Contents

1 1 Templates

1.1 Start

```
// .vimrc
syn on
set mouse=a sw=4 ts=4 ai si nu wrap
nnoremap ; :
map H :'<,'>!./hash.sh<CR>

// Hash Utility
#!/bin/bash
input=$(cat /dev/stdin)
hash=$(cpp -fpreprocessed -P <(echo "$input")|sed ':a;N;$!ba;s/[
    \t\n]//g'|md5sum)
echo "// start: ${hash}"
echo "$input"
echo "// end"

// Terminal: comparing generated output to sample output
./my_program < sample.in | diff sample.out -
```

1.2 Template - C++

```cpp
#include<bits/stdc++.h>
using namespace std;

typedef long long ll;
typedef pair<int, int> ii;
typedef vector<int> vi;
typedef vector<ii> vii;
typedef vector<vi> vvi;
typedef vector<ll> vll;
typedef vector<vll> vvll;

static bool DBG = 1;

ll mod(ll a, ll b) {
    return ((a%b)+b)%b;
}

int main() {
    ios_base::sync_with_stdio(0);
    cout << fixed << setprecision(15);
        int n;
        cin >> n;
        cout << n << endl;
    return 0;
}
```

1.3 Template - Java

```java
import java.util.*;
import java.math.*;
import java.io.*;
```

```java
class modelo {
	static final double EPS = 1.e-10;
	static final boolean DBG = true;

	private static int cmp(double x, double y = 0, double tol = EPS) {
		return (x <= y + tol)? (x + tol < y)? -1 : 0 : 1;
	}

	public static void main(String[] argv) {
		Scanner s = new Scanner(System.in);
	}
}
```

2 2 Data Structures

2.1 BIT

```cpp
template<typename T> struct BIT{
    int S;
    vector<T> v;

    BIT<T>(int _S){
        S = _S;
        v.resize(S+1);
    }

    void update(int i, T k){
        for(i++; i<=S; i+=i&-i)
            v[i] = v[i] + k;
    }

    T read(int i){
        T sum = 0;
        for(i++; i; i-=i&-i)
            sum = sum + v[i];
        return sum;
    }

    T read(int l, int r){
        return read(r) - read(l-1);
    }
};
```

2.2 KD Tree

```cpp
// -----------------------------------------------------------------
// A straightforward, but probably sub-optimal KD-tree implmentation
// that's probably good enough for most things (current it's a
// 2D-tree)
//
//  - constructs from n points in O(n lg^2 n) time
//  - handles nearest-neighbor query in O(lg n) if points are well
//    distributed
//  - worst case for nearest-neighbor may be linear in pathological
//    case
//
// Sonny Chan, Stanford University, April 2009
// -----------------------------------------------------------------

#include <iostream>
#include <vector>
#include <limits>
#include <cstdlib>

using namespace std;

// number type for coordinates, and its maximum value
typedef long long ntype;
const ntype sentry = numeric_limits<ntype>::max();

// point structure for 2D-tree, can be extended to 3D
struct point {
    ntype x, y;
    point(ntype xx = 0, ntype yy = 0) : x(xx), y(yy) {}
};

bool operator==(const point &a, const point &b) {
    return a.x == b.x && a.y == b.y; }

// sorts points on x-coordinate
bool on_x(const point &a, const point &b) {
    return a.x < b.x; }

// sorts points on y-coordinate
bool on_y(const point &a, const point &b) {
    return a.y < b.y; }

// squared distance between points
ntype pdist2(const point &a, const point &b) {
    ntype dx = a.x-b.x, dy = a.y-b.y;
    return dx*dx + dy*dy; }

// bounding box for a set of points
struct bbox {
    ntype x0, x1, y0, y1;

    bbox() : x0(sentry), x1(-sentry), y0(sentry), y1(-sentry) {}

    // computes bounding box from a bunch of points
    void compute(const vector<point> &v) {
        for (int i = 0; i < v.size(); ++i) {
            x0 = min(x0, v[i].x); x1 = max(x1, v[i].x);
            y0 = min(y0, v[i].y); y1 = max(y1, v[i].y);
        }
    }

    // squared distance between a point and this bbox, 0 if inside
    ntype distance(const point &p) {
        if (p.x < x0) {
            if (p.y < y0)      return pdist2(point(x0, y0), p);
            else if (p.y > y1) return pdist2(point(x0, y1), p);
            else               return pdist2(point(x0, p.y), p);
        }
        else if (p.x > x1) {
            if (p.y < y0)      return pdist2(point(x1, y0), p);
```

```
        else if (p.y > y1) return pdist2(point(x1, y1), p);
        else               return pdist2(point(x1, p.y), p);
      }
      else {
        if (p.y < y0)      return pdist2(point(p.x, y0), p);
        else if (p.y > y1) return pdist2(point(p.x, y1), p);
        else               return 0;
      }
    }
};

// stores a single node of the kd-tree, either internal or leaf
struct kdnode {
    bool leaf;    // true if this is a leaf node (has one point)
    point pt;     // the single point of this is a leaf
    bbox bound;   // bounding box for set of points in children

    kdnode *first, *second; // two children of this kd-node

    kdnode() : leaf(false), first(0), second(0) {}
    ~kdnode() { if (first) delete first; if (second) delete second; }

    // intersect a point with this node (returns squared distance)
    ntype intersect(const point &p) {
        return bound.distance(p);
    }

    // recursively builds a kd-tree from a given cloud of points
    void construct(vector<point> &vp) {
        // compute bounding box for points at this node
        bound.compute(vp);

        // if we're down to one point, then we're a leaf node
        if (vp.size() == 1) {
            leaf = true;
            pt = vp[0];
        } else {
            // split on x if the bbox is wider than high (not best
                heuristic...)
            if (bound.x1-bound.x0 >= bound.y1-bound.y0)
                sort(vp.begin(), vp.end(), on_x);
            // otherwise split on y-coordinate
            else sort(vp.begin(), vp.end(), on_y);

            // divide by taking half the array for each child
            // (not best performance if many duplicates in the middle)
            int half = vp.size()/2;
            vector<point> vl(vp.begin(), vp.begin()+half);
            vector<point> vr(vp.begin()+half, vp.end());
            first = new kdnode(); first->construct(vl);
            second = new kdnode(); second->construct(vr);
        }
    }
};

// simple kd-tree class to hold the tree and handle queries
struct kdtree {
    kdnode *root;

    // constructs a kd-tree from a points (copied here, as it sorts them)
    kdtree(const vector<point> &vp) {
        vector<point> v(vp.begin(), vp.end());
        root = new kdnode();
        root->construct(v);
    }
    ~kdtree() { delete root; }

    // recursive search method returns squared distance to nearest point
    ntype search(kdnode *node, const point &p) {
        if (node->leaf) {
            // commented special case tells a point not to find itself
//            if (p == node->pt) return sentry;
//            else
                return pdist2(p, node->pt);
        }

        ntype bfirst = node->first->intersect(p);
        ntype bsecond = node->second->intersect(p);

        // choose the side with the closest bounding box to search first
        // (note that the other side is also searched if needed)
        if (bfirst < bsecond) {
            ntype best = search(node->first, p);
            if (bsecond < best)
                best = min(best, search(node->second, p));
            return best;
        }
        else {
            ntype best = search(node->second, p);
            if (bfirst < best)
                best = min(best, search(node->first, p));
            return best;
        }
    }

    // squared distance to the nearest
    ntype nearest(const point &p) {
        return search(root, p);
    }
};

// some basic test code here
int main() {
    // generate some random points for a kd-tree
    vector<point> vp;
    for (int i = 0; i < 100000; ++i) {
        vp.push_back(point(rand()%100000, rand()%100000));
    }
    kdtree tree(vp);

    // query some points
    for (int i = 0; i < 10; ++i) {
        point q(rand()%100000, rand()%100000);
        cout << "Closest squared distance to (" << q.x << ", " << q.y <<
            ")"
             << " is " << tree.nearest(q) << endl;
    }
}
```

2.3 LCA

```
struct lca {
    int L, N;
    vector<int> depth, size, link;

    lca(){}
    lca(const vvi &graph, int root = 0) {
        N = graph.size();
        for (L = 0; (1 << L) <= N; L++);

        depth.resize(N);
        size.resize(N);
        link.resize(L*N);
        init(root, root, graph);
    }

    void init(int loc, int par, const vvi &graph) {
        link[loc] = par;
        for (int l = 1; l < L; l++)
            link[l*N + loc] = link[(l-1)*N + link[(l-1)*N + loc]];

        for (int nbr : graph[loc]) {
            if (nbr == par) continue;
            depth[nbr] = depth[loc] + 1;
            init(nbr, loc, graph);
            size[loc] += size[nbr];
        }

        size[loc]++;
    }

    int above(int loc, int dist) {
        for (int l = 0; l < L; l++)
            if ((dist >> l)&1)
                loc = link[l*N + loc];
        return loc;
    }

    int find(int u, int v) {
        if (depth[u] > depth[v]) swap(u, v);
        v = above(v, depth[v] - depth[u]);
        if (u == v) return u;

        for (int l = L - 1; l >= 0; l--) {
            if (link[l*N + u] != link[l*N + v])
                u = link[l*N + u], v = link[l*N + v];
        }

        return link[u];
    }
};
```

2.4 Lazy Segment Tree

```
template<typename T, typename U> struct seg_tree_lazy {
    int S, H;

    T zero;
    vector<T> value;

    U noop;
    vector<bool> dirty;
    vector<U> prop;

    seg_tree_lazy<T, U>(int _S, T _zero = T(), U _noop = U()) {
        zero = _zero, noop = _noop;
        for (S = 1, H = 1; S < _S; ) S *= 2, H++;

        value.resize(2*S, zero);
        dirty.resize(2*S, false);
        prop.resize(2*S, noop);
    }

    void set_leaves(vector<T> &leaves) {
        copy(leaves.begin(), leaves.end(), value.begin() + S);

        for (int i = S - 1; i > 0; i--)
            value[i] = value[2 * i] + value[2 * i + 1];
    }

    void apply(int i, U &update) {
        value[i] = update(value[i]);
        if(i < S) {
            prop[i] = prop[i] + update;
            dirty[i] = true;
        }
    }

    void rebuild(int i) {
        for (int l = i/2; l; l /= 2) {
            T combined = value[2*l] + value[2*l+1];
            value[l] = prop[l](combined);
        }
    }

    void propagate(int i) {
        for (int h = H; h > 0; h--) {
            int l = i >> h;

            if (dirty[l]) {
                apply(2*l, prop[l]);
                apply(2*l+1, prop[l]);

                prop[l] = noop;
                dirty[l] = false;
            }
        }
    }

    void upd(int i, int j, U update) {
        i += S, j += S;
        propagate(i), propagate(j);

        for (int l = i, r = j; l <= r; l /= 2, r /= 2) {
            if((l&1) == 1) apply(l++, update);
            if((r&1) == 0) apply(r--, update);
```

```
        }

        rebuild(i), rebuild(j);
    }

    T query(int i, int j){
        i += S, j += S;
        propagate(i), propagate(j);

        T res_left = zero, res_right = zero;
        for(; i <= j; i /= 2, j /= 2){
            if((i&1) == 1) res_left = res_left + value[i++];
            if((j&1) == 0) res_right = value[j--] + res_right;
        }
        return res_left + res_right;
    }
};
```

2.5 Segment Tree

```
template<typename T> struct seg_tree {
    int S;

    T zero;
    vector<T> value;

    seg_tree<T>(int _S, T _zero = T()) {
        S = _S, zero = _zero;
        value.resize(2*S+1, zero);
    }

    void set_leaves(vector<T> &leaves) {
        copy(leaves.begin(), leaves.end(), value.begin() + S);

        for (int i = S - 1; i > 0; i--)
            value[i] = value[2 * i] + value[2 * i + 1];
    }

    void upd(int i, T v) {
        i += S;
        value[i] = v;
        while(i>1){
            i/=2;
            value[i] = value[2*i] + value[2*i+1];
        }
    }

    T query(int i, int j) {
        T res_left = zero, res_right = zero;
        for(i += S, j += S; i <= j; i /= 2, j /= 2){
            if((i&1) == 1) res_left = res_left + value[i++];
            if((j&1) == 0) res_right = value[j--] + res_right;
        }
        return res_left + res_right;
    }
};
```

2.6 Union Find

```
// (struct) also keeps track of sizes
struct union_find {
        vector<int> P,S;

        union_find(int N) {
                P.resize(N), S.resize(N, 1);
                for(int i = 0; i < N; i++) P[i] = i;
        }

        int rep(int i) {
                return (P[i] == i) ? i : P[i] = rep(P[i]);
        }

        bool unio(int a, int b) {
                a = rep(a), b = rep(b);
                if(a == b) return false;
                P[b] = a;
                S[a] += S[b];
                return true;
        }
};

// (Shorter) union-find set: the vector/array contains the parent of each
    node
int find(vector <int>& C, int x){return (C[x]==x) ? x : C[x]=find(C,
    C[x]);} //C++
int find(int x){return (C[x]==x)?x:C[x]=find(C[x]);} //C
```

3 3 Graph

3.1 2-SAT

```
struct two_sat {
    int N;
    vector<vector<int>> impl;

    two_sat(int _N) {
        N = _N;
        impl.resize(2 * N);
    }

    void add_impl(int var1, bool neg1, int var2, bool neg2) {
        impl[2 * var1 + neg1].push_back(2 * var2 + neg2);
        impl[2 * var2 + !neg2].push_back(2 * var1 + !neg1);
    }

    void add_clause(int var1, bool neg1, int var2, bool neg2) {
        add_impl(var1, !neg1, var2, neg2);
    }

    void add_clause(int var1, bool neg1) {
        add_clause(var1, neg1, var1, neg1);
    }

    int V, L, C;
```

```
    stack<int> view;

    int dfs(int loc) {
        visit[loc] = V;
        label[loc] = L++;

        int low = label[loc];
        view.push(loc);
        in_view[loc] = true;

        for (int nbr : impl[loc]) {
            if(!visit[nbr]) low = min(low, dfs(nbr));
            else if(in_view[nbr]) low = min(low, label[nbr]);
        }

        if(low == label[loc]) {
            while (true) {
                int mem = view.top();
                comp[mem] = C;
                in_view[mem] = false;
                view.pop();
                if(mem == loc) break;
            }
            C++;
        }

        return low;
    }

    vector<int> visit, label, comp, in_view;

    void reset(vector<int> &v) {
        v.resize(2 * N);
        fill(v.begin(), v.end(), 0);
    }

    bool consistent() {
        V = 0, L = 0, C = 0;
        reset(visit), reset(label), reset(comp), reset(in_view);

        for (int i = 0; i < 2 * N; i++) {
            if(!visit[i]) {
                V++;
                dfs(i);
            }
        }

        for (int i = 0; i < N; i++)
            if(comp[2 * i] == comp[2 * i + 1]) {
                return false;
            }

        return true;
    }
};
```

3.2 Dense Dijkstra

```
void Dijkstra (const VVT &w, VT &dist, VI &prev, int start){
  int n = w.size();
  VI found (n);
  prev = VI(n, -1);
  dist = VT(n, 1000000000);
  dist[start] = 0;

  while (start != -1){
    found[start] = true;
    int best = -1;
    for (int k = 0; k < n; k++) if (!found[k]){
      if (dist[k] > dist[start] + w[start][k]){
        dist[k] = dist[start] + w[start][k];
        prev[k] = start;
      }
      if (best == -1 || dist[k] < dist[best]) best = k;
    }
    start = best;
  }
}
```

3.3 Dijkstra

```
// Implementation of Dijkstra's algorithm using adjacency lists
// and priority queue for efficiency.
//
// Running time: O(|E| log |V|)

#include <queue>
#include <stdio.h>

using namespace std;
const int INF = 2000000000;
typedef pair<int,int> PII;

int main(){

  int N, s, t;
  scanf ("%d%d%d", &N, &s, &t);
  vector<vector<PII> > edges(N);
  for (int i = 0; i < N; i++){
    int M;
    scanf ("%d", &M);
    for (int j = 0; j < M; j++){
      int vertex, dist;
      scanf ("%d%d", &vertex, &dist);
      edges[i].push_back (make_pair (dist, vertex)); // note order of
          arguments here
    }
  }

  // use priority queue in which top element has the "smallest" priority
  priority_queue<PII, vector<PII>, greater<PII> > Q;
  vector<int> dist(N, INF), dad(N, -1);
  Q.push (make_pair (0, s));
  dist[s] = 0;
  while (!Q.empty()){
```

```
    PII p = Q.top();
    if (p.second == t) break;
    Q.pop();

    int here = p.second;
    for (vector<PII>::iterator it=edges[here].begin();
         it!=edges[here].end(); it++){
      if (dist[here] + it->first < dist[it->second]){
        dist[it->second] = dist[here] + it->first;
        dad[it->second] = here;
        Q.push (make_pair (dist[it->second], it->second));
      }
    }
  }

  printf ("%d\n", dist[t]);
  if (dist[t] < INF)
    for(int i=t;i!=-1;i=dad[i])
      printf ("%d%c", i, (i==s?'\n':' '));

  return 0;
}
```

3.4 Eulerian Path

```
struct Edge;
typedef list<Edge>::iterator iter;

struct Edge
{
        int next_vertex;
        iter reverse_edge;

        Edge(int next_vertex)
                :next_vertex(next_vertex)
                { }
};

const int max_vertices = ;
int num_vertices;
list<Edge> adj[max_vertices];       // adjacency list

vector<int> path;

void find_path(int v)
{
        while(adj[v].size() > 0)
        {
                int vn = adj[v].front().next_vertex;
                adj[vn].erase(adj[v].front().reverse_edge);
                adj[v].pop_front();
                find_path(vn);
        }
        path.push_back(v);
}

void add_edge(int a, int b)
{
        adj[a].push_front(Edge(b));
        iter ita = adj[a].begin();
        adj[b].push_front(Edge(a));
        iter itb = adj[b].begin();
        ita->reverse_edge = itb;
        itb->reverse_edge = ita;
}
```

3.5 Heavy Light

```
template<typename T> struct heavy_light {
    lca links;
    seg_tree<T> st;
    vector<int> preorder, index, jump;

    heavy_light(const vvi &graph, int root) {
        links = lca(graph, 0);

        st = seg_tree<T>(graph.size());
        index.resize(graph.size()), jump.resize(graph.size());
        dfs(root, root, root, graph);
    }

    void dfs(int loc, int par, int lhv, const vvi &graph) {
        jump[loc] = lhv;
        index[loc] = preorder.size();
        preorder.push_back(loc);

        vector<int> ch = graph[loc];
        sort(ch.begin(), ch.end(), [&](int i, int j) {
            return links.size[i] > links.size[j]; });
        if (loc != par) ch.erase(ch.begin());

        for (int c = 0; c < ch.size(); c++)
            dfs(ch[c], loc, c ? ch[c] : lhv, graph);
    }

    void assign(int loc, T value) {
        st.upd(index[loc], value);
    }

    T __sum(int u, int r) {
        T res;
        while (u != r) {
            int go = max(index[r] + 1, index[jump[u]]);
            res = res + st.query(go, index[u]);
            u = links.link[preorder[go]];
        }
        return res;
    }

    T sum(int u, int v) {
        int r = links.find(u, v);
        return st.query(index[r], index[r]) + __sum(u, r) + __sum(v, r);
    }
};
```

3.6 Poset Width

```
vector<int> width(vector<vector<int>> poset) {
    int N = poset.size();
    bipartite_graph g(N, N);

    for (int i = 0; i < N; i++) {
        for (int j : poset[i])
            g.edge(j, i);
    }

    g.matching();

    vector<bool> vis[2];
    vis[false].resize(2 * N, false);
    vis[true].resize(2 * N, false);

    for (int i = 0; i < N; i++) {
        if (g.match[i] != -1) continue;
        if (vis[false][i]) continue;

        queue<pair<bool, int>> bfs;
        bfs.push(make_pair(false, i));
        vis[false][i] = true;

        while (!bfs.empty()) {
            bool inm = bfs.front().first;
            int loc = bfs.front().second;
            bfs.pop();

            for (int nbr : g.adj[loc]) {
                if (vis[!inm][nbr]) continue;
                if ((g.match[loc] == nbr) ^ inm) continue;

                vis[!inm][nbr] = true;
                bfs.push(make_pair(!inm, nbr));
            }
        }
    }

    vector<bool> inz(2 * N, false);
    for (int i = 0; i < 2 * N; i++)
        inz[i] = vis[true][i] || vis[false][i];

    vector<bool> ink(N, false);

    for (int i = 0; i < N; i++)
        if (!inz[i])
            ink[i]= true;

    for (int i = N; i < 2 * N; i++)
        if (inz[i])
            ink[i - N] = true;

    vector<int> res;
    for (int i = 0; i < N; i++) {
        if (!ink[i])
            res.push_back(i);
    }
    return res;
}
```

3.7 SCC

```
#include<memory.h>
struct edge{int e, nxt;};
int V, E;
edge e[MAXE], er[MAXE];
int sp[MAXV], spr[MAXV];
int group_cnt, group_num[MAXV];
bool v[MAXV];
int stk[MAXV];
void fill_forward(int x)
{
  int i;
  v[x]=true;
  for(i=sp[x];i;i=e[i].nxt) if(!v[e[i].e]) fill_forward(e[i].e);
  stk[++stk[0]]=x;
}
void fill_backward(int x)
{
  int i;
  v[x]=false;
  group_num[x]=group_cnt;
  for(i=spr[x];i;i=er[i].nxt) if(v[er[i].e]) fill_backward(er[i].e);
}
void add_edge(int v1, int v2) //add edge v1->v2
{
  e [++E].e=v2; e [E].nxt=sp [v1]; sp [v1]=E;
  er[ E].e=v1; er[E].nxt=spr[v2]; spr[v2]=E;
}
void SCC()
{
  int i;
  stk[0]=0;
  memset(v, false, sizeof(v));
  for(i=1;i<=V;i++) if(!v[i]) fill_forward(i);
  group_cnt=0;
  for(i=stk[0];i>=1;i--) if(v[stk[i]]){group_cnt++;
      fill_backward(stk[i]);}
}
```

3.8 Topological Sort

```
// This function uses performs a non-recursive topological sort.
//
// Running time: O(|V|^2). If you use adjacency lists (vector<map<int> >),
//               the running time is reduced to O(|E|).
//
//   INPUT:  w[i][j] = 1 if i should come before j, 0 otherwise
//   OUTPUT: a permutation of 0,...,n-1 (stored in a vector)
//           which represents an ordering of the nodes which
//           is consistent with w
//
// If no ordering is possible, false is returned.
```

```
typedef double TYPE;
typedef vector<TYPE> VT;
typedef vector<VT> VVT;

typedef vector<int> VI;
typedef vector<VI> VVI;

bool TopologicalSort (const VVI &w, VI &order){
  int n = w.size();
  VI parents (n);
  queue<int> q;
  order.clear();

  for (int i = 0; i < n; i++){
    for (int j = 0; j < n; j++)
      if (w[j][i]) parents[i]++;
    if (parents[i] == 0) q.push (i);
  }

  while (q.size() > 0){
    int i = q.front();
    q.pop();
    order.push_back (i);
    for (int j = 0; j < n; j++) if (w[i][j]){
      parents[j]--;
      if (parents[j] == 0) q.push (j);
    }
  }

  return (order.size() == n);
}
```

4 4 Combinatorial Optimization

4.1 Bipartite Graph

```
struct bipartite_graph {
    int A, B;
    vector<vector<int>> adj;

    bipartite_graph(int _A, int _B) {
        A = _A, B = _B;
        adj.resize(A + B);
    }

    void edge(int i, int j) {
        adj[i].push_back(A+j);
        adj[A+j].push_back(i);
    }

    vector<int> visit, match;

    bool augment(int loc, int run) {
        if(visit[loc] == run) return false;
        visit[loc] = run;

        for (int nbr : adj[loc]) {
            if (match[nbr] == -1 || augment(match[nbr], run)) {
                match[loc] = nbr, match[nbr] = loc;
                return true;
            }
        }

        return false;
    }

    int matching() {
        visit = vector<int>(A+B, -1);
        match = vector<int>(A+B, -1);

        int ans = 0;
        for (int i = 0; i < A; i++)
            ans += augment(i, i);
        return ans;
    }

    vector<bool> vertex_cover() {
        vector<bool> res(A + B, false);
        queue<int> bfs;

        for (int i = 0; i < A; i++) {
            if (match[i] == -1) bfs.push(i);
            else res[i] = true;
        }

        while (!bfs.empty()) {
            int loc = bfs.front();
            bfs.pop();
            for (int nbr : adj[loc]) {
                if (res[nbr]) continue;
                res[nbr] = true;
                int loc2 = match[nbr];
                if (loc2 == -1) continue;
                res[loc2] = false;
                bfs.push(loc2);
            }
        }

        return res;
    }
};
```

4.2 Max Flow - Dinic

```
// Adjacency list implementation of Dinic's blocking flow algorithm.
// This is very fast in practice, and only loses to push-relabel flow.
//
// Running time:
//     O(|V|^2 |E|)
//
// INPUT:
//     - graph, constructed using AddEdge()
//     - source and sink
//
// OUTPUT:
```

```cpp
//     - maximum flow value
//     - To obtain actual flow values, look at edges with capacity > 0
//       (zero capacity edges are residual edges).

#include <iostream>
#include <vector>

using namespace std;
typedef long long LL;

struct Edge {
  int from, to, cap, flow, index;
  Edge(int from, int to, int cap, int flow, int index) :
    from(from), to(to), cap(cap), flow(flow), index(index) {}
  LL rcap() { return cap - flow; }
};

struct Dinic {
  int N;
  vector<vector<Edge> > G;
  vector<vector<Edge *> > Lf;
  vector<int> layer;
  vector<int> Q;

  Dinic(int N) : N(N), G(N), Q(N) {}

  void AddEdge(int from, int to, int cap) {
    if (from == to) return;
    G[from].push_back(Edge(from, to, cap, 0, G[to].size()));
    G[to].push_back(Edge(to, from, 0, 0, G[from].size() - 1));
  }

  LL BlockingFlow(int s, int t) {
    layer.clear(); layer.resize(N, -1);
    layer[s] = 0;
    Lf.clear(); Lf.resize(N);

    int head = 0, tail = 0;
    Q[tail++] = s;
    while (head < tail) {
      int x = Q[head++];
      for (int i = 0; i < G[x].size(); i++) {
        Edge &e = G[x][i]; if (e.rcap() <= 0) continue;
        if (layer[e.to] == -1) {
          layer[e.to] = layer[e.from] + 1;
          Q[tail++] = e.to;
        }
        if (layer[e.to] > layer[e.from]) {
          Lf[e.from].push_back(&e);
        }
      }
    }
    if (layer[t] == -1) return 0;

    LL totflow = 0;
    vector<Edge *> P;
    while (!Lf[s].empty()) {
      int curr = P.empty() ? s : P.back()->to;
      if (curr == t) { // Augment
        LL amt = P.front()->rcap();
        for (int i = 0; i < P.size(); ++i) {
          amt = min(amt, P[i]->rcap());
        }
        totflow += amt;
        for (int i = P.size() - 1; i >= 0; --i) {
          P[i]->flow += amt;
          G[P[i]->to][P[i]->index].flow -= amt;
          if (P[i]->rcap() <= 0) {
            Lf[P[i]->from].pop_back();
            P.resize(i);
          }
        }
      } else if (Lf[curr].empty()) { // Retreat
        P.pop_back();
        for (int i = 0; i < N; ++i)
          for (int j = 0; j < Lf[i].size(); ++j)
            if (Lf[i][j]->to == curr)
              Lf[i].erase(Lf[i].begin() + j);
      } else { // Advance
        P.push_back(Lf[curr].back());
      }
    }
    return totflow;
  }

  LL GetMaxFlow(int s, int t) {
    LL totflow = 0;
    while (LL flow = BlockingFlow(s, t))
      totflow += flow;
    return totflow;
  }
};
```

4.3 Min Cost Matching

```cpp
///////////////////////////////////////////////////////////////////////
// Min cost bipartite matching via shortest augmenting paths
//
// This is an O(n^3) implementation of a shortest augmenting path
// algorithm for finding min cost perfect matchings in dense
// graphs. In practice, it solves 1000x1000 problems in around 1
// second.
//
//   cost[i][j] = cost for pairing left node i with right node j
//   Lmate[i] = index of right node that left node i pairs with
//   Rmate[j] = index of left node that right node j pairs with
//
// The values in cost[i][j] may be positive or negative. To perform
// maximization, simply negate the cost[][] matrix.
///////////////////////////////////////////////////////////////////////

#include <algorithm>
#include <cstdio>
#include <cmath>
#include <vector>

using namespace std;
```

```
typedef vector<double> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

double MinCostMatching(const VVD &cost, VI &Lmate, VI &Rmate) {
  int n = int(cost.size());

  // construct dual feasible solution
  VD u(n);
  VD v(n);
  for (int i = 0; i < n; i++) {
    u[i] = cost[i][0];
    for (int j = 1; j < n; j++) u[i] = min(u[i], cost[i][j]);
  }
  for (int j = 0; j < n; j++) {
    v[j] = cost[0][j] - u[0];
    for (int i = 1; i < n; i++) v[j] = min(v[j], cost[i][j] - u[i]);
  }

  // construct primal solution satisfying complementary slackness
  Lmate = VI(n, -1);
  Rmate = VI(n, -1);
  int mated = 0;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      if (Rmate[j] != -1) continue;
      if (fabs(cost[i][j] - u[i] - v[j]) < 1e-10) {
        Lmate[i] = j;
        Rmate[j] = i;
        mated++;
        break;
      }
    }
  }

  VD dist(n);
  VI dad(n);
  VI seen(n);

  // repeat until primal solution is feasible
  while (mated < n) {

    // find an unmatched left node
    int s = 0;
    while (Lmate[s] != -1) s++;

    // initialize Dijkstra
    fill(dad.begin(), dad.end(), -1);
    fill(seen.begin(), seen.end(), 0);
    for (int k = 0; k < n; k++)
      dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    while (true) {

      // find closest
      j = -1;
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        if (j == -1 || dist[k] < dist[j]) j = k;
      }
      seen[j] = 1;

      // termination condition
      if (Rmate[j] == -1) break;

      // relax neighbors
      const int i = Rmate[j];
      for (int k = 0; k < n; k++) {
        if (seen[k]) continue;
        const double new_dist = dist[j] + cost[i][k] - u[i] - v[k];
        if (dist[k] > new_dist) {
          dist[k] = new_dist;
          dad[k] = j;
        }
      }
    }

    // update dual variables
    for (int k = 0; k < n; k++) {
      if (k == j || !seen[k]) continue;
      const int i = Rmate[k];
      v[k] += dist[k] - dist[j];
      u[i] -= dist[k] - dist[j];
    }
    u[s] += dist[j];

    // augment along path
    while (dad[j] >= 0) {
      const int d = dad[j];
      Rmate[j] = Rmate[d];
      Lmate[Rmate[j]] = j;
      j = d;
    }
    Rmate[j] = s;
    Lmate[s] = j;

    mated++;
  }

  double value = 0;
  for (int i = 0; i < n; i++)
    value += cost[i][Lmate[i]];

  return value;
}
```

4.4 Min Cost Max Flow

```
// Generic flow using an adjacency matrix. If you
// want just regular max flow, setting all edge costs to 1 gives
// running time O(|E|^2 |V|).
//
// Running time: O(min(|V|^2 * totflow, |V|^3 * totcost))
//
// INPUT: cap -- a matrix such that cap[i][j] is the capacity of
```

```
//              a directed edge from node i to node j
//
//       cost -- a matrix such that cost[i][j] is the (positive)
//              cost of sending one unit of flow along a
//              directed edge from node i to node j
//
//       excess -- a vector such that the total flow from i == excess[i]
//
//
// OUTPUT: cost of the resulting flow; the matrix flow will contain
//        the actual flow values (all nonnegative).
//        The vector excess will contain node excesses that could not be
//        eliminated. Remember to check it.
//
// To use this, create a MinCostCirc object, and call it like this:
//
//   MinCostCirc circ(N);
//   circ.cap = <whatever>; circ.cost = <whatever>;
//   circ.excess[foo] = bar;
//   circ.flow[i][j] = something; (if you want)
//   int finalcost = circ.solve();
//
// If you want min-cost max-flow, leave excess blank and call
   min_cost_max_flow.
// Andy says to use caution in min-cost max-flow mode if you have negative
// costs.

typedef vector<int64_t> VI64;
typedef vector<int> VI;
typedef vector<VI64> VVI64;

const int64_t INF = 1LL<<60;

struct MinCostCirc {
  int N;
  VVI64 cap, flow, cost;
  VI dad, found, src, add;
  VI64 pi, dist, excess;

  MinCostCirc(int N) : N(N), cap(N, VI64(N)), flow(cap), cost(cap),
                     dad(N), found(N), src(N), add(N),
                     pi(N), dist(N+1), excess(N) {}

  void search() {
    fill(found.begin(), found.end(), false);
    fill(dist.begin(), dist.end(), INF);

    int here = N;
    for (int i = 0; i < N; i++)
      if (excess[i] > 0) {
        src[i] = i;
        dist[i] = 0;
        here = i;
      }

    while (here != N) {
      int best = N;
      found[here] = 1;
      for (int k = 0; k < N; k++) {
```

```
        if (found[k]) continue;
        int64_t x = dist[here] + pi[here] - pi[k];
        if (flow[k][here]) {
          int64_t val = x - cost[k][here];
          assert(val >= dist[here]);
          if (dist[k] > val) {
            dist[k] = val;
            dad[k] = here;
            add[k] = 0;
            src[k] = src[here];
          }
        }
        if (flow[here][k] < cap[here][k]) {
          int64_t val = x + cost[here][k];
          assert(val >= dist[here]);
          if (dist[k] > val) {
            dist[k] = val;
            dad[k] = here;
            add[k] = 1;
            src[k] = src[here];
          }
        }

        if (dist[k] < dist[best]) best = k;
      }
      here = best;
    }

    for (int k = 0; k < N; k++)
      if (found[k])
        pi[k] = min(pi[k] + dist[k], INF);
  }

  int64_t solve() {
    int64_t totcost = 0;
    int source, sink;
    for(int i = 0; i < N; i++)
      for(int j = 0; j < N; j++)
        if (cost[i][j] < 0)
          {
            flow[i][j] += cap[i][j];
            totcost += cost[i][j] * cap[i][j];
            excess[i] -= cap[i][j];
            excess[j] += cap[i][j];
          }

    bool again = true;
    while (again) {
      search();
      int64_t amt = INF;
      fill(found.begin(), found.end(), false);
      again = false;
      for(int sink = 0; sink < N; sink++)
        {
          if (excess[sink] >= 0 || dist[sink] == INF || found[src[sink]]++)
            continue;
          again = true;
          int source = src[sink];
```

```
        for (int x = sink; x != source; x = dad[x])
          amt = min(amt, flow[x][dad[x]] ? flow[x][dad[x]] :
                    cap[dad[x]][x] - flow[dad[x]][x]);
        amt = min(amt, min(excess[source], -excess[sink]));
        for (int x = sink; x != source; x = dad[x]) {
          if (add[x]) {
            flow[dad[x]][x] += amt;
            totcost += amt * cost[dad[x]][x];
          } else {
            flow[x][dad[x]] -= amt;
            totcost -= amt * cost[x][dad[x]];
          }
          excess[x] += amt;
          excess[dad[x]] -= amt;
        }
        assert(amt != 0);
        break; // Comment out at your peril if you need speed.
      }
    }

    return totcost;
  }

  // returns (flow, cost)
  pair<int,int> min_cost_max_flow(int source, int sink) {
    excess[source] = INF;
    excess[sink] = -INF;
    pair<int, int> ret;
    ret.second = solve();
    ret.first = INF - excess[source];
    return ret;
  }
};
```

4.5 Min Cut

```
// Adjacency matrix implementation of Stoer-Wagner min cut algorithm.
//
// Running time:
//     O(|V|^3)
//
// INPUT:
//     - graph, constructed using AddEdge()
//
// OUTPUT:
//     - (min cut value, nodes in half of min cut)

#include <cmath>
#include <vector>
#include <iostream>

using namespace std;

typedef vector<int> VI;
typedef vector<VI> VVI;

const int INF = 1000000000;

pair<int, VI> GetMinCut(VVI &weights) {
  int N = weights.size();
  VI used(N), cut, best_cut;
  int best_weight = -1;

  for (int phase = N-1; phase >= 0; phase--) {
    VI w = weights[0];
    VI added = used;
    int prev, last = 0;
    for (int i = 0; i < phase; i++) {
      prev = last;
      last = -1;
      for (int j = 1; j < N; j++)
        if (!added[j] && (last == -1 || w[j] > w[last])) last = j;
      if (i == phase-1) {
        for (int j = 0; j < N; j++) weights[prev][j] += weights[last][j];
        for (int j = 0; j < N; j++) weights[j][prev] = weights[prev][j];
        used[last] = true;
        cut.push_back(last);
        if (best_weight == -1 || w[last] < best_weight) {
          best_cut = cut;
          best_weight = w[last];
        }
      } else {
        for (int j = 0; j < N; j++)
          w[j] += weights[last][j];
        added[last] = true;
      }
    }
  }
  return make_pair(best_weight, best_cut);
}
```

5 5 Geometry

5.1 Convex Hull

```
// Compute the 2D convex hull of a set of points using the monotone chain
// algorithm. Eliminate redundant points from the hull if
    REMOVE_REDUNDANT is
// #defined.
//
// Running time: O(n log n)
//
//   INPUT:   a vector of input points, unordered.
//   OUTPUT: a vector of points in the convex hull, counterclockwise,
    starting
//            with bottommost/leftmost point

#include <cstdio>
#include <cassert>
#include <vector>
#include <algorithm>
#include <cmath>

using namespace std;
```

```
#define REMOVE_REDUNDANT

typedef double T;
const T EPS = 1e-7;
struct PT {
  T x, y;
  PT() {}
  PT(T x, T y) : x(x), y(y) {}
  bool operator<(const PT &rhs) const { return make_pair(y,x) <
      make_pair(rhs.y,rhs.x); }
  bool operator==(const PT &rhs) const { return make_pair(y,x) ==
      make_pair(rhs.y,rhs.x); }
};

T cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
T area2(PT a, PT b, PT c) { return cross(a,b) + cross(b,c) + cross(c,a); }

#ifdef REMOVE_REDUNDANT
bool between(const PT &a, const PT &b, const PT &c) {
  return (fabs(area2(a,b,c)) < EPS && (a.x-b.x)*(c.x-b.x) <= 0 &&
      (a.y-b.y)*(c.y-b.y) <= 0);
}
#endif

void ConvexHull(vector<PT> &pts) {
  sort(pts.begin(), pts.end());
  pts.erase(unique(pts.begin(), pts.end()), pts.end());
  vector<PT> up, dn;
  for (int i = 0; i < pts.size(); i++) {
    while (up.size() > 1 && area2(up[up.size()-2], up.back(), pts[i]) >=
        0) up.pop_back();
    while (dn.size() > 1 && area2(dn[dn.size()-2], dn.back(), pts[i]) <=
        0) dn.pop_back();
    up.push_back(pts[i]);
    dn.push_back(pts[i]);
  }
  pts = dn;
  for (int i = (int) up.size() - 2; i >= 1; i--) pts.push_back(up[i]);

#ifdef REMOVE_REDUNDANT
  if (pts.size() <= 2) return;
  dn.clear();
  dn.push_back(pts[0]);
  dn.push_back(pts[1]);
  for (int i = 2; i < pts.size(); i++) {
    if (between(dn[dn.size()-2], dn[dn.size()-1], pts[i])) dn.pop_back();
    dn.push_back(pts[i]);
  }
  if (dn.size() >= 3 && between(dn.back(), dn[0], dn[1])) {
    dn[0] = dn.back();
    dn.pop_back();
  }
  pts = dn;
#endif
}
```

5.2 Delaunay

```
// Slow but simple Delaunay triangulation. Does not handle
// degenerate cases (from O'Rourke, Computational Geometry in C)
//
// Running time: O(n^4)
//
// INPUT:   x[] = x-coordinates
//          y[] = y-coordinates
//
// OUTPUT:  triples = a vector containing m triples of indices
//                    corresponding to triangle vertices

#include<vector>
using namespace std;

typedef double T;

struct triple {
    int i, j, k;
    triple() {}
    triple(int i, int j, int k) : i(i), j(j), k(k) {}
};

vector<triple> delaunayTriangulation(vector<T>& x, vector<T>& y) {
        int n = x.size();
        vector<T> z(n);
        vector<triple> ret;

        for (int i = 0; i < n; i++)
            z[i] = x[i] * x[i] + y[i] * y[i];

        for (int i = 0; i < n-2; i++) {
            for (int j = i+1; j < n; j++) {
                for (int k = i+1; k < n; k++) {
                    if (j == k) continue;
                    double xn = (y[j]-y[i])*(z[k]-z[i]) -
                        (y[k]-y[i])*(z[j]-z[i]);
                    double yn = (x[k]-x[i])*(z[j]-z[i]) -
                        (x[j]-x[i])*(z[k]-z[i]);
                    double zn = (x[j]-x[i])*(y[k]-y[i]) -
                        (x[k]-x[i])*(y[j]-y[i]);
                    bool flag = zn < 0;
                    for (int m = 0; flag && m < n; m++)
                        flag = flag && ((x[m]-x[i])*xn +
                                        (y[m]-y[i])*yn +
                                        (z[m]-z[i])*zn <= 0);
                    if (flag) ret.push_back(triple(i, j, k));
                }
            }
        }
        return ret;
}

int main()
{
    T xs[]={0, 0, 1, 0.9};
    T ys[]={0, 1, 0, 0.9};
    vector<T> x(&xs[0], &xs[4]), y(&ys[0], &ys[4]);
    vector<triple> tri = delaunayTriangulation(x, y);
```

```
  //expected: 0 1 3
  //          0 3 2

  int i;
  for(i = 0; i < tri.size(); i++)
    printf("%d %d %d\n", tri[i].i, tri[i].j, tri[i].k);
  return 0;
}
```

5.3 Geometry

```
// C++ routines for computational geometry.

#include <iostream>
#include <vector>
#include <cmath>
#include <cassert>

using namespace std;

double INF = 1e100;
double EPS = 1e-12;

struct PT {
  double x, y;
  PT() {}
  PT(double x, double y) : x(x), y(y) {}
  PT(const PT &p) : x(p.x), y(p.y) {}
  PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
  PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
  PT operator * (double c)  const { return PT(x*c, y*c ); }
  PT operator / (double c)  const { return PT(x/c, y/c ); }
};

double dot(PT p, PT q)  { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
  os << "(" << p.x << "," << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p)  { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
  return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
  double r = dot(b-a,b-a);
  if (fabs(r) < EPS) return a;
  r = dot(c-a, b-a)/r;
  if (r < 0) return a;
  if (r > 1) return b;
  return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
  return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                          double a, double b, double c, double d)
{
  return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
  return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
  return LinesParallel(a, b, c, d)
      && fabs(cross(a-b, a-c)) < EPS
      && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
  if (LinesCollinear(a, b, c, d)) {
    if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
      dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
    if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
      return false;
    return true;
  }
  if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
  if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
  return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
  b=b-a; d=c-d; c=c-a;
  assert(dot(b, b) > EPS && dot(d, d) > EPS);
  return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
  b=(a+b)/2;
  c=(a+c)/2;
```

```
  return ComputeLineIntersection(b, b+RotateCW90(a-b), c,
      c+RotateCW90(a-c));
}

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
  for (int i = 0; i < p.size(); i++){
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||
      p[j].y <= q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y -
          p[i].y))
      c = !c;
  }
  return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
  for (int i = 0; i < p.size(); i++)
    if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
      return true;
    return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
  vector<PT> ret;
  b = b-a;
  a = a-c;
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
}

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r+R || d+min(r, R) < max(r, R)) return ret;
  double x = (d*d-R*R+r*r)/(2*d);
  double y = sqrt(r*r-x*x);
  PT v = (b-a)/d;
  ret.push_back(a+v*x + RotateCCW90(v)*y);
  if (y > 0)
    ret.push_back(a+v*x - RotateCCW90(v)*y);
  return ret;
}

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
}

// tests whether or not a given polygon (in CW or CCW order) is simple
bool IsSimple(const vector<PT> &p) {
  for (int i = 0; i < p.size(); i++) {
    for (int k = i+1; k < p.size(); k++) {
      int j = (i+1) % p.size();
      int l = (k+1) % p.size();
      if (i == l || j == k) continue;
      if (SegmentsIntersect(p[i], p[j], p[k], p[l]))
        return false;
    }
  }
  return true;
}

// Plane distance between parallel planes aX + bY + cZ + d1 = 0 and
// aX + bY + cZ + d2 = 0 is abs(d1 - d2) / sqrt(a*a + b*b + c*c)

// distance from point (px, py, pz) to line (x1, y1, z1)-(x2, y2, z2)
// (or ray, or segment; in the case of the ray, the endpoint is the
// first point)
  public static final int LINE = 0;
  public static final int SEGMENT = 1;
  public static final int RAY = 2;
  public static double ptLineDistSq(double x1, double y1, double z1,
      double x2, double y2, double z2, double px, double py, double pz,
      int type) {
    double pd2 = (x1-x2)*(x1-x2) + (y1-y2)*(y1-y2) + (z1-z2)*(z1-z2);
```

```
    double x, y, z;
    if (pd2 == 0) {
      x = x1;
      y = y1;
      z = z1;}
    else {
      double u = ((px-x1)*(x2-x1) + (py-y1)*(y2-y1) + (pz-z1)*(z2-z1)) /
           pd2;
      x = x1 + u * (x2 - x1);
      y = y1 + u * (y2 - y1);
      z = z1 + u * (z2 - z1);
      if (type != LINE && u < 0) {
        x = x1;
        y = y1;
        z = z1;}
      if (type == SEGMENT && u > 1.0) {
        x = x2;
        y = y2;
        z = z2;}
    }
    return (x-px)*(x-px) + (y-py)*(y-py) + (z-pz)*(z-pz);
  }

int main() {
  // expected: (-5,2)
  cerr << RotateCCW90(PT(2,5)) << endl;
  // expected: (5,-2)
  cerr << RotateCW90(PT(2,5)) << endl;
  // expected: (-5,2)
  cerr << RotateCCW(PT(2,5),M_PI/2) << endl;
  // expected: (5,2)
  cerr << ProjectPointLine(PT(-5,-2), PT(10,4), PT(3,7)) << endl;
  // expected: (5,2) (7.5,3) (2.5,1)
  cerr << ProjectPointSegment(PT(-5,-2), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(7.5,3), PT(10,4), PT(3,7)) << " "
       << ProjectPointSegment(PT(-5,-2), PT(2.5,1), PT(3,7)) << endl;
  // expected: 6.78903
  cerr << DistancePointPlane(4,-4,3,2,-2,5,-8) << endl;
  // expected: 1 0 1
  cerr << LinesParallel(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesParallel(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;
  // expected: 0 0 1
  cerr << LinesCollinear(PT(1,1), PT(3,5), PT(2,1), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(2,0), PT(4,5)) << " "
       << LinesCollinear(PT(1,1), PT(3,5), PT(5,9), PT(7,13)) << endl;
  // expected: 1 1 1 0
  cerr << SegmentsIntersect(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(4,3), PT(0,5)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(2,-1), PT(-2,1)) << " "
       << SegmentsIntersect(PT(0,0), PT(2,4), PT(5,5), PT(1,7)) << endl;
  // expected: (1,2)
  cerr << ComputeLineIntersection(PT(0,0), PT(2,4), PT(3,1), PT(-1,3)) <<
       endl;
  // expected: (1,1)
  cerr << ComputeCircleCenter(PT(-3,4), PT(6,1), PT(4,5)) << endl;
  vector<PT> v;
  v.push_back(PT(0,0));
  v.push_back(PT(5,0));
  v.push_back(PT(5,5));
  v.push_back(PT(0,5));
  // expected: 1 1 1 0 0
  cerr << PointInPolygon(v, PT(2,2)) << " "
       << PointInPolygon(v, PT(2,0)) << " "
       << PointInPolygon(v, PT(0,2)) << " "
       << PointInPolygon(v, PT(5,2)) << " "
       << PointInPolygon(v, PT(2,5)) << endl;
  // expected: 0 1 1 1 1
  cerr << PointOnPolygon(v, PT(2,2)) << " "
       << PointOnPolygon(v, PT(2,0)) << " "
       << PointOnPolygon(v, PT(0,2)) << " "
       << PointOnPolygon(v, PT(5,2)) << " "
       << PointOnPolygon(v, PT(2,5)) << endl;
  // expected: (1,6)
  //           (5,4) (4,5)
  //           blank line
  //           (4,5) (5,4)
  //           blank line
  //           (4,5) (5,4)
  vector<PT> u = CircleLineIntersection(PT(0,6), PT(2,6), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleLineIntersection(PT(0,9), PT(9,0), PT(1,1), 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(10,10), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(8,8), 5, 5);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 10, sqrt(2.0)/2.0);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  u = CircleCircleIntersection(PT(1,1), PT(4.5,4.5), 5, sqrt(2.0)/2.0);
  for (int i = 0; i < u.size(); i++) cerr << u[i] << " "; cerr << endl;
  // area should be 5.0
  // centroid should be (1.1666666, 1.166666)
  PT pa[] = { PT(0,0), PT(5,0), PT(1,1), PT(0,5) };
  vector<PT> p(pa, pa+4);
  PT c = ComputeCentroid(p);
  cerr << "Area: " << ComputeArea(p) << endl;
  cerr << "Centroid: " << c << endl;
}
```

6 6 Numerics

6.1 Euclid

```
// This is a collection of useful code for solving problems that
// involve modular linear equations. Note that all of the
// algorithms described here work on nonnegative integers.

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int,int> PII;
```

```
// return a % b (positive value)
int mod(int a, int b) {
  return ((a%b)+b)%b;
}

// computes gcd(a,b)
int gcd(int a, int b) {
  int tmp;
  while(b){a%=b; tmp=a; a=b; b=tmp;}
  return a;
}

// computes lcm(a,b)
int lcm(int a, int b) {
  return a/gcd(a,b)*b;
}

// returns d = gcd(a,b); finds x,y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
  int xx = y = 0;
  int yy = x = 1;
  while (b) {
    int q = a/b;
    int t = b; b = a%b; a = t;
    t = xx; xx = x-q*xx; x = t;
    t = yy; yy = y-q*yy; y = t;
  }
  return a;
}

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
  int x, y;
  VI solutions;
  int d = extended_euclid(a, n, x, y);
  if (!(b%d)) {
    x = mod (x*(b/d), n);
    for (int i = 0; i < d; i++)
      solutions.push_back(mod(x + i*(n/d), n));
  }
  return solutions;
}

// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
  int x, y;
  int d = extended_euclid(a, n, x, y);
  if (d > 1) return -1;
  return mod(x,n);
}

// Chinese remainder theorem (special case): find z such that
// z % x = a, z % y = b. Here, z is unique modulo M = lcm(x,y).
// Return (z,M). On failure, M = -1.
PII chinese_remainder_theorem(int x, int a, int y, int b) {
  int s, t;
  int d = extended_euclid(x, y, s, t);
  if (a%d != b%d) return make_pair(0, -1);
  return make_pair(mod(s*b*x+t*a*y,x*y)/d, x*y/d);
}

// Chinese remainder theorem: find z such that
// z % x[i] = a[i] for all i. Note that the solution is
// unique modulo M = lcm_i (x[i]). Return (z,M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &x, const VI &a) {
  PII ret = make_pair(a[0], x[0]);
  for (int i = 1; i < x.size(); i++) {
    ret = chinese_remainder_theorem(ret.second, ret.first, x[i], a[i]);
    if (ret.second == -1) break;
  }
  return ret;
}

// computes x and y such that ax + by = c; on failure, x = y =-1
void linear_diophantine(int a, int b, int c, int &x, int &y) {
  int d = gcd(a,b);
  if (c%d) {
    x = y = -1;
  } else {
    x = c/d * mod_inverse(a/d, b/d);
    y = (c-a*x)/b;
  }
}

int main() {

  // expected: 2
  cout << gcd(14, 30) << endl;

  // expected: 2 -2 1
  int x, y;
  int d = extended_euclid(14, 30, x, y);
  cout << d << " " << x << " " << y << endl;

  // expected: 95 45
  VI sols = modular_linear_equation_solver(14, 30, 100);
  for (int i = 0; i < (int) sols.size(); i++) cout << sols[i] << " ";
  cout << endl;

  // expected: 8
  cout << mod_inverse(8, 9) << endl;

  // expected: 23 56
  //           11 12
  int xs[] = {3, 5, 7, 4, 6};
  int as[] = {2, 3, 2, 3, 5};
  PII ret = chinese_remainder_theorem(VI (xs, xs+3), VI(as, as+3));
  cout << ret.first << " " << ret.second << endl;
  ret = chinese_remainder_theorem (VI(xs+3, xs+5), VI(as+3, as+5));
  cout << ret.first << " " << ret.second << endl;

  // expected: 5 -15
  linear_diophantine(7, 2, 5, x, y);
  cout << x << " " << y << endl;
```

```
}
```

6.2 FFT

```
namespace fft {
    struct cnum {
        double a, b;
        cnum operator+(const cnum &c) { return { a + c.a, b + c.b }; }
        cnum operator-(const cnum &c) { return { a - c.a, b - c.b }; }
        cnum operator*(const cnum &c) { return { a*c.a - b*c.b, a*c.b +
            b*c.a }; }
        cnum operator/(double d) { return { a / d, b / d }; }
    };

    const double PI = 2 * atan2(1, 0);

    int deg;
    vector<int> rev;

    void set_degree(int _deg) {
        assert(__builtin_popcount(_deg) == 1);
        deg = _deg;
        rev.resize(deg);

        for (int i = 1, j = 0; i < deg; i++) {
            int bit = deg / 2;
            for (; j >= bit; bit /= 2)
                j -= bit;
            j += bit;

            rev[i] = j;
        }
    }

    void transform(vector<cnum> &poly, bool invert) {
        if(deg != poly.size()) set_degree(poly.size());

        for (int i = 1; i < deg; i++)
            if(rev[i] > i)
                swap(poly[i], poly[rev[i]]);

        for (int len = 2; len <= deg; len *= 2) {
            double ang = 2 * PI / len * (invert ? -1 : 1);
            cnum base = { cos(ang), sin(ang) };

            for (int i = 0; i < deg; i += len) {
                cnum w = {1, 0};

                for (int j = 0; j < len / 2; j++) {
                    cnum u = poly[i+j];
                    cnum v = w * poly[i+j+len/2];

                    poly[i+j] = u + v;
                    poly[i+j+len/2] = u - v;

                    w = w * base;
                }
            }
        }

        if(invert) {
            for (int i = 0; i < deg; i++)
                poly[i] = poly[i] / double(deg);
        }
    }
};
```

6.3 Gauss-Jordan

```
// Gauss-Jordan elimination with full pivoting.
//
// Uses:
//   (1) solving systems of linear equations (AX=B)
//   (2) inverting matrices (AX=I)
//   (3) computing determinants of square matrices
//
// Running time: O(n^3)
//
// INPUT:   a[][] = an nxn matrix
//          b[][] = an nxm matrix
//
// OUTPUT:  X      = an nxm matrix (stored in b[][])
//          A^{-1} = an nxn matrix (stored in a[][])
//          returns determinant of a[][]

#include <iostream>
#include <vector>
#include <cmath>

using namespace std;

const double EPS = 1e-10;

typedef vector<int> VI;
typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

T GaussJordan(VVT &a, VVT &b) {
  const int n = a.size();
  const int m = b[0].size();
  VI irow(n), icol(n), ipiv(n);
  T det = 1;

  for (int i = 0; i < n; i++) {
    int pj = -1, pk = -1;
    for (int j = 0; j < n; j++) if (!ipiv[j])
      for (int k = 0; k < n; k++) if (!ipiv[k])
        if (pj == -1 || fabs(a[j][k]) > fabs(a[pj][pk])) { pj = j; pk = k;
            }
    if (fabs(a[pj][pk]) < EPS) { cerr << "Matrix is singular." << endl;
        exit(0); }
    ipiv[pk]++;
    swap(a[pj], a[pk]);
    swap(b[pj], b[pk]);
    if (pj != pk) det *= -1;
```

```
    irow[i] = pj;
    icol[i] = pk;

    T c = 1.0 / a[pk][pk];
    det *= a[pk][pk];
    a[pk][pk] = 1.0;
    for (int p = 0; p < n; p++) a[pk][p] *= c;
    for (int p = 0; p < m; p++) b[pk][p] *= c;
    for (int p = 0; p < n; p++) if (p != pk) {
      c = a[p][pk];
      a[p][pk] = 0;
      for (int q = 0; q < n; q++) a[p][q] -= a[pk][q] * c;
      for (int q = 0; q < m; q++) b[p][q] -= b[pk][q] * c;
    }
  }

  for (int p = n-1; p >= 0; p--) if (irow[p] != icol[p]) {
    for (int k = 0; k < n; k++) swap(a[k][irow[p]], a[k][icol[p]]);
  }

  return det;
}

int main() {
  const int n = 4;
  const int m = 2;
  double A[n][n] = { {1,2,3,4},{1,0,1,0},{5,3,2,4},{6,1,4,6} };
  double B[n][m] = { {1,2},{4,3},{5,6},{8,7} };
  VVT a(n), b(n);
  for (int i = 0; i < n; i++) {
    a[i] = VT(A[i], A[i] + n);
    b[i] = VT(B[i], B[i] + m);
  }

  double det = GaussJordan(a, b);

  // expected: 60
  cout << "Determinant: " << det << endl;

  // expected: -0.233333 0.166667 0.133333 0.0666667
  //           0.166667 0.166667 0.333333 -0.333333
  //           0.233333 0.833333 -0.133333 -0.0666667
  //           0.05 -0.75 -0.1 0.2
  cout << "Inverse: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }

  // expected: 1.63333 1.3
  //           -0.166667 0.5
  //           2.36667 1.7
  //           -1.85 -1.35
  cout << "Solution: " << endl;
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++)
      cout << b[i][j] << ' ';
    cout << endl;
  }
}
```

6.4 Matrix

```
template<typename T> struct matrix {
    int N;
    vector<T> dat;

    matrix<T> (int _N, T fill = T(0), T diag = T(0)) {
        N = _N;
        dat.resize(N * N, fill);

        for (int i = 0; i < N; i++)
            (*this)(i, i) = diag;
    }

    T& operator()(int i, int j) {
        return dat[N * i + j];
    }

    matrix<T> operator *(matrix<T> &b){
        matrix<T> r(N);

        for(int i=0; i<N; i++)
            for(int j=0; j<N; j++)
                for(int k=0; k<N; k++)
                    r(i, j) = r(i, j) + (*this)(i, k) * b(k, j);

        return r;
    }

    matrix<T> pow(ll expo){
        if(!expo) return matrix<T>(N, T(0), T(1));
        matrix<T> r = (*this * *this).pow(expo/2);
        return expo&1 ? r * *this : r;
    }

    friend ostream& operator<<(ostream &os, matrix<T> &m){
        os << "{";
        for(int i=0; i<m.N; i++){
            if(i) os << "},\n ";
            os << "{";
            for(int j=0; j<m.N; j++){
                if(j) os << ", ";
                os << setw(10) << m(i, j) << setw(0);
            }
        }
        return os << "}}";
    }
};

struct mll {
    const int MOD;

    ll val;
    mll(ll _val = 0) {
        val = _val % MOD;
```

```
        if (val < 0) val += MOD;
    }

    mll operator+(const mll &o) {
        return mll((val + o.val) % MOD);
    }

    mll operator*(const mll &o) {
        return mll((val * o.val) % MOD);
    }

    friend ostream& operator<<(ostream &os, mll &m) {
        return os << m.val;
    }
};
```

6.5 Primes

```
// O(sqrt(x)) Exhaustive Primality Test
#include <cmath>
#define EPS 1e-7
typedef long long LL;
bool IsPrimeSlow (LL x)
{
  if(x<=1) return false;
  if(x<=3) return true;
  if (!(x%2) || !(x%3)) return false;
  LL s=(LL)(sqrt((double)(x))+EPS);
  for(LL i=5;i<=s;i+=6)
  {
    if (!(x%i) || !(x%(i+2))) return false;
  }
  return true;
}
// Primes less than 1000:
//      2     3     5     7    11    13    17    19    23    29    31    37
//     41    43    47    53    59    61    67    71    73    79    83    89
//     97   101   103   107   109   113   127   131   137   139   149   151
//    157   163   167   173   179   181   191   193   197   199   211   223
//    227   229   233   239   241   251   257   263   269   271   277   281
//    283   293   307   311   313   317   331   337   347   349   353   359
//    367   373   379   383   389   397   401   409   419   421   431   433
//    439   443   449   457   461   463   467   479   487   491   499   503
//    509   521   523   541   547   557   563   569   571   577   587   593
//    599   601   607   613   617   619   631   641   643   647   653   659
//    661   673   677   683   691   701   709   719   727   733   739   743
//    751   757   761   769   773   787   797   809   811   821   823   827
//    829   839   853   857   859   863   877   881   883   887   907   911
//    919   929   937   941   947   953   967   971   977   983   991   997

// Other primes:
//    The largest prime smaller than 10 is 7.
//    The largest prime smaller than 100 is 97.
//    The largest prime smaller than 1000 is 997.
//    The largest prime smaller than 10000 is 9973.
//    The largest prime smaller than 100000 is 99991.
//    The largest prime smaller than 1000000 is 999983.
//    The largest prime smaller than 10000000 is 9999991.
//    The largest prime smaller than 100000000 is 99999989.
//    The largest prime smaller than 1000000000 is 999999937.
//    The largest prime smaller than 10000000000 is 9999999967.
//    The largest prime smaller than 100000000000 is 99999999977.
//    The largest prime smaller than 1000000000000 is 999999999989.
//    The largest prime smaller than 10000000000000 is 9999999999971.
//    The largest prime smaller than 100000000000000 is 99999999999973.
//    The largest prime smaller than 1000000000000000 is 999999999999989.
//    The largest prime smaller than 10000000000000000 is
    9999999999999937.
//    The largest prime smaller than 100000000000000000 is
    99999999999999997.
//    The largest prime smaller than 1000000000000000000 is
    999999999999999989.
```

6.6 Reduced Row Echelon Form

```
// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting. This can be used for computing
// the rank of a matrix.
//
// Running time: O(n^3)
//
// INPUT:   a[][] = an nxn matrix
//
// OUTPUT:  rref[][] = an nxm matrix (stored in a[][])
//          returns rank of a[][]

const double EPSILON = 1e-7;

typedef vector<double> VD;
typedef vector<VD> VVD;

// returns rank

int rref (VVD &a){
  int i,j,r,c;
  int n = a.size();
  int m = a[0].size();
  for (r=c=0;c<m;c++){
    j=r;
    for (i=r+1;i<n;i++) if (fabs(a[i][c])>fabs(a[j][c])) j = i;
    if (fabs(a[j][c])<EPSILON) continue;
    for (i=0;i<m;i++) swap (a[j][i],a[r][i]);

    double s = a[r][c];
    for (j=0;j<m;j++) a[r][j] /= s;
    for (i=0;i<n;i++) if (i != r){
      double t = a[i][c];
      for (j=0;j<m;j++) a[i][j] -= t*a[r][j];
    }
    r++;
  }
  return r;
}
```

6.7 Simplex

```
// Two-phase simplex algorithm for solving linear programs of the form
//
//     maximize     c^T x
//     subject to   Ax <= b
//                  x >= 0
//
// INPUT: A -- an m x n matrix
//        b -- an m-dimensional vector
//        c -- an n-dimensional vector
//        x -- a vector where the optimal solution will be stored
//
// OUTPUT: value of the optimal solution (infinity if unbounded
//         above, nan if infeasible)
//
// To use this code, create an LPSolver object with A, b, and c as
// arguments. Then, call Solve(x).

#include <iostream>
#include <iomanip>
#include <vector>
#include <cmath>
#include <limits>

using namespace std;

typedef long double DOUBLE;
typedef vector<DOUBLE> VD;
typedef vector<VD> VVD;
typedef vector<int> VI;

const DOUBLE EPS = 1e-9;

struct LPSolver {
  int m, n;
  VI B, N;
  VVD D;

  LPSolver(const VVD &A, const VD &b, const VD &c) :
    m(b.size()), n(c.size()), N(n + 1), B(m), D(m + 2, VD(n + 2)) {
    for (int i = 0; i < m; i++) for (int j = 0; j < n; j++) D[i][j] =
        A[i][j];
    for (int i = 0; i < m; i++) { B[i] = n + i; D[i][n] = -1; D[i][n + 1]
        = b[i]; }
    for (int j = 0; j < n; j++) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m + 1][n] = 1;
  }

  void Pivot(int r, int s) {
    for (int i = 0; i < m + 2; i++) if (i != r)
      for (int j = 0; j < n + 2; j++) if (j != s)
        D[i][j] -= D[r][j] * D[i][s] / D[r][s];
    for (int j = 0; j < n + 2; j++) if (j != s) D[r][j] /= D[r][s];
    for (int i = 0; i < m + 2; i++) if (i != r) D[i][s] /= -D[r][s];
    D[r][s] = 1.0 / D[r][s];
    swap(B[r], N[s]);
  }

  bool Simplex(int phase) {
    int x = phase == 1 ? m + 1 : m;
    while (true) {
      int s = -1;
      for (int j = 0; j <= n; j++) {
        if (phase == 2 && N[j] == -1) continue;
        if (s == -1 || D[x][j] < D[x][s] || D[x][j] == D[x][s] && N[j] <
            N[s]) s = j;
      }
      if (D[x][s] > -EPS) return true;
      int r = -1;
      for (int i = 0; i < m; i++) {
        if (D[i][s] < EPS) continue;
        if (r == -1 || D[i][n + 1] / D[i][s] < D[r][n + 1] / D[r][s] ||
          (D[i][n + 1] / D[i][s]) == (D[r][n + 1] / D[r][s]) && B[i] <
            B[r]) r = i;
      }
      if (r == -1) return false;
      Pivot(r, s);
    }
  }

  DOUBLE Solve(VD &x) {
    int r = 0;
    for (int i = 1; i < m; i++) if (D[i][n + 1] < D[r][n + 1]) r = i;
    if (D[r][n + 1] < -EPS) {
      Pivot(r, n);
      if (!Simplex(1) || D[m + 1][n + 1] < -EPS) return
          -numeric_limits<DOUBLE>::infinity();
      for (int i = 0; i < m; i++) if (B[i] == -1) {
        int s = -1;
        for (int j = 0; j <= n; j++)
          if (s == -1 || D[i][j] < D[i][s] || D[i][j] == D[i][s] && N[j] <
            N[s]) s = j;
        Pivot(i, s);
      }
    }
    if (!Simplex(2)) return numeric_limits<DOUBLE>::infinity();
    x = VD(n);
    for (int i = 0; i < m; i++) if (B[i] < n) x[B[i]] = D[i][n + 1];
    return D[m][n + 1];
  }
};

int main() {

  const int m = 4;
  const int n = 3;
  DOUBLE _A[m][n] = {
    { 6, -1, 0 },
    { -1, -5, 0 },
    { 1, 5, 1 },
    { -1, -5, -1 }
  };
  DOUBLE _b[m] = { 10, -4, 5, -5 };
  DOUBLE _c[n] = { 1, -1, 0 };

  VVD A(m);
  VD b(_b, _b + m);
```

```
  VD c(_c, _c + n);
  for (int i = 0; i < m; i++) A[i] = VD(_A[i], _A[i] + n);

  LPSolver solver(A, b, c);
  VD x;
  DOUBLE value = solver.Solve(x);

  cerr << "VALUE: " << value << endl; // VALUE: 1.29032
  cerr << "SOLUTION:"; // SOLUTION: 1.74194 0.451613 1
  for (size_t i = 0; i < x.size(); i++) cerr << " " << x[i];
  cerr << endl;
  return 0;
}
```

7 7 String

7.1 KMP

```
template<typename T> struct kmp {
    int M;
    vector<T> needle;
    vector<int> succ;

    kmp(vector<T> _needle) {
        needle = _needle;
        M = needle.size();

        succ.resize(M + 1);
        succ[0] = -1, succ[1] = 0;

        int cur = 0;
        for (int i = 2; i <= M; ) {
            if (needle[i-1] == needle[cur]) succ[i++] = ++cur;
            else if (cur) cur = succ[cur];
            else succ[i++] = 0;
        }
    }

    vector<bool> find(vector<T> &haystack) {
        int N = haystack.size(), i = 0;
        vector<bool> res(N);

        for (int m = 0; m + i < N; ) {
            if (i < M && needle[i] == haystack[m + i]) {
                if (i == M - 1) res[m] = true;
                i++;
            } else if (succ[i] != -1) {
                m = m + i - succ[i];
                i = succ[i];
            } else {
                i = 0;
                m++;
            }
        }

        return res;
    }
};
```

7.2 Suffix Arrays

```
// Suffix array construction in O(L log^2 L) time. Routine for
// computing the length of the longest common prefix of any two
// suffixes in O(log L) time.
//
// INPUT:  string s
//
// OUTPUT: array suffix[] such that suffix[i] = index (from 0 to L-1)
//         of substring s[i...L-1] in the list of sorted suffixes.
//         That is, if we take the inverse of the permutation suffix[],
//         we get the actual suffix array.

#include <vector>
#include <iostream>
#include <string>

using namespace std;

struct SuffixArray {
  const int L;
  string s;
  vector<vector<int> > P;
  vector<pair<pair<int,int>,int> > M;

  SuffixArray(const string &s) : L(s.length()), s(s), P(1, vector<int>(L,
      0)), M(L) {
    for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
    for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
      P.push_back(vector<int>(L, 0));
      for (int i = 0; i < L; i++)
        M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ?
            P[level-1][i + skip] : -1000), i);
      sort(M.begin(), M.end());
      for (int i = 0; i < L; i++)
        P[level][M[i].second] = (i > 0 && M[i].first == M[i-1].first) ?
            P[level][M[i-1].second] : i;
    }
  }

  vector<int> GetSuffixArray() { return P.back(); }

  // returns the length of the longest common prefix of s[i...L-1] and
      s[j...L-1]
  int LongestCommonPrefix(int i, int j) {
    int len = 0;
    if (i == j) return L - i;
    for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
      if (P[k][i] == P[k][j]) {
        i += 1 << k;
        j += 1 << k;
        len += 1 << k;
      }
    }
    return len;
  }
```

```
};

int main() {

  // bobocel is the 0'th suffix
  //  obocel is the 5'th suffix
  //   bocel is the 1'st suffix
  //    ocel is the 6'th suffix
  //     cel is the 2'nd suffix
  //      el is the 3'rd suffix
  //       l is the 4'th suffix
  SuffixArray suffix("bobocel");
  vector<int> v = suffix.GetSuffixArray();

  // Expected output: 0 5 1 6 2 3 4
  //                  2
  for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
  cout << endl;
  cout << suffix.LongestCommonPrefix(0, 2) << endl;
}
```

8 8 Misc

8.1 IO

```
#include <iostream>
#include <iomanip>

using namespace std;

int main()
{
    // Ouput a specific number of digits past the decimal point,
    // in this case 5
    cout.setf(ios::fixed); cout << setprecision(5);
    cout << 100.0/7.0 << endl;
    cout.unsetf(ios::fixed);

    // Output the decimal point and trailing zeros
    cout.setf(ios::showpoint);
    cout << 100.0 << endl;
    cout.unsetf(ios::showpoint);

    // Output a '+' before positive values
    cout.setf(ios::showpos);
    cout << 100 << " " << -100 << endl;
    cout.unsetf(ios::showpos);

    // Output numerical values in hexadecimal
    cout << hex << 100 << " " << 1000 << " " << 10000 << dec << endl;
}
```

8.2 Longest Increasing Subsequence

```
// Given a list of numbers of length n, this routine extracts a
// longest increasing subsequence.
//
// Running time: O(n log n)
//
//   INPUT: a vector of integers
//   OUTPUT: a vector containing the longest increasing subsequence

#include <iostream>
#include <vector>
#include <algorithm>

using namespace std;

typedef vector<int> VI;
typedef pair<int,int> PII;
typedef vector<PII> VPII;

#define STRICTLY_INCREASNG

VI LongestIncreasingSubsequence(VI v) {
  VPII best;
  VI dad(v.size(), -1);

  for (int i = 0; i < v.size(); i++) {
#ifdef STRICTLY_INCREASNG
    PII item = make_pair(v[i], 0);
    VPII::iterator it = lower_bound(best.begin(), best.end(), item);
    item.second = i;
#else
    PII item = make_pair(v[i], i);
    VPII::iterator it = upper_bound(best.begin(), best.end(), item);
#endif
    if (it == best.end()) {
      dad[i] = (best.size() == 0 ? -1 : best.back().second);
      best.push_back(item);
    } else {
      dad[i] = dad[it->second];
      *it = item;
    }
  }

  VI ret;
  for (int i = best.back().second; i >= 0; i = dad[i])
    ret.push_back(v[i]);
  reverse(ret.begin(), ret.end());
  return ret;
}
```

8.3 Regular Expressions - Java

```
// Code which demonstrates the use of Java's regular expression libraries.
// This is a solution for
//
//   Loglan: a logical language
//   http://acm.uva.es/p/v1/134.html
//
// In this problem, we are given a regular language, whose rules can be
// inferred directly from the code. For each sentence in the input, we
    must
```

```java
// determine whether the sentence matches the regular expression or not.
    The
// code consists of (1) building the regular expression (which is fairly
// complex) and (2) using the regex to match sentences.

import java.util.*;
import java.util.regex.*;

public class LogLan {

    public static String BuildRegex (){
        String space = " +";

        String A = "([aeiou])";
        String C = "([a-z&&[^aeiou]])";
        String MOD = "(g" + A + ")";
        String BA = "(b" + A + ")";
        String DA = "(d" + A + ")";
        String LA = "(l" + A + ")";
        String NAM = "([a-z]*" + C + ")";
        String PREDA = "(" + C + C + A + C + A + "|" + C + A + C + C + A +
            ")";

        String predstring = "(" + PREDA + "(" + space + PREDA + ")*)";
        String predname = "(" + LA + space + predstring + "|" + NAM + ")";
        String preds = "(" + predstring + "(" + space + A + space +
            predstring + ")*)";
        String predclaim = "(" + predname + space + BA + space + preds +
            "|" + DA + space +
           preds + ")";
        String verbpred = "(" + MOD + space + predstring + ")";
        String statement = "(" + predname + space + verbpred + space +
            predname + "|" +
           predname + space + verbpred + ")";
        String sentence = "(" + statement + "|" + predclaim + ")";

        return "^" + sentence + "$";
    }

    public static void main (String args[]){

        String regex = BuildRegex();
        Pattern pattern = Pattern.compile (regex);

        Scanner s = new Scanner(System.in);
        while (true) {

            // In this problem, each sentence consists of multiple lines,
                The where the last
            // line is terminated by a period. The code below reads lines
                until
            // encountering a line whose final character is a '.'. Note
                the use of
            //
            //   s.length() to get length of string
            //   s.charAt() to extract characters from a Java string
            //   s.trim() to remove whitespace from the beginning and end
                of Java string
            //
            // Other useful String manipulation methods include
            //
            //   s.compareTo(t) < 0 if s < t, lexicographically
            //   s.indexOf("apple") returns index of first occurrence of
                "apple" in s
            //   s.lastIndexOf("apple") returns index of last occurrence
                of "apple" in s
            //   s.replace(c,d) replaces occurrences of character c with d
            //   s.startsWith("apple) returns (s.indexOf("apple") == 0)
            //   s.toLowerCase() / s.toUpperCase() returns a new
                lower/uppercased string
            //
            //   Integer.parseInt(s) converts s to an integer (32-bit)
            //   Long.parseLong(s) converts s to a long (64-bit)
            //   Double.parseDouble(s) converts s to a double

            String sentence = "";
            while (true){
                sentence = (sentence + " " + s.nextLine()).trim();
                if (sentence.equals("#")) return;
                if (sentence.charAt(sentence.length()-1) == '.') break;
            }

            // now, we remove the period, and match the regular expression

            String removed_period = sentence.substring(0,
                sentence.length()-1).trim();
            if (pattern.matcher (removed_period).find()){
                System.out.println ("Good");
            } else {
                System.out.println ("Bad!");
            }
        }
    }
}
```