Bayesian Calculations: Simulation of Coin Flipping

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Contents

1 *Bayes Rule*

Mathematically Bayes Theorem is as follows:

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

In words, Bayes Theorem may be written as follows:

$$posterior = \frac{likelihood * prior}{data}$$

Our posterior beliefs are proportional to our prior beliefs, multiplied by the likelihood of those beliefs given the data.

More colloquially, perhaps, Bayes Theorem allows us to state our prior beliefs, and then to update those beliefs with data, thus arriving at an updated set of beliefs.

2 *This Example*

In this example, we provide an example of using Bayes Theorem to examine our conclusions about the proportion of heads when a coin is flipped 10 times.

Conventionally, we call this proportion that we are trying to estimate θ.

For the sake of simplicity, this example uses a relatively simple set of prior beliefs about 3 possible values for the proportion θ.

R code in this example is adapted and simplified from Kruschke (2011), p. 70

3 *Prior*

We set a simple set of prior beliefs, concerning 3 values of θ, the proportion of heads.

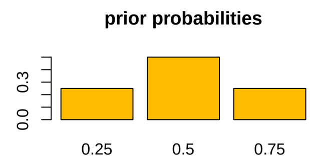
```
theta1 <- c(0.25, 0.5, 0.75)  # candidate parameter values

ptheta1 <- c(0.25, 0.5, 0.25)  # prior probabilities

ptheta1 <- ptheta1/sum(ptheta1)  # normalize
```

Our values of θ are 0.25, 0.5 and 0.75, with probabilities $P(\theta)$ of 0.25, 0.5 and 0.25.

```
barplot(names.arg = theta1, height = ptheta1,
    main = "prior probabilities", col = "#FFBB00")  # graph
```




```
myBayesianEstimates <- tibble(theta1, ptheta1)

pander(myBayesianEstimates)  # nice table
```

theta1	ptheta1
0.25	0.25
0.5	0.5
0.75	0.25

4 *The Data*

10 coin flips. 1 Heads. 9 Tails.

```
data1 <- c(1, 0, 0, 0, 0, 0, 0, 0, 0, 0)  # the data

data1_factor <- factor(data1, levels = c(0, 1),
    labels = c("T", "H"))

n_heads <- sum(data1 == 1)  # number of heads

n_tails <- sum(data1 == 0)  # number of tails

plot(x = seq(1,10), # x from 1 to 10
     y = rep(1,10), # y = 1
     col = data1_factor, # color by data
     cex = 3, # size
     lwd = 2, # line width
```

```
     axes = FALSE,
     main = "10 Coin Tosses",
     xlab = " ",
     ylab = " ")

text(x = seq(1,10), # x from 1 to 10
     y = rep(1,10), # y = 1
     labels = data1_factor) # label with data1_factor
```

5 Likelihood

The likelihood is the probability that a given value of θ would produce this number of heads.

The probability of multiple independent events A, B, C, etc. is $P(A, B, C, ...) = P(A) * P(B) * P(C) * ....$

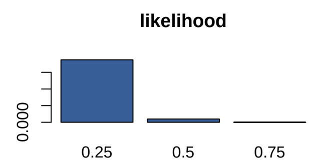
Therefore, in this case, the likelihood is $[P(heads)]^{\text{number of heads}}$ and multiply this by $[P(tails)]^{\text{number of tails}}$.

Thus:

$$\mathcal{L}(\theta) = \theta^{\text{number of heads}} * (1 - \theta)^{\text{number of tails}}$$

```
likelihood1 <- theta1^n_heads * (1 - theta1)^n_tails  # likelihood
```

```
barplot(names.arg = theta1, height = likelihood1,
    main = "likelihood", col = "#375E97")  # graph
```



At this point our estimates include not only a value of θ and $P(\theta)$, but also the likelihood, $\mathcal{L}(\theta)$.

```
myBayesianEstimates <- tibble(theta1, ptheta1,
    likelihood1)
```

```
pander(myBayesianEstimates)  # nice table
```

theta1	ptheta1	likelihood1
0.25	0.25	0.01877
0.5	0.5	0.0009766
0.75	0.25	2.861e-06

6 Posterior

We then calculate the denominator of Bayes theorem:

$$\Sigma[\mathcal{L}(\theta) * P(\theta)]$$

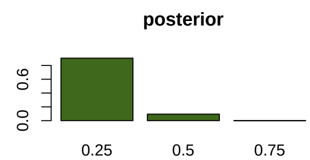
```
pdata1 <- sum(likelihood1 * ptheta1)  # normalize
```

We then use Bayes Rule to calculate the posterior:

$$P(H|D) = \frac{P(D|H)P(H)}{P(D)}$$

```
posterior1 <- likelihood1 * ptheta1/pdata1  # Bayes Rule
```

```
barplot(names.arg = theta1, height = posterior1,
    main = "posterior", col = "#3F681C")  # graph
```



Our estimates now include θ, $P(\theta)$, $\mathcal{L}(\theta)$ and $P(\theta|D)$.

```
myBayesianEstimates <- tibble(theta1, ptheta1,
    likelihood1, posterior1)
```

```
pander(myBayesianEstimates)  # nice table
```

theta1	ptheta1	likelihood1	posterior1
0.25	0.25	0.01877	0.9056
0.5	0.5	0.0009766	0.09423
0.75	0.25	2.861e-06	0.000138