FINITE DEFORMATIONS OF AN EARTHWORM SEGMENT

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The displacements of the segment

The earthworm is structured in segments aligned one behind the other. Each segment consists of an elastic membrane, which forms an elastic hollow cylinder filled with liquid. This liquid doesn't change its amount during normal locomotion and could also be considered to be incompressible. Into the elastic membrane two groups of muscles are embedded, the logitudinal and the circumferential ones. This two muscle groups form the antagonists for the motion of a segment.

If the circumferential group of muscles is contracting, while the longitudinal group relaxes, the circumference of the segment will decrease and the length will increase because the volume rests unchanged. Conversly the segment will shorten and the circumference will increase, if the longitudinal muscles contract and the circumferential muscles relax (Figure 1). Because a change of pressure in the liquid filling of the segment transfers forces between the antagonistic groups of muscles, such an arrangement is named hydrostatic skeleton.

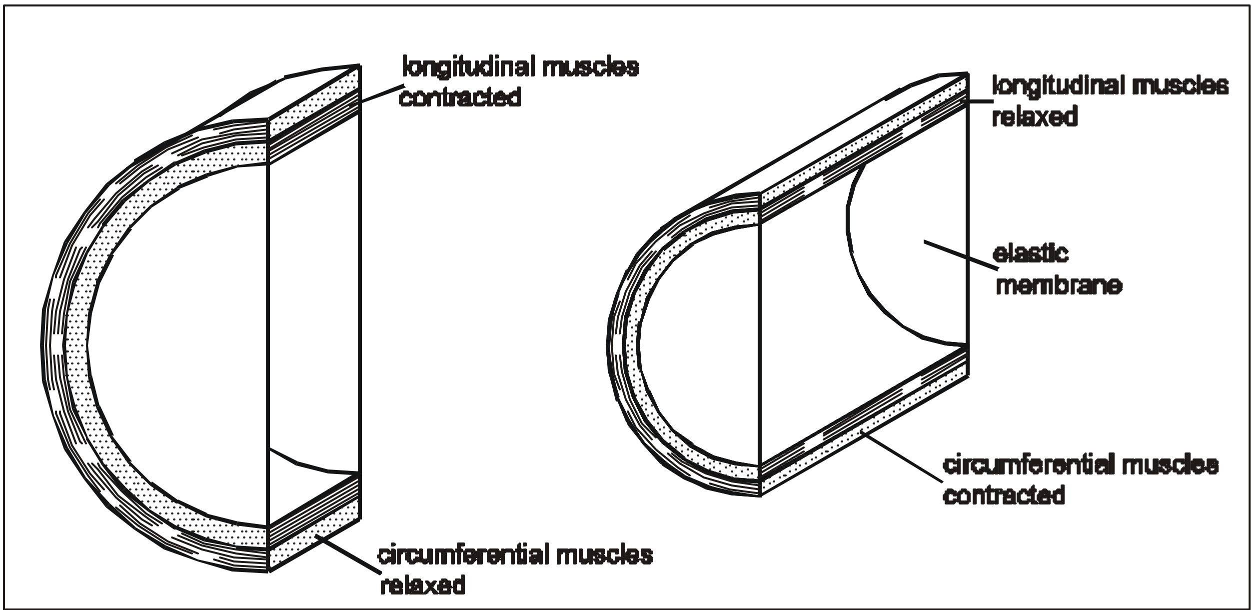

Figure 1: Simplified illustration of an earthworms segment in two states of contraction

Alternating stimulations of the two states pass undulatory the body of the earthworm. These coordinated deformations of the single segments lead to the locomotion of the worm [1], [3].

Requirements of a segment model:
A mechanical model of the segments should yield the displacements of the structure for the given loads, which can be forces or displacements. With the knowledge of this dependance a construction could be varried in order to obtain the desired function between these quantities. For these calculations the method of finite elements is a well suited tool and for the following investigations the finite element program ANSYS was available. In detail the finite element model of the segment has to meet the following conditions:

- Large elastic deformations should be taken into account. The segments of an earthworm for example shorten themselves up to ¼ of their elongated state.
- The volume of the liquid filling remains constant during deformation, so the volume of the space enclosed by the elastic membrane remains constant, too.
- Also the volume of the elastic membrane remains constant during deformation.
- Loads and displacements result in changes of pressure in the fluid and are so transfered to other parts of the structure. On the other hand no shear stresses act in the fluid and no shear stresses are transfered from the fluid to the membrane.
- This investigations will concentrate on deformations of peristalsis locomotion at low velocities, so inertia and dynamic forces of flow can be neglected.

Constitutive laws for finite elastic deformations

Constitutive laws with linear elastic characteristics like Hooks law change their volume during deformation. A simple example shows this property of Hooks law. A finite element with a linear elastic material forms a cube with an edgelength of a=1. It is stretched normal to his front side in x-direction by the amount u (Figure 2).

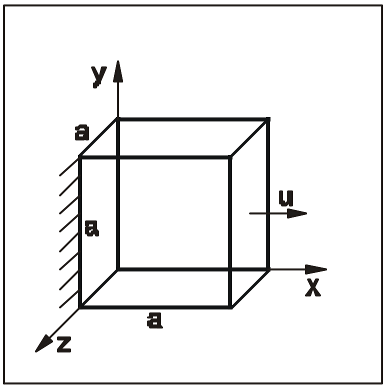

Figure 2: Stretched cube

Then the analytic calculation leads to the strains

$$\varepsilon_{xx} = \frac{u_x}{a}; \qquad \varepsilon_{yy} = \varepsilon_{zz} = -\nu \cdot \varepsilon_{xx} \tag{1}$$

with the Poisson’s ratio ν. The displacements at a right angle to the direction of stretch are obtained from

$$u_y = u_z = \int_0^a \varepsilon_{yy} dy = -\nu\varepsilon_{xx} \cdot a = -\nu u_x \quad . \tag{2}$$

The edgelength in this direction decreases linear with the amount of the supplied stretch. So the edgelength could also become negative. The same result is obtained with the finite elements method.

Because most of the technically used materials support elastic deformations in a range of only some percent, the change of volume in most technical applications rest small and can be neglected. Many biological materials and within the technically used materials the group of elastomeres like silicon and other rubber like materials allow elastic deformations of more than 100% and maintain a constant volume during deformation. For these materials adapted constitutive laws are needed.

The elastic potential:
Elastic material behaviour means that the deformations are reversible and no function of the path, on which they are applied to the structure. These characteristics require the existence of a potential.

The components of stress do work on the accompanying changes of the deformations:

$$\sigma_{ij} d\varepsilon_{ij} = dW \qquad (3)$$

$$W = \int_0^{\varepsilon_{ij}} dW = \int \sigma_{ij} d\bar{\varepsilon}_{ij} \qquad (4)$$

$$\sigma_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}} \qquad (5)$$

Where W is the specific strain energie or the specific elastic potential.

The deformation gradient:
The deformation gradient is defined by the partial derivatives of the coordinates x_i in the deformed state with respect to the coordinates of the undeformed initial state X_j

$$F_{ij} = \frac{\partial x_i}{\partial X_j} \quad . \qquad (6)$$

Changes of volume can be measured as a function of the determinant of the deformation gradient and the condition of incompressibility is

$$\det F = 1 \quad . \qquad (7)$$

The expression

$$B_{ij} = F_{ik} \cdot F_{jk} \qquad (8)$$

is called left Cauchy-Green deformation tensor.

The Mooney-Rivlin strain energy:
The Mooney-Rivlin strain energy is expressed in terms of the first two invariances of the left Cauchy-Green deformation tensor:

$$W = \frac{a}{2}(I_1 - 3) + \frac{b}{2}(I_2 - 3) \quad . \tag{9}$$

With the material constants a and b. The first two invariances are

$$I_1 = B_{ii} \tag{10}$$

$$I_2 = \frac{1}{2}\left(B_{ij}B_{ji} - B_{ii}B_{jj}\right) \tag{11}$$

Expressions of stresses:
The Cauchy components of stress τ_{ij} are measured in the deformed configuration and refer to this configuration. The components of the 1. Piola-Kirchhoff-stress tensor Tij are also measured in the deformed configuration, but they refer to the undeformed state. The following transformation exists between these formulations:

$$T_{ji} = \det F_{ij} \cdot F_{jk}^{-1} \cdot \tau_{ki} \tag{12}$$

With the expression for the Mooney-Rivlin strain energy (9) the Cauchy stresses are obtained from (5) to

$$\tau_{ij} = -p\delta_{ij} + aB_{ij} - bB_{ij}^{-1} \tag{13}$$

Where p is the hydrostatic pressure, a and b are material constants [2]. The special case with b=0 provides the Neo-Hookean constitutive law, which will be assumed in the following. Diagramm (1) shows the stress-stretch relationship of pure tension for the Neo-Hookean law and experimental data obtained from a tensile test with latex rubber. The material constant a is set to a=0.23 N/mm^2 in this example.

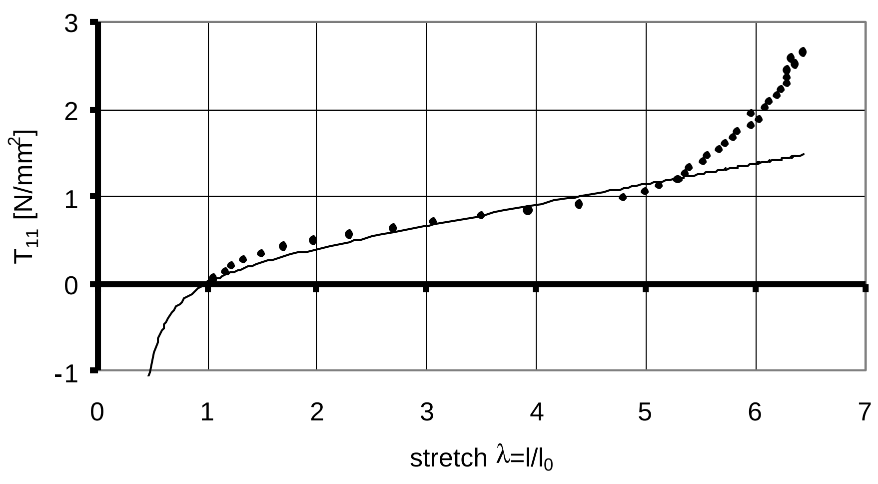

Diagram 1: Neo-Hookean law (line) and experimental data (dots)

Constitutive law of the liquid filling

Solids under shear stress deform by a finite amount. Opposite to the solids fluids under the same load deform indefinitely. This mechanical definition of a liquid assumes that no shear can be transfered by this liquid, so only pressure stress is possible in a fluid. This pressure in any point of the fluid is equal in each direction. A state, which meets these conditions, is called a hydrostatic state of stress.

Therefore a mechanical model of the segments should transfer loads to the solid structure by changes of pressure in the liquid filling. Shear stresses in the fluid and between the fluid and the adjacent solid structure should be at least small, compared with the hydrostatic pressure. The volume of the liquid filling should be constant, if the segment is subject to large deformations.

In the used finite element program no special fluid elements are available, which keep a constant volume while considering large elastic deformations. Hyperelastic elements, which are normally used to calculate constructions made of elastomers, are well suited to consider large deformations and they are able to take into account the condition of incompressibility. So this class of elements could be used to simulate a static fluid, if the material law is adapted in that way, that the loaded element provides a hydrostatic state of stress.

Equation (13) shows, in case of a Neo-Hookean constitutive law, the shear stresses are proportional to the only constant a. Therefore the choice of a small value for this constant yields to an approximation for the hydrostatic state of stress. First calculations brought out, that this statement is only valid for small amounts of deformations, because there is an increasing divergence to the hydrostatic state of stress with increasing deformation. The constant a can’t be chosen near zero as well, because for too small values numerical difficulties appear.

Coupling of liquid filling and elastic membrane

To simulate the liquid filling in the following finite element calculations, hyperelastic elements, normally intended to simulate elastomers, are used. The material constants for these elements are selected in that way, that the characteristics of a very compliant rubber are obtained, which can be considered to be an approximation of a static fluid for small strain. To point out this approximation, these elements are refered as „rubber fluids“ in the following.

To use these „rubber fluids“ a strategy is still needed to obtain a hydrostatic state of stress for large deformations. This strategy can be developed using a characteristic property of fluids. Differently to elastic solids the state of stress in a fluid can be calculated, if the load applied to the fluid and its actual configuration are known. The knowledge of an unloaded and unstressed initial configuration is not necessary. Using this property an iterative method can be proposed.

The external loads are applied to the entire structure in one step and the solution is calculated for this configuration. Subsequently the state of stress in the solid parts of the structure is conserved, while the stresses in the fluid parts are set to zero. This configuration is the new initial state for the next solution. The calculation starts with the deformed structure of the precedent iteration step. In this state the elastic solid parts are prestressed, while the fluid parts are free of stress. With the results of this calculation the next iteration loop is started. To terminate the iteration loop a convergence criterion is needed. The divergence to the hydrostatic state of stress for instance is a practical measure to terminate the iteration.

Cylindrical membrane with „rubber fluid“ filling

In the following example the segment is simplified by removing the front ends of a hollow cylinder filled with liquid. It is difficult to reproduce these theoretical arrangements in an experiment. Nevertheless it is useful to verify the finite element calculations because the deformations are rather simple.

A thin cylindrical membrane is filled with the „rubber fluid“. The boundaries of the cylindrical membrane are free and pressure is applied to the front ends of the filling. Axial and rotational symmetry can be used. So a plane quarter is sufficient if the boundary conditions and element options are adapted. Elastic membrane and liquid filling are replaced by one hyperelastic element each (Figure 3). For both elements a Neo-Hookean constitutive law is used. The material constants are chosen so that the value of the only constant for the „rubber fluid“ is 1% of the value of the elastic membrane. At the contact zone of the two elements the degree of freedom in the radial direction is coupled, so the initial nodes at the same position in this zone are subject to the same radial displacement.

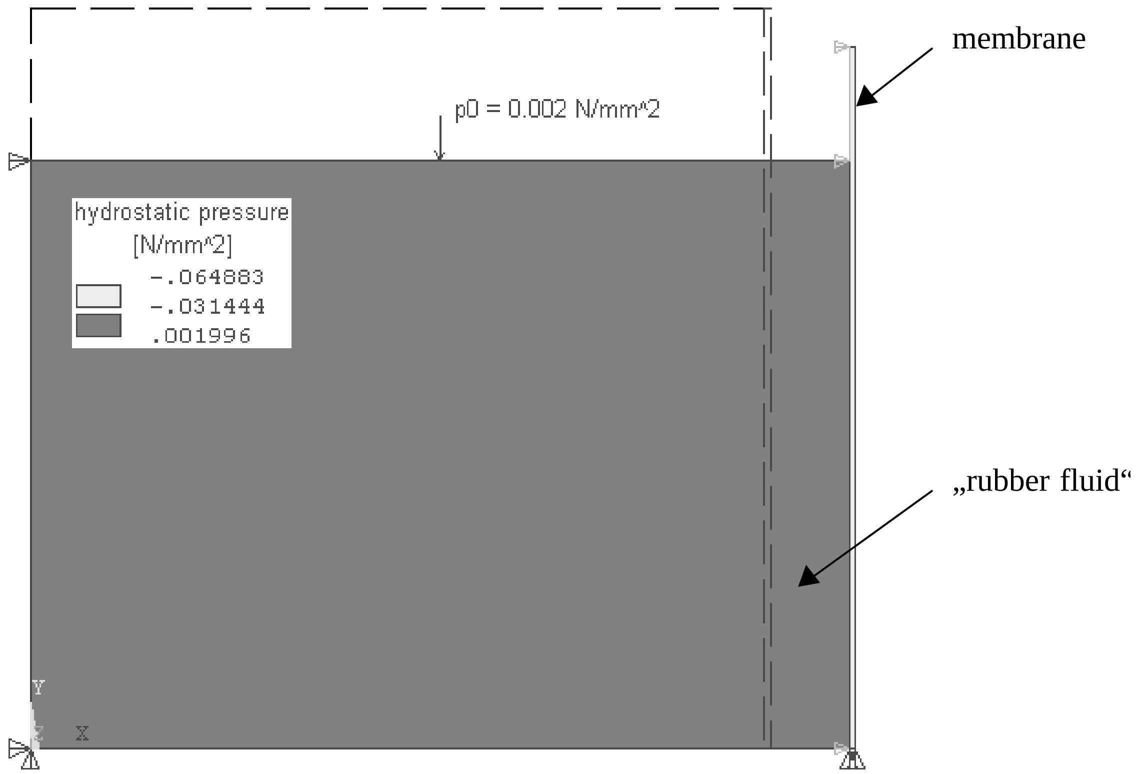

Figure 3: Deformed and undeformed (broken line) structure after 20 iterations

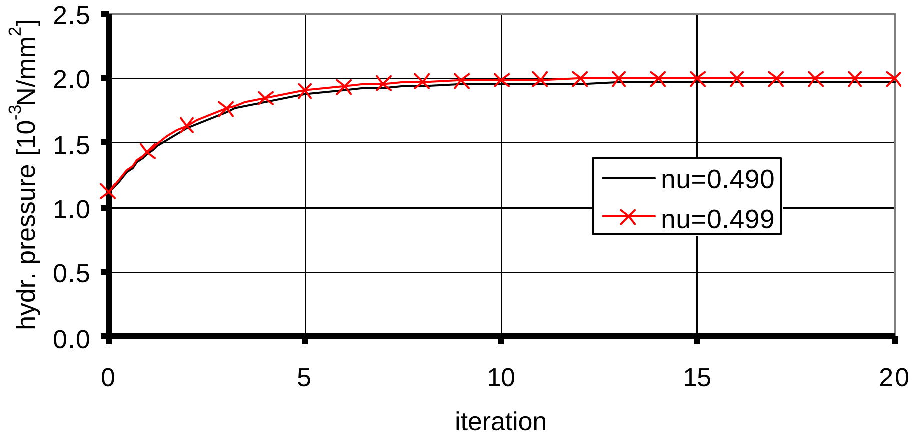

Diagram 2: Hydrostatic pressure in the „rubber fluid“ for different values of ν

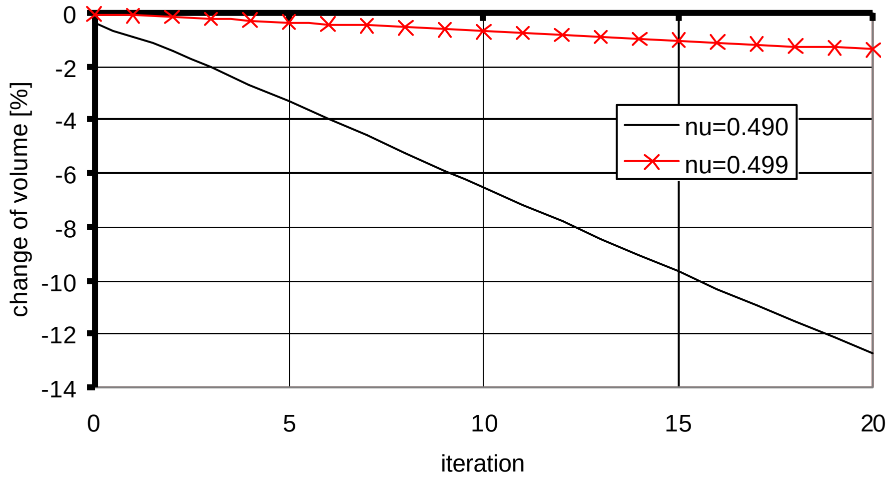

Diagram 3: Cumulative change of volume of the liquid filling for different values of ν

Results

The hydrostatic pressure in the liquid filling reaches the expected amount (Diagram 2). A defect in volume of the „rubber fluid“ accumulates with increasing number of iterations (Diagram 3). The reason for the decreasing volume is the Poisson's ratio ν. The value should be ν=0.5 to maintain a constant volume. In the finite element formulation the value must be smaller than 0.5. Therefore the choice of ν=0.499 for the Poissons's ratio seems reasonable. Furthermore it should be avoided to perform more iteration loops than needed to reach the hydrostatic state of stress.

The radial displacement of the elastic membrane related to the initial radius reaches ΔR/R0=0.115 in this example. If the „rubber fluid“ is replaced by internal pressure an analytical calculation can be performed. It leads to a change in radius of ΔR/R0=0.142. The difference between the analytical calculation and the results obtained numerically can be explained by the fact, that the contact zone in the deformed state is smaller than the inner area of the membrane (Figure 3). Thus the force acting on the membrane is also smaller than in case of directly applied internal pressure. So the obtained displacements seem to be explicable. Nevertheless it would be interesting to perform a comparing experiment.

The deformations for the entire cylinder with liquid filling are calculated using 441 elements (Figure 4).

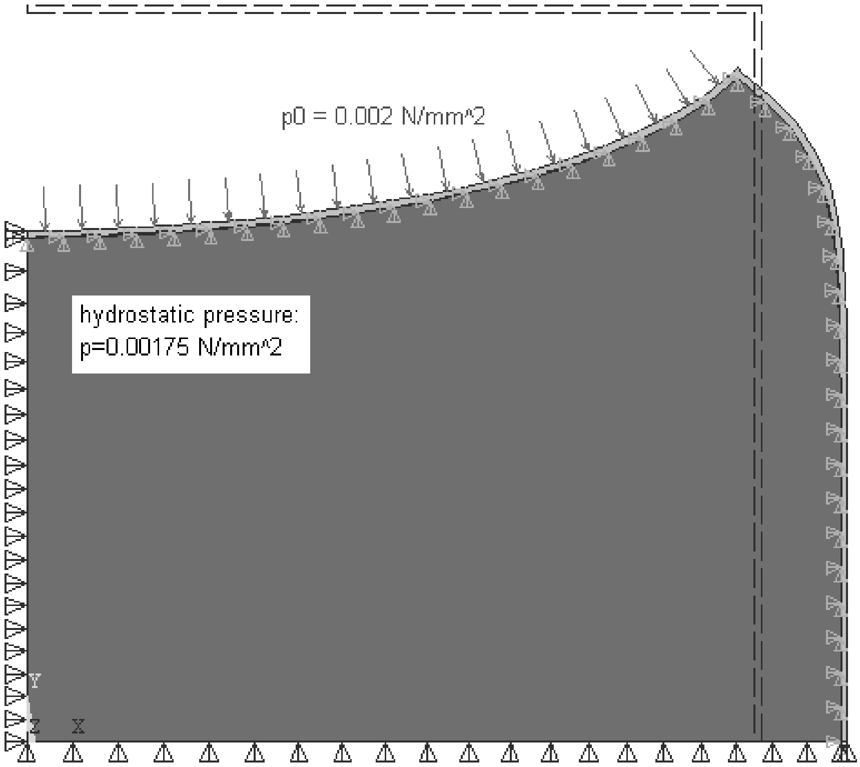

Figure 4: Deformed structure of the liquid filled cylinder after 20 iterations. The contour of the undeformed membrane is shown with broken lines.

References

[1] Gray, J.: Animal Locomotion. London, Beccles: William Clowes & Sons 1968.
[2] Green, A. E., Zerna, W.: Theoretical elasticity. Oxford: Clarendon Press 1954.
[3] Peters, W., Walldorf, V.: Der Regenwurm Lumbricus Terrestris L., Eine Praktikumsanleitung. Heidelberg, Wiesbaden: Quelle und Meyer 1986.

Summary

The body of earthworms is structured with the same kinds of segments. Each of these segments is consisting of a compliant integument filled with liquid. Their locomotion is based on coordinated deformations of their segments. A model based on finite element method is proposed to calculate deformations of such a segment. For the compliant integument of this model the constitutive equations are selected and an approximation is proposed to take into account static effects of liquid.

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