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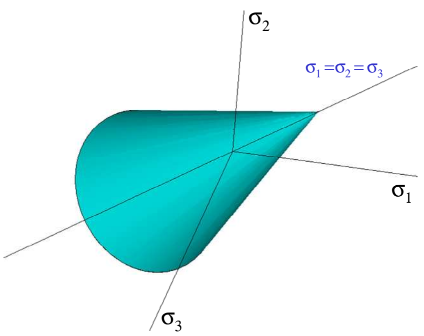
## 1. Introduction:

The Drucker-Prager material model is used for pressure-dependent inelastic behavior of materials such as soils, rock, concrete, and powder. Because ANSYS offers three different Drucker-Prager constitutive models, this memo hopes to provide a comparison of the available options.

## 2. Background on Drucker-Prager Plasticity:

The Drucker-Prager plasticity model is different from typical metal plasticity models since it contains a dependence on hydrostatic pressure. For metal plasticity (assuming Mises or similar yield surface), only the deviatoric stress is assumed to cause yielding – if we plot the yield surface in principal stress space, this results in a cylinder whose axis is the hydrostatic pressure line, indicating that yielding is independent of the hydrostatic stress state. For the Mises yield surface, theoretically, one could have infinite hydrostatic compression, and no yielding would occur.

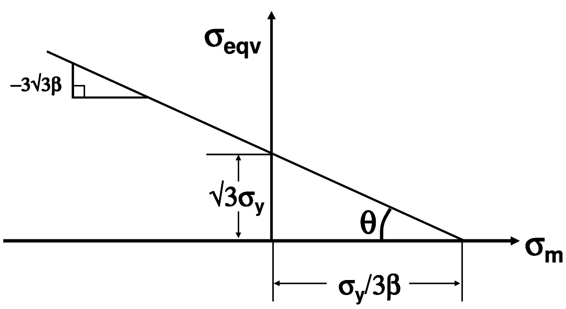



On the other hand, the Drucker-Prager plasticity model has a term that is dependent on the hydrostatic pressure. For a linear yield surface ("linear" referring to the linear shape when plotted in the plane of effective stress vs. hydrostatic pressure), this means that if there is some hydrostatic tension, the yield strength would be smaller. Conversely, as hydrostatic compression increases, so would the yield strength. When the yield surface is plotted in principal stress space, it would look like a cone, as shown in the figure on the left.

The two main characteristics that result are that (a) the yield strength changes, depending on the hydrostatic stress state and (b) some inelastic volumetric strain can occur, as defined by the flow potential. Because of these points, the Drucker-Prager material model is used for geomechanics or powder compaction or any other application where both hydrostatic dependence and inelastic volume strain are important.

## 3. Drucker-Prager Model:

Besides reviewing the yield surface in principal stress space, as shown earlier, one can also look at the yield surface along the plane defined by the effective stress (SEQV or q or $\sigma_{eqv}$) and hydrostatic pressure (HPRES or $\sigma_m$). The linear Drucker-Prager yield surface would look as illustrated on the right.



The two main items that are required for the linear Drucker-Prager case are the slope ("angle of internal friction") and the value at which it intersects the y-axis (i.e., the yield strength at zero hydrostatic pressure, related to the "cohesion value").

- The cohesion value $c$ is related to the yield strength $\sigma_y$ via the relationship $c = \dfrac{\sqrt{3}(3-\sin\theta)}{6\cos\theta}\sigma_y$. Note that the intersection occurs at $\sigma_y/\sqrt{3}$ (the `TB,DP` yield function is defined with $q/\sqrt{3}$), so sometimes this is rewritten as $c = \dfrac{(3-\sin\theta)}{6\cos\theta}\sigma_y'$
- The angle of internal friction $\theta$ describes the slope of the yield surface. One can imagine that if the angle is zero, this would imply no dependence on hydrostatic pressure – effectively, this would change the behavior to the Mises yield surface
- There is actually a third parameter for the Drucker-Prager material model – *the dilatancy angle* $\theta_f$ that describes the flow potential. If $\theta=\theta_f$, the flow is *associative*.[1] If $\theta_f=0$, no inelastic volumetric strains will be produced.

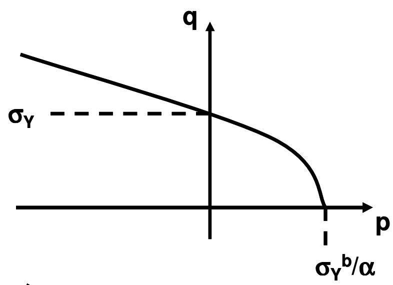
The basic Drucker-Prager material model has the following features:

- Available for `LINK1`, `LINK8`, `PIPE20`, `BEAM23`, `BEAM24`, `PLANE42`, `SHELL43`, `SOLID45`, `PIPE60`, `SOLID62`, `SOLID65`, `PLANE82`, `SHELL91`, `SOLID92`, `SHELL93`, `SOLID95`
- Assumes perfectly plastic behavior (no strain hardening)
- The equivalent stress parameter NL,SEPL provides the yield strength at the current hydrostatic stress level. When yielding occurs, SEQV should equal SEPL.

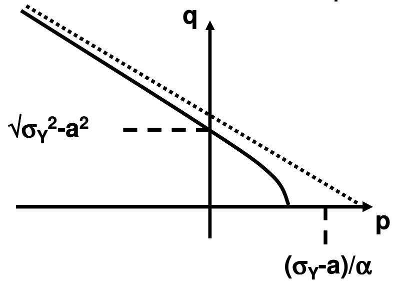
Because of the fact that perfectly plastic behavior is assumed, the user should be careful when solving models with the basic Drucker-Prager material, especially for force-loaded problems, since there would be zero stiffness at the integration point undergoing yielding.

## 4. Extended Drucker-Prager Model:

The Extended Drucker-Prager (`TB,EDP`) is meant to address some shortcomings of the basic Drucker-Prager model – namely, the use of perfectly-plastic behavior and the requirement of a linear yield surface.




The Extended Drucker-Prager model was introduced at ANSYS 10.0 for use with `PLANE182/183` (except for plane stress) and `SOLID185/186/187`.




- A linear, power law, or hyperbolic yield surface can be specified with the `TBOPT` argument of the `TB,EDP` command. This affects the shape of the shear envelope (yield surface) – note how the power law and hyperbolic yield surfaces on the right also create a 'blunt' tip of the cone, which is helpful in preventing numerical difficulties in tension.

[1] Associative flow rules result in symmetric matrices whereas a non-associative flow results in unsymmetric matrices.

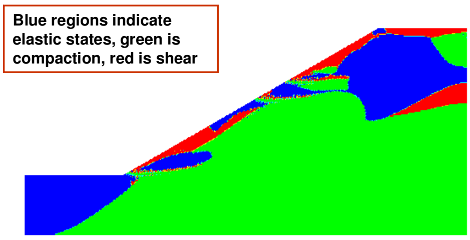
- Strain-hardening behavior is specified by adding an isotropic hardening plasticity model to the same material ID. Although a yield strength is input in `TB,EDP`, if an isotropic hardening law (`TB,BISO/MISO/NLISO/PLASTIC`) is associated with the same material, the yield strength and subsequent strain-hardening behavior is controlled by the isotropic hardening input.
- A flow potential also needs to be specified, although the user can select a different flow potential, if desired.

## 5. Drucker-Prager Cap Model:

Although this input is a subset of the aforementioned Extended Drucker-Prager material (`TB,EDP`), the Cap model should be viewed as a separate constitutive model. The Cap model was introduced at ANSYS 11.0 to give users the ability to define both an expansion and compaction cap. With the regular Drucker-Prager models, the yield strength would increase indefinitely if the hydrostatic compressive stresses kept increasing in magnitude. A compaction cap allows users to define a hydrostatic pressure at which compaction occurs – hence, instead of yielding on the shear envelope, yielding can also occur from compaction or expansion.

Below are some important points about the Cap model:

- Supported elements are the same as the Extended Drucker-Prager model
- The equations are not based on $\sigma_{eqv}$ (q) and $\sigma_m$. Instead, the equations for the Cap model are based on invariants $I_1$ and $J_2$, so the user must keep this in mind.
- The shear envelope (portion of the yield surface dominated by shearing) is defined by both linear (C7) and exponential terms (C5 & C6).
- The hardening of the shear envelope can be defined by adding an isotropic hardening law (`TB,BISO/MISO/NLISO/PLASTIC`). If no hardening law is present, the input yield strength (C4) is used, assuming perfectly-plastic behavior.
- The shape of the compaction cap is defined by $X_i$ (C3), which is 3 times the value of hydrostatic pressure where compaction starts, and by $R^Y{}_c$ (C1), which describes the ratio of $I_1$ to $J_2$.
- The hardening of the compaction cap is defined by constants C9-C11. The $W^c{}_1$ (C9) value is the limiting value of volumetric plastic strain. The $D^c{}_1$ (C10) and $D^c{}_2$ (C11) values are obtained from curve-fitting the volumetric strain vs. 3*pressure ($I_1$) experimental data.
- The expansion cap is decribed by $R^T{}_c$ (C2), similar to C1.
- The yield surface does not necessarily have to be axisymmetric about the hydrostatic pressure axis. C8 allows the user to input the ratio of triaxial expansion strength to triaxial compression strength. The C8 constant is used in the Lode angle function, which results in distorting the yield surface around the hydrostatic pressure axis. C8=1.0 gives a symmetric yield surface.
- For output, NL,SRAT does not give the "stress ratio" but, instead, allows the user to see how the material is yielding – 0 refers to elastic state (no yielding), 1 describes regions that are undergoing cap compaction, 2 shows inelastic behavior due to shearing, and 3 reflects regions undergoing cap expansion.



## 6. Determining which Drucker-Prager Material to Use:

The three Drucker-Prager options available to users have been introduced in the previous section. One can see that the Cap model provides the most features to users, although there are significantly more input parameters to deal with. In general, the following guidelines may help:

- If one requires the Drucker-Prager material with the concrete solid element `SOLID65` or with line/shell elements,[2] use `TB,DP`. In all other cases, use of `TB,EDP` is recommended (linear, power law, or hyperbolic yield functions & flow potentials).
- If modeling compaction effects is desired, use `TB,EDP,,,,CYFUN` (Cap model). Note that flow potential parameters, if non-associative, can also be input via `TB,EDP,,,,CFPOT`.

Because the equations are different between the three models, it is sometimes instructive to understand relationships between the parameters.[3] The table below summarizes the case of *linear* Drucker-Prager yield surface:

| | TB,DP | TB,EDP,,,,LYFUN | TB,EDP,,,,LFPOT | TB,EDP,,,,CYFUN | TB,EDP,,,,CFPOT |
|---|---|---|---|---|---|
| C1 | $c = \frac{(3-\sin\theta)}{6\cos\theta}\sigma_y'$ | $\alpha = \frac{6\sin\theta}{(3-\sin\theta)}$ | $\alpha_f = \frac{6\sin\theta_f}{(3-\sin\theta_f)}$ | | |
| C2 | $\theta$ | $\sigma_y'$ | | | |
| C3 | $\theta_f$ | | | (large value) | 0 |
| C4 | | | | $\sigma_i = \frac{\sigma_y'}{\sqrt{3}}$ | $\alpha_f^Y = \frac{\alpha_f}{3\sqrt{3}}$ |
| C5 | | | | 0 | |
| C6 | | | | 0 | |
| C7 | | | | $\alpha^Y = \frac{\alpha}{3\sqrt{3}}$ | |
| C8 | | | | 1 | |
| C9 | | | | 0 | |
| C10 | | | | 0 | |
| C11 | | | | 0 | |

Remember that `TB,DP` is derived from $q/\sqrt{3}$ and $\sigma_m$ while `TB,EDP` (except for Cap model) is based on $q$ and $\sigma_m$. `TB,EDP,,,,CYFUN` (Cap model) is formulated in terms of $I_1$ and $J_2$.

## 7. Conclusion:

ANSYS provides simple as well as sophisticated Drucker-Prager models to suit different users' needs. The *ANSYS Theory Reference* is the best source to understand the assumptions related to each Drucker-Prager variant, but the author hopes that this memo may have clarified some of the differences and salient points of each.

[2] The use of Drucker-Prager material with beam and shell elements is uncommon, although it is permitted.

[3] Please note that the relationships described for the Cap model are only valid for this linear case. For the nonlinear case, there is no direct relationship between Cap model parameters and other Drucker-Prager models.