MULTI-CRITERIA SHAPE OPTIMIZATION OF A FUNNEL IN CATHODE RAY TUBES USING RESPONSE SURFACE MODEL

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ABSTRACT

The ultimate goal of simulation that represents the behaviour of structures is to optimize their response performances within the specific requirements and needs with respect to the design variables. The first step of the design of cathode ray tubes is to design the glass geometry, called funnel geometry, to endure the vacuum stress because it is a main structure of cathode ray tubes. In order to create 3-dimensional funnel geometry in the cathode ray tubes, higher order response surface model is used instead of NURBS (non-uniform rational B-splines) or Bezier curve because it is more robust for understanding the geometry change in finite element analysis. By combining finite element analysis, response surface model and sequential quadratic programming within the process integration framework, the shape optimization of a funnel is successfully performed and the maximum stress is reduced to almost half of the current one.

INTRODUCTION

Multi-criteria shape optimization based on finite element analysis has been increasingly concerned in the practical applications but it is difficult to describe the continuous shape changes without the mesh distortion of finite element analysis. In order to characterize the continuous shape changes with a finite number of design variables, the reduced-basis method that a few of design vectors are often used to sufficiently describe the shape changes in finite element analysis has been implemented into finite element analysis in the literature.

In this research, higher order response surface models based on design of experiments are proposed as one of the ways to represent the continuous shape changes for multi-criteria shape optimization instead of using the reduced-basis method. Design of experiments is utilized for exploring the design space and for constructing the response surface models to facilitate the effective solution of multi-criteria optimization problems. Response surface models provide an efficient means to rapidly model the trade-off among many conflicting goals under given constraints.

Usually in most practical optimization processes, engineers want to know which goals are in their degree of uncertainty, the so-called confidence levels, or not. In order to manipulate the confidence level of multi-criteria optimization, conventional optimization method can be revised by using the fuzzy set theory that is called weighted geometric mean and product operator. The confidence level can be described by a design range and a fuzzy membership function. Fuzzy decision-making algorithm is then introduced and investigated to manipulate the engineer's confidence level in the optimization process because the fuzzy model may be used to quantify the confidence level in the range from 0 to 1. The membership function value of 1 indicates the greatest confidence and the value of 0 corresponds to the weakest belief of the objective function. Fuzzy set theory is utilized to form a realistic description of the optimization objectives. The membership function reflects the degree of certainty from the viewpoint of engineer associated with the confidence level of multi-criteria functions. For simplicity, a linear variation of membership is employed in this study. In order to combine the membership values of the individual objective functions with a single aggregated value of membership, weighted geometric mean method is used.

The proposed steps for implementing the multi-criteria optimization procedure based on the fuzzy decision-making algorithm and response surface model used in this study are explained as follows. The procedure that facilitates solving multi-criteria optimization includes three major steps:

Step 1: to eliminate relative unimportant shape design variables and make model reduction by using sensitivity and screening techniques such as stepwise regression design.
Step 2: to build the responses surface models to relate each response to all shape design variables using design of experiments for shape optimization.

Step 3: to use fuzzy geometric mean to find multi-criteria optimization solution combined with sequential quadratic programming.

To verify the developed procedure, the finite element commercial program of ANSYS, is used to explore and optimize the trade-off between weight and stress of a funnel in cathode ray tubes. In this research, The finite element analysis is automatically simulated by using the process integration framework called ModelCenter and the response surface models of funnel geometry are constructed from the design of experiments tables of ANSYS. Sequential quadratic programming algorithm with fuzzy product operator is successfully utilized for shape optimization of a funnel in cathode ray tubes and reduced the maximum stress level to almost half of the current one.

FUNNEL MODEL IN CATHODE RAY TUBES

Recently in the area of display research, Flat Panel Display (FPD) is developed together with LCD, PDP and OELD. In the research of FPD, it is required to reduce the depth of cathode ray tubes for space saving. When reducing the depth of cathode ray tubes, the most important component is a glass which should basically withstand the vacuum stress between inner and outer pressures. First an ideal arch-like shape of funnel geometry is designed to distribute vacuum stress from the shape of current product. However, in order to reduce the depth of cathode ray tubes under yield stress, the arch-like shape of funnel should be changed. As a result of that, the shape optimization of funnel geometry is necessary.

FINITE ELEMENT MODEL

The model has been constructed using commercail finite element program based upon the coordinates of a real funnel in cathode ray tubes for computer simulation [ANSYS]. Finite element modeling includes a geometrical shape construction, mesh generation, boundary conditions and loading. A three-dimensional solid element of SOLID45 is used for modeling the funnel in cathode ray tubes.

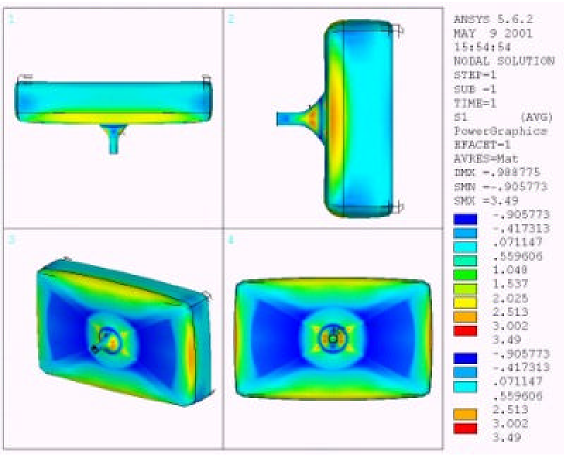


Figure 1. The 1st principal stress of a funnel

The developed funnel model has 6171 nodes and 4028 elements. The 1st principal stress of initial model is 3.49 kgf/mm^2 that is higher than the yield stress. Therefore the stress level hould be minimized under the yield stress. Figure 1 shows the 1st principal stress of funnel. Finite element analysis by solid elements is not suitable for shape optimization because it takes too much time for computer simulation. Instead of using solid elements, the simplified shell elements are recommended and implemented in this study. The meshed view of the funnel model by shell elements is shown in Figure 2. This simplified funnel model has 2665 nodes and 5168 elements.

There are three components for constrcting a cathode ray tubes such as panel, cone and funnel. Among three components, the shape of funnel should be changed to minimize the stress and weight of cathode ray tubes because cone and panel have no enough degree of freedom to change the geometry. In order to define the design variables for geometry of funnel, three ANSYS commands are used:

- SPLINE: It generates a segmented spline through a series of keypoints. It is used for defining the shape geometry of funnel.
- ASKIN: It generates an area by "skinning" a surface through guiding lines. It is used for generating Finite Element mesh of funnel together with AMESH command.
- AMESH: Generates automatically nodes and area elements of funnel within areas. By using AMESH command, shape change is automatically updated and used for shape optimization.

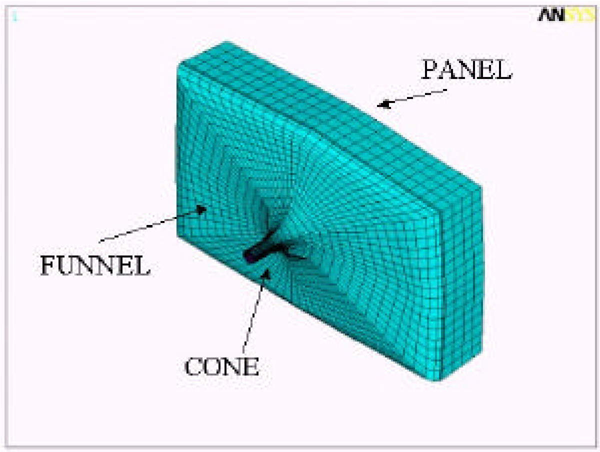


Figure 2. The meshed view of the funnel model by shell elements

Because the inside state of a cathode ray tubes forms a vacuum, boundary conditions are applied to describe this state as shown in Figure 3. The inner pressure is 1.3604e-12 kgf/mm^2 and the outer one is 1.0332e-2 kgf/mm^2 in order to represent the vacuum state. The fixed boundary conditions are also applied to the bottom of the cone, short and long axis of funnel in order to prevent movement and rotation of the model as shown in Figure 3.

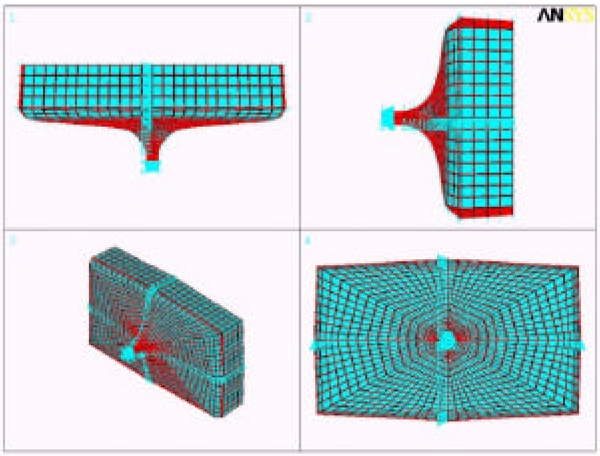


Figure 3. The boundary conditions of the funnel model

There is no difference of funnel geometry between solid and shell elements because they use the same coordinates. After finishing shape optimization by using shell elements, the optimal solution is validated from solid elements.

FUNNEL GEOMETRY DEFINITION

In order to create 3-dimensional funnel geometry in the cathode ray tubes, three axes of funnel geometry are defined as short axis, long axis and diagonal axis shown in Figure 4. Only a quarter model is generated and expanded to full model because of geometric symmetry. Each axis is composed of 16 key points to define 3-dimensional funnel geometry. The segmented splines are generated from 16 key points of each axis by SPLINE command and translated to areas for Finite Element mesh by ASKIN command. Finite Element mesh are automatically generated by AMESH command.

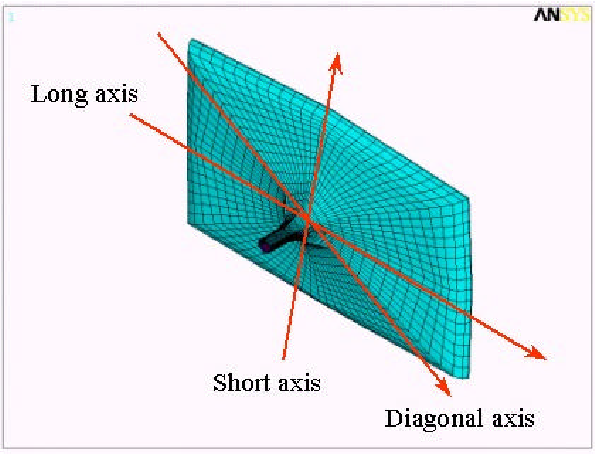


Figure 4. Three axes of funnel geometry

In order to generate 3-dimensional funnel geometry, 16 key points of each axis should be parameterized as design variables. In this study, 5^{th} order polynomial model is used instead of NURBS or Bezier curve because it is more robust for the numerical noise existed in Finite Element analysis. The coefficients of 5^{th} order polynomial model in equation (1) is calculated from ordinary least square method where x means long axis, y means short axis and z means the funnel coordinates caclulated from long axis and short axis. For shape optimization of the 3-dimensional funnel geometry, the range of allowable funnel geometry is determined by changing the coefficients of 5^{th} order polynomial model compared with space limitation of the cathode ray tubes.

$$z = b_0 + \sum_{i=1}^{5} b_{x_i} x^i + \sum_{j=1}^{5} b_{y_i} y^i \qquad (1)$$

Table 1. 7 design variables for funnel geometry

Axis	Design variables	Number of design variables
Long	l1c, l1x2	2
Diagonal	l2c, l2x2, l2y2	3
Short	l3c, l3y2	2

Table 1 shows the final 7 design variables which explain the full range of funnel geometry where l1c means a coefficient of constant and l1x2 means a coefficient of x^2 (square) in long axis.

PROCESS INTEGRATION

In order to generate 16 key points for finite element simulation and update automatically simulation input file, the framework for process integration and automation is necessary. ModelCenter process integration framework for integrating the different tools and components, called COTS (commercial-off-the-shelf), is used to perform the simulation program and reach the shape optimization by employing sequential quadratic programming, siumltaneously.

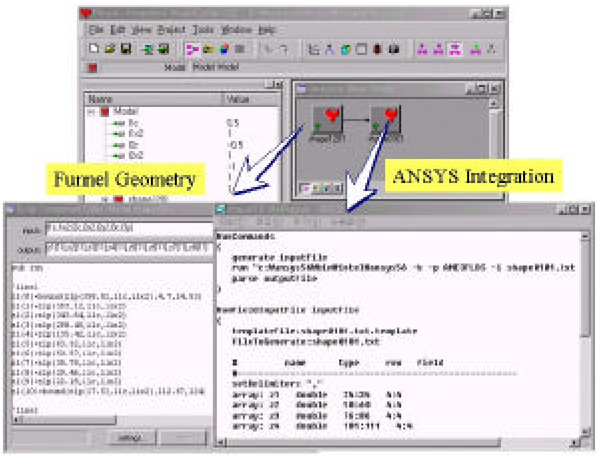


Figure 5. The integration process within ModelCenter framework

ModelCenter based on JAVA provides a highly visual environment to link applications and map key data from one analysis tool to another residing on different computers within a network [Phoenix Integration]. The 5^{th} order polynomial model is integrated within ModelCenter framework by a Visual Basic script component to generate 16 key points for finite element program input file. The integration process is shown in Figure 5 where the simulation is run in batch mode.

SCREENING DESIGN

The main purpose of screening deign is to identify the important design variables from the many possible ones with minimum numbers of simulation. Most screening designs are obtained by using fraction of the 2^p factorial design such as Plackett-Burmann, orthogonal array and fractional factorial design [Mayer & Montgomery].

In this funnel design as shown in Figure 6, the Orthogonal Array called Taguchi's $L_8 2^7$ is used for screening design because a funnel model has 7 design variables. In order to check the nonlinear characteristics, one center point is added in $L_8 2^7$ array where the responses are 1^{st} principal stress and volume. The 1^{st} order regression models of two responses, 1^{st} principal stress and volume, are generated by Eq. (2) for design space exploration where k means the number of design variables, x means design variables and y means responses.

$$y = b_0 + \sum_{i=1}^{k} b_i x_i \qquad (2)$$

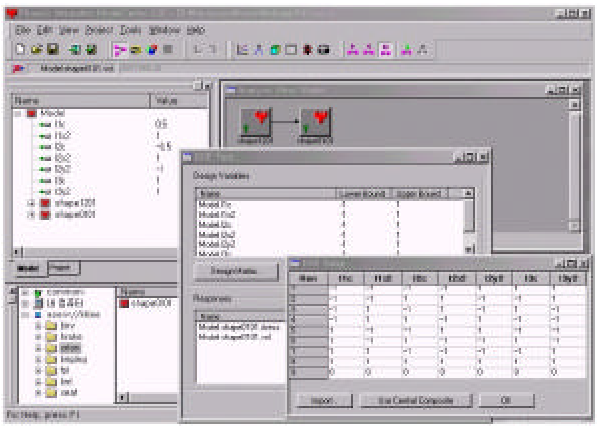
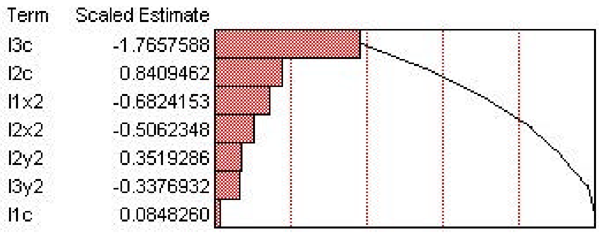


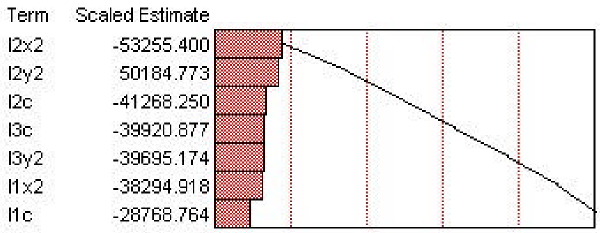
Figure 6. Orthogonal array for the screening design

Though Taguchi's $L_8 2^7$ array is used in this study, it will be better for performing a regression analysis instead of ANOVA (Analysis of variance) analysis that was widely used in industrial fields because all design variables are continuous. Figure 7 shows the Pareto plots of 1^{st} principal stress and volume [SAS Institute]. From Figure 7 (a), l3c has almost 40 % influence but l1c has no influence for 1^{st} principal stress. Figure 7 (a) shows the 4 design variables (13c, l2c, l1x2 and l2x2) have almost 80 % influence. In the case of Figure 7 (b), there is no dominant design varable for volume of funnel. It means that engineers can change any design variable for their own prurpose because all design variable have nearly equal influence.

Figure 8 shows the prediction profiler generated from 9 finite element simulations and generally used for design space exploration. Design space exploration from Figure 8 shows the 5 design variables have the same tendency and only two design variables have the trade-off between 1^{st} principal stress and volume. The 5 design variables which have the same tendency are l1x2, l2x2, l2y2, l3c and l3y2 and they are fixed to [-1, -1, -1, 1, -1] from design space exploration. The remaining 2 trade-off design variables are l1c and l2c and expanded to 2^{nd} order response surface model for shape optimization.



(a) 1^{st} principal stress



(b) volume

Figure 7. The Pareto plots of two responses

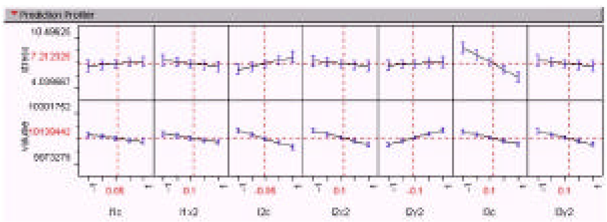


Figure 8. The prediction profiler of design variables

RESPONSE SURFACE MODEL

From above screening design, two important design variables, l1c and l2c, are selected for 2^{nd} order response surface model. By using 2^{nd} order response surface methodology in optimization, engineers can take the following benefits:

- A "snapshot" of the "surface" affected by a set of design variables over some specified design space,
- The settings of design variables which will satisfy desirable specifications, and
- The settings of design variables which will yield a maximum (or minimum) response by performing optimization analysis

The response surface model is estimated by Central Composite Design which consisted of 4 edge points, 4 star points and 1 center point [Roux & Haftka]. The 2nd order response surface is estimated from the 2nd finite element simulations as follows:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_{x_i} x_i + \sum_{i=1}^{k} \sum_{j=i}^{k} \beta_{x_i x_j} x_i x_j \qquad (3)$$

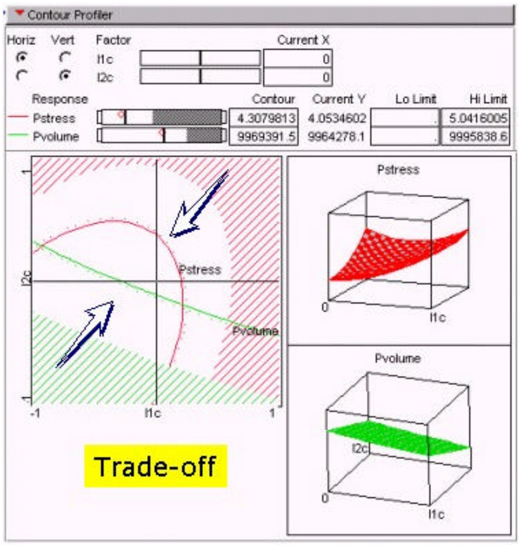


Figure 9. The contour profiler of design variables

By using contour profiler in Figure 9 generated from response surface models, the trade-off tendency between 1st principal stress and volume is explored graphically. From this contour profiler, engineers can finish design process if they are satisfied with an optimal solution from some alternatives what they generate. But it is more efficient to use the multi-creteria optimization method based on response surface model for solving trade-off problem because response surface model replaced the time consuming finite element analyses within interesting design space without any additional finite element simulation.

MULTI-CRITERIA OPTIMIZATION

Among several methods to deal with nonlinear multi-criteria optimization, the weighting, compromise and max-min programming approaches are three typical numerical techniques widely applied on engineering design. The compromise decision support problem is also one of multi-criteria mathematical construction that enables engineers to determine the values of design variables that satisfy a set of constraints to achieve a set of responses, i.e., multi-criteria. The objective in a compromise decision support problem is to minimize the deviations of different responses from target values using lexicographic minimization. In this study, the product of deviations of different responses within max-min response ranges is minimized by using fuzzy set theory called fuzzy geometric mean.

FUZZY SET THEORY

Usually in most practical optimization processes, engineers want to know which responses are in their confidence levels or not. In order to implement the confidence level of multi-criteria optimization, the widely used conventional concept of optimization could be changed by using the fuzzy set theory. The confidence level can be formulated by a fuzzy membership function which represents design space. Fuzzy set theory is used to formulate a realistic meaning of the objective function. The membership function explains the degree of certainty from the engineer's viewpoint associated with the confidence level of multi-criteria optimization. The membership function value of 1 means the greatest confidence and the value of 0 indicates to the weakest belief of the objective function. For simple explanation, only a linear variation of membership function is used in this study.

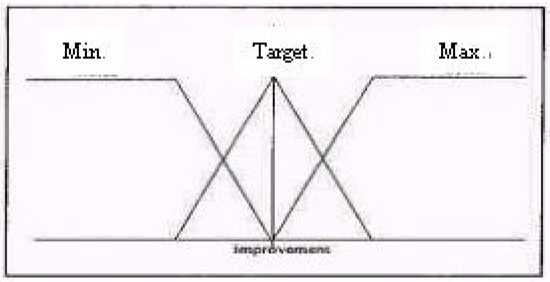


Figure 10. Three types of linear membership functions

Figure 10 explains the three types of linear membership functions that are assigned to the objective functions. The minimization of membership functions in whole engineer's design space is normalized with the $Y_i^{\max}(x)$ and $Y_i^{\min}(x)$ of each objective function, respectively, and expressed as follows, where the maximization of $Y_i(x)$ is equivalent to the minimization of $-Y_i(x)$:

$$\mu_i(x) = \begin{cases} 1 & if\ Y_i(x) \le Y_i^{\min} \\ \dfrac{Y_i^{\max} - Y_i(x)}{Y_i^{\max} - Y_i^{\min}} & if\ Y_i^{\min} < Y_i(x) < Y_i^{\max} \\ 0 & if\ Y_i(x) \ge Y_i^{\max} \end{cases} \qquad (4)$$

With a specific target of responses, two-sided transformation to create the target membership function is expressed as follows:

$$m_i(x)=\begin{cases}\dfrac{Y_i(x)-Y_i^{\min}}{Y_i^T-Y_i^{\min}} & if\, Y_i^{\min}< Y_i(x)\le Y_i^T\\ \dfrac{Y_i^{\max}-Y_i(x)}{Y_i^{\max}-Y_i^T} & if\, Y_i^T < Y_i(x) < Y_i^{\max}\\ 0 & if\, Y_i(x)\le Y_i^{\min}\, or\, Y_i(x)\ge Y_i^{\max}\end{cases} \tag{5}$$

Multi-criteria optimization integrated with the fuzzy membership function corresponding to each engineer's confidence level, $m_i(x)$, is constructed as follows by using the method of product-operator sometimes called fuzzy geometric mean operator [Shih]:

$$Maximize\,[\prod_{i=1}^{n} m_i(x)]^{\frac{1}{n}} \tag{6}$$

SHAPE OPTIMIZATION OF FUNNEL

In the case of funnel optimization, there are two responses of 1st principal stress and volume. Because the 1st principal stress should be minimized under given yield stress, the optimization problem is mathematically defined as:

$$\begin{aligned}&Maximize\, m_{stress}(x)\\ &subject\,to\;\; m_{volume}(x) > m_{volume}^{initial}\end{aligned} \tag{7}$$

In addition to equation (7) which is performed by constraint minimization, two alternatives are proposed from the view point of stress minimization by using unconstraint minimization as follows:

- Maximization of overall confidence level

$$Maximize\left(m_{stress}(x)\times m_{volume}(x)\right)^{1/2} \tag{8}$$

- Maximization of stress confidence level

$$Maximize\, m_{stress}(x) \tag{9}$$

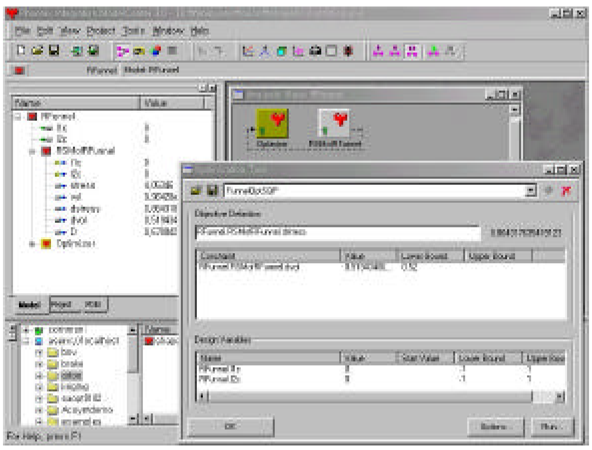


Figure 11. ModelCenter framework for shape optimization of funnel

Figure 11 shows the ModelCenter framework for shape optimization of funnel where sequential quadratic optimization algorithm is selected and multi-criteria optimization is implemented by using Visual Basic script.

Figure 12 shows the result of shape optimization from sequential quadratic programming within ModelCenter framework [Vanderplaats]. The optimization is terminated after 5 iterations and the confidence lovel is 0.9 under given volume constraint.

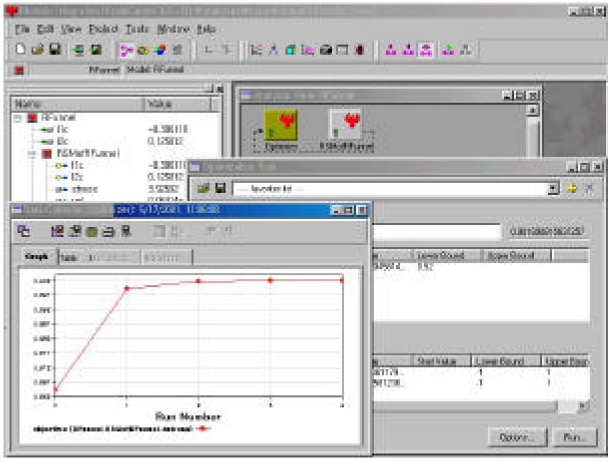


Figure 12. The result of shape optimization

Table 2 shows the comparison of response between response surface model and finite element simulation for three alternatives, where the minimum stress is 3.5929 kgf/mm^2 in equation (9). Because the difference between minimum and maximum volume is only 0.9% that means the change is negligible, the optimal solution of equation (9) is selected to perform the finite element simulation by solid elements. Table 2 shows the maximum error is 5.35 % in the case of 1st principal stress from equation (8) and it is within acceptable range.

Table 2. Comparison of three optimal solutions

Eq	Response	RSM	FEA	Error(%)
(7)	stress	4.06157	4.21282	3.59
	volume	9968486	9977940	0.095
(8)	stress	4.44995	4.70158	5.35
	volume	9939174	9940401	0.012
(9)	stress	3.5929	3.65048	1.58
	volume	10031112	10031020	0.92e-5

Figure 13 shows the optimal 1st principal stress of the finite element simulation by solid elements that are generated from the optimal funnel geometry by shell elements. The optimal stress is 1.629 kgf/mm^2 and it is under yield stress.

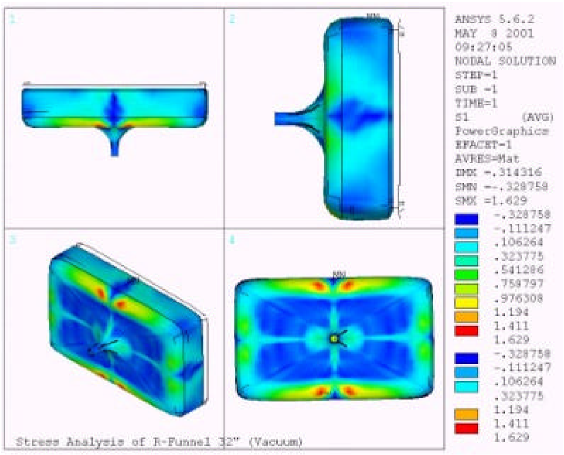


Figure 13. The optimal 1st principal stress by solid elements

Table 3 shows the comparison of initial and optimal geometry of solid elements. The optimal stress is reduced to 53.3 % within 5.87 % reduction of weight.

Table 3. Comparison of initial and optimal geometry

Response	Initial	Optimal	Reduction(%)
stress	3.49	1.629	53.3
weight	41.39	38.96	5.87

CONCLUSION

Instead of using complicate NURBS or Bezier curve representing funnel geometry, the simplified process based on response surface model is presented which follows three steps in this study. The first step is a screening design to identify the few important design variables from the many ones. The second is the construction of response surface model to analyze the few important design variables to optimize the shape of funnel geometry. The third is a multi-criteria optimization by fuzzy geometric mean. Combining these three steps within integration framework, multi-criteria shape optimization of a funnel in cathode ray tubes is successfully implemented as follows:

1. By using response surface model based on design of experiments, design space exploration is successfully performed before optimization.
2. By expanding fuzzy set theory to multi-criteria optimzation, each engineer's confidence level is transformed to a single objective function called overall confidence level ranged from 0 to 1.
3. By using proposed approach, the stress is reduced to 53.3 % within 5.87 % reduction of weight of a cathode ray tube.

ACKNOWLEDGEMENT

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