

ICPC World Finals 2019 Templates

SYSU_Balloon

2019 年 3 月 5 日

目录

1	莫队算法	3
1.1	普通莫队	3
1.2	树上莫队	3
2	字符串	4
2.1	哈希	4
2.2	KMP	4
2.3	扩展 KMP	4
2.4	Manacher	5
2.5	AC 自动机	6
2.6	后缀数组	6
2.7	后缀自动机	7
2.8	回文树	8
3	数据结构	9
3.1	ST 表	9
3.2	K-D Tree	9
3.3	左偏树	11
3.4	线段树小技巧	11
3.5	Splay	12
3.6	可持久化 Treap	13
3.7	可持久化并查集	14
4	树	15
4.1	点分治	15
4.2	Link Cut Tree	15
4.3	虚树	17
5	图	17
5.1	Tarjan 有向图强联通分量	17
5.2	Tarjan 双联通分量	18
5.3	2-SAT	19

5.4	网络流	19
5.4.1	最大流	19
5.4.2	上下界有源汇网络流	20
5.4.3	费用流	20
6	数学	21
6.1	扩展欧几里得解同余方程	21
6.2	同余方程组	21
6.3	FFT	22
6.3.1	普通 FFT	22
6.4	求原根	22
6.5	NTT	23
6.6	线性基	23
6.7	高斯消元法实数方程	24
6.8	高斯消元法模方程	24
6.9	蔡勒公式	25
6.10	皮克定理	25
7	计算几何	25
7.1	凸包	25
7.2	定义	26
7.3	半平面交	28
7.4	圆与多边形交集	28
7.5	三角形面积并	29
7.6	K 圆并	30
7.7	三维计算几何	32
8	黑科技和杂项	32
8.1	读入优化	32
9	Vim	33
10	Java	33

1 莫队算法

1.1 普通莫队

分块块数为 \sqrt{n} 是最优的。记每次进行 `add()` 操作的复杂度为 $O(A)$ ，`del()` 操作的复杂度为 $O(D)$ ，查询答案 `answer()` 的复杂度为 $O(S)$ 。则总复杂度为 $O(n\sqrt{n}(A+D) + qS)$ 。 S 可以大一点，但必须保证 A, D 尽可能小。

```
1 struct Q { int l, r, sqrtl, id; } q[N];
2 int n, m, v[N], ans[N], nowans;
3 bool cmp(const Q &a, const Q &b) { if (a.sqrtl != b.sqrtl) return a.sqrtl < b.sqrtl;
   return a.r < b.r; }
4 void change(int x) { if (!v[x]) add(x); else del(x); v[x] ^= 1; }
5
6 for (int i=1; i<=m; i++) q[i].sqrtl = q[i].l / sqrt(n), q[i].id = i;
7 sort(q+1, q+m+1, cmp);
8 int L=1, R=0;
9 memset(v, 0, sizeof(v));
10 for (int i=1; i<=m; i++) {
11     while (L<q[i].l) change(L++);
12     while (L>q[i].l) change(--L);
13     while (R<q[i].r) change(++R);
14     while (R>q[i].r) change(R--);
15     ans[q[i].id] = answer();
16 }
```

1.2 树上莫队

```
1 struct Query { int l, r, id, l_group; } query[N];
2 int v[N], ans[N];
3 bool cmp(const Query &a, const Query &b) { if (a.l_group != b.l_group) return a.
   l_group < b.l_group; return dfn[a.r] < dfn[b.r]; }
4 void upd(int x) { if (!v[x]) add(x); else del(x); v[x] ^= 1; }
5 void go(int &u, int taru, int v) {
6     int lca0 = lca(u, taru);
7     int lca1 = lca(u, v); upd(lca1);
8     int lca2 = lca(taru, v); upd(lca2);
9     for (int x=u; x!=lca0; x=father[x]) upd(x);
10    for (int x=taru; x!=lca0; x=father[x]) upd(x);
11    u = taru;
12 }
13
14 for (int i=1; i<=m; i++) {
15     if (dfn[query[i].l] > dfn[query[i].r]) swap(query[i].l, query[i].r);
16     query[i].id = i; query[i].l_group = dfn[query[i].l] / sqrt(n);
17 }
18 sort(query+1, query+m+1, cmp);
19 int L=1, R=1; upd(1);
20 for (int i=1; i<=m; i++) {
```

```

21     go(L, query[i].l, R);
22     go(R, query[i].r, L);
23     ans[query[i].id] = answer();
24 }

```

2 字符串

2.1 哈希

```

1  const int P=31,D=1000173169;
2  int hash(int l, int r) { return (LL) (f[r]-(LL)f[l-1]*pow[r-l+1]%D+D)%D; }
3  pow[0] = 1; for (int i=1;i<=n;i++) pow[i] = (LL)pow[i-1]*P%D;
4  for (int i=1;i<=n;i++) f[i] = (LL) ((LL)f[i-1]*P+a[i])%D;

```

2.2 KMP

输入：模式串长度 n ，模式串 a ，匹配串长度 m ，匹配串 b ；输出：依次输出每个匹配成功的起始位置；下标从 0 开始。

```

1  void kmp(int n, char* a, int m, char *b) {
2      int i, j;
3      for (nxt[0] = j = -1, i = 1; i < n; nxt[i++] = j) {
4          while (~j && a[j + 1] != a[i]) j = nxt[j];
5          if (a[j + 1] == a[i]) ++j;
6      }
7      for (j = -1, i = 0; i < m; ++i) {
8          while (~j && a[j + 1] != b[i]) j = nxt[j];
9          if (a[j + 1] == b[i]) ++j;
10         if (j == n - 1) {
11             printf("%d\n", i - n + 1);
12             j = nxt[j];
13         }
14     }
15 }

```

2.3 扩展 KMP

next: a 关于自己每个后缀的最长公共前缀; ret: a 关于 b 的每个后缀的最长公共前缀; EXKMP 的 next[i] 表示：从 i 到 $n-1$ 的字符串 st 前缀和原串前缀的最长重叠长度。

```

1  void get_next(char *a, int *next) {
2      int i, j, k, n = strlen(a);
3      for (j = 0; j+1<n && a[j]==a[j+1];j++);
4      next[1] = j; k = 1;
5      for (i=2;i<n;i++) {
6          int len = k+next[k], L = next[i-k];
7          if (L < len-i) {

```

```

8         next[i] = L;
9     } else {
10         for (j = max(0, len-i); i+j<n && a[j]==a[i+j]; j++);
11         next[i] = j;
12         k = i;
13     }
14 }
15 }
16 void ExtendedKMP(char *a, char *b, int *next, int *ret) {
17     get_next(a, next);
18     int n = strlen(a), m = strlen(b);
19     int i, j, k;
20     for (j=0; j<n && j<m && a[j]==b[j]; j++);
21     ret[0] = j;
22     k = 0;
23     for (i=1; i<m; i++) {
24         int len = k+ret[k], L = next[i-k];
25         if (L < len-i) {
26             ret[i] = L;
27         } else {
28             for (j = max(0, len-i); j<n && i+j<m && a[j]==b[i+j]; j++);
29             ret[i] = j;
30             k = i;
31         }
32     }
33 }

```

2.4 Manacher

$p[i]$ 表示以 i 为对称轴的最长回文串长度

```

1 char st[N*2], s[N];
2 int len, p[N*2];
3 while (scanf("%s", s) != EOF) {
4     len = strlen(s);
5     st[0] = '$', st[1] = '#';
6     for (int i=1; i<=len; i++)
7         st[i*2] = s[i-1], st[i*2+1] = '#';
8     len = len * 2 + 2;
9     int mx = 0, id = 0, ans = 0;
10    for (int i=1; i<=len; i++) {
11        p[i] = (mx > i) ? min(p[id*2-i]+1, mx-i) : 1;
12        for (; st[i+p[i]] == st[i-p[i]]; ++p[i]);
13        if (p[i]+i > mx) mx = p[i]+i, id = i;
14        p[i]--;
15        if (p[i] > ans) ans = p[i];
16    }
17    printf("%d\n", ans);
18 }

```

2.5 AC 自动机

```
1 struct Node { int next[26]; int terminal, fail; };
2 void build() {
3     head = 0, tail = 1; q[1] = 1;
4     while (head != tail) {
5         int x = q[++head];
6         /*(when necessary) node[x].terminal != node[node[x].fail].terminal; */
7         for (int i=0;i<26;i++)
8             if (node[x].next[i]) {
9                 int y = node[x].fail;
10                while (y) {
11                    if (node[y].next[i]) {
12                        node[node[x].next[i]].fail = node[y].next[i];
13                        break;
14                    }
15                    y = node[y].fail;
16                }
17                if (!node[node[x].next[i]].fail) node[node[x].next[i]].fail = 1;
18                q[++tail] = node[x].next[i];
19            }
20     }
21 }
```

2.6 后缀数组

参数 m 表示字符集的大小, 即 $0 \leq r_i < m$

```
1 int n, r[N], wa[N], wb[N], ws[N], sa[N], rank[N], height[N];
2 int cmp(int *r, int a, int b, int l, int n) { return r[a]==r[b] && a+l<n && b+l<n &&
   r[a+l]==r[b+l]; }
3 void suffix_array(int m) {
4     int i, j, p, *x=wa, *y=wb, *t;
5     for (i=0;i<m;i++) ws[i]=0; for (i=0;i<n;i++) ws[x[i]=r[i]]++;
6     for (i=1;i<m;i++) ws[i]+=ws[i-1]; for (i=n-1;i>=0;i--) sa[--ws[x[i]]]=i;
7     for (j=1,p=1;p<n;m=p,j<=1) {
8         for (p=0,i=n-j;i<n;i++) y[p++]=i;
9         for (i=0;i<n;i++) if (sa[i]>=j) y[p++]=sa[i]-j;
10        for (i=0;i<m;i++) ws[i]=0; for (i=0;i<n;i++) ws[x[y[i]]]++;
11        for (i=1;i<m;i++) ws[i]+=ws[i-1];
12        for (i=n-1;i>=0;i--) sa[--ws[x[y[i]]]]=y[i];
13        for (t=x,x=y,y=t,x[sa[0]]=0,i=1,p=1;i<n;i++)
14            x[sa[i]]=cmp(y,sa[i-1],sa[i],j,n)?p-1:p++;
15    }
16    for (i=0;i<n;i++) rank[sa[i]]=i; rank[n] = -1;
17    for (i=j=0;i<n;i++) if (rank[i]) {
18        while (r[i+j]==r[sa[rank[i]-1]+j]) j++;
19        height[rank[i]]=j;
20        if (j) j--;
21    }
```

22 }

2.7 后缀自动机

下面的代码是求两个串的 LCS（最长公共子串）。

```
1  #define M (N << 1)
2  char st[N];
3  int pre[M], son[26][M], step[M], refer[M], size[M], tmp[M], topo[M], last, total;
4  int apply(int x, int now) {
5      step[++total] = x;
6      refer[total] = now;
7      return total;
8  }
9  void extend(char x, int now) {
10     int p = last, np = apply(step[last]+1, now);
11     size[np] = 1;
12     for (; p && !son[x][p]; p=pre[p]) son[x][p] = np;
13     if (!p) pre[np] = 1;
14     else {
15         int q = son[x][p];
16         if (step[p]+1 == step[q]) pre[np] = q;
17         else {
18             int nq = apply(step[p]+1, now);
19             for (int i=0; i<26; i++) son[i][nq] = son[i][q];
20             pre[nq] = pre[q]; pre[q] = pre[np] = nq;
21             for (; p && son[x][p]==q; p=pre[p]) son[x][p] = nq;
22         }
23     }
24     last = np;
25 }
26 void init() {
27     last = total = 0;
28     last = apply(0, 0);
29     scanf("%s", st);
30     int n = strlen(st);
31     for (int i = 0; i <= n * 2; ++i) {
32         pre[i] = step[i] = refer[i] = size[i] = tmp[i] = topo[i] = 0;
33         for (int j = 0; j < 26; ++j) son[j][i] = 0;
34     }
35     for (int i = 0; i < n; ++i) extend(st[i] - 'a', i);
36     for (int i = 1; i <= total; ++i) tmp[step[i]] ++;
37     for (int i = 1; i <= n; ++i) tmp[i] += tmp[i - 1];
38     for (int i = 1; i <= total; ++i) topo[tmp[step[i]]--] = i;
39     for (int i = total; i; --i) size[pre[topo[i]]] += size[topo[i]];
40 }
41 int main() {
42     init();
43     int p = 1, now = 0, ans = 0;
44     scanf("%s", st);
```

```

45     for (int i=0; st[i]; i++) {
46         int index = st[i]-'a';
47         for (; p && !son[index][p]; p = pre[p], now = step[p]) ;
48         if (!p) p = 1;
49         if (son[index][p]) {
50             p = son[index][p]; now++;
51             if (now > ans) ans = now;
52         }
53     }
54     printf("%d\n",ans);
55     return 0;
56 }

```

一些定义和性质：① $\text{Right}(\text{str})$ 表示 str 在母串 S 中所有出现的结束位置集合；② 一个状态 s 表示的所有子串 Right 集合相同，为 $\text{Right}(s)$ ；③ $\text{Parent}(s)$ 满足 $\text{Right}(s)$ 是 $\text{Right}(\text{Parent}(s))$ 的真子集，并且 $\text{Right}(\text{Parent}(s))$ 的大小最小；④ Parent 函数可以表示一个树形结构。不妨叫它 Parent 树；⑤ 一个 Right 集合和一个长度定义了一个子串；⑥ 对于状态 s ，使得 $\text{Right}(s)$ 合法的子串长度是一个区间 $[\min(s), \max(s)]$ ；⑦ $\max(\text{Parent}(s)) = \min(s) - 1$ ；⑧ 令 $\text{refer}(s)$ 表示产生 s 状态的字符所在位置。则 $\text{Right}(s)$ 的合法子串的起始位置为 $[\text{refer}(s) - \max(s) + 1, \text{refer}(s) - \min(s) + 1]$ ，即 $[\text{refer}(s) - \max(s) + 1, \text{refer}(s) - \max(\text{Parent}(s))]$ 。

代码中变量名含义：① $\text{pre}[s]$ 为上述定义中的 $\text{Parent}(s)$ ；② $\text{step}[s]$ 为从初始状态走到 s 状态最多需要多少步；③ $\text{refer}[s]$ 为上述定义中的 $\text{refer}(s)$ ；④ $\text{size}[s]$ 为 $\text{Right}(s)$ 集合的大小；⑤ $\text{topo}[s]$ 为 Parent 树的拓扑序，根（初始状态）在前。

2.8 回文树

① $\text{len}[i]$ 表示编号为 i 的节点表示的回文串的长度（一个节点表示一个回文串）② $\text{next}[i][c]$ 表示编号为 i 的节点表示的回文串在两边添加字符 c 以后变成的回文串的编号（和字典树类似）。③ $\text{fail}[i]$ 表示节点 i 失配以后跳转不等于自身的节点 i 表示的回文串的最长后缀回文串（和 AC 自动机类似）。④ $\text{cnt}[i]$ 表示节点 i 表示的本质不同的串的个数（建树时求出的不是完全的，最后 $\text{count}()$ 函数跑一遍以后才是正确的）⑤ $\text{num}[i]$ 表示以节点 i 表示的最长回文串的最右端点为回文串结尾的回文串个数。⑥ last 指向新添加一个字母后所形成的最长回文串表示的节点。⑦ $\text{st}[i]$ 表示第 i 次添加的字符（一开始设 $\text{st}[0] = -1$ （可以是任意一个在串 S 中不会出现的字符））。⑧ tot 表示添加的节点个数。⑨ n 表示添加的字符个数。

【URAL2040】Palindromes and Super Abilities 2

逐个添加字符串 S 里的字符 S_1, S_2, \dots, S_n 。每次添加字符后，他想知道添加字符后将出现多少个新的本质不同的回文子串。字符集为 $\{a, b\}$

```

1 struct PAM {
2     int n, tot, last, len[N], fail[N], next[N][2], num[N], cnt[N];
3     void init() { n=0; tot=1; len[1]=-1; fail[1]=0; len[0]=+0; fail[0]=1; last=1; }
4     int get_fail(int x) { for (; st[n-len[x]-1]!=st[n]; x=fail[x]); return x; }
5     void insert(char c) {
6         ++n; int cur=get_fail(last); // 判断上一个串的前一个位置和新添加的位置是否相同，相同则说明构成回文。否则找 fail 指针。
7         if (!next[cur][c]) {
8             ++tot; len[tot]=len[cur]+2; fail[tot]=next[get_fail(fail[cur])][c];

```



```

9         next[cur][c]=tot; num[tot] = num[fail[tot]] + 1; answer[n]='1';
10     } else answer[n]='0';
11     last=next[cur][c]; cnt[last] ++;
12 }
13 void count () { for (int i=tot-1; i>=0; --i) cnt[fail[i]] += cnt[i]; }
14 //父亲累加儿子的cnt, 因为如果fail[v]=u, 则u一定是v的子回文串。
15 } pam;
16 n=strlen(st+1); pam.init();
17 for (int i=1;i<=n;i++) pam.insert(st[i]-'a');

```

3 数据结构

3.1 ST 表

```

1 int Log[N], f[17][N];
2 int ask(int x, int y) { int k=Log[y-x+1]; return max(f[k][x], f[k][y-(1<<k)+1]); }
3 for (int i=2; i<=n; i++) Log[i]=Log[i>>1]+1; for (int j=1; j<K; j++) for (int i=1; i+(1<<j)-1<=n; i++) f[j][i]=max(f[j-1][i], f[j-1][i+(1<<j-1)]);

```

3.2 K-D Tree

① change 将编号为 x 的点的权值增加 p ; ② euclid_lower_bound 欧几里得距离的平方, 下界; ③ euclid_upper_bound 欧几里得距离的平方, 上界; ④ manhattan_lower_bound 曼哈顿距离, 下界; ⑤ manhattan_upper_bound 曼哈顿距离, 上界; ⑥ add 添加一个点 (注意此处的添加可能导致这棵树不平衡, 慎用!); ⑦ ask(p, X, Y, ans) 询问距离点 (X, Y) 最远的一个点的距离, ans 需传入无穷小; ⑧ ask($p, x1, y1, x2, y2$) 查询矩形范围内所有点的权值和。

```

1 int n, cmp_d, root, id[N];
2 struct node { int d[2], l, r, Max[2], Min[2], val, sum, f; } t[N];
3 inline bool cmp(const node &a, const node &b) {
4     if (a.d[cmp_d] != b.d[cmp_d]) return a.d[cmp_d] < b.d[cmp_d];
5     return a.d[cmp_d ^ 1] < b.d[cmp_d ^ 1];
6 }
7 inline void umax(int &a, int b) { if (b > a) a = b; }
8 inline void umin(int &a, int b) { if (b < a) a = b; }
9 inline void up(int x, int y) { umax(t[x].Max[0], t[y].Max[0]); umin(t[x].Min[0], t[y].Min[0]); umax(t[x].Max[1], t[y].Max[1]); umin(t[x].Min[1], t[y].Min[1]); }
10 int build(int l, int r, int D, int f) {
11     int mid = (l + r) / 2; cmp_d = D;
12     nth_element(t + l + 1, t + mid + 1, t + r + 1, cmp);
13     id[t[mid].f] = mid; t[mid].f = f;
14     t[mid].Max[0] = t[mid].Min[0] = t[mid].d[0];
15     t[mid].Max[1] = t[mid].Min[1] = t[mid].d[1];
16     t[mid].val = t[mid].sum = 0;
17     if (l != mid) t[mid].l = build(l, mid - 1, !D, mid);
18     else t[mid].l = 0;
19     if (r != mid) t[mid].r = build(mid + 1, r, !D, mid);

```

```

20     else t[mid].r = 0;
21     if (t[mid].l) up(mid, t[mid].l);
22     if (t[mid].r) up(mid, t[mid].r);
23     return mid;
24 }
25 void change(int x, int p) {
26     x = id[x]; // 将点的编号映成排序后的编号
27     for (t[x].val += p; x; x = t[x].f) t[x].sum += p;
28 }
29 inline long long sqr(long long x) { return x * x; }
30 inline long long euclid_lower_bound(const node &a, int X, int Y) {
31     return sqr(max(max(X - a.Max[0], a.Min[0] - X), 0)) +
32     sqr(max(max(Y - a.Max[1], a.Min[1] - Y), 0)); }
33 inline long long euclid_upper_bound(const node &a, int X, int Y) {
34     return max(sqr(X - a.Min[0]), sqr(X - a.Max[0])) +
35     max(sqr(Y - a.Min[1]), sqr(Y - a.Max[1])); }
36 inline long long manhattan_lower_bound(const node &a, int X, int Y) {
37     return max(a.Min[0] - X, 0) + max(X - a.Max[0], 0) +
38     max(a.Min[1] - Y, 0) + max(Y - a.Max[1], 0);
39 }
40 inline long long manhattan_upper_bound(const node &a, int X, int Y) {
41     return max(abs(X - a.Max[0]), abs(a.Min[0] - X)) +
42     max(abs(Y - a.Max[1]), abs(a.Min[1] - Y));
43 }
44 void add(int k) {
45     t[k].Max[0] = t[k].Min[0] = t[k].d[0]; t[k].Max[1] = t[k].Min[1] = t[k].d[1];
46     t[k].val = t[k].sum = 0; t[k].l = t[k].r = t[k].f = 0;
47     if (!root) root = k, return;
48     int p = root, D = 0;
49     while (1) { up(p, k);
50         if (t[k].d[D] <= t[p].d[D]) { if (t[p].l) p = t[p].l; else t[p].l = k, t[k].
51             f = p, return; }
52         else { if (t[p].r) p = t[p].r; else t[p].r = k, t[k].f = p, return; }
53         D ^= 1;
54     }
55 }
56 inline long long getdis(const node &a, int X, int Y) { return sqr(a.d[0] - X) + sqr(
57     a.d[1] - Y); }
58 void ask(int p, int X, int Y, long long &ans) {
59     if (!p) return; ans = max(ans, getdis(t[p], X, Y));
60     long long dl = t[p].l ? euclid_upper_bound(t[t[p].l], X, Y) : 0;
61     long long dr = t[p].r ? euclid_upper_bound(t[t[p].r], X, Y) : 0;
62     if (dl > dr) { if (dl > ans) ask(t[p].l, X, Y, ans); if (dr > ans) ask(t[p].r, X
63         , Y, ans); }
64     else { if (dr > ans) ask(t[p].r, X, Y, ans); if (dl > ans) ask(t[p].l, X, Y, ans
65         ); }
66 }
67 int ask(int p, int x1, int y1, int x2, int y2) {
68     if (t[p].Min[0] > x2 || t[p].Max[0] < x1 || t[p].Min[1] > y2 || t[p].Max[1] < y1
69         ) return 0;

```

```

65     if (t[p].Min[0] >= x1 && t[p].Max[0] <= x2 && t[p].Min[1] >= y1 && t[p].Max[1]
        <= y2) return t[p].sum;
66     int s = 0;
67     if (t[p].d[0] >= x1 && t[p].d[0] <= x2 && t[p].d[1] >= y1 && t[p].d[1] <= y2) s
        += t[p].val;
68     if (t[p].l) s += ask(t[p].l, x1, y1, x2, y2);
69     if (t[p].r) s += ask(t[p].r, x1, y1, x2, y2);
70     return s;
71 }
72 for (int i = 1; i <= n; ++i) t[i].d[0] = x, t[i].d[1] = y;
73 root = build(1, n, 0, 0);

```

3.3 左偏树

左偏树是一个可并堆。下面的程序写的是一个小根堆，如果需要改成大根堆请在注释了 here 那行修改。接口：① push 插入一个元素；② merge 合并两个堆，注意，合并后原来那个堆将不可访问；③ top 返回堆顶元素；④ pop 删除堆顶元素；⑤ size 返回堆的大小。

```

1 template <class T> class leftist { public:
2     struct node { T key; int dist; node *l, *r; };
3     leftist() : root(NULL), s(0) {}
4     void push(const T &x) { leftist y; y.s = 1; y.root = new node; y.root -> key = x
        ; y.root -> dist = 0; y.root -> l = y.root -> r = NULL; merge(y); }
5     node* merge(node *x, node *y) {
6         if (x == NULL) return y; if (y == NULL) return x;
7         if (y -> key < x -> key) swap(x, y); //here
8         x -> r = merge(x -> r, y);
9         int ld = x -> l ? x -> l -> dist : -1;
10        int rd = x -> r ? x -> r -> dist : -1;
11        if (ld < rd) swap(x -> l, x -> r);
12        if (x -> r == NULL) x -> dist = 0;
13        else x -> dist = x -> r -> dist + 1; return x;
14    }
15    void merge(leftist &x) { root = merge(root, x.root); s += x.s; }
16    T top() const { if (root == NULL) return T(); return root -> key; }
17    void pop() { if (root == NULL) return; node *p = root; root = merge(root -> l,
        root -> r); --s; delete p; }
18    int size() const { return s; }
19 private: node* root; int s;
20 };

```

3.4 线段树小技巧

给定一个序列 a ，寻找一个最大的 i 使得 $i \leq y$ 且满足一些条件（如 $a[i] \geq w$ ，那么需要在线段树维护 a 的区间最大值）

```

1 int query1(int p, int left, int right, int y, int w) {
2     if (right <= y) {
3         if (! __condition__ ) return -1;

```

```

4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (y <= mid) return queryl(p<<1|0, left, mid, y, w);
8     int ret = queryl(p<<1|1, mid+1, right, y, w);
9     if (ret != -1) return ret;
10    return queryl(p<<1|0, left, mid, y, w);
11 }

```

给定一个序列 a ，寻找一个最小的 i 使得 $i \geq x$ 且满足一些条件（如 $a[i] \geq w$ ，那么需要在线段树维护 a 的区间最大值）

```

1 int queryr(int p, int left, int right, int x, int w) {
2     if (left >= x) {
3         if (! __condition__ ) return -1;
4         else if (left == right) return left;
5     }
6     int mid = (left + right) / 2;
7     if (x > mid) return queryr(p<<1|1, mid+1, right, x, w);
8     int ret = queryr(p<<1|0, left, mid, x, w);
9     if (ret != -1) return ret;
10    return queryr(p<<1|1, mid+1, right, x, w);
11 }

```

3.5 Splay

接口：① ADD $x \ y \ d$ 将 $[x, y]$ 的所有数加上 d ；② REVERSE $x \ y$ 将 $[x, y]$ 翻转；③ INSERT $x \ p$ 将 p 插入到第 x 个数的后面；④ DEL x 将第 x 个数删除。

```

1 int w[N], Min[N], son[N][2], size[N], father[N], rev[N], lazy[N];
2 int top, rt, q[N];
3 void pushdown(int x) {
4     if (!x) return;
5     if (rev[x]) rev[son[x][0]] ^= 1, rev[son[x][1]] ^= 1, swap(son[x][0], son[x][1]), rev[x] = 0;
6     if (lazy[x]) lazy[son[x][0]] += lazy[x], lazy[son[x][1]] += lazy[x], w[x] += lazy[x], Min[x] += lazy[x], lazy[x] = 0;
7 }
8 void pushup(int x) {
9     if (!x) return; pushdown(son[x][0]); pushdown(son[x][1]);
10    size[x] = size[son[x][0]] + size[son[x][1]] + 1; Min[x] = w[x];
11    if (son[x][0]) Min[x] = min(Min[x], Min[son[x][0]]);
12    if (son[x][1]) Min[x] = min(Min[x], Min[son[x][1]]);
13 }
14 void sc(int x, int y, int w) { son[x][w] = y; father[y] = x; pushup(x); }
15 void _ins(int w) {
16     top++; w[top] = Min[top] = w; son[top][0] = son[top][1] = 0;
17     size[top] = 1; father[top] = 0; rev[top] = 0;
18 }
19 void init() { top = 0; _ins(0); _ins(0); rt=1; sc(1, 2, 1); }

```

```

20 void rotate(int x) {
21     if (!x) return; int y = father[x], w = son[y][1]==x;
22     sc(y, son[x][w^1], w); sc(father[y], x, son[father[y]][1]==y); sc(x, y, w^1);
23 }
24 void flushdown(int x) {
25     int t=0; for (; x; x=father[x]) q[++t]=x;
26     for (; t; t--) pushdown(q[t]);
27 }
28 void Splay(int x, int root=0) {
29     flushdown(x);
30     while (father[x] != root) { int y=father[x], w=son[y][1]==x;
31         if (father[y] != root && son[father[y]][w]==y) rotate(y);
32         rotate(x); }
33 }
34 int find(int k) {
35     Splay(rt);
36     while (1) { pushdown(rt);
37         if (size[son[rt][0]]+1==k) Splay(rt), return rt;
38         else if (size[son[rt][0]]+1<k) k-=size[son[rt][0]]+1, rt=son[rt][1];
39         else rt=son[rt][0]; }
40 }
41 int split(int x, int y) {
42     int fx = find(x), fy = find(y+2); Splay(fx); Splay(fy, fx); return son[fy][0]; }
43 void add(int x, int y, int d) { //add d to each number in a[x]...a[y]
44     int t = split(x, y); lazy[t] += d; Splay(t); rt=t; }
45 void reverse(int x, int y) { // reverse the x-th to y-th elements
46     int t = split(x, y); rev[t] ^= 1; Splay(t); rt=t; }
47 void insert(int x, int p) { // insert p after the x-th element
48     int fx = find(x+1), fy = find(x+2);
49     Splay(fx); Splay(fy, fx); _ins(p); sc(fy, top, 0); Splay(top); rt=top; }
50 void del(int x) { // delete the x-th element in Splay
51     int fx = find(x), fy = find(x+2);
52     Splay(fx); Splay(fy, fx); son[fy][0] = 0; Splay(fy); rt=fy; }

```

3.6 可持久化 Treap

接口：① insert 在当前第 x 个字符后插入 c ；② del 删除第 x 个字符到第 y 个字符；③ copy 复制第 l 个字符到第 r 个字符，然后粘贴到第 x 个字符后；④ reverse 翻转第 x 个到第 y 个字符；⑤ query 表示询问当前第 x 个字符是什么。

```

1 char key[N];
2 bool rev[N];
3 int lc[N], rc[N], size[N]; // if size is long long, remember here
4 int n, root;
5 LL Rand() { return rd = (rd * 2037205211 + 2502208711) % mod; }
6 void init() { n = root = 0; }
7 inline int copy(int x) { ++ n; key[n] = key[x]; (copy rev, lc, rc, size); return n; }
8 inline void pushdown(int x) {

```

```

9      if (!rev[x]) return;
10     if (lc[x]) lc[x] = copy(lc[x]); if (rc[x]) rc[x] = copy(rc[x]);
11     swap(lc[x], rc[x]); rev[lc[x]] ^= 1; rev[rc[x]] ^= 1; rev[x] = 0;
12 }
13 inline void pushup(int x) { size[x] = size[lc[x]] + size[rc[x]] + 1; }
14 int merge(int u, int v) {
15     if (!u || !v) return u+v; pushdown(u); pushdown(v);
16     int t = Rand() % (size[u] + size[v]), r; // if size is long long, remember here
17     if (t < size[u]) r = copy(u), rc[r] = merge(rc[u], v);
18     else r = copy(v), lc[r] = merge(u, lc[v]);
19     pushup(r); return r;
20 }
21 int split(int u, int x, int y) { // if size is long long, remember here
22     if (x > y) return 0; pushdown(u);
23     if (x == 1 && y == size[u]) return copy(u);
24     if (y <= size[lc[u]]) return split(lc[u], x, y);
25     int t = size[lc[u]] + 1; // if size is long long, remember here
26     if (x > t) return split(rc[u], x-t, y-t);
27     int num = copy(u); lc[num]=split(lc[u], x, t-1); rc[num]=split(rc[u], 1, y-t);
28     pushup(num); return num;
29 }
30 void insert(int x, char c) {
31     int t1 = split(root, 1, x), t2 = split(root, x+1, size[root]);
32     key[++n] = c; lc[n] = rc[n] = rev[n] = 0; pushup(n); root = merge(merge(t1, n),
33     t2); }
34 void del(int x, int y) {
35     int t1 = split(root, 1, x-1), t2 = split(root, y+1, size[root]); root = merge(t1
36     , t2); }
37 void copy(int l, int r, int x) {
38     int t1 = split(root, 1, x), t2 = split(root, 1, r), t3 = split(root, x+1, size[
39     root]);
40     root = merge(merge(t1, t2), t3); }
41 void reverse(int x, int y) {
42     int t1 = split(root, 1, x-1), t2 = split(root, x, y), t3 = split(root, y+1, size
43     [root]);
44     rev[t2] ^= 1; root = merge(merge(t1, t2), t3); }
45 char query(int k) {
46     int x = root;
47     while (1) { pushdown(x);
48         if (k <= size[lc[x]]) x = lc[x];
49         else if (k == size[lc[x]] + 1) return key[x];
50         else k -= size[lc[x]] + 1, x = rc[x]; }
51 }

```

3.7 可持久化并查集

接口：① merge 在 time 时刻将 x 和 y 连一条边，注意加边顺序必须按 time 从小到大加边 ② GetFather 询问 time 时刻及以前的连边状态中，x 所属的集合

```
1 const int inf = 0x3f3f3f3f;
```

```

2  int father[N], Father[N], Time[N];
3  vector<int> e[N];
4  void init() { for (int i=1;i<=n;i++) father[i]=Father[i]=i,Time[i]=inf,e[i].clear(),
    e[i].push_back(i);}
5  int getfather(int x) { return (father[x]==x) ? x : father[x]=getfather(father[x]); }
6  int GetFather(int x, int time) {return (Time[x]<=time)?GetFather(Father[x],time):x;}
7  void merge(int x, int y, int time) {
8      int fx = getfather(x), fy = getfather(y); if (fx == fy) return;
9      if (e[fx].size() > e[fy].size()) swap(fx, fy);
10     father[fx] = fy; Father[fx] = fy; Time[fx] = time;
11     for (int i=0;i<e[fx].size();i++) e[fy].push_back(e[fx][i]);
12 }

```

4 树

4.1 点分治

```

1  void getsize(int x, int root = 0) {
2      size[x] = 1; son[x] = 0; int dd = 0;
3      for (int p = gh[x]; p; p = edge[p].next) {
4          int y = edge[p].adj;
5          if (y == root || !vis[y]) continue;
6          size[x] += size[y];
7          if (size[y] > dd) dd = size[y], son[x] = y;
8      }
9  }
10 int getroot(int x) {
11     int sz = size[x];
12     while (size[son[x]] > sz/2) x = son[x]; return x;
13 }
14 void dc(int x) {
15     getsize(x); x = getroot(x);
16     vis[x] = 1;
17     for (int p = gh[x]; p; p = edge[p].next) {
18         int y = edge[p].adj;
19         if (vis[y]) continue;
20         dc(y);
21     }
22     vis[x] = 0;
23 }

```

4.2 Link Cut Tree

① 注意，一开始必须调用 `lct.init(0)`，否则求出的最小值一定会是 0。② `minval` 维护的是链上 `val` 最小值。③ `sumval2` 维护的是子树 `val2` 的和。

```

1  int f[N], son[N][2], sz[N], rev[N], tot;
2  int val[N], minid[N], minval[N];

```

```

3  int val2[N], sumval2[N]; // 记得开 long long 。注意两个都要开 long long ，因为 val2
   还包含了虚儿子的子树和。
4  stack<int> s;
5  void init(int i) {
6      tot = max(tot, i); son[i][0] = son[i][1] = 0; f[i] = rev[i] = 0;
7      if (i == 0) sz[i] = 0, val[i] = minval[i] = inf, minid[i] = i, val2[i] = sumval2
        [i] = 0;
8      else sz[i] = 1, val[i] = minval[i] = VAL, minid[i] = i, val2[i] = sumval2[i] =
        VAL2;
9  }
10 bool isroot(int x) { return !f[x] || (son[f[x]][0] != x && son[f[x]][1] != x); }
11 void revl(int x) { if (!x) return; swap(son[x][0], son[x][1]); rev[x] ^= 1; }
12 void down(int x) { if (!x) return; if (rev[x]) revl(son[x][0]), revl(son[x][1]), rev
    [x] = 0; }
13 void up(int x) { if (!x) return; down(son[x][0]); down(son[x][1]);
14     sz[x] = sz[son[x][0]] + sz[son[x][1]] + 1; minval[x] = val[x]; minid[x] = x;
15     if (minval[son[x][0]] < minval[x]) minval[x] = minval[son[x][0]], minid[x] =
        minid[son[x][0]];
16     if (minval[son[x][1]] < minval[x]) minval[x] = minval[son[x][1]], minid[x] =
        minid[son[x][1]];
17     sumval2[x] = sumval2[son[x][0]] + sumval2[son[x][1]] + val2[x];
18 }
19 void rotate(int x) {
20     int y = f[x], w = son[y][1] == x; son[y][w] = son[x][w ^ 1];
21     if (son[x][w ^ 1]) f[son[x][w ^ 1]] = y;
22     if (f[y]) {
23         int z = f[y];
24         if (son[z][0] == y) son[z][0] = x;
25         else if (son[z][1] == y) son[z][1] = x;
26     }
27     f[x] = f[y]; f[y] = x; son[x][w ^ 1] = y; up(y);
28 }
29 void splay(int x) {
30     while (!s.empty()) s.pop(); s.push(x);
31     for (int i = x; !isroot(i); i = f[i]) s.push(f[i]);
32     while (!s.empty()) down(s.top()), s.pop();
33     while (!isroot(x)) {
34         int y = f[x];
35         if (!isroot(y)) {
36             if ((son[f[y]][0] == y) ^ (son[y][0] == x)) rotate(x);
37             else rotate(y);
38         }
39         rotate(x);
40     } up(x);
41 }
42 void access(int x) {for (int y = 0; x; y = x, x = f[x]) splay(x), val2[x] += sumval2
    [son[x][1]], son[x][1] = y, val2[x] -= sumval2[son[x][1]], up(x); }
43 int root(int x) { access(x); splay(x); while (son[x][0]) x = son[x][0]; return x; }
44 void makeroot(int x) { access(x); splay(x); revl(x); }
45 void link(int x, int y) {

```



```

46     makeroot(x); f[x] = y; access(x);
47     // 如果需要维护子树和 val2, sumval2, 这样是不够的。因为增加了虚边, 所以需要修改
        val2 值。将上面的代码替换为以下代码:
48     // makeroot(x); makeroot(y); f[x] = y; val2[y] += sumval2[x]; up(y); access(x);
49 }
50 void cutf(int x) { access(x); splay(x); f[son[x][0]] = 0; son[x][0] = 0; up(x); } //
        它和父亲的边
51 void cut(int x, int y) { makeroot(x); cutf(y); } // 切断 x 与 y 之间的边 (须保证 x
        与 y 相邻)
52 int ask(int x, int y) { makeroot(x); access(y); splay(y); return minid[y]; } // 询问
        x 到 y 之间取得最小值的点
53 int querymin_cut(int x, int y) { int m = ask(x, y); makeroot(x); cutf(m); makeroot(y)
        ); cutf(m); return val[m]; } // 询问 x 到 y 之间取得最小值的点, 并把它删去 (须保
        证该点在 x 和 y 之间, 且度数恰好为 2)
54 void link(int x, int y, int w) { init(++tot); val[tot] = minval[tot] = w; link(x,
        tot); link(y, tot); } // 在 x 和 y 之间添加一条权值为 w 的边 (将边视为点插入)
55 int getsumval2(int x, int y) { makeroot(x); access(y); return val2[y]; } // 令 x 为
        根, 求 y 子树的 val2 的和

```

4.3 虚树

设 $a[0 \cdots k-1]$ 为需要构建虚树的点。

构建出虚树的节点保存在 a 数组中, k 为节点个数。加边调用函数 `addege(int x, int y, int w)`。

```

1  bool cmp(int x, int y) { return dfn[x] < dfn[y]; }
2  stack<int> stk;
3  sort(a, a + k, cmp);
4  int m = k;
5  for (int j = 1; j < m; ++j)
6      a[k++] = lca(a[j - 1], a[j]);
7  sort(a, a + k, cmp);
8  k = unique(a, a + k) - a;
9  stk.push(a[0]);
10 for (int j = 1; j < k; ++j) {
11     int u = lca(stk.top(), a[j]);
12     while (dep[stk.top()] > dep[u]) --top;
13     assert(stk.top() == u);
14     stk.push(a[j]);
15     addege(u, a[j], dis[a[j]] - dis[u]);
16 }

```

5 图

5.1 Tarjan 有向图强联通分量

① 割点的判断: 一个顶点 u 是割点, 当且仅当满足 (1) 或 (2): (1) u 为树根, 且 u 有多于一个子树 (即: 存在一个儿子 v 使得 $dfn[u] + 1 \neq dfn[v]$); (2) u 不为树根, 且满足存在 (u, v) 为树枝

边 (u 为 v 的父亲), 使得 $dfn[u] \leq low[v]$ 。② 桥的判断: 一条无向边 (u, v) 是桥, 当且仅当 (u, v) 为树枝边, 满足 $dfn[u] < low[v]$ 。

```

1 struct EDGE { int adj, next; } edge[M];
2 int n, m, top, gh[N];
3 int dfn[N], low[N], cnt, ind, stop, instack[N], stack[N], belong[N];
4 void addedge(int x, int y) { edge[++top].adj = y; edge[top].next = gh[x]; gh[x] =
    top; }
5 void tarjan(int x) {
6     dfn[x] = low[x] = ++ind;
7     instack[x] = 1; stack[++stop] = x;
8     for (int p=gh[x]; p; p=edge[p].next)
9         if (!dfn[edge[p].adj]) tarjan(edge[p].adj), low[x] = min(low[x], low[edge[p]
            ].adj);
10        else if (instack[edge[p].adj]) low[x] = min(low[x], dfn[edge[p].adj]);
11    if (dfn[x] == low[x]) {
12        ++cnt; int tmp=0;
13        while (tmp!=x) tmp = stack[stop--], belong[tmp] = cnt, instack[tmp] = 0;
14    }
15 }

```

5.2 Tarjan 双联通分量

以下代码为点双联通分量。若要更改为边双联通, 在第 8 行将 $low[next] \geq dfn[x]$ 改为 $low[next] > dfn[x]$, 并将 14 行 $vec[tot].push_back(x)$ 删除。

```

1 void DFS(int x,int fa) {
2     vis[x]=true; dfn[x]=low[x]=++times; sk[++tp]=x;
3     for (int pt=first[x];pt;pt=e[pt].next) {
4         int next=e[pt].to; if (e[pt].id==fa) continue;
5         if (!vis[next]) {
6             DFS(next,e[pt].id);
7             low[x]=min(low[x],low[next]);
8             if (low[next]>=dfn[x]) {
9                 vec[++tot].clear();
10                while (tp) {
11                    vec[tot].push_back(sk[tp--]);
12                    if (sk[tp+1]==next) break;
13                }
14                vec[tot].push_back(x);
15            }
16            else if (dfn[next]>last) low[x]=min(low[x],dfn[next]);
17        }
18    }
19    for (i=1;i<=n;i++) if (!vis[i]) {
20        DFS(i,0); last=times;
21        if (tp) {
22            tot++; vec[tot].clear();
23            for (i=1;i<=tp;i++) vec[tot].push_back(sk[i]);
24            tp=0;

```

```

25     }
26 }

```

5.3 2-SAT

记 $x \rightarrow y$ 的有向边表示选了 x 就要选 y 。

```

1 struct MergePoint {
2     struct EDGE { int adj, next; } edge[M];
3     int ex[M], ey[M]; bool instack[N];
4     int gh[N], top, dfn[N], low[N], cnt, ind, stop, stack[N], belong[N];
5     void init() { cnt = ind = stop = top = 0; memset(dfn, 0, sizeof(dfn)); memset(
        instack, 0, sizeof(instack)); memset(gh, 0, sizeof(gh)); }
6     void addedge(int x, int y) { swap(x, y); edge[++top].adj = y; edge[top].next =
        gh[x]; gh[x] = top; ex[top] = x; ey[top] = y; }
7     void tarjan(int x) {}
8     void work() { for (i) if (!dfn[i]) tarjan(i); }
9 } merge;
10 struct Topsort {
11     struct EDGE { int adj, next; } edge[M];
12     int n, top, gh[N], ops[N], deg[N], ans[N]; std::queue<int> q;
13     void init() { n = merge.cnt; top = 0; memset(gh, 0, sizeof(gh)); memset(deg, 0,
        sizeof(deg)); }
14     void addedge(int x, int y) { if (x == y) return; edge[++top].adj = y; edge[top].
        next = gh[x]; gh[x] = top; ++deg[y]; }
15     void work() {
16         for (int i = 1; i <= n; ++i) if (!deg[i]) q.push(i);
17         while (!q.empty()) {
18             int x = q.front(); q.pop();
19             for (int p = gh[x]; p; p = edge[p].next) if (--deg[edge[p].adj]) q.push
                (edge[p].adj);
20             if (ans[x]) continue; ans[x] = -1; ans[ops[x]] = 1; //-1 NO, 1 YES
21         }
22     }
23 } ts;
24 merge.init(); merge.addedge(); merge.work();
25 for (int i = 1; i <= n; ++i) {
26     int x = merge.belong[U(i, 0)], y = merge.belong[U(i, 1)];
27     if (x==y) NO(); ts.ops[x]=y; ts.ops[y]=x;
28 }
29 ts.init(); ts.work();
30 puts("YES"); for (int i = 1; i <= n; ++i) select(ts.ans[merge.belong[U(i,1)]] == 1);

```

5.4 网络流

5.4.1 最大流

注意: top 要初始化为 1

```

1 struct EDGE { int adj, w, next; } edge[M];
2 int n, top, gh[N], nrl[N], dist[N], q[N];
3 void addedge(int x, int y, int w) { edge[++top].adj = y; edge[top].w = w; edge[top].
    next = gh[x]; gh[x] = top; edge[++top].adj = x; edge[top].w = 0; edge[top].next
    = gh[y]; gh[y] = top; }
4 int bfs() {
5     memset(dist, 0, sizeof(dist));
6     q[1] = S; int head = 0, tail = 1; dist[S] = 1;
7     while (head != tail) {
8         int x = q[++head];
9         for (int p=gh[x]; p; p=edge[p].next)
10             if (edge[p].w && !dist[edge[p].adj]) {
11                 dist[edge[p].adj] = dist[x] + 1;
12                 q[++tail] = edge[p].adj;
13             }
14     }
15     return dist[T];
16 }
17 int dinic(int x, int delta) {
18     if (x==T) return delta;
19     for (int& p=nrl[x]; p && delta; p=edge[p].next)
20         if (edge[p].w && dist[x]+1 == dist[edge[p].adj]) {
21             int dd = dinic(edge[p].adj, min(delta, edge[p].w));
22             if (!dd) continue;
23             edge[p].w -= dd;
24             edge[p^1].w += dd;
25             return dd;
26         }
27     return 0;
28 }
29 int ans = 0; while (bfs()) { memcpy(nrl, gh, sizeof(gh)); int t; while (t = dinic(S,
    inf)) ans += t; } return ans;

```

5.4.2 上下界有源汇网络流

① T 向 S 连容量为正无穷的边，将有源汇转化为无源汇。②每条边容量减去下界，设 $in[i]$ 表示流入 i 的下界之和减去流出 i 的下界之和。③新建超级源汇 SS, TT ，对于 $in[i] > 0$ 的点， SS 向 i 连容量为 $in[i]$ 的边。对于 $in[i] < 0$ 的点， i 向 TT 连容量为 $-in[i]$ 的边。④求出以 SS, TT 为源汇的最大流，如果等于 $\sum in[i] (in[i] > 0)$ ，则存在可行流。再求出 S, T 为源汇的最大流即为最大流。⑤费用流：建完图后等价于求以 SS, TT 为源汇的费用流。

5.4.3 费用流

注意：top 要初始化为 1

```

1 struct EDGE { int adj, w, cost, next; } edge[M*2];
2 int gh[N], q[N], dist[N], v[N], pre[N], prev[N], top, S, T;
3 void addedge(int x, int y, int w, int cost) { x->y(w, cost); y->x(0, -cost); }
4 void clear() { top = 1; memset(gh, 0, sizeof(gh)); }

```

```

5 bool spfa() {} // 从S出发, 返回dist[T] != inf
6 int ans = 0;
7 while (spfa()) {
8     int mx = inf;
9     for (int x=T;x!=S;x=pre[x]) mx = min(edge[pre[x]].w, mx);
10    ans += dist[T] * mx;
11    for (int x=T;x!=S;x=pre[x]) edge[pre[x]].w -= mx, edge[pre[x]^1].w += mx;
12 }
13 return ans;

```

6 数学

6.1 扩展欧几里得解同余方程

ans[] 保存的是循环节内所有的解

```

1 int exgcd(int a,int b,int&x,int&y){
2     if(!b) return x=1,y=0,a;
3     int d=exgcd(b,a%b,x,y),t=x;
4     return x=y,y=t-a/b*y,d;
5 }
6 void cal(ll a,ll b,ll n){//ax=b(mod n)
7     ll x,y,d=exgcd(a,n,x,y);
8     if(b%d) return;
9     x=(x%n+n)%n;
10    ans[cnt=1]=x*(b/d)%(n/d);
11    for(ll i=1;i<d;i++) ans[++cnt]=(ans[1]+i*n/d)%n;
12 }

```

6.2 同余方程组

```

1 int n,flag,k,m,a,r,d,x,y;
2 int main(){
3     scanf("%d",&n);
4     flag=k=1,m=0;
5     while(n--){
6         scanf("%d%d",&a,&r);//ans%a=r
7         if(flag){
8             d=exgcd(k,a,x,y);
9             if((r-m)%d){flag=0;continue;}
10            x=(x*((r-m)/d)+a/d)%(a/d),y=k/d*a,m=(x*k+m)%y;
11            if(m<0)m+=y;
12            k=y;
13        }
14    }
15    printf("%d",flag?m:-1);//若flag=1,说明有解,解为ki+m,i为任意整数
16 }

```

6.3 FFT

6.3.1 普通 FFT

```
1 struct comp {
2     double real , imag; comp() {}
3     comp(double real , double imag): real(real) , imag(imag) {}
4     friend inline comp operator+(const comp &a , const comp &b) { return comp(a.real
5         + b.real , a.imag + b.imag); }
6     friend inline comp operator-(const comp &a , const comp &b) { return comp(a.real
7         - b.real , a.imag - b.imag); }
8     friend inline comp operator*(const comp &a , const comp &b) { return comp(a.real
9         * b.real - a.imag * b.imag , a.real * b.imag + a.imag * b.real); }
10 };
11 comp A[maxn] , B[maxn];
12 int rev[maxn], m, len;
13 inline void init(int n) {
14     for (m = 1, len = 0; m < n + n; m <= 1 , len ++);
15     for (int i = 0; i < m; ++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len - 1))
16         ;
17     for (int i = 0; i < m; ++i) A[i] = B[i] = comp(0, 0);
18 }
19 inline void dft(comp *a , int v) {
20     for (int i = 0; i < m; ++i) if (i < rev[i]) swap(a[i] , a[rev[i]]);
21     for (int s = 2; s <= m; s <= 1) {
22         comp g(cos(2 * pi / s) , v * sin(2 * pi / s));
23         for (int k = 0; k < m; k += s) {
24             comp w(1 , 0);
25             for (int j = 0; j < s / 2; ++j) {
26                 comp &u = a[k + j + s / 2], &v = a[k + j];
27                 comp t = w * u; u = v - t; v = v + t; w = w * g;
28             }
29         }
30     }
31     if (v == -1) for (int i = 0; i < m; ++i) a[i].real /= m , a[i].imag /= m;
32 }
```

6.4 求原根

```
1 vector <LL> a;
2 bool g_test(LL g, LL p) { for (LL i = 0; i < a.size(); ++i) if (pow_mod(g, (p-1)/a[i]
3     ], p) == 1) return 0; return 1; }
4 LL p_root(LL p) {
5     LL tmp = p - 1;
6     for (LL i = 2; i <= tmp / i; ++i)
7         if (tmp % i == 0) { a.push_back(i); while (tmp % i == 0) tmp /= i; }
8     if (tmp != 1) a.push_back(tmp);
9     LL g = 1; while (1) { if (g_test(g, p)) return g; ++g; }
10 }
```

6.5 NTT

```
1 #define mod 998244353
2 #define gg 3
3 void NTT(int *x, int n, int reverse) {
4     static int rev[N];
5     int m = 1, len = 0; for (; m < n + n; m <= 1, ++len);
6     for (int i = 0; i < m; ++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (len - 1))
7     ;
8     for (int i = 0; i < m; ++i) if (i < rev[i]) swap(x[i], x[rev[i]]);
9     for (int h = 2; h <= m; h <= 1) {
10         int wn = power(gg, (mod - 1) / h);
11         if (reverse == -1) wn = power(wn, mod - 2);
12         for (int i = 0; i < m; i += h) {
13             int w = 1;
14             for (int j = i; j < i + h / 2; ++j) {
15                 int u = x[j], v = 1ll * w * x[j + h / 2] % mod;
16                 x[j] = (u + v) % mod; x[j + h / 2] = (u - v + mod) % mod;
17                 w = 1ll * w * wn % mod;
18             }
19         }
20     if (reverse == -1) { int t = power(m, mod - 2); for (int i = 0; i < m; ++i) x[i]
21         = 1ll * x[i] * t % mod; }
```

6.6 线性基

```
1 #define B 30
2 const int allset=(1<<B)-1;
3 struct LB {
4     int mat[B],cnt;
5     multiset<int> st;
6     LB(){}
7     void clear() { st.clear(); cnt=0; memset(mat,0,sizeof(mat)); }
8     void add(int x) {
9         for (int i=B-1;i>=0;i--) if ((x>>i)&1) {
10             if (mat[i]) x^=mat[i];
11             else { cnt++; mat[i]=x; break; }
12         }
13     }
14     void fix() {
15         for (int i=0;i<B;i++) if (mat[i])
16             for (int j=i+1;j<B;j++) if ((mat[j]>>i)&1) mat[j]^=mat[i];
17     }
18     void preset() { //正确性待定
19         fix(); for (int i=0;i<B;i++) if (mat[i]) st.insert(mat[i]);
20     }
21     int kth(int k) { //正确性待定
```

```

22     int i=0,ans=0; if (k<=0||k>(1<<cnt)-1) return 0;//无解
23     for (multiset<int>::iterator it=st.begin();it!=st.end();it++,i++)
24         if ((k>>i)&1) ans^=(*it);
25     return ans;
26 }
27 int getmax() {
28     fix(); int ans=0;
29     for (int i=B-1;i>=0;i--) if (ans^mat[i]>ans) ans^=mat[i];
30     return ans;
31 }
32 } tree[N*10];

```

6.7 高斯消元法实数方程

```

1 void Gauss(int n,int m) {
2     int i,j,k,t;
3     double mul;
4     for (i=j=1;i<=n&&j<=m;i++,j++) {
5         for (k=i+1;k<=n;k++)
6             if (abs(mat[k][j])>abs(mat[i][j]))
7                 for (t=1;t<=m+1;t++) swap(mat[i][t],mat[k][t]);
8         if (abs(mat[i][j])<eps) { i--; continue; }
9         for (k=i+1;k<=n;k++) {
10             mul=mat[k][j]/mat[i][j];
11             for (t=1;t<=m+1;t++) mat[k][t]-=mat[i][t]*mul;
12         }
13     }
14     for (i=n;i>=1;i--) { //solved表示那个变量是否确定
15         for (j=1;j<=m;j++) if (abs(mat[i][j])>eps) break;
16         if (j>m) continue; solved[j]=true; ans[j]=mat[i][m+1];
17         for (k=j+1;k<=m;k++)
18             if (abs(mat[i][k])>eps&&!solved[k]) solved[j]=false;
19         for (k=j+1;k<=m;k++) ans[j]-=ans[k]*mat[i][k];
20         ans[j]/=mat[i][j];
21     }
22 }

```

6.8 高斯消元法模方程

```

1 void Gauss(LL n,LL m) {
2     LL i,j,k,t,lcm,muli,mulk;
3     for (i=j=1;i<=n&&j<=m;i++,j++) {
4         for (k=i;k<=n;k++) if (mat[k][j]) {
5             for (t=1;t<=m+1;t++) swap(mat[k][t],mat[i][t]);
6             break;
7         }
8         if (mat[i][j]==0) { i--; continue; }
9         for (k=i+1;k<=n;k++) if (mat[k][j]) {

```



```

10         lcm=mat[k][j]*mat[i][j]/__gcd(mat[k][j],mat[i][j]);
11         muli=lcm/mat[i][j]; mulk=lcm/mat[k][j];
12         for (t=1;t<=m+1;t++) {
13             mat[k][t]=mat[k][t]*mulk-mat[i][t]*muli;
14             mat[k][t]=(mat[k][t]%mod+mod)%mod;
15         }
16     }
17 }
18 for (i=n;i>=1;i--) {
19     for (j=1;j<=m;j++) if (mat[i][j]) break;
20     if (j>m) continue; ans[j]=mat[i][m+1];
21     for (k=j+1;k<=m;k++) ans[j]-=ans[k]*mat[i][k];
22     ans[j]=(ans[j]*power(mat[i][j],mod-2)%mod+mod)%mod;
23 }
24 }

```

6.9 蔡勒公式

$$w = (\lfloor \frac{c}{4} \rfloor - 2c + y + \lfloor \frac{y}{4} \rfloor + \lfloor \frac{13(m+1)}{5} \rfloor + d - 1) \bmod 7$$

① w : 0 星期日, 1 星期一, \dots , 6 星期六; ② c : 年份前两位数; ③ y : 年份后两位数; ④ m : 月 ($3 \leq m \leq 14$, 即在蔡勒公式中, 1、2 月要看作上一年的 13、14 月来计算); ⑤ d : 日。

6.10 皮克定理

给定顶点坐标均是整点 (或正方形格点) 的简单多边形 (凸多边形), 皮克定理说明了其面积 S 和内部格点数目 n 、边上格点数目 s 的关系: $S = n + \frac{s}{2} + 1$ 。

7 计算几何

7.1 凸包

```

1 bool cmp(const Point &a,const Point &b) {
2     return F(a.x-b.x)<0||F(a.x-b.x)==0&&a.y<b.y; }
3 void Gram(int id[], int n) {
4     int i,mid; sort(id,id+n,cmp); tp=0; //凸包从x最小的点出发, 逆时针方向
5     for (i=0;i<n;i++) {
6         for (;tp>=2&&Cross(p[sk[tp-1]]-p[sk[tp-2]],p[id[i]]-p[sk[tp-1]])<=0;tp--);
7         //有重点必须用<=不留共线点, 无重点用<=不留共线点, 无重点用<留共线点
8         sk[tp++]=id[i];
9     }
10    mid=tp;
11    for (i=n-2;i>=0;i--) {
12        for (;tp>mid&&Cross(p[sk[tp-1]]-p[sk[tp-2]],p[id[i]]-p[sk[tp-1]])<=0;tp--);
13        //有重点必须用<=不留共线点, 无重点用<=不留共线点, 无重点用<留共线点
14        sk[tp++]=id[i];
15    }

```

```

16     if (n>1) tp--;
17 }

```

7.2 定义

```

1 struct Point { double x,y; Point(){} Point(double _x,double _y):x(_x),y(_y){} };
2 struct Seg { Point a,b; Seg(){} Seg(Point _a,Point _b):a(_a),b(_b){} };
3 struct Circle { double x,y,r;
4     Point pt() { return Point(x,y); }
5     double Area() { return pi*r*r; }
6 };
7 Point operator +(const Point &a,const Point &b);
8 Point operator -(const Point &a,const Point &b);
9 Point operator *(const Point &a,double b);
10 Point operator /(const Point &a,double b);
11 int F(double x) { if (x>eps) return 1; if (x<-eps) return -1; return 0; }
12 bool operator ==(const Point &a,const Point &b) {
13     return F(a.x-b.x)==0&&F(a.y-b.y)==0; }
14 double Dist(const Point &a) { return sqrt(a.x*a.x+a.y*a.y); }
15 double Dot(const Point &a,const Point &b) { return a.x*b.x+a.y*b.y; }
16 double Cross(const Point &a,const Point &b) { return a.x*b.y-a.y*b.x; }
17 Point Rotate(const Point &p,double a) { // 逆时针旋转
18     return Point(p.x*cos(a)-p.y*sin(a),p.x*sin(a)+p.y*cos(a)); }
19 Point Inter(Seg a,Seg b) { // 两线段相交 (前提有交点)
20     double s=Cross(a.b-a.a,b.a-a.a),t=Cross(a.b-a.a,b.b-a.a);
21     return b.a+(b.b-b.a)*s/(s-t); }
22 vector<Point> SegCir(Seg seg,Point pt,double r) { // 线圆
23     vector<Point> ans; double mul; Point vec,mid;
24     ans.clear(); vec=Rotate(seg.b-seg.a,pi/2);
25     mid=Inter(seg,Seg(pt,pt+vec));
26     if (F(Dist(pt-mid)-r)>0) return ans;
27     if (F(Dist(pt-mid)-r)==0) {
28         ans.push_back(mid); ans.push_back(mid); return ans;
29     }
30     vec=seg.b-seg.a; mul=sqrt(r*r-Dist2(mid-pt))/Dist(vec);
31     ans.push_back(mid+vec*mul); ans.push_back(mid-vec*mul);
32     return ans;
33 }
34 vector<Point> Circir(Circle a,Circle b) { // 圆圆相交
35     vector<Point> ans; double dis,dis2,alpha; Point pa,pb,vec;
36     ans.clear(); if (a.r<b.r) swap(a,b);
37     pa=a.pt(); pb=b.pt(); vec=pb-pa;
38     dis=Dist(vec); dis2=Dist2(vec);
39     if (F(dis-(a.r+b.r))>0||F(dis-(a.r-b.r))<0) return ans;
40     if (F(dis-(a.r+b.r))==0) {
41         ans.push_back(pa+vec*a.r/(a.r+b.r)); return ans;
42     }
43     if (F(dis-(a.r-b.r))==0) {
44         ans.push_back(pa+vec*a.r/(a.r-b.r)); return ans;

```

```

45     }
46     alpha=acos((a.r*a.r+dis2-b.r*b.r)/2/a.r/dis);
47     ans.push_back(pa+Rotate(vec,alpha)*a.r/dis);
48     ans.push_back(pa+Rotate(vec,-alpha)*a.r/dis);
49     return ans;
50 }
51 double Bing(double ra,double rb,double dis) {
52     double alpha,beta; if (ra<rb) swap(ra,rb);
53     if (F(dis-(ra-rb))<=0) return pi*ra*ra;
54     if (F(dis-(ra+rb))>=0) return pi*ra*ra+pi*rb*rb;
55     alpha=acos((ra*ra+dis*dis-rb*rb)/2/dis/ra);
56     beta=acos((rb*rb+dis*dis-ra*ra)/2/dis/rb);
57     return (pi-alpha)*ra*ra+(pi-beta)*rb*rb+ra*dis*sin(alpha);
58 }
59 double Jiao(double ra,double rb,double dis) {
60     return pi*ra*ra+pi*rb*rb-Bing(ra,rb,dis); }
61 Point Gongmid(Circle a,Circle b) { //正确性待定
62     Point pa=a.pt(),pb=b.pt();
63     return pa+(pb-pa)*a.r/(a.r+b.r); }
64 Point Gongright(Circle a,Circle b) {
65     Point pa=a.pt(),pb=b.pt();
66     return pa+(pb-pa)*a.r/(a.r-b.r); }
67 int Ptinpol(Point pt) {
68     int wn=0;
69     for(int i=0;i<n;i++) {
70         if(Ins(pt,Seg(p[i],p[(i+1)%n]))) return 2;
71         int k=F(Cross(p[(i+1)%n]-p[i],pt-p[i]));
72         int d1=F(p[i].y-pt.y), d2=F(p[(i+1)%n].y-pt.y);
73         if(k>0&&d1<=0&&d2>0)wn++;
74         if(k<0&&d2<=0&&d1>0)wn--;
75     }
76     return wn!=0;
77 }
78 bool Cirinpol(Point pt) { //需要点在多边形内的前提
79     double nearest=inf;
80     for (int i=0;i<n;i++) {
81         nearest=min(nearest,Dist(p[i]-pt));
82         if (F(Dot(pt-p[i],p[(i+1)%n]-p[i]))>0&&
83             F(Dot(pt-p[(i+1)%n],p[i]-p[(i+1)%n]))>0)
84             nearest=min(nearest,abs(Cross(p[i]-pt,p[(i+1)%n]-pt))/dis[i]);
85     }
86     return F(nearest-r)>=0;
87 }
88 bool Ins(const Point &p,const Seg &s) {
89     return F(Cross(s.a-p,s.b-p))==0&&F(p.x-min(s.a.x,s.b.x))>=0&&
90         F(p.x-max(s.a.x,s.b.x))<=0&&F(p.y-min(s.a.y,s.b.y))>=0&&
91         F(p.y-max(s.a.y,s.b.y))<=0; }
92 double PS(const Point &p,const Seg &s) { // 点到线段最短距离
93     if (F(Dot(p-s.a,s.b-s.a))<0||F(Dot(p-s.b,s.a-s.b))<0)
94         return min(Dist(p-s.a),Dist(p-s.b));

```

```

95     return abs(Cross(s.a-p,s.b-p))/Dist(s.a-s.b); }
96 double SS(const Seg &a,const Seg &b) { // 线段到线段最短距离
97     return min(min(PS(a.a,b),PS(a.b,b)),min(PS(b.a,a),PS(b.b,a))); }
98 double Alpha(Point a,Point b) {
99     double ans=atan2(b.y,b.x)-atan2(a.y,a.x);
100     if (ans<0) ans=-ans; if (ans>pi) ans=2*pi-ans; return ans; }
101 double Shan(Circle c,double a) { return c.r*c.r*a/2; }

```

7.3 半平面交

```

1  bool CmpHP(Seg a,Seg b) {
2      Point va=a.b-a.a, vb=b.b-b.a;
3      double dega=atan2(va.y,va.x), degb=atan2(vb.y,vb.x);
4      return F(dega-degb)<0||F(dega-degb)==0&&Cross(a.b-a.a,b.a-a.a)<0;
5  }
6  void HalfPlane(Seg hp[], int n, Point pol[], int &pols) {
7      Point mid;
8      hp[n++]=Seg(Point(-oo,-oo),Point(oo,-oo));
9      hp[n++]=Seg(Point(oo,-oo),Point(oo,oo));
10     hp[n++]=Seg(Point(oo,oo),Point(-oo,oo));
11     hp[n++]=Seg(Point(-oo,oo),Point(-oo,-oo));
12     sort(hp, hp+n, CmpHP);
13     int tp=0, low=0, high=-1; //sk 0~tp-1
14     for (int i=0;i<n;i++)
15         if (high-low+1==0||F(Cross(sk[high].b-sk[high].a, hp[i].b-hp[i].a))) {
16             for (;low<high;high--) {
17                 mid=Inter(sk[high], sk[high-1]);
18                 if (F(Cross(hp[i].b-hp[i].a, mid-hp[i].a))>0) break;
19             }
20             for (;low<high;low++) {
21                 mid=Inter(sk[low], sk[low+1]);
22                 if (F(Cross(hp[i].b-hp[i].a, mid-hp[i].a))>0) break;
23             }
24             sk[++high]=hp[i];
25         }
26     for (;low<high;high--) {
27         mid=Inter(sk[high], sk[high-1]);
28         if (Cross(sk[low].b-sk[low].a, mid-sk[low].a)>0) break;
29     }
30     tp=high-low+1; for (int i=0;i<tp;i++) sk[i]=sk[low+i];
31     pols=0; if (tp<=2) return;
32     for (int i=0;i<tp;i++) pol[pols++]=Inter(sk[i], sk[(i+1)%tp]);
33 }

```

7.4 圆与多边形交集

```

1  double CT(Circle c,Point a,Point b) { // 圆与三角形交 (多边形)
2      double da=Dist(a-c.pt()), db=Dist(b-c.pt());

```

```

3   if (da>db) { swap(a,b); swap(da,db); }
4   Seg s=Seg(a,b);
5   vector<Point> temp=CS(c,s);
6   if (F(db-c.r)<=0) return 0.5*abs(Cross(a-c.pt(),b-c.pt()));
7   if (F(da-c.r)<0) {
8       if (F(Dot(a-temp[1],b-temp[1]))<0) swap(temp[0],temp[1]);
9       return Shan(c,Alpha(temp[0]-c.pt(),b-c.pt()))+
10          0.5*abs(Cross(a-c.pt(),temp[0]-c.pt()));
11   }
12   if (!temp.size()) return Shan(c,Alpha(a-c.pt(),b-c.pt()));
13   if (Ins(temp[1],s)&&Dist2(a-temp[1])<Dist2(a-temp[0])) swap(temp[0],temp[1]);
14   if (Ins(temp[0],s)&&Ins(temp[1],s)) {
15       return Shan(c,Alpha(a-c.pt(),temp[0]-c.pt()))+
16          Shan(c,Alpha(b-c.pt(),temp[1]-c.pt()))+
17          0.5*abs(Cross(temp[0]-c.pt(),temp[1]-c.pt()));
18   }
19   return Shan(c,Alpha(a-c.pt(),b-c.pt()));
20 }

```

7.5 三角形面积并

```

1  #define pr pair<ld,ld>
2  typedef long double ld;
3  const ld EPS=1e-8, INF=1e100;
4  struct Point {
5      ld x,y; Point(){} Point(ld _,ld __):x(_),y(__){}
6      void read() { double _x,_y; scanf("%lf%lf",&_x,&_y); x=_x,y=_y; }
7      friend bool operator<(Point a,Point b) {
8          if(fabs(a.x-b.x)<EPS) return a.y<b.y;
9          return a.x<b.x;
10     }
11     friend Point operator +(Point a,Point b) { return Point(a.x+b.x,a.y+b.y); }
12     friend Point operator -(Point a,Point b) { return Point(a.x-b.x,a.y-b.y); }
13     friend Point operator *(ld a,Point b) { return Point(a*b.x,a*b.y); }
14     friend ld operator *(Point a,Point b) { return a.x*b.x+a.y*b.y; }
15     friend ld operator ^(Point a,Point b) { return a.x*b.y-a.y*b.x; }
16 } a[N][3],Poi[N*N];
17 struct Line {
18     Point p,v; Line(){} Line(Point x,Point y){p=x,v=y-x;}
19     Point operator [] (int k) { if(k) return p+v; else return p; }
20     friend bool Cross(Line a,Line b) {
21         return (a.v^b[0]-a.p)*(a.v^b[1]-a.p)<=EPS &&
22            (b.v^a[0]-b.p)*(b.v^a[1]-b.p)<=EPS;
23     }
24     friend Point getP(Line a,Line b) {
25         Point u=a.p-b.p; ld temp=(b.v^u)/(a.v^b.v);
26         return a.p+temp*a.v;
27     }
28 } l[N][3],T;

```

```

29 pr p[N];
30 int main() {
31     int n,m,i,j,k,x,y,cnt,tot;
32     ld ans,last,A,B,sum;
33     scanf("%d",&n);
34     for(i=1,tot=0;i<=n;i++) {
35         a[i][0].read(),a[i][1].read(),a[i][2].read();
36         Poi[++tot]=a[i][0],Poi[++tot]=a[i][1],Poi[++tot]=a[i][2];
37         sort(a[i],a[i]+3);
38         if((a[i][2]-a[i][0]^a[i][1]-a[i][0])>EPS)
39             l[i][0]=Line(a[i][0],a[i][2]),l[i][1]=Line(a[i][2],a[i][1]),l[i][2]=Line
                (a[i][1],a[i][0]);
40         else
41             l[i][0]=Line(a[i][2],a[i][0]),l[i][1]=Line(a[i][1],a[i][2]),l[i][2]=Line
                (a[i][0],a[i][1]);
42     }
43     for(i=1;i<=n;i++) for(j=1;j<i;j++) for(x=0;x<3;x++) for(y=0;y<3;y++)
44         if(Cross(l[i][x],l[j][y])) Poi[++tot]=getP(l[i][x],l[j][y]);
45     sort(Poi+1,Poi+tot+1);
46     ans=0,last=Poi[1].x; T=Line(Point(0,-INF),Point(0,INF));
47     for(i=2;i<=tot;i++) {
48         T.p.x=(last+Poi[i].x)/2;
49         for(j=1,cnt=0;j<=n;j++)
50             if(Cross(l[j][0],T)) {
51                 if(Cross(l[j][1],T)) B=getP(l[j][1],T).y;
52                 else B=getP(l[j][2],T).y;
53                 A=getP(l[j][0],T).y; if (A>B) swap(A,B);
54                 p[++cnt]=pr(A,B);
55             }
56         sort(p+1,p+cnt+1);
57         for(j=1,sum=0,A=-INF;j<=cnt;j++) {
58             if(p[j].first>A) sum+=p[j].second-p[j].first, A=p[j].second;
59             else if(p[j].second>A) sum+=p[j].second-A, A=p[j].second;
60         }
61         ans+=(Poi[i].x-last)*sum; last=Poi[i].x;
62     }
63     printf("%.2lf\n", (double)ans);
64 }

```

7.6 K 圓并

```

1 #define sqr(x) ((x)*(x))
2 const double eps = 1e-8;
3 double area[N]; int n;
4 int dcmp(double x) { if (x < -eps) return -1; else return x > eps; }
5 struct cp { double x, y, r, angle; int d;
6     cp(){} cp(double xx, double yy, double ang = 0, int t = 0) {
7         x = xx; y = yy; angle = ang; d = t; }
8     void get() { scanf("%lf%lf%lf", &x, &y, &r); d = 1; }

```

```

9  }cir[N], tp[N * 2];
10 double dis(cp a, cp b) { return sqrt(sqr(a.x - b.x) + sqr(a.y - b.y)); }
11 double cross(cp p0, cp p1, cp p2) {
12     return (p1.x - p0.x) * (p2.y - p0.y) - (p1.y - p0.y) * (p2.x - p0.x);
13 }
14 int CirCrossCir(cp p1, double r1, cp p2, double r2, cp &cp1, cp &cp2) {
15     double mx = p2.x - p1.x, sx = p2.x + p1.x, mx2 = mx * mx;
16     double my = p2.y - p1.y, sy = p2.y + p1.y, my2 = my * my;
17     double sq = mx2 + my2, d = -(sq - sqr(r1 - r2)) * (sq - sqr(r1 + r2));
18     if (d + eps < 0) return 0; if (d < eps) d = 0; else d = sqrt(d);
19     double x = mx * ((r1 + r2) * (r1 - r2) + mx * sx) + sx * my2;
20     double y = my * ((r1 + r2) * (r1 - r2) + my * sy) + sy * mx2;
21     double dx = mx * d, dy = my * d; sq *= 2;
22     cp1.x = (x - dy) / sq; cp1.y = (y + dx) / sq;
23     cp2.x = (x + dy) / sq; cp2.y = (y - dx) / sq;
24     if (d > eps) return 2; else return 1;
25 }
26 bool circmp(const cp& u, const cp& v) { return dcmp(u.r - v.r) < 0; }
27 bool cmp(const cp& u, const cp& v) {
28     if (dcmp(u.angle - v.angle)) return u.angle < v.angle;
29     return u.d > v.d;
30 }
31 double calc(cp cir, cp cp1, cp cp2) {
32     double ans = (cp2.angle - cp1.angle) * sqr(cir.r)
33         - cross(cir, cp1, cp2) + cross(cp(0, 0), cp1, cp2);
34     return ans / 2;
35 }
36 void CirUnion(cp cir[], int n) {
37     cp cp1, cp2; sort(cir, cir + n, circmp);
38     for (int i = 0; i < n; ++i) for (int j = i + 1; j < n; ++j)
39         if (dcmp(dis(cir[i], cir[j]) + cir[i].r - cir[j].r) <= 0) cir[i].d++;
40     for (int i = 0; i < n; ++i) {
41         int tn = 0, cnt = 0;
42         for (int j = 0; j < n; ++j) {
43             if (i == j) continue;
44             if (CirCrossCir(cir[i], cir[i].r, cir[j], cir[j].r, cp2, cp1) < 2) continue;
45             cp1.angle = atan2(cp1.y - cir[i].y, cp1.x - cir[i].x);
46             cp2.angle = atan2(cp2.y - cir[i].y, cp2.x - cir[i].x);
47             cp1.d = 1; tp[tn++] = cp1; cp2.d = -1; tp[tn++] = cp2;
48             if (dcmp(cp1.angle - cp2.angle) > 0) cnt++;
49         }
50         tp[tn++] = cp(cir[i].x - cir[i].r, cir[i].y, pi, -cnt);
51         tp[tn++] = cp(cir[i].x + cir[i].r, cir[i].y, -pi, cnt);
52         sort(tp, tp + tn, cmp);
53         int p, s = cir[i].d + tp[0].d;
54         for (int j = 1; j < tn; ++j) {
55             p = s; s += tp[j].d;
56             area[p] += calc(cir[i], tp[j - 1], tp[j]);
57         }
58     }
}

```

```

59 }
60 void solve() {
61     for (int i = 0; i < n; ++i) cir[i].get();
62     memset(area, 0, sizeof(area));
63     CirUnion(cir, n);
64     for (int i = 1; i <= n; ++i) {
65         area[i] -= area[i + 1];
66         printf("[%d] = %.3lf\n", i, area[i]);
67     }
68 }

```

7.7 三维计算几何

```

1 Point Cross(Point a, Point b) {
2     return Point(a.y*b.z-a.z*b.y, a.z*b.x-a.x*b.z, a.x*b.y-a.y*b.x); }
3 double Crossxy(Point a, Point b) { return a.x*b.y-a.y*b.x; }
4 vector<Point> SegPlane(Seg seg, Plane p) {
5     vector<Point> ans; ans.clear();
6     Point fa=Cross(p.b-p.a, p.c-p.a);
7     if (F(Dot(fa, seg.b-seg.a))==0) return ans;
8     double s=Dot(p.a-seg.a, fa)/Dist(fa), t=Dot(p.a-seg.b, fa)/Dist(fa);
9     ans.push_back(seg.a+(seg.b-seg.a)*s/(s-t));
10    return ans;
11 }
12 // mixed product
13 double Mix(Point3 a, Point3 b, Point3 c) { return Dot(Cross(a,b),c); }
14 double PP(Point3 pt, Plane pl) { // distance from point to plane
15     Point3 fa=Cross(pl.b-pl.a, pl.c-pl.a);
16     return abs(Dot(fa, pt-pl.a))/Dist(fa);
17 }
18 // get the center point from 3D (need plane well prepared)
19 Point3 Getcenter(Point3 p[], int n, Plane pp[], int nn) {
20     double sumv=0;
21     Point3 sum=Point3(0,0,0);
22     for (int i=0; i<nn; i++)
23     {
24         double tempv=Mix(pp[i].b-pp[i].a, pp[i].c-pp[i].a, Point3(0,0,0)-pp[i].a);
25         sum=sum+(pp[i].a+pp[i].b+pp[i].c)*tempv/4.0;
26         sumv+=tempv;
27     }
28     return sum/sumv;
29 }

```

8 黑科技和杂项

8.1 读入优化

int rd(int &x); 读入一个整数，保存在变量 x 中。如正常读入，返回值为 1，否则返回 EOF (-1)


```

1  #define rd RD<int>
2  #define rdll RD<long long>
3  const int S = 2000000; // 2MB
4  char s[S], *h = s+S, *t = h;
5  inline char getchr(void) {
6      if(h == t) { if (t != s + S) return EOF; t = s + fread(s, 1, S, stdin); h = s; }
7      return *h++;
8  }
9  template <class T>
10 inline int RD(T &x) {
11     char c = 0; int sign = 0;
12     for (; !isdigit(c); c = getchr()) {
13         if (c == EOF) return -1; if (c == '-') sign ^= 1;
14     }
15     x = 0; for (; isdigit(c); c = getchr()) x = x * 10 + c - '0';
16     if (sign) x = -x; return 1;
17 }

```

9 Vim

```

1  syntax on
2  set cindent
3  set nu
4  set tabstop=4
5  set shiftwidth=4
6  set background=dark
7
8  inoremap <C-j> <down>
9  inoremap <C-k> <up>
10 inoremap <C-h> <left>
11 inoremap <C-l> <right>

```

10 Java