

ICPC Templates For Africamonkey

Africamonkey

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目录

1	莫队算法	3
1.1	普通莫队	3
1.2	树上莫队	3
2	字符串	5
2.1	哈希	5
2.2	KMP	5
2.3	可动态修改的 KMP	6
2.4	扩展 KMP	6
2.5	Manacher	7
2.6	最小表示法	8
2.7	AC 自动机	8
2.8	后缀数组	9
2.9	后缀自动机	10
3	数据结构	11
3.1	ST 表	11
3.2	线段树小技巧	12
3.3	Splay	12
3.4	可持久化 Treap	15
3.5	可持久化并查集	17
4	树	18
4.1	点分治	18
4.2	动态树	19
5	图	21
5.1	欧拉回路	21
5.2	最短路径	22
5.2.1	Dijkstra	22
5.2.2	SPFA	23
5.3	K 短路	24

5.4	Tarjan	27
5.5	2-SAT	28
5.6	统治者树 (Dominator Tree)	30
5.7	网络流	31
5.7.1	最大流	31
5.7.2	上下界有源汇网络流	32
5.7.3	上下界无源汇网络流	32
5.7.4	费用流	33
5.7.5	zkw 费用流	34
6	数学	36
6.1	扩展欧几里得解同余方程	36
6.2	同余方程组	36
6.3	卡特兰数	37
6.4	斯特林数	37
6.4.1	第一类斯特林数	37
6.4.2	第二类斯特林数	37
6.5	错排公式	37
6.6	Lucas 定理	37
6.7	高斯消元	38
6.7.1	行列式	38
6.7.2	Matrix-Tree 定理	39
6.8	调和级数	39
6.9	曼哈顿距离的变换	39
6.10	线性筛素数	39
6.11	杜教筛	39
6.12	FFT	41
6.13	求原根	42
6.14	NTT	44
6.15	组合数 lcm	45
6.16	区间 lcm 的维护	45
7	几何	45
7.1	凸包	45
8	黑科技和杂项	46
8.1	找规律	46
8.2	高精度计算	49

1 莫队算法

1.1 普通莫队

```
1 struct Q { int l, r, sqrtl, id; } q[N];
2 int n, m, v[N], ans[N], nowans;
3 bool cmp(const Q &a, const Q &b) {
4     if (a.sqrtl != b.sqrtl) return a.sqrtl < b.sqrtl;
5     return a.r < b.r;
6 }
7 void change(int x) { if (!v[x]) checkin(); else checkout(); }
8 int main() {
9     .....
10    for (int i=1;i<=m;i++) q[i].sqrtl = q[i].l / sqrt(n), q[i].id = i;
11    sort(q+1, q+m+1, cmp);
12    int L=1,R=0; nowans=0;
13    memset(v, 0, sizeof(v));
14    for (int i=1;i<=m;i++) {
15        while (L<q[i].l) change(L++);
16        while (L>q[i].l) change(--L);
17        while (R<q[i].r) change(++R);
18        while (R>q[i].r) change(R--);
19        ans[q[i].id] = nowans;
20    }
21    .....
22 }
```

1.2 树上莫队

```
1 struct Query { int l, r, id, l_group; } query[N];
2 struct EDGE { int adj, next; } edge[N*2];
3 int n, m, top, gh[N], c[N], reorder[N], deep[N], father[N], size[N], son[N], Top[N];
4 void addedge(int x, int y) {
5     edge[++top].adj = y;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 void dfs(int x, int root=0) {
10    reorder[x] = ++top; father[x] = root; deep[x] = deep[root] + 1;
11    son[x] = 0; size[x] = 1; int dd = 0;
12    for (int p=gh[x]; p; p=edge[p].next)
13        if (edge[p].adj != root) {
14            dfs(edge[p].adj, x);
15            if (size[edge[p].adj] > dd) {
16                son[x] = edge[p].adj;
17                dd = size[edge[p].adj];
18            }
19            size[x] += size[edge[p].adj];
20        }
```

```

21 }
22 void split(int x, int tp) {
23     Top[x] = tp;
24     if (son[x]) split(son[x], tp);
25     for (int p=gh[x]; p; p=edge[p].next)
26         if (edge[p].adj != father[x] && edge[p].adj != son[x])
27             split(edge[p].adj, edge[p].adj);
28 }
29 int lca(int x, int y) {
30     int tx = Top[x], ty = Top[y];
31     while (tx != ty) {
32         if (deep[tx] < deep[ty]) {
33             swap(tx, ty);
34             swap(x, y);
35         }
36         x = father[tx];
37         tx = Top[x];
38     }
39     if (deep[x] < deep[y]) swap(x, y);
40     return y;
41 }
42 bool cmp(const Query &a, const Query &b) {
43     if (a.l_group != b.l_group) return a.l_group < b.l_group;
44     return reorder[a.r] < reorder[b.r];
45 }
46 int v[N], ans[N];
47 void upd(int x) { if (!v[x]) checkin(); else checkout(); }
48 void go(int &u, int taru, int v) {
49     int lca0 = lca(u, taru);
50     int lca1 = lca(u, v); upd(lca1);
51     int lca2 = lca(taru, v); upd(lca2);
52     for (int x=u; x!=lca0; x=father[x]) upd(x);
53     for (int x=taru; x!=lca0; x=father[x]) upd(x);
54     u = taru;
55 }
56 int main() {
57     memset(gh, 0, sizeof(gh));
58     scanf("%d%d", &n, &m); top = 0;
59     for (int i=1; i<n; i++) {
60         int x,y; scanf("%d%d", &x, &y);
61         addedge(x, y); addedge(y, x);
62     }
63     top = 0; dfs(1); split(1, 1);
64     for (int i=1; i<=m; i++) {
65         if (reorder[query[i].l] > reorder[query[i].r])
66             swap(query[i].l, query[i].r);
67         query[i].id = i;
68         query[i].l_group = reorder[query[i].l] / sqrt(n);
69     }
70     sort(query+1, query+m+1, cmp);

```

```

71     int L=1,R=1; upd(1);
72     for (int i=1;i<=m;i++) {
73         go(L,query[i].l,R);
74         go(R,query[i].r,L);
75         ans[query[i].id] = answer();
76     }
77     .....
78 }

```

2 字符串

2.1 哈希

```

1  const int P=31,D=1000173169;
2  int n, pow[N], f[N]; char a[N];
3  int hash(int l, int r) { return (LL) (f[r]-(LL)f[l-1]*pow[r-l+1]%D+D)%D; }
4  int main() {
5      scanf("%d%s", &n, a+1);
6      pow[0] = 1;
7      for (int i=1;i<=n;i++) pow[i] = (LL)pow[i-1]*P%D;
8      for (int i=1;i<=n;i++) f[i] = (LL) ((LL)f[i-1]*P+a[i])%D;
9  }

```

2.2 KMP

接口: `int find_substring(char *pattern, char *text, int *next, int *ret);`

输入: 模式串, 匹配串

输出: 返回值表示模式串在匹配串中出现的次数

KMP 的 `next[i]` 表示从 0 到 i 的字符串 s, 前缀和后缀的最长重叠长度。

```

1  void find_next(char *pattern, int *next) {
2      int n = strlen(pattern);
3      for (int i=1;i<n;i++) {
4          int j = i;
5          while (j > 0) {
6              j = next[j];
7              if (pattern[j] == pattern[i]) {
8                  next[i+1] = j+1;
9                  break;
10             }
11         }
12     }
13 }
14 int find_substring(char *pattern, char *text, int *next, int *ret) {
15     find_next(pattern, next);
16     int n = strlen(pattern);
17     int m = strlen(text);

```

```

18     int k = 0;
19     for (int i=0, j=0; i<m; i++) {
20         if (j<n && text[i]==pattern[j]) {
21             j++;
22         } else {
23             while (j>0) {
24                 j = next[j];
25                 if (text[i] == pattern[j]) {
26                     j++;
27                     break;
28                 }
29             }
30         }
31         if (j == n)
32             ret[k++] = i-n+1;
33     }
34     return k;
35 }

```

2.3 可动态修改的 KMP

支持：加入一个字符，删除一个字符。

时间复杂度： $O(n\alpha)$ ， α 为字符集大小。

代码中的字符为 '0' - '9'，可自行修改为 'a' - 'z'

```

1 char t[N];
2 int top, nxt[N], nxt_l[N][10];
3 inline void del_letter() { --top; }
4 inline void add_letter(char x) {
5     t[top++] = x;
6     int j = top-1;
7     memset(nxt_l[top], 0, sizeof(nxt_l[top]));
8     nxt[top] = nxt_l[top-1][x-'0'];
9     memcpy(nxt_l[top], nxt_l[nxt[top]], sizeof(nxt_l[nxt[top]]));
10    nxt_l[top][t[nxt[top]]-'0'] = nxt[top]+1;
11 }

```

2.4 扩展 KMP

接口：void ExtendedKMP(char *a, char *b, int *next, int *ret);

输出：

next: a 关于自己每个后缀的最长公共前缀

ret: a 关于 b 的每个后缀的最长公共前缀

EXKMP 的 next[i] 表示：从 i 到 n-1 的字符串 st 前缀和原串前缀的最长重叠长度。

```

1 void get_next(char *a, int *next) {
2     int i, j, k;
3     int n = strlen(a);

```

```

4     for (j = 0; j+1<n && a[j]==a[j+1];j++);
5     next[1] = j;
6     k = 1;
7     for (i=2;i<n;i++) {
8         int len = k+next[k], l = next[i-k];
9         if (l < len-i) {
10             next[i] = l;
11         } else {
12             for (j = max(0, len-i);i+j<n && a[j]==a[i+j];j++);
13             next[i] = j;
14             k = i;
15         }
16     }
17 }
18 void ExtendedKMP(char *a, char *b, int *next, int *ret) {
19     get_next(a, next);
20     int n = strlen(a), m = strlen(b);
21     int i, j, k;
22     for (j=0;j<n && j<m && a[j]==b[j];j++);
23     ret[0] = j;
24     k = 0;
25     for (i=1;i<m;i++) {
26         int len = k+ret[k], l = next[i-k];
27         if (l < len-i) {
28             ret[i] = l;
29         } else {
30             for (j = max(0, len-i);j<n && i+j<m && a[j]==b[i+j];j++);
31             ret[i] = j;
32             k = i;
33         }
34     }
35 }

```

2.5 Manacher

$p[i]$ 表示以 i 为对称轴的最长回文串长度

```

1 char st[N*2], s[N];
2 int len, p[N*2];
3
4 while (scanf("%s", s) != EOF) {
5     len = strlen(s);
6     st[0] = '$', st[1] = '#';
7     for (int i=1;i<=len;i++)
8         st[i*2] = s[i-1], st[i*2+1] = '#';
9     len = len * 2 + 2;
10    int mx = 0, id = 0, ans = 0;
11    for (int i=1;i<=len;i++) {
12        p[i] = (mx > i) ? min(p[id*2-i]+1, mx-i) : 1;
13        for (; st[i+p[i]] == st[i-p[i]]; ++p[i]) ;

```

```

14         if (p[i]+i > mx) mx = p[i]+i, id = i;
15         p[i] --;
16         if (p[i] > ans) ans = p[i];
17     }
18     printf("%d\n", ans);
19 }

```

2.6 最小表示法

```

1 string smallestRepresation(string s) {
2     int i, j, k, l;
3     int n = s.length();
4     s += s;
5     for (i=0, j=1; j<n; ) {
6         for (k=0; k<n && s[i+k]==s[j+k]; k++);
7         if (k>=n) break;
8         if (s[i+k]<s[j+k]) j+=k+1;
9         else {
10             l=i+k;
11             i=j;
12             j=max(l, j)+1;
13         }
14     }
15     return s.substr(i, n);
16 }

```

2.7 AC 自动机

```

1 struct Node {
2     int next[**Size of Alphabet**];
3     int terminal, fail;
4 } node[**Number of Nodes**];
5 int top;
6 void add(char *st) {
7     int len = strlen(st), x = 1;
8     for (int i=0; i<len; i++) {
9         int ind = trans(st[i]);
10        if (!node[x].next[ind])
11            node[x].next[ind] = ++top;
12        x = node[x].next[ind];
13    }
14    node[x].terminal = 1;
15 }
16 int q[**Number of Nodes**], head, tail;
17 void build() {
18     head = 0, tail = 1; q[1] = 1;
19     while (head != tail) {
20         int x = q[++head];

```



```

21      /*(when necessary) node[x].terminal != node[node[x].fail].terminal; */
22      for (int i=0;i<n;i++)
23          if (node[x].next[i]) {
24              if (x == 1) node[node[x].next[i]].fail = 1;
25              else {
26                  int y = node[x].fail;
27                  while (y) {
28                      if (node[y].next[i]) {
29                          node[node[x].next[i]].fail = node[y].next[i];
30                          break;
31                      }
32                      y = node[y].fail;
33                  }
34                  if (!node[node[x].next[i]].fail) node[node[x].next[i]].fail = 1;
35              }
36              q[++tail] = node[x].next[i];
37          }
38      }
39  }

```

2.8 后缀数组

参数 m 表示字符集的大小, 即 $0 \leq r_i < m$

```

1  #define rank rank2
2  int n, r[N], wa[N], wb[N], ws[N], sa[N], rank[N], height[N];
3  int cmp(int *r, int a, int b, int l, int n)
4  {
5      if (r[a]==r[b])
6      {
7          if (a+l<n && b+l<n && r[a+l]==r[b+l])
8              return 1;
9      }
10     return 0;
11 }
12 void suffix_array(int m)
13 {
14     int i, j, p, *x=wa, *y=wb, *t;
15     for (i=0;i<m;i++) ws[i]=0;
16     for (i=0;i<n;i++) ws[x[i]=r[i]]++;
17     for (i=1;i<m;i++) ws[i]+=ws[i-1];
18     for (i=n-1;i>=0;i--) sa[--ws[x[i]]]=i;
19     for (j=1,p=1;p<n;m=p, j<=<=1)
20     {
21         for (p=0,i=n-j;i<n;i++) y[p++]=i;
22         for (i=0;i<n;i++) if (sa[i]>=j) y[p++]=sa[i]-j;
23         for (i=0;i<m;i++) ws[i]=0;
24         for (i=0;i<n;i++) ws[x[y[i]]]++;
25         for (i=1;i<m;i++) ws[i]+=ws[i-1];
26         for (i=n-1;i>=0;i--) sa[--ws[x[y[i]]]]=y[i];

```

```

27     for (t=x, x=y, y=t, x[sa[0]]=0, i=1, p=1; i<n; i++)
28         x[sa[i]]=cmp(y, sa[i-1], sa[i], j, n)?p-1:p++;
29     }
30     for (i=0; i<n; i++) rank[sa[i]]=i;
31 }
32 void calc_height()
33 {
34     int j=0;
35     for (int i=0; i<n; i++)
36         if (rank[i])
37         {
38             while (r[i+j]==r[sa[rank[i]-1]+j]) j++;
39             height[rank[i]]=j;
40             if (j) j--;
41         }
42 }

```

2.9 后缀自动机

下面的代码是求两个串的 LCS（最长公共子串）。

```

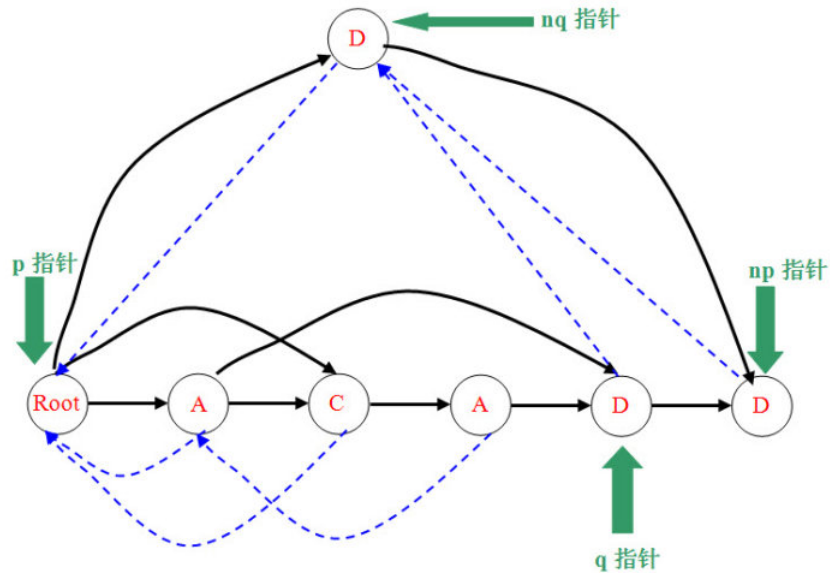
1  #include <stdio>
2  #include <stdlib>
3  #include <string>
4  #define N 500001
5  using namespace std;
6  char st[N];
7  int pre[N<<1], son[26][N<<1], step[N<<1], last, total;
8  int apply(int x) { step[++total]=x; return total; }
9  void Extend(char x) {
10     int p = last, np = apply(step[last]+1);
11     for (; p && !son[x][p]; p=pre[p]) son[x][p] = np;
12     if (!p) pre[np] = 1;
13     else {
14         int q = son[x][p];
15         if (step[p]+1 == step[q]) pre[np] = q;
16         else {
17             int nq = apply(step[p]+1);
18             for (int i=0; i<26; i++) son[i][nq] = son[i][q];
19             pre[nq] = pre[q];
20             pre[q] = pre[np] = nq;
21             for (; p && son[x][p]==q; p=pre[p]) son[x][p] = nq;
22         }
23     }
24     last = np;
25 }
26 void init() {
27     last = total = 0;
28     last = apply(0);
29     scanf("%s", st);

```

```

30     for (int i=0; st[i]; i++)
31         Extend(st[i]-'a');
32     scanf("%s",st);
33 }
34 int main() {
35     init();
36     int p = 1, now = 0, ans = 0;
37     for (int i=0; st[i]; i++) {
38         int index = st[i]-'a';
39         for (; p && !son[index][p]; p = pre[p], now = step[p]) ;
40         if (!p) p = 1;
41         if (son[index][p]) {
42             p = son[index][p];
43             now++;
44             if (now > ans) ans = now;
45         }
46     }
47     printf("%d\n",ans);
48     return 0;
49 }

```



3 数据结构

3.1 ST 表

```

1 int Log[N], f[17][N];
2 int ask(int x, int y) {
3     int k = log[y-x+1];
4     return max(f[k][x], f[k][y-(1<<k)+1]);
5 }

```

```

6 int main(){
7     for(i=2;i<=n;i++) Log[i]=Log[i>>1]+1;
8     for(j=1;j<K;j++) for(i=1;i+(1<<j-1)<=n;i++) f[j][i]=max(f[j-1][i],f[j-1][i+(1<<j-1)]);
9 }

```

3.2 线段树小技巧

给定一个序列 a ，寻找一个最大的 i 使得 $i \leq y$ 且满足一些条件（如 $a[i] \geq w$ ，那么需要在线段树维护 a 的区间最大值）

```

1 int queryl(int p, int left, int right, int y, int w) {
2     if (right <= y) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (y <= mid) return queryl(p<<1|0, left, mid, y, w);
8     int ret = queryl(p<<1|1, mid+1, right, y, w);
9     if (ret != -1) return ret;
10    return queryl(p<<1|0, left, mid, y, w);
11 }

```

给定一个序列 a ，寻找一个最小的 i 使得 $i \geq x$ 且满足一些条件（如 $a[i] \geq w$ ，那么需要在线段树维护 a 的区间最大值）

```

1 int queryr(int p, int left, int right, int y, int w) {
2     if (left >= x) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (x > mid) return queryr(p<<1|1, mid+1, right, y, w);
8     int ret = queryr(p<<1|0, left, mid, y, w);
9     if (ret != -1) return ret;
10    return queryr(p<<1|1, mid+1, right, y, w);
11 }

```

3.3 Splay

接口：

ADD $x \ y \ d$ ：将 $[x, y]$ 的所有数加上 d

REVERSE $x \ y$ ：将 $[x, y]$ 翻转

INSERT $x \ p$ ：将 p 插入到第 x 个数的后面

DEL x ：将第 x 个数删除

```

1 struct SPLAY {
2     struct NODE {
3         int w, min;

```

```

4      int son[2], size, father, rev, lazy;
5  } node[N];
6  int top, rt;
7  void pushdown(int x) {
8      if (!x) return;
9      if (node[x].rev) {
10         node[node[x].son[0]].rev ^= 1;
11         node[node[x].son[1]].rev ^= 1;
12         swap(node[x].son[0], node[x].son[1]);
13         node[x].rev = 0;
14     }
15     if (node[x].lazy) {
16         node[node[x].son[0]].lazy += node[x].lazy;
17         node[node[x].son[1]].lazy += node[x].lazy;
18         node[x].w += node[x].lazy;
19         node[x].min += node[x].lazy;
20         node[x].lazy = 0;
21     }
22 }
23 void pushup(int x) {
24     if (!x) return;
25     pushdown(node[x].son[0]);
26     pushdown(node[x].son[1]);
27     node[x].size = node[node[x].son[0]].size + node[node[x].son[1]].size + 1;
28     node[x].min = node[x].w;
29     if (node[x].son[0]) node[x].min = min(node[x].min, node[node[x].son[0]].min)
30     ;
31     if (node[x].son[1]) node[x].min = min(node[x].min, node[node[x].son[1]].min)
32     ;
33 }
34 void sc(int x, int y, int w) {
35     node[x].son[w] = y;
36     node[y].father = x;
37     pushup(x);
38 }
39 void _ins(int w) {
40     top++;
41     node[top].w = node[top].min = w;
42     node[top].son[0] = node[top].son[1] = 0;
43     node[top].size = 1; node[top].father = 0; node[top].rev = 0;
44 }
45 void init() {
46     top = 0;
47     _ins(0); _ins(0); rt=1;
48     sc(1, 2, 1);
49 }
50 void rotate(int x) {
51     if (!x) return;
52     int y = node[x].father;
53     int w = node[y].son[1]==x;

```

```

52     sc(y, node[x].son[w^1], w);
53     sc(node[y].father, x, node[node[y].father].son[1]==y);
54     sc(x, y, w^1);
55 }
56 int q[N];
57 void flushdown(int x) {
58     int t=0; for (; x; x=node[x].father) q[++t]=x;
59     for (; t; t--) pushdown(q[t]);
60 }
61 void Splay(int x, int root=0) {
62     flushdown(x);
63     while (node[x].father != root) {
64         int y=node[x].father;
65         int w=node[y].son[1]==x;
66         if (node[y].father != root && node[node[y].father].son[w]==y) rotate(y);
67         rotate(x);
68     }
69 }
70 int find(int k) {
71     Splay(rt);
72     while (1) {
73         pushdown(rt);
74         if (node[node[rt].son[0]].size+1==k) {
75             Splay(rt);
76             return rt;
77         } else
78         if (node[node[rt].son[0]].size+1<k) {
79             k-=node[node[rt].son[0]].size+1;
80             rt=node[rt].son[1];
81         } else {
82             rt=node[rt].son[0];
83         }
84     }
85 }
86 int split(int x, int y) {
87     int fx = find(x);
88     int fy = find(y+2);
89     Splay(fx);
90     Splay(fy, fx);
91     return node[fy].son[0];
92 }
93 void add(int x, int y, int d) { //add d to each number in a[x]...a[y]
94     int t = split(x, y);
95     node[t].lazy += d;
96     Splay(t); rt=t;
97 }
98 void reverse(int x, int y) { // reverse the x-th to y-th elements
99     int t = split(x, y);
100    node[t].rev ^= 1;
101    Splay(t); rt=t;

```

```

102     }
103     void insert(int x, int p) { // insert p after the x-th element
104         int fx = find(x+1);
105         int fy = find(x+2);
106         Splay(fx);
107         Splay(fy, fx);
108         _ins(p);
109         sc(fy, top, 0);
110         Splay(top); rt=top;
111     }
112     void del(int x) { // delete the x-th element in Splay
113         int fx = find(x), fy = find(x+2);
114         Splay(fx); Splay(fy, fx);
115         node[fy].son[0] = 0;
116         Splay(fy); rt=fy;
117     }
118 } tree;

```

3.4 可持久化 Treap

接口：

void insert(int x, char c); 在当前第 x 个字符后插入 c

void del(int x, int y); 删除第 x 个字符到第 y 个字符

void copy(int l, int r, int x); 复制第 l 个字符到第 r 个字符，然后粘贴到第 x 个字符后

void reverse(int x, int y); 翻转第 x 个到第 y 个字符

char query(int k); 表示询问当前第 x 个字符是什么

```

1  #define mod 1000000007
2  struct Treap {
3      struct Node {
4          char key;
5          bool reverse;
6          int lc, rc, size;
7      } node[N];
8      int n, root, rd;
9      int Rand() { rd = (rd * 20372052LL + 25022087LL) % mod; return rd; }
10     void init() { n = root = 0; }
11     inline int copy(int x) { node[++n] = node[x]; return n; }
12     inline void pushdown(int x) {
13         if (!node[x].reverse) return;
14         if (node[x].lc) node[x].lc = copy(node[x].lc);
15         if (node[x].rc) node[x].rc = copy(node[x].rc);
16         swap(node[x].lc, node[x].rc);
17         node[node[x].lc].reverse ^= 1;
18         node[node[x].rc].reverse ^= 1;
19         node[x].reverse = 0;
20     }
21     inline void pushup(int x) { node[x].size = node[node[x].lc].size + node[node[x].rc].size + 1; }

```

```

22  int merge(int u, int v) {
23      if (!u || !v) return u+v;
24      pushdown(u); pushdown(v);
25      int t = Rand() % (node[u].size + node[v].size), r;
26      if (t < node[u].size) {
27          r = copy(u);
28          node[r].rc = merge(node[u].rc, v);
29      } else {
30          r = copy(v);
31          node[r].lc = merge(u, node[v].lc);
32      }
33      pushup(r);
34      return r;
35  }
36  int split(int u, int x, int y) {
37      if (x > y) return 0;
38      pushdown(u);
39      if (x == 1 && y == node[u].size) return u;
40      if (y <= node[node[u].lc].size) return split(node[u].lc, x, y);
41      int t = node[node[u].lc].size + 1;
42      if (x > t) return split(node[u].rc, x-t, y-t);
43      int num = copy(u);
44      node[num].lc = split(node[u].lc, x, t-1);
45      node[num].rc = split(node[u].rc, 1, y-t);
46      pushup(num);
47      return num;
48  }
49  void insert(int x, char c) {
50      int t1 = split(root, 1, x), t2 = split(root, x+1, node[root].size);
51      node[++n].key = c; node[n].size = 1;
52      root = merge(merge(t1, n), t2);
53  }
54  void del(int x, int y) {
55      int t1 = split(root, 1, x-1), t2 = split(root, y+1, node[root].size);
56      root = merge(t1, t2);
57  }
58  void copy(int l, int r, int x) {
59      int t1 = split(root, 1, x), t2 = split(root, 1, r), t3 = split(root, x+1,
60          node[root].size);
61      root = merge(merge(t1, t2), t3);
62  }
63  void reverse(int x, int y) {
64      int t1 = split(root, 1, x-1), t2 = split(root, x, y), t3 = split(root, y+1,
65          node[root].size);
66      node[t2].reverse ^= 1;
67      root = merge(merge(t1, t2), t3);
68  }
69  char query(int k) {
70      int x = root;
71      while (1) {

```



```

70         pushdown(x);
71         if (k <= node[node[x].lc].size) x = node[x].lc;
72         else
73             if (k == node[node[x].lc].size + 1) return node[x].key;
74         else
75             k -= node[node[x].lc].size + 1, x = node[x].rc;
76     }
77 }
78 } treap;

```

3.5 可持久化并查集

接口：

void init() 初始化

void merge(int x, int y, int time) 在 time 时刻将 x 和 y 连一条边，注意加边顺序必须按 time 从小到大加边

void GetFather(int x, int time) 询问 time 时刻及以前的连边状态中，x 所属的集合

```

1 namespace pers_union {
2     const int inf = 0x3f3f3f3f;
3     int father[N], Father[N], Time[N];
4     vector<int> e[N];
5     void init() {
6         for (int i=1;i<=n;i++) {
7             father[i] = i;
8             Father[i] = i;
9             Time[i] = inf;
10            e[i].clear();
11            e[i].push_back(i);
12        }
13    }
14    int getfather(int x) {
15        return (father[x] == x) ? x : father[x] = getfather(father[x]);
16    }
17    int GetFather(int x, int time) {
18        return (Time[x] <= time) ? GetFather(Father[x], time) : x;
19    }
20    void merge(int x, int y, int time) {
21        int fx = getfather(x), fy = getfather(y);
22        if (fx == fy) return;
23        if (e[fx].size() > e[fy].size()) swap(fx, fy);
24        father[fx] = fy;
25        Father[fx] = fy;
26        Time[fx] = time;
27        for (int i=0;i<e[fx].size();i++) {
28            e[fy].push_back(e[fx][i]);
29        }
30    }
31 };

```

4 树

4.1 点分治

初始化时须设置 $top = 1$ 。

```
1 void addedge(int x, int y) {
2     edge[++top].adj = y;
3     edge[top].valid = 1;
4     edge[top].next = gh[x];
5     gh[x] = top;
6 }
7 void get_size(int x, int root=0) {
8     size[x] = 1; son[x] = 0;
9     int dd = 0;
10    for (int p=gh[x]; p; p=edge[p].next)
11        if (edge[p].adj != root && edge[p].valid) {
12            get_size(edge[p].adj, x);
13            size[x] += size[edge[p].adj];
14            if (size[edge[p].adj] > dd) {
15                dd = size[edge[p].adj];
16                son[x] = edge[p].adj;
17            }
18        }
19 }
20 int getroot(int x) {
21     get_size(x);
22     int sz = size[x];
23     while (size[son[x]] > sz/2)
24         x = son[x];
25     return x;
26 }
27 void dc(int x) {
28     x = getroot(x);
29     static int list[N], ltop;
30     ltop = 0;
31     for (int p=gh[x]; p; p=edge[p].next)
32         if (edge[p].valid)
33             list[++ltop] = p;
34     clear();
35     for (int i=1; i<=ltop; i++) {
36         update();
37         modify();
38     }
39     clear();
40     for (int i=ltop; i>=1; i--) {
41         update();
42         modify();
43     }
44     //be careful about the root
45     for (int p=gh[x]; p; p=edge[p].next)
```

```

46         if (edge[p].valid) {
47             edge[p].valid = 0;
48             edge[p^1].valid = 0;
49             dc(edge[p].adj);
50         }
51     }

```

4.2 动态树

接口：

command(x, y)：将 x 到 y 路径的 Splay Tree 分离出来。

linkcut(u1, v1, u2, v2)：将树中原有的边 (u1, v1) 删除，加入一条新边 (u2, v2)

```

1  struct DynamicTREE{
2      struct NODE{
3          int father, son[2], top, size, reverse;
4      } splay[N];
5      void init(int i, int fat) {
6          splay[i].father = splay[i].son[0] = splay[i].son[1] = 0;
7          splay[i].top = fat; splay[i].size = 1; splay[i].reverse = 0;
8      }
9      void pushdown(int x) {
10         if (!x) return;
11         int s0 = splay[x].son[0], s1 = splay[x].son[1];
12         if (splay[x].reverse) {
13             splay[s0].reverse ^= 1;
14             splay[s1].reverse ^= 1;
15             swap(splay[x].son[0], splay[x].son[1]);
16             splay[x].reverse = 0;
17         }
18         s0 = splay[x].son[0], s1 = splay[x].son[1];
19         splay[s0].top = splay[s1].top = splay[x].top;
20     }
21     void pushup(int x) {
22         if (!x) return;
23         pushdown(splay[x].son[0]);
24         pushdown(splay[x].son[1]);
25         splay[x].size = splay[splay[x].son[0]].size + splay[splay[x].son[1]].size +
26             1;
27     }
28     void sc(int x, int y, int w, bool Auto=true) {
29         splay[x].son[w] = y;
30         splay[y].father = x;
31         if (Auto) {
32             pushup(y);
33             pushup(x);
34         }
35     }
36     int top, tush[N];

```

```

36 void flowdown(int x) {
37     for (top=1; x; top++, x = splay[x].father) tush[top] = x;
38     for (; top; top--) pushdown(tush[top]);
39 }
40 void rotate(int x) {
41     if (!x) return;
42     int y = splay[x].father;
43     int w = splay[y].son[1] == x;
44     pushdown(y);
45     pushdown(x);
46     sc(splay[y].father, x, splay[splay[y].father].son[1]==y, false);
47     sc(y, splay[x].son[w^1], w, false);
48     sc(x, y, w^1, false);
49     pushup(y);
50     pushup(x);
51 }
52 void Splay(int x, int rt=0) {
53     if (!x) return;
54     flowdown(x);
55     while (splay[x].father != rt) {
56         int y = splay[x].father;
57         int w = splay[y].son[1]==x;
58         if (splay[y].father != rt && splay[splay[y].father].son[w] == y) rotate(
59             y);
60         rotate(x);
61     }
62 void split(int x) {
63     int y = splay[x].son[1];
64     if (!y) return;
65     splay[y].father = 0;
66     splay[x].son[1] = 0;
67     splay[y].top = x;
68     pushup(x);
69 }
70 void access(int x) {
71     int y = 0;
72     while (x) {
73         Splay(x);
74         split(x);
75         sc(x, y, 1);
76         Splay(x);
77         y = x;
78         x = splay[x].top;
79     }
80 }
81 void changeroot(int x) {
82     access(x);
83     Splay(x);
84     splay[x].reverse = 1;

```

```

85     Splay(x);
86 }
87 void command(int x, int y, ...) {
88     LL ans = 0;
89     changeroot(x);
90     access(y);
91     Splay(x);
92     //then you can modify the Splay Tree
93 }
94 void linkcut(int u1, int v1, int u2, int v2) {
95     changeroot(u1);
96     access(v1);
97     Splay(u1); split(u1);
98     splay[v1].top = 0;
99     access(u2); changeroot(u2);
100    access(v2); changeroot(v2);
101    Splay(u2); Splay(v2);
102    splay[v2].top = u2;
103 }
104 } lct;

```

5 图

5.1 欧拉回路

欧拉回路：

无向图：每个顶点的度数都是偶数，则存在欧拉回路。

有向图：每个顶点的入度 = 出度，则存在欧拉回路。

欧拉路径：

无向图：当且仅当该图所有顶点的度数为偶数，或者除了两个度数为奇数外其余的全是偶数。

有向图：当且仅当该图所有顶点出度 = 入度或者一个顶点出度 = 入度 + 1，另一个顶点入度 = 出度 + 1，其他顶点出度 = 入度。

下面 $O(n + m)$ 求欧拉回路的代码中， n 为点数， m 为边数，若有解则依次输出经过的边的编号，若是无向图，则正数表示 x 到 y ，负数表示 y 到 x 。

```

1 namespace UndirectedGraph{
2     int n,m,i,x,y,d[N],g[N],v[M<<1],w[M<<1],vis[M<<1],nxt[M<<1],ed;
3     int ans[M],cnt;
4     void add(int x,int y,int z){
5         d[x]++;
6         v[++ed]=y;w[ed]=z;nxt[ed]=g[x];g[x]=ed;
7     }
8     void dfs(int x){
9         for(int&i=g[x];i;){
10             if(vis[i]){i=nxt[i];continue;}
11             vis[i]=vis[i^1]=1;
12             int j=w[i];
13             dfs(v[i]);

```

```

14         ans[++cnt]=j;
15     }
16 }
17 void solve() {
18     scanf("%d%d",&n,&m);
19     for(i=ed=1;i<=m;i++) scanf("%d%d",&x,&y), add(x,y,i), add(y,x,-i);
20     for(i=1;i<=n;i++) if(d[i]&1) {puts("NO");return;}
21     for(i=1;i<=n;i++) if(g[i]) {dfs(i);break;}
22     for(i=1;i<=n;i++) if(g[i]) {puts("NO");return;}
23     puts("YES");
24     for(i=m;i;i--) printf("%d_",ans[i]);
25 }
26 }
27 namespace DirectedGraph{
28     int n,m,i,x,y,d[N],g[N],v[M],vis[M],nxt[M],ed;
29     int ans[M],cnt;
30     void add(int x,int y){
31         d[x]++;d[y]--;
32         v[++ed]=y;nxt[ed]=g[x];g[x]=ed;
33     }
34     void dfs(int x){
35         for(int&i=g[x];i;){
36             if(vis[i]){i=nxt[i];continue;}
37             vis[i]=1;
38             int j=i;
39             dfs(v[i]);
40             ans[++cnt]=j;
41         }
42     }
43     void solve(){
44         scanf("%d%d",&n,&m);
45         for(i=1;i<=m;i++) scanf("%d%d",&x,&y), add(x,y);
46         for(i=1;i<=n;i++) if(d[i]) {puts("NO");return;}
47         for(i=1;i<=n;i++) if(g[i]) {dfs(i);break;}
48         for(i=1;i<=n;i++) if(g[i]) {puts("NO");return;}
49         puts("YES");
50         for(i=m;i;i--) printf("%d_",ans[i]);
51     }
52 }

```

5.2 最短路径

5.2.1 Dijkstra

```

1 #include <queue>
2 using namespace std;
3 struct EDGE { int adj, w, next; } edge[M*2];
4 struct dat { int id, dist; dat(int id=0, int dist=0) : id(id), dist(dist) {} };

```

```

5 struct cmp { bool operator () (const dat &a, const dat &b) { return a.dist > b.dist;
    } };
6 priority_queue < dat, vector<dat>, cmp > q;
7 int n, top, gh[N], v[N], dist[N];
8 void addedge(int x, int y, int w) {
9     edge[++top].adj = y;
10    edge[top].w = w;
11    edge[top].next = gh[x];
12    gh[x] = top;
13 }
14 int dijkstra(int s, int t) {
15     memset(dist, 63, sizeof(dist));
16     memset(v, 0, sizeof(v));
17     dist[s] = 0;
18     q.push(dat(s, 0));
19     while (!q.empty()) {
20         dat x = q.top(); q.pop();
21         if (v[x.id]) continue; v[x.id] = 1;
22         for (int p=gh[x.id]; p; p=edge[p].next) {
23             if (x.dist + edge[p].w < dist[edge[p].adj]) {
24                 dist[edge[p].adj] = x.dist + edge[p].w;
25                 q.push(dat(edge[p].adj, dist[edge[p].adj]));
26             }
27         }
28     }
29     return dist[t];
30 }

```

5.2.2 SPFA

```

1 struct EDGE { int adj, w, next; } edge[M*2];
2 int n,m,top,gh[N],v[N],cnt[N],q[N],dist[N],head,tail;
3 void addedge(int x, int y, int w) {
4     edge[++top].adj = y;
5     edge[top].w = w;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 int spfa(int S, int T) {
10    memset(v, 0, sizeof(v));
11    memset(cnt, 0, sizeof(cnt));
12    memset(dist, 63, sizeof(dist));
13    head = 0, tail = 1;
14    dist[S] = 0; q[1] = S;
15    while (head != tail) {
16        (head += 1) %= N;
17        int x = q[head]; v[x] = 0;
18        ++cnt[x]; if (cnt[x] > n) return -1;
19        for (int p=gh[x]; p; p=edge[p].next)

```

```

20         if (dist[x] + edge[p].w < dist[edge[p].adj]) {
21             dist[edge[p].adj] = dist[x] + edge[p].w;
22             if (!v[edge[p].adj]) {
23                 v[edge[p].adj] = 1;
24                 (tail += 1) %= N;
25                 q[tail] = edge[p].adj;
26             }
27         }
28     }
29     return dist[T];
30 }

```

5.3 K 短路

接口：

kthsp::init(n)：初始化并设置节点个数为 n

kthsp::add(x, y, w)：添加一条 x 到 y 的有向边

kthsp::work(S, T, k)：求 S 到 T 的第 k 短路

```

1  #include <queue>
2
3  #define N 200020
4  #define M 400020
5  #define LOGM 20
6  #define LL long long
7  #define inf (1LL<<61)
8
9  namespace pheap {
10     struct Node {
11         int next, son[2];
12         LL val;
13     } node[M*LOGM];
14     int LOG[M];
15     int root[M], size[M*LOGM], top;
16     int add() {
17         ++top; assert(top < M*LOGM);
18         node[top].next = node[top].son[0] = node[top].son[1] = 0;
19         node[top].val = inf;
20         return top;
21     }
22     int copy(int x) { int t = add(); node[t] = node[x]; return t; }
23     void init() {
24         top = -1; add();
25         for (int i=2; i<M; i++) LOG[i] = LOG[i>>1] + 1;
26     }
27     void upd(int x, int &next, LL &val) {
28         if (val < node[x].val) {
29             swap(val, node[x].val);
30             swap(next, node[x].next);

```



```

31     }
32 }
33 void insert(int x, int next, LL val) {
34     int sz = size[root[x]] + 1;
35     root[x] = copy(root[x]);
36     size[root[x]] = sz; x = root[x];
37     upd(x, next, val);
38     for (int i=LOG[sz]-1;i>=0;i--) {
39         int ind = (sz>>i)&1;
40         node[x].son[ind] = copy(node[x].son[ind]);
41         x = node[x].son[ind];
42         upd(x, next, val);
43     }
44 }
45 };
46
47 namespace kthsp {
48     using namespace pheap;
49     struct EDGE {
50         int adj, w, next;
51     } edge[2][M];
52     struct W {
53         int x, y, w;
54     } e[M];
55     bool has_init = 0;
56     int n, m, top[2], gh[2][N], v[N];
57     LL dist[N];
58     void init(int n1) {
59         has_init = 1;
60         n = n1; m = 0;
61         memset(top, 0, sizeof(top));
62         memset(gh, 0, sizeof(gh));
63         for (int i=1;i<=n;i++) dist[i] = inf;
64     }
65     void addedge(int id, int x, int y, int w) {
66         edge[id][++top[id]].adj = y;
67         edge[id][top[id]].w = w;
68         edge[id][top[id]].next = gh[id][x];
69         gh[id][x] = top[id];
70     }
71     void add(int x, int y, int w) {
72         assert(has_init);
73         e[++m].x=x; e[m].y=y; e[m].w=w;
74     }
75     int q[N], best[N], bestw[N];
76     int deg[N];
77     void spfa(int S) {
78         for (int i=1;i<=n;i++) deg[i] = 0;
79         for (int i=1;i<=m;i++) deg[e[i].x] ++;
80         int head = 0, tail = 1;

```

```

81     dist[S] = 0; q[1] = S;
82     while (head != tail) {
83         (head += 1) %= N;
84         int x = q[head];
85         for (int p=gh[1][x]; p; p=edge[1][p].next) {
86             if (dist[x] + edge[1][p].w < dist[edge[1][p].adj]) {
87                 dist[edge[1][p].adj] = dist[x] + edge[1][p].w;
88                 best[edge[1][p].adj] = x;
89                 bestw[edge[1][p].adj] = p;
90             }
91             if (!--deg[edge[1][p].adj]) {
92                 (tail += 1) %= N;
93                 q[tail] = edge[1][p].adj;
94             }
95         }
96     }
97 }
98 void dfs(int x) {
99     if (v[x]) return; v[x] = 1;
100    if (best[x]) root[x] = root[best[x]];
101    for (int p=gh[0][x]; p; p=edge[0][p].next)
102        if (dist[edge[0][p].adj] != inf && bestw[x] != p) {
103            insert(x, edge[0][p].adj, edge[0][p].w + dist[edge[0][p].adj] - dist
104                [x]);
105        }
106    for (int p=gh[1][x]; p; p=edge[1][p].next)
107        if (best[edge[1][p].adj] == x)
108            dfs(edge[1][p].adj);
109 }
110 typedef pair<LL,int> pli;
111 priority_queue <pli, vector<pli>, greater<pli> > pq;
112 LL work(int S, int T, int k) {
113     assert(has_init);
114     n++; add(T, n, 0);
115     if (S == T) k++;
116     T = n;
117     for (int i=1; i<=m; i++) {
118         addedge(0, e[i].x, e[i].y, e[i].w);
119         addedge(1, e[i].y, e[i].x, e[i].w);
120     }
121     spfa(T);
122     root[T] = 0; pheap::init();
123     memset(v, 0, sizeof(v));
124     dfs(T);
125     while (!pq.empty()) pq.pop();
126     if (k == 1) return dist[S];
127     if (root[S]) pq.push(make_pair(dist[S] + node[root[S]].val, root[S]));
128     while (k--) {
129         if (pq.empty()) return inf;
130         pli now = pq.top(); pq.pop();

```

```

130         if (k == 1) return now.first;
131         int x = node[now.second].next, u = node[now.second].son[0], v = node[now
            .second].son[1];
132         if (root[x]) pq.push(make_pair(now.first + node[root[x]].val, root[x]));
133         if (u) pq.push(make_pair(now.first - node[now.second].val + node[u].val,
            u));
134         if (v) pq.push(make_pair(now.first - node[now.second].val + node[v].val,
            v));
135     }
136     return 0;
137 }
138 };

```

5.4 Tarjan

割点的判断：一个顶点 u 是割点，当且仅当满足 (1) 或 (2)：

(1) u 为树根，且 u 有多于一个子树

(2) u 不为树根，且满足存在 (u, v) 为树枝边 (u 为 v 的父亲)，使得 $dfn[u] \leq low[v]$

桥的判断：一条无向边 (u, v) 是桥，当且仅当 (u, v) 为树枝边，满足 $dfn[u] < low[v]$

```

1 struct EDGE { int adj, next; } edge[M];
2 int n, m, top, gh[N];
3 int dfn[N], low[N], cnt, ind, stop, instack[N], stack[N], belong[N];
4 void addedge(int x, int y) {
5     edge[++top].adj = y;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 void tarjan(int x) {
10     dfn[x] = low[x] = ++ind;
11     instack[x] = 1; stack[++stop] = x;
12     for (int p=gh[x]; p; p=edge[p].next)
13         if (!dfn[edge[p].adj]) {
14             tarjan(edge[p].adj);
15             low[x] = min(low[x], low[edge[p].adj]);
16         } else if (instack[edge[p].adj]) {
17             low[x] = min(low[x], dfn[edge[p].adj]);
18         }
19     if (dfn[x] == low[x]) {
20         ++cnt; int tmp=0;
21         while (tmp!=x) {
22             tmp = stack[stop--];
23             belong[tmp] = cnt;
24             instack[tmp] = 0;
25         }
26     }
27 }

```

5.5 2-SAT

```
1 #define N number_of_vertex
2 #define M number_of_edges
3
4 struct MergePoint {
5     struct EDGE {
6         int adj, next;
7     } edge[M];
8     int ex[M], ey[M];
9     bool instack[N];
10    int gh[N], top, dfn[N], low[N], cnt, ind, stop, stack[N], belong[N];
11    void init() {
12        cnt = ind = stop = top = 0;
13        memset(dfn, 0, sizeof(dfn));
14        memset(instack, 0, sizeof(instack));
15        memset(gh, 0, sizeof(gh));
16    }
17    void addedge(int x, int y) { //reverse
18        std::swap(x, y);
19        edge[++top].adj = y;
20        edge[top].next = gh[x];
21        gh[x] = top;
22        ex[top] = x;
23        ey[top] = y;
24    }
25    void tarjan(int x) {
26        dfn[x] = low[x] = ++ind;
27        instack[x] = 1; stack[++stop] = x;
28        for (int p=gh[x]; p; p=edge[p].next)
29            if (!dfn[edge[p].adj]) {
30                tarjan(edge[p].adj);
31                low[x] = std::min(low[x], low[edge[p].adj]);
32            } else if (instack[edge[p].adj]) {
33                low[x] = std::min(low[x], dfn[edge[p].adj]);
34            }
35        if (dfn[x] == low[x]) {
36            ++cnt; int tmp = 0;
37            while (tmp!=x) {
38                tmp = stack[stop--];
39                belong[tmp] = cnt;
40                instack[tmp] = 0;
41            }
42        }
43    }
44    void work() {
45        for (int i = (__first__); i <= (__last__); ++i)
46            if (!dfn[i])
47                tarjan(i);
48    }
```

```

49 } merge;
50
51 struct Topsort {
52     struct EDGE {
53         int adj, next;
54     } edge[M];
55     int n, top, gh[N], ops[N], deg[N], ans[N];
56     std::queue<int> q;
57     void init() {
58         n = merge.cnt; top = 0;
59         memset(gh, 0, sizeof(gh));
60         memset(deg, 0, sizeof(deg));
61     }
62     void addedge(int x, int y) {
63         if (x == y) return;
64         edge[++top].adj = y;
65         edge[top].next = gh[x];
66         gh[x] = top;
67         ++deg[y];
68     }
69     void work() {
70         for (int i = 1; i <= n; ++i)
71             if (!deg[i])
72                 q.push(i);
73         while (!q.empty()) {
74             int x = q.front();
75             q.pop();
76             for (int p = gh[x]; p; p = edge[p].next)
77                 if (--deg[edge[p].adj])
78                     q.push(edge[p].adj);
79             if (ans[x]) continue;
80             ans[x] = -1; //not selected
81             ans[ops[x]] = 1; //selected
82         }
83     }
84 } ts;

```

调用示例:

```

1  merge.init();
2  merge.addedge();
3  merge.work();
4  for (int i = 1; i <= n; ++i) {
5      if (merge.belong[U(i, 0)] == merge.belong[U(i, 1)]) {
6          puts("NO");
7          return 0;
8      }
9      ts.ops[merge.belong[U(i, 0)]] = merge.belong[U(i, 1)];
10     ts.ops[merge.belong[U(i, 1)]] = merge.belong[U(i, 0)];
11 }
12 ts.init();

```

```

13     ts.work();
14     puts("YES");
15     for (int i = 1; i <= n; ++i) {
16         int x = U(i, 0), y = U(i, 1);
17         x = merge.belong[x], y = merge.belong[y];
18         x = ts.ans[x], y = ts.ans[y];
19         if (x == 1) puts("0_is_selected");
20         if (y == 1) puts("1_is_selected");
21     }

```

5.6 统治者树 (Dominator Tree)

Dominator Tree 可以解决判断一类有向图必经点的问题。

$idom[x]$ 表示离 x 最近的必经点 (重编号后)。将 $idom[x]$ 作为 x 的父亲, 构成一棵 Dominator Tree

接口:

`void dominator::init(int n);` 初始化, 有向图节点数为 n

`void dominator::addedge(int u, int v);` 添加一条有向边 (u, v)

`void dominator::work(int root);` 以 $root$ 为根, 建立一棵 Dominator Tree

结果的返回:

在执行 `dominator::work(int root);` 后, 树边保存在 `vector<int> tree[N]` 中

```

1 namespace dominator {
2     vector<int> g[N], rg[N], bucket[N], tree[N];
3     int n, ind, idom[N], sdom[N], dfn[N], dsu[N], father[N], label[N], rev[N];
4     void dfs(int x) {
5         ++ind;
6         dfn[x] = ind; rev[ind] = x;
7         label[ind] = dsu[ind] = sdom[ind] = ind;
8         for (auto p : g[x]) {
9             if (!dfn[p]) dfs(p), father[dfn[p]] = dfn[x];
10            rg[dfn[p]].push_back(dfn[x]);
11        }
12    }
13    void init(int n1) {
14        n = n1; ind = 0;
15        for (int i = 1; i <= n; ++i) {
16            g[i].clear();
17            rg[i].clear();
18            bucket[i].clear();
19            tree[i].clear();
20            dfn[i] = 0;
21        }
22    }
23    void addedge(int u, int v) {
24        g[u].push_back(v);
25    }
26    int find(int x, int step=0) {

```

```

27     if (dsu[x] == x) return step ? -1 : x;
28     int y = find(dsu[x], 1);
29     if (y < 0) return x;
30     if (sdom[label[dsu[x]]] < sdom[label[x]])
31         label[x] = label[dsu[x]];
32     dsu[x] = y;
33     return step ? dsu[x] : label[x];
34 }
35 void work(int root) {
36     dfs(root); n = ind;
37     for (int i = n; i; --i) {
38         for (auto p : rg[i])
39             sdom[i] = min(sdom[i], sdom[find(p)]);
40         if (i > 1) bucket[sdom[i]].push_back(i);
41         for (auto p : bucket[i]) {
42             int u = find(p);
43             if (sdom[p] == sdom[u]) idom[p] = sdom[p];
44             else idom[p] = u;
45         }
46         if (i > 1) dsu[i] = father[i];
47     }
48     for (int i = 2; i <= n; ++i) {
49         if (idom[i] != sdom[i])
50             idom[i] = idom[idom[i]];
51         tree[rev[i]].push_back(rev[idom[i]]);
52         tree[rev[idom[i]]].push_back(rev[i]);
53     }
54 }
55 };

```

5.7 网络流

5.7.1 最大流

注意: *top* 要初始化为 1

```

1 struct EDGE { int adj, w, next; } edge[M];
2 int n, top, gh[N], nrl[N];
3 void addedge(int x, int y, int w) {
4     edge[++top].adj = y;
5     edge[top].w = w;
6     edge[top].next = gh[x];
7     gh[x] = top;
8     edge[++top].adj = x;
9     edge[top].w = 0;
10    edge[top].next = gh[y];
11    gh[y] = top;
12 }
13 int dist[N], q[N];
14 int bfs() {

```

```

15     memset(dist, 0, sizeof(dist));
16     q[1] = S; int head = 0, tail = 1; dist[S] = 1;
17     while (head != tail) {
18         int x = q[++head];
19         for (int p=gh[x]; p; p=edge[p].next)
20             if (edge[p].w && !dist[edge[p].adj]) {
21                 dist[edge[p].adj] = dist[x] + 1;
22                 q[++tail] = edge[p].adj;
23             }
24     }
25     return dist[T];
26 }
27 int dinic(int x, int delta) {
28     if (x==T) return delta;
29     for (int& p=nrl[x]; p && delta; p=edge[p].next)
30         if (edge[p].w && dist[x]+1 == dist[edge[p].adj]) {
31             int dd = dinic(edge[p].adj, min(delta, edge[p].w));
32             if (!dd) continue;
33             edge[p].w -= dd;
34             edge[p^1].w += dd;
35             return dd;
36         }
37     return 0;
38 }
39 int work() {
40     int ans = 0;
41     while (bfs()) {
42         memcpy(nrl, gh, sizeof(gh));
43         int t; while (t = dinic(S, inf)) ans += t;
44     }
45     return ans;
46 }

```

5.7.2 上下界有源汇网络流

T 向 S 连容量为正无穷的边，将有源汇转化为无源汇。

每条边容量减去下界，设 $in[i]$ 表示流入 i 的下界之和减去流出 i 的下界之和。

新建超级源汇 SS, TT ，对于 $in[i] > 0$ 的点， SS 向 i 连容量为 $in[i]$ 的边。对于 $in[i] < 0$ 的点， i 向 TT 连容量为 $-in[i]$ 的边。

求出以 SS, TT 为源汇的最大流，如果等于 $\sum in[i] (in[i] > 0)$ ，则存在可行流。再求出 S, T 为源汇的最大流即为最大流。

费用流：建完图后等价于求以 SS, TT 为源汇的费用流。

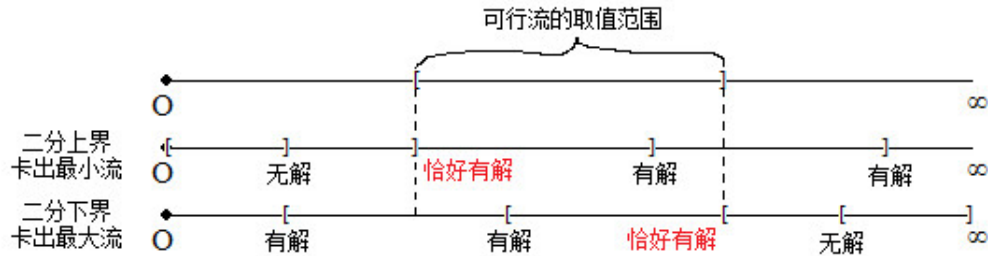
5.7.3 上下界无源汇网络流

1. 怎样求无源汇有上下界网络的可行流？

由于有源汇的网络我们先要转化成无源汇，所以本来就无源汇的网络不用再作特殊处理。

2. 怎样求无源汇有上下界网络的最大流、最小流？

一种简易的方法是采用二分思想，不断通过可行流的存在与否对 (t, s) 边的上下界 U, L 进行调整。求最大流时令 $U = \infty$ 并二分 L ；求最小流时令 $L = 0$ 并二分 U 。道理很简单，因为可行流的取值范围是一段连续的区间，我们只要通过二分找到有解和无解的分界线即可。



5.7.4 费用流

注意：top 要初始化为 1

```

1 #define inf 0x3f3f3f3f
2 struct NetWorkFlow {
3     struct EDGE {
4         int adj, w, cost, next;
5     } edge[M*2];
6     int gh[N], q[N], dist[N], v[N], pre[N], prev[N], top;
7     int S, T;
8     void addedge(int x, int y, int w, int cost) {
9         edge[++top].adj = y;
10        edge[top].w = w;
11        edge[top].cost = cost;
12        edge[top].next = gh[x];
13        gh[x] = top;
14        edge[++top].adj = x;
15        edge[top].w = 0;
16        edge[top].cost = -cost;
17        edge[top].next = gh[y];
18        gh[y] = top;
19    }
20    void clear() {
21        top = 1;
22        memset(gh, 0, sizeof(gh));
23    }
24    int spfa() {
25        memset(dist, 63, sizeof(dist));
26        memset(v, 0, sizeof(v));
27        int head = 0, tail = 1;
28        q[1] = S; v[S] = 1; dist[S] = 0;
29        while (head != tail) {
30            (head += 1) %= N;
31            int x = q[head];

```

```

32     v[x] = 0;
33     for (int p=gh[x]; p; p=edge[p].next)
34         if (edge[p].w && dist[x] + edge[p].cost < dist[edge[p].adj]) {
35             dist[edge[p].adj] = dist[x] + edge[p].cost;
36             pre[edge[p].adj] = x;
37             prev[edge[p].adj] = p;
38             if (!v[edge[p].adj]) {
39                 v[edge[p].adj] = 1;
40                 (tail += 1) %= N;
41                 q[tail] = edge[p].adj;
42             }
43         }
44     }
45     return dist[T] != inf;
46 }
47 int work() {
48     int ans = 0;
49     while (spfa()) {
50         int mx = inf;
51         for (int x=T; x!=S; x=pre[x])
52             mx = min(edge[prev[x]].w, mx);
53         ans += dist[T] * mx;
54         for (int x=T; x!=S; x=pre[x]) {
55             edge[prev[x]].w -= mx;
56             edge[prev[x]^1].w += mx;
57         }
58     }
59     return ans;
60 }
61 } nwf;

```

5.7.5 zkw 费用流

注意: *top* 要初始化为 1, 不得用于有负权的图

```

1 #define inf 0x3f3f3f3f
2 struct NetWorkFlow {
3     struct EDGE {
4         int adj, w, cost, next;
5     } edge[M*2];
6     int gh[N], top;
7     int S, T;
8     void addedge(int x, int y, int w, int cost) {
9         edge[++top].adj = y;
10        edge[top].w = w;
11        edge[top].cost = cost;
12        edge[top].next = gh[x];
13        gh[x] = top;
14        edge[++top].adj = x;
15        edge[top].w = 0;

```

```

16     edge[top].cost = -cost;
17     edge[top].next = gh[y];
18     gh[y] = top;
19 }
20 void clear() {
21     top = 1;
22     memset(gh, 0, sizeof(gh));
23 }
24 int cost, d[N], slk[N], v[N];
25 int aug(int x, int f) {
26     int left = f;
27     if (x == T) {
28         cost += f * d[S];
29         return f;
30     }
31     v[x] = true;
32     for (int p=gh[x]; p; p=edge[p].next)
33         if (edge[p].w && !v[edge[p].adj]) {
34             int t = d[edge[p].adj] + edge[p].cost - d[x];
35             if (t == 0) {
36                 int delt = aug(edge[p].adj, min(left, edge[p].w));
37                 if (delt > 0) {
38                     edge[p].w -= delt;
39                     edge[p^1].w += delt;
40                     left -= delt;
41                 }
42                 if (left == 0) return f;
43             } else {
44                 if (t < slk[edge[p].adj])
45                     slk[edge[p].adj] = t;
46             }
47         }
48     return f-left;
49 }
50 bool modlabel() {
51     int delt = inf;
52     for (int i=1; i<=T; i++)
53         if (!v[i]) {
54             if (slk[i] < delt) delt = slk[i];
55             slk[i] = inf;
56         }
57     if (delt == inf) return true;
58     for (int i=1; i<=T; i++)
59         if (v[i]) d[i] += delt;
60     return false;
61 }
62 int work() {
63     cost = 0;
64     memset(d, 0, sizeof(d));
65     memset(slk, 63, sizeof(slk));

```

```

66         do {
67             do {
68                 memset(v, 0, sizeof(v));
69             } while (aug(S, inf));
70         } while (!modlabel());
71         return cost;
72     }
73 } nwf;

```

6 数学

6.1 扩展欧几里得解同余方程

ans[] 保存的是循环节内所有的解

```

1 int exgcd(int a,int b,int&x,int&y){
2     if(!b) return x=1,y=0,a;
3     int d=exgcd(b,a%b,x,y),t=x;
4     return x=y,y=t-a/b*y,d;
5 }
6 void cal(ll a,ll b,ll n){ //ax=b(mod n)
7     ll x,y,d=exgcd(a,n,x,y);
8     if(b%d) return;
9     x=(x%n+n)%n;
10    ans[cnt=1]=x*(b/d)%(n/d);
11    for(ll i=1;i<d;i++) ans[++cnt]=(ans[1]+i*n/d)%n;
12 }

```

6.2 同余方程组

```

1 int n,flag,k,m,a,r,d,x,y;
2 int main(){
3     scanf("%d",&n);
4     flag=k=1,m=0;
5     while(n--){
6         scanf("%d%d",&a,&r); //ans%a=r
7         if(flag){
8             d=exgcd(k,a,x,y);
9             if((r-m)%d){ flag=0;continue; }
10            x=(x*(r-m)/d+a/d)%(a/d),y=k/d*a,m=((x*k+m)%y)%y;
11            if(m<0)m+=y;
12            k=y;
13        }
14    }
15    printf("%d",flag?m:-1); //若flag=1,说明有解,解为ki+m,i为任意整数
16 }

```

6.3 卡特兰数

$$h_1 = 1, h_n = \frac{h_{n-1}(4n-2)}{n+1} = \frac{C(2n,n)}{n+1} = C(2n,n) - C(2n,n-1)$$

在一个格点阵列中, 从 $(0,0)$ 点走到 (n,m) 点且不经过对角线 $x=y$ 的方案数 ($x > y$) :

$$C(n+m-1,m) - C(n+m-1,m-1)$$

在一个格点阵列中, 从 $(0,0)$ 点走到 (n,m) 点且不穿过对角线 $x=y$ 的方案数 ($x \geq y$) :

$$C(n+m,m) - C(n+m,m-1)$$

6.4 斯特林数

6.4.1 第一类斯特林数

第一类 Stirling 数 $S(p,k)$ 的一个组合学解释是: 将 p 个物体排成 k 个非空循环排列的方法数。

$S(p,k)$ 的递推公式: $S(p,k) = (p-1)S(p-1,k) + S(p-1,k-1), 1 \leq k \leq p-1$

边界条件: $S(p,0) = 0, p \geq 1$ $S(p,p) = 1, p \geq 0$

6.4.2 第二类斯特林数

第二类 Stirling 数 $S(p,k)$ 的一个组合学解释是: 将 p 个物体划分成 k 个非空的不可辨别 (可以理解为盒子没有编号) 集合的方法数。

$S(p,k)$ 的递推公式: $S(p,k) = kS(p-1,k) + S(p-1,k-1), 1 \leq k \leq p-1$

边界条件: $S(p,0) = 0, p \geq 1$ $S(p,p) = 1, p \geq 0$

也有卷积形式:

$$S(n,m) = \frac{1}{m!} \sum_{k=0}^m (-1)^k C(m,k) (m-k)^n = \sum_{k=0}^m \frac{(-1)^k (m-k)^n}{k!(m-k)!} = \sum_{k=0}^m \frac{(-1)^k}{k!} \times \frac{(m-k)^n}{(m-k)!}$$

6.5 错排公式

$$D_1 = 0, D_2 = 1, D_n = (n-1)(D_{n-2} + D_{n-1})$$

6.6 Lucas 定理

接口:

初始化: `void lucas::init();`

计算 $C(n,m) \% mod$ 的值: `LL lucas::Lucas(LL n, LL m);`

```
1 #define mod 110119
2 #define LL long long
3 namespace lucas {
4     LL fac[mod+1], facv[mod+1];
5     LL power(LL base, LL times) {
6         LL ans = 1;
7         while (times) {
8             if (times&1) (ans *= base) %= mod;
9             (base *= base) %= mod;
10            times >>= 1;
11        }
12    }
```

```

11     }
12     return ans;
13 }
14 void init() {
15     fac[0] = 1; for (int i=1;i<mod;i++) fac[i] = (fac[i-1] * i) % mod;
16     facv[mod-1] = power(fac[mod-1], mod-2);
17     for (int i=mod-2;i>=0;--i) facv[i] = (facv[i+1] * (i+1)) % mod;
18 }
19 LL C(unsigned LL n, unsigned LL m) {
20     if (n < m) return 0;
21     return (fac[n] * facv[m] % mod * facv[n-m] % mod) % mod;
22 }
23 LL Lucas(unsigned LL n, unsigned LL m)
24 {
25     if (m == 0) return 1;
26     return (C(n%mod, m%mod) * Lucas(n/mod, m/mod)) % mod;
27 }
28 };

```

6.7 高斯消元

6.7.1 行列式

```

1 int ans = 1;
2 for (int i=0;i<n;i++) {
3     for (int j=i;j<n;j++)
4         if (g[j][i]) {
5             for (int k=i;k<n;k++)
6                 swap(g[i][k], g[j][k]);
7             if (j != i) ans *= -1;
8             break;
9         }
10    if (g[i][i] == 0) {
11        ans = 0;
12        break;
13    }
14    for (int j=i+1;j<n;j++) {
15        while (g[j][i]) {
16            int t = g[i][i] / g[j][i];
17            for (int k=i;k<n;k++)
18                g[i][k] = (g[i][k] + mod - ((LL)t * g[j][k] % mod)) % mod;
19            for (int k=i;k<n;k++)
20                swap(g[i][k], g[j][k]);
21            ans *= -1;
22        }
23    }
24 }
25 for (int i=0;i<n;i++)
26     ans = ((LL)ans * g[i][i]) % mod;

```

```

27 ans = (ans % mod + mod) % mod;
28 printf("%d\n", ans);

```

6.7.2 Matrix-Tree 定理

对于一张图，建立矩阵 C ， $C[i][i]$ = i 的度数，若 i, j 之间有边，那么 $C[i][j] = -1$ ，否则为 0。这张图的生成树个数等于矩阵 C 的 $n - 1$ 阶行列式的值。

6.8 调和级数

$\sum_{i=1}^n \frac{1}{i}$ 在 n 较大时约等于 $\ln(n) + r$ ， r 为欧拉常数，约等于 0.5772156649015328。

6.9 曼哈顿距离的变换

$$|x_1 - x_2| + |y_1 - y_2| = \max(|(x_1 + y_1) - (x_2 + y_2)|, |(x_1 - y_1) - (x_2 - y_2)|)$$

6.10 线性筛素数

```

1 mu[1]=phi[1]=1;top=0;
2 for (int i=2;i<N;i++) {
3     if (!v[i]) prime[++top]=i, mu[i] = -1, phi[i] = i-1;
4     for (int j=1;i*prime[j]<N && j<=top;j++) {
5         v[i*prime[j]] = 1;
6         if (i%prime[j]) {
7             mu[i*prime[j]] = -mu[i];
8             phi[i*prime[j]] = phi[i] * (prime[j]-1);
9         } else {
10            mu[i*prime[j]] = 0;
11            phi[i*prime[j]] = phi[i] * prime[j];
12            break;
13        }
14    }
15 }

```

6.11 杜教筛

$\text{getphi}(t, x)$ 表示求 $\sum_{i=1}^x i^t \phi(i)$ 。

推导过程：

记 $S(n) = \sum_{i=1}^n f(i)$ ，取任意函数 g 有恒等式

$$S(n) = \sum_{i=1}^n (f \cdot g)(i) - \sum_{i=2}^n g(i) S\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$$

其中， $(f \cdot g)$ 表示 f 和 g 的狄利克雷卷积：即： $(f \cdot g)(n) = \sum_{d|n} f(d)g\left(\frac{n}{d}\right)$

关于恒等式的证明：

将 $\sum_{i=2}^n g(i)S(\lfloor \frac{n}{i} \rfloor)$ 移到左边去，即只需证

$$\sum_{i=1}^n (f \cdot g)(i) = \sum_{i=1}^n g(i)S(\lfloor \frac{n}{i} \rfloor)$$

将狄利克雷卷积展开，得：

$$\sum_{i=1}^n \sum_{d|i} g(d)f(\frac{i}{d}) = \sum_{i=1}^n g(i)S(\lfloor \frac{n}{i} \rfloor)$$

即：

$$\sum_{d=1}^n g(d) \sum_{i=1}^{\lfloor \frac{n}{d} \rfloor} f(i) = \sum_{i=1}^n g(i)S(\lfloor \frac{n}{i} \rfloor)$$

显然相等，恒等式证完。

取 $f(i) = i^p \phi(i), g(i) = i^p$ ，则有：

$$S(n) = \sum_{i=1}^n i^p \phi(i) = \sum_{i=1}^n i^{p+1} - \sum_{i=2}^n i^p S(\lfloor \frac{n}{i} \rfloor)$$

其中有用到等式 $\sum_{d|n} \phi(d) = n$

```

1 #include <bits/stdc++.h>
2
3 #define N 5000020
4 #define LL long long
5 #define mod 1000000007
6 #define div2 ((mod+1)/2)
7 #define div6 ((mod+1)/6)
8
9 using namespace std;
10
11 int n, prime[N], v[N];
12 LL phi[3][N];
13
14 map<int, int> mp[3];
15
16 int sum(int t, int x) { //calculate 1^t + 2^t + ... + x^t
17     if (t == 0) return x;
18     if (t == 1) return 1ll * x * (x + 1) % mod * div2 % mod;
19     if (t == 2) return 1ll * x * (x + 1) % mod * (21ll * x % mod + 1) % mod * div6 %
        mod;
20     if (t == 3) return 1ll * x * x % mod * (x + 1) % mod * (x + 1) % mod * div2 %
        mod * div2 % mod;
21 }
22
23 int getphi(int t, int x) {
24     if (x < N) return phi[t][x];
25     if (mp[t].find(x) != mp[t].end()) return mp[t][x];

```



```

26     LL ans = 0; int r = 0;
27     for (int l = 2; l <= x; l = r + 1) {
28         r = x / (x / l);
29         ans += 1ll * getphi(t, x / l) * (((LL)sum(t, r) - sum(t, l - 1) + mod) % mod
30             ) % mod;
31         ans %= mod;
32     }
33     ans = (LL)sum(t + 1, x) - ans + mod;
34     ans %= mod;
35     mp[t][x] = ans;
36     return (int)ans;
37 }
38
39 int main() {
40     memset(v, 0, sizeof(v));
41     int top = 0;
42     phi[0][1] = 1, phi[1][1] = 1, phi[2][1] = 1;
43     for (int i = 2; i < N; ++i) {
44         if (!v[i]) prime[++top] = i, phi[0][i] = i - 1, phi[1][i] = 1ll * i * phi
45             [0][i] % mod, phi[2][i] = 1ll * i * phi[1][i] % mod;
46         for (int j = 1; j <= top && prime[j] * i < N; ++j) {
47             v[i * prime[j]] = 1;
48             if (i % prime[j] == 0) {
49                 phi[0][i * prime[j]] = phi[0][i] * prime[j];
50                 phi[1][i * prime[j]] = 1ll * phi[1][i] * prime[j] % mod * prime[j] %
51                     mod;
52                 phi[2][i * prime[j]] = 1ll * phi[2][i] * prime[j] % mod * prime[j] %
53                     mod * prime[j] % mod;
54                 break;
55             } else {
56                 phi[0][i * prime[j]] = phi[0][i] * (prime[j] - 1);
57                 phi[1][i * prime[j]] = 1ll * phi[1][i] * (prime[j] - 1) % mod *
58                     prime[j] % mod;
59                 phi[2][i * prime[j]] = 1ll * phi[2][i] * (prime[j] - 1) % mod *
60                     prime[j] % mod * prime[j] % mod;
61             }
62         }
63     }

```

6.12 FFT

```

1 typedef complex<double> comp;
2 namespace FFT {

```

```

3   comp A[N], B[N], omega[N];
4   void transform(comp *x, int len) {
5       for (int i=1, j=len/2; i<len-1; i++) {
6           if (i<j) swap(x[i], x[j]);
7           int k = len/2;
8           while (j>=k) {
9               j-=k;
10              k/=2;
11          }
12          if (j<k) j+=k;
13      }
14  }
15  void fft(comp *x, int len, int reverse) {
16      transform(x, len);
17      for (int h=2; h<=len; h<=1) {
18          for (int i=0; i<h/2; i++) omega[i] = polar(1.0, 2*pi*reverse/h*i);
19          for (int i=0; i<len; i+=h) {
20              for (int j=i; j<i+h/2; j++) {
21                  comp w = omega[j-i];
22                  comp u = x[j];
23                  comp v = (w * x[j+h/2]);
24                  x[j] = u + v;
25                  x[j+h/2] = u - v;
26              }
27          }
28      }
29      if (reverse == -1) {
30          for (int i=0; i<len; i++)
31              x[i] /= len;
32      }
33  }
34  void work(int n, int *a, int *b) {
35      int len = 1;
36      while (len <= n*2) len *= 2;
37      for (int i=0; i<len; i++) A[i] = B[i] = 0;
38      for (int i=0; i<n; i++) A[i] = a[i], B[i] = b[i];
39      fft(A, len, 1); fft(B, len, 1);
40      for (int i=0; i<len; i++) A[i] = A[i] * B[i];
41      fft(A, len, -1);
42      for (int i=0; i<len; i++) {
43          LL r = round(A[i].real());
44          a[i] = r % mod;
45      }
46  }
47  };

```

6.13 求原根

接口: LL p_root(LL p);

输入：一个素数 p

输出： p 的原根

```
1 #include <bits/stdc++.h>
2 #define LL long long
3
4 using namespace std;
5
6 vector <LL> a;
7
8 LL pow_mod(LL base, LL times, LL mod) {
9     LL ret = 1;
10    while (times) {
11        if (times&1) ret = ret * base % mod;
12        base = base * base % mod;
13        times>>=1;
14    }
15    return ret;
16 }
17
18 bool g_test(LL g, LL p) {
19     for (LL i = 0; i < a.size(); ++i)
20         if (pow_mod(g, (p-1)/a[i], p) == 1) return 0;
21     return 1;
22 }
23
24 LL p_root(LL p) {
25     LL tmp = p - 1;
26     for (LL i = 2; i <= tmp / i; ++i)
27         if (tmp % i == 0) {
28             a.push_back(i);
29             while (tmp % i == 0)
30                 tmp /= i;
31         }
32     if (tmp != 1) a.push_back(tmp);
33     LL g = 1;
34     while (1) {
35         if (g_test(g, p)) return g;
36         ++g;
37     }
38 }
39
40 int main() {
41     LL p;
42     cin >> p;
43     cout << p_root(p) << endl;
44 }
```

6.14 NTT

998244353 原根为 3 , 1004535809 原根为 3 , 786433 原根为 10 , 880803841 原根为 26 。

```
1  #define mod 998244353
2  #define g 3
3  LL wi[N], wiv[N];
4  LL power(LL base, LL times) {
5      LL ans = 1;
6      while (times) {
7          if (times&1) (ans *= base) %= mod;
8          (base *= base) %= mod;
9          times >>= 1;
10     }
11     return ans;
12 }
13 void transform(LL *x, int len) {
14     for (int i=1, j=len/2; i<len-1; i++) {
15         if (i<j) swap(x[i], x[j]);
16         int k = len/2;
17         while (j>=k) {
18             j-=k;
19             k/=2;
20         }
21         if (j<k) j+=k;
22     }
23 }
24 void NTT(LL *x, int len, int reverse) {
25     transform(x, len);
26     for (int h=2; h<=len; h<=1) {
27         for (int i=0; i<len; i+=h) {
28             LL w = 1, wn;
29             if (reverse==1) wn = wi[h]; else wn = wiv[h];
30             for (int j=i; j<i+h/2; j++) {
31                 LL u = x[j];
32                 LL v = (w * x[j+h/2]) % mod;
33                 x[j] = (u + v) % mod;
34                 x[j+h/2] = (u - v + mod) % mod;
35                 (w *= wn) %= mod;
36             }
37         }
38     }
39     if (reverse == -1) {
40         LL t = power(len, mod-2);
41         for (int i=0; i<len; i++)
42             (x[i] *= t) %= mod;
43     }
44 }
45 LL A[N], B[N];
46 int main() {
47     for (int i=1; i<N; i*=2) {
```

```

48     wi[i] = power(g, (mod-1)/i);
49     wiv[i] = power(wi[i], mod-2);
50 }
51 memset(A, 0, sizeof(A));
52 memset(B, 0, sizeof(B));
53 NTT(A, len, 1); NTT(B, len, 1);
54 for (int i=0;i<len;i++) (A[i] *= B[i]) %= mod;
55 NTT(A, len, -1);
56 }

```

6.15 组合数 lcm

$$(n+1)lcm(C(n,0), C(n,1), \dots, C(n,k)) = lcm(n+1, n, n-1, \dots, n-k+1)$$

6.16 区间 lcm 的维护

对于一个数，将其分解质因数，若有因子 p^k ，那么拆分成 k 个数 p, p^2, \dots, p^k ，权值都为 p ，那么查询区间 $[l, r]$ 内所有数的 lcm 的答案 = 所有在该区间中出现过的数的权值之积，可持久化线段树维护即可。

7 几何

7.1 凸包

```

1  typedef complex<int> point;
2  #define X real()
3  #define Y imag()
4  int n;
5  long long cross(point a, point b) {
6      return 1ll * a.X * b.Y - 1ll * a.Y * b.X;
7  }
8  bool cmp(point a, point b) {
9      return make_pair(a.X, a.Y) < make_pair(b.X, b.Y);
10 }
11 int convexHull(point p[], int n, point ch[]) {
12     sort(p, p + n, cmp);
13     int m = 0;
14     for(int i = 0; i < n; ++i) {
15         while(m > 1 && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
16         ch[m++] = p[i];
17     }
18     int k = m;
19     for(int i = n - 2; i >= 0; --i) {
20         while(m > k && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
21         ch[m++] = p[i];
22     }
23     if(n > 1) m--;
24     return m;

```

8 黑科技和杂项

8.1 找规律

有些题目，只给一个正整数 n ，然后要求输出一个答案。这时，我们可以暴力得到小数据的解，用高斯消元得到递推式，然后用矩阵快速幂求解。

使用方法：

首先在 gauss.in 中输入小数据的解 ($n = 1$ 时, $n = 2$ 时, \dots)，以 EOF 结束。

依次运行 gauss.cpp, matrix.cpp，得到 matrix.out

将 matrix.out 中的文件粘贴在 main.cpp 中相应的位置中。注意模数一定要是质数。

```

1 //gauss.cpp
2 #include <bits/stdc++.h>
3 #define N 102
4 #define mod 1000000007
5 //caution: you can use this program iff mod is a prime.
6
7 using namespace std;
8
9 int n, m, k, a[N], g[N][N];
10
11 int power(int base, int times) {
12     int ret = 1;
13     while (times) {
14         if (times & 1) ret = 1ll * ret * base % mod;
15         base = 1ll * base * base % mod;
16         times >>= 1;
17     }
18     return ret;
19 }
20
21 int test() {
22     for (int i=0; i<m; i++) {
23         for (int j=i; j<=m; j++)
24             if (g[j][i]) {
25                 for (int k=i; k<=m; k++)
26                     swap(g[i][k], g[j][k]);
27                 break;
28             }
29     if (g[i][i] == 0)
30         return 0;
31     for (int j=i+1; j<n; j++) {
32         while (g[j][i]) {
33             int t = 1ll * g[i][i] * power(g[j][i], mod - 2) % mod;
34             for (int k=i; k<n; k++)
35                 g[i][k] = (g[i][k] + mod - (1ll * t * g[j][k] % mod)) % mod;

```

```

36         for (int k=i;k<=m;k++)
37             swap(g[i][k], g[j][k]);
38     }
39 }
40 int t = power(g[i][i], mod - 2);
41 for (int j = 0; j <= m; ++j)
42     g[i][j] = 1ll * g[i][j] * t % mod;
43 }
44 for (int i = m; i < n; ++i)
45     if (g[i][m]) return 0;
46 for (int i = m - 1; i >= 0; --i) {
47     int t = power(g[i][i], mod - 2);
48     g[i][i] = 1;
49     g[i][m] = 1ll * g[i][m] * t % mod;
50     for (int j = 0; j < i; ++j)
51         g[j][m] = (g[j][m] + mod - 1ll * g[i][m] * g[j][i] % mod) % mod;
52 }
53 printf("%d\n", m);
54 for (int i = 0; i < m; ++i)
55     printf("%d_", g[i][m]);
56 puts("");
57 for (int i = 0; i < m - 1; ++i)
58     printf("%d_", a[i]);
59 puts("1");
60 return 1;
61 }
62
63 int main() {
64     freopen("gauss.in", "r", stdin);
65     freopen("gauss.out", "w", stdout);
66     k = 0;
67     while (~scanf("%d", &a[k++])) ;
68     for (int sm = 1; sm <= k - sm; ++sm) {
69         n = k - sm - 1;
70         m = sm + 1;
71         for (int i = 0; i < n; ++i) {
72             for (int j = 0; j <= sm; ++j)
73                 g[i][j] = a[i + j];
74             g[i][m] = 1;
75             swap(g[i][m - 1], g[i][m]);
76         }
77         if (test()) return 0;
78     }
79     puts("no_solution");
80     return 0;
81 }

```

```

1 //matrix.cpp
2 #include <bits/stdc++.h>
3 #define N 102

```

```

4 using namespace std;
5
6 int n, a[N];
7
8 int main() {
9     freopen("gauss.out", "r", stdin);
10    freopen("matrix.out", "w", stdout);
11    scanf("%d", &n);
12    for (int i = 0; i < n; ++i) scanf("%d", &a[i]);
13    printf("#define_M_%d\n", n);
14    printf("const_int_trans[M][M]=_\n");
15    for (int i = 0; i < n; ++i) {
16        printf("\t{");
17        for (int j = 0; j < n; ++j) {
18            int t;
19            if (j < n - 2) t = i == j + 1;
20            else if (j == n - 2) t = a[i];
21            else t = i == n - 1;
22            printf("%s%d", j == 0 ? "" : ",_", t);
23        }
24        printf("}%s\n", i == n - 1 ? "" : ",");
25    }
26    printf("};\n");
27    printf("const_int_pref[M]=_\n");
28    for (int i = 0; i < n; ++i) {
29        int x;
30        scanf("%d", &x);
31        printf("%d%s", x, i == n - 1 ? "};\n" : ",_");
32    }
33    return 0;
34 }

```

```

1 //main.cpp
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 /* paste matrix.out here. */
6
7 #define mod 1000000007
8
9 struct Matrix {
10     int c[M][M];
11     void clear() { memset(c, 0, sizeof(c)); }
12     void identity() { clear(); for (int i = 0; i < M; ++i) c[i][i] = 1; }
13     void base() { memcpy(c, trans, sizeof(trans)); }
14     friend Matrix operator * (const Matrix &a, const Matrix &b) {
15         Matrix c; c.clear();
16         for (int i = 0; i < M; ++i)
17             for (int j = 0; j < M; ++j)
18                 for (int k = 0; k < M; ++k)

```



```

19         c.c[i][j] = (c.c[i][j] + 111 * a.c[i][k] * b.c[k][j] % mod) %
                mod;
20     return c;
21 }
22 } start, base;
23
24 Matrix power(Matrix base, int times) {
25     Matrix ret; ret.identity();
26     while (times) {
27         if (times & 1) ret = ret * base;
28         base = base * base;
29         times >>= 1;
30     }
31     return ret;
32 }
33
34 int main() {
35     int tot;
36     scanf("%d", &tot);
37     while (tot--) {
38         int n;
39         scanf("%d", &n);
40         start.clear();
41         for (int i = 0; i < M; ++i) start.c[0][i] = pref[i];
42         base.base();
43         base = power(base, n - 1);
44         start = start * base;
45         printf("%d\n", start.c[0][0]);
46     }
47     return 0;
48 }

```

8.2 高精度计算

```

1  #include<algorithm>
2  using namespace std;
3  const int N_huge=850,base=100000000;
4  char s[N_huge*10];
5  struct huge{
6      typedef long long value;
7      value a[N_huge];int len;
8      void clear(){len=1;a[len]=0;}
9      huge(){clear();}
10     huge(value x){*this=x;}
11     huge operator =(huge b){
12         len=b.len;for (int i=1;i<=len;++i)a[i]=b.a[i]; return *this;
13     }
14     huge operator =(value x){
15         len=0;

```

```

16     while (x) a[++len]=x%base,x/=base;
17     if (!len) a[++len]=0;
18     return *this;
19 }
20 huge operator +(huge b){
21     int L=len>b.len?len:b.len;huge tmp;
22     for (int i=1;i<=L+1;++i) tmp.a[i]=0;
23     for (int i=1;i<=L;++i){
24         if (i>len) tmp.a[i]+=b.a[i];
25         else if (i>b.len) tmp.a[i]+=a[i];
26         else {
27             tmp.a[i]+=a[i]+b.a[i];
28             if (tmp.a[i]>=base){
29                 tmp.a[i]-=base;++tmp.a[i+1];
30             }
31         }
32     }
33     if (tmp.a[L+1]) tmp.len=L+1;
34     else tmp.len=L;
35     return tmp;
36 }
37 huge operator -(huge b){
38     int L=len>b.len?len:b.len;huge tmp;
39     for (int i=1;i<=L+1;++i) tmp.a[i]=0;
40     for (int i=1;i<=L;++i){
41         if (i>b.len) b.a[i]=0;
42         tmp.a[i]+=a[i]-b.a[i];
43         if (tmp.a[i]<0){
44             tmp.a[i]+=base;--tmp.a[i+1];
45         }
46     }
47     while (L>1&&!tmp.a[L]) --L;
48     tmp.len=L;
49     return tmp;
50 }
51 huge operator *(huge b){
52     int L=len+b.len;huge tmp;
53     for (int i=1;i<=L;++i) tmp.a[i]=0;
54     for (int i=1;i<=len;++i)
55         for (int j=1;j<=b.len;++j){
56             tmp.a[i+j-1]+=a[i]*b.a[j];
57             if (tmp.a[i+j-1]>=base){
58                 tmp.a[i+j]+=tmp.a[i+j-1]/base;
59                 tmp.a[i+j-1]%=base;
60             }
61         }
62     tmp.len=len+b.len;
63     while (tmp.len>1&&!tmp.a[tmp.len]) --tmp.len;
64     return tmp;
65 }

```

```

66 pair<huge,huge> divide(huge a,huge b){
67     int L=a.len;huge c,d;
68     for (int i=L;i--;i){
69         c.a[i]=0;d=d*base;d.a[1]=a.a[i];
70         int l=0,r=base-1,mid;
71         while (l<r){
72             mid=(l+r+1)>>1;
73             if (b*mid<=d)l=mid;
74             else r=mid-1;
75         }
76         c.a[i]=1;d-=b*l;
77     }
78     while (L>1&&!c.a[L])--L;c.len=L;
79     return make_pair(c,d);
80 }
81 huge operator / (value x){
82     value d=0;huge tmp;
83     for (int i=len;i--;i){
84         d=d*base+a[i];
85         tmp.a[i]=d/x;d%=x;
86     }
87     tmp.len=len;
88     while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
89     return tmp;
90 }
91 value operator %(value x){
92     value d=0;
93     for (int i=len;i--;i)d=(d*base+a[i])%x;
94     return d;
95 }
96 huge operator / (huge b){return divide(*this,b).first;}
97 huge operator % (huge b){return divide(*this,b).second;}
98 huge &operator += (huge b) {*this=*this+b;return *this;}
99 huge &operator -= (huge b) {*this=*this-b;return *this;}
100 huge &operator *= (huge b) {*this=*this*b;return *this;}
101 huge &operator ++ () {huge T;T=1;*this=*this+T;return *this;}
102 huge &operator -- () {huge T;T=1;*this=*this-T;return *this;}
103 huge operator ++ (int) {huge T,tmp=*this;T=1;*this=*this+T;return tmp;}
104 huge operator -- (int) {huge T,tmp=*this;T=1;*this=*this-T;return tmp;}
105 huge operator + (value x) {huge T;T=x;return *this+T;}
106 huge operator - (value x) {huge T;T=x;return *this-T;}
107 huge operator * (value x) {huge T;T=x;return *this*T;}
108 huge operator *= (value x) {*this=*this*x;return *this;}
109 huge operator += (value x) {*this=*this+x;return *this;}
110 huge operator -= (value x) {*this=*this-x;return *this;}
111 huge operator /= (value x) {*this=*this/x;return *this;}
112 huge operator %=(value x) {*this=*this%x;return *this;}
113 bool operator == (value x) {huge T;T=x;return *this==T;}
114 bool operator != (value x) {huge T;T=x;return *this!=T;}
115 bool operator <= (value x) {huge T;T=x;return *this<=T;}

```

```

116     bool operator >=(value x){huge T;T=x;return *this>=T;}
117     bool operator <(value x){huge T;T=x;return *this<T;}
118     bool operator >(value x){huge T;T=x;return *this>T;}
119     bool operator <(huge b){
120         if (len<b.len)return 1;
121         if (len>b.len)return 0;
122         for (int i=len;i--i){
123             if (a[i]<b.a[i])return 1;
124             if (a[i]>b.a[i])return 0;
125         }
126         return 0;
127     }
128     bool operator ==(huge b){
129         if (len!=b.len)return 0;
130         for (int i=len;i--i){
131             if (a[i]!=b.a[i])return 0;
132         }
133         return 1;
134     }
135     bool operator !=(huge b){return !(*this==b);}
136     bool operator >(huge b){return !(*this<b||*this==b);}
137     bool operator <=(huge b){return (*this<b)||(*this==b);}
138     bool operator >=(huge b){return (*this>b)||(*this==b);}
139     void str(char s[]){
140         int l=strlen(s);value x=0,y=1;len=0;
141         for (int i=l-1;i>=0;--i){
142             x=x+(s[i]-'0')*y;y*=10;
143             if (y==base)a[++len]=x,x=0,y=1;
144         }
145         if (!len||x)a[++len]=x;
146     }
147     void read(){
148         scanf("%s",s);this->str(s);
149     }
150     void print(){
151         printf("%d", (int)a[len]);
152         for (int i=len-1;i--i){
153             for (int j=base/10;j>=10;j/=10){
154                 if (a[i]<j)printf("0");
155                 else break;
156             }
157             printf("%d", (int)a[i]);
158         }
159         printf("\n");
160     }
161     int main(){
162         f[1]=f[2]=1;
163         for(int i=3;i<=1000;i++)f[i]=f[i-1]+f[i-2];
164     }

```