

Chaboche Nonlinear Kinematic Hardening Model

Sheldon Imaoka
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1 Introduction

The Chaboche nonlinear kinematic hardening model was added in ANSYS 5.6 to complement the existing isotropic and kinematic hardening rules that users relied on. Despite its availability for nearly ten years as of the time of this writing, the Chaboche model has enjoyed limited popularity, in part because of the perceived complexity of calibrating the material parameters. This memo hopes to introduce the basics related to the Chaboche nonlinear kinematic model.

Please note that many material parameters for the examples shown in this memo were taken from Reference [5]. The author highly recommends obtaining a copy of this book, as it is a useful reference not only for the nonlinear kinematic hardening model but also for plasticity and viscoplasticity in general.

2 Background on Material Behavior

2.1 Linear vs. Nonlinear Kinematic Hardening

The yield function for the nonlinear kinematic hardening model TB, CHABOCHE is shown below:

$$F = \sqrt{\frac{3}{2} (\{s\} - \{\alpha\})^T [M] (\{s\} - \{\alpha\})} - R = 0 \quad (1)$$

In the above equation, which can be found in Section 4 of Reference [2], $\{s\}$ is the deviatoric stress, $\{\alpha\}$ refers to the back stress, and R represents the yield stress. The back stress is related to the translation of the yield surface, and there is little in Equation (1) that distinguishes it from other kinematic hardening models at this point.

The back stress $\{\alpha\}$ for the Chaboche model is calculated as follows:

$$\{\alpha\} = \sum_{i=1}^n \{\alpha_i\} \quad (2a)$$

$$\{\Delta\alpha\}_i = \frac{2}{3}C_i\{\Delta\epsilon^{pl}\} - \gamma_i\{\alpha_i\}\Delta\epsilon^{pl} + \frac{1}{C_i}\frac{dC_i}{d\theta}\Delta\theta\{\alpha\} \quad (2b)$$

where $\hat{\epsilon}^{pl}$ is the accumulated plastic strain, θ is temperature, and C_i and γ_i are the Chaboche material parameters for n number of pairs. In Equation (2b), one may note that the first term is the hardening modulus. On the other hand, the second term of the evolution of the back stress is a “recall term” that produces a nonlinear effect.

The input consists of defining the elastic properties (e.g., elastic modulus, Poisson's ratio) via `MP,EX` and `MP,NUXY`, then issuing `TB,CHABOCHE,,ntemp,n`, where *ntemp* is the number of temperature sets and n is the number of kinematic models. Any temperature-dependent group of constants are preceded with the `TBTEMP` command defining the temperature, while the material parameters for that temperature are entered via the `TBDATA` command. The first constant is R , or the yield stress of the material — this value may be overridden if an isotropic hardening model is added, as covered in Subsection 2.5. The second and third material constants are C_1 and γ_1 — these may be followed by additional pairs of C_i and γ_i , depending on the number n of kinematic models requested.

2.2 Initial Hardening Modulus

The material parameter C_i is the *initial hardening modulus*. For a single kinematic hardening model ($n = 1$), if γ_1 is set to zero, the parameter C_1 will describe the slope of stress versus equivalent plastic strain. This would represent a *linear kinematic hardening model*.¹ One can also reproduce the same behavior with the bilinear kinematic hardening model (TB,BKIN), as shown in Figure 1 — plots of a linear kinematic hardening model with TB,CHABOCHE as well as TB,BKIN are superimposed, showing identical results. However, one should note that the tangent modulus E_{tan} specified in TB,BKIN is based on total strain, whereas C_i in TB,CHABOCHE is based on equivalent plastic strain. Using a stress value $\sigma' > R$, the relationship between C_i and E_{tan} is expressed in Equation (3):

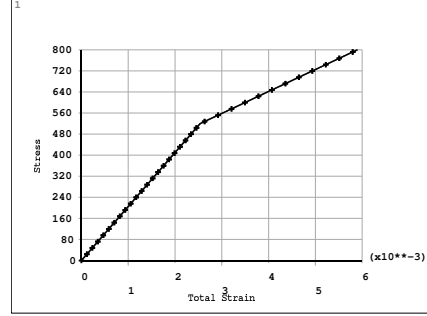


Figure 1: Linear Kinematic Hardening

$$E_{tan} = \frac{\sigma' - R}{\frac{\sigma' - R}{C_1} + \frac{\sigma'}{E_{elastic}} - \frac{R}{E_{elastic}}} \quad (3)$$

It is worth pointing out that there are various plasticity models in ANSYS, where some, such as TB,BKIN, use equivalent total strain, and others, such as TB,PLASTIC, use equivalent plastic strain. The Chaboche model in ANSYS uses equivalent plastic strain, and the author prefers this approach, as the elastic modulus and Poisson's ratio completely describe the elastic behavior, while the nonlinear constitutive model fully defines the plastic behavior.

¹The term “linear” refers to the relationship of stress and equivalent plastic strain. For ANSYS plasticity models such as “bilinear” or “multilinear” kinematic hardening, these terms describe the relationship between stress and equivalent total strain.

2.3 Nonlinear Recall Parameter

The second material parameter, γ_i , controls the rate at which the hardening modulus decreases with increasing plastic strain. By examining Equation (2b), one can see that the back stress increment $\{\dot{\alpha}\}$ becomes lower as plastic strain increases. Specifically, a limiting value of C_i/γ_i exists, indicating that the yield surface cannot translate anymore, which manifests itself as a hardening modulus of zero at large plastic strains.

A comparison of TB,BKIN with the Chaboche model, including a non-zero γ_1 parameter, is shown in Figure 2(a). Despite the Chaboche hardening modulus *initially* being the same as the linear kinematic model, the hardening modulus decreases, the rate of which is defined by γ_i .

Figure 2(b) is the same plot but at larger strains. One can see the hardening modulus for the Chaboche model decreasing to zero. For this case, the yield stress was assumed to be 520 MPa. For the Chaboche model, $C_1 = 140,600$ while $\gamma_1 = 380$. Since the limiting value of the back stress $\{\alpha\}$ is $\frac{C_1}{\gamma_1} = 370$, one would expect that the asymptotic value would be $R + \alpha = 890$ MPa, which matches with the results shown in Figure 2(b).

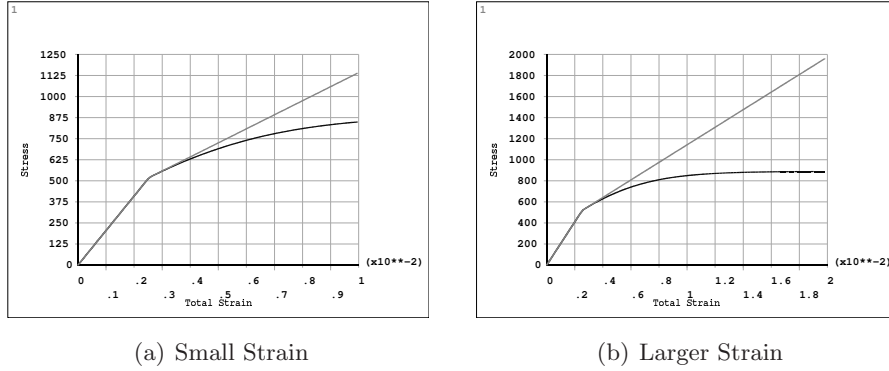


Figure 2: Comparison of Linear and Nonlinear Kinematic Hardening

2.4 Multiple Kinematic Hardening Models

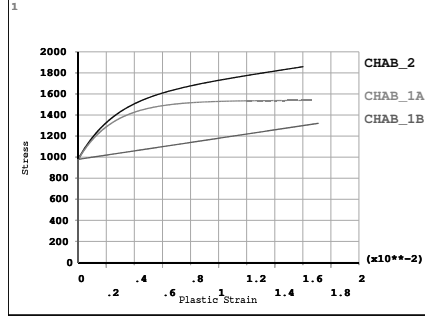


Figure 3: Two Kinematic Models

A single nonlinear kinematic hardening model is described by the two material parameters, C_1 and γ_1 , discussed in Subsections 2.2 and 2.3, respectively. However, a single kinematic model may not be sufficient to describe the complex response of a given material, so with TB,CHABOCHE, up to $n = 5$ kinematic models may be superimposed. This superposition of simple models is another salient feature of the

Chaboche model, and this idea will be expanded upon in the next subsection.

Figure 3 shows a comparison of three results of stress versus plastic strain — single kinematic models “CHAB_1A” and “CHAB_1B” are plotted with a two-model version “CHAB_2”. The “CHAB_2” model contains the same material parameters as “CHAB_1A” and “CHAB_1B,” and one may be able to visualize how these two kinematic models are superimposed (assuming constant yield stress R) to create the more complex stress-strain response of “CHAB_2.”

2.5 Combined Hardening

The main usage of the Chaboche model is for cyclic loading applications, as will be discussed shortly. Similar to other kinematic hardening models in ANSYS, the yield surface translates in principal stress space, and the elastic domain remains unchanged. For example, when the loading is reversed for a simple tensile specimen, yielding is assumed to occur at $\sigma_{max} - 2R$, where σ_{max} is the maximum stress prior to unloading — this is the well-known *Bauschinger effect*.

The Chaboche kinematic hardening model, however, can be used in conjunction with an isotropic hardening model. The isotropic hardening model describes the change in the elastic domain for the material, and the user can select from one of the following:

- TB,BISO: Bilinear isotropic hardening, where the user specifies the yield stress and tangent modulus (with respect to total strain)
- TB,MISO: Multilinear isotropic hardening, where the user specifies up

to 100 stress vs. total strain data points

- TB,PLASTIC,,,MISO: Nonlinear plasticity, where the user specifies up to 100 stress vs. plastic strain data points
- TB,NLISO,,,VOCE: Voce hardening law, where $R = k + R_o \hat{\epsilon}^{pl} + R_\infty (1 - e^{-b \hat{\epsilon}^{pl}})$. Material parameters k , R_o , R_∞ , and b are supplied by the user.

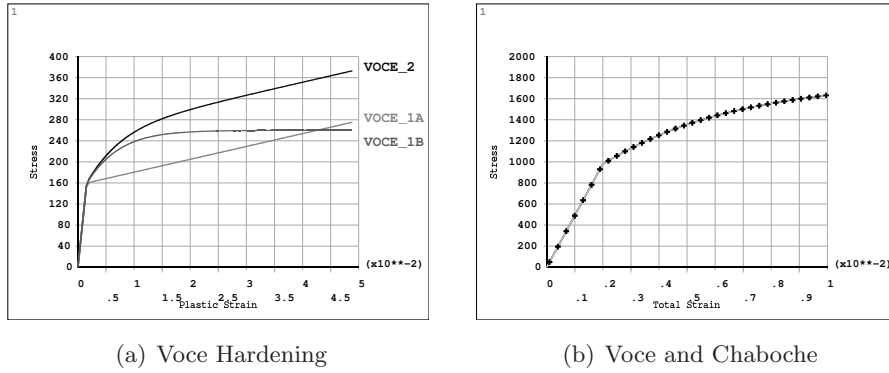


Figure 4: Combined Hardening

The Voce hardening law can be thought of as containing two terms — a linear term $R_o \hat{\epsilon}^{pl}$ and an asymptotic term $R_\infty (1 - e^{-b \hat{\epsilon}^{pl}})$. A sample model is shown in Figure 4(a), where “VOCE.1A” is the linear term, “VOCE.1B” is the asymptotic term, and “VOCE.2” is the complete model. Compare Figure 4(a) with Figure 3, and note the similarities for the case of monotonic, proportional loading.

It may be useful to compare the Voce and two-term nonlinear kinematic hardening models further for monotonic loading situations, in order to better understand the material parameters. The initial yield stress k should be defined the same for both. $R_\infty = \frac{C_1}{\gamma_1}$ describes the limiting, asymptotic value that is added to the initial yield stress k . The rate at which this decays is defined by $b = \gamma_1$. The hardening modulus at very large strains is defined by $R_o = C_2$, and for the Chaboche model, $\gamma_2 = 0$. Two “equivalent,” sample models are compared in Figure 4(b), and one can see that the stress-strain response is the same. Of course, the present discussion comparing the two constitutive models is meant to aid the reader in obtaining a better

understanding of the material parameters rather than to incorrectly imply that the two models are interchangeable for cyclic loading applications.

The combination of kinematic and isotropic hardening in the Chaboche references² typically use a form similar to the Voce hardening law but without the constant hardening modulus term:

$$R = k + Q \left(1 - e^{-b\hat{\epsilon}^{pl}} \right) \quad (4)$$

To understand how this isotropic hardening term is used, consider the case of a strain-controlled loading of a test specimen. For cyclic hardening, with $\pm\epsilon_{max}$ loading, the stress response of a sample model is shown in Figure 5. Note that with progressive number of cycles, the elastic domain expands — this difference in the change in the yield stress is described by the addition of the isotropic hardening term. Also, note that with more cycles, the change in the elastic domain stabilizes. This is why the R_o term is neglected for many situations, as the elastic domain does not keep increasing indefinitely.

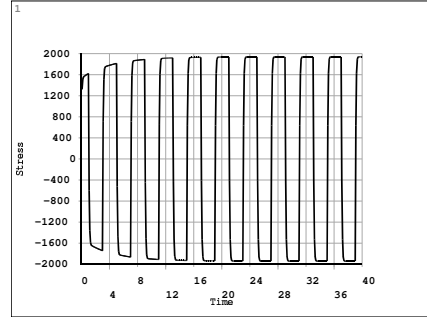


Figure 5: Cyclic Hardening

To use calibrated material constants, simply specify k , $R_\infty = Q$, and b . The user is not limited to using the Voce hardening law (TB,NLISO,,,VOCE) with the Chaboche nonlinear kinematic hardening model — use of other isotropic hardening models is permitted. Also, please keep in mind that the value of $R_\infty = Q$ is typically a function of the strain range, so the material parameters should be reflective of the expected operational strain ranges.

2.6 Anisotropic Yield Function

It is worth pointing out that the Chaboche model in ANSYS (TB,CHABOCHE) assumes a von Mises yield surface by default. The Hill anisotropic yield function (TB,HILL) may be used in conjunction with TB,CHABOCHE, although details of this will not be covered in the present memo.

²See Refs. [3], [4], and [5]

3 Calibration of Single Model

Per page 226 of Reference [5], tension-compression tests of symmetric, strain-controlled tests can be used to obtain the material parameters k , C_1 , and γ_1 . The stabilized hysteresis loops corresponding to different strain amplitudes should be used. The following steps (taken from the above reference) can be used to determine the material parameters:

1. Determine the yield stress k from the elastic domain (k is usually half the elastic domain size).
2. For a given test, determine the plastic strain range $\Delta\epsilon^{pl}$.
3. For a given test, determine the stress range $\Delta\sigma$.
4. Plotting $\frac{\Delta\sigma}{2} - k$ against $\frac{\Delta\epsilon^{pl}}{2}$ for the multiple tests, estimate the asymptotic value corresponding to $\frac{C_1}{\gamma_1}$.
5. Using the expression $\frac{\Delta\sigma}{2} - k = \frac{C_1}{\gamma_1} \tanh(\gamma_1 \frac{\Delta\epsilon^{pl}}{2})$, fit the results to solve for C_1 and γ_1 . This can be done, for example, in Microsoft Excel using the Solver Add-In. Note that curve-fitting procedures often benefit from reasonable initial values. Since C_1 is the initial hardening modulus, the slope after the yield stress can be taken as an estimate of C_1 , and through the relation of $\frac{C_1}{\gamma_1}$ determined in Step 4, an initial value of γ_1 can be obtained.

3.1 Example Using Several Stabilized Cycles

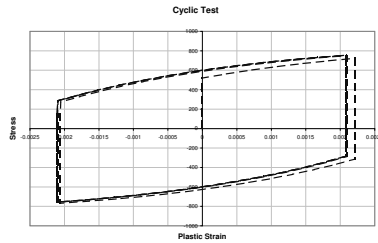


Figure 6: Strain-Controlled Test

An example of stress versus plastic strain is shown in Figure 6, where the test specimen is loaded $\pm\epsilon_a$. Looking at the stabilized hysteresis loop, one can see that the elastic domain is roughly $(300 + 760) = 1,060$ MPa, so the yield stress k can be estimated as 530 MPa. The stress range $\Delta\sigma$ is around $2 \times 760 = 1,520$ MPa. The plastic strain range $\Delta\epsilon^{pl}$ is estimated to be $2 \times .21\% = 0.42\%$.

When two additional strain-controlled, cyclic tests are performed, the following sets of data were obtained: (a) $k_2 = 500$, $\frac{\Delta\sigma_2}{2} = 840$, $\frac{\Delta\epsilon_2^{pl}}{2} = 0.37\%$

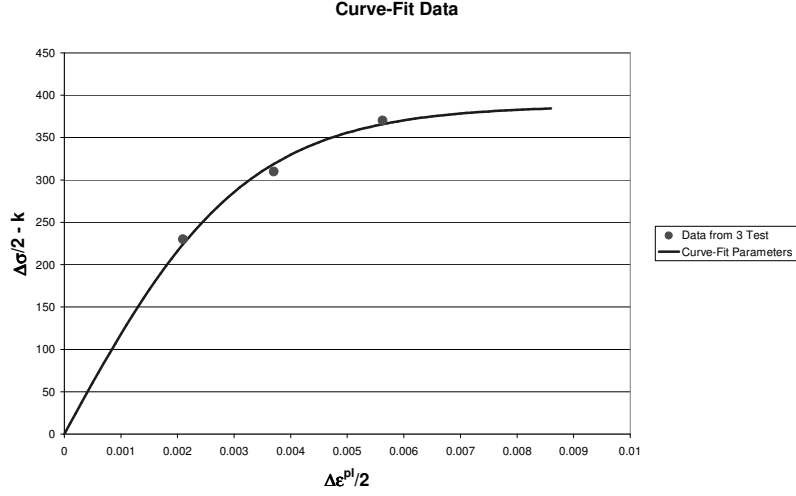


Figure 7: Curve-Fit Data

and (b) $k_3 = 540$, $\frac{\Delta\sigma_3}{2} = 900$, $\frac{\Delta\epsilon_3^{pl}}{2} = 0.5625\%$. All three data points are plotted as red dots in Figure 7, with the abscissa being $\frac{\Delta\epsilon^{pl}}{2}$ and the ordinate being $\frac{\Delta\sigma}{2} - k$. Using Step 4 in Section 3, the asymptotic value was estimated to be ≤ 400 MPa. The yield stress k was estimated to be 530 MPa based on k_1 , k_2 , and k_3 values. From Figure 6, the initial hardening modulus C_1 was estimated as 164,000 MPa, so $\gamma_1 = 410$ — these were used as starting values. The constants $C_1 = 122,000$ MPa and $\gamma_1 = 314$ were then obtained — the curve-fit data matches reasonably well with the presented data, and the parameters could be refined further, if needed.

3.2 Example Using Single Stabilized Cycle

If only data from a *single* strain-controlled test is available, material identification can still be performed, although the derived parameters are best suited for that particular strain range.

Using the example shown in Figure 6, assume that stress and plastic strain points for the stabilized loop are tabulated as follows:

ϵ_i^{pl}	σ_i (MPa)
-0.0021	300
-0.001	480
0.0	590
0.001	680
0.0021	760

Table 1: Sample Plastic Strain vs. Stress Points

There are two items that need adjustment: the plastic strain values ϵ^{pl} should start from zero, and the stress needs to be converted to the back stress $\{\alpha\}$. Shift the plastic strain values such that the first point starts from zero. For the back stress, determine the elastic range (recall from Subsection 3.1 that this was determined to be 1060 MPa), then subtract the stress by the yield stress, or half the value of the elastic domain. The resulting values are shown below:

ϵ_i^{pl}	α_i (MPa)
0.0	-230
0.0011	-50
0.0021	60
0.0031	150
0.0042	230

Table 2: Corrected Plastic Strain vs. Back Stress

To determine the C_1 and γ_1 constants, one may consider using the back stress relation $\alpha_i = \frac{C_1}{\gamma_1} \left(1 - e^{-\gamma_1 \epsilon_i^{pl}}\right)$, although this expression assumes zero initial back stress. Since the model contains non-zero back stress, the first data point is used in the following equation to solve for the other pairs:

$$\alpha_i = \frac{C_1}{\gamma_1} \left(1 - e^{-\gamma_1 \epsilon_i^{pl}}\right) + \alpha_1 e^{-\gamma_1 \epsilon_i^{pl}} \quad \text{for } i > 1 \quad (5)$$

One can use Microsoft Excel's Solver Add-In or any other means to perform a fit to determine C_1 and γ_1 . The considerations noted earlier about using the slope after yielding as the initial value of C_1 , along with estimating an initial value of γ_1 from the asymptotic value, still apply, as these will aid any curve-fitting procedure.

For this example, the author obtained values of $C_1 = 120,500$ MPa and $\gamma_1 = 280$. While these values differ slightly from $C_1 = 122,000$ MPa and $\gamma_1 = 314$ obtained in Subsection 3.1, please note that (a) the author did not use a digitizer to determine the data points but used rough, visual estimates, and (b) the curve-fit data obtained from several stabilized cycles should be more representative of behavior over a wider strain range.

As an alternative or supplement to the option presented in Subsection 3.1, the user may also use the approach outlined in this subsection to manually obtain C_1 and γ_1 coefficients for various strain ranges to better understand the variation of the material parameters that may be present.

3.3 Example Using Single Tension Curve

A user may only have data from a single tension test but may wish to use the nonlinear kinematic hardening model. While this is not recommended since there is no cyclic test data with which to correlate the material parameters, such an approach may be suitable for situations dealing with a few cycles.

Plastic strain and stress data will be estimated from the *first curve* of Figure 6, which would represent a single tensile loading case:

ϵ_i^{pl}	σ_i (MPa)
0.0	520
0.0005	580
0.001	630
0.0015	680
0.0022	720

Table 3: Plastic Strain vs. Stress for First Curve

As with the situation presented in Subsection 3.2, the stress needs to be converted to the back stress. The yield surface should be determined (for example, by using 0.2% offset), and this scalar value should be subtracted from the stress values in Table 3. Then, $\{\alpha_i\}$ values for each data point can be matched against the following equation:

$$\alpha_i = \frac{C_1}{\gamma_1} \left(1 - e^{-\gamma_1 \epsilon_i^{pl}} \right) \quad (6)$$

The user can solve for values of C_i and γ_i — the same procedure to obtain initial values and to perform the curve-fit as explained above still apply for the uniaxial tension case. For this particular set of data that

was approximated from the first curve of Figure 6, the author obtained $C_1 = 135,300$ MPa and $\gamma_1 = 380$. The use of the single uniaxial test data for this specific example resulted in higher estimates of C_1 and γ_1 compared with those obtained in Subsections 3.1 and 3.2, although the asymptotic value $\frac{C_1}{\gamma_1} = 356$ MPa is actually the smallest of the three.

As noted above, the lack of cyclic test data prevents validation of the derived material parameters for general cyclic applications. Consequently, the author *strongly* recommends creating a simple one-element model to simulate the cyclic behavior of the calculated material parameters — in this way, the user can see what the *numerical* cyclic behavior will be for the set of material parameters. With a simple model, the user may also vary the loading for larger strain ranges in order to understand the response in strain ranges for which no test data is present.

3.4 Comparison of Results

Figure 8 shows the original data compared with the three sets of calculated material parameters. As one can see, the results for this strain range match reasonably well for all three cases, despite some variation in the C_1 and γ_1 values. However, the parameters derived from multiple stabilized cycles would be expected to give the best correlation for different strain ranges.

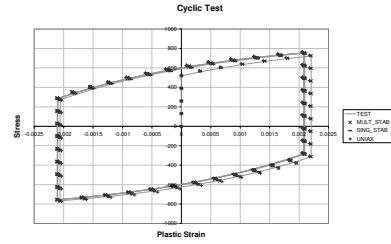


Figure 8: Comparison of Data

4 Rate-Dependent Version

In ANSYS 12.0.1, a rate-dependent form of the Chaboche model was introduced. This is accessed via adding `TB,RATE,,,CHABOCHE` in conjunction with the regular `TB,CHABOCHE` definition *without* isotropic hardening.

The rate-dependent additions to the Chaboche model (Equations (1) and (2)) are shown below:

$$\dot{\epsilon}_{pl} = \left(\frac{\sigma - R}{K} \right)^{\frac{1}{m}} \quad (7)$$

$$R = K_0 + R_0 \dot{\epsilon}_{pl} + R_\infty \left(1 - e^{-b \dot{\epsilon}_{pl}} \right) \quad (8)$$

where the first 4 constants for `TB,RATE,,,CHABOCHE` are K_0 , R_0 , R_∞ , and b (isotropic hardening Equation (8)) while the last 2 constants are m and K (rate-dependent Equation (7)). These 6 material constants are input via the `TBDATA` command.

From looking at the above equations, one can see that the rate-dependent Chaboche model incorporates a strain-rate dependent term (similar to Peirce or Perzyna options) with Voce hardening.

5 Conclusion

Background information on the Chaboche material model was presented in this memo, along with a basic discussion of the calibration of the material constants for a single nonlinear kinematic hardening model.

While phenomena such as ratchetting, shakedown, mean stress relaxation, and cyclic softening were not introduced, the information in this memo may serve as a starting point for users wishing to combine multiple nonlinear kinematic hardening models or to define combined (isotropic and kinematic) hardening laws. The reader is encouraged to review the cited references, including [1], for additional details on cyclic plasticity and its definition and usage in ANSYS.

Revisions to this Document

- STI0805A (November 14, 2009): Updated for 12.0.1. Updated Equation (2), added Section 4. Added additional information concerning input in Section 1.

References

- [1] ANSYS, Inc. *Advanced Structural Nonlinearities*, First edition, 2005. Inventory Number: 002205, ANSYS 9.0.
- [2] ANSYS, Inc. *Theory Reference for ANSYS and ANSYS Workbench 11.0*, 2007.
- [3] Jean-Louis Chaboche. Constitutive Equations for Cyclic Plasticity and Cyclic Viscoplasticity. *International Journal of Plasticity*, 5:247–302, 1989.

- [4] Jean-Louis Chaboche. On Some Modifications of Kinematic Hardening to Improve the Description of Ratchetting Effects. *International Journal of Plasticity*, 7:661–678, 1991.
- [5] Jean Lemaitre and Jean-Louis Chaboche. *Mechanics of Solid Materials*. Cambridge University Press, English edition, 1990.

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