A Lagrangian particle scheme is applied to the projection method for the incompressible Navier-Stokes equations. The approximation of spatial derivatives is obtained by the weighted least squares method. The pressure Poisson equation is solved by a local iterative procedure with the help of the least squares method. Numerical tests are performed for two dimensional cases. The Couette flow, Poiseuelle flow, decaying shear flow and the driven cavity flow are presented. The numerical solutions are obtained for stationary as well as instationary cases and are compared with the analytical solutions for channel flows. Finally, the driven cavity in a unit square is considered and the stationary solution obtained from this scheme is compared with that from the finite element method.
Keywords: Incompressible Navier-Stokes equations, Meshfree method, Projection method, Particle scheme, Least squares approximation
AMS subject classification: 76D05, 76M28
(25 pages, 2001)

31. R. Korn, M. Krekel

Optimal Portfolios with Fixed Consumption or Income Streams

We consider some portfolio optimisation problems where either the investor has a desire for an a priori specified consumption stream or/and follows a deterministic pay in scheme while also trying to maximize expected utility from final wealth. We derive explicit closed form solutions for continuous and discrete monetary streams. The mathematical method used is classical stochastic control theory.
Keywords: Portfolio optimisation, stochastic control, HJB equation, discretisation of control problems.
(23 pages, 2002)

32. M. Krekel

Optimal portfolios with a loan dependent credit spread

If an investor borrows money he generally has to pay higher interest rates than he would have received, if he had put his funds on a savings account. The classical model of continuous time portfolio optimisation ignores this effect. Since there is obviously a connection between the default probability and the total per-

centage of wealth, which the investor is in debt, we study portfolio optimisation with a control dependent interest rate. Assuming a logarithmic and a power utility function, respectively, we prove explicit formulae of the optimal control.
Keywords: Portfolio optimisation, stochastic control, HJB equation, credit spread, log utility, power utility, non-linear wealth dynamics
(25 pages, 2002)

33. J. Ohser, W. Nagel, K. Schladitz

The Euler number of discretized sets - on the choice of adjacency in homogeneous lattices

Two approaches for determining the Euler-Poincaré characteristic of a set observed on lattice points are considered in the context of image analysis { the integral geometric and the polyhedral approach. Information about the set is assumed to be available on lattice points only. In order to retain properties of the Euler number and to provide a good approximation of the true Euler number of the original set in the Euclidean space, the appropriate choice of adjacency in the lattice for the set and its background is crucial. Adjacencies are defined using tessellations of the whole space into polyhedrons. In R 3 , two new 14 adjacencies are introduced additionally to the well known 6 and 26 adjacencies. For the Euler number of a set and its complement, a consistency relation holds. Each of the pairs of adjacencies (14:1; 14:1), (14:2; 14:2), (6; 26), and (26; 6) is shown to be a pair of complementary adjacencies with respect to this relation. That is, the approximations of the Euler numbers are consistent if the set and its background (complement) are equipped with this pair of adjacencies. Furthermore, sufficient conditions for the correctness of the approximations of the Euler number are given. The analysis of selected microstructures and a simulation study illustrate how the estimated Euler number depends on the chosen adjacency. It also shows that there is not a uniquely best pair of adjacencies with respect to the estimation of the Euler number of a set in Euclidean space.
Keywords: image analysis, Euler number, neighborhod relationships, cuboidal lattice
(32 pages, 2002)

34. I. Ginzburg, K. Steiner

Lattice Boltzmann Model for Free-Surface flow and Its Application to Filling Process in Casting

A generalized lattice Boltzmann model to simulate free-surface is constructed in both two and three dimensions. The proposed model satisfies the interfacial boundary conditions accurately. A distinctive feature of the model is that the collision processes is carried out only on the points occupied partially or fully by the fluid. To maintain a sharp interfacial front, the method includes an anti-diffusion algorithm. The unknown distribution functions at the interfacial region are constructed according to the first order Chapman-Enskog analysis. The interfacial boundary conditions are satisfied exactly by the coefficients in the Chapman-Enskog expansion. The distribution functions are naturally expressed in the local interfacial coordinates. The macroscopic quantities at the interface are extracted from the least-square solutions of a locally linearized system obtained from the known distribution functions. The proposed method does not require any geometric front construction and is robust for any interfacial topology. Simulation results of realistic filling process are presented: rectangular cavity in two dimensions and Hammer box, Campbell box, Sheffield box, and Motorblock in three dimensions. To enhance the stability at high Reynolds numbers, various upwind-type schemes are developed. Free-slip and no-slip boundary conditions are also discussed.
Keywords: Lattice Boltzmann models; free-surface phenomena; interface boundary conditions; filling processes; injection molding; volume of fluid method; interface boundary conditions; advection-schemes; upwind-schemes
(54 pages, 2002)

35. M. Günther, A. Klar, T. Materne, R. Wegener

Multivalued fundamental diagrams and stop and go waves for continuum traffic equations

In the present paper a kinetic model for vehicular traffic leading to multivalued fundamental diagrams is developed and investigated in detail. For this model phase transitions can appear depending on the local density and velocity of the flow. A derivation of associated macroscopic traffic equations from the kinetic equation is given. Moreover, numerical experiments show the appearance of stop and go waves for highway traffic with a bottleneck.
Keywords: traffic flow, macroscopic equations, kinetic derivation, multivalued fundamental diagram, stop and go waves, phase transitions
(25 pages, 2002)

36. S. Feldmann, P. Lang, D. Prätzel-Wolters

Parameter influence on the zeros of network determinants

To a network N(q) with determinant D(s;q) depending on a parameter vector q Î Rr via identification of some of its vertices, a network N^ (q) is assigned. The paper deals with procedures to find N^ (q), such that its determinant D^ (s;q) admits a factorization in the determinants of appropriate subnetworks, and with the estimation of the deviation of the zeros of D^ from the zeros of D. To solve the estimation problem state space methods are applied.
Keywords: Networks, Equicofactor matrix polynomials, Realization theory, Matrix perturbation theory
(30 pages, 2002)

37. K. Koch, J. Ohser, K. Schladitz

Spectral theory for random closed sets and estimating the covariance via frequency space

A spectral theory for stationary random closed sets is developed and provided with a sound mathematical basis. Definition and proof of existence of the Bartlett spectrum of a stationary random closed set as well as the proof of a Wiener-Khintchine theorem for the power spectrum are used to two ends: First, well known second order characteristics like the covariance can be estimated faster than usual via frequency space. Second, the Bartlett spectrum and the power spectrum can be used as second order characteristics in frequency space. Examples show, that in some cases information about the random closed set is easier to obtain from these characteristics in frequency space than from their real world counterparts.
Keywords: Random set, Bartlett spectrum, fast Fourier transform, power spectrum
(28 pages, 2002)

38. D. d'Humières, I. Ginzburg

Multi-reflection boundary conditions for lattice Boltzmann models

We present a unified approach of several boundary conditions for lattice Boltzmann models. Its general framework is a generalization of previously introduced schemes such as the bounce-back rule, linear or quadratic interpolations, etc. The objectives are two fold: first to give theoretical tools to study the existing boundary conditions and their corresponding accuracy; secondly to design formally third- order accurate boundary conditions for general flows. Using these boundary conditions, Couette and Poiseuille flows are exact solution of the lattice Boltzmann models for a Reynolds number Re = 0 (Stokes limit).
Numerical comparisons are given for Stokes flows in periodic arrays of spheres and cylinders, linear periodic array of cylinders between moving plates and for Navier-Stokes flows in periodic arrays of cylinders for Re < 200. These results show a significant improvement of the overall accuracy when using the linear interpolations instead of the bounce-back reflection (up to an order of magnitude on the hydrodynamics fields). Further improvement is achieved with the new multi-reflection boundary conditions, reaching a level of accuracy close to the quasi-analytical reference solutions, even for rather modest grid resolutions and few points in the narrowest channels. More important, the pressure and velocity fields in the vicinity of the obstacles are much smoother with multi-reflection than with the other boundary conditions.
Finally the good stability of these schemes is highlighted by some simulations of moving obstacles: a cylinder between flat walls and a sphere in a cylinder.
Keywords: lattice Boltzmann equation, boudary condistions, bounce-back rule, Navier-Stokes equation
(72 pages, 2002)

39. R. Korn

Elementare Finanzmathematik

Im Rahmen dieser Arbeit soll eine elementar gehaltene Einführung in die Aufgabenstellungen und Prinzipien der modernen Finanzmathematik gegeben werden. Insbesondere werden die Grundlagen der Modellierung von Aktienkursen, der Bewertung von Optionen und der Portfolio-Optimierung vorgestellt. Natürlich können die verwendeten Methoden und die entwickelte Theorie nicht in voller Allgemeinheit für den Schulunterricht verwendet werden, doch sollen einzelne Prinzipien so heraus gearbeitet werden, dass sie auch an einfachen Beispielen verstanden werden können.
Keywords: Finanzmathematik, Aktien, Optionen, Portfolio-Optimierung, Börse, Lehrerweiterbildung, Mathematikunterricht
(98 pages, 2002)

40. J. Kallrath, M. C. Müller, S. Nickel

Batch Presorting Problems: Models and Complexity Results

In this paper we consider short term storage systems. We analyze presorting strategies to improve the effiency of these storage systems. The presorting task is called Batch PreSorting Problem (BPSP). The BPSP is a variation of an assigment problem, i. e., it has an assigment problem kernel and some additional constraints. We present different types of these presorting problems, introduce mathematical programming formulations and prove the NP-completeness for one type of the BPSP. Experiments are carried out in order to compare the different model formulations and to investigate the behavior of these models.
Keywords: Complexity theory, Integer programming, Assigment, Logistics
(19 pages, 2002)

41. J. Linn

On the frame-invariant description of the phase space of the Folgar-Tucker equation

The Folgar-Tucker equation is used in flow simulations of fiber suspensions to predict fiber orientation depending on the local flow. In this paper, a complete, frame-invariant description of the phase space of this differential equation is presented for the first time.
Key words: fiber orientation, Folgar-Tucker equation, injection molding
(5 pages, 2003)

42. T. Hanne, S. Nickel

A Multi-Objective Evolutionary Algorithm for Scheduling and Inspection Planning in Software Development Projects

In this article, we consider the problem of planning inspections and other tasks within a software development (SD) project with respect to the objectives quality (no. of defects), project duration, and costs. Based on a discrete-event simulation model of SD processes comprising the phases coding, inspection, test, and rework, we present a simplified formulation of the problem as a multiobjective optimization problem. For solving the problem (i. e. finding an approximation of the efficient set) we develop a multiobjective evolutionary algorithm. Details of the algorithm are discussed as well as results of its application to sample problems.
Key words: multiple objective programming, project management and scheduling, software development, evolutionary algorithms, efficient set
(29 pages, 2003)

43. T. Bortfeld , K.-H. Küfer, M. Monz, A. Scherrer, C. Thieke, H. Trinkaus

Intensity-Modulated Radiotherapy - A Large Scale Multi-Criteria Programming Problem -

Radiation therapy planning is always a tight rope walk between dangerous insufficient dose in the target volume and life threatening overdosing of organs at risk. Finding ideal balances between these inherently contradictory goals challenges dosimetrists and physicians in their daily practice. Today's planning systems are typically based on a single evaluation function that measures the quality of a radiation treatment plan. Unfortunately, such a one dimensional approach cannot satisfactorily map the different backgrounds of physicians and the patient dependent necessities. So, too often a time consuming iteration process between evaluation of dose distribution and redefinition of the evaluation function is needed.
In this paper we propose a generic multi-criteria approach based on Pareto's solution concept. For each entity of interest - target volume or organ at risk a structure dependent evaluation function is defined measuring deviations from ideal doses that are calculated from statistical functions. A reasonable bunch of clinically meaningful Pareto optimal solutions are stored in a data base, which can be interactively searched by physicians. The system guarantees dynamical planning as well as the discussion of tradeoffs between different entities.
Mathematically, we model the upcoming inverse problem as a multi-criteria linear programming problem. Because of the large scale nature of the problem it is not possible to solve the problem in a 3D-setting without adaptive reduction by appropriate approximation schemes.
Our approach is twofold: First, the discretization of the continuous problem is based on an adaptive hierarchical clustering process which is used for a local refinement of constraints during the optimization procedure. Second, the set of Pareto optimal solutions is approximated by an adaptive grid of representatives that are found by a hybrid process of calculating extreme compromises and interpolation methods.
Keywords: multiple criteria optimization, representative systems of Pareto solutions, adaptive triangulation, clustering and disaggregation techniques, visualization of Pareto solutions, medical physics, external beam radiotherapy planning, intensity modulated radiotherapy
(31 pages, 2003)

44. T. Halfmann, T. Wichmann

Overview of Symbolic Methods in Industrial Analog Circuit Design

Industrial analog circuits are usually designed using numerical simulation tools. To obtain a deeper circuit understanding, symbolic analysis techniques can additionally be applied. Approximation methods which reduce the complexity of symbolic expressions are needed in order to handle industrial-sized problems.
This paper will give an overview to the field of symbolic analog circuit analysis. Starting with a motivation, the state-of-the-art simplification algorithms for linear as well as for nonlinear circuits are presented. The basic ideas behind the different techniques are described, whereas the technical details can be found in the cited references. Finally, the application of linear and nonlinear symbolic analysis will be shown on two example circuits.
Keywords: CAD, automated analog circuit design, symbolic analysis, computer algebra, behavioral modeling, system simulation, circuit sizing, macro modeling, differential-algebraic equations, index
(17 pages, 2003)

45. S. E. Mikhailov, J. Orlik

Asymptotic Homogenisation in Strength and Fatigue Durability Analysis of Composites

Asymptotic homogenisation technique and two-scale convergence is used for analysis of macro-strength and fatigue durability of composites with a periodic structure under cyclic loading. The linear damage accumulation rule is employed in the phenomenological micro-durability conditions (for each component of the composite) under varying cyclic loading. Both local and non-local strength and durability conditions are analysed. The strong convergence of the strength and fatigue damage measure as the structure period tends to zero is proved and their limiting values are estimated.
Keywords: multiscale structures, asymptotic homogenization, strength, fatigue, singularity, non-local conditions
(14 pages, 2003)

46. P. Domínguez-Marín, P. Hansen, N. Mladenovi´c , S. Nickel

Heuristic Procedures for Solving the Discrete Ordered Median Problem

We present two heuristic methods for solving the Discrete Ordered Median Problem (DOMP), for which no such approaches have been developed so far. The DOMP generalizes classical discrete facility location problems, such as the p-median, p-center and Uncapacitated Facility Location problems. The first procedure proposed in this paper is based on a genetic algorithm developed by Moreno Vega [MV96] for p-median and p-center problems. Additionally, a second heuristic approach based on the Variable Neighborhood Search metaheuristic (VNS) proposed by Hansen & Mladenovic [HM97] for the p-median problem is described. An extensive numerical study is presented to show the efficiency of both heuristics and compare them.
Keywords: genetic algorithms, variable neighborhood search, discrete facility location
(31 pages, 2003)

47. N. Boland, P. Domínguez-Marín, S. Nickel, J. Puerto

Exact Procedures for Solving the Discrete Ordered Median Problem

The Discrete Ordered Median Problem (DOMP) generalizes classical discrete location problems, such as the N-median, N-center and Uncapacitated Facility Location problems. It was introduced by Nickel [16], who formulated it as both a nonlinear and a linear integer program. We propose an alternative integer linear programming formulation for the DOMP, discuss relationships between both integer linear programming formulations, and show how properties of optimal solutions can be used to strengthen these formulations. Moreover, we present a specific branch and bound procedure to solve the DOMP more efficiently. We test the integer linear programming formulations and this branch and bound method computationally on randomly generated test problems.
Keywords: discrete location, Integer programming
(41 pages, 2003)

48. S. Feldmann, P. Lang

Padé-like reduction of stable discrete linear systems preserving their stability

A new stability preserving model reduction algorithm for discrete linear SISO-systems based on their impulse response is proposed. Similar to the Padé approximation, an equation system for the Markov parameters involving the Hankel matrix is considered, that here however is chosen to be of very high dimension. Although this equation system therefore in general cannot be solved exactly, it is proved that the approximate solution, computed via the Moore-Penrose inverse, gives rise to a stability preserving reduction scheme, a property that cannot be guaranteed for the Padé approach. Furthermore, the proposed algorithm is compared to another stability preserving reduction approach, namely the balanced truncation method, showing comparable performance of the reduced systems. The balanced truncation method however starts from a state space description of the systems and in general is expected to be more computational demanding.
Keywords: Discrete linear systems, model reduction, stability, Hankel matrix, Stein equation
(16 pages, 2003)

49. J. Kallrath, S. Nickel

A Polynomial Case of the Batch Presorting Problem

This paper presents new theoretical results for a special case of the batch presorting problem (BPSP). We will show tht this case can be solved in polynomial time. Offline and online algorithms are presented for solving the BPSP. Competetive analysis is used for comparing the algorithms.
Keywords: batch presorting problem, online optimization, competetive analysis, polynomial algorithms, logistics
(17 pages, 2003)

50. T. Hanne, H. L. Trinkaus

knowCube for MCDM – Visual and Interactive Support for Multicriteria Decision Making

In this paper, we present a novel multicriteria decision support system (MCDSS), called knowCube, consisting of components for knowledge organization, generation, and navigation. Knowledge organization rests upon a database for managing qualitative and quantitative criteria, together with add-on information. Knowledge generation serves filling the database via e. g. identification, optimization, classification or simulation. For "finding needles in haycocks", the knowledge navigation component supports graphical database retrieval and interactive, goal-oriented problem solving. Navigation "helpers" are, for instance, cascading criteria aggregations, modifiable metrics, ergonomic interfaces, and customizable visualizations. Examples from real-life projects, e.g. in industrial engineering and in the life sciences, illustrate the application of our MCDSS.
Key words: Multicriteria decision making, knowledge management, decision support systems, visual interfaces, interactive navigation, real-life applications.
(26 pages, 2003)

51. O. Iliev, V. Laptev

On Numerical Simulation of Flow Through Oil Filters

This paper concerns numerical simulation of flow through oil filters. Oil filters consist of filter housing (filter box), and a porous filtering medium, which completely separates the inlet from the outlet. We discuss mathematical models, describing coupled flows in the pure liquid subregions and in the porous filter media, as well as interface conditions between them. Further, we reformulate the problem in fictitious regions method manner, and discuss peculiarities of the numerical algorithm in solving the coupled system. Next, we show numerical results, validating the model and the algorithm. Finally, we present results from simulation of 3-D oil flow through a real car filter.
Keywords: oil filters, coupled flow in plain and porous media, Navier-Stokes, Brinkman, numerical simulation
(8 pages, 2003)

52. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva

On a Multigrid Adaptive Refinement Solver for Saturated Non-Newtonian Flow in Porous Media

A multigrid adaptive refinement algorithm for non-Newtonian flow in porous media is presented. The saturated flow of a non-Newtonian fluid is described by the continuity equation and the generalized Darcy law. The resulting second order nonlinear elliptic equation is discretized by a finite volume method on a cell-centered grid. A nonlinear full-multigrid, full-approximation-storage algorithm is implemented. As a smoother, a single grid solver based on Picard linearization and Gauss-Seidel relaxation is used. Further, a local refinement multigrid algorithm on a composite grid is developed. A residual based error indicator is used in the adaptive refinement criterion. A special implementation approach is used, which allows us to perform unstructured local refinement in conjunction with the finite volume discretization. Several results from numerical experiments are presented in order to examine the performance of the solver.
Keywords: Nonlinear multigrid, adaptive refinement, non-Newtonian flow in porous media
(17 pages, 2003)

53. S. Kruse

On the Pricing of Forward Starting Options under Stochastic Volatility

We consider the problem of pricing European forward starting options in the presence of stochastic volatility. By performing a change of measure using the asset

price at the time of strike determination as a numeraire, we derive a closed-form solution based on Heston's model of stochastic volatility.
Keywords: Option pricing, forward starting options, Heston model, stochastic volatility, cliquet options
(11 pages, 2003)

54. O. Iliev, D. Stoyanov

Multigrid – adaptive local refinement solver for incompressible flows

A non-linear multigrid solver for incompressible Navier-Stokes equations, exploiting finite volume discretization of the equations, is extended by adaptive local refinement. The multigrid is the outer iterative cycle, while the SIMPLE algorithm is used as a smoothing procedure. Error indicators are used to define the refinement subdomain. A special implementation approach is used, which allows to perform unstructured local refinement in conjunction with the finite volume discretization. The multigrid - adaptive local refinement algorithm is tested on 2D Poisson equation and further is applied to a lid-driven flows in a cavity (2D and 3D case), comparing the results with bench-mark data. The software design principles of the solver are also discussed.
Keywords: Navier-Stokes equations, incompressible flow, projection-type splitting, SIMPLE, multigrid methods, adaptive local refinement, lid-driven flow in a cavity
(37 pages, 2003)

55. V. Starikovicius

The multiphase flow and heat transfer in porous media

In first part of this work, summaries of traditional Multiphase Flow Model and more recent Multiphase Mixture Model are presented. Attention is being paid to attempts include various heterogeneous aspects into models. In second part, MMM based differential model for two-phase immiscible flow in porous media is considered. A numerical scheme based on the sequential solution procedure and control volume based finite difference schemes for the pressure and saturation-conservation equations is developed. A computer simulator is built, which exploits object-oriented programming techniques. Numerical result for several test problems are reported.
Keywords: Two-phase flow in porous media, various formulations, global pressure, multiphase mixture model, numerical simulation
(30 pages, 2003)

56. P. Lang, A. Sarishvili, A. Wirsen

Blocked neural networks for knowledge extraction in the software development process

One of the main goals of an organization developing software is to increase the quality of the software while at the same time to decrease the costs and the duration of the development process. To achieve this, various decisions e.ecting this goal before and during the development process have to be made by the managers. One appropriate tool for decision support are simulation models of the software life cycle, which also help to understand the dynamics of the software development process. Building up a simulation model requires a mathematical description of the interactions between di.erent objects involved in the development process. Based on experimental data, techniques from the .eld of knowledge discovery can be used to quantify these interactions and to generate new process knowledge based on the analysis of the determined relationships. In this paper blocked neuronal networks and related relevance measures will be presented as an appropriate tool for quanti.cation and validation of qualitatively known dependencies in the software development process.
Keywords: Blocked Neural Networks, Nonlinear Regression, Knowledge Extraction, Code Inspection
(21 pages, 2003)

57. H. Knaf, P. Lang, S. Zeiser

Diagnosis aiding in Regulation Thermography using Fuzzy Logic

The objective of the present article is to give an overview of an application of Fuzzy Logic in Regulation Thermography, a method of medical diagnosis support. An introduction to this method of the complementary medical science based on temperature measurements – so-called thermograms – is provided. The process of modelling the physician's thermogram evaluation rules using the calculus of Fuzzy Logic is explained.
Keywords: fuzzy logic,knowledge representation, expert system
(22 pages, 2003)

58. M.T. Melo, S. Nickel, F. Saldanha da Gama

Largescale models for dynamic multi-commodity capacitated facility location

In this paper we focus on the strategic design of supply chain networks. We propose a mathematical modeling framework that captures many practical aspects of network design problems simultaneously but which have not received adequate attention in the literature. The aspects considered include: dynamic planning horizon, generic supply chain network structure, external supply of materials, inventory opportunities for goods, distribution of commodities, facility configuration, availability of capital for investments, and storage limitations. Moreover, network configuration decisions concerning the gradual relocation of facilities over the planning horizon are considered. To cope with fluctuating demands, capacity expansion and reduction scenarios are also analyzed as well as modular capacity shifts. The relation of the proposed modeling framework with existing models is discussed. For problems of reasonable size we report on our computational experience with standard mathematical programming software. In particular, useful insights on the impact of various factors on network design decisions are provided.
Keywords: supply chain management, strategic planning, dynamic location, modeling
(40 pages, 2003)

59. J. Orlik

Homogenization for contact problems with periodically rough surfaces

We consider the contact of two elastic bodies with rough surfaces at the interface. The size of the micro-peaks and valleys is very small compared with the macrosize of the bodies' domains. This makes the direct application of the FEM for the calculation of the contact problem prohibitively costly. A method is developed that allows deriving a macrocontact condition on the interface. The method involves the twoscale asymptotic homogenization procedure that takes into account the microgeometry of the interface layer and the stiffnesses of materials of both domains. The macrocontact condition can then be used in a FEM model for the contact problem on the macrolevel. The averaged contact stiffness obtained allows the replacement of the interface layer in the macromodel by the macrocontact condition.
Keywords: asymptotic homogenization, contact problems
(28 pages, 2004)

60. A. Scherrer, K.-H. Küfer, M. Monz, F. Alonso, T. Bortfeld

IMRT planning on adaptive volume structures – a significant advance of computational complexity

In intensity-modulated radiotherapy (IMRT) planning the oncologist faces the challenging task of finding a treatment plan that he considers to be an ideal compromise of the inherently contradictive goals of delivering a sufficiently high dose to the target while widely sparing critical structures. The search for this a priori unknown compromise typically requires the computation of several plans, i.e. the solution of several optimization problems. This accumulates to a high computational expense due to the large scale of these problems - a consequence of the discrete problem formulation. This paper presents the adaptive clustering method as a new algorithmic concept to overcome these difficulties. The computations are performed on an individually adapted structure of voxel clusters rather than on the original voxels leading to a decisively reduced computational complexity as numerical examples on real clinical data demonstrate. In contrast to many other similar concepts, the typical trade-off between a reduction in computational complexity and a loss in exactness can be avoided: the adaptive clustering method produces the optimum of the original problem. This flexible method can be applied to both single- and multi-criteria optimization methods based on most of the convex evaluation functions used in practice.
Keywords: Intensity-modulated radiation therapy (IMRT), inverse treatment planning, adaptive volume structures, hierarchical clustering, local refinement, adaptive clustering, convex programming, mesh generation, multi-grid methods
(24 pages, 2004)

61. D. Kehrwald

Parallel lattice Boltzmann simulation of complex flows

After a short introduction to the basic ideas of lattice Boltzmann methods and a brief description of a modern parallel computer, it is shown how lattice Boltzmann schemes are successfully applied for simulating fluid flow in microstructures and calculating material properties of porous media. It is explained how lattice Boltzmann schemes compute the gradient of the velocity field without numerical differentiation. This feature is then utilised for the simulation of pseudo-plastic fluids, and numerical results are presented for a simple benchmark problem as well as for the simulation of liquid composite moulding.
Keywords: Lattice Boltzmann methods, parallel computing, microstructure simulation, virtual material design, pseudo-plastic fluids, liquid composite moulding
(12 pages, 2004)

62. O. Iliev, J. Linn, M. Moog, D. Niedziela, V. Starikovicius

On the Performance of Certain Iterative Solvers for Coupled Systems Arising in Discretization of Non-Newtonian Flow Equations

Iterative solution of large scale systems arising after discretization and linearization of the unsteady non-Newtonian Navier–Stokes equations is studied. cross WLF model is used to account for the non-Newtonian behavior of the fluid. Finite volume method is used to discretize the governing system of PDEs. Viscosity is treated explicitely (e.g., it is taken from the previous time step), while other terms are treated implicitly. Different preconditioners (block–diagonal, block–triangular, relaxed incomplete LU factorization, etc.) are used in conjunction with advanced iterative methods, namely, BiCGStab, CGS, GMRES. The action of the preconditioner in fact requires inverting different blocks. For this purpose, in addition to preconditioned BiCGStab, CGS, GMRES, we use also algebraic multigrid method (AMG). The performance of the iterative solvers is studied with respect to the number of unknowns, characteristic velocity in the basic flow, time step, deviation from Newtonian behavior, etc. Results from numerical experiments are presented and discussed.
Keywords: Performance of iterative solvers, Preconditioners, Non-Newtonian flow
(17 pages, 2004)

63. R. Ciegis, O. Iliev, S. Rief, K. Steiner

On Modelling and Simulation of Different Regimes for Liquid Polymer Moulding

In this paper we consider numerical algorithms for solving a system of nonlinear PDEs arising in modeling of liquid polymer injection. We investigate the particular case when a porous preform is located within the mould, so that the liquid polymer flows through a porous medium during the filling stage. The nonlinearity of the governing system of PDEs is due to the non-Newtonian behavior of the polymer, as well as due to the moving free boundary. The latter is related to the penetration front and a Stefan type problem is formulated to account for it. A finite-volume method is used to approximate the given differential problem. Results of numerical experiments are presented.
We also solve an inverse problem and present algorithms for the determination of the absolute preform permeability coefficient in the case when the velocity of the penetration front is known from measurements. In both cases (direct and inverse problems) we emphasize on the specifics related to the non-Newtonian behavior of the polymer. For completeness, we discuss also the Newtonian case. Results of some experimental

measurements are presented and discussed.
Keywords: Liquid Polymer Moulding, Modelling, Simulation, Infiltration, Front Propagation, non-Newtonian flow in porous media
(43 pages, 2004)

64. T. Hanne, H. Neu

Simulating Human Resources in Software Development Processes

In this paper, we discuss approaches related to the explicit modeling of human beings in software development processes. While in most older simulation models of software development processes, esp. those of the system dynamics type, humans are only represented as a labor pool, more recent models of the discrete-event simulation type require representations of individual humans. In that case, particularities regarding the person become more relevant. These individual effects are either considered as stochastic variations of productivity, or an explanation is sought based on individual characteristics, such as skills for instance. In this paper, we explore such possibilities by recurring to some basic results in psychology, sociology, and labor science. Various specific models for representing human effects in software process simulation are discussed.
Keywords: Human resource modeling, software process, productivity, human factors, learning curve
(14 pages, 2004)

65. O. Iliev, A. Mikelic, P. Popov

Fluid structure interaction problems in deformable porous media: Toward permeability of deformable porous media

In this work the problem of fluid flow in deformable porous media is studied. First, the stationary fluid-structure interaction (FSI) problem is formulated in terms of incompressible Newtonian fluid and a linearized elastic solid. The flow is assumed to be characterized by very low Reynolds number and is described by the Stokes equations. The strains in the solid are small allowing for the solid to be described by the Lame equations, but no restrictions are applied on the magnitude of the displacements leading to strongly coupled, nonlinear fluid-structure problem. The FSI problem is then solved numerically by an iterative procedure which solves sequentially fluid and solid subproblems. Each of the two subproblems is discretized by finite elements and the fluid-structure coupling is reduced to an interface boundary condition. Several numerical examples are presented and the results from the numerical computations are used to perform permeability computations for different geometries.
Keywords: fluid-structure interaction, deformable porous media, upscaling, linear elasticity, stokes, finite elements
(28 pages, 2004)

66. F. Gaspar, O. Iliev, F. Lisbona, A. Naumovich, P. Vabishchevich

On numerical solution of 1-D poroelasticity equations in a multilayered domain

Finite volume discretization of Biot system of poroelasticity in a multilayered domain is presented. Staggered grid is used in order to avoid nonphysical oscillations of the numerical solution, appearing when a collocated grid is used. Various numerical experiments are presented in order to illustrate the accuracy of the finite difference scheme. In the first group of experiments, problems having analytical solutions are solved, and the order of convergence for the velocity, the pressure, the displacements, and the stresses is analyzed. In the second group of experiments numerical solution of real problems is presented.
Keywords: poroelasticity, multilayered material, finite volume discretization, MAC type grid
(41 pages, 2004)

67. J. Ohser, K. Schladitz, K. Koch, M. Nöthe

Diffraction by image processing and its application in materials science

A spectral theory for constituents of macroscopically homogeneous random microstructures modeled as homogeneous random closed sets is developed and provided with a sound mathematical basis, where the spectrum obtained by Fourier methods corresponds to the angular intensity distribution of x-rays scattered by this constituent. It is shown that the fast Fourier transform applied to three-dimensional images of microstructures obtained by micro-tomography is a powerful tool of image processing. The applicability of this technique is is demonstrated in the analysis of images of porous media.
Keywords: porous microstructure, image analysis, random set, fast Fourier transform, power spectrum, Bartlett spectrum
(13 pages, 2004)

68. H. Neunzert

Mathematics as a Technology: Challenges for the next 10 Years

No doubt: Mathematics has become a technology in its own right, maybe even a key technology. Technology may be defined as the application of science to the problems of commerce and industry. And science? Science maybe defined as developing, testing and improving models for the prediction of system behavior; the language used to describe these models is mathematics and mathematics provides methods to evaluate these models. Here we are! Why has mathematics become a technology only recently? Since it got a tool, a tool to evaluate complex, “near to reality” models: Computer! The model may be quite old – Navier-Stokes equations describe flow behavior rather well, but to solve these equations for realistic geometry and higher Reynolds numbers with sufficient precision is even for powerful parallel computing a real challenge. Make the models as simple as possible, as complex as necessary – and then evaluate them with the help of efficient and reliable algorithms: These are genuine mathematical tasks.
Keywords: applied mathematics, technology, modelling, simulation, visualization, optimization, glass processing, spinning processes, fiber-fluid interaction, trubulence effects, topological optimization, multicriteria optimization, Uncertainty and Risk, financial mathematics, Malliavin calculus, Monte-Carlo methods, virtual material design, filtration, bio-informatics, system biology
(29 pages, 2004)

69. R. Ewing, O. Iliev, R. Lazarov, A. Naumovich

On convergence of certain finite difference discretizations for 1D poroelasticity interface problems

Finite difference discretizations of 1D poroelasticity equations with discontinuous coefficients are analyzed. A recently suggested FD discretization of poroelasticity equations with constant coefficients on staggered grid, [5], is used as a basis. A careful treatment of the interfaces leads to harmonic averaging of the discontinuous coefficients. Here, convergence for the pressure and for the displacement is proven in certain norms for the scheme with harmonic averaging (HA). Order of convergence 1.5 is proven for arbitrary located interface, and second order convergence is proven for the case when the interface coincides with a grid node. Furthermore, following the ideas from [3], modified HA discretization are suggested for particular cases. The velocity and the stress are approximated with second order on the interface in this case. It is shown that for wide class of problems, the modified discretization provides better accuracy. Second order convergence for modified scheme is proven for the case when the interface coincides with a displacement grid node. Numerical experiments are presented in order to illustrate our considerations.
Keywords: poroelasticity, multilayered material, finite volume discretizations, MAC type grid, error estimates
(26 pages,2004)

70. W. Dörfler, O. Iliev, D. Stoyanov, D. Vassileva

On Efficient Simulation of Non-Newtonian Flow in Saturated Porous Media with a Multigrid Adaptive Refinement Solver

Flow of non-Newtonian in saturated porous media can be described by the continuity equation and the generalized Darcy law. Efficient solution of the resulting second order nonlinear elliptic equation is discussed here. The equation is discretized by a finite volume method on a cell-centered grid. Local adaptive refinement of the grid is introduced in order to reduce the number of unknowns. A special implementation approach is used, which allows us to perform unstructured local refinement in conjunction with the finite volume discretization. Two residual based error indicators are exploited in the adaptive refinement criterion. Second order accurate discretization on the interfaces between refined and non-refined subdomains, as well as on the boundaries with Dirichlet boundary condition, are presented here, as an essential part of the accurate and efficient algorithm. A nonlinear full approximation storage multigrid algorithm is developed especially for the above described composite (coarse plus locally refined) grid approach. In particular, second order approximation around interfaces is a result of a quadratic approximation of slave nodes in the multigrid - adaptive refinement (MG-AR) algorithm. Results from numerical solution of various academic and practice-induced problems are presented and the performance of the solver is discussed.
Keywords: Nonlinear multigrid, adaptive renement, non-Newtonian in porous media
(25 pages, 2004)

71. J. Kalcsics, S. Nickel, M. Schröder

Towards a Unified Territory Design Approach – Applications, Algorithms and GIS Integration

Territory design may be viewed as the problem of grouping small geographic areas into larger geographic clusters called territories in such a way that the latter are acceptable according to relevant planning criteria. In this paper we review the existing literature for applications of territory design problems and solution approaches for solving these types of problems. After identifying features common to all applications we introduce a basic territory design model and present in detail two approaches for solving this model: a classical location–allocation approach combined with optimal split resolution techniques and a newly developed computational geometry based method. We present computational results indicating the efficiency and suitability of the latter method for solving large–scale practical problems in an interactive environment. Furthermore, we discuss extensions to the basic model and its integration into Geographic Information Systems.
Keywords: territory desgin, political districting, sales territory alignment, optimization algorithms, Geographical Information Systems
(40 pages, 2005)

72. K. Schladitz, S. Peters, D. Reinel-Bitzer, A. Wiegmann, J. Ohser

Design of acoustic trim based on geometric modeling and flow simulation for non-woven

In order to optimize the acoustic properties of a stacked fiber non-woven, the microstructure of the non-woven is modeled by a macroscopically homogeneous random system of straight cylinders (tubes). That is, the fibers are modeled by a spatially stationary random system of lines (Poisson line process), dilated by a sphere. Pressing the non-woven causes anisotropy. In our model, this anisotropy is described by a one parametric distribution of the direction of the fibers. In the present application, the anisotropy parameter has to be estimated from 2d reflected light microscopic images of microsections of the non-woven.
After fitting the model, the flow is computed in digitized realizations of the stochastic geometric model using the lattice Boltzmann method. Based on the flow resistivity, the formulas of Delany and Bazley predict the frequency-dependent acoustic absorption of the non-woven in the impedance tube.
Using the geometric model, the description of a non-woven with improved acoustic absorption properties is obtained in the following way: First, the fiber thicknesses, porosity and anisotropy of the fiber system are modified. Then the flow and acoustics simulations are performed in the new sample. These two steps are repeatedc for various sets of parameters. Finally, the set of parameters for the geometric model leading to the best acoustic absorption is chosen.
Keywords: random system of fibers, Poisson line process, flow resistivity, acoustic absorption, Lattice-Boltzmann method, non-woven
(21 pages, 2005)

73. V. Rutka, A. Wiegmann

Explicit Jump Immersed Interface Method for virtual material design of the effective elastic moduli of composite materials

Virtual material design is the microscopic variation of materials in the computer, followed by the numerical evaluation of the effect of this variation on the material's macroscopic properties. The goal of this procedure is an in some sense improved material. Here, we give examples regarding the dependence of the effective elastic moduli of a composite material on the geometry of the shape of an inclusion. A new approach on how to solve such interface problems avoids mesh generation and gives second order accurate results even in the vicinity of the interface.
The Explicit Jump Immersed Interface Method is a finite difference method for elliptic partial differential equations that works on an equidistant Cartesian grid in spite of non-grid aligned discontinuities in equation parameters and solution. Near discontinuities, the standard finite difference approximations are modified by adding correction terms that involve jumps in the function and its derivatives. This work derives the correction terms for two dimensional linear elasticity with piecewise constant coefficients, i. e. for composite materials. It demonstrates numerically convergence and approximation properties of the method.
Keywords: virtual material design, explicit jump immersed interface method, effective elastic moduli, composite materials
(22 pages, 2005)

74. T. Hanne

Eine Übersicht zum Scheduling von Baustellen

Im diesem Dokument werden Aspekte der formalen zeitlichen Planung bzw. des Scheduling für Bauprojekte anhand ausgewählter Literatur diskutiert. Auf allgemeine Aspekte des Scheduling soll dabei nicht eingegangen werden. Hierzu seien als Standard-Referenzen nur Brucker (2004) und Pinedo (1995) genannt. Zu allgemeinen Fragen des Projekt-Managements sei auf Kerzner (2003) verwiesen.
Im Abschnitt 1 werden einige Anforderungen und Besonderheiten der Planung von Baustellen diskutiert. Diese treten allerdings auch in zahlreichen anderen Bereichen der Produktionsplanung und des Projektmanagements auf. In Abschnitt 2 werden dann Aspekte zur Formalisierung von Scheduling-Problemen in der Bauwirtschaft diskutiert, insbesondere Ziele und zu berücksichtigende Restriktionen. Auf eine mathematische Formalisierung wird dabei allerdings verzichtet. Abschnitt 3 bietet eine Übersicht über Verfahren und grundlegende Techniken für die Berechnung von Schedules. In Abschnitt 4 wird ein Überblick über vorhandene Software, zum einen verbreitete Internationale Software, zum anderen deutschsprachige Branchenlösungen, gegeben. Anschließend werden Schlussfolgerungen gezogen und es erfolgt eine Auflistung der Literaturquellen.
Keywords: Projektplanung, Scheduling, Bauplanung, Bauindustrie
(32 pages, 2005)

75. J. Linn

The Folgar–Tucker Model as a Differential Algebraic System for Fiber Orientation Calculation

The Folgar–Tucker equation (FTE) is the model most frequently used for the prediction of fiber orientation (FO) in simulations of the injection molding process for short–fiber reinforced thermoplasts. In contrast to its widespread use in injection molding simulations, little is known about the mathematical properties of the FTE: an investigation of e. g. its phase space M_{FT} has been presented only recently [12]. The restriction of the dependent variable of the FTE to the set M_{FT} turns the FTE into a differential algebraic system (DAS), a fact which is commonly neglected when devising numerical schemes for the integration of the FTE. In this article we present some recent results on the problem of trace stability as well as some introductory material which complements our recent paper [12].
Keywords: fiber orientation, Folgar–Tucker model, invariants, algebraic constraints, phase space, trace stability
(15 pages, 2005)

76. M. Speckert, K. Dreßler, H. Mauch, A. Lion, G. J. Wierda

Simulation eines neuartigen Prüfsystems für Achserprobungen durch MKS-Modellierung einschließlich Regelung

Testing new suspensions based on real load data is performed on elaborate multi channel test rigs. Usually, wheel forces and moments measured during driving maneuvers are reproduced by the test rig. Because of the complicated interaction between test rig and suspension each new rig configuration has to prove its efficiency with respect to the requirements and the configuration might be subject to optimization.
This paper deals with mathematical and physical modeling of a new concept of a test rig which is based on two hexapods. The model contains the geometric configuration as well as the hydraulics and the controller. It is implemented as an ADAMS/Car template and can be combined with different suspension models to get a complete assembly representing the entire test rig. Using this model, all steps required for a real test run such as controller adaptation, drive file iteration and simulation can be performed. Geometric or hydraulic parameters can be modified easily to improve the setup and adapt the system to the suspension and the given load data.
The model supports and accompanies the introduction of the new rig concept and can be used to prepare real tests on a virtual basis. Using both a front and a rear suspension the approach is described and the potentials coming with the simulation are pointed out.
Keywords: virtual test rig, suspension testing, multibody simulation, modeling hexapod test rig, optimization of test rig configuration
(20 pages, 2005)
In deutscher Sprache; bereits erschienen in: VDI-Berichte Nr. 1900, VDI-Verlag GmbH Düsseldorf (2005), Seiten 227-246

77. K.-H. Küfer, M. Monz, A. Scherrer, P. Süss, F. Alonso, A. S. A. Sultan, Th. Bortfeld, D. Craft, Chr. Thieke

Multicriteria optimization in intensity modulated radiotherapy planning

Inverse treatment planning of intensity modulated radiothrapy is a multicriteria optimization problem: planners have to find optimal compromises between a sufficiently highdose intumor tissuethat garantuee a high tumor control, and, dangerous overdosing of critical structures, in order to avoid high normal tissue complcication problems.
The approach presented in this work demonstrates how to state a flexible generic multicriteria model of the IMRT planning problem and how to produce clinically highly relevant Pareto-solutions. The model is imbedded in a principal concept of Reverse Engineering, a general optimization paradigm for design problems. Relevant parts of the Pareto-set are approximated by using extreme compromises as cornerstone solutions, a concept that is always feasible if box constraints for objective funtions are available. A major practical drawback of generic multicriteria concepts trying to compute or approximate parts of the Pareto-set is the high computational effort. This problem can be overcome by exploitation of an inherent asymmetry of the IMRT planning problem and an adaptive approximation scheme for optimal solutions based on an adaptive clustering preprocessing technique. Finally, a coherent approach for calculating and selecting solutions in a real-timeinteractive decision-making process is presented. The paper is concluded with clinical examples and a discussion of ongoing research topics.
Keywords: multicriteria optimization, extreme solutions, real-time decision making, adaptive approximation schemes, clustering methods, IMRT planning, reverse engineering
(51 pages, 2005)

78. S. Amstutz, H. Andrä

A new algorithm for topology optimization using a level-set method

The levelset method has been recently introduced in the field of shape optimization, enabling a smooth representation of the boundaries on a fixed mesh and therefore leading to fast numerical algorithms. However, most of these algorithms use a HamiltonJacobi equation to connect the evolution of the levelset function with the deformation of the contours, and consequently they cannot create any new holes in the domain (at least in 2D). In this work, we propose an evolution equation for the levelset function based on a generalization of the concept of topological gradient. This results in a new algorithm allowing for all kinds of topology changes.
Keywords: shape optimization, topology optimization, topological sensitivity, level-set
(22 pages, 2005)

79. N. Ettrich

Generation of surface elevation models for urban drainage simulation

Traditional methods fail for the purpose of simulating the complete flow process in urban areas as a consequence of heavy rainfall and as required by the European Standard EN-752 since the bi-directional coupling between sewer and surface is not properly handled. The methodology, developed in the BMBF/EUREKA-project RisUrSim, solves this problem by carrying out the runoff on the basis of shallow water equations solved on high-resolution surface grids. Exchange nodes between the sewer and the surface, like inlets and manholes, are located in the computational grid and water leaving the sewer in case of surcharge is further distributed on the surface.
So far, it has been a problem to get the dense topographical information needed to build models suitable for hydrodynamic runoff calculation in urban areas. Recent airborne data collection methods like laser scanning, however, offer a great chance to economically gather densely sampled input data. This paper studies the potential of such laser-scan data sets for urban water hydrodynamics.
Keywords: Flooding, simulation, urban elevation models, laser scanning
(22 pages, 2005)

80. H. Andrä, J. Linn, I. Matei, I. Shklyar, K. Steiner, E. Teichmann

OPTCAST – Entwicklung adäquater Strukturoptimierungsverfahren für Gießereien Technischer Bericht (KURZFASSUNG)

Im vorliegenden Bericht werden die Erfahrungen und Ergebnisse aus dem Projekt OptCast zusammengestellt. Das Ziel dieses Projekts bestand (a) in der Anpassung der Methodik der automatischen Strukturoptimierung für Gussteile und (b) in der Entwicklung und Bereitstellung von gießereispezifischen Optimierungstools für Gießereien und Ingenieurbüros.
Gießtechnische Restriktionen lassen sich nicht auf geometrische Restriktionen reduzieren, sondern sind nur über eine Gießsimulation (Erstarrungssimulation und Eigenspannungsanalyse) adäquat erfassbar, da die lokalen Materialeigenschaften des Gussteils nicht nur von der geometrischen Form des Teils, sondern auch vom verwendeten Material abhängen. Wegen dieser Erkenntnis wurde ein neuartiges iteratives Topologieoptimierungsverfahren unter Verwendung der Level-Set-Technik entwickelt, bei dem keine variable Dichte des Materials eingeführt wird. In jeder Iteration wird ein scharfer Rand des Bauteils berechnet. Somit ist die Gießsimulation in den iterativen Optimierungsprozess integrierbar.
Der Bericht ist wie folgt aufgebaut: In Abschnitt 2 wird der Anforderungskatalog erläutert, der sich aus der Bearbeitung von Benchmark-Problemen in der ersten Projektphase ergab. In Abschnitt 3 werden die Benchmark-Probleme und deren Lösung mit den im Projekt entwickelten Tools beschrieben. Abschnitt 4 enthält die Beschreibung der neu entwickelten Schnittstellen und die mathematische Formulierung des Topologieoptimierungsproblems. Im letzten Abschnitt wird das neue Topologieoptimierungsverfahren, das die Simulation des Gießprozesses einschließt, erläutert.
Keywords: Topologieoptimierung, Level-Set-Methode, Gießprozesssimulation, Gießtechnische Restriktionen, CAE-Kette zur Strukturoptimierung
(77 pages, 2005)

81. N. Marheineke, R. Wegener

Fiber Dynamics in Turbulent Flows Part I: General Modeling Framework

The paper at hand deals with the modeling of turbulence effects on the dynamics of a long slender elastic fiber. Independent of the choice of the drag model, a general aerodynamic force concept is derived on the basis of the velocity field for the randomly fluctuating component of the flow. Its construction as centered differentiable Gaussian field complies thereby with the requirements of the stochastic k-ε turbulence model and Kolmogorov's universal equilibrium theory on local isotropy.

Keywords: fiber-fluid interaction; Cosserat rod; turbulence modeling; Kolmogorov's energy spectrum; double-velocity correlations; differentiable Gaussian fields

Part II: Specific Taylor Drag

In [12], an aerodynamic force concept for a general air drag model is derived on top of a stochastic k-ε description for a turbulent flow field. The turbulence effects on the dynamics of a long slender elastic fiber are particularly modeled by a correlated random Gaussian force and in its asymptotic limit on a macroscopic fiber scale by Gaussian white noise with flow-dependent amplitude. The paper at hand now presents quantitative similarity estimates and numerical comparisons for the concrete choice of a Taylor drag model in a given application.

Keywords: flexible fibers; k-ε turbulence model; fiber-turbulence interaction scales; air drag; random Gaussian aerodynamic force; white noise; stochastic differential equations; ARMA process

(38 pages, 2005)

82. C. H. Lampert, O. Wirjadi

An Optimal Non-Orthogonal Separation of the Anisotropic Gaussian Convolution Filter

We give ananalytical and geometrical treatment of what it means to separate a Gaussian kernel along arbitrary axes in R^n, and we present a separation scheme that allows to efficiently implement anisotropic Gaussian convolution filters in arbitrary dimension. Based on our previous analysis we show that this scheme is optimal with regard to the number of memory accesses and interpolation operations needed.

Our method relies on non-orthogonal convolution axes and works completely in image space. Thus, it avoids the need for an FFT-subroutine. Depending on the accuracy and speed requirements, different interpolation schemes and methods to implement the one-dimensional Gaussian (FIR, IIR) can be integrated. The algorithm is also feasible for hardware that does not contain a floating-point unit.

Special emphasis is laid on analyzing the performance and accuracy of our method. In particular, we show that without anyspecial optimization of the source code, our method can perform anisotropic Gaussian filtering faster than methods relying on the Fast Fourier Transform.

Keywords: Anisotropic Gaussian filter, linear filtering, orientation space, nD image processing, separable filters

(25 pages, 2005)

83. H. Andrä, D. Stoyanov

Error indicators in the parallel finite element solver for linear elasticity DDFEM

This report discusses two approaches for a posteriori error indication in the linear elasticity solver DDFEM: An indicator based on the Richardson extrapolation and Zienkiewicz-Zhu-type indicator.

The solver handles 3D linear elasticity steady-state problems. It uses own input language to de-scribe the mesh and the boundary conditions. Finite element discretization over tetrahedral meshes with first or second order shape functions (hierarchical basis) has been used to resolve the model. The parallelization of the numerical method is based on the domain decomposition approach. DDFEM is highly portable over a set of parallel computer architectures supporting the MPI-standard.

Keywords: linear elasticity, finite element method, hierarchical shape functions, domain decom-position, parallel implementation, a posteriori error estimates

(21 pages, 2006)

84. M. Schröder, I. Solchenbach

Optimization of Transfer Quality in Regional Public Transit

In this paper we address the improvement of transfer quality in public mass transit networks. Generally there are several transit operators offering service and our work is motivated by the question how their timetables can be altered to yield optimized transfer possibilities in the over-all network. To achieve this, only small changes to the timetables are allowed.

The set-up makes it possible to use a quadratic semi-assignment model to solve the optimization problem. We apply this model, equipped with a new way to assess transfer quality, to the solu-tion of four real-world examples. It turns out that improvements in overall transfer quality can be determined by such optimization-based techniques. Therefore they can serve as a first step to-wards a decision support tool for planners of regional transit networks.

Keywords: public transit, transfer quality, quadratic assignment problem

(16 pages, 2006)

85. A. Naumovich, F. J. Gaspar

On a multigrid solver for the three-dimensional Biot poroelasticity system in multilayered domains

In this paper, we present problem-dependent prolongation and problem-dependent restriction for a multigrid solver for the three-dimensional Biot poroelasticity system, which is solved in a multilayered domain. The system is discretized on a staggered grid using the finite volume method. During the discretization, special care is taken of the discontinuous coefficients. For the efficient multigrid solver, a need in operator-dependent restriction and/or prolongation arises. We derive these operators so that they are consistent with the discretization. They account for the discontinuities of the coefficients, as well as for the coupling of the unknowns within the Biot system. A set of numerical experiments shows necessity of use of the operator-dependent restriction and prolongation in the multigrid solver for the considered class of problems.

Keywords: poroelasticity, interface problem, multigrid, operator-dependent prolongation

(11 pages, 2006)

86. S. Panda, R. Wegener, N. Marheineke

Slender Body Theory for the Dynamics of Curved Viscous Fibers

The paper at hand presents a slender body theory for the dynamics of a curved inertial viscous Newtonian fiber. Neglecting surface tension and temperature dependence, the fiber flow is modeled as a three-dimensional free boundary value problem via instationary incompressible Navier-Stokes equations. From regular asymptotic expansions in powers of the slenderness parameter leading-order balance laws for mass (cross-section) and momentum are derived that combine the unrestricted motion of the fiber center-line with the inner viscous transport. The physically reasonable form of the one-dimensional fiber model results thereby from the introduction of the intrinsic velocity that characterizes the convective terms.

Keywords: curved viscous fibers; fluid dynamics; Navier-Stokes equations; free boundary value problem; asymptotic expansions; slender body theory

(14 pages, 2006)

87. E. Ivanov, H. Andrä, A. Kudryavtsev

Domain Decomposition Approach for Automatic Parallel Generation of Tetrahedral Grids

The desire to simulate more and more geometrical and physical features of technical structures and the availability of parallel computers and parallel numerical solvers which can exploit the power of these machines have lead to a steady increase in the number of grid elements used. Memory requirements and computational time are too large for usual serial PCs. An a priori partitioning algorithm for the parallel generation of 3D nonoverlapping compatible unstructured meshes based on a CAD surface description is presented in this paper. Emphasis is given to practical issues and implementation rather than to theoretical complexity. To achieve robustness of the algorithm with respect to the geometrical shape of the structure authors propose to have several or many but relatively simple algorithmic steps. The geometrical domain decomposition approach has been applied. It allows us to use classic 2D and 3D high-quality Delaunay mesh generators for independent and simultaneous volume meshing. Different aspects of load balancing methods are also explored in the paper. The MPI library and SPMD model are used for parallel grid generator implementation. Several 3D examples are shown.

Key words: Grid Generation, Unstructured Grid, Delaunay Triangulation, Parallel Programming, Domain Decomposition, Load Balancing

(18 pages, 2006)

88. S. Tiwari, S. Antonov, D. Hietel, J. Kuhnert, R. Wegener

A Meshfree Method for Simulations of Interactions between Fluids and Flexible Structures

We present the application of a meshfree method for simulations of interaction between fluids and flexible structures. As a flexible structure we consider a sheet of paper. In a twodimensional framework this sheet can be modeled as curve by the dynamical Kirchhoff-Love theory. The external forces taken into account are gravitation and the pressure difference between upper and lower surface of the sheet. This pressure difference is computed using the Finite Pointset Method (FPM) for the incompressible Navier-Stokes equations. FPM is a meshfree, Lagrangian particle method. The dynamics of the sheet are computed by a finite difference method. We show the suitability of the meshfree method for simulations of fluidstructure interaction in several applications

Key words: Meshfree Method, FPM, Fluid Structure Interaction, Sheet of Paper, Dynamical Coupling

(16 pages, 2006)

89. R. Ciegis , O. Iliev, V. Starikovicius, K. Steiner

Numerical Algorithms for Solving Problems of Multiphase Flows in Porous Media

In this paper we discuss numerical algorithms for solving the system of nonlinear PDEs, arising in modelling of two-phase ows in porous media, as well as the proper object oriented implementation of these algorithms. Global pressure model for isothermal two-phase immiscible flow in porous media is considered in this paper. Finite-volume method is used for the space discretization of the system of PDEs. Different time stepping discretizations and linearization approaches are discussed. The main concepts of the PDE software tool MfsolverC++ are given. Numerical results for one realistic problem are presented.

Keywords: nonlinear algorithms, finite-volume method, software tools, porous media, flows

(16 pages, 2006)

90. D. Niedziela, O. Iliev, A. Latz

On 3D Numerical Simulations of Viscoelastic Fluids

In this work we present and solve a class of non–Newtonian viscoelastic fluid flow problems. Models for non-Newtonian fluids can be classified into three large groups depending on the type of the constitutive relation used: algebraic, differential and integral. The first type of models are most simple one, the last are the most physically adequate ones. Here we consider some models from the first and the third groups, and present robust and efficient algorithms for their solution. We present new mathematical model, which belongs to the class of generalized Newtonian models and is able to account for the anisotropy of the viscosity tensor observed in many real liquids. In particular, we discuss a unified model that captures both shear thinning and extensional thickening for complex flows. The resulting large variations of the viscosity tensor in space and time are leading to a strong numerical coupling of the momentum equations due to the appearance of mixed derivatives in the discretization. To treat this strong coupling appropriately, we present two modifications of classical projection methods (like e.g. SIMPLE). In the first modification all momentum equations are solved coupled (i.e. mixed derivative are discretized implicitly) but still iterations are performed between the momen-

tum equations and the continuity equation. The second one is a fully coupled method, where momentum and continuity equation are solved together using a proper preconditioner. The models involving integral constitutive relation which accounts for the history of deformations, result in a system of integro-differential equations. To solve it, we suggest a proper splitting scheme, which treats the integral and the differential parts consecutively. Integral Oldroyd B and Doi Edwards models are used to simulate flows of dilute and concentrated polymer solutions, respectively.
Keywords: non–Newtonian fluids, anisotropic viscosity, integral constitutive equation
(18 pages, 2006)

91. A. Winterfeld

Application of general semi-infinite Programming to Lapidary Cutting Problems

We consider a volume maximization problemarising in gemstone cutting industry. The problem is formulated as a general semi-infnite program(GSIP)and solved using an interior point method developed by Stein. It is shown, that the convexity assumption needed for the convergence of the algorithm can be satisfied by appropriate model-ling. Clustering techniques are used to reduce the number of container constraints, which is necessary to make the subproblems practically tractable. An iterative process consisting of GSIP optimization and adaptive refinement steps is then employed to obtain an optimal solution which is also feasible for the original problem. Some numeri-cal results based on real world data are also presented.
Keywords: large scale optimization, nonlinear programming, general semi-infinite optimization, design centering, clustering
(26 pages, 2006)

92. J. Orlik, A. Ostrovska

Space-Time Finite Element Approximation and Numerical Solution of Hereditary Linear Viscoelasticity Problems

In this paper we suggest a fast numerical approach to treat problems of the hereditary linear viscoelasticity, which results in the system of elliptic partial differential equations in space variables, who's coefficients are Volterra integral operators of the second kind in time. We propose to approximate the relaxation kernels by the product of purely time- and space-dependent terms, which is achieved by their piecewisepolynomial space-interpolation. A priori error estimate was obtained and it was shown, that such approximation does not decrease the convergence order, when an interpolation polynomial is chosen of the same order as the shape functions for the spatial finite element approximation, while the computational effort is significantly reduced.
Keywords: hereditary viscoelasticity; kern approximation by interpolation; space-time finite element approximation, stability and a priori estimate
(24 pages, 2006)

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