

String Theory Explanation of Large-Scale Anisotropy and Anomalous Alignment

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The discovery of anomalies in the cosmic microwave background (CMB) indicates large-scale anisotropies, non-Gaussian distributions, and anomalous alignments of the quadrupole and octupole modes of the anisotropy with each other and with both the ecliptic and equinoxes. Further analysis indicates that the statistical anisotropy and non-Gaussian temperature fluctuations are mainly due to long-range correlations. However, the source of the large-scale correlation and the cause of the anomalous alignment in CMB remains unknown. In this work, we show a new development in string theory, the universal wave function interpretation of string theory (UWFIST) indicates the existence of large-scale quantum vibrations. These large-scale quantum vibrations can cause the large-scale correlation and anomalous alignment observed in the background field. They can explain the observed large-scale anisotropies, non-Gaussian distributions, and anomalous alignments of the quadrupole and octupole modes in the microwave background.

Keywords: Dark energy; anomalies in cosmic microwave background; string theory; large-scale anisotropies; anomalous alignments of the quadrupole and octupole modes of the anisotropy; universal wave function interpretation of string theory.

1. Introduction

The cosmic microwave background (CMB) radiation is the thermal radiation existing in the whole universe at around the temperature of 2.725 K.^{1,2} Discovered in 1964 by radio telescope, the CMB radiation is found to be nearly isotropic. More accurate measurement reveals that the CMB is not exactly the same in all directions,

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it is anisotropic. There are small fluctuations in the temperature across the sky at the level of about 1 part in 100,000.^{3,4}

The CMB provides critical evidence for the big bang cosmology. It indicates that our universe started from a big bang.^{5,6} According to big bang cosmology, the CMB is the electromagnetic radiation left behind from the period of recombination. The recombination epoch is the time period when neutral atoms first formed. After the recombination epoch, photons started to travel freely through space. As the universe continues to expand, the CMB grows fainter and less energetic. The small anisotropies are generated by quantum fluctuations of matter in earlier time. Although many different processes might explain CMB's black body spectrum, no model other than the Big Bang has yet explained the fluctuations.

Hundreds of CMB experiments have been performed to measure and characterize signatures of the cosmic background radiation. Some of the most famous experiments are the NASA Cosmic Background Explorer (COBE) satellite which orbited in 1989–1996, the Wilkinson Microwave Anisotropy Probe (WMAP) which orbited in 2003–2010 and the European Space Agency's Planck Surveyor which orbited in 2009–2013. They detected anomalies in the CMB, including large-scale anisotropies, non-Gaussian distributions, and anomalous alignments.^{7–23} Current inflation theory can explain short-range anisotropies in the microwave background radiation, but it fails to explain large-scale anisotropies.

The most outstanding anomaly is the anomalous alignment of low- l multipole controversy.^{24–35} In the analysis on the behavior of the temperature anisotropy power spectrum, the temperature fluctuation is expanded in spherical harmonics. The first three lowest multipoles include dipole ($l = 1$), quadrupole ($l = 2$), and octupole ($l = 3$). It is found that the quadrupole and octupole ($l = 3$) modes of the anisotropy appear to have an unexplained alignment with each other and with the plane of the earth around the sun, i.e. both ecliptic, the circular course of sun, and equinox, the moment in which the plane of Earth's equator passes through the ecliptic.

Some people consider that the anomalous alignment of the low- l multipole with ecliptic and equinox violates the Copernican principle because it implies that the solar system is special. Land and Magueijo call this alignment the “axis of evil”.³⁵ A number of groups have suggested that this could be the signature of new physics at the greatest observable scales.³⁶

In this work, we propose a possible source and explanation of the large-scale anisotropies, non-Gaussian distributions, and anomalous alignments from a new development in string theory, the universal wave function interpretation of string theory (UWFIST).^{37,38}

String theory is a promising candidate for the grand unification theory.^{39,40} The grand unification theory, also called the theory of everything, is the attempt to use one mathematical formula to explain all the fundamental forces and matter discovered so far and to integrate quantum physics and general relativity together. Understanding and applying string theory has been a long-debated subject.

The currently accepted interpretation is that string theory studies the quantum dynamics of a string. The vibration of string creates particles and interactions. The scale of the string is set at the Planck scale (approximately 1.6×10^{-35} meter). String theory has the potential to predict the dimension of space-time, the particle and interaction spectrum, to unify all the forces and fundamental particles, and more. However, with all of this great potential and promise, current string theory has not yet made many testable predictions.

The UWFIST is derived from the space-time uncertainty relationship.³⁷ It suggests that the wave function of our universe is created from a string-like action. The derived wave function of the universe contains all the elementary particles, fundamental forces, as well as dark energy, dark matter, and the large structure of our universe. In our previous work,³⁷ we show that there exist long-range quantum vibrations in our universe. These long-range vibrations could be the source of dark energy. From the derived wave function of the universe, we can calculate the vacuum energy of the universe. We demonstrated that using the age of our universe and fundamental constants such as the gravitational constant G , the Planck constant \hbar , and the speed of light, UWFIST may yield an estimate of the dark energy consistent with the current experimental observation.³⁷ We also show³⁸ that the long-range vibration is the candidate for inflaton and provides the energy driving the expansion of the universe.

In this paper, we will briefly review the derivation of the UWFIST and the existence of large-scale quantum vibrations in our universe. We will show that these large-scale quantum vibrations can cause large-scale anisotropies, anomalous alignments, and non-Gaussian distributions in the CMB radiation as well as in other background matter and radiation, thus providing UWFIST an explanation for large-scale anisotropies, anomalous alignments, and non-Gaussian distributions from fundamental principles and theory.

2. A New Way to Derive and Interpret String Theory

In our previous work,³⁷ we show that when we take both the effect of quantum physics and gravity into consideration, we obtain an uncertainty relation between space measurement $\Delta\sigma$ and time measurement $\Delta\tau$ in the causal region:

$$\Delta\sigma\Delta\tau \geq l_p t_p, \quad (1)$$

where l_p is the Planck length, $l_p = (\hbar G/c^3)^{1/2} = 1.616 \times 10^{-35}$ m, and t_p is the Planck time,

$$t_p = (\hbar G/c^5)^{1/2} = 5.39 \times 10^{-44} \text{ s}.$$

We suggest that this relation indicates that space and time are quantized and the non-commutation relation between space and time:

$$[\sigma, \tau] = i l_p t_p. \quad (2)$$

Using the quantization procedure in quantum physics,⁴¹ we propose that the space time uncertainty relation (1) and non-commutation relation (2) indicates a string action A_s in the form:

$$A_s = \int d\tau d\sigma / l_p t_p. \quad (3)$$

Action A_s evolves the wave function from $\psi(0,0)$ at the space 0 and time 0 to $\psi(L, T)$ at the space L and time T in the following way:

$$\psi(L, T) = \Sigma_{\text{sum over all possible paths}} \exp\left(i \int_0^T d\tau \int_0^L d\sigma / l_p t_p\right) \psi(0, 0). \quad (4)$$

Equation (4) is the universal wave function interpretation of string theory. This unique interpretation of string theory was proposed first in our work.³⁷

UWFIST deviates from the usual interpretation of string theory in two ways. Firstly, the world-sheet space σ and time τ integration of the action (3) is over the causal region, i.e.,

$$A_s = \int_0^T d\tau \int_0^L d\sigma / l_p^2. \quad (5)$$

Here, T and L are the age and length of the causal horizon, respectively. They relate to the age of our universe. In the normal interpretation of string theory, σ is the string coordinate. L is taken to be the length of the string. It is set to be Planck length l_p . In formula (4), T and L are variables. They change as the universe evolves or as space and time pass by. The second difference between the usual string theory and UWFIST is that the universal wave function $\Psi(L, T)$ is introduced. Here, $\Psi(0,0)$ and $\Psi(L, T)$ represent the universal wave function at the initial space and time and at the space L and time T , respectively. We propose that the universal wave function could be the wave function of our universe. If we extend the string action to the cases of superstring and heterotic string with background fields, the universal wave function Ψ can include all fundamental particles, all gauge interactions, gravity, and more. It also includes the large structures of our universe up to the scale of horizon. In other words, the UWFIST could be the wave function of our universe. Except for these two major conceptual differences, the usual string theory calculation can still be applied in the UWFIST.

In the string theory, there are two sets of space-time: the 2-dimensional world-sheet (σ, τ) and the observed space-time $X^\mu(\sigma, \tau)$. The observed space-time $X^\mu(\sigma, \tau)$ is a projection from the world sheet to possibly a higher dimensional space-time. In terms of $X^\mu(\sigma, \tau)$, the action A_s in (3) becomes:

$$A_s = (1/l_p t_p) \int d\tau d\sigma g^{1/2} g^{ab} \partial a X_\mu \partial b X^\mu. \quad (6)$$

Here, $g^{\alpha\beta}$ is the metric tensor on the world-sheet and $g = -\det g^{\alpha\beta}$. As shown in string theory,^{39,40} the general form of action (6) in the presence of a massless background field is of the form:

$$A_s = \left[i\alpha \int_0^T d\tau \int_0^L d\sigma g^{1/2} (g^{\alpha\beta} G^{\mu\nu} \partial_\alpha X_\mu \partial_\beta X_\nu + \varepsilon^{\alpha\beta} B^{\mu\nu} \partial_\alpha X_\mu \partial_\beta X_\nu + 1/4\alpha\Phi R) \right]. \quad (7)$$

Here, $G^{\mu\nu}(\tau, \sigma)$, $B^{\mu\nu}(\tau, \sigma)$, $\Phi(\tau, \sigma)$ are metric tensor, anti-symmetric tensor, and scalar background fields on the observed space. And R is the scalar curvature related to $g^{\alpha\beta}$. The universal wave function in the presence of background fields is:

$$\begin{aligned} & \Psi(X^\mu(L, T), G^{\mu\nu}(L, T), B^{\mu\nu}(L, T), \Phi(L, T)) \\ &= \int DX^\mu Dg^{\mu\nu} DB^{\mu\nu} D\Phi \exp \left[i\alpha \int_0^T d\tau \int_0^L d\sigma g^{1/2} (g^{\alpha\beta} G^{\mu\nu} \partial_\alpha X_\mu \partial_\beta X_\nu \right. \\ & \quad \left. + \varepsilon^{\alpha\beta} B^{\mu\nu} \partial_\alpha X_\mu \partial_\beta X_\nu + 1/4\alpha\Phi R) \right]. \end{aligned} \quad (8)$$

We can extend $\Psi(X^\mu(L, T), G^{\mu\nu}(L, T), B^{\mu\nu}(L, T), \Phi(L, T))$ to include other massive vibrations. We can also extend the above string action to the cases of superstring and heterotic string to include all fundamental particles, all gauge interactions, gravity, and more. This universal wave function tells us the probability for our universe to be at space-time coordinate $X^\mu(L, T)$ and background fields $G^{\mu\nu}(L, T)$, $B^{\mu\nu}(L, T)$, $\Phi(L, T)$. We can do a Fourier transformation to transform the wave function $\Psi(X^\mu(L, T), G^{\mu\nu}(L, T), B^{\mu\nu}(L, T), \Phi(L, T))$ to be expressed in the vibrational space expressed by the various vibrations (ω^μ , $\kappa^{\mu\nu}$, ...). The wave function $\Psi(\omega^\mu(L, T), \kappa^{\mu\nu}(L, T), \dots)$ expresses what kind of vibrations exist in our universe and how much they exist at proper time T and horizon length L in our universe.

3. Long-Range Vibrations, Large-Distance Anisotropies and Anomalous Alignment

A natural indication and consequence of UWFIST is the existence of long-range vibrations. The space-time coordinate $X^\mu(\tau, \sigma)$ is composed of vibrations in the form $\exp[i\pi n(\sigma + c\tau)/L]$ and $\exp[i\pi n(\sigma - c\tau)/L]$. Here, n is an integer. Unlike the normal string theory, in which L is the Planck length, in UWFIST as we have explained before, L is the length of causal horizon. The largest wavelength is on the order of the length of the causal horizon. These vibrations are very fine in the sense that they vary in space and time very slowly therefore it is very difficult to detect them. In fact, to detect the vibration with the wavelength L , it takes the time L/c or it requires the detector that is at least on the order of L . If L is the horizon distance, this means that

it takes the time of the age of the visible universe or a detector as large as the whole visible universe to detect these vibrations. In our previous work,^{37,38} we showed that these long-range vibrations can be the source of dark energy and provide the way to derive and explain why the cosmological constant is so small. They are also potential candidates for inflaton and offer a new inflaton scheme that can be derived and calculated from string theory.

Here, we are going to show that these large-scale quantum vibrations can cause large-scale correlation. To demonstrate this, we calculate the correction function:

$$\langle \psi * (x, t) \psi(x + \Delta x, t + \Delta t) \rangle$$

that are created by the long-range vibrations $\exp[i\pi n(x + ct)/L]$ as an example. Since the experimental measurement is made over a range of time, let's say ΔT , we need to calculate

$$\int_0^{\Delta T} \langle \psi * (x, t) \psi(x + \Delta x, t + \Delta t) \rangle \sim e^{i\pi\Delta x/L} [e^{i\pi c\Delta T/L} - 1].$$

As L is on the scale of horizon, $c\Delta T/L$ is close to zero.

$$\int_0^{\Delta T} \langle \psi * (x, t) \psi(x + \Delta x, t + \Delta t) \rangle \sim e^{i\pi\Delta x/L} \sin(\pi c\Delta T/L) \sim (\pi c\Delta T/L) e^{i\pi\Delta x/L}. \quad (9)$$

The result (9) indicates that the correlation caused by the long-range vibration is non-zero. This correlation varies on the order of the length of the horizon L . It is proportional to the measurement time ΔT . Such correlation does not exist for the normal string theory or particle physics where L is very small. However, in the wave function interpretation of string theory, this large-distance correlation appears naturally in UWFIST. It is a direct and obvious derivation from UWFIST.

Some of the consequences of this large-distance correlation derived from UWFIST are that it can cause large-scale anisotropies, non-Gaussian distribution, and large-scale correlation in the CMB and other radiation or matter background.

It has been found that the statistical anisotropy and Gaussianity of temperature fluctuations is mainly due to the long-range correlations.^{41,42} There is also further evidence of correlations between the WMAP data and two all-sky probes of large-scale structure, the hard X-ray background observed by the HEAO-1 satellite and the NVSS survey of radio galaxies.^{41,42} This observation agrees with the UWFIST.

Because these long-range vibrations are quantum vibrations, their alignment is determined by the measurement. It is similar to the observation of the polarization of light. The observed polarization is determined by the direction of the polarizer. In the same way, the alignment of the quadrupole and octupole modes is determined by the measurement. Since all of our detectors are placed in the solar system and Earth in the ecliptic plane, the alignment of both quadrupole and octupole modes should be done with the ecliptic plane and equinox.

We conclude that UWFIST can explain the large-scale anisotropies, non-Gaussian distribution, and anomalous alignment. They are due to the large-scale quantum vibrations predicted in UWFIST. In fact, we suggest that the large-scale anisotropies, non-Gaussian distribution, and anomalous alignment provide the critical experimental evidence for UWFIST. Furthermore, UWFIST suggests that since the large-scale quantum vibrations are the source for dark energy, the large-scale anisotropies, non-Gaussian distribution, and anomalous alignment can also provide some detailed information about dark energy.

4. Discussion and Conclusion

In this paper, we show that the universal wave function formulation of string theory (UWFIST) can explain the large-scale anisotropies and the anomalous alignment of cosmic background radiation. We show that the long-range quantum vibrations, predicted in UWFIST, can cause the large-scale anisotropies and anomalous alignment in the microwave background and other matter and radiation background. The anomalous alignment with the solar system is due to that fact that the measurement is made in the solar system. The anomalous alignment does not violate the Copernican principle.

In our previous work,³⁹ we have shown that from UWFIST, we can estimate the vacuum energy and cosmological constant consistent with the experimental result using the age of the universe and the fundamental constants \hbar , G , and c . We suggest that the large-scale anisotropies and the anomalous alignment of CMB radiation and other matter and radiation provide further evidence and confirmation for the validity of UWFIST as a promising grand unified theory.

The calculation of the correlation function caused by the long-range vibration in (9) indicates that the correlation function is proportional to the measurement time ΔT . The experimental proof of this result can be used as a further validation of UWFIST as the grand unification theory. We will explore this in the future work.

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