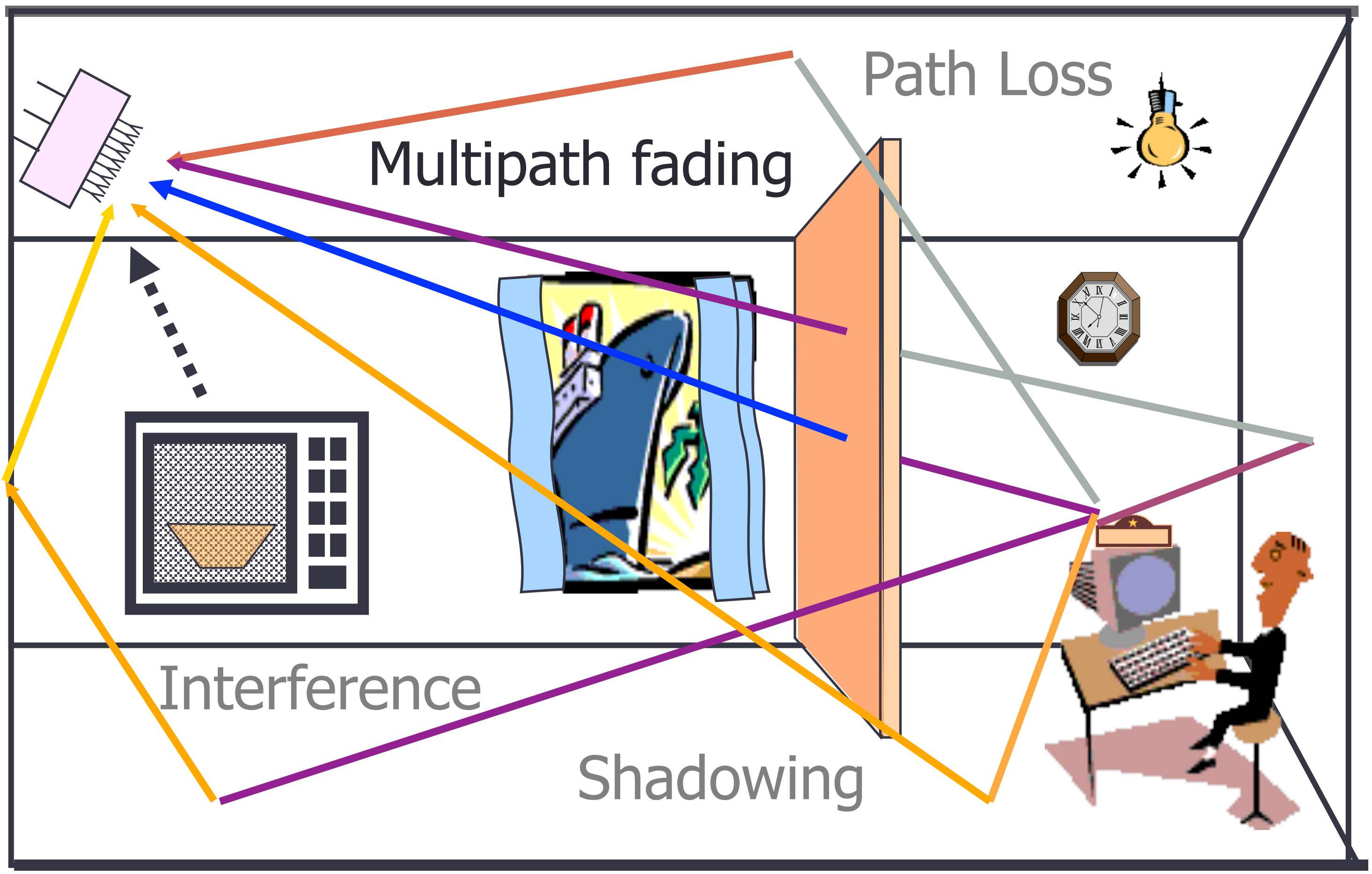
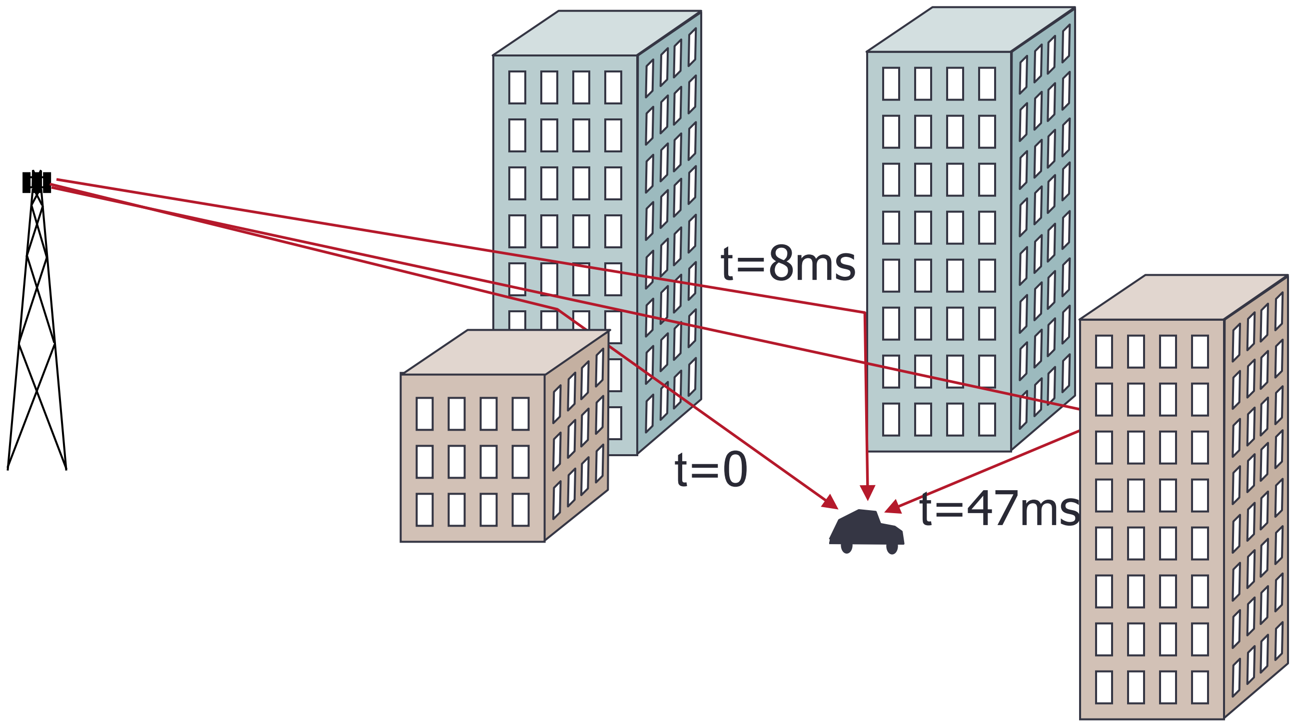
MULTIPATH FADING

Various Features in a Wireless Channel



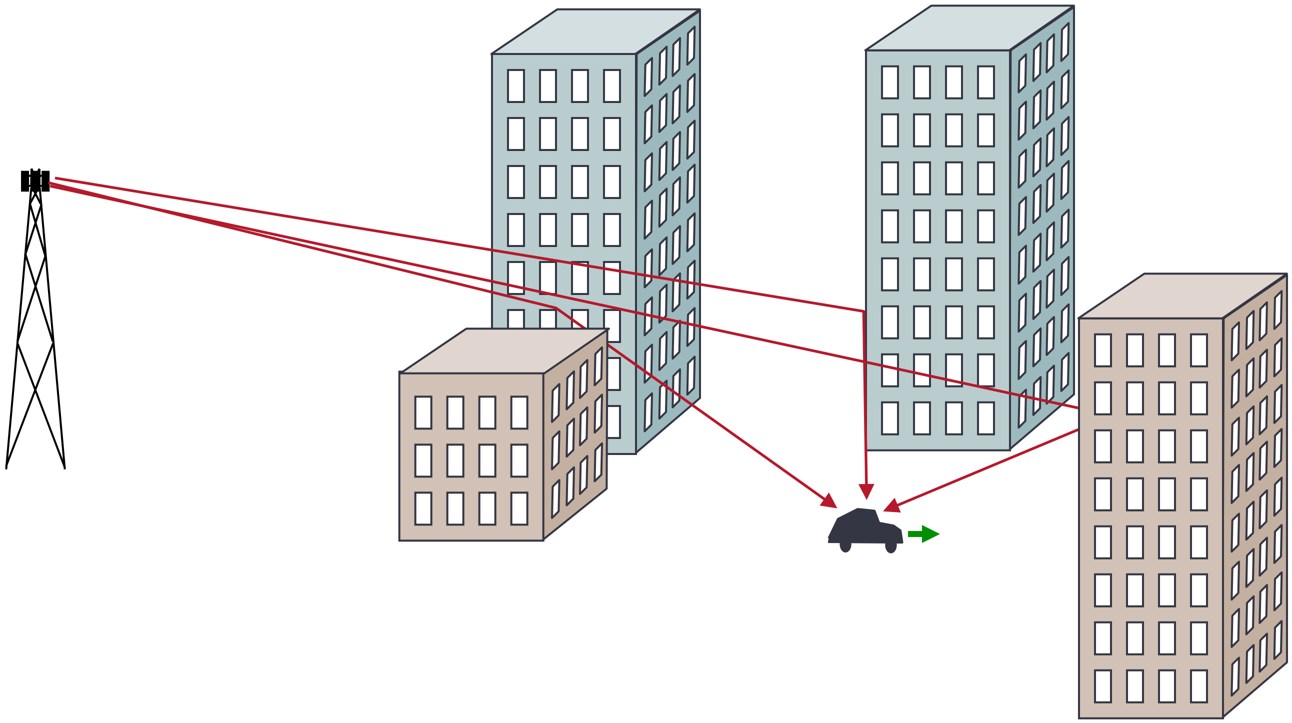
Excess Delay

- The propagation delay relative to that of the shortest path



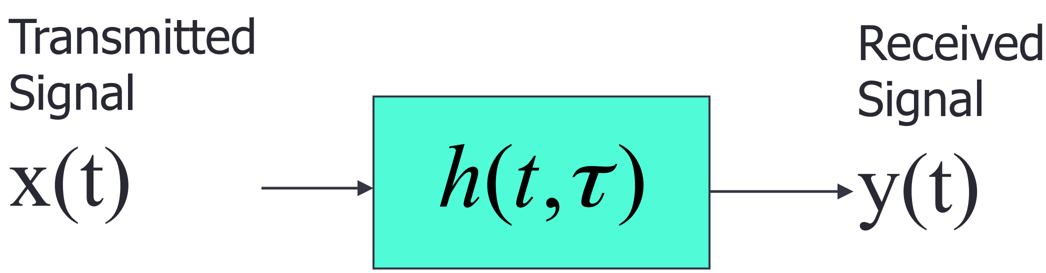
Strength Variation

- As the vehicle moves, the strength of each path varies because the surfaces are complex



The Channel is a Filter

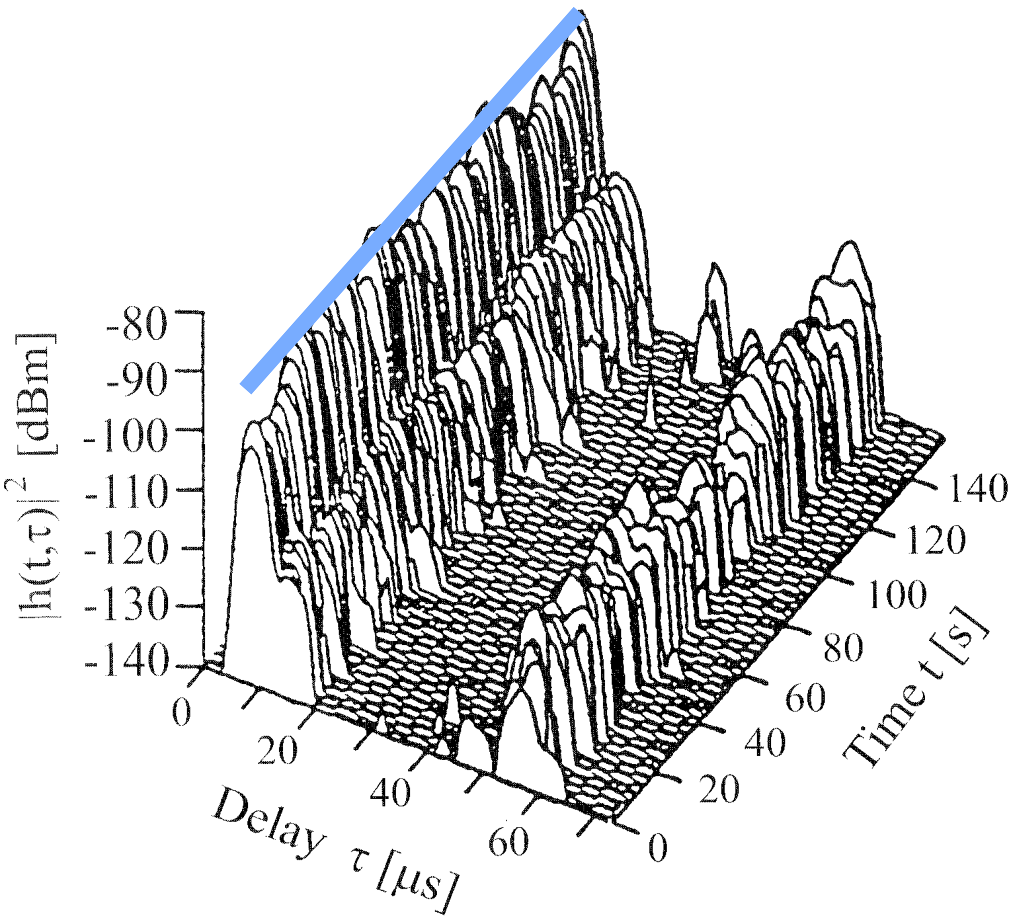
- The multipath channel can be represented as linear, time-varying bandpass filter



$$y(t) = \int_{-\infty}^{+\infty} x(t-\tau)h(t,\tau)d\tau$$

Measured Data from Darmstadt, Germany

[Molisch, ’01]



Baseband Impulse Response

- More convenient to work with baseband signals

$$h(t,\tau) = \mathrm{Re}\left\{ h_b(t,\tau)e^{j\omega_c t} \right\}$$

$$x(t) = \mathrm{Re}\left\{ c(t)e^{j\omega_c t} \right\}$$

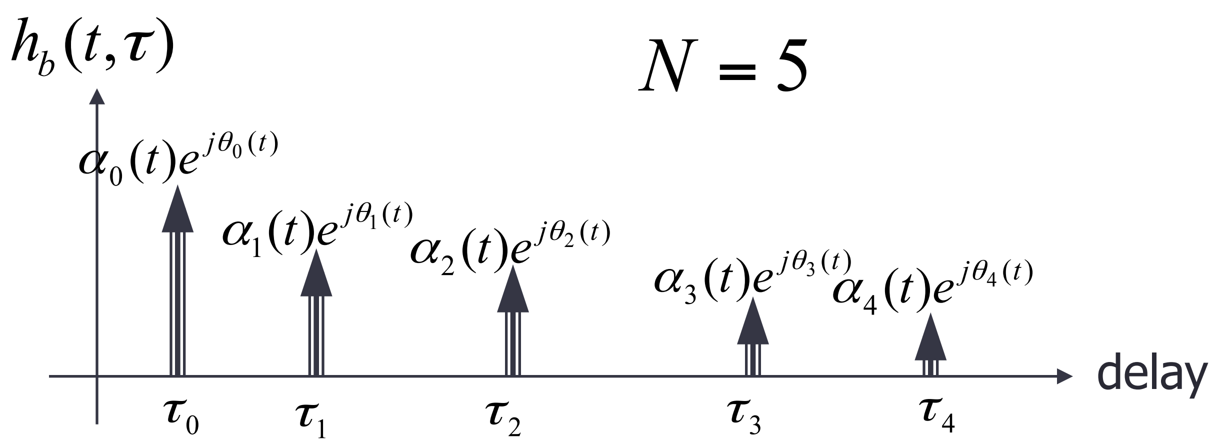
$$y(t) = \mathrm{Re}\left\{ r(t)e^{j\omega_c t} \right\}$$

$$r(t) = \frac{1}{2}\int_{-\infty}^{+\infty} c(t-\tau)h_b(t,\tau)d\tau$$

The factor of ½ ensures that baseband average power equals passband average power

Path Model

- The channel is assumed to comprise N discrete paths of propagation (rays)
- Each path has an amplitude $\alpha(t)$, a phase $\theta(t)$ and a propagation delay τ



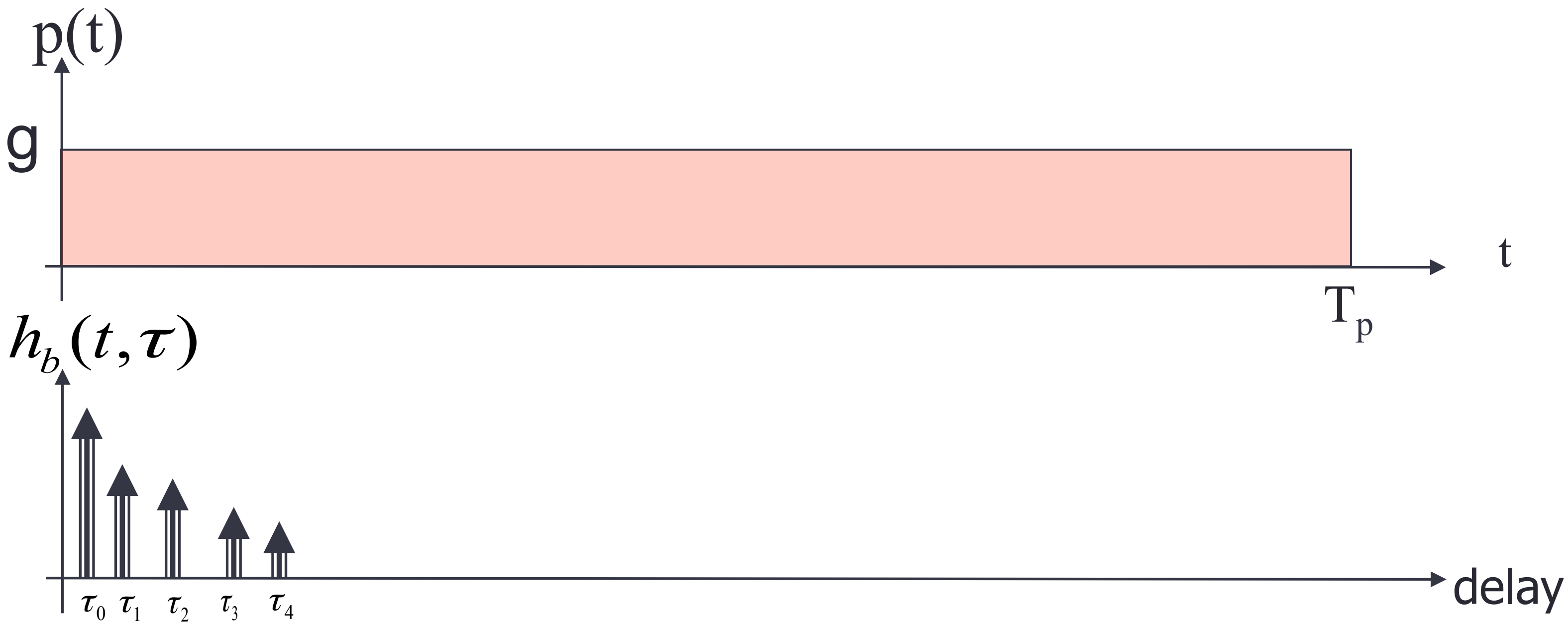
Probing the Channel

- The channel may be probed or "sounded" by transmitting a pulse $p(t)$ and recording the response at the receiver
- The response is the convolution of $p(t)$ with the channel impulse response

$$r(t) = \frac{1}{2}\sum_{i=0}^{N-1} \alpha_i(t) e^{j\theta_i(t)} p(t-\tau_i)$$

Pulse Width >> τ_{max}

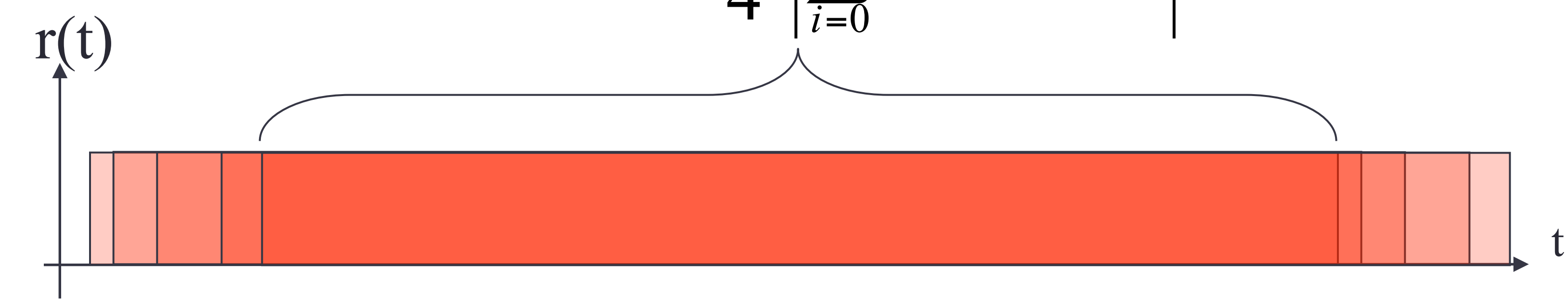
- Suppose a pulse much wider than the length of the impulse response is transmitted at time $t=0$

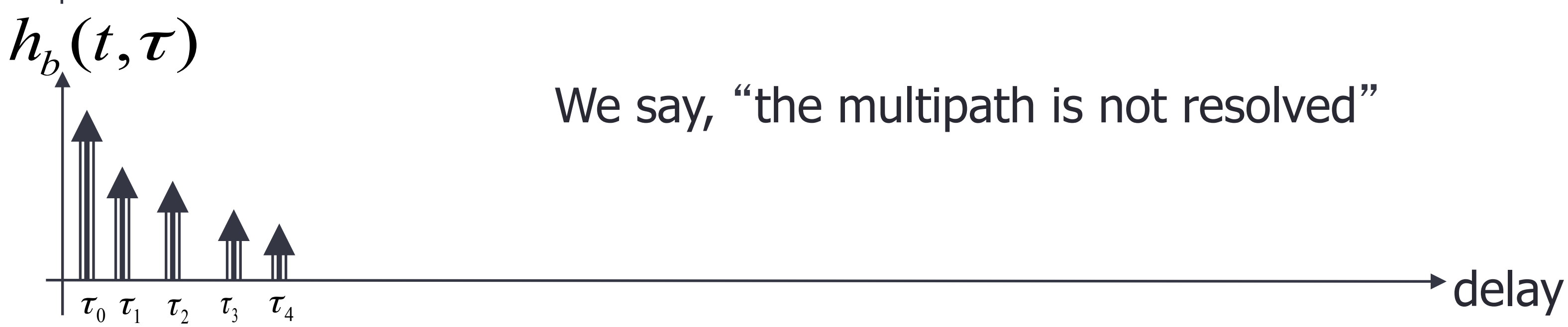


Instantaneous Power

The **magnitude squared** of any sample in the interval t_4 and $t_0 + T_p$ will equal

$$|r(t)|^2 = \frac{\gamma^2}{4}\left|\sum_{i=0}^{N-1}\alpha_i(t)e^{j\theta_i(t)}\right|^2$$

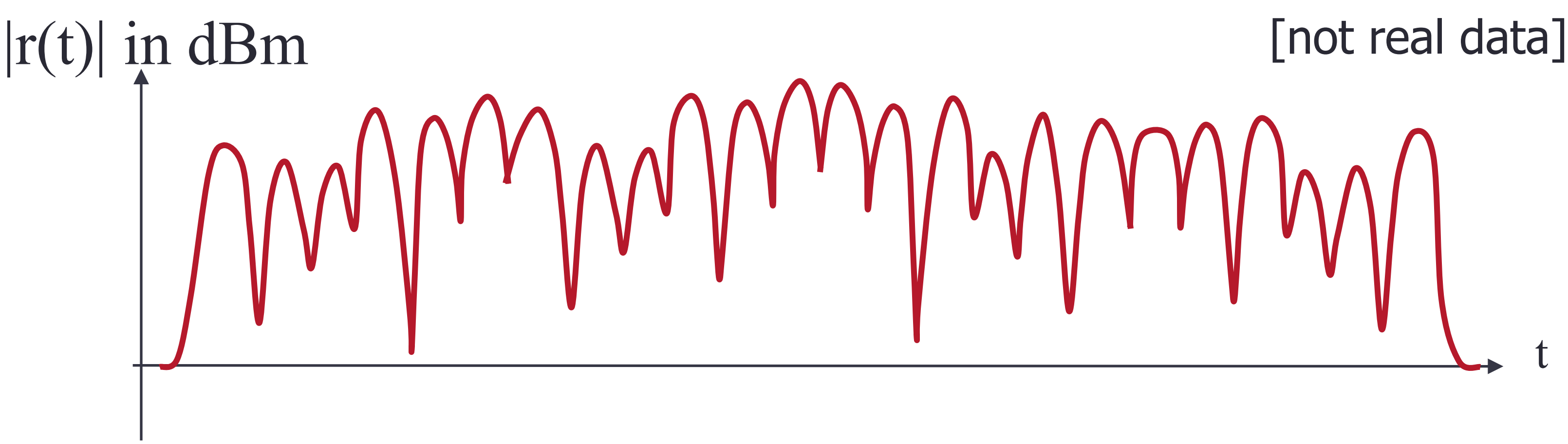




We say, "the multipath is not resolved"

Time Variation of the Probe Response

- If one or both of the terminals moves, the path phases change because the path lengths change
- The path amplitudes do not change much
- These changes yield large changes in the magnitude of the received waveform



Narrowband Fading

- This same type of fading happens to a digital waveform if the symbol period is much larger than (>10 times) the channel "length"
- Such long symbol periods correspond to "narrowband" signals

Average Power for Narrowband Signals

- Assuming the channel is ergodic, the ensemble average may be approximated by a time average:

$$\Omega = E_{\alpha\theta}\left\{ \left| r(t) \right|^2 \right\} \approx \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \frac{\gamma^2}{4} \left| \sum_{i=0}^{N} \alpha_i(s) e^{j\theta_i(s)} \right|^2 ds$$

where the interval [t-$T/2$,t+$T/2$] corresponds to a local area

Uncorrelated Scattering

- Assume that the phases of different paths are uncorrelated and that the energy of the pulse is one
- Then the time average simplifies to

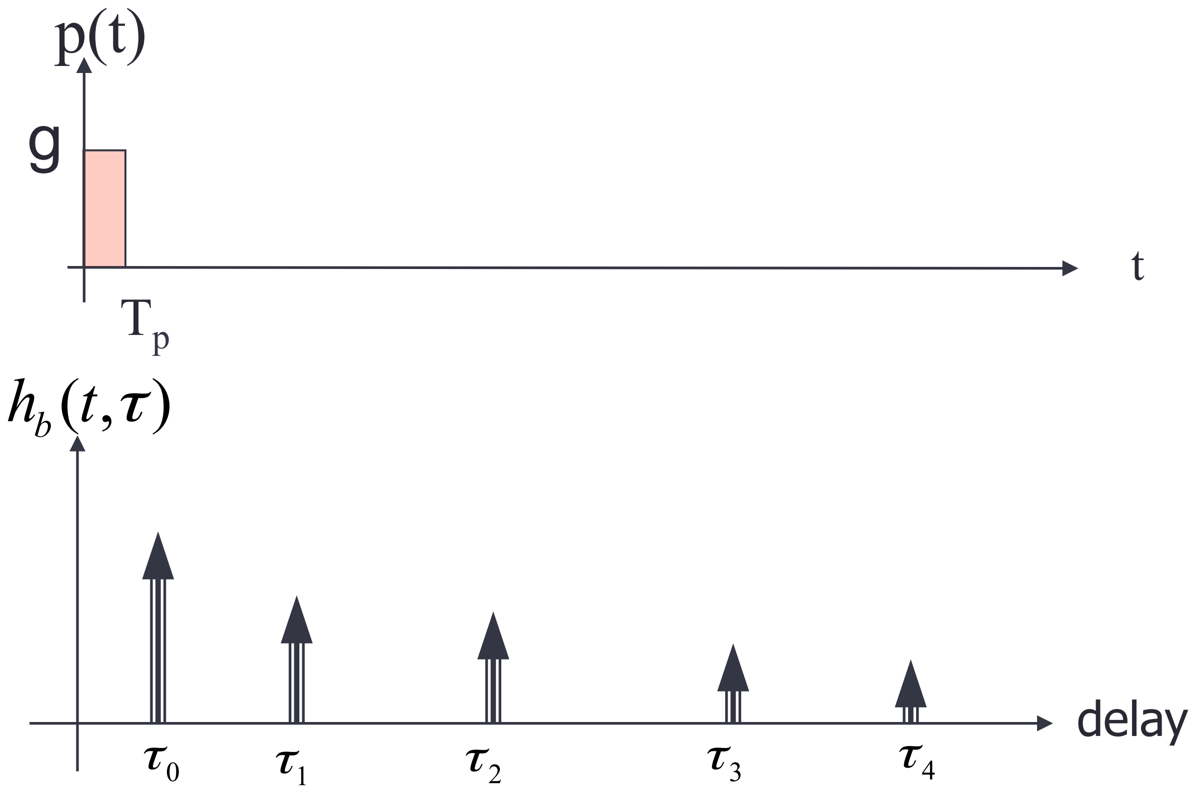
$$\Omega = E_{\alpha\theta}\left\{ \left| r(t) \right|^2 \right\} \approx \sum_{i=0}^{N} \overline{\alpha_i^2}$$

where

$$\overline{\alpha_i^2} \approx \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \alpha_i^2(s)\,ds$$

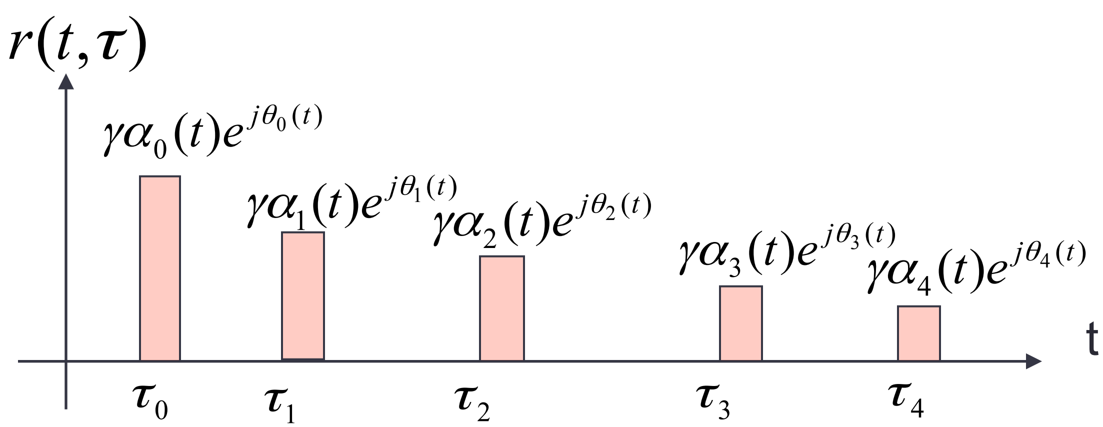
Pulse Width << τ_{max}

- Now consider a small pulse width



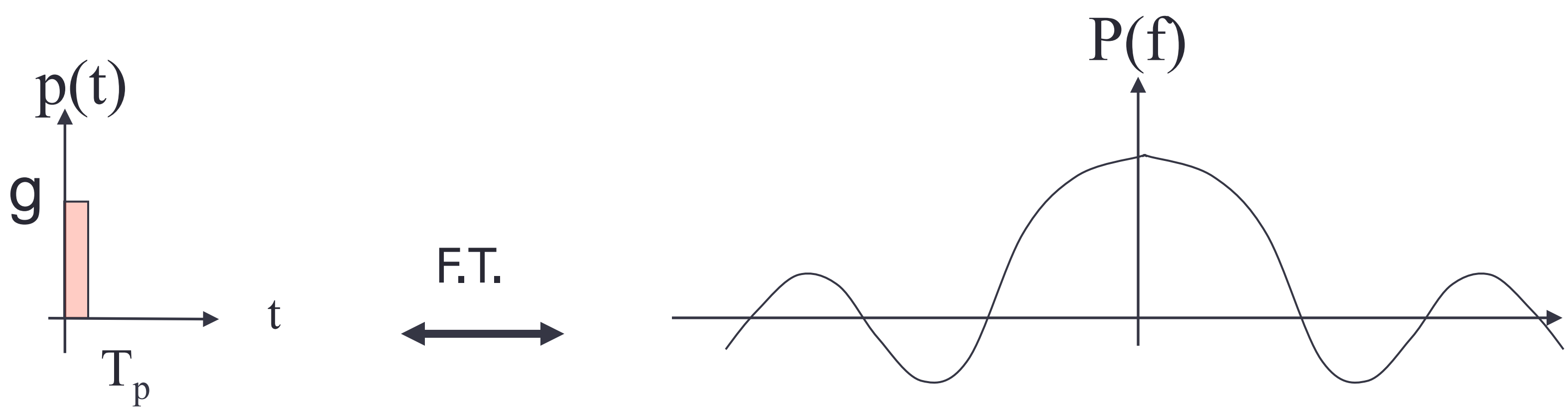
Multipath Resolved

- Pulses do not overlap



Wideband Signal

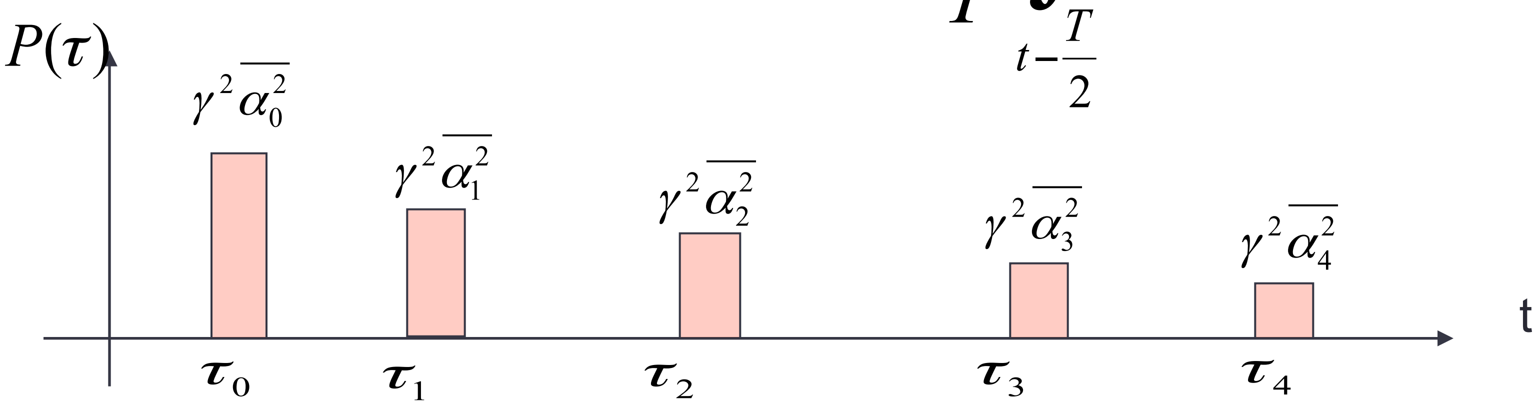
- The Fourier Transform of such a narrow pulse has a wide spectrum



Power Delay Profile (PDP)

- The PDP is a time-average of $|r(t,\tau)|^2$ over a small interval (assuming the terminal is moving)

$$P(\tau) \approx \frac{1}{T} \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} \left| r(s,\tau) \right|^2 ds$$



W. Mohr, "Modeling of wideband mobile radio channels based on propagation measurements," in Proc. 16th Int. Symp. Personal, Indoor, Mobile Radio Communications, vol. 2, pp. 397-401, 1995

Average Power for Wideband Signals

- The average power is the integral of the PDP

$$P_{AVG} = \int_0^{+\infty} P(\tau)d\tau$$

$$= \sum_{i=0}^{N-1} \overline{\alpha_i^2} = \Omega$$

Local Average Powers Are The Same

- Narrowband and Wideband averaged powers are equal

Moments of the PDP

- Channels are often described by their rms delay spread
- To compute rms delay spread, normalize the PDP to make it like a PDF for a random variable (unit area) and then find its standard deviation
- Must you use excess delay to compute rms delay spread?

Mean Delay

- Must first compute the mean delay

$$\bar{\tau} = \frac{\int_0^{+\infty} \tau P(\tau) d\tau}{\int_0^{+\infty} P(\tau) d\tau}$$

For this to be mean excess delay, the origin of the τ axis needs to be the time of the first arriving path

Second Moment

- Next need the second moment of this “PDF”

$$\overline{\tau^2} = \frac{\int_0^{+\infty} \tau^2 \, P(\tau) d\tau}{\int_0^{+\infty} P(\tau) d\tau}$$

RMS Delay Spread

- Recall that standard deviation is the square root of variance and variance is the second moment minus the first moment squared

Variance

$$\sigma_\tau^2 = \overline{\tau^2} - \left(\bar{\tau}\right)^2$$

rms delay spread

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \left(\bar{\tau}\right)^2}$$

Example Data

[Rappaport, ’02]

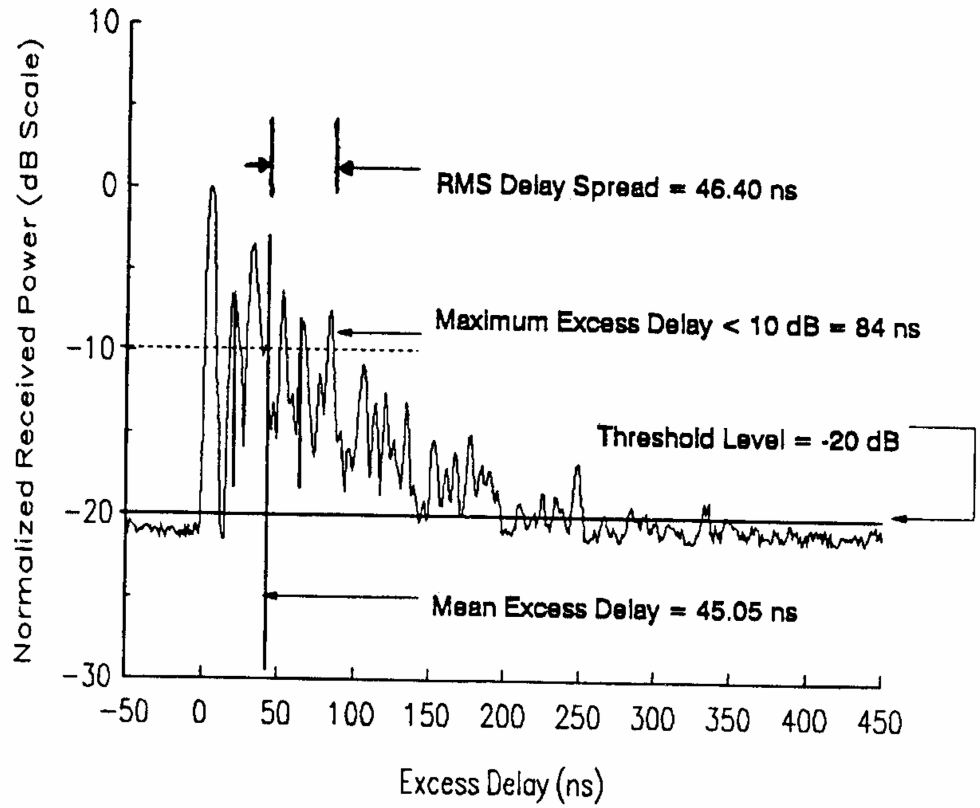


Figure 5.10 Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

How RMS Delay Spread Can Be Used

- If σ_τ<<symbol period, assume "narrowband" fading effects
- If σ_τ>>symbol period, assume "wideband" fading effects (will need an equalizer, CDMA or OFDM)

The Frequency Domain View

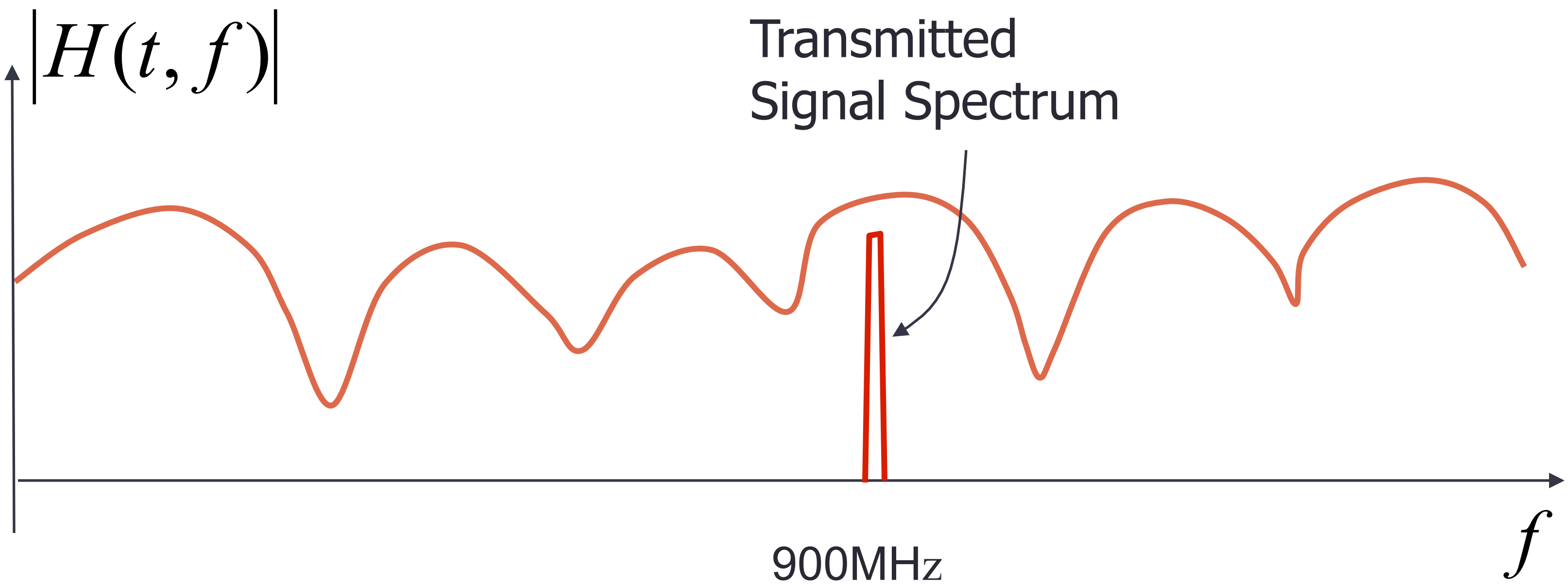
- σ_τ<<symbol period implies that the frequency response of the channel,

$$H(t,f) = \int_{-\infty}^{+\infty} h(t,\tau)\exp(-j2\pi f\tau)d\tau,$$

doesn't vary much with frequency over the bandwidth of the transmitted signal

Narrowband Case

- The channel "appears flat to the signal"

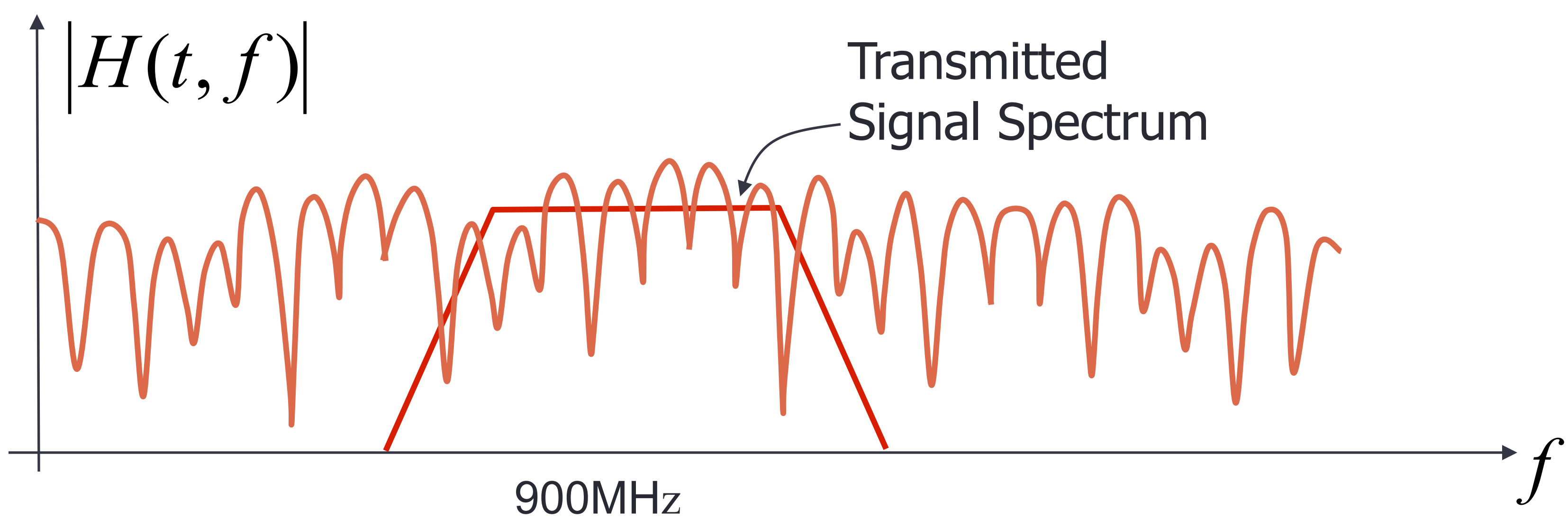


Flat Fading

- When σ_τ<<symbol period, we say the signal undergoes ***flat fading***
- The channel frequency response is approximately flat over the signal bandwidth

Wideband Case

- σ_τ>> symbol period implies that the frequency response of the channel varies significantly with frequency over the bandwidth of the transmitted signal



Frequency Selective Fading

- When σ_τ>> symbol period, we say the signal undergoes ***frequency selective fading***
- The channel frequency response is strong for some frequencies and not for others within the signal bandwidth

Complete Narrowband Statistical Fading Model

- $\alpha = |r(t)| = xy$ is the signal envelope. Recall $E\{\alpha^2\} = \Omega = \sum_{i=0}^{N-1} \overline{\alpha_i^2}$
- x represents the small-scale or multipath fading, and has either the Rayleigh or Rician distribution with unit mean square value, i.e. Therefore,

$$E\{x^2\} = 1 \qquad E\{\alpha^2\} = E\{y^2\}$$

- y represents the large-scale or shadow fading, and has a lognormal distribution. Recall $E\{y^2\} = \Omega$ is local average power
- Let $z = 10\log_{10}(y^2)$ Then z is $N(\hat{p}(d), \sigma^2)$ (the z' s are the points on the scatter plot)
- $\hat{p}(d)$ is the path loss, with the model

$$\hat{p}(d) = p(d_o) - n10\log_{10}\left(\frac{d}{d_o}\right)$$

Complete Wideband Statistical Fading Model

- Recall $\Omega = \sum_{i=0}^{N-1} \overline{\alpha_i^2}$. Each path of the wideband model has independent small-scale fading: $\alpha_i = x_i y$
- x_i is the small-scale or multipath fading on each path, and has either the Rayleigh or Rician distribution with mean square value

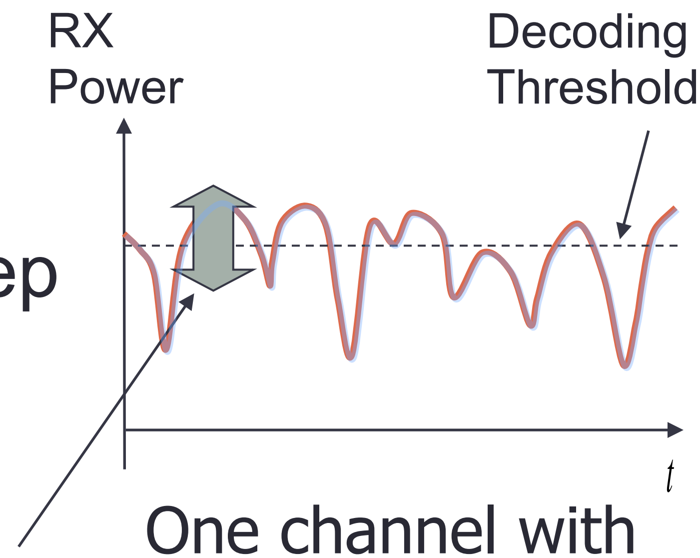
$$E\left\{x_i^2\right\} = E\left\{\alpha_i^2\right\} / \Omega$$

- y represents the large-scale or shadow fading, as before, and has a lognormal distribution.
- Let $z = 10\log_{10}\left(y^2\right)$. Then z is $N\left(\hat{p}(d), \sigma^2\right)$ (the z's are the points on the scatter plot)
- $\hat{p}(d)$ is the path loss, with the model

$$\hat{p}(d) = p(d_o) - n10\log_{10}\left(\frac{d}{d_o}\right)$$

Fade Margin

- Extra power (i.e. a fade margin) is required, to keep above the decoding threshold



Fade Margin

One channel with multipath fading

Summary

- The multipath channel model has a discrete number of propagation paths
- Each path has amplitude, phase and delay
- The PDP is the local average of the magnitude squared of the impulse response of the channel
- Average power of the channel is the integral of the PDP
- Average power is same for narrowband and wideband channels
- The fading is “flat” or “frequency selective” depending on the comparison between rms delay spread and the symbol period

References

- [Rapp, ’02] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 2002
- [Molisch, ’01] Andreas F. Molisch (ed), *Wideband Wireless Digital Communications*, Prentice Hall PTR, 2001.
- W. Mohr, “Modeling of wideband mobile radio channels based on propagation measurements,” in *Proc. 16th Int. Symp. Personal, Indoor, Mobile Radio Communications*, vol. 2, pp. 397-401, 1995