ICPC Templates For Africamonkey

Africamonkey

2017 年 8 月 19 日

目录

1 莫队算法

1.1 普通莫队

```
struct Q { int l, r, sqrtl, id; } q[N];
int n, m, v[N], ans[N], nowans;
bool cmp(const Q &a, const Q &b) {
    if (a.sqrtl != b.sqrtl) return a.sqrtl < b.sqrtl;
    return a.r < b.r;
}
void change(int x) { if (!v[x]) checkin(); else checkout(); }
int main() {
    ......
    for (int i=1;i<=m;i++) q[i].sqrtl = q[i].l / sqrt(n), q[i].id = i;
    sort(q+1, q+m+1, cmp);
    int L=1,R=0; nowans=0;
    memset(v, 0, sizeof(v));
    for (int i=1;i<=m;i++) {
        while (L<q[i].l) change(L++);
        while (L>q[i].l) change(--L);
        while (R<q[i].r) change(++R);
        while (R>q[i].r) change(R--);
        ans[q[i].id] = nowans;
    }
    ......
}
```

1.2 树上莫队

```
struct Query { int l, r, id, l_group; } query[N];
struct EDGE { int adj, next; } edge[N*2];
int n, m, top, gh[N], c[N], reorder[N], deep[N], father[N], size[N], son[N], Top[N];
void addedge(int x, int y) {
    edge[++top].adj = y;
    edge[top].next = gh[x];
    gh[x] = top;
}
void dfs(int x, int root=0) {
    reorder[x] = ++top; father[x] = root; deep[x] = deep[root] + 1;
    son[x] = 0; size[x] = 1; int dd = 0;
    for (int p=gh[x]; p; p=edge[p].next)
        if (edge[p].adj != root) {
            dfs(edge[p].adj, x);
            if (size[edge[p].adj] > dd) {
                son[x] = edge[p].adj;
                dd = size[edge[p].adj];
            }
            size[x] += size[edge[p].adj];
        }
```

```
}
void split(int x, int tp) {
    Top[x] = tp;
    if (son[x]) split(son[x], tp);
    for (int p=gh[x]; p; p=edge[p].next)
        if (edge[p].adj != father[x] && edge[p].adj != son[x])
            split(edge[p].adj, edge[p].adj);
}
int lca(int x, int y) {
    int tx = Top[x], ty = Top[y];
    while (tx != ty) {
        if (deep[tx] < deep[ty]) {
            swap(tx, ty);
            swap(x, y);
        }
        x = father[tx];
        tx = Top[x];
    }
    if (deep[x] < deep[y]) swap(x, y);
    return y;
}
bool cmp(const Query &a, const Query &b) {
    if (a.l_group != b.l_group) return a.l_group < b.l_group;
    return reorder[a.r] < reorder[b.r];
}
int v[N], ans[N];
void upd(int x) { if (!v[x]) checkin(); else checkout(); }
void go(int &u, int taru, int v) {
    int lca0 = lca(u, taru);
    int lca1 = lca(u, v);   upd(lca1);
    int lca2 = lca(taru, v); upd(lca2);
    for (int x=u; x!=lca0; x=father[x]) upd(x);
    for (int x=taru; x!=lca0; x=father[x]) upd(x);
    u = taru;
}
int main() {
    memset(gh, 0, sizeof(gh));
    scanf("%d%d", &n, &m); top = 0;
    for (int i=1;i<n;i++) {
        int x,y; scanf("%d%d", &x, &y);
        addedge(x, y); addedge(y, x);
    }
    top = 0; dfs(1); split(1, 1);
    for (int i=1;i<=m;i++) {
        if (reorder[query[i].l] > reorder[query[i].r])
            swap(query[i].l, query[i].r);
        query[i].id = i;
        query[i].l_group = reorder[query[i].l] / sqrt(n);
    }
    sort(query+1, query+m+1, cmp);
```

```
    int L=1,R=1; upd(1);
    for (int i=1;i<=m;i++) {
        go(L,query[i].l,R);
        go(R,query[i].r,L);
        ans[query[i].id] = answer();
    }
    ......
}
```

2 字符串

2.1 哈希

```
const int P=31,D=1000173169;
int n, pow[N], f[N]; char a[N];
int hash(int l, int r) { return (LL)(f[r]-(LL)f[l-1]*pow[r-l+1]%D+D)%D; }
int main() {
    scanf("%d%s", &n, a+1);
    pow[0] = 1;
    for (int i=1;i<=n;i++) pow[i] = (LL)pow[i-1]*P%D;
    for (int i=1;i<=n;i++) f[i] = (LL)((LL)f[i-1]*P+a[i])%D;
}
```

2.2 KMP

接口：void kmp(int n, char *a, int m, char *b);

输入：模式串长度 n ，模式串 a ，匹配串长度 m ，匹配串 b

输出：依次输出每个匹配成功的起始位置

下标从 0 开始。

```
void kmp(int n, char* a, int m, char *b) {
    int i, j;
    for (nxt[0] = j = -1, i = 1; i < n; nxt[i++] = j) {
        while (~j && a[j + 1] != a[i]) j = nxt[j];
        if (a[j + 1] == a[i]) ++j;
    }
    for (j = -1, i = 0; i < m; ++i) {
        while (~j && a[j + 1] != b[i]) j = nxt[j];
        if (a[j + 1] == b[i]) ++j;
        if (j == n - 1) {
            printf("%d\n", i - (n - 1) + 1);
            j = nxt[j];
        }
    }
}
```

2.3 可动态修改的 KMP

支持：加入一个字符，删除一个字符。

时间复杂度：$O(n\alpha)$ ，α 为字符集大小。

代码中的字符为 $'0' -' 9'$ ，可自行修改为 $'a' -' z'$

```
char t[N];
int top, nxt[N], nxt_l[N][10];
inline void del_letter() { --top; }
inline void add_letter(char x) {
    t[top++] = x;
    int j = top-1;
    memset(nxt_l[top], 0, sizeof(nxt_l[top]));
    nxt[top] = nxt_l[top-1][x-'0'];
    memcpy(nxt_l[top], nxt_l[nxt[top]], sizeof(nxt_l[nxt[top]]));
    nxt_l[top][t[nxt[top]]-'0'] = nxt[top]+1;
}
```

2.4 扩展 KMP

接口：void ExtendedKMP(char *a, char *b, int *next, int *ret);

输出：

next: a 关于自己每个后缀的最长公共前缀

ret: a 关于 b 的每个后缀的最长公共前缀

EXKMP 的 next[i] 表示：从 i 到 n-1 的字符串 st 前缀和原串前缀的最长重叠长度。

```
void get_next(char *a, int *next) {
    int i, j, k;
    int n = strlen(a);
    for (j = 0; j+1<n && a[j]==a[j+1];j++);
    next[1] = j;
    k = 1;
    for (i=2;i<n;i++) {
        int len = k+next[k], l = next[i-k];
        if (l < len-i) {
            next[i] = l;
        } else {
            for (j = max(0, len-i);i+j<n && a[j]==a[i+j];j++);
            next[i] = j;
            k = i;
        }
    }
}
void ExtendedKMP(char *a, char *b, int *next, int *ret) {
    get_next(a, next);
    int n = strlen(a), m = strlen(b);
    int i, j, k;
    for (j=0;j<n && j<m && a[j]==b[j];j++);
    ret[0] = j;
    k = 0;
```

```
    for (i=1;i<m;i++) {
        int len = k+ret[k], l = next[i-k];
        if (l < len-i) {
            ret[i] = l;
        } else {
            for (j = max(0, len-i);j<n && i+j<m && a[j]==b[i+j];j++);
            ret[i] = j;
            k = i;
        }
    }
}
```

2.5 Manacher

p[i] 表示以 i 为对称轴的最长回文串长度

```
char st[N*2], s[N];
int len, p[N*2];

while (scanf("%s", s) != EOF) {
    len = strlen(s);
    st[0] = '$', st[1] = '#';
    for (int i=1;i<=len;i++)
        st[i*2] = s[i-1], st[i*2+1] = '#';
    len = len * 2 + 2;
    int mx = 0, id = 0, ans = 0;
    for (int i=1;i<=len;i++) {
        p[i] = (mx > i) ? min(p[id*2-i]+1, mx-i) : 1;
        for (; st[i+p[i]] == st[i-p[i]]; ++p[i]) ;
        if (p[i]+i > mx) mx = p[i]+i, id = i;
        p[i] --;
        if (p[i] > ans) ans = p[i];
    }
    printf("%d\n", ans);
}
```

2.6 最小表示法

```
string smallestRepresation(string s) {
    int i, j, k, l;
    int n = s.length();
    s += s;
    for (i=0,j=1;j<n;) {
        for (k=0;k<n && s[i+k]==s[j+k];k++);
        if (k>=n) break;
        if (s[i+k]<s[j+k]) j+=k+1;
        else {
            l=i+k;
            i=j;
```

```
            j=max(l,j)+1;
        }
    }
    return s.substr(i, n);
}
```

2.7 AC 自动机

```
struct Node {
    int next[**Size of Alphabet**];
    int terminal, fail;
} node[**Number of Nodes**];
int top;
void add(char *st) {
    int len = strlen(st), x = 1;
    for (int i=0;i<len;i++) {
        int ind = trans(st[i]);
        if (!node[x].next[ind])
            node[x].next[ind] = ++top;
        x = node[x].next[ind];
    }
    node[x].terminal = 1;
}
int q[**Number of Nodes**], head, tail;
void build() {
    head = 0, tail = 1; q[1] = 1;
    while (head != tail) {
        int x = q[++head];
        /*(when necessary) node[x].terminal |= node[node[x].fail].terminal; */
        for (int i=0;i<n;i++)
            if (node[x].next[i]) {
                if (x == 1) node[node[x].next[i]].fail = 1;
                else {
                    int y = node[x].fail;
                    while (y) {
                        if (node[y].next[i]) {
                            node[node[x].next[i]].fail = node[y].next[i];
                            break;
                        }
                        y = node[y].fail;
                    }
                    if (!node[node[x].next[i]].fail) node[node[x].next[i]].fail = 1;
                }
                q[++tail] = node[x].next[i];
            }
    }
}
```

2.8 后缀数组

2.8.1 倍增算法

参数 m 表示字符集的大小，即 $0 \le r_i < m$

```
#define rank rank2
int n, r[N], wa[N], wb[N], ws[N], sa[N], rank[N], height[N];
int cmp(int *r, int a, int b, int l, int n) {
    if (r[a]==r[b]) {
        if (a+l<n && b+l<n && r[a+l]==r[b+l])
            return 1;
    }
    return 0;
}
void suffix_array(int m) {
    int i, j, p, *x=wa, *y=wb, *t;
    for (i=0;i<m;i++) ws[i]=0;
    for (i=0;i<n;i++) ws[x[i]=r[i]]++;
    for (i=1;i<m;i++) ws[i]+=ws[i-1];
    for (i=n-1;i>=0;i--) sa[--ws[x[i]]]=i;
    for (j=1,p=1;p<n;m=p,j<<=1) {
        for (p=0,i=n-j;i<n;i++) y[p++]=i;
        for (i=0;i<n;i++) if (sa[i]>=j) y[p++]=sa[i]-j;
        for (i=0;i<m;i++) ws[i]=0;
        for (i=0;i<n;i++) ws[x[y[i]]]++;
        for (i=1;i<m;i++) ws[i]+=ws[i-1];
        for (i=n-1;i>=0;i--) sa[--ws[x[y[i]]]]=y[i];
        for (t=x,x=y,y=t,x[sa[0]]=0,i=1,p=1;i<n;i++)
            x[sa[i]]=cmp(y,sa[i-1],sa[i],j,n)?p-1:p++;
    }
    for (i=0;i<n;i++) rank[sa[i]]=i;
    rank[n] = -1;
}
void calc_height() {
    int j=0;
    for (int i=0;i<n;i++)
        if (rank[i])
        {
            while (r[i+j]==r[sa[rank[i]-1]+j]) j++;
            height[rank[i]]=j;
            if (j) j--;
        }
}
```

2.8.2 DC3 算法

注意：

N 至少为字符串长度的 3 倍

接口：suffix_array(int *r, int *sa, int n, int m);

r 表示字符串，sa 为后缀数组输出，n 表示字符串长度，下标从 0 开始。m 为字符集大小。

```
#define F(x) ((x)/3 + ((x)%3 == 1 ? 0:tb))
#define G(x) ((x) < tb ? (x)*3+1 : ((x)-tb)*3 + 2)
#define rank rank2

int r[N], wa[N], wb[N], ws[N], wv[N], sa[N], rank[N];

int c0(int *r,int a,int b) {
    return r[a]==r[b]&&r[a+1]==r[b+1]&&r[a+2]==r[b+2];
}

int c12(int k,int *r,int a,int b) {
    if(k==2) return r[a]<r[b]||r[a]==r[b]&&c12(1,r,a+1,b+1);
    else return r[a]<r[b]||r[a]==r[b]&&wv[a+1]<wv[b+1];
}

void dsort(int *r,int *a,int *b,int n,int m) {
    int i;for(i=0;i<n;i++) wv[i]=r[a[i]];
    for(i=0;i<m;i++) ws[i]=0;
    for(i=0;i<n;i++) ws[wv[i]]++;
    for(i=1;i<m;i++) ws[i]+=ws[i-1];
    for(i=n-1;i>=0;i--) b[--ws[wv[i]]]=a[i];
}

void dc3(int *r,int *sa,int n,int m) {
    int i,j,*rn=r+n,*san=sa+n,ta=0,tb=(n+1)/3,tbc=0,p;
    r[n]=r[n+1]=0;
    for(i=0;i<n;i++) if(i%3!=0) wa[tbc++]=i;
    dsort(r+2,wa,wb,tbc,m);
    dsort(r+1,wb,wa,tbc,m);
    dsort(r,wa,wb,tbc,m);
    for(p=1,rn[F(wb[0])]=0,i=1;i<tbc;i++) rn[F(wb[i])]=c0(r,wb[i-1],wb[i])?p-1:p++;
    if(p<tbc) dc3(rn,san,tbc,p);
    else for(i=0;i<tbc;i++) san[rn[i]]=i;
    for(i=0;i<tbc;i++) if(san[i]<tb) wb[ta++]=san[i]*3;
    if(n%3==1) wb[ta++]=n-1;
    dsort(r,wb,wa,ta,m);
    for(i=0;i<tbc;i++) wv[wb[i]=G(san[i])]=i;
    for(i=0,j=0,p=0;i<ta && j<tbc;p++)sa[p]=c12(wb[j]%3,r,wa[i],wb[j])?wa[i++]:wb[j++];
    for(;i<ta;p++) sa[p]=wa[i++];
    for(;j<tbc;p++) sa[p]=wb[j++];
}

void suffix_array(int *r, int *sa, int n, int m) {
    dc3(r, sa, n + 1, m);
    int top = 0;
    for (int i = 0; i < n + 1; ++i)
        if (sa[i] < n) sa[top++] = sa[i];
    for (int i = 0; i < n; ++i) rank[sa[i]] = i;
```

```
    rank[n] = -1;
}
```

2.8.3 小技巧：拼接字符串

接口：

int gao1(int l, int r, int c, int p); 区间 $[l,r)$ 中保证第 0 位到第 $c-1$ 位都是相同的（设为字符串 s ），现在我们在 s 后面接一个字符 p ，得到一个新的字符串 s' 。返回值为最小的 k 满足后缀 $sa[k]$ 前 $c+1$ 位为 s'

int gao2(int l, int r, int c, int p); 区间 $[l,r)$ 中保证第 0 位到第 $c-1$ 位都是相同的（设为字符串 s ），现在我们在 s 后面接一个后缀 $sa[p]$ ，得到一个新的字符串 s' 。返回值为最小的 k 满足后缀 $sa[k]$ 前 $c+len(sa[p])$ 位为 s'

```
int gao1(int l,int r,int c,int p) {
        --l;
        while (l+1<r) {
                int md=(l+r)>>1;
                if (sa[md]+c<n&&s[sa[md]+c]>=p) r=md; else l=md;
        }
        return r;
}
int gao2(int l,int r,int c,int p) {
        --l;
        while (l+1<r) {
                int md=(l+r)>>1;
                if (sa[md]+c<=n&&rk[sa[md]+c]>=p) r=md; else l=md;
        }
        return r;
}
```

示例调用：

```
suf1[m] = -1, suf2[m] = n;
for (int i = m - 1; i >= 0; --i) {
    int l = gao1(0, n, 0, t[i]), r = gao1(0, n, 0, t[i]);
    suf1[i] = gao2(l, r, 1, suf1[i + 1]);
    suf2[i] = gao2(l, r, 1, suf2[i + 1]);
}
```

2.9 后缀自动机

下面的代码是求两个串的 LCS（最长公共子串）。

```
#include <cstdio>
#include <cstdlib>
#include <cstring>
#define N 500001
using namespace std;
char st[N];
```

```
int pre[N<<1], son[26][N<<1], step[N<<1], refer[N<<1], last, total;
int apply(int x, int now) { step[++total]=x; refer[total] = now; return total; }
void extend(char x, int now) {
    int p = last, np = apply(step[last]+1, now);
    for (; p && !son[x][p]; p=pre[p]) son[x][p] = np;
    if (!p) pre[np] = 1;
    else {
        int q = son[x][p];
        if (step[p]+1 == step[q]) pre[np] = q;
        else {
            int nq = apply(step[p]+1, now);
            for (int i=0;i<26;i++) son[i][nq] = son[i][q];
            pre[nq] = pre[q];
            pre[q] = pre[np] = nq;
            for (; p && son[x][p]==q; p=pre[p]) son[x][p] = nq;
        }
    }
    last = np;
}
void init() {
    last = total = 0;
    last = apply(0, 0);
    scanf("%s",st);
    for (int i=0; st[i]; i++)
        extend(st[i]-'a', i);
    scanf("%s",st);
}
int main() {
    init();
    int p = 1, now = 0, ans = 0;
    for (int i=0; st[i]; i++) {
        int index = st[i]-'a';
        for (; p && !son[index][p]; p = pre[p], now = step[p]) ;
        if (!p) p = 1;
        if (son[index][p]) {
            p = son[index][p];
            now++;
            if (now > ans) ans = now;
        }
    }
    printf("%d\n",ans);
    return 0;
}
```

一些定义和性质:

Right(str) 表示 str 在母串 S 中所有出现的结束位置集合

一个状态 s 表示的所有子串 Right 集合相同，为 Right(s)

Parent(s) 满足 Right(s) 是 Right(Parent(s)) 的真子集，并且 Right(Parent(s)) 的大小最小

Parent 函数可以表示一个树形结构。不妨叫它 Parent 树

一个 Right 集合和一个长度定义了一个子串

对于状态 s ，使得 Right(s) 合法的子串长度是一个区间 [min(s), max(s)]

max(Parent(s)) = min(s) - 1

令 refer(s) 表示产生 s 状态的字符所在位置。则 Right(s) 的合法子串的起始位置为 [refer(s) - max(s) + 1, refer(s) - min(s) + 1] ，即 [refer(s) - max(s) + 1, refer(s) - max(Parent(s))]

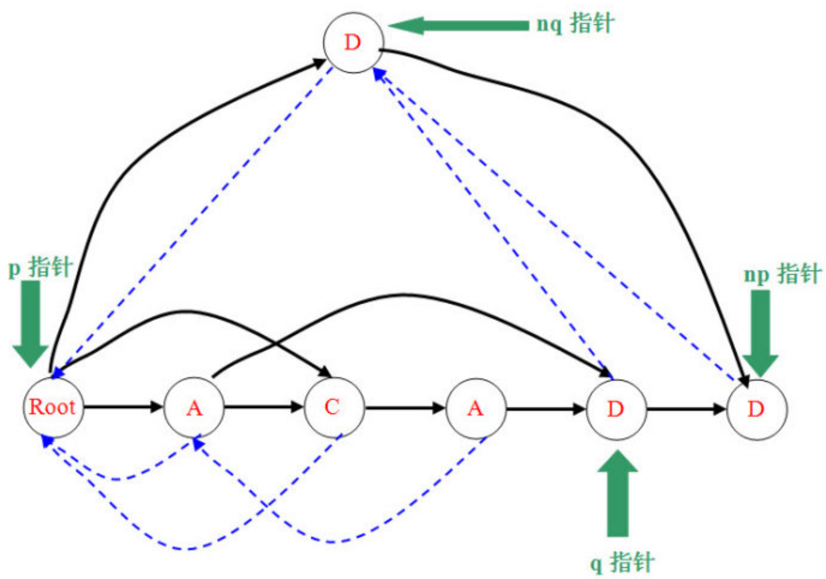


图 1: ACADD 构成的后缀自动机

我们发现 fail 构出一棵前缀树

和后缀树相同，为了使每个前缀都是叶子结点，我们不妨在串 s 前加入一个没出现的字符'#'

2.10 回文树

【URAL2040】Palindromes and Super Abilities 2

逐个添加字符串 S 里的字符 $S_1, S_2, ..., S_n$ 。每次添加字符后，他想知道添加字符后将出现多少个新的本质不同的回文子串。字符集为 $\{a, b\}$

```
#include <bits/stdc++.h>
#define N 5000020

char st[N], answer[N];
int n;

struct PAM {
    int n, tot, last;
    int len[N], fail[N], next[N][2];
    void init() {
        n=0; tot=1;
        len[1]=-1; fail[1]=0;
        len[0]=+0; fail[0]=1;
        last=1;
```

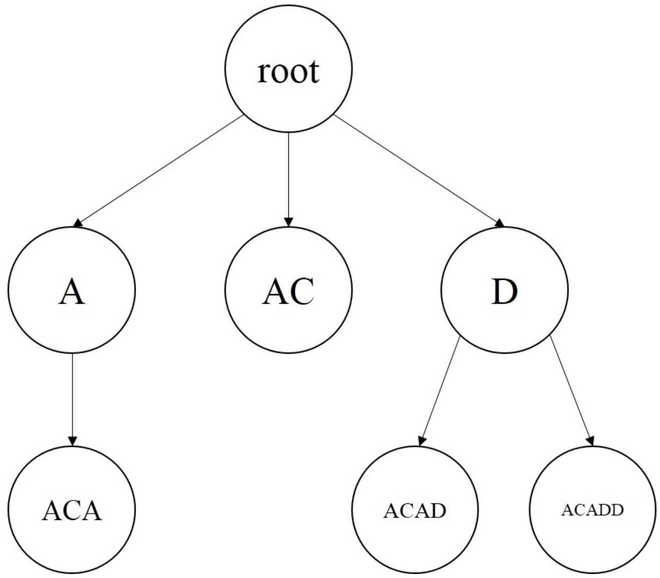


图 2: 串 ACADD 按 fail 构出的前缀树，与图 1 对应

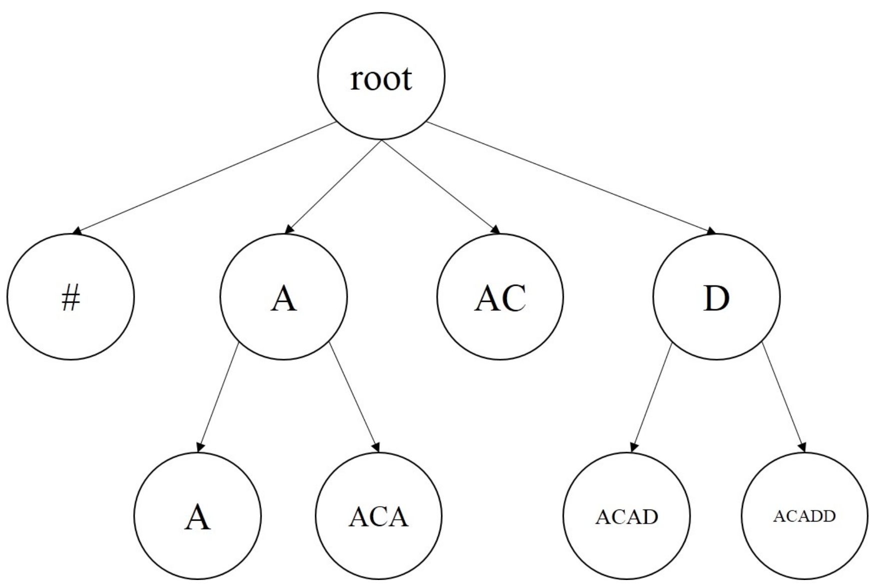


图 3: 串 #ACADD 按 fail 构出的前缀树

```
    }
    int get_fail(int x) {
        for (; st[n-len[x]-1]!=st[n]; x=fail[x]);
        return x;
    }
    void insert(char c) {
        ++n; int cur=get_fail(last); // 判断上一个串的前一个位置和新添加的位置是否相同，相同则说明构成回文。否则找 fail 指针。
        if (!next[cur][c]) {
            ++tot;
            len[tot]=len[cur]+2;
            fail[tot]=next[get_fail(fail[cur])][c];
            next[cur][c]=tot;
            answer[n]='1';
        } else {
            answer[n]='0';
        }
        last=next[cur][c];
    }
} pam;

int main() {
    scanf("%s", st+1); n=strlen(st+1);
    pam.init();
    for (int i=1;i<=n;i++) pam.insert(st[i]-'a');
    puts(answer+1);
    return 0;
}
```

3 数据结构

3.1 ST 表

```
int Log[N],f[17][N];
int ask(int x,int y){
    int k=Log[y-x+1];
    return max(f[k][x],f[k][y-(1<<k)+1]);
}
int main(){
    for (int i=2;i<=n;i++)Log[i]=Log[i>>1]+1;
    for (int j=1;j<K;j++)
        for (int i=1;i+(1<<j-1)<=n;i++)
            f[j][i]=max(f[j-1][i],f[j-1][i+(1<<j-1)]);
}
```

3.2 左偏树

左偏树是一个可并堆。

下面的程序写的是一个小根堆，如果需要改成大根堆请在注释了 here 那行修改。

接口：

void push(const T &x); 插入一个元素。

void merge(leftist &x); 合并两个堆。注意，合并后原来那个堆将不可访问。

T top() const; 返回堆顶元素。

void pop(); 删除堆顶元素。

int size() const; 返回堆的大小。

```
template <class T>
class leftist {
public:
    struct node {
        T key;
        int dist;
        node *l, *r;
    };
    leftist() : root(NULL), s(0) {}
    void push(const T &x) {
        leftist y;
        y.s = 1;
        y.root = new node;
        y.root -> key = x;
        y.root -> dist = 0;
        y.root -> l = y.root -> r = NULL;
        merge(y);
    }
    node* merge(node *x, node *y) {
        if (x == NULL) return y;
        if (y == NULL) return x;
        if (y -> key < x -> key) swap(x, y); //here
        x -> r = merge(x -> r, y);
        int ld = x -> l ? x -> l -> dist : -1;
        int rd = x -> r ? x -> r -> dist : -1;
        if (ld < rd) swap(x -> l, x -> r);
        if (x -> r == NULL) x -> dist = 0;
        else x -> dist = x -> r -> dist + 1;
        return x;
    }
    void merge(leftist &x) {
        root = merge(root, x.root);
        s += x.s;
    }
    T top() const {
        if (root == NULL) return T();
        return root -> key;
    }
    void pop() {
        if (root == NULL) return;
        node *p = root;
        root = merge(root -> l, root -> r);
```

```
        --s;
        delete p;
    }
    int size() const {
        return s;
    }
private:
    node* root;
    int s;
};
```

3.3 线段树小技巧

给定一个序列 a ，寻找一个最大的 i 使得 $i \le y$ 且满足一些条件（如 $a[i] \ge w$ ，那么需要在线段树维护 a 的区间最大值）

```
int queryl(int p, int left, int right, int y, int w) {
    if (right <= y) {
        if (! __condition__ ) return -1;
        else if (left == right) return left;
    }
    int mid = (left + right) / 2;
    if (y <= mid) return queryl(p<<1|0, left, mid, y, w);
    int ret = queryl(p<<1|1, mid+1, right, y, w);
    if (ret != -1) return ret;
    return queryl(p<<1|0, left, mid, y, w);
}
```

给定一个序列 a ，寻找一个最小的 i 使得 $i \ge x$ 且满足一些条件（如 $a[i] \ge w$ ，那么需要在线段树维护 a 的区间最大值）

```
int queryr(int p, int left, int right, int x, int w) {
    if (left >= x) {
        if (! __condition__ ) return -1;
        else if (left == right) return left;
    }
    int mid = (left + right) / 2;
    if (x > mid) return queryr(p<<1|1, mid+1, right, x, w);
    int ret = queryr(p<<1|0, left, mid, x, w);
    if (ret != -1) return ret;
    return queryr(p<<1|1, mid+1, right, x, w);
}
```

3.4 Splay

接口：

ADD x y d ： 将 $[x, y]$ 的所有数加上 d

REVERSE x y ： 将 $[x, y]$ 翻转

INSERT x p ： 将 p 插入到第 x 个数的后面

DEL x： 将第 x 个数删除

```
struct SPLAY {
    struct NODE {
        int w, min;
        int son[2], size, father, rev, lazy;
    } node[N];
    int top, rt;
    void pushdown(int x) {
        if (!x) return;
        if (node[x].rev) {
            node[node[x].son[0]].rev ^= 1;
            node[node[x].son[1]].rev ^= 1;
            swap(node[x].son[0], node[x].son[1]);
            node[x].rev = 0;
        }
        if (node[x].lazy) {
            node[node[x].son[0]].lazy += node[x].lazy;
            node[node[x].son[1]].lazy += node[x].lazy;
            node[x].w += node[x].lazy;
            node[x].min += node[x].lazy;
            node[x].lazy = 0;
        }
    }
    void pushup(int x) {
        if (!x) return;
        pushdown(node[x].son[0]);
        pushdown(node[x].son[1]);
        node[x].size = node[node[x].son[0]].size + node[node[x].son[1]].size + 1;
        node[x].min = node[x].w;
        if (node[x].son[0]) node[x].min = min(node[x].min, node[node[x].son[0]].min);
        if (node[x].son[1]) node[x].min = min(node[x].min, node[node[x].son[1]].min);
    }
    void sc(int x, int y, int w) {
        node[x].son[w] = y;
        node[y].father = x;
        pushup(x);
    }
    void _ins(int w) {
        top++;
        node[top].w = node[top].min = w;
        node[top].son[0] = node[top].son[1] = 0;
        node[top].size = 1; node[top].father = 0; node[top].rev = 0;
    }
    void init() {
        top = 0;
        _ins(0); _ins(0); rt=1;
        sc(1, 2, 1);
    }
```

```
    void rotate(int x) {
        if (!x) return;
        int y = node[x].father;
        int w = node[y].son[1]==x;
        sc(y, node[x].son[w^1], w);
        sc(node[y].father, x, node[node[y].father].son[1]==y);
        sc(x, y, w^1);
    }
    int q[N];
    void flushdown(int x) {
        int t=0; for (; x; x=node[x].father) q[++t]=x;
        for (; t; t--) pushdown(q[t]);
    }
    void Splay(int x, int root=0) {
        flushdown(x);
        while (node[x].father != root) {
            int y=node[x].father;
            int w=node[y].son[1]==x;
            if (node[y].father != root && node[node[y].father].son[w]==y) rotate(y);
            rotate(x);
        }
    }
    int find(int k) {
        Splay(rt);
        while (1) {
            pushdown(rt);
            if (node[node[rt].son[0]].size+1==k) {
                Splay(rt);
                return rt;
            } else
            if (node[node[rt].son[0]].size+1<k) {
                k-=node[node[rt].son[0]].size+1;
                rt=node[rt].son[1];
            } else {
                rt=node[rt].son[0];
            }
        }
    }
    int split(int x, int y) {
        int fx = find(x);
        int fy = find(y+2);
        Splay(fx);
        Splay(fy, fx);
        return node[fy].son[0];
    }
    void add(int x, int y, int d) { //add d to each number in a[x]...a[y]
        int t = split(x, y);
        node[t].lazy += d;
        Splay(t); rt=t;
    }
```

```
    void reverse(int x, int y) { // reverse the x-th to y-th elements
        int t = split(x, y);
        node[t].rev ^= 1;
        Splay(t); rt=t;
    }
    void insert(int x, int p) { // insert p after the x-th element
        int fx = find(x+1);
        int fy = find(x+2);
        Splay(fx);
        Splay(fy, fx);
        _ins(p);
        sc(fy, top, 0);
        Splay(top); rt=top;
    }
    void del(int x) { // delete the x-th element in Splay
        int fx = find(x), fy = find(x+2);
        Splay(fx); Splay(fy, fx);
        node[fy].son[0] = 0;
        Splay(fy); rt=fy;
    }
} tree;
```

3.5 可持久化 Treap

接口:

void insert(int x, char c); 在当前第 x 个字符后插入 c

void del(int x, int y); 删除第 x 个字符到第 y 个字符

void copy(int l, int r, int x); 复制第 l 个字符到第 r 个字符，然后粘贴到第 x 个字符后

void reverse(int x, int y); 翻转第 x 个到第 y 个字符

char query(int k); 表示询问当前第 x 个字符是什么

```
#define mod 1000000007
struct Treap {
    struct Node {
        char key;
        bool reverse;
        int lc, rc, size;
    } node[N];
    int n, root, rd;
    int Rand() { rd = (rd * 20372052LL + 25022087LL) % mod; return rd; }
    void init() { n = root = 0; }
    inline int copy(int x) { node[++n] = node[x]; return n; }
    inline void pushdown(int x) {
        if (!node[x].reverse) return;
        if (node[x].lc) node[x].lc = copy(node[x].lc);
        if (node[x].rc) node[x].rc = copy(node[x].rc);
        swap(node[x].lc, node[x].rc);
        node[node[x].lc].reverse ^= 1;
        node[node[x].rc].reverse ^= 1;
```

```
        node[x].reverse = 0;
    }
    inline void pushup(int x) { node[x].size = node[node[x].lc].size + node[node[x].rc].size + 1; }
    int merge(int u, int v) {
        if (!u || !v) return u+v;
        pushdown(u); pushdown(v);
        int t = Rand() % (node[u].size + node[v].size), r;
        if (t < node[u].size) {
            r = copy(u);
            node[r].rc = merge(node[u].rc, v);
        } else {
            r = copy(v);
            node[r].lc = merge(u, node[v].lc);
        }
        pushup(r);
        return r;
    }
    int split(int u, int x, int y) {
        if (x > y) return 0;
        pushdown(u);
        if (x == 1 && y == node[u].size) return u;
        if (y <= node[node[u].lc].size) return split(node[u].lc, x, y);
        int t = node[node[u].lc].size + 1;
        if (x > t) return split(node[u].rc, x-t, y-t);
        int num = copy(u);
        node[num].lc = split(node[u].lc, x, t-1);
        node[num].rc = split(node[u].rc, 1, y-t);
        pushup(num);
        return num;
    }
    void insert(int x, char c) {
        int t1 = split(root, 1, x), t2 = split(root, x+1, node[root].size);
        node[++n].key = c; node[n].size = 1;
        root = merge(merge(t1, n), t2);
    }
    void del(int x, int y) {
        int t1 = split(root, 1, x-1), t2 = split(root, y+1, node[root].size);
        root = merge(t1, t2);
    }
    void copy(int l, int r, int x) {
        int t1 = split(root, 1, x), t2 = split(root, l, r), t3 = split(root, x+1, node[root].size);
        root = merge(merge(t1, t2), t3);
    }
    void reverse(int x, int y) {
        int t1 = split(root, 1, x-1), t2 = split(root, x, y), t3 = split(root, y+1, node[root].size);
        node[t2].reverse ^= 1;
        root = merge(merge(t1, t2), t3);
```

```
    }
    char query(int k) {
        int x = root;
        while (1) {
            pushdown(x);
            if (k <= node[node[x].lc].size) x = node[x].lc;
            else
            if (k == node[node[x].lc].size + 1) return node[x].key;
            else
            k -= node[node[x].lc].size + 1, x = node[x].rc;
        }
    }
} treap;
```

3.6 可持久化并查集

接口：

void init() 初始化

void merge(int x, int y, int time) 在 time 时刻将 x 和 y 连一条边，注意加边顺序必须按 time 从小到大加边

void GetFather(int x, int time) 询问 time 时刻及以前的连边状态中，x 所属的集合

```
namespace pers_union {
    const int inf = 0x3f3f3f3f;
    int father[N], Father[N], Time[N];
    vector<int> e[N];
    void init() {
        for (int i=1;i<=n;i++) {
            father[i] = i;
            Father[i] = i;
            Time[i] = inf;
            e[i].clear();
            e[i].push_back(i);
        }
    }
    int getfather(int x) {
        return (father[x] == x) ? x : father[x] = getfather(father[x]);
    }
    int GetFather(int x, int time) {
        return (Time[x] <= time) ? GetFather(Father[x], time) : x;
    }
    void merge(int x, int y, int time) {
        int fx = getfather(x), fy = getfather(y);
        if (fx == fy) return;
        if (e[fx].size() > e[fy].size()) swap(fx, fy);
        father[fx] = fy;
        Father[fx] = fy;
        Time[fx] = time;
        for (int i=0;i<e[fx].size();i++) {
```

```
            e[fy].push_back(e[fx][i]);
        }
    }
};
```

4 树

4.1 点分治

初始化时须设置 $top = 1$ 。

```
void addedge(int x, int y) {
    edge[++top].adj = y;
    edge[top].valid = 1;
    edge[top].next = gh[x];
    gh[x] = top;
}
void get_size(int x, int root=0) {
    size[x] = 1; son[x] = 0;
    int dd = 0;
    for (int p=gh[x]; p; p=edge[p].next)
        if (edge[p].adj != root && edge[p].valid) {
            get_size(edge[p].adj, x);
            size[x] += size[edge[p].adj];
            if (size[edge[p].adj] > dd) {
                dd = size[edge[p].adj];
                son[x] = edge[p].adj;
            }
        }
}
int getroot(int x) {
    get_size(x);
    int sz = size[x];
    while (size[son[x]] > sz/2)
        x = son[x];
    return x;
}
void dc(int x) {
    x = getroot(x);
    static int list[N], ltop;
    ltop = 0;
    for (int p=gh[x]; p; p=edge[p].next)
        if (edge[p].valid)
            list[++ltop] = p;
    clear();
    for (int i=1;i<=ltop;i++) {
        update();
        modify();
    }
```

```
    clear();
    for (int i=ltop;i>=1;i--) {
        update();
        modify();
    }
    //be careful about the root
    for (int p=gh[x]; p; p=edge[p].next)
        if (edge[p].valid) {
            edge[p].valid = 0;
            edge[p^1].valid = 0;
            dc(edge[p].adj);
        }
}
```

4.2 Link Cut Tree

接口:

command(x, y)：将 x 到 y 路径的 Splay Tree 分离出来。

linkcut(u1, v1, u2, v2)：将树中原有的边 (u1, v1) 删除，加入一条新边 (u2, v2)

```
struct DynamicTREE{
    struct NODE{
        int father, son[2], top, size, reverse;
    } splay[N];
    void init(int i, int fat) {
        splay[i].father = splay[i].son[0] = splay[i].son[1] = 0;
        splay[i].top = fat; splay[i].size = 1; splay[i].reverse = 0;
    }
    void pushdown(int x) {
        if (!x) return;
        int s0 = splay[x].son[0], s1 = splay[x].son[1];
        if (splay[x].reverse) {
            splay[s0].reverse ^= 1;
            splay[s1].reverse ^= 1;
            swap(splay[x].son[0], splay[x].son[1]);
            splay[x].reverse = 0;
        }
        s0 = splay[x].son[0], s1 = splay[x].son[1];
        splay[s0].top = splay[s1].top = splay[x].top;
    }
    void pushup(int x) {
        if (!x) return;
        pushdown(splay[x].son[0]);
        pushdown(splay[x].son[1]);
        splay[x].size = splay[splay[x].son[0]].size + splay[splay[x].son[1]].size +
            1;
    }
    void sc(int x, int y, int w, bool Auto=true) {
        splay[x].son[w] = y;
```

```
        splay[y].father = x;
        if (Auto) {
            pushup(y);
            pushup(x);
        }
    }
    int top, tush[N];
    void flowdown(int x) {
        for (top=1; x; top++, x = splay[x].father) tush[top] = x;
        for (; top; top--) pushdown(tush[top]);
    }
    void rotate(int x) {
        if (!x) return;
        int y = splay[x].father;
        int w = splay[y].son[1] == x;
        pushdown(y);
        pushdown(x);
        sc(splay[y].father, x, splay[splay[y].father].son[1]==y, false);
        sc(y, splay[x].son[w^1], w, false);
        sc(x, y, w^1, false);
        pushup(y);
        pushup(x);
    }
    void Splay(int x, int rt=0) {
        if (!x) return;
        flowdown(x);
        while (splay[x].father != rt) {
            int y = splay[x].father;
            int w = splay[y].son[1]==x;
            if (splay[y].father != rt && splay[splay[y].father].son[w] == y) rotate(
                y);
            rotate(x);
        }
    }
    void split(int x) {
        int y = splay[x].son[1];
        if (!y) return;
        splay[y].father = 0;
        splay[x].son[1] = 0;
        splay[y].top = x;
        pushup(x);
    }
    void access(int x) {
        int y = 0;
        while (x) {
            Splay(x);
            split(x);
            sc(x, y, 1);
            Splay(x);
            y = x;
```

```
            x = splay[x].top;
        }
    }
    void changeroot(int x) {
        access(x);
        Splay(x);
        splay[x].reverse = 1;
        Splay(x);
    }
    void command(int x, int y, ...) {
        LL ans = 0;
        changeroot(x);
        access(y);
        Splay(x);
        //then you can modify the Splay Tree
    }
    void linkcut(int u1, int v1, int u2, int v2) {
        changeroot(u1);
        access(v1);
        Splay(u1); split(u1);
        splay[v1].top = 0;
        access(u2); changeroot(u2);
        access(v2); changeroot(v2);
        Splay(u2); Splay(v2);
        splay[v2].top = u2;
    }
} lct;
```

5 图

5.1 欧拉回路

欧拉回路:

无向图：每个顶点的度数都是偶数，则存在欧拉回路。

有向图：每个顶点的入度 = 出度，则存在欧拉回路。

欧拉路径:

无向图：当且仅当该图所有顶点的度数为偶数，或者除了两个度数为奇数外其余的全是偶数。

有向图：当且仅当该图所有顶点出度 = 入度或者一个顶点出度 = 入度 + 1，另一个顶点入度 = 出度 + 1，其他顶点出度 = 入度。

下面 $O(n+m)$ 求欧拉回路的代码中，n 为点数，m 为边数，若有解则依次输出经过的边的编号, 若是无向图, 则正数表示 x 到 y ，负数表示 y 到 x 。

```
namespace UndirectedGraph{
    int n,m,i,x,y,d[N],g[N],v[M<<1],w[M<<1],vis[M<<1],nxt[M<<1],ed;
    int ans[M],cnt;
    void add(int x,int y,int z){
        d[x]++;
        v[++ed]=y;w[ed]=z;nxt[ed]=g[x];g[x]=ed;
```

```
    }
    void dfs(int x){
        for(int&i=g[x];i;){
            if(vis[i]){i=nxt[i];continue;}
            vis[i]=vis[i^1]=1;
            int j=w[i];
            dfs(v[i]);
            ans[++cnt]=j;
        }
    }
    void solve(){
        scanf("%d%d",&n,&m);
        for(i=ed=1;i<=m;i++)scanf("%d%d",&x,&y),add(x,y,i),add(y,x,-i);
        for(i=1;i<=n;i++)if(d[i]&1){puts("NO");return;}
        for(i=1;i<=n;i++)if(g[i]){dfs(i);break;}
        for(i=1;i<=n;i++)if(g[i]){puts("NO");return;}
        puts("YES");
        for(i=m;i;i--)printf("%d ",ans[i]);
    }
}
namespace DirectedGraph{
    int n,m,i,x,y,d[N],g[N],v[M],vis[M],nxt[M],ed;
    int ans[M],cnt;
    void add(int x,int y){
        d[x]++;d[y]--;
        v[++ed]=y;nxt[ed]=g[x];g[x]=ed;
    }
    void dfs(int x){
        for(int&i=g[x];i;){
            if(vis[i]){i=nxt[i];continue;}
            vis[i]=1;
            int j=i;
            dfs(v[i]);
            ans[++cnt]=j;
        }
    }
    void solve(){
        scanf("%d%d",&n,&m);
        for(i=1;i<=m;i++)scanf("%d%d",&x,&y),add(x,y);
        for(i=1;i<=n;i++)if(d[i]){puts("NO");return;}
        for(i=1;i<=n;i++)if(g[i]){dfs(i);break;}
        for(i=1;i<=n;i++)if(g[i]){puts("NO");return;}
        puts("YES");
        for(i=m;i;i--)printf("%d ",ans[i]);
    }
}
```

5.2 最短路径

5.2.1 Dijkstra

```
#include <queue>
using namespace std;
struct EDGE { int adj, w, next; } edge[M*2];
struct dat { int id, dist; dat(int id=0, int dist=0) : id(id), dist(dist) {} };
struct cmp { bool operator () (const dat &a, const dat &b) { return a.dist > b.dist;
    } };
priority_queue < dat, vector<dat>, cmp > q;
int n, top, gh[N], v[N], dist[N];
void addedge(int x, int y, int w) {
    edge[++top].adj = y;
    edge[top].w = w;
    edge[top].next = gh[x];
    gh[x] = top;
}
int dijkstra(int s, int t) {
    memset(dist, 63, sizeof(dist));
    memset(v, 0, sizeof(v));
    dist[s] = 0;
    q.push(dat(s, 0));
    while (!q.empty()) {
        dat x = q.top(); q.pop();
        if (v[x.id]) continue; v[x.id] = 1;
        for (int p=gh[x.id]; p; p=edge[p].next) {
            if (x.dist + edge[p].w < dist[edge[p].adj]) {
                dist[edge[p].adj] = x.dist + edge[p].w;
                q.push(dat(edge[p].adj, dist[edge[p].adj]));
            }
        }
    }
    return dist[t];
}
```

5.2.2 SPFA

```
struct EDGE { int adj, w, next; } edge[M*2];
int n,m,top,gh[N],v[N],cnt[N],q[N],dist[N],head,tail;
void addedge(int x, int y, int w) {
    edge[++top].adj = y;
    edge[top].w = w;
    edge[top].next = gh[x];
    gh[x] = top;
}
int spfa(int S, int T) {
    memset(v, 0, sizeof(v));
    memset(cnt, 0, sizeof(cnt));
```

```
    memset(dist, 63, sizeof(dist));
    head = 0, tail = 1;
    dist[S] = 0; q[1] = S;
    while (head != tail) {
        (head += 1) %= N;
        int x = q[head]; v[x] = 0;
        ++cnt[x]; if (cnt[x] > n) return -1;
        for (int p=gh[x]; p; p=edge[p].next)
            if (dist[x] + edge[p].w < dist[edge[p].adj]) {
                dist[edge[p].adj] = dist[x] + edge[p].w;
                if (!v[edge[p].adj]) {
                    v[edge[p].adj] = 1;
                    (tail += 1) %= N;
                    q[tail] = edge[p].adj;
                }
            }
    }
    return dist[T];
}
```

5.3 K 短路

接口:

kthsp::init(n) ： 初始化并设置节点个数为 n

kthsp::add(x, y, w) ： 添加一条 x 到 y 的有向边

kthsp::work(S, T, k) ： 求 S 到 T 的第 k 短路

```
#include <queue>

#define N 200020
#define M 400020
#define LOGM 20
#define LL long long
#define inf (1LL<<61)

namespace pheap {
    struct Node {
        int next, son[2];
        LL val;
    } node[M*LOGM];
    int LOG[M];
    int root[M], size[M*LOGM], top;
    int add() {
        ++top; assert(top < M*LOGM);
        node[top].next = node[top].son[0] = node[top].son[1] = 0;
        node[top].val = inf;
        return top;
    }
    int copy(int x) { int t = add(); node[t] = node[x]; return t; }
```

```
    void init() {
        top = -1; add();
        for (int i=2;i<M;i++) LOG[i] = LOG[i>>1] + 1;
    }
    void upd(int x, int &next, LL &val) {
        if (val < node[x].val) {
            swap(val, node[x].val);
            swap(next, node[x].next);
        }
    }
    void insert(int x, int next, LL val) {
        int sz = size[root[x]] + 1;
        root[x] = copy(root[x]);
        size[root[x]] = sz; x = root[x];
        upd(x, next, val);
        for (int i=LOG[sz]-1;i>=0;i--) {
            int ind = (sz>>i)&1;
            node[x].son[ind] = copy(node[x].son[ind]);
            x = node[x].son[ind];
            upd(x, next, val);
        }
    }
};

namespace kthsp {
    using namespace pheap;
    struct EDGE {
        int adj, w, next;
    } edge[2][M];
    struct W {
        int x, y, w;
    } e[M];
    bool has_init = 0;
    int n, m, top[2], gh[2][N], v[N];
    LL dist[N];
    void init(int n1) {
        has_init = 1;
        n = n1; m = 0;
        memset(top, 0, sizeof(top));
        memset(gh, 0, sizeof(gh));
        for (int i=1;i<=n;i++) dist[i] = inf;
    }
    void addedge(int id, int x, int y, int w) {
        edge[id][++top[id]].adj = y;
        edge[id][top[id]].w = w;
        edge[id][top[id]].next = gh[id][x];
        gh[id][x] = top[id];
    }
    void add(int x, int y, int w) {
        assert(has_init);
```

```
        e[++m].x=x; e[m].y=y; e[m].w=w;
    }
    int q[N], best[N], bestw[N];
    int deg[N];
    void spfa(int S) {
        for (int i=1;i<=n;i++) deg[i] = 0;
        for (int i=1;i<=m;i++) deg[e[i].x] ++;
        int head = 0, tail = 1;
        dist[S] = 0; q[1] = S;
        while (head != tail) {
            (head += 1) %= N;
            int x = q[head];
            for (int p=gh[1][x]; p; p=edge[1][p].next) {
                if (dist[x] + edge[1][p].w < dist[edge[1][p].adj]) {
                    dist[edge[1][p].adj] = dist[x] + edge[1][p].w;
                    best[edge[1][p].adj] = x;
                    bestw[edge[1][p].adj] = p;
                }
                if (!--deg[edge[1][p].adj]) {
                    (tail += 1) %= N;
                    q[tail] = edge[1][p].adj;
                }
            }
        }
    }
    void dfs(int x) {
        if (v[x]) return; v[x] = 1;
        if (best[x]) root[x] = root[best[x]];
        for (int p=gh[0][x]; p; p=edge[0][p].next)
            if (dist[edge[0][p].adj] != inf && bestw[x] != p) {
                insert(x, edge[0][p].adj, edge[0][p].w + dist[edge[0][p].adj] - dist
                    [x]);
            }
        for (int p=gh[1][x]; p; p=edge[1][p].next)
            if (best[edge[1][p].adj] == x)
                dfs(edge[1][p].adj);
    }
    typedef pair<LL,int> pli;
    priority_queue <pli, vector<pli>, greater<pli> > pq;
    LL work(int S, int T, int k) {
        assert(has_init);
        n++; add(T, n, 0);
        if (S == T) k ++;
        T = n;
        for (int i=1;i<=m;i++) {
            addedge(0, e[i].x, e[i].y, e[i].w);
            addedge(1, e[i].y, e[i].x, e[i].w);
        }
        spfa(T);
        root[T] = 0; pheap::init();
```

```
        memset(v, 0, sizeof(v));
        dfs(T);
        while (!pq.empty()) pq.pop();
        if (k == 1) return dist[S];
        if (root[S]) pq.push(make_pair(dist[S] + node[root[S]].val, root[S]));
        while (k--) {
            if (pq.empty()) return inf;
            pli now = pq.top(); pq.pop();
            if (k == 1) return now.first;
            int x = node[now.second].next, u = node[now.second].son[0], v = node[now
                .second].son[1];
            if (root[x]) pq.push(make_pair(now.first + node[root[x]].val, root[x]));
            if (u) pq.push(make_pair(now.first - node[now.second].val + node[u].val,
                 u));
            if (v) pq.push(make_pair(now.first - node[now.second].val + node[v].val,
                 v));
        }
        return 0;
    }
};
```

5.4 Tarjan

割点的判断：一个顶点 u 是割点，当且仅当满足 (1) 或 (2)：
(1) u 为树根，且 u 有多于一个子树（即：存在一个儿子 v 使得 $dfn[u]+1 \neq dfn[v]$ ）
(2) u 不为树根，且满足存在 (u,v) 为树枝边（u 为 v 的父亲），使得 $dfn[u] \leq low[v]$
桥的判断：一条无向边 (u,v) 是桥，当且仅当 (u,v) 为树枝边，满足 $dfn[u] < low[v]$

```
struct EDGE { int adj, next; } edge[M];
int n, m, top, gh[N];
int dfn[N], low[N], cnt, ind, stop, instack[N], stack[N], belong[N];
void addedge(int x, int y) {
    edge[++top].adj = y;
    edge[top].next = gh[x];
    gh[x] = top;
}
void tarjan(int x) {
    dfn[x] = low[x] = ++ind;
    instack[x] = 1; stack[++stop] = x;
    for (int p=gh[x]; p; p=edge[p].next)
        if (!dfn[edge[p].adj]) {
            tarjan(edge[p].adj);
            low[x] = min(low[x], low[edge[p].adj]);
        } else if (instack[edge[p].adj]) {
            low[x] = min(low[x], dfn[edge[p].adj]);
        }
    if (dfn[x] == low[x]) {
        ++cnt; int tmp=0;
        while (tmp!=x) {
```

```
            tmp = stack[stop--];
            belong[tmp] = cnt;
            instack[tmp] = 0;
        }
    }
}
```

5.5 2-SAT

```
#define N number_of_vertex
#define M number_of_edges

struct MergePoint {
    struct EDGE {
        int adj, next;
    } edge[M];
    int ex[M], ey[M];
    bool instack[N];
    int gh[N], top, dfn[N], low[N], cnt, ind, stop, stack[N], belong[N];
    void init() {
        cnt = ind = stop = top = 0;
        memset(dfn, 0, sizeof(dfn));
        memset(instack, 0, sizeof(instack));
        memset(gh, 0, sizeof(gh));
    }
    void addedge(int x, int y) { //reverse
        std::swap(x, y);
        edge[++top].adj = y;
        edge[top].next = gh[x];
        gh[x] = top;
        ex[top] = x;
        ey[top] = y;
    }
    void tarjan(int x) {
        dfn[x] = low[x] = ++ind;
        instack[x] = 1; stack[++stop] = x;
        for (int p=gh[x]; p; p=edge[p].next)
            if (!dfn[edge[p].adj]) {
                tarjan(edge[p].adj);
                low[x] = std::min(low[x], low[edge[p].adj]);
            } else if (instack[edge[p].adj]) {
                low[x] = std::min(low[x], dfn[edge[p].adj]);
            }
        if (dfn[x] == low[x]) {
            ++cnt; int tmp = 0;
            while (tmp!=x) {
                tmp = stack[stop--];
                belong[tmp] = cnt;
                instack[tmp] = 0;
```

```
            }
        }
    }
    void work() {
        for (int i = (__first__); i <= (__last__); ++i)
            if (!dfn[i])
                tarjan(i);
    }
} merge;

struct Topsort {
    struct EDGE {
        int adj, next;
    } edge[M];
    int n, top, gh[N], ops[N], deg[N], ans[N];
    std::queue<int> q;
    void init() {
        n = merge.cnt; top = 0;
        memset(gh, 0, sizeof(gh));
        memset(deg, 0, sizeof(deg));
    }
    void addedge(int x, int y) {
        if (x == y) return;
        edge[++top].adj = y;
        edge[top].next = gh[x];
        gh[x] = top;
        ++deg[y];
    }
    void work() {
        for (int i = 1; i <= n; ++i)
            if (!deg[i])
                q.push(i);
        while (!q.empty()) {
            int x = q.front();
            q.pop();
            for (int p = gh[x]; p; p = edge[p].next)
                if (!--deg[edge[p].adj])
                    q.push(edge[p].adj);
            if (ans[x]) continue;
            ans[x] = -1; //not selected
            ans[ops[x]] = 1; //selected
        }
    }
} ts;
```

调用示例:

```
    merge.init();
    merge.addedge();
    merge.work();
    for (int i = 1; i <= n; ++i) {
```

```
        if (merge.belong[U(i, 0)] == merge.belong[U(i, 1)]) {
            puts("NO");
            return 0;
        }
        ts.ops[merge.belong[U(i, 0)]] = merge.belong[U(i, 1)];
        ts.ops[merge.belong[U(i, 1)]] = merge.belong[U(i, 0)];
    }
    ts.init();
    ts.work();
    puts("YES");
    for (int i = 1; i <= n; ++i) {
        int x = U(i, 0), y = U(i, 1);
        x = merge.belong[x], y = merge.belong[y];
        x = ts.ans[x], y = ts.ans[y];
        if (x == 1) puts("0␣is␣selected");
        if (y == 1) puts("1␣is␣selected");
    }
```

5.6 统治者树（Dominator Tree）

Dominator Tree 可以解决判断一类有向图必经点的问题。

idom[x] 表示离 x 最近的必经点（重编号后）。将 idom[x] 作为 x 的父亲，构成一棵 Dominator Tree

接口：

void dominator::init(int n); 初始化，有向图节点数为 n

void dominator::addedge(int u, int v); 添加一条有向边 (u, v)

void dominator::work(int root); 以 root 为根，建立一棵 Dominator Tree

结果的返回：

在执行 dominator::work(int root); 后，树边保存在 vector <int> tree[N] 中

```
namespace dominator {
    vector <int> g[N], rg[N], bucket[N], tree[N];
    int n, ind, idom[N], sdom[N], dfn[N], dsu[N], father[N], label[N], rev[N];
    void dfs(int x) {
        ++ind;
        dfn[x] = ind; rev[ind] = x;
        label[ind] = dsu[ind] = sdom[ind] = ind;
        for (auto p : g[x]) {
            if (!dfn[p]) dfs(p), father[dfn[p]] = dfn[x];
            rg[dfn[p]].push_back(dfn[x]);
        }
    }
    void init(int n1) {
        n = n1; ind = 0;
        for (int i = 1; i <= n; ++i) {
            g[i].clear();
            rg[i].clear();
            bucket[i].clear();
```

```
            tree[i].clear();
            dfn[i] = 0;
        }
    }
    void addedge(int u, int v) {
        g[u].push_back(v);
    }
    int find(int x, int step=0) {
        if (dsu[x] == x) return step ? -1 : x;
        int y = find(dsu[x], 1);
        if (y < 0) return x;
        if (sdom[label[dsu[x]]] < sdom[label[x]])
            label[x] = label[dsu[x]];
        dsu[x] = y;
        return step ? dsu[x] : label[x];
    }
    void work(int root) {
        dfs(root); n = ind;
        for (int i = n; i; --i) {
            for (auto p : rg[i])
                sdom[i] = min(sdom[i], sdom[find(p)]);
            if (i > 1) bucket[sdom[i]].push_back(i);
            for (auto p : bucket[i]) {
                int u = find(p);
                if (sdom[p] == sdom[u]) idom[p] = sdom[p];
                else idom[p] = u;
            }
            if (i > 1) dsu[i] = father[i];
        }
        for (int i = 2; i <= n; ++i) {
            if (idom[i] != sdom[i])
                idom[i] = idom[idom[i]];
            tree[rev[i]].push_back(rev[idom[i]]);
            tree[rev[idom[i]]].push_back(rev[i]);
        }
    }
};
```

5.7 网络流

5.7.1 最大流

注意：*top 要初始化为 1*

```
struct EDGE { int adj, w, next; } edge[M];
int n, top, gh[N], nrl[N];
void addedge(int x, int y, int w) {
    edge[++top].adj = y;
    edge[top].w = w;
    edge[top].next = gh[x];
```

```
    gh[x] = top;
    edge[++top].adj = x;
    edge[top].w = 0;
    edge[top].next = gh[y];
    gh[y] = top;
}
int dist[N], q[N];
int bfs() {
    memset(dist, 0, sizeof(dist));
    q[1] = S; int head = 0, tail = 1; dist[S] = 1;
    while (head != tail) {
        int x = q[++head];
        for (int p=gh[x]; p; p=edge[p].next)
            if (edge[p].w && !dist[edge[p].adj]) {
                dist[edge[p].adj] = dist[x] + 1;
                q[++tail] = edge[p].adj;
            }
    }
    return dist[T];
}
int dinic(int x, int delta) {
    if (x==T) return delta;
    for (int& p=nrl[x]; p && delta; p=edge[p].next)
        if (edge[p].w && dist[x]+1 == dist[edge[p].adj]) {
            int dd = dinic(edge[p].adj, min(delta, edge[p].w));
            if (!dd) continue;
            edge[p].w -= dd;
            edge[p^1].w += dd;
            return dd;
        }
    return 0;
}
int work() {
    int ans = 0;
    while (bfs()) {
        memcpy(nrl, gh, sizeof(gh));
        int t; while (t = dinic(S, inf)) ans += t;
    }
    return ans;
}
```

5.7.2 上下界有源汇网络流

T 向 S 连容量为正无穷的边，将有源汇转化为无源汇。

每条边容量减去下界，设 $in[i]$ 表示流入 i 的下界之和减去流出 i 的下界之和。

新建超级源汇 SS,TT ，对于 $in[i] > 0$ 的点，SS 向 i 连容量为 $in[i]$ 的边。对于 $in[i] < 0$ 的点，i 向 TT 连容量为 $-in[i]$ 的边。

求出以 SS,TT 为源汇的最大流，如果等于 $\Sigma in[i](in[i] > 0)$ ，则存在可行流。再求出 S,T 为源汇的最大流即为最大流。

费用流：建完图后等价于求以 SS,TT 为源汇的费用流。

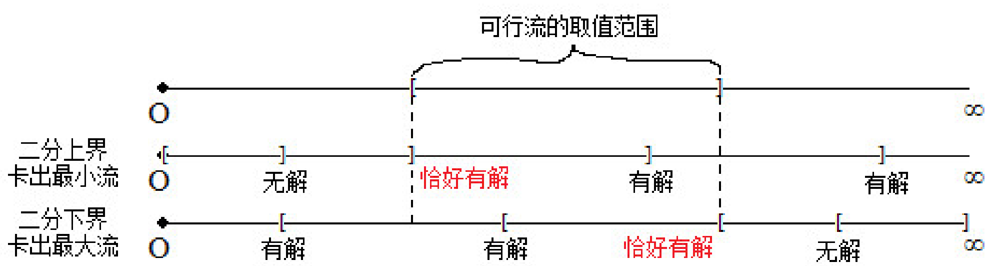
5.7.3 上下界无源汇网络流

1. 怎样求无源汇有上下界网络的可行流？

由于有源汇的网络我们先要转化成无源汇，所以本来就无源汇的网络不用再作特殊处理。

2. 怎样求无源汇有上下界网络的最大流、最小流？

一种简易的方法是采用二分的思想，不断通过可行流的存在与否对 (t,s) 边的上下界 U,L 进行调整。求最大流时令 $U=\infty$ 并二分 L；求最小流时令 $L=0$ 并二分 U 。道理很简单，因为可行流的取值范围是一段连续的区间，我们只要通过二分找到有解和无解的分界线即可。



5.7.4 费用流

注意：*top 要初始化为 1*

```
#define inf 0x3f3f3f3f
struct NetWorkFlow {
    struct EDGE {
        int adj, w, cost, next;
    } edge[M*2];
    int gh[N], q[N], dist[N], v[N], pre[N], prev[N], top;
    int S, T;
    void addedge(int x, int y, int w, int cost) {
        edge[++top].adj = y;
        edge[top].w = w;
        edge[top].cost = cost;
        edge[top].next = gh[x];
        gh[x] = top;
        edge[++top].adj = x;
        edge[top].w = 0;
        edge[top].cost = -cost;
        edge[top].next = gh[y];
        gh[y] = top;
    }
    void clear() {
        top = 1;
        memset(gh, 0, sizeof(gh));
    }
```

```
    int spfa() {
        memset(dist, 63, sizeof(dist));
        memset(v, 0, sizeof(v));
        int head = 0, tail = 1;
        q[1] = S; v[S] = 1; dist[S] = 0;
        while (head != tail) {
            (head += 1) %= N;
            int x = q[head];
            v[x] = 0;
            for (int p=gh[x]; p; p=edge[p].next)
                if (edge[p].w && dist[x] + edge[p].cost < dist[edge[p].adj]) {
                    dist[edge[p].adj] = dist[x] + edge[p].cost;
                    pre[edge[p].adj] = x;
                    prev[edge[p].adj] = p;
                    if (!v[edge[p].adj]) {
                        v[edge[p].adj] = 1;
                        (tail += 1) %= N;
                        q[tail] = edge[p].adj;
                    }
                }
        }
        return dist[T] != inf;
    }
    int work() {
        int ans = 0;
        while (spfa()) {
            int mx = inf;
            for (int x=T;x!=S;x=pre[x])
                mx = min(edge[prev[x]].w, mx);
            ans += dist[T] * mx;
            for (int x=T;x!=S;x=pre[x]) {
                edge[prev[x]].w -= mx;
                edge[prev[x]^1].w += mx;
            }
        }
        return ans;
    }
} nwf;
```

5.7.5 zkw 费用流

注意：*top* 要初始化为 *1*，不得用于有负权的图

```
#define inf 0x3f3f3f3f //modify if you use long long or double
template <class _tp>
struct NetWorkFlow {
    struct EDGE {
        int adj, next;
        _tp w, cost;
    } edge[M*2];
```

```
    int gh[N], top;
    int S, T;
    void addedge(int x, int y, _tp w, _tp cost) {
        edge[++top].adj = y;
        edge[top].w = w;
        edge[top].cost = cost;
        edge[top].next = gh[x];
        gh[x] = top;
        edge[++top].adj = x;
        edge[top].w = 0;
        edge[top].cost = -cost;
        edge[top].next = gh[y];
        gh[y] = top;
    }
    void clear() {
        top = 1;
        memset(gh, 0, sizeof(gh));
    }
    int v[N];
    _tp cost, d[N], slk[N];
    _tp aug(int x, _tp f) {
        _tp left = f;
        if (x == T) {
            cost += f * d[S];
            return f;
        }
        v[x] = true;
        for (int p=gh[x]; p; p=edge[p].next)
            if (edge[p].w && !v[edge[p].adj]) {
                _tp t = d[edge[p].adj] + edge[p].cost - d[x];
                if (t == 0) {
                    _tp delt = aug(edge[p].adj, min(left, edge[p].w));
                    if (delt > 0) {
                        edge[p].w -= delt;
                        edge[p^1].w += delt;
                        left -= delt;
                    }
                    if (left == 0) return f;
                } else {
                if (t < slk[edge[p].adj])
                    slk[edge[p].adj] = t;
                }
            }
        return f-left;
    }
    bool modlabel() {
        _tp delt = inf;
        for (int i=1;i<=T;i++)
            if (!v[i]) {
                if (slk[i] < delt) delt = slk[i];
```

```
                slk[i] = inf;
            }
        if (delt == inf) return true;
        for (int i=1;i<=T;i++)
            if (v[i]) d[i] += delt;
        return false;
    }
    _tp work() {
        cost = 0;
        memset(d, 0, sizeof(d));
        memset(slk, 63, sizeof(slk));
        do {
            do {
                memset(v, 0, sizeof(v));
            } while (aug(S, inf));
        } while (!modlabel());
        return cost;
    }
};
NetWorkFlow<int> nwf;
```

6 数学

6.1 扩展欧几里得解同余方程

ans[] 保存的是循环节内所有的解

```
int exgcd(int a,int b,int&x,int&y){
    if(!b)return x=1,y=0,a;
    int d=exgcd(b,a%b,x,y),t=x;
    return x=y,y=t-a/b*y,d;
}
void cal(ll a,ll b,ll n){//ax=b(mod n)
    ll x,y,d=exgcd(a,n,x,y);
    if(b%d)return;
    x=(x%n+n)%n;
    ans[cnt=1]=x*(b/d)%(n/d);
    for(ll i=1;i<d;i++)ans[++cnt]=(ans[1]+i*n/d)%n;
}
```

6.2 同余方程组

```
int n,flag,k,m,a,r,d,x,y;
int main(){
    scanf("%d",&n);
    flag=k=1,m=0;
    while(n--){
        scanf("%d%d",&a,&r);//ans%a=r
```

```
        if(flag){
            d=exgcd(k,a,x,y);
            if((r-m)%d){flag=0;continue;}
            x=(x*(r-m)/d+a/d)%(a/d),y=k/d*a,m=((x*k+m)%y)%y;
            if(m<0)m+=y;
            k=y;
        }
    }
    printf("%d",flag?m:-1);//若flag=1,说明有解,解为ki+m,i为任意整数
}
```

6.3 卡特兰数

$h_1 = 1, h_n = \frac{h_{n-1}(4n-2)}{n+1} = \frac{C(2n,n)}{n+1} = C(2n, n) - C(2n, n-1)$
在一个格点阵列中，从 $(0,0)$ 点走到 (n,m) 点且不经过对角线 $x = y$ 的方案数 $(x > y)$：
$C(n+m-1, m) - C(n+m-1, m-1)$
在一个格点阵列中，从 $(0,0)$ 点走到 (n,m) 点且不穿过对角线 $x = y$ 的方案数 $(x \geq y)$：
$C(n+m, m) - C(n+m, m-1)$

6.4 斯特林数

6.4.1 第一类斯特林数

第一类 Stirling 数 $S(p,k)$ 的一个组合学解释是：将 p 个物体排成 k 个非空循环排列的方法数。
$S(p,k)$ 的递推公式：$S(p,k) = (p-1)S(p-1,k) + S(p-1,k-1), 1 \leq k \leq p-1$
边界条件：$S(p,0) = 0, p \geq 1$ $S(p,p) = 1, p \geq 0$

6.4.2 第二类斯特林数

第二类 Stirling 数 $S(p,k)$ 的一个组合学解释是：将 p 个物体划分成 k 个非空的不可辨别（可以理解为盒子没有编号）集合的方法数。
$S(p,k)$ 的递推公式：$S(p,k) = kS(p-1,k) + S(p-1,k-1), 1 \leq k \leq p-1$
边界条件：$S(p,0) = 0, p \geq 1$ $S(p,p) = 1, p \geq 0$
也有卷积形式：

$$S(n,m) = \frac{1}{m!}\sum_{k=0}^{m}(-1)^k C(m,k)(m-k)^n = \sum_{k=0}^{m}\frac{(-1)^k(m-k)^n}{k!(m-k)!} = \sum_{k=0}^{m}\frac{(-1)^k}{k!} \times \frac{(m-k)^n}{(m-k)!}$$

6.5 错排公式

$$D_1 = 0, D_2 = 1, D_n = (n-1)(D_{n-2} + D_{n-1})$$

6.6 Lucas 定理

接口：

初始化：void lucas::init();

计算 $C(n, m)\% mod$ 的值：LL lucas::Lucas(LL n, LL m);

```
#define mod 110119
#define LL long long
namespace lucas {
    LL fac[mod+1], facv[mod+1];
    LL power(LL base, LL times) {
        LL ans = 1;
        while (times) {
            if (times&1) (ans *= base) %= mod;
            (base *= base) %= mod;
            times >>= 1;
        }
        return ans;
    }
    void init() {
        fac[0] = 1; for (int i=1;i<mod;i++) fac[i] = (fac[i-1] * i) % mod;
        facv[mod-1] = power(fac[mod-1], mod-2);
        for (int i=mod-2;i>=0;--i) facv[i] = (facv[i+1] * (i+1)) % mod;
    }
    LL C(unsigned LL n, unsigned LL m) {
        if (n < m) return 0;
        return (fac[n] * facv[m] % mod * facv[n-m] % mod) % mod;
    }
    LL Lucas(unsigned LL n, unsigned LL m)
    {
        if (m == 0) return 1;
        return (C(n%mod, m%mod) * Lucas(n/mod, m/mod)) %mod;
    }
};
```

6.7 高斯消元

6.7.1 行列式

```
int ans = 1;
for (int i=0;i<n;i++) {
    for (int j=i;j<n;j++)
        if (g[j][i]) {
            for (int k=i;k<n;k++)
                swap(g[i][k], g[j][k]);
            if (j != i) ans *= -1;
            break;
        }
    if (g[i][i] == 0) {
        ans = 0;
```

```
            break;
        }
    for (int j=i+1;j<n;j++) {
        while (g[j][i]) {
            int t = g[i][i] / g[j][i];
            for (int k=i;k<n;k++)
                g[i][k] = (g[i][k] + mod - ((LL)t * g[j][k] % mod)) % mod;
            for (int k=i;k<n;k++)
                swap(g[i][k], g[j][k]);
            ans *= -1;
        }
    }
}
for (int i=0;i<n;i++)
    ans = ((LL)ans * g[i][i]) % mod;
ans = (ans % mod + mod) % mod;
printf("%d\n", ans);
```

6.7.2 Matrix-Tree 定理

对于一张图，建立矩阵 C ，$C[i][i] = \mathrm{i}$ 的度数，若 i, j 之间有边，那么 $C[i][j] = -1$ ，否则为 0 。这张图的生成树个数等于矩阵 C 的 $n-1$ 阶行列式的值。

6.8 调和级数

$\sum\limits_{i=1}^{n} \frac{1}{i}$ 在 n 较大时约等于 $ln(n) + r$ ，r 为欧拉常数，约等于 0.5772156649015328 。

6.9 曼哈顿距离的变换

$|x_1 - x_2| + |y_1 - y_2| = max(|(x_1 + y_1) - (x_2 + y_2)|, |(x_1 - y_1) - (x_2 - y_2)|)$

6.10 线性筛素数

```
mu[1]=phi[1]=1;top=0;
for (int i=2;i<N;i++) {
    if (!v[i]) prime[++top]=i, mu[i] = -1, phi[i] = i-1;
    for (int j=1;i*prime[j]<N && j<=top;j++) {
        v[i*prime[j]] = 1;
        if (i%prime[j]) {
            mu[i*prime[j]] = -mu[i];
            phi[i*prime[j]] = phi[i] * (prime[j]-1);
        } else {
            mu[i*prime[j]] = 0;
            phi[i*prime[j]] = phi[i] * prime[j];
            break;
        }
    }
}
```

6.11 杜教筛

getphi(t, x) 表示求 $\sum_{i=1}^{x} i^t\phi(i)$ 。

推导过程:

记 $S(n)=\sum_{i=1}^{n} f(i)$ ，取任意函数 g 有恒等式

$$S(n)=\sum_{i=1}^{n}(f\cdot g)(i)-\sum_{i=2}^{n}g(i)S(\lfloor\frac{n}{i}\rfloor)$$

其中，$(f\cdot g)$ 表示 f 和 g 的狄利克雷卷积：即：$(f\cdot g)(n)=\sum_{d|n} f(d)g(\frac{n}{d})$

关于恒等式的证明:

将 $\sum_{i=2}^{n} g(i)S(\lfloor\frac{n}{i}\rfloor)$ 移到左边去，即只需证

$$\sum_{i=1}^{n}(f\cdot g)(i)=\sum_{i=1}^{n}g(i)S(\lfloor\frac{n}{i}\rfloor)$$

将狄利克雷卷积展开，得:

$$\sum_{i=1}^{n}\sum_{d|i}g(d)f(\frac{i}{d})=\sum_{i=1}^{n}g(i)S(\lfloor\frac{n}{i}\rfloor)$$

即:

$$\sum_{d=1}^{n}g(d)\sum_{i=1}^{\lfloor\frac{n}{d}\rfloor}f(i)=\sum_{i=1}^{n}g(i)S(\lfloor\frac{n}{i}\rfloor)$$

显然相等，恒等式证完。

取 $f(i)=i^p\phi(i),g(i)=i^p$ ，则有:

$$S(n)=\sum_{i=1}^{n}i^p\phi(i)=\sum_{i=1}^{n}i^{p+1}-\sum_{i=2}^{n}i^pS(\lfloor\frac{n}{i}\rfloor)$$

其中有用到等式 $\sum_{d|n}\phi(d)=n$

另外，莫比乌斯函数 $\mu(n)=\begin{cases}1, & 若n=1\\ (-1)^k, & 若n无平方数因数，且n=p_1p_2\cdots p_k\\ 0, & 若n有大于\ 1\ 的平方数因数\end{cases}$ 也可以使用杜教筛求前缀和，记 $S'(n)=\sum_{i=1}^{n}\mu(i)$ ，则 $S'(n)=1-\sum_{i=2}^{n}S'(\lfloor\frac{n}{i}\rfloor)$

```
#include <bits/stdc++.h>

#define N 5000020
#define LL long long
#define mod 1000000007
#define div2 ((mod+1)/2)
#define div6 ((mod+1)/6)

using namespace std;
```

```
int n, prime[N], v[N];
LL phi[3][N];

map<int, int> mp[3];

int sum(int t, int x) { //calculate 1^t + 2^t + ... + x^t
    if (t == 0) return x;
    if (t == 1) return 1ll * x * (x + 1) % mod * div2 % mod;
    if (t == 2) return 1ll * x * (x + 1) % mod * (2ll * x % mod + 1) % mod * div6 %
        mod;
    if (t == 3) return 1ll * x * x % mod * (x + 1) % mod * (x + 1) % mod * div2 %
        mod * div2 % mod;
}

int getphi(int t, int x) {
    if (x < N) return phi[t][x];
    if (mp[t].find(x) != mp[t].end()) return mp[t][x];
    LL ans = 0; int r = 0;
    for (int l = 2; l <= x; l = r + 1) {
        r = x / (x / l);
        ans += 1ll * getphi(t, x / l) * (((LL)sum(t, r) - sum(t, l - 1) + mod) % mod
            ) % mod;
        ans %= mod;
    }
    ans = (LL)sum(t + 1, x) - ans + mod;
    ans %= mod;
    mp[t][x] = ans;
    return (int)ans;
}

int main() {
    memset(v, 0, sizeof(v));
    int top = 0;
    phi[0][1] = 1, phi[1][1] = 1, phi[2][1] = 1;
    for (int i = 2; i < N; ++i) {
        if (!v[i]) prime[++top] = i, phi[0][i] = i - 1, phi[1][i] = 1ll * i * phi
            [0][i] % mod, phi[2][i] = 1ll * i * phi[1][i] % mod;
        for (int j = 1; j <= top && prime[j] * i < N; ++j) {
            v[i * prime[j]] = 1;
            if (i % prime[j] == 0) {
                phi[0][i * prime[j]] = phi[0][i] * prime[j];
                phi[1][i * prime[j]] = 1ll * phi[1][i] * prime[j] % mod * prime[j] %
                    mod;
                phi[2][i * prime[j]] = 1ll * phi[2][i] * prime[j] % mod * prime[j] %
                    mod * prime[j] % mod;
                break;
            } else {
                phi[0][i * prime[j]] = phi[0][i] * (prime[j] - 1);
                phi[1][i * prime[j]] = 1ll * phi[1][i] * (prime[j] - 1) % mod *
```

```
                prime[j] % mod;
            phi[2][i * prime[j]] = 1ll * phi[2][i] * (prime[j] - 1) % mod *
                prime[j] % mod * prime[j] % mod;
            }
        }
    }
    for (int i = 2; i < N; ++i) {
        phi[0][i] = (phi[0][i] + phi[0][i - 1]) % mod;
        phi[1][i] = (phi[1][i] + phi[1][i - 1]) % mod;
        phi[2][i] = (phi[2][i] + phi[2][i - 1]) % mod;
    }
}
```

6.12 FFT

```
namespace FFT {
    const int maxn = 65537;
    const double pi = acos(-1.0);

    struct comp {
        double real , imag;
        comp() {}
        comp(double real , double imag): real(real) , imag(imag) {}
        friend inline comp operator+(const comp &a , const comp &b) {
            return comp(a.real + b.real , a.imag + b.imag);
        }
        friend inline comp operator-(const comp &a , const comp &b) {
            return comp(a.real - b.real , a.imag - b.imag);
        }
        friend inline comp operator*(const comp &a , const comp &b) {
            return comp(a.real * b.real - a.imag * b.imag , a.real * b.imag + a.imag
                 * b.real);
        }
    };

    comp A[maxn] , B[maxn];
    int rev[maxn], m, len;

    inline void init(int n) {
        for (m = 1, len = 0; m < n + n; m <<= 1 , len ++);
        for (int i = 0; i < m; ++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len -
             1));
        for (int i = 0; i < m; ++i) A[i] = B[i] = comp(0, 0);
    }

    inline void dft(comp *a , int v) {
        for (int i = 0; i < m; ++i) if (i < rev[i]) swap(a[i] , a[rev[i]]);
        for (int s = 2; s <= m; s <<= 1) {
            comp g(cos(2 * pi / s) , v * sin(2 * pi / s));
```

```
            for (int k = 0; k < m; k += s) {
                comp w(1 , 0);
                for (int j = 0; j < s / 2; ++j) {
                    comp u = a[k + j] , t = w * a[k + j + s / 2];
                    a[k + j] = u + t;
                    a[k + j + s / 2] = u - t;
                    w = w * g;
                }
            }
        }
        if (v == -1)
            for (int i = 0; i < m; ++i) a[i].real /= m , a[i].imag /= m;
    }
}
```

6.13 FWT

给定长度为 2^n 的序列 $A[0\cdots 2^n-1], B[0\cdots 2^n-1]$ ，求这两序列的

or 卷积：$C_k = \sum\limits_{i\ or\ j=k} A_i B_j$

and 卷积：$C_k = \sum\limits_{i\ and\ j=k} A_i B_j$

xor 卷积：$C_k = \sum\limits_{i\ xor\ j=k} A_i B_j$

序列对 10^9+7 取模

```
#include <bits/stdc++.h>
using namespace std;
inline int read() {
    int s = 0; char c; while((c=getchar())<'0'||c>'9');
    do{s=s*10+c-'0';}while((c=getchar())>='0'&&c<='9');
    return s;
}
typedef long long lint;
typedef pair<lint,lint> pii;
const int N = (1<<18)+37, MO = 1e9+7;
int n;
pii tand(lint a,lint b) { return (pii){a+b,b}; }
pii iand(lint a,lint b) { return (pii){a-b,b}; }
pii tor(lint a,lint b) { return (pii){a,a+b}; }
pii ior(lint a,lint b) { return (pii){a,b-a}; }
pii txor(lint a,lint b) { return (pii){a+b,a-b}; }
pii ixor(lint a,lint b) { return (pii){(a+b)/2,(a-b)/2}; }
pii (*tr)(lint,lint);
pii (*trs[3])(lint,lint)={tor,tand,txor};
pii (*ivs[3])(lint,lint)={ior,iand,ixor};
lint a[N],b[N],x[N],y[N];
inline void fwt(lint *a) {
    int s,k,j,je;
    for(s=2;s<=n;s<<=1) for(k=0;k<n;k+=s)
        for(j=0,je=s>>1;j<je;j++) {
```

```
            pii t = tr(a[k+j],a[k+j+je]);
            a[k+j] = t.first;
            a[k+j+je] = t.second;
        }
}
int main() {
    int i;
    n = read(); n = 1<<n;
    for(i=0;i<n;i++) a[i] = read();
    for(i=0;i<n;i++) b[i] = read();
    for(int k=0;k<3;k++) {
        for(i=0;i<n;i++) x[i] = a[i], y[i] = b[i];
        tr = trs[k]; fwt(x), fwt(y);
        for(i=0;i<n;i++) x[i] *= y[i];
        tr = ivs[k]; fwt(x);
        for(i=0;i<n;i++) printf("%d ",x[i]%MO); puts("");
    }
    return 0;
}
```

6.14 求原根

接口：LL p_root(LL p);

输入：一个素数 p

输出：p 的原根

```
#include <bits/stdc++.h>
#define LL long long

using namespace std;

vector <LL> a;

LL pow_mod(LL base, LL times, LL mod) {
    LL ret = 1;
    while (times) {
        if (times&1) ret = ret * base % mod;
        base = base * base % mod;
        times>>=1;
    }
    return ret;
}

bool g_test(LL g, LL p) {
    for (LL i = 0; i < a.size(); ++i)
        if (pow_mod(g, (p-1)/a[i], p) == 1) return 0;
    return 1;
}

```

```
LL p_root(LL p) {
    LL tmp = p - 1;
    for (LL i = 2; i <= tmp / i; ++i)
        if (tmp % i == 0) {
            a.push_back(i);
            while (tmp % i == 0)
                tmp /= i;
        }
    if (tmp != 1) a.push_back(tmp);
    LL g = 1;
    while (1) {
        if (g_test(g, p)) return g;
        ++g;
    }
}

int main() {
    LL p;
    cin >> p;
    cout << p_root(p) << endl;
}
```

6.15 NTT

998244353 原根为 3 ， 1004535809 原根为 3 ， 786433 原根为 10 ， 880803841 原根为 26 。

NTT 公式:

$$y_n = \sum_{i=0}^{d-1} x_i (g^{\frac{P-1}{d}})^{in} \bmod P$$

```
#define mod 998244353
#define g 3
LL wi[N], wiv[N];
LL power(LL base, LL times) {
    LL ans = 1;
    while (times) {
        if (times&1) (ans *= base) %= mod;
        (base *= base) %= mod;
        times >>= 1;
    }
    return ans;
}
void transform(LL *x, int len) {
    for (int i=1,j=len/2;i<len-1;i++) {
        if (i<j) swap(x[i], x[j]);
        int k = len/2;
        while (j>=k) {
            j-=k;
            k/=2;
        }
```

```
        if (j<k) j+=k;
    }
}
void NTT(LL *x, int len, int reverse) {
    transform(x, len);
    for (int h=2;h<=len;h<<=1) {
        for (int i=0;i<len;i+=h) {
            LL w = 1, wn;
            if (reverse==1) wn = wi[h]; else wn = wiv[h];
            for (int j=i;j<i+h/2;j++) {
                LL u = x[j];
                LL v = (w * x[j+h/2]) % mod;
                x[j ] = (u + v) % mod;
                x[j+h/2] = (u - v + mod) % mod;
                (w *= wn) %= mod;
            }
        }
    }
    if (reverse == -1) {
        LL t = power(len, mod-2);
        for (int i=0;i<len;i++)
            (x[i] *= t) %= mod;
    }
}
LL A[N], B[N];
int main() {
    for (int i=1;i<N;i*=2) {
        wi[i] = power(g, (mod-1)/i);
        wiv[i] = power(wi[i], mod-2);
    }
    memset(A, 0, sizeof(A));
    memset(B, 0, sizeof(B));
    NTT(A, len, 1); NTT(B, len, 1);
    for (int i=0;i<len;i++) (A[i] *= B[i]) %= mod;
    NTT(A, len, -1);
}
```

6.16 组合数 lcm

$$(n+1)lcm(C(n,0),C(n,1),...,C(n,k)) = lcm(n+1,n,n-1,...,n-k+1)$$

6.17 区间 lcm 的维护

对于一个数，将其分解质因数，若有因子 p^k ，那么拆分出 k 个数 $p,p^2,...,p^k$ ，权值都为 p ，那么查询区间 $[l,r]$ 内所有数的 lcm 的答案 = 所有在该区间中出现过的数的权值之积，可持久化线段树维护即可。

7 几何

7.1 凸包

```cpp
typedef complex<int> point;
#define X real()
#define Y imag()
int n;
long long cross(point a,point b) {
    return 1ll * a.X * b.Y - 1ll * a.Y * b.X;
}
bool cmp(point a,point b) {
    return make_pair(a.X, a.Y) < make_pair(b.X, b.Y);
}
int convexHull(point p[],int n,point ch[]) {
    sort(p, p + n, cmp);
    int m = 0;
    for(int i = 0; i < n; ++i) {
        while(m > 1 && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
        ch[m++] = p[i];
    }
    int k = m;
    for(int i = n - 2; i >= 0; --i) {
        while(m > k && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
        ch[m++] = p[i];
    }
    if(n > 1) m--;
    return m;
}
```

8 黑科技和杂项

8.1 找规律

有些题目，只给一个正整数 n ，然后要求输出一个答案。这时，我们可以暴力得到小数据的解，用高斯消元得到递推式，然后用矩阵快速幂求解。

使用方法：

首先在 gauss.in 中输入小数据的解（$n = 1$ 时，$n = 2$ 时，$\cdots$），以 EOF 结束。

依次运行 gauss.cpp , matrix.cpp ， 得到 matrix.out

将 matrix.out 中的文件粘贴在 main.cpp 中相应的位置中。注意模数一定要是质数。

```cpp
//gauss.cpp
#include <bits/stdc++.h>
#define N 102
#define mod 1000000007
//caution: you can use this program iff mod is a prime.

using namespace std;
```

```

int n, m, k, a[N], g[N][N];

int power(int base, int times) {
    int ret = 1;
    while (times) {
        if (times & 1) ret = 1ll * ret * base % mod;
        base = 1ll * base * base % mod;
        times >>= 1;
    }
    return ret;
}

int test() {
    for (int i=0;i<m;i++) {
        for (int j=i;j<=m;j++)
            if (g[j][i]) {
                for (int k=i;k<=m;k++)
                    swap(g[i][k], g[j][k]);
                break;
            }
        if (g[i][i] == 0)
            return 0;
        for (int j=i+1;j<n;j++) {
            while (g[j][i]) {
                int t = 1ll * g[i][i] * power(g[j][i], mod - 2) % mod;
                for (int k=i;k<n;k++)
                    g[i][k] = (g[i][k] + mod - (1ll * t * g[j][k] % mod)) % mod;
                for (int k=i;k<=m;k++)
                    swap(g[i][k], g[j][k]);
            }
        }
        int t = power(g[i][i], mod - 2);
        for (int j = 0; j <= m; ++j)
            g[i][j] = 1ll * g[i][j] * t % mod;
    }
    for (int i = m; i < n; ++i)
        if (g[i][m]) return 0;
    for (int i = m - 1; i >= 0; --i) {
        int t = power(g[i][i], mod - 2);
        g[i][i] = 1;
        g[i][m] = 1ll * g[i][m] * t % mod;
        for (int j = 0; j < i; ++j)
            g[j][m] = (g[j][m] + mod - 1ll * g[i][m] * g[j][i] % mod) % mod;
    }
    printf("%d\n", m);
    for (int i = 0; i < m; ++i)
        printf("%d ", g[i][m]);
    puts("");
    for (int i = 0; i < m - 1; ++i)
```

```
        printf("%d␣", a[i]);
    puts("1");
    return 1;
}

int main() {
    freopen("gauss.in", "r", stdin);
    freopen("gauss.out", "w", stdout);
    k = 0;
    while (~scanf("%d", &a[k++])) ;
    for (int sm = 1; sm <= k - sm; ++sm) {
        n = k - sm - 1;
        m = sm + 1;
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j <= sm; ++j)
                g[i][j] = a[i + j];
            g[i][m] = 1;
            swap(g[i][m - 1], g[i][m]);
        }
        if (test()) return 0;
    }
    puts("no␣solution");
    return 0;
}
```

```
//matrix.cpp
#include <bits/stdc++.h>
#define N 102
using namespace std;

int n, a[N];

int main() {
    freopen("gauss.out", "r", stdin);
    freopen("matrix.out", "w", stdout);
    scanf("%d", &n);
    for (int i = 0; i < n; ++i) scanf("%d", &a[i]);
    printf("#define␣M␣%d\n", n);
    printf("const␣int␣trans[M][M]␣=␣{\n");
    for (int i = 0; i < n; ++i) {
        printf("\t{");
        for (int j = 0; j < n; ++j) {
            int t;
            if (j < n - 2) t = i == j + 1;
            else if (j == n - 2) t = a[i];
            else t = i == n - 1;
            printf("%s%d", j == 0 ? "" : ",␣", t);
        }
        printf("}%s\n", i == n - 1 ? "" : ",");
    }
```

```
    printf("};\n");
    printf("const int pref[M] = {");
    for (int i = 0; i < n; ++i) {
        int x;
        scanf("%d", &x);
        printf("%d%s", x, i == n - 1 ? "};\n" : ", ");
    }
    return 0;
}
```

```
//main.cpp
#include <bits/stdc++.h>
using namespace std;

/* paste matrix.out here. */

#define mod 1000000007

struct Matrix {
    int c[M][M];
    void clear() { memset(c, 0, sizeof(c)); }
    void identity() { clear(); for (int i = 0; i < M; ++i) c[i][i] = 1; }
    void base() { memcpy(c, trans, sizeof(trans)); }
    friend Matrix operator * (const Matrix &a, const Matrix &b) {
        Matrix c; c.clear();
        for (int i = 0; i < M; ++i)
            for (int j = 0; j < M; ++j)
                for (int k = 0; k < M; ++k)
                    c.c[i][j] = (c.c[i][j] + 1ll * a.c[i][k] * b.c[k][j] % mod) % mod;
        return c;
    }
} start, base;

Matrix power(Matrix base, int times) {
    Matrix ret; ret.identity();
    while (times) {
        if (times & 1) ret = ret * base;
        base = base * base;
        times >>= 1;
    }
    return ret;
}

int main() {
    int tot;
    scanf("%d", &tot);
    while (tot--) {
        int n;
        scanf("%d", &n);
```

```
        start.clear();
        for (int i = 0; i < M; ++i) start.c[0][i] = pref[i];
        base.base();
        base = power(base, n - 1);
        start = start * base;
        printf("%d\n", start.c[0][0]);
    }
    return 0;
}
```

8.2 高精度计算

```
#include<algorithm>
using namespace std;
const int N_huge=850,base=100000000;
char s[N_huge*10];
struct huge{
    typedef long long value;
    value a[N_huge];int len;
    void clear(){len=1;a[len]=0;}
    huge(){clear();}
    huge(value x){*this=x;}
    huge operator =(huge b){
        len=b.len;for (int i=1;i<=len;++i)a[i]=b.a[i]; return *this;
    }
    huge operator =(value x){
        len=0;
        while (x)a[++len]=x%base,x/=base;
        if (!len)a[++len]=0;
        return *this;
    }
    huge operator +(huge b){
        int L=len>b.len?len:b.len;huge tmp;
        for (int i=1;i<=L+1;++i)tmp.a[i]=0;
        for (int i=1;i<=L;++i){
            if (i>len)tmp.a[i]+=b.a[i];
            else if (i>b.len)tmp.a[i]+=a[i];
            else {
                tmp.a[i]+=a[i]+b.a[i];
                if (tmp.a[i]>=base){
                    tmp.a[i]-=base;++tmp.a[i+1];
                }
            }
        }
        if (tmp.a[L+1])tmp.len=L+1;
            else tmp.len=L;
        return tmp;
    }
    huge operator -(huge b){
```

```
        int L=len>b.len?len:b.len;huge tmp;
        for (int i=1;i<=L+1;++i)tmp.a[i]=0;
        for (int i=1;i<=L;++i){
            if (i>b.len)b.a[i]=0;
            tmp.a[i]+=a[i]-b.a[i];
            if (tmp.a[i]<0){
                tmp.a[i]+=base;--tmp.a[i+1];
            }
        }
        while (L>1&&!tmp.a[L])--L;
        tmp.len=L;
        return tmp;
    }
    huge operator *(huge b){
        int L=len+b.len;huge tmp;
        for (int i=1;i<=L;++i)tmp.a[i]=0;
        for (int i=1;i<=len;++i)
            for (int j=1;j<=b.len;++j){
                tmp.a[i+j-1]+=a[i]*b.a[j];
                if (tmp.a[i+j-1]>=base){
                    tmp.a[i+j]+=tmp.a[i+j-1]/base;
                    tmp.a[i+j-1]%=base;
                }
            }
        tmp.len=len+b.len;
        while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
        return tmp;
    }
    pair<huge,huge> divide(huge a,huge b){
        int L=a.len;huge c,d;
        for (int i=L;i;--i){
        c.a[i]=0;d=d*base;d.a[1]=a.a[i];
            int l=0,r=base-1,mid;
            while (l<r){
                mid=(l+r+1)>>1;
                if (b*mid<=d)l=mid;
                    else r=mid-1;
            }
            c.a[i]=l;d-=b*l;
        }
        while (L>1&&!c.a[L])--L;c.len=L;
        return make_pair(c,d);
    }
    huge operator /(value x){
        value d=0;huge tmp;
        for (int i=len;i;--i){
            d=d*base+a[i];
            tmp.a[i]=d/x;d%=x;
        }
        tmp.len=len;
```

```
        while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
        return tmp;
    }
    value operator %(value x){
        value d=0;
        for (int i=len;i;--i)d=(d*base+a[i])%x;
        return d;
    }
    huge operator /(huge b){return divide(*this,b).first;}
    huge operator %(huge b){return divide(*this,b).second;}
    huge &operator +=(huge b){*this=*this+b;return *this;}
    huge &operator -=(huge b){*this=*this-b;return *this;}
    huge &operator *=(huge b){*this=*this*b;return *this;}
    huge &operator ++(){huge T;T=1;*this=*this+T;return *this;}
    huge &operator --(){huge T;T=1;*this=*this-T;return *this;}
    huge operator ++(int){huge T,tmp=*this;T=1;*this=*this+T;return tmp;}
    huge operator --(int){huge T,tmp=*this;T=1;*this=*this-T;return tmp;}
    huge operator +(value x){huge T;T=x;return *this+T;}
    huge operator -(value x){huge T;T=x;return *this-T;}
    huge operator *(value x){huge T;T=x;return *this*T;}
    huge operator *=(value x){*this=*this*x;return *this;}
    huge operator +=(value x){*this=*this+x;return *this;}
    huge operator -=(value x){*this=*this-x;return *this;}
    huge operator /=(value x){*this=*this/x;return *this;}
    huge operator %=(value x){*this=*this%x;return *this;}
    bool operator ==(value x){huge T;T=x;return *this==T;}
    bool operator !=(value x){huge T;T=x;return *this!=T;}
    bool operator <=(value x){huge T;T=x;return *this<=T;}
    bool operator >=(value x){huge T;T=x;return *this>=T;}
    bool operator <(value x){huge T;T=x;return *this<T;}
    bool operator >(value x){huge T;T=x;return *this>T;}
    bool operator <(huge b){
        if (len<b.len)return 1;
        if (len>b.len)return 0;
        for (int i=len;i;--i){
            if (a[i]<b.a[i])return 1;
            if (a[i]>b.a[i])return 0;
        }
        return 0;
    }
    bool operator ==(huge b){
        if (len!=b.len)return 0;
        for (int i=len;i;--i)
            if (a[i]!=b.a[i])return 0;
        return 1;
    }
    bool operator !=(huge b){return !(*this==b);}
    bool operator >(huge b){return !(*this<b||*this==b);}
    bool operator <=(huge b){return (*this<b)||(*this==b);}
    bool operator >=(huge b){return (*this>b)||(*this==b);}
```

```
    void str(char s[]){
        int l=strlen(s);value x=0,y=1;len=0;
        for (int i=l-1;i>=0;--i){
            x=x+(s[i]-'0')*y;y*=10;
            if (y==base)a[++len]=x,x=0,y=1;
        }
        if (!len||x)a[++len]=x;
    }
    void read(){
        scanf("%s",s);this->str(s);
    }
    void print(){
        printf("%d",(int)a[len]);
        for (int i=len-1;i;--i){
            for (int j=base/10;j>=10;j/=10){
                if (a[i]<j)printf("0");
                    else break;
            }
            printf("%d",(int)a[i]);
        }
        printf("\n");
    }
}f[1005];
int main(){
    f[1]=f[2]=1;
    for(int i=3;i<=1000;i++)f[i]=f[i-1]+f[i-2];
}
```

8.3 读入优化

```
#define rd RD<int>
#define rdll RD<long long>
template <typename Type>
inline Type RD() {
    Type x = 0;
    int flag = 0;
    char c = getchar();
    while (!isdigit(c) && c != '-')
        c = getchar();
    (c == '-') ? (flag = 1) : (x = c - '0');
    while (isdigit(c = getchar()))
        x = x * 10 + c - '0';
    return flag ? -x : x;
}
inline char rdch() {
    char c = getchar();
    while (!isalpha(c)) c = getchar();
    return c;
}
```

8.4 位运算及其运用

8.4.1 枚举子集

枚举 i 的非空子集 j

```
for (int j = i; j; j = (j - 1) & i);
```

8.4.2 求 1 的个数

```
int __builtin_popcount(unsigned int x);
```

8.4.3 求前缀 0 的个数

```
int __builtin_clz(unsigned int x);
```

8.4.4 求后缀 0 的个数

```
int __builtin_ctz(unsigned int x);
```