ICPC Templates For Africamonkey

Africamonkey

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目录

1 莫队算法

1.1 普通莫队

```
1  struct Q { int l, r, sqrtl, id; } q[N];
2  int n, m, v[N], ans[N], nowans;
3  bool cmp(const Q &a, const Q &b) {
4      if (a.sqrtl != b.sqrtl) return a.sqrtl < b.sqrtl;
5      return a.r < b.r;
6  }
7  void change(int x) { if (!v[x]) checkin(); else checkout(); }
8  int main() {
9      ......
10     for (int i=1;i<=m;i++) q[i].sqrtl = q[i].l / sqrt(n), q[i].id = i;
11     sort(q+1, q+m+1, cmp);
12     int L=1,R=0; nowans=0;
13     memset(v, 0, sizeof(v));
14     for (int i=1;i<=m;i++) {
15         while (L<q[i].l) change(L++);
16         while (L>q[i].l) change(--L);
17         while (R<q[i].r) change(++R);
18         while (R>q[i].r) change(R--);
19         ans[q[i].id] = nowans;
20     }
21     ......
22 }
```

1.2 树上莫队

```
1  struct Query { int l, r, id, l_group; } query[N];
2  struct EDGE { int adj, next; } edge[N*2];
3  int n, m, top, gh[N], c[N], reorder[N], deep[N], father[N], size[N], son[N], Top[N];
4  void addedge(int x, int y) {
5      edge[++top].adj = y;
6      edge[top].next = gh[x];
7      gh[x] = top;
8  }
9  void dfs(int x, int root=0) {
10     reorder[x] = ++top; father[x] = root; deep[x] = deep[root] + 1;
11     son[x] = 0; size[x] = 1; int dd = 0;
12     for (int p=gh[x]; p; p=edge[p].next)
13         if (edge[p].adj != root) {
14             dfs(edge[p].adj, x);
15             if (size[edge[p].adj] > dd) {
16                 son[x] = edge[p].adj;
17                 dd = size[edge[p].adj];
18             }
19             size[x] += size[edge[p].adj];
20         }
```

```
21  }
22  void split(int x, int tp) {
23      Top[x] = tp;
24      if (son[x]) split(son[x], tp);
25      for (int p=gh[x]; p; p=edge[p].next)
26          if (edge[p].adj != father[x] && edge[p].adj != son[x])
27              split(edge[p].adj, edge[p].adj);
28  }
29  int lca(int x, int y) {
30      int tx = Top[x], ty = Top[y];
31      while (tx != ty) {
32          if (deep[tx] < deep[ty]) {
33              swap(tx, ty);
34              swap(x, y);
35          }
36          x = father[tx];
37          tx = Top[x];
38      }
39      if (deep[x] < deep[y]) swap(x, y);
40      return y;
41  }
42  bool cmp(const Query &a, const Query &b) {
43      if (a.l_group != b.l_group) return a.l_group < b.l_group;
44      return reorder[a.r] < reorder[b.r];
45  }
46  int v[N], ans[N];
47  void upd(int x) { if (!v[x]) checkin(); else checkout(); }
48  void go(int &u, int taru, int v) {
49      int lca0 = lca(u, taru);
50      int lca1 = lca(u, v);    upd(lca1);
51      int lca2 = lca(taru, v); upd(lca2);
52      for (int x=u; x!=lca0; x=father[x]) upd(x);
53      for (int x=taru; x!=lca0; x=father[x]) upd(x);
54      u = taru;
55  }
56  int main() {
57      memset(gh, 0, sizeof(gh));
58      scanf("%d%d", &n, &m); top = 0;
59      for (int i=1;i<n;i++) {
60          int x,y; scanf("%d%d", &x, &y);
61          addedge(x, y); addedge(y, x);
62      }
63      top = 0; dfs(1); split(1, 1);
64      for (int i=1;i<=m;i++) {
65          if (reorder[query[i].l] > reorder[query[i].r])
66              swap(query[i].l, query[i].r);
67          query[i].id = i;
68          query[i].l_group = reorder[query[i].l] / sqrt(n);
69      }
70      sort(query+1, query+m+1, cmp);
```

```
71    int L=1,R=1; upd(1);
72    for (int i=1;i<=m;i++) {
73        go(L,query[i].l,R);
74        go(R,query[i].r,L);
75        ans[query[i].id] = answer();
76    }
77    ......
78 }
```

2 字符串

2.1 哈希

```
1 const int P=31,D=1000173169;
2 int n, pow[N], f[N]; char a[N];
3 int hash(int l, int r) { return (LL)(f[r]-(LL)f[l-1]*pow[r-l+1]%D+D)%D; }
4 int main() {
5     scanf("%d%s", &n, a+1);
6     pow[0] = 1;
7     for (int i=1;i<=n;i++) pow[i] = (LL)pow[i-1]*P%D;
8     for (int i=1;i<=n;i++) f[i] = (LL)((LL)f[i-1]*P+a[i])%D;
9 }
```

2.2 KMP

接口：int find_substring(char *pattern, char *text, int *next, int *ret);

输入：模式串，匹配串

输出：返回值表示模式串在匹配串中出现的次数

KMP 的 next[i] 表示从 0 到 i 的字符串 s，前缀和后缀的最长重叠长度。

```
1 void find_next(char *pattern, int *next) {
2     int n = strlen(pattern);
3     for (int i=1;i<n;i++) {
4         int j = i;
5         while (j > 0) {
6             j = next[j];
7             if (pattern[j] == pattern[i]) {
8                 next[i+1] = j+1;
9                 break;
10            }
11        }
12    }
13 }
14 int find_substring(char *pattern, char *text, int *next, int *ret) {
15     find_next(pattern, next);
16     int n = strlen(pattern);
17     int m = strlen(text);
```

```
18    int k = 0;
19    for (int i=0,j=0;i<m;i++) {
20        if (j<n && text[i]==pattern[j]) {
21            j++;
22        } else {
23            while (j>0) {
24                j = next[j];
25                if (text[i] == pattern[j]) {
26                    j++;
27                    break;
28                }
29            }
30        }
31        if (j == n)
32            ret[k++] = i-n+1;
33    }
34    return k;
35 }
```

2.3 可动态修改的 KMP

支持：加入一个字符，删除一个字符。

时间复杂度：$O(n\alpha)$，α 为字符集大小。

代码中的字符为 $'0'-'9'$，可自行修改为 $'a'-'z'$

```
1  char t[N];
2  int top, nxt[N], nxt_l[N][10];
3  inline void del_letter() { --top; }
4  inline void add_letter(char x) {
5      t[top++] = x;
6      int j = top-1;
7      memset(nxt_l[top], 0, sizeof(nxt_l[top]));
8      nxt[top] = nxt_l[top-1][x-'0'];
9      memcpy(nxt_l[top], nxt_l[nxt[top]], sizeof(nxt_l[nxt[top]]));
10     nxt_l[top][t[nxt[top]]-'0'] = nxt[top]+1;
11 }
```

2.4 扩展 KMP

接口：void ExtendedKMP(char *a, char *b, int *next, int *ret);

输出：

next: a 关于自己每个后缀的最长公共前缀

ret: a 关于 b 的每个后缀的最长公共前缀

EXKMP 的 next[i] 表示：从 i 到 n-1 的字符串 st 前缀和原串前缀的最长重叠长度。

```
1  void get_next(char *a, int *next) {
2      int i, j, k;
3      int n = strlen(a);
```

```
4      for (j = 0; j+1<n && a[j]==a[j+1];j++);
5      next[1] = j;
6      k = 1;
7      for (i=2;i<n;i++) {
8          int len = k+next[k], l = next[i-k];
9          if (l < len-i) {
10             next[i] = l;
11         } else {
12             for (j = max(0, len-i);i+j<n && a[j]==a[i+j];j++);
13             next[i] = j;
14             k = i;
15         }
16     }
17 }
18 void ExtendedKMP(char *a, char *b, int *next, int *ret) {
19     get_next(a, next);
20     int n = strlen(a), m = strlen(b);
21     int i, j, k;
22     for (j=0;j<n && j<m && a[j]==b[j];j++);
23     ret[0] = j;
24     k = 0;
25     for (i=1;i<m;i++) {
26         int len = k+ret[k], l = next[i-k];
27         if (l < len-i) {
28             ret[i] = l;
29         } else {
30             for (j = max(0, len-i);j<n && i+j<m && a[j]==b[i+j];j++);
31             ret[i] = j;
32             k = i;
33         }
34     }
35 }
```

2.5 Manacher

p[i] 表示以 i 为对称轴的最长回文串长度

```
1  char st[N*2], s[N];
2  int len, p[N*2];
3
4  while (scanf("%s", s) != EOF) {
5      len = strlen(s);
6      st[0] = '$', st[1] = '#';
7      for (int i=1;i<=len;i++)
8          st[i*2] = s[i-1], st[i*2+1] = '#';
9      len = len * 2 + 2;
10     int mx = 0, id = 0, ans = 0;
11     for (int i=1;i<=len;i++) {
12         p[i] = (mx > i) ? min(p[id*2-i]+1, mx-i) : 1;
13         for (; st[i+p[i]] == st[i-p[i]]; ++p[i]) ;
```

```
14          if (p[i]+i > mx) mx = p[i]+i, id = i;
15          p[i] --;
16          if (p[i] > ans) ans = p[i];
17      }
18      printf("%d\n", ans);
19  }
```

2.6 最小表示法

```
1   string smallestRepresation(string s) {
2       int i, j, k, l;
3       int n = s.length();
4       s += s;
5       for (i=0,j=1;j<n;) {
6           for (k=0;k<n && s[i+k]==s[j+k];k++);
7           if (k>=n) break;
8           if (s[i+k]<s[j+k]) j+=k+1;
9           else {
10              l=i+k;
11              i=j;
12              j=max(l,j)+1;
13          }
14      }
15      return s.substr(i, n);
16  }
```

2.7 AC 自动机

```
1   struct Node {
2       int next[**Size of Alphabet**];
3       int terminal, fail;
4   } node[**Number of Nodes**];
5   int top;
6   void add(char *st) {
7       int len = strlen(st), x = 1;
8       for (int i=0;i<len;i++) {
9           int ind = trans(st[i]);
10          if (!node[x].next[ind])
11              node[x].next[ind] = ++top;
12          x = node[x].next[ind];
13      }
14      node[x].terminal = 1;
15  }
16  int q[**Number of Nodes**], head, tail;
17  void build() {
18      head = 0, tail = 1; q[1] = 1;
19      while (head != tail) {
20          int x = q[++head];
```

```
21      /*(when necessary) node[x].terminal |= node[node[x].fail].terminal; */
22      for (int i=0;i<n;i++)
23          if (node[x].next[i]) {
24              if (x == 1) node[node[x].next[i]].fail = 1;
25              else {
26                  int y = node[x].fail;
27                  while (y) {
28                      if (node[y].next[i]) {
29                          node[node[x].next[i]].fail = node[y].next[i];
30                          break;
31                      }
32                      y = node[y].fail;
33                  }
34                  if (!node[node[x].next[i]].fail) node[node[x].next[i]].fail = 1;
35              }
36              q[++tail] = node[x].next[i];
37          }
38      }
39  }
```

2.8 后缀数组

2.8.1 倍增算法

参数 m 表示字符集的大小，即 $0 \le r_i < m$

```
1   #define rank rank2
2   int n, r[N], wa[N], wb[N], ws[N], sa[N], rank[N], height[N];
3   int cmp(int *r, int a, int b, int l, int n) {
4       if (r[a]==r[b]) {
5           if (a+l<n && b+l<n && r[a+l]==r[b+l])
6               return 1;
7       }
8       return 0;
9   }
10  void suffix_array(int m) {
11      int i, j, p, *x=wa, *y=wb, *t;
12      for (i=0;i<m;i++) ws[i]=0;
13      for (i=0;i<n;i++) ws[x[i]=r[i]]++;
14      for (i=1;i<m;i++) ws[i]+=ws[i-1];
15      for (i=n-1;i>=0;i--) sa[--ws[x[i]]]=i;
16      for (j=1,p=1;p<n;m=p,j<<=1) {
17          for (p=0,i=n-j;i<n;i++) y[p++]=i;
18          for (i=0;i<n;i++) if (sa[i]>=j) y[p++]=sa[i]-j;
19          for (i=0;i<m;i++) ws[i]=0;
20          for (i=0;i<n;i++) ws[x[y[i]]]++;
21          for (i=1;i<m;i++) ws[i]+=ws[i-1];
22          for (i=n-1;i>=0;i--) sa[--ws[x[y[i]]]]=y[i];
23          for (t=x,x=y,y=t,x[sa[0]]=0,i=1,p=1;i<n;i++)
24              x[sa[i]]=cmp(y,sa[i-1],sa[i],j,n)?p-1:p++;
```

```
25            }
26       for (i=0;i<n;i++) rank[sa[i]]=i;
27       rank[n] = -1;
28   }
29   void calc_height() {
30       int j=0;
31       for (int i=0;i<n;i++)
32           if (rank[i])
33           {
34               while (r[i+j]==r[sa[rank[i]-1]+j]) j++;
35               height[rank[i]]=j;
36               if (j) j--;
37           }
38   }
```

2.8.2 DC3 算法

注意:

N 至少为字符串长度的 3 倍

接口: suffix_array(int *r, int *sa, int n, int m);

r 表示字符串, sa 为后缀数组输出, n 表示字符串长度, 下标从 0 开始。m 为字符集大小。

```
1    #define F(x) ((x)/3 + ((x)%3 == 1 ? 0:tb))
2    #define G(x) ((x) < tb ? (x)*3+1 : ((x)-tb)*3 + 2)
3    #define rank rank2
4
5    int r[N], wa[N], wb[N], ws[N], wv[N], sa[N], rank[N];
6
7    int c0(int *r,int a,int b) {
8        return r[a]==r[b]&&r[a+1]==r[b+1]&&r[a+2]==r[b+2];
9    }
10
11   int c12(int k,int *r,int a,int b) {
12       if(k==2) return r[a]<r[b]||r[a]==r[b]&&c12(1,r,a+1,b+1);
13       else return r[a]<r[b]||r[a]==r[b]&&wv[a+1]<wv[b+1];
14   }
15
16   void dsort(int *r,int *a,int *b,int n,int m) {
17       int i;for(i=0;i<n;i++) wv[i]=r[a[i]];
18       for(i=0;i<m;i++) ws[i]=0;
19       for(i=0;i<n;i++) ws[wv[i]]++;
20       for(i=1;i<m;i++) ws[i]+=ws[i-1];
21       for(i=n-1;i>=0;i--) b[--ws[wv[i]]]=a[i];
22   }
23
24   void dc3(int *r,int *sa,int n,int m) {
25       int i,j,*rn=r+n,*san=sa+n,ta=0,tb=(n+1)/3,tbc=0,p;
26       r[n]=r[n+1]=0;
27       for(i=0;i<n;i++) if(i%3!=0) wa[tbc++]=i;
```

```
28    dsort(r+2,wa,wb,tbc,m);
29    dsort(r+1,wb,wa,tbc,m);
30    dsort(r,wa,wb,tbc,m);
31    for(p=1,rn[F(wb[0])]=0,i=1;i<tbc;i++) rn[F(wb[i])]=c0(r,wb[i-1],wb[i])?p-1:p++;
32    if(p<tbc) dc3(rn,san,tbc,p);
33    else for(i=0;i<tbc;i++) san[rn[i]]=i;
34    for(i=0;i<tbc;i++) if(san[i]<tb) wb[ta++]=san[i]*3;
35    if(n%3==1) wb[ta++]=n-1;
36    dsort(r,wb,wa,ta,m);
37    for(i=0;i<tbc;i++) wv[wb[i]=G(san[i])]=i;
38    for(i=0,j=0,p=0;i<ta && j<tbc;p++)sa[p]=c12(wb[j]%3,r,wa[i],wb[j])?wa[i++]:wb[j
        ++];
39    for(;i<ta;p++) sa[p]=wa[i++];
40    for(;j<tbc;p++) sa[p]=wb[j++];
41 }
42
43 void suffix_array(int *r, int *sa, int n, int m) {
44    dc3(r, sa, n + 1, m);
45    int top = 0;
46    for (int i = 0; i < n + 1; ++i)
47        if (sa[i] < n) sa[top++] = sa[i];
48    for (int i = 0; i < n; ++i) rank[sa[i]] = i;
49    rank[n] = -1;
50 }
```

2.8.3 小技巧：拼接字符串

接口：

int gao1(int l, int r, int c, int p); 区间 $[l, r)$ 中保证第 0 位到第 $c-1$ 位都是相同的（设为字符串 s ），现在我们在 s 后面接一个字符 p ，得到一个新的字符串 s' 。返回值为最小的 k 满足后缀 $sa[k]$ 前 $c+1$ 位为 s'

int gao2(int l, int r, int c, int p); 区间 $[l, r)$ 中保证第 0 位到第 $c-1$ 位都是相同的（设为字符串 s ），现在我们在 s 后面接一个后缀 $sa[p]$ ，得到一个新的字符串 s' 。返回值为最小的 k 满足后缀 $sa[k]$ 前 $c+len(sa[p])$ 位为 s'

```
1  int gao1(int l,int r,int c,int p) {
2      --l;
3      while (l+1<r) {
4          int md=(l+r)>>1;
5          if (sa[md]+c<n&&s[sa[md]+c]>=p) r=md; else l=md;
6      }
7      return r;
8  }
9  int gao2(int l,int r,int c,int p) {
10     --l;
11     while (l+1<r) {
12         int md=(l+r)>>1;
13         if (sa[md]+c<=n&&rk[sa[md]+c]>=p) r=md; else l=md;
14     }
```

```
15        return r;
16  }
```

示例调用：

```
1   suf1[m] = -1, suf2[m] = n;
2   for (int i = m - 1; i >= 0; --i) {
3       int l = gao1(0, n, 0, t[i]), r = gao1(0, n, 0, t[i]);
4       suf1[i] = gao2(l, r, 1, suf1[i + 1]);
5       suf2[i] = gao2(l, r, 1, suf2[i + 1]);
6   }
```

2.9 后缀自动机

下面的代码是求两个串的 LCS（最长公共子串）。

```
1   #include <cstdio>
2   #include <cstdlib>
3   #include <cstring>
4   #define N 500001
5   using namespace std;
6   char st[N];
7   int pre[N<<1], son[26][N<<1], step[N<<1], refer[N<<1], last, total;
8   int apply(int x, int now) { step[++total]=x; refer[total] = now; return total; }
9   void extend(char x, int now) {
10      int p = last, np = apply(step[last]+1, now);
11      for (; p && !son[x][p]; p=pre[p]) son[x][p] = np;
12      if (!p) pre[np] = 1;
13      else {
14          int q = son[x][p];
15          if (step[p]+1 == step[q]) pre[np] = q;
16          else {
17              int nq = apply(step[p]+1, now);
18              for (int i=0;i<26;i++) son[i][nq] = son[i][q];
19              pre[nq] = pre[q];
20              pre[q] = pre[np] = nq;
21              for (; p && son[x][p]==q; p=pre[p]) son[x][p] = nq;
22          }
23      }
24      last = np;
25  }
26  void init() {
27      last = total = 0;
28      last = apply(0, 0);
29      scanf("%s",st);
30      for (int i=0; st[i]; i++)
31          extend(st[i]-'a', i);
32      scanf("%s",st);
33  }
34  int main() {
```

```
35      init();
36      int p = 1, now = 0, ans = 0;
37      for (int i=0; st[i]; i++) {
38          int index = st[i]-'a';
39          for (; p && !son[index][p]; p = pre[p], now = step[p]) ;
40          if (!p) p = 1;
41          if (son[index][p]) {
42              p = son[index][p];
43              now++;
44              if (now > ans) ans = now;
45          }
46      }
47      printf("%d\n",ans);
48      return 0;
49  }
```

一些定义和性质:

Right(str) 表示 str 在母串 S 中所有出现的结束位置集合

一个状态 s 表示的所有子串 Right 集合相同,为 Right(s)

Parent(s) 满足 Right(s) 是 Right(Parent(s)) 的真子集,并且 Right(Parent(s)) 的大小最小

Parent 函数可以表示一个树形结构。不妨叫它 Parent 树

一个 Right 集合和一个长度定义了一个子串

对于状态 s ,使得 Right(s) 合法的子串长度是一个区间 [min(s), max(s)]

$max(Parent(s)) = min(s) - 1$

令 refer(s) 表示产生 s 状态的字符所在位置。则 Right(s) 的合法子串的起始位置为 [refer(s) - max(s) + 1, refer(s) - min(s) + 1] ,即 [refer(s) - max(s) + 1, refer(s) - max(Parent(s))]

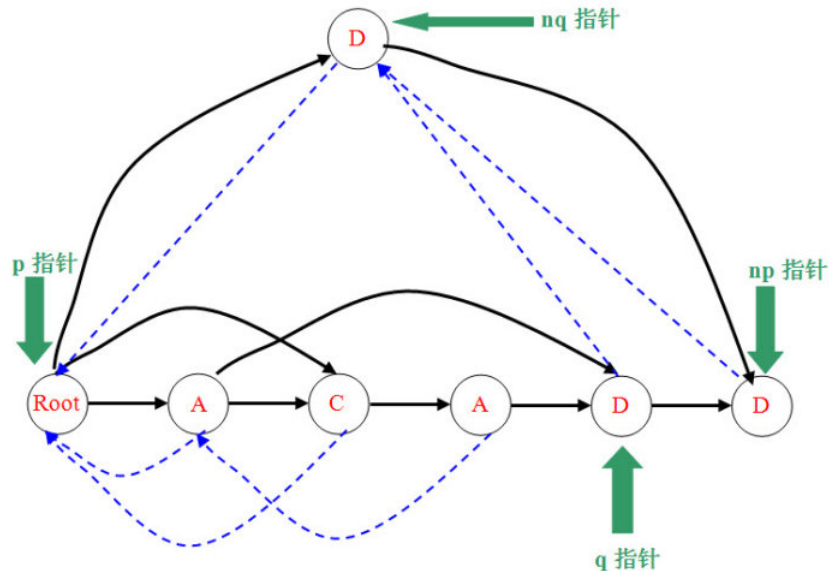

图 1: ACADD 构成的后缀自动机

我们发现 fail 构出一棵前缀树

和后缀树相同,为了使每个前缀都是叶子结点,我们不妨在串 s 前加入一个没出现的字符'#'

14
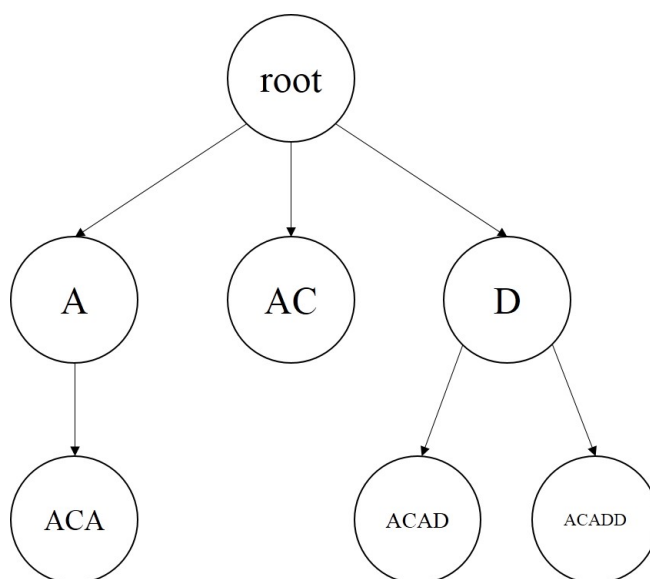
图 2: 串 ACADD 按 fail 构出的前缀树，与图 1 对应
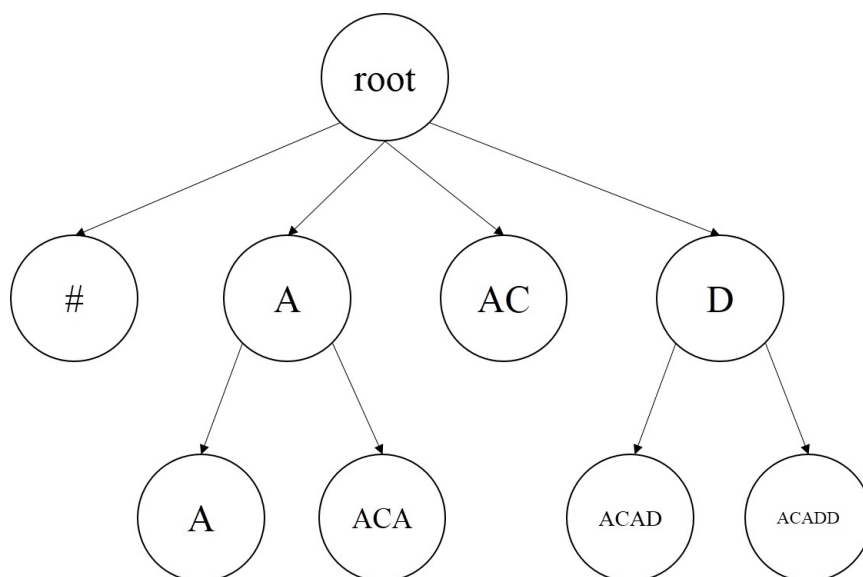
图 3: 串 #ACADD 按 fail 构出的前缀树

3 数据结构

3.1 ST 表

```
1  int Log[N],f[17][N];
2  int ask(int x,int y){
3      int k=Log[y-x+1];
4      return max(f[k][x],f[k][y-(1<<k)+1]);
5  }
6  int main(){
7      for (int i=2;i<=n;i++)Log[i]=Log[i>>1]+1;
8      for (int j=1;j<K;j++)
9          for (int i=1;i+(1<<j-1)<=n;i++)
10             f[j][i]=max(f[j-1][i],f[j-1][i+(1<<j-1)]);
11 }
```

3.2 左偏树

左偏树是一个可并堆。

下面的程序写的是一个小根堆，如果需要改成大根堆请在注释了 here 那行修改。

接口：

void push(const T &x); 插入一个元素。

void merge(leftist &x); 合并两个堆。注意，合并后原来那个堆将不可访问。

T top() const; 返回堆顶元素。

void pop(); 删除堆顶元素。

int size() const; 返回堆的大小。

```
1  template <class T>
2  class leftist {
3  public:
4      struct node {
5          T key;
6          int dist;
7          node *l, *r;
8      };
9      leftist() : root(NULL), s(0) {}
10     void push(const T &x) {
11         leftist y;
12         y.s = 1;
13         y.root = new node;
14         y.root -> key = x;
15         y.root -> dist = 0;
16         y.root -> l = y.root -> r = NULL;
17         merge(y);
18     }
19     node* merge(node *x, node *y) {
20         if (x == NULL) return y;
21         if (y == NULL) return x;
```

```
22          if (y -> key < x -> key) swap(x, y); //here
23          x -> r = merge(x -> r, y);
24          int ld = x -> l ? x -> l -> dist : -1;
25          int rd = x -> r ? x -> r -> dist : -1;
26          if (ld < rd) swap(x -> l, x -> r);
27          if (x -> r == NULL) x -> dist = 0;
28          else x -> dist = x -> r -> dist + 1;
29          return x;
30      }
31      void merge(leftist &x) {
32          root = merge(root, x.root);
33          s += x.s;
34      }
35      T top() const {
36          if (root == NULL) return T();
37          return root -> key;
38      }
39      void pop() {
40          if (root == NULL) return;
41          node *p = root;
42          root = merge(root -> l, root -> r);
43          --s;
44          delete p;
45      }
46      int size() const {
47          return s;
48      }
49 private:
50      node* root;
51      int s;
52 };
```

3.3 线段树小技巧

给定一个序列 a，寻找一个最大的 i 使得 $i \le y$ 且满足一些条件（如 $a[i] \ge w$，那么需要在线段树维护 a 的区间最大值）

```
1  int queryl(int p, int left, int right, int y, int w) {
2      if (right <= y) {
3          if (! __condition__ ) return -1;
4          else if (left == right) return left;
5      }
6      int mid = (left + right) / 2;
7      if (y <= mid) return queryl(p<<1|0, left, mid, y, w);
8      int ret = queryl(p<<1|1, mid+1, right, y, w);
9      if (ret != -1) return ret;
10     return queryl(p<<1|0, left, mid, y, w);
11 }
```

给定一个序列 a，寻找一个最小的 i 使得 $i \ge x$ 且满足一些条件（如 $a[i] \ge w$，那么需要在线段树

维护 a 的区间最大值）

```c
int queryr(int p, int left, int right, int x, int w) {
    if (left >= x) {
        if (! __condition__ ) return -1;
        else if (left == right) return left;
    }
    int mid = (left + right) / 2;
    if (x > mid) return queryr(p<<1|1, mid+1, right, x, w);
    int ret = queryr(p<<1|0, left, mid, x, w);
    if (ret != -1) return ret;
    return queryr(p<<1|1, mid+1, right, x, w);
}
```

3.4 Splay

接口：

ADD x y d： 将 $[x,y]$ 的所有数加上 d

REVERSE x y： 将 $[x,y]$ 翻转

INSERT x p： 将 p 插入到第 x 个数的后面

DEL x： 将第 x 个数删除

```c
struct SPLAY {
    struct NODE {
        int w, min;
        int son[2], size, father, rev, lazy;
    } node[N];
    int top, rt;
    void pushdown(int x) {
        if (!x) return;
        if (node[x].rev) {
            node[node[x].son[0]].rev ^= 1;
            node[node[x].son[1]].rev ^= 1;
            swap(node[x].son[0], node[x].son[1]);
            node[x].rev = 0;
        }
        if (node[x].lazy) {
            node[node[x].son[0]].lazy += node[x].lazy;
            node[node[x].son[1]].lazy += node[x].lazy;
            node[x].w += node[x].lazy;
            node[x].min += node[x].lazy;
            node[x].lazy = 0;
        }
    }
    void pushup(int x) {
        if (!x) return;
        pushdown(node[x].son[0]);
        pushdown(node[x].son[1]);
        node[x].size = node[node[x].son[0]].size + node[node[x].son[1]].size + 1;
        node[x].min = node[x].w;
```

```
29          if (node[x].son[0]) node[x].min = min(node[x].min, node[node[x].son[0]].min)
                ;
30          if (node[x].son[1]) node[x].min = min(node[x].min, node[node[x].son[1]].min)
                ;
31      }
32      void sc(int x, int y, int w) {
33          node[x].son[w] = y;
34          node[y].father = x;
35          pushup(x);
36      }
37      void _ins(int w) {
38          top++;
39          node[top].w = node[top].min = w;
40          node[top].son[0] = node[top].son[1] = 0;
41          node[top].size = 1; node[top].father = 0; node[top].rev = 0;
42      }
43      void init() {
44          top = 0;
45          _ins(0); _ins(0); rt=1;
46          sc(1, 2, 1);
47      }
48      void rotate(int x) {
49          if (!x) return;
50          int y = node[x].father;
51          int w = node[y].son[1]==x;
52          sc(y, node[x].son[w^1], w);
53          sc(node[y].father, x, node[node[y].father].son[1]==y);
54          sc(x, y, w^1);
55      }
56      int q[N];
57      void flushdown(int x) {
58          int t=0; for (; x; x=node[x].father) q[++t]=x;
59          for (; t; t--) pushdown(q[t]);
60      }
61      void Splay(int x, int root=0) {
62          flushdown(x);
63          while (node[x].father != root) {
64              int y=node[x].father;
65              int w=node[y].son[1]==x;
66              if (node[y].father != root && node[node[y].father].son[w]==y) rotate(y);
67              rotate(x);
68          }
69      }
70      int find(int k) {
71          Splay(rt);
72          while (1) {
73              pushdown(rt);
74              if (node[node[rt].son[0]].size+1==k) {
75                  Splay(rt);
76                  return rt;
```

```
77              } else
78              if (node[node[rt].son[0]].size+1<k) {
79                  k-=node[node[rt].son[0]].size+1;
80                  rt=node[rt].son[1];
81              } else {
82                  rt=node[rt].son[0];
83              }
84          }
85      }
86      int split(int x, int y) {
87          int fx = find(x);
88          int fy = find(y+2);
89          Splay(fx);
90          Splay(fy, fx);
91          return node[fy].son[0];
92      }
93      void add(int x, int y, int d) { //add d to each number in a[x]...a[y]
94          int t = split(x, y);
95          node[t].lazy += d;
96          Splay(t); rt=t;
97      }
98      void reverse(int x, int y) { // reverse the x-th to y-th elements
99          int t = split(x, y);
100         node[t].rev ^= 1;
101         Splay(t); rt=t;
102     }
103     void insert(int x, int p) { // insert p after the x-th element
104         int fx = find(x+1);
105         int fy = find(x+2);
106         Splay(fx);
107         Splay(fy, fx);
108         _ins(p);
109         sc(fy, top, 0);
110         Splay(top); rt=top;
111     }
112     void del(int x) { // delete the x-th element in Splay
113         int fx = find(x), fy = find(x+2);
114         Splay(fx); Splay(fy, fx);
115         node[fy].son[0] = 0;
116         Splay(fy); rt=fy;
117     }
118 } tree;
```

3.5 可持久化 Treap

接口：

void insert(int x, char c); 在当前第 x 个字符后插入 c

void del(int x, int y); 删除第 x 个字符到第 y 个字符

void copy(int l, int r, int x); 复制第 l 个字符到第 r 个字符，然后粘贴到第 x 个字符后

void reverse(int x, int y); 翻转第 x 个到第 y 个字符

char query(int k); 表示询问当前第 x 个字符是什么

```
1   #define mod 1000000007
2   struct Treap {
3       struct Node {
4           char key;
5           bool reverse;
6           int lc, rc, size;
7       } node[N];
8       int n, root, rd;
9       int Rand() { rd = (rd * 20372052LL + 25022087LL) % mod; return rd; }
10      void init() { n = root = 0; }
11      inline int copy(int x) { node[++n] = node[x]; return n; }
12      inline void pushdown(int x) {
13          if (!node[x].reverse) return;
14          if (node[x].lc) node[x].lc = copy(node[x].lc);
15          if (node[x].rc) node[x].rc = copy(node[x].rc);
16          swap(node[x].lc, node[x].rc);
17          node[node[x].lc].reverse ^= 1;
18          node[node[x].rc].reverse ^= 1;
19          node[x].reverse = 0;
20      }
21      inline void pushup(int x) { node[x].size = node[node[x].lc].size + node[node[x].
            rc].size + 1; }
22      int merge(int u, int v) {
23          if (!u || !v) return u+v;
24          pushdown(u); pushdown(v);
25          int t = Rand() % (node[u].size + node[v].size), r;
26          if (t < node[u].size) {
27              r = copy(u);
28              node[r].rc = merge(node[u].rc, v);
29          } else {
30              r = copy(v);
31              node[r].lc = merge(u, node[v].lc);
32          }
33          pushup(r);
34          return r;
35      }
36      int split(int u, int x, int y) {
37          if (x > y) return 0;
38          pushdown(u);
39          if (x == 1 && y == node[u].size) return u;
40          if (y <= node[node[u].lc].size) return split(node[u].lc, x, y);
41          int t = node[node[u].lc].size + 1;
42          if (x > t) return split(node[u].rc, x-t, y-t);
43          int num = copy(u);
44          node[num].lc = split(node[u].lc, x, t-1);
45          node[num].rc = split(node[u].rc, 1, y-t);
46          pushup(num);
```

```
47          return num;
48      }
49      void insert(int x, char c) {
50          int t1 = split(root, 1, x), t2 = split(root, x+1, node[root].size);
51          node[++n].key = c; node[n].size = 1;
52          root = merge(merge(t1, n), t2);
53      }
54      void del(int x, int y) {
55          int t1 = split(root, 1, x-1), t2 = split(root, y+1, node[root].size);
56          root = merge(t1, t2);
57      }
58      void copy(int l, int r, int x) {
59          int t1 = split(root, 1, x), t2 = split(root, l, r), t3 = split(root, x+1,
                  node[root].size);
60          root = merge(merge(t1, t2), t3);
61      }
62      void reverse(int x, int y) {
63          int t1 = split(root, 1, x-1), t2 = split(root, x, y), t3 = split(root, y+1,
                  node[root].size);
64          node[t2].reverse ^= 1;
65          root = merge(merge(t1, t2), t3);
66      }
67      char query(int k) {
68          int x = root;
69          while (1) {
70              pushdown(x);
71              if (k <= node[node[x].lc].size) x = node[x].lc;
72              else
73              if (k == node[node[x].lc].size + 1) return node[x].key;
74              else
75              k -= node[node[x].lc].size + 1, x = node[x].rc;
76          }
77      }
78 } treap;
```

3.6 可持久化并查集

接口：

void init() 初始化

void merge(int x, int y, int time) 在 time 时刻将 x 和 y 连一条边，注意加边顺序必须按 time 从小到大加边

void GetFather(int x, int time) 询问 time 时刻及以前的连边状态中，x 所属的集合

```
1 namespace pers_union {
2      const int inf = 0x3f3f3f3f;
3      int father[N], Father[N], Time[N];
4      vector<int> e[N];
5      void init() {
6          for (int i=1;i<=n;i++) {
```

```
 7            father[i] = i;
 8            Father[i] = i;
 9            Time[i] = inf;
10            e[i].clear();
11            e[i].push_back(i);
12        }
13    }
14    int getfather(int x) {
15        return (father[x] == x) ? x : father[x] = getfather(father[x]);
16    }
17    int GetFather(int x, int time) {
18        return (Time[x] <= time) ? GetFather(Father[x], time) : x;
19    }
20    void merge(int x, int y, int time) {
21        int fx = getfather(x), fy = getfather(y);
22        if (fx == fy) return;
23        if (e[fx].size() > e[fy].size()) swap(fx, fy);
24        father[fx] = fy;
25        Father[fx] = fy;
26        Time[fx] = time;
27        for (int i=0;i<e[fx].size();i++) {
28            e[fy].push_back(e[fx][i]);
29        }
30    }
31 };
```

4 树

4.1 点分治

初始化时须设置 $top = 1$ 。

```
 1 void addedge(int x, int y) {
 2     edge[++top].adj = y;
 3     edge[top].valid = 1;
 4     edge[top].next = gh[x];
 5     gh[x] = top;
 6 }
 7 void get_size(int x, int root=0) {
 8     size[x] = 1; son[x] = 0;
 9     int dd = 0;
10     for (int p=gh[x]; p; p=edge[p].next)
11         if (edge[p].adj != root && edge[p].valid) {
12             get_size(edge[p].adj, x);
13             size[x] += size[edge[p].adj];
14             if (size[edge[p].adj] > dd) {
15                 dd = size[edge[p].adj];
16                 son[x] = edge[p].adj;
17             }
```

```
18          }
19  }
20  int getroot(int x) {
21      get_size(x);
22      int sz = size[x];
23      while (size[son[x]] > sz/2)
24          x = son[x];
25      return x;
26  }
27  void dc(int x) {
28      x = getroot(x);
29      static int list[N], ltop;
30      ltop = 0;
31      for (int p=gh[x]; p; p=edge[p].next)
32          if (edge[p].valid)
33              list[++ltop] = p;
34      clear();
35      for (int i=1;i<=ltop;i++) {
36          update();
37          modify();
38      }
39      clear();
40      for (int i=ltop;i>=1;i--) {
41          update();
42          modify();
43      }
44      //be careful about the root
45      for (int p=gh[x]; p; p=edge[p].next)
46          if (edge[p].valid) {
47              edge[p].valid = 0;
48              edge[p^1].valid = 0;
49              dc(edge[p].adj);
50          }
51  }
```

4.2   Link Cut Tree

接口：

command(x, y)：将 x 到 y 路径的 Splay Tree 分离出来。

linkcut(u1, v1, u2, v2)：将树中原有的边 (u1, v1) 删除，加入一条新边 (u2, v2)

```
1  struct DynamicTREE{
2      struct NODE{
3          int father, son[2], top, size, reverse;
4      } splay[N];
5      void init(int i, int fat) {
6          splay[i].father = splay[i].son[0] = splay[i].son[1] = 0;
7          splay[i].top = fat; splay[i].size = 1; splay[i].reverse = 0;
8      }
```

```
 9    void pushdown(int x) {
10        if (!x) return;
11        int s0 = splay[x].son[0], s1 = splay[x].son[1];
12        if (splay[x].reverse) {
13            splay[s0].reverse ^= 1;
14            splay[s1].reverse ^= 1;
15            swap(splay[x].son[0], splay[x].son[1]);
16            splay[x].reverse = 0;
17        }
18        s0 = splay[x].son[0], s1 = splay[x].son[1];
19        splay[s0].top = splay[s1].top = splay[x].top;
20    }
21    void pushup(int x) {
22        if (!x) return;
23        pushdown(splay[x].son[0]);
24        pushdown(splay[x].son[1]);
25        splay[x].size = splay[splay[x].son[0]].size + splay[splay[x].son[1]].size +
                1;
26    }
27    void sc(int x, int y, int w, bool Auto=true) {
28        splay[x].son[w] = y;
29        splay[y].father = x;
30        if (Auto) {
31            pushup(y);
32            pushup(x);
33        }
34    }
35    int top, tush[N];
36    void flowdown(int x) {
37        for (top=1; x; top++, x = splay[x].father) tush[top] = x;
38        for (; top; top--) pushdown(tush[top]);
39    }
40    void rotate(int x) {
41        if (!x) return;
42        int y = splay[x].father;
43        int w = splay[y].son[1] == x;
44        pushdown(y);
45        pushdown(x);
46        sc(splay[y].father, x, splay[splay[y].father].son[1]==y, false);
47        sc(y, splay[x].son[w^1], w, false);
48        sc(x, y, w^1, false);
49        pushup(y);
50        pushup(x);
51    }
52    void Splay(int x, int rt=0) {
53        if (!x) return;
54        flowdown(x);
55        while (splay[x].father != rt) {
56            int y = splay[x].father;
57            int w = splay[y].son[1]==x;
```

```
58          if (splay[y].father != rt && splay[splay[y].father].son[w] == y) rotate(
                y);
59          rotate(x);
60      }
61  }
62  void split(int x) {
63      int y = splay[x].son[1];
64      if (!y) return;
65      splay[y].father = 0;
66      splay[x].son[1] = 0;
67      splay[y].top = x;
68      pushup(x);
69  }
70  void access(int x) {
71      int y = 0;
72      while (x) {
73          Splay(x);
74          split(x);
75          sc(x, y, 1);
76          Splay(x);
77          y = x;
78          x = splay[x].top;
79      }
80  }
81  void changeroot(int x) {
82      access(x);
83      Splay(x);
84      splay[x].reverse = 1;
85      Splay(x);
86  }
87  void command(int x, int y, ...) {
88      LL ans = 0;
89      changeroot(x);
90      access(y);
91      Splay(x);
92      //then you can modify the Splay Tree
93  }
94  void linkcut(int u1, int v1, int u2, int v2) {
95      changeroot(u1);
96      access(v1);
97      Splay(u1); split(u1);
98      splay[v1].top = 0;
99      access(u2); changeroot(u2);
100     access(v2); changeroot(v2);
101     Splay(u2); Splay(v2);
102     splay[v2].top = u2;
103 }
104 } lct;
```

5 图

5.1 欧拉回路

欧拉回路:

无向图: 每个顶点的度数都是偶数, 则存在欧拉回路。

有向图: 每个顶点的入度 = 出度, 则存在欧拉回路。

欧拉路径:

无向图: 当且仅当该图所有顶点的度数为偶数, 或者除了两个度数为奇数外其余的全是偶数。

有向图: 当且仅当该图所有顶点出度 = 入度或者一个顶点出度 = 入度 + 1, 另一个顶点入度 = 出度 + 1, 其他顶点出度 = 入度。

下面 $O(n+m)$ 求欧拉回路的代码中, n 为点数, m 为边数, 若有解则依次输出经过的边的编号, 若是无向图, 则正数表示 x 到 y, 负数表示 y 到 x。

```
 1  namespace UndirectedGraph{
 2      int n,m,i,x,y,d[N],g[N],v[M<<1],w[M<<1],vis[M<<1],nxt[M<<1],ed;
 3      int ans[M],cnt;
 4      void add(int x,int y,int z){
 5          d[x]++;
 6          v[++ed]=y;w[ed]=z;nxt[ed]=g[x];g[x]=ed;
 7      }
 8      void dfs(int x){
 9          for(int&i=g[x];i;){
10              if(vis[i]){i=nxt[i];continue;}
11              vis[i]=vis[i^1]=1;
12              int j=w[i];
13              dfs(v[i]);
14              ans[++cnt]=j;
15          }
16      }
17      void solve(){
18          scanf("%d%d",&n,&m);
19          for(i=ed=1;i<=m;i++)scanf("%d%d",&x,&y),add(x,y,i),add(y,x,-i);
20          for(i=1;i<=n;i++)if(d[i]&1){puts("NO");return;}
21          for(i=1;i<=n;i++)if(g[i]){dfs(i);break;}
22          for(i=1;i<=n;i++)if(g[i]){puts("NO");return;}
23          puts("YES");
24          for(i=m;i;i--)printf("%d ",ans[i]);
25      }
26  }
27  namespace DirectedGraph{
28      int n,m,i,x,y,d[N],g[N],v[M],vis[M],nxt[M],ed;
29      int ans[M],cnt;
30      void add(int x,int y){
31          d[x]++;d[y]--;
32          v[++ed]=y;nxt[ed]=g[x];g[x]=ed;
33      }
34      void dfs(int x){
35          for(int&i=g[x];i;){
```

```
36          if(vis[i]){i=nxt[i];continue;}
37          vis[i]=1;
38          int j=i;
39          dfs(v[i]);
40          ans[++cnt]=j;
41      }
42  }
43  void solve(){
44      scanf("%d%d",&n,&m);
45      for(i=1;i<=m;i++)scanf("%d%d",&x,&y),add(x,y);
46      for(i=1;i<=n;i++)if(d[i]){puts("NO");return;}
47      for(i=1;i<=n;i++)if(g[i]){dfs(i);break;}
48      for(i=1;i<=n;i++)if(g[i]){puts("NO");return;}
49      puts("YES");
50      for(i=m;i;i--)printf("%d␣",ans[i]);
51  }
52  }
```

5.2 最短路径

5.2.1 Dijkstra

```cpp
1  #include <queue>
2  using namespace std;
3  struct EDGE { int adj, w, next; } edge[M*2];
4  struct dat { int id, dist; dat(int id=0, int dist=0) : id(id), dist(dist) {} };
5  struct cmp { bool operator () (const dat &a, const dat &b) { return a.dist > b.dist;
       } };
6  priority_queue < dat, vector<dat>, cmp > q;
7  int n, top, gh[N], v[N], dist[N];
8  void addedge(int x, int y, int w) {
9      edge[++top].adj = y;
10     edge[top].w = w;
11     edge[top].next = gh[x];
12     gh[x] = top;
13  }
14  int dijkstra(int s, int t) {
15      memset(dist, 63, sizeof(dist));
16      memset(v, 0, sizeof(v));
17      dist[s] = 0;
18      q.push(dat(s, 0));
19      while (!q.empty()) {
20          dat x = q.top(); q.pop();
21          if (v[x.id]) continue; v[x.id] = 1;
22          for (int p=gh[x.id]; p; p=edge[p].next) {
23              if (x.dist + edge[p].w < dist[edge[p].adj]) {
24                  dist[edge[p].adj] = x.dist + edge[p].w;
25                  q.push(dat(edge[p].adj, dist[edge[p].adj]));
26              }
```

```
27          }
28      }
29      return dist[t];
30  }
```

5.2.2 SPFA

```
1   struct EDGE { int adj, w, next; } edge[M*2];
2   int n,m,top,gh[N],v[N],cnt[N],q[N],dist[N],head,tail;
3   void addedge(int x, int y, int w) {
4       edge[++top].adj = y;
5       edge[top].w = w;
6       edge[top].next = gh[x];
7       gh[x] = top;
8   }
9   int spfa(int S, int T) {
10      memset(v, 0, sizeof(v));
11      memset(cnt, 0, sizeof(cnt));
12      memset(dist, 63, sizeof(dist));
13      head = 0, tail = 1;
14      dist[S] = 0; q[1] = S;
15      while (head != tail) {
16          (head += 1) %= N;
17          int x = q[head]; v[x] = 0;
18          ++cnt[x]; if (cnt[x] > n) return -1;
19          for (int p=gh[x]; p; p=edge[p].next)
20              if (dist[x] + edge[p].w < dist[edge[p].adj]) {
21                  dist[edge[p].adj] = dist[x] + edge[p].w;
22                  if (!v[edge[p].adj]) {
23                      v[edge[p].adj] = 1;
24                      (tail += 1) %= N;
25                      q[tail] = edge[p].adj;
26                  }
27              }
28      }
29      return dist[T];
30  }
```

5.3 K 短路

接口：

kthsp::init(n)：初始化并设置节点个数为 n

kthsp::add(x, y, w)：添加一条 x 到 y 的有向边

kthsp::work(S, T, k)：求 S 到 T 的第 k 短路

```
1   #include <queue>
2
3   #define N 200020
```

```
 4   #define M 400020
 5   #define LOGM 20
 6   #define LL long long
 7   #define inf (1LL<<61)
 8
 9   namespace pheap {
10       struct Node {
11           int next, son[2];
12           LL val;
13       } node[M*LOGM];
14       int LOG[M];
15       int root[M], size[M*LOGM], top;
16       int add() {
17           ++top; assert(top < M*LOGM);
18           node[top].next = node[top].son[0] = node[top].son[1] = 0;
19           node[top].val = inf;
20           return top;
21       }
22       int copy(int x) { int t = add(); node[t] = node[x]; return t; }
23       void init() {
24           top = -1; add();
25           for (int i=2;i<M;i++) LOG[i] = LOG[i>>1] + 1;
26       }
27       void upd(int x, int &next, LL &val) {
28           if (val < node[x].val) {
29               swap(val, node[x].val);
30               swap(next, node[x].next);
31           }
32       }
33       void insert(int x, int next, LL val) {
34           int sz = size[root[x]] + 1;
35           root[x] = copy(root[x]);
36           size[root[x]] = sz; x = root[x];
37           upd(x, next, val);
38           for (int i=LOG[sz]-1;i>=0;i--) {
39               int ind = (sz>>i)&1;
40               node[x].son[ind] = copy(node[x].son[ind]);
41               x = node[x].son[ind];
42               upd(x, next, val);
43           }
44       }
45   };
46
47   namespace kthsp {
48       using namespace pheap;
49       struct EDGE {
50           int adj, w, next;
51       } edge[2][M];
52       struct W {
53           int x, y, w;
```

```
54    } e[M];
55    bool has_init = 0;
56    int n, m, top[2], gh[2][N], v[N];
57    LL dist[N];
58    void init(int n1) {
59        has_init = 1;
60        n = n1; m = 0;
61        memset(top, 0, sizeof(top));
62        memset(gh, 0, sizeof(gh));
63        for (int i=1;i<=n;i++) dist[i] = inf;
64    }
65    void addedge(int id, int x, int y, int w) {
66        edge[id][++top[id]].adj = y;
67        edge[id][top[id]].w = w;
68        edge[id][top[id]].next = gh[id][x];
69        gh[id][x] = top[id];
70    }
71    void add(int x, int y, int w) {
72        assert(has_init);
73        e[++m].x=x; e[m].y=y; e[m].w=w;
74    }
75    int q[N], best[N], bestw[N];
76    int deg[N];
77    void spfa(int S) {
78        for (int i=1;i<=n;i++) deg[i] = 0;
79        for (int i=1;i<=m;i++) deg[e[i].x] ++;
80        int head = 0, tail = 1;
81        dist[S] = 0; q[1] = S;
82        while (head != tail) {
83            (head += 1) %= N;
84            int x = q[head];
85            for (int p=gh[1][x]; p; p=edge[1][p].next) {
86                if (dist[x] + edge[1][p].w < dist[edge[1][p].adj]) {
87                    dist[edge[1][p].adj] = dist[x] + edge[1][p].w;
88                    best[edge[1][p].adj] = x;
89                    bestw[edge[1][p].adj] = p;
90                }
91                if (!--deg[edge[1][p].adj]) {
92                    (tail += 1) %= N;
93                    q[tail] = edge[1][p].adj;
94                }
95            }
96        }
97    }
98    void dfs(int x) {
99        if (v[x]) return; v[x] = 1;
100       if (best[x]) root[x] = root[best[x]];
101       for (int p=gh[0][x]; p; p=edge[0][p].next)
102           if (dist[edge[0][p].adj] != inf && bestw[x] != p) {
103               insert(x, edge[0][p].adj, edge[0][p].w + dist[edge[0][p].adj] - dist
```

```
                              [x]);
104                    }
105            for (int p=gh[1][x]; p; p=edge[1][p].next)
106                if (best[edge[1][p].adj] == x)
107                    dfs(edge[1][p].adj);
108        }
109    typedef pair<LL,int> pli;
110    priority_queue <pli, vector<pli>, greater<pli> > pq;
111    LL work(int S, int T, int k) {
112        assert(has_init);
113        n++; add(T, n, 0);
114        if (S == T) k ++;
115        T = n;
116        for (int i=1;i<=m;i++) {
117            addedge(0, e[i].x, e[i].y, e[i].w);
118            addedge(1, e[i].y, e[i].x, e[i].w);
119        }
120        spfa(T);
121        root[T] = 0; pheap::init();
122        memset(v, 0, sizeof(v));
123        dfs(T);
124        while (!pq.empty()) pq.pop();
125        if (k == 1) return dist[S];
126        if (root[S]) pq.push(make_pair(dist[S] + node[root[S]].val, root[S]));
127        while (k--) {
128            if (pq.empty()) return inf;
129            pli now = pq.top(); pq.pop();
130            if (k == 1) return now.first;
131            int x = node[now.second].next, u = node[now.second].son[0], v = node[now
                    .second].son[1];
132            if (root[x]) pq.push(make_pair(now.first + node[root[x]].val, root[x]));
133            if (u) pq.push(make_pair(now.first - node[now.second].val + node[u].val,
                    u));
134            if (v) pq.push(make_pair(now.first - node[now.second].val + node[v].val,
                    v));
135        }
136        return 0;
137    }
138 };
```

5.4 Tarjan

   割点的判断：一个顶点 u 是割点，当且仅当满足 (1) 或 (2)：
(1) u 为树根，且 u 有多于一个子树
(2) u 不为树根，且满足存在 (u,v) 为树枝边（u 为 v 的父亲），使得 $dfn[u] \leq low[v]$
桥的判断：一条无向边 (u,v) 是桥，当且仅当 (u,v) 为树枝边，满足 $dfn[u] < low[v]$

```
1 struct EDGE { int adj, next; } edge[M];
2 int n, m, top, gh[N];
```

```
3   int dfn[N], low[N], cnt, ind, stop, instack[N], stack[N], belong[N];
4   void addedge(int x, int y) {
5       edge[++top].adj = y;
6       edge[top].next = gh[x];
7       gh[x] = top;
8   }
9   void tarjan(int x) {
10      dfn[x] = low[x] = ++ind;
11      instack[x] = 1; stack[++stop] = x;
12      for (int p=gh[x]; p; p=edge[p].next)
13          if (!dfn[edge[p].adj]) {
14              tarjan(edge[p].adj);
15              low[x] = min(low[x], low[edge[p].adj]);
16          } else if (instack[edge[p].adj]) {
17              low[x] = min(low[x], dfn[edge[p].adj]);
18          }
19      if (dfn[x] == low[x]) {
20          ++cnt; int tmp=0;
21          while (tmp!=x) {
22              tmp = stack[stop--];
23              belong[tmp] = cnt;
24              instack[tmp] = 0;
25          }
26      }
27  }
```

5.5 2-SAT

```
1   #define N number_of_vertex
2   #define M number_of_edges
3
4   struct MergePoint {
5       struct EDGE {
6           int adj, next;
7       } edge[M];
8       int ex[M], ey[M];
9       bool instack[N];
10      int gh[N], top, dfn[N], low[N], cnt, ind, stop, stack[N], belong[N];
11      void init() {
12          cnt = ind = stop = top = 0;
13          memset(dfn, 0, sizeof(dfn));
14          memset(instack, 0, sizeof(instack));
15          memset(gh, 0, sizeof(gh));
16      }
17      void addedge(int x, int y) { //reverse
18          std::swap(x, y);
19          edge[++top].adj = y;
20          edge[top].next = gh[x];
21          gh[x] = top;
```

```
22          ex[top] = x;
23          ey[top] = y;
24      }
25      void tarjan(int x) {
26          dfn[x] = low[x] = ++ind;
27          instack[x] = 1; stack[++stop] = x;
28          for (int p=gh[x]; p; p=edge[p].next)
29              if (!dfn[edge[p].adj]) {
30                  tarjan(edge[p].adj);
31                  low[x] = std::min(low[x], low[edge[p].adj]);
32              } else if (instack[edge[p].adj]) {
33                  low[x] = std::min(low[x], dfn[edge[p].adj]);
34              }
35          if (dfn[x] == low[x]) {
36              ++cnt; int tmp = 0;
37              while (tmp!=x) {
38                  tmp = stack[stop--];
39                  belong[tmp] = cnt;
40                  instack[tmp] = 0;
41              }
42          }
43      }
44      void work() {
45          for (int i = (__first__); i <= (__last__); ++i)
46              if (!dfn[i])
47                  tarjan(i);
48      }
49  } merge;
50
51  struct Topsort {
52      struct EDGE {
53          int adj, next;
54      } edge[M];
55      int n, top, gh[N], ops[N], deg[N], ans[N];
56      std::queue<int> q;
57      void init() {
58          n = merge.cnt; top = 0;
59          memset(gh, 0, sizeof(gh));
60          memset(deg, 0, sizeof(deg));
61      }
62      void addedge(int x, int y) {
63          if (x == y) return;
64          edge[++top].adj = y;
65          edge[top].next = gh[x];
66          gh[x] = top;
67          ++deg[y];
68      }
69      void work() {
70          for (int i = 1; i <= n; ++i)
71              if (!deg[i])
```

```
72                    q.push(i);
73            while (!q.empty()) {
74                int x = q.front();
75                q.pop();
76                for (int p = gh[x]; p; p = edge[p].next)
77                    if (!--deg[edge[p].adj])
78                        q.push(edge[p].adj);
79                if (ans[x]) continue;
80                ans[x] = -1; //not selected
81                ans[ops[x]] = 1; //selected
82            }
83        }
84  } ts;
```

调用示例：

```
1       merge.init();
2       merge.addedge();
3       merge.work();
4       for (int i = 1; i <= n; ++i) {
5           if (merge.belong[U(i, 0)] == merge.belong[U(i, 1)]) {
6               puts("NO");
7               return 0;
8           }
9           ts.ops[merge.belong[U(i, 0)]] = merge.belong[U(i, 1)];
10          ts.ops[merge.belong[U(i, 1)]] = merge.belong[U(i, 0)];
11      }
12      ts.init();
13      ts.work();
14      puts("YES");
15      for (int i = 1; i <= n; ++i) {
16          int x = U(i, 0), y = U(i, 1);
17          x = merge.belong[x], y = merge.belong[y];
18          x = ts.ans[x], y = ts.ans[y];
19          if (x == 1) puts("0 is selected");
20          if (y == 1) puts("1 is selected");
21      }
```

5.6 统治者树 (Dominator Tree)

Dominator Tree 可以解决判断一类有向图必经点的问题。

idom[x] 表示离 x 最近的必经点（重编号后）。将 idom[x] 作为 x 的父亲，构成一棵 Dominator Tree

接口：

void dominator::init(int n); 初始化，有向图节点数为 n

void dominator::addedge(int u, int v); 添加一条有向边 (u, v)

void dominator::work(int root); 以 root 为根，建立一棵 Dominator Tree

结果的返回：

在执行 dominator::work(int root); 后，树边保存在 vector <int> tree[N] 中

35

```cpp
namespace dominator {
    vector <int> g[N], rg[N], bucket[N], tree[N];
    int n, ind, idom[N], sdom[N], dfn[N], dsu[N], father[N], label[N], rev[N];
    void dfs(int x) {
        ++ind;
        dfn[x] = ind; rev[ind] = x;
        label[ind] = dsu[ind] = sdom[ind] = ind;
        for (auto p : g[x]) {
            if (!dfn[p]) dfs(p), father[dfn[p]] = dfn[x];
            rg[dfn[p]].push_back(dfn[x]);
        }
    }
    void init(int n1) {
        n = n1; ind = 0;
        for (int i = 1; i <= n; ++i) {
            g[i].clear();
            rg[i].clear();
            bucket[i].clear();
            tree[i].clear();
            dfn[i] = 0;
        }
    }
    void addedge(int u, int v) {
        g[u].push_back(v);
    }
    int find(int x, int step=0) {
        if (dsu[x] == x) return step ? -1 : x;
        int y = find(dsu[x], 1);
        if (y < 0) return x;
        if (sdom[label[dsu[x]]] < sdom[label[x]])
            label[x] = label[dsu[x]];
        dsu[x] = y;
        return step ? dsu[x] : label[x];
    }
    void work(int root) {
        dfs(root); n = ind;
        for (int i = n; i; --i) {
            for (auto p : rg[i])
                sdom[i] = min(sdom[i], sdom[find(p)]);
            if (i > 1) bucket[sdom[i]].push_back(i);
            for (auto p : bucket[i]) {
                int u = find(p);
                if (sdom[p] == sdom[u]) idom[p] = sdom[p];
                else idom[p] = u;
            }
            if (i > 1) dsu[i] = father[i];
        }
        for (int i = 2; i <= n; ++i) {
            if (idom[i] != sdom[i])
```

```
50          idom[i] = idom[idom[i]];
51          tree[rev[i]].push_back(rev[idom[i]]);
52          tree[rev[idom[i]]].push_back(rev[i]);
53        }
54      }
55  };
```

5.7 网络流

5.7.1 最大流

注意：*top 要初始化为 1*

```
 1  struct EDGE { int adj, w, next; } edge[M];
 2  int n, top, gh[N], nrl[N];
 3  void addedge(int x, int y, int w) {
 4      edge[++top].adj = y;
 5      edge[top].w = w;
 6      edge[top].next = gh[x];
 7      gh[x] = top;
 8      edge[++top].adj = x;
 9      edge[top].w = 0;
10      edge[top].next = gh[y];
11      gh[y] = top;
12  }
13  int dist[N], q[N];
14  int bfs() {
15      memset(dist, 0, sizeof(dist));
16      q[1] = S; int head = 0, tail = 1; dist[S] = 1;
17      while (head != tail) {
18          int x = q[++head];
19          for (int p=gh[x]; p; p=edge[p].next)
20              if (edge[p].w && !dist[edge[p].adj]) {
21                  dist[edge[p].adj] = dist[x] + 1;
22                  q[++tail] = edge[p].adj;
23              }
24      }
25      return dist[T];
26  }
27  int dinic(int x, int delta) {
28      if (x==T) return delta;
29      for (int& p=nrl[x]; p && delta; p=edge[p].next)
30          if (edge[p].w && dist[x]+1 == dist[edge[p].adj]) {
31              int dd = dinic(edge[p].adj, min(delta, edge[p].w));
32              if (!dd) continue;
33              edge[p].w -= dd;
34              edge[p^1].w += dd;
35              return dd;
36          }
37      return 0;
```

```
38  }
39  int work() {
40      int ans = 0;
41      while (bfs()) {
42          memcpy(nrl, gh, sizeof(gh));
43          int t; while (t = dinic(S, inf)) ans += t;
44      }
45      return ans;
46  }
```

5.7.2 上下界有源汇网络流

T 向 S 连容量为正无穷的边，将有源汇转化为无源汇。

每条边容量减去下界，设 $in[i]$ 表示流入 i 的下界之和减去流出 i 的下界之和。

新建超级源汇 SS, TT，对于 $in[i] > 0$ 的点，SS 向 i 连容量为 $in[i]$ 的边。对于 $in[i] < 0$ 的点，i 向 TT 连容量为 $-in[i]$ 的边。

求出以 SS, TT 为源汇的最大流，如果等于 $\Sigma in[i](in[i] > 0)$，则存在可行流。再求出 S, T 为源汇的最大流即为最大流。

费用流：建完图后等价于求以 SS, TT 为源汇的费用流。

5.7.3 上下界无源汇网络流

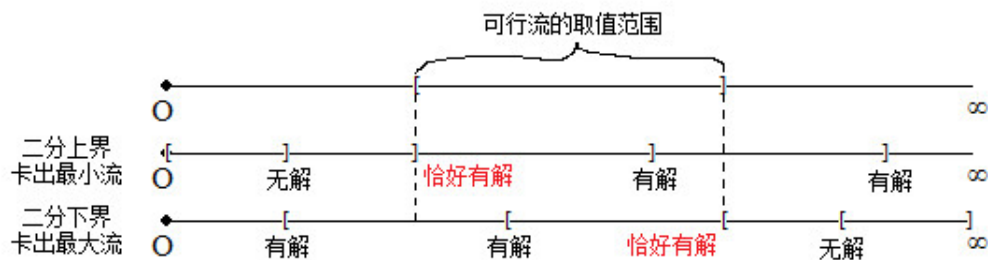1. 怎样求无源汇有上下界网络的可行流？

由于有源汇的网络我们先要转化成无源汇，所以本来就无源汇的网络不用再作特殊处理。

2. 怎样求无源汇有上下界网络的最大流、最小流？

一种简易的方法是采用二分的思想，不断通过可行流的存在与否对 (t, s) 边的上下界 U, L 进行调整。求最大流时令 $U = \infty$ 并二分 L；求最小流时令 $L = 0$ 并二分 U。道理很简单，因为可行流的取值范围是一段连续的区间，我们只要通过二分找到有解和无解的分界线即可。

5.7.4 费用流

注意：*top 要初始化为 1*

```
1  #define inf 0x3f3f3f3f
2  struct NetWorkFlow {
3      struct EDGE {
4          int adj, w, cost, next;
```

38

```
5        } edge[M*2];
6    int gh[N], q[N], dist[N], v[N], pre[N], prev[N], top;
7    int S, T;
8    void addedge(int x, int y, int w, int cost) {
9        edge[++top].adj = y;
10       edge[top].w = w;
11       edge[top].cost = cost;
12       edge[top].next = gh[x];
13       gh[x] = top;
14       edge[++top].adj = x;
15       edge[top].w = 0;
16       edge[top].cost = -cost;
17       edge[top].next = gh[y];
18       gh[y] = top;
19   }
20   void clear() {
21       top = 1;
22       memset(gh, 0, sizeof(gh));
23   }
24   int spfa() {
25       memset(dist, 63, sizeof(dist));
26       memset(v, 0, sizeof(v));
27       int head = 0, tail = 1;
28       q[1] = S; v[S] = 1; dist[S] = 0;
29       while (head != tail) {
30           (head += 1) %= N;
31           int x = q[head];
32           v[x] = 0;
33           for (int p=gh[x]; p; p=edge[p].next)
34               if (edge[p].w && dist[x] + edge[p].cost < dist[edge[p].adj]) {
35                   dist[edge[p].adj] = dist[x] + edge[p].cost;
36                   pre[edge[p].adj] = x;
37                   prev[edge[p].adj] = p;
38                   if (!v[edge[p].adj]) {
39                       v[edge[p].adj] = 1;
40                       (tail += 1) %= N;
41                       q[tail] = edge[p].adj;
42                   }
43               }
44       }
45       return dist[T] != inf;
46   }
47   int work() {
48       int ans = 0;
49       while (spfa()) {
50           int mx = inf;
51           for (int x=T;x!=S;x=pre[x])
52               mx = min(edge[prev[x]].w, mx);
53           ans += dist[T] * mx;
54           for (int x=T;x!=S;x=pre[x]) {
```

```
55            edge[prev[x]].w -= mx;
56            edge[prev[x]^1].w += mx;
57          }
58        }
59        return ans;
60      }
61  } nwf;
```

5.7.5 zkw 费用流

注意: *top* 要初始化为 *1*, 不得用于有负权的图

```
1   #define inf 0x3f3f3f3f
2   struct NetWorkFlow {
3       struct EDGE {
4           int adj, w, cost, next;
5       } edge[M*2];
6       int gh[N], top;
7       int S, T;
8       void addedge(int x, int y, int w, int cost) {
9           edge[++top].adj = y;
10          edge[top].w = w;
11          edge[top].cost = cost;
12          edge[top].next = gh[x];
13          gh[x] = top;
14          edge[++top].adj = x;
15          edge[top].w = 0;
16          edge[top].cost = -cost;
17          edge[top].next = gh[y];
18          gh[y] = top;
19      }
20      void clear() {
21          top = 1;
22          memset(gh, 0, sizeof(gh));
23      }
24      int cost, d[N], slk[N], v[N];
25      int aug(int x, int f) {
26          int left = f;
27          if (x == T) {
28              cost += f * d[S];
29              return f;
30          }
31          v[x] = true;
32          for (int p=gh[x]; p; p=edge[p].next)
33              if (edge[p].w && !v[edge[p].adj]) {
34                  int t = d[edge[p].adj] + edge[p].cost - d[x];
35                  if (t == 0) {
36                      int delt = aug(edge[p].adj, min(left, edge[p].w));
37                      if (delt > 0) {
38                          edge[p].w -= delt;
```

```
39                      edge[p^1].w += delt;
40                      left -= delt;
41                  }
42              if (left == 0) return f;
43          } else {
44              if (t < slk[edge[p].adj])
45                  slk[edge[p].adj] = t;
46          }
47      }
48      return f-left;
49  }
50  bool modlabel() {
51      int delt = inf;
52      for (int i=1;i<=T;i++)
53          if (!v[i]) {
54              if (slk[i] < delt) delt = slk[i];
55              slk[i] = inf;
56          }
57      if (delt == inf) return true;
58      for (int i=1;i<=T;i++)
59          if (v[i]) d[i] += delt;
60      return false;
61  }
62  int work() {
63      cost = 0;
64      memset(d, 0, sizeof(d));
65      memset(slk, 63, sizeof(slk));
66      do {
67          do {
68              memset(v, 0, sizeof(v));
69          } while (aug(S, inf));
70      } while (!modlabel());
71      return cost;
72  }
73  } nwf;
```

6 数学

6.1 扩展欧几里得解同余方程

ans[] 保存的是循环节内所有的解

```
1  int exgcd(int a,int b,int&x,int&y){
2      if(!b)return x=1,y=0,a;
3      int d=exgcd(b,a%b,x,y),t=x;
4      return x=y,y=t-a/b*y,d;
5  }
6  void cal(ll a,ll b,ll n){//ax=b(mod n)
7      ll x,y,d=exgcd(a,n,x,y);
```

```
8      if(b%d)return;
9      x=(x%n+n)%n;
10     ans[cnt=1]=x*(b/d)%(n/d);
11     for(ll i=1;i<d;i++)ans[++cnt]=(ans[1]+i*n/d)%n;
12  }
```

6.2 同余方程组

```
1   int n,flag,k,m,a,r,d,x,y;
2   int main(){
3       scanf("%d",&n);
4       flag=k=1,m=0;
5       while(n--){
6           scanf("%d%d",&a,&r);//ans%a=r
7           if(flag){
8               d=exgcd(k,a,x,y);
9               if((r-m)%d){flag=0;continue;}
10              x=(x*(r-m)/d+a/d)%(a/d),y=k/d*a,m=((x*k+m)%y)%y;
11              if(m<0)m+=y;
12              k=y;
13          }
14      }
15      printf("%d",flag?m:-1);//若flag=1,说明有解,解为ki+m,i为任意整数
16  }
```

6.3 卡特兰数

$h_1 = 1, h_n = \frac{h_{n-1}(4n-2)}{n+1} = \frac{C(2n,n)}{n+1} = C(2n,n) - C(2n,n-1)$

在一个格点阵列中，从 $(0,0)$ 点走到 (n,m) 点且不经过对角线 $x = y$ 的方案数 $(x > y)$：
$C(n + m - 1, m) - C(n + m - 1, m - 1)$

在一个格点阵列中，从 $(0,0)$ 点走到 (n,m) 点且不穿过对角线 $x = y$ 的方案数 $(x \geq y)$：
$C(n + m, m) - C(n + m, m - 1)$

6.4 斯特林数

6.4.1 第一类斯特林数

第一类 Stirling 数 $S(p, k)$ 的一个组合学解释是：将 p 个物体排成 k 个非空循环排列的方法数。

$S(p, k)$ 的递推公式：$S(p, k) = (p - 1)S(p - 1, k) + S(p - 1, k - 1), 1 \leq k \leq p - 1$

边界条件：$S(p, 0) = 0, p \geq 1$ $S(p, p) = 1, p \geq 0$

6.4.2 第二类斯特林数

第二类 Stirling 数 $S(p, k)$ 的一个组合学解释是：将 p 个物体划分成 k 个非空的不可辨别（可以理解为盒子没有编号）集合的方法数。

$S(p, k)$ 的递推公式：$S(p, k) = kS(p - 1, k) + S(p - 1, k - 1), 1 \leq k \leq p - 1$

边界条件：$S(p, 0) = 0, p \geq 1$ $S(p, p) = 1, p \geq 0$

也有卷积形式：

$$S(n, m) = \frac{1}{m!} \sum_{k=0}^{m} (-1)^k C(m, k)(m-k)^n = \sum_{k=0}^{m} \frac{(-1)^k (m-k)^n}{k!(m-k)!} = \sum_{k=0}^{m} \frac{(-1)^k}{k!} \times \frac{(m-k)^n}{(m-k)!}$$

6.5 错排公式

$$D_1 = 0, D_2 = 1, D_n = (n-1)(D_{n-2} + D_{n-1})$$

6.6 Lucas 定理

接口：
初始化：void lucas::init();
计算 $C(n, m)\% mod$ 的值：LL lucas::Lucas(LL n, LL m);

```
#define mod 110119
#define LL long long
namespace lucas {
    LL fac[mod+1], facv[mod+1];
    LL power(LL base, LL times) {
        LL ans = 1;
        while (times) {
            if (times&1) (ans *= base) %= mod;
            (base *= base) %= mod;
            times >>= 1;
        }
        return ans;
    }
    void init() {
        fac[0] = 1; for (int i=1;i<mod;i++) fac[i] = (fac[i-1] * i) % mod;
        facv[mod-1] = power(fac[mod-1], mod-2);
        for (int i=mod-2;i>=0;--i) facv[i] = (facv[i+1] * (i+1)) % mod;
    }
    LL C(unsigned LL n, unsigned LL m) {
        if (n < m) return 0;
        return (fac[n] * facv[m] % mod * facv[n-m] % mod) % mod;
    }
    LL Lucas(unsigned LL n, unsigned LL m)
    {
        if (m == 0) return 1;
        return (C(n%mod, m%mod) * Lucas(n/mod, m/mod)) %mod;
    }
};
```

6.7 高斯消元

6.7.1 行列式

```
1  int ans = 1;
2  for (int i=0;i<n;i++) {
3      for (int j=i;j<n;j++)
4          if (g[j][i]) {
5              for (int k=i;k<n;k++)
6                  swap(g[i][k], g[j][k]);
7              if (j != i) ans *= -1;
8              break;
9          }
10     if (g[i][i] == 0) {
11         ans = 0;
12         break;
13     }
14     for (int j=i+1;j<n;j++) {
15         while (g[j][i]) {
16             int t = g[i][i] / g[j][i];
17             for (int k=i;k<n;k++)
18                 g[i][k] = (g[i][k] + mod - ((LL)t * g[j][k] % mod)) % mod;
19             for (int k=i;k<n;k++)
20                 swap(g[i][k], g[j][k]);
21             ans *= -1;
22         }
23     }
24 }
25 for (int i=0;i<n;i++)
26     ans = ((LL)ans * g[i][i]) % mod;
27 ans = (ans % mod + mod) % mod;
28 printf("%d\n", ans);
```

6.7.2 Matrix-Tree 定理

对于一张图，建立矩阵 C ，$C[i][i] = i$ 的度数，若 i, j 之间有边，那么 $C[i][j] = -1$ ，否则为 0 。这张图的生成树个数等于矩阵 C 的 $n-1$ 阶行列式的值。

6.8 调和级数

$\sum\limits_{i=1}^{n} \frac{1}{i}$ 在 n 较大时约等于 $ln(n) + r$ ，r 为欧拉常数，约等于 0.5772156649015328 。

6.9 曼哈顿距离的变换

$$|x_1 - x_2| + |y_1 - y_2| = max(|(x_1 + y_1) - (x_2 + y_2)|, |(x_1 - y_1) - (x_2 - y_2)|)$$

6.10 线性筛素数

```
1  mu[1]=phi[1]=1;top=0;
2  for (int i=2;i<N;i++) {
3      if (!v[i]) prime[++top]=i, mu[i] = -1, phi[i] = i-1;
```

```
4      for (int j=1;i*prime[j]<N && j<=top;j++) {
5          v[i*prime[j]] = 1;
6          if (i%prime[j]) {
7              mu[i*prime[j]] = -mu[i];
8              phi[i*prime[j]] = phi[i] * (prime[j]-1);
9          } else {
10             mu[i*prime[j]] = 0;
11             phi[i*prime[j]] = phi[i] * prime[j];
12             break;
13         }
14     }
15 }
```

6.11 杜教筛

getphi(t, x) 表示求 $\sum\limits_{i=1}^{x} i^t \phi(i)$ 。

推导过程:

记 $S(n) = \sum\limits_{i=1}^{n} f(i)$, 取任意函数 g 有恒等式

$$S(n) = \sum_{i=1}^{n} (f \cdot g)(i) - \sum_{i=2}^{n} g(i)S(\lfloor \frac{n}{i} \rfloor)$$

其中, $(f \cdot g)$ 表示 f 和 g 的狄利克雷卷积: 即: $(f \cdot g)(n) = \sum\limits_{d|n} f(d)g(\frac{n}{d})$

关于恒等式的证明:

将 $\sum\limits_{i=2}^{n} g(i)S(\lfloor \frac{n}{i} \rfloor)$ 移到左边去, 即只需证

$$\sum_{i=1}^{n} (f \cdot g)(i) = \sum_{i=1}^{n} g(i)S(\lfloor \frac{n}{i} \rfloor)$$

将狄利克雷卷积展开, 得:

$$\sum_{i=1}^{n} \sum_{d|i} g(d)f(\frac{i}{d}) = \sum_{i=1}^{n} g(i)S(\lfloor \frac{n}{i} \rfloor)$$

即:

$$\sum_{d=1}^{n} g(d) \sum_{i=1}^{\lfloor \frac{n}{d} \rfloor} f(i) = \sum_{i=1}^{n} g(i)S(\lfloor \frac{n}{i} \rfloor)$$

显然相等, 恒等式证完。

取 $f(i) = i^p \phi(i), g(i) = i^p$, 则有:

$$S(n) = \sum_{i=1}^{n} i^p \phi(i) = \sum_{i=1}^{n} i^{p+1} - \sum_{i=2}^{n} i^p S(\lfloor \frac{n}{i} \rfloor)$$

其中有用到等式 $\sum\limits_{d|n} \phi(d) = n$

另外，莫比乌斯函数 $\mu(n) = \begin{cases} 1, & \text{若} n = 1 \\ (-1)^k, & \text{若} n \text{无平方数因数，且} n = p_1 p_2 \cdots p_k \\ 0, & \text{若} n \text{有大于 1 的平方数因数} \end{cases}$ 也可以使用杜教

筛求前缀和，记 $S'(n) = \sum_{i=1}^{n} \mu(i)$，则 $S'(n) = 1 - \sum_{i=2}^{n} S'(\lfloor \frac{n}{i} \rfloor)$

```cpp
#include <bits/stdc++.h>

#define N 5000020
#define LL long long
#define mod 1000000007
#define div2 ((mod+1)/2)
#define div6 ((mod+1)/6)

using namespace std;

int n, prime[N], v[N];
LL phi[3][N];

map<int, int> mp[3];

int sum(int t, int x) { //calculate 1^t + 2^t + ... + x^t
    if (t == 0) return x;
    if (t == 1) return 1ll * x * (x + 1) % mod * div2 % mod;
    if (t == 2) return 1ll * x * (x + 1) % mod * (2ll * x % mod + 1) % mod * div6 %
        mod;
    if (t == 3) return 1ll * x * x % mod * (x + 1) % mod * (x + 1) % mod * div2 %
        mod * div2 % mod;
}

int getphi(int t, int x) {
    if (x < N) return phi[t][x];
    if (mp[t].find(x) != mp[t].end()) return mp[t][x];
    LL ans = 0; int r = 0;
    for (int l = 2; l <= x; l = r + 1) {
        r = x / (x / l);
        ans += 1ll * getphi(t, x / l) * (((LL)sum(t, r) - sum(t, l - 1) + mod) % mod
            ) % mod;
        ans %= mod;
    }
    ans = (LL)sum(t + 1, x) - ans + mod;
    ans %= mod;
    mp[t][x] = ans;
    return (int)ans;
}

int main() {
    memset(v, 0, sizeof(v));
    int top = 0;
    phi[0][1] = 1, phi[1][1] = 1, phi[2][1] = 1;
    for (int i = 2; i < N; ++i) {
```

```
43         if (!v[i]) prime[++top] = i, phi[0][i] = i - 1, phi[1][i] = 1ll * i * phi
               [0][i] % mod, phi[2][i] = 1ll * i * phi[1][i] % mod;
44         for (int j = 1; j <= top && prime[j] * i < N; ++j) {
45             v[i * prime[j]] = 1;
46             if (i % prime[j] == 0) {
47                 phi[0][i * prime[j]] = phi[0][i] * prime[j];
48                 phi[1][i * prime[j]] = 1ll * phi[1][i] * prime[j] % mod * prime[j] %
                       mod;
49                 phi[2][i * prime[j]] = 1ll * phi[2][i] * prime[j] % mod * prime[j] %
                       mod * prime[j] % mod;
50                 break;
51             } else {
52                 phi[0][i * prime[j]] = phi[0][i] * (prime[j] - 1);
53                 phi[1][i * prime[j]] = 1ll * phi[1][i] * (prime[j] - 1) % mod *
                       prime[j] % mod;
54                 phi[2][i * prime[j]] = 1ll * phi[2][i] * (prime[j] - 1) % mod *
                       prime[j] % mod * prime[j] % mod;
55             }
56         }
57     }
58     for (int i = 2; i < N; ++i) {
59         phi[0][i] = (phi[0][i] + phi[0][i - 1]) % mod;
60         phi[1][i] = (phi[1][i] + phi[1][i - 1]) % mod;
61         phi[2][i] = (phi[2][i] + phi[2][i - 1]) % mod;
62     }
63 }
```

6.12 FFT

```
1  typedef complex<double> comp;
2  namespace FFT {
3      comp A[N], B[N],omega[N];
4      void transform(comp *x, int len) {
5          for (int i=1,j=len/2;i<len-1;i++) {
6              if (i<j) swap(x[i], x[j]);
7              int k = len/2;
8              while (j>=k) {
9                  j-=k;
10                 k/=2;
11             }
12             if (j<k) j+=k;
13         }
14     }
15     void fft(comp *x, int len, int reverse) {
16         transform(x, len);
17         for (int h=2;h<=len;h<<=1) {
18             for (int i=0;i<h/2;i++) omega[i] = polar(1.0, 2*pi*reverse/h*i);
19             for (int i=0;i<len;i+=h) {
20                 for (int j=i;j<i+h/2;j++) {
```

```
21                     comp w = omega[j-i];
22                     comp u = x[j];
23                     comp v = (w * x[j+h/2]);
24                     x[j] = u + v;
25                     x[j+h/2] = u - v;
26                 }
27             }
28         }
29         if (reverse == -1) {
30             for (int i=0;i<len;i++)
31                 x[i] /= len;
32         }
33     }
34     void work(int n, int *a, int *b) {
35         int len = 1;
36         while (len <= n*2) len *= 2;
37         for (int i=0;i<len;i++) A[i] = B[i] = 0;
38         for (int i=0;i<n;i++) A[i] = a[i], B[i] = b[i];
39         fft(A, len, 1); fft(B, len, 1);
40         for (int i=0;i<len;i++) A[i] = A[i] * B[i];
41         fft(A, len, -1);
42         for (int i=0;i<len;i++) {
43             LL r = round(A[i].real());
44             a[i] = r % mod;
45         }
46     }
47 };
```

6.13 FWT

给定长度为 2^n 的序列 $A[0\cdots 2^n-1], B[0\cdots 2^n-1]$ ，求这两序列的

or 卷积: $C_k = \sum\limits_{i\ or\ j=k} A_i B_j$

and 卷积: $C_k = \sum\limits_{i\ and\ j=k} A_i B_j$

xor 卷积: $C_k = \sum\limits_{i\ xor\ j=k} A_i B_j$

序列对 $10^9 + 7$ 取模

```
1  #include <bits/stdc++.h>
2  using namespace std;
3  inline int read() {
4      int s = 0; char c; while((c=getchar())<'0'||c>'9');
5      do{s=s*10+c-'0';}while((c=getchar())>='0'&&c<='9');
6      return s;
7  }
8  typedef long long lint;
9  typedef pair<lint,lint> pii;
10 const int N = (1<<18)+37, MO = 1e9+7;
11 int n;
12 pii tand(lint a,lint b) { return (pii){a+b,b}; }
```

```
13  pii iand(lint a,lint b) { return (pii){a-b,b}; }
14  pii tor(lint a,lint b) { return (pii){a,a+b}; }
15  pii ior(lint a,lint b) { return (pii){a,b-a}; }
16  pii txor(lint a,lint b) { return (pii){a+b,a-b}; }
17  pii ixor(lint a,lint b) { return (pii){(a+b)/2,(a-b)/2}; }
18  pii (*tr)(lint,lint);
19  pii (*trs[3])(lint,lint)={tor,tand,txor};
20  pii (*ivs[3])(lint,lint)={ior,iand,ixor};
21  lint a[N],b[N],x[N],y[N];
22  inline void fwt(lint *a) {
23      int s,k,j,je;
24      for(s=2;s<=n;s<<=1) for(k=0;k<n;k+=s)
25          for(j=0,je=s>>1;j<je;j++) {
26              pii t = tr(a[k+j],a[k+j+je]);
27              a[k+j] = t.first;
28              a[k+j+je] = t.second;
29          }
30  }
31  int main() {
32      int i;
33      n = read(); n = 1<<n;
34      for(i=0;i<n;i++) a[i] = read();
35      for(i=0;i<n;i++) b[i] = read();
36      for(int k=0;k<3;k++) {
37          for(i=0;i<n;i++) x[i] = a[i], y[i] = b[i];
38          tr = trs[k]; fwt(x), fwt(y);
39          for(i=0;i<n;i++) x[i] *= y[i];
40          tr = ivs[k]; fwt(x);
41          for(i=0;i<n;i++) printf("%d ",x[i]%MO); puts("");
42      }
43      return 0;
44  }
```

6.14 求原根

接口：LL p_root(LL p);
输入：一个素数 p
输出：p 的原根

```
1   #include <bits/stdc++.h>
2   #define LL long long
3
4   using namespace std;
5
6   vector <LL> a;
7
8   LL pow_mod(LL base, LL times, LL mod) {
9       LL ret = 1;
10      while (times) {
```

```
11          if (times&1) ret = ret * base % mod;
12          base = base * base % mod;
13          times>>=1;
14      }
15      return ret;
16  }
17
18  bool g_test(LL g, LL p) {
19      for (LL i = 0; i < a.size(); ++i)
20          if (pow_mod(g, (p-1)/a[i], p) == 1) return 0;
21      return 1;
22  }
23
24  LL p_root(LL p) {
25      LL tmp = p - 1;
26      for (LL i = 2; i <= tmp / i; ++i)
27          if (tmp % i == 0) {
28              a.push_back(i);
29              while (tmp % i == 0)
30                  tmp /= i;
31          }
32      if (tmp != 1) a.push_back(tmp);
33      LL g = 1;
34      while (1) {
35          if (g_test(g, p)) return g;
36          ++g;
37      }
38  }
39
40  int main() {
41      LL p;
42      cin >> p;
43      cout << p_root(p) << endl;
44  }
```

6.15 NTT

998244353 原根为 3 ，1004535809 原根为 3 ，786433 原根为 10 ，880803841 原根为 26 。

```
1   #define mod 998244353
2   #define g 3
3   LL wi[N], wiv[N];
4   LL power(LL base, LL times) {
5       LL ans = 1;
6       while (times) {
7           if (times&1) (ans *= base) %= mod;
8           (base *= base) %= mod;
9           times >>= 1;
10      }
11      return ans;
```

```
12  }
13  void transform(LL *x, int len) {
14      for (int i=1,j=len/2;i<len-1;i++) {
15          if (i<j) swap(x[i], x[j]);
16          int k = len/2;
17          while (j>=k) {
18              j-=k;
19              k/=2;
20          }
21          if (j<k) j+=k;
22      }
23  }
24  void NTT(LL *x, int len, int reverse) {
25      transform(x, len);
26      for (int h=2;h<=len;h<<=1) {
27          for (int i=0;i<len;i+=h) {
28              LL w = 1, wn;
29              if (reverse==1) wn = wi[h]; else wn = wiv[h];
30              for (int j=i;j<i+h/2;j++) {
31                  LL u = x[j];
32                  LL v = (w * x[j+h/2]) % mod;
33                  x[j ] = (u + v) % mod;
34                  x[j+h/2] = (u - v + mod) % mod;
35                  (w *= wn) %= mod;
36              }
37          }
38      }
39      if (reverse == -1) {
40          LL t = power(len, mod-2);
41          for (int i=0;i<len;i++)
42              (x[i] *= t) %= mod;
43      }
44  }
45  LL A[N], B[N];
46  int main() {
47      for (int i=1;i<N;i*=2) {
48          wi[i] = power(g, (mod-1)/i);
49          wiv[i] = power(wi[i], mod-2);
50      }
51      memset(A, 0, sizeof(A));
52      memset(B, 0, sizeof(B));
53      NTT(A, len, 1); NTT(B, len, 1);
54      for (int i=0;i<len;i++) (A[i] *= B[i]) %= mod;
55      NTT(A, len, -1);
56  }
```

6.16 组合数 lcm

$$(n+1)lcm(C(n,0), C(n,1), ..., C(n,k)) = lcm(n+1, n, n-1, ..., n-k+1)$$

6.17 区间 lcm 的维护

对于一个数，将其分解质因数，若有因子 p^k，那么拆分出 k 个数 $p, p^2, ..., p^k$，权值都为 p，那么查询区间 $[l, r]$ 内所有数的 lcm 的答案 = 所有在该区间中出现过的数的权值之积，可持久化线段树维护即可。

7 几何

7.1 凸包

```cpp
typedef complex<int> point;
#define X real()
#define Y imag()
int n;
long long cross(point a,point b) {
    return 1ll * a.X * b.Y - 1ll * a.Y * b.X;
}
bool cmp(point a,point b) {
    return make_pair(a.X, a.Y) < make_pair(b.X, b.Y);
}
int convexHull(point p[],int n,point ch[]) {
    sort(p, p + n, cmp);
    int m = 0;
    for(int i = 0; i < n; ++i) {
        while(m > 1 && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
        ch[m++] = p[i];
    }
    int k = m;
    for(int i = n - 2; i >= 0; --i) {
        while(m > k && cross(ch[m-1] - ch[m-2], p[i] - ch[m-2]) <= 0) m--;
        ch[m++] = p[i];
    }
    if(n > 1) m--;
    return m;
}
```

8 黑科技和杂项

8.1 找规律

有些题目，只给一个正整数 n，然后要求输出一个答案。这时，我们可以暴力得到小数据的解，用高斯消元得到递推式，然后用矩阵快速幂求解。

使用方法：

首先在 gauss.in 中输入小数据的解（$n = 1$ 时，$n = 2$ 时，$\cdots$），以 *EOF* 结束。

依次运行 gauss.cpp，matrix.cpp，得到 matrix.out

将 matrix.out 中的文件粘贴在 main.cpp 中相应的位置中。注意模数一定要是质数。

```cpp
//gauss.cpp
#include <bits/stdc++.h>
#define N 102
#define mod 1000000007
//caution: you can use this program iff mod is a prime.

using namespace std;

int n, m, k, a[N], g[N][N];

int power(int base, int times) {
    int ret = 1;
    while (times) {
        if (times & 1) ret = 1ll * ret * base % mod;
        base = 1ll * base * base % mod;
        times >>= 1;
    }
    return ret;
}

int test() {
    for (int i=0;i<m;i++) {
        for (int j=i;j<=m;j++)
            if (g[j][i]) {
                for (int k=i;k<=m;k++)
                    swap(g[i][k], g[j][k]);
                break;
            }
        if (g[i][i] == 0)
            return 0;
        for (int j=i+1;j<n;j++) {
            while (g[j][i]) {
                int t = 1ll * g[i][i] * power(g[j][i], mod - 2) % mod;
                for (int k=i;k<n;k++)
                    g[i][k] = (g[i][k] + mod - (1ll * t * g[j][k] % mod)) % mod;
                for (int k=i;k<=m;k++)
                    swap(g[i][k], g[j][k]);
            }
        }
        int t = power(g[i][i], mod - 2);
        for (int j = 0; j <= m; ++j)
            g[i][j] = 1ll * g[i][j] * t % mod;
    }
    for (int i = m; i < n; ++i)
        if (g[i][m]) return 0;
    for (int i = m - 1; i >= 0; --i) {
        int t = power(g[i][i], mod - 2);
        g[i][i] = 1;
        g[i][m] = 1ll * g[i][m] * t % mod;
```

```
50          for (int j = 0; j < i; ++j)
51              g[j][m] = (g[j][m] + mod - 1ll * g[i][m] * g[j][i] % mod) % mod;
52      }
53      printf("%d\n", m);
54      for (int i = 0; i < m; ++i)
55          printf("%d ", g[i][m]);
56      puts("");
57      for (int i = 0; i < m - 1; ++i)
58          printf("%d ", a[i]);
59      puts("1");
60      return 1;
61  }
62
63  int main() {
64      freopen("gauss.in", "r", stdin);
65      freopen("gauss.out", "w", stdout);
66      k = 0;
67      while (~scanf("%d", &a[k++])) ;
68      for (int sm = 1; sm <= k - sm; ++sm) {
69          n = k - sm - 1;
70          m = sm + 1;
71          for (int i = 0; i < n; ++i) {
72              for (int j = 0; j <= sm; ++j)
73                  g[i][j] = a[i + j];
74              g[i][m] = 1;
75              swap(g[i][m - 1], g[i][m]);
76          }
77          if (test()) return 0;
78      }
79      puts("no solution");
80      return 0;
81  }
```

```
1   //matrix.cpp
2   #include <bits/stdc++.h>
3   #define N 102
4   using namespace std;
5
6   int n, a[N];
7
8   int main() {
9       freopen("gauss.out", "r", stdin);
10      freopen("matrix.out", "w", stdout);
11      scanf("%d", &n);
12      for (int i = 0; i < n; ++i) scanf("%d", &a[i]);
13      printf("#define M %d\n", n);
14      printf("const int trans[M][M] = {\n");
15      for (int i = 0; i < n; ++i) {
16          printf("\t{");
17          for (int j = 0; j < n; ++j) {
```

```cpp
            int t;
            if (j < n - 2) t = i == j + 1;
            else if (j == n - 2) t = a[i];
            else t = i == n - 1;
            printf("%s%d", j == 0 ? "" : ",␣", t);
        }
        printf("}%s\n", i == n - 1 ? "" : ",");
    }
    printf("};\n");
    printf("const␣int␣pref[M]␣=␣{");
    for (int i = 0; i < n; ++i) {
        int x;
        scanf("%d", &x);
        printf("%d%s", x, i == n - 1 ? "};\n" : ",␣");
    }
    return 0;
}
```

```cpp
//main.cpp
#include <bits/stdc++.h>
using namespace std;

/* paste matrix.out here. */

#define mod 1000000007

struct Matrix {
    int c[M][M];
    void clear() { memset(c, 0, sizeof(c)); }
    void identity() { clear(); for (int i = 0; i < M; ++i) c[i][i] = 1; }
    void base() { memcpy(c, trans, sizeof(trans)); }
    friend Matrix operator * (const Matrix &a, const Matrix &b) {
        Matrix c; c.clear();
        for (int i = 0; i < M; ++i)
            for (int j = 0; j < M; ++j)
                for (int k = 0; k < M; ++k)
                    c.c[i][j] = (c.c[i][j] + 1ll * a.c[i][k] * b.c[k][j] % mod) %
                        mod;
        return c;
    }
} start, base;

Matrix power(Matrix base, int times) {
    Matrix ret; ret.identity();
    while (times) {
        if (times & 1) ret = ret * base;
        base = base * base;
        times >>= 1;
    }
    return ret;
```

```
32  }
33
34  int main() {
35      int tot;
36      scanf("%d", &tot);
37      while (tot--) {
38          int n;
39          scanf("%d", &n);
40          start.clear();
41          for (int i = 0; i < M; ++i) start.c[0][i] = pref[i];
42          base.base();
43          base = power(base, n - 1);
44          start = start * base;
45          printf("%d\n", start.c[0][0]);
46      }
47      return 0;
48  }
```

8.2 高精度计算

```
1   #include<algorithm>
2   using namespace std;
3   const int N_huge=850,base=100000000;
4   char s[N_huge*10];
5   struct huge{
6       typedef long long value;
7       value a[N_huge];int len;
8       void clear(){len=1;a[len]=0;}
9       huge(){clear();}
10      huge(value x){*this=x;}
11      huge operator =(huge b){
12          len=b.len;for (int i=1;i<=len;++i)a[i]=b.a[i]; return *this;
13      }
14      huge operator =(value x){
15          len=0;
16          while (x)a[++len]=x%base,x/=base;
17          if (!len)a[++len]=0;
18          return *this;
19      }
20      huge operator +(huge b){
21          int L=len>b.len?len:b.len;huge tmp;
22          for (int i=1;i<=L+1;++i)tmp.a[i]=0;
23          for (int i=1;i<=L;++i){
24              if (i>len)tmp.a[i]+=b.a[i];
25              else if (i>b.len)tmp.a[i]+=a[i];
26              else {
27                  tmp.a[i]+=a[i]+b.a[i];
28                  if (tmp.a[i]>=base){
29                      tmp.a[i]-=base;++tmp.a[i+1];
```

```
30                                    }
31                                }
32                            }
33              if (tmp.a[L+1])tmp.len=L+1;
34                  else tmp.len=L;
35              return tmp;
36          }
37      huge operator -(huge b){
38              int L=len>b.len?len:b.len;huge tmp;
39              for (int i=1;i<=L+1;++i)tmp.a[i]=0;
40              for (int i=1;i<=L;++i){
41                  if (i>b.len)b.a[i]=0;
42                  tmp.a[i]+=a[i]-b.a[i];
43                  if (tmp.a[i]<0){
44                      tmp.a[i]+=base;--tmp.a[i+1];
45                  }
46              }
47              while (L>1&&!tmp.a[L])--L;
48              tmp.len=L;
49              return tmp;
50          }
51      huge operator *(huge b){
52              int L=len+b.len;huge tmp;
53              for (int i=1;i<=L;++i)tmp.a[i]=0;
54              for (int i=1;i<=len;++i)
55                  for (int j=1;j<=b.len;++j){
56                      tmp.a[i+j-1]+=a[i]*b.a[j];
57                      if (tmp.a[i+j-1]>=base){
58                          tmp.a[i+j]+=tmp.a[i+j-1]/base;
59                          tmp.a[i+j-1]%=base;
60                      }
61                  }
62              tmp.len=len+b.len;
63              while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
64              return tmp;
65          }
66      pair<huge,huge> divide(huge a,huge b){
67              int L=a.len;huge c,d;
68              for (int i=L;i;--i){
69          c.a[i]=0;d=d*base;d.a[1]=a.a[i];
70                  int l=0,r=base-1,mid;
71                  while (l<r){
72                      mid=(l+r+1)>>1;
73                      if (b*mid<=d)l=mid;
74                          else r=mid-1;
75                  }
76                  c.a[i]=l;d-=b*l;
77              }
78              while (L>1&&!c.a[L])--L;c.len=L;
79              return make_pair(c,d);
```

```
 80          }
 81          huge operator /(value x){
 82              value d=0;huge tmp;
 83              for (int i=len;i;--i){
 84                  d=d*base+a[i];
 85                  tmp.a[i]=d/x;d%=x;
 86              }
 87              tmp.len=len;
 88              while (tmp.len>1&&!tmp.a[tmp.len])--tmp.len;
 89              return tmp;
 90          }
 91          value operator %(value x){
 92              value d=0;
 93              for (int i=len;i;--i)d=(d*base+a[i])%x;
 94              return d;
 95          }
 96          huge operator /(huge b){return divide(*this,b).first;}
 97          huge operator %(huge b){return divide(*this,b).second;}
 98          huge &operator +=(huge b){*this=*this+b;return *this;}
 99          huge &operator -=(huge b){*this=*this-b;return *this;}
100          huge &operator *=(huge b){*this=*this*b;return *this;}
101          huge &operator ++(){huge T;T=1;*this=*this+T;return *this;}
102          huge &operator --(){huge T;T=1;*this=*this-T;return *this;}
103          huge operator ++(int){huge T,tmp=*this;T=1;*this=*this+T;return tmp;}
104          huge operator --(int){huge T,tmp=*this;T=1;*this=*this-T;return tmp;}
105          huge operator +(value x){huge T;T=x;return *this+T;}
106          huge operator -(value x){huge T;T=x;return *this-T;}
107          huge operator *(value x){huge T;T=x;return *this*T;}
108          huge operator *=(value x){*this=*this*x;return *this;}
109          huge operator +=(value x){*this=*this+x;return *this;}
110          huge operator -=(value x){*this=*this-x;return *this;}
111          huge operator /=(value x){*this=*this/x;return *this;}
112          huge operator %=(value x){*this=*this%x;return *this;}
113          bool operator ==(value x){huge T;T=x;return *this==T;}
114          bool operator !=(value x){huge T;T=x;return *this!=T;}
115          bool operator <=(value x){huge T;T=x;return *this<=T;}
116          bool operator >=(value x){huge T;T=x;return *this>=T;}
117          bool operator <(value x){huge T;T=x;return *this<T;}
118          bool operator >(value x){huge T;T=x;return *this>T;}
119          bool operator <(huge b){
120              if (len<b.len)return 1;
121              if (len>b.len)return 0;
122              for (int i=len;i;--i){
123                  if (a[i]<b.a[i])return 1;
124                  if (a[i]>b.a[i])return 0;
125              }
126              return 0;
127          }
128          bool operator ==(huge b){
129              if (len!=b.len)return 0;
```

```
130        for (int i=len;i;--i)
131            if (a[i]!=b.a[i])return 0;
132        return 1;
133    }
134    bool operator !=(huge b){return !(*this==b);}
135    bool operator >(huge b){return !(*this<b||*this==b);}
136    bool operator <=(huge b){return (*this<b)||(*this==b);}
137    bool operator >=(huge b){return (*this>b)||(*this==b);}
138    void str(char s[]){
139        int l=strlen(s);value x=0,y=1;len=0;
140        for (int i=l-1;i>=0;--i){
141            x=x+(s[i]-'0')*y;y*=10;
142            if (y==base)a[++len]=x,x=0,y=1;
143        }
144        if (!len||x)a[++len]=x;
145    }
146    void read(){
147        scanf("%s",s);this->str(s);
148    }
149    void print(){
150        printf("%d",(int)a[len]);
151        for (int i=len-1;i;--i){
152            for (int j=base/10;j>=10;j/=10){
153                if (a[i]<j)printf("0");
154                    else break;
155            }
156            printf("%d",(int)a[i]);
157        }
158        printf("\n");
159    }
160 }f[1005];
161 int main(){
162    f[1]=f[2]=1;
163    for(int i=3;i<=1000;i++)f[i]=f[i-1]+f[i-2];
164 }
```

8.3 读入优化

```
1 #define rd RD<int>
2 #define rdll RD<long long>
3 template <typename Type>
4 inline Type RD() {
5     Type x = 0;
6     int flag = 0;
7     char c = getchar();
8     while (!isdigit(c) && c != '-')
9         c = getchar();
10    (c == '-') ? (flag = 1) : (x = c - '0');
11    while (isdigit(c = getchar()))
```

```
12          x = x * 10 + c - '0';
13      return flag ? -x : x;
14  }
15  inline char rdch() {
16      char c = getchar();
17      while (!isalpha(c)) c = getchar();
18      return c;
19  }
```

8.4   位运算及其运用

8.4.1   枚举子集

枚举 i 的非空子集 j

```
1  for (int j = i; j; j = (j - 1) & i);
```

8.4.2   求 1 的个数

```
1  int __builtin_popcount(unsigned int x);
```

8.4.3   求前缀 0 的个数

```
1  int __builtin_clz(unsigned int x);
```

8.4.4   求后缀 0 的个数

```
1  int __builtin_ctz(unsigned int x);
```