

# ICPC Templates For Africamonkey

Africamonkey

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# 1 莫队算法

## 1.1 普通莫队

```
1 struct Q { int l, r, sqrtl, id; } q[N];
2 int n, m, v[N], ans[N], nowans;
3 bool cmp(const Q &a, const Q &b) {
4     if (a.sqrtl != b.sqrtl) return a.sqrtl < b.sqrtl;
5     return a.r < b.r;
6 }
7 void change(int x) { if (!v[x]) checkin(); else checkout(); }
8 int main() {
9     .....
10    for (int i=1;i<=m;i++) q[i].sqrtl = q[i].l / sqrt(n), q[i].id = i;
11    sort(q+1, q+m+1, cmp);
12    int L=1,R=0; nowans=0;
13    memset(v, 0, sizeof(v));
14    for (int i=1;i<=m;i++) {
15        while (L<q[i].l) change(L++);
16        while (L>q[i].l) change(--L);
17        while (R<q[i].r) change(++R);
18        while (R>q[i].r) change(R--);
19        ans[q[i].id] = nowans;
20    }
21    .....
22 }
```

## 1.2 树上莫队

```
1 struct Query { int l, r, id, l_group; } query[N];
2 struct EDGE { int adj, next; } edge[N*2];
3 int n, m, top, gh[N], c[N], reorder[N], deep[N], father[N], size[N], son[N], Top[N];
4 void addedge(int x, int y) {
5     edge[++top].adj = y;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 void dfs(int x, int root=0) {
10    reorder[x] = ++top; father[x] = root; deep[x] = deep[root] + 1;
11    son[x] = 0; size[x] = 1; int dd = 0;
12    for (int p=gh[x]; p; p=edge[p].next)
13        if (edge[p].adj != root) {
14            dfs(edge[p].adj, x);
15            if (size[edge[p].adj] > dd) {
16                son[x] = edge[p].adj;
17                dd = size[edge[p].adj];
18            }
19            size[x] += size[edge[p].adj];
20        }
```

```

21 }
22 void split(int x, int tp) {
23     Top[x] = tp;
24     if (son[x]) split(son[x], tp);
25     for (int p=gh[x]; p; p=edge[p].next)
26         if (edge[p].adj != father[x] && edge[p].adj != son[x])
27             split(edge[p].adj, edge[p].adj);
28 }
29 int lca(int x, int y) {
30     int tx = Top[x], ty = Top[y];
31     while (tx != ty) {
32         if (deep[tx] < deep[ty]) {
33             swap(tx, ty);
34             swap(x, y);
35         }
36         x = father[tx];
37         tx = Top[x];
38     }
39     if (deep[x] < deep[y]) swap(x, y);
40     return y;
41 }
42 bool cmp(const Query &a, const Query &b) {
43     if (a.l_group != b.l_group) return a.l_group < b.l_group;
44     return reorder[a.r] < reorder[b.r];
45 }
46 int v[N], ans[N];
47 void upd(int x) { if (!v[x]) checkin(); else checkout(); }
48 void go(int &u, int taru, int v) {
49     int lca0 = lca(u, taru);
50     int lca1 = lca(u, v); upd(lca1);
51     int lca2 = lca(taru, v); upd(lca2);
52     for (int x=u; x!=lca0; x=father[x]) upd(x);
53     for (int x=taru; x!=lca0; x=father[x]) upd(x);
54     u = taru;
55 }
56 int main() {
57     memset(gh, 0, sizeof(gh));
58     scanf("%d%d", &n, &m); top = 0;
59     for (int i=1; i<n; i++) {
60         int x,y; scanf("%d%d", &x, &y);
61         addedge(x, y); addedge(y, x);
62     }
63     top = 0; dfs(1); split(1, 1);
64     for (int i=1; i<=m; i++) {
65         if (reorder[query[i].l] > reorder[query[i].r])
66             swap(query[i].l, query[i].r);
67         query[i].id = i;
68         query[i].l_group = reorder[query[i].l] / sqrt(n);
69     }
70     sort(query+1, query+m+1, cmp);

```

```

71     int L=1,R=1; upd(1);
72     for (int i=1;i<=m;i++) {
73         go(L,query[i].l,R);
74         go(R,query[i].r,L);
75         ans[query[i].id] = answer();
76     }
77     .....
78 }

```

## 2 字符串

### 2.1 哈希

```

1  const int P=31,D=1000173169;
2  int n, pow[N], f[N]; char a[N];
3  int hash(int l, int r) { return (LL) (f[r]-(LL)f[l-1]*pow[r-l+1]%D+D)%D; }
4  int main() {
5      scanf("%d%s", &n, a+1);
6      pow[0] = 1;
7      for (int i=1;i<=n;i++) pow[i] = (LL)pow[i-1]*P%D;
8      for (int i=1;i<=n;i++) f[i] = (LL) ((LL)f[i-1]*P+a[i])%D;
9  }

```

### 2.2 KMP

接口: `int find_substring(char *pattern, char *text, int *next, int *ret);`

输入: 模式串, 匹配串

输出: 返回值表示模式串在匹配串中出现的次数

KMP 的 `next[i]` 表示从 0 到 i 的字符串 s, 前缀和后缀的最长重叠长度。

```

1  void find_next(char *pattern, int *next) {
2      int n = strlen(pattern);
3      for (int i=1;i<n;i++) {
4          int j = i;
5          while (j > 0) {
6              j = next[j];
7              if (pattern[j] == pattern[i]) {
8                  next[i+1] = j+1;
9                  break;
10             }
11         }
12     }
13 }
14 int find_substring(char *pattern, char *text, int *next, int *ret) {
15     find_next(pattern, next);
16     int n = strlen(pattern);
17     int m = strlen(text);

```

```

18     int k = 0;
19     for (int i=0, j=0; i<m; i++) {
20         if (j<n && text[i]==pattern[j]) {
21             j++;
22         } else {
23             while (j>0) {
24                 j = next[j];
25                 if (text[i] == pattern[j]) {
26                     j++;
27                     break;
28                 }
29             }
30         }
31         if (j == n)
32             ret[k++] = i-n+1;
33     }
34     return k;
35 }

```

## 2.3 可动态修改的 KMP

支持：加入一个字符，删除一个字符。

时间复杂度： $O(n\alpha)$ ， $\alpha$  为字符集大小。

代码中的字符为 '0' - '9'，可自行修改为 'a' - 'z'

```

1 char t[N];
2 int top, nxt[N], nxt_l[N][10];
3 inline void del_letter() { --top; }
4 inline void add_letter(char x) {
5     t[top++] = x;
6     int j = top-1;
7     memset(nxt_l[top], 0, sizeof(nxt_l[top]));
8     nxt[top] = nxt_l[top-1][x-'0'];
9     memcpy(nxt_l[top], nxt_l[nxt[top]], sizeof(nxt_l[nxt[top]]));
10    nxt_l[top][t[nxt[top]]-'0'] = nxt[top]+1;
11 }

```

## 2.4 扩展 KMP

接口：void ExtendedKMP(char \*a, char \*b, int \*next, int \*ret);

输出：

next: a 关于自己每个后缀的最长公共前缀

ret: a 关于 b 的每个后缀的最长公共前缀

EXKMP 的 next[i] 表示：从 i 到 n-1 的字符串 st 前缀和原串前缀的最长重叠长度。

```

1 void get_next(char *a, int *next) {
2     int i, j, k;
3     int n = strlen(a);

```

```

4     for (j = 0; j+1<n && a[j]==a[j+1];j++);
5     next[1] = j;
6     k = 1;
7     for (i=2;i<n;i++) {
8         int len = k+next[k], l = next[i-k];
9         if (l < len-i) {
10             next[i] = l;
11         } else {
12             for (j = max(0, len-i);i+j<n && a[j]==a[i+j];j++);
13             next[i] = j;
14             k = i;
15         }
16     }
17 }
18 void ExtendedKMP(char *a, char *b, int *next, int *ret) {
19     get_next(a, next);
20     int n = strlen(a), m = strlen(b);
21     int i, j, k;
22     for (j=0;j<n && j<m && a[j]==b[j];j++);
23     ret[0] = j;
24     k = 0;
25     for (i=1;i<m;i++) {
26         int len = k+ret[k], l = next[i-k];
27         if (l < len-i) {
28             ret[i] = l;
29         } else {
30             for (j = max(0, len-i);j<n && i+j<m && a[j]==b[i+j];j++);
31             ret[i] = j;
32             k = i;
33         }
34     }
35 }

```

## 2.5 Manacher

$p[i]$  表示以  $i$  为对称轴的最长回文串长度

```

1 char st[N*2], s[N];
2 int len, p[N*2];
3
4 while (scanf("%s", s) != EOF) {
5     len = strlen(s);
6     st[0] = '$', st[1] = '#';
7     for (int i=1;i<=len;i++)
8         st[i*2] = s[i-1], st[i*2+1] = '#';
9     len = len * 2 + 2;
10    int mx = 0, id = 0, ans = 0;
11    for (int i=1;i<=len;i++) {
12        p[i] = (mx > i) ? min(p[id*2-i]+1, mx-i) : 1;
13        for (; st[i+p[i]] == st[i-p[i]]; ++p[i]) ;

```



```

14         if (p[i]+i > mx) mx = p[i]+i, id = i;
15         p[i] --;
16         if (p[i] > ans) ans = p[i];
17     }
18     printf("%d\n", ans);
19 }

```

## 2.6 最小表示法

```

1 string smallestRepresation(string s) {
2     int i, j, k, l;
3     int n = s.length();
4     s += s;
5     for (i=0, j=1; j<n; ) {
6         for (k=0; k<n && s[i+k]==s[j+k]; k++);
7         if (k>=n) break;
8         if (s[i+k]<s[j+k]) j+=k+1;
9         else {
10             l=i+k;
11             i=j;
12             j=max(l, j)+1;
13         }
14     }
15     return s.substr(i, n);
16 }

```

## 2.7 AC 自动机

```

1 struct Node {
2     int next[**Size of Alphabet**];
3     int terminal, fail;
4 } node[**Number of Nodes**];
5 int top;
6 void add(char *st) {
7     int len = strlen(st), x = 1;
8     for (int i=0; i<len; i++) {
9         int ind = trans(st[i]);
10        if (!node[x].next[ind])
11            node[x].next[ind] = ++top;
12        x = node[x].next[ind];
13    }
14    node[x].terminal = 1;
15 }
16 int q[**Number of Nodes**], head, tail;
17 void build() {
18     head = 0, tail = 1; q[1] = 1;
19     while (head != tail) {
20         int x = q[++head];

```

```

21      /*(when necessary) node[x].terminal != node[node[x].fail].terminal; */
22      for (int i=0;i<n;i++)
23          if (node[x].next[i]) {
24              if (x == 1) node[node[x].next[i]].fail = 1;
25              else {
26                  int y = node[x].fail;
27                  while (y) {
28                      if (node[y].next[i]) {
29                          node[node[x].next[i]].fail = node[y].next[i];
30                          break;
31                      }
32                      y = node[y].fail;
33                  }
34                  if (!node[node[x].next[i]].fail) node[node[x].next[i]].fail = 1;
35              }
36              q[++tail] = node[x].next[i];
37          }
38      }
39  }

```

## 2.8 后缀数组

### 2.8.1 倍增算法

参数  $m$  表示字符集的大小, 即  $0 \leq r_i < m$

```

1  #define rank rank2
2  int n, r[N], wa[N], wb[N], ws[N], sa[N], rank[N], height[N];
3  int cmp(int *r, int a, int b, int l, int n) {
4      if (r[a]==r[b]) {
5          if (a+l<n && b+l<n && r[a+l]==r[b+l])
6              return 1;
7      }
8      return 0;
9  }
10 void suffix_array(int m) {
11     int i, j, p, *x=wa, *y=wb, *t;
12     for (i=0;i<m;i++) ws[i]=0;
13     for (i=0;i<n;i++) ws[x[i]=r[i]]++;
14     for (i=1;i<m;i++) ws[i]+=ws[i-1];
15     for (i=n-1;i>=0;i--) sa[--ws[x[i]]]=i;
16     for (j=1;p=1;p<n;m=p,j<=1) {
17         for (p=0,i=n-j;i<n;i++) y[p++]=i;
18         for (i=0;i<n;i++) if (sa[i]>=j) y[p++]=sa[i]-j;
19         for (i=0;i<m;i++) ws[i]=0;
20         for (i=0;i<n;i++) ws[x[y[i]]]++;
21         for (i=1;i<m;i++) ws[i]+=ws[i-1];
22         for (i=n-1;i>=0;i--) sa[--ws[x[y[i]]]]=y[i];
23         for (t=x,x=y,y=t,x[sa[0]]=0,i=1,p=1;i<n;i++)
24             x[sa[i]]=cmp(y,sa[i-1],sa[i],j,n)?p-1:p++;

```

```

25     }
26     for (i=0;i<n;i++) rank[sa[i]]=i;
27     rank[n] = -1;
28 }
29 void calc_height() {
30     int j=0;
31     for (int i=0;i<n;i++)
32         if (rank[i])
33             {
34                 while (r[i+j]==r[sa[rank[i]-1]+j]) j++;
35                 height[rank[i]]=j;
36                 if (j) j--;
37             }
38 }

```

## 2.8.2 DC3 算法

注意：

$N$  至少为字符串长度的 3 倍

接口：suffix\_array(int \*r, int \*sa, int n, int m);

$r$  表示字符串， $sa$  为后缀数组输出， $n$  表示字符串长度，下标从 0 开始。 $m$  为字符集大小。

```

1  #define F(x) ((x)/3 + ((x)%3 == 1 ? 0:tb))
2  #define G(x) ((x) < tb ? (x)*3+1 : ((x)-tb)*3 + 2)
3  #define rank rank2
4
5  int r[N], wa[N], wb[N], ws[N], wv[N], sa[N], rank[N];
6
7  int c0(int *r,int a,int b) {
8      return r[a]==r[b]&&r[a+1]==r[b+1]&&r[a+2]==r[b+2];
9  }
10
11 int c12(int k,int *r,int a,int b) {
12     if(k==2) return r[a]<r[b] || r[a]==r[b]&&c12(1,r,a+1,b+1);
13     else return r[a]<r[b] || r[a]==r[b]&&wv[a+1]<wv[b+1];
14 }
15
16 void dsort(int *r,int *a,int *b,int n,int m) {
17     int i;for(i=0;i<n;i++) wv[i]=r[a[i]];
18     for(i=0;i<m;i++) ws[i]=0;
19     for(i=0;i<n;i++) ws[wv[i]]++;
20     for(i=1;i<m;i++) ws[i]+=ws[i-1];
21     for(i=n-1;i>=0;i--) b[--ws[wv[i]]]=a[i];
22 }
23
24 void dc3(int *r,int *sa,int n,int m) {
25     int i,j,*rn=r+n,*san=sa+n,ta=0,tb=(n+1)/3,tbc=0,p;
26     r[n]=r[n+1]=0;
27     for(i=0;i<n;i++) if(i%3!=0) wa[tbc++]=i;

```

```

28     dsort(r+2, wa, wb, tbc, m);
29     dsort(r+1, wb, wa, tbc, m);
30     dsort(r, wa, wb, tbc, m);
31     for(p=1, rn[F(wb[0])]=0, i=1; i<tbc; i++) rn[F(wb[i])]=c0(r, wb[i-1], wb[i])?p-1:p++;
32     if(p<tbc) dc3(rn, san, tbc, p);
33     else for(i=0; i<tbc; i++) san[rn[i]]=i;
34     for(i=0; i<tbc; i++) if(san[i]<tb) wb[ta++]=san[i]*3;
35     if(n%3==1) wb[ta++]=n-1;
36     dsort(r, wb, wa, ta, m);
37     for(i=0; i<tbc; i++) wv[wb[i]=G(san[i])]=i;
38     for(i=0, j=0, p=0; i<ta && j<tbc; p++) sa[p]=c12(wb[j]%3, r, wa[i], wb[j])?wa[i++]:wb[j
    ++];
39     for(; i<ta; p++) sa[p]=wa[i++];
40     for(; j<tbc; p++) sa[p]=wb[j++];
41 }
42
43 void suffix_array(int *r, int *sa, int n, int m) {
44     dc3(r, sa, n+1, m);
45     int top = 0;
46     for (int i = 0; i < n+1; ++i)
47         if (sa[i] < n) sa[top++] = sa[i];
48     for (int i = 0; i < n; ++i) rank[sa[i]] = i;
49     rank[n] = -1;
50 }

```

### 2.8.3 小技巧：拼接字符串

接口：

`int gao1(int l, int r, int c, int p)`; 区间  $[l, r)$  中保证第 0 位到第  $c-1$  位都是相同的（设为字符串  $s$ ），现在我们在  $s$  后面接一个字符  $p$ ，得到一个新的字符串  $s'$ 。返回值为最小的  $k$  满足后缀  $sa[k]$  前  $c+1$  位为  $s'$

`int gao2(int l, int r, int c, int p)`; 区间  $[l, r)$  中保证第 0 位到第  $c-1$  位都是相同的（设为字符串  $s$ ），现在我们在  $s$  后面接一个后缀  $sa[p]$ ，得到一个新的字符串  $s'$ 。返回值为最小的  $k$  满足后缀  $sa[k]$  前  $c+len(sa[p])$  位为  $s'$

```

1 int gao1(int l, int r, int c, int p) {
2     --l;
3     while (l+1<r) {
4         int md=(l+r)>>1;
5         if (sa[md]+c<n&&s[sa[md]+c]>=p) r=md; else l=md;
6     }
7     return r;
8 }
9 int gao2(int l, int r, int c, int p) {
10    --l;
11    while (l+1<r) {
12        int md=(l+r)>>1;
13        if (sa[md]+c<n&&rk[sa[md]+c]>=p) r=md; else l=md;
14    }

```

```

15     return r;
16 }

```

示例调用：

```

1  suf1[m] = -1, suf2[m] = n;
2  for (int i = m - 1; i >= 0; --i) {
3      int l = gao1(0, n, 0, t[i]), r = gao1(0, n, 0, t[i]);
4      suf1[i] = gao2(l, r, l, suf1[i + 1]);
5      suf2[i] = gao2(l, r, l, suf2[i + 1]);
6  }

```

## 2.9 后缀自动机

下面的代码是求两个串的 LCS（最长公共子串）。

```

1  #include <cstdio>
2  #include <cstdlib>
3  #include <cstring>
4  #define N 500001
5  using namespace std;
6  char st[N];
7  int pre[N<<1], son[26][N<<1], step[N<<1], refer[N<<1], last, total;
8  int apply(int x, int now) { step[++total]=x; refer[total] = now; return total; }
9  void extend(char x, int now) {
10     int p = last, np = apply(step[last]+1, now);
11     for (; p && !son[x][p]; p=pre[p]) son[x][p] = np;
12     if (!p) pre[np] = 1;
13     else {
14         int q = son[x][p];
15         if (step[p]+1 == step[q]) pre[np] = q;
16         else {
17             int nq = apply(step[p]+1, now);
18             for (int i=0; i<26; i++) son[i][nq] = son[i][q];
19             pre[nq] = pre[q];
20             pre[q] = pre[np] = nq;
21             for (; p && son[x][p]==q; p=pre[p]) son[x][p] = nq;
22         }
23     }
24     last = np;
25 }
26 void init() {
27     last = total = 0;
28     last = apply(0, 0);
29     scanf("%s", st);
30     for (int i=0; st[i]; i++)
31         extend(st[i]-'a', i);
32     scanf("%s", st);
33 }
34 int main() {

```

```

35  init();
36  int p = 1, now = 0, ans = 0;
37  for (int i=0; st[i]; i++) {
38      int index = st[i]-'a';
39      for (; p && !son[index][p]; p = pre[p], now = step[p]) ;
40      if (!p) p = 1;
41      if (son[index][p]) {
42          p = son[index][p];
43          now++;
44          if (now > ans) ans = now;
45      }
46  }
47  printf("%d\n",ans);
48  return 0;
49  }

```

一些定义和性质:

$\text{Right}(\text{str})$  表示  $\text{str}$  在母串  $S$  中所有出现的结束位置集合

一个状态  $s$  表示的所有子串  $\text{Right}$  集合相同, 为  $\text{Right}(s)$

$\text{Parent}(s)$  满足  $\text{Right}(s)$  是  $\text{Right}(\text{Parent}(s))$  的真子集, 并且  $\text{Right}(\text{Parent}(s))$  的大小最小

$\text{Parent}$  函数可以表示一个树形结构。不妨叫它  $\text{Parent}$  树

一个  $\text{Right}$  集合和一个长度定义了一个子串

对于状态  $s$ , 使得  $\text{Right}(s)$  合法的子串长度是一个区间  $[\min(s), \max(s)]$

$\max(\text{Parent}(s)) = \min(s) - 1$

令  $\text{refer}(s)$  表示产生  $s$  状态的字符所在位置。则  $\text{Right}(s)$  的合法子串的起始位置为  $[\text{refer}(s) - \max(s) + 1, \text{refer}(s) - \min(s) + 1]$ , 即  $[\text{refer}(s) - \max(s) + 1, \text{refer}(s) - \max(\text{Parent}(s))]$

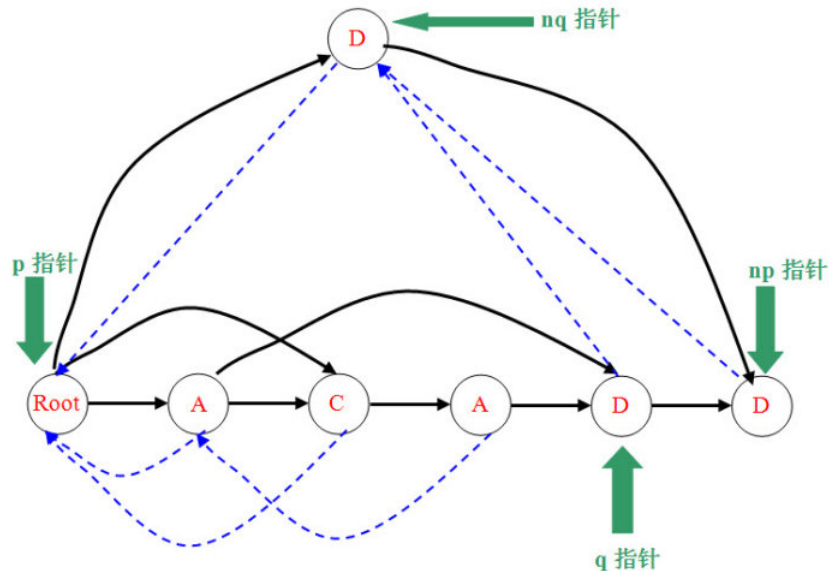


图 1: ACADD 构成的后缀自动机

我们发现 fail 构出一棵前缀树

和后缀树相同, 为了使每个前缀都是叶子结点, 我们不妨在串  $s$  前加入一个没出现的字符 '#'

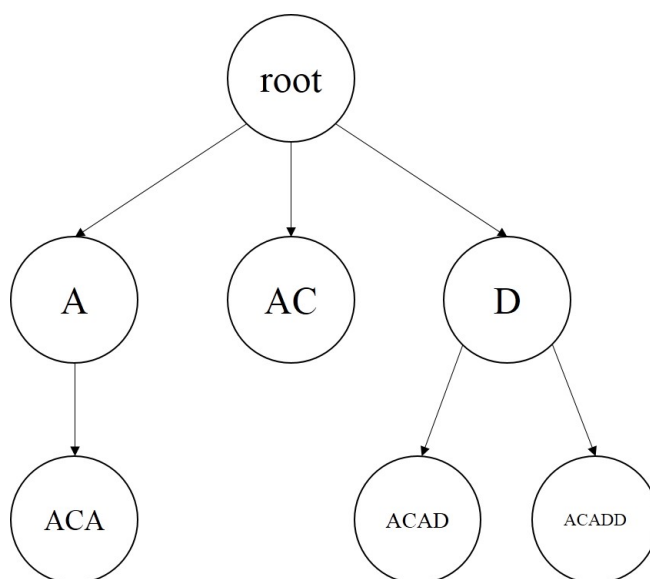


图 2: 串 ACADD 按 fail 构出的前缀树, 与图 1 对应

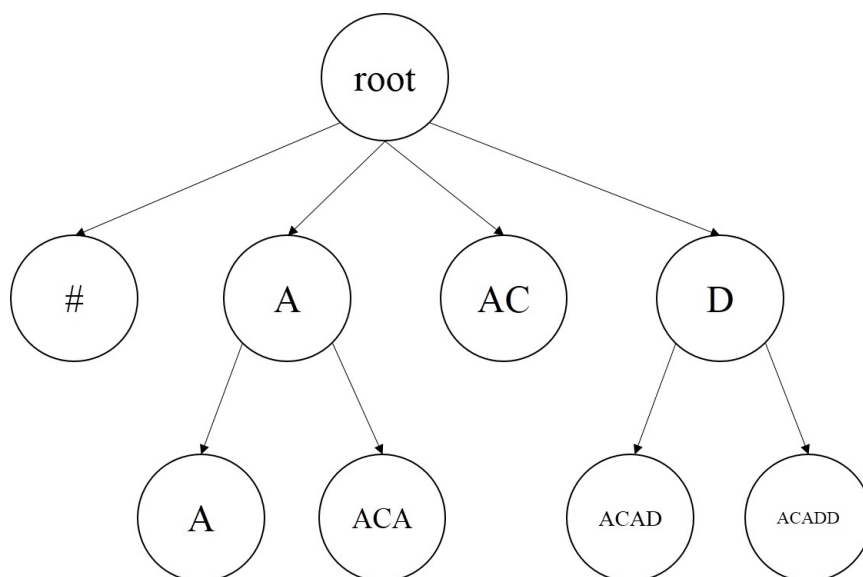


图 3: 串 #ACADD 按 fail 构出的前缀树

## 2.10 回文树

### 【URAL2040】Palindromes and Super Abilities 2

逐个添加字符串  $S$  里的字符  $S_1, S_2, \dots, S_n$ 。每次添加字符后，他想知道添加字符后将出现多少个新的本质不同的回文子串。字符集为  $\{a, b\}$

```
1 #include <bits/stdc++.h>
2 #define N 5000020
3
4 char st[N], answer[N];
5 int n;
6
7 struct PAM {
8     int n, tot, last;
9     int len[N], fail[N], next[N][2];
10    void init() {
11        n=0; tot=1;
12        len[1]=-1; fail[1]=0;
13        len[0]=+0; fail[0]=1;
14        last=1;
15    }
16    int get_fail(int x) {
17        for (; st[n-len[x]-1]!=st[n]; x=fail[x]);
18        return x;
19    }
20    void insert(char c) {
21        ++n; int cur=get_fail(last); // 判断上一个串的前一个位置和新添加的位置是否相
22        同，相同则说明构成回文。否则找 fail 指针。
23        if (!next[cur][c]) {
24            ++tot;
25            len[tot]=len[cur]+2;
26            fail[tot]=next[get_fail(fail[cur])][c];
27            next[cur][c]=tot;
28            answer[n]='1';
29        } else {
30            answer[n]='0';
31        }
32        last=next[cur][c];
33    } pam;
34
35 int main() {
36     scanf("%s", st+1); n=strlen(st+1);
37     pam.init();
38     for (int i=1; i<=n; i++) pam.insert(st[i]-'a');
39     puts(answer+1);
40     return 0;
41 }
```



## 3 数据结构

### 3.1 ST 表

```
1 int Log[N], f[17][N];
2 int ask(int x, int y) {
3     int k = Log[y - x + 1];
4     return max(f[k][x], f[k][y - (1 << k) + 1]);
5 }
6 int main() {
7     for (int i = 2; i <= n; i++) Log[i] = Log[i >> 1] + 1;
8     for (int j = 1; j < K; j++)
9         for (int i = 1; i + (1 << j - 1) <= n; i++)
10             f[j][i] = max(f[j - 1][i], f[j - 1][i + (1 << j - 1)]);
11 }
```

### 3.2 左偏树

左偏树是一个可并堆。

下面的程序写的是一个小根堆，如果需要改成大根堆请在注释了 here 那行修改。

接口：

void push(const T &x); 插入一个元素。

void merge(leftist &x); 合并两个堆。注意，合并后原来那个堆将不可访问。

T top() const; 返回堆顶元素。

void pop(); 删除堆顶元素。

int size() const; 返回堆的大小。

```
1 template <class T>
2 class leftist {
3 public:
4     struct node {
5         T key;
6         int dist;
7         node *l, *r;
8     };
9     leftist() : root(NULL), s(0) {}
10    void push(const T &x) {
11        leftist y;
12        y.s = 1;
13        y.root = new node;
14        y.root -> key = x;
15        y.root -> dist = 0;
16        y.root -> l = y.root -> r = NULL;
17        merge(y);
18    }
19    node* merge(node *x, node *y) {
20        if (x == NULL) return y;
21        if (y == NULL) return x;
```

```

22     if (y -> key < x -> key) swap(x, y); //here
23     x -> r = merge(x -> r, y);
24     int ld = x -> l ? x -> l -> dist : -1;
25     int rd = x -> r ? x -> r -> dist : -1;
26     if (ld < rd) swap(x -> l, x -> r);
27     if (x -> r == NULL) x -> dist = 0;
28     else x -> dist = x -> r -> dist + 1;
29     return x;
30 }
31 void merge(leftist &x) {
32     root = merge(root, x.root);
33     s += x.s;
34 }
35 T top() const {
36     if (root == NULL) return T();
37     return root -> key;
38 }
39 void pop() {
40     if (root == NULL) return;
41     node *p = root;
42     root = merge(root -> l, root -> r);
43     --s;
44     delete p;
45 }
46 int size() const {
47     return s;
48 }
49 private:
50     node* root;
51     int s;
52 };

```

### 3.3 线段树小技巧

给定一个序列  $a$ ，寻找一个最大的  $i$  使得  $i \leq y$  且满足一些条件（如  $a[i] \geq w$ ，那么需要在线段树维护  $a$  的区间最大值）

```

1  int queryl(int p, int left, int right, int y, int w) {
2      if (right <= y) {
3          if (! __condition__ ) return -1;
4          else if (left == right) return left;
5      }
6      int mid = (left + right) / 2;
7      if (y <= mid) return queryl(p<<1|0, left, mid, y, w);
8      int ret = queryl(p<<1|1, mid+1, right, y, w);
9      if (ret != -1) return ret;
10     return queryl(p<<1|0, left, mid, y, w);
11 }

```

给定一个序列  $a$ ，寻找一个最小的  $i$  使得  $i \geq x$  且满足一些条件（如  $a[i] \geq w$ ，那么需要在线段树

维护  $a$  的区间最大值)

```
1 int queryr(int p, int left, int right, int x, int w) {
2     if (left >= x) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (x > mid) return queryr(p<<1|1, mid+1, right, x, w);
8     int ret = queryr(p<<1|0, left, mid, x, w);
9     if (ret != -1) return ret;
10    return queryr(p<<1|1, mid+1, right, x, w);
11 }
```

### 3.4 Splay

接口:

ADD  $x\ y\ d$ : 将  $[x, y]$  的所有数加上  $d$

REVERSE  $x\ y$ : 将  $[x, y]$  翻转

INSERT  $x\ p$ : 将  $p$  插入到第  $x$  个数的后面

DEL  $x$ : 将第  $x$  个数删除

```
1 struct SPLAY {
2     struct NODE {
3         int w, min;
4         int son[2], size, father, rev, lazy;
5     } node[N];
6     int top, rt;
7     void pushdown(int x) {
8         if (!x) return;
9         if (node[x].rev) {
10            node[node[x].son[0]].rev ^= 1;
11            node[node[x].son[1]].rev ^= 1;
12            swap(node[x].son[0], node[x].son[1]);
13            node[x].rev = 0;
14        }
15        if (node[x].lazy) {
16            node[node[x].son[0]].lazy += node[x].lazy;
17            node[node[x].son[1]].lazy += node[x].lazy;
18            node[x].w += node[x].lazy;
19            node[x].min += node[x].lazy;
20            node[x].lazy = 0;
21        }
22    }
23    void pushup(int x) {
24        if (!x) return;
25        pushdown(node[x].son[0]);
26        pushdown(node[x].son[1]);
27        node[x].size = node[node[x].son[0]].size + node[node[x].son[1]].size + 1;
28        node[x].min = node[x].w;
```

```

29     if (node[x].son[0]) node[x].min = min(node[x].min, node[node[x].son[0]].min)
30     ;
31     if (node[x].son[1]) node[x].min = min(node[x].min, node[node[x].son[1]].min)
32     ;
33 }
34 void sc(int x, int y, int w) {
35     node[x].son[w] = y;
36     node[y].father = x;
37     pushup(x);
38 }
39 void _ins(int w) {
40     top++;
41     node[top].w = node[top].min = w;
42     node[top].son[0] = node[top].son[1] = 0;
43     node[top].size = 1; node[top].father = 0; node[top].rev = 0;
44 }
45 void init() {
46     top = 0;
47     _ins(0); _ins(0); rt=1;
48     sc(1, 2, 1);
49 }
50 void rotate(int x) {
51     if (!x) return;
52     int y = node[x].father;
53     int w = node[y].son[1]==x;
54     sc(y, node[x].son[w^1], w);
55     sc(node[y].father, x, node[node[y].father].son[1]==y);
56     sc(x, y, w^1);
57 }
58 int q[N];
59 void flushdown(int x) {
60     int t=0; for (; x; x=node[x].father) q[++t]=x;
61     for (; t; t--) pushdown(q[t]);
62 }
63 void Splay(int x, int root=0) {
64     flushdown(x);
65     while (node[x].father != root) {
66         int y=node[x].father;
67         int w=node[y].son[1]==x;
68         if (node[y].father != root && node[node[y].father].son[w]==y) rotate(y);
69         rotate(x);
70     }
71 }
72 int find(int k) {
73     Splay(rt);
74     while (1) {
75         pushdown(rt);
76         if (node[node[rt].son[0]].size+1==k) {
77             Splay(rt);
78             return rt;
79         }
80     }
81 }

```

```

77         } else
78         if (node[node[rt].son[0]].size+1<k) {
79             k-=node[node[rt].son[0]].size+1;
80             rt=node[rt].son[1];
81         } else {
82             rt=node[rt].son[0];
83         }
84     }
85 }
86 int split(int x, int y) {
87     int fx = find(x);
88     int fy = find(y+2);
89     Splay(fx);
90     Splay(fy, fx);
91     return node[fy].son[0];
92 }
93 void add(int x, int y, int d) { //add d to each number in a[x]...a[y]
94     int t = split(x, y);
95     node[t].lazy += d;
96     Splay(t); rt=t;
97 }
98 void reverse(int x, int y) { // reverse the x-th to y-th elements
99     int t = split(x, y);
100    node[t].rev ^= 1;
101    Splay(t); rt=t;
102 }
103 void insert(int x, int p) { // insert p after the x-th element
104     int fx = find(x+1);
105     int fy = find(x+2);
106     Splay(fx);
107     Splay(fy, fx);
108     _ins(p);
109     sc(fy, top, 0);
110     Splay(top); rt=top;
111 }
112 void del(int x) { // delete the x-th element in Splay
113     int fx = find(x), fy = find(x+2);
114     Splay(fx); Splay(fy, fx);
115     node[fy].son[0] = 0;
116     Splay(fy); rt=fy;
117 }
118 } tree;

```

### 3.5 可持久化 Treap

接口:

void insert(int x, char c); 在当前第  $x$  个字符后插入  $c$

void del(int x, int y); 删除第  $x$  个字符到第  $y$  个字符

void copy(int l, int r, int x); 复制第  $l$  个字符到第  $r$  个字符, 然后粘贴到第  $x$  个字符后

void reverse(int x, int y); 翻转第  $x$  个到第  $y$  个字符  
char query(int k); 表示询问当前第  $x$  个字符是什么

```

1 #define mod 1000000007
2 struct Treap {
3     struct Node {
4         char key;
5         bool reverse;
6         int lc, rc, size;
7     } node[N];
8     int n, root, rd;
9     int Rand() { rd = (rd * 20372052LL + 25022087LL) % mod; return rd; }
10    void init() { n = root = 0; }
11    inline int copy(int x) { node[++n] = node[x]; return n; }
12    inline void pushdown(int x) {
13        if (!node[x].reverse) return;
14        if (node[x].lc) node[x].lc = copy(node[x].lc);
15        if (node[x].rc) node[x].rc = copy(node[x].rc);
16        swap(node[x].lc, node[x].rc);
17        node[node[x].lc].reverse ^= 1;
18        node[node[x].rc].reverse ^= 1;
19        node[x].reverse = 0;
20    }
21    inline void pushup(int x) { node[x].size = node[node[x].lc].size + node[node[x].rc].size + 1; }
22    int merge(int u, int v) {
23        if (!u || !v) return u+v;
24        pushdown(u); pushdown(v);
25        int t = Rand() % (node[u].size + node[v].size), r;
26        if (t < node[u].size) {
27            r = copy(u);
28            node[r].rc = merge(node[u].rc, v);
29        } else {
30            r = copy(v);
31            node[r].lc = merge(u, node[v].lc);
32        }
33        pushup(r);
34        return r;
35    }
36    int split(int u, int x, int y) {
37        if (x > y) return 0;
38        pushdown(u);
39        if (x == 1 && y == node[u].size) return u;
40        if (y <= node[node[u].lc].size) return split(node[u].lc, x, y);
41        int t = node[node[u].lc].size + 1;
42        if (x > t) return split(node[u].rc, x-t, y-t);
43        int num = copy(u);
44        node[num].lc = split(node[u].lc, x, t-1);
45        node[num].rc = split(node[u].rc, 1, y-t);
46        pushup(num);

```

```

47     return num;
48 }
49 void insert(int x, char c) {
50     int t1 = split(root, 1, x), t2 = split(root, x+1, node[root].size);
51     node[++n].key = c; node[n].size = 1;
52     root = merge(merge(t1, n), t2);
53 }
54 void del(int x, int y) {
55     int t1 = split(root, 1, x-1), t2 = split(root, y+1, node[root].size);
56     root = merge(t1, t2);
57 }
58 void copy(int l, int r, int x) {
59     int t1 = split(root, 1, x), t2 = split(root, l, r), t3 = split(root, x+1,
60         node[root].size);
61     root = merge(merge(t1, t2), t3);
62 }
63 void reverse(int x, int y) {
64     int t1 = split(root, 1, x-1), t2 = split(root, x, y), t3 = split(root, y+1,
65         node[root].size);
66     node[t2].reverse ^= 1;
67     root = merge(merge(t1, t2), t3);
68 }
69 char query(int k) {
70     int x = root;
71     while (1) {
72         pushdown(x);
73         if (k <= node[node[x].lc].size) x = node[x].lc;
74         else
75             if (k == node[node[x].lc].size + 1) return node[x].key;
76             else
77                 k -= node[node[x].lc].size + 1, x = node[x].rc;
78     }
79 }
80 } treap;

```

### 3.6 可持久化并查集

接口：

void init() 初始化

void merge(int x, int y, int time) 在 time 时刻将 x 和 y 连一条边，注意加边顺序必须按 time 从小到大加边

void GetFather(int x, int time) 询问 time 时刻及以前的连边状态中，x 所属的集合

```

1 namespace pers_union {
2     const int inf = 0x3f3f3f3f;
3     int father[N], Father[N], Time[N];
4     vector<int> e[N];
5     void init() {
6         for (int i=1; i<=n; i++) {

```

```

7         father[i] = i;
8         Father[i] = i;
9         Time[i] = inf;
10        e[i].clear();
11        e[i].push_back(i);
12    }
13 }
14 int getfather(int x) {
15     return (father[x] == x) ? x : father[x] = getfather(father[x]);
16 }
17 int GetFather(int x, int time) {
18     return (Time[x] <= time) ? GetFather(Father[x], time) : x;
19 }
20 void merge(int x, int y, int time) {
21     int fx = getfather(x), fy = getfather(y);
22     if (fx == fy) return;
23     if (e[fx].size() > e[fy].size()) swap(fx, fy);
24     father[fx] = fy;
25     Father[fx] = fy;
26     Time[fx] = time;
27     for (int i=0; i<e[fx].size(); i++) {
28         e[fy].push_back(e[fx][i]);
29     }
30 }
31 };

```

## 4 树

### 4.1 点分治

初始化时须设置  $top = 1$  。

```

1 void addedge(int x, int y) {
2     edge[++top].adj = y;
3     edge[top].valid = 1;
4     edge[top].next = gh[x];
5     gh[x] = top;
6 }
7 void get_size(int x, int root=0) {
8     size[x] = 1; son[x] = 0;
9     int dd = 0;
10    for (int p=gh[x]; p; p=edge[p].next)
11        if (edge[p].adj != root && edge[p].valid) {
12            get_size(edge[p].adj, x);
13            size[x] += size[edge[p].adj];
14            if (size[edge[p].adj] > dd) {
15                dd = size[edge[p].adj];
16                son[x] = edge[p].adj;
17            }

```



```

18     }
19 }
20 int getroot(int x) {
21     get_size(x);
22     int sz = size[x];
23     while (size[son[x]] > sz/2)
24         x = son[x];
25     return x;
26 }
27 void dc(int x) {
28     x = getroot(x);
29     static int list[N], ltop;
30     ltop = 0;
31     for (int p=gh[x]; p; p=edge[p].next)
32         if (edge[p].valid)
33             list[++ltop] = p;
34     clear();
35     for (int i=1;i<=ltop;i++) {
36         update();
37         modify();
38     }
39     clear();
40     for (int i=ltop;i>=1;i--) {
41         update();
42         modify();
43     }
44     //be careful about the root
45     for (int p=gh[x]; p; p=edge[p].next)
46         if (edge[p].valid) {
47             edge[p].valid = 0;
48             edge[p^1].valid = 0;
49             dc(edge[p].adj);
50         }
51 }

```

## 4.2 Link Cut Tree

接口：

command(x, y)：将 x 到 y 路径的 Splay Tree 分离出来。

linkcut(u1, v1, u2, v2)：将树中原有的边 (u1, v1) 删除，加入一条新边 (u2, v2)

```

1 struct DynamicTREE{
2     struct NODE{
3         int father, son[2], top, size, reverse;
4     } splay[N];
5     void init(int i, int fat) {
6         splay[i].father = splay[i].son[0] = splay[i].son[1] = 0;
7         splay[i].top = fat; splay[i].size = 1; splay[i].reverse = 0;
8     }

```

```

9   void pushdown(int x) {
10       if (!x) return;
11       int s0 = splay[x].son[0], s1 = splay[x].son[1];
12       if (splay[x].reverse) {
13           splay[s0].reverse ^= 1;
14           splay[s1].reverse ^= 1;
15           swap(splay[x].son[0], splay[x].son[1]);
16           splay[x].reverse = 0;
17       }
18       s0 = splay[x].son[0], s1 = splay[x].son[1];
19       splay[s0].top = splay[s1].top = splay[x].top;
20   }
21   void pushup(int x) {
22       if (!x) return;
23       pushdown(splay[x].son[0]);
24       pushdown(splay[x].son[1]);
25       splay[x].size = splay[splay[x].son[0]].size + splay[splay[x].son[1]].size +
26           1;
27   }
28   void sc(int x, int y, int w, bool Auto=true) {
29       splay[x].son[w] = y;
30       splay[y].father = x;
31       if (Auto) {
32           pushup(y);
33           pushup(x);
34       }
35   }
36   int top, tush[N];
37   void flowdown(int x) {
38       for (top=1; x; top++, x = splay[x].father) tush[top] = x;
39       for (; top; top--) pushdown(tush[top]);
40   }
41   void rotate(int x) {
42       if (!x) return;
43       int y = splay[x].father;
44       int w = splay[y].son[1] == x;
45       pushdown(y);
46       pushdown(x);
47       sc(splay[y].father, x, splay[splay[y].father].son[1]==y, false);
48       sc(y, splay[x].son[w^1], w, false);
49       sc(x, y, w^1, false);
50       pushup(y);
51       pushup(x);
52   }
53   void Splay(int x, int rt=0) {
54       if (!x) return;
55       flowdown(x);
56       while (splay[x].father != rt) {
57           int y = splay[x].father;
58           int w = splay[y].son[1]==x;

```

```

58         if (splay[y].father != rt && splay[splay[y].father].son[w] == y) rotate(
59             y);
60         rotate(x);
61     }
62     void split(int x) {
63         int y = splay[x].son[1];
64         if (!y) return;
65         splay[y].father = 0;
66         splay[x].son[1] = 0;
67         splay[y].top = x;
68         pushup(x);
69     }
70     void access(int x) {
71         int y = 0;
72         while (x) {
73             Splay(x);
74             split(x);
75             sc(x, y, 1);
76             Splay(x);
77             y = x;
78             x = splay[x].top;
79         }
80     }
81     void changeroot(int x) {
82         access(x);
83         Splay(x);
84         splay[x].reverse = 1;
85         Splay(x);
86     }
87     void command(int x, int y, ...) {
88         LL ans = 0;
89         changeroot(x);
90         access(y);
91         Splay(x);
92         //then you can modify the Splay Tree
93     }
94     void linkcut(int u1, int v1, int u2, int v2) {
95         changeroot(u1);
96         access(v1);
97         Splay(u1); split(u1);
98         splay[v1].top = 0;
99         access(u2); changeroot(u2);
100        access(v2); changeroot(v2);
101        Splay(u2); Splay(v2);
102        splay[v2].top = u2;
103    }
104 } lct;

```

## 5 图

### 5.1 欧拉回路

欧拉回路：

无向图：每个顶点的度数都是偶数，则存在欧拉回路。

有向图：每个顶点的入度 = 出度，则存在欧拉回路。

欧拉路径：

无向图：当且仅当该图所有顶点的度数为偶数，或者除了两个度数为奇数外其余的全是偶数。

有向图：当且仅当该图所有顶点出度 = 入度或者一个顶点出度 = 入度 + 1，另一个顶点入度 = 出度 + 1，其他顶点出度 = 入度。

下面  $O(n + m)$  求欧拉回路的代码中， $n$  为点数， $m$  为边数，若有解则依次输出经过的边的编号，若是无向图，则正数表示  $x$  到  $y$ ，负数表示  $y$  到  $x$ 。

```
1 namespace UndirectedGraph{
2     int n,m,i,x,y,d[N],g[N],v[M<<1],w[M<<1],vis[M<<1],nxt[M<<1],ed;
3     int ans[M],cnt;
4     void add(int x,int y,int z){
5         d[x]++;
6         v[++ed]=y;w[ed]=z;nxt[ed]=g[x];g[x]=ed;
7     }
8     void dfs(int x){
9         for(int&i=g[x];i;){
10             if(vis[i]){i=nxt[i];continue;}
11             vis[i]=vis[i^1]=1;
12             int j=w[i];
13             dfs(v[i]);
14             ans[++cnt]=j;
15         }
16     }
17     void solve(){
18         scanf("%d%d",&n,&m);
19         for(i=ed=1;i<=m;i++)scanf("%d%d",&x,&y),add(x,y,i),add(y,x,-i);
20         for(i=1;i<=n;i++)if(d[i]&1){puts("NO");return;}
21         for(i=1;i<=n;i++)if(g[i]){dfs(i);break;}
22         for(i=1;i<=n;i++)if(g[i]){puts("NO");return;}
23         puts("YES");
24         for(i=m;i;i--)printf("%d_",ans[i]);
25     }
26 }
27 namespace DirectedGraph{
28     int n,m,i,x,y,d[N],g[N],v[M],vis[M],nxt[M],ed;
29     int ans[M],cnt;
30     void add(int x,int y){
31         d[x]++;d[y]--;
32         v[++ed]=y;nxt[ed]=g[x];g[x]=ed;
33     }
34     void dfs(int x){
35         for(int&i=g[x];i;){
```

```

36         if(vis[i]){i=nxt[i];continue;}
37         vis[i]=1;
38         int j=i;
39         dfs(v[i]);
40         ans[++cnt]=j;
41     }
42 }
43 void solve(){
44     scanf("%d%d",&n,&m);
45     for(i=1;i<=m;i++)scanf("%d%d",&x,&y),add(x,y);
46     for(i=1;i<=n;i++)if(d[i]){puts("NO");return;}
47     for(i=1;i<=n;i++)if(g[i]){dfs(i);break;}
48     for(i=1;i<=n;i++)if(g[i]){puts("NO");return;}
49     puts("YES");
50     for(i=m;i;i--)printf("%d_",ans[i]);
51 }
52 }

```

## 5.2 最短路径

### 5.2.1 Dijkstra

```

1  #include <queue>
2  using namespace std;
3  struct EDGE { int adj, w, next; } edge[M*2];
4  struct dat { int id, dist; dat(int id=0, int dist=0) : id(id), dist(dist) {} };
5  struct cmp { bool operator () (const dat &a, const dat &b) { return a.dist > b.dist;
6  } };
7  priority_queue < dat, vector<dat>, cmp > q;
8  int n, top, gh[N], v[N], dist[N];
9  void addedge(int x, int y, int w) {
10     edge[++top].adj = y;
11     edge[top].w = w;
12     edge[top].next = gh[x];
13     gh[x] = top;
14 }
15 int dijkstra(int s, int t) {
16     memset(dist, 63, sizeof(dist));
17     memset(v, 0, sizeof(v));
18     dist[s] = 0;
19     q.push(dat(s, 0));
20     while (!q.empty()) {
21         dat x = q.top(); q.pop();
22         if (v[x.id]) continue; v[x.id] = 1;
23         for (int p=gh[x.id]; p; p=edge[p].next) {
24             if (x.dist + edge[p].w < dist[edge[p].adj]) {
25                 dist[edge[p].adj] = x.dist + edge[p].w;
26                 q.push(dat(edge[p].adj, dist[edge[p].adj]));

```

```

27     }
28 }
29 return dist[t];
30 }

```

### 5.2.2 SPFA

```

1 struct EDGE { int adj, w, next; } edge[M*2];
2 int n,m,top,gh[N],v[N],cnt[N],q[N],dist[N],head,tail;
3 void addedge(int x, int y, int w) {
4     edge[++top].adj = y;
5     edge[top].w = w;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 int spfa(int S, int T) {
10     memset(v, 0, sizeof(v));
11     memset(cnt, 0, sizeof(cnt));
12     memset(dist, 63, sizeof(dist));
13     head = 0, tail = 1;
14     dist[S] = 0; q[1] = S;
15     while (head != tail) {
16         (head += 1) %= N;
17         int x = q[head]; v[x] = 0;
18         ++cnt[x]; if (cnt[x] > n) return -1;
19         for (int p=gh[x]; p; p=edge[p].next)
20             if (dist[x] + edge[p].w < dist[edge[p].adj]) {
21                 dist[edge[p].adj] = dist[x] + edge[p].w;
22                 if (!v[edge[p].adj]) {
23                     v[edge[p].adj] = 1;
24                     (tail += 1) %= N;
25                     q[tail] = edge[p].adj;
26                 }
27             }
28     }
29     return dist[T];
30 }

```

## 5.3 K 短路

接口：

kthsp::init(n)：初始化并设置节点个数为 n

kthsp::add(x, y, w)：添加一条 x 到 y 的有向边

kthsp::work(S, T, k)：求 S 到 T 的第 k 短路

```

1 #include <queue>
2
3 #define N 200020

```

```

4  #define M 400020
5  #define LOGM 20
6  #define LL long long
7  #define inf (1LL<<61)
8
9  namespace pheap {
10     struct Node {
11         int next, son[2];
12         LL val;
13     } node[M*LOGM];
14     int LOG[M];
15     int root[M], size[M*LOGM], top;
16     int add() {
17         ++top; assert(top < M*LOGM);
18         node[top].next = node[top].son[0] = node[top].son[1] = 0;
19         node[top].val = inf;
20         return top;
21     }
22     int copy(int x) { int t = add(); node[t] = node[x]; return t; }
23     void init() {
24         top = -1; add();
25         for (int i=2;i<M;i++) LOG[i] = LOG[i>>1] + 1;
26     }
27     void upd(int x, int &next, LL &val) {
28         if (val < node[x].val) {
29             swap(val, node[x].val);
30             swap(next, node[x].next);
31         }
32     }
33     void insert(int x, int next, LL val) {
34         int sz = size[root[x]] + 1;
35         root[x] = copy(root[x]);
36         size[root[x]] = sz; x = root[x];
37         upd(x, next, val);
38         for (int i=LOG[sz]-1;i>=0;i--) {
39             int ind = (sz>>i)&1;
40             node[x].son[ind] = copy(node[x].son[ind]);
41             x = node[x].son[ind];
42             upd(x, next, val);
43         }
44     }
45 };
46
47 namespace kthsp {
48     using namespace pheap;
49     struct EDGE {
50         int adj, w, next;
51     } edge[2][M];
52     struct W {
53         int x, y, w;

```

```

54     } e[M];
55     bool has_init = 0;
56     int n, m, top[2], gh[2][N], v[N];
57     LL dist[N];
58     void init(int n1) {
59         has_init = 1;
60         n = n1; m = 0;
61         memset(top, 0, sizeof(top));
62         memset(gh, 0, sizeof(gh));
63         for (int i=1;i<=n;i++) dist[i] = inf;
64     }
65     void addedge(int id, int x, int y, int w) {
66         edge[id][++top[id]].adj = y;
67         edge[id][top[id]].w = w;
68         edge[id][top[id]].next = gh[id][x];
69         gh[id][x] = top[id];
70     }
71     void add(int x, int y, int w) {
72         assert(has_init);
73         e[++m].x=x; e[m].y=y; e[m].w=w;
74     }
75     int q[N], best[N], bestw[N];
76     int deg[N];
77     void spfa(int S) {
78         for (int i=1;i<=n;i++) deg[i] = 0;
79         for (int i=1;i<=m;i++) deg[e[i].x] ++;
80         int head = 0, tail = 1;
81         dist[S] = 0; q[1] = S;
82         while (head != tail) {
83             (head += 1) %= N;
84             int x = q[head];
85             for (int p=gh[1][x]; p; p=edge[1][p].next) {
86                 if (dist[x] + edge[1][p].w < dist[edge[1][p].adj]) {
87                     dist[edge[1][p].adj] = dist[x] + edge[1][p].w;
88                     best[edge[1][p].adj] = x;
89                     bestw[edge[1][p].adj] = p;
90                 }
91                 if (!--deg[edge[1][p].adj]) {
92                     (tail += 1) %= N;
93                     q[tail] = edge[1][p].adj;
94                 }
95             }
96         }
97     }
98     void dfs(int x) {
99         if (v[x]) return; v[x] = 1;
100         if (best[x]) root[x] = root[best[x]];
101         for (int p=gh[0][x]; p; p=edge[0][p].next)
102             if (dist[edge[0][p].adj] != inf && bestw[x] != p) {
103                 insert(x, edge[0][p].adj, edge[0][p].w + dist[edge[0][p].adj] - dist

```



```

103         [x]);
104     }
105     for (int p=gh[1][x]; p; p=edge[1][p].next)
106         if (best[edge[1][p].adj] == x)
107             dfs(edge[1][p].adj);
108 }
109 typedef pair<LL,int> pli;
110 priority_queue <pli, vector<pli>, greater<pli> > pq;
111 LL work(int S, int T, int k) {
112     assert(has_init);
113     n++; add(T, n, 0);
114     if (S == T) k ++;
115     T = n;
116     for (int i=1;i<=m;i++) {
117         addedge(0, e[i].x, e[i].y, e[i].w);
118         addedge(1, e[i].y, e[i].x, e[i].w);
119     }
120     spfa(T);
121     root[T] = 0; pheap::init();
122     memset(v, 0, sizeof(v));
123     dfs(T);
124     while (!pq.empty()) pq.pop();
125     if (k == 1) return dist[S];
126     if (root[S]) pq.push(make_pair(dist[S] + node[root[S]].val, root[S]));
127     while (k--) {
128         if (pq.empty()) return inf;
129         pli now = pq.top(); pq.pop();
130         if (k == 1) return now.first;
131         int x = node[now.second].next, u = node[now.second].son[0], v = node[now
            .second].son[1];
132         if (root[x]) pq.push(make_pair(now.first + node[root[x]].val, root[x]));
133         if (u) pq.push(make_pair(now.first - node[now.second].val + node[u].val,
            u));
134         if (v) pq.push(make_pair(now.first - node[now.second].val + node[v].val,
            v));
135     }
136     return 0;
137 }
138 };

```

## 5.4 Tarjan

割点的判断：一个顶点  $u$  是割点，当且仅当满足 (1) 或 (2)：

- (1)  $u$  为树根，且  $u$  有多于一个子树（即：存在一个儿子  $v$  使得  $dfn[u] + 1 \neq dfn[v]$ ）
- (2)  $u$  不为树根，且满足存在  $(u, v)$  为树枝边（ $u$  为  $v$  的父亲），使得  $dfn[u] \leq low[v]$

桥的判断：一条无向边  $(u, v)$  是桥，当且仅当  $(u, v)$  为树枝边，满足  $dfn[u] < low[v]$

```

1 struct EDGE { int adj, next; } edge[M];
2 int n, m, top, gh[N];

```

```

3  int dfn[N], low[N], cnt, ind, stop, instack[N], stack[N], belong[N];
4  void addedge(int x, int y) {
5      edge[++top].adj = y;
6      edge[top].next = gh[x];
7      gh[x] = top;
8  }
9  void tarjan(int x) {
10     dfn[x] = low[x] = ++ind;
11     instack[x] = 1; stack[++stop] = x;
12     for (int p=gh[x]; p; p=edge[p].next)
13         if (!dfn[edge[p].adj]) {
14             tarjan(edge[p].adj);
15             low[x] = min(low[x], low[edge[p].adj]);
16         } else if (instack[edge[p].adj]) {
17             low[x] = min(low[x], dfn[edge[p].adj]);
18         }
19     if (dfn[x] == low[x]) {
20         ++cnt; int tmp=0;
21         while (tmp!=x) {
22             tmp = stack[stop--];
23             belong[tmp] = cnt;
24             instack[tmp] = 0;
25         }
26     }
27 }

```

## 5.5 2-SAT

```

1  #define N number_of_vertex
2  #define M number_of_edges
3
4  struct MergePoint {
5      struct EDGE {
6          int adj, next;
7      } edge[M];
8      int ex[M], ey[M];
9      bool instack[N];
10     int gh[N], top, dfn[N], low[N], cnt, ind, stop, stack[N], belong[N];
11     void init() {
12         cnt = ind = stop = top = 0;
13         memset(dfn, 0, sizeof(dfn));
14         memset(instack, 0, sizeof(instack));
15         memset(gh, 0, sizeof(gh));
16     }
17     void addedge(int x, int y) { //reverse
18         std::swap(x, y);
19         edge[++top].adj = y;
20         edge[top].next = gh[x];
21         gh[x] = top;

```

```

22     ex[top] = x;
23     ey[top] = y;
24 }
25 void tarjan(int x) {
26     dfn[x] = low[x] = ++ind;
27     instack[x] = 1; stack[++stop] = x;
28     for (int p=gh[x]; p; p=edge[p].next)
29         if (!dfn[edge[p].adj]) {
30             tarjan(edge[p].adj);
31             low[x] = std::min(low[x], low[edge[p].adj]);
32         } else if (instack[edge[p].adj]) {
33             low[x] = std::min(low[x], dfn[edge[p].adj]);
34         }
35     if (dfn[x] == low[x]) {
36         ++cnt; int tmp = 0;
37         while (tmp!=x) {
38             tmp = stack[stop--];
39             belong[tmp] = cnt;
40             instack[tmp] = 0;
41         }
42     }
43 }
44 void work() {
45     for (int i = (__first__); i <= (__last__); ++i)
46         if (!dfn[i])
47             tarjan(i);
48 }
49 } merge;
50
51 struct Topsort {
52     struct EDGE {
53         int adj, next;
54     } edge[M];
55     int n, top, gh[N], ops[N], deg[N], ans[N];
56     std::queue<int> q;
57     void init() {
58         n = merge.cnt; top = 0;
59         memset(gh, 0, sizeof(gh));
60         memset(deg, 0, sizeof(deg));
61     }
62     void addedge(int x, int y) {
63         if (x == y) return;
64         edge[++top].adj = y;
65         edge[top].next = gh[x];
66         gh[x] = top;
67         ++deg[y];
68     }
69     void work() {
70         for (int i = 1; i <= n; ++i)
71             if (!deg[i])

```

```

72         q.push(i);
73     while (!q.empty()) {
74         int x = q.front();
75         q.pop();
76         for (int p = gh[x]; p; p = edge[p].next)
77             if (!--deg[edge[p].adj])
78                 q.push(edge[p].adj);
79         if (ans[x]) continue;
80         ans[x] = -1; //not selected
81         ans[ops[x]] = 1; //selected
82     }
83 }
84 } ts;

```

调用示例：

```

1     merge.init();
2     merge.addedge();
3     merge.work();
4     for (int i = 1; i <= n; ++i) {
5         if (merge.belong[U(i, 0)] == merge.belong[U(i, 1)]) {
6             puts("NO");
7             return 0;
8         }
9         ts.ops[merge.belong[U(i, 0)]] = merge.belong[U(i, 1)];
10        ts.ops[merge.belong[U(i, 1)]] = merge.belong[U(i, 0)];
11    }
12    ts.init();
13    ts.work();
14    puts("YES");
15    for (int i = 1; i <= n; ++i) {
16        int x = U(i, 0), y = U(i, 1);
17        x = merge.belong[x], y = merge.belong[y];
18        x = ts.ans[x], y = ts.ans[y];
19        if (x == 1) puts("0_is_selected");
20        if (y == 1) puts("1_is_selected");
21    }

```

## 5.6 统治者树 (Dominator Tree)

Dominator Tree 可以解决判断一类有向图必经点的问题。

$idom[x]$  表示离  $x$  最近的必经点（重编号后）。将  $idom[x]$  作为  $x$  的父亲，构成一棵 Dominator Tree

接口：

`void dominator::init(int n)`; 初始化，有向图节点数为  $n$

`void dominator::addedge(int u, int v)`; 添加一条有向边  $(u, v)$

`void dominator::work(int root)`; 以  $root$  为根，建立一棵 Dominator Tree

结果的返回：

在执行 `dominator::work(int root)`; 后，树边保存在 `vector<int> tree[N]` 中

```

1 namespace dominator {
2     vector<int> g[N], rg[N], bucket[N], tree[N];
3     int n, ind, idom[N], sdom[N], dfn[N], dsu[N], father[N], label[N], rev[N];
4     void dfs(int x) {
5         ++ind;
6         dfn[x] = ind; rev[ind] = x;
7         label[ind] = dsu[ind] = sdom[ind] = ind;
8         for (auto p : g[x]) {
9             if (!dfn[p]) dfs(p), father[dfn[p]] = dfn[x];
10            rg[dfn[p]].push_back(dfn[x]);
11        }
12    }
13    void init(int n1) {
14        n = n1; ind = 0;
15        for (int i = 1; i <= n; ++i) {
16            g[i].clear();
17            rg[i].clear();
18            bucket[i].clear();
19            tree[i].clear();
20            dfn[i] = 0;
21        }
22    }
23    void addedge(int u, int v) {
24        g[u].push_back(v);
25    }
26    int find(int x, int step=0) {
27        if (dsu[x] == x) return step ? -1 : x;
28        int y = find(dsu[x], 1);
29        if (y < 0) return x;
30        if (sdom[label[dsu[x]]] < sdom[label[x]])
31            label[x] = label[dsu[x]];
32        dsu[x] = y;
33        return step ? dsu[x] : label[x];
34    }
35    void work(int root) {
36        dfs(root); n = ind;
37        for (int i = n; i; --i) {
38            for (auto p : rg[i])
39                sdom[i] = min(sdom[i], sdom[find(p)]);
40            if (i > 1) bucket[sdom[i]].push_back(i);
41            for (auto p : bucket[i]) {
42                int u = find(p);
43                if (sdom[p] == sdom[u]) idom[p] = sdom[p];
44                else idom[p] = u;
45            }
46            if (i > 1) dsu[i] = father[i];
47        }
48        for (int i = 2; i <= n; ++i) {
49            if (idom[i] != sdom[i])

```

```

50         idom[i] = idom[idom[i]];
51         tree[rev[i]].push_back(rev[idom[i]]);
52         tree[rev[idom[i]]].push_back(rev[i]);
53     }
54 }
55 };

```

## 5.7 网络流

### 5.7.1 最大流

注意: *top* 要初始化为 1

```

1  struct EDGE { int adj, w, next; } edge[M];
2  int n, top, gh[N], nrl[N];
3  void addedge(int x, int y, int w) {
4      edge[++top].adj = y;
5      edge[top].w = w;
6      edge[top].next = gh[x];
7      gh[x] = top;
8      edge[++top].adj = x;
9      edge[top].w = 0;
10     edge[top].next = gh[y];
11     gh[y] = top;
12 }
13 int dist[N], q[N];
14 int bfs() {
15     memset(dist, 0, sizeof(dist));
16     q[1] = S; int head = 0, tail = 1; dist[S] = 1;
17     while (head != tail) {
18         int x = q[++head];
19         for (int p=gh[x]; p; p=edge[p].next)
20             if (edge[p].w && !dist[edge[p].adj]) {
21                 dist[edge[p].adj] = dist[x] + 1;
22                 q[++tail] = edge[p].adj;
23             }
24     }
25     return dist[T];
26 }
27 int dinic(int x, int delta) {
28     if (x==T) return delta;
29     for (int& p=nrl[x]; p && delta; p=edge[p].next)
30         if (edge[p].w && dist[x]+1 == dist[edge[p].adj]) {
31             int dd = dinic(edge[p].adj, min(delta, edge[p].w));
32             if (!dd) continue;
33             edge[p].w -= dd;
34             edge[p^1].w += dd;
35             return dd;
36         }
37     return 0;

```

```

38 }
39 int work() {
40     int ans = 0;
41     while (bfs()) {
42         memcpy(nrl, gh, sizeof(gh));
43         int t; while (t = dinic(S, inf)) ans += t;
44     }
45     return ans;
46 }

```

### 5.7.2 上下界有源汇网络流

$T$  向  $S$  连容量为正无穷的边，将有源汇转化为无源汇。

每条边容量减去下界，设  $in[i]$  表示流入  $i$  的下界之和减去流出  $i$  的下界之和。

新建超级源汇  $SS, TT$ ，对于  $in[i] > 0$  的点， $SS$  向  $i$  连容量为  $in[i]$  的边。对于  $in[i] < 0$  的点， $i$  向  $TT$  连容量为  $-in[i]$  的边。

求出以  $SS, TT$  为源汇的最大流，如果等于  $\sum in[i] (in[i] > 0)$ ，则存在可行流。再求出  $S, T$  为源汇的最大流即为最大流。

费用流：建完图后等价于求以  $SS, TT$  为源汇的费用流。

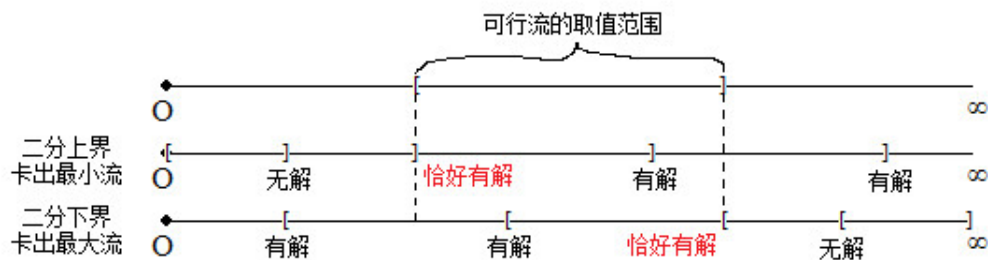
### 5.7.3 上下界无源汇网络流

1. 怎样求无源汇有上下界网络的可行流？

由于有源汇的网络我们先要转化成无源汇，所以本来就无源汇的网络不用再作特殊处理。

2. 怎样求无源汇有上下界网络的最大流、最小流？

一种简易的方法是采用二分思想，不断通过可行流的存在与否对  $(t, s)$  边的上下界  $U, L$  进行调整。求最大流时令  $U = \infty$  并二分  $L$ ；求最小流时令  $L = 0$  并二分  $U$ 。道理很简单，因为可行流的取值范围是一段连续的区间，我们只要通过二分找到有解和无解的分界线即可。



### 5.7.4 费用流

注意： $top$  要初始化为 1

```

1 #define inf 0x3f3f3f3f
2 struct NetWorkFlow {
3     struct EDGE {
4         int adj, w, cost, next;

```

```

5     } edge[M*2];
6     int gh[N], q[N], dist[N], v[N], pre[N], prev[N], top;
7     int S, T;
8     void addedge(int x, int y, int w, int cost) {
9         edge[++top].adj = y;
10        edge[top].w = w;
11        edge[top].cost = cost;
12        edge[top].next = gh[x];
13        gh[x] = top;
14        edge[++top].adj = x;
15        edge[top].w = 0;
16        edge[top].cost = -cost;
17        edge[top].next = gh[y];
18        gh[y] = top;
19    }
20    void clear() {
21        top = 1;
22        memset(gh, 0, sizeof(gh));
23    }
24    int spfa() {
25        memset(dist, 63, sizeof(dist));
26        memset(v, 0, sizeof(v));
27        int head = 0, tail = 1;
28        q[1] = S; v[S] = 1; dist[S] = 0;
29        while (head != tail) {
30            (head += 1) %= N;
31            int x = q[head];
32            v[x] = 0;
33            for (int p=gh[x]; p; p=edge[p].next)
34                if (edge[p].w && dist[x] + edge[p].cost < dist[edge[p].adj]) {
35                    dist[edge[p].adj] = dist[x] + edge[p].cost;
36                    pre[edge[p].adj] = x;
37                    prev[edge[p].adj] = p;
38                    if (!v[edge[p].adj]) {
39                        v[edge[p].adj] = 1;
40                        (tail += 1) %= N;
41                        q[tail] = edge[p].adj;
42                    }
43                }
44        }
45        return dist[T] != inf;
46    }
47    int work() {
48        int ans = 0;
49        while (spfa()) {
50            int mx = inf;
51            for (int x=T; x!=S; x=pre[x])
52                mx = min(edge[prev[x]].w, mx);
53            ans += dist[T] * mx;
54            for (int x=T; x!=S; x=pre[x]) {

```



```

55         edge[prev[x]].w -= mx;
56         edge[prev[x]^1].w += mx;
57     }
58 }
59 return ans;
60 }
61 } nwf;

```

### 5.7.5 zkw 费用流

注意: *top* 要初始化为 1, 不得用于有负权的图

```

1 #define inf 0x3f3f3f3f //modify if you use long long or double
2 template <class _tp>
3 struct NetWorkFlow {
4     struct EDGE {
5         int adj, next;
6         _tp w, cost;
7     } edge[M*2];
8     int gh[N], top;
9     int S, T;
10    void addedge(int x, int y, _tp w, _tp cost) {
11        edge[++top].adj = y;
12        edge[top].w = w;
13        edge[top].cost = cost;
14        edge[top].next = gh[x];
15        gh[x] = top;
16        edge[++top].adj = x;
17        edge[top].w = 0;
18        edge[top].cost = -cost;
19        edge[top].next = gh[y];
20        gh[y] = top;
21    }
22    void clear() {
23        top = 1;
24        memset(gh, 0, sizeof(gh));
25    }
26    int v[N];
27    _tp cost, d[N], slk[N];
28    _tp aug(int x, _tp f) {
29        _tp left = f;
30        if (x == T) {
31            cost += f * d[S];
32            return f;
33        }
34        v[x] = true;
35        for (int p=gh[x]; p; p=edge[p].next)
36            if (edge[p].w && !v[edge[p].adj]) {
37                _tp t = d[edge[p].adj] + edge[p].cost - d[x];
38                if (t == 0) {

```

```

39         _tp delt = aug(edge[p].adj, min(left, edge[p].w));
40         if (delt > 0) {
41             edge[p].w -= delt;
42             edge[p^1].w += delt;
43             left -= delt;
44         }
45         if (left == 0) return f;
46     } else {
47         if (t < slk[edge[p].adj])
48             slk[edge[p].adj] = t;
49     }
50 }
51 return f-left;
52 }
53 bool modlabel() {
54     _tp delt = inf;
55     for (int i=1;i<=T;i++)
56         if (!v[i]) {
57             if (slk[i] < delt) delt = slk[i];
58             slk[i] = inf;
59         }
60     if (delt == inf) return true;
61     for (int i=1;i<=T;i++)
62         if (v[i]) d[i] += delt;
63     return false;
64 }
65 _tp work() {
66     cost = 0;
67     memset(d, 0, sizeof(d));
68     memset(slk, 63, sizeof(slk));
69     do {
70         do {
71             memset(v, 0, sizeof(v));
72         } while (aug(S, inf));
73     } while (!modlabel());
74     return cost;
75 }
76 };
77 NetWorkFlow<int> nwf;

```

## 6 数学

### 6.1 扩展欧几里得解同余方程

ans[] 保存的是循环节内所有的解

```

1 int exgcd(int a,int b,int&x,int&y){
2     if(!b) return x=1,y=0,a;
3     int d=exgcd(b,a%b,x,y),t=x;

```

```

4     return x=y,y=t-a/b*y,d;
5 }
6 void cal(ll a,ll b,ll n){//ax=b(mod n)
7     ll x,y,d=exgcd(a,n,x,y);
8     if(b%d) return;
9     x=(x%n+n)%n;
10    ans[cnt=1]=x*(b/d)%(n/d);
11    for(ll i=1;i<d;i++) ans[++cnt]=(ans[1]+i*n/d)%n;
12 }

```

## 6.2 同余方程组

```

1 int n,flag,k,m,a,r,d,x,y;
2 int main(){
3     scanf("%d",&n);
4     flag=k=1,m=0;
5     while(n--){
6         scanf("%d%d",&a,&r);//ans%a=r
7         if(flag){
8             d=exgcd(k,a,x,y);
9             if((r-m)%d){flag=0;continue;}
10            x=(x*(r-m)/d+a/d)%(a/d),y=k/d*a,m=((x*k+m)%y)%y;
11            if(m<0)m+=y;
12            k=y;
13        }
14    }
15    printf("%d",flag?m:-1);//若flag=1,说明有解,解为ki+m,i为任意整数
16 }

```

## 6.3 卡特兰数

$$h_1 = 1, h_n = \frac{h_{n-1}(4n-2)}{n+1} = \frac{C(2n,n)}{n+1} = C(2n,n) - C(2n,n-1)$$

在一个格点阵列中,从  $(0,0)$  点走到  $(n,m)$  点且不经过对角线  $x=y$  的方案数  $(x>y)$  :

$$C(n+m-1,m) - C(n+m-1,m-1)$$

在一个格点阵列中,从  $(0,0)$  点走到  $(n,m)$  点且不穿过对角线  $x=y$  的方案数  $(x\geq y)$  :

$$C(n+m,m) - C(n+m,m-1)$$

## 6.4 斯特林数

### 6.4.1 第一类斯特林数

第一类 Stirling 数  $S(p,k)$  的一个组合学解释是:将  $p$  个物体排成  $k$  个非空循环排列的方法数。

$S(p,k)$  的递推公式:  $S(p,k) = (p-1)S(p-1,k) + S(p-1,k-1), 1 \leq k \leq p-1$

边界条件:  $S(p,0) = 0, p \geq 1, S(p,p) = 1, p \geq 0$

### 6.4.2 第二类斯特林数

第二类 Stirling 数  $S(p, k)$  的一个组合学解释是：将  $p$  个物体划分成  $k$  个非空的不可辨别（可以理解为盒子没有编号）集合的方法数。

$S(p, k)$  的递推公式：  $S(p, k) = kS(p-1, k) + S(p-1, k-1), 1 \leq k \leq p-1$

边界条件：  $S(p, 0) = 0, p \geq 1$   $S(p, p) = 1, p \geq 0$

也有卷积形式：

$$S(n, m) = \frac{1}{m!} \sum_{k=0}^m (-1)^k C(m, k) (m-k)^n = \sum_{k=0}^m \frac{(-1)^k (m-k)^n}{k! (m-k)!} = \sum_{k=0}^m \frac{(-1)^k}{k!} \times \frac{(m-k)^n}{(m-k)!}$$

### 6.5 错排公式

$$D_1 = 0, D_2 = 1, D_n = (n-1)(D_{n-2} + D_{n-1})$$

### 6.6 Lucas 定理

接口：

初始化： `void lucas::init();`

计算  $C(n, m) \% \text{mod}$  的值： `LL lucas::Lucas(LL n, LL m);`

```
1 #define mod 110119
2 #define LL long long
3 namespace lucas {
4     LL fac[mod+1], facv[mod+1];
5     LL power(LL base, LL times) {
6         LL ans = 1;
7         while (times) {
8             if (times&1) (ans *= base) %= mod;
9             (base *= base) %= mod;
10            times >>= 1;
11        }
12        return ans;
13    }
14    void init() {
15        fac[0] = 1; for (int i=1; i<mod; i++) fac[i] = (fac[i-1] * i) % mod;
16        facv[mod-1] = power(fac[mod-1], mod-2);
17        for (int i=mod-2; i>=0; --i) facv[i] = (facv[i+1] * (i+1)) % mod;
18    }
19    LL C(unsigned LL n, unsigned LL m) {
20        if (n < m) return 0;
21        return (fac[n] * facv[m] % mod * facv[n-m] % mod) % mod;
22    }
23    LL Lucas(unsigned LL n, unsigned LL m)
24    {
25        if (m == 0) return 1;
26        return (C(n%mod, m%mod) * Lucas(n/mod, m/mod)) % mod;
27    }
```

28 };

## 6.7 高斯消元

### 6.7.1 行列式

```
1 int ans = 1;
2 for (int i=0; i<n; i++) {
3     for (int j=i; j<n; j++)
4         if (g[j][i]) {
5             for (int k=i; k<n; k++)
6                 swap(g[i][k], g[j][k]);
7             if (j != i) ans *= -1;
8             break;
9         }
10    if (g[i][i] == 0) {
11        ans = 0;
12        break;
13    }
14    for (int j=i+1; j<n; j++) {
15        while (g[j][i]) {
16            int t = g[i][i] / g[j][i];
17            for (int k=i; k<n; k++)
18                g[i][k] = (g[i][k] + mod - ((LL)t * g[j][k] % mod)) % mod;
19            for (int k=i; k<n; k++)
20                swap(g[i][k], g[j][k]);
21            ans *= -1;
22        }
23    }
24 }
25 for (int i=0; i<n; i++)
26     ans = ((LL)ans * g[i][i]) % mod;
27 ans = (ans % mod + mod) % mod;
28 printf("%d\n", ans);
```

### 6.7.2 Matrix-Tree 定理

对于一张图，建立矩阵  $C$ ， $C[i][i]$  =  $i$  的度数，若  $i, j$  之间有边，那么  $C[i][j] = -1$ ，否则为 0。这张图的生成树个数等于矩阵  $C$  的  $n-1$  阶行列式的值。

## 6.8 调和级数

$\sum_{i=1}^n \frac{1}{i}$  在  $n$  较大时约等于  $\ln(n) + r$ ， $r$  为欧拉常数，约等于 0.5772156649015328。

## 6.9 曼哈顿距离的变换

$$|x_1 - x_2| + |y_1 - y_2| = \max(|(x_1 + y_1) - (x_2 + y_2)|, |(x_1 - y_1) - (x_2 - y_2)|)$$

## 6.10 线性筛素数

```

1 mu[1]=phi[1]=1;top=0;
2 for (int i=2;i<N;i++) {
3     if (!v[i]) prime[++top]=i, mu[i] = -1, phi[i] = i-1;
4     for (int j=1;i*prime[j]<N && j<=top;j++) {
5         v[i*prime[j]] = 1;
6         if (i%prime[j]) {
7             mu[i*prime[j]] = -mu[i];
8             phi[i*prime[j]] = phi[i] * (prime[j]-1);
9         } else {
10            mu[i*prime[j]] = 0;
11            phi[i*prime[j]] = phi[i] * prime[j];
12            break;
13        }
14    }
15 }

```

## 6.11 杜教筛

getphi(t, x) 表示求  $\sum_{i=1}^x i^t \phi(i)$ 。

推导过程：

记  $S(n) = \sum_{i=1}^n f(i)$ ，取任意函数  $g$  有恒等式

$$S(n) = \sum_{i=1}^n (f \cdot g)(i) - \sum_{i=2}^n g(i) S(\lfloor \frac{n}{i} \rfloor)$$

其中， $(f \cdot g)$  表示  $f$  和  $g$  的狄利克雷卷积：即： $(f \cdot g)(n) = \sum_{d|n} f(d)g(\frac{n}{d})$

关于恒等式的证明：

将  $\sum_{i=2}^n g(i) S(\lfloor \frac{n}{i} \rfloor)$  移到左边去，即只需证

$$\sum_{i=1}^n (f \cdot g)(i) = \sum_{i=1}^n g(i) S(\lfloor \frac{n}{i} \rfloor)$$

将狄利克雷卷积展开，得：

$$\sum_{i=1}^n \sum_{d|i} g(d) f(\frac{i}{d}) = \sum_{i=1}^n g(i) S(\lfloor \frac{n}{i} \rfloor)$$

即：

$$\sum_{d=1}^n g(d) \sum_{i=1}^{\lfloor \frac{n}{d} \rfloor} f(i) = \sum_{i=1}^n g(i) S(\lfloor \frac{n}{i} \rfloor)$$

显然相等，恒等式证完。

取  $f(i) = i^p \phi(i)$ ,  $g(i) = i^p$ ，则有：

$$S(n) = \sum_{i=1}^n i^p \phi(i) = \sum_{i=1}^n i^{p+1} - \sum_{i=2}^n i^p S(\lfloor \frac{n}{i} \rfloor)$$

其中有用到等式  $\sum_{d|n} \phi(d) = n$

另外，莫比乌斯函数  $\mu(n) = \begin{cases} 1, & \text{若 } n = 1 \\ (-1)^k, & \text{若 } n \text{ 无平方数因数, 且 } n = p_1 p_2 \cdots p_k \\ 0, & \text{若 } n \text{ 有大于 } 1 \text{ 的平方数因数} \end{cases}$  也可以使用杜教

筛求前缀和，记  $S'(n) = \sum_{i=1}^n \mu(i)$ ，则  $S'(n) = 1 - \sum_{i=2}^n S'(\lfloor \frac{n}{i} \rfloor)$

```

1 #include <bits/stdc++.h>
2
3 #define N 5000020
4 #define LL long long
5 #define mod 1000000007
6 #define div2 ((mod+1)/2)
7 #define div6 ((mod+1)/6)
8
9 using namespace std;
10
11 int n, prime[N], v[N];
12 LL phi[3][N];
13
14 map<int, int> mp[3];
15
16 int sum(int t, int x) { //calculate 1^t + 2^t + ... + x^t
17     if (t == 0) return x;
18     if (t == 1) return 1ll * x * (x + 1) % mod * div2 % mod;
19     if (t == 2) return 1ll * x * (x + 1) % mod * (21ll * x % mod + 1) % mod * div6 %
        mod;
20     if (t == 3) return 1ll * x * x % mod * (x + 1) % mod * (x + 1) % mod * div2 %
        mod * div2 % mod;
21 }
22
23 int getphi(int t, int x) {
24     if (x < N) return phi[t][x];
25     if (mp[t].find(x) != mp[t].end()) return mp[t][x];
26     LL ans = 0; int r = 0;
27     for (int l = 2; l <= x; l = r + 1) {
28         r = x / (x / l);
29         ans += 1ll * getphi(t, x / l) * (((LL)sum(t, r) - sum(t, l - 1) + mod) % mod
            ) % mod;
30         ans %= mod;
31     }
32     ans = (LL)sum(t + 1, x) - ans + mod;
33     ans %= mod;
34     mp[t][x] = ans;
35     return (int)ans;
36 }

```

```

37
38 int main() {
39     memset(v, 0, sizeof(v));
40     int top = 0;
41     phi[0][1] = 1, phi[1][1] = 1, phi[2][1] = 1;
42     for (int i = 2; i < N; ++i) {
43         if (!v[i]) prime[++top] = i, phi[0][i] = i - 1, phi[1][i] = 1ll * i * phi
            [0][i] % mod, phi[2][i] = 1ll * i * phi[1][i] % mod;
44         for (int j = 1; j <= top && prime[j] * i < N; ++j) {
45             v[i * prime[j]] = 1;
46             if (i % prime[j] == 0) {
47                 phi[0][i * prime[j]] = phi[0][i] * prime[j];
48                 phi[1][i * prime[j]] = 1ll * phi[1][i] * prime[j] % mod * prime[j] %
                    mod;
49                 phi[2][i * prime[j]] = 1ll * phi[2][i] * prime[j] % mod * prime[j] %
                    mod * prime[j] % mod;
50                 break;
51             } else {
52                 phi[0][i * prime[j]] = phi[0][i] * (prime[j] - 1);
53                 phi[1][i * prime[j]] = 1ll * phi[1][i] * (prime[j] - 1) % mod *
                    prime[j] % mod;
54                 phi[2][i * prime[j]] = 1ll * phi[2][i] * (prime[j] - 1) % mod *
                    prime[j] % mod * prime[j] % mod;
55             }
56         }
57     }
58     for (int i = 2; i < N; ++i) {
59         phi[0][i] = (phi[0][i] + phi[0][i - 1]) % mod;
60         phi[1][i] = (phi[1][i] + phi[1][i - 1]) % mod;
61         phi[2][i] = (phi[2][i] + phi[2][i - 1]) % mod;
62     }
63 }

```

## 6.12 FFT

```

1 namespace FFT {
2     const int maxn = 65537;
3     const double pi = acos(-1.0);
4
5     struct comp {
6         double real , imag;
7         comp() {}
8         comp(double real , double imag): real(real) , imag(imag) {}
9         friend inline comp operator+(const comp &a , const comp &b) {
10             return comp(a.real + b.real , a.imag + b.imag);
11         }
12         friend inline comp operator-(const comp &a , const comp &b) {
13             return comp(a.real - b.real , a.imag - b.imag);
14         }

```



```

15     friend inline comp operator*(const comp &a , const comp &b) {
16         return comp(a.real * b.real - a.imag * b.imag , a.real * b.imag + a.imag
17             * b.real);
18     };
19
20     comp A[maxn] , B[maxn];
21     int rev[maxn], m, len;
22
23     inline void init(int n) {
24         for (m = 1, len = 0; m < n + n; m <= 1 , len ++);
25         for (int i = 0; i < m; ++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len -
26             1));
27         for (int i = 0; i < m; ++i) A[i] = B[i] = comp(0, 0);
28     }
29
30     inline void dft(comp *a , int v) {
31         for (int i = 0; i < m; ++i) if (i < rev[i]) swap(a[i] , a[rev[i]]);
32         for (int s = 2; s <= m; s <= 1) {
33             comp g(cos(2 * pi / s) , v * sin(2 * pi / s));
34             for (int k = 0; k < m; k += s) {
35                 comp w(1 , 0);
36                 for (int j = 0; j < s / 2; ++j) {
37                     comp u = a[k + j] , t = w * a[k + j + s / 2];
38                     a[k + j] = u + t;
39                     a[k + j + s / 2] = u - t;
40                     w = w * g;
41                 }
42             }
43             if (v == -1)
44                 for (int i = 0; i < m; ++i) a[i].real /= m , a[i].imag /= m;
45         }
46     }

```

### 6.13 FWT

给定长度为  $2^n$  的序列  $A[0 \cdots 2^n - 1], B[0 \cdots 2^n - 1]$  , 求这两序列的

or 卷积:  $C_k = \sum_{i \text{ or } j = k} A_i B_j$

and 卷积:  $C_k = \sum_{i \text{ and } j = k} A_i B_j$

xor 卷积:  $C_k = \sum_{i \text{ xor } j = k} A_i B_j$

序列对  $10^9 + 7$  取模

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 inline int read() {
4     int s = 0; char c; while((c=getchar())<'0' || c>'9');
5     do{s=s*10+c-'0';}while((c=getchar())>='0' && c<='9');

```

```

6     return s;
7 }
8 typedef long long lint;
9 typedef pair<lint,lint> pii;
10 const int N = (1<<18)+37, MO = 1e9+7;
11 int n;
12 pii tand(lint a,lint b) { return (pii){a+b,b}; }
13 pii iand(lint a,lint b) { return (pii){a-b,b}; }
14 pii tor(lint a,lint b) { return (pii){a,a+b}; }
15 pii ior(lint a,lint b) { return (pii){a,b-a}; }
16 pii txor(lint a,lint b) { return (pii){a+b,a-b}; }
17 pii ixor(lint a,lint b) { return (pii){(a+b)/2,(a-b)/2}; }
18 pii (*tr)(lint,lint);
19 pii (*trs[3])(lint,lint)={tor,tand,txor};
20 pii (*ivs[3])(lint,lint)={ior,iand,ixor};
21 lint a[N],b[N],x[N],y[N];
22 inline void fwt(lint *a) {
23     int s,k,j,je;
24     for(s=2;s<=n;s<=1) for(k=0;k<n;k+=s)
25         for(j=0,je=s>>1;j<je;j++) {
26             pii t = tr(a[k+j],a[k+j+je]);
27             a[k+j] = t.first;
28             a[k+j+je] = t.second;
29         }
30 }
31 int main() {
32     int i;
33     n = read(); n = 1<<n;
34     for(i=0;i<n;i++) a[i] = read();
35     for(i=0;i<n;i++) b[i] = read();
36     for(int k=0;k<3;k++) {
37         for(i=0;i<n;i++) x[i] = a[i], y[i] = b[i];
38         tr = trs[k]; fwt(x), fwt(y);
39         for(i=0;i<n;i++) x[i] *= y[i];
40         tr = ivs[k]; fwt(x);
41         for(i=0;i<n;i++) printf("%d_",x[i]%MO); puts("");
42     }
43     return 0;
44 }

```

## 6.14 求原根

接口: LL p\_root(LL p);

输入: 一个素数  $p$

输出:  $p$  的原根

```

1 #include <bits/stdc++.h>
2 #define LL long long
3

```

```

4 using namespace std;
5
6 vector <LL> a;
7
8 LL pow_mod(LL base, LL times, LL mod) {
9     LL ret = 1;
10    while (times) {
11        if (times&1) ret = ret * base % mod;
12        base = base * base % mod;
13        times>>=1;
14    }
15    return ret;
16 }
17
18 bool g_test(LL g, LL p) {
19     for (LL i = 0; i < a.size(); ++i)
20         if (pow_mod(g, (p-1)/a[i], p) == 1) return 0;
21     return 1;
22 }
23
24 LL p_root(LL p) {
25     LL tmp = p - 1;
26     for (LL i = 2; i <= tmp / i; ++i)
27         if (tmp % i == 0) {
28             a.push_back(i);
29             while (tmp % i == 0)
30                 tmp /= i;
31         }
32     if (tmp != 1) a.push_back(tmp);
33     LL g = 1;
34     while (1) {
35         if (g_test(g, p)) return g;
36         ++g;
37     }
38 }
39
40 int main() {
41     LL p;
42     cin >> p;
43     cout << p_root(p) << endl;
44 }

```

## 6.15 NTT

998244353 原根为 3，1004535809 原根为 3，786433 原根为 10，880803841 原根为 26。

```

1 #define mod 998244353
2 #define g 3
3 LL wi[N], wiv[N];
4 LL power(LL base, LL times) {

```

```

5     LL ans = 1;
6     while (times) {
7         if (times&1) (ans *= base) %= mod;
8         (base *= base) %= mod;
9         times >>= 1;
10    }
11    return ans;
12 }
13 void transform(LL *x, int len) {
14     for (int i=1,j=len/2;i<len-1;i++) {
15         if (i<j) swap(x[i], x[j]);
16         int k = len/2;
17         while (j>=k) {
18             j-=k;
19             k/=2;
20         }
21         if (j<k) j+=k;
22     }
23 }
24 void NTT(LL *x, int len, int reverse) {
25     transform(x, len);
26     for (int h=2;h<=len;h<=1) {
27         for (int i=0;i<len;i+=h) {
28             LL w = 1, wn;
29             if (reverse==1) wn = wi[h]; else wn = wiv[h];
30             for (int j=i;j<i+h/2;j++) {
31                 LL u = x[j];
32                 LL v = (w * x[j+h/2]) % mod;
33                 x[j] = (u + v) % mod;
34                 x[j+h/2] = (u - v + mod) % mod;
35                 (w *= wn) %= mod;
36             }
37         }
38     }
39     if (reverse == -1) {
40         LL t = power(len, mod-2);
41         for (int i=0;i<len;i++)
42             (x[i] *= t) %= mod;
43     }
44 }
45 LL A[N], B[N];
46 int main() {
47     for (int i=1;i<N;i*=2) {
48         wi[i] = power(g, (mod-1)/i);
49         wiv[i] = power(wi[i], mod-2);
50     }
51     memset(A, 0, sizeof(A));
52     memset(B, 0, sizeof(B));
53     NTT(A, len, 1); NTT(B, len, 1);
54     for (int i=0;i<len;i++) (A[i] *= B[i]) %= mod;

```

```

55     NTT(A, len, -1);
56 }

```

## 6.16 组合数 lcm

$$(n+1)lcm(C(n,0), C(n,1), \dots, C(n,k)) = lcm(n+1, n, n-1, \dots, n-k+1)$$

## 6.17 区间 lcm 的维护

对于一个数，将其分解质因数，若有因子  $p^k$ ，那么拆分成  $k$  个数  $p, p^2, \dots, p^k$ ，权值都为  $p$ ，那么查询区间  $[l, r]$  内所有数的 lcm 的答案 = 所有在该区间中出现过的数的权值之积，可持久化线段树维护即可。

# 7 几何

## 7.1 凸包

```

1  typedef complex<int> point;
2  #define X real()
3  #define Y imag()
4  int n;
5  long long cross(point a, point b) {
6      return 1ll * a.X * b.Y - 1ll * a.Y * b.X;
7  }
8  bool cmp(point a, point b) {
9      return make_pair(a.X, a.Y) < make_pair(b.X, b.Y);
10 }
11 int convexHull(point p[], int n, point ch[]) {
12     sort(p, p + n, cmp);
13     int m = 0;
14     for(int i = 0; i < n; ++i) {
15         while(m > 1 && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
16         ch[m++] = p[i];
17     }
18     int k = m;
19     for(int i = n - 2; i >= 0; --i) {
20         while(m > k && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
21         ch[m++] = p[i];
22     }
23     if(n > 1) m--;
24     return m;
25 }

```

## 8 黑科技和杂项

### 8.1 找规律

有些题目，只给一个正整数  $n$ ，然后要求输出一个答案。这时，我们可以暴力得到小数据的解，用高斯消元得到递推式，然后用矩阵快速幂求解。

使用方法：

首先在 gauss.in 中输入小数据的解 ( $n = 1$  时,  $n = 2$  时,  $\dots$ )，以 EOF 结束。

依次运行 gauss.cpp，matrix.cpp，得到 matrix.out

将 matrix.out 中的文件粘贴在 main.cpp 中相应的位置中。注意模数一定要是质数。

```
1 //gauss.cpp
2 #include <bits/stdc++.h>
3 #define N 102
4 #define mod 1000000007
5 //caution: you can use this program iff mod is a prime.
6
7 using namespace std;
8
9 int n, m, k, a[N], g[N][N];
10
11 int power(int base, int times) {
12     int ret = 1;
13     while (times) {
14         if (times & 1) ret = 1ll * ret * base % mod;
15         base = 1ll * base * base % mod;
16         times >>= 1;
17     }
18     return ret;
19 }
20
21 int test() {
22     for (int i=0; i<m; i++) {
23         for (int j=i; j<=m; j++)
24             if (g[j][i]) {
25                 for (int k=i; k<=m; k++)
26                     swap(g[i][k], g[j][k]);
27                 break;
28             }
29     if (g[i][i] == 0)
30         return 0;
31     for (int j=i+1; j<n; j++) {
32         while (g[j][i]) {
33             int t = 1ll * g[i][i] * power(g[j][i], mod - 2) % mod;
34             for (int k=i; k<n; k++)
35                 g[i][k] = (g[i][k] + mod - (1ll * t * g[j][k] % mod)) % mod;
36             for (int k=i; k<=m; k++)
37                 swap(g[i][k], g[j][k]);
38         }
39     }
```

```

40     int t = power(g[i][i], mod - 2);
41     for (int j = 0; j <= m; ++j)
42         g[i][j] = 111 * g[i][j] * t % mod;
43 }
44 for (int i = m; i < n; ++i)
45     if (g[i][m]) return 0;
46 for (int i = m - 1; i >= 0; --i) {
47     int t = power(g[i][i], mod - 2);
48     g[i][i] = 1;
49     g[i][m] = 111 * g[i][m] * t % mod;
50     for (int j = 0; j < i; ++j)
51         g[j][m] = (g[j][m] + mod - 111 * g[i][m] * g[j][i] % mod) % mod;
52 }
53 printf("%d\n", m);
54 for (int i = 0; i < m; ++i)
55     printf("%d_", g[i][m]);
56 puts("");
57 for (int i = 0; i < m - 1; ++i)
58     printf("%d_", a[i]);
59 puts("1");
60 return 1;
61 }
62
63 int main() {
64     freopen("gauss.in", "r", stdin);
65     freopen("gauss.out", "w", stdout);
66     k = 0;
67     while (~scanf("%d", &a[k++])) ;
68     for (int sm = 1; sm <= k - sm; ++sm) {
69         n = k - sm - 1;
70         m = sm + 1;
71         for (int i = 0; i < n; ++i) {
72             for (int j = 0; j <= sm; ++j)
73                 g[i][j] = a[i + j];
74             g[i][m] = 1;
75             swap(g[i][m - 1], g[i][m]);
76         }
77         if (test()) return 0;
78     }
79     puts("no_solution");
80     return 0;
81 }

```

```

1 //matrix.cpp
2 #include <bits/stdc++.h>
3 #define N 102
4 using namespace std;
5
6 int n, a[N];
7

```

```

8  int main() {
9      freopen("gauss.out", "r", stdin);
10     freopen("matrix.out", "w", stdout);
11     scanf("%d", &n);
12     for (int i = 0; i < n; ++i) scanf("%d", &a[i]);
13     printf("#define_M_%d\n", n);
14     printf("const_int_trans[M][M]_={\n");
15     for (int i = 0; i < n; ++i) {
16         printf("\t{");
17         for (int j = 0; j < n; ++j) {
18             int t;
19             if (j < n - 2) t = i == j + 1;
20             else if (j == n - 2) t = a[i];
21             else t = i == n - 1;
22             printf("%s%d", j == 0 ? "" : ",_", t);
23         }
24         printf("%s\n", i == n - 1 ? "" : ",");
25     }
26     printf("};\n");
27     printf("const_int_pref[M]_={");
28     for (int i = 0; i < n; ++i) {
29         int x;
30         scanf("%d", &x);
31         printf("%d%s", x, i == n - 1 ? "};\n" : ",_");
32     }
33     return 0;
34 }

```

```

1  //main.cpp
2  #include <bits/stdc++.h>
3  using namespace std;
4
5  /* paste matrix.out here. */
6
7  #define mod 1000000007
8
9  struct Matrix {
10     int c[M][M];
11     void clear() { memset(c, 0, sizeof(c)); }
12     void identity() { clear(); for (int i = 0; i < M; ++i) c[i][i] = 1; }
13     void base() { memcpy(c, trans, sizeof(trans)); }
14     friend Matrix operator * (const Matrix &a, const Matrix &b) {
15         Matrix c; c.clear();
16         for (int i = 0; i < M; ++i)
17             for (int j = 0; j < M; ++j)
18                 for (int k = 0; k < M; ++k)
19                     c.c[i][j] = (c.c[i][j] + 1ll * a.c[i][k] * b.c[k][j] % mod) %
20                                     mod;
21         return c;
22     }
23 }

```



```

22 } start, base;
23
24 Matrix power(Matrix base, int times) {
25     Matrix ret; ret.identity();
26     while (times) {
27         if (times & 1) ret = ret * base;
28         base = base * base;
29         times >>= 1;
30     }
31     return ret;
32 }
33
34 int main() {
35     int tot;
36     scanf("%d", &tot);
37     while (tot--) {
38         int n;
39         scanf("%d", &n);
40         start.clear();
41         for (int i = 0; i < M; ++i) start.c[0][i] = pref[i];
42         base.base();
43         base = power(base, n - 1);
44         start = start * base;
45         printf("%d\n", start.c[0][0]);
46     }
47     return 0;
48 }

```

## 8.2 高精度计算

```

1  #include<algorithm>
2  using namespace std;
3  const int N_huge=850,base=1000000000;
4  char s[N_huge*10];
5  struct huge{
6      typedef long long value;
7      value a[N_huge];int len;
8      void clear(){len=1;a[len]=0;}
9      huge(){clear();}
10     huge(value x){*this=x;}
11     huge operator =(huge b){
12         len=b.len;for (int i=1;i<=len;++i)a[i]=b.a[i]; return *this;
13     }
14     huge operator =(value x){
15         len=0;
16         while (x)a[++len]=x%base,x/=base;
17         if (!len)a[++len]=0;
18         return *this;
19     }

```

```

20 huge operator +(huge b){
21     int L=len>b.len?len:b.len;huge tmp;
22     for (int i=1;i<=L+1;++i)tmp.a[i]=0;
23     for (int i=1;i<=L;++i){
24         if (i>len)tmp.a[i]+=b.a[i];
25         else if (i>b.len)tmp.a[i]+=a[i];
26         else {
27             tmp.a[i]+=a[i]+b.a[i];
28             if (tmp.a[i]>=base){
29                 tmp.a[i]-=base;++tmp.a[i+1];
30             }
31         }
32     }
33     if (tmp.a[L+1])tmp.len=L+1;
34     else tmp.len=L;
35     return tmp;
36 }
37 huge operator -(huge b){
38     int L=len>b.len?len:b.len;huge tmp;
39     for (int i=1;i<=L+1;++i)tmp.a[i]=0;
40     for (int i=1;i<=L;++i){
41         if (i>b.len)b.a[i]=0;
42         tmp.a[i]+=a[i]-b.a[i];
43         if (tmp.a[i]<0){
44             tmp.a[i]+=base;--tmp.a[i+1];
45         }
46     }
47     while (L>1&&!tmp.a[L])--L;
48     tmp.len=L;
49     return tmp;
50 }
51 huge operator *(huge b){
52     int L=len+b.len;huge tmp;
53     for (int i=1;i<=L;++i)tmp.a[i]=0;
54     for (int i=1;i<=len;++i)
55         for (int j=1;j<=b.len;++j){
56             tmp.a[i+j-1]+=a[i]*b.a[j];
57             if (tmp.a[i+j-1]>=base){
58                 tmp.a[i+j]+=tmp.a[i+j-1]/base;
59                 tmp.a[i+j-1]%=base;
60             }
61         }
62     tmp.len=len+b.len;
63     while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
64     return tmp;
65 }
66 pair<huge,huge> divide(huge a,huge b){
67     int L=a.len;huge c,d;
68     for (int i=L;i--i){
69         c.a[i]=0;d=d*base;d.a[1]=a.a[i];

```

```

70         int l=0,r=base-1,mid;
71         while (l<r){
72             mid=(l+r+1)>>1;
73             if (b*mid<=d)l=mid;
74             else r=mid-1;
75         }
76         c.a[i]=l;d-=b*l;
77     }
78     while (L>1&&!c.a[L])--L;c.len=L;
79     return make_pair(c,d);
80 }
81 huge operator / (value x){
82     value d=0;huge tmp;
83     for (int i=len;i--i){
84         d=d*base+a[i];
85         tmp.a[i]=d/x;d%=x;
86     }
87     tmp.len=len;
88     while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
89     return tmp;
90 }
91 value operator % (value x){
92     value d=0;
93     for (int i=len;i--i)d=(d*base+a[i])%x;
94     return d;
95 }
96 huge operator / (huge b){return divide(*this,b).first;}
97 huge operator % (huge b){return divide(*this,b).second;}
98 huge &operator += (huge b){*this=*this+b;return *this;}
99 huge &operator -= (huge b){*this=*this-b;return *this;}
100 huge &operator *= (huge b){*this=*this*b;return *this;}
101 huge &operator ++(){huge T;T=1;*this=*this+T;return *this;}
102 huge &operator --(){huge T;T=1;*this=*this-T;return *this;}
103 huge operator ++(int){huge T,tmp=*this;T=1;*this=*this+T;return tmp;}
104 huge operator --(int){huge T,tmp=*this;T=1;*this=*this-T;return tmp;}
105 huge operator +(value x){huge T;T=x;return *this+T;}
106 huge operator -(value x){huge T;T=x;return *this-T;}
107 huge operator *(value x){huge T;T=x;return *this*T;}
108 huge operator *=(value x){*this=*this*x;return *this;}
109 huge operator +=(value x){*this=*this+x;return *this;}
110 huge operator -=(value x){*this=*this-x;return *this;}
111 huge operator /=(value x){*this=*this/x;return *this;}
112 huge operator %=(value x){*this=*this%x;return *this;}
113 bool operator ==(value x){huge T;T=x;return *this==T;}
114 bool operator !=(value x){huge T;T=x;return *this!=T;}
115 bool operator <=(value x){huge T;T=x;return *this<=T;}
116 bool operator >=(value x){huge T;T=x;return *this>=T;}
117 bool operator <(value x){huge T;T=x;return *this<T;}
118 bool operator >(value x){huge T;T=x;return *this>T;}
119 bool operator <(huge b){

```

```

120     if (len<b.len)return 1;
121     if (len>b.len)return 0;
122     for (int i=len;i--;i){
123         if (a[i]<b.a[i])return 1;
124         if (a[i]>b.a[i])return 0;
125     }
126     return 0;
127 }
128 bool operator ==(huge b){
129     if (len!=b.len)return 0;
130     for (int i=len;i--;i){
131         if (a[i]!=b.a[i])return 0;
132     }
133     return 1;
134 bool operator !=(huge b){return !(*this==b);}
135 bool operator >(huge b){return !(*this<b||*this==b);}
136 bool operator <=(huge b){return (*this<b)||(*this==b);}
137 bool operator >=(huge b){return (*this>b)||(*this==b);}
138 void str(char s[]){
139     int l=strlen(s);value x=0,y=1;len=0;
140     for (int i=l-1;i>=0;--i){
141         x=x+(s[i]-'0')*y;y*=10;
142         if (y==base)a[++len]=x,x=0,y=1;
143     }
144     if (!len||x)a[++len]=x;
145 }
146 void read(){
147     scanf("%s",s);this->str(s);
148 }
149 void print(){
150     printf("%d", (int)a[len]);
151     for (int i=len-1;i--;i){
152         for (int j=base/10;j>=10;j/=10){
153             if (a[i]<j)printf("0");
154             else break;
155         }
156         printf("%d", (int)a[i]);
157     }
158     printf("\n");
159 }
160 }f[1005];
161 int main(){
162     f[1]=f[2]=1;
163     for(int i=3;i<=1000;i++)f[i]=f[i-1]+f[i-2];
164 }

```

### 8.3 读入优化

```
1 #define rd RD<int>
```

```

2 #define rdll RD<long long>
3 template <typename Type>
4 inline Type RD() {
5     Type x = 0;
6     int flag = 0;
7     char c = getchar();
8     while (!isdigit(c) && c != '-')
9         c = getchar();
10    (c == '-') ? (flag = 1) : (x = c - '0');
11    while (isdigit(c = getchar()))
12        x = x * 10 + c - '0';
13    return flag ? -x : x;
14 }
15 inline char rdch() {
16     char c = getchar();
17     while (!isalpha(c)) c = getchar();
18     return c;
19 }

```

## 8.4 位运算及其运用

### 8.4.1 枚举子集

枚举  $i$  的非空子集  $j$

```

1 for (int j = i; j; j = (j - 1) & i);

```

### 8.4.2 求 1 的个数

```

1 int __builtin_popcount(unsigned int x);

```

### 8.4.3 求前缀 0 的个数

```

1 int __builtin_clz(unsigned int x);

```

### 8.4.4 求后缀 0 的个数

```

1 int __builtin_ctz(unsigned int x);

```