

# ICPC Templates For Africamonkey

Africamonkey

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## Contents

<b>1</b>	<b>莫队算法</b>	<b>3</b>
1.1	普通莫队	3
1.2	树上莫队	3
<b>2</b>	<b>字符串</b>	<b>5</b>
2.1	哈希	5
2.2	KMP	5
2.3	可动态修改的KMP	6
2.4	扩展KMP	6
2.5	Manacher	7
2.6	最小表示法	7
2.7	AC自动机	8
2.8	后缀数组	9
2.9	后缀自动机	10
<b>3</b>	<b>数据结构</b>	<b>11</b>
3.1	ST表	11
3.2	线段树小技巧	11
3.3	Splay	12
3.4	可持久化Treap	15
<b>4</b>	<b>树</b>	<b>17</b>
4.1	动态树	17
<b>5</b>	<b>图</b>	<b>19</b>
5.1	欧拉回路	19
5.2	最短路径	20
5.2.1	Dijkstra	20
5.2.2	SPFA	20
5.3	K 短路	21
5.4	Tarjan	24
5.5	2-SAT	24
5.6	统治者树 (Dominator Tree)	26
5.7	网络流	27

5.7.1	最大流	27
5.7.2	上下界有源汇网络流	28
5.7.3	上下界无源汇网络流	29
5.7.4	费用流	29
5.7.5	zkw费用流	30
<b>6</b>	<b>数学</b>	<b>33</b>
6.1	扩展欧几里得解同余方程	33
6.2	同余方程组	33
6.3	卡特兰数	33
6.4	斯特林数	33
6.4.1	第一类斯特林数	33
6.4.2	第二类斯特林数	34
6.5	错排公式	34
6.6	Lucas定理	34
6.7	高斯消元	34
6.7.1	行列式	34
6.7.2	Matrix-Tree定理	35
6.8	调和级数	35
6.9	曼哈顿距离的变换	35
6.10	线性筛素数	35
6.11	FFT	36
6.12	求原根	37
6.13	NTT	38
6.14	组合数 lcm	39
6.15	区间 lcm 的维护	39
<b>7</b>	<b>几何</b>	<b>40</b>
7.1	凸包	40
<b>8</b>	<b>黑科技和杂项</b>	<b>40</b>
8.1	找规律	40
8.2	高精度计算	43

# 1 莫队算法

## 1.1 普通莫队

```
1 struct Q { int l, r, sqrtl, id; } q[N];
2 int n, m, v[N], ans[N], nowans;
3 bool cmp(const Q &a, const Q &b) {
4     if (a.sqrtl != b.sqrtl) return a.sqrtl < b.sqrtl;
5     return a.r < b.r;
6 }
7 void change(int x) { if (!v[x]) checkin(); else checkout(); }
8 int main() {
9     .....
10    for (int i=1;i<=m;i++) q[i].sqrtl = q[i].l / sqrt(n), q[i].id = i;
11    sort(q+1, q+m+1, cmp);
12    int L=1,R=0; nowans=0;
13    memset(v, 0, sizeof(v));
14    for (int i=1;i<=m;i++) {
15        while (L<q[i].l) change(L++);
16        while (L>q[i].l) change(--L);
17        while (R<q[i].r) change(++R);
18        while (R>q[i].r) change(R--);
19        ans[q[i].id] = nowans;
20    }
21    .....
22 }
```

## 1.2 树上莫队

```
1 struct Query { int l, r, id, l_group; } query[N];
2 struct EDGE { int adj, next; } edge[N*2];
3 int n, m, top, gh[N], c[N], reorder[N], deep[N], father[N], size[N], son[N], Top[N];
4 void addedge(int x, int y) {
5     edge[++top].adj = y;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 void dfs(int x, int root=0) {
10    reorder[x] = ++top; father[x] = root; deep[x] = deep[root] + 1;
11    son[x] = 0; size[x] = 1; int dd = 0;
12    for (int p=gh[x]; p; p=edge[p].next)
13        if (edge[p].adj != root) {
14            dfs(edge[p].adj, x);
15            if (size[edge[p].adj] > dd) {
16                son[x] = edge[p].adj;
17                dd = size[edge[p].adj];
18            }
19            size[x] += size[edge[p].adj];
20        }
21 }
22 void split(int x, int tp) {
23     Top[x] = tp;
24     if (son[x]) split(son[x], tp);
25     for (int p=gh[x]; p; p=edge[p].next)
26         if (edge[p].adj != father[x] && edge[p].adj != son[x])
```

```

27         split(edge[p].adj, edge[p].adj);
28     }
29     int lca(int x, int y) {
30         int tx = Top[x], ty = Top[y];
31         while (tx != ty) {
32             if (deep[tx] < deep[ty]) {
33                 swap(tx, ty);
34                 swap(x, y);
35             }
36             x = father[tx];
37             tx = Top[x];
38         }
39         if (deep[x] < deep[y]) swap(x, y);
40         return y;
41     }
42     bool cmp(const Query &a, const Query &b) {
43         if (a.l_group != b.l_group) return a.l_group < b.l_group;
44         return reorder[a.r] < reorder[b.r];
45     }
46     int v[N], ans[N];
47     void upd(int x) { if (!v[x]) checkin(); else checkout(); }
48     void go(int &u, int taru, int v) {
49         int lca0 = lca(u, taru);
50         int lca1 = lca(u, v);   upd(lca1);
51         int lca2 = lca(taru, v); upd(lca2);
52         for (int x=u; x!=lca0; x=father[x]) upd(x);
53         for (int x=taru; x!=lca0; x=father[x]) upd(x);
54         u = taru;
55     }
56     int main() {
57         memset(gh, 0, sizeof(gh));
58         scanf("%d%d", &n, &m); top = 0;
59         for (int i=1;i<n;i++) {
60             int x,y; scanf("%d%d", &x, &y);
61             addedge(x, y); addedge(y, x);
62         }
63         top = 0; dfs(1); split(1, 1);
64         for (int i=1;i<=m;i++) {
65             if (reorder[query[i].l] > reorder[query[i].r])
66                 swap(query[i].l, query[i].r);
67             query[i].id = i;
68             query[i].l_group = reorder[query[i].l] / sqrt(n);
69         }
70         sort(query+1, query+m+1, cmp);
71         int L=1,R=1; upd(1);
72         for (int i=1;i<=m;i++) {
73             go(L,query[i].l,R);
74             go(R,query[i].r,L);
75             ans[query[i].id] = answer();
76         }
77         .....
78     }

```

## 2 字符串

### 2.1 哈希

```
1 const int P=31,D=1000173169;
2 int n, pow[N], f[N]; char a[N];
3 int hash(int l, int r) { return (LL)(f[r]-(LL)f[l-1]*pow[r-l+1]%D+D)%D; }
4 int main() {
5     scanf("%d%s", &n, a+1);
6     pow[0] = 1;
7     for (int i=1;i<=n;i++) pow[i] = (LL)pow[i-1]*P%D;
8     for (int i=1;i<=n;i++) f[i] = (LL)((LL)f[i-1]*P+a[i])%D;
9 }
```

### 2.2 KMP

接口: `int find_substring(char *pattern, char *text, int *next, int *ret);`

输入: 模式串, 匹配串

输出: 返回值表示模式串在匹配串中出现的次数

KMP的`next[i]`表示从0到i的字符串s, 前缀和后缀的最长重叠长度。

```
1 void find_next(char *pattern, int *next) {
2     int n = strlen(pattern);
3     for (int i=1;i<n;i++) {
4         int j = i;
5         while (j > 0) {
6             j = next[j];
7             if (pattern[j] == pattern[i]) {
8                 next[i+1] = j+1;
9                 break;
10            }
11        }
12    }
13 }
14 int find_substring(char *pattern, char *text, int *next, int *ret) {
15     find_next(pattern, next);
16     int n = strlen(pattern);
17     int m = strlen(text);
18     int k = 0;
19     for (int i=0,j=0;i<m;i++) {
20         if (j<n && text[i]==pattern[j]) {
21             j++;
22         } else {
23             while (j>0) {
24                 j = next[j];
25                 if (text[i] == pattern[j]) {
26                     j++;
27                     break;
28                 }
29             }
30         }
31         if (j == n)
32             ret[k++] = i-n+1;
33     }
34     return k;
35 }
```

35 }

## 2.3 可动态修改的KMP

支持：加入一个字符，删除一个字符。

时间复杂度：  $O(n\alpha)$ ， $\alpha$  为字符集大小。

代码中的字符为 '0' - '9'，可自行修改为 'a' - 'z'

```
1 char t[N];
2 int top, nxt[N], nxt_l[N][10];
3 inline void del_letter() { --top; }
4 inline void add_letter(char x) {
5     t[top++] = x;
6     int j = top-1;
7     memset(nxt_l[top], 0, sizeof(nxt_l[top]));
8     nxt[top] = nxt_l[top-1][x-'0'];
9     memcpy(nxt_l[top], nxt_l[nxt[top]], sizeof(nxt_l[nxt[top]]));
10    nxt_l[top][t[nxt[top]]-'0'] = nxt[top]+1;
11 }
```

## 2.4 扩展KMP

接口： void ExtendedKMP(char \*a, char \*b, int \*next, int \*ret);

输出：

next: a 关于自己每个后缀的最长公共前缀

ret: a 关于 b 的每个后缀的最长公共前缀

EXKMP的next[i]表示：从i到n-1的字符串st前缀和原串前缀的最长重叠长度。

```
1 void get_next(char *a, int *next) {
2     int i, j, k;
3     int n = strlen(a);
4     for (j = 0; j+1<n && a[j]==a[j+1];j++);
5     next[1] = j;
6     k = 1;
7     for (i=2;i<n;i++) {
8         int len = k+next[k], l = next[i-k];
9         if (l < len-i) {
10             next[i] = l;
11         } else {
12             for (j = max(0, len-i);i+j<n && a[j]==a[i+j];j++);
13             next[i] = j;
14             k = i;
15         }
16     }
17 }
18 void ExtendedKMP(char *a, char *b, int *next, int *ret) {
19     get_next(a, next);
20     int n = strlen(a), m = strlen(b);
21     int i, j, k;
22     for (j=0;j<n && j<m && a[j]==b[j];j++);
23     ret[0] = j;
24     k = 0;
```

```

25     for (i=1;i<m;i++) {
26         int len = k+ret[k], l = next[i-k];
27         if (l < len-i) {
28             ret[i] = l;
29         } else {
30             for (j = max(0, len-i); j<n && i+j<m && a[j]==b[i+j]; j++);
31             ret[i] = j;
32             k = i;
33         }
34     }
35 }

```

## 2.5 Manacher

$p[i]$  表示以  $i$  为对称轴的最长回文串长度

```

1  char st[N*2], s[N];
2  int len, p[N*2];
3
4  while (scanf("%s", s) != EOF) {
5      len = strlen(s);
6      st[0] = '$', st[1] = '#';
7      for (int i=1;i<=len;i++)
8          st[i*2] = s[i-1], st[i*2+1] = '#';
9      len = len * 2 + 2;
10     int mx = 0, id = 0, ans = 0;
11     for (int i=1;i<=len;i++) {
12         p[i] = (mx > i) ? min(p[id*2-i]+1, mx-i) : 1;
13         for (; st[i+p[i]] == st[i-p[i]]; ++p[i]) ;
14         if (p[i]+i > mx) mx = p[i]+i, id = i;
15         p[i] --;
16         if (p[i] > ans) ans = p[i];
17     }
18     printf("%d\n", ans);
19 }

```

## 2.6 最小表示法

```

1  string smallestRepresation(string s) {
2      int i, j, k, l;
3      int n = s.length();
4      s += s;
5      for (i=0,j=1;j<n;) {
6          for (k=0;k<n && s[i+k]==s[j+k];k++);
7          if (k>=n) break;
8          if (s[i+k]<s[j+k]) j+=k+1;
9          else {
10             l=i+k;
11             i=j;
12             j=max(l,j)+1;
13         }
14     }
15     return s.substr(i, n);
16 }

```

## 2.7 AC自动机

```
1 struct Node {
2     int next[**Size of Alphabet**];
3     int terminal, fail;
4 } node[**Number of Nodes**];
5 int top;
6 void add(char *st) {
7     int len = strlen(st), x = 1;
8     for (int i=0;i<len;i++) {
9         int ind = trans(st[i]);
10        if (!node[x].next[ind])
11            node[x].next[ind] = ++top;
12        x = node[x].next[ind];
13    }
14    node[x].terminal = 1;
15 }
16 int q[**Number of Nodes**], head, tail;
17 void build() {
18     head = 0, tail = 1; q[1] = 1;
19     while (head != tail) {
20         int x = q[++head];
21         /*(when necessary) node[x].terminal != node[node[x].fail].terminal; */
22         for (int i=0;i<n;i++)
23             if (node[x].next[i]) {
24                 if (x == 1) node[node[x].next[i]].fail = 1;
25                 else {
26                     int y = node[x].fail;
27                     while (y) {
28                         if (node[y].next[i]) {
29                             node[node[x].next[i]].fail = node[y].next[i];
30                             break;
31                         }
32                         y = node[y].fail;
33                     }
34                     if (!node[node[x].next[i]].fail) node[node[x].next[i]].fail = 1;
35                 }
36                 q[++tail] = node[x].next[i];
37             }
38     }
39 }
```



## 2.8 后缀数组

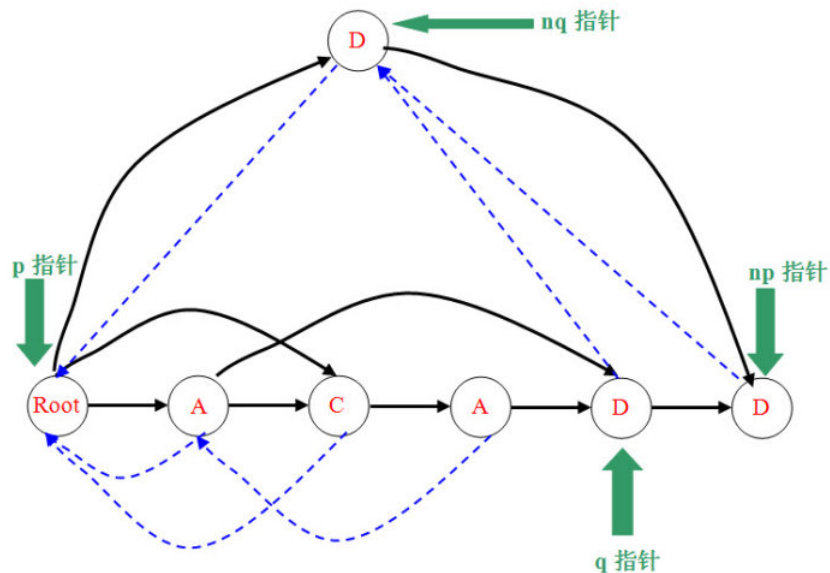
参数  $m$  表示字符集的大小, 即  $0 \leq r_i < m$

```
1  #define rank rank2
2  int n, r[N], wa[N], wb[N], ws[N], sa[N], rank[N], height[N];
3  int cmp(int *r, int a, int b, int l, int n)
4  {
5      if (r[a]==r[b])
6      {
7          if (a+l<n && b+l<n && r[a+l]==r[b+l])
8              return 1;
9      }
10     return 0;
11 }
12 void suffix_array(int m)
13 {
14     int i, j, p, *x=wa, *y=wb, *t;
15     for (i=0;i<m;i++) ws[i]=0;
16     for (i=0;i<n;i++) ws[x[i]=r[i]]++;
17     for (i=1;i<m;i++) ws[i]+=ws[i-1];
18     for (i=n-1;i>=0;i--) sa[--ws[x[i]]]=i;
19     for (j=1,p=1;p<n;m=p,j<=1)
20     {
21         for (p=0,i=n-j;i<n;i++) y[p++]=i;
22         for (i=0;i<n;i++) if (sa[i]>=j) y[p++]=sa[i]-j;
23         for (i=0;i<m;i++) ws[i]=0;
24         for (i=0;i<n;i++) ws[x[y[i]]]++;
25         for (i=1;i<m;i++) ws[i]+=ws[i-1];
26         for (i=n-1;i>=0;i--) sa[--ws[x[y[i]]]]=y[i];
27         for (t=x,x=y,y=t,x[sa[0]]=0,i=1,p=1;i<n;i++)
28             x[sa[i]]=cmp(y,sa[i-1],sa[i],j,n)?p-1:p++;
29     }
30     for (i=0;i<n;i++) rank[sa[i]]=i;
31 }
32 void calc_height()
33 {
34     int j=0;
35     for (int i=0;i<n;i++)
36         if (rank[i])
37         {
38             while (r[i+j]==r[sa[rank[i]-1]+j]) j++;
39             height[rank[i]]=j;
40             if (j) j--;
41         }
42 }
```

## 2.9 后缀自动机

下面的代码是求两个串的LCS（最长公共子串）。

```
1  #include <stdio>
2  #include <stdlib>
3  #include <string>
4  #define N 500001
5  using namespace std;
6  char st[N];
7  int pre[N<<1], son[26][N<<1], step[N<<1], last, total;
8  int apply(int x) { step[++total]=x; return total; }
9  void Extend(char x) {
10     int p = last, np = apply(step[last]+1);
11     for (; p && !son[x][p]; p=pre[p]) son[x][p] = np;
12     if (!p) pre[np] = 1;
13     else {
14         int q = son[x][p];
15         if (step[p]+1 == step[q]) pre[np] = q;
16         else {
17             int nq = apply(step[p]+1);
18             for (int i=0;i<26;i++) son[i][nq] = son[i][q];
19             pre[nq] = pre[q];
20             pre[q] = pre[np] = nq;
21             for (; p && son[x][p]==q; p=pre[p]) son[x][p] = nq;
22         }
23     }
24     last = np;
25 }
26 void init() {
27     last = total = 0;
28     last = apply(0);
29     scanf("%s",st);
30     for (int i=0; st[i]; i++)
31         Extend(st[i]-'a');
32     scanf("%s",st);
33 }
34 int main() {
35     init();
36     int p = 1, now = 0, ans = 0;
37     for (int i=0; st[i]; i++) {
38         int index = st[i]-'a';
39         for (; p && !son[index][p]; p = pre[p], now = step[p]) ;
40         if (!p) p = 1;
41         if (son[index][p]) {
42             p = son[index][p];
43             now++;
44             if (now > ans) ans = now;
45         }
46     }
47     printf("%d\n",ans);
48     return 0;
49 }
```



## 3 数据结构

### 3.1 ST表

```

1 int Log[N], f[17][N];
2 int ask(int x, int y) {
3     int k = log[y - x + 1];
4     return max(f[k][x], f[k][y - (1 << k) + 1]);
5 }
6 int main() {
7     for (i = 2; i <= n; i++) Log[i] = Log[i >> 1] + 1;
8     for (j = 1; j < K; j++) for (i = 1; i + (1 << j - 1) <= n; i++) f[j][i] = max(f[j - 1][i], f[j - 1][i + (1 << j - 1)]);
9 }

```

### 3.2 线段树小技巧

给定一个序列  $a$ ，寻找一个最大的  $i$  使得  $i \leq y$  且满足一些条件（如  $a[i] \geq w$ ，那么需要在线段树维护  $a$  的区间最大值）

```

1 int queryl(int p, int left, int right, int y, int w) {
2     if (right <= y) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (y <= mid) return queryl(p << 1 | 0, left, mid, y, w);
8     int ret = queryl(p << 1 | 1, mid + 1, right, y, w);
9     if (ret != -1) return ret;
10    return queryl(p << 1 | 0, left, mid, y, w);
11 }

```

给定一个序列  $a$ ，寻找一个最小的  $i$  使得  $i \geq x$  且满足一些条件（如  $a[i] \geq w$ ，那么需要在线段树维护  $a$  的区间最大值）

```

1 int queryr(int p, int left, int right, int y, int w) {
2     if (left >= x) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (x > mid) return queryr(p<<1|1, mid+1, right, y, w);
8     int ret = queryr(p<<1|0, left, mid, y, w);
9     if (ret != -1) return ret;
10    return queryr(p<<1|1, mid+1, right, y, w);
11 }

```

### 3.3 Splay

接口:

ADD  $x\ y\ d$ : 将  $[x, y]$  的所有数加上  $d$

REVERSE  $x\ y$ : 将  $[x, y]$  翻转

INSERT  $x\ p$ : 将  $p$  插入到第  $x$  个数的后面

DEL  $x$ : 将第  $x$  个数删除

```

1 struct SPLAY {
2     struct NODE {
3         int w, min;
4         int son[2], size, father, rev, lazy;
5     } node[N];
6     int top, rt;
7     void pushdown(int x) {
8         if (!x) return;
9         if (node[x].rev) {
10            node[node[x].son[0]].rev ^= 1;
11            node[node[x].son[1]].rev ^= 1;
12            swap(node[x].son[0], node[x].son[1]);
13            node[x].rev = 0;
14        }
15        if (node[x].lazy) {
16            node[node[x].son[0]].lazy += node[x].lazy;
17            node[node[x].son[1]].lazy += node[x].lazy;
18            node[x].w += node[x].lazy;
19            node[x].min += node[x].lazy;
20            node[x].lazy = 0;
21        }
22    }
23    void pushup(int x) {
24        if (!x) return;
25        pushdown(node[x].son[0]);
26        pushdown(node[x].son[1]);
27        node[x].size = node[node[x].son[0]].size + node[node[x].son[1]].size + 1;
28        node[x].min = node[x].w;
29        if (node[x].son[0]) node[x].min = min(node[x].min, node[node[x].son[0]].min);
30        if (node[x].son[1]) node[x].min = min(node[x].min, node[node[x].son[1]].min);
31    }
32    void sc(int x, int y, int w) {
33        node[x].son[w] = y;
34        node[y].father = x;

```

```

35     pushup(x);
36 }
37 void _ins(int w) {
38     top++;
39     node[top].w = node[top].min = w;
40     node[top].son[0] = node[top].son[1] = 0;
41     node[top].size = 1; node[top].father = 0; node[top].rev = 0;
42 }
43 void init() {
44     top = 0;
45     _ins(0); _ins(0); rt=1;
46     sc(1, 2, 1);
47 }
48 void rotate(int x) {
49     if (!x) return;
50     int y = node[x].father;
51     int w = node[y].son[1]==x;
52     sc(y, node[x].son[w^1], w);
53     sc(node[y].father, x, node[node[y].father].son[1]==y);
54     sc(x, y, w^1);
55 }
56 int q[N];
57 void flushdown(int x) {
58     int t=0; for (; x; x=node[x].father) q[++t]=x;
59     for (; t; t--) pushdown(q[t]);
60 }
61 void Splay(int x, int root=0) {
62     flushdown(x);
63     while (node[x].father != root) {
64         int y=node[x].father;
65         int w=node[y].son[1]==x;
66         if (node[y].father != root && node[node[y].father].son[w]==y) rotate(y);
67         rotate(x);
68     }
69 }
70 int find(int k) {
71     Splay(rt);
72     while (1) {
73         pushdown(rt);
74         if (node[node[rt].son[0]].size+1==k) {
75             Splay(rt);
76             return rt;
77         } else
78         if (node[node[rt].son[0]].size+1<k) {
79             k-=node[node[rt].son[0]].size+1;
80             rt=node[rt].son[1];
81         } else {
82             rt=node[rt].son[0];
83         }
84     }
85 }
86 int split(int x, int y) {
87     int fx = find(x);
88     int fy = find(y+2);
89     Splay(fx);
90     Splay(fy, fx);
91     return node[fy].son[0];

```

```

92     }
93     void add(int x, int y, int d) { //add d to each number in a[x]...a[y]
94         int t = split(x, y);
95         node[t].lazy += d;
96         Splay(t); rt=t;
97     }
98     void reverse(int x, int y) { // reverse the x-th to y-th elements
99         int t = split(x, y);
100        node[t].rev ^= 1;
101        Splay(t); rt=t;
102    }
103    void insert(int x, int p) { // insert p after the x-th element
104        int fx = find(x+1);
105        int fy = find(x+2);
106        Splay(fx);
107        Splay(fy, fx);
108        _ins(p);
109        sc(fy, top, 0);
110        Splay(top); rt=top;
111    }
112    void del(int x) { // delete the x-th element in Splay
113        int fx = find(x), fy = find(x+2);
114        Splay(fx); Splay(fy, fx);
115        node[fy].son[0] = 0;
116        Splay(fy); rt=fy;
117    }
118 } tree;

```

### 3.4 可持久化Treap

接口:

void insert(int x, char c); 在当前第  $x$  个字符后插入  $c$

void del(int x, int y); 删除第  $x$  个字符到第  $y$  个字符

void copy(int l, int r, int x); 复制第  $l$  个字符到第  $r$  个字符, 然后粘贴到第  $x$  个字符后

void reverse(int x, int y); 翻转第  $x$  个到第  $y$  个字符

char query(int k); 表示询问当前第  $x$  个字符是什么

```
1 #define mod 1000000007
2 struct Treap {
3     struct Node {
4         char key;
5         bool reverse;
6         int lc, rc, size;
7     } node[N];
8     int n, root, rd;
9     int Rand() { rd = (rd * 20372052LL + 25022087LL) % mod; return rd; }
10    void init() { n = root = 0; }
11    inline int copy(int x) { node[++n] = node[x]; return n; }
12    inline void pushdown(int x) {
13        if (!node[x].reverse) return;
14        if (node[x].lc) node[x].lc = copy(node[x].lc);
15        if (node[x].rc) node[x].rc = copy(node[x].rc);
16        swap(node[x].lc, node[x].rc);
17        node[node[x].lc].reverse ^= 1;
18        node[node[x].rc].reverse ^= 1;
19        node[x].reverse = 0;
20    }
21    inline void pushup(int x) { node[x].size = node[node[x].lc].size + node[node[x].rc].size
22        + 1; }
23    int merge(int u, int v) {
24        if (!u || !v) return u+v;
25        pushdown(u); pushdown(v);
26        int t = Rand() % (node[u].size + node[v].size), r;
27        if (t < node[u].size) {
28            r = copy(u);
29            node[r].rc = merge(node[u].rc, v);
30        } else {
31            r = copy(v);
32            node[r].lc = merge(u, node[v].lc);
33        }
34        pushup(r);
35        return r;
36    }
37    int split(int u, int x, int y) {
38        if (x > y) return 0;
39        pushdown(u);
40        if (x == 1 && y == node[u].size) return u;
41        if (y <= node[node[u].lc].size) return split(node[u].lc, x, y);
42        int t = node[node[u].lc].size + 1;
43        if (x > t) return split(node[u].rc, x-t, y-t);
44        int num = copy(u);
45        node[num].lc = split(node[u].lc, x, t-1);
46        node[num].rc = split(node[u].rc, 1, y-t);
47        pushup(num);
```

```

47     return num;
48 }
49 void insert(int x, char c) {
50     int t1 = split(root, 1, x), t2 = split(root, x+1, node[root].size);
51     node[++n].key = c; node[n].size = 1;
52     root = merge(merge(t1, n), t2);
53 }
54 void del(int x, int y) {
55     int t1 = split(root, 1, x-1), t2 = split(root, y+1, node[root].size);
56     root = merge(t1, t2);
57 }
58 void copy(int l, int r, int x) {
59     int t1 = split(root, 1, x), t2 = split(root, 1, r), t3 = split(root, x+1, node[root].
60         size);
61     root = merge(merge(t1, t2), t3);
62 }
63 void reverse(int x, int y) {
64     int t1 = split(root, 1, x-1), t2 = split(root, x, y), t3 = split(root, y+1, node[root].
65         size);
66     node[t2].reverse ^= 1;
67     root = merge(merge(t1, t2), t3);
68 }
69 char query(int k) {
70     int x = root;
71     while (1) {
72         pushdown(x);
73         if (k <= node[node[x].lc].size) x = node[x].lc;
74         else
75             if (k == node[node[x].lc].size + 1) return node[x].key;
76             else
77                 k -= node[node[x].lc].size + 1, x = node[x].rc;
78     }
79 }
80 } treap;

```



## 4 树

### 4.1 动态树

接口:

command(x, y) : 将 x 到 y 路径的 Splay Tree 分离出来。

linkcut(u1, v1, u2, v2) : 将树中原有的边 (u1, v1) 删除, 加入一条新边 (u2, v2)

```
1 struct DynamicTREE{
2     struct NODE{
3         int father, son[2], top, size, reverse;
4     } splay[N];
5     void init(int i, int fat) {
6         splay[i].father = splay[i].son[0] = splay[i].son[1] = 0;
7         splay[i].top = fat; splay[i].size = 1; splay[i].reverse = 0;
8     }
9     void pushdown(int x) {
10        if (!x) return;
11        int s0 = splay[x].son[0], s1 = splay[x].son[1];
12        if (splay[x].reverse) {
13            splay[s0].reverse ^= 1;
14            splay[s1].reverse ^= 1;
15            swap(splay[x].son[0], splay[x].son[1]);
16            splay[x].reverse = 0;
17        }
18        s0 = splay[x].son[0], s1 = splay[x].son[1];
19        splay[s0].top = splay[s1].top = splay[x].top;
20    }
21    void pushup(int x) {
22        if (!x) return;
23        pushdown(splay[x].son[0]);
24        pushdown(splay[x].son[1]);
25        splay[x].size = splay[splay[x].son[0]].size + splay[splay[x].son[1]].size + 1;
26    }
27    void sc(int x, int y, int w, bool Auto=true) {
28        splay[x].son[w] = y;
29        splay[y].father = x;
30        if (Auto) {
31            pushup(y);
32            pushup(x);
33        }
34    }
35    int top, tush[N];
36    void flowdown(int x) {
37        for (top=1; x; top++, x = splay[x].father) tush[top] = x;
38        for (; top; top--) pushdown(tush[top]);
39    }
40    void rotate(int x) {
41        if (!x) return;
42        int y = splay[x].father;
43        int w = splay[y].son[1] == x;
44        pushdown(y);
45        pushdown(x);
46        sc(splay[y].father, x, splay[splay[y].father].son[1]==y, false);
47        sc(y, splay[x].son[w^1], w, false);
48        sc(x, y, w^1, false);
49        pushup(y);
```

```

50     pushup(x);
51 }
52 void Splay(int x, int rt=0) {
53     if (!x) return;
54     flowdown(x);
55     while (splay[x].father != rt) {
56         int y = splay[x].father;
57         int w = splay[y].son[1]==x;
58         if (splay[y].father != rt && splay[splay[y].father].son[w] == y) rotate(y);
59         rotate(x);
60     }
61 }
62 void split(int x) {
63     int y = splay[x].son[1];
64     if (!y) return;
65     splay[y].father = 0;
66     splay[x].son[1] = 0;
67     splay[y].top = x;
68     pushup(x);
69 }
70 void access(int x) {
71     int y = 0;
72     while (x) {
73         Splay(x);
74         split(x);
75         sc(x, y, 1);
76         Splay(x);
77         y = x;
78         x = splay[x].top;
79     }
80 }
81 void changeroot(int x) {
82     access(x);
83     Splay(x);
84     splay[x].reverse = 1;
85     Splay(x);
86 }
87 void command(int x, int y, ...) {
88     LL ans = 0;
89     changeroot(x);
90     access(y);
91     Splay(x);
92     //then you can modify the Splay Tree
93 }
94 void linkcut(int u1, int v1, int u2, int v2) {
95     changeroot(u1);
96     access(v1);
97     Splay(u1); split(u1);
98     splay[v1].top = 0;
99     access(u2); changeroot(u2);
100    access(v2); changeroot(v2);
101    Splay(u2); Splay(v2);
102    splay[v2].top = u2;
103 }
104 } lct;

```

## 5 图

### 5.1 欧拉回路

欧拉回路:

无向图: 每个顶点的度数都是偶数, 则存在欧拉回路。

有向图: 每个顶点的入度 = 出度, 则存在欧拉回路。

欧拉路径:

无向图: 当且仅当该图所有顶点的度数为偶数, 或者除了两个度数为奇数外其余的全是偶数。

有向图: 当且仅当该图所有顶点出度 = 入度或者一个顶点出度 = 入度 + 1, 另一个顶点入度 = 出度 + 1, 其他顶点出度 = 入度。下面  $O(n+m)$  求欧拉回路的代码中,  $n$  为点数,  $m$  为边数, 若有解则依次输出经过的边的编号, 若是无向图, 则正数表示  $x$  到  $y$ , 负数表示  $y$  到  $x$ 。

```
1 namespace UndirectedGraph{
2     int n,m,i,x,y,d[N],g[N],v[M<<1],w[M<<1],vis[M<<1],nxt[M<<1],ed;
3     int ans[M],cnt;
4     void add(int x,int y,int z){
5         d[x]++;
6         v[++ed]=y;w[ed]=z;nxt[ed]=g[x];g[x]=ed;
7     }
8     void dfs(int x){
9         for(int&i=g[x];i;){
10             if(vis[i]){i=nxt[i];continue;}
11             vis[i]=vis[i^1]=1;
12             int j=w[i];
13             dfs(v[i]);
14             ans[++cnt]=j;
15         }
16     }
17     void solve(){
18         scanf("%d%d",&n,&m);
19         for(i=ed=1;i<=m;i++) scanf("%d%d",&x,&y),add(x,y,i),add(y,x,-i);
20         for(i=1;i<=n;i++) if(d[i]&1){puts("NO");return;}
21         for(i=1;i<=n;i++) if(g[i]){dfs(i);break;}
22         for(i=1;i<=n;i++) if(g[i]){puts("NO");return;}
23         puts("YES");
24         for(i=m;i;i--)printf("%d_",ans[i]);
25     }
26 }
27 namespace DirectedGraph{
28     int n,m,i,x,y,d[N],g[N],v[M],vis[M],nxt[M],ed;
29     int ans[M],cnt;
30     void add(int x,int y){
31         d[x]++;d[y]--;
32         v[++ed]=y;nxt[ed]=g[x];g[x]=ed;
33     }
34     void dfs(int x){
35         for(int&i=g[x];i;){
36             if(vis[i]){i=nxt[i];continue;}
37             vis[i]=1;
38             int j=i;
39             dfs(v[i]);
40             ans[++cnt]=j;
41         }
42     }
```

```

43     void solve() {
44         scanf("%d%d", &n, &m);
45         for(i=1; i<=m; i++) scanf("%d%d", &x, &y), add(x, y);
46         for(i=1; i<=n; i++) if(d[i]) {puts("NO"); return;}
47         for(i=1; i<=n; i++) if(g[i]) {dfs(i); break;}
48         for(i=1; i<=n; i++) if(g[i]) {puts("NO"); return;}
49         puts("YES");
50         for(i=m; i; i--) printf("%d_", ans[i]);
51     }
52 }

```

## 5.2 最短路径

### 5.2.1 Dijkstra

```

1  #include <queue>
2  using namespace std;
3  struct EDGE { int adj, w, next; } edge[M*2];
4  struct dat { int id, dist; dat(int id=0, int dist=0) : id(id), dist(dist) {} };
5  struct cmp { bool operator () (const dat &a, const dat &b) { return a.dist > b.dist; } };
6  priority_queue < dat, vector<dat>, cmp > q;
7  int n, top, gh[N], v[N], dist[N];
8  void addedge(int x, int y, int w) {
9      edge[++top].adj = y;
10     edge[top].w = w;
11     edge[top].next = gh[x];
12     gh[x] = top;
13 }
14 int dijkstra(int s, int t) {
15     memset(dist, 63, sizeof(dist));
16     memset(v, 0, sizeof(v));
17     dist[s] = 0;
18     q.push(dat(s, 0));
19     while (!q.empty()) {
20         dat x = q.top(); q.pop();
21         if (v[x.id]) continue; v[x.id] = 1;
22         for (int p=gh[x.id]; p; p=edge[p].next) {
23             if (x.dist + edge[p].w < dist[edge[p].adj]) {
24                 dist[edge[p].adj] = x.dist + edge[p].w;
25                 q.push(dat(edge[p].adj, dist[edge[p].adj]));
26             }
27         }
28     }
29     return dist[t];
30 }

```

### 5.2.2 SPFA

```

1  struct EDGE { int adj, w, next; } edge[M*2];
2  int n, m, top, gh[N], v[N], cnt[N], q[N], dist[N], head, tail;
3  void addedge(int x, int y, int w) {
4      edge[++top].adj = y;
5      edge[top].w = w;
6      edge[top].next = gh[x];

```

```

7     gh[x] = top;
8 }
9 int spfa(int S, int T) {
10     memset(v, 0, sizeof(v));
11     memset(cnt, 0, sizeof(cnt));
12     memset(dist, 63, sizeof(dist));
13     head = 0, tail = 1;
14     dist[S] = 0; q[1] = S;
15     while (head != tail) {
16         (head += 1) %= N;
17         int x = q[head]; v[x] = 0;
18         ++cnt[x]; if (cnt[x] > n) return -1;
19         for (int p=gh[x]; p; p=edge[p].next)
20             if (dist[x] + edge[p].w < dist[edge[p].adj]) {
21                 dist[edge[p].adj] = dist[x] + edge[p].w;
22                 if (!v[edge[p].adj]) {
23                     v[edge[p].adj] = 1;
24                     (tail += 1) %= N;
25                     q[tail] = edge[p].adj;
26                 }
27             }
28     }
29     return dist[T];
30 }

```

## 5.3 K 短路

接口:

kthsp::init(n) : 初始化并设置节点个数为n

kthsp::add(x, y, w) : 添加一条x到y的有向边

kthsp::work(S, T, k) : 求S到T的第k短路

```

1 #include <queue>
2
3 #define N 200020
4 #define M 400020
5 #define LOGM 20
6 #define LL long long
7 #define inf (1LL<<61)
8
9 namespace pheap {
10     struct Node {
11         int next, son[2];
12         LL val;
13     } node[M*LOGM];
14     int LOG[M];
15     int root[M], size[M*LOGM], top;
16     int add() {
17         ++top; assert(top < M*LOGM);
18         node[top].next = node[top].son[0] = node[top].son[1] = 0;
19         node[top].val = inf;
20         return top;
21     }
22     int copy(int x) { int t = add(); node[t] = node[x]; return t; }
23     void init() {

```

```

24     top = -1; add();
25     for (int i=2;i<M;i++) LOG[i] = LOG[i>>1] + 1;
26 }
27 void upd(int x, int &next, LL &val) {
28     if (val < node[x].val) {
29         swap(val, node[x].val);
30         swap(next, node[x].next);
31     }
32 }
33 void insert(int x, int next, LL val) {
34     int sz = size[root[x]] + 1;
35     root[x] = copy(root[x]);
36     size[root[x]] = sz; x = root[x];
37     upd(x, next, val);
38     for (int i=LOG[sz]-1;i>=0;i--) {
39         int ind = (sz>>i)&1;
40         node[x].son[ind] = copy(node[x].son[ind]);
41         x = node[x].son[ind];
42         upd(x, next, val);
43     }
44 }
45 };
46
47 namespace kthsp {
48     using namespace pheap;
49     struct EDGE {
50         int adj, w, next;
51     } edge[2][M];
52     struct W {
53         int x, y, w;
54     } e[M];
55     bool has_init = 0;
56     int n, m, top[2], gh[2][N], v[N];
57     LL dist[N];
58     void init(int n1) {
59         has_init = 1;
60         n = n1; m = 0;
61         memset(top, 0, sizeof(top));
62         memset(gh, 0, sizeof(gh));
63         for (int i=1;i<=n;i++) dist[i] = inf;
64     }
65     void addedge(int id, int x, int y, int w) {
66         edge[id][++top[id]].adj = y;
67         edge[id][top[id]].w = w;
68         edge[id][top[id]].next = gh[id][x];
69         gh[id][x] = top[id];
70     }
71     void add(int x, int y, int w) {
72         assert(has_init);
73         e[++m].x=x; e[m].y=y; e[m].w=w;
74     }
75     int q[N], best[N], bestw[N];
76     int deg[N];
77     void spfa(int S) {
78         for (int i=1;i<=n;i++) deg[i] = 0;
79         for (int i=1;i<=m;i++) deg[e[i].x] ++;
80         int head = 0, tail = 1;

```

```

81     dist[S] = 0; q[1] = S;
82     while (head != tail) {
83         (head += 1) %= N;
84         int x = q[head];
85         for (int p=gh[1][x]; p; p=edge[1][p].next) {
86             if (dist[x] + edge[1][p].w < dist[edge[1][p].adj]) {
87                 dist[edge[1][p].adj] = dist[x] + edge[1][p].w;
88                 best[edge[1][p].adj] = x;
89                 bestw[edge[1][p].adj] = p;
90             }
91             if (!--deg[edge[1][p].adj]) {
92                 (tail += 1) %= N;
93                 q[tail] = edge[1][p].adj;
94             }
95         }
96     }
97 }
98 void dfs(int x) {
99     if (v[x]) return; v[x] = 1;
100    if (best[x]) root[x] = root[best[x]];
101    for (int p=gh[0][x]; p; p=edge[0][p].next)
102        if (dist[edge[0][p].adj] != inf && bestw[x] != p) {
103            insert(x, edge[0][p].adj, edge[0][p].w + dist[edge[0][p].adj] - dist[x]);
104        }
105    for (int p=gh[1][x]; p; p=edge[1][p].next)
106        if (best[edge[1][p].adj] == x)
107            dfs(edge[1][p].adj);
108 }
109 typedef pair<LL,int> pli;
110 priority_queue <pli, vector<pli>, greater<pli> > pq;
111 LL work(int S, int T, int k) {
112     assert(has_init);
113     n++; add(T, n, 0);
114     if (S == T) k ++;
115     T = n;
116     for (int i=1;i<=m;i++) {
117         addedge(0, e[i].x, e[i].y, e[i].w);
118         addedge(1, e[i].y, e[i].x, e[i].w);
119     }
120     spfa(T);
121     root[T] = 0; pheap::init();
122     memset(v, 0, sizeof(v));
123     dfs(T);
124     while (!pq.empty()) pq.pop();
125     if (k == 1) return dist[S];
126     if (root[S]) pq.push(make_pair(dist[S] + node[root[S]].val, root[S]));
127     while (k--) {
128         if (pq.empty()) return inf;
129         pli now = pq.top(); pq.pop();
130         if (k == 1) return now.first;
131         int x = node[now.second].next, u = node[now.second].son[0], v = node[now.second].
            son[1];
132         if (root[x]) pq.push(make_pair(now.first + node[root[x]].val, root[x]));
133         if (u) pq.push(make_pair(now.first - node[now.second].val + node[u].val, u));
134         if (v) pq.push(make_pair(now.first - node[now.second].val + node[v].val, v));
135     }
136     return 0;

```

```

137     }
138 };

```

## 5.4 Tarjan

割点的判断：一个顶点  $u$  是割点，当且仅当满足 (1) 或 (2)：

(1)  $u$  为树根，且  $u$  有多于一个子树

(2)  $u$  不为树根，且满足存在  $(u, v)$  为树枝边（ $u$  为  $v$  的父亲），使得  $dfn[u] \leq low[v]$

桥的判断：一条无向边  $(u, v)$  是桥，当且仅当  $(u, v)$  为树枝边，满足  $dfn[u] < low[v]$

```

1 struct EDGE { int adj, next; } edge[M];
2 int n, m, top, gh[N];
3 int dfn[N], low[N], cnt, ind, stop, instack[N], stack[N], belong[N];
4 void addedge(int x, int y) {
5     edge[++top].adj = y;
6     edge[top].next = gh[x];
7     gh[x] = top;
8 }
9 void tarjan(int x) {
10     dfn[x] = low[x] = ++ind;
11     instack[x] = 1; stack[++stop] = x;
12     for (int p=gh[x]; p; p=edge[p].next)
13         if (!dfn[edge[p].adj]) {
14             tarjan(edge[p].adj);
15             low[x] = min(low[x], low[edge[p].adj]);
16         } else if (instack[edge[p].adj]) {
17             low[x] = min(low[x], dfn[edge[p].adj]);
18         }
19     if (dfn[x] == low[x]) {
20         ++cnt; int tmp=0;
21         while (tmp!=x) {
22             tmp = stack[stop--];
23             belong[tmp] = cnt;
24             instack[tmp] = 0;
25         }
26     }
27 }

```

## 5.5 2-SAT

```

1 #define N number_of_vertex
2 #define M number_of_edges
3
4 struct MergePoint {
5     struct EDGE {
6         int adj, next;
7     } edge[M];
8     int ex[M], ey[M];
9     bool instack[N];
10    int gh[N], top, dfn[N], low[N], cnt, ind, stop, stack[N], belong[N];
11    void init() {
12        cnt = ind = stop = top = 0;
13        memset(dfn, 0, sizeof(dfn));
14        memset(instack, 0, sizeof(instack));

```



```

15     memset(gh, 0, sizeof(gh));
16 }
17 void addedge(int x, int y) { //reverse
18     std::swap(x, y);
19     edge[++top].adj = y;
20     edge[top].next = gh[x];
21     gh[x] = top;
22     ex[top] = x;
23     ey[top] = y;
24 }
25 void tarjan(int x) {
26     dfn[x] = low[x] = ++ind;
27     instack[x] = 1; stack[++stop] = x;
28     for (int p=gh[x]; p; p=edge[p].next)
29         if (!dfn[edge[p].adj]) {
30             tarjan(edge[p].adj);
31             low[x] = std::min(low[x], low[edge[p].adj]);
32         } else if (instack[edge[p].adj]) {
33             low[x] = std::min(low[x], dfn[edge[p].adj]);
34         }
35     if (dfn[x] == low[x]) {
36         ++cnt; int tmp = 0;
37         while (tmp!=x) {
38             tmp = stack[stop--];
39             belong[tmp] = cnt;
40             instack[tmp] = 0;
41         }
42     }
43 }
44 void work() {
45     for (int i = (__first__); i <= (__last__); ++i)
46         if (!dfn[i])
47             tarjan(i);
48 }
49 } merge;
50
51 struct Topsort {
52     struct EDGE {
53         int adj, next;
54     } edge[M];
55     int n, top, gh[N], ops[N], deg[N], ans[N];
56     std::queue<int> q;
57     void init() {
58         n = merge.cnt; top = 0;
59         memset(gh, 0, sizeof(gh));
60         memset(deg, 0, sizeof(deg));
61     }
62     void addedge(int x, int y) {
63         if (x == y) return;
64         edge[++top].adj = y;
65         edge[top].next = gh[x];
66         gh[x] = top;
67         ++deg[y];
68     }
69     void work() {
70         for (int i = 1; i <= n; ++i)
71             if (!deg[i])

```

```

72         q.push(i);
73     while (!q.empty()) {
74         int x = q.front();
75         q.pop();
76         for (int p = gh[x]; p; p = edge[p].next)
77             if (--deg[edge[p].adj])
78                 q.push(edge[p].adj);
79         if (ans[x]) continue;
80         ans[x] = -1; //not selected
81         ans[ops[x]] = 1; //selected
82     }
83 }
84 } ts;

```

调用示例:

```

1     merge.init();
2     merge.addedge();
3     merge.work();
4     for (int i = 1; i <= n; ++i) {
5         if (merge.belong[U(i, 0)] == merge.belong[U(i, 1)]) {
6             puts("NO");
7             return 0;
8         }
9         ts.ops[merge.belong[U(i, 0)]] = merge.belong[U(i, 1)];
10        ts.ops[merge.belong[U(i, 1)]] = merge.belong[U(i, 0)];
11    }
12    ts.init();
13    ts.work();
14    puts("YES");
15    for (int i = 1; i <= n; ++i) {
16        int x = U(i, 0), y = U(i, 1);
17        x = merge.belong[x], y = merge.belong[y];
18        x = ts.ans[x], y = ts.ans[y];
19        if (x == 1) puts("0_is_selected");
20        if (y == 1) puts("1_is_selected");
21    }

```

## 5.6 统治者树 (Dominator Tree)

Dominator Tree 可以解决判断一类有向图必经点的问题。

$idom[x]$  表示离  $x$  最近的必经点 (重编号后)。将  $idom[x]$  作为  $x$  的父亲, 构成一棵 Dominator Tree 接口:

`void dominator::init(int n);` 初始化, 有向图节点数为  $n$

`void dominator::addedge(int u, int v);` 添加一条有向边  $(u, v)$

`void dominator::work(int root);` 以  $root$  为根, 建立一棵 Dominator Tree

结果的返回:

在执行 `dominator::work(int root);` 后, 树边保存在 `vector<int> tree[N]` 中

```

1 namespace dominator {
2     vector<int> g[N], rg[N], bucket[N], tree[N];
3     int n, ind, idom[N], sdom[N], dfn[N], dsu[N], father[N], label[N], rev[N];
4     void dfs(int x) {
5         ++ind;
6         dfn[x] = ind; rev[ind] = x;

```

```

7      label[ind] = dsu[ind] = sdom[ind] = ind;
8      for (auto p : g[x]) {
9          if (!dfn[p]) dfs(p), father[dfn[p]] = dfn[x];
10         rg[dfn[p]].push_back(dfn[x]);
11     }
12 }
13 void init(int n1) {
14     n = n1; ind = 0;
15     for (int i = 1; i <= n; ++i) {
16         g[i].clear();
17         rg[i].clear();
18         bucket[i].clear();
19         tree[i].clear();
20         dfn[i] = 0;
21     }
22 }
23 void addedge(int u, int v) {
24     g[u].push_back(v);
25 }
26 int find(int x, int step=0) {
27     if (dsu[x] == x) return step ? -1 : x;
28     int y = find(dsu[x], 1);
29     if (y < 0) return x;
30     if (sdom[label[dsu[x]]] < sdom[label[x]])
31         label[x] = label[dsu[x]];
32     dsu[x] = y;
33     return step ? dsu[x] : label[x];
34 }
35 void work(int root) {
36     dfs(root); n = ind;
37     for (int i = n; i; --i) {
38         for (auto p : rg[i])
39             sdom[i] = min(sdom[i], sdom[find(p)]);
40         if (i > 1) bucket[sdom[i]].push_back(i);
41         for (auto p : bucket[i]) {
42             int u = find(p);
43             if (sdom[p] == sdom[u]) idom[p] = sdom[p];
44             else idom[p] = u;
45         }
46         if (i > 1) dsu[i] = father[i];
47     }
48     for (int i = 2; i <= n; ++i) {
49         if (idom[i] != sdom[i])
50             idom[i] = idom[idom[i]];
51         tree[rev[i]].push_back(rev[idom[i]]);
52         tree[rev[idom[i]]].push_back(rev[i]);
53     }
54 }
55 };

```

## 5.7 网络流

### 5.7.1 最大流

注意: *top* 要初始化为 1

```

1 struct EDGE { int adj, w, next; } edge[M];
2 int n, top, gh[N], nrl[N];
3 void addedge(int x, int y, int w) {
4     edge[++top].adj = y;
5     edge[top].w = w;
6     edge[top].next = gh[x];
7     gh[x] = top;
8     edge[++top].adj = x;
9     edge[top].w = 0;
10    edge[top].next = gh[y];
11    gh[y] = top;
12 }
13 int dist[N], q[N];
14 int bfs() {
15     memset(dist, 0, sizeof(dist));
16     q[1] = S; int head = 0, tail = 1; dist[S] = 1;
17     while (head != tail) {
18         int x = q[++head];
19         for (int p=gh[x]; p; p=edge[p].next)
20             if (edge[p].w && !dist[edge[p].adj]) {
21                 dist[edge[p].adj] = dist[x] + 1;
22                 q[++tail] = edge[p].adj;
23             }
24     }
25     return dist[T];
26 }
27 int dinic(int x, int delta) {
28     if (x==T) return delta;
29     for (int& p=nrl[x]; p && delta; p=edge[p].next)
30         if (edge[p].w && dist[x]+1 == dist[edge[p].adj]) {
31             int dd = dinic(edge[p].adj, min(delta, edge[p].w));
32             if (!dd) continue;
33             edge[p].w -= dd;
34             edge[p^1].w += dd;
35             return dd;
36         }
37     return 0;
38 }
39 int work() {
40     int ans = 0;
41     while (bfs()) {
42         memcpy(nrl, gh, sizeof(gh));
43         int t; while (t = dinic(S, inf)) ans += t;
44     }
45     return ans;
46 }

```

### 5.7.2 上下界有源汇网络流

$T$  向  $S$  连容量为正无穷的边，将有源汇转化为无源汇。

每条边容量减去下界，设  $in[i]$  表示流入  $i$  的下界之和减去流出  $i$  的下界之和。

新建超级源汇  $SS, TT$ ，对于  $in[i] > 0$  的点， $SS$  向  $i$  连容量为  $in[i]$  的边。对于  $in[i] < 0$  的点， $i$  向  $TT$  连容量为  $-in[i]$  的边。

求出以  $SS, TT$  为源汇的最大流，如果等于  $\sum in[i] (in[i] > 0)$ ，则存在可行流。再求出  $S, T$  为源汇的最

大流即为最大流。

费用流：建完图后等价于求以  $SS, TT$  为源汇的费用流。

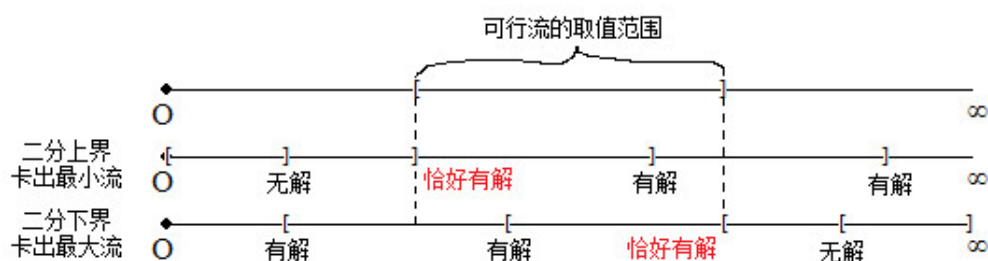
### 5.7.3 上下界无源汇网络流

1. 怎样求无源汇有上下界网络的可行流？

由于有源汇的网络我们先要转化成无源汇，所以本来就无源汇的网络不用再做特殊处理。

2. 怎样求无源汇有上下界网络的最大流、最小流？

一种简易的方法是采用二分思想，不断通过可行流的存在与否对  $(t, s)$  边的上下界  $U, L$  进行调整。求最大流时令  $U = \infty$  并二分  $L$ ；求最小流时令  $L = 0$  并二分  $U$ 。道理很简单，因为可行流的取值范围是一段连续的区间，我们只要通过二分找到有解和无解的分界线即可。



### 5.7.4 费用流

注意：  $top$  要初始化为 1

```
1 #define inf 0x3f3f3f3f
2 struct NetWorkFlow {
3     struct EDGE {
4         int adj, w, cost, next;
5     } edge[M*2];
6     int gh[N], q[N], dist[N], v[N], pre[N], prev[N], top;
7     int S, T;
8     void addedge(int x, int y, int w, int cost) {
9         edge[++top].adj = y;
10        edge[top].w = w;
11        edge[top].cost = cost;
12        edge[top].next = gh[x];
13        gh[x] = top;
14        edge[++top].adj = x;
15        edge[top].w = 0;
16        edge[top].cost = -cost;
17        edge[top].next = gh[y];
18        gh[y] = top;
19    }
20    void clear() {
21        top = 1;
22        memset(gh, 0, sizeof(gh));
23    }
24    int spfa() {
25        memset(dist, 63, sizeof(dist));
```

```

26     memset(v, 0, sizeof(v));
27     int head = 0, tail = 1;
28     q[1] = S; v[S] = 1; dist[S] = 0;
29     while (head != tail) {
30         (head += 1) %= N;
31         int x = q[head];
32         v[x] = 0;
33         for (int p=gh[x]; p; p=edge[p].next)
34             if (edge[p].w && dist[x] + edge[p].cost < dist[edge[p].adj]) {
35                 dist[edge[p].adj] = dist[x] + edge[p].cost;
36                 pre[edge[p].adj] = x;
37                 prev[edge[p].adj] = p;
38                 if (!v[edge[p].adj]) {
39                     v[edge[p].adj] = 1;
40                     (tail += 1) %= N;
41                     q[tail] = edge[p].adj;
42                 }
43             }
44     }
45     return dist[T] != inf;
46 }
47 int work() {
48     int ans = 0;
49     while (spfa()) {
50         int mx = inf;
51         for (int x=T; x!=S; x=pre[x])
52             mx = min(edge[prev[x]].w, mx);
53         ans += dist[T] * mx;
54         for (int x=T; x!=S; x=pre[x]) {
55             edge[prev[x]].w -= mx;
56             edge[prev[x]^1].w += mx;
57         }
58     }
59     return ans;
60 }
61 } nwf;

```

### 5.7.5 zkw费用流

注意: *top* 要初始化为 1, 不得用于有负权的图

```

1 #define inf 0x3f3f3f3f
2 struct NetWorkFlow {
3     struct EDGE {
4         int adj, w, cost, next;
5     } edge[M*2];
6     int gh[N], top;
7     int S, T;
8     void addedge(int x, int y, int w, int cost) {
9         edge[++top].adj = y;
10        edge[top].w = w;
11        edge[top].cost = cost;
12        edge[top].next = gh[x];
13        gh[x] = top;
14        edge[++top].adj = x;
15        edge[top].w = 0;

```

```

16     edge[top].cost = -cost;
17     edge[top].next = gh[y];
18     gh[y] = top;
19 }
20 void clear() {
21     top = 1;
22     memset(gh, 0, sizeof(gh));
23 }
24 int cost, d[N], slk[N], v[N];
25 int aug(int x, int f) {
26     int left = f;
27     if (x == T) {
28         cost += f * d[S];
29         return f;
30     }
31     v[x] = true;
32     for (int p=gh[x]; p; p=edge[p].next)
33         if (edge[p].w && !v[edge[p].adj]) {
34             int t = d[edge[p].adj] + edge[p].cost - d[x];
35             if (t == 0) {
36                 int delt = aug(edge[p].adj, min(left, edge[p].w));
37                 if (delt > 0) {
38                     edge[p].w -= delt;
39                     edge[p^1].w += delt;
40                     left -= delt;
41                 }
42                 if (left == 0) return f;
43             } else {
44                 if (t < slk[edge[p].adj])
45                     slk[edge[p].adj] = t;
46             }
47         }
48     return f-left;
49 }
50 bool modlabel() {
51     int delt = inf;
52     for (int i=1; i<=T; i++)
53         if (!v[i]) {
54             if (slk[i] < delt) delt = slk[i];
55             slk[i] = inf;
56         }
57     if (delt == inf) return true;
58     for (int i=1; i<=T; i++)
59         if (v[i]) d[i] += delt;
60     return false;
61 }
62 int work() {
63     cost = 0;
64     memset(d, 0, sizeof(d));
65     memset(slk, 63, sizeof(slk));
66     do {
67         do {
68             memset(v, 0, sizeof(v));
69         } while (aug(S, inf));
70     } while (!modlabel());
71     return cost;
72 }

```

73 | } nwf;

---



## 6 数学

### 6.1 扩展欧几里得解同余方程

ans[] 保存的是循环节内所有的解

```
1 int exgcd(int a,int b,int&x,int&y){
2     if(!b) return x=1,y=0,a;
3     int d=exgcd(b,a%b,x,y),t=x;
4     return x=y,y=t-a/b*y,d;
5 }
6 void cal(ll a,ll b,ll n){//ax=b(mod n)
7     ll x,y,d=exgcd(a,n,x,y);
8     if(b%d) return;
9     x=(x%n+n)%n;
10    ans[cnt=1]=x*(b/d)%(n/d);
11    for(ll i=1;i<d;i++) ans[++cnt]=(ans[1]+i*n/d)%n;
12 }
```

### 6.2 同余方程组

```
1 int n,flag,k,m,a,r,d,x,y;
2 int main(){
3     scanf("%d",&n);
4     flag=k=1,m=0;
5     while(n--){
6         scanf("%d%d",&a,&r);//ans%a=r
7         if(flag){
8             d=exgcd(k,a,x,y);
9             if((r-m)%d){flag=0;continue;}
10            x=(x*(r-m)/d+a/d)%(a/d),y=k/d*a,m=(x*k+m)%y;
11            if(m<0)m+=y;
12            k=y;
13        }
14    }
15    printf("%d",flag?m:-1);//若flag说明有解解为=l,,ki+m,为任意整数i
16 }
```

### 6.3 卡特兰数

$$h_1 = 1, h_n = \frac{h_{n-1}(4n-2)}{n+1} = \frac{C(2n,n)}{n+1} = C(2n,n) - C(2n,n-1)$$

在一个格点阵列中, 从  $(0,0)$  点走到  $(n,m)$  点且不过对角线  $x=y$  的方案数  $(x>y)$  :

$$C(n+m-1,m) - C(n+m-1,m-1)$$

在一个格点阵列中, 从  $(0,0)$  点走到  $(n,m)$  点且不穿过对角线  $x=y$  的方案数  $(x\geq y)$  :

$$C(n+m,m) - C(n+m,m-1)$$

### 6.4 斯特林数

#### 6.4.1 第一类斯特林数

第一类 Stirling 数  $S(p,k)$  的一个组合学解释是: 将  $p$  个物体排成  $k$  个非空循环排列的方法数。

$S(p,k)$  的递推公式:  $S(p,k) = (p-1)S(p-1,k) + S(p-1,k-1), 1 \leq k \leq p-1$

边界条件:  $S(p,0) = 0, p \geq 1$   $S(p,p) = 1, p \geq 0$

### 6.4.2 第二类斯特林数

第二类 Stirling 数  $S(p, k)$  的一个组合学解释是：将  $p$  个物体划分成  $k$  个非空的不可辨别（可以理解为盒子没有编号）集合的方法数。

$S(p, k)$  的递推公式：  $S(p, k) = kS(p-1, k) + S(p-1, k-1), 1 \leq k \leq p-1$

边界条件：  $S(p, 0) = 0, p \geq 1$   $S(p, p) = 1, p \geq 0$

也有卷积形式：

$$S(n, m) = \frac{1}{m!} \sum_{k=0}^m (-1)^k C(m, k) (m-k)^n = \sum_{k=0}^m \frac{(-1)^k (m-k)^n}{k! (m-k)!} = \sum_{k=0}^m \frac{(-1)^k}{k!} \times \frac{(m-k)^n}{(m-k)!}$$

### 6.5 错排公式

$$D_1 = 0, D_2 = 1, D_n = (n-1)(D_{n-2} + D_{n-1})$$

### 6.6 Lucas定理

接口：

初始化： `void lucas::init();`

计算  $C(n, m) \% \text{mod}$  的值： `LL lucas::Lucas(LL n, LL m);`

```

1  #define mod 110119
2  #define LL long long
3  namespace lucas {
4      LL fac[mod+1], facv[mod+1];
5      LL power(LL base, LL times) {
6          LL ans = 1;
7          while (times) {
8              if (times&1) (ans *= base) %= mod;
9              (base *= base) %= mod;
10             times >>= 1;
11         }
12         return ans;
13     }
14     void init() {
15         fac[0] = 1; for (int i=1; i<mod; i++) fac[i] = (fac[i-1] * i) % mod;
16         facv[mod-1] = power(fac[mod-1], mod-2);
17     }
18     LL C(unsigned LL n, unsigned LL m) {
19         if (n < m) return 0;
20         return (fac[n] * facv[m] % mod * facv[n-m] % mod) % mod;
21     }
22     LL Lucas(unsigned LL n, unsigned LL m)
23     {
24         if (m == 0) return 1;
25         return (C(n%mod, m%mod) * Lucas(n/mod, m/mod)) % mod;
26     }
27 };

```

### 6.7 高斯消元

#### 6.7.1 行列式

```

1  int ans = 1;
2  for (int i=0;i<n;i++) {
3      for (int j=i;j<n;j++)
4          if (g[j][i]) {
5              for (int k=i;k<n;k++)
6                  swap(g[i][k], g[j][k]);
7              if (j != i) ans *= -1;
8              break;
9          }
10     if (g[i][i] == 0) {
11         ans = 0;
12         break;
13     }
14     for (int j=i+1;j<n;j++) {
15         while (g[j][i]) {
16             int t = g[i][i] / g[j][i];
17             for (int k=i;k<n;k++)
18                 g[i][k] = (g[i][k] + mod - ((LL)t * g[j][k] % mod)) % mod;
19             for (int k=i;k<n;k++)
20                 swap(g[i][k], g[j][k]);
21             ans *= -1;
22         }
23     }
24 }
25 for (int i=0;i<n;i++)
26     ans = ((LL)ans * g[i][i]) % mod;
27 ans = (ans % mod + mod) % mod;
28 printf("%d\n", ans);

```

### 6.7.2 Matrix-Tree定理

对于一张图，建立矩阵  $C$ ， $C[i][i] = i$  的度数，若  $i, j$  之间有边，那么  $C[i][j] = -1$ ，否则为 0。这张图的生成树个数等于矩阵  $C$  的  $n-1$  阶行列式的值。

## 6.8 调和级数

$\sum_{i=1}^n \frac{1}{i}$  在  $n$  较大时约等于  $\ln(n) + r$ ， $r$  为欧拉常数，约等于 0.5772156649015328。

## 6.9 曼哈顿距离的变换

$$|x_1 - x_2| + |y_1 - y_2| = \max(|(x_1 + y_1) - (x_2 + y_2)|, |(x_1 - y_1) - (x_2 - y_2)|)$$

## 6.10 线性筛素数

```

1  mu[1]=phi[1]=1;top=0;
2  for (int i=2;i<N;i++) {
3      if (!v[i]) prime[++top]=i, mu[i] = -1, phi[i] = i-1;
4      for (int j=1;i*prime[j]<N && j<=top;j++) {
5          v[i*prime[j]] = 1;
6          if (i%prime[j]) {
7              mu[i*prime[j]] = -mu[i];
8              phi[i*prime[j]] = phi[i] * (prime[j]-1);

```

```

9         } else {
10             mu[i*prime[j]] = 0;
11             phi[i*prime[j]] = phi[i] * prime[j];
12             break;
13         }
14     }
15 }

```

## 6.11 FFT

```

1  typedef complex<double> comp;
2  namespace FFT {
3      comp A[N], B[N], omega[N];
4      void transform(comp *x, int len) {
5          for (int i=1, j=len/2; i<len-1; i++) {
6              if (i<j) swap(x[i], x[j]);
7              int k = len/2;
8              while (j>=k) {
9                  j-=k;
10                 k/=2;
11             }
12             if (j<k) j+=k;
13         }
14     }
15     void fft(comp *x, int len, int reverse) {
16         transform(x, len);
17         for (int h=2; h<=len; h<=1) {
18             for (int i=0; i<h/2; i++) omega[i] = polar(1.0, 2*pi*reverse/h*i);
19             for (int i=0; i<len; i+=h) {
20                 for (int j=i; j<i+h/2; j++) {
21                     comp w = omega[j-i];
22                     comp u = x[j];
23                     comp v = (w * x[j+h/2]);
24                     x[j] = u + v;
25                     x[j+h/2] = u - v;
26                 }
27             }
28         }
29         if (reverse == -1) {
30             for (int i=0; i<len; i++)
31                 x[i] /= len;
32         }
33     }
34     void work(int n, int *a, int *b) {
35         int len = 1;
36         while (len <= n*2) len *= 2;
37         for (int i=0; i<len; i++) A[i] = B[i] = 0;
38         for (int i=0; i<n; i++) A[i] = a[i], B[i] = b[i];
39         fft(A, len, 1); fft(B, len, 1);
40         for (int i=0; i<len; i++) A[i] = A[i] * B[i];
41         fft(A, len, -1);
42         for (int i=0; i<len; i++) {
43             LL r = round(A[i].real());
44             a[i] = r % mod;
45         }

```

```
46     }
47 };
```

## 6.12 求原根

接口: `LL p_root(LL p);`

输入: 一个素数  $p$

输出:  $p$  的原根

```
1  #include <bits/stdc++.h>
2  #define LL long long
3
4  using namespace std;
5
6  vector <LL> a;
7
8  LL pow_mod(LL base, LL times, LL mod) {
9      LL ret = 1;
10     while (times) {
11         if (times&1) ret = ret * base % mod;
12         base = base * base % mod;
13         times>>=1;
14     }
15     return ret;
16 }
17
18 bool g_test(LL g, LL p) {
19     for (LL i = 0; i < a.size(); ++i)
20         if (pow_mod(g, (p-1)/a[i], p) == 1) return 0;
21     return 1;
22 }
23
24 LL p_root(LL p) {
25     LL tmp = p - 1;
26     for (LL i = 2; i <= tmp / i; ++i)
27         if (tmp % i == 0) {
28             a.push_back(i);
29             while (tmp % i == 0)
30                 tmp /= i;
31         }
32     if (tmp != 1) a.push_back(tmp);
33     LL g = 1;
34     while (1) {
35         if (g_test(g, p)) return g;
36         ++g;
37     }
38 }
39
40 int main() {
41     LL p;
42     cin >> p;
43     cout << p_root(p) << endl;
44 }
```

## 6.13 NTT

998244353 原根为 3， 1004535809 原根为 3， 786433 原根为 10， 880803841 原根为 26。

```
1 #define mod 998244353
2 #define g 3
3 LL wi[N], wiv[N];
4 LL power(LL base, LL times) {
5     LL ans = 1;
6     while (times) {
7         if (times&1) (ans *= base) %= mod;
8         (base *= base) %= mod;
9         times >>= 1;
10    }
11    return ans;
12 }
13 void transform(LL *x, int len) {
14     for (int i=1, j=len/2; i<len-1; i++) {
15         if (i<j) swap(x[i], x[j]);
16         int k = len/2;
17         while (j>=k) {
18             j-=k;
19             k/=2;
20         }
21         if (j<k) j+=k;
22     }
23 }
24 void NTT(LL *x, int len, int reverse) {
25     transform(x, len);
26     for (int h=2; h<=len; h<=1) {
27         for (int i=0; i<len; i+=h) {
28             LL w = 1, wn;
29             if (reverse==1) wn = wi[h]; else wn = wiv[h];
30             for (int j=i; j<i+h/2; j++) {
31                 LL u = x[j];
32                 LL v = (w * x[j+h/2]) % mod;
33                 x[j] = (u + v) % mod;
34                 x[j+h/2] = (u - v + mod) % mod;
35                 (w *= wn) %= mod;
36             }
37         }
38     }
39     if (reverse == -1) {
40         LL t = power(len, mod-2);
41         for (int i=0; i<len; i++)
42             (x[i] *= t) %= mod;
43     }
44 }
45 LL A[N], B[N];
46 int main() {
47     for (int i=1; i<N; i*=2) {
48         wi[i] = power(g, (mod-1)/i);
49         wiv[i] = power(wi[i], mod-2);
50     }
51     memset(A, 0, sizeof(A));
52     memset(B, 0, sizeof(B));
53     NTT(A, len, 1); NTT(B, len, 1);
54     for (int i=0; i<len; i++) (A[i] *= B[i]) %= mod;
```

```
55     NTT(A, len, -1);  
56 }
```

### 6.14 组合数 lcm

$$(n+1)lcm(C(n,0), C(n,1), \dots, C(n,k)) = lcm(n+1, n, n-1, \dots, n-k+1)$$

### 6.15 区间 lcm 的维护

对于一个数，将其分解质因数，若有因子  $p^k$ ，那么拆分成  $k$  个数  $p, p^2, \dots, p^k$ ，权值都为  $p$ ，那么查询区间  $[l, r]$  内所有数的 lcm 的答案 = 所有在该区间中出现过的数的权值之积，可持久化线段树维护即可。

## 7 几何

### 7.1 凸包

```
1 typedef complex<int> point;
2 #define X real()
3 #define Y imag()
4 int n;
5 long long cross(point a, point b) {
6     return 1ll * a.X * b.Y - 1ll * a.Y * b.X;
7 }
8 bool cmp(point a, point b) {
9     return make_pair(a.X, a.Y) < make_pair(b.X, b.Y);
10 }
11 int convexHull(point p[], int n, point ch[]) {
12     sort(p, p + n, cmp);
13     int m = 0;
14     for(int i = 0; i < n; ++i) {
15         while(m > 1 && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
16         ch[m++] = p[i];
17     }
18     int k = m;
19     for(int i = n - 2; i >= 0; --i) {
20         while(m > k && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
21         ch[m++] = p[i];
22     }
23     if(n > 1) m--;
24     return m;
25 }
```

## 8 黑科技和杂项

### 8.1 找规律

有些题目，只给一个正整数  $n$ ，然后要求输出一个答案。这时，我们可以暴力得到小数据的解，用高斯消元得到递推式，然后用矩阵快速幂求解。

使用方法：

首先在 gauss.in 中输入小数据的解（ $n = 1$  时， $n = 2$  时， $\dots$ ），以 EOF 结束。

依次运行 gauss.cpp，matrix.cpp，得到 matrix.out

将 matrix.out 中的文件粘贴在 main.cpp 中相应的位置中。注意模数一定要是质数。

```
1 //gauss.cpp
2 #include <bits/stdc++.h>
3 #define N 102
4 #define mod 1000000007
5 //caution: you can use this program iff mod is a prime.
6
7 using namespace std;
8
9 int n, m, k, a[N], g[N][N];
10
11 int power(int base, int times) {
```



```

12     int ret = 1;
13     while (times) {
14         if (times & 1) ret = 111 * ret * base % mod;
15         base = 111 * base * base % mod;
16         times >>= 1;
17     }
18     return ret;
19 }
20
21 int test() {
22     for (int i=0; i<m; i++) {
23         for (int j=i; j<=m; j++)
24             if (g[j][i]) {
25                 for (int k=i; k<=m; k++)
26                     swap(g[i][k], g[j][k]);
27                 break;
28             }
29         if (g[i][i] == 0)
30             return 0;
31         for (int j=i+1; j<n; j++) {
32             while (g[j][i]) {
33                 int t = 111 * g[i][i] * power(g[j][i], mod - 2) % mod;
34                 for (int k=i; k<n; k++)
35                     g[i][k] = (g[i][k] + mod - (111 * t * g[j][k] % mod)) % mod;
36                 for (int k=i; k<=m; k++)
37                     swap(g[i][k], g[j][k]);
38             }
39         }
40         int t = power(g[i][i], mod - 2);
41         for (int j = 0; j <= m; ++j)
42             g[i][j] = 111 * g[i][j] * t % mod;
43     }
44     for (int i = m; i < n; ++i)
45         if (g[i][m]) return 0;
46     for (int i = m - 1; i >= 0; --i) {
47         int t = power(g[i][i], mod - 2);
48         g[i][i] = 1;
49         g[i][m] = 111 * g[i][m] * t % mod;
50         for (int j = 0; j < i; ++j)
51             g[j][m] = (g[j][m] + mod - 111 * g[i][m] * g[j][i] % mod) % mod;
52     }
53     printf("%d\n", m);
54     for (int i = 0; i < m; ++i)
55         printf("%d_", g[i][m]);
56     puts("");
57     for (int i = 0; i < m - 1; ++i)
58         printf("%d_", a[i]);
59     puts("1");
60     return 1;
61 }
62
63 int main() {
64     freopen("gauss.in", "r", stdin);
65     freopen("gauss.out", "w", stdout);
66     k = 0;
67     while (~scanf("%d", &a[k++])) ;
68     for (int sm = 1; sm <= k - sm; ++sm) {

```

```

69     n = k - sm - 1;
70     m = sm + 1;
71     for (int i = 0; i < n; ++i) {
72         for (int j = 0; j <= sm; ++j)
73             g[i][j] = a[i + j];
74         g[i][m] = 1;
75         swap(g[i][m - 1], g[i][m]);
76     }
77     if (test()) return 0;
78 }
79 puts("no_solution");
80 return 0;
81 }

```

```

1  //matrix.cpp
2  #include <bits/stdc++.h>
3  #define N 102
4  using namespace std;
5
6  int n, a[N];
7
8  int main() {
9      freopen("gauss.out", "r", stdin);
10     freopen("matrix.out", "w", stdout);
11     scanf("%d", &n);
12     for (int i = 0; i < n; ++i) scanf("%d", &a[i]);
13     printf("#define_M_%d\n", n);
14     printf("const_int_trans[M][M]_={\n");
15     for (int i = 0; i < n; ++i) {
16         printf("\t{");
17         for (int j = 0; j < n; ++j) {
18             int t;
19             if (j < n - 2) t = i == j + 1;
20             else if (j == n - 2) t = a[i];
21             else t = i == n - 1;
22             printf("%s%d", j == 0 ? "" : ", ", t);
23         }
24         printf("}%s\n", i == n - 1 ? "" : ",");
25     }
26     printf(");\n");
27     printf("const_int_pref[M]_={");
28     for (int i = 0; i < n; ++i) {
29         int x;
30         scanf("%d", &x);
31         printf("%d%s", x, i == n - 1 ? "};\n" : ",");
32     }
33     return 0;
34 }

```

```

1  //main.cpp
2  #include <bits/stdc++.h>
3  using namespace std;
4
5  /* paste matrix.out here. */
6
7  #define mod 1000000007
8

```

```

9 struct Matrix {
10     int c[M][M];
11     void clear() { memset(c, 0, sizeof(c)); }
12     void identity() { clear(); for (int i = 0; i < M; ++i) c[i][i] = 1; }
13     void base() { memcpy(c, trans, sizeof(trans)); }
14     friend Matrix operator * (const Matrix &a, const Matrix &b) {
15         Matrix c; c.clear();
16         for (int i = 0; i < M; ++i)
17             for (int j = 0; j < M; ++j)
18                 for (int k = 0; k < M; ++k)
19                     c.c[i][j] = (c.c[i][j] + 1ll * a.c[i][k] * b.c[k][j] % mod) % mod;
20         return c;
21     }
22 } start, base;
23
24 Matrix power(Matrix base, int times) {
25     Matrix ret; ret.identity();
26     while (times) {
27         if (times & 1) ret = ret * base;
28         base = base * base;
29         times >>= 1;
30     }
31     return ret;
32 }
33
34 int main() {
35     int tot;
36     scanf("%d", &tot);
37     while (tot--) {
38         int n;
39         scanf("%d", &n);
40         start.clear();
41         for (int i = 0; i < M; ++i) start.c[0][i] = pref[i];
42         base.base();
43         base = power(base, n - 1);
44         start = start * base;
45         printf("%d\n", start.c[0][0]);
46     }
47     return 0;
48 }

```

## 8.2 高精度计算

```

1 #include<algorithm>
2 using namespace std;
3 const int N_huge=850,base=100000000;
4 char s[N_huge*10];
5 struct huge{
6     typedef long long value;
7     value a[N_huge];int len;
8     void clear(){len=1;a[len]=0;}
9     huge(){clear();}
10    huge(value x){*this=x;}
11    huge operator =(huge b){
12        len=b.len;for (int i=1;i<=len;++i)a[i]=b.a[i]; return *this;

```

```

13     }
14     huge operator =(value x){
15         len=0;
16         while (x)a[++len]=x%base,x/=base;
17         if (!len)a[++len]=0;
18         return *this;
19     }
20     huge operator +(huge b){
21         int L=len>b.len?len:b.len;huge tmp;
22         for (int i=1;i<=L+1;++i)tmp.a[i]=0;
23         for (int i=1;i<=L;++i){
24             if (i>len)tmp.a[i]+=b.a[i];
25             else if (i>b.len)tmp.a[i]+=a[i];
26             else {
27                 tmp.a[i]+=a[i]+b.a[i];
28                 if (tmp.a[i]>=base){
29                     tmp.a[i]-=base;++tmp.a[i+1];
30                 }
31             }
32         }
33         if (tmp.a[L+1])tmp.len=L+1;
34         else tmp.len=L;
35         return tmp;
36     }
37     huge operator -(huge b){
38         int L=len>b.len?len:b.len;huge tmp;
39         for (int i=1;i<=L+1;++i)tmp.a[i]=0;
40         for (int i=1;i<=L;++i){
41             if (i>b.len)b.a[i]=0;
42             tmp.a[i]+=a[i]-b.a[i];
43             if (tmp.a[i]<0){
44                 tmp.a[i]+=base;--tmp.a[i+1];
45             }
46         }
47         while (L>1&&!tmp.a[L])--L;
48         tmp.len=L;
49         return tmp;
50     }
51     huge operator *(huge b){
52         int L=len+b.len;huge tmp;
53         for (int i=1;i<=L;++i)tmp.a[i]=0;
54         for (int i=1;i<=len;++i)
55             for (int j=1;j<=b.len;++j){
56                 tmp.a[i+j-1]+=a[i]*b.a[j];
57                 if (tmp.a[i+j-1]>=base){
58                     tmp.a[i+j]+=tmp.a[i+j-1]/base;
59                     tmp.a[i+j-1]%=base;
60                 }
61             }
62         tmp.len=len+b.len;
63         while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
64         return tmp;
65     }
66     pair<huge,huge> divide(huge a,huge b){
67         int L=a.len;huge c,d;
68         for (int i=L;i;--i){
69             c.a[i]=0;d=d*base;d.a[1]=a.a[i];

```

```

70         int l=0,r=base-1,mid;
71         while (l<r){
72             mid=(l+r+1)>>1;
73             if (b*mid<=d)l=mid;
74             else r=mid-1;
75         }
76         c.a[i]=l;d-=b*l;
77     }
78     while (L>1&&!c.a[L])--L;c.len=L;
79     return make_pair(c,d);
80 }
81 huge operator / (value x){
82     value d=0;huge tmp;
83     for (int i=len;i--i){
84         d=d*base+a[i];
85         tmp.a[i]=d/x;d%=x;
86     }
87     tmp.len=len;
88     while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
89     return tmp;
90 }
91 value operator % (value x){
92     value d=0;
93     for (int i=len;i--i)d=(d*base+a[i])%x;
94     return d;
95 }
96 huge operator / (huge b){return divide(*this,b).first;}
97 huge operator % (huge b){return divide(*this,b).second;}
98 huge &operator += (huge b){*this=*this+b;return *this;}
99 huge &operator -= (huge b){*this=*this-b;return *this;}
100 huge &operator *= (huge b){*this=*this*b;return *this;}
101 huge &operator ++() {huge T;T=1;*this=*this+T;return *this;}
102 huge &operator --() {huge T;T=1;*this=*this-T;return *this;}
103 huge operator ++(int) {huge T,tmp=*this;T=1;*this=*this+T;return tmp;}
104 huge operator --(int) {huge T,tmp=*this;T=1;*this=*this-T;return tmp;}
105 huge operator + (value x) {huge T;T=x;return *this+T;}
106 huge operator - (value x) {huge T;T=x;return *this-T;}
107 huge operator * (value x) {huge T;T=x;return *this*T;}
108 huge operator *= (value x) {*this=*this*x;return *this;}
109 huge operator += (value x) {*this=*this+x;return *this;}
110 huge operator -= (value x) {*this=*this-x;return *this;}
111 huge operator /= (value x) {*this=*this/x;return *this;}
112 huge operator %= (value x) {*this=*this%x;return *this;}
113 bool operator == (value x) {huge T;T=x;return *this==T;}
114 bool operator != (value x) {huge T;T=x;return *this!=T;}
115 bool operator <= (value x) {huge T;T=x;return *this<=T;}
116 bool operator >= (value x) {huge T;T=x;return *this>=T;}
117 bool operator < (value x) {huge T;T=x;return *this<T;}
118 bool operator > (value x) {huge T;T=x;return *this>T;}
119 bool operator < (huge b) {
120     if (len<b.len)return 1;
121     if (len>b.len)return 0;
122     for (int i=len;i--i){
123         if (a[i]<b.a[i])return 1;
124         if (a[i]>b.a[i])return 0;
125     }
126     return 0;

```

```

127     }
128     bool operator ==(huge b) {
129         if (len!=b.len) return 0;
130         for (int i=len;i--i)
131             if (a[i]!=b.a[i]) return 0;
132         return 1;
133     }
134     bool operator !=(huge b) {return !(*this==b);}
135     bool operator >(huge b) {return !(*this<b||*this==b);}
136     bool operator <=(huge b) {return (*this<b)||(*this==b);}
137     bool operator >=(huge b) {return (*this>b)||(*this==b);}
138     void str(char s[]) {
139         int l=strlen(s);value x=0,y=1;len=0;
140         for (int i=l-1;i>=0;--i) {
141             x=x+(s[i-' ']-0)*y;y*=10;
142             if (y==base) a[++len]=x,x=0,y=1;
143         }
144         if (!len||x) a[++len]=x;
145     }
146     void read() {
147         scanf("%s",s);this->str(s);
148     }
149     void print() {
150         printf("%d", (int) a[len]);
151         for (int i=len-1;i--i) {
152             for (int j=base/10;j>=10;j/=10) {
153                 if (a[i]<j) printf("0");
154                 else break;
155             }
156             printf("%d", (int) a[i]);
157         }
158         printf("\n");
159     }
160 }f[1005];
161 int main() {
162     f[1]=f[2]=1;
163     for(int i=3;i<=1000;i++) f[i]=f[i-1]+f[i-2];
164 }

```