

ICPC Templates For Africamonkey

Africamonkey

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1 莫队算法

1.1 普通莫队

```
1 struct Q { int l, r, sqrtl, id; } q[N];
2 int n, m, v[N], ans[N], nowans;
3 bool cmp(const Q &a, const Q &b) {
4     if (a.sqrtl != b.sqrtl) return a.sqrtl < b.sqrtl;
5     return a.r < b.r;
6 }
7 void change(int x) { if (!v[x]) checkin(); else checkout(); }
8 int main() {
9     .....
10    for (int i=1;i<=m;i++) q[i].sqrtl = q[i].l / sqrt(n), q[i].id = i;
11    sort(q+1, q+m+1, cmp);
12    int L=1,R=0; nowans=0;
13    memset(v, 0, sizeof(v));
14    for (int i=1;i<=m;i++) {
15        while (L<q[i].l) change(L++);
16        while (L>q[i].l) change(--L);
17        while (R<q[i].r) change(++R);
18        while (R>q[i].r) change(R--);
19        ans[q[i].id] = nowans;
20    }
21    .....
22 }
```

1.2 树上莫队

```
1 struct Query { int l, r, id, l_group; } query[N];
2 struct EDGE { int adj, next; } edge[N*2];
3 int n, m, top, gh[N], c[N], reorder[N], deep[N], father[N], size[N], son[N], Top[N];
4 void addedge(int x, int y) {
5     edge[++top].adj = y;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 void dfs(int x, int root=0) {
10    reorder[x] = ++top; father[x] = root; deep[x] = deep[root] + 1;
11    son[x] = 0; size[x] = 1; int dd = 0;
12    for (int p=gh[x]; p; p=edge[p].next)
13        if (edge[p].adj != root) {
14            dfs(edge[p].adj, x);
15            if (size[edge[p].adj] > dd) {
16                son[x] = edge[p].adj;
17                dd = size[edge[p].adj];
18            }
19            size[x] += size[edge[p].adj];
20        }
```

```

21 }
22 void split(int x, int tp) {
23     Top[x] = tp;
24     if (son[x]) split(son[x], tp);
25     for (int p=gh[x]; p; p=edge[p].next)
26         if (edge[p].adj != father[x] && edge[p].adj != son[x])
27             split(edge[p].adj, edge[p].adj);
28 }
29 int lca(int x, int y) {
30     int tx = Top[x], ty = Top[y];
31     while (tx != ty) {
32         if (deep[tx] < deep[ty]) {
33             swap(tx, ty);
34             swap(x, y);
35         }
36         x = father[tx];
37         tx = Top[x];
38     }
39     if (deep[x] < deep[y]) swap(x, y);
40     return y;
41 }
42 bool cmp(const Query &a, const Query &b) {
43     if (a.l_group != b.l_group) return a.l_group < b.l_group;
44     return reorder[a.r] < reorder[b.r];
45 }
46 int v[N], ans[N];
47 void upd(int x) { if (!v[x]) checkin(); else checkout(); }
48 void go(int &u, int taru, int v) {
49     int lca0 = lca(u, taru);
50     int lca1 = lca(u, v); upd(lca1);
51     int lca2 = lca(taru, v); upd(lca2);
52     for (int x=u; x!=lca0; x=father[x]) upd(x);
53     for (int x=taru; x!=lca0; x=father[x]) upd(x);
54     u = taru;
55 }
56 int main() {
57     memset(gh, 0, sizeof(gh));
58     scanf("%d%d", &n, &m); top = 0;
59     for (int i=1; i<n; i++) {
60         int x,y; scanf("%d%d", &x, &y);
61         addedge(x, y); addedge(y, x);
62     }
63     top = 0; dfs(1); split(1, 1);
64     for (int i=1; i<=m; i++) {
65         if (reorder[query[i].l] > reorder[query[i].r])
66             swap(query[i].l, query[i].r);
67         query[i].id = i;
68         query[i].l_group = reorder[query[i].l] / sqrt(n);
69     }
70     sort(query+1, query+m+1, cmp);

```

```

71     int L=1,R=1; upd(1);
72     for (int i=1;i<=m;i++) {
73         go(L,query[i].l,R);
74         go(R,query[i].r,L);
75         ans[query[i].id] = answer();
76     }
77     .....
78 }

```

2 字符串

2.1 哈希

```

1  const int P=31,D=1000173169;
2  int n, pow[N], f[N]; char a[N];
3  int hash(int l, int r) { return (LL) (f[r]-(LL)f[l-1]*pow[r-l+1]%D+D)%D; }
4  int main() {
5      scanf("%d%s", &n, a+1);
6      pow[0] = 1;
7      for (int i=1;i<=n;i++) pow[i] = (LL)pow[i-1]*P%D;
8      for (int i=1;i<=n;i++) f[i] = (LL) ((LL)f[i-1]*P+a[i])%D;
9  }

```

2.2 KMP

接口: `int find_substring(char *pattern, char *text, int *next, int *ret);`

输入: 模式串, 匹配串

输出: 返回值表示模式串在匹配串中出现的次数

KMP 的 `next[i]` 表示从 0 到 i 的字符串 s, 前缀和后缀的最长重叠长度。

```

1  void find_next(char *pattern, int *next) {
2      int n = strlen(pattern);
3      for (int i=1;i<n;i++) {
4          int j = i;
5          while (j > 0) {
6              j = next[j];
7              if (pattern[j] == pattern[i]) {
8                  next[i+1] = j+1;
9                  break;
10             }
11         }
12     }
13 }
14 int find_substring(char *pattern, char *text, int *next, int *ret) {
15     find_next(pattern, next);
16     int n = strlen(pattern);
17     int m = strlen(text);

```

```

18     int k = 0;
19     for (int i=0, j=0; i<m; i++) {
20         if (j<n && text[i]==pattern[j]) {
21             j++;
22         } else {
23             while (j>0) {
24                 j = next[j];
25                 if (text[i] == pattern[j]) {
26                     j++;
27                     break;
28                 }
29             }
30         }
31         if (j == n)
32             ret[k++] = i-n+1;
33     }
34     return k;
35 }

```

2.3 可动态修改的 KMP

支持：加入一个字符，删除一个字符。

时间复杂度： $O(n\alpha)$ ， α 为字符集大小。

代码中的字符为 '0' - '9'，可自行修改为 'a' - 'z'

```

1 char t[N];
2 int top, nxt[N], nxt_l[N][10];
3 inline void del_letter() { --top; }
4 inline void add_letter(char x) {
5     t[top++] = x;
6     int j = top-1;
7     memset(nxt_l[top], 0, sizeof(nxt_l[top]));
8     nxt[top] = nxt_l[top-1][x-'0'];
9     memcpy(nxt_l[top], nxt_l[nxt[top]], sizeof(nxt_l[nxt[top]]));
10    nxt_l[top][t[nxt[top]]-'0'] = nxt[top]+1;
11 }

```

2.4 扩展 KMP

接口：void ExtendedKMP(char *a, char *b, int *next, int *ret);

输出：

next: a 关于自己每个后缀的最长公共前缀

ret: a 关于 b 的每个后缀的最长公共前缀

EXKMP 的 next[i] 表示：从 i 到 n-1 的字符串 st 前缀和原串前缀的最长重叠长度。

```

1 void get_next(char *a, int *next) {
2     int i, j, k;
3     int n = strlen(a);

```

```

4     for (j = 0; j+1<n && a[j]==a[j+1];j++);
5     next[1] = j;
6     k = 1;
7     for (i=2;i<n;i++) {
8         int len = k+next[k], l = next[i-k];
9         if (l < len-i) {
10             next[i] = l;
11         } else {
12             for (j = max(0, len-i);i+j<n && a[j]==a[i+j];j++);
13             next[i] = j;
14             k = i;
15         }
16     }
17 }
18 void ExtendedKMP(char *a, char *b, int *next, int *ret) {
19     get_next(a, next);
20     int n = strlen(a), m = strlen(b);
21     int i, j, k;
22     for (j=0;j<n && j<m && a[j]==b[j];j++);
23     ret[0] = j;
24     k = 0;
25     for (i=1;i<m;i++) {
26         int len = k+ret[k], l = next[i-k];
27         if (l < len-i) {
28             ret[i] = l;
29         } else {
30             for (j = max(0, len-i);j<n && i+j<m && a[j]==b[i+j];j++);
31             ret[i] = j;
32             k = i;
33         }
34     }
35 }

```

2.5 Manacher

$p[i]$ 表示以 i 为对称轴的最长回文串长度

```

1 char st[N*2], s[N];
2 int len, p[N*2];
3
4 while (scanf("%s", s) != EOF) {
5     len = strlen(s);
6     st[0] = '$', st[1] = '#';
7     for (int i=1;i<=len;i++)
8         st[i*2] = s[i-1], st[i*2+1] = '#';
9     len = len * 2 + 2;
10    int mx = 0, id = 0, ans = 0;
11    for (int i=1;i<=len;i++) {
12        p[i] = (mx > i) ? min(p[id*2-i]+1, mx-i) : 1;
13        for (; st[i+p[i]] == st[i-p[i]]; ++p[i]) ;

```

```

14         if (p[i]+i > mx) mx = p[i]+i, id = i;
15         p[i] --;
16         if (p[i] > ans) ans = p[i];
17     }
18     printf("%d\n", ans);
19 }

```

2.6 最小表示法

```

1 string smallestRepresation(string s) {
2     int i, j, k, l;
3     int n = s.length();
4     s += s;
5     for (i=0, j=1; j<n; ) {
6         for (k=0; k<n && s[i+k]==s[j+k]; k++);
7         if (k>=n) break;
8         if (s[i+k]<s[j+k]) j+=k+1;
9         else {
10             l=i+k;
11             i=j;
12             j=max(l, j)+1;
13         }
14     }
15     return s.substr(i, n);
16 }

```

2.7 AC 自动机

```

1 struct Node {
2     int next[**Size of Alphabet**];
3     int terminal, fail;
4 } node[**Number of Nodes**];
5 int top;
6 void add(char *st) {
7     int len = strlen(st), x = 1;
8     for (int i=0; i<len; i++) {
9         int ind = trans(st[i]);
10        if (!node[x].next[ind])
11            node[x].next[ind] = ++top;
12        x = node[x].next[ind];
13    }
14    node[x].terminal = 1;
15 }
16 int q[**Number of Nodes**], head, tail;
17 void build() {
18     head = 0, tail = 1; q[1] = 1;
19     while (head != tail) {
20         int x = q[++head];

```



```

21      /*(when necessary) node[x].terminal != node[node[x].fail].terminal; */
22      for (int i=0;i<n;i++)
23          if (node[x].next[i]) {
24              if (x == 1) node[node[x].next[i]].fail = 1;
25              else {
26                  int y = node[x].fail;
27                  while (y) {
28                      if (node[y].next[i]) {
29                          node[node[x].next[i]].fail = node[y].next[i];
30                          break;
31                      }
32                      y = node[y].fail;
33                  }
34                  if (!node[node[x].next[i]].fail) node[node[x].next[i]].fail = 1;
35              }
36              q[++tail] = node[x].next[i];
37          }
38      }
39  }

```

2.8 后缀数组

2.8.1 倍增算法

参数 m 表示字符集的大小, 即 $0 \leq r_i < m$

```

1  #define rank rank2
2  int n, r[N], wa[N], wb[N], ws[N], sa[N], rank[N], height[N];
3  int cmp(int *r, int a, int b, int l, int n) {
4      if (r[a]==r[b]) {
5          if (a+l<n && b+l<n && r[a+l]==r[b+l])
6              return 1;
7      }
8      return 0;
9  }
10 void suffix_array(int m) {
11     int i, j, p, *x=wa, *y=wb, *t;
12     for (i=0;i<m;i++) ws[i]=0;
13     for (i=0;i<n;i++) ws[x[i]=r[i]]++;
14     for (i=1;i<m;i++) ws[i]+=ws[i-1];
15     for (i=n-1;i>=0;i--) sa[--ws[x[i]]]=i;
16     for (j=1;p=1;p<n;m=p,j<=1) {
17         for (p=0,i=n-j;i<n;i++) y[p++]=i;
18         for (i=0;i<n;i++) if (sa[i]>=j) y[p++]=sa[i]-j;
19         for (i=0;i<m;i++) ws[i]=0;
20         for (i=0;i<n;i++) ws[x[y[i]]]++;
21         for (i=1;i<m;i++) ws[i]+=ws[i-1];
22         for (i=n-1;i>=0;i--) sa[--ws[x[y[i]]]]=y[i];
23         for (t=x,x=y,y=t,x[sa[0]]=0,i=1,p=1;i<n;i++)
24             x[sa[i]]=cmp(y,sa[i-1],sa[i],j,n)?p-1:p++;

```

```

25     }
26     for (i=0;i<n;i++) rank[sa[i]]=i;
27     rank[n] = -1;
28 }
29 void calc_height() {
30     int j=0;
31     for (int i=0;i<n;i++)
32         if (rank[i])
33             {
34                 while (r[i+j]==r[sa[rank[i]-1]+j]) j++;
35                 height[rank[i]]=j;
36                 if (j) j--;
37             }
38 }

```

2.8.2 DC3 算法

注意：

N 至少为字符串长度的 3 倍

接口：suffix_array(int *r, int *sa, int n, int m);

r 表示字符串， sa 为后缀数组输出， n 表示字符串长度，下标从 0 开始。 m 为字符集大小。

```

1  #define F(x) ((x)/3 + ((x)%3 == 1 ? 0:tb))
2  #define G(x) ((x) < tb ? (x)*3+1 : ((x)-tb)*3 + 2)
3  #define rank rank2
4
5  int r[N], wa[N], wb[N], ws[N], wv[N], sa[N], rank[N];
6
7  int c0(int *r,int a,int b) {
8      return r[a]==r[b]&&r[a+1]==r[b+1]&&r[a+2]==r[b+2];
9  }
10
11 int c12(int k,int *r,int a,int b) {
12     if(k==2) return r[a]<r[b] || r[a]==r[b]&&c12(1,r,a+1,b+1);
13     else return r[a]<r[b] || r[a]==r[b]&&wv[a+1]<wv[b+1];
14 }
15
16 void dsort(int *r,int *a,int *b,int n,int m) {
17     int i;for(i=0;i<n;i++) wv[i]=r[a[i]];
18     for(i=0;i<m;i++) ws[i]=0;
19     for(i=0;i<n;i++) ws[wv[i]]++;
20     for(i=1;i<m;i++) ws[i]+=ws[i-1];
21     for(i=n-1;i>=0;i--) b[--ws[wv[i]]]=a[i];
22 }
23
24 void dc3(int *r,int *sa,int n,int m) {
25     int i,j,*rn=r+n,*san=sa+n,ta=0,tb=(n+1)/3,tbc=0,p;
26     r[n]=r[n+1]=0;
27     for(i=0;i<n;i++) if(i%3!=0) wa[tbc++]=i;

```

```

28     dsort(r+2, wa, wb, tbc, m);
29     dsort(r+1, wb, wa, tbc, m);
30     dsort(r, wa, wb, tbc, m);
31     for(p=1, rn[F(wb[0])]=0, i=1; i<tbc; i++) rn[F(wb[i])]=c0(r, wb[i-1], wb[i])?p-1:p++;
32     if(p<tbc) dc3(rn, san, tbc, p);
33     else for(i=0; i<tbc; i++) san[rn[i]]=i;
34     for(i=0; i<tbc; i++) if(san[i]<tb) wb[ta++]=san[i]*3;
35     if(n%3==1) wb[ta++]=n-1;
36     dsort(r, wb, wa, ta, m);
37     for(i=0; i<tbc; i++) wv[wb[i]=G(san[i])]=i;
38     for(i=0, j=0, p=0; i<ta && j<tbc; p++) sa[p]=c12(wb[j]%3, r, wa[i], wb[j])?wa[i++]:wb[j
    ++];
39     for(; i<ta; p++) sa[p]=wa[i++];
40     for(; j<tbc; p++) sa[p]=wb[j++];
41 }
42
43 void suffix_array(int *r, int *sa, int n, int m) {
44     dc3(r, sa, n+1, m);
45     int top = 0;
46     for (int i = 0; i < n+1; ++i)
47         if (sa[i] < n) sa[top++] = sa[i];
48     for (int i = 0; i < n; ++i) rank[sa[i]] = i;
49     rank[n] = -1;
50 }

```

2.8.3 小技巧：拼接字符串

接口：

`int gao1(int l, int r, int c, int p)`; 区间 $[l, r)$ 中保证第 0 位到第 $c-1$ 位都是相同的（设为字符串 s ），现在我们在 s 后面接一个字符 p ，得到一个新的字符串 s' 。返回值为最小的 k 满足后缀 $sa[k]$ 前 $c+1$ 位为 s'

`int gao2(int l, int r, int c, int p)`; 区间 $[l, r)$ 中保证第 0 位到第 $c-1$ 位都是相同的（设为字符串 s ），现在我们在 s 后面接一个后缀 $sa[p]$ ，得到一个新的字符串 s' 。返回值为最小的 k 满足后缀 $sa[k]$ 前 $c+len(sa[p])$ 位为 s'

```

1  int gao1(int l, int r, int c, int p) {
2      --l;
3      while (l+1<r) {
4          int md=(l+r)>>1;
5          if (sa[md]+c<n&&s[sa[md]+c]>=p) r=md; else l=md;
6      }
7      return r;
8  }
9  int gao2(int l, int r, int c, int p) {
10     --l;
11     while (l+1<r) {
12         int md=(l+r)>>1;
13         if (sa[md]+c<n&&rk[sa[md]+c]>=p) r=md; else l=md;
14     }

```

```

15     return r;
16 }

```

示例调用：

```

1  suf1[m] = -1, suf2[m] = n;
2  for (int i = m - 1; i >= 0; --i) {
3      int l = gao1(0, n, 0, t[i]), r = gao1(0, n, 0, t[i]);
4      suf1[i] = gao2(l, r, 1, suf1[i + 1]);
5      suf2[i] = gao2(l, r, 1, suf2[i + 1]);
6  }

```

2.9 后缀自动机

下面的代码是求两个串的 LCS（最长公共子串）。

```

1  #include <cstdio>
2  #include <cstdlib>
3  #include <cstring>
4  #define N 500001
5  using namespace std;
6  char st[N];
7  int pre[N<<1], son[26][N<<1], step[N<<1], last, total;
8  int apply(int x) { step[++total]=x; return total; }
9  void Extend(char x) {
10     int p = last, np = apply(step[last]+1);
11     for (; p && !son[x][p]; p=pre[p]) son[x][p] = np;
12     if (!p) pre[np] = 1;
13     else {
14         int q = son[x][p];
15         if (step[p]+1 == step[q]) pre[np] = q;
16         else {
17             int nq = apply(step[p]+1);
18             for (int i=0;i<26;i++) son[i][nq] = son[i][q];
19             pre[nq] = pre[q];
20             pre[q] = pre[np] = nq;
21             for (; p && son[x][p]==q; p=pre[p]) son[x][p] = nq;
22         }
23     }
24     last = np;
25 }
26 void init() {
27     last = total = 0;
28     last = apply(0);
29     scanf("%s",st);
30     for (int i=0; st[i]; i++)
31         Extend(st[i]-'a');
32     scanf("%s",st);
33 }
34 int main() {

```

```

35     init();
36     int p = 1, now = 0, ans = 0;
37     for (int i=0; st[i]; i++) {
38         int index = st[i]-'a';
39         for (; p && !son[index][p]; p = pre[p], now = step[p]) ;
40         if (!p) p = 1;
41         if (son[index][p]) {
42             p = son[index][p];
43             now++;
44             if (now > ans) ans = now;
45         }
46     }
47     printf("%d\n",ans);
48     return 0;
49 }

```

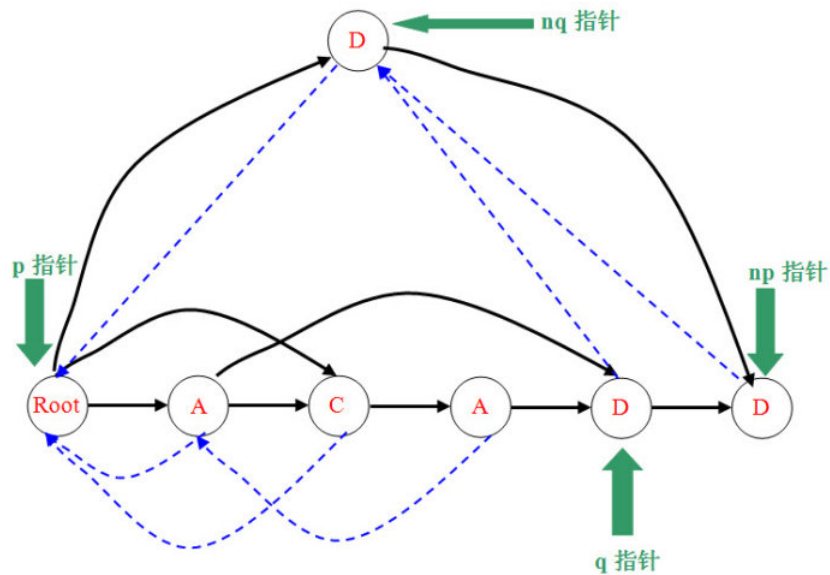


图 1: ACADD 构成的后缀自动机

我们发现 fail 构出一棵前缀树

和后缀树相同，为了使每个前缀都是叶子结点，我们不妨在串 s 前加入一个没出现的字符'#'

3 数据结构

3.1 ST 表

```

1  int Log[N], f[17][N];
2  int ask(int x, int y) {
3      int k=log[y-x+1];
4      return max(f[k][x], f[k][y-(1<<k)+1]);
5  }

```

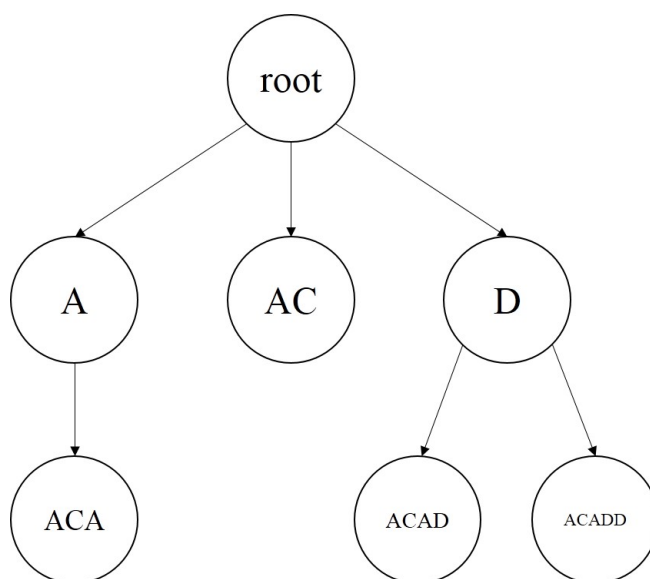


图 2: 串 ACADD 按 fail 构出的前缀树, 与图 1 对应

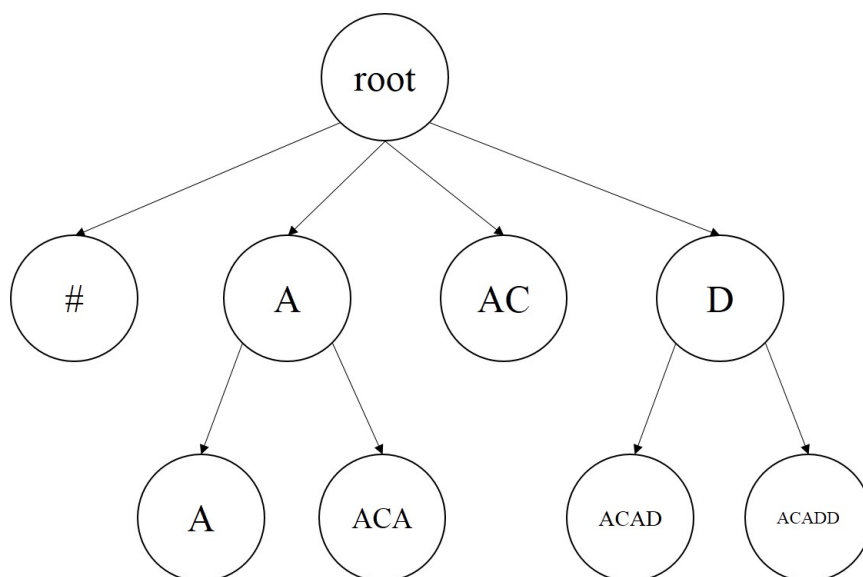


图 3: 串 #ACADD 按 fail 构出的前缀树

```

6 int main() {
7     for (i=2; i<=n; i++) Log[i]=Log[i>>1]+1;
8     for (j=1; j<K; j++) for (i=1; i+(1<<j-1)<=n; i++) f[j][i]=max(f[j-1][i], f[j-1][i+(1<<j-1)]);
9 }

```

3.2 线段树小技巧

给定一个序列 a ，寻找一个最大的 i 使得 $i \leq y$ 且满足一些条件（如 $a[i] \geq w$ ，那么需要在线段树维护 a 的区间最大值）

```

1 int queryl(int p, int left, int right, int y, int w) {
2     if (right <= y) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (y <= mid) return queryl(p<<1|0, left, mid, y, w);
8     int ret = queryl(p<<1|1, mid+1, right, y, w);
9     if (ret != -1) return ret;
10    return queryl(p<<1|0, left, mid, y, w);
11 }

```

给定一个序列 a ，寻找一个最小的 i 使得 $i \geq x$ 且满足一些条件（如 $a[i] \geq w$ ，那么需要在线段树维护 a 的区间最大值）

```

1 int queryr(int p, int left, int right, int y, int w) {
2     if (left >= x) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (x > mid) return queryr(p<<1|1, mid+1, right, y, w);
8     int ret = queryr(p<<1|0, left, mid, y, w);
9     if (ret != -1) return ret;
10    return queryr(p<<1|1, mid+1, right, y, w);
11 }

```

3.3 Splay

接口：

ADD $x \ y \ d$ ：将 $[x, y]$ 的所有数加上 d

REVERSE $x \ y$ ：将 $[x, y]$ 翻转

INSERT $x \ p$ ：将 p 插入到第 x 个数的后面

DEL x ：将第 x 个数删除

```

1 struct SPLAY {
2     struct NODE {
3         int w, min;

```

```

4      int son[2], size, father, rev, lazy;
5  } node[N];
6  int top, rt;
7  void pushdown(int x) {
8      if (!x) return;
9      if (node[x].rev) {
10         node[node[x].son[0]].rev ^= 1;
11         node[node[x].son[1]].rev ^= 1;
12         swap(node[x].son[0], node[x].son[1]);
13         node[x].rev = 0;
14     }
15     if (node[x].lazy) {
16         node[node[x].son[0]].lazy += node[x].lazy;
17         node[node[x].son[1]].lazy += node[x].lazy;
18         node[x].w += node[x].lazy;
19         node[x].min += node[x].lazy;
20         node[x].lazy = 0;
21     }
22 }
23 void pushup(int x) {
24     if (!x) return;
25     pushdown(node[x].son[0]);
26     pushdown(node[x].son[1]);
27     node[x].size = node[node[x].son[0]].size + node[node[x].son[1]].size + 1;
28     node[x].min = node[x].w;
29     if (node[x].son[0]) node[x].min = min(node[x].min, node[node[x].son[0]].min)
30     ;
31     if (node[x].son[1]) node[x].min = min(node[x].min, node[node[x].son[1]].min)
32     ;
33 }
34 void sc(int x, int y, int w) {
35     node[x].son[w] = y;
36     node[y].father = x;
37     pushup(x);
38 }
39 void _ins(int w) {
40     top++;
41     node[top].w = node[top].min = w;
42     node[top].son[0] = node[top].son[1] = 0;
43     node[top].size = 1; node[top].father = 0; node[top].rev = 0;
44 }
45 void init() {
46     top = 0;
47     _ins(0); _ins(0); rt=1;
48     sc(1, 2, 1);
49 }
50 void rotate(int x) {
51     if (!x) return;
52     int y = node[x].father;
53     int w = node[y].son[1]==x;

```



```

52     sc(y, node[x].son[w^1], w);
53     sc(node[y].father, x, node[node[y].father].son[1]==y);
54     sc(x, y, w^1);
55 }
56 int q[N];
57 void flushdown(int x) {
58     int t=0; for (; x; x=node[x].father) q[++t]=x;
59     for (; t; t--) pushdown(q[t]);
60 }
61 void Splay(int x, int root=0) {
62     flushdown(x);
63     while (node[x].father != root) {
64         int y=node[x].father;
65         int w=node[y].son[1]==x;
66         if (node[y].father != root && node[node[y].father].son[w]==y) rotate(y);
67         rotate(x);
68     }
69 }
70 int find(int k) {
71     Splay(rt);
72     while (1) {
73         pushdown(rt);
74         if (node[node[rt].son[0]].size+1==k) {
75             Splay(rt);
76             return rt;
77         } else
78         if (node[node[rt].son[0]].size+1<k) {
79             k-=node[node[rt].son[0]].size+1;
80             rt=node[rt].son[1];
81         } else {
82             rt=node[rt].son[0];
83         }
84     }
85 }
86 int split(int x, int y) {
87     int fx = find(x);
88     int fy = find(y+2);
89     Splay(fx);
90     Splay(fy, fx);
91     return node[fy].son[0];
92 }
93 void add(int x, int y, int d) { //add d to each number in a[x]...a[y]
94     int t = split(x, y);
95     node[t].lazy += d;
96     Splay(t); rt=t;
97 }
98 void reverse(int x, int y) { // reverse the x-th to y-th elements
99     int t = split(x, y);
100    node[t].rev ^= 1;
101    Splay(t); rt=t;

```

```

102     }
103     void insert(int x, int p) { // insert p after the x-th element
104         int fx = find(x+1);
105         int fy = find(x+2);
106         Splay(fx);
107         Splay(fy, fx);
108         _ins(p);
109         sc(fy, top, 0);
110         Splay(top); rt=top;
111     }
112     void del(int x) { // delete the x-th element in Splay
113         int fx = find(x), fy = find(x+2);
114         Splay(fx); Splay(fy, fx);
115         node[fy].son[0] = 0;
116         Splay(fy); rt=fy;
117     }
118 } tree;

```

3.4 可持久化 Treap

接口：

void insert(int x, char c); 在当前第 x 个字符后插入 c

void del(int x, int y); 删除第 x 个字符到第 y 个字符

void copy(int l, int r, int x); 复制第 l 个字符到第 r 个字符，然后粘贴到第 x 个字符后

void reverse(int x, int y); 翻转第 x 个到第 y 个字符

char query(int k); 表示询问当前第 x 个字符是什么

```

1  #define mod 1000000007
2  struct Treap {
3      struct Node {
4          char key;
5          bool reverse;
6          int lc, rc, size;
7      } node[N];
8      int n, root, rd;
9      int Rand() { rd = (rd * 20372052LL + 25022087LL) % mod; return rd; }
10     void init() { n = root = 0; }
11     inline int copy(int x) { node[++n] = node[x]; return n; }
12     inline void pushdown(int x) {
13         if (!node[x].reverse) return;
14         if (node[x].lc) node[x].lc = copy(node[x].lc);
15         if (node[x].rc) node[x].rc = copy(node[x].rc);
16         swap(node[x].lc, node[x].rc);
17         node[node[x].lc].reverse ^= 1;
18         node[node[x].rc].reverse ^= 1;
19         node[x].reverse = 0;
20     }
21     inline void pushup(int x) { node[x].size = node[node[x].lc].size + node[node[x].rc].size + 1; }

```

```

22  int merge(int u, int v) {
23      if (!u || !v) return u+v;
24      pushdown(u); pushdown(v);
25      int t = Rand() % (node[u].size + node[v].size), r;
26      if (t < node[u].size) {
27          r = copy(u);
28          node[r].rc = merge(node[u].rc, v);
29      } else {
30          r = copy(v);
31          node[r].lc = merge(u, node[v].lc);
32      }
33      pushup(r);
34      return r;
35  }
36  int split(int u, int x, int y) {
37      if (x > y) return 0;
38      pushdown(u);
39      if (x == 1 && y == node[u].size) return u;
40      if (y <= node[node[u].lc].size) return split(node[u].lc, x, y);
41      int t = node[node[u].lc].size + 1;
42      if (x > t) return split(node[u].rc, x-t, y-t);
43      int num = copy(u);
44      node[num].lc = split(node[u].lc, x, t-1);
45      node[num].rc = split(node[u].rc, 1, y-t);
46      pushup(num);
47      return num;
48  }
49  void insert(int x, char c) {
50      int t1 = split(root, 1, x), t2 = split(root, x+1, node[root].size);
51      node[++n].key = c; node[n].size = 1;
52      root = merge(merge(t1, n), t2);
53  }
54  void del(int x, int y) {
55      int t1 = split(root, 1, x-1), t2 = split(root, y+1, node[root].size);
56      root = merge(t1, t2);
57  }
58  void copy(int l, int r, int x) {
59      int t1 = split(root, 1, x), t2 = split(root, 1, r), t3 = split(root, x+1,
60          node[root].size);
61      root = merge(merge(t1, t2), t3);
62  }
63  void reverse(int x, int y) {
64      int t1 = split(root, 1, x-1), t2 = split(root, x, y), t3 = split(root, y+1,
65          node[root].size);
66      node[t2].reverse ^= 1;
67      root = merge(merge(t1, t2), t3);
68  }
69  char query(int k) {
70      int x = root;
71      while (1) {

```

```

70         pushdown(x);
71         if (k <= node[node[x].lc].size) x = node[x].lc;
72         else
73             if (k == node[node[x].lc].size + 1) return node[x].key;
74         else
75             k -= node[node[x].lc].size + 1, x = node[x].rc;
76     }
77 }
78 } treap;

```

3.5 可持久化并查集

接口：

void init() 初始化

void merge(int x, int y, int time) 在 time 时刻将 x 和 y 连一条边，注意加边顺序必须按 time 从小到大加边

void GetFather(int x, int time) 询问 time 时刻及以前的连边状态中，x 所属的集合

```

1 namespace pers_union {
2     const int inf = 0x3f3f3f3f;
3     int father[N], Father[N], Time[N];
4     vector<int> e[N];
5     void init() {
6         for (int i=1;i<=n;i++) {
7             father[i] = i;
8             Father[i] = i;
9             Time[i] = inf;
10            e[i].clear();
11            e[i].push_back(i);
12        }
13    }
14    int getfather(int x) {
15        return (father[x] == x) ? x : father[x] = getfather(father[x]);
16    }
17    int GetFather(int x, int time) {
18        return (Time[x] <= time) ? GetFather(Father[x], time) : x;
19    }
20    void merge(int x, int y, int time) {
21        int fx = getfather(x), fy = getfather(y);
22        if (fx == fy) return;
23        if (e[fx].size() > e[fy].size()) swap(fx, fy);
24        father[fx] = fy;
25        Father[fx] = fy;
26        Time[fx] = time;
27        for (int i=0;i<e[fx].size();i++) {
28            e[fy].push_back(e[fx][i]);
29        }
30    }
31 };

```

4 树

4.1 点分治

初始化时须设置 $top = 1$ 。

```
1 void addedge(int x, int y) {
2     edge[++top].adj = y;
3     edge[top].valid = 1;
4     edge[top].next = gh[x];
5     gh[x] = top;
6 }
7 void get_size(int x, int root=0) {
8     size[x] = 1; son[x] = 0;
9     int dd = 0;
10    for (int p=gh[x]; p; p=edge[p].next)
11        if (edge[p].adj != root && edge[p].valid) {
12            get_size(edge[p].adj, x);
13            size[x] += size[edge[p].adj];
14            if (size[edge[p].adj] > dd) {
15                dd = size[edge[p].adj];
16                son[x] = edge[p].adj;
17            }
18        }
19 }
20 int getroot(int x) {
21     get_size(x);
22     int sz = size[x];
23     while (size[son[x]] > sz/2)
24         x = son[x];
25     return x;
26 }
27 void dc(int x) {
28     x = getroot(x);
29     static int list[N], ltop;
30     ltop = 0;
31     for (int p=gh[x]; p; p=edge[p].next)
32         if (edge[p].valid)
33             list[++ltop] = p;
34     clear();
35     for (int i=1; i<=ltop; i++) {
36         update();
37         modify();
38     }
39     clear();
40     for (int i=ltop; i>=1; i--) {
41         update();
42         modify();
43     }
44     //be careful about the root
45     for (int p=gh[x]; p; p=edge[p].next)
```

```

46         if (edge[p].valid) {
47             edge[p].valid = 0;
48             edge[p^1].valid = 0;
49             dc(edge[p].adj);
50         }
51     }

```

4.2 动态树

接口：

command(x, y)：将 x 到 y 路径的 Splay Tree 分离出来。

linkcut(u1, v1, u2, v2)：将树中原有的边 (u1, v1) 删除，加入一条新边 (u2, v2)

```

1  struct DynamicTREE{
2      struct NODE{
3          int father, son[2], top, size, reverse;
4      } splay[N];
5      void init(int i, int fat) {
6          splay[i].father = splay[i].son[0] = splay[i].son[1] = 0;
7          splay[i].top = fat; splay[i].size = 1; splay[i].reverse = 0;
8      }
9      void pushdown(int x) {
10         if (!x) return;
11         int s0 = splay[x].son[0], s1 = splay[x].son[1];
12         if (splay[x].reverse) {
13             splay[s0].reverse ^= 1;
14             splay[s1].reverse ^= 1;
15             swap(splay[x].son[0], splay[x].son[1]);
16             splay[x].reverse = 0;
17         }
18         s0 = splay[x].son[0], s1 = splay[x].son[1];
19         splay[s0].top = splay[s1].top = splay[x].top;
20     }
21     void pushup(int x) {
22         if (!x) return;
23         pushdown(splay[x].son[0]);
24         pushdown(splay[x].son[1]);
25         splay[x].size = splay[splay[x].son[0]].size + splay[splay[x].son[1]].size +
26             1;
27     }
28     void sc(int x, int y, int w, bool Auto=true) {
29         splay[x].son[w] = y;
30         splay[y].father = x;
31         if (Auto) {
32             pushup(y);
33             pushup(x);
34         }
35     }
36     int top, tush[N];

```

```

36 void flowdown(int x) {
37     for (top=1; x; top++, x = splay[x].father) tush[top] = x;
38     for (; top; top--) pushdown(tush[top]);
39 }
40 void rotate(int x) {
41     if (!x) return;
42     int y = splay[x].father;
43     int w = splay[y].son[1] == x;
44     pushdown(y);
45     pushdown(x);
46     sc(splay[y].father, x, splay[splay[y].father].son[1]==y, false);
47     sc(y, splay[x].son[w^1], w, false);
48     sc(x, y, w^1, false);
49     pushup(y);
50     pushup(x);
51 }
52 void Splay(int x, int rt=0) {
53     if (!x) return;
54     flowdown(x);
55     while (splay[x].father != rt) {
56         int y = splay[x].father;
57         int w = splay[y].son[1]==x;
58         if (splay[y].father != rt && splay[splay[y].father].son[w] == y) rotate(
59             y);
60         rotate(x);
61     }
62 }
63 void split(int x) {
64     int y = splay[x].son[1];
65     if (!y) return;
66     splay[y].father = 0;
67     splay[x].son[1] = 0;
68     splay[y].top = x;
69     pushup(x);
70 }
71 void access(int x) {
72     int y = 0;
73     while (x) {
74         Splay(x);
75         split(x);
76         sc(x, y, 1);
77         Splay(x);
78         y = x;
79         x = splay[x].top;
80     }
81 }
82 void changeroot(int x) {
83     access(x);
84     Splay(x);
85     splay[x].reverse = 1;

```

```

85     Splay(x);
86 }
87 void command(int x, int y, ...) {
88     LL ans = 0;
89     changeroot(x);
90     access(y);
91     Splay(x);
92     //then you can modify the Splay Tree
93 }
94 void linkcut(int u1, int v1, int u2, int v2) {
95     changeroot(u1);
96     access(v1);
97     Splay(u1); split(u1);
98     splay[v1].top = 0;
99     access(u2); changeroot(u2);
100    access(v2); changeroot(v2);
101    Splay(u2); Splay(v2);
102    splay[v2].top = u2;
103 }
104 } lct;

```

5 图

5.1 欧拉回路

欧拉回路：

无向图：每个顶点的度数都是偶数，则存在欧拉回路。

有向图：每个顶点的入度 = 出度，则存在欧拉回路。

欧拉路径：

无向图：当且仅当该图所有顶点的度数为偶数，或者除了两个度数为奇数外其余的全是偶数。

有向图：当且仅当该图所有顶点出度 = 入度或者一个顶点出度 = 入度 + 1，另一个顶点入度 = 出度 + 1，其他顶点出度 = 入度。

下面 $O(n + m)$ 求欧拉回路的代码中， n 为点数， m 为边数，若有解则依次输出经过的边的编号，若是无向图，则正数表示 x 到 y ，负数表示 y 到 x 。

```

1 namespace UndirectedGraph{
2     int n,m,i,x,y,d[N],g[N],v[M<<1],w[M<<1],vis[M<<1],nxt[M<<1],ed;
3     int ans[M],cnt;
4     void add(int x,int y,int z){
5         d[x]++;
6         v[++ed]=y;w[ed]=z;nxt[ed]=g[x];g[x]=ed;
7     }
8     void dfs(int x){
9         for(int&i=g[x];i;){
10             if(vis[i]){i=nxt[i];continue;}
11             vis[i]=vis[i^1]=1;
12             int j=w[i];
13             dfs(v[i]);

```



```

14         ans[++cnt]=j;
15     }
16 }
17 void solve() {
18     scanf("%d%d",&n,&m);
19     for(i=ed=1;i<=m;i++) scanf("%d%d",&x,&y), add(x,y,i), add(y,x,-i);
20     for(i=1;i<=n;i++) if(d[i]&1) {puts("NO");return;}
21     for(i=1;i<=n;i++) if(g[i]) {dfs(i);break;}
22     for(i=1;i<=n;i++) if(g[i]) {puts("NO");return;}
23     puts("YES");
24     for(i=m;i;i--) printf("%d_",ans[i]);
25 }
26 }
27 namespace DirectedGraph{
28     int n,m,i,x,y,d[N],g[N],v[M],vis[M],nxt[M],ed;
29     int ans[M],cnt;
30     void add(int x,int y){
31         d[x]++;d[y]--;
32         v[++ed]=y;nxt[ed]=g[x];g[x]=ed;
33     }
34     void dfs(int x){
35         for(int&i=g[x];i;){
36             if(vis[i]){i=nxt[i];continue;}
37             vis[i]=1;
38             int j=i;
39             dfs(v[i]);
40             ans[++cnt]=j;
41         }
42     }
43     void solve(){
44         scanf("%d%d",&n,&m);
45         for(i=1;i<=m;i++) scanf("%d%d",&x,&y), add(x,y);
46         for(i=1;i<=n;i++) if(d[i]) {puts("NO");return;}
47         for(i=1;i<=n;i++) if(g[i]) {dfs(i);break;}
48         for(i=1;i<=n;i++) if(g[i]) {puts("NO");return;}
49         puts("YES");
50         for(i=m;i;i--) printf("%d_",ans[i]);
51     }
52 }

```

5.2 最短路径

5.2.1 Dijkstra

```

1 #include <queue>
2 using namespace std;
3 struct EDGE { int adj, w, next; } edge[M*2];
4 struct dat { int id, dist; dat(int id=0, int dist=0) : id(id), dist(dist) {} };

```

```

5 struct cmp { bool operator () (const dat &a, const dat &b) { return a.dist > b.dist;
    } };
6 priority_queue < dat, vector<dat>, cmp > q;
7 int n, top, gh[N], v[N], dist[N];
8 void addedge(int x, int y, int w) {
9     edge[++top].adj = y;
10    edge[top].w = w;
11    edge[top].next = gh[x];
12    gh[x] = top;
13 }
14 int dijkstra(int s, int t) {
15     memset(dist, 63, sizeof(dist));
16     memset(v, 0, sizeof(v));
17     dist[s] = 0;
18     q.push(dat(s, 0));
19     while (!q.empty()) {
20         dat x = q.top(); q.pop();
21         if (v[x.id]) continue; v[x.id] = 1;
22         for (int p=gh[x.id]; p; p=edge[p].next) {
23             if (x.dist + edge[p].w < dist[edge[p].adj]) {
24                 dist[edge[p].adj] = x.dist + edge[p].w;
25                 q.push(dat(edge[p].adj, dist[edge[p].adj]));
26             }
27         }
28     }
29     return dist[t];
30 }

```

5.2.2 SPFA

```

1 struct EDGE { int adj, w, next; } edge[M*2];
2 int n,m,top,gh[N],v[N],cnt[N],q[N],dist[N],head,tail;
3 void addedge(int x, int y, int w) {
4     edge[++top].adj = y;
5     edge[top].w = w;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 int spfa(int S, int T) {
10    memset(v, 0, sizeof(v));
11    memset(cnt, 0, sizeof(cnt));
12    memset(dist, 63, sizeof(dist));
13    head = 0, tail = 1;
14    dist[S] = 0; q[1] = S;
15    while (head != tail) {
16        (head += 1) %= N;
17        int x = q[head]; v[x] = 0;
18        ++cnt[x]; if (cnt[x] > n) return -1;
19        for (int p=gh[x]; p; p=edge[p].next)

```

```

20         if (dist[x] + edge[p].w < dist[edge[p].adj]) {
21             dist[edge[p].adj] = dist[x] + edge[p].w;
22             if (!v[edge[p].adj]) {
23                 v[edge[p].adj] = 1;
24                 (tail += 1) %= N;
25                 q[tail] = edge[p].adj;
26             }
27         }
28     }
29     return dist[T];
30 }

```

5.3 K 短路

接口：

kthsp::init(n)：初始化并设置节点个数为 n

kthsp::add(x, y, w)：添加一条 x 到 y 的有向边

kthsp::work(S, T, k)：求 S 到 T 的第 k 短路

```

1  #include <queue>
2
3  #define N 200020
4  #define M 400020
5  #define LOGM 20
6  #define LL long long
7  #define inf (1LL<<61)
8
9  namespace pheap {
10     struct Node {
11         int next, son[2];
12         LL val;
13     } node[M*LOGM];
14     int LOG[M];
15     int root[M], size[M*LOGM], top;
16     int add() {
17         ++top; assert(top < M*LOGM);
18         node[top].next = node[top].son[0] = node[top].son[1] = 0;
19         node[top].val = inf;
20         return top;
21     }
22     int copy(int x) { int t = add(); node[t] = node[x]; return t; }
23     void init() {
24         top = -1; add();
25         for (int i=2; i<M; i++) LOG[i] = LOG[i>>1] + 1;
26     }
27     void upd(int x, int &next, LL &val) {
28         if (val < node[x].val) {
29             swap(val, node[x].val);
30             swap(next, node[x].next);

```

```

31     }
32 }
33 void insert(int x, int next, LL val) {
34     int sz = size[root[x]] + 1;
35     root[x] = copy(root[x]);
36     size[root[x]] = sz; x = root[x];
37     upd(x, next, val);
38     for (int i=LOG[sz]-1;i>=0;i--) {
39         int ind = (sz>>i)&1;
40         node[x].son[ind] = copy(node[x].son[ind]);
41         x = node[x].son[ind];
42         upd(x, next, val);
43     }
44 }
45 };
46
47 namespace kthsp {
48     using namespace pheap;
49     struct EDGE {
50         int adj, w, next;
51     } edge[2][M];
52     struct W {
53         int x, y, w;
54     } e[M];
55     bool has_init = 0;
56     int n, m, top[2], gh[2][N], v[N];
57     LL dist[N];
58     void init(int n1) {
59         has_init = 1;
60         n = n1; m = 0;
61         memset(top, 0, sizeof(top));
62         memset(gh, 0, sizeof(gh));
63         for (int i=1;i<=n;i++) dist[i] = inf;
64     }
65     void addedge(int id, int x, int y, int w) {
66         edge[id][++top[id]].adj = y;
67         edge[id][top[id]].w = w;
68         edge[id][top[id]].next = gh[id][x];
69         gh[id][x] = top[id];
70     }
71     void add(int x, int y, int w) {
72         assert(has_init);
73         e[++m].x=x; e[m].y=y; e[m].w=w;
74     }
75     int q[N], best[N], bestw[N];
76     int deg[N];
77     void spfa(int S) {
78         for (int i=1;i<=n;i++) deg[i] = 0;
79         for (int i=1;i<=m;i++) deg[e[i].x] ++;
80         int head = 0, tail = 1;

```

```

81     dist[S] = 0; q[1] = S;
82     while (head != tail) {
83         (head += 1) %= N;
84         int x = q[head];
85         for (int p=gh[1][x]; p; p=edge[1][p].next) {
86             if (dist[x] + edge[1][p].w < dist[edge[1][p].adj]) {
87                 dist[edge[1][p].adj] = dist[x] + edge[1][p].w;
88                 best[edge[1][p].adj] = x;
89                 bestw[edge[1][p].adj] = p;
90             }
91             if (!--deg[edge[1][p].adj]) {
92                 (tail += 1) %= N;
93                 q[tail] = edge[1][p].adj;
94             }
95         }
96     }
97 }
98 void dfs(int x) {
99     if (v[x]) return; v[x] = 1;
100    if (best[x]) root[x] = root[best[x]];
101    for (int p=gh[0][x]; p; p=edge[0][p].next)
102        if (dist[edge[0][p].adj] != inf && bestw[x] != p) {
103            insert(x, edge[0][p].adj, edge[0][p].w + dist[edge[0][p].adj] - dist
104                [x]);
105        }
106    for (int p=gh[1][x]; p; p=edge[1][p].next)
107        if (best[edge[1][p].adj] == x)
108            dfs(edge[1][p].adj);
109 }
110 typedef pair<LL,int> pli;
111 priority_queue <pli, vector<pli>, greater<pli> > pq;
112 LL work(int S, int T, int k) {
113     assert(has_init);
114     n++; add(T, n, 0);
115     if (S == T) k++;
116     T = n;
117     for (int i=1; i<=m; i++) {
118         addedge(0, e[i].x, e[i].y, e[i].w);
119         addedge(1, e[i].y, e[i].x, e[i].w);
120     }
121     spfa(T);
122     root[T] = 0; pheap::init();
123     memset(v, 0, sizeof(v));
124     dfs(T);
125     while (!pq.empty()) pq.pop();
126     if (k == 1) return dist[S];
127     if (root[S]) pq.push(make_pair(dist[S] + node[root[S]].val, root[S]));
128     while (k--) {
129         if (pq.empty()) return inf;
130         pli now = pq.top(); pq.pop();

```

```

130         if (k == 1) return now.first;
131         int x = node[now.second].next, u = node[now.second].son[0], v = node[now
            .second].son[1];
132         if (root[x]) pq.push(make_pair(now.first + node[root[x]].val, root[x]));
133         if (u) pq.push(make_pair(now.first - node[now.second].val + node[u].val,
            u));
134         if (v) pq.push(make_pair(now.first - node[now.second].val + node[v].val,
            v));
135     }
136     return 0;
137 }
138 };

```

5.4 Tarjan

割点的判断：一个顶点 u 是割点，当且仅当满足 (1) 或 (2)：

(1) u 为树根，且 u 有多于一个子树

(2) u 不为树根，且满足存在 (u, v) 为树枝边 (u 为 v 的父亲)，使得 $dfn[u] \leq low[v]$

桥的判断：一条无向边 (u, v) 是桥，当且仅当 (u, v) 为树枝边，满足 $dfn[u] < low[v]$

```

1 struct EDGE { int adj, next; } edge[M];
2 int n, m, top, gh[N];
3 int dfn[N], low[N], cnt, ind, stop, instack[N], stack[N], belong[N];
4 void addedge(int x, int y) {
5     edge[++top].adj = y;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 void tarjan(int x) {
10     dfn[x] = low[x] = ++ind;
11     instack[x] = 1; stack[++stop] = x;
12     for (int p=gh[x]; p; p=edge[p].next)
13         if (!dfn[edge[p].adj]) {
14             tarjan(edge[p].adj);
15             low[x] = min(low[x], low[edge[p].adj]);
16         } else if (instack[edge[p].adj]) {
17             low[x] = min(low[x], dfn[edge[p].adj]);
18         }
19     if (dfn[x] == low[x]) {
20         ++cnt; int tmp=0;
21         while (tmp!=x) {
22             tmp = stack[stop--];
23             belong[tmp] = cnt;
24             instack[tmp] = 0;
25         }
26     }
27 }

```

5.5 2-SAT

```
1 #define N number_of_vertex
2 #define M number_of_edges
3
4 struct MergePoint {
5     struct EDGE {
6         int adj, next;
7     } edge[M];
8     int ex[M], ey[M];
9     bool instack[N];
10    int gh[N], top, dfn[N], low[N], cnt, ind, stop, stack[N], belong[N];
11    void init() {
12        cnt = ind = stop = top = 0;
13        memset(dfn, 0, sizeof(dfn));
14        memset(instack, 0, sizeof(instack));
15        memset(gh, 0, sizeof(gh));
16    }
17    void addedge(int x, int y) { //reverse
18        std::swap(x, y);
19        edge[++top].adj = y;
20        edge[top].next = gh[x];
21        gh[x] = top;
22        ex[top] = x;
23        ey[top] = y;
24    }
25    void tarjan(int x) {
26        dfn[x] = low[x] = ++ind;
27        instack[x] = 1; stack[++stop] = x;
28        for (int p=gh[x]; p; p=edge[p].next)
29            if (!dfn[edge[p].adj]) {
30                tarjan(edge[p].adj);
31                low[x] = std::min(low[x], low[edge[p].adj]);
32            } else if (instack[edge[p].adj]) {
33                low[x] = std::min(low[x], dfn[edge[p].adj]);
34            }
35        if (dfn[x] == low[x]) {
36            ++cnt; int tmp = 0;
37            while (tmp!=x) {
38                tmp = stack[stop--];
39                belong[tmp] = cnt;
40                instack[tmp] = 0;
41            }
42        }
43    }
44    void work() {
45        for (int i = (__first__); i <= (__last__); ++i)
46            if (!dfn[i])
47                tarjan(i);
48    }
```

```

49 } merge;
50
51 struct Topsort {
52     struct EDGE {
53         int adj, next;
54     } edge[M];
55     int n, top, gh[N], ops[N], deg[N], ans[N];
56     std::queue<int> q;
57     void init() {
58         n = merge.cnt; top = 0;
59         memset(gh, 0, sizeof(gh));
60         memset(deg, 0, sizeof(deg));
61     }
62     void addedge(int x, int y) {
63         if (x == y) return;
64         edge[++top].adj = y;
65         edge[top].next = gh[x];
66         gh[x] = top;
67         ++deg[y];
68     }
69     void work() {
70         for (int i = 1; i <= n; ++i)
71             if (!deg[i])
72                 q.push(i);
73         while (!q.empty()) {
74             int x = q.front();
75             q.pop();
76             for (int p = gh[x]; p; p = edge[p].next)
77                 if (--deg[edge[p].adj])
78                     q.push(edge[p].adj);
79             if (ans[x]) continue;
80             ans[x] = -1; //not selected
81             ans[ops[x]] = 1; //selected
82         }
83     }
84 } ts;

```

调用示例:

```

1 merge.init();
2 merge.addedge();
3 merge.work();
4 for (int i = 1; i <= n; ++i) {
5     if (merge.belong[U(i, 0)] == merge.belong[U(i, 1)]) {
6         puts("NO");
7         return 0;
8     }
9     ts.ops[merge.belong[U(i, 0)]] = merge.belong[U(i, 1)];
10    ts.ops[merge.belong[U(i, 1)]] = merge.belong[U(i, 0)];
11 }
12 ts.init();

```



```

13     ts.work();
14     puts("YES");
15     for (int i = 1; i <= n; ++i) {
16         int x = U(i, 0), y = U(i, 1);
17         x = merge.belong[x], y = merge.belong[y];
18         x = ts.ans[x], y = ts.ans[y];
19         if (x == 1) puts("0_is_selected");
20         if (y == 1) puts("1_is_selected");
21     }

```

5.6 统治者树 (Dominator Tree)

Dominator Tree 可以解决判断一类有向图必经点的问题。

$idom[x]$ 表示离 x 最近的必经点 (重编号后)。将 $idom[x]$ 作为 x 的父亲, 构成一棵 Dominator Tree

接口:

`void dominator::init(int n);` 初始化, 有向图节点数为 n

`void dominator::addedge(int u, int v);` 添加一条有向边 (u, v)

`void dominator::work(int root);` 以 $root$ 为根, 建立一棵 Dominator Tree

结果的返回:

在执行 `dominator::work(int root);` 后, 树边保存在 `vector<int> tree[N]` 中

```

1 namespace dominator {
2     vector<int> g[N], rg[N], bucket[N], tree[N];
3     int n, ind, idom[N], sdom[N], dfn[N], dsu[N], father[N], label[N], rev[N];
4     void dfs(int x) {
5         ++ind;
6         dfn[x] = ind; rev[ind] = x;
7         label[ind] = dsu[ind] = sdom[ind] = ind;
8         for (auto p : g[x]) {
9             if (!dfn[p]) dfs(p), father[dfn[p]] = dfn[x];
10            rg[dfn[p]].push_back(dfn[x]);
11        }
12    }
13    void init(int n1) {
14        n = n1; ind = 0;
15        for (int i = 1; i <= n; ++i) {
16            g[i].clear();
17            rg[i].clear();
18            bucket[i].clear();
19            tree[i].clear();
20            dfn[i] = 0;
21        }
22    }
23    void addedge(int u, int v) {
24        g[u].push_back(v);
25    }
26    int find(int x, int step=0) {

```

```

27     if (dsu[x] == x) return step ? -1 : x;
28     int y = find(dsu[x], 1);
29     if (y < 0) return x;
30     if (sdom[label[dsu[x]]] < sdom[label[x]])
31         label[x] = label[dsu[x]];
32     dsu[x] = y;
33     return step ? dsu[x] : label[x];
34 }
35 void work(int root) {
36     dfs(root); n = ind;
37     for (int i = n; i; --i) {
38         for (auto p : rg[i])
39             sdom[i] = min(sdom[i], sdom[find(p)]);
40         if (i > 1) bucket[sdom[i]].push_back(i);
41         for (auto p : bucket[i]) {
42             int u = find(p);
43             if (sdom[p] == sdom[u]) idom[p] = sdom[p];
44             else idom[p] = u;
45         }
46         if (i > 1) dsu[i] = father[i];
47     }
48     for (int i = 2; i <= n; ++i) {
49         if (idom[i] != sdom[i])
50             idom[i] = idom[idom[i]];
51         tree[rev[i]].push_back(rev[idom[i]]);
52         tree[rev[idom[i]]].push_back(rev[i]);
53     }
54 }
55 };

```

5.7 网络流

5.7.1 最大流

注意: *top* 要初始化为 1

```

1 struct EDGE { int adj, w, next; } edge[M];
2 int n, top, gh[N], nrl[N];
3 void addedge(int x, int y, int w) {
4     edge[++top].adj = y;
5     edge[top].w = w;
6     edge[top].next = gh[x];
7     gh[x] = top;
8     edge[++top].adj = x;
9     edge[top].w = 0;
10    edge[top].next = gh[y];
11    gh[y] = top;
12 }
13 int dist[N], q[N];
14 int bfs() {

```

```

15     memset(dist, 0, sizeof(dist));
16     q[1] = S; int head = 0, tail = 1; dist[S] = 1;
17     while (head != tail) {
18         int x = q[++head];
19         for (int p=gh[x]; p; p=edge[p].next)
20             if (edge[p].w && !dist[edge[p].adj]) {
21                 dist[edge[p].adj] = dist[x] + 1;
22                 q[++tail] = edge[p].adj;
23             }
24     }
25     return dist[T];
26 }
27 int dinic(int x, int delta) {
28     if (x==T) return delta;
29     for (int& p=nrl[x]; p && delta; p=edge[p].next)
30         if (edge[p].w && dist[x]+1 == dist[edge[p].adj]) {
31             int dd = dinic(edge[p].adj, min(delta, edge[p].w));
32             if (!dd) continue;
33             edge[p].w -= dd;
34             edge[p^1].w += dd;
35             return dd;
36         }
37     return 0;
38 }
39 int work() {
40     int ans = 0;
41     while (bfs()) {
42         memcpy(nrl, gh, sizeof(gh));
43         int t; while (t = dinic(S, inf)) ans += t;
44     }
45     return ans;
46 }

```

5.7.2 上下界有源汇网络流

T 向 S 连容量为正无穷的边，将有源汇转化为无源汇。

每条边容量减去下界，设 $in[i]$ 表示流入 i 的下界之和减去流出 i 的下界之和。

新建超级源汇 SS, TT ，对于 $in[i] > 0$ 的点， SS 向 i 连容量为 $in[i]$ 的边。对于 $in[i] < 0$ 的点， i 向 TT 连容量为 $-in[i]$ 的边。

求出以 SS, TT 为源汇的最大流，如果等于 $\sum in[i] (in[i] > 0)$ ，则存在可行流。再求出 S, T 为源汇的最大流即为最大流。

费用流：建完图后等价于求以 SS, TT 为源汇的费用流。

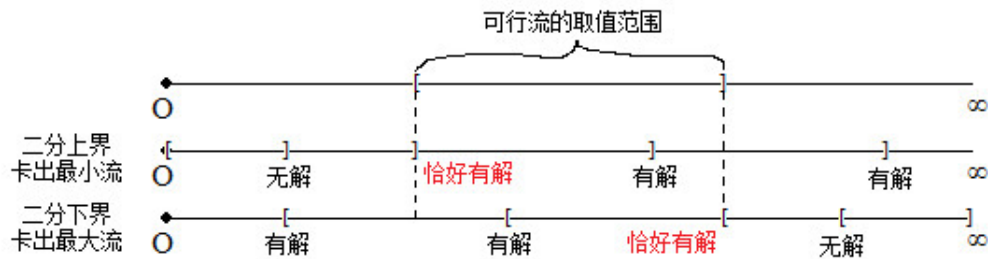
5.7.3 上下界无源汇网络流

1. 怎样求无源汇有上下界网络的可行流？

由于有源汇的网络我们先要转化成无源汇，所以本来就无源汇的网络不用再做特殊处理。

2. 怎样求无源汇有上下界网络的最大流、最小流？

一种简易的方法是采用二分思想，不断通过可行流的存在与否对 (t, s) 边的上下界 U, L 进行调整。求最大流时令 $U = \infty$ 并二分 L ；求最小流时令 $L = 0$ 并二分 U 。道理很简单，因为可行流的取值范围是一段连续的区间，我们只要通过二分找到有解和无解的分界线即可。



5.7.4 费用流

注意：top 要初始化为 1

```

1 #define inf 0x3f3f3f3f
2 struct NetWorkFlow {
3     struct EDGE {
4         int adj, w, cost, next;
5     } edge[M*2];
6     int gh[N], q[N], dist[N], v[N], pre[N], prev[N], top;
7     int S, T;
8     void addedge(int x, int y, int w, int cost) {
9         edge[++top].adj = y;
10        edge[top].w = w;
11        edge[top].cost = cost;
12        edge[top].next = gh[x];
13        gh[x] = top;
14        edge[++top].adj = x;
15        edge[top].w = 0;
16        edge[top].cost = -cost;
17        edge[top].next = gh[y];
18        gh[y] = top;
19    }
20    void clear() {
21        top = 1;
22        memset(gh, 0, sizeof(gh));
23    }
24    int spfa() {
25        memset(dist, 63, sizeof(dist));
26        memset(v, 0, sizeof(v));
27        int head = 0, tail = 1;
28        q[1] = S; v[S] = 1; dist[S] = 0;
29        while (head != tail) {
30            (head += 1) %= N;
31            int x = q[head];

```

```

32     v[x] = 0;
33     for (int p=gh[x]; p; p=edge[p].next)
34         if (edge[p].w && dist[x] + edge[p].cost < dist[edge[p].adj]) {
35             dist[edge[p].adj] = dist[x] + edge[p].cost;
36             pre[edge[p].adj] = x;
37             prev[edge[p].adj] = p;
38             if (!v[edge[p].adj]) {
39                 v[edge[p].adj] = 1;
40                 (tail += 1) %= N;
41                 q[tail] = edge[p].adj;
42             }
43         }
44     }
45     return dist[T] != inf;
46 }
47 int work() {
48     int ans = 0;
49     while (spfa()) {
50         int mx = inf;
51         for (int x=T; x!=S; x=pre[x])
52             mx = min(edge[prev[x]].w, mx);
53         ans += dist[T] * mx;
54         for (int x=T; x!=S; x=pre[x]) {
55             edge[prev[x]].w -= mx;
56             edge[prev[x]^1].w += mx;
57         }
58     }
59     return ans;
60 }
61 } nwf;

```

5.7.5 zkw 费用流

注意: *top* 要初始化为 1, 不得用于有负权的图

```

1 #define inf 0x3f3f3f3f
2 struct NetWorkFlow {
3     struct EDGE {
4         int adj, w, cost, next;
5     } edge[M*2];
6     int gh[N], top;
7     int S, T;
8     void addedge(int x, int y, int w, int cost) {
9         edge[++top].adj = y;
10        edge[top].w = w;
11        edge[top].cost = cost;
12        edge[top].next = gh[x];
13        gh[x] = top;
14        edge[++top].adj = x;
15        edge[top].w = 0;

```

```

16     edge[top].cost = -cost;
17     edge[top].next = gh[y];
18     gh[y] = top;
19 }
20 void clear() {
21     top = 1;
22     memset(gh, 0, sizeof(gh));
23 }
24 int cost, d[N], slk[N], v[N];
25 int aug(int x, int f) {
26     int left = f;
27     if (x == T) {
28         cost += f * d[S];
29         return f;
30     }
31     v[x] = true;
32     for (int p=gh[x]; p; p=edge[p].next)
33         if (edge[p].w && !v[edge[p].adj]) {
34             int t = d[edge[p].adj] + edge[p].cost - d[x];
35             if (t == 0) {
36                 int delt = aug(edge[p].adj, min(left, edge[p].w));
37                 if (delt > 0) {
38                     edge[p].w -= delt;
39                     edge[p^1].w += delt;
40                     left -= delt;
41                 }
42                 if (left == 0) return f;
43             } else {
44                 if (t < slk[edge[p].adj])
45                     slk[edge[p].adj] = t;
46             }
47         }
48     return f-left;
49 }
50 bool modlabel() {
51     int delt = inf;
52     for (int i=1; i<=T; i++)
53         if (!v[i]) {
54             if (slk[i] < delt) delt = slk[i];
55             slk[i] = inf;
56         }
57     if (delt == inf) return true;
58     for (int i=1; i<=T; i++)
59         if (v[i]) d[i] += delt;
60     return false;
61 }
62 int work() {
63     cost = 0;
64     memset(d, 0, sizeof(d));
65     memset(slk, 63, sizeof(slk));

```

```

66         do {
67             do {
68                 memset(v, 0, sizeof(v));
69             } while (aug(S, inf));
70         } while (!modlabel());
71         return cost;
72     }
73 } nwf;

```

6 数学

6.1 扩展欧几里得解同余方程

ans[] 保存的是循环节内所有的解

```

1 int exgcd(int a,int b,int&x,int&y){
2     if(!b) return x=1,y=0,a;
3     int d=exgcd(b,a%b,x,y),t=x;
4     return x=y,y=t-a/b*y,d;
5 }
6 void cal(ll a,ll b,ll n){ //ax=b(mod n)
7     ll x,y,d=exgcd(a,n,x,y);
8     if(b%d) return;
9     x=(x%n+n)%n;
10    ans[cnt=1]=x*(b/d)%(n/d);
11    for(ll i=1;i<d;i++) ans[++cnt]=(ans[1]+i*n/d)%n;
12 }

```

6.2 同余方程组

```

1 int n,flag,k,m,a,r,d,x,y;
2 int main(){
3     scanf("%d",&n);
4     flag=k=1,m=0;
5     while(n--){
6         scanf("%d%d",&a,&r); //ans%a=r
7         if(flag){
8             d=exgcd(k,a,x,y);
9             if((r-m)%d){ flag=0;continue; }
10            x=(x*(r-m)/d+a/d)%(a/d), y=k/d*a, m=((x*k+m)%y)%y;
11            if(m<0) m+=y;
12            k=y;
13        }
14    }
15    printf("%d",flag?m:-1); //若 flag=1,说明有解,解为ki+m,i为任意整数
16 }

```

6.3 卡特兰数

$$h_1 = 1, h_n = \frac{h_{n-1}(4n-2)}{n+1} = \frac{C(2n,n)}{n+1} = C(2n,n) - C(2n,n-1)$$

在一个格点阵列中, 从 $(0,0)$ 点走到 (n,m) 点且不经过对角线 $x=y$ 的方案数 ($x > y$) :

$$C(n+m-1,m) - C(n+m-1,m-1)$$

在一个格点阵列中, 从 $(0,0)$ 点走到 (n,m) 点且不穿过对角线 $x=y$ 的方案数 ($x \geq y$) :

$$C(n+m,m) - C(n+m,m-1)$$

6.4 斯特林数

6.4.1 第一类斯特林数

第一类 Stirling 数 $S(p,k)$ 的一个组合学解释是: 将 p 个物体排成 k 个非空循环排列的方法数。

$S(p,k)$ 的递推公式: $S(p,k) = (p-1)S(p-1,k) + S(p-1,k-1), 1 \leq k \leq p-1$

边界条件: $S(p,0) = 0, p \geq 1$ $S(p,p) = 1, p \geq 0$

6.4.2 第二类斯特林数

第二类 Stirling 数 $S(p,k)$ 的一个组合学解释是: 将 p 个物体划分成 k 个非空的不可辨别 (可以理解为盒子没有编号) 集合的方法数。

$S(p,k)$ 的递推公式: $S(p,k) = kS(p-1,k) + S(p-1,k-1), 1 \leq k \leq p-1$

边界条件: $S(p,0) = 0, p \geq 1$ $S(p,p) = 1, p \geq 0$

也有卷积形式:

$$S(n,m) = \frac{1}{m!} \sum_{k=0}^m (-1)^k C(m,k) (m-k)^n = \sum_{k=0}^m \frac{(-1)^k (m-k)^n}{k!(m-k)!} = \sum_{k=0}^m \frac{(-1)^k}{k!} \times \frac{(m-k)^n}{(m-k)!}$$

6.5 错排公式

$$D_1 = 0, D_2 = 1, D_n = (n-1)(D_{n-2} + D_{n-1})$$

6.6 Lucas 定理

接口:

初始化: `void lucas::init();`

计算 $C(n,m) \% mod$ 的值: `LL lucas::Lucas(LL n, LL m);`

```
1 #define mod 110119
2 #define LL long long
3 namespace lucas {
4     LL fac[mod+1], facv[mod+1];
5     LL power(LL base, LL times) {
6         LL ans = 1;
7         while (times) {
8             if (times&1) (ans *= base) %= mod;
9             (base *= base) %= mod;
10            times >>= 1;
11        }
12    }
```



```

11     }
12     return ans;
13 }
14 void init() {
15     fac[0] = 1; for (int i=1;i<mod;i++) fac[i] = (fac[i-1] * i) % mod;
16     facv[mod-1] = power(fac[mod-1], mod-2);
17     for (int i=mod-2;i>=0;--i) facv[i] = (facv[i+1] * (i+1)) % mod;
18 }
19 LL C(unsigned LL n, unsigned LL m) {
20     if (n < m) return 0;
21     return (fac[n] * facv[m] % mod * facv[n-m] % mod) % mod;
22 }
23 LL Lucas(unsigned LL n, unsigned LL m)
24 {
25     if (m == 0) return 1;
26     return (C(n%mod, m%mod) * Lucas(n/mod, m/mod)) % mod;
27 }
28 };

```

6.7 高斯消元

6.7.1 行列式

```

1 int ans = 1;
2 for (int i=0;i<n;i++) {
3     for (int j=i;j<n;j++)
4         if (g[j][i]) {
5             for (int k=i;k<n;k++)
6                 swap(g[i][k], g[j][k]);
7             if (j != i) ans *= -1;
8             break;
9         }
10    if (g[i][i] == 0) {
11        ans = 0;
12        break;
13    }
14    for (int j=i+1;j<n;j++) {
15        while (g[j][i]) {
16            int t = g[i][i] / g[j][i];
17            for (int k=i;k<n;k++)
18                g[i][k] = (g[i][k] + mod - ((LL)t * g[j][k] % mod)) % mod;
19            for (int k=i;k<n;k++)
20                swap(g[i][k], g[j][k]);
21            ans *= -1;
22        }
23    }
24 }
25 for (int i=0;i<n;i++)
26     ans = ((LL)ans * g[i][i]) % mod;

```

```

27 ans = (ans % mod + mod) % mod;
28 printf("%d\n", ans);

```

6.7.2 Matrix-Tree 定理

对于一张图，建立矩阵 C ， $C[i][i]$ = i 的度数，若 i, j 之间有边，那么 $C[i][j] = -1$ ，否则为 0。这张图的生成树个数等于矩阵 C 的 $n - 1$ 阶行列式的值。

6.8 调和级数

$\sum_{i=1}^n \frac{1}{i}$ 在 n 较大时约等于 $\ln(n) + r$ ， r 为欧拉常数，约等于 0.5772156649015328。

6.9 曼哈顿距离的变换

$$|x_1 - x_2| + |y_1 - y_2| = \max(|(x_1 + y_1) - (x_2 + y_2)|, |(x_1 - y_1) - (x_2 - y_2)|)$$

6.10 线性筛素数

```

1 mu[1]=phi[1]=1;top=0;
2 for (int i=2;i<N;i++) {
3     if (!v[i]) prime[++top]=i, mu[i] = -1, phi[i] = i-1;
4     for (int j=1;i*prime[j]<N && j<=top;j++) {
5         v[i*prime[j]] = 1;
6         if (i%prime[j]) {
7             mu[i*prime[j]] = -mu[i];
8             phi[i*prime[j]] = phi[i] * (prime[j]-1);
9         } else {
10            mu[i*prime[j]] = 0;
11            phi[i*prime[j]] = phi[i] * prime[j];
12            break;
13        }
14    }
15 }

```

6.11 杜教筛

$\text{getphi}(t, x)$ 表示求 $\sum_{i=1}^x i^t \phi(i)$ 。

推导过程：

记 $S(n) = \sum_{i=1}^n f(i)$ ，取任意函数 g 有恒等式

$$S(n) = \sum_{i=1}^n (f \cdot g)(i) - \sum_{i=2}^n g(i) S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$$

其中， $(f \cdot g)$ 表示 f 和 g 的狄利克雷卷积：即： $(f \cdot g)(n) = \sum_{d|n} f(d)g\left(\frac{n}{d}\right)$

关于恒等式的证明：

将 $\sum_{i=2}^n g(i)S(\lfloor \frac{n}{i} \rfloor)$ 移到左边去，即只需证

$$\sum_{i=1}^n (f \cdot g)(i) = \sum_{i=1}^n g(i)S(\lfloor \frac{n}{i} \rfloor)$$

将狄利克雷卷积展开，得：

$$\sum_{i=1}^n \sum_{d|i} g(d)f(\frac{i}{d}) = \sum_{i=1}^n g(i)S(\lfloor \frac{n}{i} \rfloor)$$

即：

$$\sum_{d=1}^n g(d) \sum_{i=1}^{\lfloor \frac{n}{d} \rfloor} f(i) = \sum_{i=1}^n g(i)S(\lfloor \frac{n}{i} \rfloor)$$

显然相等，恒等式证完。

取 $f(i) = i^p \phi(i), g(i) = i^p$ ，则有：

$$S(n) = \sum_{i=1}^n i^p \phi(i) = \sum_{i=1}^n i^{p+1} - \sum_{i=2}^n i^p S(\lfloor \frac{n}{i} \rfloor)$$

其中有用到等式 $\sum_{d|n} \phi(d) = n$

```

1 #include <bits/stdc++.h>
2
3 #define N 5000020
4 #define LL long long
5 #define mod 1000000007
6 #define div2 ((mod+1)/2)
7 #define div6 ((mod+1)/6)
8
9 using namespace std;
10
11 int n, prime[N], v[N];
12 LL phi[3][N];
13
14 map<int, int> mp[3];
15
16 int sum(int t, int x) { //calculate 1^t + 2^t + ... + x^t
17     if (t == 0) return x;
18     if (t == 1) return 1ll * x * (x + 1) % mod * div2 % mod;
19     if (t == 2) return 1ll * x * (x + 1) % mod * (21ll * x % mod + 1) % mod * div6 %
        mod;
20     if (t == 3) return 1ll * x * x % mod * (x + 1) % mod * (x + 1) % mod * div2 %
        mod * div2 % mod;
21 }
22
23 int getphi(int t, int x) {
24     if (x < N) return phi[t][x];
25     if (mp[t].find(x) != mp[t].end()) return mp[t][x];

```

```

26     LL ans = 0; int r = 0;
27     for (int l = 2; l <= x; l = r + 1) {
28         r = x / (x / l);
29         ans += 1ll * getphi(t, x / l) * (((LL)sum(t, r) - sum(t, l - 1) + mod) % mod
30             ) % mod;
31         ans %= mod;
32     }
33     ans = (LL)sum(t + 1, x) - ans + mod;
34     ans %= mod;
35     mp[t][x] = ans;
36     return (int)ans;
37 }
38 int main() {
39     memset(v, 0, sizeof(v));
40     int top = 0;
41     phi[0][1] = 1, phi[1][1] = 1, phi[2][1] = 1;
42     for (int i = 2; i < N; ++i) {
43         if (!v[i]) prime[++top] = i, phi[0][i] = i - 1, phi[1][i] = 1ll * i * phi
44             [0][i] % mod, phi[2][i] = 1ll * i * phi[1][i] % mod;
45         for (int j = 1; j <= top && prime[j] * i < N; ++j) {
46             v[i * prime[j]] = 1;
47             if (i % prime[j] == 0) {
48                 phi[0][i * prime[j]] = phi[0][i] * prime[j];
49                 phi[1][i * prime[j]] = 1ll * phi[1][i] * prime[j] % mod * prime[j] %
50                     mod;
51                 phi[2][i * prime[j]] = 1ll * phi[2][i] * prime[j] % mod * prime[j] %
52                     mod * prime[j] % mod;
53                 break;
54             } else {
55                 phi[0][i * prime[j]] = phi[0][i] * (prime[j] - 1);
56                 phi[1][i * prime[j]] = 1ll * phi[1][i] * (prime[j] - 1) % mod *
57                     prime[j] % mod;
58                 phi[2][i * prime[j]] = 1ll * phi[2][i] * (prime[j] - 1) % mod *
59                     prime[j] % mod * prime[j] % mod;
60             }
61         }
62     }
63 }

```

6.12 FFT

```

1 typedef complex<double> comp;
2 namespace FFT {

```

```

3   comp A[N], B[N], omega[N];
4   void transform(comp *x, int len) {
5       for (int i=1, j=len/2; i<len-1; i++) {
6           if (i<j) swap(x[i], x[j]);
7           int k = len/2;
8           while (j>=k) {
9               j-=k;
10              k/=2;
11          }
12          if (j<k) j+=k;
13      }
14  }
15  void fft(comp *x, int len, int reverse) {
16      transform(x, len);
17      for (int h=2; h<=len; h<=1) {
18          for (int i=0; i<h/2; i++) omega[i] = polar(1.0, 2*pi*reverse/h*i);
19          for (int i=0; i<len; i+=h) {
20              for (int j=i; j<i+h/2; j++) {
21                  comp w = omega[j-i];
22                  comp u = x[j];
23                  comp v = (w * x[j+h/2]);
24                  x[j] = u + v;
25                  x[j+h/2] = u - v;
26              }
27          }
28      }
29      if (reverse == -1) {
30          for (int i=0; i<len; i++)
31              x[i] /= len;
32      }
33  }
34  void work(int n, int *a, int *b) {
35      int len = 1;
36      while (len <= n*2) len *= 2;
37      for (int i=0; i<len; i++) A[i] = B[i] = 0;
38      for (int i=0; i<n; i++) A[i] = a[i], B[i] = b[i];
39      fft(A, len, 1); fft(B, len, 1);
40      for (int i=0; i<len; i++) A[i] = A[i] * B[i];
41      fft(A, len, -1);
42      for (int i=0; i<len; i++) {
43          LL r = round(A[i].real());
44          a[i] = r % mod;
45      }
46  }
47  };

```

6.13 求原根

接口: LL p_root(LL p);

输入：一个素数 p

输出： p 的原根

```
1 #include <bits/stdc++.h>
2 #define LL long long
3
4 using namespace std;
5
6 vector <LL> a;
7
8 LL pow_mod(LL base, LL times, LL mod) {
9     LL ret = 1;
10    while (times) {
11        if (times&1) ret = ret * base % mod;
12        base = base * base % mod;
13        times>>=1;
14    }
15    return ret;
16 }
17
18 bool g_test(LL g, LL p) {
19     for (LL i = 0; i < a.size(); ++i)
20         if (pow_mod(g, (p-1)/a[i], p) == 1) return 0;
21     return 1;
22 }
23
24 LL p_root(LL p) {
25     LL tmp = p - 1;
26     for (LL i = 2; i <= tmp / i; ++i)
27         if (tmp % i == 0) {
28             a.push_back(i);
29             while (tmp % i == 0)
30                 tmp /= i;
31         }
32     if (tmp != 1) a.push_back(tmp);
33     LL g = 1;
34     while (1) {
35         if (g_test(g, p)) return g;
36         ++g;
37     }
38 }
39
40 int main() {
41     LL p;
42     cin >> p;
43     cout << p_root(p) << endl;
44 }
```

6.14 NTT

998244353 原根为 3 , 1004535809 原根为 3 , 786433 原根为 10 , 880803841 原根为 26 。

```
1  #define mod 998244353
2  #define g 3
3  LL wi[N], wiv[N];
4  LL power(LL base, LL times) {
5      LL ans = 1;
6      while (times) {
7          if (times&1) (ans *= base) %= mod;
8          (base *= base) %= mod;
9          times >>= 1;
10     }
11     return ans;
12 }
13 void transform(LL *x, int len) {
14     for (int i=1, j=len/2; i<len-1; i++) {
15         if (i<j) swap(x[i], x[j]);
16         int k = len/2;
17         while (j>=k) {
18             j-=k;
19             k/=2;
20         }
21         if (j<k) j+=k;
22     }
23 }
24 void NTT(LL *x, int len, int reverse) {
25     transform(x, len);
26     for (int h=2; h<=len; h<=1) {
27         for (int i=0; i<len; i+=h) {
28             LL w = 1, wn;
29             if (reverse==1) wn = wi[h]; else wn = wiv[h];
30             for (int j=i; j<i+h/2; j++) {
31                 LL u = x[j];
32                 LL v = (w * x[j+h/2]) % mod;
33                 x[j] = (u + v) % mod;
34                 x[j+h/2] = (u - v + mod) % mod;
35                 (w *= wn) %= mod;
36             }
37         }
38     }
39     if (reverse == -1) {
40         LL t = power(len, mod-2);
41         for (int i=0; i<len; i++)
42             (x[i] *= t) %= mod;
43     }
44 }
45 LL A[N], B[N];
46 int main() {
47     for (int i=1; i<N; i*=2) {
```

```

48     wi[i] = power(g, (mod-1)/i);
49     wiv[i] = power(wi[i], mod-2);
50 }
51 memset(A, 0, sizeof(A));
52 memset(B, 0, sizeof(B));
53 NTT(A, len, 1); NTT(B, len, 1);
54 for (int i=0;i<len;i++) (A[i] *= B[i]) %= mod;
55 NTT(A, len, -1);
56 }

```

6.15 组合数 lcm

$$(n+1)lcm(C(n,0), C(n,1), \dots, C(n,k)) = lcm(n+1, n, n-1, \dots, n-k+1)$$

6.16 区间 lcm 的维护

对于一个数，将其分解质因数，若有因子 p^k ，那么拆分成 k 个数 p, p^2, \dots, p^k ，权值都为 p ，那么查询区间 $[l, r]$ 内所有数的 lcm 的答案 = 所有在该区间中出现过的数的权值之积，可持久化线段树维护即可。

7 几何

7.1 凸包

```

1  typedef complex<int> point;
2  #define X real()
3  #define Y imag()
4  int n;
5  long long cross(point a, point b) {
6      return 1ll * a.X * b.Y - 1ll * a.Y * b.X;
7  }
8  bool cmp(point a, point b) {
9      return make_pair(a.X, a.Y) < make_pair(b.X, b.Y);
10 }
11 int convexHull(point p[], int n, point ch[]) {
12     sort(p, p + n, cmp);
13     int m = 0;
14     for(int i = 0; i < n; ++i) {
15         while(m > 1 && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
16         ch[m++] = p[i];
17     }
18     int k = m;
19     for(int i = n - 2; i >= 0; --i) {
20         while(m > k && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
21         ch[m++] = p[i];
22     }
23     if(n > 1) m--;
24     return m;

```


8 黑科技和杂项

8.1 找规律

有些题目，只给一个正整数 n ，然后要求输出一个答案。这时，我们可以暴力得到小数据的解，用高斯消元得到递推式，然后用矩阵快速幂求解。

使用方法：

首先在 gauss.in 中输入小数据的解 ($n = 1$ 时, $n = 2$ 时, \dots)，以 EOF 结束。

依次运行 gauss.cpp, matrix.cpp，得到 matrix.out

将 matrix.out 中的文件粘贴在 main.cpp 中相应的位置中。注意模数一定要是质数。

```

1 //gauss.cpp
2 #include <bits/stdc++.h>
3 #define N 102
4 #define mod 1000000007
5 //caution: you can use this program iff mod is a prime.
6
7 using namespace std;
8
9 int n, m, k, a[N], g[N][N];
10
11 int power(int base, int times) {
12     int ret = 1;
13     while (times) {
14         if (times & 1) ret = 1ll * ret * base % mod;
15         base = 1ll * base * base % mod;
16         times >>= 1;
17     }
18     return ret;
19 }
20
21 int test() {
22     for (int i=0; i<m; i++) {
23         for (int j=i; j<=m; j++)
24             if (g[j][i]) {
25                 for (int k=i; k<=m; k++)
26                     swap(g[i][k], g[j][k]);
27                 break;
28             }
29     if (g[i][i] == 0)
30         return 0;
31     for (int j=i+1; j<n; j++) {
32         while (g[j][i]) {
33             int t = 1ll * g[i][i] * power(g[j][i], mod - 2) % mod;
34             for (int k=i; k<n; k++)
35                 g[i][k] = (g[i][k] + mod - (1ll * t * g[j][k] % mod)) % mod;

```

```

36         for (int k=i;k<=m;k++)
37             swap(g[i][k], g[j][k]);
38     }
39 }
40 int t = power(g[i][i], mod - 2);
41 for (int j = 0; j <= m; ++j)
42     g[i][j] = 1ll * g[i][j] * t % mod;
43 }
44 for (int i = m; i < n; ++i)
45     if (g[i][m]) return 0;
46 for (int i = m - 1; i >= 0; --i) {
47     int t = power(g[i][i], mod - 2);
48     g[i][i] = 1;
49     g[i][m] = 1ll * g[i][m] * t % mod;
50     for (int j = 0; j < i; ++j)
51         g[j][m] = (g[j][m] + mod - 1ll * g[i][m] * g[j][i] % mod) % mod;
52 }
53 printf("%d\n", m);
54 for (int i = 0; i < m; ++i)
55     printf("%d_", g[i][m]);
56 puts("");
57 for (int i = 0; i < m - 1; ++i)
58     printf("%d_", a[i]);
59 puts("1");
60 return 1;
61 }
62
63 int main() {
64     freopen("gauss.in", "r", stdin);
65     freopen("gauss.out", "w", stdout);
66     k = 0;
67     while (~scanf("%d", &a[k++])) ;
68     for (int sm = 1; sm <= k - sm; ++sm) {
69         n = k - sm - 1;
70         m = sm + 1;
71         for (int i = 0; i < n; ++i) {
72             for (int j = 0; j <= sm; ++j)
73                 g[i][j] = a[i + j];
74             g[i][m] = 1;
75             swap(g[i][m - 1], g[i][m]);
76         }
77         if (test()) return 0;
78     }
79     puts("no_solution");
80     return 0;
81 }

```

```

1 //matrix.cpp
2 #include <bits/stdc++.h>
3 #define N 102

```

```

4 using namespace std;
5
6 int n, a[N];
7
8 int main() {
9     freopen("gauss.out", "r", stdin);
10    freopen("matrix.out", "w", stdout);
11    scanf("%d", &n);
12    for (int i = 0; i < n; ++i) scanf("%d", &a[i]);
13    printf("#define_M_%d\n", n);
14    printf("const_int_trans[M][M]=_\n");
15    for (int i = 0; i < n; ++i) {
16        printf("\t{");
17        for (int j = 0; j < n; ++j) {
18            int t;
19            if (j < n - 2) t = i == j + 1;
20            else if (j == n - 2) t = a[i];
21            else t = i == n - 1;
22            printf("%s%d", j == 0 ? "" : ",_", t);
23        }
24        printf("}%s\n", i == n - 1 ? "" : ",");
25    }
26    printf("};\n");
27    printf("const_int_pref[M]=_\n");
28    for (int i = 0; i < n; ++i) {
29        int x;
30        scanf("%d", &x);
31        printf("%d%s", x, i == n - 1 ? "};\n" : ",_");
32    }
33    return 0;
34 }

```

```

1 //main.cpp
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 /* paste matrix.out here. */
6
7 #define mod 1000000007
8
9 struct Matrix {
10     int c[M][M];
11     void clear() { memset(c, 0, sizeof(c)); }
12     void identity() { clear(); for (int i = 0; i < M; ++i) c[i][i] = 1; }
13     void base() { memcpy(c, trans, sizeof(trans)); }
14     friend Matrix operator * (const Matrix &a, const Matrix &b) {
15         Matrix c; c.clear();
16         for (int i = 0; i < M; ++i)
17             for (int j = 0; j < M; ++j)
18                 for (int k = 0; k < M; ++k)

```

```

19         c.c[i][j] = (c.c[i][j] + 111 * a.c[i][k] * b.c[k][j] % mod) %
                mod;
20     return c;
21 }
22 } start, base;
23
24 Matrix power(Matrix base, int times) {
25     Matrix ret; ret.identity();
26     while (times) {
27         if (times & 1) ret = ret * base;
28         base = base * base;
29         times >>= 1;
30     }
31     return ret;
32 }
33
34 int main() {
35     int tot;
36     scanf("%d", &tot);
37     while (tot--) {
38         int n;
39         scanf("%d", &n);
40         start.clear();
41         for (int i = 0; i < M; ++i) start.c[0][i] = pref[i];
42         base.base();
43         base = power(base, n - 1);
44         start = start * base;
45         printf("%d\n", start.c[0][0]);
46     }
47     return 0;
48 }

```

8.2 高精度计算

```

1 #include<algorithm>
2 using namespace std;
3 const int N_huge=850,base=100000000;
4 char s[N_huge*10];
5 struct huge{
6     typedef long long value;
7     value a[N_huge];int len;
8     void clear(){len=1;a[len]=0;}
9     huge(){clear();}
10    huge(value x){*this=x;}
11    huge operator =(huge b){
12        len=b.len;for (int i=1;i<=len;++i)a[i]=b.a[i]; return *this;
13    }
14    huge operator =(value x){
15        len=0;

```

```

16     while (x) a[++len]=x%base,x/=base;
17     if (!len) a[++len]=0;
18     return *this;
19 }
20 huge operator +(huge b){
21     int L=len>b.len?len:b.len;huge tmp;
22     for (int i=1;i<=L+1;++i) tmp.a[i]=0;
23     for (int i=1;i<=L;++i){
24         if (i>len) tmp.a[i]+=b.a[i];
25         else if (i>b.len) tmp.a[i]+=a[i];
26         else {
27             tmp.a[i]+=a[i]+b.a[i];
28             if (tmp.a[i]>=base){
29                 tmp.a[i]-=base;++tmp.a[i+1];
30             }
31         }
32     }
33     if (tmp.a[L+1]) tmp.len=L+1;
34     else tmp.len=L;
35     return tmp;
36 }
37 huge operator -(huge b){
38     int L=len>b.len?len:b.len;huge tmp;
39     for (int i=1;i<=L+1;++i) tmp.a[i]=0;
40     for (int i=1;i<=L;++i){
41         if (i>b.len) b.a[i]=0;
42         tmp.a[i]+=a[i]-b.a[i];
43         if (tmp.a[i]<0){
44             tmp.a[i]+=base;--tmp.a[i+1];
45         }
46     }
47     while (L>1&&!tmp.a[L]) --L;
48     tmp.len=L;
49     return tmp;
50 }
51 huge operator *(huge b){
52     int L=len+b.len;huge tmp;
53     for (int i=1;i<=L;++i) tmp.a[i]=0;
54     for (int i=1;i<=len;++i)
55         for (int j=1;j<=b.len;++j){
56             tmp.a[i+j-1]+=a[i]*b.a[j];
57             if (tmp.a[i+j-1]>=base){
58                 tmp.a[i+j]+=tmp.a[i+j-1]/base;
59                 tmp.a[i+j-1]%=base;
60             }
61         }
62     tmp.len=len+b.len;
63     while (tmp.len>1&&!tmp.a[tmp.len]) --tmp.len;
64     return tmp;
65 }

```

```

66 pair<huge,huge> divide(huge a,huge b){
67     int L=a.len;huge c,d;
68     for (int i=L;i--;i){
69         c.a[i]=0;d=d*base;d.a[1]=a.a[i];
70         int l=0,r=base-1,mid;
71         while (l<r){
72             mid=(l+r+1)>>1;
73             if (b*mid<=d)l=mid;
74             else r=mid-1;
75         }
76         c.a[i]=1;d-=b*l;
77     }
78     while (L>1&&!c.a[L])--L;c.len=L;
79     return make_pair(c,d);
80 }
81 huge operator / (value x){
82     value d=0;huge tmp;
83     for (int i=len;i--;i){
84         d=d*base+a[i];
85         tmp.a[i]=d/x;d%=x;
86     }
87     tmp.len=len;
88     while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
89     return tmp;
90 }
91 value operator %(value x){
92     value d=0;
93     for (int i=len;i--;i)d=(d*base+a[i])%x;
94     return d;
95 }
96 huge operator / (huge b){return divide(*this,b).first;}
97 huge operator % (huge b){return divide(*this,b).second;}
98 huge &operator += (huge b) {*this=*this+b;return *this;}
99 huge &operator -= (huge b) {*this=*this-b;return *this;}
100 huge &operator *= (huge b) {*this=*this*b;return *this;}
101 huge &operator ++ () {huge T;T=1;*this=*this+T;return *this;}
102 huge &operator -- () {huge T;T=1;*this=*this-T;return *this;}
103 huge operator ++ (int) {huge T,tmp=*this;T=1;*this=*this+T;return tmp;}
104 huge operator -- (int) {huge T,tmp=*this;T=1;*this=*this-T;return tmp;}
105 huge operator + (value x) {huge T;T=x;return *this+T;}
106 huge operator - (value x) {huge T;T=x;return *this-T;}
107 huge operator * (value x) {huge T;T=x;return *this*T;}
108 huge operator *= (value x) {*this=*this*x;return *this;}
109 huge operator += (value x) {*this=*this+x;return *this;}
110 huge operator -= (value x) {*this=*this-x;return *this;}
111 huge operator /= (value x) {*this=*this/x;return *this;}
112 huge operator %=(value x) {*this=*this%x;return *this;}
113 bool operator == (value x) {huge T;T=x;return *this==T;}
114 bool operator != (value x) {huge T;T=x;return *this!=T;}
115 bool operator <= (value x) {huge T;T=x;return *this<=T;}

```

```

116     bool operator >=(value x){huge T;T=x;return *this>=T;}
117     bool operator <(value x){huge T;T=x;return *this<T;}
118     bool operator >(value x){huge T;T=x;return *this>T;}
119     bool operator <(huge b){
120         if (len<b.len)return 1;
121         if (len>b.len)return 0;
122         for (int i=len;i--i){
123             if (a[i]<b.a[i])return 1;
124             if (a[i]>b.a[i])return 0;
125         }
126         return 0;
127     }
128     bool operator ==(huge b){
129         if (len!=b.len)return 0;
130         for (int i=len;i--i){
131             if (a[i]!=b.a[i])return 0;
132         }
133         return 1;
134     }
135     bool operator !=(huge b){return !(*this==b);}
136     bool operator >(huge b){return !(*this<b||*this==b);}
137     bool operator <=(huge b){return (*this<b)||(*this==b);}
138     bool operator >=(huge b){return (*this>b)||(*this==b);}
139     void str(char s[]){
140         int l=strlen(s);value x=0,y=1;len=0;
141         for (int i=l-1;i>=0;--i){
142             x=x+(s[i]-'0')*y;y*=10;
143             if (y==base)a[++len]=x,x=0,y=1;
144         }
145         if (!len||x)a[++len]=x;
146     }
147     void read(){
148         scanf("%s",s);this->str(s);
149     }
150     void print(){
151         printf("%d", (int)a[len]);
152         for (int i=len-1;i--i){
153             for (int j=base/10;j>=10;j/=10){
154                 if (a[i]<j)printf("0");
155                 else break;
156             }
157             printf("%d", (int)a[i]);
158         }
159         printf("\n");
160     }
161     int main(){
162         f[1]=f[2]=1;
163         for(int i=3;i<=1000;i++)f[i]=f[i-1]+f[i-2];
164     }

```