Using a Heat Transfer Analogy to Solve for Squeeze Film Damping and Stiffness Coefficients in MEMS Structures

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January 23, 2003

MEMS structures undergoing transverse motion to a fixed wall exhibit damping effects that must be considered in a dynamics simulation. Existing heat transfer finite elements can be used to compute equivalent damping coefficients and squeeze stiffness coefficients. These coefficients can be used as input for lumped-parameter damping and spring stiffness elements in a dynamics model. This paper reviews the theory and application of heat transfer elements for simulating squeeze-film damping effects.

The paper applies this field analogy to the calculation of damping and squeeze stiffness coefficients to rigid structures moving normal to a fixed wall (electrode). This approach is valid for small deflections of the structure. For large deflections, this method will break down because the assumption of rigid movement is invalid. In this case, the method should be applied using modal basis functions similar to the approach used for electrostatic-structural coupling in the ROM144 element. This technique is currently being developed at ANSYS for damping characterization and will be released in the near future.

Theoretical Background

Reynolds equation known from lubrication technology and theory of rarified gas physics are the theoretical background to analyze fluid structural interactions of microstructures [1], [2], [3]. This happens, for example, in the case of accelerometers where the seismic mass moves perpendicular to a fixed wall, in the case of micromirror displays where the mirror plate tilts around a horizontal axis, and for clamped beams in the case of RF filters where a structure moves with respect to a fixed wall. Other examples are published in literature [4].

Reynolds squeeze film equations are restricted to structures with lateral dimensions much larger than the gap separation. Furthermore the pressure change p must be small compared to ambient pressure p_0 and viscose friction may not cause a significant temperature change.

PLANE55 and PLANE77 thermal elements be used in an analogous way to determine the fluidic response for given wall velocities $\dot{u}_z = v_z$. Both elements allow for static, harmonic and transient types of analyses. Static analyses can be used to compute damping parameter for low driving frequencies (compression effects are neglected), harmonic response analysis covers damping and squeeze effects at the operating point and transient analysis holds for non-harmonic load functions.

Reynolds squeeze film equation is often written in the following form [1], [5]

Eq. 1
$$\frac{p_0\, d^2}{12\,\eta}\left[\frac{\partial^2}{\partial\, x^2}\left(\frac{p}{p_0}\right)+\frac{\partial^2}{\partial\, y^2}\left(\frac{p}{p_0}\right)\right]=\frac{\partial}{\partial\, t}\left(\frac{u_z}{d}\right)+\frac{\partial}{\partial\, t}\left(\frac{p}{p_0}\right)$$

which is equivalent to

Eq. 2
$$\frac{d^3}{12\,\eta}\left(\frac{\partial^2\ p}{\partial\, x^2}+\frac{\partial^2\ p}{\partial\, y^2}\right)=\frac{d}{p_0}\,\frac{\partial\ p}{\partial\, t}+v_z$$

whereby η is the dynamic viscosity, *d* the local gap separation and v_z the wall velocity in normal direction.

Obviously one can recognize the analogy relationship to the 2-D heat flow equation implemented for thermal solids PLANE55 and PLANE77:

Eq. 3
$$\lambda\left(\frac{\partial^2\ T}{\partial\, x^2}+\frac{\partial^2\ T}{\partial\, y^2}\right)=c\rho\,\frac{\partial\, T}{\partial\, t}-Q$$

Both elements can be used to solve the Reynolds squeeze film equation by making use of appropriate substitution of material properties such that thermal conductivity, specific heat and density are defined as:

Eq. 4
$$\lambda=\frac{d^3}{12\,\eta}\qquad c=\frac{d}{p_0}\qquad \rho=1$$

The degree of freedom TEMP therefore is analogous to pressure. Likewise, a negative heat generation rate is analogous to velocity (out of plane).

Following an analysis, the pressure can be integrated over the surface of the structure to obtain the force. The force can be divided by the applied velocity to obtain the damping coefficient. For a time-harmonic analysis, the damping coefficient is obtained by dividing the Real component of the force by the applied normal velocity. The squeeze stiffness is obtained by dividing the Imaginary component of the force by the displacement;

Eq. 5
$$C=\frac{F^{\mathrm{Re}}}{v_z}\qquad K=\frac{F^{\mathrm{Im}}\omega}{v_z}$$

Whereby F^{Re} is the Real component of the force on the structure, F^{Im} is the Imaginary component of the force, and ω is the frequency (rad/sec).

Reynolds squeeze film equation assumes a continuous fluid flow regime. This happens if the gap thickness *d* (i.e. characteristic length) is more than one hundred times larger than the mean free path of the fluid particles L_m. The mean free path of a gas is inversely proportional to pressure and is given by

Eq. 6
$$L_m(p_0)=\frac{L_0\ P_0}{p_0}$$

where L_0 is the mean free path at pressure P_0. Assuming P_0 to be 1 atm which is equal to $1.01325 \cdot 10^5$ Pa the number for L_0 is about 64 nm. Consequently continuum theory can be applied for squeeze film problems with gap separation more than 6.4 µm without modification but this number decreases strongly for evacuated systems. A general measure of the acceptance of the continuous flow assumption with respect to pressure and gap separation is the Knudsen number *Kn*

Eq. 7

$$Kn = \frac{L_m}{d} = \frac{L_0\ P_0}{p_0\ d}$$

which should be smaller than 0.01 when using continuum theory.

Fluid flow behavior at higher Knudsen numbers degenerates and requires special treatments such as considering slip flow boundary conditions at the wall interface or models derived from the Boltzmann equation. A convenient way to adjust the results of continuous flow to the results of transition or molecular regime is based on a modification of the dynamic viscosity *VISC*. Veijola [6] and other authors introduced a fit function of the effective viscosity η_{eff}

Eq. 8

$$\eta_{eff} = \frac{\eta}{1 + 9.638\ Kn^{1.159}} = \frac{\eta}{1 + 9.638 \left(\frac{L_0 P_0}{p_0\ d} \right)^{1.159}}$$

which is valid for Knudsen numbers between 0 and 880 with relative error less than ±5%.

Example Problem: Rectangular plate with transverse plate motion

According to Blech [1] an analytical solution for the damping and squeeze coefficient for a rigid plate moving with a transverse motion is given by:

Eq. 9

$$C(\Omega) = \frac{64\ \sigma(\Omega)\ p_0\ A}{\pi^6\ d\ \Omega} \sum_{m=odd} \sum_{n=odd} \frac{m^2 + n^2\ c^2}{(m\ n)^2 \left[\left(m^2 + n^2\ c^2 \right)^2 + \frac{\sigma(\Omega)^2}{\pi^4} \right]}$$

Eq. 10

$$K_S(\Omega) = \frac{64\ \sigma(\Omega)^2\ p_0\ A}{\pi^8\ d} \sum_{m=odd} \sum_{n=odd} \frac{1}{(m\ n)^2 \left[\left(m^2 + n^2\ c^2 \right)^2 + \frac{\sigma(\Omega)^2}{\pi^4} \right]}$$

where $C(\Omega)$ is the frequency dependent damping coefficient, $K_S(\Omega)$ is the squeeze stiffness coefficient, p_0 the ambient pressure, A the surface area, c the ratio of plate length

a divided by plate width b, d the film thickness, Ω the response frequency and σ the squeeze number of the system. The squeeze number is given by

Eq. 11
$$\sigma(\Omega) = \frac{12\,\eta_{eff}\;a^2}{p_0\;d^2}\Omega$$

for rectangular plates where η_{eff} is the effective viscosity.

For the example problem, the following input quantities are used ;

$a = 0.001;\; b = 0.002;\; d = 5\,10^{-6};\; p_0 = 10^5;\; \eta_{eff} = 18.3\,10^{-6}$ (SI units)

Table 1: Output quantities: Mesh density 40x40 equally spaced

Frequency f	Damping coefficient		Squeeze stiffness coefficient	
	PLANE55	Analytical	PLANE55	Analytical
1 Hz	0.2007	0.2008	$53.72\ 10^{-6}$	$53.66\ 10^{-6}$
20 kHz	0.1164	0.1167	$12.07\ 10^{3}$	$12.07\ 10^{3}$
50 kHz	0.0408	0.0411	$23.56\ 10^{3}$	$23.61\ 10^{3}$
100 kHz	$15.14\ 10^{-3}$	$15.29\ 10^{-3}$	$28.58\ 10^{3}$	$28.65\ 10^{3}$
500 kHz	$1.441\ 10^{-3}$	$1.508\ 10^{-3}$	$34.70\ 10^{3}$	$34.89\ 10^{3}$
1000 kHz	$0.499\ 10^{-3}$	$0.545\ 10^{-3}$	$36.09\ 10^{3}$	$36.39\ 10^{3}$

Note that the deviations can be lowered in case of a better discretization. On the other hand the analytical solution is based on a series representation which approximates the exact solution too. So we expect small deviations in both, the analytical and numerical solution.

Figure 1 and Figure 2 show the pressure distribution of the squeeze film at the midsurface plane at low and high response frequencies. At high frequencies with respect to cut-off the viscous friction hinders the gas flow and compression effects become important. The pressure change is almost constant underneath the entire plate as known from piston engines.

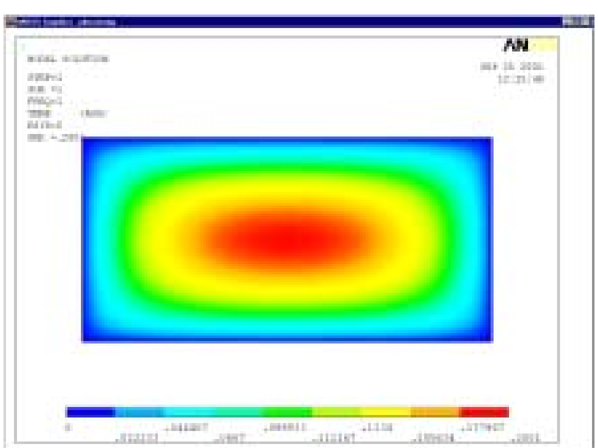

Figure 1: Pressure distribution at low frequencies

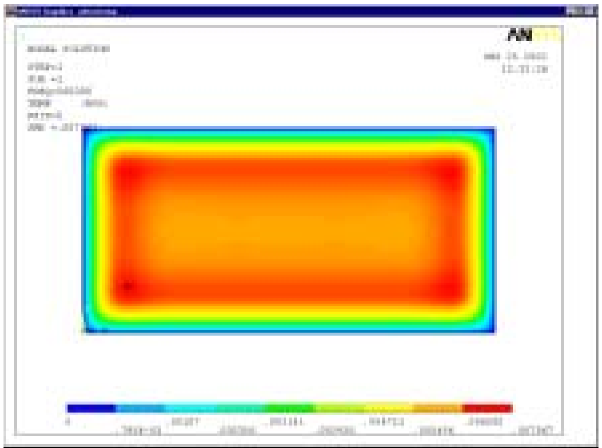
Figure 2: Pressure distribution at high frequencies

The input file for this example is shown below for the 100 kHz. Case:

```
/batch,list
/prep7
/title, Heat transfer analogy for squeeze film damping and stiffness
/com,   coefficient extraction
/com,   Ref: J.J. Blech: "On Isothermal Squeeze Films", J. Lubrication
/com,   Technology, Vol. 5 pp. 615-520, 1983
/com,
/com,   Using a thermal analogy, solve for the  equivalent damping
/com,   coefficient and squeeze stiffness
/com,       Temperature > Pressure
/com,       Velocity    > Heat generate rate
/com,       Heat flow  >  Fluid flow

et,1,55

a=.001              ! plate width (m)
b=.002              ! plate length (m)
d=5e-6              ! gap (m)
po=1e5              ! normnal pressure (N/m**2)
visc=18.3e-6        ! viscosity (kg/m/s)
velo=0.002          ! arbitrary uniform velocity (m/sec)
area=a*b            ! plate area
freq=100000         ! Operating frequency (Hz.)
omega=2*3.14159*freq

keqv=d**3/12/visc ! Equivalent thermal conductivity
ceqv=d/po           ! Equivalent specific heat
rhoeqv=1            ! Equivalent density

mp,kxx,1,keqv
mp,c,1,ceqv
mp,dens,1,rhoeqv

rectng,0,b,0,a
esize,,40
amesh,all
```

```
nsel,ext
d,all,temp,0            ! Set temperature (pressure) to zero
nsel,all

bfe,all,hgen,,-velo

finish
/solu
antyp,harm              ! Harmonic Thermal analysis
harfrq,freq
solve
finish
/post1
set,1,1
etable,presR,temp             ! extract "Real" pressure
etable,earea,volu
smult,forR,presR,earea        ! compute "Real" force
ssum
*get,Fre,ssum,,item,forR
set,1,1,,1
etable,presI,temp             ! extract "Imaginary" pressure
smult,forI,presI,earea        ! compute "Imaginary" pressure
ssum
*get,Fim,ssum,,item,forI

K=Fim*omega/velo              ! Compute equivalent stiffness
C=abs(Fre/velo)               ! Compute equivalent damping

/com, ** Equivalent stiffness: Analytic solution = 28650 **
*stat,K
/com, ** Equivalent damping: Analytic solution = .01529 **
*stat,C
finish
```

Considering Circular Perforated Holes in MEMS Structures

The heat transfer analogy may be extended in the case of circular holes in plates as is often the case in MEMS structures to reduce damping effects (see Figure 3). In general there are 3 different approaches to modeling the holes. First, one can neglect the fluid resistance across the hole. This can be done by applying zero pressure boundary conditions at the hole circumference. Such a model is correct for large hole diameters compared to the hole length. Second, one can assume a very high flow resistance, which happens in case of narrow and long holes. In this very special case one may not apply any pressure BC at the holes. No fluid flow can pass the hole. Third and most accurate is to consider the true hole resistance by using fluid link elements. A fluid link element can be modeled via thermal analogy using the LINK33 element.

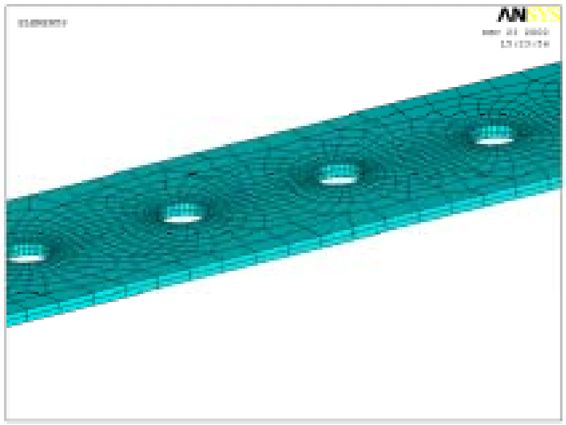

Figure 3: Perforation holes etched into a clamped beam to lower dissipative effects

When modeling the third case, a single LINK33 would follow the length of the hole, perpendicular to the plate surface. At the intersecting plate surface, place a node at the center of the hole and couple all the nodes around the hole radius to the node. At the node representing the other end of the hole, set the pressure (temperature) to zero.

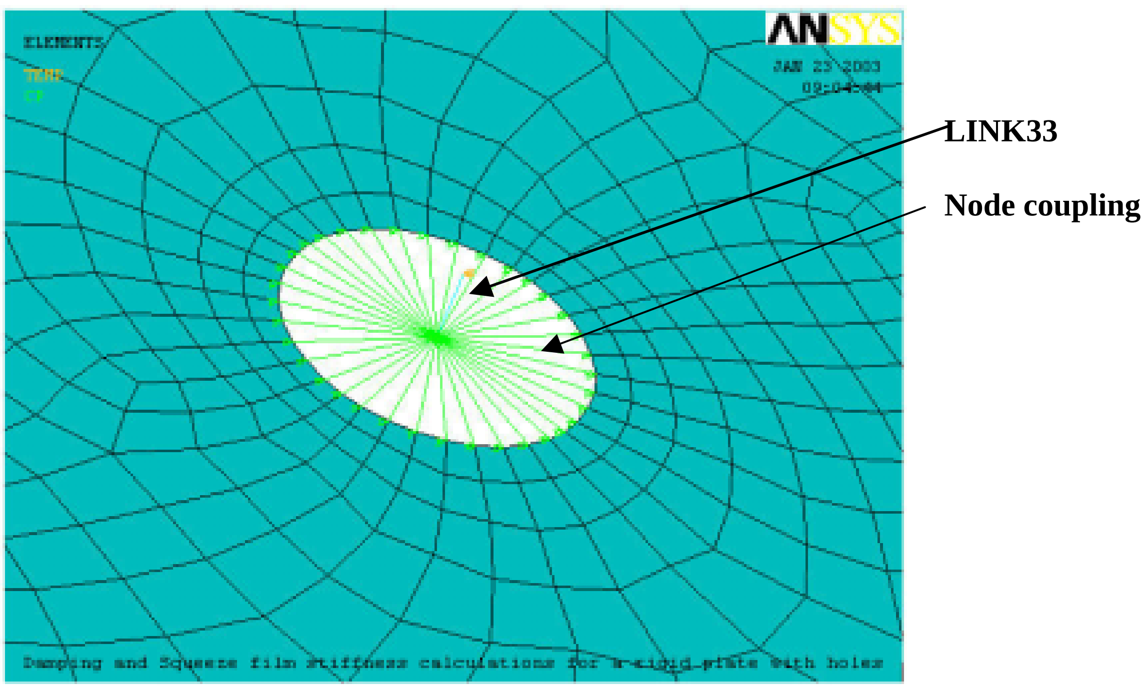

Figure 4: **Link32 element through plate hole (shown with node coupling to hole)**

For continuum theory using LINK33, the flow rate Q through a circular channel with $l >> r$, $Re < 2320$ and $r > 100L_m$ is defined by

Eq. 12
$$Q = \frac{r^4 \pi}{8\,\eta\, l}\,\Delta p = \frac{r^2}{8\,\eta}\,\frac{A}{l}\Delta p$$

where r is the channel's radius, η the dynamic viscosity, l the channel length, A the cross sectional area and Δp the pressure drop across the element. In terms of heat flow analogy

Eq. 13
$$Q = \frac{\lambda\, A}{l}\,\Delta T$$

we replace the thermal conductivity λ by

Eq. 14
$$\lambda = \frac{r^2}{8\,\eta}$$

For high Knudsen numbers, the effective viscocity must be modified. The following equations are based on several articles published by Sharipov [7].

The relative channel length is defined by

Eq. 15
$$\Gamma = \frac{l}{r}$$

The measure of the gas rarefaction is not only the Knudsen number itself but also the inverse Knudsen number D

Eq. 16
$$Kn = \frac{r}{L_m} \qquad D = \frac{\sqrt{\pi}\, r}{2\, L_m}$$

where the mean free path L_m is a function of ambient pressure.

The flow conductance is expressed by the relative flow rate coefficient Q_R which considers the influence of channel diameter and gas pressure to the fluid flow

Eq. 17
$$Q_R = \frac{D}{4} + 1.485\,\frac{1.78\,D + 1}{2.625\,D + 1}$$

Fringing effects at the inlet and outlet increase the pressure drop along the channel considerable. This effect is approximated by a relative channel elongation defined by

Eq. 18
$$\Delta\Gamma = \frac{3\,\pi}{8}\,\frac{1 + 1.7\,D^{-0.858}}{1 + 0.688\,D^{-0.858}\Gamma^{-0.125}}$$

Finally the effective viscosity is

Eq. 19
$$\eta_{eff} = \frac{D\,\eta}{4\,Q_R}$$

and the total flow rate

Eq. 20

$$Q = \frac{r^2}{8\,\eta_{eff}\left(1+\dfrac{\Delta\Gamma}{\Gamma}\right)}\frac{A}{l}\Delta p$$

Hence, the analgous thermal conductivity for LINK33 becomes

Eq. 20

$$\lambda = \frac{r^2}{8\,\eta_{eff}\left(1+\dfrac{\Delta\Gamma}{\Gamma}\right)}$$

Example Problem: Rectangular beam with holes: Transverse motion

A rectangular beam with perforated holes under transverse motion is modeled to compute the effective damping and squeeze stiffness coefficients. Three cases were considered:

1. Holes modeled with no resistance (pressure=0 in hole)
2. Holes modeled with infinite resistance (pressure left unspecified)
3. Holes modeled with finite resistance (pressure set to zero at top of hole)

Table 2 lists the damping and squeeze coefficient results. Figures 5 and 6 illustrate the Real and Imaginary pressure distribution. The input file for case 3 is listed.

Table2: **Beam Model results considering perforated holes**

Frequency (kHz.)	Hole option	Damping Coefficient	Squeeze stiffness coefficient
150	Infinite resistance	1.517e-6	.02257
150	Finite resistance	1.208e-6	.01433
150	No resistance	0.8819e-6	.00889

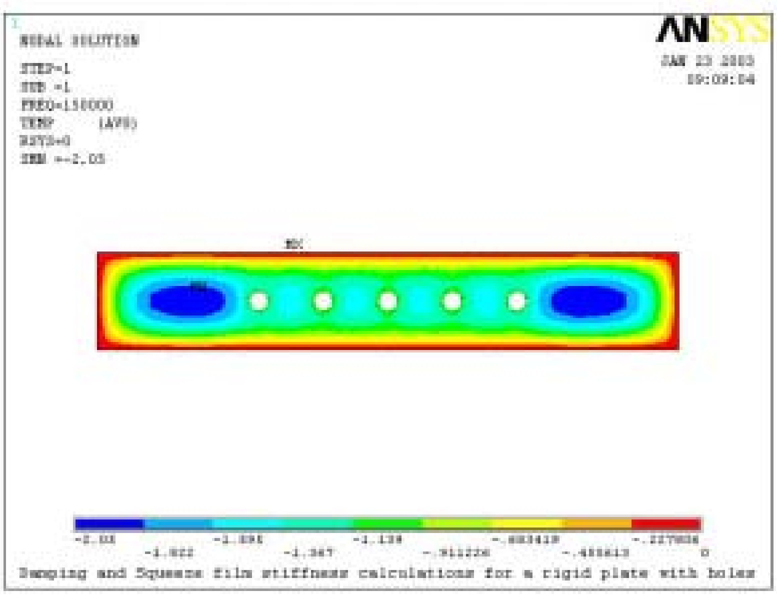

Figure 5: Pressure distribution (Real component)

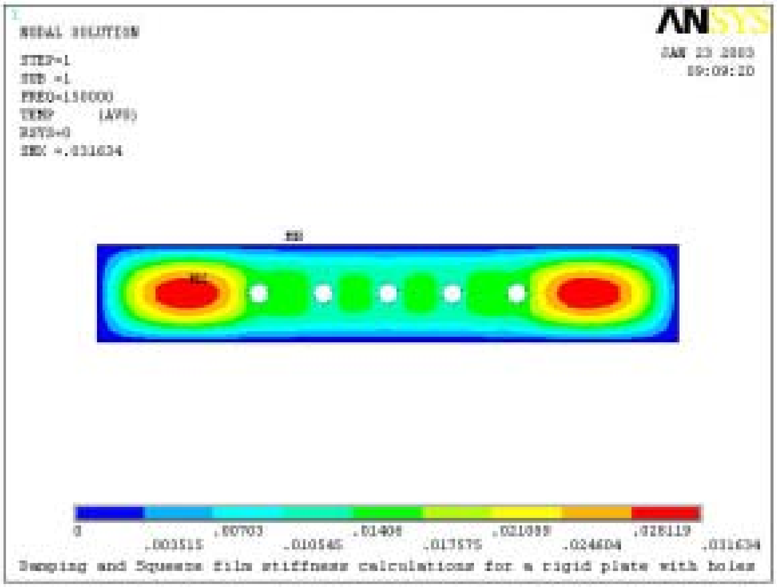

Figure 6: Pressure distribution (Imaginary component)

The input file for this example is shown below for the finite-resistance Case:

```
/batch,list
 /PREP7
/title, Damping and Squeeze film stiffness calculations for a rigid
/com,    plate with holes
/com,    Thermal analogy using PLANE55 and LINK33
/com,    Assume High Knudsen number theory

ET, 1, 55               ! 4 node thermal element (Plate region)
ET, 2, 33               ! 2 node thermal link (Hole region)

s_l=60e-6                       ! Plate length
s_l1=40e-6                      ! Plate hole location
s_w=10e-6                       ! Plate width
s_t=1e-6                        ! Plate thickness
c_r=2e-6                        ! Hole radius
d_el=2e-6                       ! Gap
pamb=1e5                        ! ambient pressure
visc=18.3e-6                    ! viscosity
pref=1e5                        ! reference pressure
mfp=64e-9                       ! mean free path
velo=.002                       ! arbitrary velocity
pi=3.14157
freq=150000                     ! Frequency (Hz.)
omega=2*pi*freq

/com,  Compute effective viscosity for plate region
Knp=mfp/d_el
visceffp=visc/(1+9.638*(Knp**1.159))

/com,  Compute effective viscosity for hole region
gamma=s_t/c_r
Kn=c_r/mfp
D=sqrt(pi)*c_r/2/mfp
```

```
Qr=D/4+1.485*((1.78*D+1)/(2.625*D+1))
denom=1+.688*(D**(-.858))*(gamma**(-.125))
dgamma=3*pi/8*(1+1.7*(D**(-.858)))/denom
visceff=D*visc/4/Qr
keqv=c_r**2/(8*visceff*(1+dgamma/gamma))

/com, Define effetive material properties for plate

mp,kxx,1,d_el**3/12/visceffp
mp,c,1,d_el/pamb
mp,dens,1,1

/com, Define effective material properties for hole

mp,kxx,2,keqv
mp,c,2,d_el/pamb
mp,dens,2,1

/com, Build model

area=pi*c_r**2
r,2,area

rectng,-s_l,s_l,-s_w,s_w            ! Plate domain
pcirc,c_r                           ! Hole domain
agen,3,2,,,-s_l1/3
agen,3,2,,,s_l1/3
ASBA, 1, all

TYPE, 1
MAT, 1
smrtsize,4
AMESH, all                          ! Mesh plate domain


! Begin Hole generation
*do,i,1,5
 nsel,all
 *GET, numb, node, , num, max       ! Create nodes for link elements
 N, numb+1,-s_l1+i*s_l1/3,,
 N, numb+2,-s_l1+i*s_l1/3,, s_t
 TYPE,2
 MAT, 2
 REAL,2
 NSEL, all
 E, numb+1, numb+2                  ! Define 2-D link element
 ESEL, s, type,,1
 NSLE,s,1
 local,11,1,-s_l1+i*s_l1/3
 csys,11
 NSEL,r, loc, x, c_r       ! Select all nodes on the hole circumference
 NSEL,a, node, ,numb+1
 *GET, next, node, , num, min
 CP, i, temp, numb+1, next
 nsel,u,node, ,numb+1
 nsel,u,node, ,next
 CP, i, temp,all !Define a coupled DOF set to realize constant pressure
```

```
 csys,0
*enddo
 ! End hole generation

nsel,s,loc,x,-s_l
nsel,a,loc,x,s_l
nsel,a,loc,y,-s_w
nsel,a,loc,y,s_w
nsel,r,loc,z,-1e-9,1e-9
d,all,temp              ! Fix pressure at outer plate boundary
nsel,all

esel,s,type,,2
nsle,s,1
nsel,r,loc,z,s_t
d,all,temp,0            ! Set pressure to zero at top of hole

allsel

bfe,all,hgen,,-velo     ! Apply arbitrary velocity
fini

finish
/solu
antyp,harm              ! Harmonic Thermal analysis
harfrq,freq
solve
finish
/post1
esel,s,type,,1
set,1,1
etable,presR,temp       ! extract "Real" pressure
etable,earea,volu
smult,forR,presR,earea  ! compute "Real" force
ssum
*get,Fre,ssum,,item,forR
set,1,1,,1
etable,presI,temp       ! extract "Imaginary" pressure
smult,forI,presI,earea  ! compute "Imaginary" pressure
ssum
*get,Fim,ssum,,item,forI

K=Fim*omega/velo        ! Compute equivalent stiffness
C=abs(Fre/velo)         ! Compute equivalent damping

/com, ******* Equivalent stiffness  ************************
*stat,K
/com, ******* Equivalent damping  *************************
*stat,C
finish
save
```

Literature:
(alphabetic order)

[1] J. J. Blech: *On Isothermal Squeeze Films*, Journal of Lubrication Technology, Vol.105, pp. 615-620, 1983

[2] W. S. Griffin, et. al.: *A Study of Squeeze-film Damping*, J. of Basic Engineering, pp. 451-456, 1966

[3] W. E. Langlois: *Isothermal Squeeze Films*, Quarterly Applied Mathematics, Vol. 20, No. 2, pp. 131-150, 1962

[4] J. E. Mehner, et. al.: *Simulation of Gas Film Damping on Microstructures with Nontrivial Geometries*, Proc. of the MEMS Conference, Heidelberg, Germany, 1998

[5] Y. J. Yang: *Squeeze-Film Damping for MEMS Structures*, Master Theses, Massachusetts Institute of Technology, 1998

[6] T. Veijola: *Equivalent Circuit Models for Micromechanical Inertial Sensors*, Circuit Theory Laboratory Report Series CT-39, Helsinki University of Technology, 1999

[7] F. Sharipov: *Rarefied Gas Flow Through a Long Rectangular Channel*, J. Vac. Sci. Technol., A17(5), pp. 3062-3066, 1999