

(a) Give the 95% confidence interval.

(b) If you wanted 99% confidence for the same study, would your margin of error be greater than, equal to, or less than 8? Explain your answer.

6.13 Changing the sample size. Consider the setting of the previous exercise. Suppose that the sample mean is again 73 and the population standard deviation is 28. Make a diagram similar to Figure 6.5 (page 361) that illustrates the effect of sample size on the width of a 95% interval. Use the following sample sizes: 10, 20, 40, and 80. Summarize what the diagram shows.

6.14 Changing the confidence level. Consider the setting of the previous two exercises. Suppose that the sample mean is still 73, the sample size is 49, and the population standard deviation is 28. Make a diagram similar to Figure 6.6 (page 363) that illustrates the effect of the confidence level on the width of the interval. Use 80%, 90%, 95%, and 99%. Summarize what the diagram shows.

6.15 Confidence interval mistakes and misunderstandings. Suppose that 500 randomly selected alumni of the University of Okoboji were asked to rate the university's academic advising services on a 1 to 10 scale. The sample mean \bar{x} was found to be 8.6. Assume that the population standard deviation is known to be $\sigma = 2.2$.

(a) Ima Bitlost computes the 95% confidence interval for the average satisfaction score as $8.6 \pm 1.96(2.2)$. What is her mistake?

(b) After correcting her mistake in part (a), she states, "I am 95% confident that the sample mean falls between 8.4 and 8.8." What is wrong with this statement?

(c) She quickly realizes her mistake in part (b) and instead states, "The probability that the true mean is between 8.4 and 8.8 is 0.95." What misinterpretation is she making now?

(d) Finally, in her defense for using the Normal distribution to determine the confidence interval she says, "Because the sample size is quite large, the population of alumni ratings will be approximately Normal." Explain to Ima her misunderstanding and correct this statement.

6.16 More confidence interval mistakes and misunderstandings. Suppose that 100 randomly selected members of the Karaoke Channel were asked how much time they typically spend on the site during the week.⁷ The sample mean \bar{x} was found to be 3.8 hours. Assume that the population standard deviation is known to be $\sigma = 2.9$.

(a) Cary Oakey computes the 95% confidence interval for the average time on the site as $3.8 \pm 1.96(2.9/100)$. What is his mistake?

(b) He corrects this mistake and then states that "95% of the members spend between 3.23 and 4.37 hours a week

on the site." What is wrong with his interpretation of this interval?

(c) The margin of error is slightly larger than half an hour. To reduce this to roughly 15 minutes, Cary says that the sample size needs to be doubled to 200. What is wrong with this statement?

6.17 The state of stress in the United States. Since 2007, the American Psychological Association has supported an annual nationwide survey to examine stress across the United States.⁸ A total of 340 Millennials (18- to 33-year-olds) were asked to indicate their average stress level (on a 10-point scale) during the past month. The mean score was 5.4. Assume that the population standard deviation is 2.3.

(a) Give the margin of error and find the 95% confidence interval for this sample.

(b) Repeat these calculations for a 99% confidence interval. How do the results compare with those in part (a)?

6.18 Inference based on integer values. Refer to Exercise 6.17. The data for this study are integer values between 1 and 10. Explain why the confidence interval based on the Normal distribution should be a good approximation.

6.19 Mean TRAP in young women. For many important processes that occur in the body, direct measurement of characteristics of the process is not possible. In many cases, however, we can measure a *biomarker*, a biochemical substance that is relatively easy to measure and is associated with the process of interest. Bone turnover is the net effect of two processes: the breaking down of old bone, called resorption, and the building of new bone, called formation. One biochemical measure of bone resorption is tartrate-resistant acid phosphatase (TRAP), which can be measured in blood. In a study of bone turnover in young women, serum TRAP was measured in 31 subjects.⁹ The mean was 13.2 units per liter (U/l). Assume that the standard deviation is known to be 6.5 U/l. Give the margin of error and find a 95% confidence interval for the mean TRAP amount in young women represented by this sample.

6.20 Mean OC in young women. Refer to the previous exercise. A biomarker for bone formation measured in the same study was osteocalcin (OC), measured in the blood. For the 31 subjects in the study, the mean was 33.4 nanograms per milliliter (ng/ml). Assume that the standard deviation is known to be 19.6 ng/ml. Report the 95% confidence interval.

6.21 Populations sampled and margins of error.

Consider the following two scenarios. (A) Take a simple random sample of 100 sophomore students at your college or university. (B) Take a simple random sample of 100 students at your college or university. For each of

these samples you will record the amount spent on textbooks used for classes during the fall semester. Which sample should have the smaller margin of error? Explain your answer.

 **6.22 Average starting salary.** The National Association of Colleges and Employers (NACE) Fall Salary Survey shows that the current class of college graduates received an average starting-salary offer of \$44,259.¹⁰ Your institution collected an SRS ($n = 400$) of its recent graduates and obtained a 95% confidence interval of (\$44,793, \$47,157). What can we conclude about the *difference* between the average starting salary of recent graduates at your institution and the overall NACE average? Write a short summary.

6.23 Consumption of sugar-sweetened beverages. A recent study estimated that the U.S. per capita consumption of sugar-sweetened beverages among adults aged 20 to 34 years is 338 kilocalories per day (kcal/d).¹¹ Suppose that the population distribution is heavily skewed, with a standard deviation equal to 300 kcal/d. If you plan to take an SRS of 1000 young adults,

- (a) the 68–95–99.7 rule says that the probability is about 0.95 that \bar{x} is within _____ kcal/d of the population mean μ . (Fill in the blank.)
- (b) about 95% of all samples will capture the true mean of kilocalories consumed per day in the interval \bar{x} plus or minus _____ kcal/d. (Fill in the blank.)

6.24 Apartment rental rates. You want to rent an unfurnished two-bedroom apartment in Dallas next year. The mean monthly rent for a random sample of 10 apartments advertised in the local newspaper is \$1050. Assume that the monthly rents in Dallas follow a Normal distribution with a standard deviation of \$220. Find a 95% confidence interval for the mean monthly rent for unfurnished two-bedroom apartments available in Dallas.

6.25 More on apartment rental rates. Refer to the previous exercise. Will the 95% confidence interval include approximately 95% of the rents for all unfurnished two-bedroom apartments in this area? Explain why or why not.

 **6.26 Inference based on skewed data.** The mean OC for the 31 subjects in Exercise 6.20 was 33.4 ng/ml. In our calculations, we assumed that the standard deviation was known to be 19.6 ng/ml. Use the 68–95–99.7 rule from Chapter 1 (page xx) to find the approximate bounds on the values of OC that would include these percents of the population. If the assumed standard deviation is correct, this distribution may be highly skewed. Why? (Hint: The measured values for a variable such as this are all positive.) Do you think that this skewness will invalidate the use of the Normal confidence interval in this case? Explain your answer.

6.27 Average hours per week listening to the radio. The *Student Monitor* surveys 1200 undergraduates from four-year colleges and universities throughout the United States semiannually to understand trends among college students.¹² Recently, the *Student Monitor* reported that the average amount of time listening to the radio per week was 11.5 hours. Of the 1200 students surveyed, 83% said that they listened to the radio, so this collection of listening times has around 204 ($17\% \times 1200$) zeros. Assume that the standard deviation is 8.3 hours.

- (a) Give a 95% confidence interval for the mean time spent per week listening to the radio.
- (b) Is it true that 95% of the 1200 students reported weekly times that lie in the interval you found in part (a)? Explain your answer.
- (c) It appears that the population distribution has many zeros and is skewed to the right. Explain why the confidence interval based on the Normal distribution should nevertheless be a good approximation.

6.28 Average minutes per week listening to the radio. Refer to the previous exercise.

- (a) Give the mean and standard deviation in minutes.
- (b) Calculate the 95% confidence interval in minutes from your answer to part (a).
- (c) Explain how you could have directly calculated this interval from the 95% interval that you calculated in the previous exercise.

6.29 Satisfied with your job? Job satisfaction is one of four workplace measures that the Gallup-Healthways Well-Being Index tracks among U.S. workers. The question asked is “Are you satisfied or dissatisfied with your job or the work that you do?” In 2011, 87.5% responded that they were satisfied. Material provided with the results of the poll noted:

Results are based on telephone interviews conducted as part of the Gallup-Healthways Well-Being Index survey Jan. 1–April 30, 2011, with a random sample of 61,889 adults, aged 18 and older, living in all 50 U.S. states and the District of Columbia, selected using random-digit-dial sampling.

For results based on the total sample of national adults, one can say with 95% confidence that the maximum margin of sampling error is 1 percentage point.¹³

The poll uses a complex multistage sample design, but the sample percent has approximately a Normal sampling distribution.

- (a) The announced poll result was $87.5\% \pm 1\%$. Can we be certain that the true population percent falls in this interval? Explain your answer.

- (c) A study summary says that the results are statistically significant and the P -value is 0.98.
- (d) The z test statistic is equal to 0.018. Because this is less than $\alpha = 0.05$, the null hypothesis was rejected.

6.54 Determining hypotheses. State the appropriate null hypothesis H_0 and alternative hypothesis H_a in each of the following cases.

- (a) A 2010 study reported that 88% of students owned a cell phone. You plan to take an SRS of students to see if the percent has increased.
- (b) The examinations in a large freshman chemistry class are scaled after grading so that the mean score is 75. The professor thinks that students who attend early-morning recitation sections will have a higher mean score than the class as a whole. Her students in these sections this semester can be considered a sample from the population of all students who might attend an early-morning section, so she compares their mean score with 75.
- (c) The student newspaper at your college recently changed the format of its opinion page. You want to test whether students find the change an improvement. You take a random sample of students and select those who regularly read the newspaper. They are asked to indicate their opinions on the changes using a five-point scale: -2 if the new format is much worse than the old, -1 if the new format is somewhat worse than the old, 0 if the new format is the same as the old, $+1$ if the new format is somewhat better than the old, and $+2$ if the new format is much better than the old.

6.55 More on determining hypotheses. State the null hypothesis H_0 and the alternative hypothesis H_a in each case. Be sure to identify the parameters that you use to state the hypotheses.

- (a) A university gives credit in first-year calculus to students who pass a placement test. The mathematics department wants to know if students who get credit in this way differ in their success with second-year calculus. Scores in second-year calculus are scaled so the average each year is equivalent to a 77. This year 21 students who took second-year calculus passed the placement test.
- (b) Experiments on learning in animals sometimes measure how long it takes a mouse to find its way through a maze. The mean time is 20 seconds for one particular maze. A researcher thinks that playing rap music will cause the mice to complete the maze more slowly. She measures how long each of 12 mice takes with the rap music as a stimulus.
- (c) The average square footage of one-bedroom apartments in a new student-housing development is advertised to be 880 square feet. A student group thinks that the apartments are smaller than advertised. They hire an

engineer to measure a sample of apartments to test their suspicion.

6.56 Even more on determining hypotheses. In each of the following situations, state an appropriate null hypothesis H_0 and alternative hypothesis H_a . Be sure to identify the parameters that you use to state the hypotheses. (We have not yet learned how to test these hypotheses.)

- (a) A sociologist asks a large sample of high school students which television channel they like best. She suspects that a higher percent of males than of females will name MTV as their favorite channel.
- (b) An education researcher randomly divides sixth-grade students into two groups for physical education class. He teaches both groups basketball skills, using the same methods of instruction in both classes. He encourages Group A with compliments and other positive behavior but acts cool and neutral toward Group B. He hopes to show that positive teacher attitudes result in a higher mean score on a test of basketball skills than do neutral attitudes.
- (c) An education researcher believes that among college students there is a negative correlation between time spent at social network sites and self-esteem, measured on a 0 to 100 scale. To test this, she gathers social-networking information and self-esteem data from a sample of students at your college.

6.57 Translating research questions into hypotheses. Translate each of the following research questions into appropriate H_0 and H_a .

(a) U.S. Census Bureau data show that the mean household income in the area served by a shopping mall is \$42,800 per year. A market research firm questions shoppers at the mall to find out whether the mean household income of mall shoppers is higher than that of the general population.

(b) Last year, your online registration technicians took an average of 0.4 hours to respond to trouble calls from students trying to register. Do this year's data show a different average response time?

6.58 Computing the P -value. A test of the null hypothesis $H_0: \mu = \mu_0$ gives test statistic $z = 1.77$.

- (a) What is the P -value if the alternative is $H_a: \mu > \mu_0$?
 (b) What is the P -value if the alternative is $H_a: \mu < \mu_0$?
 (c) What is the P -value if the alternative is $H_a: \mu \neq \mu_0$?

6.59 More on computing the P -value. A test of the null hypothesis $H_0: \mu = \mu_0$ gives test statistic $z = -1.69$.

- (a) What is the P -value if the alternative is $H_a: \mu > \mu_0$?
 (b) What is the P -value if the alternative is $H_a: \mu < \mu_0$?
 (c) What is the P -value if the alternative is $H_a: \mu \neq \mu_0$?

Test your hypotheses, report your results, and write a short summary of what you have found.

 **6.69 Are the pine trees randomly distributed from east to west?** Answer the questions in the previous exercise for the east–west direction, for which the sample mean is 113.8.

6.70 Who is the author? Statistics can help decide the authorship of literary works. Sonnets by a certain Elizabethan poet are known to contain an average of $\mu = 8.9$ new words (words not used in the poet's other works). The standard deviation of the number of new words is $\sigma = 2.5$. Now a manuscript with six new sonnets has come to light, and scholars are debating whether it is the poet's work. The new sonnets contain an average of $\bar{x} = 10.2$ words not used in the poet's known works. We expect poems by another author to contain more new words, so to see if we have evidence that the new sonnets are not by our poet we test

$$H_0: \mu = 8.9$$

$$H_a: \mu > 8.9$$

Give the z test statistic and its P -value. What do you conclude about the authorship of the new poems?

6.71 Attitudes toward school. The Survey of Study Habits and Attitudes (SSHA) is a psychological test that measures the motivation, attitude toward school, and study habits of students. Scores range from 0 to 200. The mean score for U.S. college students is about 115, and the standard deviation is about 30. A teacher who suspects that older students have better attitudes toward school gives the SSHA to 25 students who are at least 30 years of age. Their mean score is $\bar{x} = 127.8$.

(a) Assuming that $\sigma = 30$ for the population of older students, carry out a test of

$$H_0: \mu = 115$$

$$H_a: \mu > 115$$

Report the P -value of your test, and state your conclusion clearly.

(b) Your test in part (a) required two important assumptions in addition to the assumption that the value of σ is known. What are they? Which of these assumptions is most important to the validity of your conclusion in part (a)?

6.72 Nutritional intake among Canadian high-performance athletes. Since previous studies have reported that elite athletes are often deficient in their nutritional intake (for example, total calories, carbohydrates, protein), a group of researchers decided to evaluate Canadian high-performance athletes.²⁴ A total of $n = 324$ athletes from eight Canadian sports centers participated in the study. One reported finding was that

the average caloric intake among the $n = 201$ women was 2403.7 kilocalories per day (kcal/d). The recommended amount is 2811.5 kcal/d. Is there evidence that female Canadian athletes are deficient in caloric intake?

- (a) State the appropriate H_0 and H_a to test this.
 (b) Assuming a standard deviation of 880 kcal/d, carry out the test. Give the P -value, and then interpret the result in plain language.

6.73 Are the measurements similar? Refer to Exercise 6.30 (page 371). In addition to the computer's calculations of miles per gallon, the driver also recorded the miles per gallon by dividing the miles driven by the number of gallons at each fill-up. The following data are the differences between the computer's and the driver's calculations for that random sample of 20 records. The driver wants to determine if these calculations are different. Assume that the standard deviation of a difference is $\sigma = 3.0$. 

5.0	6.5	-0.6	1.7	3.7	4.5	8.0	2.2	4.9	3.0
4.4	0.1	3.0	1.1	1.1	5.0	2.1	3.7	-0.6	-4.2

- (a) State the appropriate H_0 and H_a to test this suspicion.
 (b) Carry out the test. Give the P -value, and then interpret the result in plain language.

6.74 Adjusting for the cost of living. In Example 6.9 (page 373), we compared the average credit card balance between undergraduates in the Midwest and the South. In testing the difference, we considered a one-sided test because the cost of living is higher in the South (Example 6.14). Assuming that \$1 in the Midwest is worth about \$1.09 in the South, test whether there is a difference between the average balances in the two regions using South dollars. For simplicity, assume that the standard deviation is unchanged.

6.75 Nicotine content in cigarettes. According to data from the Tobacco Institute Testing Laboratory, Camel Lights king size cigarettes contain an average of 0.61 milligrams of nicotine. An advocacy group commissions an independent test to see if the mean nicotine content is higher than the industry laboratory claims.

- (a) What are H_0 and H_a ?
 (b) Suppose that the test statistic is $z = 1.72$. Is this result significant at the 5% level?
 (c) Is the result significant at the 1% level?

 **6.76 Impact of \bar{x} on significance.** The *Statistical Significance* applet illustrates statistical tests with a fixed level of significance for Normally distributed data with known standard deviation. Open the applet and keep the default settings for the null ($\mu = 0$) and the alternative ($\mu > 0$) hypotheses, the sample size ($n = 10$), the

the vitamin C group. Can we conclude that vitamin C has a strong effect in preventing colds? Explain your answer.

6.97 How far do rich parents take us? How much education children get is strongly associated with the wealth and social status of their parents, termed “socio-economic status,” or SES. The SES of parents, however, has little influence on whether children who have graduated from college continue their education. One study looked at whether college graduates took the graduate admissions tests for business, law, and other graduate programs. The effects of the parents’ SES on taking the LSAT test for law school were “both statistically insignificant and small.”

- (a) What does “statistically insignificant” mean?
- (b) Why is it important that the effects were small in size as well as statistically insignificant?

6.98 Do you agree? State whether or not you agree with each of the following statements and provide a short summary of the reasons for your answers.

- (a) If the P -value is larger than 0.05, the null hypothesis is true.
- (b) Practical significance is not the same as statistical significance.
- (c) We can perform a statistical analysis using any set of data.
- (d) If you find an interesting pattern in a set of data, it is appropriate to then use a significance test to determine its significance.
- (e) It’s always better to use a significance level of $\alpha = 0.05$ than to use $\alpha = 0.01$ because it is easier to find statistical significance.

6.99 Practical significance and sample size. Every user of statistics should understand the distinction between statistical significance and practical importance. A sufficiently large sample will declare very small effects statistically significant. Consider the study of elite female Canadian athletes in Exercise 6.72 (page 393). Female athletes were consuming an average of 2403.7 kcal/d with a standard deviation of 880 kcal/d. Suppose that a nutritionist is brought in to implement a new health program for these athletes. This program should increase mean caloric intake but not change the standard deviation. Given the standard deviation and how calorie deficient these athletes are, a change in the mean of 50 kcal/d to 2453.7 is of little importance. However, with a large enough sample, this change can be significant. To see this, calculate the P -value for the test of

$$H_0: \mu = 2403.7$$

$$H_a: \mu > 2403.7$$

in each of the following situations:

- (a) A sample of 100 athletes; their average caloric intake is $\bar{x} = 2453.7$.
- (b) A sample of 500 athletes; their average caloric intake is $\bar{x} = 2453.7$.
- (c) A sample of 2500 athletes; their average caloric intake is $\bar{x} = 2453.7$.

6.100 Statistical versus practical significance. A study with 7500 subjects reported a result that was statistically significant at the 5% level. Explain why this result might not be particularly important.

6.101 More on statistical versus practical significance. A study with 14 subjects reported a result that failed to achieve statistical significance at the 5% level. The P -value was 0.051. Write a short summary of how you would interpret these findings.

 **6.102 Find journal articles.** Find two journal articles that report results with statistical analyses. For each article, summarize how the results are reported and write a critique of the presentation. Be sure to include details regarding use of significance testing at a particular level of significance, P -values, and confidence intervals.

6.103 Create an example of your own. For each of the following cases, provide an example and an explanation as to why it is appropriate.

- (a) A set of data or an experiment for which statistical inference is not valid.
- (b) A set of data or an experiment for which statistical inference is valid.

 **6.104 Predicting success of trainees.** What distinguishes managerial trainees who eventually become executives from those who, after expensive training, don’t succeed and leave the company? We have abundant data on past trainees—data on their personalities and goals, their college preparation and performance, even their family backgrounds and their hobbies. Statistical software makes it easy to perform dozens of significance tests on these dozens of variables to see which ones best predict later success. We find that future executives are significantly more likely than washouts to have an urban or suburban upbringing and an undergraduate degree in a technical field.

Explain clearly why using these “significant” variables to select future trainees is not wise. Then suggest a follow-up study using this year’s trainees as subjects that should clarify the importance of the variables identified by the first study.

6.105 Searching for significance. Give an example of a situation where searching for significance would lead to misleading conclusions.

changes, explain what happens to the power for each alternative μ in the table.

(a) Change to the two-sided alternative.

(b) Increase σ to 2.

(c) Increase n from 10 to 20.

6.116 Power of the random north-south distribution of trees test.

In Exercise 6.68 (page 392) you performed a two-sided significance test of the null hypothesis that the average north-south location of the longleaf pine trees sampled in the Wade Tract was $\mu = 100$. There were 584 trees in the sample and the standard deviation was assumed to be 58. The sample mean in that analysis was $\bar{x} = 99.74$. Use the *Statistical Power* applet to compute the power for the alternative $\mu = 99$ using a two-sided test at the 5% level of significance.

6.117 Power of the random east-west distribution of trees test.

Refer to the previous exercise. Note that in the east-west direction, the average location was 113.8. Use the *Statistical Power* applet to find the power for the alternative $\mu = 110$.

6.118 Planning another test to compare consumption.

Example 6.15 (page 383) gives a test of a hypothesis about the mean consumption of sugar-sweetened beverages at your university based on a sample of size $n = 100$. The hypotheses are

$$H_0: \mu = 286$$

$$H_a: \mu \neq 286$$

While the result was not statistically significant, it did provide some evidence that the mean was smaller than 286. Thus, you plan to recruit another sample of students from your university but this time use a one-sided alternative. You were thinking of surveying $n = 100$ students but now wonder if this sample size gives adequate power to detect a decrease of 15 calories per day to $\mu = 271$.

(a) Given $\alpha = 0.05$, for what values of z will you reject the null hypothesis?

(b) Using $\sigma = 155$ and $\mu = 286$, for what values of \bar{x} will you reject H_0 ?

(c) Using $\sigma = 155$ and $\mu = 271$, what is the probability that \bar{x} will fall in the region defined in part (b)?

(d) Will a sample size of $n = 100$ give you adequate power? Or do you need to find ways to increase the power? Explain your answer.

(e) Use the *Statistical Power* applet to determine the sample size n that gives you power near 0.80.

6.119 Power of the mean SATM score test. Example 6.16 (page 384) gives a test of a hypothesis about the

SATM scores of California high school students based on an SRS of 500 students. The hypotheses are

$$H_0: \mu = 475$$

$$H_a: \mu > 475$$

Assume that the population standard deviation is $\sigma = 100$. The test rejects H_0 at the 1% level of significance when $z \geq 2.326$, where

$$z = \frac{\bar{x} - 475}{100/\sqrt{500}}$$

Is this test sufficiently sensitive to usually detect an increase of 10 points in the population mean SATM score? Answer this question by calculating the power of the test to detect the alternative $\mu = 485$.

6.120 Choose the appropriate distribution.

You must decide which of two discrete distributions a random variable X has. We will call the distributions p_0 and p_1 . Here are the probabilities they assign to the values x of X :

x	0	1	2	3	4	5	6
p_0	0.1	0.1	0.2	0.1	0.1	0.1	0.3
p_1	0.2	0.2	0.2	0.1	0.1	0.1	0.1

You have a single observation on X and wish to test

$$H_0: p_0 \text{ is correct}$$

$$H_a: p_1 \text{ is correct}$$

One possible decision procedure is to reject H_0 only if $X \leq 2$.

(a) Find the probability of a Type I error, that is, the probability that you reject H_0 when p_0 is the correct distribution.

(b) Find the probability of a Type II error.

6.121 Computer-assisted career guidance systems.

A wide variety of computer-assisted career guidance systems have been developed over the last decade. These programs use factors such as student interests, aptitude, skills, personality, and family history to recommend a career path. For simplicity, suppose that a program recommends a high school graduate either to go to college or to join the workforce.

(a) What are the two hypotheses and the two types of error that the program can make?

(b) The program can be adjusted to decrease one error probability at the cost of an increase in the other error probability. Which error probability would you choose to make smaller, and why? (This is a matter of judgment. There is no single correct answer.)

7.20 One-sided versus two-sided P -values. Computer software reports $\bar{x} = 11.2$ and $P = 0.068$ for a t test of $H_0: \mu = 0$ versus $H_a: \mu \neq 0$. Based on prior knowledge, you justified testing the alternative $H_a: \mu > 0$. What is the P -value for your significance test?

7.21 More on one-sided versus two-sided P -values.

Suppose that computer software reports $\bar{x} = -11.2$ and $P = 0.068$ for a t test of $H_0: \mu = 0$ versus $H_a: \mu \neq 0$. Would this change your P -value for the alternative hypothesis in the previous exercise? Use a sketch of the distribution of the test statistic under the null hypothesis to illustrate and explain your answer.

7.22 A one-sample t test. The one-sample t statistic for testing

$$\begin{aligned} H_0: \mu &= 8 \\ H_a: \mu &> 8 \end{aligned}$$

from a sample of $n = 16$ observations has the value $t = 2.15$.

- (a) What are the degrees of freedom for this statistic?
- (b) Give the two critical values t^* from Table D that bracket t .
- (c) Between what two values does the P -value of the test fall?
- (d) Is the value $t = 2.15$ significant at the 5% level? Is it significant at the 1% level?
- (e) If you have software available, find the exact P -value.

7.23 Another one-sample t test. The one-sample t statistic for testing

$$\begin{aligned} H_0: \mu &= 40 \\ H_a: \mu &\neq 40 \end{aligned}$$

from a sample of $n = 27$ observations has the value $t = 2.01$.

- (a) What are the degrees of freedom for t ?
- (b) Locate the two critical values t^* from Table D that bracket t .
- (c) Between what two values does the P -value of the test fall?
- (d) Is the value $t = 2.01$ statistically significant at the 5% level? At the 1% level?
- (e) If you have software available, find the exact P -value.

7.24 A final one-sample t test. The one-sample t statistic for testing

$$\begin{aligned} H_0: \mu &= 20 \\ H_a: \mu &< 20 \end{aligned}$$

based on $n = 11$ observations has the value $t = -1.85$.

- (a) What are the degrees of freedom for this statistic?
- (b) Between what two values does the P -value of the test fall?
- (c) If you have software available, find the exact P -value.

7.25 Two-sided to one-sided P -value. Most software gives P -values for two-sided alternatives. Explain why you cannot always divide these P -values by 2 to obtain P -values for one-sided alternatives.

7.26 Number of friends on Facebook. Facebook recently examined all active Facebook users (more than 10% of the global population) and determined that the average user has 190 friends. This distribution takes only integer values, so it is certainly not Normal. It is also highly skewed to the right, with a median of 100 friends. Consider the following SRS of $n = 30$ Facebook users from your large university.  FACEFR

594	60	417	120	132	176	516	319	734	8
31	325	52	63	537	27	368	11	12	190
85	165	288	65	57	81	257	24	297	148

- (a) Are these data also heavily skewed? Use graphical methods to examine the distribution. Write a short summary of your findings.
- (b) Do you think it is appropriate to use the t methods of this section to compute a 95% confidence interval for the mean number of friends that Facebook users at your large university have? Explain why or why not.
- (c) Compute the sample mean and standard deviation, the standard error of the mean, and the margin of error for 95% confidence.
- (d) Report the 95% confidence interval for μ , the average number of friends for Facebook users at your large university.

7.27 Alcohol content in beer. In February 2013, two California residents filed a class-action lawsuit against Anheuser-Busch, alleging the company was watering down beers to boost profits.⁹ They argued that because water was being added, the true alcohol content of the beer by volume is less than the advertised amount. For example, they alleged that Budweiser beer has an alcohol content by volume of 4.7% instead of the stated 5%. Several media outlets picked up on this suit and hired independent labs to test samples of Budweiser beer and find the alcohol content. Below is a summary of these tests, each done on a single can.  BUD

- | | | |
|------|------|------|
| 4.94 | 5.00 | 4.99 |
|------|------|------|
- (a) Even though we have a very small sample, test the null hypothesis that the alcohol content is 4.7% by volume. Do the data provide evidence against the claim of the two residents?