Metaheurística GRASP com refinamento por busca local para o Flowshop Permutacional

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1. Modelagem matemática com programação linear inteira

O modelo utilizado foi descrito por [Tseng et al. 2004].

Considera M máquinas, N tarefas. $T_{rj} \geqslant 0$ representa o tempo de processamento da tarefa j na máquina r, para toda tarefa e máquina.

Variável $C_{ri} \ge 0$ indica o tempo que a tarefa i completou na máquina r.

Variável $D_{ik} \in \{0,1\}$ (=1) indica se a tarefa i é executada em algum momento antes da tarefa k; (=0) caso contrário.

Parâmetro "big-M" P.

$$Minimize C_{max}$$
 (1)

Sujeito a:

$$C_{1i} \geqslant T_{1i} \qquad 1 \leqslant i \leqslant N \tag{2}$$

$$C_{ri} - C_{r-1,i} \geqslant T_{ri} \qquad 2 \leqslant r \leqslant M, 1 \leqslant i \leqslant N \tag{3}$$

$$C_{ri} - C_{rk} + PD_{ik} \geqslant T_{ri} \qquad 1 \leqslant r \leqslant M, 1 \leqslant i < k \leqslant N$$
 (4)

$$C_{ri} - C_{rk} + PD_{ik} \leqslant P - T_{rk} \qquad 1 \leqslant r \leqslant M, 1 \leqslant i < k \leqslant N$$
 (5)

$$C_{\text{max}} \geqslant C_{Mi}$$
 $1 \leqslant i \leqslant N$ (6)

2. Resultados computacionais

Instância	BKS	α	Valor F.O.	$GAP_{\mathrm{BKS}}\left(\%\right)$	Tempo (s.)
VFR10_15_1	1307.00	0.00	1339.6 ± 18.319	2.49	1.5 ± 0.04
VFR10_15_1	1307.00	0.20	1354.2 ± 23.011	3.61	1.4 ± 0.03
VFR10_15_1	1307.00	0.40	1364.2 ± 28.944	4.38	1.5 ± 0.04
VFR10_15_1	1307.00	0.60	1346.1 ± 42.331	2.99	1.4 ± 0.03
VFR10_15_1	1307.00	0.80	1362.9 ± 30.205	4.28	1.5 ± 0.04
VFR10_15_1	1307.00	1.00	1342.2 ± 28.867	2.69	1.5 ± 0.03
VFR100_60_1	9395.00	0.00	10008.8 ± 47.123	6.53	57.7 ± 0.59
VFR100 60 1	9395.00	0.20	10054.5 ± 70.099	7.02	57.7 ± 0.42
VFR100_60_1	9395.00	0.40	10039.1 ± 54.017	6.86	57.9 ± 0.52
VFR100 60 1	9395.00	0.60	10040.9 ± 73.843	6.87	58.5 ± 0.87
VFR100 60 1	9395.00	0.80	10048.8 ± 69.904	6.96	58 ± 1
VFR100_60_1	9395.00	1.00	10057.8 ± 55.519	7.05	58.2 ± 0.99
VFR20 10 3	1592.00	0.00	1687.5 ± 29.304	6.00	2.1 ± 0.05
VFR20_10_3	1592.00	0.20	1685.8 ± 23.223	5.89	2 ± 0.03
VFR20_10_3	1592.00	0.40	1682 ± 21.417	5.65	2 ± 0.03
VFR20 10 3	1592.00	0.60	1690.8 ± 39.6	6.21	2 ± 0.04
VFR20_10_3	1592.00	0.80	1692.3 ± 32.094	6.30	2 ± 0.02
VFR20_10_3	1592.00	1.00	1682.7 ± 24.157	5.70	2 ± 0.04
VFR20_20_1	2270.00	0.00	2360.1 ± 33.478	3.97	3.9 ± 0.07
VFR20_20_1	2270.00	0.20	2355.8 ± 41.214	3.78	3.9 ± 0.08

Instância	BKS	α	Valor F.O.	${ m GAP}_{ m BKS}$ (%)	Tempo (s.)
VFR20_20_1	2270.00	0.40	2350 ± 25.573	3.52	3.9 ± 0.08
VFR20_20_1	2270.00	0.60	2376.6 ± 31.178	4.70	3.9 ± 0.06
VFR20_20_1	2270.00	0.80	2362.9 ± 26.236	4.09	3.8 ± 0.05
VFR20_20_1	2270.00	1.00	2366.9 ± 38.766	4.27	3.9 ± 0.07
VFR500_40_1	28548.00	0.00	30640.6 ± 67.832	7.33	200.4 ± 8.47
VFR500_40_1	28548.00	0.20	30753.7 ± 111.634	7.73	200 ± 4.51
VFR500_40_1	28548.00	0.40	30697.4 ± 107.934	7.53	197.2 ± 1.52
VFR500_40_1	28548.00	0.60	30681.7 ± 127.513	7.47	198.4 ± 1.59
VFR500_40_1	28548.00	0.80	30688.4 ± 101.606	7.50	199.6 ± 3.45
VFR500_40_1	28548.00	1.00	30741.5 ± 113.56	7.68	200.9 ± 7.53
VFR500_60_3	31125.00	0.00	33539.6 ± 106.966	7.76	298.5 ± 4.31
VFR500_60_3	31125.00	0.20	33624.6 ± 167.947	8.03	300.7 ± 3.79
VFR500_60_3	31125.00	0.40	33535.1 ± 81.036	7.74	299.2 ± 3.89
VFR500_60_3	31125.00	0.60	33576.6 ± 71.104	7.88	300.6 ± 3.38
VFR500_60_3	31125.00	0.80	33490.7 ± 96.158	7.60	298.3 ± 3.3
VFR500_60_3	31125.00	1.00	33530.5 ± 65.58	7.73	298.7 ± 2.61
VFR60_10_3	3423.00	0.00	3632.6 ± 62.45	6.12	6 ± 0.06
VFR60_10_3	3423.00	0.20	3637.4 ± 67.612	6.26	6 ± 0.14
VFR60_10_3	3423.00	0.40	3630.7 ± 55.041	6.07	6 ± 0.08
VFR60_10_3	3423.00	0.60	3608.3 ± 50.557	5.41	5.9 ± 0.11
VFR60_10_3	3423.00	0.80	3603.6 ± 72.537	5.28	6 ± 0.08
VFR60_10_3	3423.00	1.00	3626.3 ± 54.214	5.94	6 ± 0.09
VFR60_5_10	3663.00	0.00	3668.4 ± 7.291	0.15	3.2 ± 0.09
VFR60_5_10	3663.00	0.20	3667.9 ± 5.971	0.13	3.2 ± 0.13
VFR60_5_10	3663.00	0.40	3672.2 ± 8.574	0.25	3.1 ± 0.05
VFR60_5_10	3663.00	0.60	3674.4 ± 8.03	0.31	3.2 ± 0.06
VFR60_5_10	3663.00	0.80	3668.6 ± 7.152	0.15	3.2 ± 0.03
VFR60_5_10	3663.00	1.00	3665.6 ± 1.897	0.07	3.1 ± 0.05
VFR600_20_1	31433.00	0.00	32904.4 ± 69.306	4.68	118.4 ± 1.86
VFR600_20_1	31433.00	0.20	32930 ± 65.09	4.76	121.1 ± 5.56
VFR600_20_1	31433.00	0.40	32999.7 ± 123.094	4.98	119.3 ± 1.99
VFR600_20_1	31433.00	0.60	32982.4 ± 68.39	4.93	119.2 ± 1.82
VFR600_20_1	31433.00	0.80	32932.5 ± 134.142	4.77	123.1 ± 9.14
VFR600_20_1	31433.00	1.00	32990.1 ± 97.588	4.95	122.6 ± 7.68
VFR700_20_10	36417.00	0.00	37857.4 ± 114.996	3.96	140.6 ± 2.03
VFR700_20_10	36417.00	0.20	37792.3 ± 93.295	3.78	140 ± 3.16
VFR700_20_10	36417.00	0.40	37865.9 ± 79.689	3.98	139 ± 2.11
VFR700_20_10	36417.00	0.60	37798.9 ± 87.46	3.79	142.6 ± 9.19
VFR700_20_10	36417.00	0.80	37882.2 ± 110.235	4.02	140.3 ± 3.43
VFR700_20_10	36417.00	1.00	37807.6 ± 124.189	3.82	139.8 ± 2.51

Instância	BKS	Valor relaxação	Obj. solução inteira	GAP _{BKS} (%)
VFR10_15_1	1307	880.0	1307	0.0
VFR10_10_3	1592	687.0	1873	56.9
VFR_20_20_1	2270	1391.0	2573	42.6
VFR60_5_10	3663	382.0	3878	89.3
VFR100_60_1	9395	TL	_	∞
VFR500_40_1	28548	TL	_	∞
VFR500_60_3	31125	TL	_	∞
VFR600_20_1	31433	TL	_	∞
VFR700_20_10	36417	TL	_	∞

Referências

Tseng, F. T., Stafford Jr, E. F., and Gupta, J. N. (2004). An empirical analysis of integer programming formulations for the permutation flowshop. *Omega*, 32(4):285–293.

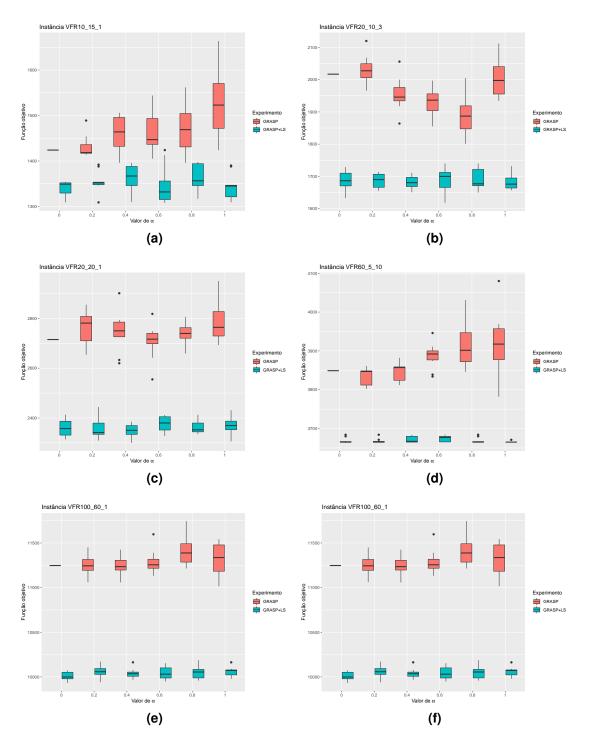


Figura 1. Boxplot relacionando valor médio da função objetivo para as diversas instâncias de testes, com vários valores α e 10 replicações por caso de teste.

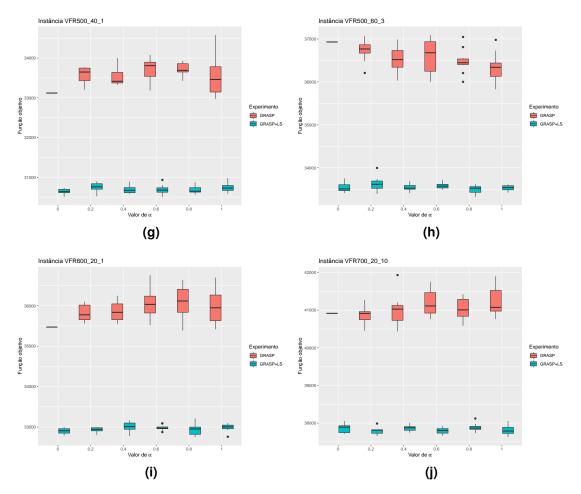


Figura 1. Boxplot relacionando valor médio da função objetivo para as diversas instâncias de testes, com vários valores α e 10 replicações por caso de teste. Continuação da figura anterior.