

ICPC World Finals 2019 Templates

SYSU_Balloon

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1 数学

1.1 Miller Rabin

```

1 // 2,7,61 : < 4759123141
2 // 2,3,5,7,11,13,17 : < 341550071728320
3 // 2,3,7,61,24251 : < 10^16 only 46856248255981
4
5 //BZOJ-3667
6 #include<cstdio>
7 #include<cstdlib>
8 typedef long long ll;
9 ll _,n,x,ans,st;
10 ll gcd(ll x,ll y){return y==0?x:gcd(y,x%y);}
11 #define abs(x) (x>0?x:-(x))
12 #define cmax(a,b) (a<b?a=b:1)

```

```

13 ll mul(ll a,ll b,ll p){
14     ll tmp=(a*b-(ll)((long double)a/p*b+1e-7)*p);
15     return tmp<0?tmp+p:tmp;
16 }
17 ll power(ll t,ll k,ll p){
18     ll f=1;
19     for(;k>=1,t=mul(t,t,p))if(k&1)f=mul(f,t,p);
20     return f;
21 }
22 bool check(ll a,int k,ll p,ll q){
23     ll t=power(a,q,p);
24     if(t==1||t==p-1)return 1;
25     for(;k--){
26         t=mul(t,t,p);
27         if(t==p-1)return 1;
28     }
29     return 0;
30 }
31 bool mr(ll p){
32     if(p<=1)return 0;
33     if(p==2)return 1;
34     if(~p&1)return 0;
35     ll q=p-1;int i,k=0;
36     while(~q&1)q>>=1,k++;
37     for(i=0;i<5;i++)
38         if(!check(rand()%(p-1)+1,k,p,q))return 0;
39     return 1;
40 }
41 ll rho(ll n,ll c){
42     ll x=rand()%n,y=x,p=1;
43     while(p==1)
44         x=(mul(x,x,n)+c)%n,
45         y=(mul(y,y,n)+c)%n,
46         y=(mul(y,y,n)+c)%n,
47         p=gcd(n,abs(x-y));
48     return p;
49 }
50 void solve(ll n){
51     if(n==1)return;
52     if(mr(n)){cmax(ans,n);return;}
53     if(~n&1)cmax(ans,2),solve(n>>1);
54     else{
55         ll t=n;
56         while(t==n)t=rho(n,rand()%(n-1)+1);
57         solve(t),solve(n/t);
58     }

```

```

59 }
60 int main(){
61     for(srand(1626),scanf("%lld",&_);_--;){
62         scanf("%lld",&x),ans=0;solve(x);
63         if(ans==x)puts("Prime");
64         else printf("%lld\n",ans);
65     }
66 }

```

1.2 同余方程

```

1 void gcd(LL a, LL b, LL &d, LL &x, LL &y){
2     if (!b){ d=a; x=1; y=0; return; }
3     gcd(b,a%b,d,y,x); y-=x*(a/b);
4 }
5 IL void sim(LL &a, LL n) { a%=n; if (a<0) a+=n; }
6 IL LL solve(LL a, LL b, LL n){ // a*x==b (mod n)
7     sim(a,n); sim(b,n); // optional
8     static LL d,x,y;
9     gcd(a,n,d,x,y);
10    if (b%d) return -1;
11    b/=d; n/=d;
12    if (x<0) x+=n;
13    return b*x%n;
14 }
15 // x==a1 (mod n1); x==a2 (mod n2);
16 // passing gcd in solve can reduce time
17 void merge(LL a1, LL n1, LL a2, LL n2, LL &x, LL &n){
18     n=lcm(n1,n2);
19     LL k=solve(n1,a2-a1,n2);
20     if (k==-1) { x=-1; return; }
21     sim(x=n1*k+a1,n);
22 }
23 // getinv , gcd(a,n) must be 1
24 IL LL getinv(LL a, LL n){
25     static LL d,x,y;
26     gcd(a,n,d,x,y);
27     // if (d!=1) return -1;
28     return x<0?x+n:x;
29 }

```

1.3 线性筛法

```

1 const int N=100050;
2 int b[N],a[N],cnt,mx[N],phi[N],mu[N];
3
4 void getprime(int n=100000){
5     memset(b+2,1,sizeof(b[0])*(n-1));
6     mu[1]=1;
7     ft(i,2,n){
8         if (b[i]){
9             a[mx[i]=++cnt]=i;
10            phi[i]=i-1; mu[i]=-1;
11        }
12        ft(j,1,mx[i]){
13            int k=i*a[j];
14            if (k>n) break;
15            b[k]=0; mx[k]=j;
16            phi[k]= phi[i]*(a[j]-(j!=mx[i]));
17            mu[k]= j==mx[i] ? 0 : -mu[i];
18        }
19    }
20 }

```

1.4 离散对数

```

1 // BSGS , a^x==b (mod n) , n is a prime
2 LL bsgs(LL a, LL b, LL n){
3     int m=sqrt(n+0.5);
4     LL p=power(a,m,n);
5     LL v=getinv(p,n);
6     static hash_map x;
7     x.clear();
8     LL e=1; x[e]=0;
9     ft(i,1,m){
10        e=e*a%n;
11        if (!x.count(e)) x[e]=i;
12    }
13    for(LL i=0;i<n;i+=m){
14        if (x.count(b)) return i+x[b];
15        b=b*v%n;
16    }
17    return -1;
18 }
19

```

```

20 //BSGS
21 //y^x==z (mod p) ->x=?
22 scanf ("%d%d%d", &y, &z, &p), y%=p, z%=p; j=z;
23 if (y==0) {puts ("Cannot_find_x"); continue;}
24 for (k=s=1; k*k<=p; k++);
25 std::map<int, int> hash; flag=0;
26 for (int i=0; i<k; i++, s=1LL*s*y%p, j=1LL*j*y%p) hash[j]=i;
27 for (int i=1, j=s; i<=k&&!flag; i++, j=1LL*j*s%p)
28 if (hash.count(j)) ans=i*k-hash[j], flag=1;
29 if (flag==0) puts ("Cannot_find_x");
30 else printf ("%d\n", ans);
31
32 //exBSGS
33 int bsgs(int a, ll b, int p) {
34     if (a%p, b%p, b==1) return 0;
35     ll t=1; int f, g, delta=0, m=sqrt(p)+1, i;
36     for (g=gcd(a, p); g!=1; g=gcd(a, p)) {
37         if (b%g) return -1;
38         b/=g, p/=g, t=t*(a/g)%p, delta++;
39         if (b==t) return delta;
40     }
41     std::map<int, int> hash;
42     for (i=0; i<m; i++, b=b*a%p) hash[b]=i;
43     for (i=1, f=power(a, m); i<=m+1; i++)
44         if (t=f*t%p, hash.count(t)) return i*m-hash[t]+delta;
45     return -1;
46 }

```

1.5 Lucas

```

1 void init_Lucas() {
2     fac[0]=1; ft(1, 1, P-1) fac[i]=fac[i-1]*i%P;
3     inv[1]=1; ft(1, 2, P-1) inv[i]=(P-P/i)*inv[P%i]%P;
4     inv[0]=1; ft(1, 1, P-1) inv[i]=inv[i-1]*inv[i]%P;
5 }
6 IL LL C(int n, int m) {
7     LL ans=1;
8     while (n|m) {
9         int a=n%P, b=m%P;
10        if (a<b) return 0;
11        n/=P; m/=P;
12        ans= ans *fac[a]%P *inv[b]%P *inv[a-b]%P;
13    }
14    return ans;

```

```

15 }

```

1.6 高斯消元

```

1 int i=1, j=1;
2 while (i<=n && j<=m) {
3     int k=i;
4     while (k<=n && a[k][j].x==0) k++;
5     if (k>n) { j++; continue; }
6     if (k!=i) ft(t, j, m) swap(a[i][t], a[k][t]);
7
8     ft(p, j+1, m) a[i][p]=a[i][p]/a[i][j];
9     a[i][j]=(R){1, 1};
10
11     ft(t, 1, n) if (t!=i && a[t][j].x) {
12         ft(p, j+1, m) a[t][p]=a[t][p]-a[i][p]*a[t][j];
13         a[t][j]=(R){0, 1};
14     }
15     i++; j++;
16 }
17
18 // xor equations
19 int n, m;
20 bitset<N> a[N];
21 bool solve() {
22     int i=1, j=1;
23     while (i<=n && j<=m) {
24         int k=i;
25         while (k<=n && !a[k][j]) k++;
26         if (k>n) { j++; continue; }
27         if (j==m) return false; // no solution
28         if (k!=i) swap(a[i], a[k]);
29         ft(t, 1, n) if (t!=i && a[t][j]) a[t]^=a[i];
30         i++; j++;
31     }
32     return true; // have solution (but may have 0==0)
33 }
34 // no solution : have 1==0
35 // have solution : no 1==0 (but may have 0==0)
36 // have solution can be dependent or independent

```

1.7 瀚之的莫比乌斯

```

1 void getprime(int n){
2     miu[1]=pre[1]=b[1]=1;
3     for(int i=2;i<=n;i++){
4         if(!b[i]) p[mx[i]++]=i, miu[i]=-1;
5         for(int j=1;j<=mx[i];j++){
6             int k=i*p[j]; if(k>n) break;
7             b[k]=1; mx[k]=j;
8             if(j==mx[i]) miu[k]=0;
9             else miu[k]=miu[i]*miu[p[j]];
10        }
11        pre[i]=pre[i-1]+i*miu[i];
12    }
13 }
14 ll f(int n,int m){return 1ll*n*m*(n+m+2)/2;}
15 ll calc(int n,int m){ // sigma{ i+j | i<=n, j<=m, gcd(i,j)=1 }
16     if(n>m) swap(n,m); ll ans=0;
17     for(int i=1,j=0,k=0;i<=n;i=min(j,k)+1)
18         ans+=(pre[min(j=n/(n/i),k=m/(m/i))]-pre[i-1])*f(n/i,m/i);
19     return ans;
20 }

```

1.8 FFT|NTT

```

1 typedef complex<double> comp;
2 comp A[N], B[N];
3 int rev[N], m, len;
4 inline void init(int n) {
5     for (m = 1, len = 0; m < n + n; m <= 1, len ++);
6     for (int i = 0; i < m; ++i) rev[i]=(rev[i>>1]>>1) | ((i&1)<<(len-1));
7     for (int i = 0; i < m; ++i) A[i] = B[i] = comp(0, 0);
8 }
9 inline void dft(comp *a, int v) {
10     for (int i = 0; i < m; ++i) if (i < rev[i]) swap(a[i], a[rev[i]]);
11     for (int s = 2; s <= m; s <= 1) {
12         comp g(cos(2 * pi / s), v * sin(2 * pi / s));
13         // NTT: int g = power(gg, (mod - 1) / s);
14         // NTT: if (v == -1) g = power(g, mod - 2);
15         for (int k = 0; k < m; k += s) {
16             comp w(1, 0);
17             // NTT: int w = 1;
18             for (int j = 0; j < s / 2; ++j) {
19                 comp &u = a[k + j + s / 2], &v = a[k + j];

```

```

20                 comp t = w * u; u = v - t; v = v + t; w = w * g;
21                 // NTT: be aware of "+-*"
22             }
23         }
24     }
25     if (v == -1) for (int i = 0; i < m; ++i) a[i] /= m;
26     // NTT: be aware of "/"
27 }

```

1.9 求原根

```

1 vector <LL> a;
2 bool g_test(LL g, LL p) { for (LL i = 0; i < a.size(); ++i) if (pow_mod(g, (p
3     -1)/a[i], p) == 1) return 0; return 1; }
4 LL p_root(LL p) {
5     LL tmp = p - 1;
6     for (LL i = 2; i <= tmp / i; ++i)
7         if (tmp % i == 0) { a.push_back(i); while (tmp % i == 0) tmp /= i; }
8     if (tmp != 1) a.push_back(tmp);
9     LL g = 1; while (1) { if (g_test(g, p)) return g; ++g; }
10 }

```

1.10 线性基

```

1 #define B 30
2 const int allset=(1<<B)-1;
3 struct LB {
4     int mat[B],cnt;
5     multiset<int> st;
6     LB(){}
7     void clear() { st.clear(); cnt=0; memset(mat,0,sizeof(mat)); }
8     void add(int x) {
9         for (int i=B-1;i>=0;i--) if ((x>>i)&1) {
10             if (mat[i]) x^=mat[i];
11             else { cnt++; mat[i]=x; break; }
12         }
13     }
14     void fix() {
15         for (int i=0;i<B;i++) if (mat[i])
16             for (int j=i+1;j<B;j++) if ((mat[j]>>i)&1) mat[j]^=mat[i];
17     }
18     void preset() { //正确性待定

```

```

19     fix(); for (int i=0;i<B;i++) if (mat[i]) st.insert(mat[i]);
20 }
21 int kth(int k) { //正确性待定
22     int i=0,ans=0; if (k<=0||k>(1<<cnt)-1) return 0;//无解
23     for (multiset<int>::iterator it=st.begin();it!=st.end();it++,i++)
24         if ((k>>i)&1) ans^=(*it);
25     return ans;
26 }
27 int getMax() {
28     fix(); int ans=0;
29     for (int i=B-1;i>=0;i--) if (ans^mat[i]>ans) ans^=mat[i];
30     return ans;
31 }
32 } tree[N*10];

```

1.11 高斯消元法实数方程

```

1 void Gauss(int n,int m) {
2     int i,j,k,t;
3     double mul;
4     for (i=j=1;i<=n&&j<=m;i++,j++) {
5         for (k=i+1;k<=n;k++)
6             if (abs(mat[k][j])>abs(mat[i][j]))
7                 for (t=1;t<=m+1;t++) swap(mat[i][t],mat[k][t]);
8         if (abs(mat[i][j])<eps) { i--; continue; }
9         for (k=i+1;k<=n;k++) {
10             mul=mat[k][j]/mat[i][j];
11             for (t=1;t<=m+1;t++) mat[k][t]-=mat[i][t]*mul;
12         }
13     }
14     for (i=n;i>=1;i--) { //solved表示那个变量是否确定
15         for (j=1;j<=m;j++) if (abs(mat[i][j])>eps) break;
16         if (j>m) continue; solved[j]=true; ans[j]=mat[i][m+1];
17         for (k=j+1;k<=m;k++)
18             if (abs(mat[i][k])>eps&&!solved[k]) solved[j]=false;
19         for (k=j+1;k<=m;k++) ans[j]-=ans[k]*mat[i][k];
20         ans[j]/=mat[i][j];
21     }
22 }

```

1.12 高斯消元法模方程

```

1 void Gauss(LL n,LL m) {
2     LL i,j,k,t,lcm,muli,mulk;
3     for (i=j=1;i<=n&&j<=m;i++,j++) {
4         for (k=i;k<=n;k++) if (mat[k][j]) {
5             for (t=1;t<=m+1;t++) swap(mat[k][t],mat[i][t]);
6             break;
7         }
8         if (mat[i][j]==0) { i--; continue; }
9         for (k=i+1;k<=n;k++) if (mat[k][j]) {
10             lcm=mat[k][j]*mat[i][j]/__gcd(mat[k][j],mat[i][j]);
11             muli=lcm/mat[i][j]; mulk=lcm/mat[k][j];
12             for (t=1;t<=m+1;t++) {
13                 mat[k][t]=mat[k][t]*mulk-mat[i][t]*muli;
14                 mat[k][t]=(mat[k][t]%mod+mod)%mod;
15             }
16         }
17     }
18     for (i=n;i>=1;i--) {
19         for (j=1;j<=m;j++) if (mat[i][j]) break;
20         if (j>m) continue; ans[j]=mat[i][m+1];
21         for (k=j+1;k<=m;k++) ans[j]-=ans[k]*mat[i][k];
22         ans[j]=(ans[j]*power(mat[i][j],mod-2)%mod+mod)%mod;
23     }
24 }

```

1.13 蔡勒公式

$$w = (\lfloor \frac{c}{4} \rfloor - 2c + y + \lfloor \frac{y}{4} \rfloor + \lfloor \frac{13(m+1)}{5} \rfloor + d - 1) \bmod 7$$

① w : 0 星期日, 1 星期一, \dots , 6 星期六; ② c : 年份前两位数; ③ y : 年份后两位数; ④ m : 月 ($3 \leq m \leq 14$, 即在蔡勒公式中, 1、2 月要看作上一年的 13、14 月来计算); ⑤ d : 日。

1.14 皮克定理

给定顶点坐标均是整点 (或正方形格点) 的简单多边形 (凸多边形), 皮克定理说明了其面积 S 和内部格点数目 n 、边上格点数目 s 的关系: $S = n + \frac{s}{2} + 1$

。

2 计算几何

2.1 凸包

```

1 bool cmp(const Point &a,const Point &b) {
2     return F(a.x-b.x)<0||F(a.x-b.x)==0&&a.y<b.y; }
3 void Gram(int id[], int n) {
4     int i,mid; sort(id,id+n,cmp); tp=0; //凸包从x最小的点出发, 逆时针方向
5     for (i=0;i<n;i++) {
6         for (;tp>=2&&Cross(p[sk[tp-1]]-p[sk[tp-2]],p[id[i]]-p[sk[tp-1]])<=0;tp--);
7         //有重点必须用<=不留共线点, 无重点用<=不留共线点, 无重点用<留共线点
8         sk[tp++]=id[i];
9     }
10    mid=tp;
11    for (i=n-2;i>=0;i--) {
12        for (;tp>mid&&Cross(p[sk[tp-1]]-p[sk[tp-2]],p[id[i]]-p[sk[tp-1]])<=0;tp--);
13        //有重点必须用<=不留共线点, 无重点用<=不留共线点, 无重点用<留共线点
14        sk[tp++]=id[i];
15    }
16    if (n>1) tp--;
17 }

```

2.2 定义

```

1 struct Point { double x,y; Point(){} Point(double _x,double _y):x(_x),y(_y){}
2     };
3 struct Seg { Point a,b; Seg(){} Seg(Point _a,Point _b):a(_a),b(_b){} };
4 struct Circle { double x,y,r;
5     Point pt() { return Point(x,y); }
6     double Area() { return pi*r*r; }
7 };
8 Point operator +(const Point &a,const Point &b);
9 Point operator -(const Point &a,const Point &b);
10 Point operator *(const Point &a,double b);
11 Point operator /(const Point &a,double b);
12 int F(double x) { if (x>eps) return 1; if (x<-eps) return -1; return 0; }
13 bool operator ==(const Point &a,const Point &b) {
14     return F(a.x-b.x)==0&&F(a.y-b.y)==0; }
15 double Dist(const Point &a) { return sqrt(a.x*a.x+a.y*a.y); }
16 double Dot(const Point &a,const Point &b) { return a.x*b.x+a.y*b.y; }
17 double Cross(const Point &a,const Point &b) { return a.x*b.y-a.y*b.x; }

```

```

17 Point Rotate(const Point &p,double a) { // 逆时针旋转
18     return Point(p.x*cos(a)-p.y*sin(a),p.x*sin(a)+p.y*cos(a)); }
19 Point Inter(Seg a,Seg b) { // 两线段相交 (前提有交点)
20     double s=Cross(a.b-a.a,b.a-a.a),t=Cross(a.b-a.a,b.b-a.a);
21     return b.a+(b.b-b.a)*s/(s-t); }
22 vector<Point> SegCir(Seg seg,Point pt,double r) { //线圆
23     vector<Point> ans; double mul; Point vec,mid;
24     ans.clear(); vec=Rotate(seg.b-seg.a,pi/2);
25     mid=Inter(seg,Seg(pt,pt+vec));
26     if (F(Dist(pt-mid)-r)>0) return ans;
27     if (F(Dist(pt-mid)-r)==0) {
28         ans.push_back(mid); ans.push_back(mid); return ans;
29     }
30     vec=seg.b-seg.a; mul=sqrt(r*r-Dist2(mid-pt))/Dist(vec);
31     ans.push_back(mid+vec*mul); ans.push_back(mid-vec*mul);
32     return ans;
33 }
34 vector<Point> Circir(Circle a,Circle b) { //圆圆相交
35     vector<Point> ans; double dis,dis2,alpha; Point pa,pb,vec;
36     ans.clear(); if (a.r<b.r) swap(a,b);
37     pa=a.pt(); pb=b.pt(); vec=pb-pa;
38     dis=Dist(vec); dis2=Dist2(vec);
39     if (F(dis-(a.r+b.r))>0||F(dis-(a.r-b.r)<0) return ans;
40     if (F(dis-(a.r+b.r))==0) {
41         ans.push_back(pa+vec*a.r/(a.r+b.r)); return ans;
42     }
43     if (F(dis-(a.r-b.r))==0) {
44         ans.push_back(pa+vec*a.r/(a.r-b.r)); return ans;
45     }
46     alpha=acos((a.r*a.r+dis2-b.r*b.r)/2/a.r/dis);
47     ans.push_back(pa+Rotate(vec,alpha)*a.r/dis);
48     ans.push_back(pa+Rotate(vec,-alpha)*a.r/dis);
49     return ans;
50 }
51 double Bing(double ra,double rb,double dis) {
52     double alpha,beta; if (ra<rb) swap(ra,rb);
53     if (F(dis-(ra-rb))<=0) return pi*ra*ra;
54     if (F(dis-(ra+rb))>=0) return pi*ra*ra+pi*rb*rb;
55     alpha=acos((ra*ra+dis*dis-rb*rb)/2/dis/ra);
56     beta=acos((rb*rb+dis*dis-ra*ra)/2/dis/rb);
57     return (pi-alpha)*ra*ra+(pi-beta)*rb*rb+ra*dis*sin(alpha);
58 }
59 double Jiao(double ra,double rb,double dis) {
60     return pi*ra*ra+pi*rb*rb-Bing(ra,rb,dis); }
61 Point Gongmid(Circle a,Circle b) { //正确性待定
62     Point pa=a.pt(),pb=b.pt();

```

```

63     return pa+(pb-pa)*a.r/(a.r+b.r); }
64 Point Gongright(Circle a,Circle b) {
65     Point pa=a.pt(),pb=b.pt();
66     return pa+(pb-pa)*a.r/(a.r-b.r); }
67 int Ptinpol(Point pt) {
68     int wn=0;
69     for(int i=0;i<n;i++) {
70         if(Ins(pt,Seg(p[i],p[(i+1)%n]))) return 2;
71         int k=F(Cross(p[(i+1)%n]-p[i],pt-p[i]));
72         int d1=F(p[i].y-pt.y), d2=F(p[(i+1)%n].y-pt.y);
73         if(k>0&&d1<=0&&d2>0)wn++;
74         if(k<0&&d2<=0&&d1>0)wn--;
75     }
76     return wn!=0;
77 }
78 bool Cirinpol(Point pt) { //需要点在多边形内的前提
79     double nearest=inf;
80     for (int i=0;i<n;i++) {
81         nearest=min(nearest,Dist(p[i]-pt));
82         if (F(Dot(pt-p[i],p[(i+1)%n]-p[i]))>0&&
83             F(Dot(pt-p[(i+1)%n],p[i]-p[(i+1)%n]))>0)
84             nearest=min(nearest,abs(Cross(p[i]-pt,p[(i+1)%n]-pt))/dis[i]);
85     }
86     return F(nearest-r)>=0;
87 }
88 bool Ins(const Point &p,const Seg &s) {
89     return F(Cross(s.a-p,s.b-p))==0&&F(p.x-min(s.a.x,s.b.x))>=0&&
90         F(p.x-max(s.a.x,s.b.x))<=0&&F(p.y-min(s.a.y,s.b.y))>=0&&
91         F(p.y-max(s.a.y,s.b.y))<=0; }
92 double PS(const Point &p,const Seg &s) { // 点到线段最短距离
93     if (F(Dot(p-s.a,s.b-s.a))<0||F(Dot(p-s.b,s.a-s.b))<0)
94         return min(Dist(p-s.a),Dist(p-s.b));
95     return abs(Cross(s.a-p,s.b-p))/Dist(s.a-s.b); }
96 double SS(const Seg &a,const Seg &b) { // 线段到线段最短距离
97     return min(min(PS(a.a,b),PS(a.b,b)),min(PS(b.a,a),PS(b.b,a))); }
98 double Alpha(Point a,Point b) {
99     double ans=atan2(b.y,b.x)-atan2(a.y,a.x);
100     if (ans<0) ans=-ans; if (ans>pi) ans=2*pi-ans; return ans; }
101 double Shan(Circle c,double a) { return c.r*c.r*a/2; }

```

2.3 半平面交

```

1 bool Cmphp(Seg a,Seg b) {
2     Point va=a.b-a.a, vb=b.b-b.a;

```

```

3     double dega=atan2(va.y,va.x), degb=atan2(vb.y,vb.x);
4     return F(dega-degb)<0||F(dega-degb)==0&&Cross(a.b-a.a,b.a-a.a)<0;
5 }
6 void HalfPlane(Seg hp[], int n, Point pol[], int &pols) {
7     Point mid;
8     hp[n++]=Seg(Point(-oo,-oo),Point(oo,-oo));
9     hp[n++]=Seg(Point(oo,-oo),Point(oo,oo));
10    hp[n++]=Seg(Point(oo,oo),Point(-oo,oo));
11    hp[n++]=Seg(Point(-oo,oo),Point(-oo,-oo));
12    sort(hp,hp+n,Cmphp);
13    int tp=0, low=0, high=-1; //sk 0~tp-1
14    for (int i=0;i<n;i++)
15        if (high-low+1==0||F(Cross(sk[high].b-sk[high].a,hp[i].b-hp[i].a))) {
16            for (;low<high;high--) {
17                mid=Inter(sk[high],sk[high-1]);
18                if (F(Cross(hp[i].b-hp[i].a,mid-hp[i].a))>0) break;
19            }
20            for (;low<high;low++) {
21                mid=Inter(sk[low],sk[low+1]);
22                if (F(Cross(hp[i].b-hp[i].a,mid-hp[i].a))>0) break;
23            }
24            sk[++high]=hp[i];
25        }
26    for (;low<high;high--) {
27        mid=Inter(sk[high],sk[high-1]);
28        if (Cross(sk[low].b-sk[low].a,mid-sk[low].a)>0) break;
29    }
30    tp=high-low+1; for (int i=0;i<tp;i++) sk[i]=sk[low+i];
31    pols=0; if (tp<=2) return;
32    for (int i=0;i<tp;i++) pol[pols++]=Inter(sk[i],sk[(i+1)%tp]);
33 }

```

2.4 圆与多边形交集

```

1 double CT(Circle c,Point a,Point b) { // 圆与三角形交 (多边形)
2     double da=Dist(a-c.pt()), db=Dist(b-c.pt());
3     if (da>db) { swap(a,b); swap(da,db); }
4     Seg s=Seg(a,b);
5     vector<Point> temp=CS(c,s);
6     if (F(db-c.r)<=0) return 0.5*abs(Cross(a-c.pt(),b-c.pt()));
7     if (F(da-c.r)<0) {
8         if (F(Dot(a-temp[1],b-temp[1]))<0) swap(temp[0],temp[1]);
9         return Shan(c,Alpha(temp[0]-c.pt(),b-c.pt()))+
10            0.5*abs(Cross(a-c.pt(),temp[0]-c.pt()));

```



```

11     }
12     if (!temp.size()) return Shan(c, Alpha(a-c.pt(), b-c.pt()));
13     if (Ins(temp[1], s) && Dist2(a-temp[1]) < Dist2(a-temp[0])) swap(temp[0], temp
14         [1]);
15     if (Ins(temp[0], s) && Ins(temp[1], s)) {
16         return Shan(c, Alpha(a-c.pt(), temp[0]-c.pt())) +
17             Shan(c, Alpha(b-c.pt(), temp[1]-c.pt())) +
18             0.5 * abs(Cross(temp[0]-c.pt(), temp[1]-c.pt()));
19     }
20     return Shan(c, Alpha(a-c.pt(), b-c.pt()));
}

```

2.5 三角形面积并

```

1  #define pr pair<ld, ld>
2  typedef long double ld;
3  const ld EPS=1e-8, INF=1e100;
4  struct Point {
5      ld x, y; Point(){} Point(ld __, ld __):x(__), y(__){}
6      void read() { double __x, __y; scanf("%lf%lf", &__x, &__y); x=__x, y=__y; }
7      friend bool operator<(Point a, Point b) {
8          if(fabs(a.x-b.x)<EPS) return a.y<b.y;
9          return a.x<b.x;
10     }
11     friend Point operator +(Point a, Point b) { return Point(a.x+b.x, a.y+b.y); }
12     friend Point operator -(Point a, Point b) { return Point(a.x-b.x, a.y-b.y); }
13     friend Point operator *(ld a, Point b) { return Point(a*b.x, a*b.y); }
14     friend ld operator *(Point a, Point b) { return a.x*b.x+a.y*b.y; }
15     friend ld operator ^(Point a, Point b) { return a.x*b.y-a.y*b.x; }
16 } a[N][3], Poi[N*N];
17 struct Line {
18     Point p, v; Line(){} Line(Point x, Point y){p=x, v=y-x;}
19     Point operator [] (int k) { if(k) return p+v; else return p; }
20     friend bool Cross(Line a, Line b) {
21         return (a.v^b[0]-a.p)*(a.v^b[1]-a.p)<=-EPS &&
22             (b.v^a[0]-b.p)*(b.v^a[1]-b.p)<=-EPS;
23     }
24     friend Point getP(Line a, Line b) {
25         Point u=a.p-b.p; ld temp=(b.v^u)/(a.v^b.v);
26         return a.p+temp*a.v;
27     }
28 } l[N][3], T;
29 pr p[N];
30 int main() {

```

```

31     int n, m, i, j, k, x, y, cnt, tot;
32     ld ans, last, A, B, sum;
33     scanf("%d", &n);
34     for(i=1, tot=0; i<=n; i++) {
35         a[i][0].read(), a[i][1].read(), a[i][2].read();
36         Poi[++tot]=a[i][0], Poi[++tot]=a[i][1], Poi[++tot]=a[i][2];
37         sort(a[i], a[i]+3);
38         if((a[i][2]-a[i][0]^a[i][1]-a[i][0])>EPS)
39             l[i][0]=Line(a[i][0], a[i][2]), l[i][1]=Line(a[i][2], a[i][1]), l[i
40                 ] [2]=Line(a[i][1], a[i][0]);
41         else
42             l[i][0]=Line(a[i][2], a[i][0]), l[i][1]=Line(a[i][1], a[i][2]), l[i
43                 ] [2]=Line(a[i][0], a[i][1]);
44     }
45     for(i=1; i<=n; i++) for(j=1; j<i; j++) for(x=0; x<3; x++) for(y=0; y<3; y++)
46         if(Cross(l[i][x], l[j][y])) Poi[++tot]=getP(l[i][x], l[j][y]);
47     sort(Poi+1, Poi+tot+1);
48     ans=0, last=Poi[1].x; T=Line(Point(0, -INF), Point(0, INF));
49     for(i=2; i<=tot; i++) {
50         T.p.x=(last+Poi[i].x)/2;
51         for(j=1, cnt=0; j<=n; j++)
52             if(Cross(l[j][0], T)) {
53                 if(Cross(l[j][1], T)) B=getP(l[j][1], T).y;
54                 else B=getP(l[j][2], T).y;
55                 A=getP(l[j][0], T).y; if (A>B) swap(A, B);
56                 p[++cnt]=pr(A, B);
57             }
58     sort(p+1, p+cnt+1);
59     for(j=1, sum=0, A=-INF; j<=cnt; j++) {
60         if(p[j].first>A) sum+=p[j].second-p[j].first, A=p[j].second;
61         else if(p[j].second>A) sum+=p[j].second-A, A=p[j].second;
62     }
63     ans+=(Poi[i].x-last)*sum; last=Poi[i].x;
64 }

```

2.6 K 圆并

```

1  #define sqr(x) ((x)*(x))
2  const double eps = 1e-8;
3  double area[N]; int n;
4  int dcmp(double x) { if (x < -eps) return -1; else return x > eps; }
5  struct cp { double x, y, r, angle; int d;

```

```

6   cp(){} cp(double xx, double yy, double ang = 0, int t = 0) {
7       x = xx; y = yy; angle = ang; d = t; }
8   void get() { scanf("%lf%lf%lf", &x, &y, &r); d = 1; }
9   }cir[N], tp[N * 2];
10  double dis(cp a, cp b) { return sqrt(sqr(a.x - b.x) + sqr(a.y - b.y)); }
11  double cross(cp p0, cp p1, cp p2) {
12      return (p1.x - p0.x) * (p2.y - p0.y) - (p1.y - p0.y) * (p2.x - p0.x);
13  }
14  int CirCrossCir(cp p1, double r1, cp p2, double r2, cp &cp1, cp &cp2) {
15      double mx = p2.x - p1.x, sx = p2.x + p1.x, mx2 = mx * mx;
16      double my = p2.y - p1.y, sy = p2.y + p1.y, my2 = my * my;
17      double sq = mx2 + my2, d = -(sq - sqr(r1 - r2)) * (sq - sqr(r1 + r2));
18      if (d + eps < 0) return 0; if (d < eps) d = 0; else d = sqrt(d);
19      double x = mx * ((r1 + r2) * (r1 - r2) + mx * sx) + sx * my2;
20      double y = my * ((r1 + r2) * (r1 - r2) + my * sy) + sy * mx2;
21      double dx = mx * d, dy = my * d; sq *= 2;
22      cp1.x = (x - dy) / sq; cp1.y = (y + dx) / sq;
23      cp2.x = (x + dy) / sq; cp2.y = (y - dx) / sq;
24      if (d > eps) return 2; else return 1;
25  }
26  bool circmp(const cp& u, const cp& v) { return dcmp(u.r - v.r) < 0; }
27  bool cmp(const cp& u, const cp& v) {
28      if (dcmp(u.angle - v.angle)) return u.angle < v.angle;
29      return u.d > v.d;
30  }
31  double calc(cp cir, cp cp1, cp cp2) {
32      double ans = (cp2.angle - cp1.angle) * sqr(cir.r)
33          - cross(cir, cp1, cp2) + cross(cp(0, 0), cp1, cp2);
34      return ans / 2;
35  }
36  void CirUnion(cp cir[], int n) {
37      cp cp1, cp2; sort(cir, cir + n, circmp);
38      for (int i = 0; i < n; ++i) for (int j = i + 1; j < n; ++j)
39          if (dcmp(dis(cir[i], cir[j]) + cir[i].r - cir[j].r) <= 0) cir[i].d++;
40      for (int i = 0; i < n; ++i) {
41          int tn = 0, cnt = 0;
42          for (int j = 0; j < n; ++j) {
43              if (i == j) continue;
44              if (CirCrossCir(cir[i], cir[i].r, cir[j], cir[j].r, cp2, cp1) < 2)
45                  continue;
46              cp1.angle = atan2(cp1.y - cir[i].y, cp1.x - cir[i].x);
47              cp2.angle = atan2(cp2.y - cir[i].y, cp2.x - cir[i].x);
48              cp1.d = 1; tp[tn++] = cp1; cp2.d = -1; tp[tn++] = cp2;
49              if (dcmp(cp1.angle - cp2.angle) > 0) cnt++;
50          }
51          tp[tn++] = cp(cir[i].x - cir[i].r, cir[i].y, pi, -cnt);

```

```

51      tp[tn++] = cp(cir[i].x - cir[i].r, cir[i].y, -pi, cnt);
52      sort(tp, tp + tn, cmp);
53      int p, s = cir[i].d + tp[0].d;
54      for (int j = 1; j < tn; ++j) {
55          p = s; s += tp[j].d;
56          area[p] += calc(cir[i], tp[j - 1], tp[j]);
57      }
58  }
59  }
60  void solve() {
61      for (int i = 0; i < n; ++i) cir[i].get();
62      memset(area, 0, sizeof(area));
63      CirUnion(cir, n);
64      for (int i = 1; i <= n; ++i) {
65          area[i] -= area[i + 1];
66          printf("[%d]_=%.3lf\n", i, area[i]);
67      }
68  }

```

2.7 三维计算几何

```

1  Point Cross(Point a, Point b) {
2      return Point(a.y*b.z - a.z*b.y, a.z*b.x - a.x*b.z, a.x*b.y - a.y*b.x); }
3  double Crossxy(Point a, Point b) { return a.x*b.y - a.y*b.x; }
4  vector<Point> SegPlane(Seg seg, Plane p) {
5      vector<Point> ans; ans.clear();
6      Point fa = Cross(p.b - p.a, p.c - p.a);
7      if (F(Dot(fa, seg.b - seg.a)) == 0) return ans;
8      double s = Dot(p.a - seg.a, fa) / Dist(fa), t = Dot(p.a - seg.b, fa) / Dist(fa);
9      ans.push_back(seg.a + (seg.b - seg.a) * s / (s - t));
10     return ans;
11 }
12 // mixed product
13 double Mix(Point3 a, Point3 b, Point3 c) { return Dot(Cross(a, b), c); }
14 double PP(Point3 pt, Plane pl) { // distance from point to plane
15     Point3 fa = Cross(pl.b - pl.a, pl.c - pl.a);
16     return abs(Dot(fa, pt - pl.a)) / Dist(fa);
17 }
18 // get the center point from 3D (need plane well prepared)
19 Point3 Getcenter(Point3 p[], int n, Plane pp[], int nn) {
20     double sumv = 0;
21     Point3 sum = Point3(0, 0, 0);
22     for (int i = 0; i < nn; i++)
23     {

```

```

24     double tempv=Mix(pp[i].b-pp[i].a,pp[i].c-pp[i].a,Point3(0,0,0)-pp[i].a)
25     ;
26     sum=sum+(pp[i].a+pp[i].b+pp[i].c)*tempv/4.0;
27     sumv+=tempv;
28 }
29 return sum/sumv;

```

3 字符串

3.1 哈希

```

1 const int P=31,D=1000173169;
2 int hash(int l, int r) { return (LL)(f[r]-(LL)f[l-1]*pow[r-l+1]%D+D)%D; }
3 pow[0] = 1; for (int i=1;i<=n;i++) pow[i] = (LL)pow[i-1]*P%D;
4 for (int i=1;i<=n;i++) f[i] = (LL)((LL)f[i-1]*P+a[i])%D;

```

3.2 KMP

输入：模式串长度 n ，模式串 a ，匹配串长度 m ，匹配串 b ；输出：依次输出每个匹配成功的起始位置；下标从 0 开始。

```

1 void kmp(int n, char* a, int m, char *b) {
2     int i, j;
3     for (nxt[0] = j = -1, i = 1; i < n; nxt[i++] = j) {
4         while (~j && a[j + 1] != a[i]) j = nxt[j];
5         if (a[j + 1] == a[i]) ++j;
6     }
7     for (j = -1, i = 0; i < m; ++i) {
8         while (~j && a[j + 1] != b[i]) j = nxt[j];
9         if (a[j + 1] == b[i]) ++j;
10        if (j == n - 1) {
11            printf("%d\n", i - n + 1);
12            j = nxt[j];
13        }
14    }
15 }

```

3.3 扩展 KMP

next: a 关于自己每个后缀的最长公共前缀；ret: a 关于 b 的每个后缀的最长公共前缀；EXKMP 的 next[i] 表示：从 i 到 $n-1$ 的字符串 st 前缀和原串前缀的最长重叠长度。

```

1 void get_next(char *a, int *next) {
2     int i, j, k, n = strlen(a);
3     for (j = 0; j+1<n && a[j]==a[j+1];j++);
4     next[1] = j; k = 1;
5     for (i=2;i<n;i++) {
6         int len = k+next[k], L = next[i-k];
7         if (L < len-i) {
8             next[i] = L;
9         } else {
10            for (j = max(0, len-i);i+j<n && a[j]==a[i+j];j++);
11            next[i] = j;
12            k = i;
13        }
14    }
15 }
16 void ExtendedKMP(char *a, char *b, int *next, int *ret) {
17     get_next(a, next);
18     int n = strlen(a), m = strlen(b);
19     int i, j, k;
20     for (j=0;j<n && j<m && a[j]==b[j];j++);
21     ret[0] = j;
22     k = 0;
23     for (i=1;i<m;i++) {
24         int len = k+ret[k], L = next[i-k];
25         if (L < len-i) {
26             ret[i] = L;
27         } else {
28             for (j = max(0, len-i);j<n && i+j<m && a[j]==b[i+j];j++);
29             ret[i] = j;
30             k = i;
31        }
32    }
33 }

```

3.4 Manacher

$p[i]$ 表示以 i 为对称轴的最长回文串长度

```

1 char st[N*2], s[N];
2 int len, p[N*2];
3 while (scanf("%s", s) != EOF) {
4     len = strlen(s);
5     st[0] = '$', st[1] = '#';
6     for (int i=1;i<=len;i++)
7         st[i*2] = s[i-1], st[i*2+1] = '#';
8     len = len * 2 + 2;
9     int mx = 0, id = 0, ans = 0;
10    for (int i=1;i<=len;i++) {
11        p[i] = (mx > i) ? min(p[id*2-i]+1, mx-i) : 1;
12        for (; st[i+p[i]] == st[i-p[i]]; ++p[i]);
13        if (p[i]+i > mx) mx = p[i]+i, id = i;
14        p[i]--;
15        if (p[i] > ans) ans = p[i];
16    }
17    printf("%d\n", ans);
18 }

```

3.5 AC 自动机

```

1 struct Node { int next[26]; int terminal, fail; };
2 void build() {
3     head = 0, tail = 1; q[1] = 1;
4     while (head != tail) {
5         int x = q[++head];
6         /*(when necessary) node[x].terminal != node[node[x].fail].terminal; */
7         for (int i=0;i<26;i++)
8             if (node[x].next[i]) {
9                 int y = node[x].fail;
10                while (y) {
11                    if (node[y].next[i]) {
12                        node[node[x].next[i]].fail = node[y].next[i];
13                        break;
14                    }
15                    y = node[y].fail;
16                }
17                if (!node[node[x].next[i]].fail) node[node[x].next[i]].fail = 1;
18                q[++tail] = node[x].next[i];
19            }
20     }
21 }

```

3.6 后缀数组

参数 m 表示字符集的大小, 即 $0 \leq r_i < m$

```

1 int n, r[N], wa[N], wb[N], ws[N], sa[N], rank[N], height[N];
2 int cmp(int *r, int a, int b, int l, int n) { return r[a]==r[b] && a+1<n && b+1<n && r[a+1]==r[b+1]; }
3 void suffix_array(int m) {
4     int i, j, p, *x=wa, *y=wb, *t;
5     for (i=0;i<m;i++) ws[i]=0; for (i=0;i<n;i++) ws[x[i]=r[i]]++;
6     for (i=1;i<m;i++) ws[i]+=ws[i-1]; for (i=n-1;i>=0;i--) sa[--ws[x[i]]]=i;
7     for (j=1,p=1;p<n;m=p,j<=1) {
8         for (p=0,i=n-j;i<n;i++) y[p++]=i;
9         for (i=0;i<n;i++) if (sa[i]>=j) y[p++]=sa[i]-j;
10        for (i=0;i<m;i++) ws[i]=0; for (i=0;i<n;i++) ws[x[y[i]]]++;
11        for (i=1;i<m;i++) ws[i]+=ws[i-1];
12        for (i=n-1;i>=0;i--) sa[--ws[x[y[i]]]]=y[i];
13        for (t=x,x=y,y=t,x[sa[0]]=0,i=1,p=1;i<n;i++)
14            x[sa[i]]=cmp(y,sa[i-1],sa[i],j,n)?p-1:p++;
15    }
16    for (i=0;i<n;i++) rank[sa[i]]=i; rank[n] = -1;
17    for (i=j=0;i<n;i++) if (rank[i]) {
18        while (r[i+j]==r[sa[rank[i]-1]+j]) j++;
19        height[rank[i]]=j;
20        if (j) j--;
21    }
22 }

```

3.7 后缀自动机

下面的代码是求两个串的 LCS (最长公共子串)。

```

1 #define M (N << 1)
2 char st[N];
3 int pre[M], son[26][M], step[M], refer[M], size[M], tmp[M], topo[M], last, total;
4 int apply(int x, int now) {
5     step[++total] = x;
6     refer[total] = now;
7     return total;
8 }
9 void extend(char x, int now) {
10    int p = last, np = apply(step[last]+1, now);
11    size[np] = 1;
12    for (; p && !son[x][p]; p=pre[p]) son[x][p] = np;

```

```

13     if (!p) pre[np] = 1;
14     else {
15         int q = son[x][p];
16         if (step[p]+1 == step[q]) pre[np] = q;
17         else {
18             int nq = apply(step[p]+1, now);
19             for (int i=0; i<26; i++) son[i][nq] = son[i][q];
20             pre[nq] = pre[q]; pre[q] = pre[np] = nq;
21             for (; p && son[x][p]==q; p=pre[p]) son[x][p] = nq;
22         }
23     }
24     last = np;
25 }
26 void init() {
27     last = total = 0;
28     last = apply(0, 0);
29     scanf("%s", st);
30     int n = strlen(st);
31     for (int i = 0; i <= n * 2; ++i) {
32         pre[i] = step[i] = refer[i] = size[i] = tmp[i] = topo[i] = 0;
33         for (int j = 0; j < 26; ++j) son[j][i] = 0;
34     }
35     for (int i = 0; i < n; ++i) extend(st[i] - 'a', i);
36     for (int i = 1; i <= total; ++i) tmp[step[i]] ++;
37     for (int i = 1; i <= n; ++i) tmp[i] += tmp[i - 1];
38     for (int i = 1; i <= total; ++i) topo[tmp[step[i]]--] = i;
39     for (int i = total; i; --i) size[pre[topo[i]]] += size[topo[i]];
40 }
41 int main() {
42     init();
43     int p = 1, now = 0, ans = 0;
44     scanf("%s", st);
45     for (int i=0; st[i]; i++) {
46         int index = st[i] - 'a';
47         for (; p && !son[index][p]; p = pre[p], now = step[p]);
48         if (!p) p = 1;
49         if (son[index][p]) {
50             p = son[index][p]; now++;
51             if (now > ans) ans = now;
52         }
53     }
54     printf("%d\n", ans);
55     return 0;
56 }

```

一些定义和性质: ① $\text{Right}(\text{str})$ 表示 str 在母串 S 中所有出现的结束位置集

合; ② 一个状态 s 表示的所有子串 Right 集合相同, 为 $\text{Right}(s)$; ③ $\text{Parent}(s)$ 满足 $\text{Right}(s)$ 是 $\text{Right}(\text{Parent}(s))$ 的真子集, 并且 $\text{Right}(\text{Parent}(s))$ 的大小最小; ④ Parent 函数可以表示一个树形结构。不妨叫它 Parent 树; ⑤ 一个 Right 集合和一个长度定义了一个子串; ⑥ 对于状态 s , 使得 $\text{Right}(s)$ 合法的子串长度是一个区间 $[\min(s), \max(s)]$; ⑦ $\max(\text{Parent}(s)) = \min(s) - 1$; ⑧ 令 $\text{refer}(s)$ 表示产生 s 状态的字符所在位置。则 $\text{Right}(s)$ 的合法子串的起始位置为 $[\text{refer}(s) - \max(s) + 1, \text{refer}(s) - \min(s) + 1]$, 即 $[\text{refer}(s) - \max(s) + 1, \text{refer}(s) - \max(\text{Parent}(s))]$ 。

代码中变量含义: ① $\text{pre}[s]$ 为上述定义中的 $\text{Parent}(s)$; ② $\text{step}[s]$ 为从初始状态走到 s 状态最多需要多少步; ③ $\text{refer}[s]$ 为上述定义中的 $\text{refer}(s)$; ④ $\text{size}[s]$ 为 $\text{Right}(s)$ 集合的大小; ⑤ $\text{topo}[s]$ 为 Parent 树的拓扑序, 根 (初始状态) 在前。

3.8 回文树

① $\text{len}[i]$ 表示编号为 i 的节点表示的回文串的长度 (一个节点表示一个回文串) ② $\text{next}[i][c]$ 表示编号为 i 的节点表示的回文串在两边添加字符 c 以后变成回文串的编号 (和字典树类似)。③ $\text{fail}[i]$ 表示节点 i 失配以后跳转不等于自身的节点 i 表示的回文串的最长后缀回文串 (和 AC 自动机类似)。④ $\text{cnt}[i]$ 表示节点 i 表示的本质不同的串的个数 (建树时求出的不是完全的, 最后 $\text{count}()$ 函数跑一遍以后才是正确的) ⑤ $\text{num}[i]$ 表示以节点 i 表示的最长回文串的最右端点为回文串结尾的回文串个数。⑥ last 指向新添加一个字母后所形成的最长回文串表示的节点。⑦ $\text{st}[i]$ 表示第 i 次添加的字符 (一开始设 $\text{st}[0] = -1$ (可以是任意一个在串 S 中不会出现的字符))。⑧ tot 表示添加的节点个数。⑨ n 表示添加的字符个数。

【URAL2040】Palindromes and Super Abilities 2

逐个添加字符串 S 里的字符 S_1, S_2, \dots, S_n 。每次添加字符后, 他想知道添加字符后将出现多少个新的本质不同的回文子串。字符集为 $\{a, b\}$

```

1 struct PAM {
2     int n, tot, last, len[N], fail[N], next[N][2], num[N], cnt[N];
3     void init() { n=0; tot=1; len[1]=-1; fail[1]=0; len[0]=+0; fail[0]=1; last
        =1; }
4     int get_fail(int x) { for (; st[n-len[x]-1]!=st[n]; x=fail[x]); return x; }
5     void insert(char c) {

```

```

6      ++n; int cur=get_fail(last); // 判断上一个串的前一个位置和新添加的位置
      是否相同, 相同则说明构成回文。否则找 fail 指针。
7      if (!next[cur][c]) {
8          ++tot; len[tot]=len[cur]+2; fail[tot]=next[get_fail(fail[cur])][c];
9          next[cur][c]=tot; num[tot] = num[fail[tot]] + 1; answer[n]='1';
10     } else answer[n]='0';
11     last=next[cur][c]; cnt[last] ++;
12 }
13 void count () { for (int i=tot-1; i>=0; --i) cnt[fail[i]] += cnt[i]; }
14 // 父亲累加儿子的 cnt, 因为如果 fail[v]=u, 则 u 一定是 v 的子回文串。
15 } pam;
16 n=strlen(st+1); pam.init();
17 for (int i=1; i<=n; i++) pam.insert(st[i]-'a');

```

4 数据结构

4.1 ST 表

```

1 int Log[N], f[17][N];
2 int ask(int x, int y) { int k=Log[y-x+1]; return max(f[k][x], f[k][y-(1<<k)+1]); }
3 for (int i=2; i<=n; i++) Log[i]=Log[i>>1]+1; for (int j=1; j<K; j++) for (int i=1; i
    +(1<<j-1)<=n; i++) f[j][i]=max(f[j-1][i], f[j-1][i+(1<<j-1)]);

```

4.2 K-D Tree

① change 将编号为 x 的点的权值增加 p ; ② euclid_lower_bound 欧几里得距离的平方, 下界; ③ euclid_upper_bound 欧几里得距离的平方, 上界; ④ manhattan_lower_bound 曼哈顿距离, 下界; ⑤ manhattan_upper_bound 曼哈顿距离, 上界; ⑥ add 添加一个点 (注意此处的添加可能导致这棵树不平衡, 慎用!); ⑦ ask(p, X, Y, ans) 询问距离点 (X, Y) 最远的一个点的距离, ans 需传入无穷小; ⑧ ask($p, x1, y1, x2, y2$) 查询矩形范围内所有点的权值和。

```

1 int n, cmp_d, root, id[N];
2 struct node { int d[2], l, r, Max[2], Min[2], val, sum, f; } t[N];
3 inline bool cmp(const node &a, const node &b) {
4     if (a.d[cmp_d] != b.d[cmp_d]) return a.d[cmp_d] < b.d[cmp_d];
5     return a.d[cmp_d ^ 1] < b.d[cmp_d ^ 1];
6 }
7 inline void umax(int &a, int b) { if (b > a) a = b; }
8 inline void umin(int &a, int b) { if (b < a) a = b; }

```

```

9 inline void up(int x, int y) { umax(t[x].Max[0], t[y].Max[0]); umin(t[x].Min
    [0], t[y].Min[0]); umax(t[x].Max[1], t[y].Max[1]); umin(t[x].Min[1], t[y].
    Min[1]); }
10 int build(int l, int r, int D, int f) {
11     int mid = (l + r) / 2; cmp_d = D;
12     nth_element(t + l + 1, t + mid + 1, t + r + 1, cmp);
13     id[t[mid].f] = mid; t[mid].f = f;
14     t[mid].Max[0] = t[mid].Min[0] = t[mid].d[0];
15     t[mid].Max[1] = t[mid].Min[1] = t[mid].d[1];
16     t[mid].val = t[mid].sum = 0;
17     if (l != mid) t[mid].l = build(l, mid - 1, !D, mid);
18     else t[mid].l = 0;
19     if (r != mid) t[mid].r = build(mid + 1, r, !D, mid);
20     else t[mid].r = 0;
21     if (t[mid].l) up(mid, t[mid].l);
22     if (t[mid].r) up(mid, t[mid].r);
23     return mid;
24 }
25 void change(int x, int p) {
26     x = id[x]; // 将点的编号映成排序后的编号
27     for (t[x].val += p; x; x = t[x].f) t[x].sum += p;
28 }
29 inline long long sqr(long long x) { return x * x; }
30 inline long long euclid_lower_bound(const node &a, int X, int Y) {
31     return sqr(max(max(X - a.Max[0], a.Min[0] - X), 0)) +
32     sqr(max(max(Y - a.Max[1], a.Min[1] - Y), 0)); }
33 inline long long euclid_upper_bound(const node &a, int X, int Y) {
34     return max(sqr(X - a.Min[0]), sqr(X - a.Max[0])) +
35     max(sqr(Y - a.Min[1]), sqr(Y - a.Max[1])); }
36 inline long long manhattan_lower_bound(const node &a, int X, int Y) {
37     return max(a.Min[0] - X, 0) + max(X - a.Max[0], 0) +
38     max(a.Min[1] - Y, 0) + max(Y - a.Max[1], 0);
39 }
40 inline long long manhattan_upper_bound(const node &a, int X, int Y) {
41     return max(abs(X - a.Max[0]), abs(a.Min[0] - X)) +
42     max(abs(Y - a.Max[1]), abs(a.Min[1] - Y));
43 }
44 void add(int k) {
45     t[k].Max[0] = t[k].Min[0] = t[k].d[0]; t[k].Max[1] = t[k].Min[1] = t[k].d
        [1];
46     t[k].val = t[k].sum = 0; t[k].l = t[k].r = t[k].f = 0;
47     if (!root) root = k, return;
48     int p = root, D = 0;
49     while (1) { up(p, k);
50         if (t[k].d[D] <= t[p].d[D]) { if (t[p].l) p = t[p].l; else t[p].l = k,
            t[k].f = p, return; }

```

```

51     else { if (t[p].r) p = t[p].r; else t[p].r = k, t[k].f = p, return; }
52     D ^= 1;
53 }
54 }
55 inline long long getdis(const node &a, int X, int Y) { return sqr(a.d[0] - X) +
    sqr(a.d[1] - Y); }
56 void ask(int p, int X, int Y, long long &ans) {
57     if (!p) return; ans = max(ans, getdis(t[p], X, Y));
58     long long dl = t[p].l ? euclid_upper_bound(t[t[p].l], X, Y) : 0;
59     long long dr = t[p].r ? euclid_upper_bound(t[t[p].r], X, Y) : 0;
60     if (dl > dr) { if (dl > ans) ask(t[p].l, X, Y, ans); if (dr > ans) ask(t[p]
        ].r, X, Y, ans); }
61     else { if (dr > ans) ask(t[p].r, X, Y, ans); if (dl > ans) ask(t[p].l, X, Y
        , ans); }
62 }
63 int ask(int p, int x1, int y1, int x2, int y2) {
64     if (t[p].Min[0] > x2 || t[p].Max[0] < x1 || t[p].Min[1] > y2 || t[p].Max[1]
        < y1) return 0;
65     if (t[p].Min[0] >= x1 && t[p].Max[0] <= x2 && t[p].Min[1] >= y1 && t[p].Max
        [1] <= y2) return t[p].sum;
66     int s = 0;
67     if (t[p].d[0] >= x1 && t[p].d[0] <= x2 && t[p].d[1] >= y1 && t[p].d[1] <=
        y2) s += t[p].val;
68     if (t[p].l) s += ask(t[p].l, x1, y1, x2, y2);
69     if (t[p].r) s += ask(t[p].r, x1, y1, x2, y2);
70     return s;
71 }
72 for (int i = 1; i <= n; ++i) t[i].d[0] = x, t[i].d[1] = y;
73 root = build(1, n, 0, 0);

```

4.3 左偏树

左偏树是一个可并堆。下面的程序写的是一个小根堆，如果需要改成大根堆请在注释了 here 那行修改。接口：① push 插入一个元素；② merge 合并两个堆，注意，合并后原来那个堆将不可访问；③ top 返回堆顶元素；④ pop 删除堆顶元素；⑤ size 返回堆的大小。

```

1 template <class T> class leftist { public:
2     struct node { T key; int dist; node *l, *r; };
3     leftist() : root(NULL), s(0) {}
4     void push(const T &x) { leftist y; y.s = 1; y.root = new node; y.root ->
        key = x; y.root -> dist = 0; y.root -> l = y.root -> r = NULL; merge(y
        ); }

```

```

5     node* merge(node *x, node *y) {
6         if (x == NULL) return y; if (y == NULL) return x;
7         if (y -> key < x -> key) swap(x, y); //here
8         x -> r = merge(x -> r, y);
9         int ld = x -> l ? x -> l -> dist : -1;
10        int rd = x -> r ? x -> r -> dist : -1;
11        if (ld < rd) swap(x -> l, x -> r);
12        if (x -> r == NULL) x -> dist = 0;
13        else x -> dist = x -> r -> dist + 1; return x;
14    }
15    void merge(leftist &x) { root = merge(root, x.root); s += x.s; }
16    T top() const { if (root == NULL) return T(); return root -> key; }
17    void pop() { if (root == NULL) return; node *p = root; root = merge(root ->
        l, root -> r); --s; delete p; }
18    int size() const { return s; }
19 private: node* root; int s;
20 };

```

4.4 线段树小技巧

给定一个序列 a ，寻找一个最大的 i 使得 $i \leq y$ 且满足一些条件（如 $a[i] \geq w$ ，那么需要在线段树维护 a 的区间最大值）

```

1 int queryl(int p, int left, int right, int y, int w) {
2     if (right <= y) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (y <= mid) return queryl(p<<1|0, left, mid, y, w);
8     int ret = queryl(p<<1|1, mid+1, right, y, w);
9     if (ret != -1) return ret;
10    return queryl(p<<1|0, left, mid, y, w);
11 }

```

给定一个序列 a ，寻找一个最小的 i 使得 $i \geq x$ 且满足一些条件（如 $a[i] \geq w$ ，那么需要在线段树维护 a 的区间最大值）

```

1 int queryr(int p, int left, int right, int x, int w) {
2     if (left >= x) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;

```

```

7   if (x > mid) return queryr(p<<1|1, mid+1, right, x, w);
8   int ret = queryr(p<<1|0, left, mid, x, w);
9   if (ret != -1) return ret;
10  return queryr(p<<1|1, mid+1, right, x, w);
11 }

```

4.5 Splay

接口：① ADD x y d 将 $[x, y]$ 的所有数加上 d ；② REVERSE x y 将 $[x, y]$ 翻转；③ INSERT x p 将 p 插入到第 x 个数的后面；④ DEL x 将第 x 个数删除。

```

1  int w[N], Min[N], son[N][2], size[N], father[N], rev[N], lazy[N];
2  int top, rt, q[N];
3  void pushdown(int x) {
4      if (!x) return;
5      if (rev[x]) rev[son[x][0]] ^= 1, rev[son[x][1]] ^= 1, swap(son[x][0], son[x][1]), rev[x] = 0;
6      if (lazy[x]) lazy[son[x][0]] += lazy[x], lazy[son[x][1]] += lazy[x], w[x] += lazy[x], Min[x] += lazy[x], lazy[x] = 0;
7  }
8  void pushup(int x) {
9      if (!x) return; pushdown(son[x][0]); pushdown(son[x][1]);
10     size[x] = size[son[x][0]] + size[son[x][1]] + 1; Min[x] = w[x];
11     if (son[x][0]) Min[x] = min(Min[x], Min[son[x][0]]);
12     if (son[x][1]) Min[x] = min(Min[x], Min[son[x][1]]);
13 }
14 void sc(int x, int y, int w) { son[x][w] = y; father[y] = x; pushup(x); }
15 void _ins(int w) {
16     top++; w[top] = Min[top] = w; son[top][0] = son[top][1] = 0;
17     size[top] = 1; father[top] = 0; rev[top] = 0;
18 }
19 void init() { top = 0; _ins(0); _ins(0); rt=1; sc(1, 2, 1); }
20 void rotate(int x) {
21     if (!x) return; int y = father[x], w = son[y][1]==x;
22     sc(y, son[x][w^1], w); sc(father[y], x, son[father[y]][1]==y); sc(x, y, w^1);
23 }
24 void flushdown(int x) {
25     int t=0; for (; x; x=father[x]) q[++t]=x;
26     for (; t; t--) pushdown(q[t]);
27 }
28 void Splay(int x, int root=0) {
29     flushdown(x);

```

```

30 while (father[x] != root) { int y=father[x], w=son[y][1]==x;
31     if (father[y] != root && son[father[y]][w]==y) rotate(y);
32     rotate(x); }
33 }
34 int find(int k) {
35     Splay(rt);
36     while (1) { pushdown(rt);
37         if (size[son[rt][0]]+1==k) Splay(rt), return rt;
38         else if (size[son[rt][0]]+1<k) k-=size[son[rt][0]]+1, rt=son[rt][1];
39         else rt=son[rt][0]; }
40 }
41 int split(int x, int y) {
42     int fx = find(x), fy = find(y+2); Splay(fx); Splay(fy, fx); return son[fy][0]; }
43 void add(int x, int y, int d) { //add d to each number in a[x]...a[y]
44     int t = split(x, y); lazy[t] += d; Splay(t); rt=t; }
45 void reverse(int x, int y) { // reverse the x-th to y-th elements
46     int t = split(x, y); rev[t] ^= 1; Splay(t); rt=t; }
47 void insert(int x, int p) { // insert p after the x-th element
48     int fx = find(x+1), fy = find(x+2);
49     Splay(fx); Splay(fy, fx); _ins(p); sc(fy, top, 0); Splay(top); rt=top; }
50 void del(int x) { // delete the x-th element in Splay
51     int fx = find(x), fy = find(x+2);
52     Splay(fx); Splay(fy, fx); son[fy][0] = 0; Splay(fy); rt=fy; }

```

4.6 可持久化 Treap

接口：① insert 在当前第 x 个字符后插入 c ；② del 删除第 x 个字符到第 y 个字符；③ copy 复制第 l 个字符到第 r 个字符，然后粘贴到第 x 个字符后；④ reverse 翻转第 x 个到第 y 个字符；⑤ query 表示询问当前第 x 个字符是什么。

```

1  char key[N];
2  bool rev[N];
3  int lc[N], rc[N], size[N]; // if size is long long, remember here
4  int n, root;
5  LL Rand() { return rd = (rd * 2037205211 + 2502208711) % mod; }
6  void init() { n = root = 0; }
7  inline int copy(int x) { ++n; key[n] = key[x]; (copy rev, lc, rc, size);
8      return n; }
9  inline void pushdown(int x) {
10     if (!rev[x]) return;
11     if (lc[x]) lc[x] = copy(lc[x]); if (rc[x]) rc[x] = copy(rc[x]);

```



```

11     swap(lc[x], rc[x]); rev[lc[x]] ^= 1; rev[rc[x]] ^= 1; rev[x] = 0;
12 }
13 inline void pushup(int x) { size[x] = size[lc[x]] + size[rc[x]] + 1; }
14 int merge(int u, int v) {
15     if (!u || !v) return u+v; pushdown(u); pushdown(v);
16     int t = Rand() % (size[u] + size[v]), r; // if size is long long, remember
        here
17     if (t < size[u]) r = copy(u), rc[r] = merge(rc[u], v);
18     else r = copy(v), lc[r] = merge(u, lc[v]);
19     pushup(r); return r;
20 }
21 int split(int u, int x, int y) { // if size is long long, remember here
22     if (x > y) return 0; pushdown(u);
23     if (x == 1 && y == size[u]) return copy(u);
24     if (y <= size[lc[u]]) return split(lc[u], x, y);
25     int t = size[lc[u]] + 1; // if size is long long, remember here
26     if (x > t) return split(rc[u], x-t, y-t);
27     int num = copy(u); lc[num]=split(lc[u], x, t-1); rc[num]=split(rc[u], 1, y-
        t);
28     pushup(num); return num;
29 }
30 void insert(int x, char c) {
31     int t1 = split(root, 1, x), t2 = split(root, x+1, size[root]);
32     key[++n] = c; lc[n] = rc[n] = rev[n] = 0; pushup(n); root = merge(merge(t1,
        n), t2); }
33 void del(int x, int y) {
34     int t1 = split(root, 1, x-1), t2 = split(root, y+1, size[root]); root =
        merge(t1, t2); }
35 void copy(int l, int r, int x) {
36     int t1 = split(root, 1, x), t2 = split(root, 1, r), t3 = split(root, x+1,
        size[root]);
37     root = merge(merge(t1, t2), t3); }
38 void reverse(int x, int y) {
39     int t1 = split(root, 1, x-1), t2 = split(root, x, y), t3 = split(root, y+1,
        size[root]);
40     rev[t2] ^= 1; root = merge(merge(t1, t2), t3); }
41 char query(int k) {
42     int x = root;
43     while (1) { pushdown(x);
44         if (k <= size[lc[x]]) x = lc[x];
45         else if (k == size[lc[x]] + 1) return key[x];
46         else k -= size[lc[x]] + 1, x = rc[x]; }
47 }

```

4.7 可持久化并查集

接口：① merge 在 time 时刻将 x 和 y 连一条边，注意加边顺序必须按 time 从小到大加边 ② GetFather 询问 time 时刻及以前的连边状态中，x 所属的集合

```

1  const int inf = 0x3f3f3f3f;
2  int father[N], Father[N], Time[N];
3  vector<int> e[N];
4  void init() { for (int i=1;i<=n;i++) father[i]=Father[i]=i,Time[i]=inf,e[i].
        clear(),e[i].push_back(i); }
5  int getfather(int x) { return (father[x]==x) ? x : father[x]=getfather(father[x]
        ); }
6  int GetFather(int x, int time) {return (Time[x]<=time)?GetFather(Father[x],time
        ):x;}
7  void merge(int x, int y, int time) {
8      int fx = getfather(x), fy = getfather(y); if (fx == fy) return;
9      if (e[fx].size() > e[fy].size()) swap(fx, fy);
10     father[fx] = fy; Father[fx] = fy; Time[fx] = time;
11     for (int i=0;i<e[fx].size();i++) e[fy].push_back(e[fx][i]);
12 }

```

4.8 普通莫队

分块块数为 \sqrt{n} 是最优的。记每次进行 add() 操作的复杂度为 $O(A)$, del() 操作的复杂度为 $O(D)$, 查询答案 answer() 的复杂度为 $O(S)$ 。则总复杂度为 $O(n\sqrt{n}(A + D) + qS)$ 。S 可以大一点，但必须保证 A, D 尽可能小。

```

1  struct Q { int l, r, sqrtl, id; } q[N];
2  int n, m, v[N], ans[N], nowans;
3  bool cmp(const Q &a, const Q &b) { if (a.sqrtl != b.sqrtl) return a.sqrtl < b.
        sqrtl; return a.r < b.r; }
4  void change(int x) { if (!v[x]) add(x); else del(x); v[x] ^= 1; }
5
6  for (int i=1;i<=m;i++) q[i].sqrtl = q[i].l / sqrt(n), q[i].id = i;
7  sort(q+1, q+m+1, cmp);
8  int L=1, R=0;
9  memset(v, 0, sizeof(v));
10 for (int i=1;i<=m;i++) {
11     while (L<q[i].l) change(L++);
12     while (L>q[i].l) change(--L);
13     while (R<q[i].r) change(++R);
14     while (R>q[i].r) change(R--);

```

```

15     ans[q[i].id] = answer();
16 }

```

4.9 树上莫队

```

1 struct Query { int l, r, id, l_group; } query[N];
2 int v[N], ans[N];
3 bool cmp(const Query &a, const Query &b) { if (a.l_group != b.l_group) return a
    .l_group < b.l_group; return dfn[a.r] < dfn[b.r]; }
4 void upd(int x) { if (!v[x]) add(x); else del(x); v[x] ^= 1; }
5 void go(int &u, int taru, int v) {
6     int lca0 = lca(u, taru);
7     int lca1 = lca(u, v); upd(lca1);
8     int lca2 = lca(taru, v); upd(lca2);
9     for (int x=u; x!=lca0; x=father[x]) upd(x);
10    for (int x=taru; x!=lca0; x=father[x]) upd(x);
11    u = taru;
12 }
13
14 for (int i=1;i<=m;i++) {
15     if (dfn[query[i].l] > dfn[query[i].r]) swap(query[i].l, query[i].r);
16     query[i].id = i; query[i].l_group = dfn[query[i].l] / sqrt(n);
17 }
18 sort(query+1, query+m+1, cmp);
19 int L=1,R=1; upd(1);
20 for (int i=1;i<=m;i++) {
21     go(L,query[i].l,R);
22     go(R,query[i].r,L);
23     ans[query[i].id] = answer();
24 }

```

5 树

5.1 点分治

```

1 void getsize(int x, int root = 0) {
2     size[x] = 1; son[x] = 0; int dd = 0;
3     for (int p = gh[x]; p; p = edge[p].next) {
4         int y = edge[p].adj;
5         if (y == root || !vis[y]) continue;
6         size[x] += size[y];

```

```

7         if (size[y] > dd) dd = size[y], son[x] = y;
8     }
9 }
10 int getroot(int x) {
11     int sz = size[x];
12     while (size[son[x]] > sz/2) x = son[x]; return x;
13 }
14 void dc(int x) {
15     getsize(x); x = getroot(x);
16     vis[x] = 1;
17     for (int p = gh[x]; p; p = edge[p].next) {
18         int y = edge[p].adj;
19         if (vis[y]) continue;
20         dc(y);
21     }
22     vis[x] = 0;
23 }

```

5.2 Link Cut Tree

① 注意，一开始必须调用 `lct.init(0)`，否则求出的最小值一定会是 0。② `minval` 维护的是链上 `val` 最小值。③ `sumval2` 维护的是子树 `val2` 的和。

```

1 int f[N], son[N][2], sz[N], rev[N], tot;
2 int val[N], minid[N], minval[N];
3 int val2[N], sumval2[N]; // 记得开 long long。注意两个都要开 long long，因为
    val2 还包含了虚儿子的子树和。
4 stack<int> s;
5 void init(int i) {
6     tot = max(tot, i); son[i][0] = son[i][1] = 0; f[i] = rev[i] = 0;
7     if (i == 0) sz[i] = 0, val[i] = minval[i] = inf, minid[i] = i, val2[i] =
        sumval2[i] = 0;
8     else sz[i] = 1, val[i] = minval[i] = VAL, minid[i] = i, val2[i] = sumval2[i]
        ] = VAL2;
9 }
10 bool isroot(int x) { return !f[x] || (son[f[x]][0] != x && son[f[x]][1] != x);
    }
11 void revl(int x) { if (!x) return; swap(son[x][0], son[x][1]); rev[x] ^= 1; }
12 void down(int x) { if (!x) return; if (rev[x]) revl(son[x][0]), revl(son[x][1])
    , rev[x] = 0; }
13 void up(int x) { if (!x) return; down(son[x][0]); down(son[x][1]);
14     sz[x] = sz[son[x][0]] + sz[son[x][1]] + 1; minval[x] = val[x]; minid[x] = x
        ;

```

```

15     if (minval[son[x][0]] < minval[x]) minval[x] = minval[son[x][0]], minid[x]
        = minid[son[x][0]];
16     if (minval[son[x][1]] < minval[x]) minval[x] = minval[son[x][1]], minid[x]
        = minid[son[x][1]];
17     sumval2[x] = sumval2[son[x][0]] + sumval2[son[x][1]] + val2[x];
18 }
19 void rotate(int x) {
20     int y = f[x], w = son[y][1] == x; son[y][w] = son[x][w ^ 1];
21     if (son[x][w ^ 1]) f[son[x][w ^ 1]] = y;
22     if (f[y]) {
23         int z = f[y];
24         if (son[z][0] == y) son[z][0] = x;
25         else if (son[z][1] == y) son[z][1] = x;
26     }
27     f[x] = f[y]; f[y] = x; son[x][w ^ 1] = y; up(y);
28 }
29 void splay(int x) {
30     while (!s.empty()) s.pop(); s.push(x);
31     for (int i = x; !isroot(i); i = f[i]) s.push(f[i]);
32     while (!s.empty()) down(s.top()), s.pop();
33     while (!isroot(x)) {
34         int y = f[x];
35         if (!isroot(y)) {
36             if ((son[f[y]][0] == y) ^ (son[y][0] == x)) rotate(x);
37             else rotate(y);
38         }
39         rotate(x);
40     } up(x);
41 }
42 void access(int x) {for (int y = 0; x; y = x, x = f[x]) splay(x), val2[x] +=
        sumval2[son[x][1]], son[x][1] = y, val2[x] -= sumval2[son[x][1]], up(x); }
43 int root(int x) { access(x); splay(x); while (son[x][0]) x = son[x][0]; return
        x; }
44 void makeroot(int x) { access(x); splay(x); rev1(x); }
45 void link(int x, int y) {
46     makeroot(x); f[x] = y; access(x);
47     // 如果需要维护子树和 val2, sumval2, 这样是不够的。因为增加了虚边, 所以需要修改 val2 值。将上面的代码替换为以下代码:
48     // makeroot(x); makeroot(y); f[x] = y; val2[y] += sumval2[x]; up(y); access
        (x);
49 }
50 void cutf(int x) { access(x); splay(x); f[son[x][0]] = 0; son[x][0] = 0; up(x);
        } // 它和父亲的边
51 void cut(int x, int y) { makeroot(x); cutf(y); } // 切断 x 与 y 之间的边 (须保证 x 与 y 相邻)
52 int ask(int x, int y) { makeroot(x); access(y); splay(y); return minid[y]; } //

```

询问 x 到 y 之间取得最小值的点

```

53 int querymin_cut(int x, int y) { int m = ask(x, y); makeroot(x); cutf(m);
        makeroot(y); cutf(m); return val[m]; } // 询问 x 到 y 之间取得最小值的点,
        并把它删去 (须保证该点在 x 和 y 之间, 且度数恰好为 2)
54 void link(int x, int y, int w) { init(++tot); val[tot] = minval[tot] = w; link(
        x, tot); link(y, tot); } // 在 x 和 y 之间添加一条权值为 w 的边 (将边视为
        点插入)
55 int getsumval2(int x, int y) { makeroot(x); access(y); return val2[y]; } // 令
        x 为根, 求 y 子树的 val2 的和

```

5.3 虚树

设 $a[0 \cdots k-1]$ 为需要构建虚树的点。

构建出虚树的节点保存在 a 数组中, k 为节点个数。加边调用函数 `addege(int x, int y, int w)`。

```

1 bool cmp(int x, int y) { return dfn[x] < dfn[y]; }
2 stack<int> stk;
3 sort(a, a + k, cmp);
4 int m = k;
5 for (int j = 1; j < m; ++j)
6     a[k++] = lca(a[j-1], a[j]);
7 sort(a, a + k, cmp);
8 k = unique(a, a + k) - a;
9 stk.push(a[0]);
10 for (int j = 1; j < k; ++j) {
11     int u = lca(stk.top(), a[j]);
12     while (dep[stk.top()] > dep[u]) --top;
13     assert(stk.top() == u);
14     stk.push(a[j]);
15     addege(u, a[j], dis[a[j]] - dis[u]);
16 }

```

6 图

6.1 Tarjan 有向图强联通分量

① 割点的判断: 一个顶点 u 是割点, 当且仅当满足 (1) 或 (2): (1) u 为树根, 且 u 有多于一个子树 (即: 存在一个儿子 v 使得 $dfn[u]+1 \neq dfn[v]$); (2) u 不为树根, 且满足存在 (u, v) 为树枝边 (u 为 v 的父亲), 使得 $dfn[u] \leq low[v]$ 。② 桥

的判断:一条无向边 (u, v) 是桥,当且仅当 (u, v) 为树枝边,满足 $dfn[u] < low[v]$

。

```

1 struct EDGE { int adj, next; } edge[M];
2 int n, m, top, gh[N];
3 int dfn[N], low[N], cnt, ind, stop, instack[N], stack[N], belong[N];
4 void addedge(int x, int y) { edge[++top].adj = y; edge[top].next = gh[x]; gh[x]
    = top; }
5 void tarjan(int x) {
6     dfn[x] = low[x] = ++ind;
7     instack[x] = 1; stack[++stop] = x;
8     for (int p=gh[x]; p; p=edge[p].next)
9         if (!dfn[edge[p].adj]) tarjan(edge[p].adj), low[x] = min(low[x], low[
            edge[p].adj]);
10        else if (instack[edge[p].adj]) low[x] = min(low[x], dfn[edge[p].adj]);
11    if (dfn[x] == low[x]) {
12        ++cnt; int tmp=0;
13        while (tmp!=x) tmp = stack[stop--], belong[tmp] = cnt, instack[tmp] =
            0;
14    }
15 }

```

6.2 Tarjan 双联通分量

以下代码为点双联通分量。若要更改为边双联通,在第8行将 $low[next] \geq dfn[x]$ 改为 $low[next] > dfn[x]$,并将14行 $vec[tot].push_back(x)$ 删除。

```

1 void DFS(int x,int fa) {
2     vis[x]=true; dfn[x]=low[x]=++times; sk[++tp]=x;
3     for (int pt=first[x];pt;pt=e[pt].next) {
4         int next=e[pt].to; if (e[pt].id==fa) continue;
5         if (!vis[next]) {
6             DFS(next,e[pt].id);
7             low[x]=min(low[x],low[next]);
8             if (low[next]>=dfn[x]) { // ***
9                 vec[++tot].clear();
10                while (tp) {
11                    vec[tot].push_back(sk[tp--]);
12                    if (sk[tp+1]==next) break;
13                }
14                vec[tot].push_back(x); // ***
15            }
16        } else if (dfn[next]>last) low[x]=min(low[x],dfn[next]);
17    }

```

```

18 }
19 for (i=1;i<=n;i++) if (!vis[i]) {
20     DFS(i,0); last=times;
21     if (tp) {
22         tot++; vec[tot].clear();
23         for (i=1;i<=tp;i++) vec[tot].push_back(sk[i]);
24         tp=0;
25     }
26 }

```

6.3 欧拉回路

```

1 struct E { int to,ne; } e[M<<1];
2 int t,n,m,la[N],e_top;
3 int in[N],out[N];
4 void add(int x, int y){
5     out[x]++; in[y]++;
6     e[++e_top]=(E){y,la[x]}; la[x]=e_top;
7 }
8 int sta[M],top;
9 bool vis[M<<1];
10 void dfs(int x){
11     for(int i=la[x]; i; i=la[x]){
12         la[x]=e[i].ne;
13         if (vis[i]) continue;
14         vis[i]=true; if (t==1) vis[i^1]=true;
15         dfs(e[i].to);
16         if (t==2) sta[++top]=i;
17         else sta[++top]=(i&1)?(-(i>>1)):(i>>1);
18     }
19 }
20 int main(){
21     scanf("%d%d%d",&t,&n,&m);
22     if (m==0) YES(); if (t==1) e_top=1;
23     ft(1,1,m){ scanf("%d%d",&x,&y); add(x,y); if (t==1) add(y,x); }
24     if (t==1) ft(1,1,n) if (in[i]&1) NO();
25     if (t==2) ft(1,1,n) if (in[i]!=out[i]) NO();
26     dfs(e[3-t].to); if (top!=m) NO();
27     YES(); fd(1,top,1) printf("%d_",sta[i]);
28 }

```

6.4 带花树

```

1  const int N=550;
2  struct E { int to,ne; } e[N*N];
3  int n,m,la[N],e_top,f[N];
4  int find(int x) { return f[x]=f[x]==x?x:find(f[x]); }
5  int mat[N],pre[N],cond[N],q[N],l,r,vis[N],vt;
6  int lca(int x, int y){
7      vt++; x=find(x); y=find(y);
8      while (vis[x]!=vt){ if(x){vis[x]=vt;x=find(pre[mat[x]]);} swap(x,y); }
9      return x;
10 }
11 void blossom(int x, int y, int g){
12     while (find(x)!=g){
13         pre[x]=y; if (cond[mat[x]]==1) cond[q[++r]=mat[x]]=0;
14         if (f[x]==x) f[x]=g; if (f[mat[x]]==mat[x]) f[mat[x]]=g;
15         y=mat[x]; x=pre[y];
16     }
17 }
18 int match(int s){
19     forto(i,1,n){ cond[i]=-1; pre[i]=0; f[i]=i; }
20     cond[q[l=r=1]=s]=0;
21     while (l<=r){ int x=q[l++];
22         forE(i,x){
23             int y=e[i].to;
24             if (cond[y]==-1){
25                 if (mat[y]==0){
26                     while (x){
27                         int t=mat[x]; mat[x]=y; mat[y]=x; y=t; x=pre[y];
28                     }
29                     return true;
30                 }
31                 cond[y]=1; pre[y]=x; cond[q[++r]=mat[y]]=0;
32             } else if (find(x)!=find(y) && cond[y]==0){
33                 int g=lca(x,y); blossom(x,y,g); blossom(y,x,g);
34             }
35         }
36     }
37     return false;
38 }
39 int main(){
40     scanf("%d%d",&n,&m); int ans=0;
41     while (m--){ scanf("%d%d",&x,&y); add(x,y); add(y,x); }
42     forto(i,1,n) if (!mat[i] && match(i)) ans++;
43     printf("%d\n",ans); forto(i,1,n) printf("%d_",mat[i]);

```

44 }

6.5 KM 算法

```

1  const int N=500, inf=0x7fffffff;
2  int n,fx[N],fy[N],pre[N];
3  LL w[N][N],lx[N],ly[N],sla[N];
4  bool vx[N],vy[N],a[N][N];
5  int q[N],l,r;
6  bool check(int x, int y){
7      if (!fy[y]){
8          while (x){ int t=fx[x]; fx[x]=y; fy[y]=x; y=t; x=pre[y]; }
9          return true;
10     }
11     vy[y]=true; pre[y]=x; vx[q[++r]=fy[y]]=true; return false;
12 }
13 void bfs(int s){
14     ft(i,1,n) { vx[i]=vy[i]=false; sla[i]=inf; }
15     vx[q[l=r=1]=s]=true;
16     while (true){
17         while (l<=r){
18             int x=q[l++];
19             ft(y,1,n) if (!vy[y]){
20                 LL t=lx[x]+ly[y]-w[x][y];
21                 if (t==0 && check(x,y)) return;
22                 if (t && t<sla[y]) { sla[y]=t; pre[y]=x; }
23             }
24         }
25         int d=inf;
26         ft(y,1,n) if (!vy[y]) cmin(d,sla[y]);
27         ft(x,1,n) if (vx[x]) lx[x]-=d;
28         ft(y,1,n) if (vy[y]) ly[y]+=d; else sla[y]-=d;
29         ft(y,1,n) if (!vy[y] && !sla[y] && check(pre[y],y)) return;
30     }
31 }
32 void KM(){
33     ft(x,1,n) { lx[x]=w[x][1]; ft(y,2,n) cmax(lx[x],w[x][y]); }
34     ft(s,1,n) bfs(s);
35 }
36 int main(){
37     int nl,nr,m; scanf("%d%d%d",&nl,&nr,&m);
38     while (m--){ scanf("%d%d%d",&x,&y,&z); w[x][y]=z; a[x][y]=true; }
39     n=MAX(nl,nr); KM();
40     LL ans=0; ft(i,1,n) ans+=lx[i]; ft(j,1,n) ans+=ly[j];

```

```

41     printf("%lld\n",ans);
42     ft(i,1,nl) printf("%d_",a[i][fx[i]]?fx[i]:0);
43 }

```

6.6 2-SAT

记 $x \rightarrow y$ 的有向边表示选了 x 就要选 y 。

```

1 struct MergePoint {
2     struct EDGE { int adj, next; } edge[M];
3     int ex[M], ey[M]; bool instack[N];
4     int gh[N], top, dfn[N], low[N], cnt, ind, stop, stack[N], belong[N];
5     void init() { cnt = ind = stop = top = 0; memset(dfn, 0, sizeof(dfn));
6         memset(instack, 0, sizeof(instack)); memset(gh, 0, sizeof(gh)); }
7     void addedge(int x, int y) { swap(x, y); edge[++top].adj = y; edge[top].
8         next = gh[x]; gh[x] = top; ex[top] = x; ey[top] = y; }
9     void tarjan(int x) {}
10    void work() { for (i) if (!dfn[i]) tarjan(i); }
11 } merge;
12 struct Topsort {
13     struct EDGE { int adj, next; } edge[M];
14     int n, top, gh[N], ops[N], deg[N], ans[N]; std::queue<int> q;
15     void init() { n = merge.cnt; top = 0; memset(gh, 0, sizeof(gh)); memset(deg
16         , 0, sizeof(deg)); }
17     void addedge(int x, int y) { if (x == y) return; edge[++top].adj = y; edge[
18         top].next = gh[x]; gh[x] = top; ++deg[y]; }
19     void work() {
20         for (int i = 1; i <= n; ++i) if (!deg[i]) q.push(i);
21         while (!q.empty()) {
22             int x = q.front(); q.pop();
23             for (int p = gh[x]; p; p = edge[p].next) if (!--deg[edge[p].adj]) q
24                 .push(edge[p].adj);
25             if (ans[x]) continue; ans[x] = -1; ans[ops[x]] = 1; //-1 NO, 1 YES
26         }
27     }
28 } ts;
29 merge.init(); merge.addedge(); merge.work();
30 for (int i = 1; i <= n; ++i) {
31     int x = merge.belong[U(i, 0)], y = merge.belong[U(i, 1)];
32     if (x==y) NO(); ts.ops[x]=y; ts.ops[y]=x;
33 }
34 ts.init(); ts.work();
35 puts("YES"); for (int i = 1; i <= n; ++i) select(ts.ans[merge.belong[U(i,1)]] ==
36     1);

```

6.7 网络流

6.7.1 最大流

注意: top 要初始化为 1

```

1 struct EDGE { int adj, w, next; } edge[M];
2 int n, top, gh[N], nrl[N], dist[N], q[N];
3 void addedge(int x, int y, int w) { edge[++top].adj = y; edge[top].w = w; edge[
4     top].next = gh[x]; gh[x] = top; edge[++top].adj = x; edge[top].w = 0; edge
5     [top].next = gh[y]; gh[y] = top; }
6 int bfs() {
7     memset(dist, 0, sizeof(dist));
8     q[1] = S; int head = 0, tail = 1; dist[S] = 1;
9     while (head != tail) {
10         int x = q[++head];
11         for (int p=gh[x]; p; p=edge[p].next)
12             if (edge[p].w && !dist[edge[p].adj]) {
13                 dist[edge[p].adj] = dist[x] + 1;
14                 q[++tail] = edge[p].adj;
15             }
16     }
17     return dist[T];
18 }
19 int dinic(int x, int delta) {
20     if (x==T) return delta;
21     for (int& p=nrl[x]; p && delta; p=edge[p].next)
22         if (edge[p].w && dist[x]+1 == dist[edge[p].adj]) {
23             int dd = dinic(edge[p].adj, min(delta, edge[p].w));
24             if (!dd) continue;
25             edge[p].w -= dd;
26             edge[p^1].w += dd;
27             return dd;
28         }
29     return 0;
30 }
31 int ans = 0; while (bfs()) { memcpy(nrl, gh, sizeof(gh)); int t; while (t =
32     dinic(S, inf)) ans += t; } return ans;

```

6.7.2 上下界有源汇网络流

① T 向 S 连容量为正无穷的边, 将有源汇转化为无源汇。② 每条边容量减去下界, 设 $in[i]$ 表示流入 i 的下界之和减去流出 i 的下界之和。③ 新建超级源汇 SS, TT , 对于 $in[i] > 0$ 的点, SS 向 i 连容量为 $in[i]$ 的边。对于 $in[i] < 0$

的点, i 向 TT 连容量为 $-in[i]$ 的边。④求出以 SS, TT 为源汇的最大流, 如果等于 $\sum in[i] (in[i] > 0)$, 则存在可行流。再求出 S, T 为源汇的最大流即为最大流。⑤费用流: 建完图后等价于求以 SS, TT 为源汇的费用流。

6.7.3 费用流

注意: top 要初始化为 1

```
1 struct EDGE { int adj, w, cost, next; } edge[M*2];
2 int gh[N], q[N], dist[N], v[N], pre[N], prev[N], top, S, T;
3 void addedge(int x, int y, int w, int cost) {x->y(w, cost); y->x(0, -cost);}
4 void clear() { top = 1; memset(gh, 0, sizeof(gh)); }
5 bool spfa() {} // 从S出发, 返回dist[T] != inf
6 int ans = 0;
7 while (spfa()) {
8     int mx = inf;
9     for (int x=T; x!=S; x=pre[x]) mx = min(edge[prev[x]].w, mx);
10    ans += dist[T] * mx;
11    for (int x=T; x!=S; x=pre[x]) edge[prev[x]].w -= mx, edge[prev[x]^1].w += mx;
12 }
13 return ans;
```

7 杂项

7.1 读入优化

`int rd(int &x);` 读入一个整数, 保存在变量 x 中。如正常读入, 返回值为 1, 否则返回 EOF (-1)

```
1 #define rd RD<int>
2 #define rdll RD<long long>
3 const int S = 2000000; // 2MB
4 char s[S], *h = s+S, *t = h;
5 inline char getchrr(void) {
6     if(h == t) { if (t != s + S) return EOF; t = s + fread(s, 1, S, stdin); h = s; }
7     return *h++;
8 }
9 template <class T>
10 inline int RD(T &x) {
11     char c = 0; int sign = 0;
12     for (; !isdigit(c); c = getchrr()) {
```

```
13         if (c == EOF) return -1; if (c == '-') sign ^= 1;
14     }
15     x = 0; for (; isdigit(c); c = getchrr()) x = x * 10 + c - '0';
16     if (sign) x = -x; return 1;
17 }
```

7.2 Vim

```
1 syntax on
2 set cindent
3 set nu
4 set tabstop=4
5 set shiftwidth=4
6 set background=dark
7
8 inoremap <C-j> <down>
9 inoremap <C-k> <up>
10 inoremap <C-h> <left>
11 inoremap <C-l> <right>
```

7.3 Java

```
1 头文件
2 import java.math.*;
3 import java.util.*;
4 public class Main {
5     public static void main(String []args) {
6     }
7 }
8 输入输出
9 Scanner cin = new Scanner(System.in);
10 int a = cin.nextInt();
11 BigDecimal a = cin.nextBigDecimal();
12 while (cin.hasNext()) {} // 输入到 EOF 结束
13 System.out.println(str); // 有换行
14 System.out.print(str); // 无换行
15 System.out.println("Hello, _%s._Next_year, _you'll_be_%d", name, age); // C风格输出
16 大数类
17 BigInteger a = BigInteger.valueOf(12);
18 BigInteger b = new BigInteger(String.valueOf(12));
19 BigDecimal c = BigDecimal.valueOf(12.0);
```

```
20 BigDecimal d = new BigDecimal("12.0"); // 字符串防止double精度误差
21 大数比较
22 c.compareTo(BigDecimal.ZERO)==0 //判断相等, c==0
23 c.compareTo(BigDecimal.ZERO)>0 //判断大于, c>0
24 c.compareTo(BigDecimal.ZERO)<0 //判断小于, c<0
25 大数基本运算
26 Big*** add(Big*** b) // 加上b
27 Big*** subtract(Big*** b) // 减去b
28 Big*** multiply(Big*** b) // 乘b
29 Big*** divide(Big*** b) // 除以b
30 BigDecimal divide(BigDecimal b, int 精确位数, BigDecimal.ROUND_HALF_UP); // 除以b, 保留小数
31 Big*** pow(int b) // this^b
32 Big*** remainder(Big*** b) // mod b
33 Big*** abs() // 绝对值
34 Big*** negate() // 取负号
35 Big*** max(Big*** b) // 返回this和b中的最大值
36 Big*** min(Big*** b) // 返回this和b中的最小值
37 BigInteger gcd(BigInteger val) // 返回abs(this)和abs(val)的最大公约数
38 BigInteger mod(BigInteger val) // 求 this mod val
39 BigInteger modInverse(BigInteger val) // 求逆元, 返回 this^(-1) mod val
40 大数格式控制
41 toString()将BigDecimal转成字符串, 然后配合一些字符串函数进行处理:
42 str.startsWith("0"); // 以0开始
43 str.endsWith("0"); // 以0结束
44 str.substring(int x, int y); // 从x到y的str的子串
45 str.substring(int x); // 从x到结尾的子串
46 c.stripTrailingZeros().toPlainString(); // c去除末尾0, 转成普通字符串
47 setScale(int newScale, RoundingMode roundingMode) 返回BigDecimal。newScale表示保留位数。CEILING/DOWN/FLOOR/HALF_DOWN/HALF_UP。
48 大数进制转换
49 支持2~36进制 (0-9 + 小写a-z)
50 BigInteger a=cin.nextBigInteger(2); // 读入一个二进制数
51 System.out.println(a.toString(2)); // 输出二进制
```