ICPC Templates For Africamonkey

Africamonkey

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1 莫队算法

1.1 普通莫队

分块块数为 $\sqrt{n}$ 是最优的。

记每次进行 add() 操作的复杂度为 $O(A)$ ，del() 操作的复杂度为 $O(D)$ ，查询答案 answer() 的复杂度为 $O(S)$ 。

则总复杂度为 $O(n\sqrt{n}(A+D)+qS)$ 。

S 可以大一点，但必须保证 A, D 尽可能小。

```
struct Q { int l, r, sqrtl, id; } q[N];
int n, m, v[N], ans[N], nowans;
bool cmp(const Q &a, const Q &b) {
    if (a.sqrtl != b.sqrtl) return a.sqrtl < b.sqrtl;
    return a.r < b.r;
}
void change(int x) {
    if (!v[x]) add(x);
    else del(x);
    v[x] ^= 1;
}
int main() {
    ......
    for (int i=1;i<=m;i++) q[i].sqrtl = q[i].l / sqrt(n), q[i].id = i;
    sort(q+1, q+m+1, cmp);
    int L=1, R=0;
    memset(v, 0, sizeof(v));
    for (int i=1;i<=m;i++) {
        while (L<q[i].l) change(L++);
        while (L>q[i].l) change(--L);
        while (R<q[i].r) change(++R);
        while (R>q[i].r) change(R--);
        ans[q[i].id] = answer();
    }
    ......
}
```

1.2 树上莫队

分块块数为 $\sqrt{n}$ 是最优的。

记每次进行 add() 操作的复杂度为 $O(A)$ ，del() 操作的复杂度为 $O(D)$ ，查询答案 answer() 的复杂度为 $O(S)$ 。

则总复杂度为 $O(n\sqrt{n}(A+D)+qS)$ 。

S 可以大一点，但必须保证 A, D 尽可能小。

```
struct Query { int l, r, id, l_group; } query[N];
struct EDGE { int adj, next; } edge[N*2];
int n, m, top, gh[N], c[N], reorder[N], deep[N], father[N], size[N], son[N], Top[N];
void addedge(int x, int y) {
```

```
    edge[++top].adj = y;
    edge[top].next = gh[x];
    gh[x] = top;
}
void dfs(int x, int root=0) {
    reorder[x] = ++top; father[x] = root; deep[x] = deep[root] + 1;
    son[x] = 0; size[x] = 1; int dd = 0;
    for (int p=gh[x]; p; p=edge[p].next)
        if (edge[p].adj != root) {
            dfs(edge[p].adj, x);
            if (size[edge[p].adj] > dd) {
                son[x] = edge[p].adj;
                dd = size[edge[p].adj];
            }
            size[x] += size[edge[p].adj];
        }
}
void split(int x, int tp) {
    Top[x] = tp;
    if (son[x]) split(son[x], tp);
    for (int p=gh[x]; p; p=edge[p].next)
        if (edge[p].adj != father[x] && edge[p].adj != son[x])
            split(edge[p].adj, edge[p].adj);
}
int lca(int x, int y) {
    int tx = Top[x], ty = Top[y];
    while (tx != ty) {
        if (deep[tx] < deep[ty]) {
            swap(tx, ty);
            swap(x, y);
        }
        x = father[tx];
        tx = Top[x];
    }
    if (deep[x] < deep[y]) swap(x, y);
    return y;
}
bool cmp(const Query &a, const Query &b) {
    if (a.l_group != b.l_group) return a.l_group < b.l_group;
    return reorder[a.r] < reorder[b.r];
}
int v[N], ans[N];

void upd(int x) {
    if (!v[x]) add(x);
    else del(x);
    v[x] ^= 1;
}

void go(int &u, int taru, int v) {
```

```
    int lca0 = lca(u, taru);
    int lca1 = lca(u, v);   upd(lca1);
    int lca2 = lca(taru, v); upd(lca2);
    for (int x=u; x!=lca0; x=father[x]) upd(x);
    for (int x=taru; x!=lca0; x=father[x]) upd(x);
    u = taru;
}
int main() {
    memset(gh, 0, sizeof(gh));
    scanf("%d%d", &n, &m); top = 0;
    for (int i=1;i<n;i++) {
        int x,y; scanf("%d%d", &x, &y);
        addedge(x, y); addedge(y, x);
    }
    top = 0; dfs(1); split(1, 1);
    for (int i=1;i<=m;i++) {
        if (reorder[query[i].l] > reorder[query[i].r])
            swap(query[i].l, query[i].r);
        query[i].id = i;
        query[i].l_group = reorder[query[i].l] / sqrt(n);
    }
    sort(query+1, query+m+1, cmp);
    int L=1,R=1; upd(1);
    for (int i=1;i<=m;i++) {
        go(L,query[i].l,R);
        go(R,query[i].r,L);
        ans[query[i].id] = answer();
    }
    ......
}
```

2 字符串

2.1 哈希

```
const int P=31,D=1000173169;
int n, pow[N], f[N]; char a[N];
int hash(int l, int r) { return (LL)(f[r]-(LL)f[l-1]*pow[r-l+1]%D+D)%D; }
int main() {
    scanf("%d%s", &n, a+1);
    pow[0] = 1;
    for (int i=1;i<=n;i++) pow[i] = (LL)pow[i-1]*P%D;
    for (int i=1;i<=n;i++) f[i] = (LL)((LL)f[i-1]*P+a[i])%D;
}
```

2.2 KMP

接口：void kmp(int n, char *a, int m, char *b);

输入：模式串长度 n ，模式串 a ，匹配串长度 m ，匹配串 b

输出：依次输出每个匹配成功的起始位置

下标从 0 开始。

```
void kmp(int n, char* a, int m, char *b) {
    int i, j;
    for (nxt[0] = j = -1, i = 1; i < n; nxt[i++] = j) {
        while (~j && a[j + 1] != a[i]) j = nxt[j];
        if (a[j + 1] == a[i]) ++j;
    }
    for (j = -1, i = 0; i < m; ++i) {
        while (~j && a[j + 1] != b[i]) j = nxt[j];
        if (a[j + 1] == b[i]) ++j;
        if (j == n - 1) {
            printf("%d\n", i - n + 1);
            j = nxt[j];
        }
    }
}
```

2.3 可动态修改的 KMP

支持：加入一个字符，删除一个字符。

时间复杂度：$O(n\alpha)$ ，α 为字符集大小。

代码中的字符为 $'0' -' 9'$ ，可自行修改为 $'a' -' z'$

```
char t[N];
int top, nxt[N], nxt_l[N][10];
inline void del_letter() { --top; }
inline void add_letter(char x) {
    t[top++] = x;
    int j = top-1;
    memset(nxt_l[top], 0, sizeof(nxt_l[top]));
    nxt[top] = nxt_l[top-1][x-'0'];
    memcpy(nxt_l[top], nxt_l[nxt[top]], sizeof(nxt_l[nxt[top]]));
    nxt_l[top][t[nxt[top]]-'0'] = nxt[top]+1;
}
```

2.4 扩展 KMP

接口：void ExtendedKMP(char *a, char *b, int *next, int *ret);

输出：

next: a 关于自己每个后缀的最长公共前缀

ret: a 关于 b 的每个后缀的最长公共前缀

EXKMP 的 next[i] 表示：从 i 到 n-1 的字符串 st 前缀和原串前缀的最长重叠长度。

```
void get_next(char *a, int *next) {
    int i, j, k;
    int n = strlen(a);
```

```
    for (j = 0; j+1<n && a[j]==a[j+1];j++);
    next[1] = j;
    k = 1;
    for (i=2;i<n;i++) {
        int len = k+next[k], l = next[i-k];
        if (l < len-i) {
            next[i] = l;
        } else {
            for (j = max(0, len-i);i+j<n && a[j]==a[i+j];j++);
            next[i] = j;
            k = i;
        }
    }
}
void ExtendedKMP(char *a, char *b, int *next, int *ret) {
    get_next(a, next);
    int n = strlen(a), m = strlen(b);
    int i, j, k;
    for (j=0;j<n && j<m && a[j]==b[j];j++);
    ret[0] = j;
    k = 0;
    for (i=1;i<m;i++) {
        int len = k+ret[k], l = next[i-k];
        if (l < len-i) {
            ret[i] = l;
        } else {
            for (j = max(0, len-i);j<n && i+j<m && a[j]==b[i+j];j++);
            ret[i] = j;
            k = i;
        }
    }
}
```

2.5 Manacher

p[i] 表示以 i 为对称轴的最长回文串长度

```
char st[N*2], s[N];
int len, p[N*2];

while (scanf("%s", s) != EOF) {
    len = strlen(s);
    st[0] = '$', st[1] = '#';
    for (int i=1;i<=len;i++)
        st[i*2] = s[i-1], st[i*2+1] = '#';
    len = len * 2 + 2;
    int mx = 0, id = 0, ans = 0;
    for (int i=1;i<=len;i++) {
        p[i] = (mx > i) ? min(p[id*2-i]+1, mx-i) : 1;
        for (; st[i+p[i]] == st[i-p[i]]; ++p[i]) ;
```

```
        if (p[i]+i > mx) mx = p[i]+i, id = i;
        p[i] --;
        if (p[i] > ans) ans = p[i];
    }
    printf("%d\n", ans);
}
```

2.6 最小表示法

```
string smallestRepresation(string s) {
    int i, j, k, l;
    int n = s.length();
    s += s;
    for (i=0,j=1;j<n;) {
        for (k=0;k<n && s[i+k]==s[j+k];k++);
        if (k>=n) break;
        if (s[i+k]<s[j+k]) j+=k+1;
        else {
            l=i+k;
            i=j;
            j=max(l,j)+1;
        }
    }
    return s.substr(i, n);
}
```

2.7 AC 自动机

```
struct Node {
    int next[**Size of Alphabet**];
    int terminal, fail;
} node[**Number of Nodes**];
int top;
void add(char *st) {
    int len = strlen(st), x = 1;
    for (int i=0;i<len;i++) {
        int ind = trans(st[i]);
        if (!node[x].next[ind])
            node[x].next[ind] = ++top;
        x = node[x].next[ind];
    }
    node[x].terminal = 1;
}
int q[**Number of Nodes**], head, tail;
void build() {
    head = 0, tail = 1; q[1] = 1;
    while (head != tail) {
        int x = q[++head];
```

```
        /*(when necessary) node[x].terminal |= node[node[x].fail].terminal; */
        for (int i=0;i<n;i++)
            if (node[x].next[i]) {
                if (x == 1) node[node[x].next[i]].fail = 1;
                else {
                    int y = node[x].fail;
                    while (y) {
                        if (node[y].next[i]) {
                            node[node[x].next[i]].fail = node[y].next[i];
                            break;
                        }
                        y = node[y].fail;
                    }
                    if (!node[node[x].next[i]].fail) node[node[x].next[i]].fail = 1;
                }
                q[++tail] = node[x].next[i];
            }
    }
}
```

2.8 后缀数组

2.8.1 倍增算法

参数 m 表示字符集的大小，即 $0 \le r_i < m$

```
#define rank rank2
int n, r[N], wa[N], wb[N], ws[N], sa[N], rank[N], height[N];
int cmp(int *r, int a, int b, int l, int n) {
    if (r[a]==r[b]) {
        if (a+l<n && b+l<n && r[a+l]==r[b+l])
            return 1;
    }
    return 0;
}
void suffix_array(int m) {
    int i, j, p, *x=wa, *y=wb, *t;
    for (i=0;i<m;i++) ws[i]=0;
    for (i=0;i<n;i++) ws[x[i]=r[i]]++;
    for (i=1;i<m;i++) ws[i]+=ws[i-1];
    for (i=n-1;i>=0;i--) sa[--ws[x[i]]]=i;
    for (j=1,p=1;p<n;m=p,j<<=1) {
        for (p=0,i=n-j;i<n;i++) y[p++]=i;
        for (i=0;i<n;i++) if (sa[i]>=j) y[p++]=sa[i]-j;
        for (i=0;i<m;i++) ws[i]=0;
        for (i=0;i<n;i++) ws[x[y[i]]]++;
        for (i=1;i<m;i++) ws[i]+=ws[i-1];
        for (i=n-1;i>=0;i--) sa[--ws[x[y[i]]]]=y[i];
        for (t=x,x=y,y=t,x[sa[0]]=0,i=1,p=1;i<n;i++)
            x[sa[i]]=cmp(y,sa[i-1],sa[i],j,n)?p-1:p++;
```

```
    }
    for (i=0;i<n;i++) rank[sa[i]]=i;
    rank[n] = -1;
}
void calc_height() {
    int j=0;
    for (int i=0;i<n;i++)
        if (rank[i])
        {
            while (r[i+j]==r[sa[rank[i]-1]+j]) j++;
            height[rank[i]]=j;
            if (j) j--;
        }
}
```

2.8.2 DC3 算法

感谢浙江大学陈靖邦提供本模板。

```
namespace SA {
int sa[N], rk[N], ht[N], s[N<<1], t[N<<1], p[N], cnt[N], cur[N];
#define pushS(x) sa[cur[s[x]]--] = x
#define pushL(x) sa[cur[s[x]]++] = x
#define inducedSort(v) fill_n(sa, n, -1); fill_n(cnt, m, 0);                  \
    for (int i = 0; i < n; i++) cnt[s[i]]++;                                  \
    for (int i = 1; i < m; i++) cnt[i] += cnt[i-1];                           \
    for (int i = 0; i < m; i++) cur[i] = cnt[i]-1;                            \
    for (int i = n1-1; ~i; i--) pushS(v[i]);                                  \
    for (int i = 1; i < m; i++) cur[i] = cnt[i-1];                            \
    for (int i = 0; i < n; i++) if (sa[i] > 0 &&  t[sa[i]-1]) pushL(sa[i]-1); \
    for (int i = 0; i < m; i++) cur[i] = cnt[i]-1;                            \
    for (int i = n-1;  ~i; i--) if (sa[i] > 0 && !t[sa[i]-1]) pushS(sa[i]-1)
void sais(int n, int m, int *s, int *t, int *p) {
    int n1 = t[n-1] = 0, ch = rk[0] = -1, *s1 = s+n;
    for (int i = n-2; ~i; i--) t[i] = s[i] == s[i+1] ? t[i+1] : s[i] > s[i+1];
    for (int i = 1; i < n; i++) rk[i] = t[i-1] && !t[i] ? (p[n1] = i, n1++) : -1;
    inducedSort(p);
    for (int i = 0, x, y; i < n; i++) if (~(x = rk[sa[i]])) {
        if (ch < 1 || p[x+1] - p[x] != p[y+1] - p[y]) ch++;
        else for (int j = p[x], k = p[y]; j <= p[x+1]; j++, k++)
            if ((s[j]<<1|t[j]) != (s[k]<<1|t[k])) {ch++; break;}
        s1[y = x] = ch;
    }
    if (ch+1 < n1) sais(n1, ch+1, s1, t+n, p+n1);
    else for (int i = 0; i < n1; i++) sa[s1[i]] = i;
    for (int i = 0; i < n1; i++) s1[i] = p[sa[i]];
    inducedSort(s1);
}
template<typename T>
int mapCharToInt(int n, const T *str) {
```

```
    int m = *max_element(str, str+n);
    fill_n(rk, m+1, 0);
    for (int i = 0; i < n; i++) rk[str[i]] = 1;
    for (int i = 0; i < m; i++) rk[i+1] += rk[i];
    for (int i = 0; i < n; i++) s[i] = rk[str[i]] - 1;
    return rk[m];
}
// Ensure that str[n] is the unique lexicographically smallest character in str.
template<typename T>
void suffixArray(int n, const T *str) {
    int m = mapCharToInt(++n, str);
    sais(n, m, s, t, p);
    for (int i = 0; i < n; i++) rk[sa[i]] = i;
    for (int i = 0, h = ht[0] = 0; i < n-1; i++) {
        int j = sa[rk[i]-1];
        while (i+h < n && j+h < n && s[i+h] == s[j+h]) h++;
        if (ht[rk[i]] = h) h--;
    }
}
};
```

2.8.3 小技巧：拼接字符串

接口：

int gao1(int l, int r, int c, int p); 区间 $[l,r)$ 中保证第 0 位到第 $c-1$ 位都是相同的（设为字符串 s ），现在我们在 s 后面接一个字符 p ，得到一个新的字符串 s' 。返回值为最小的 k 满足后缀 $sa[k]$ 前 $c+1$ 位为 s'

int gao2(int l, int r, int c, int p); 区间 $[l,r)$ 中保证第 0 位到第 $c-1$ 位都是相同的（设为字符串 s ），现在我们在 s 后面接一个后缀 $sa[p]$ ，得到一个新的字符串 s' 。返回值为最小的 k 满足后缀 $sa[k]$ 前 $c+len(sa[p])$ 位为 s'

```
int gao1(int l,int r,int c,int p) {
        --l;
        while (l+1<r) {
                int md=(l+r)>>1;
                if (sa[md]+c<n&&s[sa[md]+c]>=p) r=md; else l=md;
        }
        return r;
}
int gao2(int l,int r,int c,int p) {
        --l;
        while (l+1<r) {
                int md=(l+r)>>1;
                if (sa[md]+c<=n&&rk[sa[md]+c]>=p) r=md; else l=md;
        }
        return r;
}
```

示例调用：

```
suf1[m] = -1, suf2[m] = n;
for (int i = m - 1; i >= 0; --i) {
    int l = gao1(0, n, 0, t[i]), r = gao1(0, n, 0, t[i]);
    suf1[i] = gao2(l, r, 1, suf1[i + 1]);
    suf2[i] = gao2(l, r, 1, suf2[i + 1]);
}
```

2.9 后缀自动机

下面的代码是求两个串的 LCS（最长公共子串）。

```
#include <bits/stdc++.h>

#define N 500001
#define M (N << 1)

using namespace std;

char st[N];
int pre[M], son[26][M], step[M], refer[M], size[M], tmp[M], topo[M], last, total;

int apply(int x, int now) {
    step[++total] = x;
    refer[total] = now;
    return total;
}

void extend(char x, int now) {
    int p = last, np = apply(step[last]+1, now);
    size[np] = 1;
    for (; p && !son[x][p]; p=pre[p]) son[x][p] = np;
    if (!p) pre[np] = 1;
    else {
        int q = son[x][p];
        if (step[p]+1 == step[q]) pre[np] = q;
        else {
            int nq = apply(step[p]+1, now);
            for (int i=0;i<26;i++) son[i][nq] = son[i][q];
            pre[nq] = pre[q];
            pre[q] = pre[np] = nq;
            for (; p && son[x][p]==q; p=pre[p]) son[x][p] = nq;
        }
    }
    last = np;
}
void init() {
    last = total = 0;
    last = apply(0, 0);
    scanf("%s",st);
```

```
    int n = strlen(st);
    for (int i = 0; i <= n * 2; ++i) {
        pre[i] = step[i] = refer[i] = size[i] = tmp[i] = topo[i] = 0;
        for (int j = 0; j < 26; ++j)
            son[j][i] = 0;
    }
    for (int i = 0; i < n; ++i)
        extend(st[i] - ’a’, i);
    for (int i = 1; i <= total; ++i)
        tmp[step[i]] ++;
    for (int i = 1; i <= n; ++i)
        tmp[i] += tmp[i - 1];
    for (int i = 1; i <= total; ++i)
        topo[tmp[step[i]]--] = i;
    for (int i = total; i; --i)
        size[pre[topo[i]]] += size[topo[i]];
}
int main() {
    init();
    int p = 1, now = 0, ans = 0;
    scanf("%s", st);
    for (int i=0; st[i]; i++) {
        int index = st[i]-’a’;
        for (; p && !son[index][p]; p = pre[p], now = step[p]) ;
        if (!p) p = 1;
        if (son[index][p]) {
            p = son[index][p];
            now++;
            if (now > ans) ans = now;
        }
    }
    printf("%d\n",ans);
    return 0;
}
```

一些定义和性质 Right(str) 表示 str 在母串 S 中所有出现的结束位置集合

一个状态 s 表示的所有子串 Right 集合相同，为 Right(s)

Parent(s) 满足 Right(s) 是 Right(Parent(s)) 的真子集，并且 Right(Parent(s)) 的大小最小

Parent 函数可以表示一个树形结构。不妨叫它 Parent 树

一个 Right 集合和一个长度定义了一个子串

对于状态 s ，使得 Right(s) 合法的子串长度是一个区间 [min(s), max(s)]

max(Parent(s)) = min(s) - 1

令 refer(s) 表示产生 s 状态的字符所在位置。则 Right(s) 的合法子串的起始位置为 [refer(s) - max(s) + 1, refer(s) - min(s) + 1] ，即 [refer(s) - max(s) + 1, refer(s) - max(Parent(s))]

代码中变量名含义 pre[s] 为上述定义中的 Parent(s)

step[s] 为从初始状态走到 s 状态最多需要多少步

refer[s] 为上述定义中的 refer(s)
size[s] 为 Right(s) 集合的大小
topo[s] 为 Parent 树的拓扑序，根（初始状态）在前

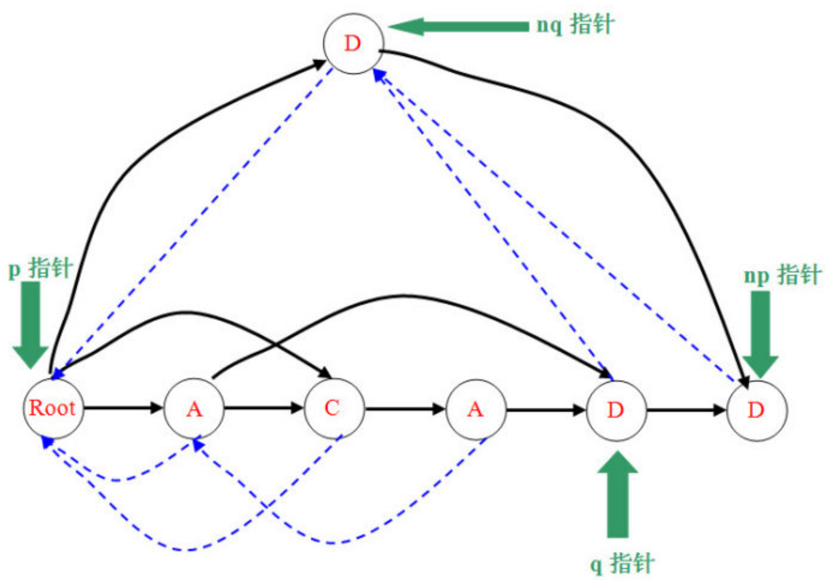

图 1: ACADD 构成的后缀自动机

我们发现 fail 构出一棵前缀树
和后缀树相同，为了使每个前缀都是叶子结点，我们不妨在串 s 前加入一个没出现的字符'#'

2.9.1 广义后缀自动机

先建 Trie ，再按照 *BFS* 序建后缀自动机。从节点 x 开始向子树更新时，其所有儿子都从同一个 last ，即 $last[x]$ 更新。

2.10 回文树

- len[i] 表示编号为 i 的节点表示的回文串的长度（一个节点表示一个回文串）
- next[i][c] 表示编号为 i 的节点表示的回文串在两边添加字符 c 以后变成的回文串的编号（和字典树类似）。
- fail[i] 表示节点 i 失配以后跳转不等于自身的节点 i 表示的回文串的最长后缀回文串（和 AC 自动机类似）。
- cnt[i] 表示节点 i 表示的本质不同的串的个数（建树时求出的不是完全的，最后 count() 函数跑一遍以后才是正确的）
- num[i] 表示以节点 i 表示的最长回文串的最右端点为回文串结尾的回文串个数。
- last 指向新添加一个字母后所形成的最长回文串表示的节点。
- st[i] 表示第 i 次添加的字符（一开始设 st[0] = -1（可以是任意一个在串 S 中不会出现的字符））。

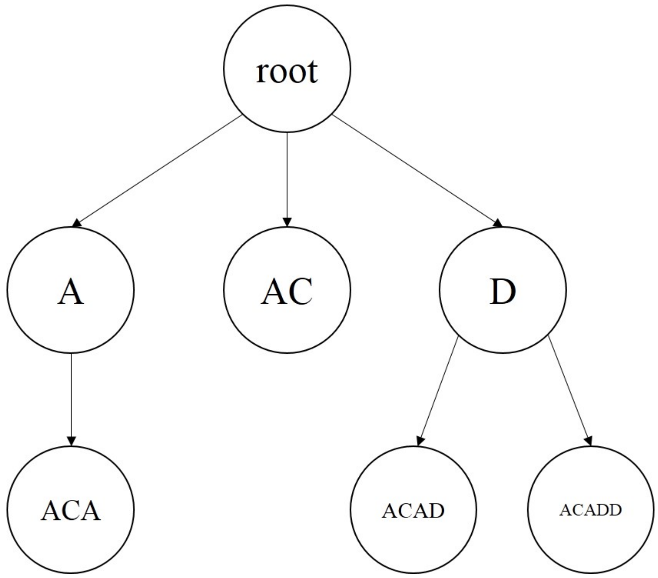

图 2: 串 ACADD 按 fail 构出的前缀树，与图 1 对应

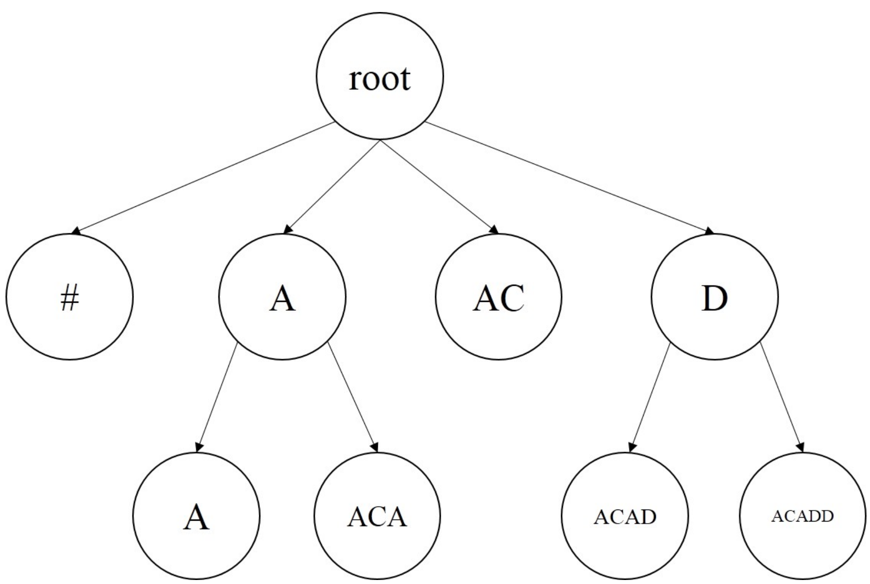

图 3: 串 #ACADD 按 fail 构出的前缀树

- tot 表示添加的节点个数。
- n 表示添加的字符个数。

【URAL2040】Palindromes and Super Abilities 2

逐个添加字符串 S 里的字符 $S_1, S_2, ..., S_n$ 。每次添加字符后，他想知道添加字符后将出现多少个新的本质不同的回文子串。字符集为 $\{a, b\}$

```
#include <bits/stdc++.h>
#define N 5000020

char st[N], answer[N];
int n;

struct PAM {
    int n, tot, last;
    int len[N], fail[N], next[N][2], num[N], cnt[N];
    void init() {
        n=0; tot=1;
        len[1]=-1; fail[1]=0;
        len[0]=+0; fail[0]=1;
        last=1;
    }
    int get_fail(int x) {
        for (; st[n-len[x]-1]!=st[n]; x=fail[x]);
        return x;
    }
    void insert(char c) {
        ++n; int cur=get_fail(last); // 判断上一个串的前一个位置和新添加的位置是否相同，相同则说明构成回文。否则找 fail 指针。
        if (!next[cur][c]) {
            ++tot;
            len[tot]=len[cur]+2;
            fail[tot]=next[get_fail(fail[cur])][c];
            next[cur][c]=tot;
            num[tot] = num[fail[tot]] + 1;
            answer[n]='1';
        } else {
            answer[n]='0';
        }
        last=next[cur][c];
        cnt[last] ++;
    }
    void count () {
        for ( int i = tot - 1 ; i >= 0 ; -- i ) cnt[fail[i]] += cnt[i] ;
        //父亲累加儿子的cnt，因为如果fail[v]=u，则u一定是v的子回文串！
    }

} pam;

int main() {
```

```
    scanf("%s", st+1); n=strlen(st+1);
    pam.init();
    for (int i=1;i<=n;i++) pam.insert(st[i]-'a');
    puts(answer+1);
    return 0;
}
```

3 数据结构

3.1 ST 表

```
int Log[N],f[17][N];
int ask(int x,int y){
    int k=Log[y-x+1];
    return max(f[k][x],f[k][y-(1<<k)+1]);
}
int main(){
    for (int i=2;i<=n;i++)Log[i]=Log[i>>1]+1;
    for (int j=1;j<K;j++)
        for (int i=1;i+(1<<j-1)<=n;i++)
            f[j][i]=max(f[j-1][i],f[j-1][i+(1<<j-1)]);
}
```

3.2 K-D Tree

```
int n, cmp_d, root, id[N];

struct node {
    int d[2], l, r, Max[2], Min[2], val, sum, f;
} t[N];

inline bool cmp(const node &a, const node &b) {
    if (a.d[cmp_d] != b.d[cmp_d]) return a.d[cmp_d] < b.d[cmp_d];
    return a.d[cmp_d ^ 1] < b.d[cmp_d ^ 1];
}

inline void umax(int &a, int b) {
    if (b > a) a = b;
}

inline void umin(int &a, int b) {
    if (b < a) a = b;
}

inline void up(int x, int y) {
    umax(t[x].Max[0], t[y].Max[0]);
    umin(t[x].Min[0], t[y].Min[0]);
```

```
    umax(t[x].Max[1], t[y].Max[1]);
    umin(t[x].Min[1], t[y].Min[1]);
}

int build(int l, int r, int D, int f) {
    int mid = (l + r) / 2;
    cmp_d = D;
    nth_element(t + l + 1, t + mid + 1, t + r + 1, cmp);
    id[t[mid].f] = mid;
    t[mid].f = f;
    t[mid].Max[0] = t[mid].Min[0] = t[mid].d[0];
    t[mid].Max[1] = t[mid].Min[1] = t[mid].d[1];
    t[mid].val = t[mid].sum = 0;
    if (l != mid) t[mid].l = build(l, mid - 1, !D, mid);
    else t[mid].l = 0;
    if (r != mid) t[mid].r = build(mid + 1, r, !D, mid);
    else t[mid].r = 0;
    if (t[mid].l) up(mid, t[mid].l);
    if (t[mid].r) up(mid, t[mid].r);
    return mid;
}

// 将编号为 x 的点的权值增加 p
// 请注意，此处的 x 是经过排序的。你需要将点的坐标先作映射。
void change(int x, int p) {
    x = id[x];
    for (t[x].val += p; x; x = t[x].f)
        t[x].sum += p;
}

inline long long sqr(long long x) {
    return x * x;
}

// 欧几里得距离的平方，下界
inline long long euclid_lower_bound(const node &a, int X, int Y) {
    return sqr(max(max(X - a.Max[0], a.Min[0] - X), 0)) +
        sqr(max(max(Y - a.Max[1], a.Min[1] - Y), 0));
}

// 欧几里得距离的平方，上界
inline long long euclid_upper_bound(const node &a, int X, int Y) {
    return max(sqr(X - a.Min[0]), sqr(X - a.Max[0])) +
        max(sqr(Y - a.Min[1]), sqr(Y - a.Max[1]));
}

// 曼哈顿距离，下界
inline long long manhattan_lower_bound(const node &a, int X, int Y) {
    return max(a.Min[0] - X, 0) + max(X - a.Max[0], 0) +
        max(a.Min[1] - Y, 0) + max(Y - a.Max[1], 0);
```

```cpp
}

// 曼哈顿距离，上界
inline long long manhattan_upper_bound(const node &a, int X, int Y) {
    return max(abs(X - a.Max[0]), abs(a.Min[0] - X)) +
        max(abs(Y - a.Max[1]), abs(a.Min[1] - Y));
}

// 添加一个点（注意此处的添加可能导致这棵树不平衡，慎用！）
void add(int k) {
    t[k].Max[0] = t[k].Min[0] = t[k].d[0];
    t[k].Max[1] = t[k].Min[1] = t[k].d[1];
    t[k].val = t[k].sum = 0;
    t[k].l = t[k].r = t[k].f = 0;
    if (!root) {
        root = k;
        return;
    }
    int p = root;
    int D = 0;
    while (1) {
        up(p, k);
        if (t[k].d[D] <= t[p].d[D]) {
            if (t[p].l) p = t[p].l;
            else {
                t[p].l = k;
                t[k].f = p;
                return;
            }
        } else {
            if (t[p].r) p = t[p].r;
            else {
                t[p].r = k;
                t[k].f = p;
                return;
            }
        }
        D ^= 1;
    }
}

inline long long getdis(const node &a, int X, int Y) {
    return sqr(a.d[0] - X) + sqr(a.d[1] - Y);
}

// 此处询问距离点 (X, Y) 最远的一个点的距离， ans 需传入无穷小
void ask(int p, int X, int Y, long long &ans) {
    if (!p) return;
    ans = max(ans, getdis(t[p], X, Y));
    long long dl = t[p].l ? euclid_upper_bound(t[t[p].l], X, Y) : 0;
```

```
    long long dr = t[p].r ? euclid_upper_bound(t[t[p].r], X, Y) : 0;
    if (dl > dr) {
        if (dl > ans) ask(t[p].l, X, Y, ans);
        if (dr > ans) ask(t[p].r, X, Y, ans);
    } else {
        if (dr > ans) ask(t[p].r, X, Y, ans);
        if (dl > ans) ask(t[p].l, X, Y, ans);
    }
}

// 查询矩形范围内所有点的权值和
int ask(int p, int x1, int y1, int x2, int y2) {
    if (t[p].Min[0] > x2 || t[p].Max[0] < x1 || t[p].Min[1] > y2 || t[p].Max[1] < y1
        ) return 0;
    if (t[p].Min[0] >= x1 && t[p].Max[0] <= x2 && t[p].Min[1] >= y1 && t[p].Max[1]
        <= y2) return t[p].sum;
    int s = 0;
    if (t[p].d[0] >= x1 && t[p].d[0] <= x2 && t[p].d[1] >= y1 && t[p].d[1] <= y2) s
        += t[p].val;
    if (t[p].l) s += ask(t[p].l, x1, y1, x2, y2);
    if (t[p].r) s += ask(t[p].r, x1, y1, x2, y2);
    return s;
}

int main() {
    while (~scanf("%d", &n)) {
        for (int i = 1; i <= n; ++i) {
            int x, y, type;
            scanf("%d%d%d", &x, &y, &type);
            t[i].d[0] = x;
            t[i].d[1] = y;
        }
        root = build(1, n, 0, 0);
    }
}
```

3.3 左偏树

左偏树是一个可并堆。

下面的程序写的是一个小根堆，如果需要改成大根堆请在注释了 here 那行修改。

接口：

void push(const T &x); 插入一个元素。

void merge(leftist &x); 合并两个堆。注意，合并后原来那个堆将不可访问。

T top() const; 返回堆顶元素。

void pop(); 删除堆顶元素。

int size() const; 返回堆的大小。

```
template <class T>
class leftist {
```

```
public:
    struct node {
        T key;
        int dist;
        node *l, *r;
    };
    leftist() : root(NULL), s(0) {}
    void push(const T &x) {
        leftist y;
        y.s = 1;
        y.root = new node;
        y.root -> key = x;
        y.root -> dist = 0;
        y.root -> l = y.root -> r = NULL;
        merge(y);
    }
    node* merge(node *x, node *y) {
        if (x == NULL) return y;
        if (y == NULL) return x;
        if (y -> key < x -> key) swap(x, y); //here
        x -> r = merge(x -> r, y);
        int ld = x -> l ? x -> l -> dist : -1;
        int rd = x -> r ? x -> r -> dist : -1;
        if (ld < rd) swap(x -> l, x -> r);
        if (x -> r == NULL) x -> dist = 0;
        else x -> dist = x -> r -> dist + 1;
        return x;
    }
    void merge(leftist &x) {
        root = merge(root, x.root);
        s += x.s;
    }
    T top() const {
        if (root == NULL) return T();
        return root -> key;
    }
    void pop() {
        if (root == NULL) return;
        node *p = root;
        root = merge(root -> l, root -> r);
        --s;
        delete p;
    }
    int size() const {
        return s;
    }
private:
    node* root;
    int s;
};
```

3.4 线段树小技巧

给定一个序列 a ，寻找一个最大的 i 使得 $i \le y$ 且满足一些条件（如 $a[i] \ge w$ ，那么需要在线段树维护 a 的区间最大值）

```
int queryl(int p, int left, int right, int y, int w) {
    if (right <= y) {
        if (! __condition__ ) return -1;
        else if (left == right) return left;
    }
    int mid = (left + right) / 2;
    if (y <= mid) return queryl(p<<1|0, left, mid, y, w);
    int ret = queryl(p<<1|1, mid+1, right, y, w);
    if (ret != -1) return ret;
    return queryl(p<<1|0, left, mid, y, w);
}
```

给定一个序列 a ，寻找一个最小的 i 使得 $i \ge x$ 且满足一些条件（如 $a[i] \ge w$ ，那么需要在线段树维护 a 的区间最大值）

```
int queryr(int p, int left, int right, int x, int w) {
    if (left >= x) {
        if (! __condition__ ) return -1;
        else if (left == right) return left;
    }
    int mid = (left + right) / 2;
    if (x > mid) return queryr(p<<1|1, mid+1, right, x, w);
    int ret = queryr(p<<1|0, left, mid, x, w);
    if (ret != -1) return ret;
    return queryr(p<<1|1, mid+1, right, x, w);
}
```

3.5 Splay

接口：

ADD x y d ： 将 $[x,y]$ 的所有数加上 d

REVERSE x y ： 将 $[x,y]$ 翻转

INSERT x p ： 将 p 插入到第 x 个数的后面

DEL x ： 将第 x 个数删除

```
struct SPLAY {
    struct NODE {
        int w, min;
        int son[2], size, father, rev, lazy;
    } node[N];
    int top, rt;
    void pushdown(int x) {
```

```
        if (!x) return;
        if (node[x].rev) {
            node[node[x].son[0]].rev ^= 1;
            node[node[x].son[1]].rev ^= 1;
            swap(node[x].son[0], node[x].son[1]);
            node[x].rev = 0;
        }
        if (node[x].lazy) {
            node[node[x].son[0]].lazy += node[x].lazy;
            node[node[x].son[1]].lazy += node[x].lazy;
            node[x].w += node[x].lazy;
            node[x].min += node[x].lazy;
            node[x].lazy = 0;
        }
    }
    void pushup(int x) {
        if (!x) return;
        pushdown(node[x].son[0]);
        pushdown(node[x].son[1]);
        node[x].size = node[node[x].son[0]].size + node[node[x].son[1]].size + 1;
        node[x].min = node[x].w;
        if (node[x].son[0]) node[x].min = min(node[x].min, node[node[x].son[0]].min);
        if (node[x].son[1]) node[x].min = min(node[x].min, node[node[x].son[1]].min);
    }
    void sc(int x, int y, int w) {
        node[x].son[w] = y;
        node[y].father = x;
        pushup(x);
    }
    void _ins(int w) {
        top++;
        node[top].w = node[top].min = w;
        node[top].son[0] = node[top].son[1] = 0;
        node[top].size = 1; node[top].father = 0; node[top].rev = 0;
    }
    void init() {
        top = 0;
        _ins(0); _ins(0); rt=1;
        sc(1, 2, 1);
    }
    void rotate(int x) {
        if (!x) return;
        int y = node[x].father;
        int w = node[y].son[1]==x;
        sc(y, node[x].son[w^1], w);
        sc(node[y].father, x, node[node[y].father].son[1]==y);
        sc(x, y, w^1);
    }
```

```
    int q[N];
    void flushdown(int x) {
        int t=0; for (; x; x=node[x].father) q[++t]=x;
        for (; t; t--) pushdown(q[t]);
    }
    void Splay(int x, int root=0) {
        flushdown(x);
        while (node[x].father != root) {
            int y=node[x].father;
            int w=node[y].son[1]==x;
            if (node[y].father != root && node[node[y].father].son[w]==y) rotate(y);
            rotate(x);
        }
    }
    int find(int k) {
        Splay(rt);
        while (1) {
            pushdown(rt);
            if (node[node[rt].son[0]].size+1==k) {
                Splay(rt);
                return rt;
            } else
            if (node[node[rt].son[0]].size+1<k) {
                k-=node[node[rt].son[0]].size+1;
                rt=node[rt].son[1];
            } else {
                rt=node[rt].son[0];
            }
        }
    }
    int split(int x, int y) {
        int fx = find(x);
        int fy = find(y+2);
        Splay(fx);
        Splay(fy, fx);
        return node[fy].son[0];
    }
    void add(int x, int y, int d) { //add d to each number in a[x]...a[y]
        int t = split(x, y);
        node[t].lazy += d;
        Splay(t); rt=t;
    }
    void reverse(int x, int y) { // reverse the x-th to y-th elements
        int t = split(x, y);
        node[t].rev ^= 1;
        Splay(t); rt=t;
    }
    void insert(int x, int p) { // insert p after the x-th element
        int fx = find(x+1);
        int fy = find(x+2);
```

```
        Splay(fx);
        Splay(fy, fx);
        _ins(p);
        sc(fy, top, 0);
        Splay(top); rt=top;
    }
    void del(int x) { // delete the x-th element in Splay
        int fx = find(x), fy = find(x+2);
        Splay(fx); Splay(fy, fx);
        node[fy].son[0] = 0;
        Splay(fy); rt=fy;
    }
} tree;
```

3.6 可持久化 Treap

接口:

void insert(int x, char c); 在当前第 x 个字符后插入 c

void del(int x, int y); 删除第 x 个字符到第 y 个字符

void copy(int l, int r, int x); 复制第 l 个字符到第 r 个字符，然后粘贴到第 x 个字符后

void reverse(int x, int y); 翻转第 x 个到第 y 个字符

char query(int k); 表示询问当前第 x 个字符是什么

```
#define mod 1000000007
struct Treap {
    struct Node {
        char key;
        bool reverse;
        int lc, rc, size; // if size is long long, remember here
    } node[N];
    int n, root, rd;
    int Rand() { rd = (rd * 20372052LL + 25022087LL) % mod; return rd; }

    /*
    LL Rand() {
        LL t1 = rand() % 32768;
        LL t2 = rand() % 32768;
        LL t3 = rand() % 32768;
        LL t4 = rand() % 32768;
        return (((t1 * 32768) + t2) * 32768 + t3) * 32768 + t4;
    }
    */

    void init() {
        n = root = 0;
    }
    inline int copy(int x) {
        node[++n] = node[x]; return n;
    }
```

```
    inline void pushdown(int x) {
        if (!node[x].reverse) return;
        if (node[x].lc) node[x].lc = copy(node[x].lc);
        if (node[x].rc) node[x].rc = copy(node[x].rc);
        swap(node[x].lc, node[x].rc);
        node[node[x].lc].reverse ^= 1;
        node[node[x].rc].reverse ^= 1;
        node[x].reverse = 0;
    }
    inline void pushup(int x) {
        node[x].size = node[node[x].lc].size + node[node[x].rc].size + 1;
    }
    int merge(int u, int v) {
        if (!u || !v) return u+v;
        pushdown(u); pushdown(v);
        int t = Rand() % (node[u].size + node[v].size), r;  // if size is long long, remember here
        if (t < node[u].size) {
            r = copy(u);
            node[r].rc = merge(node[u].rc, v);
        } else {
            r = copy(v);
            node[r].lc = merge(u, node[v].lc);
        }
        pushup(r);
        return r;
    }
    int split(int u, int x, int y) { // if size is long long, remember here
        if (x > y) return 0;
        pushdown(u);
        if (x == 1 && y == node[u].size) return copy(u);
        if (y <= node[node[u].lc].size) return split(node[u].lc, x, y);
        int t = node[node[u].lc].size + 1; // if size is long long, remember here
        if (x > t) return split(node[u].rc, x-t, y-t);
        int num = copy(u);
        node[num].lc = split(node[u].lc, x, t-1);
        node[num].rc = split(node[u].rc, 1, y-t);
        pushup(num);
        return num;
    }
    void insert(int x, char c) {
        int t1 = split(root, 1, x), t2 = split(root, x+1, node[root].size);
        node[++n].key = c;
        node[n].lc = node[n].rc = 0;
        node[n].reverse = 0;
        pushup(n);
        root = merge(merge(t1, n), t2);
    }
    void del(int x, int y) {
        int t1 = split(root, 1, x-1), t2 = split(root, y+1, node[root].size);
```

```
        root = merge(t1, t2);
    }
    void copy(int l, int r, int x) {
        int t1 = split(root, 1, x), t2 = split(root, l, r), t3 = split(root, x+1,
            node[root].size);
        root = merge(merge(t1, t2), t3);
    }
    void reverse(int x, int y) {
        int t1 = split(root, 1, x-1), t2 = split(root, x, y), t3 = split(root, y+1,
            node[root].size);
        node[t2].reverse ^= 1;
        root = merge(merge(t1, t2), t3);
    }
    char query(int k) {
        int x = root;
        while (1) {
            pushdown(x);
            if (k <= node[node[x].lc].size) x = node[x].lc;
            else
            if (k == node[node[x].lc].size + 1) return node[x].key;
            else
            k -= node[node[x].lc].size + 1, x = node[x].rc;
        }
    }
} treap;
```

3.7 可持久化并查集

接口:

void init() 初始化

void merge(int x, int y, int time) 在 time 时刻将 x 和 y 连一条边，注意加边顺序必须按 time 从小到大加边

void GetFather(int x, int time) 询问 time 时刻及以前的连边状态中，x 所属的集合

```
namespace pers_union {
    const int inf = 0x3f3f3f3f;
    int father[N], Father[N], Time[N];
    vector<int> e[N];
    void init() {
        for (int i=1;i<=n;i++) {
            father[i] = i;
            Father[i] = i;
            Time[i] = inf;
            e[i].clear();
            e[i].push_back(i);
        }
    }
    int getfather(int x) {
        return (father[x] == x) ? x : father[x] = getfather(father[x]);
```

```
    }
    int GetFather(int x, int time) {
        return (Time[x] <= time) ? GetFather(Father[x], time) : x;
    }
    void merge(int x, int y, int time) {
        int fx = getfather(x), fy = getfather(y);
        if (fx == fy) return;
        if (e[fx].size() > e[fy].size()) swap(fx, fy);
        father[fx] = fy;
        Father[fx] = fy;
        Time[fx] = time;
        for (int i=0;i<e[fx].size();i++) {
            e[fy].push_back(e[fx][i]);
        }
    }
};
```

4 树

4.1 树链剖分

接口：

void addedge(int x, int y);	将 x 到 y 连边，注意这是单向边
void dfs(int x, int root = 0);	从 x 开始遍历整棵树
void split(int x, int tp);	划分轻重链
int lca(int x, int y);	求 x 和 y 的 lca
int query(int x, int y);	求 x 到 y 经过的点数
int skip(int x, int k);	求从 x 向根方向跳 k 步到达的节点（若超出根，则返回 0）
void get_data(int x, int y);	将 x 到 y 路径上的重链找出来，存在 seg[0] 中

Debug 技巧：换一个根来 dfs 以测试程序是否能通过 $father[i] > i$ 的数据

```
struct EDGE {
    int adj, next;
} edge[N * 2];

int n, gh[N], top, s_top;
int father[N], deep[N], son[N], size[N], Top[N], dfn[N], rdfn[N];

void addedge(int x, int y) {
    edge[++top].adj = y;
    edge[top].next = gh[x];
    gh[x] = top;
}

void dfs(int x, int root = 0) {
    father[x] = root;
    deep[x] = deep[root] + 1;
    son[x] = 0;
```

```
    size[x] = 1;
    int dd = 0;
    for (int p = gh[x]; p; p = edge[p].next)
        if (edge[p].adj != root) {
            dfs(edge[p].adj, x);
            if (size[edge[p].adj] > dd) {
                dd = size[edge[p].adj];
                son[x] = edge[p].adj;
            }
            size[x] += size[edge[p].adj];
        }
}

void split(int x, int tp) {
    Top[x] = tp; dfn[x] = ++s_top; rdfn[s_top] = x;
    if (son[x]) split(son[x], tp);
    for (int p = gh[x]; p; p = edge[p].next)
        if (edge[p].adj != father[x] && edge[p].adj != son[x])
            split(edge[p].adj, edge[p].adj);
}

int lca(int x, int y) {
    int tx = Top[x], ty = Top[y];
    while (tx != ty) {
        if (deep[tx] < deep[ty]) {
            swap(tx, ty);
            swap(x, y);
        }
        x = father[tx];
        tx = Top[x];
    }
    if (deep[x] < deep[y])
        swap(x, y);
    return y;
}

int query(int x, int y) {
    int tx = Top[x], ty = Top[y];
    int ans = 0;
    while (tx != ty) {
        if (deep[tx] < deep[ty]) {
            swap(tx, ty);
            swap(x, y);
        }
        ans += dfn[x] - dfn[tx] + 1;
        x = father[tx];
        tx = Top[x];
    }
    if (deep[x] < deep[y])
        swap(x, y);
```

```
    ans += dfn[x] - dfn[y] + 1;
    return ans;
}

int skip(int x, int k) {
    int tx = Top[x];
    while (tx) {
        if (k < dfn[x] - dfn[tx] + 1) {
            return rdfn[ dfn[x] - k ];
        } else {
            k -= dfn[x] - dfn[tx] + 1;
            x = father[tx];
            tx = Top[x];
        }
    }
    return 0;
}

struct segment {
    int l, r;
    data d;
    segment(int _l, int _r) { // from _l to _r
        l = _l, r = _r;
        if (l <= r) d = query(l, r, 0);
        else d = query(r, l, 1); //reverse
    }
};

vector&lt;segment&gt; seg[2];

void get_data(int x, int y) {
    seg[0].clear(); seg[1].clear();
    int tx = Top[x], ty = Top[y];
    int s = 0;
    while (tx != ty) {
        if (deep[tx] < deep[ty]) {
            swap(tx, ty);
            swap(x, y);
            s ^= 1;
        }
        if (s == 0)
            seg[s].push_back(segment(w[x], w[tx]));
        else
            seg[s].push_back(segment(w[tx], w[x]));
        x = father[tx];
        tx = Top[x];
    }
    if (x != y) {
        if (deep[x] < deep[y]) {
            swap(x, y);
```

```
            s ^= 1;
        }
        if (s == 0)
            seg[s].push_back(segment(w[x], w[y] + 1));
        else
            seg[s].push_back(segment(w[y] + 1, w[x]));
    }
    reverse(seg[1].begin(), seg[1].end());
    for (int i = 0; i < seg[1].size(); ++i)
        seg[0].push_back(seg[1][i]);
    // saved to seg[0]
}

void init() {
    top = s_top = 0;
    for (int i = 1; i <= n; ++i) gh[i] = 0;
}
```

4.2 点分治

初始化时须设置 $top = 1$ 。

```
void addedge(int x, int y) {
    edge[++top].adj = y;
    edge[top].valid = 1;
    edge[top].next = gh[x];
    gh[x] = top;
}
void get_size(int x, int root=0) {
    size[x] = 1; son[x] = 0;
    int dd = 0;
    for (int p=gh[x]; p; p=edge[p].next)
        if (edge[p].adj != root && edge[p].valid) {
            get_size(edge[p].adj, x);
            size[x] += size[edge[p].adj];
            if (size[edge[p].adj] > dd) {
                dd = size[edge[p].adj];
                son[x] = edge[p].adj;
            }
        }
}
int getroot(int x) {
    get_size(x);
    int sz = size[x];
    while (size[son[x]] > sz/2)
        x = son[x];
    return x;
}
void dc(int x) {
    x = getroot(x);
```

```
    static int list[N], ltop;
    ltop = 0;
    for (int p=gh[x]; p; p=edge[p].next)
        if (edge[p].valid)
            list[++ltop] = edge[p].adj;
    clear();
    for (int i=1;i<=ltop;i++) {
        update();
        modify();
    }
    clear();
    for (int i=ltop;i>=1;i--) {
        update();
        modify();
    }
    //be careful about the root
    for (int p=gh[x]; p; p=edge[p].next)
        if (edge[p].valid) {
            edge[p].valid = 0;
            edge[p^1].valid = 0;
            dc(edge[p].adj);
        }
}
```

4.3 Link Cut Tree

请注意，一开始必须调用 lct.init(0) ，否则求出的最小值一定会是 0 。
minval 维护的是链上 val 最小值。
sumval2 维护的是子树 val2 的和。

```
struct DTree {
    int f[N], son[N][2], sz[N], rev[N];
    int val[N], minid[N], minval[N];
    int val2[N], sumval2[N]; // 记得开 long long 。注意两个都要开 long long ，因为
        val2 还包含了虚儿子的子树和。
    int tot;
    stack<int> s;
    void init(int i) {
        tot = max(tot, i);
        son[i][0] = son[i][1] = 0;
        f[i] = rev[i] = 0;
        if (i == 0) {
            sz[i] = 0;
            val[i] = minval[i] = inf;
            minid[i] = i;
            val2[i] = sumval2[i] = 0;
        } else {
            sz[i] = 1;
            val[i] = minval[i] = VAL;
```

```
        minid[i] = i;
        val2[i] = sumval2[i] = VAL2;
    }
}
bool isroot(int x) {
    return !f[x] || (son[f[x]][0] != x && son[f[x]][1] != x);
}
void rev1(int x) {
    if (!x) return;
    swap(son[x][0], son[x][1]);
    rev[x] ^= 1;
}
void down(int x) {
    if (!x) return;
    if (rev[x]) rev1(son[x][0]), rev1(son[x][1]), rev[x] = 0;
}
void up(int x) {
    if (!x) return;
    down(son[x][0]); down(son[x][1]);
    sz[x] = sz[son[x][0]] + sz[son[x][1]] + 1;
    minval[x] = val[x]; minid[x] = x;
    if (minval[son[x][0]] < minval[x]) minval[x] = minval[son[x][0]], minid[x] = minid[son[x][0]];
    if (minval[son[x][1]] < minval[x]) minval[x] = minval[son[x][1]], minid[x] = minid[son[x][1]];
    sumval2[x] = sumval2[son[x][0]] + sumval2[son[x][1]] + val2[x];
}
void rotate(int x) {
    int y = f[x], w = son[y][1] == x;
    son[y][w] = son[x][w ^ 1];
    if (son[x][w ^ 1]) f[son[x][w ^ 1]] = y;
    if (f[y]) {
        int z = f[y];
        if (son[z][0] == y) son[z][0] = x;
        else if (son[z][1] == y) son[z][1] = x;
    }
    f[x] = f[y]; f[y] = x; son[x][w ^ 1] = y;
    up(y);
}
void splay(int x) {
    while (!s.empty()) s.pop();
    s.push(x);
    for (int i = x; !isroot(i); i = f[i]) s.push(f[i]);
    while (!s.empty()) down(s.top()), s.pop();
    while (!isroot(x)) {
        int y = f[x];
        if (!isroot(y)) {
            if ((son[f[y]][0] == y) ^ (son[y][0] == x))
                rotate(x);
            else
```

```
                    rotate(y);
                }
                rotate(x);
            }
            up(x);
        }
        void access(int x) {
            for (int y = 0; x; y = x, x = f[x]) {
                splay(x);
                val2[x] += sumval2[son[x][1]];
                son[x][1] = y;
                val2[x] -= sumval2[son[x][1]];
                up(x);
            }
        }
        int root(int x) {
            access(x);
            splay(x);
            while (son[x][0]) x = son[x][0];
            return x;
        }
        void makeroot(int x) {
            access(x);
            splay(x);
            rev1(x);
        }
        void link(int x, int y) {
            makeroot(x);
            f[x] = y;
            access(x);
            // 如果需要维护子树和 val2, sumval2 ，这样是不够的。因为增加了虚边，所以需要
                修改 val2 值。将上面的三行代码替换为以下代码:
            // makeroot(x);
            // makeroot(y);
            // f[x] = y;
            // val2[y] += sumval2[x];
            // up(y);
            // access(x);
        }
        void cutf(int x) { // 它和父亲的边
            access(x);
            splay(x);
            f[son[x][0]] = 0;
            son[x][0] = 0;
            up(x);
        }
        void cut(int x, int y) { // 切断 x 与 y 之间的边（须保证 x 与 y 相邻）
            makeroot(x);
            cutf(y);
        }
```

```
    int ask(int x, int y) { // 询问 x 到 y 之间取得最小值的点
        makeroot(x);
        access(y);
        splay(y);
        return minid[y];
    }
    int querymin_cut(int x, int y) { // 询问 x 到 y 之间取得最小值的点，并把它删去 (须保证该点在 x 和 y 之间，且度数恰好为 2)
        int m = ask(x, y);
        makeroot(x);
        cutf(m);
        makeroot(y);
        cutf(m);
        return val[m];
    }
    void link(int x, int y, int w) { // 在 x 和 y 之间添加一条权值为 w 的边 (将边视为点插入)
        init(++tot);
        val[tot] = minval[tot] = w;
        link(x, tot);
        link(y, tot);
    }
    int getsumval2(int x, int y) { // 令 x 为根，求 y 子树的 val2 的和
        makeroot(x);
        access(y);
        return val2[y];
    }
} lct;
```

4.4 树形 DP

适用于每个点都要求 dp 值的题目。

示例代码的状态转移方程:

$$f(x) = \sum_{y=son(x)} f(y) + (\sum_{y=son(x)} sz(y))^2 - (\sum_{y=son(x)} sz(y)^2) + (\sum_{y=son(x)} sz(y))$$

```
#include <bits/stdc++.h>

#define N 1000020
#define LL long long

using namespace std;

int n;
LL usz[N], uf[N];
LL dsz[N], df[N];
LL dp[N];
vector<int> g[N];

```

```
void dfs1(int x, int root) {
    if (root) g[x].erase(find(g[x].begin(), g[x].end(), root));
    LL s1 = 0, s2 = 0;
    LL sf = 0;
    uf[x] = 0;
    for (auto y : g[x]) {
        dfs1(y, x);
        s1 += usz[y];
        s2 += 1ll * usz[y] * usz[y];
        sf += uf[y];
    }
    usz[x] = s1 + 1;
    uf[x] += sf + (s1 * s1 - s2) + s1;
}

void dfs2(int x, int root) {
    LL s1 = dsz[x], s2 = dsz[x] * dsz[x];
    LL sf = df[x];
    for (auto y : g[x]) {
        s1 += usz[y];
        s2 += usz[y] * usz[y];
        sf += uf[y];
    }
    for (auto y : g[x]) {
        dsz[y] = s1 + 1 - usz[y];
        df[y] = (sf - uf[y]) + (s1 - usz[y]) * (s1 - usz[y]) - (s2 - usz[y] * usz[y
            ]) + (s1 - usz[y]);
        dfs2(y, x);
    }
    dp[x] = min(ans, sf + s1 * s1 - s2 + s1);
}

int main() {
    scanf("%d", &n);
    for (int i = 1; i < n; ++i) {
        int x, y;
        scanf("%d%d", &x, &y);
        g[x].push_back(y);
        g[y].push_back(x);
    }
    dfs1(1, 0);
    dfs2(1, 0);
}
```

4.5 求子树的直径

树形 DP 。

答案保存在 u, d 数组中。

$u[x].exc$ 表示切断 x 与 $father[x]$ 的边，$father[x]$ 表示的那颗子树的直径。

$d[x].exc$ 表示切断 x 与 $father[x]$ 的边，x 表示的那颗子树的直径。

```cpp
#include <bits/stdc++.h>

#define N 200020

using namespace std;

vector<int> g[N];
int n, q, top;
int deep[N], father[N], son[N], size[N], Top[N], dfn[N], rdfn[N];

void dfs(int x, int root = 0) {
    deep[x] = deep[root] + 1;
    father[x] = root;
    son[x] = 0; size[x] = 1;
    if (root) g[x].erase(lower_bound(g[x].begin(), g[x].end(), root));
    // 去根
    int dd = 0;
    for (int i = 0; i < g[x].size(); ++i) {
        dfs(g[x][i], x);
        if (size[g[x][i]] > dd) {
            dd = size[g[x][i]];
            son[x] = g[x][i];
        }
        size[x] += size[g[x][i]];
    }
}

void split(int x, int tp) {
    dfn[x] = ++top; rdfn[top] = x; Top[x] = tp;
    if (son[x]) split(son[x], tp);
    for (int i = 0; i < g[x].size(); ++i)
        if (g[x][i] != son[x])
            split(g[x][i], g[x][i]);
}

struct data {
    int inc, inc_id;
    int exc, exc_l, exc_r;
    //inc 表示从该点出发可以走到的最远距离
    //inc_id 表示从该点出发可以走到的最远点的编号
    //exc 表示子树中两点最远距离
    //exc_l, exc_r 表示子树中两点取得最远距离的两点的编号
    data() {
        inc = inc_id = 0;
        exc = exc_l = exc_r = 0;
    }
} u[N], d[N];

int safe(int x, int y) {
```

```
    // 防止 inc_id = 0 的情况
    if (x) return x;
    return y;
}

void dfs1(int x) {
    d[x].inc = 1; d[x].inc_id = x;
    data mx1 = data(), mx2 = data();
    // mx1, mx2 表示儿子 inc 最大、第2大值，用于更新该点 exc
    for (int i = 0; i < g[x].size(); ++i) {
        dfs1(g[x][i]);
        if (d[g[x][i]].inc + 1 > d[x].inc) {
            d[x].inc = d[g[x][i]].inc + 1;
            d[x].inc_id = d[g[x][i]].inc_id;
        }
        if (d[g[x][i]].inc > mx1.inc) {
            mx2 = mx1;
            mx1 = d[g[x][i]];
        } else
        if (d[g[x][i]].inc > mx2.inc) {
            mx2 = d[g[x][i]];
        }
    }
    d[x].exc = mx1.inc + mx2.inc + 1;
    d[x].exc_l = safe(mx1.inc_id, x);
    d[x].exc_r = safe(mx2.inc_id, x);
    for (int i = 0; i < g[x].size(); ++i)
        if (d[g[x][i]].exc > d[x].exc) {
            d[x].exc = d[g[x][i]].exc;
            d[x].exc_l = d[g[x][i]].exc_l;
            d[x].exc_r = d[g[x][i]].exc_r;
        }
}

void dfs2(int x, data y) {
    u[x] = y;
    if (!y.exc) y.exc = 1, y.exc_l = y.exc_r = x;
    data mx1 = y, mx2 = data(), mx3 = data(), mxe1 = y, mxe2 = data();
    // mx1, mx2, mx3 表示根过来的子树中 inc 的最大、第2大、第3大值
    // mxe1, mxe2 表示根过来的子树中 exc 的最大、第2大值
    int mx1_id = -1, mx2_id = -1, mx3_id = -1, mxe1_id = -1, mxe2_id = -1;
    for (int i = 0; i < g[x].size(); ++i) {
        if (d[g[x][i]].inc > mx1.inc) {
            mx3 = mx2; mx3_id = mx2_id;
            mx2 = mx1; mx2_id = mx1_id;
            mx1 = d[g[x][i]]; mx1_id = i;
        } else
        if (d[g[x][i]].inc > mx2.inc) {
            mx3 = mx2; mx3_id = mx2_id;
            mx2 = d[g[x][i]]; mx2_id = i;
```

```
        } else
        if (d[g[x][i]].inc > mx3.inc) {
            mx3 = d[g[x][i]]; mx3_id = i;
        }
        if (d[g[x][i]].exc > mxe1.exc) {
            mxe2 = mxe1; mxe2_id = mxe1_id;
            mxe1 = d[g[x][i]]; mxe1_id = i;
        } else
        if (d[g[x][i]].exc > mxe2.exc) {
            mxe2 = d[g[x][i]]; mxe2_id = i;
        }
    }
    for (int i = 0; i < g[x].size(); ++i) {
        data z = data();
        if (i == mx1_id) {
            z.exc = mx2.inc + mx3.inc + 1;
            z.exc_l = safe(mx2.inc_id, x);
            z.exc_r = safe(mx3.inc_id, x);
        } else
        if (i == mx2_id) {
            z.exc = mx1.inc + mx3.inc + 1;
            z.exc_l = safe(mx1.inc_id, x);
            z.exc_r = safe(mx3.inc_id, x);
        } else {
            z.exc = mx1.inc + mx2.inc + 1;
            z.exc_l = safe(mx1.inc_id, x);
            z.exc_r = safe(mx2.inc_id, x);
        }
        if (i == mxe1_id) {
            if (mxe2.exc > z.exc) z = mxe2;
        } else {
            if (mxe1.exc > z.exc) z = mxe1;
        }
        if (i == mx1_id) {
            z.inc = mx2.inc + 1;
            z.inc_id = safe(mx2.inc_id, x);
        } else {
            z.inc = mx1.inc + 1;
            z.inc_id = safe(mx1.inc_id, x);
        }
        dfs2(g[x][i], z);
    }
}
```

4.6 虚树

设 $a[0 \cdots k-1]$ 为需要构建虚树的点。

构建出虚树的节点保存在 a 数组中，k 为节点个数。加边调用函数 addedge(int x, int y, int w) 。

```
bool cmp(int x, int y) {
```

```
    return dfn[x] < dfn[y];
}

stack<int> stk;

void solve() {
    sort(a, a + k, cmp);
    int m = k;
    for (int j = 1; j < m; ++j)
        a[k++] = lca(a[j - 1], a[j]);
    sort(a, a + k, cmp);
    k = unique(a, a + k) - a;
    stk.push(a[0]);
    for (int j = 1; j < k; ++j) {
        int u = lca(stk.top(), a[j]);
        while (dep[stk.top()] > dep[u]) --top;
        assert(stk.top() == u);
        stk.push(a[j]);
        addedge(u, a[j], dis[a[j]] - dis[u]);
    }
}
```

5 图

5.1 欧拉回路

欧拉回路:

无向图：每个顶点的度数都是偶数，则存在欧拉回路。

有向图：每个顶点的入度 = 出度，则存在欧拉回路。

欧拉路径:

无向图：当且仅当该图所有顶点的度数为偶数，或者除了两个度数为奇数外其余的全是偶数。

有向图：当且仅当该图所有顶点出度 = 入度或者一个顶点出度 = 入度 + 1，另一个顶点入度 = 出度 + 1，其他顶点出度 = 入度。

下面 $O(n+m)$ 求欧拉回路的代码中，n 为点数，m 为边数，若有解则依次输出经过的边的编号, 若是无向图, 则正数表示 x 到 y ，负数表示 y 到 x 。

```
namespace UndirectedGraph{
    int n,m,i,x,y,d[N],g[N],v[M<<1],w[M<<1],vis[M<<1],nxt[M<<1],ed;
    int ans[M],cnt;
    void add(int x,int y,int z){
        d[x]++;
        v[++ed]=y;w[ed]=z;nxt[ed]=g[x];g[x]=ed;
    }
    void dfs(int x){
        for(int&i=g[x];i;){
            if(vis[i]){i=nxt[i];continue;}
            vis[i]=vis[i^1]=1;
            int j=w[i];
```

```
            dfs(v[i]);
            ans[++cnt]=j;
        }
    }
    void solve(){
        scanf("%d%d",&n,&m);
        for(i=ed=1;i<=m;i++)scanf("%d%d",&x,&y),add(x,y,i),add(y,x,-i);
        for(i=1;i<=n;i++)if(d[i]&1){puts("NO");return;}
        for(i=1;i<=n;i++)if(g[i]){dfs(i);break;}
        for(i=1;i<=n;i++)if(g[i]){puts("NO");return;}
        puts("YES");
        for(i=m;i;i--)printf("%d ",ans[i]);
    }
}
namespace DirectedGraph{
    int n,m,i,x,y,d[N],g[N],v[M],vis[M],nxt[M],ed;
    int ans[M],cnt;
    void add(int x,int y){
        d[x]++;d[y]--;
        v[++ed]=y;nxt[ed]=g[x];g[x]=ed;
    }
    void dfs(int x){
        for(int&i=g[x];i;){
            if(vis[i]){i=nxt[i];continue;}
            vis[i]=1;
            int j=i;
            dfs(v[i]);
            ans[++cnt]=j;
        }
    }
    void solve(){
        scanf("%d%d",&n,&m);
        for(i=1;i<=m;i++)scanf("%d%d",&x,&y),add(x,y);
        for(i=1;i<=n;i++)if(d[i]){puts("NO");return;}
        for(i=1;i<=n;i++)if(g[i]){dfs(i);break;}
        for(i=1;i<=n;i++)if(g[i]){puts("NO");return;}
        puts("YES");
        for(i=m;i;i--)printf("%d ",ans[i]);
    }
}
```

5.2 最短路径

5.2.1 Dijkstra

```
#define LL long long

struct EDGE {
    int adj, w, next;
```

```
} edge[M*2];

typedef pair<LL, int> pli;
priority_queue <pli, vector<pli>, greater<pli> > q;

int n, top, gh[N];
LL dist[N];

void addedge(int x, int y, int w) {
    edge[++top].adj = y;
    edge[top].w = w;
    edge[top].next = gh[x];
    gh[x] = top;
}

LL dijkstra(int s, int t) {
    memset(dist, 63, sizeof(dist));
    memset(v, 0, sizeof(v));
    dist[s] = 0;
    q.push(make_pair(dist[s], s));
    while (!q.empty()) {
        LL dis = q.top().first;
        int x = q.top().second;
        q.pop();
        if (dis != dist[x]) continue;
        for (int p=gh[x]; p; p=edge[p].next) {
            if (dis + edge[p].w < dist[edge[p].adj]) {
                dist[edge[p].adj] = dis + edge[p].w;
                q.push(make_pair(dist[edge[p].adj], edge[p].adj));
            }
        }
    }
    return dist[t];
}
```

5.2.2 SPFA

```
struct EDGE {
    int adj, w, next;
} edge[M*2];

int n,m,top,gh[N],v[N],cnt[N],q[N],dist[N],head,tail;

void addedge(int x, int y, int w) {
    edge[++top].adj = y;
    edge[top].w = w;
    edge[top].next = gh[x];
    gh[x] = top;
}
```

```

int spfa(int S, int T) {
    memset(v, 0, sizeof(v));
    memset(cnt, 0, sizeof(cnt));
    memset(dist, 63, sizeof(dist));
    head = 0, tail = 1;
    dist[S] = 0; q[1] = S;
    while (head != tail) {
        (head += 1) %= N;
        int x = q[head]; v[x] = 0;
        ++cnt[x]; if (cnt[x] > n) return -1;
        for (int p=gh[x]; p; p=edge[p].next)
            if (dist[x] + edge[p].w < dist[edge[p].adj]) {
                dist[edge[p].adj] = dist[x] + edge[p].w;
                if (!v[edge[p].adj]) {
                    v[edge[p].adj] = 1;
                    (tail += 1) %= N;
                    q[tail] = edge[p].adj;
                }
            }
    }
    return dist[T];
}
```

5.3 K 短路

接口：

kthsp::init(n)： 初始化并设置节点个数为 n

kthsp::add(x, y, w)： 添加一条 x 到 y 的有向边

kthsp::work(S, T, k)： 求 S 到 T 的第 k 短路

```
#define N 200020
#define M 400020
#define LOGM 20
#define LL long long
#define inf (1LL<<61)

namespace pheap {
    struct Node {
        int next, son[2];
        LL val;
    } node[M*LOGM];
    int LOG[M];
    int root[M], size[M*LOGM], top;
    int add() {
        ++top; assert(top < M*LOGM);
        node[top].next = node[top].son[0] = node[top].son[1] = 0;
        node[top].val = inf;
        return top;
```

```
    }
    int copy(int x) {
        int t = add();
        node[t] = node[x];
        return t;
    }
    void init() {
        memset(root, 0, sizeof(root));
        top = -1; add();
        LOG[1] = 0;
        for (int i=2;i<M;i++) LOG[i] = LOG[i>>1] + 1;
    }
    void upd(int x, int &next, LL &val) {
        if (val < node[x].val) {
            swap(val, node[x].val);
            swap(next, node[x].next);
        }
    }
    void insert(int x, int next, LL val) {
        int sz = size[root[x]] + 1;
        root[x] = copy(root[x]);
        size[root[x]] = sz; x = root[x];
        upd(x, next, val);
        for (int i=LOG[sz]-1;i>=0;i--) {
            int ind = (sz>>i)&1;
            node[x].son[ind] = copy(node[x].son[ind]);
            x = node[x].son[ind];
            upd(x, next, val);
        }
    }
};

namespace kthsp {
    using namespace pheap;
    struct EDGE {
        int adj, w, next;
    } edge[2][M];
    struct W {
        int x, y, w;
    } e[M];
    bool has_init = 0;
    int n, m, top[2], gh[2][N], v[N];
    LL dist[N];
    void init(int n1) {
        has_init = 1;
        n = n1; m = 0;
        memset(top, 0, sizeof(top));
        memset(gh, 0, sizeof(gh));
        for (int i=1;i<=n;i++) dist[i] = inf;
    }
```

```cpp
    void addedge(int id, int x, int y, int w) {
        edge[id][++top[id]].adj = y;
        edge[id][top[id]].w = w;
        edge[id][top[id]].next = gh[id][x];
        gh[id][x] = top[id];
    }
    void add(int x, int y, int w) {
        assert(has_init);
        e[++m].x=x; e[m].y=y; e[m].w=w;
    }
    int best[N], bestw[N];
    typedef pair<LL, int> pli;
    priority_queue <pli, vector<pli>, greater<pli> > q;

    // you can replace dijkstra with SPFA or TOPSORT(DAG)
    void dijkstra(int S) {
        while (!q.empty()) q.pop();
        dist[S] = 0; q.push(make_pair(dist[S], S));
        while (!q.empty()) {
            LL dis = q.top().first;
            int x = q.top().second;
            q.pop();
            if (dist[x] != dis) continue;
            for (int p=gh[1][x]; p; p=edge[1][p].next) {
                int y = edge[1][p].adj;
                if (dist[x] + edge[1][p].w < dist[y]) {
                    dist[y] = dist[x] + edge[1][p].w;
                    best[y] = x;
                    bestw[y] = p;
                    q.push(make_pair(dist[y], y));
                }
            }
        }
    }
    void dfs(int x) {
        if (v[x]) return;
        v[x] = 1;
        if (best[x]) root[x] = root[best[x]];
        for (int p=gh[0][x]; p; p=edge[0][p].next)
            if (dist[edge[0][p].adj] != inf && bestw[x] != p) {
                insert(x, edge[0][p].adj, edge[0][p].w + dist[edge[0][p].adj] - dist
                    [x]);
            }
        for (int p=gh[1][x]; p; p=edge[1][p].next)
            if (best[edge[1][p].adj] == x)
                dfs(edge[1][p].adj);
    }
    LL work(int S, int T, int k) {
        assert(has_init);
        n++; add(T, n, 0);
```

```
        if (S == T) k ++;
        T = n;
        for (int i=1;i<=m;i++) {
            addedge(0, e[i].x, e[i].y, e[i].w);
            addedge(1, e[i].y, e[i].x, e[i].w);
        }
        dijkstra(T);
        root[T] = 0; pheap::init();
        memset(v, 0, sizeof(v));
        dfs(T);
        while (!q.empty()) q.pop();
        if (k == 1) return dist[S];
        if (root[S]) q.push(make_pair(dist[S] + node[root[S]].val, root[S]));
        while (k--) {
            if (q.empty()) return inf;
            pli now = q.top(); q.pop();
            if (k == 1) return now.first;
            int x = node[now.second].next, u = node[now.second].son[0], v = node[now
                .second].son[1];
            if (root[x]) q.push(make_pair(now.first + node[root[x]].val, root[x]));
            if (u) q.push(make_pair(now.first - node[now.second].val + node[u].val,
                u));
            if (v) q.push(make_pair(now.first - node[now.second].val + node[v].val,
                v));
        }
        return 0;
    }
};
```

5.4 Tarjan

割点的判断：一个顶点 u 是割点，当且仅当满足 (1) 或 (2)：
(1) u 为树根，且 u 有多于一个子树（即：存在一个儿子 v 使得 $dfn[u]+1 \neq dfn[v]$ ）
(2) u 不为树根，且满足存在 (u,v) 为树枝边（u 为 v 的父亲），使得 $dfn[u] \leq low[v]$
桥的判断：一条无向边 (u,v) 是桥，当且仅当 (u,v) 为树枝边，满足 $dfn[u] < low[v]$

```
struct EDGE { int adj, next; } edge[M];
int n, m, top, gh[N];
int dfn[N], low[N], cnt, ind, stop, instack[N], stack[N], belong[N];
void addedge(int x, int y) {
    edge[++top].adj = y;
    edge[top].next = gh[x];
    gh[x] = top;
}
void tarjan(int x) {
    dfn[x] = low[x] = ++ind;
    instack[x] = 1; stack[++stop] = x;
    for (int p=gh[x]; p; p=edge[p].next)
        if (!dfn[edge[p].adj]) {
```

```
            tarjan(edge[p].adj);
            low[x] = min(low[x], low[edge[p].adj]);
        } else if (instack[edge[p].adj]) {
            low[x] = min(low[x], dfn[edge[p].adj]);
        }
    if (dfn[x] == low[x]) {
        ++cnt; int tmp=0;
        while (tmp!=x) {
            tmp = stack[stop--];
            belong[tmp] = cnt;
            instack[tmp] = 0;
        }
    }
}
```

5.5 2-SAT

```
#define N number_of_vertex
#define M number_of_edges

struct MergePoint {
    struct EDGE {
        int adj, next;
    } edge[M];
    int ex[M], ey[M];
    bool instack[N];
    int gh[N], top, dfn[N], low[N], cnt, ind, stop, stack[N], belong[N];
    void init() {
        cnt = ind = stop = top = 0;
        memset(dfn, 0, sizeof(dfn));
        memset(instack, 0, sizeof(instack));
        memset(gh, 0, sizeof(gh));
    }
    void addedge(int x, int y) { //reverse
        std::swap(x, y);
        edge[++top].adj = y;
        edge[top].next = gh[x];
        gh[x] = top;
        ex[top] = x;
        ey[top] = y;
    }
    void tarjan(int x) {
        dfn[x] = low[x] = ++ind;
        instack[x] = 1; stack[++stop] = x;
        for (int p=gh[x]; p; p=edge[p].next)
            if (!dfn[edge[p].adj]) {
                tarjan(edge[p].adj);
                low[x] = std::min(low[x], low[edge[p].adj]);
            } else if (instack[edge[p].adj]) {
```

```
                low[x] = std::min(low[x], dfn[edge[p].adj]);
            }
        if (dfn[x] == low[x]) {
            ++cnt; int tmp = 0;
            while (tmp!=x) {
                tmp = stack[stop--];
                belong[tmp] = cnt;
                instack[tmp] = 0;
            }
        }
    }
    void work() {
        for (int i = (__first__); i <= (__last__); ++i)
            if (!dfn[i])
                tarjan(i);
    }
} merge;

struct Topsort {
    struct EDGE {
        int adj, next;
    } edge[M];
    int n, top, gh[N], ops[N], deg[N], ans[N];
    std::queue<int> q;
    void init() {
        n = merge.cnt; top = 0;
        memset(gh, 0, sizeof(gh));
        memset(deg, 0, sizeof(deg));
    }
    void addedge(int x, int y) {
        if (x == y) return;
        edge[++top].adj = y;
        edge[top].next = gh[x];
        gh[x] = top;
        ++deg[y];
    }
    void work() {
        for (int i = 1; i <= n; ++i)
            if (!deg[i])
                q.push(i);
        while (!q.empty()) {
            int x = q.front();
            q.pop();
            for (int p = gh[x]; p; p = edge[p].next)
                if (!--deg[edge[p].adj])
                    q.push(edge[p].adj);
            if (ans[x]) continue;
            ans[x] = -1; //not selected
            ans[ops[x]] = 1; //selected
        }
```

```
    }
} ts;
```

调用示例:

```
    merge.init();
    merge.addedge();
    merge.work();
    for (int i = 1; i <= n; ++i) {
        if (merge.belong[U(i, 0)] == merge.belong[U(i, 1)]) {
            puts("NO");
            return 0;
        }
        ts.ops[merge.belong[U(i, 0)]] = merge.belong[U(i, 1)];
        ts.ops[merge.belong[U(i, 1)]] = merge.belong[U(i, 0)];
    }
    ts.init();
    ts.work();
    puts("YES");
    for (int i = 1; i <= n; ++i) {
        int x = U(i, 0), y = U(i, 1);
        x = merge.belong[x], y = merge.belong[y];
        x = ts.ans[x], y = ts.ans[y];
        if (x == 1) puts("0␣is␣selected");
        if (y == 1) puts("1␣is␣selected");
    }
```

5.6 统治者树（Dominator Tree）

Dominator Tree 可以解决判断一类有向图必经点的问题。

idom[x] 表示离 x 最近的必经点（重编号后）。将 idom[x] 作为 x 的父亲，构成一棵 Dominator Tree

接口:

void dominator::init(int n); 初始化，有向图节点数为 n

void dominator::addedge(int u, int v); 添加一条有向边 (u, v)

void dominator::work(int root); 以 root 为根，建立一棵 Dominator Tree

结果的返回:

在执行 dominator::work(int root); 后，树边保存在 vector <int> tree[N] 中

```
namespace dominator {
    vector <int> g[N], rg[N], bucket[N], tree[N];
    int n, ind, idom[N], sdom[N], dfn[N], dsu[N], father[N], label[N], rev[N];
    void dfs(int x) {
        ++ind;
        dfn[x] = ind; rev[ind] = x;
        label[ind] = dsu[ind] = sdom[ind] = ind;
        for (auto p : g[x]) {
            if (!dfn[p]) dfs(p), father[dfn[p]] = dfn[x];
            rg[dfn[p]].push_back(dfn[x]);
```

```
        }
    }
    void init(int n1) {
        n = n1; ind = 0;
        for (int i = 1; i <= n; ++i) {
            g[i].clear();
            rg[i].clear();
            bucket[i].clear();
            tree[i].clear();
            dfn[i] = 0;
        }
    }
    void addedge(int u, int v) {
        g[u].push_back(v);
    }
    int find(int x, int step=0) {
        if (dsu[x] == x) return step ? -1 : x;
        int y = find(dsu[x], 1);
        if (y < 0) return x;
        if (sdom[label[dsu[x]]] < sdom[label[x]])
            label[x] = label[dsu[x]];
        dsu[x] = y;
        return step ? dsu[x] : label[x];
    }
    void work(int root) {
        dfs(root); n = ind;
        for (int i = n; i; --i) {
            for (auto p : rg[i])
                sdom[i] = min(sdom[i], sdom[find(p)]);
            if (i > 1) bucket[sdom[i]].push_back(i);
            for (auto p : bucket[i]) {
                int u = find(p);
                if (sdom[p] == sdom[u]) idom[p] = sdom[p];
                else idom[p] = u;
            }
            if (i > 1) dsu[i] = father[i];
        }
        for (int i = 2; i <= n; ++i) {
            if (idom[i] != sdom[i])
                idom[i] = idom[idom[i]];
            tree[rev[i]].push_back(rev[idom[i]]);
            tree[rev[idom[i]]].push_back(rev[i]);
        }
    }
};
```

5.7 网络流

5.7.1 最大流

注意：*top 要初始化为 1*

```
struct EDGE { int adj, w, next; } edge[M];
int n, top, gh[N], nrl[N];
void addedge(int x, int y, int w) {
    edge[++top].adj = y;
    edge[top].w = w;
    edge[top].next = gh[x];
    gh[x] = top;
    edge[++top].adj = x;
    edge[top].w = 0;
    edge[top].next = gh[y];
    gh[y] = top;
}
int dist[N], q[N];
int bfs() {
    memset(dist, 0, sizeof(dist));
    q[1] = S; int head = 0, tail = 1; dist[S] = 1;
    while (head != tail) {
        int x = q[++head];
        for (int p=gh[x]; p; p=edge[p].next)
            if (edge[p].w && !dist[edge[p].adj]) {
                dist[edge[p].adj] = dist[x] + 1;
                q[++tail] = edge[p].adj;
            }
    }
    return dist[T];
}
int dinic(int x, int delta) {
    if (x==T) return delta;
    for (int& p=nrl[x]; p && delta; p=edge[p].next)
        if (edge[p].w && dist[x]+1 == dist[edge[p].adj]) {
            int dd = dinic(edge[p].adj, min(delta, edge[p].w));
            if (!dd) continue;
            edge[p].w -= dd;
            edge[p^1].w += dd;
            return dd;
        }
    return 0;
}
int work() {
    int ans = 0;
    while (bfs()) {
        memcpy(nrl, gh, sizeof(gh));
        int t; while (t = dinic(S, inf)) ans += t;
    }
    return ans;
```

```
}
```

5.7.2 上下界有源汇网络流

T 向 S 连容量为正无穷的边，将有源汇转化为无源汇。

每条边容量减去下界，设 $in[i]$ 表示流入 i 的下界之和减去流出 i 的下界之和。

新建超级源汇 SS,TT ，对于 $in[i]>0$ 的点，SS 向 i 连容量为 $in[i]$ 的边。对于 $in[i]<0$ 的点，i 向 TT 连容量为 $-in[i]$ 的边。

求出以 SS,TT 为源汇的最大流，如果等于 $\Sigma in[i](in[i]>0)$ ，则存在可行流。再求出 S,T 为源汇的最大流即为最大流。

费用流：建完图后等价于求以 SS,TT 为源汇的费用流。

5.7.3 上下界无源汇网络流

1. 怎样求无源汇有上下界网络的可行流？

由于有源汇的网络我们先要转化成无源汇，所以本来就无源汇的网络不用再作特殊处理。

2. 怎样求无源汇有上下界网络的最大流、最小流？

一种简易的方法是采用二分的思想，不断通过可行流的存在与否对 (t,s) 边的上下界 U,L 进行调整。求最大流时令 $U=\infty$ 并二分 L ；求最小流时令 $L=0$ 并二分 U 。道理很简单，因为可行流的取值范围是一段连续的区间，我们只要通过二分找到有解和无解的分界线即可。

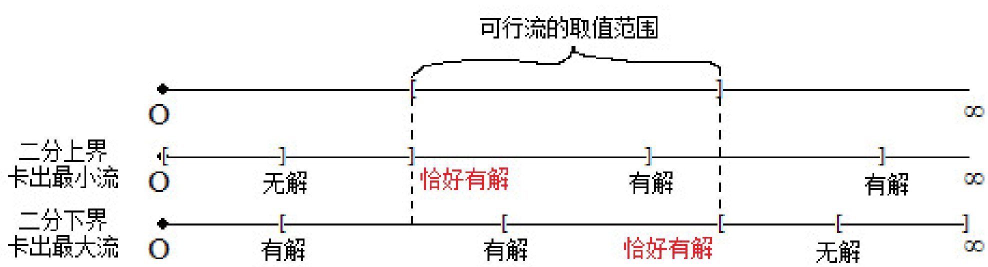

图 4: 可行流取值范围的讨论

5.7.4 费用流

注意：*top 要初始化为 1*

```
#define inf 0x3f3f3f3f
struct NetWorkFlow {
    struct EDGE {
        int adj, w, cost, next;
    } edge[M*2];
    int gh[N], q[N], dist[N], v[N], pre[N], prev[N], top;
    int S, T;
    void addedge(int x, int y, int w, int cost) {
        edge[++top].adj = y;
        edge[top].w = w;
```

```
        edge[top].cost = cost;
        edge[top].next = gh[x];
        gh[x] = top;
        edge[++top].adj = x;
        edge[top].w = 0;
        edge[top].cost = -cost;
        edge[top].next = gh[y];
        gh[y] = top;
    }
    void clear() {
        top = 1;
        memset(gh, 0, sizeof(gh));
    }
    int spfa() {
        memset(dist, 63, sizeof(dist));
        memset(v, 0, sizeof(v));
        int head = 0, tail = 1;
        q[1] = S; v[S] = 1; dist[S] = 0;
        while (head != tail) {
            (head += 1) %= N;
            int x = q[head];
            v[x] = 0;
            for (int p=gh[x]; p; p=edge[p].next)
                if (edge[p].w && dist[x] + edge[p].cost < dist[edge[p].adj]) {
                    dist[edge[p].adj] = dist[x] + edge[p].cost;
                    pre[edge[p].adj] = x;
                    prev[edge[p].adj] = p;
                    if (!v[edge[p].adj]) {
                        v[edge[p].adj] = 1;
                        (tail += 1) %= N;
                        q[tail] = edge[p].adj;
                    }
                }
        }
        return dist[T] != inf;
    }
    int work() {
        int ans = 0;
        while (spfa()) {
            int mx = inf;
            for (int x=T;x!=S;x=pre[x])
                mx = min(edge[prev[x]].w, mx);
            ans += dist[T] * mx;
            for (int x=T;x!=S;x=pre[x]) {
                edge[prev[x]].w -= mx;
                edge[prev[x]^1].w += mx;
            }
        }
        return ans;
    }
```

```
} nwf;
```

5.7.5 zkw 费用流

注意：*top* 要初始化为 *1*，不得用于有负权的图

```
#define inf 0x3f3f3f3f //modify if you use long long or double
template <class _tp>
struct NetWorkFlow {
    struct EDGE {
        int adj, next;
        _tp w, cost;
    } edge[M*2];
    int gh[N], top;
    int S, T;
    void addedge(int x, int y, _tp w, _tp cost) {
        edge[++top].adj = y;
        edge[top].w = w;
        edge[top].cost = cost;
        edge[top].next = gh[x];
        gh[x] = top;
        edge[++top].adj = x;
        edge[top].w = 0;
        edge[top].cost = -cost;
        edge[top].next = gh[y];
        gh[y] = top;
    }
    void clear() {
        top = 1;
        memset(gh, 0, sizeof(gh));
    }
    int v[N];
    _tp cost, d[N], slk[N];
    _tp aug(int x, _tp f) {
        _tp left = f;
        if (x == T) {
            cost += f * d[S];
            return f;
        }
        v[x] = true;
        for (int p=gh[x]; p; p=edge[p].next)
            if (edge[p].w && !v[edge[p].adj]) {
                _tp t = d[edge[p].adj] + edge[p].cost - d[x];
                if (t == 0) {
                    _tp delt = aug(edge[p].adj, min(left, edge[p].w));
                    if (delt > 0) {
                        edge[p].w -= delt;
                        edge[p^1].w += delt;
                        left -= delt;
                    }
```

```
                    if (left == 0) return f;
                } else {
                if (t < slk[edge[p].adj])
                    slk[edge[p].adj] = t;
                }
            }
        return f-left;
    }
    bool modlabel() {
        _tp delt = inf;
        for (int i=1;i<=T;i++)
            if (!v[i]) {
                if (slk[i] < delt) delt = slk[i];
                slk[i] = inf;
            }
        if (delt == inf) return true;
        for (int i=1;i<=T;i++)
            if (v[i]) d[i] += delt;
        return false;
    }
    _tp work() {
        cost = 0;
        memset(d, 0, sizeof(d));
        memset(slk, 63, sizeof(slk));
        do {
            do {
                memset(v, 0, sizeof(v));
            } while (aug(S, inf));
        } while (!modlabel());
        return cost;
    }
};
NetWorkFlow<int> nwf;
```

6 数学

6.1 扩展欧几里得解同余方程

ans[] 保存的是循环节内所有的解

```
int exgcd(int a,int b,int&x,int&y){
    if(!b)return x=1,y=0,a;
    int d=exgcd(b,a%b,x,y),t=x;
    return x=y,y=t-a/b*y,d;
}
void cal(ll a,ll b,ll n){//ax=b(mod n)
    ll x,y,d=exgcd(a,n,x,y);
    if(b%d)return;
    x=(x%n+n)%n;
```

```
    ans[cnt=1]=x*(b/d)%(n/d);
    for(ll i=1;i<d;i++)ans[++cnt]=(ans[1]+i*n/d)%n;
}
```

6.1.1 扩展欧几里得特殊解和解的个数

求满足 $\begin{cases} ax+by=c(a\geq 0,b\geq 0,c\geq 0) \\ x_1\leq x\leq x_2 \\ y_1\leq y\leq y_2 \end{cases}$ 的二元组 (x,y) 的个数。

```
int calc(int a, int b, int c, int x1, int x2, int y1, int y2) {
    if (a == 0 && b == 0) return c == 0 && x1 <= 0 && 0 <= x2 && y1 <= 0 && 0 <= y2;
    if (a == 0) return c % b == 0 && y1 <= c / b && c / b <= y2;
    if (b == 0) return c % a == 0 && x1 <= c / a && c / a <= x2;
    int x, y, t;
    int g = exgcd(a, b, x, y);
    if (c % g) return 0;
    x *= c / g; y *= c / g;
    int dx = b / g, dy = a / g;

    if (x > x1) t = (x - x1) / dx + 1, x = x - t * dx, y = y + t * dy;
    t = (x1 - x) / dx; if ((x1 - x) % dx) ++ t;
    x = x + t * dx, y = y - t * dy;
    x1 = max(x1, x), y2 = min(y2, y);

    if (x < x2) t = (x2 - x) / dx + 1, x = x + t * dx, y = y - t * dy;
    t = (x - x2) / dx; if ((x - x2) % dx) ++ t;
    x = x - t * dx, y = y + t * dy;
    x2 = min(x2, x), y1 = max(y1, y);

    if (y > y1) t = (y - y1) / dy + 1, x = x + t * dx, y = y - t * dy;
    t = (y1 - y) / dy; if ((y1 - y) % dy) ++ t;
    x = x - t * dx, y = y + t * dy;
    x2 = min(x2, x), y1 = max(y1, y);

    if (y < y2) t = (y2 - y) / dy + 1, x = x - t * dx, y = y + t * dy;
    t = (y - y2) / dy; if ((y - y2) % dy) ++ t;
    x = x + t * dx, y = y - t * dy;
    x1 = max(x1, x), y2 = min(y2, y);

    if (x1 > x2 && y1 > y2) return 0;
    assert(x2 - x1 == y2 - y1);
    return x2 - x1 + 1;
}
```

6.2 同余方程组

```
int n,flag,k,m,a,r,d,x,y;
```

```
int main(){
    scanf("%d",&n);
    flag=k=1,m=0;
    while(n--){
        scanf("%d%d",&a,&r);//ans%a=r
        if(flag){
            d=exgcd(k,a,x,y);
            if((r-m)%d){flag=0;continue;}
            x=(x*((r-m)/d)+a/d)%(a/d),y=k/d*a,m=((x*k+m)%y)%y;
            if(m<0)m+=y;
            k=y;
        }
    }
    printf("%d",flag?m:-1);//若flag=1,说明有解,解为ki+m,i为任意整数
}
```

6.3 类欧几里得算法

类欧几里得模板有三种形式:

$$f(a,b,c,n)=\sum_{i=0}^{n}\lfloor\frac{ai+b}{c}\rfloor$$

$$g(a,b,c,n)=\sum_{i=0}^{n}i\lfloor\frac{ai+b}{c}\rfloor$$

$$h(a,b,c,n)=\sum_{i=0}^{n}\lfloor\frac{ai+b}{c}\rfloor^2$$

```
#define LL long long

const int P = 1000000007;
int Inv(int x) {
    return x == 1 ? 1 : 1ll * (P - P / x) * Inv(P % x) % P;
}
const int i2 = Inv(2);
const int i6 = Inv(6);

struct ifo {
    int f, g, h;
    ifo(int f, int g, int h) : f(f), g(g), h(h) {}
};

int S1(int n) {
    return 1ll * n * (n + 1) % P * i2 % P;
}

int S2(int n) {
    return 1ll * n * (n + 1) % P * (2 * n + 1) % P * i6 % P;
```

```
}

ifo Get(int n, int A, int B, int C) {
    if (!A) {
        int t = B / C;
        int f = 1ll * (n + 1) * t % P;
        int g = 1ll * S1(n) * t % P;
        int h = 1ll * (n + 1) * t % P * t % P;
        return ifo(f, g, h);
    } else if (A >= C || B >= C) {
        ifo Nx = Get(n, A % C, B % C, C);
        int p = A / C, q = B / C;
        int f = (1ll * p * S1(n) + 1ll * q * (n + 1) + Nx.f) % P;
        int g = (1ll * p * S2(n) + 1ll * q * S1(n) + Nx.g) % P;
        int h = (1ll * p * p % P * S2(n) + 2ll * p * q % P * S1(n) + 1ll * (n + 1) *
             q % P * q + 2ll * p * Nx.g % P + 2ll * q * Nx.f % P + Nx.h) % P;
        return ifo(f, g, h);
    } else {
        int m = (1ll * A * n + B) / C;
        ifo Nx = Get(m - 1, C, C - B - 1, A);
        int f = (1ll * n * m - Nx.f) % P;
        int g = (1ll * m * S1(n) - 1ll * i2 * Nx.h - 1ll * i2 * Nx.f) % P;
        int h = (2ll * n * S1(m - 1) % P + 1ll * n * m - 2ll * Nx.g - Nx.f) % P;
        return ifo(f, g, h);
    }
}
```

6.4 卡特兰数

$h_1 = 1, h_n = \frac{h_{n-1}(4n-2)}{n+1} = \frac{C(2n,n)}{n+1} = C(2n,n) - C(2n,n-1)$

在一个格点阵列中，从 $(0,0)$ 点走到 (n,m) 点且不经过对角线 $x=y$ 的方案数 $(x>y)$：

$C(n+m-1,m) - C(n+m-1,m-1)$

在一个格点阵列中，从 $(0,0)$ 点走到 (n,m) 点且不穿过对角线 $x=y$ 的方案数 $(x \geq y)$：

$C(n+m,m) - C(n+m,m-1)$

6.5 斯特林数

6.5.1 第一类斯特林数

第一类 Stirling 数 $S(p,k)$ 的一个组合学解释是：将 p 个物体排成 k 个非空循环排列的方法数。

$S(p,k)$ 的递推公式：$S(p,k) = (p-1)S(p-1,k) + S(p-1,k-1), 1 \leq k \leq p-1$

边界条件：$S(p,0) = 0, p \geq 1$ $S(p,p) = 1, p \geq 0$

6.5.2 第二类斯特林数

第二类 Stirling 数 $S(p,k)$ 的一个组合学解释是：将 p 个物体划分成 k 个非空的不可辨别（可以理解为盒子没有编号）集合的方法数。

$S(p,k)$ 的递推公式：$S(p,k) = kS(p-1,k) + S(p-1,k-1), 1 \leq k \leq p-1$

边界条件：$S(p,0)=0, p\geq 1$ $S(p,p)=1, p\geq 0$
也有卷积形式：

$$S(n,m)=\frac{1}{m!}\sum_{k=0}^{m}(-1)^k C(m,k)(m-k)^n=\sum_{k=0}^{m}\frac{(-1)^k(m-k)^n}{k!(m-k)!}=\sum_{k=0}^{m}\frac{(-1)^k}{k!}\times\frac{(m-k)^n}{(m-k)!}$$

6.6 错排公式

$$D_1=0, D_2=1, D_n=(n-1)(D_{n-2}+D_{n-1})$$

6.7 Lucas 定理

接口：
初始化：void lucas::init();
计算 $C(n,m)\%mod$ 的值：LL lucas::Lucas(LL n, LL m);

```
#define mod 110119
#define LL long long
namespace lucas {
    LL fac[mod+1], facv[mod+1];
    LL power(LL base, LL times) {
        LL ans = 1;
        while (times) {
            if (times&1) (ans *= base) %= mod;
            (base *= base) %= mod;
            times >>= 1;
        }
        return ans;
    }
    void init() {
        fac[0] = 1; for (int i=1;i<mod;i++) fac[i] = (fac[i-1] * i) % mod;
        facv[mod-1] = power(fac[mod-1], mod-2);
        for (int i=mod-2;i>=0;--i) facv[i] = (facv[i+1] * (i+1)) % mod;
    }
    LL C(unsigned LL n, unsigned LL m) {
        if (n < m) return 0;
        return (fac[n] * facv[m] % mod * facv[n-m] % mod) % mod;
    }
    LL Lucas(unsigned LL n, unsigned LL m)
    {
        if (m == 0) return 1;
        return (C(n%mod, m%mod) * Lucas(n/mod, m/mod)) %mod;
    }
};
```

6.8 线性规划

6.8.1 单纯形法

单纯形法用于解决线性规划问题：

$$\max_{x_1,x_2,\cdots,x_n} x_0 = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

$$s.t. \begin{cases} A_{i1}x_1 + A_{i2}x_2 + \cdots + A_{in}x_n \le b_i, & i = 1,2,\cdots,m \\ x_j \ge 0, & j = 1,2,\cdots,n \end{cases}$$

小心：单纯形法通常能解决 $n \le 500, m \le 500$ 的数据规模的问题。若规模过大，可能导致精度爆炸。

小心：单纯形法只能解决一般线性规划问题，不能解决整数规划问题（NP Hard）。若要用单纯形法解决整数规划问题，必须先证明一般线性规划的解不比整数规划好。

若 $b_i \ge 0, i = 1,2,\cdots,n$ ，则不需要执行 init ，因为至少有一组解 $x_1 = x_2 = \cdots = x_n = 0$ 。

输入格式 .

$$\begin{array}{llllll} n & m & & & & \\ c_1 & c_2 & c_3 & \cdots & c_n & \\ A_{11} & A_{12} & A_{13} & \cdots & A_{1n} & b_1 \\ A_{21} & A_{22} & A_{23} & \cdots & A_{2n} & b_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ A_{m1} & A_{m2} & A_{m3} & \cdots & A_{mn} & b_m \end{array}$$

输出格式 .

若无解，输出 Infeasible 。

若 x_0 无界，输出 Unbounded 。

第一行输出答案 x_0 。

接下来一行输出 n 个实数表示 $x_1, x_2, \cdots, x_n$ 。

```
#include <bits/stdc++.h>

#define N 25
#define M 25

using namespace std;

const double eps = 1e-8, INF = 1e15;

int n, m;
double a[M][N], ans[N + M];
int id[N + M];

void pivot(int l, int e) {
    swap(id[n + l], id[e]);
```

```cpp
    double t = a[l][e];
    a[l][e] = 1;
    for (int j = 0; j <= n; ++j) a[l][j] /= t;
    for (int i = 0; i <= m; ++i)
        if (i != l && abs(a[i][e]) > eps) {
            t = a[i][e];
            a[i][e] = 0;
            for (int j = 0; j <= n; ++j) a[i][j] -= a[l][j] * t;
        }
}

bool init() {
    while (1) {
        int e = 0, l = 0;
        for (int i = 1; i <= m; ++i)
            if (a[i][0] < -eps && (!l || (rand() & 1)))
                l = i;
        if (!l) break;
        for (int j = 1; j <= n; ++j)
            if (a[l][j] < -eps && (!e || (rand() & 1)))
                e = j;
        if (!e) return false; // Infeasible
        pivot(l, e);
    }
    return true;
}

bool simplex() {
    while (1) {
        int l = 0, e = 0;
        double mn = INF;
        for (int j = 1; j <= n; ++j)
            if (a[0][j] > eps) {
                e = j;
                break;
            }
        if (!e) break;
        for (int i = 1; i <= m; ++i)
            if (a[i][e] > eps && a[i][0] / a[i][e] < mn) {
                mn = a[i][0] / a[i][e];
                l = i;
            }
        if (!l) return false; // Unbounded
        pivot(l, e);
    }
    return true;
}

int main() {
    scanf("%d%d", &n, &m);
```

```
    for (int i = 1; i <= n; ++i) scanf("%lf", &a[0][i]);
    for (int i = 1; i <= m; ++i) {
        for (int j = 1; j <= n; ++j) scanf("%lf", &a[i][j]);
        scanf("%lf", &a[i][0]);
    }
    for (int i = 0; i <= n + m; ++i) id[i] = 0;
    for (int i = 1; i <= n; ++i) id[i] = i;
    if (!init()) {
        puts("Infeasible");
        return 0;
    }
    if (!simplex()) {
        puts("Unbounded");
        return 0;
    }
    printf("%.10lf\n", -a[0][0]);
    for (int i = 0; i <= n + m; ++i) ans[i] = 0;
    for (int i = 1; i <= m; ++i) ans[id[n + i]] = a[i][0];
    for (int i = 1; i <= n; ++i) printf("%.10lf ", ans[i]);
    puts("");
}
```

6.8.2 对偶理论

原始问题:

$$\max_{x_1,x_2,\cdots,x_n} x_0 = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

$$s.t. \begin{cases} A_{i1}x_1 + A_{i2}x_2 + \cdots + A_{in}x_n \leq b_i, & i = 1,2,\cdots,m \\ x_j \geq 0, & j = 1,2,\cdots,n \end{cases}$$

对偶问题:

$$\min_{w_1,w_2,\cdots,w_m} w_0 = b_1x_1 + b_2x_2 + \cdots + b_mx_m$$

$$s.t. \begin{cases} A_{i1}^Tw_1 + A_{i2}^Tw_2 + \cdots A_{im}^Tw_m \geq c_i, & i = 1,2,\cdots,n \\ w_j \geq 0, & j = 1,2,\cdots,m \end{cases}$$

6.9 高斯消元

6.9.1 行列式

```
int ans = 1;
for (int i=0;i<n;i++) {
    for (int j=i;j<n;j++)
        if (g[j][i]) {
            for (int k=i;k<n;k++)
                swap(g[i][k], g[j][k]);
```

```
            if (j != i) ans *= -1;
            break;
        }
    if (g[i][i] == 0) {
        ans = 0;
        break;
    }
    for (int j=i+1;j<n;j++) {
        while (g[j][i]) {
            int t = g[i][i] / g[j][i];
            for (int k=i;k<n;k++)
                g[i][k] = (g[i][k] + mod - ((LL)t * g[j][k] % mod)) % mod;
            for (int k=i;k<n;k++)
                swap(g[i][k], g[j][k]);
            ans *= -1;
        }
    }
}
for (int i=0;i<n;i++)
    ans = ((LL)ans * g[i][i]) % mod;
ans = (ans % mod + mod) % mod;
printf("%d\n", ans);
```

6.9.2 Matrix-Tree 定理

对于一张图，建立矩阵 C ，$C[i][i] = \text{i}$ 的度数，若 i, j 之间有边，那么 $C[i][j] = -1$ ，否则为 0 。这张图的生成树个数等于矩阵 C 的 $n-1$ 阶行列式的值。

6.10 调和级数

$\sum\limits_{i=1}^{n} \frac{1}{i}$ 在 n 较大时约等于 $ln(n) + r$ ， r 为欧拉常数，约等于 0.5772156649015328 。

6.11 曼哈顿距离的变换

$|x_1 - x_2| + |y_1 - y_2| = max(|(x_1 + y_1) - (x_2 + y_2)|, |(x_1 - y_1) - (x_2 - y_2)|)$

6.12 数论函数变换

常见积性函数：

欧拉函数 $\phi(n)$ 为不超过 n 的与 n 互质的正整数个数

莫比乌斯函数 $\mu(n) = \begin{cases} 1, & \text{若} n = 1 \\ (-1)^k, & \text{若} n \text{无平方数因数，且} n = p_1 p_2 \cdots p_k \\ 0, & \text{若} n \text{有大于 1 的平方数因数} \end{cases}$

莫比乌斯函数的一次方前缀和见“杜教筛”。

莫比乌斯函数的二次方前缀和

$$\sum_{i=1}^{n} \mu(i)^2 = \sum_{d=1}^{\lfloor \sqrt{n} \rfloor} \mu(d) \lfloor \frac{n}{d^2} \rfloor$$

常见积性函数的性质:

$$n = \sum_{d|n} \phi(d)$$

$$\sum_{d|n} \mu(d) = \begin{cases} 1, & n = 1 \\ 0, & n > 1 \end{cases}$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} i \times j[gcd(i,j) = d] = \sum_{i=1}^{\lfloor \frac{n}{d} \rfloor} \sum_{j=1}^{\lfloor \frac{m}{d} \rfloor} id \times jd[gcd(i,j) = 1]$$

$$\phi(n) = \sum_{d|n} \mu(d) \frac{n}{d}$$

6.13 莫比乌斯反演

$F(n)$ 和 $f(n)$ 是定义在非负整数集合上的两个函数，则:

$$F(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} \mu(d) F(\frac{n}{d})$$

$$F(n) = \sum_{n|d} f(d) \Rightarrow f(n) = \sum_{n|d} \mu(\frac{d}{n}) F(d)$$

6.14 线性筛素数

```
mu[1]=phi[1]=1;top=0;
for (int i=2;i<N;i++) {
    if (!v[i]) prime[++top]=i, mu[i] = -1, phi[i] = i-1;
    for (int j=1;i*prime[j]<N && j<=top;j++) {
        v[i*prime[j]] = 1;
        if (i%prime[j]) {
            mu[i*prime[j]] = -mu[i];
            phi[i*prime[j]] = phi[i] * (prime[j]-1);
        } else {
            mu[i*prime[j]] = 0;
            phi[i*prime[j]] = phi[i] * prime[j];
            break;
        }
    }
}
```

6.15 杜教筛

getphi(t, x) 表示求 $\sum_{i=1}^{x} i^t \phi(i)$ 。

推导过程:

记 $S(n)=\sum_{i=1}^{n}f(i)$ ，取任意函数 g 有恒等式

$$S(n)=\sum_{i=1}^{n}(f\cdot g)(i)-\sum_{i=2}^{n}g(i)S(\lfloor\frac{n}{i}\rfloor)$$

其中，$(f\cdot g)$ 表示 f 和 g 的狄利克雷卷积：即：$(f\cdot g)(n)=\sum_{d|n}f(d)g(\frac{n}{d})$

关于恒等式的证明：

将 $\sum_{i=2}^{n}g(i)S(\lfloor\frac{n}{i}\rfloor)$ 移到左边去，即只需证

$$\sum_{i=1}^{n}(f\cdot g)(i)=\sum_{i=1}^{n}g(i)S(\lfloor\frac{n}{i}\rfloor)$$

将狄利克雷卷积展开，得：

$$\sum_{i=1}^{n}\sum_{d|i}g(d)f(\frac{i}{d})=\sum_{i=1}^{n}g(i)S(\lfloor\frac{n}{i}\rfloor)$$

即：

$$\sum_{d=1}^{n}g(d)\sum_{i=1}^{\lfloor\frac{n}{d}\rfloor}f(i)=\sum_{i=1}^{n}g(i)S(\lfloor\frac{n}{i}\rfloor)$$

显然相等，恒等式证完。

取 $f(i)=i^p\phi(i),g(i)=i^p$ ，则有：

$$S(n)=\sum_{i=1}^{n}i^p\phi(i)=\sum_{i=1}^{n}i^{p+1}-\sum_{i=2}^{n}i^pS(\lfloor\frac{n}{i}\rfloor)$$

其中有用到等式 $\sum_{d|n}\phi(d)=n$

另外，莫比乌斯函数 $\mu(n)$ 也可以使用杜教筛求前缀和，记 $S'(n)=\sum_{i=1}^{n}\mu(i)$ ，则 $S'(n)=1-\sum_{i=2}^{n}S'(\lfloor\frac{n}{i}\rfloor)$

```
#include <bits/stdc++.h>

#define N 5000020
#define LL long long
#define mod 1000000007
#define div2 ((mod+1)/2)
#define div6 ((mod+1)/6)

using namespace std;

int n, prime[N], v[N];
LL phi[3][N];

map<int, int> mp[3];

```

```
int sum(int t, int x) { //calculate 1^t + 2^t + ... + x^t
    if (t == 0) return x;
    if (t == 1) return 1ll * x * (x + 1) % mod * div2 % mod;
    if (t == 2) return 1ll * x * (x + 1) % mod * (2ll * x % mod + 1) % mod * div6 %
        mod;
    if (t == 3) return 1ll * x * x % mod * (x + 1) % mod * (x + 1) % mod * div2 %
        mod * div2 % mod;
}

int getphi(int t, int x) {
    if (x < N) return phi[t][x];
    if (mp[t].find(x) != mp[t].end()) return mp[t][x];
    LL ans = 0; int r = 0;
    for (int l = 2; l <= x; l = r + 1) {
        r = x / (x / l);
        ans += 1ll * getphi(t, x / l) * (((LL)sum(t, r) - sum(t, l - 1) + mod) % mod
            ) % mod;
        ans %= mod;
    }
    ans = (LL)sum(t + 1, x) - ans + mod;
    ans %= mod;
    mp[t][x] = ans;
    return (int)ans;
}

int main() {
    memset(v, 0, sizeof(v));
    int top = 0;
    phi[0][1] = 1, phi[1][1] = 1, phi[2][1] = 1;
    for (int i = 2; i < N; ++i) {
        if (!v[i]) prime[++top] = i, phi[0][i] = i - 1, phi[1][i] = 1ll * i * phi
            [0][i] % mod, phi[2][i] = 1ll * i * phi[1][i] % mod;
        for (int j = 1; j <= top && prime[j] * i < N; ++j) {
            v[i * prime[j]] = 1;
            if (i % prime[j] == 0) {
                phi[0][i * prime[j]] = phi[0][i] * prime[j];
                phi[1][i * prime[j]] = 1ll * phi[1][i] * prime[j] % mod * prime[j] %
                     mod;
                phi[2][i * prime[j]] = 1ll * phi[2][i] * prime[j] % mod * prime[j] %
                     mod * prime[j] % mod;
                break;
            } else {
                phi[0][i * prime[j]] = phi[0][i] * (prime[j] - 1);
                phi[1][i * prime[j]] = 1ll * phi[1][i] * (prime[j] - 1) % mod *
                    prime[j] % mod;
                phi[2][i * prime[j]] = 1ll * phi[2][i] * (prime[j] - 1) % mod *
                    prime[j] % mod * prime[j] % mod;
            }
        }
    }
```

```
    for (int i = 2; i < N; ++i) {
        phi[0][i] = (phi[0][i] + phi[0][i - 1]) % mod;
        phi[1][i] = (phi[1][i] + phi[1][i - 1]) % mod;
        phi[2][i] = (phi[2][i] + phi[2][i - 1]) % mod;
    }
}
```

6.16 洲阁筛

一种在 $O(\frac{n^{\frac{3}{4}}}{\log n})$ 的时间中求出大多数积性函数的前缀和的方法。

原文链接：http://debug18.com/posts/calculate-the-sum-of-multiplicative-function/

引入

求

$$\sum_{i=1}^{n} F(i)$$

其中 $F(x)$ 是一个积性函数，满足当 p 为质数的时候，$F(p^c)$ 是一个关于 p 的低阶多项式。

转化

我们将 $[1,n]$ 的所有数按照是否有 $> \sqrt{n}$ 的质因子分为两类, 那么显然有

$$\begin{aligned}\sum_{i=1}^{n} F(i) &= \sum_{\substack{1\le i\le n \\ i \text{ have no prime factors } >\sqrt{n}}} F(i)\left(1+\sum_{\substack{\sqrt{n}<j\le\lfloor\frac{n}{i}\rfloor \\ j \text{ is prime}}} F(j)\right) \\ &= \sum_{\substack{1\le i<\sqrt{n} \\ i \text{ have no prime factors } >\sqrt{n}}} F(i)\left(1+\sum_{\substack{\sqrt{n}<j\le\lfloor\frac{n}{i}\rfloor \\ j \text{ is prime}}} F(j)\right) + \sum_{\substack{\sqrt{n}\le i\le n \\ i \text{ have no prime factors } >\sqrt{n}}} F(i)\end{aligned}$$

现在需要计算两个东西：

1. 对于每个 $1 \le i < \sqrt{n}$ ，计算

$$\sum_{\substack{\sqrt{n}<j\le\lfloor\frac{n}{i}\rfloor \\ j \text{ is prime}}} F(j)$$

2.

$$\sum_{\substack{\sqrt{n}\le i\le n \\ i \text{ have no prime factors } >\sqrt{n}}} F(i)$$

Part 1 (calcG)

设 $g_k(i,j)$ 表示 $[1,j]$ 中与前 i 个质数互质的数的 k 次幂和. 显然有转移

$$g_k(i,j) = g_k(i-1,j) - p_i^k g_k(i-1,\lfloor\frac{j}{p_i}\rfloor)$$

观察到 j 的取值只有 $\sqrt{n}$ 种, 于是直接暴力计算的复杂度为 $O(\frac{n}{\log n})$.

如果 $p_i > \lfloor \frac{j}{p_i} \rfloor$ 即 $p_i^2 > j$ 时, $g_k(i,j)$ 的转移变为:

$$g_k(i,j) = g_k(i-1,j) - p_i^k$$

我们从小到大枚举 i, 对于某个 j 一旦 $p_{i_0}^2 > j$ 便可以不再转移, 之后如果其他的值需要使用到它在 i_1 时的值, 直接用 $g_k(i_0,j) - \sum_{l=i_0}^{i_1-1} p_l^k$ 即可.

此时的复杂度可以简单地用积分近似为 $O(\frac{n^{\frac{3}{4}}}{\log n})$.

Part 2 (calcF)

设 $f(i,j)$ 表示 $[1,j]$ 中仅由小于 $\sqrt{n}$ 的后 i 个质数组成的数的 $F(x)$ 之和. 此时当 $p_i > j$ 时, 一定有 $f(i,j) = 1$. 类似地, 当 $p_i^2 > j$ 时转移变为:

$$f(i,j) = f(i-1,j) + F(p_i)$$

所以可以从大到小枚举 i , 如果对于某个 j 有 $p_i^2 > j$, 可以不转移, 每次用的时候加入 $[p_i, \min(j, \sqrt{n})]$ 这一段的质数的 $F(p)$ 就可以了.

应用 1

求 $\sum\limits_{i=1}^{n} d(i^3)$, 其中 $d(x)$ 表示 x 的约数个数。$n \le 10^{11}$ 。

令 $F(p^c) = 3c+1$, 上洲阁筛。

sump[i] 表示小于等于 i 的质数之和

d3[i] 表示 $d(i^3)$

g0[i] 表示 $[1,i]$ 中与不超过 $\sqrt{n}$ 都互质的数的 0 次幂和。

g[i] 表示 $[1,\lfloor \frac{n}{i} \rfloor]$ 中与不超过 $\sqrt{n}$ 都互质的数的 0 次幂和。

```
LL N;
int pbnd;
int vbnd;
int l0[SIZE], l[SIZE];
LL g0[SIZE], g[SIZE], f0[SIZE], f[SIZE];

void calcG()
{
    for (int i = 1; i < vbnd; ++i) {
        g0[i] = i;
    }
    for (int i = vbnd - 1; i >= 1; --i) {
        g[i] = N / i;
    }
    for (int i = 0; i < pbnd; ++i) {
        int p = primes[i];
        for (int j = 1; j < vbnd && i < l[j]; ++j) {
            LL y = (N / j) / p;
            g[j] -= (y < vbnd ? g0[y] - std::max(0, i - l0[y]) : g[N / y] - std::max
                (0, i - l[N / y]));
        }
```

```
        for (int j = vbnd - 1; j >= 1 && i < l0[j]; --j) {
            LL y = j / p;
            g0[j] -= g0[y] - std::max(0, i - l0[y]);
        }
    }
    for (int i = 1; i < vbnd; ++i) {
        g[i] -= pbnd - l[i];
    }
}

void calcF()
{
    std::fill(f0 + 1, f0 + vbnd, 1);
    std::fill(f + 1, f + vbnd, 1);
    for (int i = pbnd - 1; i >= 0; --i) {
        int p = primes[i];
        for (int j = 1; j < vbnd && i < l[j]; ++j) {
            LL y = (N / j) / p;
            for (int z = 1; y; ++z, y /= p) {
                f[j] += (3 * z + 1) * (y < vbnd ?
                           f0[y] + 4 * std::max(0, sump[y] - std::max(l0[y], i + 1)) :
                           f[N / y] + 4 * (pbnd - std::max(l[N / y], i + 1)));
            }
        }
        for (int j = vbnd - 1; j >= 1 && i < l0[j]; --j) {
            int y = j / p;
            for (int z = 1; y; ++z, y /= p) {
                f0[j] += (3 * z + 1) * (f0[y] + 4 * std::max(0, sump[y] - std::max(l0[y], i + 1)));
            }
        }
    }
    for (int i = 1; i < vbnd; ++i) {
        f[i] += 4 * (pbnd - l[i]);
    }
}

LL calcSumS3()
{
    for (vbnd = 1; (LL)vbnd * vbnd <= N; ++vbnd) { }
    for (pbnd = 0; (LL)primes[pbnd] * primes[pbnd] <= N; ++pbnd) { }
    for (int i = 1; i < vbnd; ++i) {
        for (l0[i] = l0[i - 1]; (LL)primes[l0[i]] * primes[l0[i]] <= i; ++l0[i]) { }
    }
    l[vbnd] = 0;
    for (int i = vbnd - 1; i >= 1; --i) {
        LL x = N / i;
        for (l[i] = l[i + 1]; (LL)primes[l[i]] * primes[l[i]] <= x; ++l[i]) { }
    }
```

```
    calcG();
    calcF();

    LL ret = f[1];
    for (int i = 1; i < vbnd; ++i) {
        ret += d3[i] * 4 * (g[i] - 1);
        // 取 c = 1, 3c + 1 = 4. g[i] - 1 的原因是除去 F(1).
    }
    return ret;
}
```

应用 2 求素数的 k 次方前缀和

我们只需使用洲阁筛的 Part 1，计算出来的就是 $(\sqrt{n}, n]$ 中素书的 k 次幂和。

g0[i] 表示 $[1, i]$ 中与不超过 $\sqrt{n}$ 都互质的数的 0 次幂和。

G0[i] 表示 $[1, \lfloor \frac{n}{i} \rfloor]$ 中与不超过 $\sqrt{n}$ 都互质的数的 0 次幂和。

g1[i] 表示 $[1, i]$ 中与不超过 $\sqrt{n}$ 都互质的数的 1 次幂和。

G1[i] 表示 $[1, \lfloor \frac{n}{i} \rfloor]$ 中与不超过 $\sqrt{n}$ 都互质的数的 1 次幂和。

```
#include <bits/stdc++.h>

#define N 1000000 // sqrt(n)
#define mod 1000000007
#define LL long long

using namespace std;

int prime[N], sum1[N], v[N], l0[N], L0[N], pbnd, vbnd, top;
LL g0[N], G0[N];
int g1[N], G1[N];

void init() {
    top = 0;
    for (int i = 2; i < N; ++i) {
        if (!v[i]) prime[top++] = i;
        for (int j = 0; j < top && i * prime[j] < N; ++j) {
            v[i * prime[j]] = 1;
            if (i % prime[j] == 0) break;
        }
    }
    sum1[0] = 0;
    for (int i = 1; i <= top; ++i)
        sum1[i] = (sum1[i - 1] + prime[i - 1]) % mod;
}

int S1(long long x) {
    x %= mod;
    return x * (x + 1) / 2 % mod;
```

```
}

long long calc(long long n) {
    for (vbnd = 1; 1ll * vbnd * vbnd <= n; ++vbnd);
    for (pbnd = 0; pbnd < top && 1ll * prime[pbnd] * prime[pbnd] <= n; ++pbnd);
    l0[0] = 0;
    for (int i = 1; i < vbnd; ++i)
        for (l0[i] = l0[i - 1]; l0[i] < top && 1ll * prime[l0[i]] * prime[l0[i]] <= i; ++l0[i]);
    L0[vbnd] = 0;
    for (int i = vbnd - 1; i >= 1; --i) {
        LL x = n / i;
        for (L0[i] = L0[i + 1]; L0[i] < top && 1ll * prime[L0[i]] * prime[L0[i]] <= x; ++L0[i]);
    }
    for (int i = 1; i < vbnd; ++i) {
        g0[i] = i;
        g1[i] = S1(i);
    }
    for (int i = vbnd - 1; i >= 1; --i) {
        G0[i] = n / i;
        G1[i] = S1(n / i);
    }
    for (int i = 0; i < pbnd; ++i) {
        int p = prime[i];
        for (int j = 1; j < vbnd && i < L0[j]; ++j) {
            LL y = (n / j) / p;
            if (y < vbnd) {
                if (i - l0[y] < 0) {
                    G0[j] -= g0[y];
                    G1[j] -= 1ll * p * g1[y] % mod;
                    if (G1[j] < 0) G1[j] += mod;
                } else {
                    G0[j] -= g0[y] - (i - l0[y]);
                    G1[j] -= 1ll * p * (g1[y] % mod - (sum1[i] - sum1[l0[y]]) % mod) % mod;
                    G1[j] = ((G1[j] % mod) + mod) % mod;
                }
            } else {
                if (i - L0[n / y] < 0) {
                    G0[j] -= G0[n / y];
                    G1[j] -= 1ll * p * G1[n / y] % mod;
                    if (G1[j] < 0) G1[j] += mod;
                } else {
                    G0[j] -= G0[n / y] - (i - L0[n / y]);
                    G1[j] -= 1ll * p * (G1[n / y] % mod - (sum1[i] - sum1[L0[n / y]]) % mod) % mod;
                    G1[j] = ((G1[j] % mod) + mod) % mod;
                }
            }
```

```
        }
        for (int j = vbnd - 1; j >= 1 && i < l0[j]; --j) {
            LL y = j / p;
            if (i - l0[y] < 0) {
                g0[j] -= g0[y];
                g1[j] -= 1ll * p * g1[y] % mod;
                if (g1[j] < 0) g1[j] += mod;
            } else {
                g0[j] -= g0[y] - (i - l0[y]);
                g1[j] -= 1ll * p * (g1[y] % mod - (sum1[i] - sum1[l0[y]]) % mod) % mod;
                g1[j] = ((g1[j] % mod) + mod) % mod;
            }
        }
    }
    for (int i = 1; i < vbnd; ++i) {
        G0[i] -= pbnd - L0[i];
        G1[i] -= (sum1[pbnd] - sum1[L0[i]]) % mod;
        G1[i] = ((G1[i] % mod) + mod) % mod;
    }
    return G0[1] - 1 + pbnd; // 不超过 n 的素数个数
    return ((G1[1] + mod - 1) % mod + sum1[pbnd]) % mod; // 不超过 n 的素数和
    return ((g1[j] - 1 + sum1[l0[j]]) % mod + mod) % mod; // 不超过 j 的素数和
    return ((G1[n / j] - 1 + sum1[pbnd]) % mod + mod) % mod; // 不超过 n / j 的素数和
}

int main() {
    init();
    long long n;
    while (~scanf("%lld", &n)) {
        long long ans = calc(n);
        printf("%lld\n", ans);
    }
}

// hdu5901
```

6.17 FFT

6.17.1 普通 FFT

```
namespace FFT {
    const int maxn = 65537;
    const double pi = acos(-1.0);

    struct comp {
        double real , imag;
        comp() {}
```

```
        comp(double real , double imag): real(real) , imag(imag) {}
        friend inline comp operator+(const comp &a , const comp &b) {
            return comp(a.real + b.real , a.imag + b.imag);
        }
        friend inline comp operator-(const comp &a , const comp &b) {
            return comp(a.real - b.real , a.imag - b.imag);
        }
        friend inline comp operator*(const comp &a , const comp &b) {
            return comp(a.real * b.real - a.imag * b.imag , a.real * b.imag + a.imag
                 * b.real);
        }
    };

    comp A[maxn] , B[maxn];
    int rev[maxn], m, len;

    inline void init(int n) {
        for (m = 1, len = 0; m < n + n; m <<= 1 , len ++);
        for (int i = 0; i < m; ++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len -
             1));
        for (int i = 0; i < m; ++i) A[i] = B[i] = comp(0, 0);
    }

    inline void dft(comp *a , int v) {
        for (int i = 0; i < m; ++i) if (i < rev[i]) swap(a[i] , a[rev[i]]);
        for (int s = 2; s <= m; s <<= 1) {
            comp g(cos(2 * pi / s) , v * sin(2 * pi / s));
            for (int k = 0; k < m; k += s) {
                comp w(1 , 0);
                for (int j = 0; j < s / 2; ++j) {
                    comp &u = a[k + j + s / 2], &v = a[k + j];
                    comp t = w * u;
                    u = v - t;
                    v = v + t;
                    w = w * g;
                }
            }
        }
        if (v == -1)
            for (int i = 0; i < m; ++i) a[i].real /= m , a[i].imag /= m;
    }
}
```

6.17.2 模任意素数 FFT

注意：调用 *mulmod* 前先调用 *init* 。调用 *mulmod* 前请确保 a,b 数组足够大（比 $2n$ 大的 2 的整数次幂）且经过初始化。

```
namespace FFT {
    const long double pi = acos(-1.0);
```

```cpp

struct comp {
    long double real, imag;
    comp() {}
    comp(long double real, long double imag) : real(real), imag(imag) {}
    friend inline comp operator + (const comp &a, const comp &b) {
        return comp(a.real + b.real, a.imag + b.imag);
    }
    friend inline comp operator - (const comp &a, const comp &b) {
        return comp(a.real - b.real, a.imag - b.imag);
    }
    friend inline comp operator * (const comp &a, const comp &b) {
        return comp(a.real * b.real - a.imag * b.imag, a.real * b.imag + a.imag
            * b.real);
    }
    inline comp conj() {
        return comp(real, -imag);
    }
};

comp A[maxn], B[maxn];
int rev[maxn], m, len;

inline void init(int n) {
    for (m = 1, len = 0; m < n + n; m <<= 1, ++len);
    for (int i = 0; i < m; ++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len -
        1));
    for (int i = 0; i < m; ++i) A[i] = B[i] = comp(0, 0);
}

inline void dft(comp *a, int v) {
    for (int i = 0; i < m; ++i) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int s = 2; s <= m; s <<= 1) {
        comp g(cos(2 * pi / s), v * sin(2 * pi / s));
        for (int k = 0; k < m; k += s) {
            comp w(1, 0);
            for (int j = 0; j < s / 2; ++j) {
                comp &u = a[k + j + s / 2], &v = a[k + j];
                comp t = w * u;
                u = v - t;
                v = v + t;
                w = w * g;
            }
        }
    }
    if (v == -1)
        for (int i = 0; i < m; ++i) a[i].real /= m, a[i].imag /= m;
}

inline void mulmod(int *a, int *b, int *c) { // c = a * b % mod, c不能为a或b
```

```
        int M = sqrt(mod);
        for (int i = 0; i < m; ++i) {
            A[i] = comp(a[i] / M, a[i] % M);
            B[i] = comp(b[i] / M, b[i] % M);
        }
        dft(A, 1); dft(B, 1);
        static comp t[maxn];
        for (int i = 0; i < m; ++i) {
            int j = i ? m - i : 0;
            t[i] = ((A[i] + A[j].conj()) * (B[j].conj() - B[i]) + (A[j].conj() - A[i
                ]) * (B[i] + B[j].conj())) * comp(0, 0.25);
        }
        dft(t, -1);
        for (int i = 0; i < m; ++i)
            c[i] = (LL)(t[i].real + 0.5) % mod * M % mod;
        for (int i = 0; i < m; ++i) {
            int j = i ? m - i : 0;
            t[i] = (A[j].conj() - A[i]) * (B[j].conj() - B[i]) * comp(-0.25, 0) +
                comp(0, 0.25) * (A[i] + A[j].conj()) * (B[i] + B[j].conj());
        }
        dft(t, -1);
        for (int i = 0; i < m; ++i)
            c[i] = (1ll * c[i] + (LL)(t[i].real + 0.5) + (LL)(t[i].imag + 0.5) % mod
                 * M * M % mod) % mod;
    }
};
```

6.18 FWT

给定长度为 2^n 的序列 $A[0 \cdots 2^n - 1], B[0 \cdots 2^n - 1]$ ，求这两序列的

or 卷积：$C_k = \sum\limits_{i \text{ or } j = k} A_i B_j$

and 卷积：$C_k = \sum\limits_{i \text{ and } j = k} A_i B_j$

xor 卷积：$C_k = \sum\limits_{i \text{ xor } j = k} A_i B_j$

```
void FWT(int *a, int n) {
    for (int d = 1; d < n; d <<= 1)
        for (int m = d << 1, i = 0; i < n; i += m)
            for (int j = 0; j < d; ++j) {
                int x = a[i + j], y = a[i + j + d];
                //or: a[i + j + d] = x + y;
                //and: a[i + j] = x + y;
                //xor: a[i + j] = x + y, a[i + j + d] = x - y;
                // 如答案要求取模，此处记得取模
            }
}

void UFWT(int *a, int n) {
    for (int d = 1; d < n; d <<= 1)
```

```
        for (int m = d << 1, i = 0; i < n; i += m)
            for (int j = 0; j < d; ++j) {
                int x = a[i + j], y = a[i + j + d];
                //or: a[i + j + d] = y - x;
                //and: a[i + j] = x - y;
                //xor: a[i + j] = (x + y) / 2, a[i + j + d] = (x - y) / 2;
                // 如答案要求取模，此处记得取模
            }
}
```

6.19 求原根

接口：LL p_root(LL p);

输入：一个素数 p

输出：p 的原根

```
#include <bits/stdc++.h>
#define LL long long

using namespace std;

vector <LL> a;

LL pow_mod(LL base, LL times, LL mod) {
    LL ret = 1;
    while (times) {
        if (times&1) ret = ret * base % mod;
        base = base * base % mod;
        times>>=1;
    }
    return ret;
}

bool g_test(LL g, LL p) {
    for (LL i = 0; i < a.size(); ++i)
        if (pow_mod(g, (p-1)/a[i], p) == 1) return 0;
    return 1;
}

LL p_root(LL p) {
    LL tmp = p - 1;
    for (LL i = 2; i <= tmp / i; ++i)
        if (tmp % i == 0) {
            a.push_back(i);
            while (tmp % i == 0)
                tmp /= i;
        }
    if (tmp != 1) a.push_back(tmp);
    LL g = 1;
```

```
    while (1) {
        if (g_test(g, p)) return g;
        ++g;
    }
}

int main() {
    LL p;
    cin >> p;
    cout << p_root(p) << endl;
}
```

6.20 NTT

NTT 公式:

$$y_n = \sum_{i=0}^{d-1} x_i (g^{\frac{P-1}{d}})^{in} \bmod P$$

```
#define mod 998244353
#define gg 3

int power(int base, int times) {
    int ans = 1;
    while (times) {
        if (times & 1) ans = 1ll * ans * base % mod;
        base = 1ll * base * base % mod;
        times >>= 1;
    }
    return ans;
}

void NTT(int *x, int n, int reverse) {
    static int rev[N];
    int m = 1, len = 0;
    for (; m < n + n; m <<= 1, ++len);
    for (int i = 0; i < m; ++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len - 1))
        ;
    for (int i = 0; i < m; ++i)
        if (i < rev[i])
            swap(x[i], x[rev[i]]);
    for (int h = 2; h <= m; h <<= 1) {
        int wn = power(gg, (mod - 1) / h);
        if (reverse == -1) wn = power(wn, mod - 2);
        for (int i = 0; i < m; i += h) {
            int w = 1;
            for (int j = i; j < i + h / 2; ++j) {
                int u = x[j];
                int v = 1ll * w * x[j + h / 2] % mod;
                x[j] = (u + v) % mod;
```

```
                x[j + h / 2] = (u - v + mod) % mod;
                w = 1ll * w * wn % mod;
            }
        }
    }
    if (reverse == -1) {
        int t = power(m, mod - 2);
        for (int i = 0; i < m; ++i)
            x[i] = 1ll * x[i] * t % mod;
    }
}

int A[N], B[N];
int main() {
    memset(A, 0, sizeof(A));
    memset(B, 0, sizeof(B));
    NTT(A, len, 1); NTT(B, len, 1);
    for (int i=0;i<len;i++) A[i] = 1ll * A[i] * B[i] % mod;
    NTT(A, len, -1);
}
```

6.20.1 NTT 常用原根表

这张表格仅包含 $2^{18}k+1$ 的质数。

模数	最大长度	原根	模数	最大长度	原根	模数	最大长度	原根
786433	262144	10	5767169	524288	3	7340033	1048576	3
8650753	262144	10	10223617	262144	5	11272193	262144	3
13631489	1048576	15	14155777	524288	7	14942209	262144	11
16515073	262144	5	21495809	524288	3	22806529	262144	13
23068673	2097152	3	26214401	1048576	3	27000833	262144	3
28311553	1048576	5	29884417	524288	5	33292289	262144	3
35389441	262144	7	36175873	524288	7	37224449	524288	3
38535169	262144	11	40370177	524288	3	41680897	262144	5
42729473	262144	3	52166657	262144	3	63700993	262144	5
64749569	262144	6	68681729	524288	3	69206017	2097152	5
70254593	1048576	3	72613889	262144	3	74711041	262144	7
77070337	524288	13	81788929	2097152	7	83361793	524288	5
83623937	262144	3	85196801	262144	3	87293953	262144	7
90439681	262144	7	93585409	262144	13	93847553	524288	3
100139009	524288	3	101711873	1048576	3	103284737	524288	3
104857601	4194304	3	107216897	262144	3	111149057	2097152	3
113246209	4194304	7	114032641	262144	11	115081217	262144	3
117964801	524288	14	118751233	262144	5	120324097	262144	7
120586241	1048576	6	125042689	262144	13	126615553	262144	10

127664129	262144	3	130809857	262144	3	132120577	2097152	5
136314881	2097152	3	138412033	4194304	5	140771329	262144	7
141557761	1048576	26	142344193	262144	7	145489921	262144	7
147849217	1048576	5	151257089	262144	3	155189249	4194304	6
156499969	262144	7	158072833	262144	5	158334977	1048576	3
159645697	262144	15	163577857	4194304	23	167510017	262144	5
167772161	33554432	3	169869313	2097152	5	173801473	262144	5
175636481	524288	3	178782209	524288	3	184811521	262144	13
185597953	1048576	5	186646529	2097152	3	187432961	262144	6
189530113	262144	5	191365121	524288	3	199229441	2097152	3
200540161	262144	7	204472321	1048576	19	206307329	262144	3
207880193	262144	3	211025921	262144	6	211812353	2097152	3
214171649	262144	3	215482369	524288	13	215744513	262144	3
217317377	262144	3	218628097	524288	5	219676673	524288	3
221249537	1048576	3	222035969	262144	3	224133121	262144	23
228065281	524288	7	230424577	262144	5	230686721	4194304	6
231473153	262144	3	234356737	524288	7	236716033	262144	5
239337473	262144	3	239861761	262144	11	240648193	524288	5
244842497	524288	3	246415361	1048576	3	249561089	2097152	3
253493249	262144	3	254279681	524288	3	256376833	524288	7
257949697	2097152	5	260571137	524288	3	261881857	262144	7
263454721	262144	11	269221889	262144	3	270532609	2097152	22
270794753	262144	3	274726913	2097152	3	276037633	262144	15
277086209	262144	6	284950529	262144	3	285474817	262144	7
288882689	524288	3	290455553	1048576	3	302252033	262144	3
302776321	262144	17	305135617	1048576	5	306708481	524288	19
311427073	1048576	7	319291393	524288	5	323223553	262144	5
325844993	262144	3	328728577	524288	10	329515009	262144	13
329777153	524288	5	330301441	1048576	22	332660737	262144	10
336068609	524288	3	336855041	262144	3	340000769	262144	3
347078657	1048576	3	349962241	262144	7	351797249	524288	3
359661569	1048576	3	360972289	262144	7	361758721	1048576	29
371458049	262144	3	374603777	262144	3	376963073	524288	3
377487361	8388608	7	383778817	2097152	5	384040961	262144	3
386400257	524288	3	387186689	262144	3	387973121	2097152	6
390332417	262144	3	391643137	524288	5	395837441	524288	6
399507457	1048576	5	404226049	524288	7	409993217	1048576	3
413925377	262144	3	415236097	4194304	5	416808961	524288	37
424148993	524288	3	429391873	524288	10	433586177	524288	3
434896897	262144	15	438829057	524288	5	442761217	262144	5
444334081	262144	37	447741953	1048576	3	452198401	262144	11

455344129	262144	13	458752001	524288	6	459276289	2097152	11
460849153	524288	5	462684161	262144	3	463470593	2097152	3
464781313	262144	5	466354177	262144	10	468713473	1048576	5
469762049	67108864	3	471072769	262144	7	473694209	262144	6
475267073	262144	3	478937089	262144	13	483131393	262144	3
483655681	262144	14	487063553	524288	3	489422849	262144	3
493879297	1048576	10	495452161	524288	11	498597889	524288	7
500432897	262144	5	511967233	262144	5	517472257	524288	5
518520833	524288	3	524812289	524288	3	526123009	262144	7
529268737	262144	5	531628033	1048576	5	533463041	262144	3
536608769	262144	3	537133057	262144	5	539754497	262144	3
540540929	524288	3	541327361	262144	3	549978113	524288	3
551288833	262144	5	552861697	262144	5	555220993	524288	7
561774593	262144	3	564658177	524288	5	568066049	262144	3
569638913	262144	3	570163201	262144	7	570949633	524288	5
576454657	262144	10	576716801	2097152	6	581959681	1048576	11
582746113	262144	5	583794689	262144	3	584581121	524288	3
590872577	524288	3	595591169	8388608	3	597688321	2097152	11
605028353	1048576	3	605552641	524288	17	606339073	262144	5
607911937	262144	7	608698369	524288	7	611844097	524288	5
612892673	524288	3	615776257	262144	5	619184129	524288	3
621281281	524288	7	626262017	262144	3	629932033	262144	14
632553473	262144	3	635437057	2097152	11	637009921	524288	17
638058497	524288	3	639369217	262144	5	639631361	2097152	6
644087809	262144	11	645922817	8388608	3	648019969	2097152	17
649592833	524288	5	651952129	262144	7	655360001	1048576	3
657719297	262144	3	660078593	524288	3	663224321	524288	3
665583617	262144	3	666894337	4194304	5	675545089	262144	11
675807233	524288	3	681312257	262144	3	683409409	262144	13
683671553	4194304	3	684982273	262144	5	687603713	262144	3
690749441	262144	3	692846593	262144	5	699138049	262144	19
699924481	524288	17	703070209	524288	11	704905217	262144	3
710410241	524288	3	710934529	2097152	17	712769537	262144	3
714342401	262144	3	715128833	2097152	3	717488129	262144	3
718274561	1048576	3	720633857	262144	3	725876737	262144	7
730595329	262144	17	734527489	524288	7	737673217	524288	11
740294657	2097152	3	741605377	262144	11	745537537	1048576	5
748158977	524288	3	753664001	262144	3	754974721	16777216	11
758906881	262144	11	759693313	524288	5	760741889	524288	3
763887617	524288	3	769130497	524288	15	770703361	1048576	11
771489793	262144	10	772538369	262144	6	775421953	524288	5

781975553	262144	3	782499841	262144	11	786432001	2097152	7
790364161	262144	14	792199169	524288	3	793509889	262144	11
795082753	262144	5	798228481	262144	13	799014913	2097152	13
800063489	1048576	3	800849921	262144	6	801374209	262144	14
802160641	1048576	11	808714241	262144	3	810024961	524288	13
811859969	262144	3	813170689	524288	13	813432833	262144	3
818937857	1048576	5	820248577	262144	5	820510721	524288	3
821297153	262144	3	824180737	2097152	5	824442881	262144	3
825753601	524288	23	828112897	262144	10	829685761	262144	19
833617921	1048576	13	835452929	262144	3	839385089	524288	3
842530817	524288	3	844627969	524288	17	844890113	262144	3
848560129	262144	22	850395137	1048576	3	851705857	262144	5
860618753	262144	3	862978049	1048576	3	863764481	262144	3
864550913	524288	3	867434497	262144	5	872153089	262144	7
873725953	262144	10	875298817	262144	5	879230977	524288	15
880803841	8388608	26	881590273	262144	5	883949569	1048576	7
885522433	524288	5	888668161	524288	14	889454593	262144	15
894959617	524288	10	896008193	524288	3	897318913	262144	5
897581057	8388608	3	899678209	2097152	7	900464641	262144	7
903086081	262144	3	907018241	1048576	3	907542529	524288	7
907804673	262144	3	908328961	262144	26	909377537	262144	3
913309697	1048576	3	914096129	262144	3	918552577	4194304	5
919339009	262144	59	919601153	1048576	3	924844033	2097152	5
925892609	1048576	3	932970497	262144	3	935329793	4194304	3
938475521	1048576	3	940572673	1048576	7	943718401	4194304	7
946339841	524288	3	948699137	262144	3	950009857	2097152	7
951582721	524288	14	957349889	1048576	6	958136321	262144	3
958922753	524288	3	962592769	2097152	7	962854913	262144	3
969146369	262144	3	971243521	262144	28	972029953	1048576	10
975175681	2097152	17	976224257	1048576	3	977534977	262144	5
979107841	262144	11	980156417	262144	3	983826433	262144	11
985661441	4194304	3	993263617	262144	5	995622913	524288	5
998244353	8388608	3	1004535809	2097152	3	1005060097	524288	5
1006108673	524288	3	1007681537	1048576	3	1010565121	262144	7
1012924417	2097152	5	1015283713	262144	5	1018429441	262144	11
1019478017	262144	3	1023148033	262144	7	1036779521	262144	3
1037303809	262144	21	1045430273	1048576	3	1049100289	524288	7
1051721729	1048576	6	1052508161	262144	3	1053818881	1048576	7
1056178177	262144	5	1056440321	524288	3	1062469633	262144	5
1068236801	262144	3	1073479681	262144	11			

6.20.2 多项式求逆元

对于一个多项式 $A(x)$ ，如果存在 $B(x)$ 满足 $\deg(B) \leq \deg(A)$ 并且 $A(x)B(x) \equiv 1 (\bmod x^n)$ ，那么称 $B(x)$ 为 $A(x)$ 在 $\bmod x^n$ 意义下的逆元，记为 $A^{-1}(x)$ 。

```
// x := 1 / y
void inverse(int n0, int *x, const int *y) {
    static int fy[N];
    x[0] = power(y[0], mod - 2);
    for (int i = 1; i < n0; i <<= 1) {
        for (int j = 0; j < 4 * i; ++j) {
            fy[j] = (j < 2 * i) ? y[j] : 0;
            if (j >= i) x[j] = 0;
        }
        NTT(fy, 2 * i, 1);
        NTT(x, 2 * i, 1);
        for (int j = 0; j < 4 * i; ++j) {
            x[j] = (2 * x[j] - 1ll * x[j] * x[j] % mod * fy[j]) % mod;
            if (x[j] < 0) x[j] += mod;
        }
        NTT(x, 2 * i, -1);
    }
}
```

6.20.3 多项式取对数

```
// x := log(y)
void logarithm(int n0, int *x, int *y) {
    static int tmp[N];
    static int invs[N];
    inverse(n0, x, y);
    for (int i = 0; i < n0 * 2; ++i) {
        tmp[i] = i < n0 - 1 ? 1ll * y[i + 1] * (i + 1) % mod : 0;
        if (i >= n0) x[i] = 0;
    }
    NTT(tmp, n0, 1);
    NTT(x, n0, 1);
    for (int i = 0; i < n0 * 2; ++i)
        x[i] = 1ll * x[i] * tmp[i] % mod;
    NTT(x, n0, -1);
    invs[1] = 1;
    for (int i = 2; i < n0; ++i)
        invs[i] = (mod - 1ll * mod / i * invs[mod % i] % mod) % mod;
    for (int i = n0 - 1; i; --i)
        x[i] = 1ll * x[i - 1] * invs[i] % mod;
    x[0] = 0;
}
```

6.20.4 多项式取指数

```
// a := exp(b)
void exponent(int n0, int *a, int *b) {
    static int fb[N], x[N], y[N];
    a[0] = 1;
    for (int i = 1; i < n0; i <<= 1) {
        for (int j = 0; j < i * 2; ++j)
            y[j] = (j < i) ? a[j] : 0;
        logarithm(i * 2, x, y);
        for (int j = 0; j < 4 * i; ++j) {
            fb[j] = !j;
            if (j < 2 * i) {
                fb[j] = (fb[j] + b[j]) % mod;
                fb[j] = (fb[j] + mod - x[j]) % mod;
            }
            if (j >= i) a[j] = 0;
        }
        NTT(a, 2 * i, 1);
        NTT(fb, 2 * i, 1);
        for (int j = 0; j < 4 * i; ++j)
            a[j] = 1ll * a[j] * fb[j] % mod;
        NTT(a, 2 * i, -1);
    }
}
```

6.21 Berlekamp Messay 算法求线性递推式

适合所有 $S_n = \sum\limits_{i=1}^{L} a_i S_{n-i}$ 的递推式。只需在 $vector < int > t$ 中输入前 $2L$ 项，即可计算出第 m 项的值 modulo MOD 。

时间复杂度 $O(L^2 \log(m))$ 。

异常处理：若提示 48 行 assertion error (assert(l * 2 + 1 < s.size()) ，则表示输入项数不足 $2L+2$ 项，需要更多的项来确定线性递推式。

```
1  #include <bits/stdc++.h>
2  
3  using namespace std;
4  typedef long long ll;
5  
6  int MOD;
7  
8  int bin(int a, int n) {
9      int res = 1;
10     while (n) {
11         if (n & 1) res = 1LL * res * a % MOD;
12         a = 1LL * a * a % MOD;
13         n >>= 1;
14     }
```

```
    return res;
}

int inv(int x) {
    return bin(x, MOD - 2);
}

vector<int> berlekamp(vector<int> s) {
    int l = 0;
    vector<int> la(1, 1);
    vector<int> b(1, 1);
    for (int r = 1; r <= (int)s.size(); r++) {
        int delta = 0;
        for (int j = 0; j <= l; j++) {
            delta = (delta + 1LL * s[r - 1 - j] * la[j]) % MOD;
        }
        b.insert(b.begin(), 0);
        if (delta != 0) {
            vector<int> t(max(la.size(), b.size()));
            for (int i = 0; i < (int)t.size(); i++) {
                if (i < (int)la.size()) t[i] = (t[i] + la[i]) % MOD;
                if (i < (int)b.size()) t[i] = (t[i] - 1LL * delta * b[i] % MOD + MOD
                    ) % MOD;
            }
            if (2 * l <= r - 1) {
                b = la;
                int od = inv(delta);
                for (int &x : b) x = 1LL * x * od % MOD;
                l = r - l;
            }
            la = t;
        }
    }
    assert(la.size() == l + 1);
    assert(l * 2 + 1 < s.size());
    reverse(la.begin(), la.end());
    return la;
}

vector<int> mul(vector<int> a, vector<int> b) {
    vector<int> c(a.size() + b.size() - 1);
    for (int i = 0; i < (int)a.size(); i++) {
        for (int j = 0; j < (int)b.size(); j++) {
            c[i + j] = (c[i + j] + 1LL * a[i] * b[j]) % MOD;
        }
    }
    vector<int> res(c.size());
    for (int i = 0; i < (int)res.size(); i++) res[i] = c[i] % MOD;
    return res;
}
```

```
vector<int> mod(vector<int> a, vector<int> b) {
    if (a.size() < b.size()) a.resize(b.size() - 1);

    int o = inv(b.back());
    for (int i = (int)a.size() - 1; i >= b.size() - 1; i--) {
        if (a[i] == 0) continue;
        int coef = 1LL * o * (MOD - a[i]) % MOD;
        for (int j = 0; j < (int)b.size(); j++) {
            a[i - (int)b.size() + 1 + j] = (a[i - (int)b.size() + 1 + j] + 1LL *
                coef * b[j]) % MOD;
        }
    }
    while (a.size() >= b.size()) {
        assert(a.back() == 0);
        a.pop_back();
    }
    return a;
}

vector<int> bin(int n, vector<int> p) {
    vector<int> res(1, 1);
    vector<int> a(2); a[1] = 1;
    while (n) {
        if (n & 1) res = mod(mul(res, a), p);
        a = mod(mul(a, a), p);
        n >>= 1;
    }
    return res;
}

void solve() {
    int m = 22;
    vector<int> t;
    t.push_back(1);
    t.push_back(9);
    t.push_back(41);
    t.push_back(109);
    t.push_back(205);
    t.push_back(325);
    t.push_back(473);
    t.push_back(649);
    t.push_back(853);
    t.push_back(1085);
    t.push_back(1345);
    t.push_back(1633);
    t.push_back(1949);
    t.push_back(2293);

    MOD = 998244353;
```

```
    vector<int> v = berlekamp(t);
    vector<int> o = bin(m - 1, v);
    int res = 0;
    for (int i = 0; i < (int)o.size(); i++) res = (res + 1LL * o[i] * t[i]) % MOD;
    printf("%d\n", res);
}

int main() {
    solve();
    return 0;
}
```

6.22 幂和

$$\sum_{i=1}^{n} i^1 = \frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$$

$$\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n$$

$$\sum_{i=1}^{n} i^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12} = \frac{1}{6}n^6 + \frac{1}{2}n^5 + \frac{5}{12}n^4 - \frac{1}{12}n^2$$

$$\sum_{i=1}^{n} i^6 = \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42} = \frac{1}{7}n^7 + \frac{1}{2}n^6 + \frac{1}{2}n^5 - \frac{1}{6}n^3 + \frac{1}{42}n$$

6.23 蔡勒公式

$$w = (\lfloor \frac{c}{4} \rfloor - 2c + y + \lfloor \frac{y}{4} \rfloor + \lfloor \frac{13(m+1)}{5} \rfloor + d - 1) \bmod 7$$

w ： 0 星期日，1 星期一，2 星期二，3 星期三，4 星期四，5 星期五，6 星期六
c ： 年份前两位数
y ： 年份后两位数
m ： 月（$3 \le m \le 14$ ，即在蔡勒公式中，1、2 月要看作上一年的 13、14 月来计算）
d ： 日

6.24 皮克定理

给定顶点坐标均是整点（或正方形格点）的简单多边形（凸多边形），皮克定理说明了其面积 S 和内部格点数目 n 、边上格点数目 s 的关系：$S = n + \frac{s}{2} + 1$ 。

6.25 组合数 lcm

$$(n+1)lcm(C(n,0),C(n,1),...,C(n,k)) = lcm(n+1,n,n-1,...,n-k+1)$$

6.26 区间 lcm 的维护

对于一个数，将其分解质因数，若有因子 p^k ，那么拆分出 k 个数 $p, p^2, ..., p^k$ ，权值都为 p ，那么查询区间 $[l,r]$ 内所有数的 lcm 的答案 = 所有在该区间中出现过的数的权值之积，可持久化线段树维护即可。

7 几何

7.1 二维计算几何

7.1.1 计算几何误差修正

```
const double pi = acos(-1.0);
const double eps = 1e-8;

inline double sqr(double x) {
    return x * x;
}

inline int sgn(double x) {
    if (x < -eps) return -1;
    return x > eps;
}

inline int cmp(double x, double y) {
    return sgn(x - y);
}
```

7.1.2 计算几何点类

成员函数：

read()	输入一个点
norm()	计算向量的模长

相关函数：

double sqr(double x)	计算一个数的平方
double det(const point &a, const point &b)	计算两个向量的叉积
double dot(const point &a, const point &b)	计算两个向量的点积
double dis(const point &a, const point &b)	计算两个点的距离
point rotate_point(const point &p, double A)	$\overrightarrow{OP}$绕原点逆时针旋转 A 弧度

```
struct point {
    double x, y;
    point() : x(0), y(0) {}
    point(double a, double b) : x(a), y(b) {}
```

```cpp
    inline void read() {
        scanf("%lf%lf", &x, &y);
    }
    inline friend point operator + (const point &a, const point &b) {
        return point(a.x + b.x, a.y + b.y);
    }
    inline friend point operator - (const point &a, const point &b) {
        return point(a.x - b.x, a.y - b.y);
    }
    inline friend bool operator == (const point &a, const point &b) {
        return cmp(a.x, b.x) == 0 && cmp(a.y, b.y) == 0;
    }
    inline friend point operator * (const double &a, const point &b) {
        return point(a * b.x, a * b.y);
    }
    inline friend point operator / (const point &a, const double &b) {
        return point(a.x / b, a.y / b);
    }
    inline double norm() const {
        return sqrt(sqr(x) + sqr(y));
    }
};

inline double det(const point &a, const point &b) {
    return a.x * b.y - a.y * b.x;
}

inline double dot(const point &a, const point &b) {
    return a.x * b.x + a.y * b.y;
}

inline double dis(const point &a, const point &b) {
    return (a - b).norm();
}

inline point rotate_point(const point &p, double A) {
    double tx = p.x, ty = p.y;
    return point(tx * cos(A) - ty * sin(A), tx * sin(A) + ty * cos(A));
}
```

7.1.3 计算几何线段类

相关函数:

bool point_on_segment(const point &p, const segment &l) 判断点 p 是否在线段 l 上 (含端点)

double point_to_segment_dist(const point &p, const segment &l) 求点 p 到线段 l 的距离

point sym_point(const point &p, const segment &l) 求点 p 关于线段 l 的对称点

point point_proj_line(const point &p, const segment &l) 求点 p 到线段 l 的垂足

bool parallel(const segment &a, const segment &b) 判断线段 a 和线段 b 是否平行

point intersect_point(const segment &a, const segment &b) 求直线 a 与直线 b 的交点（如要求线段 a 与线段 b 的交点，应先判断是否有）

bool is_segment_intersect(const segment &l1, const segment &l2) 判断线段 a 与线段 b 是否相交（含端点）（如不含端点，将 $\leq$ 改为 $<$ ）

bool is_line_intersect_segment(const point &p1, const point &p2, const segment &l) 判断直线 p_1p_2 是否与线段 l 相交

bool is_half_line_intersect_segment(const point &p1, const point &p2, const segment &l) 判断射线 p_1p_2 是否与线段 l 相交（含端点 p_1 ）（如不含端点 p_1 ，将 $\geq$ 改为 $>$ ）

```
struct segment {
    point a, b;
    segment() {}
    segment(point x, point y) : a(x), b(y) {}
    void read() {
        a.read(); b.read();
    }
};

// determine whether point p is on segment l
bool point_on_segment(const point &p, const segment &l) {
    if ((cmp(l.a.x, p.x) <= 0 || cmp(l.b.x, p.x) <= 0) &&
        (cmp(l.a.x, p.x) >= 0 || cmp(l.b.x, p.x) >= 0) &&
        (cmp(l.a.y, p.y) <= 0 || cmp(l.b.y, p.y) <= 0) &&
        (cmp(l.a.y, p.y) >= 0 || cmp(l.b.y, p.y) >= 0)) {
        return sgn(det(p - l.a, l.b - l.a)) == 0;
    }
    return 0;
}

// determine the distance from the point p to segment l
double point_to_segment_dist(const point &p, const segment &l) {
    if (dis(l.a, l.b) < eps) return dis(p, l.a);
    if (sgn(dot(l.b - l.a, p - l.a)) < 0) return dis(l.a, p);
    if (sgn(dot(l.a - l.b, p - l.b)) < 0) return dis(l.b, p);
    return fabs(det(l.b - l.a, p - l.a)) / dis(l.b, l.a);
}

// determine the symmetrical point of point p on segment l
point sym_point(const point &p, const segment &l) {
    double a = l.b.x - l.a.x;
    double b = l.b.y - l.a.y;
    double t = ((p.x - l.a.x) * a + (p.y - l.a.y) * b) / (a * a + b * b);
    return point(2 * l.a.x + 2 * a * t - p.x, 2 * l.a.y + 2 * b * t - p.y);
}

point point_proj_line(const point &p, const segment &l) {
    double r = dot((l.b - l.a), (p - l.a)) / dot(l.b - l.a, l.b - l.a);
    return l.a + r * (l.b - l.a);
}
```

```

bool parallel(const segment &a, const segment &b) {
    return sgn(det(a.a - a.b, b.a - b.b)) == 0;
}

point intersect_point(const segment &a, const segment &b) {
    double s1 = det(a.a - b.a, b.b - b.a);
    double s2 = det(a.b - b.a, b.b - b.a);
    return (s1 * a.b - s2 * a.a) / (s1 - s2);
}

// determine whether segment l1 intersects with segment l2
bool is_segment_intersect(const segment &l1, const segment &l2) {
    const point &s1 = l1.a, &e1 = l1.b;
    const point &s2 = l2.a, &e2 = l2.b;
    if ( cmp( min(s1.x, e1.x), max(s2.x, e2.x) ) <= 0 &&
         cmp( min(s1.y, e1.y), max(s2.y, e2.y) ) <= 0 &&
         cmp( min(s2.x, e2.x), max(s1.x, e1.x) ) <= 0 &&
         cmp( min(s2.y, e2.y), max(s1.y, e1.y) ) <= 0 &&
         sgn( det(s2 - s1, e2 - s1) ) * sgn( det(s2 - e1, e2 - e1) ) <= 0 &&
         sgn( det(s1 - s2, e1 - s2) ) * sgn( det(s1 - e2, e1 - e2) ) <= 0)
        return 1;
    return 0;
}

// determine whether line p1p2 intersects with segment l
bool is_line_intersect_segment(const point &p1, const point &p2, const segment &l) {
    assert(!(p1 == p2));
    return sgn( det(p1 - l.a, p2 - l.a) ) * sgn( det(p1 - l.b, p2 - l.b) ) <= 0;
}

// determine whether half-line p1p2 intersects with segment l
bool is_half_line_intersect_segment(const point &p1, const point &p2, const segment &l) {
    return is_line_intersect_segment(p1, p2, l) && sgn( det(p1 - l.a, p2 - l.a) ) * sgn( det(p1 - l.a, l.b - l.a) ) >= 0;
}
```

7.2 凸包

```
typedef complex<int> point;
#define X real()
#define Y imag()
int n;
long long cross(point a,point b) {
    return 1ll * a.X * b.Y - 1ll * a.Y * b.X;
}
bool cmp(point a,point b) {
    return make_pair(a.X, a.Y) < make_pair(b.X, b.Y);
```

```
}
int convexHull(point p[],int n,point ch[]) {
    sort(p, p + n, cmp);
    int m = 0;
    for(int i = 0; i < n; ++i) {
        while(m > 1 && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
        ch[m++] = p[i];
    }
    int k = m;
    for(int i = n - 2; i >= 0; --i) {
        while(m > k && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
        ch[m++] = p[i];
    }
    if(n > 1) m--;
    return m;
}
```

7.3 半平面交

输入 vec1 表示所有的半平面 y >= kx + b 的参数 k 和 b 。

输出 vec2 表示下凸壳（对应 y >= kx + b）或者上凸壳（对应 y <= kx + b）。

```
vector< pair< LL, LL > > vec1, vec2;

LL getval(int t, LL x) {
    return vec2[t].first * x + vec2[t].second;
}

void solve() {
    // vec1 stores pair< k, b > for all plane y >=(or <=) kx + b
    sort(vec1.begin(), vec1.end());
    // reverse(vec1.begin(), vec1.end()); // if y <= kx + b
    for (int i = 0; i < vec1.size(); ++i) {
        while (vec2.size() >= 2) {
            LL k1 = vec2[vec2.size() - 2].first;
            LL b1 = vec2[vec2.size() - 2].second;
            LL k2 = vec2[vec2.size() - 1].first;
            LL b2 = vec2[vec2.size() - 1].second;
            LL k3 = vec1[i].first;
            LL b3 = vec1[i].second;
            if ((b2 - b1) * (k2 - k3) >= (b3 - b2) * (k1 - k2))
                vec2.pop_back();
            else
                break;
        }
        vec2.push_back(vec1[i]);
    }
}
```

8 黑科技和杂项

8.1 找规律

此方法已过时，请参照“数学 > Berlekamp Messay 算法求线性递推式”。本法使用矩阵快速幂，效率 $O(L^3 \log(m))$ ，而用 Berlekamp 加多项式快速幂可以做到 $O(L^2 \log(m))$ ，故不推荐使用本法。

~~有些题目，只给一个正整数~~$\cancel{n}$~~，然后要求输出一个答案。这时，我们可以暴力得到小数据的解，用高斯消元得到递推式，然后用矩阵快速幂求解。~~

~~使用方法：~~

~~首先在 gauss.in 中输入小数据的解（~~$\cancel{n = 1}$ ~~时，~~$\cancel{n = 2}$ ~~时，~~$\cancel{\cdots}$ ~~），以~~$\cancel{EOF}$ ~~结束。~~

~~依次运行 gauss.cpp，matrix.cpp，得到 matrix.out~~

~~将 matrix.out 中的文件粘贴在 main.cpp 中相应的位置中。注意模数一定要是质数。~~

```
//gauss.cpp
#include <bits/stdc++.h>
#define N 102
#define mod 1000000007
//caution: you can use this program iff mod is a prime.

using namespace std;

int n, m, k, a[N], g[N][N];

int power(int base, int times) {
    int ret = 1;
    while (times) {
        if (times & 1) ret = 1ll * ret * base % mod;
        base = 1ll * base * base % mod;
        times >>= 1;
    }
    return ret;
}

int test() {
    for (int i=0;i<m;i++) {
        for (int j=i;j<=m;j++)
            if (g[j][i]) {
                for (int k=i;k<=m;k++)
                    swap(g[i][k], g[j][k]);
                break;
            }
        if (g[i][i] == 0)
            return 0;
        for (int j=i+1;j<n;j++) {
            while (g[j][i]) {
                int t = 1ll * g[i][i] * power(g[j][i], mod - 2) % mod;
                for (int k=i;k<n;k++)
                    g[i][k] = (g[i][k] + mod - (1ll * t * g[j][k] % mod)) % mod;
                for (int k=i;k<=m;k++)
```

```
                    swap(g[i][k], g[j][k]);
            }
        }
        int t = power(g[i][i], mod - 2);
        for (int j = 0; j <= m; ++j)
            g[i][j] = 1ll * g[i][j] * t % mod;
    }
    for (int i = m; i < n; ++i)
        if (g[i][m]) return 0;
    for (int i = m - 1; i >= 0; --i) {
        int t = power(g[i][i], mod - 2);
        g[i][i] = 1;
        g[i][m] = 1ll * g[i][m] * t % mod;
        for (int j = 0; j < i; ++j)
            g[j][m] = (g[j][m] + mod - 1ll * g[i][m] * g[j][i] % mod) % mod;
    }
    printf("%d\n", m);
    for (int i = 0; i < m; ++i)
        printf("%d ", g[i][m]);
    puts("");
    for (int i = 0; i < m - 1; ++i)
        printf("%d ", a[i]);
    puts("1");
    return 1;
}

int main() {
    freopen("gauss.in", "r", stdin);
    freopen("gauss.out", "w", stdout);
    k = 0;
    while (~scanf("%d", &a[k++])) ;
    for (int sm = 1; sm <= k - sm; ++sm) {
        n = k - sm - 1;
        m = sm + 1;
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j <= sm; ++j)
                g[i][j] = a[i + j];
            g[i][m] = 1;
            swap(g[i][m - 1], g[i][m]);
        }
        if (test()) return 0;
    }
    puts("no solution");
    return 0;
}
```

```
//matrix.cpp
#include <bits/stdc++.h>
#define N 102
using namespace std;
```

```

int n, a[N];

int main() {
    freopen("gauss.out", "r", stdin);
    freopen("matrix.out", "w", stdout);
    scanf("%d", &n);
    for (int i = 0; i < n; ++i) scanf("%d", &a[i]);
    printf("#define␣M␣%d\n", n);
    printf("const␣int␣trans[M][M]␣=␣{\n");
    for (int i = 0; i < n; ++i) {
        printf("\t{");
        for (int j = 0; j < n; ++j) {
            int t;
            if (j < n - 2) t = i == j + 1;
            else if (j == n - 2) t = a[i];
            else t = i == n - 1;
            printf("%s%d", j == 0 ? "" : ",␣", t);
        }
        printf("}%s\n", i == n - 1 ? "" : ",");
    }
    printf("};\n");
    printf("const␣int␣pref[M]␣=␣{");
    for (int i = 0; i < n; ++i) {
        int x;
        scanf("%d", &x);
        printf("%d%s", x, i == n - 1 ? "};\n" : ",␣");
    }
    return 0;
}
```

```
//main.cpp
#include <bits/stdc++.h>
using namespace std;

/* paste matrix.out here. */

#define mod 1000000007

struct Matrix {
    int c[M][M];
    void clear() { memset(c, 0, sizeof(c)); }
    void identity() { clear(); for (int i = 0; i < M; ++i) c[i][i] = 1; }
    void base() { memcpy(c, trans, sizeof(trans)); }
    friend Matrix operator * (const Matrix &a, const Matrix &b) {
        Matrix c; c.clear();
        for (int i = 0; i < M; ++i)
            for (int j = 0; j < M; ++j)
                for (int k = 0; k < M; ++k)
                    c.c[i][j] = (c.c[i][j] + 1ll * a.c[i][k] * b.c[k][j] % mod) %
```

```
                        mod;
        return c;
    }
} start, base;

Matrix power(Matrix base, int times) {
    Matrix ret; ret.identity();
    while (times) {
        if (times & 1) ret = ret * base;
        base = base * base;
        times >>= 1;
    }
    return ret;
}

int main() {
    int tot;
    scanf("%d", &tot);
    while (tot--) {
        int n;
        scanf("%d", &n);
        start.clear();
        for (int i = 0; i < M; ++i) start.c[0][i] = pref[i];
        base.base();
        base = power(base, n - 1);
        start = start * base;
        printf("%d\n", start.c[0][0]);
    }
    return 0;
}
```

8.2 分数类

```
#define LL long long

struct frac {
    LL x, y;
    frac(LL _x = 0, LL _y = 1) {
        x = _x;
        y = _y;
        LL g = __gcd(abs(x), abs(y));
        x /= g;
        y /= g;
        if (y < 0) {
            x = -x;
            y = -y;
        }
    }

```

```
    inline friend frac operator + (const frac &lhs, const frac &rhs) {
        return frac(lhs.x * rhs.y + rhs.x * lhs.y, lhs.y * rhs.y);
    }

    inline friend frac operator - (const frac &lhs, const frac &rhs) {
        return frac(lhs.x * rhs.y - rhs.x * lhs.y, lhs.y * rhs.y);
    }

    inline friend frac operator - (const frac &lhs) {
        return frac(-lhs.x, lhs.y);
    }

    inline friend frac operator * (const frac &lhs, const frac &rhs) {
        return frac(lhs.x * rhs.x, lhs.y * rhs.y);
    }

    inline friend frac operator / (const frac &lhs, const frac &rhs) {
        return frac(lhs.x * rhs.y, lhs.y * rhs.x);
    }

    inline friend bool operator == (const frac &lhs, const frac &rhs) {
        return lhs.x * rhs.y == rhs.x * lhs.y;
    }

    inline friend bool operator != (const frac &lhs, const frac &rhs) {
        return lhs.x * rhs.y != rhs.x * lhs.y;
    }

    inline friend bool operator < (const frac &lhs, const frac &rhs) {
        return lhs.x * rhs.y < rhs.x * lhs.y;
    }

    inline friend bool operator > (const frac &lhs, const frac &rhs) {
        return lhs.x * rhs.y > rhs.x * lhs.y;
    }

    inline friend bool operator <= (const frac &lhs, const frac &rhs) {
        return lhs.x * rhs.y <= rhs.x * lhs.y;
    }

    inline friend bool operator >= (const frac &lhs, const frac &rhs) {
        return lhs.x * rhs.y >= rhs.x * lhs.y;
    }

    inline void print() const {
        printf("%lld/%lld\n", x, y);
    }
};
```

8.3 取模整数类

如果需要用模意义下的除法，需定义常量 D 为除数的最大值，并执行 init_inv() 。

```cpp
struct mod;
mod* inv;

struct mod {
    static constexpr int MOD = 1000 * 1000 * 1000 + 7; // std=c++11
    mod(int x_) : x((x_ % MOD + MOD) % MOD) {}
    mod() = default;
    int x = 0;
    inline mod operator *(mod other) const {
        return ((long long)x * other.x) % MOD;
    }
    inline mod& operator *=(mod other) {
        x = ((long long)x * other.x) % MOD;
        return *this;
    }
    inline mod operator +(mod other) const {
        int res = x + other.x;
        if (res >= MOD) {
            res -= MOD;
        }
        return res;
    }
    inline mod& operator +=(mod other) {
        if ((x += other.x) >= MOD) {
            x -= MOD;
        }
        return *this;
    }
    inline mod operator -(mod other) const {
        int res = x - other.x;
        if (res < 0) {
            res += MOD;
        }
        return res;
    }
    inline mod& operator -=(mod other) {
        if ((x -= other.x) < 0) {
            x += MOD;
        }
        return *this;
    }
    inline mod operator /(mod other) const {
        return (*this) * inv[other.x];
    }
    inline mod& operator /=(mod other) {
        return *this *= inv[other.x];
    }
```

```cpp
    inline bool operator ==(mod other) const {
        return x == other.x;
    }
    inline mod operator -() const {
        return x != 0 ? MOD - x : 0;
    }
};

void init_inv() {
    inv = new mod[D];
    inv[1] = 1;
    for (int i = 2; i < D; i++) {
        inv[i] = (mod::MOD - (long long)mod::MOD / i * inv[mod::MOD % i].x % mod::
            MOD) % mod::MOD;
    }
}
```

8.4 多项式类

```cpp
struct poly {
    vector<mod> C;
    poly() {}
    explicit poly(const vector<mod> &C_) : C(C_) {}
    static const poly zero;
    inline int deg() const {
        return (int)C.size() - 1;
    }
    inline mod operator[](int x) const {
        return (x < 0 || x > deg()) ? mod(0) : C[x];
    }
    inline mod& operator[](int x) {
        if (x > deg()) {
            C.resize(x + 1);
        }
        return C[x];
    }
    inline friend poly operator +(const poly& a, const poly& b) {
        vector<mod> c(max(a.deg(), b.deg()) + 1);
        for (int i = 0; i < c.size(); i++) {
            c[i] = a[i] + b[i];
        }
        return poly(c);
    }
    inline friend poly operator -(const poly& a, const poly& b) {
        vector<mod> c(max(a.deg(), b.deg()) + 1);
        for (int i = 0; i < c.size(); i++) {
            c[i] = a[i] - b[i];
        }
        return poly(c);
```

```cpp
    }
    inline bool isZero() const {
        return C.empty();
    }
    inline friend poly operator *(const poly& a, const poly& b) {
        if (a.isZero() || b.isZero()) {
            return zero;
        }
        vector<mod> c(1 + a.deg() + b.deg());
        for (int i = 0; i <= a.deg(); i++) {
            for (int j = 0; j <= b.deg(); j++) {
                c[i + j] += a[i] * b[j];
            }
        }
        return poly(c);
    }
    inline poly derivative() const {
        if (isZero()) {
            return zero;
        }
        vector<mod> res(deg());
        for (int i = 0; i < deg(); i++) {
            res[i] = C[i + 1] * (i + 1);
        }
        return poly(res);
    }
    inline poly primitive() const {
        if (isZero()) {
            return zero;
        }
        vector<mod> res(2 + deg());
        for (int i = 1; i <= 1 + deg(); i++) {
            res[i] = C[i - 1] / i;
        }
        return poly(res);
    }
    inline mod operator()(mod x) const {
        mod res = 0;
        for (int i = deg(); i >= 0; i--) {
            res = res * x + C[i];
        }
        return res;
    }
    // Expand P(x+t).
    inline poly shift(int t) const {
        poly res;
        res[deg()];
        vector<mod> binomial(deg() + 1, 0);
        binomial[0] = 1;
        for (int i = 0; i <= deg(); i++) {
```

```
            mod cur = 1;
            for (int j = i; j >= 0; j--) {
                res[j] += C[i] * binomial[j] * cur;
                cur *= t;
            }
            if (i == deg()) {
                break;
            }
            for (int j = i + 1; j > 0; j--) {
                binomial[j] += binomial[j - 1];
            }
        }
        return res;
    }
};
```

8.5 求数列中每个位置到前面任意位置的 gcd

maxgcd[i] 表示从第 i 个位置往前出发，每个 $gcd(a_j, \cdots, a_i)$ 值最靠后的位置 (first 表示 gcd 值，second 表示位置)。

例如：

```
输入数据：
5
2 4 2 3 3

maxgcd[1] = {{first = 2, second = 1}}
maxgcd[2] = {{first = 2, second = 1}, {first = 4, second = 2}}
maxgcd[3] = {{first = 2, second = 3}}
maxgcd[4] = {{first = 1, second = 3}, {first = 3, second = 4}}
maxgcd[5] = {{first = 1, second = 3}, {first = 3, second = 5}}
```

```
int n, a[N];
vector< pair<int, int> > maxgcd[N];

void insert(int x, int val, int mx) {
    for (int i = 0; i < (int)maxgcd[x].size(); ++i)
        if (maxgcd[x][i].first == val) {
            maxgcd[x][i].second = max(maxgcd[x][i].second, mx);
            return;
        }
    maxgcd[x].push_back( make_pair(val, mx) );
}

int main() {
    scanf("%d", &n);
    for (int i = 1; i <= n; ++i) scanf("%d", &a[i]);
    for (int i = 0; i <= n + 1; ++i) maxgcd[i].clear();
    for (int i = 1; i <= n; ++i) {
```

```
        for (int j = 0; j < (int)maxgcd[i - 1].size(); ++j)
            insert(i, __gcd(maxgcd[i-1][j].first, a[i]), maxgcd[i-1][j].second);
        insert(i, a[i], i);
    }
}
```

8.6 高精度计算

```
#include<algorithm>
using namespace std;
const int N_huge=850,base=100000000;
char s[N_huge*10];
struct huge{
    typedef long long value;
    value a[N_huge];int len;
    void clear(){len=1;a[len]=0;}
    huge(){clear();}
    huge(value x){*this=x;}
    huge operator =(huge b){
        len=b.len;for (int i=1;i<=len;++i)a[i]=b.a[i]; return *this;
    }
    huge operator =(value x){
        len=0;
        while (x)a[++len]=x%base,x/=base;
        if (!len)a[++len]=0;
        return *this;
    }
    huge operator +(huge b){
        int L=len>b.len?len:b.len;huge tmp;
        for (int i=1;i<=L+1;++i)tmp.a[i]=0;
        for (int i=1;i<=L;++i){
            if (i>len)tmp.a[i]+=b.a[i];
            else if (i>b.len)tmp.a[i]+=a[i];
            else {
                tmp.a[i]+=a[i]+b.a[i];
                if (tmp.a[i]>=base){
                    tmp.a[i]-=base;++tmp.a[i+1];
                }
            }
        }
        if (tmp.a[L+1])tmp.len=L+1;
            else tmp.len=L;
        return tmp;
    }
    huge operator -(huge b){
        int L=len>b.len?len:b.len;huge tmp;
        for (int i=1;i<=L+1;++i)tmp.a[i]=0;
        for (int i=1;i<=L;++i){
            if (i>b.len)b.a[i]=0;
```

```
            tmp.a[i]+=a[i]-b.a[i];
            if (tmp.a[i]<0){
                tmp.a[i]+=base;--tmp.a[i+1];
            }
        }
        while (L>1&&!tmp.a[L])--L;
        tmp.len=L;
        return tmp;
    }
    huge operator *(huge b){
        int L=len+b.len;huge tmp;
        for (int i=1;i<=L;++i)tmp.a[i]=0;
        for (int i=1;i<=len;++i)
            for (int j=1;j<=b.len;++j){
                tmp.a[i+j-1]+=a[i]*b.a[j];
                if (tmp.a[i+j-1]>=base){
                    tmp.a[i+j]+=tmp.a[i+j-1]/base;
                    tmp.a[i+j-1]%=base;
                }
            }
        tmp.len=len+b.len;
        while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
        return tmp;
    }
    pair<huge,huge> divide(huge a,huge b){
        int L=a.len;huge c,d;
        for (int i=L;i;--i){
        c.a[i]=0;d=d*base;d.a[1]=a.a[i];
            int l=0,r=base-1,mid;
            while (l<r){
                mid=(l+r+1)>>1;
                if (b*mid<=d)l=mid;
                    else r=mid-1;
            }
            c.a[i]=l;d-=b*l;
        }
        while (L>1&&!c.a[L])--L;c.len=L;
        return make_pair(c,d);
    }
    huge operator /(value x){
        value d=0;huge tmp;
        for (int i=len;i;--i){
            d=d*base+a[i];
            tmp.a[i]=d/x;d%=x;
        }
        tmp.len=len;
        while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
        return tmp;
    }
    value operator %(value x){
```

```
        value d=0;
        for (int i=len;i;--i)d=(d*base+a[i])%x;
        return d;
    }
    huge operator /(huge b){return divide(*this,b).first;}
    huge operator %(huge b){return divide(*this,b).second;}
    huge &operator +=(huge b){*this=*this+b;return *this;}
    huge &operator -=(huge b){*this=*this-b;return *this;}
    huge &operator *=(huge b){*this=*this*b;return *this;}
    huge &operator ++(){huge T;T=1;*this=*this+T;return *this;}
    huge &operator --(){huge T;T=1;*this=*this-T;return *this;}
    huge operator ++(int){huge T,tmp=*this;T=1;*this=*this+T;return tmp;}
    huge operator --(int){huge T,tmp=*this;T=1;*this=*this-T;return tmp;}
    huge operator +(value x){huge T;T=x;return *this+T;}
    huge operator -(value x){huge T;T=x;return *this-T;}
    huge operator *(value x){huge T;T=x;return *this*T;}
    huge operator *=(value x){*this=*this*x;return *this;}
    huge operator +=(value x){*this=*this+x;return *this;}
    huge operator -=(value x){*this=*this-x;return *this;}
    huge operator /=(value x){*this=*this/x;return *this;}
    huge operator %=(value x){*this=*this%x;return *this;}
    bool operator ==(value x){huge T;T=x;return *this==T;}
    bool operator !=(value x){huge T;T=x;return *this!=T;}
    bool operator <=(value x){huge T;T=x;return *this<=T;}
    bool operator >=(value x){huge T;T=x;return *this>=T;}
    bool operator <(value x){huge T;T=x;return *this<T;}
    bool operator >(value x){huge T;T=x;return *this>T;}
    bool operator <(huge b){
        if (len<b.len)return 1;
        if (len>b.len)return 0;
        for (int i=len;i;--i){
            if (a[i]<b.a[i])return 1;
            if (a[i]>b.a[i])return 0;
        }
        return 0;
    }
    bool operator ==(huge b){
        if (len!=b.len)return 0;
        for (int i=len;i;--i)
            if (a[i]!=b.a[i])return 0;
        return 1;
    }
    bool operator !=(huge b){return !(*this==b);}
    bool operator >(huge b){return !(*this<b||*this==b);}
    bool operator <=(huge b){return (*this<b)||(*this==b);}
    bool operator >=(huge b){return (*this>b)||(*this==b);}
    void str(char s[]){
        int l=strlen(s);value x=0,y=1;len=0;
        for (int i=l-1;i>=0;--i){
            x=x+(s[i]-'0')*y;y*=10;
```

```
            if (y==base)a[++len]=x,x=0,y=1;
        }
        if (!len||x)a[++len]=x;
    }
    void read(){
        scanf("%s",s);this->str(s);
    }
    void print(){
        printf("%d",(int)a[len]);
        for (int i=len-1;i;--i){
            for (int j=base/10;j>=10;j/=10){
                if (a[i]<j)printf("0");
                    else break;
            }
            printf("%d",(int)a[i]);
        }
        printf("\n");
    }
}f[1005];
int main(){
    f[1]=f[2]=1;
    for(int i=3;i<=1000;i++)f[i]=f[i-1]+f[i-2];
}
```

8.7 读入优化

8.7.1 普通读入优化

```
#define rd RD<int>
#define rdll RD<long long>
template <typename Type>
inline Type RD() {
    Type x = 0;
    int flag = 0;
    char c = getchar();
    while (!isdigit(c) && c != '-')
        c = getchar();
    (c == '-') ? (flag = 1) : (x = c - '0');
    while (isdigit(c = getchar()))
        x = x * 10 + c - '0';
    return flag ? -x : x;
}
inline char rdch() {
    char c = getchar();
    while (!isalpha(c)) c = getchar();
    return c;
}
```

8.7.2 HDU 专用读入优化

接口：

int rd(int &x); 读入一个整数，保存在变量 x 中。如正常读入，返回值为 1 ，否则返回 EOF(-1)

int rdll(long long &x);

```
#define rd RD<int>
#define rdll RD<long long>

const int S = 2000000; // 2MB

char s[S], *h = s+S, *t = h;

inline char getchr(void) {
    if(h == t) {
        if (t != s + S) return EOF;
        t = s + fread(s, 1, S, stdin);
        h = s;
    }
    return *h++;
}

template <class T>
inline int RD(T &x) {
    char c = 0;
    int sign = 0;
    for (; !isdigit(c); c = getchr()) {
        if (c == EOF)
            return -1;
        if (c == '-')
            sign ^= 1;
    }
    x = 0;
    for (; isdigit(c); c = getchr())
        x = x * 10 + c - '0';
    if (sign) x = -x;
    return 1;
}
```

8.8 O2 优化

```
#define OPTIM __attribute__((optimize("-O2")))
```

8.9 正方形展开图

如图 5。

8.10 位运算及其运用

8.10.1 枚举子集

枚举 i 的非空子集 j

```
for (int j = i; j; j = (j - 1) & i);
```

8.10.2 求 1 的个数

```
int __builtin_popcount(unsigned int x);
```

8.10.3 求前缀 0 的个数

```
int __builtin_clz(unsigned int x);
```

8.10.4 求后缀 0 的个数

```
int __builtin_ctz(unsigned int x);
```

9 Vim

```
syntax on
set cindent
set nu
set tabstop=4
set shiftwidth=4
set background=dark
```

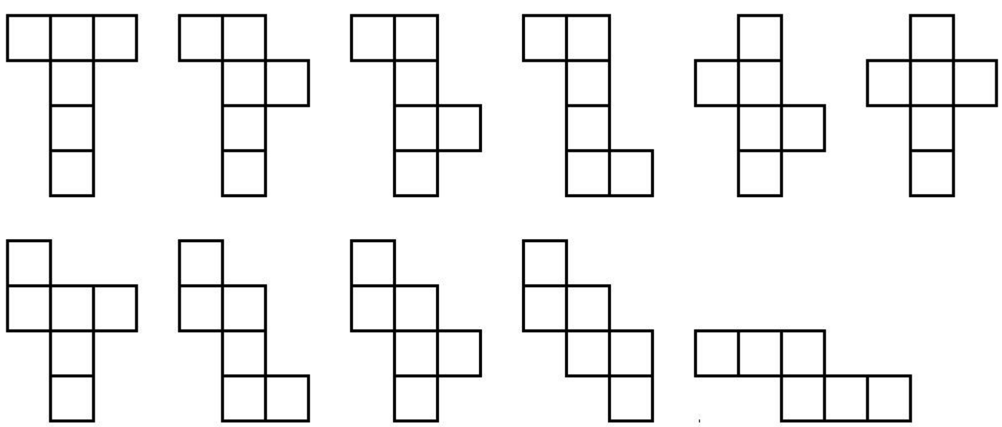

图 5: 正方形展开图

```

inoremap <C-j> <down>
inoremap <C-k> <up>
inoremap <C-h> <left>
inoremap <C-l> <right>
```