Matemática III – Primer Parcial Módulo I (17/05/2023) – Turno Miércoles

Ejercicio 1

a) $P(A \cup B) = \frac{7}{8} \quad P(A \cap B) = \frac{1}{4} \quad P(A^c) = \frac{5}{8}$

a_1) $P(A) = 1 - P(A^c) = 1 - \frac{5}{8} = \frac{3}{8}$

a_2) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Despejando:

$P(B) = P(A \cup B) - P(A) + P(A \cap B)$

$P(B) = \frac{7}{8} - \frac{3}{8} + \frac{1}{4} = \frac{3}{4}$

a_3) $P(A \cap B^c) = P(A) - P(A \cap B)$

$P(A \cap B^c) = \frac{3}{8} - \frac{1}{4} = \frac{1}{8}$

b) $P(A \cap B) \neq P(A) \cdot P(B)$

$\frac{1}{4} \neq \frac{3}{8} \cdot \frac{3}{4}$ → A y B no son independientes → A y B^c tampoco son independientes

Ejercicio 2

a) A: "La carta proviene de la caja A " → $P(A) = 1/2$

B: "La carta proviene de la caja B" → $P(B) = 1/2$

$A \cup B = S \quad A \cap B = \emptyset \quad P(A) > 0 \quad P(B) > 0$

C: "La carta es par"

$P(C/A) = \frac{4}{9} \quad P(C/B) = \frac{2}{5} \quad P(C) > 0$

$$P(A/C) = \frac{P(C/A) \cdot P(A)}{P(C/A) \cdot P(A) + P(C/B) \cdot P(B)} = \frac{4/9 \cdot 1/2}{4/9 \cdot 1/2 + 2/5 \cdot 1/2} = \frac{10}{19} = 0{,}5263$$

↓

Teorema de Bayes

b) X: "n° de veces que el equipo gana entre 2"

$X \sim B(n, p) \quad n = 2 \quad p = 0{,}5$

$$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - \left[\binom{2}{0} \cdot 0{,}5^0 \cdot (1 - 0{,}5)^{2-0}\right] = 1 - 0{,}5^2 = 0{,}75$$

Ejercicio 3

X: " Presion de aire de un neumatico (lb/pulg^2)"

$X \sim N(\mu, \sigma^2)$ $\mu = 31$ $\sigma = 0{,}2$

a) $P(X < 30{,}4) = P\left(\frac{X-\mu}{\sigma} < \frac{30{,}4-31}{0{,}2}\right) = P(Z < -3) = \Phi(-3) = 0{,}00135$

Estandarizo — App

b) Y: "n° de neumaticos con presion baja entre 4"

$Y \sim B(n, p)$ $n = 4$ $p = 0{,}00135$

$P(Y = 2) = \binom{4}{2} \cdot 0{,}00135^2 \cdot (1 - 0{,}00135)^2 = 0{,}00001$

App

Ejercicio 4

X: "Tiempo que la maquina no funciona en (hs)"

$$f_x(x) = \begin{cases} 2(1-x) & 0 \le x \le 1 \\ 0 & \text{cc} \end{cases}$$

C = costo por inactividad → $C = 10 + 20x + 4x^2$

$$E(C) = E(10 + 20x + 4x^2) = 10 + 20E(x) + 4E(x^2) = 10 + 20 \cdot \frac{1}{3} + 4 \cdot \frac{1}{6} = \frac{52}{3} = 17,\hat{3}$$

Linealidad

Cálculos Auxiliares:

$$E(x) = \int_0^1 x \cdot 2(1-x)dx = 2\int_0^1 (x - x^2)dx = 2 \cdot \left(\frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1\right) = 2 \cdot \left(\frac{1}{2} - \frac{1}{3}\right) = 2 \cdot \frac{1}{6} = \frac{1}{3}$$

$$E(x^2) = \int_0^1 x^2 \cdot 2(1-x)dx = 2\int_0^1 (x^2 - x^3)dx = 2 \cdot \left(\frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1\right) = 2 \cdot \left(\frac{1}{3} - \frac{1}{4}\right) = 2 \cdot \frac{1}{12} = \frac{1}{6}$$

$$V(C) = E(C^2) - [E(C)]^2 = \frac{1652}{5} - \left(\frac{52}{3}\right)^2 = \frac{1348}{45} = 29{,}9556$$

Cálculos Auxiliares:

$$C^2 = C \cdot C = (10 + 20x + 4x^2) \cdot (10 + 20x + 4x^2) = 100 + 400x + 480x^2 + 160x^3 + 16x^4$$

$$E(100 + 400x + 480x^2 + 160x^3 + 16x^4) = 100 + 400E(x) + 480E(x^2) + 160E(x^3) + 16E(x^4)$$

↓ Linealidad

$$E(x^3) = \int_0^1 x^3 \cdot 2(1-x)dx = 2\int_0^1 (x^3 - x^4)dx = 2 \cdot \left(\frac{x^4}{4} - \frac{x^5}{5}\Big|_0^1\right) = 2 \cdot \left(\frac{1}{4} - \frac{1}{5}\right) = 2 \cdot \frac{1}{20} = \frac{1}{10}$$

$$E(x^4) = \int_0^1 x^4 \cdot 2(1-x)dx = 2\int_0^1 (x^4 - x^5)dx = 2 \cdot \left(\frac{x^5}{5} - \frac{x^6}{6}\Big|_0^1\right) = 2 \cdot \left(\frac{1}{5} - \frac{1}{6}\right) = 2 \cdot \frac{1}{30} = \frac{1}{15}$$

Remplazando los valores:

$$E(C^2) = 100 + 400 \cdot \frac{1}{3} + 480 \cdot \frac{1}{6} + 160 \cdot \frac{1}{10} + 16 \cdot \frac{1}{15} = \frac{1652}{5} = 330{,}4$$

Ejercicio 5

X: "n° de personas que se presentan entre 200"

$X \sim B(n, p)$ $n = 200$ $p = 0{,}95$

$np = 190 > 5$

$n(1-p) = 10 > 5$

$$P(X \leq 190) \underset{\text{Corr. por cont.}}{\approx} P(X \leq 190{,}5) \underset{\text{Estandarizo}}{=} P\left(\overbrace{\frac{X - np}{\sqrt{np(1-p)}}}^{Z \approx N(0,1)} \leq \frac{190{,}5 - 190}{\sqrt{9{,}5}}\right) \underset{\text{TCL}}{\approx} \Phi(0{,}16) \underset{\text{App}}{=} 0{,}56356$$