Review of Yield/Failure Criteria

Anisotropic Behavior

Yield Criterion

Failure Criteria

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ANSYS.NET Newsletter

Ductile Materials

- **With ductile materials such as most common metals, the elastic and inelastic behavior is usually separated by a scalar value, the *yield strength* of the material.**
    - **The *yield point* and *proportional limit* are assumed to be the same in ANSYS**
    - **The uniaxial tensile test data is compared with this scalar yield strength to evaluate when yielding occurs**
    - **A particular yield criterion must be chosen to evaluate the scalar value, such as the von Mises or Hill criterion. This is done with data tables (TB) when specifying *plasticity*.**
        - **Most ANSYS plasticity rules assume von Mises yield criterion.**
        - **Use of TB,HILL allows users to specify Hill potential instead**
        - **Certain plasticity models have a built-in yield criterion such as Drucker-Prager (Mises cone) or ANISO (generalized Hill)**
        - ***The subsequent discussion will focus on Mises and Hill yield criteria rather than hardening laws or flow rules.***

Brittle Materials

- **A *failure criterion* is more commonly used to evaluate brittle, orthotropic, or composite materials**
  - **The failure criterion can describe when brittle failure (fracture) or ductile failure (yielding) occurs.**
  - **Because ANSYS is used as a design tool, most failure criteria in ANSYS do not account for post-failure response**
  - **Composite elements such as SHELL91/99 and SOLID46/191 can use data table TB,FAIL**
  - **Concrete element SOLID65 can include cracking/crushing behavior with TB,CONCR**
  - **At 6.0, the FC family of commands have been introduced for all PLANE/SOLID and 3D SHELL structural elements**
    - **Maximum stress, maximum strain, or Tsai-Wu failure criteria can be used (similar to TB,FAIL)**
    - ***This memo will focus on the "FC" command to specify failure criterion. Other features will not be presently discussed.***

Linear Elastic Behavior

- **Before discussing details of yield/failure criteria, it may be useful to cover some basic definitions:**

  - **Hooke's Law (compliance form) can be expressed as** $\varepsilon_{ij} = S_{ijkl}\sigma_{kl}$

  - ***Compliance* is the inverse of *stiffness* and is used here for simplicity.**

  - **Although compliance S_{ij} is a fourth-order tensor, because stress and strain tensors are symmetric, we usually describe stress and strain as vectors and compliance as a second-order tensor (i.e., matrix).**

  - **This leaves 36 independent constants in S_{ij}. However, since the stiffness matrix is also symmetric (conservative material), this leaves 21 independent constants.**

Anisotropic Behavior

- **A *general anisotropic linear elastic* solid therefore has 21 independent constants.**

  – **In ANSYS, 21 anisotropic elastic constants are input via TB,ANEL for SOLID64, coupled-field elements PLANE13 and SOLID5/98, and 18x PLANE/SOLID elements.**

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ & & S_{33} & S_{34} & S_{35} & S_{36} \\ & & & S_{44} & S_{45} & S_{46} \\ & & & & S_{55} & S_{56} \\ & & & & & S_{66} \end{bmatrix}$$

Orthotropic Behavior

- **An *orthotropic* material has three orthogonal planes of symmetry. In this situation, there are 9 independent constants which need to be input.**
  - **In ANSYS, *all nine orthotropic constants are input* via MP family of commands: elastic moduli EX/Y/Z, Poisson's ratios PRXY/YZ/XZ (or NUXY/YZ/XZ), and shear moduli GXY/YZ/XZ.**

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{22} & S_{23} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{55} & 0 \\ & & & & & S_{66} \end{bmatrix}$$

$$S_{11} = \frac{1}{E_x}$$

$$S_{12} = -\frac{\nu_{yx}}{E_y}$$

$$S_{44} = \frac{1}{G_{xy}}$$

...et cetera

Transversely Isotropic

- *Transversely isotropic* materials have the same behavior in a plane (e.g., xy) but different in a perpendicular direction (e.g., z direction)
  - There are five independent elastic constants for this case, and it can be considered as a subset of the orthotropic case.
  - *All nine values must be input*, but they follow the relation as implied below:

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{11} & S_{13} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & 2(S_{11} - S_{12}) & 0 & 0 \\ & & & & S_{66} & 0 \\ & & & & & S_{66} \end{bmatrix}$$

Cubic Material

- *Cubic* materials have same properties in x, y, z directions
    - There are three independent constants for cubic materials which can be input via **MP** command (EX, NUXY, GXY).
    - As with orthotropic material, *all nine constants must be input*, even though EX=EY=EZ, etc. Nine input constants are needed to distinguish cubic from isotropic material behavior (next).

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ & S_{11} & S_{12} & 0 & 0 & 0 \\ & & S_{11} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 0 \\ & & & & & S_{44} \end{bmatrix}$$

Isotropic material

- **An *isotropic* material is one which the material behavior does not rely on *any* orientation. Only elastic modulus EX and Poisson's ratio NUXY (or PRXY) need to be input. Shear modulus is calculated to be GXY=EX/(2*(1+NUXY))**
  - **There are only two independent constants which are input**
  - **Material is input via MP,EX and MP,NUXY (or MP,PRXY)**

$$S_{ij} = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ & S_{11} & S_{12} & 0 & 0 & 0 \\ & & S_{11} & 0 & 0 & 0 \\ & & & 2(S_{11} - S_{12}) & 0 & 0 \\ & & & & 2(S_{11} - S_{12}) & 0 \\ & & & & & 2(S_{11} - S_{12}) \end{bmatrix}$$

Procedure for Input of Materials

- **Linear elastic material properties (anisotropic, orthotropic, or isotropic) are input through the Materials GUI**
  - **Main Menu > Preprocessor > Material Props > Material Models…**
  - **Note that, as mentioned earlier, for transversely isotropic and cubic materials, material input is done as "orthotropic," with all nine material parameters entered. Some material constants will be the same, as shown in the previous slides.**
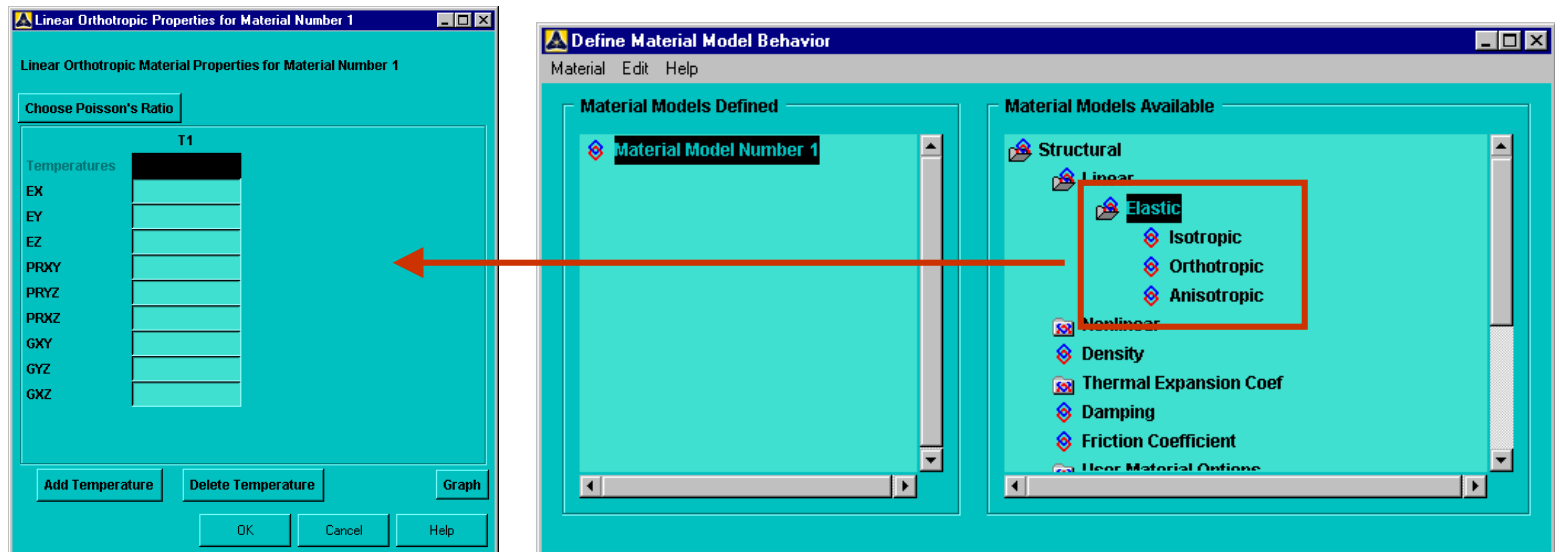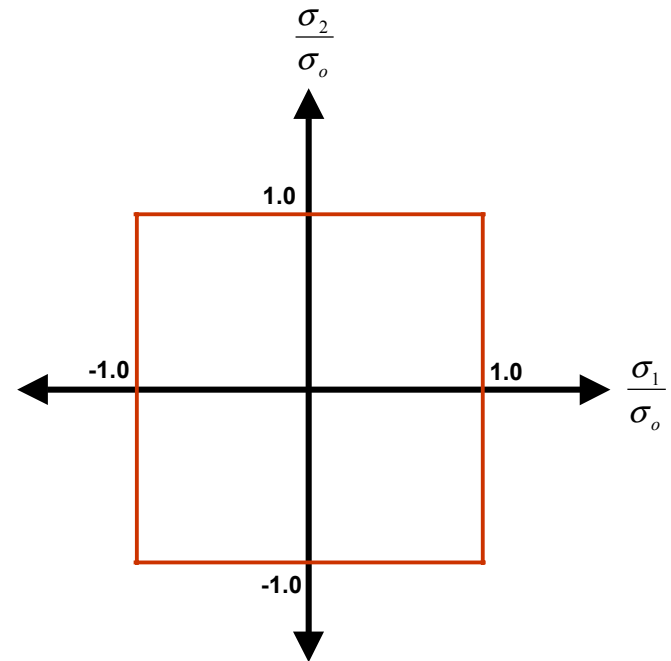
Yield/Failure Criteria

Isotropic Case

Maximum Normal Stress Failure Criterion

- **The *maximum normal stress* failure criterion (Rankine) states that failure is expected when the maximum principal stress exceeds a specified value. In this case, if the reference stress was σ_0, a ratio greater than 1.0 would cause failure.**

The failure surface is plotted in 2D principal stress space, as shown on the right.

- **Inside of the red region is linear elastic material behavior**
- **The red square defines the failure surface.**
- **No stress state can exist outside of the failure surface.**

$$\frac{\sigma_2}{\sigma_o}$$

1.0

-1.0   1.0 $\frac{\sigma_1}{\sigma_o}$

-1.0

Maximum Normal Strain Failure Criterion

- **The *maximum normal strain* failure criterion (St. Venant) specifies a limiting value of the principal strains.**
  - **Since principal strain is defined as** $\varepsilon_1 = \dfrac{1}{E}[\sigma_1 - \nu(\sigma_2 + \sigma_3)]$ **the limiting strain value can be converted to a limiting stress value in 2D plane stress case as:** $\sigma_1 = \sigma_o + \nu\sigma_2$
  - **In other words, in 2D principal stress space, the failure surface is defined by a sloped line, as shown in blue.**
  - **As with all failure/yield surfaces, the region inside the failure surface is linear elastic behavior. No stress state can exist outside of the failure surface.**
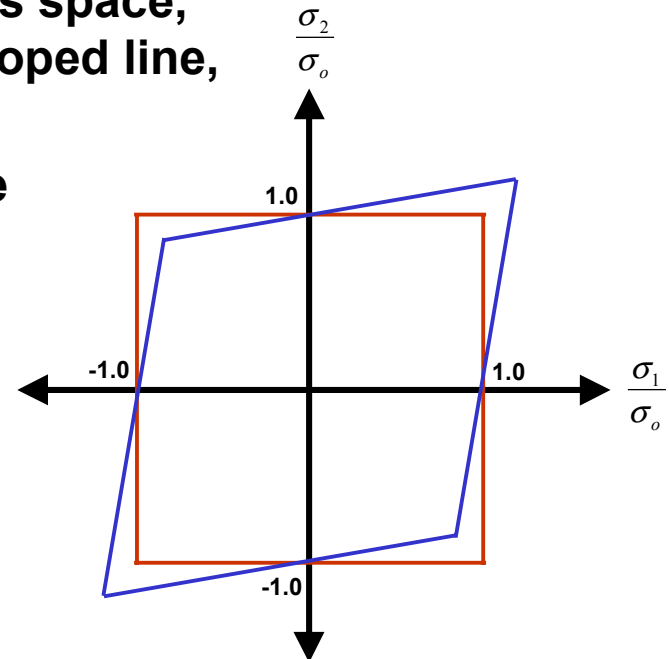
Maximum Shear Stress Yield Criterion

- **The *maximum shear stress* yield criterion (Tresca) states that yielding occurs when the maximum shear stress reaches a certain value τ_{max}.**

  - **The principal shear stresses are obtained from the principal normal stresses as follows:**

  $$\tau_1 = \frac{(\sigma_2 - \sigma_3)}{2}$$

  - **Hence, the yield surface in 2D is defined by**

  $$\sigma_o = \sigma_1 \qquad \sigma_o = (\sigma_1 - \sigma_2) \qquad \sigma_o = \sigma_2$$

  and is shown in purple on the right.

  - **Note that if both principal stresses have the same sign, the Tresca yield criterion is similar to the Rankine failure surface**

  - **This, of course, is only true for 2D plane stress, *not* for any 3D state of stress since the Rankine cube is dependent on hydrostatic pressure whereas the Tresca hexagonal tube is not.**

ANSYS.NET Tips & Tricks
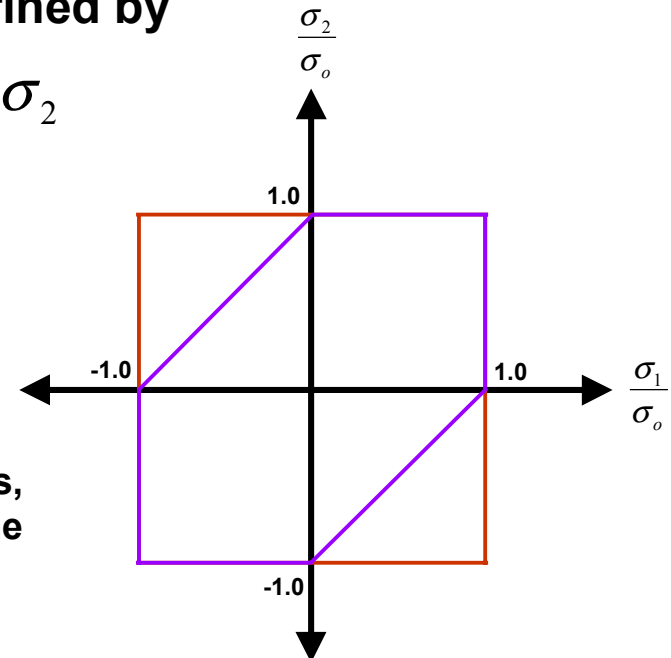
Octahedral Shear Stress Yield Criterion

- **The *octahedral shear stress* yield criterion (von Mises), also known as the *distortion energy* criterion, states that yielding occurs when**

$$\sigma_o = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

- **In 2D principal stress space, this yield surface is an ellipse, as shown in green. Note that it is similar to the Tresca yield criterion.**

- **The Mises yield surface tends to be more representative of most ductile material behavior.**

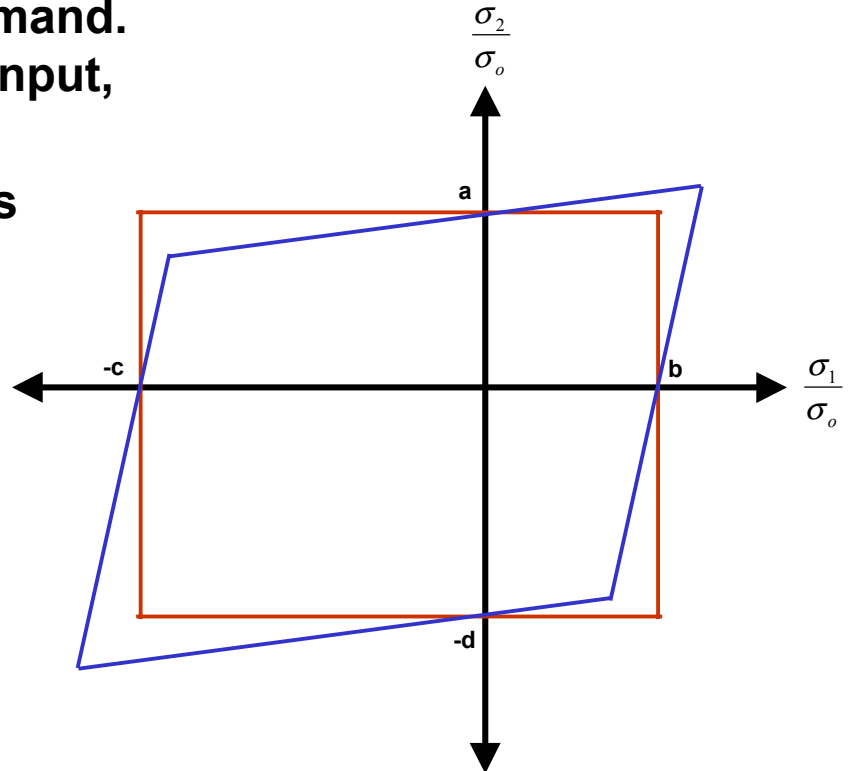- **The Tresca criterion is, at most, about 15% more conservative than the Mises criterion.**
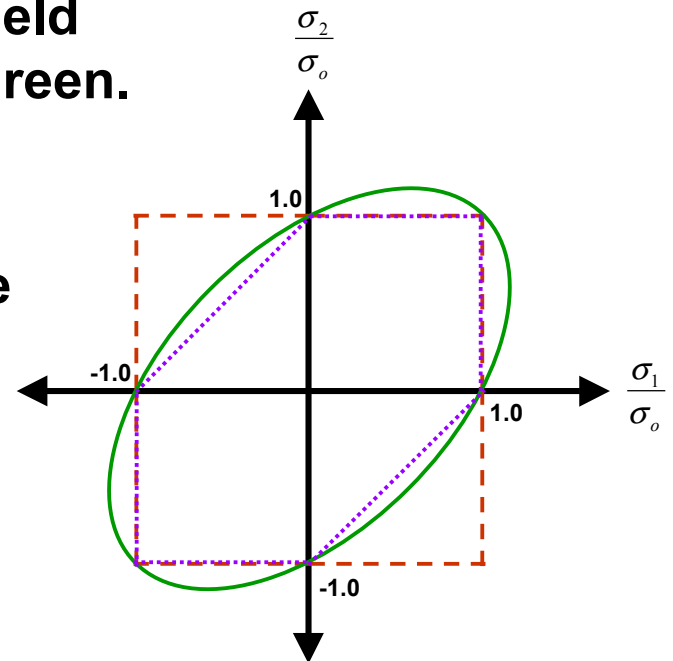
Yield/Failure Criteria

Anisotropic Case

Maximum Stress/Strain Failure Criteria

- **Although presented in the previous chapter for a symmetric case, the maximum stress and maximum strain failure criteria are often used in orthotropic or anisotropic cases.**
  - **The user should provide tensile and compressive stress or strain limits via the FC command. Compressive values, if not input, default to -(tensile).**
  - **Shear stress or strain values should also be input via the FC command (9 values for either maximum stress or maximum strain failure criterion)**
  - **Up to six temperature-dependent sets of failure criteria can be input**

The diagram shows axes labeled $\dfrac{\sigma_2}{\sigma_o}$ (vertical) and $\dfrac{\sigma_1}{\sigma_o}$ (horizontal), with points **a**, **b**, **-c**, and **-d** marking the boundaries.

Tsai-Wu Failure Criterion

- **The *Tsai-Wu* failure criterion, also known as the *interactive tensor polynomial theory* or *quadratic polynomial failure theory*, is also commonly used in composites. In 2D principal stress space, an example of the Tsai-Wu failure envelope is shown below in brown.**
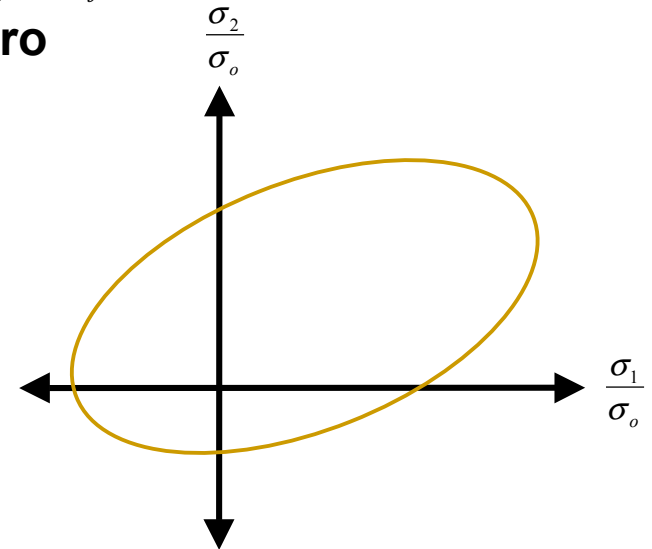 - **The Tsai-Wu criterion accounts for stress interactions and, in its general form, is written as** $F_i \sigma_i + F_{ij} \sigma_i \sigma_j = 1$
  - **Only F_1, F_2, F_{11}, F_{22}, F_{12}, F_{66} are non-zero**

$$F_1 = \sigma_{xt}^{-1} + \sigma_{xc}^{-1} \quad F_{11} = -\sigma_{xt}^{-1}\sigma_{xc}^{-1}$$

$$F_2 = \sigma_{yt}^{-1} + \sigma_{yc}^{-1} \quad F_{22} = -\sigma_{yt}^{-1}\sigma_{yc}^{-1}$$

$$F_{12} = C_{xy}\left(\sigma_{xt}\sigma_{xc}\sigma_{yt}\sigma_{yc}\right)^{-0.5}$$

$$F_{66} = \sigma_{xyf}^{-2}$$

Tsai-Wu Failure Criterion (cont'd)

- **In 2D plane stress case (or transversely isotropic case)**
  - **As shown from the equations in the previous slide, F_1, F_{11} can be determined from longitudinal tension & compression tests, F_2, F_{22} can similarly be determined from transverse tension and compression tests. F_{66} can be determined from shear tests. C_{xy} is usually assumed to be -1.0 but can be more (between 0 and -1.0, default is -1.0).**
  - **The Cxy constant is twice the value F_{12}^{*} used by Tsai and Hanh. Also, unlike some notations for Tsai-Wu, compressive values are input as negative.**
  - **One can expand this to deal with an arbitrary 3D stress state. For a general 3D case, the Tsai-Wu criterion requires 12 constants per temperature (up to 6 temperature sets) via the FC command.**

Hill Yield Criterion

- **The Mises yield criterion is the default for most of the plasticity models in ANSYS. Use of the *Hill potential* allows specification for anisotropic yield criteria.**

  - **Six constants are required for Hill specification. Unlike Tsai-Wu or ANISO, *yield ratios* are input for x, y, z normal directions and xy, yz, and xz shear (three symmetric orthogonal planes).**

  - **Note that unlike Tsai-Wu or ANISO, tension and compression behavior are assumed to be the same.**

  - **TB,HILL defines yield surface only. It must be combined with a plasticity model to define hardening (isotropic or kinematic hardening). It could also be used with creep to model anisotropic creep.**

  - **Generalized Hill (TB,ANISO) is also available to model different tension/compression behavior.**

$\dfrac{\sigma_2}{\sigma_o}$

$\dfrac{\sigma_1}{\sigma_o}$
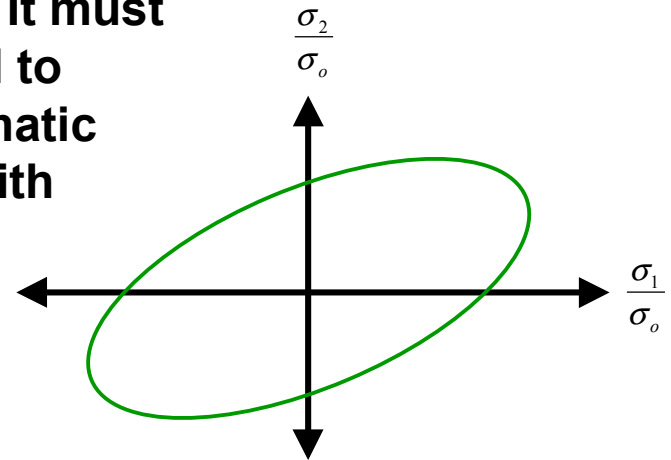
Hill Yield Criterion (cont'd)

- **The Hill yield criterion is a modified Mises surface, defined as the following:**

$$\sigma_o = \sqrt{H(\sigma_x - \sigma_y)^2 + F(\sigma_y - \sigma_z)^2 + G(\sigma_x - \sigma_z)^2 + 2N\tau_{xy}^2 + 2L\tau_{yz}^2 + 2M\tau_{xz}^2}$$

There are six constants, R_{xx}, R_{yy}, R_{zz}, R_{xy}, R_{yz}, R_{xz}, which characterize the Hill yield criterion:

$$G + H = \frac{1}{R_{xx}^2} \qquad N = \frac{3}{2R_{xy}^2} \qquad R_{xx} = \frac{\sigma_{xx}^y}{\sigma_o} \qquad R_{xy} = \sqrt{3}\frac{\sigma_{xy}^y}{\sigma_o}$$

$$F + H = \frac{1}{R_{yy}^2} \qquad L = \frac{3}{2R_{yz}^2} \qquad R_{yy} = \frac{\sigma_{yy}^y}{\sigma_o} \qquad R_{yz} = \sqrt{3}\frac{\sigma_{yz}^y}{\sigma_o}$$

$$F + G = \frac{1}{R_{zz}^2} \qquad M = \frac{3}{2R_{xz}^2} \qquad R_{zz} = \frac{\sigma_{zz}^y}{\sigma_o} \qquad R_{xz} = \sqrt{3}\frac{\sigma_{xz}^y}{\sigma_o}$$

These constants can be determined from tests performed in the six directions.

# Procedure

Failure Criterion

- **The aforementioned failure criteria – Maximum normal stress, maximum normal strain, Tsai-Wu – are accessible in the General Postprocessor.**

  - **Main Menu > General Postproc > Failure Criteria**
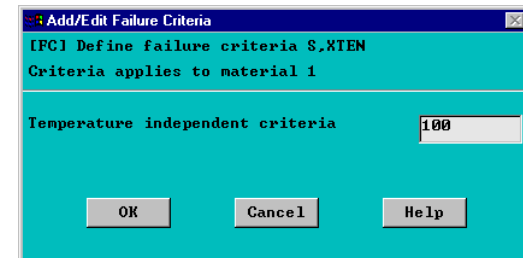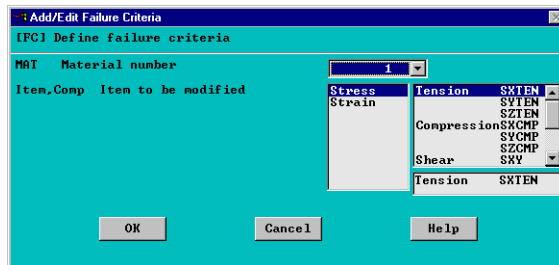
  - **The user defines the constants for each temperature and material property, as needed**

  - **Equivalent commands are FC, FCLIST, and FCDELE.**

Failure Criterion (cont'd)

– **The user can then plot max stress/strain failure as well as Tsai-Wu strength index and inverse strength ratio**

– **Main Menu > General Postproc > Plot Results > -Contour Plot-**
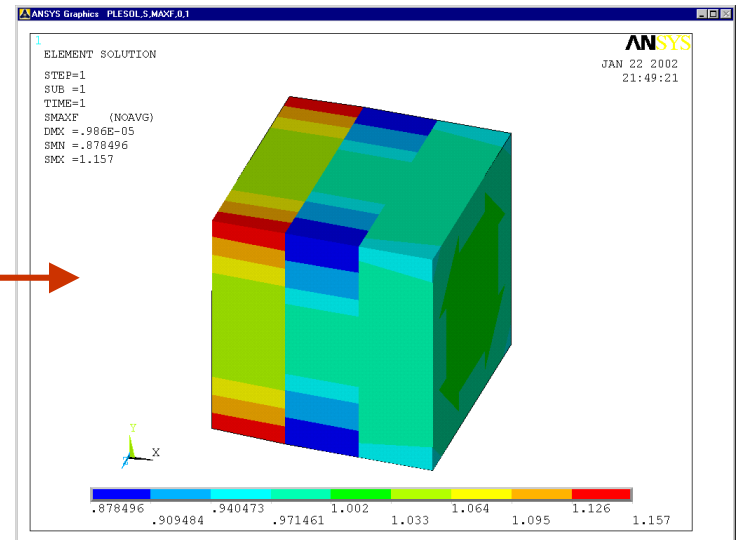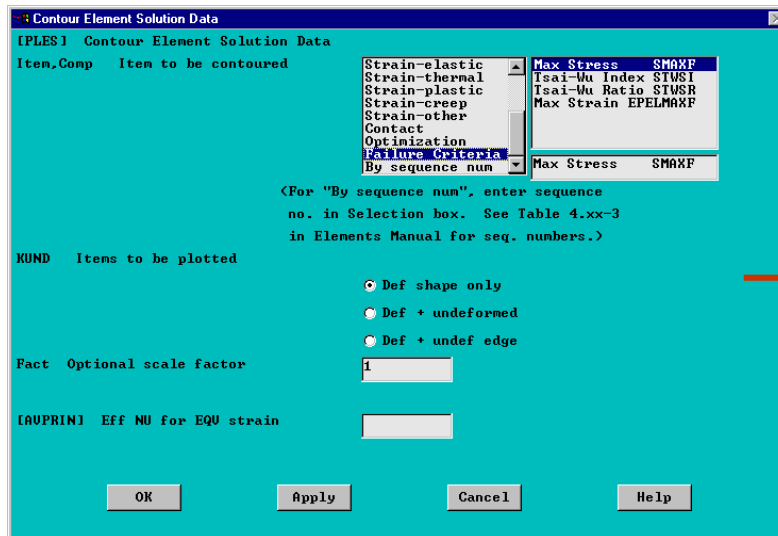
Failure Criterion (cont'd)

- **Additional notes on failure criteria:**
  - The selection of a particular failure criterion is dependent on the material. This aspect is too large of a topic to discuss, and selection of the proper failure criterion is left up to the user's engineering judgement.
  - While a *failure 'material table'* can be defined in Preprocessing, this is only for the composite elements (not covered in this memo). This "FC" method via postprocessing is available to <u>all</u> solid/plane and 3D shell structural elements.
  - At 6.0, SHELL181 has advanced composite features, including easy-to-use preprocessing tools (shell section)
  - Not covered here is the concrete feature in ANSYS. This allows for crushing and cracking in up to three orthogonal directions (i.e., actual failure of material), including postprocessing features.
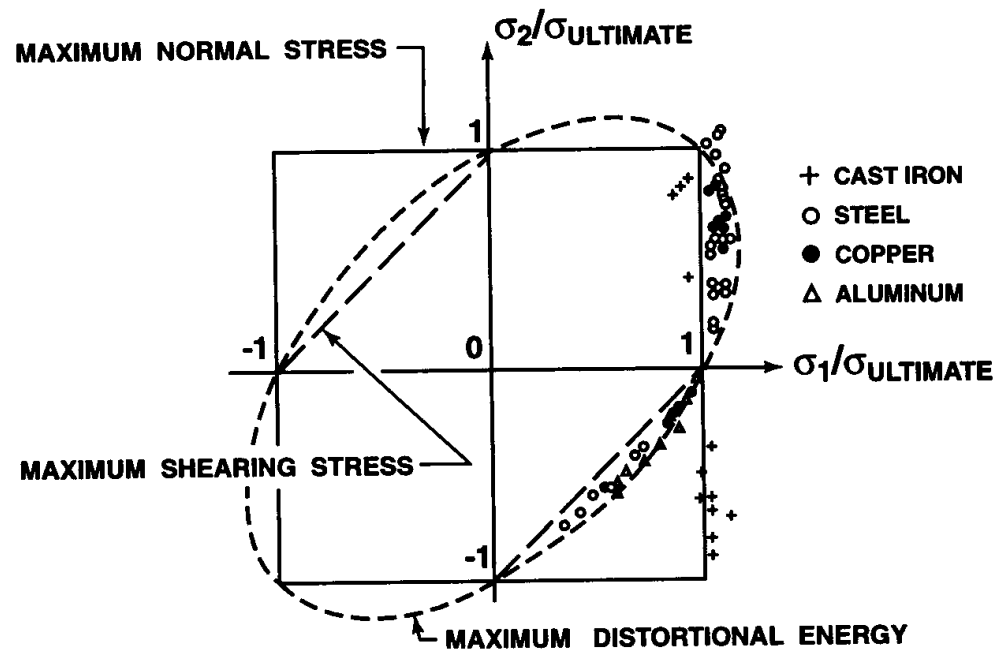
Yield Criterion

- **The two yield criteria presented in this memo – von Mises and Hill – are accessible in the Materials GUI. When selecting a particular rate-independent model, the choice of Mises or Hill yield criterion is shown.**

  - **Main Menu > Preprocessor > Material Props > Material Models…**

  - **Available with all 'core' and 18x elements for use with isotropic (BISO, MISO, NLISO) and kinematic (BKIN, KINH/MKIN, CHAB) hardening models**

  - **Use TB,HILL to specify Hill yield criterion, if command method is preferred. The default (i.e., unspecified) is Mises yield surface.**
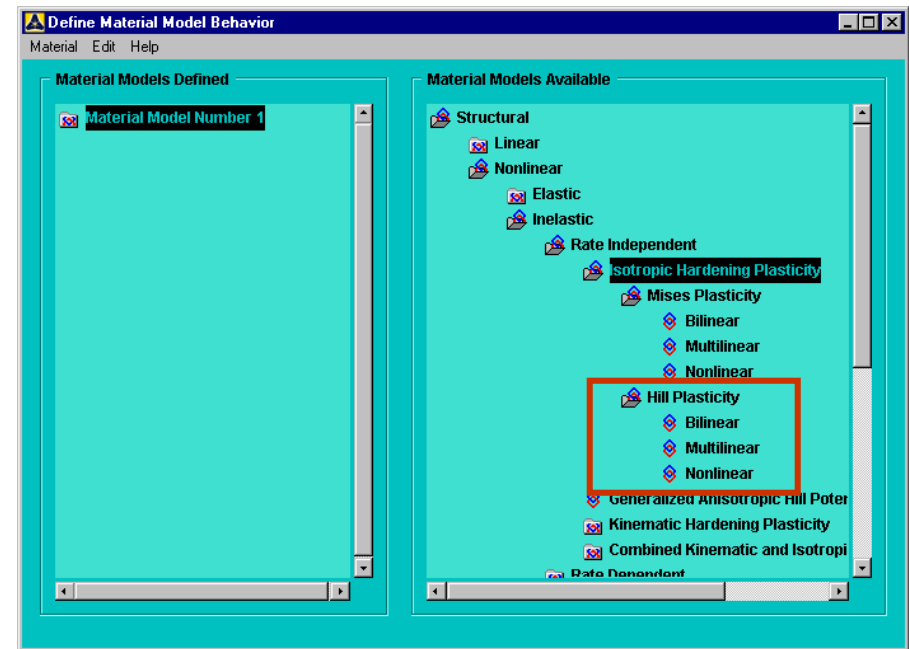
Yield Criterion (cont'd)

- **Different metals behave in a different manner. While most isotropic responses are sufficiently represented with von Mises yield criterion, some like cast iron require special models, as tensile behavior is very different from compressive.**

- **Selection of the proper yield criterion requires that the user is familiar with the material he/she is utilizing.**

Yield Criterion (cont'd)

- **Additional notes on yield criterion:**

  - **The Hill yield criterion can also be used with implicit creep and rate-dependent plasticity models (TB,RATE).**

  - **Not covered here are Drucker-Prager (TB,DP) and Generalized Hill (TB,ANISO) models. DP is a modified Mises cone (hydrostatic dependence) in principal stress space. Generalized Hill accounts for different tension/compression behavior.**

  - **Beta in 6.0 and planned to be introduced in 6.1 is a cast iron model, which is a Rankine cube (Maximum stress criterion) in tension and a Mises cylinder (Octahedral shear stress criterion) in compression. Inelastic volume change can also be considered with this model.**

Summary

- **This memo hoped to provide a *review* of some basics on anisotropy, yield criteria for ductile materials, and failure criteria for both brittle and ductile materials.**

- **The 18x family of elements have a wealth of nonlinear features for modeling nonlinear elastic and inelastic response.**
  - **The element and material technology are being separated to provide users a 'toolkit' of features from which to select.**
  - **The supported material models include the Hill and Mises yield criteria which can be used on almost all of the rate-independent and rate-dependent inelastic models.**
  - **Many of the new inelastic models, including Hill yield criterion, are also supported for 'core' elements (e.g., 42, 45, 92, 95).**

Summary (cont'd)

- **The FC command, introduced at 6.0, also tries to separate failure criteria from composite elements in order to make them available to all structural plane/solid/shell elements.**
  - **Failure criteria need not be specified before solution in preprocessing. Now, it is done in postprocessing, making it more convenient for users.**
    - **No planning required beforehand (in the older method, if one forgot to define the failure criteria before solution, the solution would need to be re-run)**
    - **Results file is not huge because failure information is not stored in results file but is calculated on-the-fly.**
  - **Also useful for evaluation of non-composite structures, such as orthotropic materials**
  - **Failure criterion is *not* used to determine post-failure response (i.e., material will not actually 'fail' during the analysis) but is used to simply evaluate the model from a design standpoint.**