

# Bayes Theorem

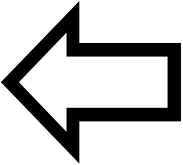
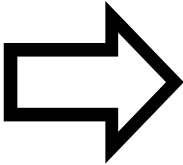
## Applied To Data Analysis

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2022-03-10

# How To Navigate This Presentation

- Use the  and  keys to move through the presentation.
- Press *o* for *panel overview*.

# Derivation

	D=1	D=0
H=1	$P(D, H)$	$P(\text{not}D, H)$
H=0	$P(D, \text{not}H)$	$P(\text{not}D, \text{not}H)$

From the definition of conditional probability:

$$P(D|H) = P(D, H)/P(H)$$

$$P(H|D) = P(D, H)/P(D)$$

Then:

$$P(D|H)P(H) = P(D, H)$$

$$P(H|D)P(D) = P(D, H)$$

Then:

$$P(D|H)P(H) = P(H|D)P(D)$$

# Bayes Theorem:

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

## In Words:

posterior  $\propto$  likelihood  $\times$  prior

# Example

Consider an example using 1,000 hypothetical studies. We imagine that only 10% of interventions are likely to have results. We adopt standard assumptions of adopting an  $\alpha$ , or chance of detecting an effect when one is not there of 5%. We similarly assume 80% power  $\beta$ , or a 20% chance of failing to detect an effect when it is not present.

Data (D)	D=1 (effect)	D=0 (no effect)
Hypothesis (H)	100 effects	900 non-effects
H=1 (conclude effect)	80 true positives	45 false positives
H=0 (conclude no effect)	20 false negatives	855 true negatives

With thanks to the [Wikipedia article](#) on this topic for inspiration for this example.

# Visualization

# Calculations

$$\begin{aligned} P(H=1|D=1) &= \frac{P(D=1|H=1)P(H=1)}{P(D)} \\ &= \frac{P(D=1|H=1)P(H=1)}{P(D=1|H=1)P(H=1) + P(D=0|H=1)P(H=1)} \\ P(H=1|D=1) &= \frac{.8 \times .1}{.08 + .045} = .64 \end{aligned}$$

See also [Thinking Through Bayesian Ideas](#)



# Discussion

- Calculations suggest that a true effect is likely in 64% of the cases where one concludes the presence of an effect.
- Consequently, calculations suggest that 36% of the time, when one concludes there is an effect, there is actually no effect.
- Put another way, despite setting  $\alpha = .05$ , 36% of cases result in a false positive.
- Notice how the Bayesian approach lets us estimate the probability of different hypotheses given the data. In some cases, this may afford us the opportunity to accept our null hypothesis  $H_0$ .