Lecture 3: Union-ization (Post-Labor Day Weekend)

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CS 334 Automata and Computation
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OUTLINE: Putting DFAs Together

DFA A_1

DFA A_2

Definition of Computation

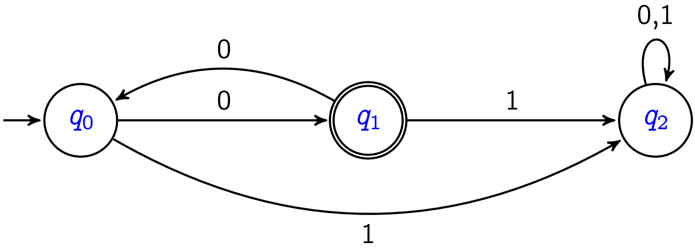
Regular Languages

DFA $A_1 \cup A_2$

Union-zation

A_1

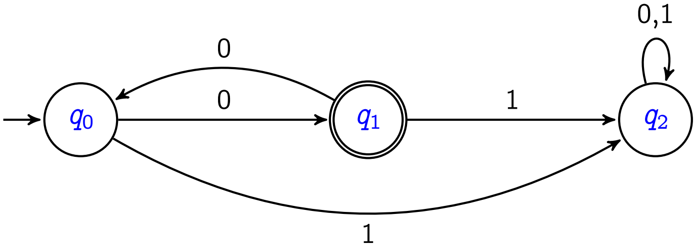
$$A_1 = (Q, \Sigma, \delta, q_0, F)$$



What is the language recognized by A_1?

A_1

$$A_1 = (Q, \Sigma, \delta, q_0, F)$$

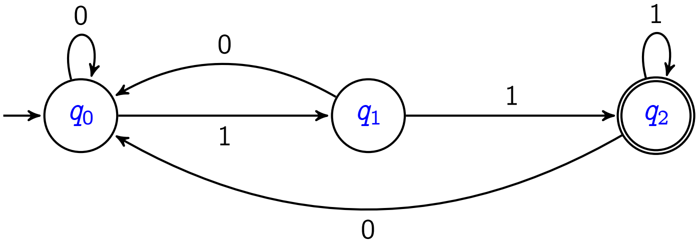


What is the language recognized by A_1? $L_1 = 0(00)^* = (00)^*0$

L_1 is the language of strings consisting only of an odd number of 0s.

A_2

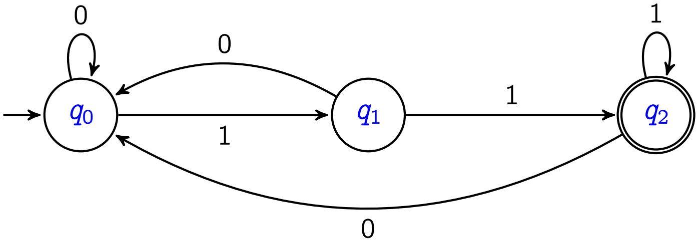
$$A_2 = (Q, \Sigma, \delta, q_0, F)$$



What is the language recognized by A_2?

A_2

$$A_2 = (Q, \Sigma, \delta, q_0, F)$$



What is the language recognized by A_2? $L_2 = \Sigma^* 11$

L_2 is the language of strings with 11 as suffix.

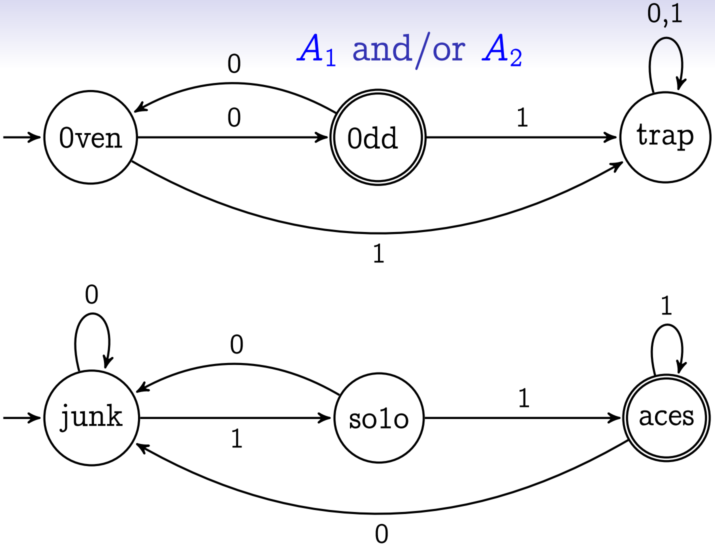
Definition of Computation with DFAs
Regular Languages

We can formalize our notion of *computation* much the way we formalized our notion of an automaton:
If $M = (Q, \Sigma, \delta, q_0, F)$ is a finite autoton, and $w = w_1 w_2 ... w_n$ is a string with $w_i \in \Sigma \forall i \in \{1, 2, ... n\}$, then M *accepts* w if a sequence of states $r_0 r_1 ... r_n$ exists in Q such that:

1. $r_0 = q_0$ (We start in the start state.),
2. $\delta(r_i, w_{i+1}) = r_{i+1} \quad \forall i \in \{0, 1, ..., n-1\}$, and
3. $r_n \in F$ (We end in a final (accepting) state.)

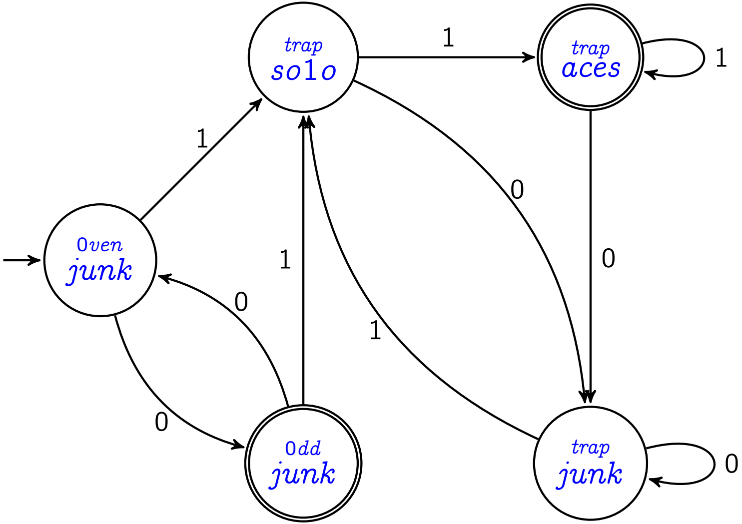
Definition: A language is a called a **regular language** if some finite automaton recognizes it.

A_1 and/or A_2



Can we put these together somehow to create a DFA that recognizes $A_1 \cup A_2$? That is, is $L_1 \cup L_2$ a regular language?

$A_1 \cup A_2$


trap
so1o
trap
aces
Oven
junk
Odd
junk
trap
junk
1
1
0
0
1
0
0
1
1
1
0

Union-ization

Given $A_1 = (Q_1, \Sigma, \delta_1, q_0^{A_1}, F_1)$ and $A_2 = (Q_2, \Sigma, \delta_2, q_0^{A_2}, F_2)$
We can form "$A_1 \cup A_2$" recognizing $L_1 \cup L_2$:
$A_1 \cup A_2 =$ (as a 5-tuple)
$(Q = Q_1 \times Q_2,$
$\Sigma = \Sigma$[1],
$\delta = \delta((s_1, s_2), a) = (\delta_1(s_1, a), \delta_2(s_2, a))$ with $a \in \Sigma$
$q_0 = (q_0^{A_1}, q_0^{A_2}),$
$F = \{(s_1, s_2) | s_1 \in F_1$ or $s_2 \in F_2\})$

Thus $L_1 \cup L_2$ is a regular language.
Note that if we take $F = F_1 \times F_2$ we get $L_1 \cap L_2$ is a regular language.

We have thus given constructions showing that the regular languages are closed under union and also under intersection.

[1] Here we assume the alphabets of both automata are the same. If they differ, we take their union: $\Sigma = \Sigma_{A_1} \cup \Sigma_{A_2}$