

ICPC Templates For Africamonkey

Africamonkey

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1 莫队算法

1.1 普通莫队

分块块数为 \sqrt{n} 是最优的。

记每次进行 `add()` 操作的复杂度为 $O(A)$ ，`del()` 操作的复杂度为 $O(D)$ ，查询答案 `answer()` 的复杂度为 $O(S)$ 。

则总复杂度为 $O(n\sqrt{n}(A + D) + qS)$ 。

S 可以大一点，但必须保证 A, D 尽可能小。

```
1 struct Q { int l, r, sqrtl, id; } q[N];
2 int n, m, v[N], ans[N], nowans;
3 bool cmp(const Q &a, const Q &b) {
4     if (a.sqrtl != b.sqrtl) return a.sqrtl < b.sqrtl;
5     return a.r < b.r;
6 }
7 void change(int x) {
8     if (!v[x]) add(x);
9     else del(x);
10    v[x] ^= 1;
11 }
12 int main() {
13     .....
14     for (int i=1;i<=m;i++) q[i].sqrtl = q[i].l / sqrt(n), q[i].id = i;
15     sort(q+1, q+m+1, cmp);
16     int L=1, R=0;
17     memset(v, 0, sizeof(v));
18     for (int i=1;i<=m;i++) {
19         while (L<q[i].l) change(L++);
20         while (L>q[i].l) change(--L);
21         while (R<q[i].r) change(++R);
22         while (R>q[i].r) change(R--);
23         ans[q[i].id] = answer();
24     }
25     .....
26 }
```

1.2 树上莫队

分块块数为 \sqrt{n} 是最优的。

记每次进行 `add()` 操作的复杂度为 $O(A)$ ，`del()` 操作的复杂度为 $O(D)$ ，查询答案 `answer()` 的复杂度为 $O(S)$ 。

则总复杂度为 $O(n\sqrt{n}(A + D) + qS)$ 。

S 可以大一点，但必须保证 A, D 尽可能小。

```
1 struct Query { int l, r, id, l_group; } query[N];
2 struct EDGE { int adj, next; } edge[N*2];
3 int n, m, top, gh[N], c[N], reorder[N], deep[N], father[N], size[N], son[N], Top[N];
4 void addedge(int x, int y) {
```

```

5     edge[++top].adj = y;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 void dfs(int x, int root=0) {
10     reorder[x] = ++top; father[x] = root; deep[x] = deep[root] + 1;
11     son[x] = 0; size[x] = 1; int dd = 0;
12     for (int p=gh[x]; p; p=edge[p].next)
13         if (edge[p].adj != root) {
14             dfs(edge[p].adj, x);
15             if (size[edge[p].adj] > dd) {
16                 son[x] = edge[p].adj;
17                 dd = size[edge[p].adj];
18             }
19             size[x] += size[edge[p].adj];
20         }
21 }
22 void split(int x, int tp) {
23     Top[x] = tp;
24     if (son[x]) split(son[x], tp);
25     for (int p=gh[x]; p; p=edge[p].next)
26         if (edge[p].adj != father[x] && edge[p].adj != son[x])
27             split(edge[p].adj, edge[p].adj);
28 }
29 int lca(int x, int y) {
30     int tx = Top[x], ty = Top[y];
31     while (tx != ty) {
32         if (deep[tx] < deep[ty]) {
33             swap(tx, ty);
34             swap(x, y);
35         }
36         x = father[tx];
37         tx = Top[x];
38     }
39     if (deep[x] < deep[y]) swap(x, y);
40     return y;
41 }
42 bool cmp(const Query &a, const Query &b) {
43     if (a.l_group != b.l_group) return a.l_group < b.l_group;
44     return reorder[a.r] < reorder[b.r];
45 }
46 int v[N], ans[N];
47
48 void upd(int x) {
49     if (!v[x]) add(x);
50     else del(x);
51     v[x] ^= 1;
52 }
53
54 void go(int &u, int taru, int v) {

```

```

55     int lca0 = lca(u, taru);
56     int lca1 = lca(u, v);   upd(lca1);
57     int lca2 = lca(taru, v); upd(lca2);
58     for (int x=u; x!=lca0; x=father[x]) upd(x);
59     for (int x=taru; x!=lca0; x=father[x]) upd(x);
60     u = taru;
61 }
62 int main() {
63     memset(gh, 0, sizeof(gh));
64     scanf("%d%d", &n, &m); top = 0;
65     for (int i=1;i<n;i++) {
66         int x,y; scanf("%d%d", &x, &y);
67         addedge(x, y); addedge(y, x);
68     }
69     top = 0; dfs(1); split(1, 1);
70     for (int i=1;i<=m;i++) {
71         if (reorder[query[i].l] > reorder[query[i].r])
72             swap(query[i].l, query[i].r);
73         query[i].id = i;
74         query[i].l_group = reorder[query[i].l] / sqrt(n);
75     }
76     sort(query+1, query+m+1, cmp);
77     int L=1,R=1; upd(1);
78     for (int i=1;i<=m;i++) {
79         go(L,query[i].l,R);
80         go(R,query[i].r,L);
81         ans[query[i].id] = answer();
82     }
83     .....
84 }

```

2 字符串

2.1 哈希

```

1  const int P=31,D=1000173169;
2  int n, pow[N], f[N]; char a[N];
3  int hash(int l, int r) { return (LL) (f[r]-(LL)f[l-1]*pow[r-l+1]%D+D)%D; }
4  int main() {
5      scanf("%d%s", &n, a+1);
6      pow[0] = 1;
7      for (int i=1;i<=n;i++) pow[i] = (LL)pow[i-1]*P%D;
8      for (int i=1;i<=n;i++) f[i] = (LL) ((LL)f[i-1]*P+a[i])%D;
9  }

```

2.2 KMP

接口: void kmp(int n, char *a, int m, char *b);

输入：模式串长度 n ，模式串 a ，匹配串长度 m ，匹配串 b

输出：依次输出每个匹配成功的起始位置

下标从 0 开始。

```
1 void kmp(int n, char* a, int m, char *b) {
2     int i, j;
3     for (nxt[0] = j = -1, i = 1; i < n; nxt[i++] = j) {
4         while (~j && a[j + 1] != a[i]) j = nxt[j];
5         if (a[j + 1] == a[i]) ++j;
6     }
7     for (j = -1, i = 0; i < m; ++i) {
8         while (~j && a[j + 1] != b[i]) j = nxt[j];
9         if (a[j + 1] == b[i]) ++j;
10        if (j == n - 1) {
11            printf("%d\n", i - n + 1);
12            j = nxt[j];
13        }
14    }
15 }
```

2.3 可动态修改的 KMP

支持：加入一个字符，删除一个字符。

时间复杂度： $O(n\alpha)$ ， α 为字符集大小。

代码中的字符为 '0' - '9'，可自行修改为 'a' - 'z'

```
1 char t[N];
2 int top, nxt[N], nxt_l[N][10];
3 inline void del_letter() { --top; }
4 inline void add_letter(char x) {
5     t[top++] = x;
6     int j = top-1;
7     memset(nxt_l[top], 0, sizeof(nxt_l[top]));
8     nxt[top] = nxt_l[top-1][x-'0'];
9     memcpy(nxt_l[top], nxt_l[nxt[top]], sizeof(nxt_l[nxt[top]]));
10    nxt_l[top][t[nxt[top]]-'0'] = nxt[top]+1;
11 }
```

2.4 扩展 KMP

接口：void ExtendedKMP(char *a, char *b, int *next, int *ret);

输出：

next: a 关于自己每个后缀的最长公共前缀

ret: a 关于 b 的每个后缀的最长公共前缀

EXKMP 的 next[i] 表示：从 i 到 n-1 的字符串 st 前缀和原串前缀的最长重叠长度。

```
1 void get_next(char *a, int *next) {
2     int i, j, k;
3     int n = strlen(a);
```



```

4     for (j = 0; j+1<n && a[j]==a[j+1];j++);
5     next[1] = j;
6     k = 1;
7     for (i=2;i<n;i++) {
8         int len = k+next[k], l = next[i-k];
9         if (l < len-i) {
10             next[i] = l;
11         } else {
12             for (j = max(0, len-i);i+j<n && a[j]==a[i+j];j++);
13             next[i] = j;
14             k = i;
15         }
16     }
17 }
18 void ExtendedKMP(char *a, char *b, int *next, int *ret) {
19     get_next(a, next);
20     int n = strlen(a), m = strlen(b);
21     int i, j, k;
22     for (j=0;j<n && j<m && a[j]==b[j];j++);
23     ret[0] = j;
24     k = 0;
25     for (i=1;i<m;i++) {
26         int len = k+ret[k], l = next[i-k];
27         if (l < len-i) {
28             ret[i] = l;
29         } else {
30             for (j = max(0, len-i);j<n && i+j<m && a[j]==b[i+j];j++);
31             ret[i] = j;
32             k = i;
33         }
34     }
35 }

```

2.5 Manacher

$p[i]$ 表示以 i 为对称轴的最长回文串长度

```

1 char st[N*2], s[N];
2 int len, p[N*2];
3
4 while (scanf("%s", s) != EOF) {
5     len = strlen(s);
6     st[0] = '$', st[1] = '#';
7     for (int i=1;i<=len;i++)
8         st[i*2] = s[i-1], st[i*2+1] = '#';
9     len = len * 2 + 2;
10    int mx = 0, id = 0, ans = 0;
11    for (int i=1;i<=len;i++) {
12        p[i] = (mx > i) ? min(p[id*2-i]+1, mx-i) : 1;
13        for (; st[i+p[i]] == st[i-p[i]]; ++p[i]) ;

```

```

14         if (p[i]+i > mx) mx = p[i]+i, id = i;
15         p[i] --;
16         if (p[i] > ans) ans = p[i];
17     }
18     printf("%d\n", ans);
19 }

```

2.6 最小表示法

```

1 string smallestRepresation(string s) {
2     int i, j, k, l;
3     int n = s.length();
4     s += s;
5     for (i=0, j=1; j<n; ) {
6         for (k=0; k<n && s[i+k]==s[j+k]; k++);
7         if (k>=n) break;
8         if (s[i+k]<s[j+k]) j+=k+1;
9         else {
10             l=i+k;
11             i=j;
12             j=max(l, j)+1;
13         }
14     }
15     return s.substr(i, n);
16 }

```

2.7 AC 自动机

```

1 struct Node {
2     int next[**Size of Alphabet**];
3     int terminal, fail;
4 } node[**Number of Nodes**];
5 int top;
6 void add(char *st) {
7     int len = strlen(st), x = 1;
8     for (int i=0; i<len; i++) {
9         int ind = trans(st[i]);
10        if (!node[x].next[ind])
11            node[x].next[ind] = ++top;
12        x = node[x].next[ind];
13    }
14    node[x].terminal = 1;
15 }
16 int q[**Number of Nodes**], head, tail;
17 void build() {
18     head = 0, tail = 1; q[1] = 1;
19     while (head != tail) {
20         int x = q[++head];

```

```

21      /*(when necessary) node[x].terminal != node[node[x].fail].terminal; */
22      for (int i=0;i<n;i++)
23          if (node[x].next[i]) {
24              if (x == 1) node[node[x].next[i]].fail = 1;
25              else {
26                  int y = node[x].fail;
27                  while (y) {
28                      if (node[y].next[i]) {
29                          node[node[x].next[i]].fail = node[y].next[i];
30                          break;
31                      }
32                      y = node[y].fail;
33                  }
34                  if (!node[node[x].next[i]].fail) node[node[x].next[i]].fail = 1;
35              }
36              q[++tail] = node[x].next[i];
37          }
38      }
39  }

```

2.8 后缀数组

2.8.1 倍增算法

参数 m 表示字符集的大小, 即 $0 \leq r_i < m$

```

1  #define rank rank2
2  int n, r[N], wa[N], wb[N], ws[N], sa[N], rank[N], height[N];
3  int cmp(int *r, int a, int b, int l, int n) {
4      if (r[a]==r[b]) {
5          if (a+l<n && b+l<n && r[a+l]==r[b+l])
6              return 1;
7      }
8      return 0;
9  }
10 void suffix_array(int m) {
11     int i, j, p, *x=wa, *y=wb, *t;
12     for (i=0;i<m;i++) ws[i]=0;
13     for (i=0;i<n;i++) ws[x[i]=r[i]]++;
14     for (i=1;i<m;i++) ws[i]+=ws[i-1];
15     for (i=n-1;i>=0;i--) sa[--ws[x[i]]]=i;
16     for (j=1;p=1;p<n;m=p,j<=1) {
17         for (p=0,i=n-j;i<n;i++) y[p++]=i;
18         for (i=0;i<n;i++) if (sa[i]>=j) y[p++]=sa[i]-j;
19         for (i=0;i<m;i++) ws[i]=0;
20         for (i=0;i<n;i++) ws[x[y[i]]]++;
21         for (i=1;i<m;i++) ws[i]+=ws[i-1];
22         for (i=n-1;i>=0;i--) sa[--ws[x[y[i]]]]=y[i];
23         for (t=x,x=y,y=t,x[sa[0]]=0,i=1,p=1;i<n;i++)
24             x[sa[i]]=cmp(y,sa[i-1],sa[i],j,n)?p-1:p++;

```

```

25     }
26     for (i=0;i<n;i++) rank[sa[i]]=i;
27     rank[n] = -1;
28 }
29 void calc_height() {
30     int j=0;
31     for (int i=0;i<n;i++)
32         if (rank[i])
33             {
34                 while (r[i+j]==r[sa[rank[i]-1]+j]) j++;
35                 height[rank[i]]=j;
36                 if (j) j--;
37             }
38 }

```

2.8.2 DC3 算法

感谢浙江大学陈靖邦提供本模板。

```

1 namespace SA {
2 int sa[N], rk[N], ht[N], s[N<<1], t[N<<1], p[N], cnt[N], cur[N];
3 #define pushS(x) sa[cur[s[x]]--] = x
4 #define pushL(x) sa[cur[s[x]]++] = x
5 #define inducedSort(v) fill_n(sa, n, -1); fill_n(cnt, m, 0); \
6     for (int i = 0; i < n; i++) cnt[s[i]]++; \
7     for (int i = 1; i < m; i++) cnt[i] += cnt[i-1]; \
8     for (int i = 0; i < m; i++) cur[i] = cnt[i]-1; \
9     for (int i = n1-1; ~i; i--) pushS(v[i]); \
10    for (int i = 1; i < m; i++) cur[i] = cnt[i-1]; \
11    for (int i = 0; i < n; i++) if (sa[i] > 0 && t[sa[i]-1]) pushL(sa[i]-1); \
12    for (int i = 0; i < m; i++) cur[i] = cnt[i]-1; \
13    for (int i = n-1; ~i; i--) if (sa[i] > 0 && !t[sa[i]-1]) pushS(sa[i]-1)
14 void sais(int n, int m, int *s, int *t, int *p) {
15     int n1 = t[n-1] = 0, ch = rk[0] = -1, *s1 = s+n;
16     for (int i = n-2; ~i; i--) t[i] = s[i] == s[i+1] ? t[i+1] : s[i] > s[i+1];
17     for (int i = 1; i < n; i++) rk[i] = t[i-1] && !t[i] ? (p[n1] = i, n1++) : -1;
18     inducedSort(p);
19     for (int i = 0, x, y; i < n; i++) if (~x = rk[sa[i]]) {
20         if (ch < 1 || p[x+1] - p[x] != p[y+1] - p[y]) ch++;
21         else for (int j = p[x], k = p[y]; j <= p[x+1]; j++, k++)
22             if ((s[j]<<1|t[j]) != (s[k]<<1|t[k])) {ch++; break;}
23         s1[y = x] = ch;
24     }
25     if (ch+1 < n1) sais(n1, ch+1, s1, t+n, p+n1);
26     else for (int i = 0; i < n1; i++) sa[s1[i]] = i;
27     for (int i = 0; i < n1; i++) s1[i] = p[sa[i]];
28     inducedSort(s1);
29 }
30 template<typename T>
31 int mapCharToInt(int n, const T *str) {

```

```

32     int m = *max_element(str, str+n);
33     fill_n(rk, m+1, 0);
34     for (int i = 0; i < n; i++) rk[str[i]] = 1;
35     for (int i = 0; i < m; i++) rk[i+1] += rk[i];
36     for (int i = 0; i < n; i++) s[i] = rk[str[i]] - 1;
37     return rk[m];
38 }
39 // Ensure that str[n] is the unique lexicographically smallest character in str.
40 template<typename T>
41 void suffixArray(int n, const T *str) {
42     int m = mapCharToInt(++n, str);
43     sais(n, m, s, t, p);
44     for (int i = 0; i < n; i++) rk[sa[i]] = i;
45     for (int i = 0, h = ht[0] = 0; i < n-1; i++) {
46         int j = sa[rk[i]-1];
47         while (i+h < n && j+h < n && s[i+h] == s[j+h]) h++;
48         if (ht[rk[i]] = h) h--;
49     }
50 }
51 };

```

2.8.3 小技巧：拼接字符串

接口：

int gao1(int l, int r, int c, int p); 区间 $[l, r)$ 中保证第 0 位到第 $c-1$ 位都是相同的（设为字符串 s ），现在我们在 s 后面接一个字符 p ，得到一个新的字符串 s' 。返回值为最小的 k 满足后缀 $sa[k]$ 前 $c+1$ 位为 s'

int gao2(int l, int r, int c, int p); 区间 $[l, r)$ 中保证第 0 位到第 $c-1$ 位都是相同的（设为字符串 s ），现在我们在 s 后面接一个后缀 $sa[p]$ ，得到一个新的字符串 s' 。返回值为最小的 k 满足后缀 $sa[k]$ 前 $c + \text{len}(sa[p])$ 位为 s'

```

1  int gao1(int l, int r, int c, int p) {
2      --l;
3      while (l+1 < r) {
4          int md = (l+r) >> 1;
5          if (sa[md] + c < n && s[sa[md] + c] >= p) r = md; else l = md;
6      }
7      return r;
8  }
9  int gao2(int l, int r, int c, int p) {
10     --l;
11     while (l+1 < r) {
12         int md = (l+r) >> 1;
13         if (sa[md] + c < n && rk[sa[md] + c] >= p) r = md; else l = md;
14     }
15     return r;
16 }

```

示例调用：

```

1 suf1[m] = -1, suf2[m] = n;
2 for (int i = m - 1; i >= 0; --i) {
3     int l = gao1(0, n, 0, t[i]), r = gao1(0, n, 0, t[i]);
4     suf1[i] = gao2(l, r, 1, suf1[i + 1]);
5     suf2[i] = gao2(l, r, 1, suf2[i + 1]);
6 }

```

2.9 后缀自动机

下面的代码是求两个串的 LCS（最长公共子串）。

```

1 #include <bits/stdc++.h>
2
3 #define N 500001
4 #define M (N << 1)
5
6 using namespace std;
7
8 char st[N];
9 int pre[M], son[26][M], step[M], refer[M], size[M], tmp[M], topo[M], last, total;
10
11 int apply(int x, int now) {
12     step[++total] = x;
13     refer[total] = now;
14     return total;
15 }
16
17 void extend(char x, int now) {
18     int p = last, np = apply(step[p]+1, now);
19     size[np] = 1;
20     for (; p && !son[x][p]; p=pre[p]) son[x][p] = np;
21     if (!p) pre[np] = 1;
22     else {
23         int q = son[x][p];
24         if (step[p]+1 == step[q]) pre[np] = q;
25         else {
26             int nq = apply(step[p]+1, now);
27             for (int i=0; i<26; i++) son[i][nq] = son[i][q];
28             pre[nq] = pre[q];
29             pre[q] = pre[np] = nq;
30             for (; p && son[x][p]==q; p=pre[p]) son[x][p] = nq;
31         }
32     }
33     last = np;
34 }
35 void init() {
36     last = total = 0;
37     last = apply(0, 0);
38     scanf("%s", st);

```

```

39     int n = strlen(st);
40     for (int i = 0; i <= n * 2; ++i) {
41         pre[i] = step[i] = refer[i] = size[i] = tmp[i] = topo[i] = 0;
42         for (int j = 0; j < 26; ++j)
43             son[j][i] = 0;
44     }
45     for (int i = 0; i < n; ++i)
46         extend(st[i] - 'a', i);
47     for (int i = 1; i <= total; ++i)
48         tmp[step[i]] ++;
49     for (int i = 1; i <= n; ++i)
50         tmp[i] += tmp[i - 1];
51     for (int i = 1; i <= total; ++i)
52         topo[tmp[step[i]]--] = i;
53     for (int i = total; i; --i)
54         size[pre[topo[i]]] += size[topo[i]];
55 }
56 int main() {
57     init();
58     int p = 1, now = 0, ans = 0;
59     scanf("%s", st);
60     for (int i=0; st[i]; i++) {
61         int index = st[i] - 'a';
62         for (; p && !son[index][p]; p = pre[p], now = step[p]) ;
63         if (!p) p = 1;
64         if (son[index][p]) {
65             p = son[index][p];
66             now++;
67             if (now > ans) ans = now;
68         }
69     }
70     printf("%d\n", ans);
71     return 0;
72 }

```

一些定义和性质 $\text{Right}(\text{str})$ 表示 str 在母串 S 中所有出现的结束位置集合

一个状态 s 表示的所有子串 Right 集合相同，为 $\text{Right}(s)$

$\text{Parent}(s)$ 满足 $\text{Right}(s)$ 是 $\text{Right}(\text{Parent}(s))$ 的真子集，并且 $\text{Right}(\text{Parent}(s))$ 的大小最小

Parent 函数可以表示一个树形结构。不妨叫它 Parent 树

一个 Right 集合和一个长度定义了一个子串

对于状态 s ，使得 $\text{Right}(s)$ 合法的子串长度是一个区间 $[\min(s), \max(s)]$

$\max(\text{Parent}(s)) = \min(s) - 1$

令 $\text{refer}(s)$ 表示产生 s 状态的字符所在位置。则 $\text{Right}(s)$ 的合法子串的起始位置为 $[\text{refer}(s) - \max(s) + 1, \text{refer}(s) - \min(s) + 1]$ ，即 $[\text{refer}(s) - \max(s) + 1, \text{refer}(s) - \max(\text{Parent}(s))]$

代码中变量含义 $\text{pre}[s]$ 为上述定义中的 $\text{Parent}(s)$

$\text{step}[s]$ 为从初始状态走到 s 状态最多需要多少步

refer[s] 为上述定义中的 refer(s)
size[s] 为 Right(s) 集合的大小
topo[s] 为 Parent 树的拓扑序，根（初始状态）在前

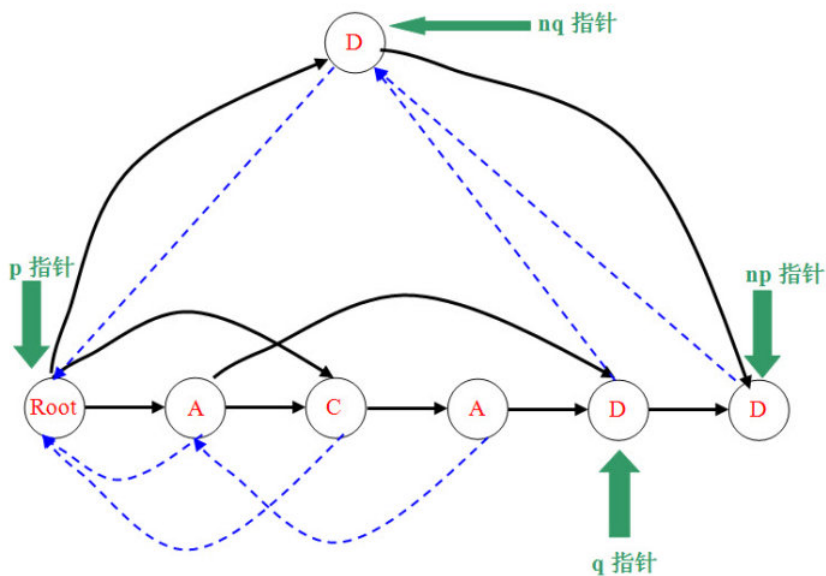


图 1: ACADD 构成的后缀自动机

我们发现 fail 构出一棵前缀树
和后缀树相同，为了使每个前缀都是叶子结点，我们不妨在串 s 前加入一个没出现的字符'#'

2.9.1 广义后缀自动机

先建 Trie，再按照 BFS 序建后缀自动机。从节点 x 开始向子树更新时，其所有儿子都从同一个 last，即 $last[x]$ 更新。

2.10 回文树

- $len[i]$ 表示编号为 i 的节点表示的回文串的长度（一个节点表示一个回文串）
- $next[i][c]$ 表示编号为 i 的节点表示的回文串在两边添加字符 c 以后变成的回文串的编号（和字典树类似）。
- $fail[i]$ 表示节点 i 失配以后跳转不等于自身的节点 i 表示的回文串的最长后缀回文串（和 AC 自动机类似）。
- $cnt[i]$ 表示节点 i 表示的本质不同的串的个数（建树时求出的不是完全的，最后 $count()$ 函数跑一遍以后才是正确的）
- $num[i]$ 表示以节点 i 表示的最长回文串的最右端点为回文串结尾的回文串个数。
- $last$ 指向新添加一个字母后所形成的最长回文串表示的节点。
- $st[i]$ 表示第 i 次添加的字符（一开始设 $st[0] = -1$ （可以是任意一个在串 S 中不会出现的字符））。

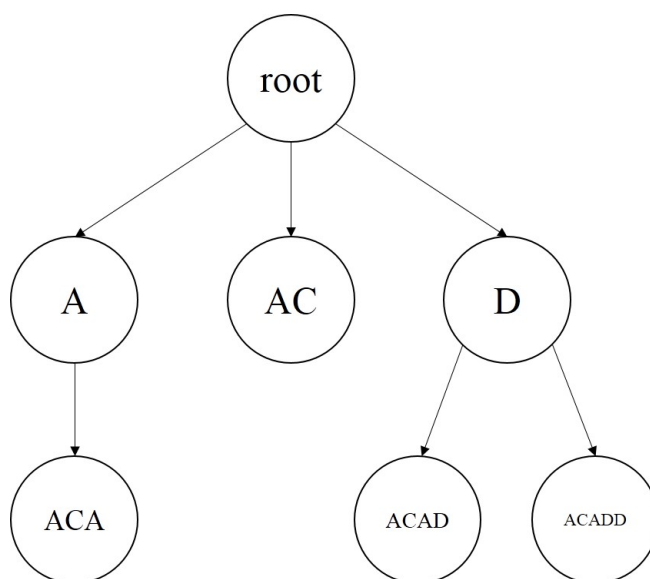


图 2: 串 ACADD 按 fail 构出的前缀树, 与图 1 对应

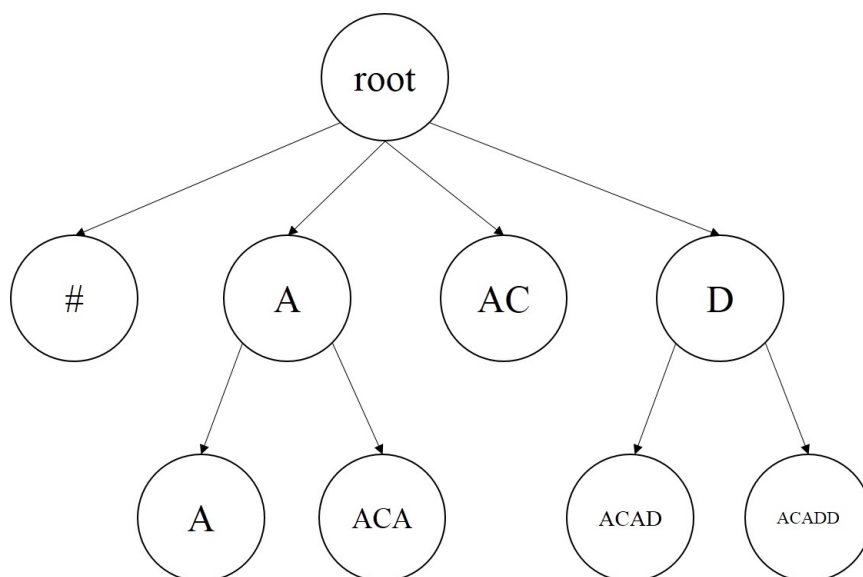


图 3: 串 #ACADD 按 fail 构出的前缀树

- tot 表示添加的节点个数。
- n 表示添加的字符个数。

【URAL2040】Palindromes and Super Abilities 2

逐个添加字符串 S 里的字符 S_1, S_2, \dots, S_n 。每次添加字符后，他想知道添加字符后将出现多少个新的本质不同的回文子串。字符集为 $\{a, b\}$

```

1  #include <bits/stdc++.h>
2  #define N 5000020
3
4  char st[N], answer[N];
5  int n;
6
7  struct PAM {
8      int n, tot, last;
9      int len[N], fail[N], next[N][2], num[N], cnt[N];
10     void init() {
11         n=0; tot=1;
12         len[1]=-1; fail[1]=0;
13         len[0]=+0; fail[0]=1;
14         last=1;
15     }
16     int get_fail(int x) {
17         for (; st[n-len[x]-1]!=st[n]; x=fail[x]);
18         return x;
19     }
20     void insert(char c) {
21         ++n; int cur=get_fail(last); // 判断上一个串的前一个位置和新添加的位置是否相
           同，相同则说明构成回文。否则找 fail 指针。
22         if (!next[cur][c]) {
23             ++tot;
24             len[tot]=len[cur]+2;
25             fail[tot]=next[get_fail(fail[cur])][c];
26             next[cur][c]=tot;
27             num[tot] = num[fail[tot]] + 1;
28             answer[n]='1';
29         } else {
30             answer[n]='0';
31         }
32         last=next[cur][c];
33         cnt[last] ++;
34     }
35     void count () {
36         for ( int i = tot - 1 ; i >= 0 ; -- i ) cnt[fail[i]] += cnt[i] ;
37         //父亲累加儿子的cnt，因为如果fail[v]=u，则u一定是v的子回文串！
38     }
39
40 } pam;
41
42 int main() {

```

```

43     scanf("%s", st+1); n=strlen(st+1);
44     pam.init();
45     for (int i=1;i<=n;i++) pam.insert(st[i]-'a');
46     puts(answer+1);
47     return 0;
48 }

```

3 数据结构

3.1 ST 表

```

1  int Log[N], f[17][N];
2  int ask(int x, int y) {
3      int k=Log[y-x+1];
4      return max(f[k][x], f[k][y-(1<<k)+1]);
5  }
6  int main() {
7      for (int i=2; i<=n; i++) Log[i]=Log[i>>1]+1;
8      for (int j=1; j<K; j++)
9          for (int i=1; i+(1<<j-1)<=n; i++)
10             f[j][i]=max(f[j-1][i], f[j-1][i+(1<<j-1)]);
11 }

```

3.2 K-D Tree

```

1  int n, cmp_d, root, id[N];
2
3  struct node {
4      int d[2], l, r, Max[2], Min[2], val, sum, f;
5  } t[N];
6
7  inline bool cmp(const node &a, const node &b) {
8      if (a.d[cmp_d] != b.d[cmp_d]) return a.d[cmp_d] < b.d[cmp_d];
9      return a.d[cmp_d ^ 1] < b.d[cmp_d ^ 1];
10 }
11
12 inline void umax(int &a, int b) {
13     if (b > a) a = b;
14 }
15
16 inline void umin(int &a, int b) {
17     if (b < a) a = b;
18 }
19
20 inline void up(int x, int y) {
21     umax(t[x].Max[0], t[y].Max[0]);
22     umin(t[x].Min[0], t[y].Min[0]);

```

```

23     umax(t[x].Max[1], t[y].Max[1]);
24     umin(t[x].Min[1], t[y].Min[1]);
25 }
26
27 int build(int l, int r, int D, int f) {
28     int mid = (l + r) / 2;
29     cmp_d = D;
30     nth_element(t + l + 1, t + mid + 1, t + r + 1, cmp);
31     id[t[mid].f] = mid;
32     t[mid].f = f;
33     t[mid].Max[0] = t[mid].Min[0] = t[mid].d[0];
34     t[mid].Max[1] = t[mid].Min[1] = t[mid].d[1];
35     t[mid].val = t[mid].sum = 0;
36     if (l != mid) t[mid].l = build(l, mid - 1, !D, mid);
37     else t[mid].l = 0;
38     if (r != mid) t[mid].r = build(mid + 1, r, !D, mid);
39     else t[mid].r = 0;
40     if (t[mid].l) up(mid, t[mid].l);
41     if (t[mid].r) up(mid, t[mid].r);
42     return mid;
43 }
44
45 // 将编号为 x 的点的权值增加 p
46 // 请注意，此处的 x 是经过排序的。你需要将点的坐标先作映射。
47 void change(int x, int p) {
48     x = id[x];
49     for (t[x].val += p; x; x = t[x].f)
50         t[x].sum += p;
51 }
52
53 inline long long sqr(long long x) {
54     return x * x;
55 }
56
57 // 欧几里得距离的平方，下界
58 inline long long euclid_lower_bound(const node &a, int X, int Y) {
59     return sqr(max(max(X - a.Max[0], a.Min[0] - X), 0)) +
60         sqr(max(max(Y - a.Max[1], a.Min[1] - Y), 0));
61 }
62
63 // 欧几里得距离的平方，上界
64 inline long long euclid_upper_bound(const node &a, int X, int Y) {
65     return max(sqr(X - a.Min[0]), sqr(X - a.Max[0])) +
66         max(sqr(Y - a.Min[1]), sqr(Y - a.Max[1]));
67 }
68
69 // 曼哈顿距离，下界
70 inline long long manhattan_lower_bound(const node &a, int X, int Y) {
71     return max(a.Min[0] - X, 0) + max(X - a.Max[0], 0) +
72         max(a.Min[1] - Y, 0) + max(Y - a.Max[1], 0);

```

```

73 }
74
75 // 曼哈顿距离, 上界
76 inline long long manhattan_upper_bound(const node &a, int X, int Y) {
77     return max(abs(X - a.Max[0]), abs(a.Min[0] - X)) +
78         max(abs(Y - a.Max[1]), abs(a.Min[1] - Y));
79 }
80
81 // 添加一个点 (注意此处的添加可能导致这棵树不平衡, 慎用!)
82 void add(int k) {
83     t[k].Max[0] = t[k].Min[0] = t[k].d[0];
84     t[k].Max[1] = t[k].Min[1] = t[k].d[1];
85     t[k].val = t[k].sum = 0;
86     t[k].l = t[k].r = t[k].f = 0;
87     if (!root) {
88         root = k;
89         return;
90     }
91     int p = root;
92     int D = 0;
93     while (1) {
94         up(p, k);
95         if (t[k].d[D] <= t[p].d[D]) {
96             if (t[p].l) p = t[p].l;
97             else {
98                 t[p].l = k;
99                 t[k].f = p;
100                 return;
101             }
102         } else {
103             if (t[p].r) p = t[p].r;
104             else {
105                 t[p].r = k;
106                 t[k].f = p;
107                 return;
108             }
109         }
110         D ^= 1;
111     }
112 }
113
114 inline long long getdis(const node &a, int X, int Y) {
115     return sqr(a.d[0] - X) + sqr(a.d[1] - Y);
116 }
117
118 // 此处询问距离点 (X, Y) 最远的一个点的距离, ans 需传入无穷小
119 void ask(int p, int X, int Y, long long &ans) {
120     if (!p) return;
121     ans = max(ans, getdis(t[p], X, Y));
122     long long dl = t[p].l ? euclid_upper_bound(t[t[p].l], X, Y) : 0;

```

```

123     long long dr = t[p].r ? euclid_upper_bound(t[t[p].r], X, Y) : 0;
124     if (dl > dr) {
125         if (dl > ans) ask(t[p].l, X, Y, ans);
126         if (dr > ans) ask(t[p].r, X, Y, ans);
127     } else {
128         if (dr > ans) ask(t[p].r, X, Y, ans);
129         if (dl > ans) ask(t[p].l, X, Y, ans);
130     }
131 }
132
133 // 查询矩形范围内所有点的权值和
134 int ask(int p, int x1, int y1, int x2, int y2) {
135     if (t[p].Min[0] > x2 || t[p].Max[0] < x1 || t[p].Min[1] > y2 || t[p].Max[1] < y1
136         ) return 0;
137     if (t[p].Min[0] >= x1 && t[p].Max[0] <= x2 && t[p].Min[1] >= y1 && t[p].Max[1]
138         <= y2) return t[p].sum;
139     int s = 0;
140     if (t[p].d[0] >= x1 && t[p].d[0] <= x2 && t[p].d[1] >= y1 && t[p].d[1] <= y2) s
141         += t[p].val;
142     if (t[p].l) s += ask(t[p].l, x1, y1, x2, y2);
143     if (t[p].r) s += ask(t[p].r, x1, y1, x2, y2);
144     return s;
145 }
146
147 int main() {
148     while (~scanf("%d", &n)) {
149         for (int i = 1; i <= n; ++i) {
150             int x, y, type;
151             scanf("%d%d%d", &x, &y, &type);
152             t[i].d[0] = x;
153             t[i].d[1] = y;
154         }
155         root = build(1, n, 0, 0);
156     }
157 }

```

3.3 左偏树

左偏树是一个可并堆。

下面的程序写的是一个小根堆，如果需要改成大根堆请在注释了 here 那行修改。

接口：

void push(const T &x); 插入一个元素。

void merge(leftist &x); 合并两个堆。注意，合并后原来那个堆将不可访问。

T top() const; 返回堆顶元素。

void pop(); 删除堆顶元素。

int size() const; 返回堆的大小。

```

1 template <class T>
2 class leftist {

```

```

3 public:
4     struct node {
5         T key;
6         int dist;
7         node *l, *r;
8     };
9     leftist() : root(NULL), s(0) {}
10    void push(const T &x) {
11        leftist y;
12        y.s = 1;
13        y.root = new node;
14        y.root -> key = x;
15        y.root -> dist = 0;
16        y.root -> l = y.root -> r = NULL;
17        merge(y);
18    }
19    node* merge(node *x, node *y) {
20        if (x == NULL) return y;
21        if (y == NULL) return x;
22        if (y -> key < x -> key) swap(x, y); //here
23        x -> r = merge(x -> r, y);
24        int ld = x -> l ? x -> l -> dist : -1;
25        int rd = x -> r ? x -> r -> dist : -1;
26        if (ld < rd) swap(x -> l, x -> r);
27        if (x -> r == NULL) x -> dist = 0;
28        else x -> dist = x -> r -> dist + 1;
29        return x;
30    }
31    void merge(leftist &x) {
32        root = merge(root, x.root);
33        s += x.s;
34    }
35    T top() const {
36        if (root == NULL) return T();
37        return root -> key;
38    }
39    void pop() {
40        if (root == NULL) return;
41        node *p = root;
42        root = merge(root -> l, root -> r);
43        --s;
44        delete p;
45    }
46    int size() const {
47        return s;
48    }
49 private:
50     node* root;
51     int s;
52 };

```

3.4 线段树小技巧

给定一个序列 a ，寻找一个最大的 i 使得 $i \leq y$ 且满足一些条件（如 $a[i] \geq w$ ，那么需要在线段树维护 a 的区间最大值）

```
1 int queryl(int p, int left, int right, int y, int w) {
2     if (right <= y) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (y <= mid) return queryl(p<<1|0, left, mid, y, w);
8     int ret = queryl(p<<1|1, mid+1, right, y, w);
9     if (ret != -1) return ret;
10    return queryl(p<<1|0, left, mid, y, w);
11 }
```

给定一个序列 a ，寻找一个最小的 i 使得 $i \geq x$ 且满足一些条件（如 $a[i] \geq w$ ，那么需要在线段树维护 a 的区间最大值）

```
1 int queryr(int p, int left, int right, int x, int w) {
2     if (left >= x) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (x > mid) return queryr(p<<1|1, mid+1, right, x, w);
8     int ret = queryr(p<<1|0, left, mid, x, w);
9     if (ret != -1) return ret;
10    return queryr(p<<1|1, mid+1, right, x, w);
11 }
```

3.5 Splay

接口：

ADD $x\ y\ d$ ：将 $[x, y]$ 的所有数加上 d

REVERSE $x\ y$ ：将 $[x, y]$ 翻转

INSERT $x\ p$ ：将 p 插入到第 x 个数的后面

DEL x ：将第 x 个数删除

```
1 struct SPLAY {
2     struct NODE {
3         int w, min;
4         int son[2], size, father, rev, lazy;
5     } node[N];
6     int top, rt;
7     void pushdown(int x) {
```



```

8      if (!x) return;
9      if (node[x].rev) {
10         node[node[x].son[0]].rev ^= 1;
11         node[node[x].son[1]].rev ^= 1;
12         swap(node[x].son[0], node[x].son[1]);
13         node[x].rev = 0;
14     }
15     if (node[x].lazy) {
16         node[node[x].son[0]].lazy += node[x].lazy;
17         node[node[x].son[1]].lazy += node[x].lazy;
18         node[x].w += node[x].lazy;
19         node[x].min += node[x].lazy;
20         node[x].lazy = 0;
21     }
22 }
23 void pushup(int x) {
24     if (!x) return;
25     pushdown(node[x].son[0]);
26     pushdown(node[x].son[1]);
27     node[x].size = node[node[x].son[0]].size + node[node[x].son[1]].size + 1;
28     node[x].min = node[x].w;
29     if (node[x].son[0]) node[x].min = min(node[x].min, node[node[x].son[0]].min)
30     ;
31     if (node[x].son[1]) node[x].min = min(node[x].min, node[node[x].son[1]].min)
32     ;
33 }
34 void sc(int x, int y, int w) {
35     node[x].son[w] = y;
36     node[y].father = x;
37     pushup(x);
38 }
39 void _ins(int w) {
40     top++;
41     node[top].w = node[top].min = w;
42     node[top].son[0] = node[top].son[1] = 0;
43     node[top].size = 1; node[top].father = 0; node[top].rev = 0;
44 }
45 void init() {
46     top = 0;
47     _ins(0); _ins(0); rt=1;
48     sc(1, 2, 1);
49 }
50 void rotate(int x) {
51     if (!x) return;
52     int y = node[x].father;
53     int w = node[y].son[1]==x;
54     sc(y, node[x].son[w^1], w);
55     sc(node[y].father, x, node[node[y].father].son[1]==y);
56     sc(x, y, w^1);
57 }

```

```

56  int q[N];
57  void flushdown(int x) {
58      int t=0; for (; x; x=node[x].father) q[++t]=x;
59      for (; t; t--) pushdown(q[t]);
60  }
61  void Splay(int x, int root=0) {
62      flushdown(x);
63      while (node[x].father != root) {
64          int y=node[x].father;
65          int w=node[y].son[1]==x;
66          if (node[y].father != root && node[node[y].father].son[w]==y) rotate(y);
67          rotate(x);
68      }
69  }
70  int find(int k) {
71      Splay(rt);
72      while (1) {
73          pushdown(rt);
74          if (node[node[rt].son[0]].size+1==k) {
75              Splay(rt);
76              return rt;
77          } else
78          if (node[node[rt].son[0]].size+1<k) {
79              k-=node[node[rt].son[0]].size+1;
80              rt=node[rt].son[1];
81          } else {
82              rt=node[rt].son[0];
83          }
84      }
85  }
86  int split(int x, int y) {
87      int fx = find(x);
88      int fy = find(y+2);
89      Splay(fx);
90      Splay(fy, fx);
91      return node[fy].son[0];
92  }
93  void add(int x, int y, int d) { //add d to each number in a[x]...a[y]
94      int t = split(x, y);
95      node[t].lazy += d;
96      Splay(t); rt=t;
97  }
98  void reverse(int x, int y) { // reverse the x-th to y-th elements
99      int t = split(x, y);
100     node[t].rev ^= 1;
101     Splay(t); rt=t;
102 }
103 void insert(int x, int p) { // insert p after the x-th element
104     int fx = find(x+1);
105     int fy = find(x+2);

```

```

106     Splay(fx);
107     Splay(fy, fx);
108     _ins(p);
109     sc(fy, top, 0);
110     Splay(top); rt=top;
111 }
112 void del(int x) { // delete the x-th element in Splay
113     int fx = find(x), fy = find(x+2);
114     Splay(fx); Splay(fy, fx);
115     node[fy].son[0] = 0;
116     Splay(fy); rt=fy;
117 }
118 } tree;

```

3.6 可持久化 Treap

接口：

void insert(int x, char c); 在当前第 x 个字符后插入 c

void del(int x, int y); 删除第 x 个字符到第 y 个字符

void copy(int l, int r, int x); 复制第 l 个字符到第 r 个字符，然后粘贴到第 x 个字符后

void reverse(int x, int y); 翻转第 x 个到第 y 个字符

char query(int k); 表示询问当前第 x 个字符是什么

```

1  #define mod 1000000007
2  struct Treap {
3      struct Node {
4          char key;
5          bool reverse;
6          int lc, rc, size; // if size is long long, remember here
7      } node[N];
8      int n, root, rd;
9      int Rand() { rd = (rd * 20372052LL + 25022087LL) % mod; return rd; }
10
11     /*
12     LL Rand() {
13         LL t1 = rand() % 32768;
14         LL t2 = rand() % 32768;
15         LL t3 = rand() % 32768;
16         LL t4 = rand() % 32768;
17         return ((t1 * 32768) + t2) * 32768 + t3) * 32768 + t4;
18     }
19     */
20
21     void init() {
22         n = root = 0;
23     }
24     inline int copy(int x) {
25         node[++n] = node[x]; return n;
26     }

```

```

27 inline void pushdown(int x) {
28     if (!node[x].reverse) return;
29     if (node[x].lc) node[x].lc = copy(node[x].lc);
30     if (node[x].rc) node[x].rc = copy(node[x].rc);
31     swap(node[x].lc, node[x].rc);
32     node[node[x].lc].reverse ^= 1;
33     node[node[x].rc].reverse ^= 1;
34     node[x].reverse = 0;
35 }
36 inline void pushup(int x) {
37     node[x].size = node[node[x].lc].size + node[node[x].rc].size + 1;
38 }
39 int merge(int u, int v) {
40     if (!u || !v) return u+v;
41     pushdown(u); pushdown(v);
42     int t = Rand() % (node[u].size + node[v].size), r; // if size is long long,
        remember here
43     if (t < node[u].size) {
44         r = copy(u);
45         node[r].rc = merge(node[u].rc, v);
46     } else {
47         r = copy(v);
48         node[r].lc = merge(u, node[v].lc);
49     }
50     pushup(r);
51     return r;
52 }
53 int split(int u, int x, int y) { // if size is long long, remember here
54     if (x > y) return 0;
55     pushdown(u);
56     if (x == 1 && y == node[u].size) return copy(u);
57     if (y <= node[node[u].lc].size) return split(node[u].lc, x, y);
58     int t = node[node[u].lc].size + 1; // if size is long long, remember here
59     if (x > t) return split(node[u].rc, x-t, y-t);
60     int num = copy(u);
61     node[num].lc = split(node[u].lc, x, t-1);
62     node[num].rc = split(node[u].rc, 1, y-t);
63     pushup(num);
64     return num;
65 }
66 void insert(int x, char c) {
67     int t1 = split(root, 1, x), t2 = split(root, x+1, node[root].size);
68     node[++n].key = c;
69     node[n].lc = node[n].rc = 0;
70     node[n].reverse = 0;
71     pushup(n);
72     root = merge(merge(t1, n), t2);
73 }
74 void del(int x, int y) {
75     int t1 = split(root, 1, x-1), t2 = split(root, y+1, node[root].size);

```

```

76     root = merge(t1, t2);
77 }
78 void copy(int l, int r, int x) {
79     int t1 = split(root, l, x), t2 = split(root, l, r), t3 = split(root, x+1,
80         node[root].size);
81     root = merge(merge(t1, t2), t3);
82 }
83 void reverse(int x, int y) {
84     int t1 = split(root, l, x-1), t2 = split(root, x, y), t3 = split(root, y+1,
85         node[root].size);
86     node[t2].reverse ^= 1;
87     root = merge(merge(t1, t2), t3);
88 }
89 char query(int k) {
90     int x = root;
91     while (1) {
92         pushdown(x);
93         if (k <= node[node[x].lc].size) x = node[x].lc;
94         else
95             if (k == node[node[x].lc].size + 1) return node[x].key;
96         else
97             k -= node[node[x].lc].size + 1, x = node[x].rc;
98     }
99 }
100 } treap;

```

3.7 可持久化并查集

接口:

void init() 初始化

void merge(int x, int y, int time) 在 time 时刻将 x 和 y 连一条边, 注意加边顺序必须按 time 从小到大加边

void GetFather(int x, int time) 询问 time 时刻及以前的连边状态中, x 所属的集合

```

1 namespace pers_union {
2     const int inf = 0x3f3f3f3f;
3     int father[N], Father[N], Time[N];
4     vector<int> e[N];
5     void init() {
6         for (int i=1;i<=n;i++) {
7             father[i] = i;
8             Father[i] = i;
9             Time[i] = inf;
10            e[i].clear();
11            e[i].push_back(i);
12        }
13    }
14    int getfather(int x) {
15        return (father[x] == x) ? x : father[x] = getfather(father[x]);

```

```

16     }
17     int GetFather(int x, int time) {
18         return (Time[x] <= time) ? GetFather(Father[x], time) : x;
19     }
20     void merge(int x, int y, int time) {
21         int fx = getfather(x), fy = getfather(y);
22         if (fx == fy) return;
23         if (e[fx].size() > e[fy].size()) swap(fx, fy);
24         father[fx] = fy;
25         Father[fx] = fy;
26         Time[fx] = time;
27         for (int i=0; i<e[fx].size(); i++) {
28             e[fy].push_back(e[fx][i]);
29         }
30     }
31 };

```

4 树

4.1 树链剖分

接口：

void addedge(int x, int y); 将 x 到 y 连边，注意这是单向边
 void dfs(int x, int root = 0); 从 x 开始遍历整棵树
 void split(int x, int tp); 划分轻重链
 int lca(int x, int y); 求 x 和 y 的 lca
 int query(int x, int y); 求 x 到 y 经过的点数
 int skip(int x, int k); 求从 x 向根方向跳 k 步到达的节点（若超出根，则返回 0）
 void get_data(int x, int y); 将 x 到 y 路径上的重链找出来，存在 seg[0] 中
Debug 技巧： 换一个根来 dfs 以测试程序是否能通过 $father[i] > i$ 的数据

```

1 struct EDGE {
2     int adj, next;
3 } edge[N * 2];
4
5 int n, gh[N], top, s_top;
6 int father[N], deep[N], son[N], size[N], Top[N], dfn[N], rdfs[N];
7
8 void addedge(int x, int y) {
9     edge[++top].adj = y;
10    edge[top].next = gh[x];
11    gh[x] = top;
12 }
13
14 void dfs(int x, int root = 0) {
15     father[x] = root;
16     deep[x] = deep[root] + 1;
17     son[x] = 0;

```

```

18     size[x] = 1;
19     int dd = 0;
20     for (int p = gh[x]; p; p = edge[p].next)
21         if (edge[p].adj != root) {
22             dfs(edge[p].adj, x);
23             if (size[edge[p].adj] > dd) {
24                 dd = size[edge[p].adj];
25                 son[x] = edge[p].adj;
26             }
27             size[x] += size[edge[p].adj];
28         }
29 }
30
31 void split(int x, int tp) {
32     Top[x] = tp; dfn[x] = ++s_top; rdfs[s_top] = x;
33     if (son[x]) split(son[x], tp);
34     for (int p = gh[x]; p; p = edge[p].next)
35         if (edge[p].adj != father[x] && edge[p].adj != son[x])
36             split(edge[p].adj, edge[p].adj);
37 }
38
39 int lca(int x, int y) {
40     int tx = Top[x], ty = Top[y];
41     while (tx != ty) {
42         if (deep[tx] < deep[ty]) {
43             swap(tx, ty);
44             swap(x, y);
45         }
46         x = father[tx];
47         tx = Top[x];
48     }
49     if (deep[x] < deep[y])
50         swap(x, y);
51     return y;
52 }
53
54 int query(int x, int y) {
55     int tx = Top[x], ty = Top[y];
56     int ans = 0;
57     while (tx != ty) {
58         if (deep[tx] < deep[ty]) {
59             swap(tx, ty);
60             swap(x, y);
61         }
62         ans += dfn[x] - dfn[tx] + 1;
63         x = father[tx];
64         tx = Top[x];
65     }
66     if (deep[x] < deep[y])
67         swap(x, y);

```

```

68     ans += dfn[x] - dfn[y] + 1;
69     return ans;
70 }
71
72 int skip(int x, int k) {
73     int tx = Top[x];
74     while (tx) {
75         if (k < dfn[x] - dfn[tx] + 1) {
76             return rdfs[ dfn[x] - k ];
77         } else {
78             k -= dfn[x] - dfn[tx] + 1;
79             x = father[tx];
80             tx = Top[x];
81         }
82     }
83     return 0;
84 }
85
86 struct segment {
87     int l, r;
88     data d;
89     segment(int _l, int _r) { // from _l to _r
90         l = _l, r = _r;
91         if (l <= r) d = query(l, r, 0);
92         else d = query(r, l, 1); //reverse
93     }
94 };
95
96 vector<segment> seg[2];
97
98 void get_data(int x, int y) {
99     seg[0].clear(); seg[1].clear();
100    int tx = Top[x], ty = Top[y];
101    int s = 0;
102    while (tx != ty) {
103        if (deep[tx] < deep[ty]) {
104            swap(tx, ty);
105            swap(x, y);
106            s ^= 1;
107        }
108        if (s == 0)
109            seg[s].push_back(segment(w[x], w[tx]));
110        else
111            seg[s].push_back(segment(w[tx], w[x]));
112        x = father[tx];
113        tx = Top[x];
114    }
115    if (x != y) {
116        if (deep[x] < deep[y]) {
117            swap(x, y);

```



```

118         s ^= 1;
119     }
120     if (s == 0)
121         seg[s].push_back(segment(w[x], w[y] + 1));
122     else
123         seg[s].push_back(segment(w[y] + 1, w[x]));
124 }
125 reverse(seg[1].begin(), seg[1].end());
126 for (int i = 0; i < seg[1].size(); ++i)
127     seg[0].push_back(seg[1][i]);
128 // saved to seg[0]
129 }
130
131 void init() {
132     top = s_top = 0;
133     for (int i = 1; i <= n; ++i) gh[i] = 0;
134 }

```

4.2 点分治

初始化时须设置 $top = 1$ 。

```

1 void addedge(int x, int y) {
2     edge[++top].adj = y;
3     edge[top].valid = 1;
4     edge[top].next = gh[x];
5     gh[x] = top;
6 }
7 void get_size(int x, int root=0) {
8     size[x] = 1; son[x] = 0;
9     int dd = 0;
10    for (int p=gh[x]; p; p=edge[p].next)
11        if (edge[p].adj != root && edge[p].valid) {
12            get_size(edge[p].adj, x);
13            size[x] += size[edge[p].adj];
14            if (size[edge[p].adj] > dd) {
15                dd = size[edge[p].adj];
16                son[x] = edge[p].adj;
17            }
18        }
19 }
20 int getroot(int x) {
21     get_size(x);
22     int sz = size[x];
23     while (size[son[x]] > sz/2)
24         x = son[x];
25     return x;
26 }
27 void dc(int x) {
28     x = getroot(x);

```

```

29     static int list[N], ltop;
30     ltop = 0;
31     for (int p=gh[x]; p; p=edge[p].next)
32         if (edge[p].valid)
33             list[++ltop] = edge[p].adj;
34     clear();
35     for (int i=1; i<=ltop; i++) {
36         update();
37         modify();
38     }
39     clear();
40     for (int i=ltop; i>=1; i--) {
41         update();
42         modify();
43     }
44     //be careful about the root
45     for (int p=gh[x]; p; p=edge[p].next)
46         if (edge[p].valid) {
47             edge[p].valid = 0;
48             edge[p^1].valid = 0;
49             dc(edge[p].adj);
50         }
51 }

```

4.3 Link Cut Tree

请注意，一开始必须调用 `lct.init(0)`，否则求出的最小值一定会是 0。

`minval` 维护的是链上 `val` 最小值。

`sumval2` 维护的是子树 `val2` 的和。

```

1 struct DTree {
2     int f[N], son[N][2], sz[N], rev[N];
3     int val[N], minid[N], minval[N];
4     int val2[N], sumval2[N]; // 记得开 long long。注意两个都要开 long long，因为
5                               // val2 还包含了虚儿子的子树和。
6     int tot;
7     stack<int> s;
8     void init(int i) {
9         tot = max(tot, i);
10        son[i][0] = son[i][1] = 0;
11        f[i] = rev[i] = 0;
12        if (i == 0) {
13            sz[i] = 0;
14            val[i] = minval[i] = inf;
15            minid[i] = i;
16            val2[i] = sumval2[i] = 0;
17        } else {
18            sz[i] = 1;
19            val[i] = minval[i] = VAL;

```

```

19         minid[i] = i;
20         val2[i] = sumval2[i] = VAL2;
21     }
22 }
23 bool isroot(int x) {
24     return !f[x] || (son[f[x]][0] != x && son[f[x]][1] != x);
25 }
26 void revl(int x) {
27     if (!x) return;
28     swap(son[x][0], son[x][1]);
29     rev[x] ^= 1;
30 }
31 void down(int x) {
32     if (!x) return;
33     if (rev[x]) revl(son[x][0]), revl(son[x][1]), rev[x] = 0;
34 }
35 void up(int x) {
36     if (!x) return;
37     down(son[x][0]); down(son[x][1]);
38     sz[x] = sz[son[x][0]] + sz[son[x][1]] + 1;
39     minval[x] = val[x]; minid[x] = x;
40     if (minval[son[x][0]] < minval[x]) minval[x] = minval[son[x][0]], minid[x] =
        minid[son[x][0]];
41     if (minval[son[x][1]] < minval[x]) minval[x] = minval[son[x][1]], minid[x] =
        minid[son[x][1]];
42     sumval2[x] = sumval2[son[x][0]] + sumval2[son[x][1]] + val2[x];
43 }
44 void rotate(int x) {
45     int y = f[x], w = son[y][1] == x;
46     son[y][w] = son[x][w ^ 1];
47     if (son[x][w ^ 1]) f[son[x][w ^ 1]] = y;
48     if (f[y]) {
49         int z = f[y];
50         if (son[z][0] == y) son[z][0] = x;
51         else if (son[z][1] == y) son[z][1] = x;
52     }
53     f[x] = f[y]; f[y] = x; son[x][w ^ 1] = y;
54     up(y);
55 }
56 void splay(int x) {
57     while (!s.empty()) s.pop();
58     s.push(x);
59     for (int i = x; !isroot(i); i = f[i]) s.push(f[i]);
60     while (!s.empty()) down(s.top()), s.pop();
61     while (!isroot(x)) {
62         int y = f[x];
63         if (!isroot(y)) {
64             if ((son[f[y]][0] == y) ^ (son[y][0] == x))
65                 rotate(x);
66             else

```

```

67         rotate(y);
68     }
69     rotate(x);
70 }
71 up(x);
72 }
73 void access(int x) {
74     for (int y = 0; x; y = x, x = f[x]) {
75         splay(x);
76         val2[x] += sumval2[son[x][1]];
77         son[x][1] = y;
78         val2[x] -= sumval2[son[x][1]];
79         up(x);
80     }
81 }
82 int root(int x) {
83     access(x);
84     splay(x);
85     while (son[x][0]) x = son[x][0];
86     return x;
87 }
88 void makeroot(int x) {
89     access(x);
90     splay(x);
91     rev1(x);
92 }
93 void link(int x, int y) {
94     makeroot(x);
95     f[x] = y;
96     access(x);
97     // 如果需要维护子树和 val2, sumval2, 这样是不够的。因为增加了虚边, 所以需要
98     // 修改 val2 值。将上面的三行代码替换为以下代码:
99     // makeroot(x);
100    // makeroot(y);
101    // f[x] = y;
102    // val2[y] += sumval2[x];
103    // up(y);
104    // access(x);
105 }
106 void cutf(int x) { // 它和父亲的边
107     access(x);
108     splay(x);
109     f[son[x][0]] = 0;
110     son[x][0] = 0;
111     up(x);
112 }
113 void cut(int x, int y) { // 切断 x 与 y 之间的边 (须保证 x 与 y 相邻)
114     makeroot(x);
115     cutf(y);
116 }

```

```

116     int ask(int x, int y) { // 询问 x 到 y 之间取得最小值的点
117         makeroot(x);
118         access(y);
119         splay(y);
120         return minid[y];
121     }
122     int querymin_cut(int x, int y) { // 询问 x 到 y 之间取得最小值的点，并把它删去
        (须保证该点在 x 和 y 之间，且度数恰好为 2)
123         int m = ask(x, y);
124         makeroot(x);
125         cutf(m);
126         makeroot(y);
127         cutf(m);
128         return val[m];
129     }
130     void link(int x, int y, int w) { // 在 x 和 y 之间添加一条权值为 w 的边 (将边视
        为点插入)
131         init(++tot);
132         val[tot] = minval[tot] = w;
133         link(x, tot);
134         link(y, tot);
135     }
136     int getsumval2(int x, int y) { // 令 x 为根，求 y 子树的 val2 的和
137         makeroot(x);
138         access(y);
139         return val2[y];
140     }
141 } lct;

```

4.4 求子树的直径

树形 DP。

答案保存在 u, d 数组中。

$u[x].exc$ 表示切断 x 与 $father[x]$ 的边， $father[x]$ 表示的那颗子树的直径。

$d[x].exc$ 表示切断 x 与 $father[x]$ 的边， x 表示的那颗子树的直径。

```

1  #include <bits/stdc++.h>
2
3  #define N 200020
4
5  using namespace std;
6
7  vector<int> g[N];
8  int n, q, top;
9  int deep[N], father[N], son[N], size[N], Top[N], dfn[N], rdfn[N];
10
11 void dfs(int x, int root = 0) {
12     deep[x] = deep[root] + 1;
13     father[x] = root;

```

```

14     son[x] = 0; size[x] = 1;
15     if (root) g[x].erase(lower_bound(g[x].begin(), g[x].end(), root));
16     // 去根
17     int dd = 0;
18     for (int i = 0; i < g[x].size(); ++i) {
19         dfs(g[x][i], x);
20         if (size[g[x][i]] > dd) {
21             dd = size[g[x][i]];
22             son[x] = g[x][i];
23         }
24         size[x] += size[g[x][i]];
25     }
26 }
27
28 void split(int x, int tp) {
29     dfn[x] = ++top; rdn[top] = x; Top[x] = tp;
30     if (son[x]) split(son[x], tp);
31     for (int i = 0; i < g[x].size(); ++i)
32         if (g[x][i] != son[x])
33             split(g[x][i], g[x][i]);
34 }
35
36 struct data {
37     int inc, inc_id;
38     int exc, exc_l, exc_r;
39     //inc 表示从该点出发可以走到的最远距离
40     //inc_id 表示从该点出发可以走到的最远点的编号
41     //exc 表示子树中两点最远距离
42     //exc_l, exc_r 表示子树中两点取得最远距离的两点的编号
43     data() {
44         inc = inc_id = 0;
45         exc = exc_l = exc_r = 0;
46     }
47 } u[N], d[N];
48
49 int safe(int x, int y) {
50     // 防止 inc_id = 0 的情况
51     if (x) return x;
52     return y;
53 }
54
55 void dfs1(int x) {
56     d[x].inc = 1; d[x].inc_id = x;
57     data mx1 = data(), mx2 = data();
58     // mx1, mx2 表示儿子 inc 最大、第2大值，用于更新该点 exc
59     for (int i = 0; i < g[x].size(); ++i) {
60         dfs1(g[x][i]);
61         if (d[g[x][i]].inc + 1 > d[x].inc) {
62             d[x].inc = d[g[x][i]].inc + 1;
63             d[x].inc_id = d[g[x][i]].inc_id;

```

```

64     }
65     if (d[g[x][i]].inc > mx1.inc) {
66         mx2 = mx1;
67         mx1 = d[g[x][i]];
68     } else
69     if (d[g[x][i]].inc > mx2.inc) {
70         mx2 = d[g[x][i]];
71     }
72 }
73 d[x].exc = mx1.inc + mx2.inc + 1;
74 d[x].exc_l = safe(mx1.inc_id, x);
75 d[x].exc_r = safe(mx2.inc_id, x);
76 for (int i = 0; i < g[x].size(); ++i)
77     if (d[g[x][i]].exc > d[x].exc) {
78         d[x].exc = d[g[x][i]].exc;
79         d[x].exc_l = d[g[x][i]].exc_l;
80         d[x].exc_r = d[g[x][i]].exc_r;
81     }
82 }
83
84 void dfs2(int x, data y) {
85     u[x] = y;
86     if (!y.exc) y.exc = 1, y.exc_l = y.exc_r = x;
87     data mx1 = y, mx2 = data(), mx3 = data(), mxel = y, mxe2 = data();
88     // mx1, mx2, mx3 表示根过来的子树中 inc 的最大、第2大、第3大值
89     // mxel, mxe2 表示根过来的子树中 exc 的最大、第2大值
90     int mx1_id = -1, mx2_id = -1, mx3_id = -1, mxel_id = -1, mxe2_id = -1;
91     for (int i = 0; i < g[x].size(); ++i) {
92         if (d[g[x][i]].inc > mx1.inc) {
93             mx3 = mx2; mx3_id = mx2_id;
94             mx2 = mx1; mx2_id = mx1_id;
95             mx1 = d[g[x][i]]; mx1_id = i;
96         } else
97         if (d[g[x][i]].inc > mx2.inc) {
98             mx3 = mx2; mx3_id = mx2_id;
99             mx2 = d[g[x][i]]; mx2_id = i;
100         } else
101         if (d[g[x][i]].inc > mx3.inc) {
102             mx3 = d[g[x][i]]; mx3_id = i;
103         }
104         if (d[g[x][i]].exc > mxel.exc) {
105             mxe2 = mxel; mxe2_id = mxel_id;
106             mxel = d[g[x][i]]; mxel_id = i;
107         } else
108         if (d[g[x][i]].exc > mxe2.exc) {
109             mxe2 = d[g[x][i]]; mxe2_id = i;
110         }
111     }
112     for (int i = 0; i < g[x].size(); ++i) {
113         data z = data();

```

```

114     if (i == mx1_id) {
115         z.exc = mx2.inc + mx3.inc + 1;
116         z.exc_l = safe(mx2.inc_id, x);
117         z.exc_r = safe(mx3.inc_id, x);
118     } else
119     if (i == mx2_id) {
120         z.exc = mx1.inc + mx3.inc + 1;
121         z.exc_l = safe(mx1.inc_id, x);
122         z.exc_r = safe(mx3.inc_id, x);
123     } else {
124         z.exc = mx1.inc + mx2.inc + 1;
125         z.exc_l = safe(mx1.inc_id, x);
126         z.exc_r = safe(mx2.inc_id, x);
127     }
128     if (i == mxel_id) {
129         if (mxe2.exc > z.exc) z = mxe2;
130     } else {
131         if (mxel.exc > z.exc) z = mxel;
132     }
133     if (i == mx1_id) {
134         z.inc = mx2.inc + 1;
135         z.inc_id = safe(mx2.inc_id, x);
136     } else {
137         z.inc = mx1.inc + 1;
138         z.inc_id = safe(mx1.inc_id, x);
139     }
140     dfs2(g[x][i], z);
141 }
142 }

```

4.5 虚树

设 $a[0 \cdots k-1]$ 为需要构建虚树的点。

构建出虚树的节点保存在 a 数组中, k 为节点个数。加边调用函数 `addedge(int x, int y, int w)`。

```

1 bool cmp(int x, int y) {
2     return dfn[x] < dfn[y];
3 }
4
5 stack<int> stk;
6
7 void solve() {
8     sort(a, a + k, cmp);
9     int m = k;
10    for (int j = 1; j < m; ++j)
11        a[k++] = lca(a[j - 1], a[j]);
12    sort(a, a + k, cmp);
13    k = unique(a, a + k) - a;
14    stk.push(a[0]);
15    for (int j = 1; j < k; ++j) {

```



```

16     int u = lca(stk.top(), a[j]);
17     while (dep[stk.top()] > dep[u]) --top;
18     assert(stk.top() == u);
19     stk.push(a[j]);
20     addedge(u, a[j], dis[a[j]] - dis[u]);
21 }
22 }

```

5 图

5.1 欧拉回路

欧拉回路：

无向图：每个顶点的度数都是偶数，则存在欧拉回路。

有向图：每个顶点的入度 = 出度，则存在欧拉回路。

欧拉路径：

无向图：当且仅当该图所有顶点的度数为偶数，或者除了两个度数为奇数外其余的全是偶数。

有向图：当且仅当该图所有顶点出度 = 入度或者一个顶点出度 = 入度 + 1，另一个顶点入度 = 出度 + 1，其他顶点出度 = 入度。

下面 $O(n + m)$ 求欧拉回路的代码中， n 为点数， m 为边数，若有解则依次输出经过的边的编号，若是无向图，则正数表示 x 到 y ，负数表示 y 到 x 。

```

1 namespace UndirectedGraph{
2     int n,m,i,x,y,d[N],g[N],v[M<<1],w[M<<1],vis[M<<1],nxt[M<<1],ed;
3     int ans[M],cnt;
4     void add(int x,int y,int z){
5         d[x]++;
6         v[++ed]=y;w[ed]=z;nxt[ed]=g[x];g[x]=ed;
7     }
8     void dfs(int x){
9         for(int&i=g[x];i;){
10             if(vis[i]){i=nxt[i];continue;}
11             vis[i]=vis[i^1]=1;
12             int j=w[i];
13             dfs(v[i]);
14             ans[++cnt]=j;
15         }
16     }
17     void solve(){
18         scanf("%d%d",&n,&m);
19         for(i=ed=1;i<=m;i++)scanf("%d%d",&x,&y),add(x,y,i),add(y,x,-i);
20         for(i=1;i<=n;i++)if(d[i]&1){puts("NO");return;}
21         for(i=1;i<=n;i++)if(g[i]){dfs(i);break;}
22         for(i=1;i<=n;i++)if(g[i]){puts("NO");return;}
23         puts("YES");
24         for(i=m;i;i--)printf("%d_",ans[i]);
25     }
26 }

```

```

27 namespace DirectedGraph{
28     int n,m,i,x,y,d[N],g[N],v[M],vis[M],nxt[M],ed;
29     int ans[M],cnt;
30     void add(int x,int y){
31         d[x]++;d[y]--;
32         v[++ed]=y;nxt[ed]=g[x];g[x]=ed;
33     }
34     void dfs(int x){
35         for(int&i=g[x];i;){
36             if(vis[i]){i=nxt[i];continue;}
37             vis[i]=1;
38             int j=i;
39             dfs(v[i]);
40             ans[++cnt]=j;
41         }
42     }
43     void solve(){
44         scanf("%d%d",&n,&m);
45         for(i=1;i<=m;i++)scanf("%d%d",&x,&y),add(x,y);
46         for(i=1;i<=n;i++)if(d[i]){puts("NO");return;}
47         for(i=1;i<=n;i++)if(g[i]){dfs(i);break;}
48         for(i=1;i<=n;i++)if(g[i]){puts("NO");return;}
49         puts("YES");
50         for(i=m;i;i--)printf("%d_",ans[i]);
51     }
52 }

```

5.2 最短路径

5.2.1 Dijkstra

```

1  #define LL long long
2
3  struct EDGE {
4      int adj, w, next;
5  } edge[M*2];
6
7  typedef pair<LL, int> pli;
8  priority_queue <pli, vector<pli>, greater<pli> > q;
9
10 int n, top, gh[N];
11 LL dist[N];
12
13 void addedge(int x, int y, int w) {
14     edge[++top].adj = y;
15     edge[top].w = w;
16     edge[top].next = gh[x];
17     gh[x] = top;
18 }

```

```

19
20 LL dijkstra(int s, int t) {
21     memset(dist, 63, sizeof(dist));
22     memset(v, 0, sizeof(v));
23     dist[s] = 0;
24     q.push(make_pair(dist[s], s));
25     while (!q.empty()) {
26         LL dis = q.top().first;
27         int x = q.top().second;
28         q.pop();
29         if (dis != dist[x]) continue;
30         for (int p=gh[x]; p; p=edge[p].next) {
31             if (dis + edge[p].w < dist[edge[p].adj]) {
32                 dist[edge[p].adj] = dis + edge[p].w;
33                 q.push(make_pair(dist[edge[p].adj], edge[p].adj));
34             }
35         }
36     }
37     return dist[t];
38 }

```

5.2.2 SPFA

```

1 struct EDGE {
2     int adj, w, next;
3 } edge[M*2];
4
5 int n,m,top,gh[N],v[N],cnt[N],q[N],dist[N],head,tail;
6
7 void addedge(int x, int y, int w) {
8     edge[++top].adj = y;
9     edge[top].w = w;
10    edge[top].next = gh[x];
11    gh[x] = top;
12 }
13
14 int spfa(int S, int T) {
15     memset(v, 0, sizeof(v));
16     memset(cnt, 0, sizeof(cnt));
17     memset(dist, 63, sizeof(dist));
18     head = 0, tail = 1;
19     dist[S] = 0; q[1] = S;
20     while (head != tail) {
21         (head += 1) %= N;
22         int x = q[head]; v[x] = 0;
23         ++cnt[x]; if (cnt[x] > n) return -1;
24         for (int p=gh[x]; p; p=edge[p].next)
25             if (dist[x] + edge[p].w < dist[edge[p].adj]) {
26                 dist[edge[p].adj] = dist[x] + edge[p].w;

```

```

27         if (!v[edge[p].adj]) {
28             v[edge[p].adj] = 1;
29             (tail += 1) %= N;
30             q[tail] = edge[p].adj;
31         }
32     }
33 }
34 return dist[T];
35 }

```

5.3 K 短路

接口：

kthsp::init(n)：初始化并设置节点个数为 n

kthsp::add(x, y, w)：添加一条 x 到 y 的有向边

kthsp::work(S, T, k)：求 S 到 T 的第 k 短路

```

1  #define N 200020
2  #define M 400020
3  #define LOGM 20
4  #define LL long long
5  #define inf (1LL<<61)
6
7  namespace pheap {
8      struct Node {
9          int next, son[2];
10         LL val;
11     } node[M*LOGM];
12     int LOG[M];
13     int root[M], size[M*LOGM], top;
14     int add() {
15         ++top; assert(top < M*LOGM);
16         node[top].next = node[top].son[0] = node[top].son[1] = 0;
17         node[top].val = inf;
18         return top;
19     }
20     int copy(int x) {
21         int t = add();
22         node[t] = node[x];
23         return t;
24     }
25     void init() {
26         memset(root, 0, sizeof(root));
27         top = -1; add();
28         LOG[1] = 0;
29         for (int i=2; i<M; i++) LOG[i] = LOG[i>>1] + 1;
30     }
31     void upd(int x, int &next, LL &val) {
32         if (val < node[x].val) {

```

```

33         swap(val, node[x].val);
34         swap(next, node[x].next);
35     }
36 }
37 void insert(int x, int next, LL val) {
38     int sz = size[root[x]] + 1;
39     root[x] = copy(root[x]);
40     size[root[x]] = sz; x = root[x];
41     upd(x, next, val);
42     for (int i=LOG[sz]-1;i>=0;i--) {
43         int ind = (sz>>i)&1;
44         node[x].son[ind] = copy(node[x].son[ind]);
45         x = node[x].son[ind];
46         upd(x, next, val);
47     }
48 }
49 };
50
51 namespace kthsp {
52     using namespace pheap;
53     struct EDGE {
54         int adj, w, next;
55     } edge[2][M];
56     struct W {
57         int x, y, w;
58     } e[M];
59     bool has_init = 0;
60     int n, m, top[2], gh[2][N], v[N];
61     LL dist[N];
62     void init(int n1) {
63         has_init = 1;
64         n = n1; m = 0;
65         memset(top, 0, sizeof(top));
66         memset(gh, 0, sizeof(gh));
67         for (int i=1;i<=n;i++) dist[i] = inf;
68     }
69     void addedge(int id, int x, int y, int w) {
70         edge[id][++top[id]].adj = y;
71         edge[id][top[id]].w = w;
72         edge[id][top[id]].next = gh[id][x];
73         gh[id][x] = top[id];
74     }
75     void add(int x, int y, int w) {
76         assert(has_init);
77         e[++m].x=x; e[m].y=y; e[m].w=w;
78     }
79     int best[N], bestw[N];
80     typedef pair<LL, int> pli;
81     priority_queue <pli, vector<pli>, greater<pli> > q;
82

```

```

83 // you can replace dijkstra with SPFA or TOPSORT(DAG)
84 void dijkstra(int S) {
85     while (!q.empty()) q.pop();
86     dist[S] = 0; q.push(make_pair(dist[S], S));
87     while (!q.empty()) {
88         LL dis = q.top().first;
89         int x = q.top().second;
90         q.pop();
91         if (dist[x] != dis) continue;
92         for (int p=gh[1][x]; p; p=edge[1][p].next) {
93             int y = edge[1][p].adj;
94             if (dist[x] + edge[1][p].w < dist[y]) {
95                 dist[y] = dist[x] + edge[1][p].w;
96                 best[y] = x;
97                 bestw[y] = p;
98                 q.push(make_pair(dist[y], y));
99             }
100         }
101     }
102 }
103 void dfs(int x) {
104     if (v[x]) return;
105     v[x] = 1;
106     if (best[x]) root[x] = root[best[x]];
107     for (int p=gh[0][x]; p; p=edge[0][p].next)
108         if (dist[edge[0][p].adj] != inf && bestw[x] != p) {
109             insert(x, edge[0][p].adj, edge[0][p].w + dist[edge[0][p].adj] - dist
110                 [x]);
111         }
112     for (int p=gh[1][x]; p; p=edge[1][p].next)
113         if (best[edge[1][p].adj] == x)
114             dfs(edge[1][p].adj);
115 }
116 LL work(int S, int T, int k) {
117     assert(has_init);
118     n++; add(T, n, 0);
119     if (S == T) k++;
120     T = n;
121     for (int i=1; i<=m; i++) {
122         addedge(0, e[i].x, e[i].y, e[i].w);
123         addedge(1, e[i].y, e[i].x, e[i].w);
124     }
125     dijkstra(T);
126     root[T] = 0; pheap::init();
127     memset(v, 0, sizeof(v));
128     dfs(T);
129     while (!q.empty()) q.pop();
130     if (k == 1) return dist[S];
131     if (root[S]) q.push(make_pair(dist[S] + node[root[S]].val, root[S]));
132     while (k--) {

```

```

132         if (q.empty()) return inf;
133         pli now = q.top(); q.pop();
134         if (k == 1) return now.first;
135         int x = node[now.second].next, u = node[now.second].son[0], v = node[now
            .second].son[1];
136         if (root[x]) q.push(make_pair(now.first + node[root[x]].val, root[x]));
137         if (u) q.push(make_pair(now.first - node[now.second].val + node[u].val,
            u));
138         if (v) q.push(make_pair(now.first - node[now.second].val + node[v].val,
            v));
139     }
140     return 0;
141 }
142 };

```

5.4 Tarjan

割点的判断：一个顶点 u 是割点，当且仅当满足 (1) 或 (2)：

- (1) u 为树根，且 u 有多于一个子树（即：存在一个儿子 v 使得 $dfn[u] + 1 \neq dfn[v]$ ）
- (2) u 不为树根，且满足存在 (u, v) 为树枝边（ u 为 v 的父亲），使得 $dfn[u] \leq low[v]$

桥的判断：一条无向边 (u, v) 是桥，当且仅当 (u, v) 为树枝边，满足 $dfn[u] < low[v]$

```

1 struct EDGE { int adj, next; } edge[M];
2 int n, m, top, gh[N];
3 int dfn[N], low[N], cnt, ind, stop, instack[N], stack[N], belong[N];
4 void addedge(int x, int y) {
5     edge[++top].adj = y;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 void tarjan(int x) {
10     dfn[x] = low[x] = ++ind;
11     instack[x] = 1; stack[++stop] = x;
12     for (int p=gh[x]; p; p=edge[p].next)
13         if (!dfn[edge[p].adj]) {
14             tarjan(edge[p].adj);
15             low[x] = min(low[x], low[edge[p].adj]);
16         } else if (instack[edge[p].adj]) {
17             low[x] = min(low[x], dfn[edge[p].adj]);
18         }
19     if (dfn[x] == low[x]) {
20         ++cnt; int tmp=0;
21         while (tmp!=x) {
22             tmp = stack[stop--];
23             belong[tmp] = cnt;
24             instack[tmp] = 0;
25         }
26     }
27 }

```

5.5 2-SAT

```
1 #define N number_of_vertex
2 #define M number_of_edges
3
4 struct MergePoint {
5     struct EDGE {
6         int adj, next;
7     } edge[M];
8     int ex[M], ey[M];
9     bool instack[N];
10    int gh[N], top, dfn[N], low[N], cnt, ind, stop, stack[N], belong[N];
11    void init() {
12        cnt = ind = stop = top = 0;
13        memset(dfn, 0, sizeof(dfn));
14        memset(instack, 0, sizeof(instack));
15        memset(gh, 0, sizeof(gh));
16    }
17    void addedge(int x, int y) { //reverse
18        std::swap(x, y);
19        edge[++top].adj = y;
20        edge[top].next = gh[x];
21        gh[x] = top;
22        ex[top] = x;
23        ey[top] = y;
24    }
25    void tarjan(int x) {
26        dfn[x] = low[x] = ++ind;
27        instack[x] = 1; stack[++stop] = x;
28        for (int p=gh[x]; p; p=edge[p].next)
29            if (!dfn[edge[p].adj]) {
30                tarjan(edge[p].adj);
31                low[x] = std::min(low[x], low[edge[p].adj]);
32            } else if (instack[edge[p].adj]) {
33                low[x] = std::min(low[x], dfn[edge[p].adj]);
34            }
35        if (dfn[x] == low[x]) {
36            ++cnt; int tmp = 0;
37            while (tmp!=x) {
38                tmp = stack[stop--];
39                belong[tmp] = cnt;
40                instack[tmp] = 0;
41            }
42        }
43    }
44    void work() {
45        for (int i = (__first__); i <= (__last__); ++i)
46            if (!dfn[i])
47                tarjan(i);
48    }
```



```

49 } merge;
50
51 struct Topsort {
52     struct EDGE {
53         int adj, next;
54     } edge[M];
55     int n, top, gh[N], ops[N], deg[N], ans[N];
56     std::queue<int> q;
57     void init() {
58         n = merge.cnt; top = 0;
59         memset(gh, 0, sizeof(gh));
60         memset(deg, 0, sizeof(deg));
61     }
62     void addedge(int x, int y) {
63         if (x == y) return;
64         edge[++top].adj = y;
65         edge[top].next = gh[x];
66         gh[x] = top;
67         ++deg[y];
68     }
69     void work() {
70         for (int i = 1; i <= n; ++i)
71             if (!deg[i])
72                 q.push(i);
73         while (!q.empty()) {
74             int x = q.front();
75             q.pop();
76             for (int p = gh[x]; p; p = edge[p].next)
77                 if (--deg[edge[p].adj])
78                     q.push(edge[p].adj);
79             if (ans[x]) continue;
80             ans[x] = -1; //not selected
81             ans[ops[x]] = 1; //selected
82         }
83     }
84 } ts;

```

调用示例:

```

1  merge.init();
2  merge.addedge();
3  merge.work();
4  for (int i = 1; i <= n; ++i) {
5      if (merge.belong[U(i, 0)] == merge.belong[U(i, 1)]) {
6          puts("NO");
7          return 0;
8      }
9      ts.ops[merge.belong[U(i, 0)]] = merge.belong[U(i, 1)];
10     ts.ops[merge.belong[U(i, 1)]] = merge.belong[U(i, 0)];
11 }
12 ts.init();

```

```

13     ts.work();
14     puts("YES");
15     for (int i = 1; i <= n; ++i) {
16         int x = U(i, 0), y = U(i, 1);
17         x = merge.belong[x], y = merge.belong[y];
18         x = ts.ans[x], y = ts.ans[y];
19         if (x == 1) puts("0_is_selected");
20         if (y == 1) puts("1_is_selected");
21     }

```

5.6 统治者树 (Dominator Tree)

Dominator Tree 可以解决判断一类有向图必经点的问题。

$idom[x]$ 表示离 x 最近的必经点 (重编号后)。将 $idom[x]$ 作为 x 的父亲, 构成一棵 Dominator Tree

接口:

`void dominator::init(int n);` 初始化, 有向图节点数为 n

`void dominator::addedge(int u, int v);` 添加一条有向边 (u, v)

`void dominator::work(int root);` 以 $root$ 为根, 建立一棵 Dominator Tree

结果的返回:

在执行 `dominator::work(int root);` 后, 树边保存在 `vector<int> tree[N]` 中

```

1 namespace dominator {
2     vector<int> g[N], rg[N], bucket[N], tree[N];
3     int n, ind, idom[N], sdom[N], dfn[N], dsu[N], father[N], label[N], rev[N];
4     void dfs(int x) {
5         ++ind;
6         dfn[x] = ind; rev[ind] = x;
7         label[ind] = dsu[ind] = sdom[ind] = ind;
8         for (auto p : g[x]) {
9             if (!dfn[p]) dfs(p), father[dfn[p]] = dfn[x];
10            rg[dfn[p]].push_back(dfn[x]);
11        }
12    }
13    void init(int n1) {
14        n = n1; ind = 0;
15        for (int i = 1; i <= n; ++i) {
16            g[i].clear();
17            rg[i].clear();
18            bucket[i].clear();
19            tree[i].clear();
20            dfn[i] = 0;
21        }
22    }
23    void addedge(int u, int v) {
24        g[u].push_back(v);
25    }
26    int find(int x, int step=0) {

```

```

27     if (dsu[x] == x) return step ? -1 : x;
28     int y = find(dsu[x], 1);
29     if (y < 0) return x;
30     if (sdom[label[dsu[x]]] < sdom[label[x]])
31         label[x] = label[dsu[x]];
32     dsu[x] = y;
33     return step ? dsu[x] : label[x];
34 }
35 void work(int root) {
36     dfs(root); n = ind;
37     for (int i = n; i; --i) {
38         for (auto p : rg[i])
39             sdom[i] = min(sdom[i], sdom[find(p)]);
40         if (i > 1) bucket[sdom[i]].push_back(i);
41         for (auto p : bucket[i]) {
42             int u = find(p);
43             if (sdom[p] == sdom[u]) idom[p] = sdom[p];
44             else idom[p] = u;
45         }
46         if (i > 1) dsu[i] = father[i];
47     }
48     for (int i = 2; i <= n; ++i) {
49         if (idom[i] != sdom[i])
50             idom[i] = idom[idom[i]];
51         tree[rev[i]].push_back(rev[idom[i]]);
52         tree[rev[idom[i]]].push_back(rev[i]);
53     }
54 }
55 };

```

5.7 网络流

5.7.1 最大流

注意: *top* 要初始化为 1

```

1 struct EDGE { int adj, w, next; } edge[M];
2 int n, top, gh[N], nrl[N];
3 void addedge(int x, int y, int w) {
4     edge[++top].adj = y;
5     edge[top].w = w;
6     edge[top].next = gh[x];
7     gh[x] = top;
8     edge[++top].adj = x;
9     edge[top].w = 0;
10    edge[top].next = gh[y];
11    gh[y] = top;
12 }
13 int dist[N], q[N];
14 int bfs() {

```

```

15     memset(dist, 0, sizeof(dist));
16     q[1] = S; int head = 0, tail = 1; dist[S] = 1;
17     while (head != tail) {
18         int x = q[++head];
19         for (int p=gh[x]; p; p=edge[p].next)
20             if (edge[p].w && !dist[edge[p].adj]) {
21                 dist[edge[p].adj] = dist[x] + 1;
22                 q[++tail] = edge[p].adj;
23             }
24     }
25     return dist[T];
26 }
27 int dinic(int x, int delta) {
28     if (x==T) return delta;
29     for (int& p=nrl[x]; p && delta; p=edge[p].next)
30         if (edge[p].w && dist[x]+1 == dist[edge[p].adj]) {
31             int dd = dinic(edge[p].adj, min(delta, edge[p].w));
32             if (!dd) continue;
33             edge[p].w -= dd;
34             edge[p^1].w += dd;
35             return dd;
36         }
37     return 0;
38 }
39 int work() {
40     int ans = 0;
41     while (bfs()) {
42         memcpy(nrl, gh, sizeof(gh));
43         int t; while (t = dinic(S, inf)) ans += t;
44     }
45     return ans;
46 }

```

5.7.2 上下界有源汇网络流

T 向 S 连容量为正无穷的边，将有源汇转化为无源汇。

每条边容量减去下界，设 $in[i]$ 表示流入 i 的下界之和减去流出 i 的下界之和。

新建超级源汇 SS, TT ，对于 $in[i] > 0$ 的点， SS 向 i 连容量为 $in[i]$ 的边。对于 $in[i] < 0$ 的点， i 向 TT 连容量为 $-in[i]$ 的边。

求出以 SS, TT 为源汇的最大流，如果等于 $\sum in[i] (in[i] > 0)$ ，则存在可行流。再求出 S, T 为源汇的最大流即为最大流。

费用流：建完图后等价于求以 SS, TT 为源汇的费用流。

5.7.3 上下界无源汇网络流

1. 怎样求无源汇有上下界网络的可行流？

由于有源汇的网络我们先要转化成无源汇，所以本来就无源汇的网络不用再做特殊处理。

2. 怎样求无源汇有上下界网络的最大流、最小流？

一种简易的方法是采用二分思想，不断通过可行流的存在与否对 (t, s) 边的上下界 U, L 进行调整。求最大流时令 $U = \infty$ 并二分 L ；求最小流时令 $L = 0$ 并二分 U 。道理很简单，因为可行流的取值范围是一段连续的区间，我们只要通过二分找到有解和无解的分界线即可。

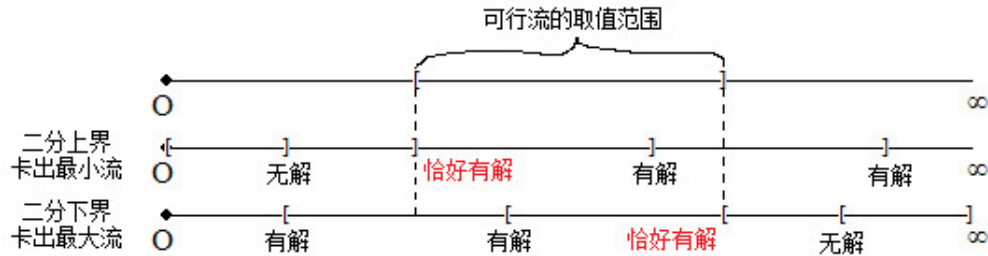


图 4: 可行流取值范围的讨论

5.7.4 费用流

注意: top 要初始化为 1

```

1 #define inf 0x3f3f3f3f
2 struct NetWorkFlow {
3     struct EDGE {
4         int adj, w, cost, next;
5     } edge[M*2];
6     int gh[N], q[N], dist[N], v[N], pre[N], prev[N], top;
7     int S, T;
8     void addedge(int x, int y, int w, int cost) {
9         edge[++top].adj = y;
10        edge[top].w = w;
11        edge[top].cost = cost;
12        edge[top].next = gh[x];
13        gh[x] = top;
14        edge[++top].adj = x;
15        edge[top].w = 0;
16        edge[top].cost = -cost;
17        edge[top].next = gh[y];
18        gh[y] = top;
19    }
20    void clear() {
21        top = 1;
22        memset(gh, 0, sizeof(gh));
23    }
24    int spfa() {
25        memset(dist, 63, sizeof(dist));
26        memset(v, 0, sizeof(v));
27        int head = 0, tail = 1;
28        q[1] = S; v[S] = 1; dist[S] = 0;
29        while (head != tail) {

```

```

30         (head += 1) %= N;
31         int x = q[head];
32         v[x] = 0;
33         for (int p=gh[x]; p; p=edge[p].next)
34             if (edge[p].w && dist[x] + edge[p].cost < dist[edge[p].adj]) {
35                 dist[edge[p].adj] = dist[x] + edge[p].cost;
36                 pre[edge[p].adj] = x;
37                 prev[edge[p].adj] = p;
38                 if (!v[edge[p].adj]) {
39                     v[edge[p].adj] = 1;
40                     (tail += 1) %= N;
41                     q[tail] = edge[p].adj;
42                 }
43             }
44     }
45     return dist[T] != inf;
46 }
47 int work() {
48     int ans = 0;
49     while (spfa()) {
50         int mx = inf;
51         for (int x=T; x!=S; x=pre[x])
52             mx = min(edge[prev[x]].w, mx);
53         ans += dist[T] * mx;
54         for (int x=T; x!=S; x=pre[x]) {
55             edge[prev[x]].w -= mx;
56             edge[prev[x]^1].w += mx;
57         }
58     }
59     return ans;
60 }
61 } nwf;

```

5.7.5 zkw 费用流

注意: *top* 要初始化为 1, 不得用于有负权的图

```

1  #define inf 0x3f3f3f3f //modify if you use long long or double
2  template <class _tp>
3  struct NetWorkFlow {
4      struct EDGE {
5          int adj, next;
6          _tp w, cost;
7      } edge[M*2];
8      int gh[N], top;
9      int S, T;
10     void addedge(int x, int y, _tp w, _tp cost) {
11         edge[++top].adj = y;
12         edge[top].w = w;
13         edge[top].cost = cost;

```

```

14     edge[top].next = gh[x];
15     gh[x] = top;
16     edge[++top].adj = x;
17     edge[top].w = 0;
18     edge[top].cost = -cost;
19     edge[top].next = gh[y];
20     gh[y] = top;
21 }
22 void clear() {
23     top = 1;
24     memset(gh, 0, sizeof(gh));
25 }
26 int v[N];
27 _tp cost, d[N], slk[N];
28 _tp aug(int x, _tp f) {
29     _tp left = f;
30     if (x == T) {
31         cost += f * d[S];
32         return f;
33     }
34     v[x] = true;
35     for (int p=gh[x]; p; p=edge[p].next)
36         if (edge[p].w && !v[edge[p].adj]) {
37             _tp t = d[edge[p].adj] + edge[p].cost - d[x];
38             if (t == 0) {
39                 _tp delt = aug(edge[p].adj, min(left, edge[p].w));
40                 if (delt > 0) {
41                     edge[p].w -= delt;
42                     edge[p^1].w += delt;
43                     left -= delt;
44                 }
45                 if (left == 0) return f;
46             } else {
47                 if (t < slk[edge[p].adj])
48                     slk[edge[p].adj] = t;
49             }
50         }
51     return f-left;
52 }
53 bool modlabel() {
54     _tp delt = inf;
55     for (int i=1; i<=T; i++)
56         if (!v[i]) {
57             if (slk[i] < delt) delt = slk[i];
58             slk[i] = inf;
59         }
60     if (delt == inf) return true;
61     for (int i=1; i<=T; i++)
62         if (v[i]) d[i] += delt;
63     return false;

```

```

64     }
65     _tp work() {
66         cost = 0;
67         memset(d, 0, sizeof(d));
68         memset(slk, 63, sizeof(slk));
69         do {
70             do {
71                 memset(v, 0, sizeof(v));
72             } while (aug(S, inf));
73         } while (!modlabel());
74         return cost;
75     }
76 };
77 NetWorkFlow<int> nwf;

```

6 数学

6.1 扩展欧几里得解同余方程

ans[] 保存的是循环节内所有的解

```

1 int exgcd(int a,int b,int&x,int&y){
2     if(!b) return x=1,y=0,a;
3     int d=exgcd(b,a%b,x,y),t=x;
4     return x=y,y=t-a/b*y,d;
5 }
6 void cal(ll a,ll b,ll n){//ax=b(mod n)
7     ll x,y,d=exgcd(a,n,x,y);
8     if(b%d) return;
9     x=(x%n+n)%n;
10    ans[cnt=1]=x*(b/d)%(n/d);
11    for(ll i=1;i<d;i++) ans[++cnt]=(ans[1]+i*n/d)%n;
12 }

```

6.1.1 扩展欧几里得特殊解和解的个数

求满足 $\begin{cases} ax + by = c (a \geq 0, b \geq 0, c \geq 0) \\ x_1 \leq x \leq x_2 \\ y_1 \leq y \leq y_2 \end{cases}$ 的二元组 (x, y) 的个数。

```

1 int calc(int a, int b, int c, int x1, int x2, int y1, int y2) {
2     if (a == 0 && b == 0) return c == 0 && x1 <= 0 && 0 <= x2 && y1 <= 0 && 0 <= y2;
3     if (a == 0) return c % b == 0 && y1 <= c / b && c / b <= y2;
4     if (b == 0) return c % a == 0 && x1 <= c / a && c / a <= x2;
5     int x, y, t;
6     int g = exgcd(a, b, x, y);
7     if (c % g) return 0;
8     x *= c / g; y *= c / g;

```



```

9      int dx = b / g, dy = a / g;
10
11      if (x > x1) t = (x - x1) / dx + 1, x = x - t * dx, y = y + t * dy;
12      t = (x1 - x) / dx; if ((x1 - x) % dx) ++ t;
13      x = x + t * dx, y = y - t * dy;
14      x1 = max(x1, x), y2 = min(y2, y);
15
16      if (x < x2) t = (x2 - x) / dx + 1, x = x + t * dx, y = y - t * dy;
17      t = (x - x2) / dx; if ((x - x2) % dx) ++ t;
18      x = x - t * dx, y = y + t * dy;
19      x2 = min(x2, x), y1 = max(y1, y);
20
21      if (y > y1) t = (y - y1) / dy + 1, x = x + t * dx, y = y - t * dy;
22      t = (y1 - y) / dy; if ((y1 - y) % dy) ++ t;
23      x = x - t * dx, y = y + t * dy;
24      x2 = min(x2, x), y1 = max(y1, y);
25
26      if (y < y2) t = (y2 - y) / dy + 1, x = x - t * dx, y = y + t * dy;
27      t = (y - y2) / dy; if ((y - y2) % dy) ++ t;
28      x = x + t * dx, y = y - t * dy;
29      x1 = max(x1, x), y2 = min(y2, y);
30
31      if (x1 > x2 && y1 > y2) return 0;
32      assert(x2 - x1 == y2 - y1);
33      return x2 - x1 + 1;
34  }

```

6.2 同余方程组

```

1  int n, flag, k, m, a, r, d, x, y;
2  int main() {
3      scanf("%d", &n);
4      flag = k = 1, m = 0;
5      while (n--) {
6          scanf("%d%d", &a, &r); // ans % a = r
7          if (flag) {
8              d = exgcd(k, a, x, y);
9              if ((r - m) % d) { flag = 0; continue; }
10             x = (x * ((r - m) / d) + a / d) % (a / d), y = k / d * a, m = ((x * k + m) % y) % y;
11             if (m < 0) m += y;
12             k = y;
13         }
14     }
15     printf("%d", flag ? m : -1); // 若 flag = 1, 说明有解, 解为 ki + m, i 为任意整数
16 }

```

6.3 类欧几里得算法

类欧几里得模板有三种形式：

$$f(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai + b}{c} \rfloor$$

$$g(a, b, c, n) = \sum_{i=0}^n i \lfloor \frac{ai + b}{c} \rfloor$$

$$h(a, b, c, n) = \sum_{i=0}^n \lfloor \frac{ai + b}{c} \rfloor^2$$

```
1 #define LL long long
2
3 const int P = 1000000007;
4 int Inv(int x) {
5     return x == 1 ? 1 : 1ll * (P - P / x) * Inv(P % x) % P;
6 }
7 const int i2 = Inv(2);
8 const int i6 = Inv(6);
9
10 struct ifo {
11     int f, g, h;
12     ifo(int f, int g, int h) : f(f), g(g), h(h) {}
13 };
14
15 int S1(int n) {
16     return 1ll * n * (n + 1) % P * i2 % P;
17 }
18
19 int S2(int n) {
20     return 1ll * n * (n + 1) % P * (2 * n + 1) % P * i6 % P;
21 }
22
23 ifo Get(int n, int A, int B, int C) {
24     if (!A) {
25         int t = B / C;
26         int f = 1ll * (n + 1) * t % P;
27         int g = 1ll * S1(n) * t % P;
28         int h = 1ll * (n + 1) * t % P * t % P;
29         return ifo(f, g, h);
30     } else if (A >= C || B >= C) {
31         ifo Nx = Get(n, A % C, B % C, C);
32         int p = A / C, q = B / C;
33         int f = (1ll * p * S1(n) + 1ll * q * (n + 1) + Nx.f) % P;
34         int g = (1ll * p * S2(n) + 1ll * q * S1(n) + Nx.g) % P;
35         int h = (1ll * p * p % P * S2(n) + 2ll * p * q % P * S1(n) + 1ll * (n + 1) *
36                 q % P * q + 2ll * p * Nx.g % P + 2ll * q * Nx.f % P + Nx.h) % P;
37         return ifo(f, g, h);
38     } else {
39         int t = A / B;
40         int f = 1ll * t * S1(n) % P;
41         int g = 1ll * t * S2(n) % P;
42         int h = 1ll * t * t * S1(n) % P;
43         return ifo(f, g, h);
44     }
45 }
```

```

38     int m = (1ll * A * n + B) / C;
39     ifo Nx = Get(m - 1, C, C - B - 1, A);
40     int f = (1ll * n * m - Nx.f) % P;
41     int g = (1ll * m * S1(n) - 1ll * i2 * Nx.h - 1ll * i2 * Nx.f) % P;
42     int h = (2ll * n * S1(m - 1) % P + 1ll * n * m - 2ll * Nx.g - Nx.f) % P;
43     return ifo(f, g, h);
44 }
45 }

```

6.4 卡特兰数

$$h_1 = 1, h_n = \frac{h_{n-1}(4n-2)}{n+1} = \frac{C(2n,n)}{n+1} = C(2n,n) - C(2n,n-1)$$

在一个格点阵列中，从 $(0,0)$ 点走到 (n,m) 点且不经过对角线 $x=y$ 的方案数 $(x > y)$:

$$C(n+m-1, m) - C(n+m-1, m-1)$$

在一个格点阵列中，从 $(0,0)$ 点走到 (n,m) 点且不穿过对角线 $x=y$ 的方案数 $(x \geq y)$:

$$C(n+m, m) - C(n+m, m-1)$$

6.5 斯特林数

6.5.1 第一类斯特林数

第一类 Stirling 数 $S(p, k)$ 的一个组合学解释是：将 p 个物体排成 k 个非空循环排列的方法数。

$S(p, k)$ 的递推公式： $S(p, k) = (p-1)S(p-1, k) + S(p-1, k-1), 1 \leq k \leq p-1$

边界条件： $S(p, 0) = 0, p \geq 1$ $S(p, p) = 1, p \geq 0$

6.5.2 第二类斯特林数

第二类 Stirling 数 $S(p, k)$ 的一个组合学解释是：将 p 个物体划分成 k 个非空的不可辨别（可以理解为盒子没有编号）集合的方法数。

$S(p, k)$ 的递推公式： $S(p, k) = kS(p-1, k) + S(p-1, k-1), 1 \leq k \leq p-1$

边界条件： $S(p, 0) = 0, p \geq 1$ $S(p, p) = 1, p \geq 0$

也有卷积形式：

$$S(n, m) = \frac{1}{m!} \sum_{k=0}^m (-1)^k C(m, k) (m-k)^n = \sum_{k=0}^m \frac{(-1)^k (m-k)^n}{k! (m-k)!} = \sum_{k=0}^m \frac{(-1)^k}{k!} \times \frac{(m-k)^n}{(m-k)!}$$

6.6 错排公式

$$D_1 = 0, D_2 = 1, D_n = (n-1)(D_{n-2} + D_{n-1})$$

6.7 Lucas 定理

接口：

初始化： `void lucas::init();`

计算 $C(n, m) \% mod$ 的值： `LL lucas::Lucas(LL n, LL m);`

```

1 #define mod 110119
2 #define LL long long
3 namespace lucas {
4     LL fac[mod+1], facv[mod+1];
5     LL power(LL base, LL times) {
6         LL ans = 1;
7         while (times) {
8             if (times&1) (ans *= base) %= mod;
9             (base *= base) %= mod;
10            times >>= 1;
11        }
12        return ans;
13    }
14    void init() {
15        fac[0] = 1; for (int i=1;i<mod;i++) fac[i] = (fac[i-1] * i) % mod;
16        facv[mod-1] = power(fac[mod-1], mod-2);
17        for (int i=mod-2;i>=0;--i) facv[i] = (facv[i+1] * (i+1)) % mod;
18    }
19    LL C(unsigned LL n, unsigned LL m) {
20        if (n < m) return 0;
21        return (fac[n] * facv[m] % mod * facv[n-m] % mod) % mod;
22    }
23    LL Lucas(unsigned LL n, unsigned LL m)
24    {
25        if (m == 0) return 1;
26        return (C(n%mod, m%mod) * Lucas(n/mod, m/mod)) %mod;
27    }
28 };

```

6.8 线性规划

6.8.1 单纯形法

单纯形法用于解决线性规划问题：

$$\begin{aligned}
 & \max_{x_1, x_2, \dots, x_n} x_0 = c_1 x_1 + c_2 x_2 + \dots + c_n x_n \\
 & s.t. \begin{cases} A_{i1}x_1 + A_{i2}x_2 + \dots + A_{in}x_n \leq b_i, & i = 1, 2, \dots, m \\ x_j \geq 0, & j = 1, 2, \dots, n \end{cases}
 \end{aligned}$$

小心：单纯形法通常能解决 $n \leq 500, m \leq 500$ 的数据规模的问题。若规模过大，可能导致精度爆炸。

小心：单纯形法只能解决一般线性规划问题，不能解决整数规划问题（NP Hard）。若要用单纯形法解决整数规划问题，必须先证明一般线性规划的解不比整数规划好。

若 $b_i \geq 0, i = 1, 2, \dots, n$ ，则不需要执行 init，因为至少有一组解 $x_1 = x_2 = \dots = x_n = 0$ 。

输入格式 .

$$\begin{array}{cccccc}
& n & & m & & \\
c_1 & c_2 & c_3 & \cdots & c_n & \\
A_{11} & A_{12} & A_{13} & \cdots & A_{1n} & b_1 \\
A_{21} & A_{22} & A_{23} & \cdots & A_{2n} & b_2 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
A_{m1} & A_{m2} & A_{m3} & \cdots & A_{mn} & b_m
\end{array}$$

输出格式

若无解，输出 Infeasible 。

若 x_0 无界，输出 Unbounded 。

第一行输出答案 x_0 。

接下来一行输出 n 个实数表示 x_1, x_2, \dots, x_n 。

```

1 #include <bits/stdc++.h>
2
3 #define N 25
4 #define M 25
5
6 using namespace std;
7
8 const double eps = 1e-8, INF = 1e15;
9
10 int n, m;
11 double a[M][N], ans[N + M];
12 int id[N + M];
13
14 void pivot(int l, int e) {
15     swap(id[n + 1], id[e]);
16     double t = a[l][e];
17     a[l][e] = 1;
18     for (int j = 0; j <= n; ++j) a[l][j] /= t;
19     for (int i = 0; i <= m; ++i)
20         if (i != l && abs(a[i][e]) > eps) {
21             t = a[i][e];
22             a[i][e] = 0;
23             for (int j = 0; j <= n; ++j) a[i][j] -= a[l][j] * t;
24         }
25 }
26
27 bool init() {
28     while (1) {
29         int e = 0, l = 0;
30         for (int i = 1; i <= m; ++i)
31             if (a[i][0] < -eps && (!l || (rand() & 1)))
32                 l = i;
33         if (!l) break;
34         for (int j = 1; j <= n; ++j)
35             if (a[l][j] < -eps && (!e || (rand() & 1)))

```

```

36         e = j;
37         if (!e) return false; // Infeasible
38         pivot(l, e);
39     }
40     return true;
41 }
42
43 bool simplex() {
44     while (1) {
45         int l = 0, e = 0;
46         double mn = INF;
47         for (int j = 1; j <= n; ++j)
48             if (a[0][j] > eps) {
49                 e = j;
50                 break;
51             }
52         if (!e) break;
53         for (int i = 1; i <= m; ++i)
54             if (a[i][e] > eps && a[i][0] / a[i][e] < mn) {
55                 mn = a[i][0] / a[i][e];
56                 l = i;
57             }
58         if (!l) return false; // Unbounded
59         pivot(l, e);
60     }
61     return true;
62 }
63
64 int main() {
65     scanf("%d%d", &n, &m);
66     for (int i = 1; i <= n; ++i) scanf("%lf", &a[0][i]);
67     for (int i = 1; i <= m; ++i) {
68         for (int j = 1; j <= n; ++j) scanf("%lf", &a[i][j]);
69         scanf("%lf", &a[i][0]);
70     }
71     for (int i = 0; i <= n + m; ++i) id[i] = 0;
72     for (int i = 1; i <= n; ++i) id[i] = i;
73     if (!init()) {
74         puts("Infeasible");
75         return 0;
76     }
77     if (!simplex()) {
78         puts("Unbounded");
79         return 0;
80     }
81     printf("%.10lf\n", -a[0][0]);
82     for (int i = 0; i <= n + m; ++i) ans[i] = 0;
83     for (int i = 1; i <= m; ++i) ans[id[n + i]] = a[i][0];
84     for (int i = 1; i <= n; ++i) printf("%.10lf_", ans[i]);
85     puts("");

```

6.8.2 对偶理论

原始问题：

$$\begin{aligned} \max_{x_1, x_2, \dots, x_n} \quad & x_0 = c_1x_1 + c_2x_2 + \dots + c_nx_n \\ \text{s.t.} \quad & \begin{cases} A_{i1}x_1 + A_{i2}x_2 + \dots + A_{in}x_n \leq b_i, & i = 1, 2, \dots, m \\ x_j \geq 0, & j = 1, 2, \dots, n \end{cases} \end{aligned}$$

对偶问题：

$$\begin{aligned} \min_{w_1, w_2, \dots, w_m} \quad & w_0 = b_1x_1 + b_2x_2 + \dots + b_mx_m \\ \text{s.t.} \quad & \begin{cases} A_{i1}^T w_1 + A_{i2}^T w_2 + \dots + A_{im}^T w_m \geq c_i, & i = 1, 2, \dots, n \\ w_j \geq 0, & j = 1, 2, \dots, m \end{cases} \end{aligned}$$

6.9 高斯消元

6.9.1 行列式

```

1  int ans = 1;
2  for (int i=0; i<n; i++) {
3      for (int j=i; j<n; j++)
4          if (g[j][i]) {
5              for (int k=i; k<n; k++)
6                  swap(g[i][k], g[j][k]);
7              if (j != i) ans *= -1;
8              break;
9          }
10     if (g[i][i] == 0) {
11         ans = 0;
12         break;
13     }
14     for (int j=i+1; j<n; j++) {
15         while (g[j][i]) {
16             int t = g[i][i] / g[j][i];
17             for (int k=i; k<n; k++)
18                 g[i][k] = (g[i][k] + mod - ((LL)t * g[j][k] % mod)) % mod;
19             for (int k=i; k<n; k++)
20                 swap(g[i][k], g[j][k]);
21             ans *= -1;
22         }
23     }
24 }
25 for (int i=0; i<n; i++)
26     ans = ((LL)ans * g[i][i]) % mod;

```

```

27 ans = (ans % mod + mod) % mod;
28 printf("%d\n", ans);

```

6.9.2 Matrix-Tree 定理

对于一张图，建立矩阵 C ， $C[i][i]$ = i 的度数，若 i, j 之间有边，那么 $C[i][j] = -1$ ，否则为 0。这张图的生成树个数等于矩阵 C 的 $n - 1$ 阶行列式的值。

6.10 调和级数

$\sum_{i=1}^n \frac{1}{i}$ 在 n 较大时约等于 $\ln(n) + r$ ， r 为欧拉常数，约等于 0.5772156649015328。

6.11 曼哈顿距离的变换

$$|x_1 - x_2| + |y_1 - y_2| = \max(|(x_1 + y_1) - (x_2 + y_2)|, |(x_1 - y_1) - (x_2 - y_2)|)$$

6.12 数论函数变换

常见积性函数：

欧拉函数 $\phi(n)$ 为不超过 n 的与 n 互质的正整数个数

$$\text{莫比乌斯函数 } \mu(n) = \begin{cases} 1, & \text{若 } n = 1 \\ (-1)^k, & \text{若 } n \text{ 无平方数因数, 且 } n = p_1 p_2 \cdots p_k \\ 0, & \text{若 } n \text{ 有大于 } 1 \text{ 的平方数因数} \end{cases}$$

莫比乌斯函数的一次方前缀和见“杜教筛”。

莫比乌斯函数的二次方前缀和

$$\sum_{i=1}^n \mu(i)^2 = \sum_{d=1}^{\lfloor \sqrt{n} \rfloor} \mu(d) \lfloor \frac{n}{d^2} \rfloor$$

常见积性函数的性质：

$$n = \sum_{d|n} \phi(d)$$

$$\sum_{d|n} \mu(d) = \begin{cases} 1, & n = 1 \\ 0, & n > 1 \end{cases}$$

$$\sum_{i=1}^n \sum_{j=1}^m i \times j [\gcd(i, j) = d] = \sum_{i=1}^{\lfloor \frac{n}{d} \rfloor} \sum_{j=1}^{\lfloor \frac{m}{d} \rfloor} id \times jd [\gcd(i, j) = 1]$$

$$\phi(n) = \sum_{d|n} \mu(d) \frac{n}{d}$$

6.13 莫比乌斯反演

$F(n)$ 和 $f(n)$ 是定义在非负整数集合上的两个函数，则：

$$F(n) = \sum_{d|n} f(d) \Rightarrow f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right)$$

$$F(n) = \sum_{n|d} f(d) \Rightarrow f(n) = \sum_{n|d} \mu\left(\frac{d}{n}\right) F(d)$$

6.14 线性筛素数

```

1 mu[1]=phi[1]=1;top=0;
2 for (int i=2;i<N;i++) {
3     if (!v[i]) prime[++top]=i, mu[i] = -1, phi[i] = i-1;
4     for (int j=1;i*prime[j]<N && j<=top;j++) {
5         v[i*prime[j]] = 1;
6         if (i%prime[j]) {
7             mu[i*prime[j]] = -mu[i];
8             phi[i*prime[j]] = phi[i] * (prime[j]-1);
9         } else {
10            mu[i*prime[j]] = 0;
11            phi[i*prime[j]] = phi[i] * prime[j];
12            break;
13        }
14    }
15 }
```

6.15 杜教筛

$\text{getphi}(t, x)$ 表示求 $\sum_{i=1}^x i^t \phi(i)$ 。

推导过程：

记 $S(n) = \sum_{i=1}^n f(i)$ ，取任意函数 g 有恒等式

$$S(n) = \sum_{i=1}^n (f \cdot g)(i) - \sum_{i=2}^n g(i) S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$$

其中， $(f \cdot g)$ 表示 f 和 g 的狄利克雷卷积：即： $(f \cdot g)(n) = \sum_{d|n} f(d)g\left(\frac{n}{d}\right)$

关于恒等式的证明：

将 $\sum_{i=2}^n g(i) S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$ 移到左边去，即只需证

$$\sum_{i=1}^n (f \cdot g)(i) = \sum_{i=1}^n g(i) S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$$

将狄利克雷卷积展开，得：

$$\sum_{i=1}^n \sum_{d|i} g(d) f\left(\frac{i}{d}\right) = \sum_{i=1}^n g(i) S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$$

即：

$$\sum_{d=1}^n g(d) \sum_{i=1}^{\lfloor \frac{n}{d} \rfloor} f(i) = \sum_{i=1}^n g(i) S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$$

显然相等，恒等式证完。

取 $f(i) = i^p \phi(i)$, $g(i) = i^p$ ，则有：

$$S(n) = \sum_{i=1}^n i^p \phi(i) = \sum_{i=1}^n i^{p+1} - \sum_{i=2}^n i^p S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$$

其中有用到等式 $\sum_{d|n} \phi(d) = n$

另外，莫比乌斯函数 $\mu(n)$ 也可以使用杜教筛求前缀和，记 $S'(n) = \sum_{i=1}^n \mu(i)$ ，则 $S'(n) = 1 -$

$$\sum_{i=2}^n S'\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$$

```

1  #include <bits/stdc++.h>
2
3  #define N 5000020
4  #define LL long long
5  #define mod 1000000007
6  #define div2 ((mod+1)/2)
7  #define div6 ((mod+1)/6)
8
9  using namespace std;
10
11 int n, prime[N], v[N];
12 LL phi[3][N];
13
14 map<int, int> mp[3];
15
16 int sum(int t, int x) { //calculate 1^t + 2^t + ... + x^t
17     if (t == 0) return x;
18     if (t == 1) return 1ll * x * (x + 1) % mod * div2 % mod;
19     if (t == 2) return 1ll * x * (x + 1) % mod * (2ll * x % mod + 1) % mod * div6 %
20         mod;
21     if (t == 3) return 1ll * x * x % mod * (x + 1) % mod * (x + 1) % mod * div2 %
22         mod * div2 % mod;
23 }
24
25 int getphi(int t, int x) {
26     if (x < N) return phi[t][x];
27     if (mp[t].find(x) != mp[t].end()) return mp[t][x];
28     LL ans = 0; int r = 0;
29     for (int l = 2; l <= x; l = r + 1) {
30         r = x / (x / l);
31         int sum_t = sum(t, r);
32         int phi_t = getphi(t, r);
33         int cnt = r - l + 1;
34         ans += 1ll * sum_t * phi_t % mod * cnt % mod;
35     }
36     mp[t][x] = ans;
37     return ans;
38 }
39
40 int main() {
41     int t;
42     while (t = read(), t) {
43         int x;
44         while (x = read(), x) {
45             int ans = getphi(t, x);
46             printf("%d\n", ans);
47         }
48     }
49     return 0;
50 }

```

```

29         ans += 1ll * getphi(t, x / 1) * (((LL)sum(t, r) - sum(t, 1 - 1) + mod) % mod
30         ) % mod;
31     ans %= mod;
32 }
33 ans = (LL)sum(t + 1, x) - ans + mod;
34 ans %= mod;
35 mp[t][x] = ans;
36 return (int)ans;
37 }
38 int main() {
39     memset(v, 0, sizeof(v));
40     int top = 0;
41     phi[0][1] = 1, phi[1][1] = 1, phi[2][1] = 1;
42     for (int i = 2; i < N; ++i) {
43         if (!v[i]) prime[++top] = i, phi[0][i] = i - 1, phi[1][i] = 1ll * i * phi
44             [0][i] % mod, phi[2][i] = 1ll * i * phi[1][i] % mod;
45         for (int j = 1; j <= top && prime[j] * i < N; ++j) {
46             v[i * prime[j]] = 1;
47             if (i % prime[j] == 0) {
48                 phi[0][i * prime[j]] = phi[0][i] * prime[j];
49                 phi[1][i * prime[j]] = 1ll * phi[1][i] * prime[j] % mod * prime[j] %
50                     mod;
51                 phi[2][i * prime[j]] = 1ll * phi[2][i] * prime[j] % mod * prime[j] %
52                     mod * prime[j] % mod;
53                 break;
54             } else {
55                 phi[0][i * prime[j]] = phi[0][i] * (prime[j] - 1);
56                 phi[1][i * prime[j]] = 1ll * phi[1][i] * (prime[j] - 1) % mod *
57                     prime[j] % mod;
58                 phi[2][i * prime[j]] = 1ll * phi[2][i] * (prime[j] - 1) % mod *
59                     prime[j] % mod * prime[j] % mod;
60             }
61         }
62     }
63     for (int i = 2; i < N; ++i) {
64         phi[0][i] = (phi[0][i] + phi[0][i - 1]) % mod;
65         phi[1][i] = (phi[1][i] + phi[1][i - 1]) % mod;
66         phi[2][i] = (phi[2][i] + phi[2][i - 1]) % mod;
67     }
68 }

```

6.16 洲阁筛

一种在 $O(\frac{n^{\frac{3}{4}}}{\log n})$ 的时间中求出大多数积性函数的前缀和的方法。

原文链接：<http://debug18.com/posts/calculate-the-sum-of-multiplicative-function/>

引入

求

$$\sum_{i=1}^n F(i)$$

其中 $F(x)$ 是一个积性函数, 满足当 p 为质数的时候, $F(p^c)$ 是一个关于 p 的低阶多项式。

转化

我们将 $[1, n]$ 的所有数按照是否有 $> \sqrt{n}$ 的质因子分为两类, 那么显然有

$$\begin{aligned} \sum_{i=1}^n F(i) &= \sum_{\substack{1 \leq i \leq n \\ i \text{ have no prime factors } > \sqrt{n}}} F(i) \left(1 + \sum_{\substack{\sqrt{n} < j \leq \lfloor \frac{n}{i} \rfloor \\ j \text{ is prime}}} F(j) \right) \\ &= \sum_{\substack{1 \leq i < \sqrt{n} \\ i \text{ have no prime factors } > \sqrt{n}}} F(i) \left(1 + \sum_{\substack{\sqrt{n} < j \leq \lfloor \frac{n}{i} \rfloor \\ j \text{ is prime}}} F(j) \right) + \sum_{\substack{\sqrt{n} \leq i \leq n \\ i \text{ have no prime factors } > \sqrt{n}}} F(i) \end{aligned}$$

现在需要计算两个东西:

1. 对于每个 $1 \leq i < \sqrt{n}$, 计算

$$\sum_{\substack{\sqrt{n} < j \leq \lfloor \frac{n}{i} \rfloor \\ j \text{ is prime}}} F(j)$$

2.

$$\sum_{\substack{\sqrt{n} \leq i \leq n \\ i \text{ have no prime factors } > \sqrt{n}}} F(i)$$

Part 1 (calcG)

设 $g_k(i, j)$ 表示 $[1, j]$ 中与前 i 个质数互质的数的 k 次幂和. 显然有转移

$$g_k(i, j) = g_k(i-1, j) - p_i^k g_k(i-1, \lfloor \frac{j}{p_i} \rfloor)$$

观察到 j 的取值只有 \sqrt{n} 种, 于是直接暴力计算的复杂度为 $O(\frac{n}{\log n})$.

如果 $p_i > \lfloor \frac{j}{p_i} \rfloor$ 即 $p_i^2 > j$ 时, $g_k(i, j)$ 的转移变为:

$$g_k(i, j) = g_k(i-1, j) - p_i^k$$

我们从小到大枚举 i , 对于某个 j 一旦 $p_{i_0}^2 > j$ 便可以不再转移, 之后如果其他的值需要使用到它在 i_1 时的值, 直接用 $g_k(i_0, j) - \sum_{l=i_0}^{i_1-1} p_l^k$ 即可.

此时的复杂度可以简单地用积分近似为 $O(\frac{n^{\frac{3}{4}}}{\log n})$.

Part 2 (calcF)

设 $f(i, j)$ 表示 $[1, j]$ 中仅由小于 \sqrt{n} 的后 i 个质数组成的数的 $F(x)$ 之和. 此时当 $p_i > j$ 时, 一定有 $f(i, j) = 1$. 类似地, 当 $p_i^2 > j$ 时转移变为:

$$f(i, j) = f(i - 1, j) + F(p_i)$$

所以可以从大到小枚举 i , 如果对于某个 j 有 $p_i^2 > j$, 可以不转移, 每次用的时候加入 $[p_i, \min(j, \sqrt{n})]$ 这一段的质数的 $F(p)$ 就可以了.

应用 1

求 $\sum_{i=1}^n d(i^3)$, 其中 $d(x)$ 表示 x 的约数个数. $n \leq 10^{11}$.

令 $F(p^c) = 3c + 1$, 上洲阁筛.

sump[i] 表示小于等于 i 的质数之和

d3[i] 表示 $d(i^3)$

g0[i] 表示 $[1, i]$ 中与不超过 \sqrt{n} 都互质的数的 0 次幂和.

g[i] 表示 $[1, \lfloor \frac{n}{i} \rfloor]$ 中与不超过 \sqrt{n} 都互质的数的 0 次幂和.

```
1 LL N;
2 int pbnd;
3 int vbnd;
4 int l0[SIZE], l[SIZE];
5 LL g0[SIZE], g[SIZE], f0[SIZE], f[SIZE];
6
7 void calcG()
8 {
9     for (int i = 1; i < vbnd; ++i) {
10         g0[i] = i;
11     }
12     for (int i = vbnd - 1; i >= 1; --i) {
13         g[i] = N / i;
14     }
15     for (int i = 0; i < pbnd; ++i) {
16         int p = primes[i];
17         for (int j = 1; j < vbnd && i < l[j]; ++j) {
18             LL y = (N / j) / p;
19             g[j] -= (y < vbnd ? g0[y] - std::max(0, i - l0[y]) : g[N / y] - std::max(0, i - l[N / y]));
20         }
21         for (int j = vbnd - 1; j >= 1 && i < l0[j]; --j) {
22             LL y = j / p;
23             g0[j] -= g0[y] - std::max(0, i - l0[y]);
24         }
25     }
26     for (int i = 1; i < vbnd; ++i) {
27         g[i] -= pbnd - l[i];
28     }
29 }
30
```

```

31 void calcF()
32 {
33     std::fill(f0 + 1, f0 + vbnd, 1);
34     std::fill(f + 1, f + vbnd, 1);
35     for (int i = pbnd - 1; i >= 0; --i) {
36         int p = primes[i];
37         for (int j = 1; j < vbnd && i < l[j]; ++j) {
38             LL y = (N / j) / p;
39             for (int z = 1; y; ++z, y /= p) {
40                 f[j] += (3 * z + 1) * (y < vbnd ?
41                     f0[y] + 4 * std::max(0, sump[y] - std::max(l0[y], i + 1))
42                     :
43                     f[N / y] + 4 * (pbnd - std::max(l[N / y], i + 1)));
44             }
45         }
46         for (int j = vbnd - 1; j >= 1 && i < l0[j]; --j) {
47             int y = j / p;
48             for (int z = 1; y; ++z, y /= p) {
49                 f0[j] += (3 * z + 1) * (f0[y] + 4 * std::max(0, sump[y] - std::max(
50                     l0[y], i + 1)));
51             }
52         }
53         for (int i = 1; i < vbnd; ++i) {
54             f[i] += 4 * (pbnd - l[i]);
55         }
56     }
57 LL calcSumS3()
58 {
59     for (vbnd = 1; (LL)vbnd * vbnd <= N; ++vbnd) { }
60     for (pbnd = 0; (LL)primes[pbnd] * primes[pbnd] <= N; ++pbnd) { }
61     for (int i = 1; i < vbnd; ++i) {
62         for (l0[i] = l0[i - 1]; (LL)primes[l0[i]] * primes[l0[i]] <= i; ++l0[i]) { }
63     }
64     l[vbnd] = 0;
65     for (int i = vbnd - 1; i >= 1; --i) {
66         LL x = N / i;
67         for (l[i] = l[i + 1]; (LL)primes[l[i]] * primes[l[i]] <= x; ++l[i]) { }
68     }
69
70     calcG();
71     calcF();
72
73     LL ret = f[1];
74     for (int i = 1; i < vbnd; ++i) {
75         ret += d3[i] * 4 * (g[i] - 1);
76         // 取  $c = 1$ ,  $3c + 1 = 4$ .  $g[i] - 1$  的原因是除去  $F(1)$ .
77     }
78     return ret;

```

应用 2 求素数的 k 次方前缀和

我们只需使用洲阁筛的 Part 1，计算出来的就是 $(\sqrt{n}, n]$ 中素数的 k 次幂和。

$g0[i]$ 表示 $[1, i]$ 中与不超过 \sqrt{n} 都互质的数的 0 次幂和。

$G0[i]$ 表示 $[1, \lfloor \frac{n}{i} \rfloor]$ 中与不超过 \sqrt{n} 都互质的数的 0 次幂和。

$g1[i]$ 表示 $[1, i]$ 中与不超过 \sqrt{n} 都互质的数的 1 次幂和。

$G1[i]$ 表示 $[1, \lfloor \frac{n}{i} \rfloor]$ 中与不超过 \sqrt{n} 都互质的数的 1 次幂和。

```

1  #include <bits/stdc++.h>
2
3  #define N 1000000 // sqrt(n)
4  #define mod 1000000007
5  #define LL long long
6
7  using namespace std;
8
9  int prime[N], sum1[N], v[N], l0[N], L0[N], pbnd, vbnd, top;
10 LL g0[N], G0[N];
11 int g1[N], G1[N];
12
13 void init() {
14     top = 0;
15     for (int i = 2; i < N; ++i) {
16         if (!v[i]) prime[top++] = i;
17         for (int j = 0; j < top && i * prime[j] < N; ++j) {
18             v[i * prime[j]] = 1;
19             if (i % prime[j] == 0) break;
20         }
21     }
22     sum1[0] = 0;
23     for (int i = 1; i <= top; ++i)
24         sum1[i] = (sum1[i - 1] + prime[i - 1]) % mod;
25 }
26
27 int S1(long long x) {
28     x %= mod;
29     return x * (x + 1) / 2 % mod;
30 }
31
32 long long calc(long long n) {
33     for (vbnd = 1; 1ll * vbnd * vbnd <= n; ++vbnd);
34     for (pbnd = 0; pbnd < top && 1ll * prime[pbnd] * prime[pbnd] <= n; ++pbnd);
35     l0[0] = 0;
36     for (int i = 1; i < vbnd; ++i)
37         for (l0[i] = l0[i - 1]; l0[i] < top && 1ll * prime[l0[i]] * prime[l0[i]] <=
             i; ++l0[i]);
38     L0[vbnd] = 0;

```

```

39     for (int i = vbnd - 1; i >= 1; --i) {
40         LL x = n / i;
41         for (L0[i] = L0[i + 1]; L0[i] < top && 11l * prime[L0[i]] * prime[L0[i]] <=
            x; ++L0[i]);
42     }
43     for (int i = 1; i < vbnd; ++i) {
44         g0[i] = i;
45         g1[i] = S1(i);
46     }
47     for (int i = vbnd - 1; i >= 1; --i) {
48         G0[i] = n / i;
49         G1[i] = S1(n / i);
50     }
51     for (int i = 0; i < pbnd; ++i) {
52         int p = prime[i];
53         for (int j = 1; j < vbnd && i < L0[j]; ++j) {
54             LL y = (n / j) / p;
55             if (y < vbnd) {
56                 if (i - l0[y] < 0) {
57                     G0[j] -= g0[y];
58                     G1[j] -= 11l * p * g1[y] % mod;
59                     if (G1[j] < 0) G1[j] += mod;
60                 } else {
61                     G0[j] -= g0[y] - (i - l0[y]);
62                     G1[j] -= 11l * p * (g1[y] % mod - (sum1[i] - sum1[l0[y]]) % mod)
                        % mod;
63                     G1[j] = ((G1[j] % mod) + mod) % mod;
64                 }
65             } else {
66                 if (i - L0[n / y] < 0) {
67                     G0[j] -= G0[n / y];
68                     G1[j] -= 11l * p * G1[n / y] % mod;
69                     if (G1[j] < 0) G1[j] += mod;
70                 } else {
71                     G0[j] -= G0[n / y] - (i - L0[n / y]);
72                     G1[j] -= 11l * p * (G1[n / y] % mod - (sum1[i] - sum1[L0[n / y]
                        ])) % mod) % mod;
73                     G1[j] = ((G1[j] % mod) + mod) % mod;
74                 }
75             }
76         }
77         for (int j = vbnd - 1; j >= 1 && i < l0[j]; --j) {
78             LL y = j / p;
79             if (i - l0[y] < 0) {
80                 g0[j] -= g0[y];
81                 g1[j] -= 11l * p * g1[y] % mod;
82                 if (g1[j] < 0) g1[j] += mod;
83             } else {
84                 g0[j] -= g0[y] - (i - l0[y]);
85                 g1[j] -= 11l * p * (g1[y] % mod - (sum1[i] - sum1[l0[y]])) % mod) %

```



```

86         mod;
87         g1[j] = ((g1[j] % mod) + mod) % mod;
88     }
89 }
90 for (int i = 1; i < vbnd; ++i) {
91     G0[i] -= pbnd - L0[i];
92     G1[i] -= (sum1[pbnd] - sum1[L0[i]]) % mod;
93     G1[i] = ((G1[i] % mod) + mod) % mod;
94 }
95 return G0[1] - 1 + pbnd; // 不超过 n 的素数个数
96 return ((G1[1] + mod - 1) % mod + sum1[pbnd]) % mod; // 不超过 n 的素数和
97 return ((g1[j] - 1 + sum1[L0[j]]) % mod + mod) % mod; // 不超过 j 的素数和
98 return ((G1[n / j] - 1 + sum1[pbnd]) % mod + mod) % mod; // 不超过 n / j 的素数
    和
99 }
100
101 int main() {
102     init();
103     long long n;
104     while (~scanf("%lld", &n)) {
105         long long ans = calc(n);
106         printf("%lld\n", ans);
107     }
108 }
109
110 // hdu5901

```

6.17 FFT

6.17.1 普通 FFT

```

1 namespace FFT {
2     const int maxn = 65537;
3     const double pi = acos(-1.0);
4
5     struct comp {
6         double real , imag;
7         comp() {}
8         comp(double real , double imag): real(real) , imag(imag) {}
9         friend inline comp operator+(const comp &a , const comp &b) {
10             return comp(a.real + b.real , a.imag + b.imag);
11         }
12         friend inline comp operator-(const comp &a , const comp &b) {
13             return comp(a.real - b.real , a.imag - b.imag);
14         }
15         friend inline comp operator*(const comp &a , const comp &b) {
16             return comp(a.real * b.real - a.imag * b.imag , a.real * b.imag + a.imag
                * b.real);

```

```

17     }
18 };
19
20 comp A[maxn] , B[maxn];
21 int rev[maxn], m, len;
22
23 inline void init(int n) {
24     for (m = 1, len = 0; m < n + n; m <= 1 , len ++);
25     for (int i = 0; i < m; ++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len -
26         1));
27     for (int i = 0; i < m; ++i) A[i] = B[i] = comp(0, 0);
28 }
29
30 inline void dft(comp *a , int v) {
31     for (int i = 0; i < m; ++i) if (i < rev[i]) swap(a[i] , a[rev[i]]);
32     for (int s = 2; s <= m; s <= 1) {
33         comp g(cos(2 * pi / s) , v * sin(2 * pi / s));
34         for (int k = 0; k < m; k += s) {
35             comp w(1 , 0);
36             for (int j = 0; j < s / 2; ++j) {
37                 comp &u = a[k + j + s / 2], &v = a[k + j];
38                 comp t = w * u;
39                 u = v - t;
40                 v = v + t;
41                 w = w * g;
42             }
43         }
44         if (v == -1)
45             for (int i = 0; i < m; ++i) a[i].real /= m , a[i].imag /= m;
46     }
47 }

```

6.17.2 模任意素数 FFT

注意：调用 *mulmod* 前先调用 *init* 。调用 *mulmod* 前请确保 *a, b* 数组足够大（比 $2n$ 大的 2 的整数次幂）且经过初始化。

```

1 namespace FFT {
2     const long double pi = acos(-1.0);
3
4     struct comp {
5         long double real, imag;
6         comp() {}
7         comp(long double real, long double imag) : real(real), imag(imag) {}
8         friend inline comp operator + (const comp &a, const comp &b) {
9             return comp(a.real + b.real, a.imag + b.imag);
10        }
11        friend inline comp operator - (const comp &a, const comp &b) {
12            return comp(a.real - b.real, a.imag - b.imag);

```

```

13     }
14     friend inline comp operator * (const comp &a, const comp &b) {
15         return comp(a.real * b.real - a.imag * b.imag, a.real * b.imag + a.imag
16                     * b.real);
17     }
18     inline comp conj() {
19         return comp(real, -imag);
20     }
21 };
22
23 comp A[maxn], B[maxn];
24 int rev[maxn], m, len;
25
26 inline void init(int n) {
27     for (m = 1, len = 0; m < n + n; m <= 1, ++len);
28     for (int i = 0; i < m; ++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len -
29         1));
30     for (int i = 0; i < m; ++i) A[i] = B[i] = comp(0, 0);
31 }
32
33 inline void dft(comp *a, int v) {
34     for (int i = 0; i < m; ++i) if (i < rev[i]) swap(a[i], a[rev[i]]);
35     for (int s = 2; s <= m; s <= 1) {
36         comp g(cos(2 * pi / s), v * sin(2 * pi / s));
37         for (int k = 0; k < m; k += s) {
38             comp w(1, 0);
39             for (int j = 0; j < s / 2; ++j) {
40                 comp &u = a[k + j + s / 2], &v = a[k + j];
41                 comp t = w * u;
42                 u = v - t;
43                 v = v + t;
44                 w = w * g;
45             }
46         }
47     }
48     if (v == -1)
49         for (int i = 0; i < m; ++i) a[i].real /= m, a[i].imag /= m;
50 }
51
52 inline void mulmod(int *a, int *b, int *c) { // c = a * b % mod, c不能为a或b
53     int M = sqrt(mod);
54     for (int i = 0; i < m; ++i) {
55         A[i] = comp(a[i] / M, a[i] % M);
56         B[i] = comp(b[i] / M, b[i] % M);
57     }
58     dft(A, 1); dft(B, 1);
59     static comp t[maxn];
60     for (int i = 0; i < m; ++i) {
61         int j = i ? m - i : 0;
62         t[i] = ((A[i] + A[j].conj()) * (B[j].conj() - B[i]) + (A[j].conj() - A[i]

```

```

        ]) * (B[i] + B[j].conj())) * comp(0, 0.25);
61     }
62     dft(t, -1);
63     for (int i = 0; i < m; ++i)
64         c[i] = (LL)(t[i].real + 0.5) % mod * M % mod;
65     for (int i = 0; i < m; ++i) {
66         int j = i ? m - i : 0;
67         t[i] = (A[j].conj() - A[i]) * (B[j].conj() - B[i]) * comp(-0.25, 0) +
                comp(0, 0.25) * (A[i] + A[j].conj()) * (B[i] + B[j].conj());
68     }
69     dft(t, -1);
70     for (int i = 0; i < m; ++i)
71         c[i] = (1ll * c[i] + (LL)(t[i].real + 0.5) + (LL)(t[i].imag + 0.5) % mod
                * M * M % mod) % mod;
72     }
73 };

```

6.18 FWT

给定长度为 2^n 的序列 $A[0 \cdots 2^n - 1], B[0 \cdots 2^n - 1]$ ，求这两序列的

or 卷积: $C_k = \sum_{i \text{ or } j=k} A_i B_j$

and 卷积: $C_k = \sum_{i \text{ and } j=k} A_i B_j$

xor 卷积: $C_k = \sum_{i \text{ xor } j=k} A_i B_j$

```

1 void FWT(int *a, int n) {
2     for (int d = 1; d < n; d <= 1)
3         for (int m = d << 1, i = 0; i < n; i += m)
4             for (int j = 0; j < d; ++j) {
5                 int x = a[i + j], y = a[i + j + d];
6                 //or: a[i + j + d] = x + y;
7                 //and: a[i + j] = x + y;
8                 //xor: a[i + j] = x + y, a[i + j + d] = x - y;
9                 // 如答案要求取模, 此处记得取模
10            }
11 }
12
13 void UFWT(int *a, int n) {
14     for (int d = 1; d < n; d <= 1)
15         for (int m = d << 1, i = 0; i < n; i += m)
16             for (int j = 0; j < d; ++j) {
17                 int x = a[i + j], y = a[i + j + d];
18                 //or: a[i + j + d] = y - x;
19                 //and: a[i + j] = x - y;
20                 //xor: a[i + j] = (x + y) / 2, a[i + j + d] = (x - y) / 2;
21                 // 如答案要求取模, 此处记得取模
22            }
23 }

```

6.19 求原根

接口: LL p_root(LL p);

输入: 一个素数 p

输出: p 的原根

```
1  #include <bits/stdc++.h>
2  #define LL long long
3
4  using namespace std;
5
6  vector <LL> a;
7
8  LL pow_mod(LL base, LL times, LL mod) {
9      LL ret = 1;
10     while (times) {
11         if (times&1) ret = ret * base % mod;
12         base = base * base % mod;
13         times>>=1;
14     }
15     return ret;
16 }
17
18 bool g_test(LL g, LL p) {
19     for (LL i = 0; i < a.size(); ++i)
20         if (pow_mod(g, (p-1)/a[i], p) == 1) return 0;
21     return 1;
22 }
23
24 LL p_root(LL p) {
25     LL tmp = p - 1;
26     for (LL i = 2; i <= tmp / i; ++i)
27         if (tmp % i == 0) {
28             a.push_back(i);
29             while (tmp % i == 0)
30                 tmp /= i;
31         }
32     if (tmp != 1) a.push_back(tmp);
33     LL g = 1;
34     while (1) {
35         if (g_test(g, p)) return g;
36         ++g;
37     }
38 }
39
40 int main() {
41     LL p;
42     cin >> p;
43     cout << p_root(p) << endl;
44 }
```

6.20 NTT

NTT 公式:

$$y_n = \sum_{i=0}^{d-1} x_i (g^{\frac{P-1}{d}})^{in} \bmod P$$

```
1 #define mod 998244353
2 #define gg 3
3
4 int power(int base, int times) {
5     int ans = 1;
6     while (times) {
7         if (times & 1) ans = 1ll * ans * base % mod;
8         base = 1ll * base * base % mod;
9         times >>= 1;
10    }
11    return ans;
12 }
13
14 void NTT(int *x, int n, int reverse) {
15     static int rev[N];
16     int m = 1, len = 0;
17     for (; m < n + n; m <= 1, ++len);
18     for (int i = 0; i < m; ++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len - 1))
19     ;
20     for (int i = 0; i < m; ++i)
21         if (i < rev[i])
22             swap(x[i], x[rev[i]]);
23     for (int h = 2; h <= m; h <= 1) {
24         int wn = power(gg, (mod - 1) / h);
25         if (reverse == -1) wn = power(wn, mod - 2);
26         for (int i = 0; i < m; i += h) {
27             int w = 1;
28             for (int j = i; j < i + h / 2; ++j) {
29                 int u = x[j];
30                 int v = 1ll * w * x[j + h / 2] % mod;
31                 x[j] = (u + v) % mod;
32                 x[j + h / 2] = (u - v + mod) % mod;
33                 w = 1ll * w * wn % mod;
34             }
35         }
36     }
37     if (reverse == -1) {
38         int t = power(m, mod - 2);
39         for (int i = 0; i < m; ++i)
40             x[i] = 1ll * x[i] * t % mod;
41     }
42 }
43 int A[N], B[N];
44 int main() {
```

```

45     memset(A, 0, sizeof(A));
46     memset(B, 0, sizeof(B));
47     NTT(A, len, 1); NTT(B, len, 1);
48     for (int i=0;i<len;i++) A[i] = 1ll * A[i] * B[i] % mod;
49     NTT(A, len, -1);
50 }

```

6.20.1 NTT 常用原根表

这张表格仅包含 $2^{18}k + 1$ 的质数。

模数	最大长度	原根	模数	最大长度	原根	模数	最大长度	原根
786433	262144	10	5767169	524288	3	7340033	1048576	3
8650753	262144	10	10223617	262144	5	11272193	262144	3
13631489	1048576	15	14155777	524288	7	14942209	262144	11
16515073	262144	5	21495809	524288	3	22806529	262144	13
23068673	2097152	3	26214401	1048576	3	27000833	262144	3
28311553	1048576	5	29884417	524288	5	33292289	262144	3
35389441	262144	7	36175873	524288	7	37224449	524288	3
38535169	262144	11	40370177	524288	3	41680897	262144	5
42729473	262144	3	52166657	262144	3	63700993	262144	5
64749569	262144	6	68681729	524288	3	69206017	2097152	5
70254593	1048576	3	72613889	262144	3	74711041	262144	7
77070337	524288	13	81788929	2097152	7	83361793	524288	5
83623937	262144	3	85196801	262144	3	87293953	262144	7
90439681	262144	7	93585409	262144	13	93847553	524288	3
100139009	524288	3	101711873	1048576	3	103284737	524288	3
104857601	4194304	3	107216897	262144	3	111149057	2097152	3
113246209	4194304	7	114032641	262144	11	115081217	262144	3
117964801	524288	14	118751233	262144	5	120324097	262144	7
120586241	1048576	6	125042689	262144	13	126615553	262144	10
127664129	262144	3	130809857	262144	3	132120577	2097152	5
136314881	2097152	3	138412033	4194304	5	140771329	262144	7
141557761	1048576	26	142344193	262144	7	145489921	262144	7
147849217	1048576	5	151257089	262144	3	155189249	4194304	6
156499969	262144	7	158072833	262144	5	158334977	1048576	3
159645697	262144	15	163577857	4194304	23	167510017	262144	5
167772161	33554432	3	169869313	2097152	5	173801473	262144	5
175636481	524288	3	178782209	524288	3	184811521	262144	13
185597953	1048576	5	186646529	2097152	3	187432961	262144	6
189530113	262144	5	191365121	524288	3	199229441	2097152	3
200540161	262144	7	204472321	1048576	19	206307329	262144	3
207880193	262144	3	211025921	262144	6	211812353	2097152	3

214171649	262144	3	215482369	524288	13	215744513	262144	3
217317377	262144	3	218628097	524288	5	219676673	524288	3
221249537	1048576	3	222035969	262144	3	224133121	262144	23
228065281	524288	7	230424577	262144	5	230686721	4194304	6
231473153	262144	3	234356737	524288	7	236716033	262144	5
239337473	262144	3	239861761	262144	11	240648193	524288	5
244842497	524288	3	246415361	1048576	3	249561089	2097152	3
253493249	262144	3	254279681	524288	3	256376833	524288	7
257949697	2097152	5	260571137	524288	3	261881857	262144	7
263454721	262144	11	269221889	262144	3	270532609	2097152	22
270794753	262144	3	274726913	2097152	3	276037633	262144	15
277086209	262144	6	284950529	262144	3	285474817	262144	7
288882689	524288	3	290455553	1048576	3	302252033	262144	3
302776321	262144	17	305135617	1048576	5	306708481	524288	19
311427073	1048576	7	319291393	524288	5	323223553	262144	5
325844993	262144	3	328728577	524288	10	329515009	262144	13
329777153	524288	5	330301441	1048576	22	332660737	262144	10
336068609	524288	3	336855041	262144	3	340000769	262144	3
347078657	1048576	3	349962241	262144	7	351797249	524288	3
359661569	1048576	3	360972289	262144	7	361758721	1048576	29
371458049	262144	3	374603777	262144	3	376963073	524288	3
377487361	8388608	7	383778817	2097152	5	384040961	262144	3
386400257	524288	3	387186689	262144	3	387973121	2097152	6
390332417	262144	3	391643137	524288	5	395837441	524288	6
399507457	1048576	5	404226049	524288	7	409993217	1048576	3
413925377	262144	3	415236097	4194304	5	416808961	524288	37
424148993	524288	3	429391873	524288	10	433586177	524288	3
434896897	262144	15	438829057	524288	5	442761217	262144	5
444334081	262144	37	447741953	1048576	3	452198401	262144	11
455344129	262144	13	458752001	524288	6	459276289	2097152	11
460849153	524288	5	462684161	262144	3	463470593	2097152	3
464781313	262144	5	466354177	262144	10	468713473	1048576	5
469762049	67108864	3	471072769	262144	7	473694209	262144	6
475267073	262144	3	478937089	262144	13	483131393	262144	3
483655681	262144	14	487063553	524288	3	489422849	262144	3
493879297	1048576	10	495452161	524288	11	498597889	524288	7
500432897	262144	5	511967233	262144	5	517472257	524288	5
518520833	524288	3	524812289	524288	3	526123009	262144	7
529268737	262144	5	531628033	1048576	5	533463041	262144	3
536608769	262144	3	537133057	262144	5	539754497	262144	3
540540929	524288	3	541327361	262144	3	549978113	524288	3

551288833	262144	5	552861697	262144	5	555220993	524288	7
561774593	262144	3	564658177	524288	5	568066049	262144	3
569638913	262144	3	570163201	262144	7	570949633	524288	5
576454657	262144	10	576716801	2097152	6	581959681	1048576	11
582746113	262144	5	583794689	262144	3	584581121	524288	3
590872577	524288	3	595591169	8388608	3	597688321	2097152	11
605028353	1048576	3	605552641	524288	17	606339073	262144	5
607911937	262144	7	608698369	524288	7	611844097	524288	5
612892673	524288	3	615776257	262144	5	619184129	524288	3
621281281	524288	7	626262017	262144	3	629932033	262144	14
632553473	262144	3	635437057	2097152	11	637009921	524288	17
638058497	524288	3	639369217	262144	5	639631361	2097152	6
644087809	262144	11	645922817	8388608	3	648019969	2097152	17
649592833	524288	5	651952129	262144	7	655360001	1048576	3
657719297	262144	3	660078593	524288	3	663224321	524288	3
665583617	262144	3	666894337	4194304	5	675545089	262144	11
675807233	524288	3	681312257	262144	3	683409409	262144	13
683671553	4194304	3	684982273	262144	5	687603713	262144	3
690749441	262144	3	692846593	262144	5	699138049	262144	19
699924481	524288	17	703070209	524288	11	704905217	262144	3
710410241	524288	3	710934529	2097152	17	712769537	262144	3
714342401	262144	3	715128833	2097152	3	717488129	262144	3
718274561	1048576	3	720633857	262144	3	725876737	262144	7
730595329	262144	17	734527489	524288	7	737673217	524288	11
740294657	2097152	3	741605377	262144	11	745537537	1048576	5
748158977	524288	3	753664001	262144	3	754974721	16777216	11
758906881	262144	11	759693313	524288	5	760741889	524288	3
763887617	524288	3	769130497	524288	15	770703361	1048576	11
771489793	262144	10	772538369	262144	6	775421953	524288	5
781975553	262144	3	782499841	262144	11	786432001	2097152	7
790364161	262144	14	792199169	524288	3	793509889	262144	11
795082753	262144	5	798228481	262144	13	799014913	2097152	13
800063489	1048576	3	800849921	262144	6	801374209	262144	14
802160641	1048576	11	808714241	262144	3	810024961	524288	13
811859969	262144	3	813170689	524288	13	813432833	262144	3
818937857	1048576	5	820248577	262144	5	820510721	524288	3
821297153	262144	3	824180737	2097152	5	824442881	262144	3
825753601	524288	23	828112897	262144	10	829685761	262144	19
833617921	1048576	13	835452929	262144	3	839385089	524288	3
842530817	524288	3	844627969	524288	17	844890113	262144	3
848560129	262144	22	850395137	1048576	3	851705857	262144	5

860618753	262144	3	862978049	1048576	3	863764481	262144	3
864550913	524288	3	867434497	262144	5	872153089	262144	7
873725953	262144	10	875298817	262144	5	879230977	524288	15
880803841	8388608	26	881590273	262144	5	883949569	1048576	7
885522433	524288	5	888668161	524288	14	889454593	262144	15
894959617	524288	10	896008193	524288	3	897318913	262144	5
897581057	8388608	3	899678209	2097152	7	900464641	262144	7
903086081	262144	3	907018241	1048576	3	907542529	524288	7
907804673	262144	3	908328961	262144	26	909377537	262144	3
913309697	1048576	3	914096129	262144	3	918552577	4194304	5
919339009	262144	59	919601153	1048576	3	924844033	2097152	5
925892609	1048576	3	932970497	262144	3	935329793	4194304	3
938475521	1048576	3	940572673	1048576	7	943718401	4194304	7
946339841	524288	3	948699137	262144	3	950009857	2097152	7
951582721	524288	14	957349889	1048576	6	958136321	262144	3
958922753	524288	3	962592769	2097152	7	962854913	262144	3
969146369	262144	3	971243521	262144	28	972029953	1048576	10
975175681	2097152	17	976224257	1048576	3	977534977	262144	5
979107841	262144	11	980156417	262144	3	983826433	262144	11
985661441	4194304	3	993263617	262144	5	995622913	524288	5
998244353	8388608	3	1004535809	2097152	3	1005060097	524288	5
1006108673	524288	3	1007681537	1048576	3	1010565121	262144	7
1012924417	2097152	5	1015283713	262144	5	1018429441	262144	11
1019478017	262144	3	1023148033	262144	7	1036779521	262144	3
1037303809	262144	21	1045430273	1048576	3	1049100289	524288	7
1051721729	1048576	6	1052508161	262144	3	1053818881	1048576	7
1056178177	262144	5	1056440321	524288	3	1062469633	262144	5
1068236801	262144	3	1073479681	262144	11			

6.20.2 多项式求逆元

对于一个多项式 $A(x)$ ，如果存在 $B(x)$ 满足 $\deg(B) \leq \deg(A)$ 并且 $A(x)B(x) \equiv 1 \pmod{x^n}$ ，那么称 $B(x)$ 为 $A(x)$ 在 $\text{mod } x^n$ 意义下的逆元，记为 $A^{-1}(x)$ 。

```

1 // x := 1 / y
2 void inverse(int n0, int *x, const int *y) {
3     static int fy[N];
4     x[0] = power(y[0], mod - 2);
5     for (int i = 1; i < n0; i <= 1) {
6         for (int j = 0; j < 4 * i; ++j) {
7             fy[j] = (j < 2 * i) ? y[j] : 0;
8             if (j >= i) x[j] = 0;
9         }
10        NTT(fy, 2 * i, 1);

```

```

11     NTT(x, 2 * i, 1);
12     for (int j = 0; j < 4 * i; ++j) {
13         x[j] = (2 * x[j] - 111 * x[j] * x[j] % mod * fy[j]) % mod;
14         if (x[j] < 0) x[j] += mod;
15     }
16     NTT(x, 2 * i, -1);
17 }
18 }

```

6.20.3 多项式取对数

```

1 // x := log(y)
2 void logarithm(int n0, int *x, int *y) {
3     static int tmp[N];
4     static int invs[N];
5     inverse(n0, x, y);
6     for (int i = 0; i < n0 * 2; ++i) {
7         tmp[i] = i < n0 - 1 ? 111 * y[i + 1] * (i + 1) % mod : 0;
8         if (i >= n0) x[i] = 0;
9     }
10    NTT(tmp, n0, 1);
11    NTT(x, n0, 1);
12    for (int i = 0; i < n0 * 2; ++i)
13        x[i] = 111 * x[i] * tmp[i] % mod;
14    NTT(x, n0, -1);
15    invs[1] = 1;
16    for (int i = 2; i < n0; ++i)
17        invs[i] = (mod - 111 * mod / i * invs[mod % i] % mod) % mod;
18    for (int i = n0 - 1; i; --i)
19        x[i] = 111 * x[i - 1] * invs[i] % mod;
20    x[0] = 0;
21 }

```

6.20.4 多项式取指数

```

1 // a := exp(b)
2 void exponent(int n0, int *a, int *b) {
3     static int fb[N], x[N], y[N];
4     a[0] = 1;
5     for (int i = 1; i < n0; i <= 1) {
6         for (int j = 0; j < i * 2; ++j)
7             y[j] = (j < i) ? a[j] : 0;
8         logarithm(i * 2, x, y);
9         for (int j = 0; j < 4 * i; ++j) {
10             fb[j] = !j;
11             if (j < 2 * i) {
12                 fb[j] = (fb[j] + b[j]) % mod;
13                 fb[j] = (fb[j] + mod - x[j]) % mod;

```

```

14         }
15         if (j >= i) a[j] = 0;
16     }
17     NTT(a, 2 * i, 1);
18     NTT(fb, 2 * i, 1);
19     for (int j = 0; j < 4 * i; ++j)
20         a[j] = 1ll * a[j] * fb[j] % mod;
21     NTT(a, 2 * i, -1);
22 }
23 }

```

6.21 Berlekamp Messay 算法求线性递推式

适合所有 $S_n = \sum_{i=1}^L a_i S_{n-i}$ 的递推式。只需在 $vector<int> t$ 中输入前 $2L$ 项，即可计算出第 m 项的值 modulo MOD。

时间复杂度 $O(L^2 \log(m))$ 。

异常处理：若提示 48 行 assertion error (`assert(1 * 2 + 1 < s.size())`)，则表示输入项数不足 $2L + 2$ 项，需要更多的项来确定线性递推式。

```

1  #include <bits/stdc++.h>
2
3  using namespace std;
4  typedef long long ll;
5
6  int MOD;
7
8  int bin(int a, int n) {
9      int res = 1;
10     while (n) {
11         if (n & 1) res = 1LL * res * a % MOD;
12         a = 1LL * a * a % MOD;
13         n >>= 1;
14     }
15     return res;
16 }
17
18 int inv(int x) {
19     return bin(x, MOD - 2);
20 }
21
22 vector<int> berlekamp(vector<int> s) {
23     int l = 0;
24     vector<int> la(1, 1);
25     vector<int> b(1, 1);
26     for (int r = 1; r <= (int)s.size(); r++) {
27         int delta = 0;
28         for (int j = 0; j <= l; j++) {
29             delta = (delta + 1LL * s[r - 1 - j] * la[j]) % MOD;

```

```

30     }
31     b.insert(b.begin(), 0);
32     if (delta != 0) {
33         vector<int> t(max(la.size(), b.size()));
34         for (int i = 0; i < (int)t.size(); i++) {
35             if (i < (int)la.size()) t[i] = (t[i] + la[i]) % MOD;
36             if (i < (int)b.size()) t[i] = (t[i] - 1LL * delta * b[i] % MOD + MOD
37                 ) % MOD;
38         }
39         if (2 * l <= r - 1) {
40             b = la;
41             int od = inv(delta);
42             for (int &x : b) x = 1LL * x * od % MOD;
43             l = r - 1;
44         }
45         la = t;
46     }
47     assert(la.size() == l + 1);
48     assert(l * 2 + 1 < s.size());
49     reverse(la.begin(), la.end());
50     return la;
51 }
52
53 vector<int> mul(vector<int> a, vector<int> b) {
54     vector<int> c(a.size() + b.size() - 1);
55     for (int i = 0; i < (int)a.size(); i++) {
56         for (int j = 0; j < (int)b.size(); j++) {
57             c[i + j] = (c[i + j] + 1LL * a[i] * b[j]) % MOD;
58         }
59     }
60     vector<int> res(c.size());
61     for (int i = 0; i < (int)res.size(); i++) res[i] = c[i] % MOD;
62     return res;
63 }
64
65 vector<int> mod(vector<int> a, vector<int> b) {
66     if (a.size() < b.size()) a.resize(b.size() - 1);
67
68     int o = inv(b.back());
69     for (int i = (int)a.size() - 1; i >= b.size() - 1; i--) {
70         if (a[i] == 0) continue;
71         int coef = 1LL * o * (MOD - a[i]) % MOD;
72         for (int j = 0; j < (int)b.size(); j++) {
73             a[i - (int)b.size() + 1 + j] = (a[i - (int)b.size() + 1 + j] + 1LL *
74                 coef * b[j]) % MOD;
75         }
76     }
77     while (a.size() >= b.size()) {
78         assert(a.back() == 0);

```

```

78         a.pop_back();
79     }
80     return a;
81 }
82
83 vector<int> bin(int n, vector<int> p) {
84     vector<int> res(1, 1);
85     vector<int> a(2); a[1] = 1;
86     while (n) {
87         if (n & 1) res = mod(mul(res, a), p);
88         a = mod(mul(a, a), p);
89         n >>= 1;
90     }
91     return res;
92 }
93
94 void solve() {
95     int m = 22;
96     vector<int> t;
97     t.push_back(1);
98     t.push_back(9);
99     t.push_back(41);
100    t.push_back(109);
101    t.push_back(205);
102    t.push_back(325);
103    t.push_back(473);
104    t.push_back(649);
105    t.push_back(853);
106    t.push_back(1085);
107    t.push_back(1345);
108    t.push_back(1633);
109    t.push_back(1949);
110    t.push_back(2293);
111
112    MOD = 998244353;
113    vector<int> v = berlekamp(t);
114    vector<int> o = bin(m - 1, v);
115    int res = 0;
116    for (int i = 0; i < (int)o.size(); i++) res = (res + 1LL * o[i] * t[i]) % MOD;
117    printf("%d\n", res);
118 }
119
120 int main() {
121     solve();
122     return 0;
123 }

```

6.22 幂和

$$\begin{aligned}\sum_{i=1}^n i^1 &= \frac{n(n+1)}{2} = \frac{1}{2}n^2 + \frac{1}{2}n \\ \sum_{i=1}^n i^2 &= \frac{n(n+1)(2n+1)}{6} = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n \\ \sum_{i=1}^n i^3 &= \frac{n^2(n+1)^2}{4} = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2 \\ \sum_{i=1}^n i^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} = \frac{1}{5}n^5 + \frac{1}{2}n^4 + \frac{1}{3}n^3 - \frac{1}{30}n \\ \sum_{i=1}^n i^5 &= \frac{n^2(n+1)^2(2n^2+2n-1)}{12} = \frac{1}{6}n^6 + \frac{1}{2}n^5 + \frac{5}{12}n^4 - \frac{1}{12}n^2 \\ \sum_{i=1}^n i^6 &= \frac{n(n+1)(2n+1)(3n^4+6n^3-3n+1)}{42} = \frac{1}{7}n^7 + \frac{1}{2}n^6 + \frac{1}{2}n^5 - \frac{1}{6}n^3 + \frac{1}{42}n\end{aligned}$$

6.23 蔡勒公式

$$w = (\lfloor \frac{c}{4} \rfloor - 2c + y + \lfloor \frac{y}{4} \rfloor + \lfloor \frac{13(m+1)}{5} \rfloor + d - 1) \bmod 7$$

w : 0 星期日, 1 星期一, 2 星期二, 3 星期三, 4 星期四, 5 星期五, 6 星期六

c : 年份前两位数

y : 年份后两位数

m : 月 ($3 \leq m \leq 14$, 即在蔡勒公式中, 1、2 月要看作上一年的 13、14 月来计算)

d : 日

6.24 皮克定理

给定顶点坐标均是整点 (或正方形格点) 的简单多边形 (凸多边形), 皮克定理说明了其面积 S 和内部格点数目 n 、边上格点数目 s 的关系: $S = n + \frac{s}{2} + 1$ 。

6.25 组合数 lcm

$$(n+1)lcm(C(n,0), C(n,1), \dots, C(n,k)) = lcm(n+1, n, n-1, \dots, n-k+1)$$

6.26 区间 lcm 的维护

对于一个数, 将其分解质因数, 若有因子 p^k , 那么拆分出 k 个数 p, p^2, \dots, p^k , 权值都为 p , 那么查询区间 $[l, r]$ 内所有数的 lcm 的答案 = 所有在该区间中出现过的数的权值之积, 可持久化线段树维护即可。

7 几何

7.1 二维计算几何

7.1.1 计算几何误差修正

```
1 const double pi = acos(-1.0);
2 const double eps = 1e-8;
3
4 inline double sqr(double x) {
5     return x * x;
6 }
7
8 inline int sgn(double x) {
9     if (x < -eps) return -1;
10    return x > eps;
11 }
12
13 inline int cmp(double x, double y) {
14     return sgn(x - y);
15 }
```

7.1.2 计算几何点类

成员函数:

read() 输入一个点

norm() 计算向量的模长

相关函数:

double sqr(double x)

计算一个数的平方

double det(const point &a, const point &b)

计算两个向量的叉积

double dot(const point &a, const point &b)

计算两个向量的点积

double dis(const point &a, const point &b)

计算两个点的距离

point rotate_point(const point &p, double A) \overrightarrow{OP} 绕原点逆时针旋转 A 弧度

```
1 struct point {
2     double x, y;
3     point() : x(0), y(0) {}
4     point(double a, double b) : x(a), y(b) {}
5     inline void read() {
6         scanf("%lf%lf", &x, &y);
7     }
8     inline friend point operator + (const point &a, const point &b) {
9         return point(a.x + b.x, a.y + b.y);
10    }
11    inline friend point operator - (const point &a, const point &b) {
12        return point(a.x - b.x, a.y - b.y);
13    }
14    inline friend bool operator == (const point &a, const point &b) {
15        return cmp(a.x, b.x) == 0 && cmp(a.y, b.y) == 0;
16    }
```



```

16     }
17     inline friend point operator * (const double &a, const point &b) {
18         return point(a * b.x, a * b.y);
19     }
20     inline friend point operator / (const point &a, const double &b) {
21         return point(a.x / b, a.y / b);
22     }
23     inline double norm() const {
24         return sqrt(sqr(x) + sqr(y));
25     }
26 };
27
28 inline double det(const point &a, const point &b) {
29     return a.x * b.y - a.y * b.x;
30 }
31
32 inline double dot(const point &a, const point &b) {
33     return a.x * b.x + a.y * b.y;
34 }
35
36 inline double dis(const point &a, const point &b) {
37     return (a - b).norm();
38 }
39
40 inline point rotate_point(const point &p, double A) {
41     double tx = p.x, ty = p.y;
42     return point(tx * cos(A) - ty * sin(A), tx * sin(A) + ty * cos(A));
43 }

```

7.1.3 计算几何线段类

相关函数：

bool point_on_segment(const point &p, const segment &l) 判断点 p 是否在线段 l 上（含端点）

double point_to_segment_dist(const point &p, const segment &l) 求点 p 到线段 l 的距离

point sym_point(const point &p, const segment &l) 求点 p 关于线段 l 的对称点

point point_proj_line(const point &p, const segment &l) 求点 p 到线段 l 的垂足

bool parallel(const segment &a, const segment &b) 判断线段 a 和线段 b 是否平行

point intersect_point(const segment &a, const segment &b) 求直线 a 与直线 b 的交点（如要求线段 a 与线段 b 的交点，应先判断是否有）

bool is_segment_intersect(const segment &l1, const segment &l2) 判断线段 a 与线段 b 是否相交（含端点）（如不含端点，将 \leq 改为 $<$ ）

bool is_line_intersect_segment(const point &p1, const point &p2, const segment &l) 判断直线 p_1p_2 是否与线段 l 相交

bool is_half_line_intersect_segment(const point &p1, const point &p2, const segment &l) 判断射线 p_1p_2 是否与线段 l 相交（含端点 p_1 ）（如不含端点 p_1 ，将 \geq 改为 $>$ ）

```

1 struct segment {
2     point a, b;

```

```

3     segment() {}
4     segment(point x, point y) : a(x), b(y) {}
5     void read() {
6         a.read(); b.read();
7     }
8 };
9
10    // determine whether point p is on segment l
11    bool point_on_segment(const point &p, const segment &l) {
12        if ((cmp(l.a.x, p.x) <= 0 || cmp(l.b.x, p.x) <= 0) &&
13            (cmp(l.a.x, p.x) >= 0 || cmp(l.b.x, p.x) >= 0) &&
14            (cmp(l.a.y, p.y) <= 0 || cmp(l.b.y, p.y) <= 0) &&
15            (cmp(l.a.y, p.y) >= 0 || cmp(l.b.y, p.y) >= 0)) {
16            return sgn(det(p - l.a, l.b - l.a)) == 0;
17        }
18        return 0;
19    }
20
21    // determine the distance from the point p to segment l
22    double point_to_segment_dist(const point &p, const segment &l) {
23        if (dis(l.a, l.b) < eps) return dis(p, l.a);
24        if (sgn(dot(l.b - l.a, p - l.a)) < 0) return dis(l.a, p);
25        if (sgn(dot(l.a - l.b, p - l.b)) < 0) return dis(l.b, p);
26        return fabs(det(l.b - l.a, p - l.a)) / dis(l.b, l.a);
27    }
28
29    // determine the symmetrical point of point p on segment l
30    point sym_point(const point &p, const segment &l) {
31        double a = l.b.x - l.a.x;
32        double b = l.b.y - l.a.y;
33        double t = ((p.x - l.a.x) * a + (p.y - l.a.y) * b) / (a * a + b * b);
34        return point(2 * l.a.x + 2 * a * t - p.x, 2 * l.a.y + 2 * b * t - p.y);
35    }
36
37    point point_proj_line(const point &p, const segment &l) {
38        double r = dot((l.b - l.a), (p - l.a)) / dot(l.b - l.a, l.b - l.a);
39        return l.a + r * (l.b - l.a);
40    }
41
42    bool parallel(const segment &a, const segment &b) {
43        return sgn(det(a.a - a.b, b.a - b.b)) == 0;
44    }
45
46    point intersect_point(const segment &a, const segment &b) {
47        double s1 = det(a.a - b.a, b.b - b.a);
48        double s2 = det(a.b - b.a, b.b - b.a);
49        return (s1 * a.b - s2 * a.a) / (s1 - s2);
50    }
51
52    // determine whether segment l1 intersects with segment l2

```

```

53 bool is_segment_intersect(const segment &l1, const segment &l2) {
54     const point &s1 = l1.a, &e1 = l1.b;
55     const point &s2 = l2.a, &e2 = l2.b;
56     if ( cmp( min(s1.x, e1.x), max(s2.x, e2.x) ) <= 0 &&
57         cmp( min(s1.y, e1.y), max(s2.y, e2.y) ) <= 0 &&
58         cmp( min(s2.x, e2.x), max(s1.x, e1.x) ) <= 0 &&
59         cmp( min(s2.y, e2.y), max(s1.y, e1.y) ) <= 0 &&
60         sgn( det(s2 - s1, e2 - s1) ) * sgn( det(s2 - e1, e2 - e1) ) <= 0 &&
61         sgn( det(s1 - s2, e1 - s2) ) * sgn( det(s1 - e2, e1 - e2) ) <= 0)
62         return 1;
63     return 0;
64 }
65
66 // determine whether line plp2 intersects with segment l
67 bool is_line_intersect_segment(const point &p1, const point &p2, const segment &l) {
68     assert(!(p1 == p2));
69     return sgn( det(p1 - l.a, p2 - l.a) ) * sgn( det(p1 - l.b, p2 - l.b) ) <= 0;
70 }
71
72 // determine whether half-line plp2 intersects with segment l
73 bool is_half_line_intersect_segment(const point &p1, const point &p2, const segment
    &l) {
74     return is_line_intersect_segment(p1, p2, l) && sgn( det(p1 - l.a, p2 - l.a) ) *
        sgn( det(p1 - l.a, l.b - l.a) ) >= 0;
75 }

```

7.2 凸包

```

1  typedef complex<int> point;
2  #define X real()
3  #define Y imag()
4  int n;
5  long long cross(point a, point b) {
6      return 1ll * a.X * b.Y - 1ll * a.Y * b.X;
7  }
8  bool cmp(point a, point b) {
9      return make_pair(a.X, a.Y) < make_pair(b.X, b.Y);
10 }
11 int convexHull(point p[], int n, point ch[]) {
12     sort(p, p + n, cmp);
13     int m = 0;
14     for(int i = 0; i < n; ++i) {
15         while(m > 1 && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
16         ch[m++] = p[i];
17     }
18     int k = m;
19     for(int i = n - 2; i >= 0; --i) {
20         while(m > k && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
21         ch[m++] = p[i];

```

```

22     }
23     if(n > 1) m--;
24     return m;
25 }

```

7.3 半平面交

输入 vec1 表示所有的半平面 $y \geq kx + b$ 的参数 k 和 b 。

输出 vec2 表示下凸壳 (对应 $y \geq kx + b$) 或者上凸壳 (对应 $y \leq kx + b$)。

```

1  vector< pair< LL, LL > > vec1, vec2;
2
3  LL getval(int t, LL x) {
4      return vec2[t].first * x + vec2[t].second;
5  }
6
7  void solve() {
8      // vec1 stores pair< k, b > for all plane y >=(or <=) kx + b
9      sort(vec1.begin(), vec1.end());
10     // reverse(vec1.begin(), vec1.end()); // if y <= kx + b
11     for (int i = 0; i < vec1.size(); ++i) {
12         while (vec2.size() >= 2) {
13             LL k1 = vec2[vec2.size() - 2].first;
14             LL b1 = vec2[vec2.size() - 2].second;
15             LL k2 = vec2[vec2.size() - 1].first;
16             LL b2 = vec2[vec2.size() - 1].second;
17             LL k3 = vec1[i].first;
18             LL b3 = vec1[i].second;
19             if ((b2 - b1) * (k2 - k3) >= (b3 - b2) * (k1 - k2))
20                 vec2.pop_back();
21             else
22                 break;
23         }
24         vec2.push_back(vec1[i]);
25     }
26 }

```

8 黑科技和杂项

8.1 找规律

此方法已过时，请参照“数学 > Berlekamp Messay 算法求线性递推式”。本法使用矩阵快速幂，效率 $O(L^3 \log(m))$ ，而用 Berlekamp 加多项式快速幂可以做到 $O(L^2 \log(m))$ ，故不推荐使用本法。

有些题目，只给一个正整数 n ，然后要求输出一个答案。这时，我们可以暴力得到小数据的解，用高斯消元得到递推式，然后用矩阵快速幂求解。

使用方法：

首先在 gauss.in 中输入小数据的解 ($n=1$ 时, $n=2$ 时, ...)，以 EOF 结束。

依次运行 gauss.epp, matrix.epp, 得到 matrix.out
将 matrix.out 中的文件粘贴在 main.epp 中相应的位置中。注意模数一定要是质数。

```
1 //gauss.cpp
2 #include <bits/stdc++.h>
3 #define N 102
4 #define mod 1000000007
5 //caution: you can use this program iff mod is a prime.
6
7 using namespace std;
8
9 int n, m, k, a[N], g[N][N];
10
11 int power(int base, int times) {
12     int ret = 1;
13     while (times) {
14         if (times & 1) ret = 1ll * ret * base % mod;
15         base = 1ll * base * base % mod;
16         times >>= 1;
17     }
18     return ret;
19 }
20
21 int test() {
22     for (int i=0; i<m; i++) {
23         for (int j=i; j<=m; j++)
24             if (g[j][i]) {
25                 for (int k=i; k<=m; k++)
26                     swap(g[i][k], g[j][k]);
27                 break;
28             }
29     if (g[i][i] == 0)
30         return 0;
31     for (int j=i+1; j<n; j++) {
32         while (g[j][i]) {
33             int t = 1ll * g[i][i] * power(g[j][i], mod - 2) % mod;
34             for (int k=i; k<n; k++)
35                 g[i][k] = (g[i][k] + mod - (1ll * t * g[j][k] % mod)) % mod;
36             for (int k=i; k<=m; k++)
37                 swap(g[i][k], g[j][k]);
38         }
39     }
40     int t = power(g[i][i], mod - 2);
41     for (int j = 0; j <= m; ++j)
42         g[i][j] = 1ll * g[i][j] * t % mod;
43 }
44 for (int i = m; i < n; ++i)
45     if (g[i][m]) return 0;
46 for (int i = m - 1; i >= 0; --i) {
47     int t = power(g[i][i], mod - 2);
```

```

48     g[i][i] = 1;
49     g[i][m] = 111 * g[i][m] * t % mod;
50     for (int j = 0; j < i; ++j)
51         g[j][m] = (g[j][m] + mod - 111 * g[i][m] * g[j][i] % mod) % mod;
52     }
53     printf("%d\n", m);
54     for (int i = 0; i < m; ++i)
55         printf("%d_", g[i][m]);
56     puts("");
57     for (int i = 0; i < m - 1; ++i)
58         printf("%d_", a[i]);
59     puts("1");
60     return 1;
61 }
62
63 int main() {
64     freopen("gauss.in", "r", stdin);
65     freopen("gauss.out", "w", stdout);
66     k = 0;
67     while (~scanf("%d", &a[k++])) ;
68     for (int sm = 1; sm <= k - sm; ++sm) {
69         n = k - sm - 1;
70         m = sm + 1;
71         for (int i = 0; i < n; ++i) {
72             for (int j = 0; j <= sm; ++j)
73                 g[i][j] = a[i + j];
74             g[i][m] = 1;
75             swap(g[i][m - 1], g[i][m]);
76         }
77         if (test()) return 0;
78     }
79     puts("no_solution");
80     return 0;
81 }

```

```

1 //matrix.cpp
2 #include <bits/stdc++.h>
3 #define N 102
4 using namespace std;
5
6 int n, a[N];
7
8 int main() {
9     freopen("gauss.out", "r", stdin);
10    freopen("matrix.out", "w", stdout);
11    scanf("%d", &n);
12    for (int i = 0; i < n; ++i) scanf("%d", &a[i]);
13    printf("#define_M_%d\n", n);
14    printf("const_int_trans[M][M]_=_{\n");
15    for (int i = 0; i < n; ++i) {

```

```

16     printf("\t{");
17     for (int j = 0; j < n; ++j) {
18         int t;
19         if (j < n - 2) t = i == j + 1;
20         else if (j == n - 2) t = a[i];
21         else t = i == n - 1;
22         printf("%s%d", j == 0 ? "" : ",_", t);
23     }
24     printf("}%s\n", i == n - 1 ? "" : ",");
25 }
26 printf("};\n");
27 printf("const_int_pref[M]_={");
28 for (int i = 0; i < n; ++i) {
29     int x;
30     scanf("%d", &x);
31     printf("%d%s", x, i == n - 1 ? "};\n" : ",_");
32 }
33 return 0;
34 }

```

```

1 //main.cpp
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 /* paste matrix.out here. */
6
7 #define mod 1000000007
8
9 struct Matrix {
10     int c[M][M];
11     void clear() { memset(c, 0, sizeof(c)); }
12     void identity() { clear(); for (int i = 0; i < M; ++i) c[i][i] = 1; }
13     void base() { memcpy(c, trans, sizeof(trans)); }
14     friend Matrix operator * (const Matrix &a, const Matrix &b) {
15         Matrix c; c.clear();
16         for (int i = 0; i < M; ++i)
17             for (int j = 0; j < M; ++j)
18                 for (int k = 0; k < M; ++k)
19                     c.c[i][j] = (c.c[i][j] + 1ll * a.c[i][k] * b.c[k][j] % mod) %
20                                     mod;
21         return c;
22     }
23 } start, base;
24
25 Matrix power(Matrix base, int times) {
26     Matrix ret; ret.identity();
27     while (times) {
28         if (times & 1) ret = ret * base;
29         base = base * base;
30         times >>= 1;
31     }
32     return ret;
33 }

```

```

30     }
31     return ret;
32 }
33
34 int main() {
35     int tot;
36     scanf("%d", &tot);
37     while (tot--) {
38         int n;
39         scanf("%d", &n);
40         start.clear();
41         for (int i = 0; i < M; ++i) start.c[0][i] = pref[i];
42         base.base();
43         base = power(base, n - 1);
44         start = start * base;
45         printf("%d\n", start.c[0][0]);
46     }
47     return 0;
48 }

```

8.2 分数类

```

1  #define LL long long
2
3  struct frac {
4      LL x, y;
5      frac(LL _x = 0, LL _y = 1) {
6          x = _x;
7          y = _y;
8          LL g = __gcd(abs(x), abs(y));
9          x /= g;
10         y /= g;
11         if (y < 0) {
12             x = -x;
13             y = -y;
14         }
15     }
16
17     inline friend frac operator + (const frac &lhs, const frac &rhs) {
18         return frac(lhs.x * rhs.y + rhs.x * lhs.y, lhs.y * rhs.y);
19     }
20
21     inline friend frac operator - (const frac &lhs, const frac &rhs) {
22         return frac(lhs.x * rhs.y - rhs.x * lhs.y, lhs.y * rhs.y);
23     }
24
25     inline friend frac operator - (const frac &lhs) {
26         return frac(-lhs.x, lhs.y);
27     }

```



```

28
29     inline friend frac operator * (const frac &lhs, const frac &rhs) {
30         return frac(lhs.x * rhs.x, lhs.y * rhs.y);
31     }
32
33     inline friend frac operator / (const frac &lhs, const frac &rhs) {
34         return frac(lhs.x * rhs.y, lhs.y * rhs.x);
35     }
36
37     inline friend bool operator == (const frac &lhs, const frac &rhs) {
38         return lhs.x * rhs.y == rhs.x * lhs.y;
39     }
40
41     inline friend bool operator != (const frac &lhs, const frac &rhs) {
42         return lhs.x * rhs.y != rhs.x * lhs.y;
43     }
44
45     inline friend bool operator < (const frac &lhs, const frac &rhs) {
46         return lhs.x * rhs.y < rhs.x * lhs.y;
47     }
48
49     inline friend bool operator > (const frac &lhs, const frac &rhs) {
50         return lhs.x * rhs.y > rhs.x * lhs.y;
51     }
52
53     inline friend bool operator <= (const frac &lhs, const frac &rhs) {
54         return lhs.x * rhs.y <= rhs.x * lhs.y;
55     }
56
57     inline friend bool operator >= (const frac &lhs, const frac &rhs) {
58         return lhs.x * rhs.y >= rhs.x * lhs.y;
59     }
60
61     inline void print() const {
62         printf("%lld/%lld\n", x, y);
63     }
64 };

```

8.3 取模整数类

如果需要用模意义下的除法，需定义常量 D 为除数的最大值，并执行 `init_inv()`。

```

1 struct mod;
2 mod* inv;
3
4 struct mod {
5     static constexpr int MOD = 1000 * 1000 * 1000 + 7; // std=c++11
6     mod(int x_) : x((x_ % MOD + MOD) % MOD) {}
7     mod() = default;
8     int x = 0;

```

```

9      inline mod operator *(mod other) const {
10          return ((long long)x * other.x) % MOD;
11      }
12      inline mod& operator *=(mod other) {
13          x = ((long long)x * other.x) % MOD;
14          return *this;
15      }
16      inline mod operator +(mod other) const {
17          int res = x + other.x;
18          if (res >= MOD) {
19              res -= MOD;
20          }
21          return res;
22      }
23      inline mod& operator +=(mod other) {
24          if ((x += other.x) >= MOD) {
25              x -= MOD;
26          }
27          return *this;
28      }
29      inline mod operator -(mod other) const {
30          int res = x - other.x;
31          if (res < 0) {
32              res += MOD;
33          }
34          return res;
35      }
36      inline mod& operator -=(mod other) {
37          if ((x -= other.x) < 0) {
38              x += MOD;
39          }
40          return *this;
41      }
42      inline mod operator /(mod other) const {
43          return (*this) * inv[other.x];
44      }
45      inline mod& operator /=(mod other) {
46          return *this *= inv[other.x];
47      }
48      inline bool operator ==(mod other) const {
49          return x == other.x;
50      }
51      inline mod operator -() const {
52          return x != 0 ? MOD - x : 0;
53      }
54 };
55
56 void init_inv() {
57     inv = new mod[D];
58     inv[1] = 1;

```

```

59     for (int i = 2; i < D; i++) {
60         inv[i] = (mod::MOD - (long long)mod::MOD / i * inv[mod::MOD % i].x % mod::
            MOD) % mod::MOD;
61     }
62 }

```

8.4 多项式类

```

1  struct poly {
2      vector<mod> C;
3      poly() {}
4      explicit poly(const vector<mod> &C_) : C(C_) {}
5      static const poly zero;
6      inline int deg() const {
7          return (int)C.size() - 1;
8      }
9      inline mod operator[](int x) const {
10         return (x < 0 || x > deg()) ? mod(0) : C[x];
11     }
12     inline mod& operator[](int x) {
13         if (x > deg()) {
14             C.resize(x + 1);
15         }
16         return C[x];
17     }
18     inline friend poly operator +(const poly& a, const poly& b) {
19         vector<mod> c(max(a.deg(), b.deg()) + 1);
20         for (int i = 0; i < c.size(); i++) {
21             c[i] = a[i] + b[i];
22         }
23         return poly(c);
24     }
25     inline friend poly operator -(const poly& a, const poly& b) {
26         vector<mod> c(max(a.deg(), b.deg()) + 1);
27         for (int i = 0; i < c.size(); i++) {
28             c[i] = a[i] - b[i];
29         }
30         return poly(c);
31     }
32     inline bool isZero() const {
33         return C.empty();
34     }
35     inline friend poly operator *(const poly& a, const poly& b) {
36         if (a.isZero() || b.isZero()) {
37             return zero;
38         }
39         vector<mod> c(1 + a.deg() + b.deg());
40         for (int i = 0; i <= a.deg(); i++) {
41             for (int j = 0; j <= b.deg(); j++) {

```

```

42         c[i + j] += a[i] * b[j];
43     }
44 }
45 return poly(c);
46 }
47 inline poly derivative() const {
48     if (isZero()) {
49         return zero;
50     }
51     vector<mod> res(deg());
52     for (int i = 0; i < deg(); i++) {
53         res[i] = C[i + 1] * (i + 1);
54     }
55     return poly(res);
56 }
57 inline poly primitive() const {
58     if (isZero()) {
59         return zero;
60     }
61     vector<mod> res(2 + deg());
62     for (int i = 1; i <= 1 + deg(); i++) {
63         res[i] = C[i - 1] / i;
64     }
65     return poly(res);
66 }
67 inline mod operator()(mod x) const {
68     mod res = 0;
69     for (int i = deg(); i >= 0; i--) {
70         res = res * x + C[i];
71     }
72     return res;
73 }
74 // Expand P(x+t).
75 inline poly shift(int t) const {
76     poly res;
77     res[deg()];
78     vector<mod> binomial(deg() + 1, 0);
79     binomial[0] = 1;
80     for (int i = 0; i <= deg(); i++) {
81         mod cur = 1;
82         for (int j = i; j >= 0; j--) {
83             res[j] += C[i] * binomial[j] * cur;
84             cur *= t;
85         }
86         if (i == deg()) {
87             break;
88         }
89         for (int j = i + 1; j > 0; j--) {
90             binomial[j] += binomial[j - 1];
91         }

```

```

92     }
93     return res;
94 }
95 };

```

8.5 求数列中每个位置到前面任意位置的 gcd

$\text{maxgcd}[i]$ 表示从第 i 个位置往前出发, 每个 $\text{gcd}(a_j, \dots, a_i)$ 值最靠后的位置 (first 表示 gcd 值, second 表示位置)。

例如:

```

1  输入数据:
2  5
3  2 4 2 3 3
4
5  maxgcd[1] = {{first = 2, second = 1}}
6  maxgcd[2] = {{first = 2, second = 1}, {first = 4, second = 2}}
7  maxgcd[3] = {{first = 2, second = 3}}
8  maxgcd[4] = {{first = 1, second = 3}, {first = 3, second = 4}}
9  maxgcd[5] = {{first = 1, second = 3}, {first = 3, second = 5}}

```

```

1  int n, a[N];
2  vector< pair<int, int> > maxgcd[N];
3
4  void insert(int x, int val, int mx) {
5      for (int i = 0; i < (int)maxgcd[x].size(); ++i)
6          if (maxgcd[x][i].first == val) {
7              maxgcd[x][i].second = max(maxgcd[x][i].second, mx);
8              return;
9          }
10     maxgcd[x].push_back( make_pair(val, mx) );
11 }
12
13 int main() {
14     scanf("%d", &n);
15     for (int i = 1; i <= n; ++i) scanf("%d", &a[i]);
16     for (int i = 0; i <= n + 1; ++i) maxgcd[i].clear();
17     for (int i = 1; i <= n; ++i) {
18         for (int j = 0; j < (int)maxgcd[i - 1].size(); ++j)
19             insert(i, __gcd(maxgcd[i-1][j].first, a[i]), maxgcd[i-1][j].second);
20         insert(i, a[i], i);
21     }
22 }

```

8.6 高精度计算

```

1  #include<algorithm>
2  using namespace std;

```

```

3  const int N_huge=850,base=100000000;
4  char s[N_huge*10];
5  struct huge{
6      typedef long long value;
7      value a[N_huge];int len;
8      void clear(){len=1;a[len]=0;}
9      huge(){clear();}
10     huge(value x){*this=x;}
11     huge operator =(huge b){
12         len=b.len;for (int i=1;i<=len;++i)a[i]=b.a[i]; return *this;
13     }
14     huge operator =(value x){
15         len=0;
16         while (x)a[++len]=x%base,x/=base;
17         if (!len)a[++len]=0;
18         return *this;
19     }
20     huge operator +(huge b){
21         int L=len>b.len?len:b.len;huge tmp;
22         for (int i=1;i<=L+1;++i)tmp.a[i]=0;
23         for (int i=1;i<=L;++i){
24             if (i>len)tmp.a[i]+=b.a[i];
25             else if (i>b.len)tmp.a[i]+=a[i];
26             else {
27                 tmp.a[i]+=a[i]+b.a[i];
28                 if (tmp.a[i]>=base){
29                     tmp.a[i]-=base;++tmp.a[i+1];
30                 }
31             }
32         }
33         if (tmp.a[L+1])tmp.len=L+1;
34         else tmp.len=L;
35         return tmp;
36     }
37     huge operator -(huge b){
38         int L=len>b.len?len:b.len;huge tmp;
39         for (int i=1;i<=L+1;++i)tmp.a[i]=0;
40         for (int i=1;i<=L;++i){
41             if (i>b.len)b.a[i]=0;
42             tmp.a[i]+=a[i]-b.a[i];
43             if (tmp.a[i]<0){
44                 tmp.a[i]+=base;--tmp.a[i+1];
45             }
46         }
47         while (L>1&&!tmp.a[L])--L;
48         tmp.len=L;
49         return tmp;
50     }
51     huge operator *(huge b){
52         int L=len+b.len;huge tmp;

```

```

53     for (int i=1;i<=L;++i)tmp.a[i]=0;
54     for (int i=1;i<=len;++i)
55         for (int j=1;j<=b.len;++j){
56             tmp.a[i+j-1]+=a[i]*b.a[j];
57             if (tmp.a[i+j-1]>=base){
58                 tmp.a[i+j]+=tmp.a[i+j-1]/base;
59                 tmp.a[i+j-1]%=base;
60             }
61         }
62     tmp.len=len+b.len;
63     while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
64     return tmp;
65 }
66 pair<huge,huge> divide(huge a,huge b){
67     int L=a.len;huge c,d;
68     for (int i=L;i--i){
69         c.a[i]=0;d=d*base;d.a[1]=a.a[i];
70         int l=0,r=base-1,mid;
71         while (l<r){
72             mid=(l+r+1)>>1;
73             if (b*mid<=d)l=mid;
74             else r=mid-1;
75         }
76         c.a[i]=1;d-=b*l;
77     }
78     while (L>1&&!c.a[L])--L;c.len=L;
79     return make_pair(c,d);
80 }
81 huge operator /(value x){
82     value d=0;huge tmp;
83     for (int i=len;i--i){
84         d=d*base+a[i];
85         tmp.a[i]=d/x;d%=x;
86     }
87     tmp.len=len;
88     while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
89     return tmp;
90 }
91 value operator %(value x){
92     value d=0;
93     for (int i=len;i--i)d=(d*base+a[i])%x;
94     return d;
95 }
96 huge operator /(huge b){return divide(*this,b).first;}
97 huge operator %(huge b){return divide(*this,b).second;}
98 huge &operator +=(huge b){*this=*this+b;return *this;}
99 huge &operator -=(huge b){*this=*this-b;return *this;}
100 huge &operator *=(huge b){*this=*this*b;return *this;}
101 huge &operator ++(){huge T;T=1;*this=*this+T;return *this;}
102 huge &operator --(){huge T;T=1;*this=*this-T;return *this;}

```

```

103 huge operator ++(int){huge T,tmp=*this;T=1;*this=*this+T;return tmp;}
104 huge operator --(int){huge T,tmp=*this;T=1;*this=*this-T;return tmp;}
105 huge operator +(value x){huge T;T=x;return *this+T;}
106 huge operator -(value x){huge T;T=x;return *this-T;}
107 huge operator *(value x){huge T;T=x;return *this*T;}
108 huge operator *=(value x){*this=*this*x;return *this;}
109 huge operator +=(value x){*this=*this+x;return *this;}
110 huge operator -=(value x){*this=*this-x;return *this;}
111 huge operator /=(value x){*this=*this/x;return *this;}
112 huge operator %=(value x){*this=*this%x;return *this;}
113 bool operator ==(value x){huge T;T=x;return *this==T;}
114 bool operator !=(value x){huge T;T=x;return *this!=T;}
115 bool operator <=(value x){huge T;T=x;return *this<=T;}
116 bool operator >=(value x){huge T;T=x;return *this>=T;}
117 bool operator <(value x){huge T;T=x;return *this<T;}
118 bool operator >(value x){huge T;T=x;return *this>T;}
119 bool operator <(huge b){
120     if (len<b.len)return 1;
121     if (len>b.len)return 0;
122     for (int i=len;i--i){
123         if (a[i]<b.a[i])return 1;
124         if (a[i]>b.a[i])return 0;
125     }
126     return 0;
127 }
128 bool operator ==(huge b){
129     if (len!=b.len)return 0;
130     for (int i=len;i--i){
131         if (a[i]!=b.a[i])return 0;
132     }
133     return 1;
134 }
135 bool operator !=(huge b){return !(*this==b);}
136 bool operator >(huge b){return !(*this<b||*this==b);}
137 bool operator <=(huge b){return (*this<b)||(*this==b);}
138 bool operator >=(huge b){return (*this>b)||(*this==b);}
139 void str(char s[]){
140     int l=strlen(s);value x=0,y=1;len=0;
141     for (int i=l-1;i>=0;--i){
142         x=x+(s[i]-'0')*y;y*=10;
143         if (y==base)a[++len]=x,x=0,y=1;
144     }
145     if (!len||x)a[++len]=x;
146 }
147 void read(){
148     scanf("%s",s);this->str(s);
149 }
150 void print(){
151     printf("%d",(int)a[len]);
152     for (int i=len-1;i--i){
153         for (int j=base/10;j>=10;j/=10){

```



```

153         if (a[i]<j)printf("0");
154         else break;
155     }
156     printf("%d", (int)a[i]);
157 }
158 printf("\n");
159 }
160 }f[1005];
161 int main(){
162     f[1]=f[2]=1;
163     for(int i=3;i<=1000;i++)f[i]=f[i-1]+f[i-2];
164 }

```

8.7 读入优化

8.7.1 普通读入优化

```

1 #define rd RD<int>
2 #define rdll RD<long long>
3 template <typename Type>
4 inline Type RD() {
5     Type x = 0;
6     int flag = 0;
7     char c = getchar();
8     while (!isdigit(c) && c != '-')
9         c = getchar();
10    (c == '-') ? (flag = 1) : (x = c - '0');
11    while (isdigit(c = getchar()))
12        x = x * 10 + c - '0';
13    return flag ? -x : x;
14 }
15 inline char rdch() {
16     char c = getchar();
17     while (!isalpha(c)) c = getchar();
18     return c;
19 }

```

8.7.2 HDU 专用读入优化

接口：

int rd(int &x); 读入一个整数，保存在变量 x 中。如正常读入，返回值为 1，否则返回 EOF(-1)
int rdll(long long &x);

```

1 #define rd RD<int>
2 #define rdll RD<long long>
3
4 const int S = 2000000; // 2MB
5
6 char s[S], *h = s+S, *t = h;

```

```

7
8 inline char getchrl(void) {
9     if(h == t) {
10         if (t != s + S) return EOF;
11         t = s + fread(s, 1, S, stdin);
12         h = s;
13     }
14     return *h++;
15 }
16
17 template <class T>
18 inline int RD(T &x) {
19     char c = 0;
20     int sign = 0;
21     for (; !isdigit(c); c = getchrl()) {
22         if (c == EOF)
23             return -1;
24         if (c == '-')
25             sign ^= 1;
26     }
27     x = 0;
28     for (; isdigit(c); c = getchrl())
29         x = x * 10 + c - '0';
30     if (sign) x = -x;
31     return 1;
32 }

```

8.8 O2 优化

```

1 #define OPTIM __attribute__((optimize("-O2")))

```

8.9 正方形展开图

如图 5。

8.10 位运算及其运用

8.10.1 枚举子集

枚举 i 的非空子集 j

```

1 for (int j = i; j; j = (j - 1) & i);

```

8.10.2 求 1 的个数

```

1 int __builtin_popcount(unsigned int x);

```

8.10.3 求前缀 0 的个数

```
1 int __builtin_clz(unsigned int x);
```

8.10.4 求后缀 0 的个数

```
1 int __builtin_ctz(unsigned int x);
```

9 Vim

```
1 syntax on
2 set cindent
3 set nu
4 set tabstop=4
5 set shiftwidth=4
6 set background=dark
7
8 inoremap <C-j> <down>
9 inoremap <C-k> <up>
10 inoremap <C-h> <left>
11 inoremap <C-l> <right>
```

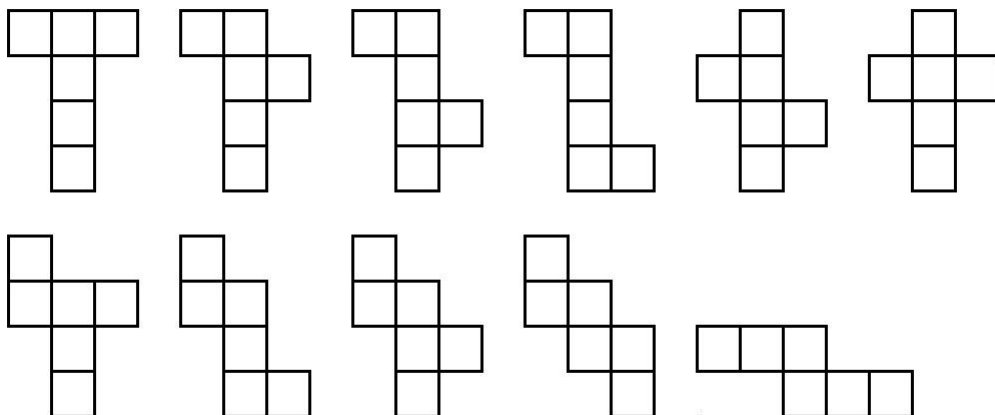


图 5: 正方形展开图