Introduction to Computer Science—Honors I

CS181—Fall 2014

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Programming assignment # 4

In this programming assignment, you will implement an image warping transformation by manipulating the pixels that make up a digital image.

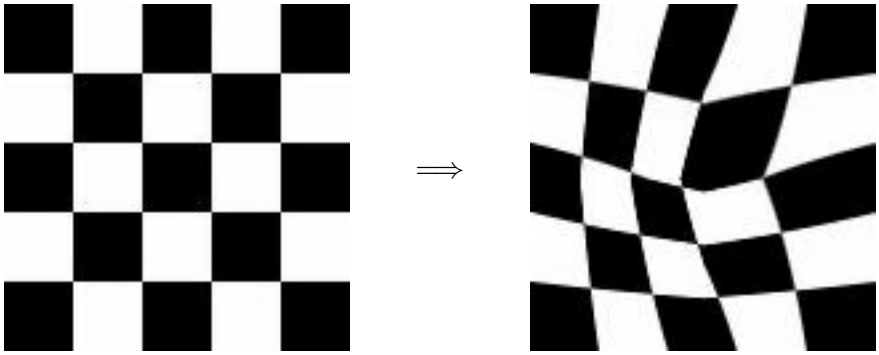
1 Requirement Specification

Your program shall take three command-line arguments: a String, representing the name of a file containing a JPEG image, and two integers, representing the (x, y) coordinates of a pixel in the image. Your program will manipulate the pixel array for the image (according to the algorithm described in the next section), and write the result in a file whose name is formed by prefixing the first command-line argument with the following string: "warped_".

Your program shall consists of a package named `assign4` that defines a class called `Main`, a class called `ImageTransform`, and possibly other auxiliary classes as you see fit. A sample invocation of your program should look like:

```
% java assign4.Main checkers.jpg 90 45
```

If "checkers.jpg" contains the image on the left below, then your program should create a file (named "warped_checkers.jpg") that contain an image like the one on the right:



To get you started with the program, we provide you with a (functionally complete) implementation of the `Main` class, and with a partial implementation of the `ImageTransform` class at:

```
http://www.cs.stevens.edu/~nicolosi/classes/14fa-cs181/assign4/Main.java
http://www.cs.stevens.edu/~nicolosi/classes/14fa-cs181/assign4/ImageTransform.java
```

Your main task is to provide the implementation of the following two methods:

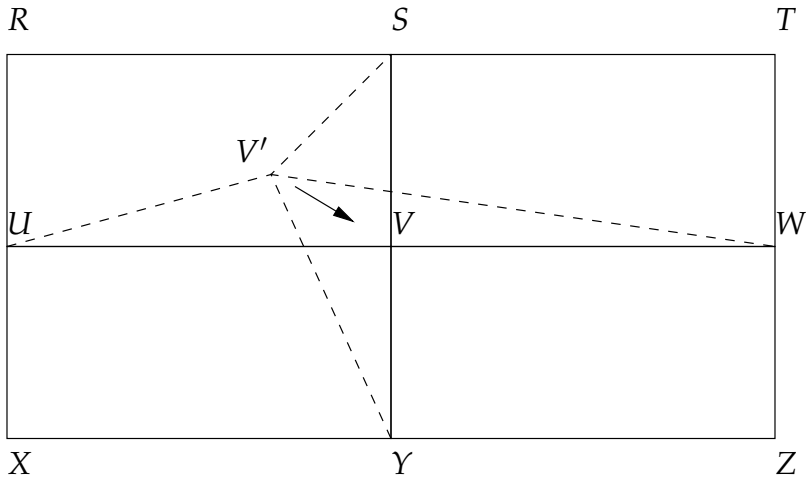
```
public int[] transform();
protected void warpRegion(int[] dstPixels, Rectangle r,
  Point nw, Point ne, Point sw, Point se);
```

As extra credit 1, additionally re-implement the `getPixel(double, double)` method to do "color interpolation" (*cf.* comments for `getPixel(double, double)`).

As extra credit 2, change your program so that the command line also specify a second (x, y) point to control the partitioning of the image into quadrants (*cf.* comments in `transform()`).

2 The Algorithm: Basic idea

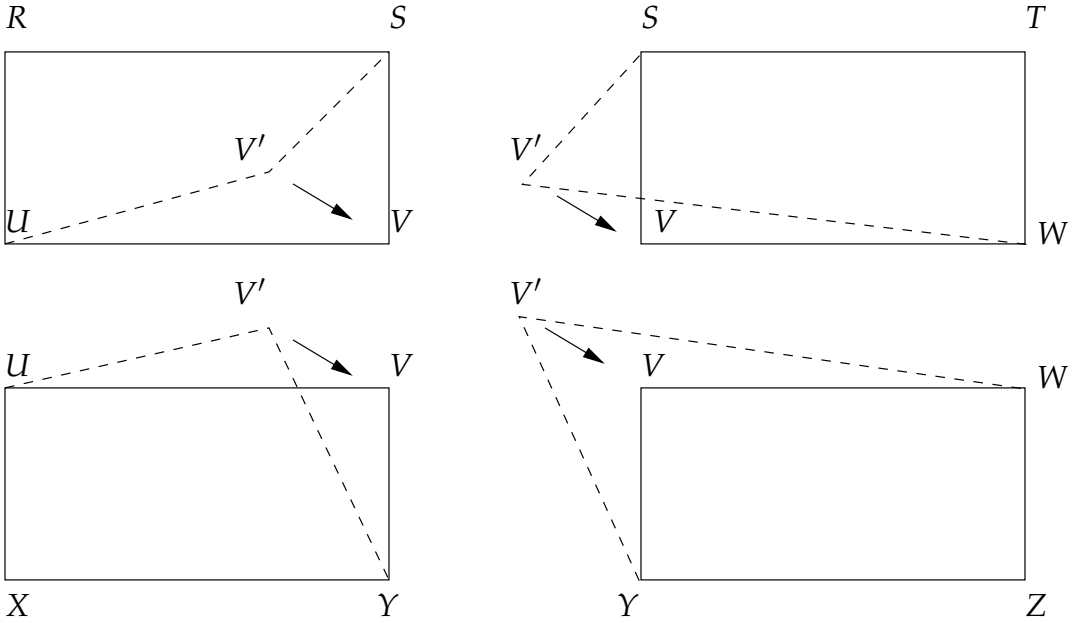
The idea behind image warping is to transform quadrilateral pixel regions into rectangular pixel regions, thereby distorting the original image.



To apply the transformation to a given image, the first step is to define the relevant quadrilaterals and the corresponding rectangles. The figure above depicts these. $RTZX$ denotes the pixel (sub-)region which will be warped. (In this assignment, this region will correspond to the entire image, but the transformation also makes sense if $RTZX$ does not cover the whole image area.) The point V determines how $RTZX$ is partitioned into four sub-rectangles, namely $RSVU$, $STWV$, $VWZY$, and $UVYX$. By default, we will assume that the partitioning point V is the midpoint of $RTZX$. (Extra-credit 2 asks you to read the partitioning point from the command line.)

The warping is also controlled by an arbitrary point V' inside the $RTZX$ rectangle.[1] The control point V' partitions $RTZX$ into four other quadrilaterals: $RSV'U$, $STWV'$, $V'WZY$, and $UV'YX$. Intuitively, the essence of warping is to stretch things so that V' gets "moved" toward V.

The picture below illustrates how this setup decomposes as four quadrilateral-to-rectangle mappings. The direction of the mappings is represented by the arrow. In detail, these mappings are: $RSV'U \mapsto RSVU$, $STWV' \mapsto STWV$, $V'WZY \mapsto VWZY$, and $UV'YX \mapsto UVYX$.



To summarize, to warp the pixel region $RTZX$, we simply warp each of the four sub-regions.

[1] The warping transformation can be generalized so that it makes sense also when the control point V' is outside of $RTZX$, but we will not need that for this assignment.

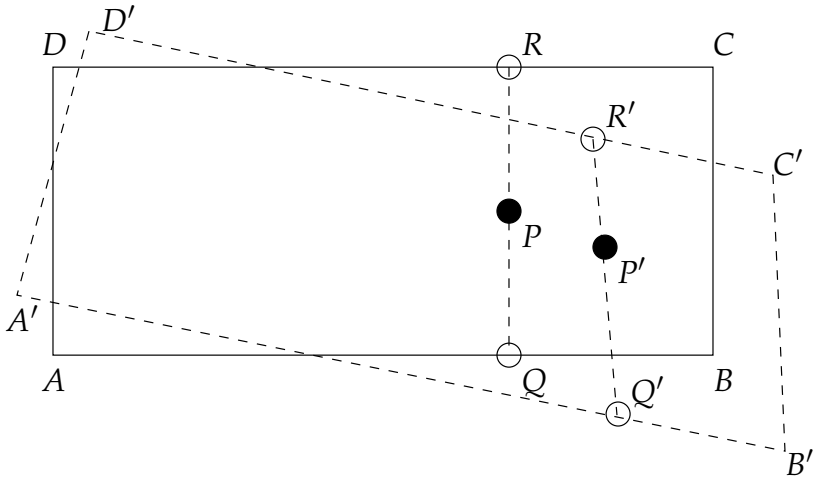
3 The Algorithm: The details

Your code for the `warpRegion(...)` method should implement the quadrilateral-to-rectangle transformation. Suppose we want to perform the warping transformation $V'WZY \mapsto VWZY$ (all other cases are analogous). Then, we must map each pixel of $V'WZY$ into a corresponding pixel of $VWZY$. But how do we iterate over each pixel of $V'WZY$? This seems like a difficult task, since image pixels are typically positioned on a grid. Taking the path of least resistance, we will iterate over the pixels in $VWZY$, and compute the corresponding pixel in $V'WZY$.

The next subsection derives this mapping, which aims at expressing a pixel in the destination shape (the rectangle) in terms of a (fractionary) pixel in the origin shape (the quadrilateral). You might want to skip it upon a first reading, and return to this after seeing the following subsection about how to use the mapping in your code.

3.1 Deriving the mapping

For completeness, we consider the general case of a quadrilateral-to-rectangle mapping, where all the corners of the quadrilateral might be distinct from those in the rectangle.



Given a pixel $P = (x, y)$ inside $DCBA$, what should the location of the corresponding pixel inside $D'C'B'A'$ be? Let it be $P' = (x', y')$, and we will compute it using linear interpolation. First, the easy cases. If the point happens to be D, C, B, or A, then the corresponding point is D', C', B', or A', respectively. If the point happens to lie anywhere directly on the $\overline{DC}$ segment (denoted as R in the diagram), then the corresponding point R' must lie directly on the $\overline{D'C'}$ segment. Using linear interpolation (*e.g.*, see `http://en.wikipedia.org/wiki/Linear_interpolation`), we obtain:

$$R' = D' + (C' - D')\frac{|\overline{RD}|}{|\overline{CD}|}$$

where $|\cdot|$ denotes the length of the given segment. (Note the above equation is stated in terms of points rather than coordinates. This is just a short-hand notation—if $P_1 = (x_1, y_1)$, $P_2 = (x_2, y_2)$, then $P_1 = P_2$ is equivalent to $x_1 = x_2$ and $y_1 = y_2$.)

Similarly, if the point happens to lie anywhere directly on the segment $\overline{AB}$ (denoted as Q in the diagram), then:

$$Q' = A' + (B' - A')\frac{|\overline{QA}|}{|\overline{BA}|}$$

Now for the generic case. If P does not lie directly on either $\overline{DC}$ or $\overline{AB}$, then let R and Q be the projections of P onto $\overline{DC}$, and $\overline{AB}$, respectively. Using the above two equations, first we compute the

corresponding R and Q inside $D'C'B'A'$. Combining the equations for R' and Q', and performing yet another linear interpolation along the $\overline{R'Q'}$ segment, we obtain:

$$P' = R' + (Q' - R')\frac{|\overline{PR}|}{|\overline{QR}|}$$

Now, let us do a bit of simplification. Observe that $|\overline{RD}| = |\overline{QA}|$, $|\overline{CD}| = |\overline{BA}|$, and $|\overline{QR}| = |\overline{CB}|$, since $DCBA$ is a rectangle. Let $\alpha = \frac{|\overline{RD}|}{|\overline{CD}|}$, and $\beta = \frac{|\overline{PR}|}{|\overline{CB}|}$, The above equations for R', Q' and P' can then be rewritten as:

$$R' = D' + (C' - D')\alpha \tag{1}$$
$$Q' = A' + (B' - A')\alpha \tag{2}$$
$$P' = R' + (Q' - R')\beta \tag{3}$$

Since $|\overline{RD}| \leq |CD|$ and $|\overline{PR}| \leq |\overline{CB}|$, we have that $\alpha, \beta \in [0, 1]$. Furthermore, α and β are the only quantities in the above equations that change with P—everything else is independent of the specific point P at hand.

3.2 Using the mapping

Substituting Equation (1) and Equation (2) into Equation (3), we get:

$$P' = D' + (C' - D')\alpha + (A' - D')\beta + (D' - C' + B' - A')\alpha\beta$$

Now to map each pixel from $D'C'B'A'$ into $DCBA$, iterate over each pixel P in $DCBA$, compute P' and store the color value of the pixel at P' at the location P . If we let c run over pixel columns and r run over pixel rows, then $\alpha = c/n$, and $\beta = r/m$, where n, m are the number of columns and rows, respectively. Therefore, the color of pixel $P = (D.x + c, D.y + r)$ should be set to the color of pixel $(P'.x, P'.y)$ as given by the above formula. (Again, recall that an equation over points really just amounts to two equations—one in the x's and one in the y's.)