Viscoelasticity

Background on Viscoelasticity

- There are two aspects common to viscoelasticity, which involve *time- and temperature-dependency*:

  – Comprised of elastic and anelastic response for deviatoric and/or volumetric strains.

  – Possible temperature-dependecy introduced via

    - Temperature-dependent relaxation constants
    - Thermorheologically simple (TRS) assumption
    - No temperature-dependency

New Viscoelasticity Capabilities

- In 6.1, a new viscoelasticity capability has been introduced
  - SHELL181, PLANE182/183, and SOLID185-187 support new viscoelastic behavior
  - Viscoelastic capability can be combined with both hypo- and hyperelasticity. This provides user with *finite-strain viscoelastic capabilities.*
  - Input is in a much easier, flexible format than previous viscoelastic capabilities (VISCO88/89).
  - Time-dependent response in the form of Prony constants is input with TB,PRONY table.
  - Possible temperature-dependency input:
    - Temperature-dependent Prony constants via TB,PRONY.
   - TRS behavior via shift function, specified with TB,SHIFT.

Anelastic Response
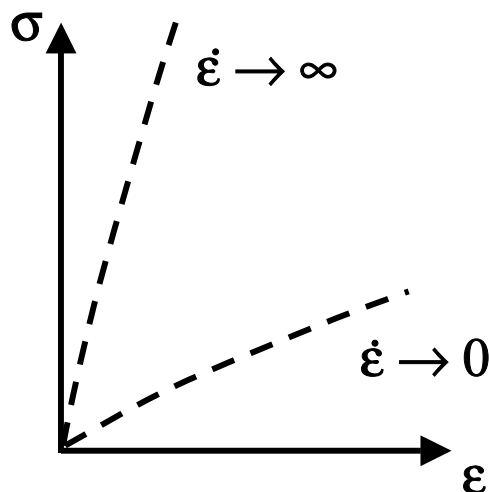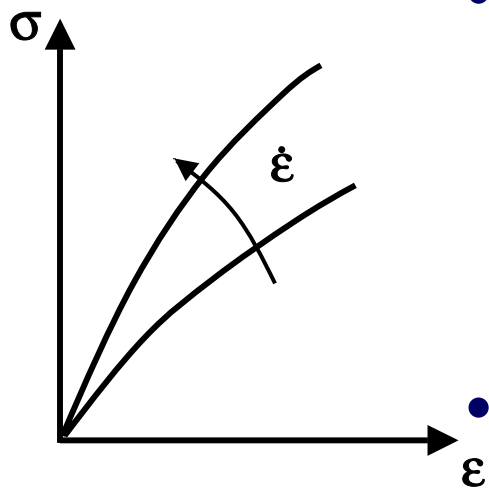
- **Viscoelasticity describes material response which contains an elastic and viscous part**
  - The elastic response is instantaneous
  - Viscous response occurs over time (anelastic)

- **The *rate effect* is such that there is limiting behavior for fast and slow loading**
  - As strain rate decreases, the bulk/shear moduli also decreases
  - For high strain rates, the elastic response is the limiting behavior
  - For low strain rates, the 'viscous' response is the limiting behavior

4

Anelastic Response (cont'd)
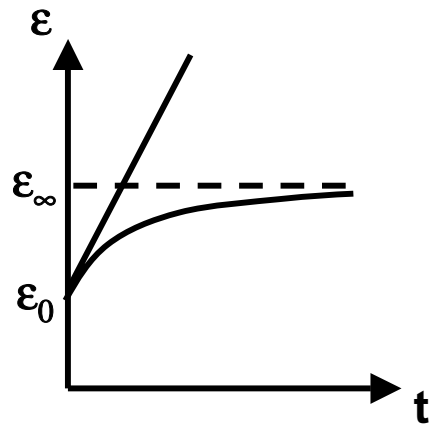
- **Creep**
  - Under constant applied stress, strain increases monotonically.
  - Cases of linear and exponential creep shown on right
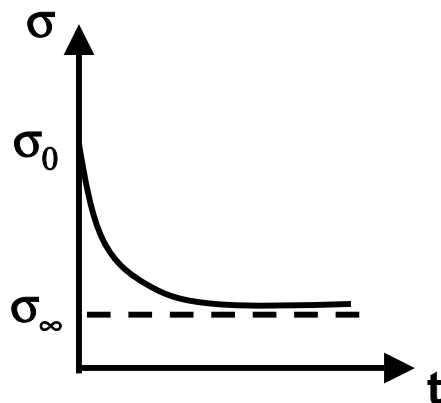
- **Stress Relaxation**
  - Under constant applied strain, stress decreases asymptotically.

Anelastic Response (cont'd)

- **Shear and bulk moduli are functions of time and are represented with Prony series.**

$$G(t) = G_\infty + \sum_{i=1}^{N} G_i \exp\left(-\frac{t}{\tau_i}\right)$$

where τ_i is the relaxation time for each Prony component G_i.

- As with other material behavior, volumetric and deviatoric terms are separated. Shown on right is shear modulus, but similar behavior can be defined for bulk modulus with separate relaxation values.

$$G(t) = G_\infty \left[\alpha_\infty + \sum_{i=1}^{N} \alpha_i \exp\left(-\frac{t}{\tau_i}\right) \right]$$

- Instead of inputting values as shown, we introduce relative moduli $\alpha_i = G_i/G_o$. N values of α_i and τ_i are input for shear and/or bulk moduli.
- Relative moduli and relaxation times can be input as temperature-dependent constants.

TRS Behavior

- **Thermorheologically Simple Behavior**
  - As mentioned earlier, viscoelastic materials can be *time- and temperature-dependent*. Both dependencies may need to be accounted for.
  - Thermorheologically simple (TRS) behavior means that time & temperature are the same phenomenon. This means that the viscoelastic response vs. log(time) function translates with change in temperature.
  - Another way of stating the above is that the material response to a load at a high temperature over a short duration is the same as the response at a lower temperature over a longer duration.

Shifting of Relaxation Modulus with Change in Temperature

G

G(0)

$T_0 < T_1 < T_2$

T_2 T_1

T_0

G(∞)

ln(t)
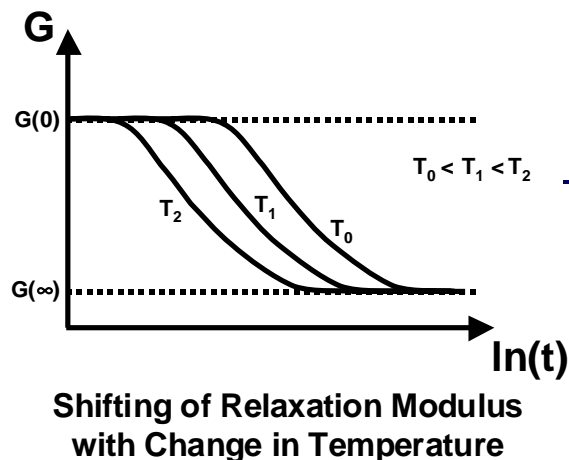
TRS Behavior (cont'd)

- ANSYS's elements which use viscoelastic capabilities can be specified to have TRS behavior. This assumption allows for a relationship with time and temperature dependency. This adequately describes many amorphous polymers.

- A consequence of this is that the short-term $G(t=0)$ and long-term $G(t=\infty)$ moduli will remain the same, regardless of the temperature (i.e., the upper and lower limits on the previous graph will remain the same for any translational shift).

- This allows for the definition of viscoelastic behavior at a one temperature yet captures the response at other temperatures. Using the concepts of reduced time and the shift function (discussed next), the viscoelastic response is 'shifted' to account for behavior at another temperature. Depending on the material, different shift functions are used.

TRS Behavior (cont'd)

- Mathematically, the aforementioned TRS behavior is expressed by the change in the relaxation times of the Prony components

$$t_i(T) = t_i(T_r) \frac{1}{A(T, T_r)}$$

  - The relaxation times for shear and/or bulk moduli must obey a scaling law as shown on the right, where T is the temperature and T_r is the 'reference' or 'base' temperature

  - The *shift function*, A(T), describes the shifting of the response curve. There are two predefined shift functions which can be used in ANSYS, *William-Landel-Ferry* and *Tool-Narayanaswamy* shift functions. A user-defined shift function may also be specified.

TRS Behavior (cont'd)

- In ANSYS, numerical integration is performed with respect to *time.*

$$t_i(T) = t_i(T_r) \frac{1}{A(T,T_r)}$$

$$\xi = \int_0^t \frac{t_i(T_r)}{t_i(T)} dt'$$

$$\xi = \int_0^t \frac{t_i(T_r)}{t_i(T_r)} A(T(t')) dt'$$

$$\xi = \int_0^t A(T(t')) dt'$$

  - Hence, the Prony representation is rewritten with time instead of temperature.

  - It is important to note that this shifting of the response curve is done with *reduced or pseudo time* (ξ). Using reduced time, isothermal equations can now be used to describe non-isothermal processes.

Usage of Viscoelasticity

- Various materials may exhibit viscoelastic response
  - Polymers
    - Usually described by WLF shift function
    - Elastomers (rubber industry)
    - Underfill, mold compound (electronics packaging)
  - Glass
    - Described by TN shift function
    - Volumetric change often important
  - Metals
    - Usually, metal anelastic response is negligible and not considered
  - Other
    - Wood, concrete (Note that SOLID65 does not support PRONY/SHIFT or EVISC)
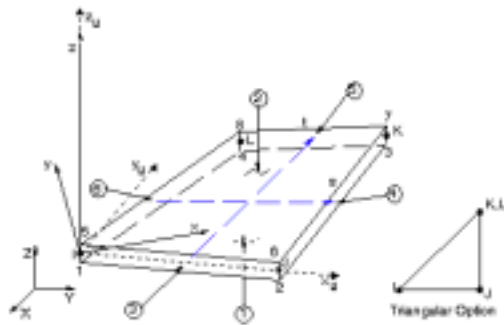
Example of Rubber Bushing
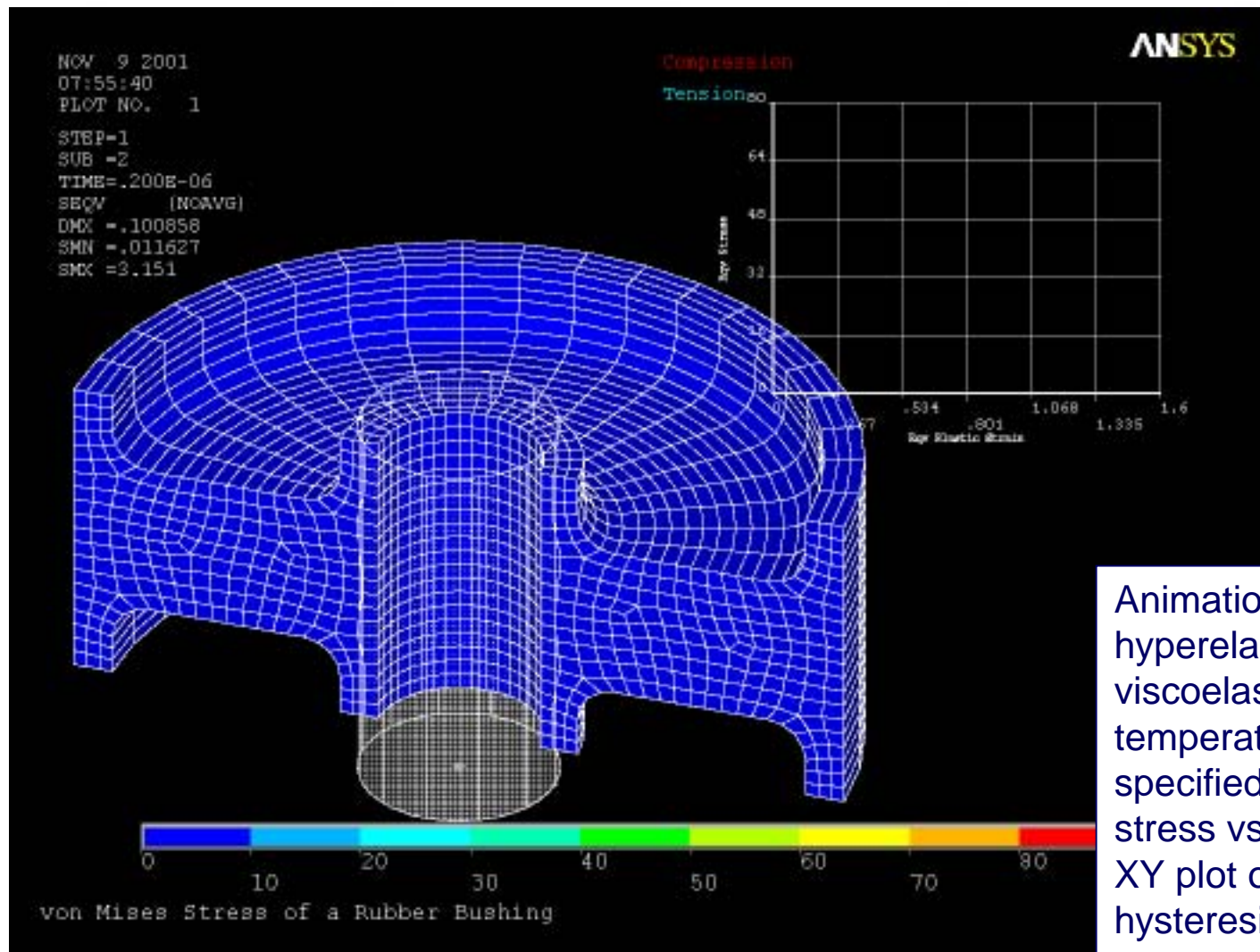
Animation of SOLID185 with hyperelasticity and viscoelasticity. No temperature-dependency specified (isothermal). True stress vs. strain shown in XY plot on top. Note hysteresis during unloading.
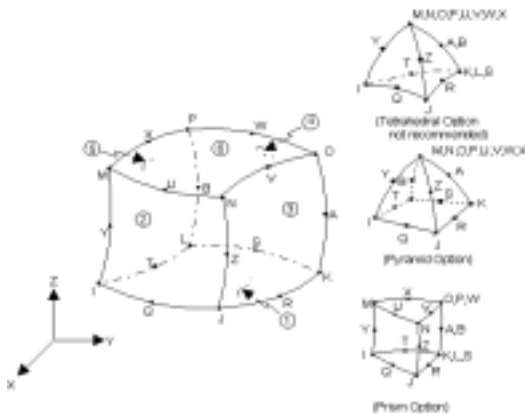
Viscoelasticity Procedure

Supported Element Types

- The new viscoelasticity material model is supported by SHELL181-SOLID187
  - LINK180 and BEAM188/189 are not supported
  - VISCO88/89 still use their own viscoelastic input with TB,EVISC and are not discussed here.

- Large-deformation behavior is supported
  - All of the element options are applicable for viscoelasticity as well, including B-Bar, URI, Enhanced Strain, Mixed U-P.
  - All states of stress for 2D elements are supported: plane strain, plane stress, axisymmetry

Procedure for Viscoelastic Input

- The Materials GUI allows specification of viscoelastic material parameters.
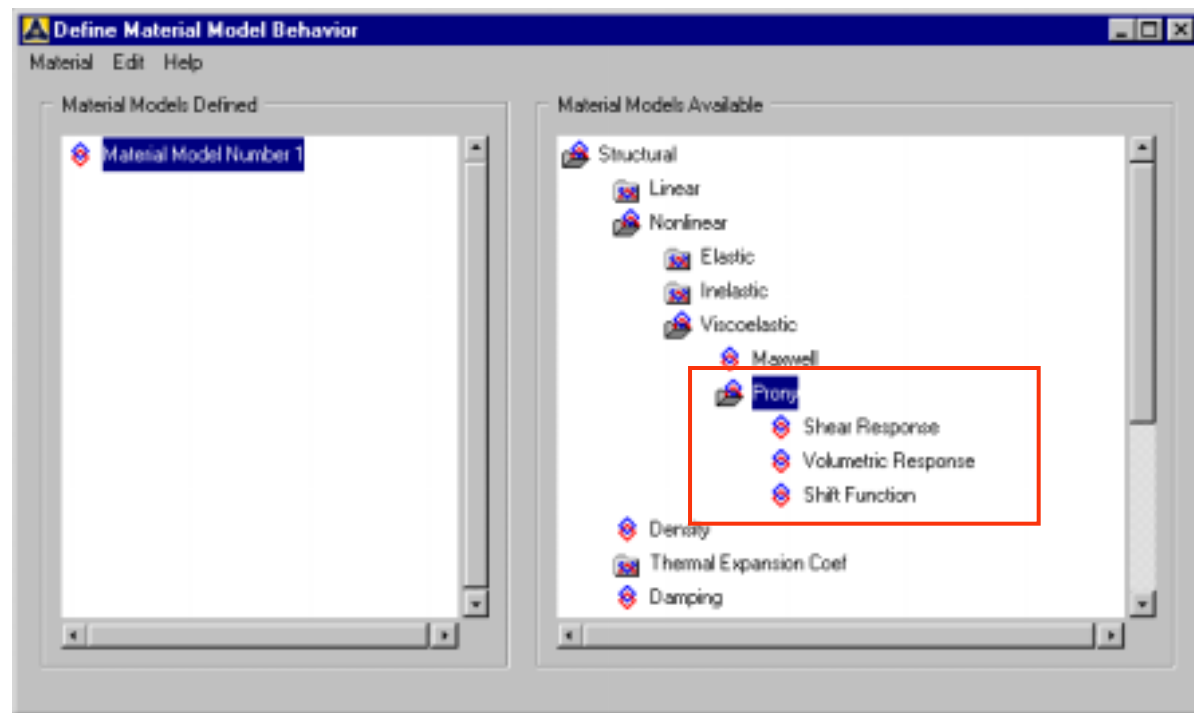
Main Menu > Preprocessor > Material Props > Material Models…

Materials GUI > Structural > Nonlinear > Viscoelastic > Prony

```
MP,EX   ,1,10e6
MP,NUXY,1,0.3
```



```
TB,HYPER,1,1,2,OGDEN
TBDATA,1,5.6,2,-3.8,-2
```
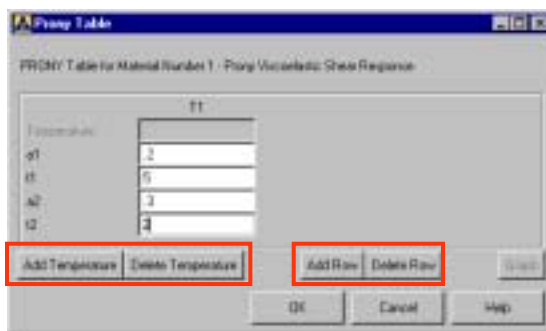
1. Isotropic linear elastic materials (MP,EX & NUXY) or nonlinear hyperelastic materials (TB,HYPER) must be input first.

   - For *hypoelastic* behavior, material response must be isotropic.
     - Currently, orthotropic (MP,EX/EY/EZ) and anisotropic elastic behavior (TB,ANEL) are not supported.

   - For *hyperelastic* behavior, all hyperelastic material models are supported.
     - These include Ogden, Polynomial, Mooney-Rivlin, Neo-Hookean, Arruda-Boyce.

   - All elastic input values can be temperature-dependent

```
TB,PRONY,1,1,4,SHEAR
TBDATA,1,0.2,5
TBDATA,3,0.3,2
```



```
TB,PRONY,1,1,2,BULK
TBDATA,1,0.05,2
```

2. Shear and volumetric response can be input by specifying relative moduli and relaxation times via TB,PRONY.

- Note that shear and volumetric response do *not* have to have same number of Prony constants
- User does not have to input both shear and volumetric response. It will depend on material characteristics.
  - For example, volumetric relaxation is often negligible in many materials. In this situation, only shear response will be specified with TB,PRONY.
- Up to 6 temperature-dependent sets of 6 pairs of constants can be input for deviatoric and volumetric response.
  - Simply use buttons in Materials GUI to add/delete temperature sets or rows (pairs).
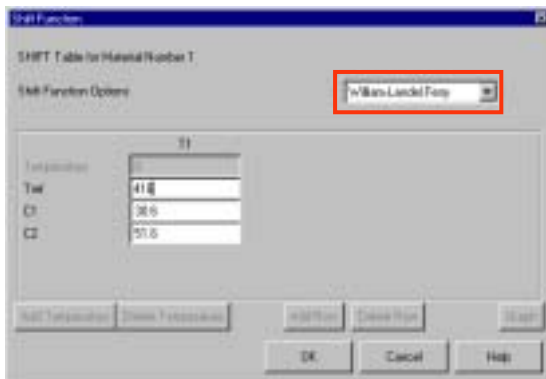
```
TB,SHIFT,1,1,3,1
TBDATA,1,418,30.6,51.6
```

3. The shift function can then be defined with TB,SHIFT. Selection of WLF, TN, or User-defined shift function can be made through the pop-up menu.

- Shift function constants are not temperature-dependent

- Generally speaking, WLF is suitable for polymers while TN is used for glass

- For isothermal case (anelastic behavior only) or temperature-dependent Prony constant input, shift function is not required.

Summary of Material Input

- The below table summarizes the material combinations possible with the new viscoelasticity capability:

	Hypoelasticity	Hyperelasticity
Isothermal Case	MP,EX + TB,PRONY	TB,HYPER + TB,PRONY
Temperature-dependent Prony Constants	MP,EX + TB,PRONY (Up to 6 sets)	TB,HYPER + TB,PRONY (Up to 6 sets)
TRS Assumption	MP,EX + TB,PRONY + TB,SHIFT	TB,HYPER + TB,PRONY + TB,SHIFT

  – With the exception of hyperelasticity, viscoelasticity cannot presently be combined with any other nonlinear material model.
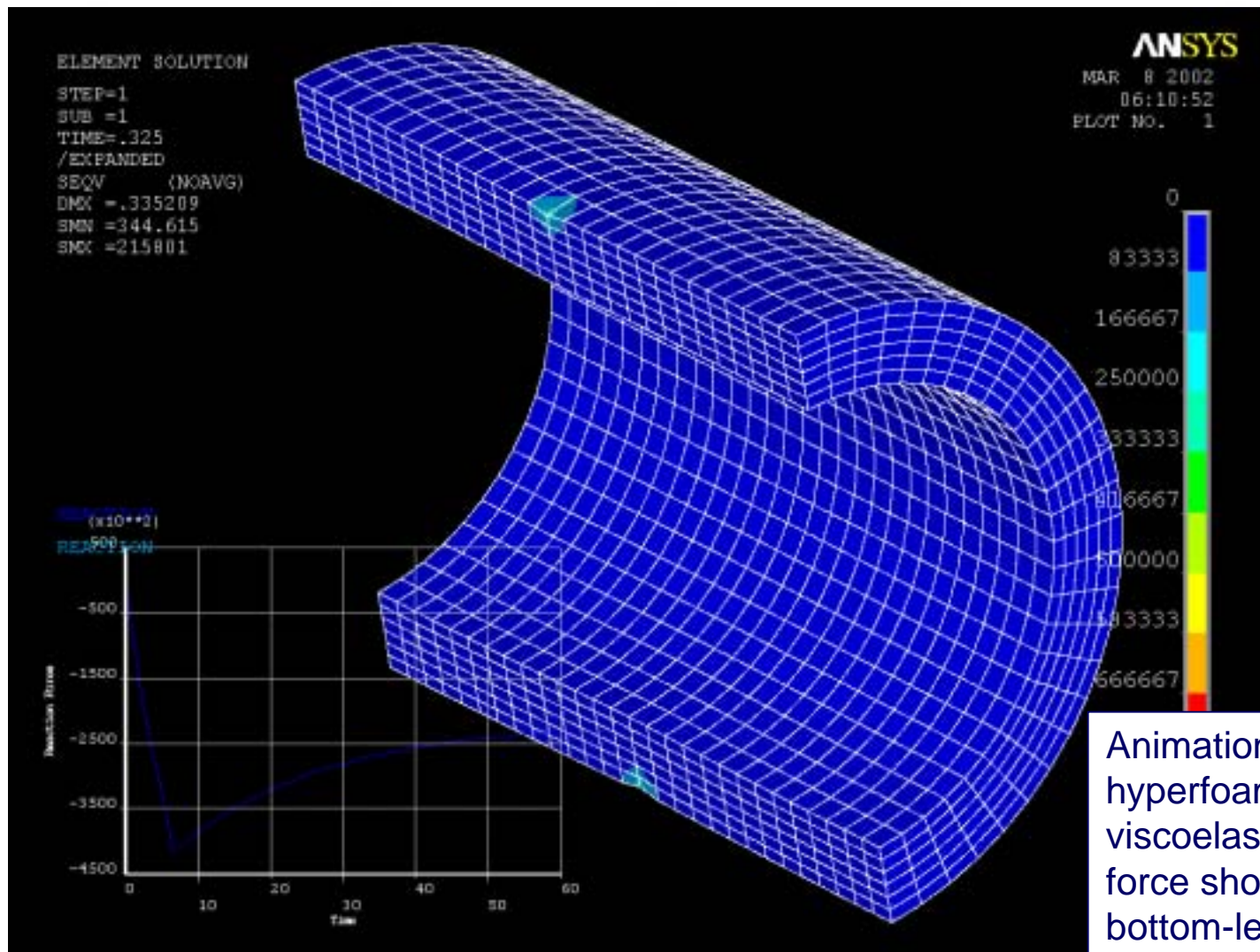
Solving Viscoelastic Problems

- Solution of viscoelastic problems are similar to most general nonlinear problems
  - Use NLGEOM,ON, as needed
  - CUTCONTROL will not take viscoelastic strains into consideration, so user must verify that time step is small enough in transition region
  - For continuum elements, if shear and/or volumetric locking may be a problem, use appropriate element technologies (e.g., B-Bar, Enhanced Strain, Mixed U/P, URI)

Postprocessing Considerations

- Postprocessing considerations are similar to any other material model.
  - Note that if volumetric relaxation is assumed, use care in evaluating equivalent elastic strain results.

Example of Pinched Cylinder

Animation of SOLID185 with hyperfoam model and viscoelasticity. Reaction force shown in XY plot on bottom-left.

Differences with VISCO88/89

For users familiar with older VISCO88/89 elements, the following comparison may be useful:

	SHELL181, PLANE18x, SOLID18x	VISCO88/89
Element Types Supported	SHELL, PLANE, SOLID	PLANE (except plane stress), SOLID
Large Deformation	Yes	No
Viscoelasticity + Hypoelasticity	Yes	Yes
Viscoelasticity + Hyperelasticity	Yes	No
Material Input	TB,PRONY for Prony constants and TB,SHIFT for shift function	TB,EVISC with 95 constants
Shear/Bulk Moduli Input	Instantaenous value determined from MP or TB,HYPER	Instantaneous and long-term shear/bulk moduli need to be specified
Prony Constants	Relative moduli input as $\alpha_i=G_i/G_o$ (Standard, widely-used input format)	Coefficients input as $C_i=G_i/(G_o-G_\infty)$ (Input not as common as other FEA vendors)
Number of Prony Constants	Up to 6 constants per temperature	Up to 10 constants
Temperature-dependent Prony Constant Input	Up to 6 temperature-dependent sets for PRONY (SHIFT constants not temperature-dependent)	No
Shift Functions Available (TRS)	WLF, TN, and User-defined	WLF, TN, and User-defined
Relative Temperature	As input to TB,SHIFT	As input to TB,EVISC for WLF. Use TREF for TN.
Fictive Temperature	No	Yes, and output available
Growth Strain	No	Yes, and output available
Effective Bulk/Shear Moduli Output	No	Yes