HOJA N°

FECHA

PRACTICA MATEMATICA

1) $E(X) = \mu \qquad V(X) = \sigma^2$

$$\overline{X}_1 = \frac{1}{n-1} \sum_{i=1}^{n-1} \qquad \overline{X}_2 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$ECM(\hat{\theta}) = V(\theta) + b((\hat{\theta}))^2 \qquad E(\overline{X}_2) = \mu \qquad \therefore b(\overline{X}_2) = 0$$

$$\therefore ECM(\overline{X}_2) = \frac{\sigma^2}{n}$$

$$ECM(\overline{X}_1) = V(\overline{X}_1) + b((\overline{X}_1))^2 \qquad E(\overline{X}_1) = \mu \qquad \therefore b(\overline{X}_2) =$$

$$\therefore ECM(\overline{X}_1) = \frac{\sigma^2}{n-1}$$

$ECM(\overline{X}_1) > ECM(\overline{X}_2) \quad \therefore \quad \overline{X}_2$ es mejor estimador puntual que $\overline{X}_1$

calc. aux

$$E(\overline{X}_1) = E\left(\frac{1}{n-1} \sum_{i=1}^{n-1} x_i\right) = \frac{1}{n-1} E\left(\sum_{i=1}^{n-1} x_i\right) =$$

$$\frac{1}{n-1} \sum_{i=1}^{n-1} \underbrace{E(x_i)}_{\mu} = \frac{1}{n-1} \sum_{i=1}^{n-1} \mu = \frac{1}{\cancel{n-1}} \cancel{n-1}\, \mu = \mu$$

(creo que bastaba solo calcular varianza)

2) $x_1, x_2, \ldots, x_7$ muestra aleatoria de v.a X

$\mu = E(X) \qquad \sigma^2 = V(X)$

$$\hat{\mu}_1 = \frac{x_1 + x_2 + \ldots x_7}{7}$$

$$\hat{\mu}_2 = \frac{2x_1 - x_6 + x_4}{2}$$

$$\hat{\mu}_3 = \frac{2x_1 - x_7 + x_5}{3}$$

NOTA

a)

- $E(\hat{\mu}_1) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = E(\bar{X}) = \mu$

$\therefore \hat{\theta}_1$ es insesgado

- $E(\hat{\mu}_2) = E\left(\frac{2X_1 - X_6 + X_4}{2}\right) = \frac{1}{2}E(2X_1 - X_6 + X_4) =$

$= \frac{1}{2} \cdot \left(2E(X_1) - E(X_6) + E(X_4)\right) = \frac{1}{2} \cdot (2\mu - \mu + \mu) =$

$= \mu - \cancel{\frac{1}{2}\mu} + \cancel{\frac{1}{2}\mu} = \mu \quad \therefore \hat{\theta}_2$ es insesgado

- $E(\hat{\mu}_3) = E\left(\frac{2X_1 - X_7 + X_3}{3}\right) = \frac{1}{3}E(2X_1 - X_7 + X_3) =$

$= \frac{1}{3}\left[2E(X_1) - E(X_7) + E(X_3)\right] = \frac{1}{3}\left[2\mu - \mu + \mu\right] =$

$= \frac{2}{3}\mu - \cancel{\frac{1}{3}\mu} + \cancel{\frac{1}{3}\mu} = \frac{2}{3}\mu$

b) $ECM(\hat{\theta}) = V(\hat{\theta}) + (b(\hat{\theta}))^2$

$\therefore ECM(\hat{\mu}_1) = V(\hat{\mu}_1) + (b(\hat{\mu}_1))^2 =$

$= \boxed{\frac{\sigma^2}{7}}$

- $V(\hat{\mu}_1) = \frac{\sigma^2}{7}$
- $b(\hat{\mu}_1) = 0$ ↓ es insesgado

- $V(\hat{\mu}_2) = V\left(\frac{2X_1 - X_6 + X_4}{2}\right) = \left(\frac{1}{2}\right)^2 V(2X_1 - X_6 + X_4) =$

$= \frac{1}{4} \cdot \left[4V(X_1) + V(X_6) + V(X_4)\right] = \frac{1}{4} \cdot \left[4\sigma^2 + \sigma^2 + \sigma^2\right] =$

$= \sigma^2 + \frac{1}{4}\sigma^2 + \frac{1}{4}\sigma^2 = \frac{3}{2}\sigma^2$

NOTA

$b(\hat{\mu}_2) = 0 \rightarrow$ es insesgado

$\therefore \ ECM(\hat{\mu}_2) = V(\hat{\mu}_2) = \boxed{\frac{3}{2}\sigma^2}$

$\cdot \ V(\hat{\mu}_3) = V\left(\frac{2X_1 - X_7 + X_3}{3}\right) = \left(\frac{1}{3}\right)^2 V(2X_1 - X_7 + X_3) =$

$= \frac{1}{9}\left[4V(X_1) + V(X_7) + V(X_3)\right] = \frac{1}{9}\left[4\sigma^2 + \sigma^2 + \sigma^2\right] =$

$= \frac{4}{9}\sigma^2 + \frac{1}{9}\sigma^2 + \frac{1}{9}\sigma^2 = \frac{6}{9}\sigma^2 = \frac{2}{3}\sigma^2$

$b(\hat{\mu}_3) = E(\hat{\mu}_3) - \mu = \frac{2}{3}\mu - \mu = -\frac{1}{3}\mu$

$\therefore \ ECM(\hat{\mu}_3) = V(\hat{\mu}_3) + (b(\hat{\mu}_3))^2 = \frac{2}{3}\sigma^2 + \left(-\frac{1}{3}\mu\right)^2 =$

$= \boxed{\frac{2}{3}\sigma^2 + \frac{\mu^2}{9}}$

c) $ECM(\hat{\mu}_1) = \frac{1}{7}\sigma^2 \quad ECM(\hat{\mu}_2) = \frac{3}{2}\sigma^2 \quad ECM(\hat{\mu}_3) = \frac{2}{3}\sigma^2 + \frac{\mu^2}{9}$

Notar que $\frac{2}{3}\sigma^2 > \frac{1}{7}\sigma^2 \quad \therefore \ \frac{2}{3}\sigma^2 + \frac{\mu^2}{9} > \frac{1}{7}\sigma^2$

y que $\frac{3}{2}\sigma^2 > \frac{1}{7}\sigma^2$

$\therefore$ $\hat{\mu}_1$ es mejor estimador puntual que $\hat{\mu}_2$ y $\hat{\mu}_3$
mejor en el sentido de que todos los valores se acercan más a μ

3) Sea $X_1, X_2, \ldots, X_n$

a) $\hat{\mu}^2 = \overline{X}^2$ es estimador insesgado de μ^2

$$E(\hat{\mu}^2) = E(\overline{X}^2) = \underbrace{V(\overline{X})}_{\frac{\sigma^2}{n}} + \underbrace{(E(X))^2}_{\mu} = \frac{\sigma^2}{n} + \mu^2 \neq \mu^2$$

b) $b(\hat{\mu}^2) = b(\overline{X}^2) = E(\overline{X}^2) - \mu^2 = \boxed{\frac{\sigma^2}{n}}$

c) a medida que aumenta n, disminuye el sesgo puesto que el denominador sería cada vez más grande (con el mismo numerador) y se obtienen valores más pequeños, es decir, es más consistente porque se acerca cada vez más a 0

4) a) Sea $X_1, X_2, \dots, X_5$ una muestra aleatoria de un v.a X, donde $X \sim P(\lambda)$. X "nro diario de desconexiones"

$$\cdot L(x_1, x_2, x_3, \dots, x_n, \lambda) \overset{\text{indep.}}{=} P(X_1 = x_1) \cdot P(X_2 = x_2) \cdot P(X_3 = x_3) \cdot$$

$$\dots P(X_n = x_n) = \frac{e^{-\lambda} \lambda^{x_1}}{x_1!} \cdot \frac{e^{-\lambda} \lambda^{x_2}}{x_2!} \dots \frac{e^{-\lambda} \lambda^{x_n}}{x_n!} =$$

$$= \frac{(e^{-\lambda})^n \lambda^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!} = \frac{e^{-\lambda n} \lambda^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!}.$$

Para facilitar cálculos se toma $\ln L(x_1, x_2, \dots, x_n, \lambda)$.

$$\ln L(x_1, x_2, \dots, x_n) = \ln \left(\frac{e^{-\lambda n} \lambda^{\sum_{i=1}^{n} x_i}}{\prod_{i=1}^{n} x_i!} \right) =$$

$$= \ln \left(e^{-\lambda n} \lambda^{\sum_{i=1}^{n} x_i} \right) - \ln \left(\prod_{i=1}^{n} x_i! \right) = \ln \left(e^{-\lambda n} \right) + \ln \left(\lambda^{\sum_{i=1}^{n} x_i} \right) -$$

$$- \ln \left(\prod_{i=1}^{n} x_i! \right) = (-\lambda n) \cdot \underbrace{\ln e}_{=1} + \left(\sum_{i=1}^{n} x_i \right) \ln \lambda - \ln \left(\prod_{i=1}^{n} x_i! \right) =$$

$$= (-\lambda n) + \left(\sum_{i=1}^{n} x_i \right) \ln \lambda - \ln \left(\prod_{i=1}^{n} x_i! \right) =$$

$$\frac{d}{d\lambda} \ln L(x_1, x_2, \dots, x_n, \lambda) = 0 \Rightarrow -n + \left(\sum_{i=1}^{n} x_i \right) \frac{1}{\lambda} - 0 = 0$$

$$\therefore n = \left(\sum_{i=1}^{n} x_i \right) \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{n} \sum_{i=1}^{n} x_i = \overline{x}$$ se puede verificar que $L(x_1, x_2, \dots, x_5, \lambda)$ tiene un

$$\therefore \lambda = \frac{1}{5} \sum_{i=1}^{5} x_i$$

$\lambda = \overline{x}$ un máximo

NOTA

HOJA N°

FECHA

es insesgado por ser el $\bar{X}$, y es consistente porque el sesgo es 0 $\therefore$ $b(\bar{X}) \xrightarrow[m\to\infty]{} 0$ $\quad V(\bar{X}) = \frac{\sigma^2}{m} \xrightarrow[m\to\infty]{} 0$

b) $\lambda = \bar{X} = \frac{1}{5}\sum^{5} x_i = \frac{2+5+3+3+7}{5} = \boxed{4}$

c) Hallar EMV de $P(X \geq 3) = \frac{e^{-\lambda}\lambda^3}{3!}$

el EMV sería (prop. invarianza)

$$g(\hat{\lambda}) = \frac{e^{-\hat{\lambda}}\,\hat{\lambda}^3}{3!} = \frac{e^{-\bar{X}}\,\bar{X}^3}{3!}$$

estimación $\rightarrow$ $\frac{e^{-4}\,4^3}{3!} = 0,195367$

$$X_i \begin{cases} 1 & \text{si es OK} \\ 0 & \text{c.c} \end{cases}$$

5) a) · $X_1, X_2, \dots, X_n$ una muestra aleatoria de una v.a $B(1,p)$

E.M.V

$$L(x_1, x_2, \dots, x_n, p) \overset{\text{indep.}}{=} P(X_1 = x_1)\cdot P(X_2 = x_2)\cdots P(X_n = x_n) =$$

$$= p^{x_1}(1-p)^{1-x_1} \cdot p^{x_2}(1-p)^{1-x_2} \cdots p^{x_3}(1-p)^{1-x_3} =$$

$$= p^{\sum_{i=1}^{n} x_i}\cdot(1-p)^{\sum_{i=1}^{n}(1-x_i)}$$

si x_i es $\neq$ a 1 o a 0 ao indep. $\binom{1}{x_i}$

$$\ln L(x_1, x_2, \dots, x_n, p) = \ln\left(p^{\sum_{i=1}^{n} x_i}\right) + \ln\left((1-p)^{\sum_{i=1}^{n}(1-x_i)}\right) =$$

$$= \sum_{i=1}^{n} x_i \ln p + \left(n - \sum_{i=1}^{n} x_i\right)\ln(1-p) =$$

$$\frac{d}{dp}\ln L(x_1, x_2, \dots, x_n, p) = 0 \Rightarrow \left(\sum_{i=1}^{n} x_i\right)\cdot\frac{1}{p} + \left(n - \sum_{i=1}^{n} x_i\right)\cdot\frac{1}{1-p}(-1) = 0$$

$$\Rightarrow \frac{\sum_{i=1}^{n} x_i}{p} - \frac{n - \sum_{i=1}^{n} x_i}{1-p} = 0 \Rightarrow \frac{\sum_{i=1}^{n} x_i}{p} = \frac{n - \sum_{i=1}^{n} x_i}{1-p}$$

$$\Rightarrow \frac{1-p}{p} = \frac{n - \sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i} \Rightarrow \frac{1}{p} - 1 = \frac{n}{\sum_{i=1}^{n} x_i} - 1 \Rightarrow p = \frac{\sum_{i=1}^{n} x_i}{n} = \bar{X}$$

NOTA

$\therefore \quad \hat{p} = \overline{X}$

b) b1) $\quad n = 100 \quad x = 5 \longrightarrow \sum_{i=1}^{100} x_i = 5$

estimación de p es $\hat{p} = \overline{X} = \frac{1}{100} \cdot \underbrace{\sum_{i=1}^{100} x_i}_{5} = \frac{5}{100} = 0{,}05$

b2) $\quad n = 100 \quad \sum_{i=1}^{100} x_i = 5 \quad \hat{p} = \overline{X}$

$$g(p) = (1-p)^6$$

· EMV $(1-p)^6$

$$g(\hat{p}) = g(\overline{X}) = (1-\overline{X})^6$$

estimación $\Rightarrow (1-0{,}05)^6 = \boxed{0{,}735092}$

6) X "proporción de tiempo que un estudiante emplea trabajando su prueba de actitud"

$$f(x) = \begin{cases} (2\theta+1)\, x^{2\theta} & , 0 \leq x \leq 1 \\ 0 & , \text{cc} \end{cases} \qquad \text{donde } \theta > -\frac{1}{2}$$

a) método de los momentos

$$\mu_1 = E(x) = \int_0^1 x \cdot \left[(2\theta+1) \cdot x^{2\theta}\right] = (1+2\theta)\, \frac{1}{2\theta+2}$$

$$\frac{1}{n} \sum_{i=1}^{n} x_i = \frac{1+2\theta}{2\theta+2} \Rightarrow$$

$$\Rightarrow \overline{X} = \frac{1+2\theta}{2\theta+2} \Rightarrow 1+2\theta = \overline{X} \cdot (2\theta+2) \Rightarrow$$

$$\Rightarrow 1+2\theta = 2\theta \cdot \overline{X} + 2\overline{X} \Rightarrow 2\theta - 2\theta\overline{X} + 2\overline{X} - 1$$

$$\Rightarrow 2\theta - 2\theta\overline{X} = 2\overline{X} - 1 \Rightarrow (2-2\overline{X})\theta = 2\overline{X} - 1 \Rightarrow$$

NOTA:

HOJA N°

FECHA

$$\theta = \frac{2\overline{x} - 1}{2 - 2\overline{x}}$$

$$\therefore \hat{\theta} = \frac{2 \cdot \left[\frac{1}{n} \sum_{i=1}^{n} x_i\right] - 1}{2 - 2\left[\frac{1}{n} \sum_{i=1}^{n} x_i\right]} \longleftarrow \text{estimador}$$

estimación

$$\frac{1}{n} \cdot \sum_{i=1}^{n} x_i = \frac{1}{10} \cdot \sum_{i=1}^{10} x_i = \frac{4}{5} = \overline{x}$$

$$\hat{\theta} = \frac{2 \cdot \frac{4}{5} - 1}{2 - 2 \cdot \frac{4}{5}} = \frac{\frac{3}{5}}{\frac{2}{5}} = \frac{3}{2} = 1,5$$

$\therefore \hat{\theta} = 1,5$ (estimación)

b) EMV

$$L(x_1, x_2, \ldots x_n, \theta) = f(x_1) \cdot f(x_2) \ldots f(x_n) =$$

$$= (2\theta + 1) x_1^{2\theta} \cdot (2\theta + 1) x_2^{2\theta} \cdot \ldots \cdot (2\theta + 1) x_n^{2\theta} =$$

$$= (2\theta + 1)^n \cdot \prod_{i=1}^{n} x_i^{2\theta}$$

$$\ln L(x_1, x_2, \ldots, x_n, \theta) = \ln\left((2\theta + 1)^n\right) + \sum_{i=1}^{n} \ln\left(x_i^{2\theta}\right) =$$

$$= n \cdot \ln(2\theta + 1) + 2\theta \sum_{i=1}^{n} \ln(x_i) =$$

$$\frac{d}{d\theta} \ln L(x_1, x_2, \ldots, x_n, \theta) = 0$$

$$\frac{2n}{2\theta + 1} + 2 \sum_{i=1}^{n} \ln(x_i) = 0$$

$$\Rightarrow \frac{2n}{2\theta + 1} = -2\sum_{i=1}^{n} \ln(x_i) \Rightarrow 2\theta + 1 = -\frac{n}{\sum_{i=1}^{n} \ln(x_i)}$$

$$\Rightarrow 2\theta = -\frac{n}{\sum_{i=1}^{n} \ln(x_i)} - 1 \Rightarrow \theta = \left[-\frac{n}{\sum_{i=1}^{n} \ln(x_i)} - 1\right] : 2$$

NOTA

$$\therefore \hat{\theta} = \left[\frac{n}{-\sum_{i=1}^{n} \ln(x_i)} - 1 \right] : 2$$ ← estimador

estimacion

$$-\sum_{i=1}^{10} \ln(x_i) = 2,4295 \qquad n=10$$

$$\hat{\theta} = \frac{\frac{10}{2,4295} - 1}{2} = \boxed{1,55804}$$

b) a y b hechos en teoria (σ^2 no es insesgado, tambien demostrado en teoria)

$$\hat{\mu} = \bar{X} \qquad \hat{\sigma}^2 = \frac{1}{m} \sum_{i=1}^{m} (x_i - \bar{x})^2$$

c) $\mu = \bar{x} = 384,4$ estimacion

$$\sigma = \sqrt{\frac{1}{m} \sum_{i=1}^{m} (x_i - \bar{x})^2} = 18,8584\,1987$$

→ desviacion estandar poblacional

d) $P(X < 420) = ?$ $\cdot\ \mu = 384,4$ $\cdot\ \sigma = 18,85841987$

$$P(X < 420) = \boxed{0,97047}$$ estimación

NOTA