Convolutional Neural Networks
CS-477 Computer Vision

Dr. Mohsin Kamal
Associate Professor
dr.mohsinkamal@seecs.edu.pk

School of Electrical Engineering and Computer Science (SEECS)
National University of Sciences and Technology (NUST), Pakistan

1 Introduction

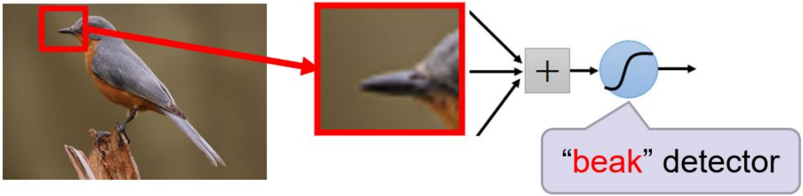
2 Convolutional Neural Network

- A convolutional neural network (CNN) is a type of artificial neural network used primarily for image recognition and processing, due to its ability to recognize patterns in images.
- FNN could not scale up to image and video processing tasks.
- CNN specifically tailored for image and video processing tasks.

Consider learning an image:

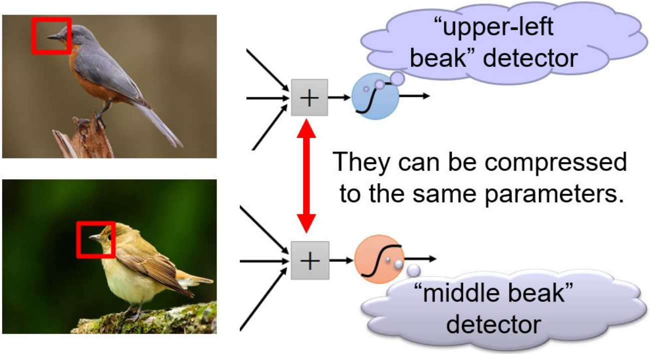
Some patterns are much smaller than the whole image

Can represent a small region with fewer parameters

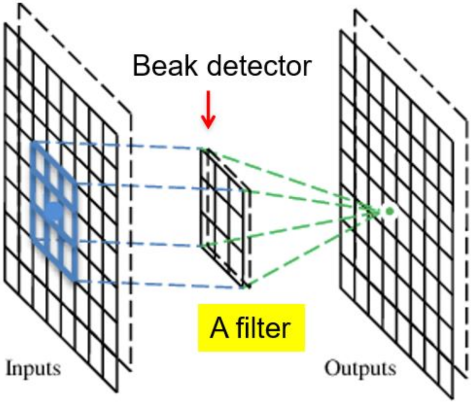


Same pattern appears in different places:

- They can be compressed!
- What about training a lot of such "small" detectors and each detector must "move around".



A CNN is a neural network with some convolutional layers (and some other layers). A convolutional layer has a number of filters that does convolutional operation.

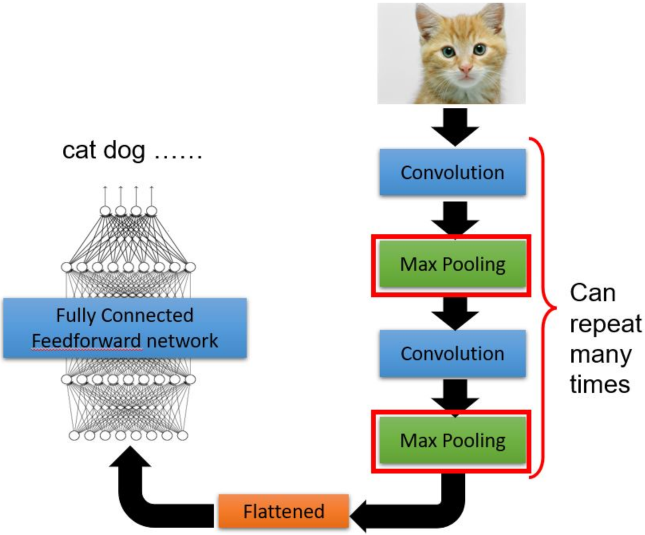


Architecture of CNN

A typical CNN has 4 layers

- Input layer
- Convolution layer
- Pooling layer
- Fully connected layer

Architecture of CNN



Input layer

- Example input a 28 pixel by 28 pixel grayscale image
- Unlike FNN, we do not "flatten" the input to a 1D vector
  - Input is presented to network in 2D as 28 x 28 matrix

Convolution

These are the network parameters to be learned.

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

Filter 1

1	-1	-1
-1	1	-1
-1	-1	1

Filter 2

-1	1	-1
-1	1	-1
-1	1	-1

⋮ ⋮

Each filter detects a small pattern (3 x 3).

Convolution

stride=1

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

Dot product → 3 -1

Filter 1

1	-1	-1
-1	1	-1
-1	-1	1

Convolution

If stride=2

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

3 -3

Filter 1

1	-1	-1
-1	1	-1
-1	-1	1

Convolution

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

stride=1

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

3	-1	-3	-1
-3	1	0	-3
-3	-3	0	1
3	-2	-2	-1

Convolution

-1	1	-1
-1	1	-1
-1	1	-1

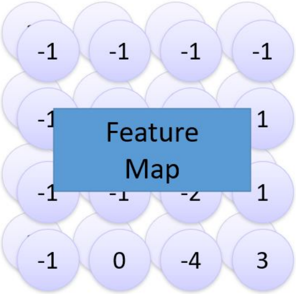
Filter 2

stride=1

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

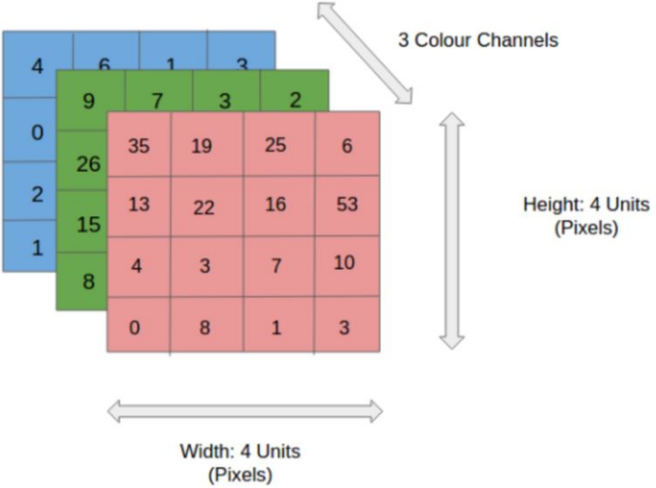
6 x 6 image

Repeat this for each filter

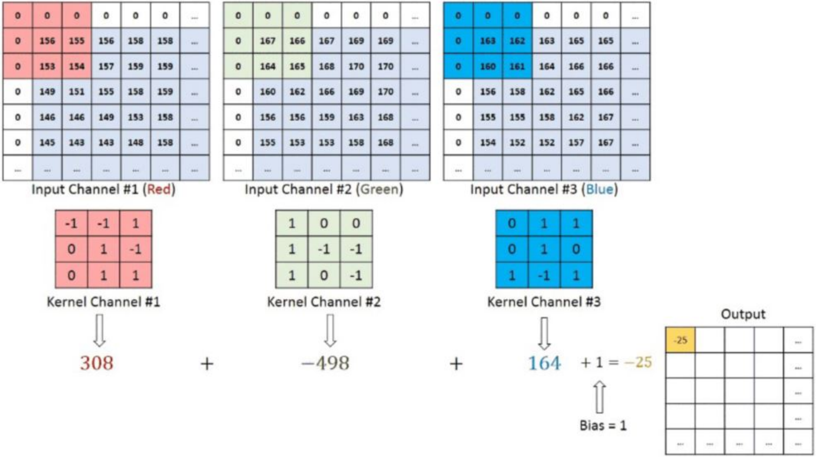


Convolution in RGB

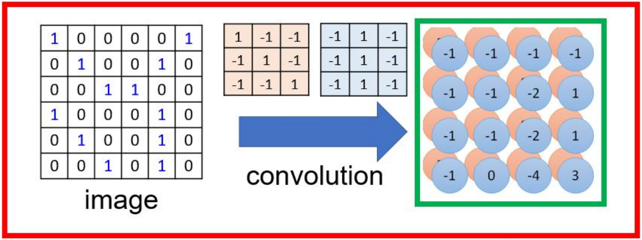
An RGB image is of the form

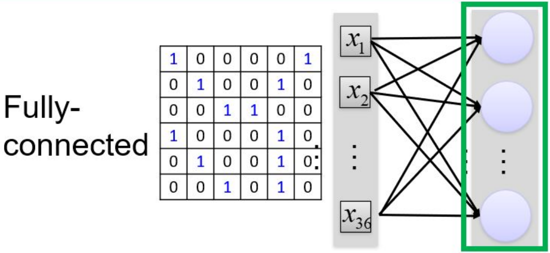


Convolution in RGB

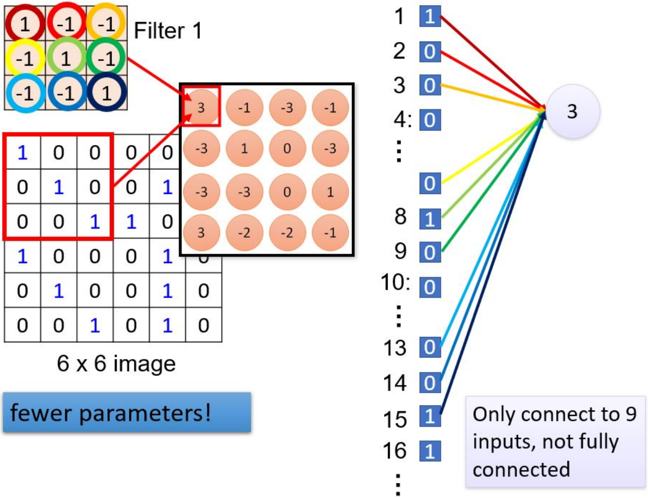


Convolution vs. Fully Connected NN

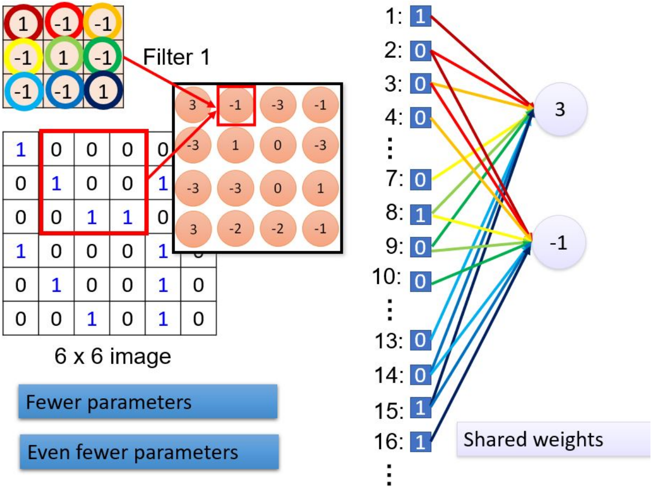




Convolution vs. Fully Connected NN

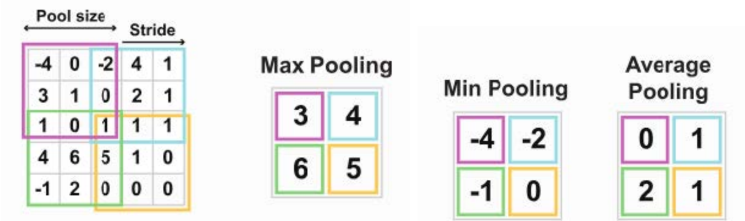


Convolution vs. Fully Connected NN



Pooling

- In convolutional neural networks (CNNs), pooling is a down-sampling operation commonly used to reduce the spatial dimensions of the input volume.
- The most common type of pooling is called max pooling and average pooling.



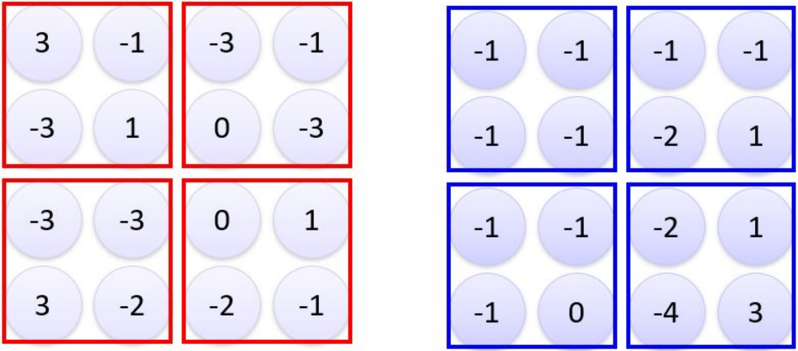
Max pooling

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

-1	1	-1
-1	1	-1
-1	1	-1

Filter 2



Why pooling

- Subsampling pixels will not change the object
- We can subsample the pixels to make image smaller
- fewer parameters to characterize the image

Max pooling

A CNN compresses a fully connected network in two ways:

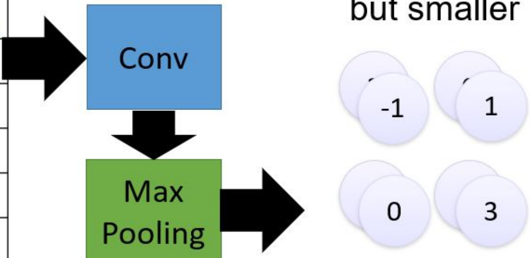
- Reducing number of connections
- Shared weights on the edges
- Max pooling further reduces the complexity

Max pooling

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

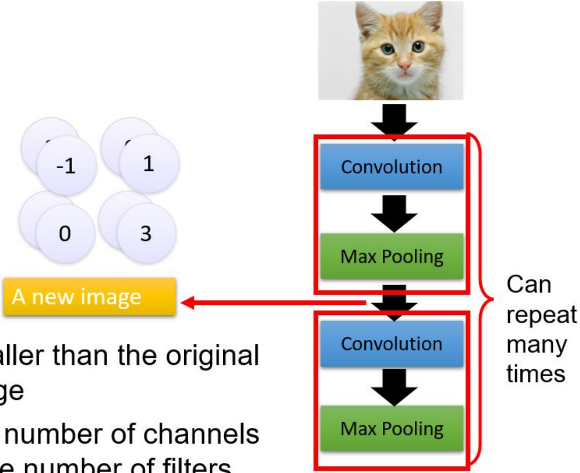
New image but smaller



2 x 2 image

Each filter is a channel

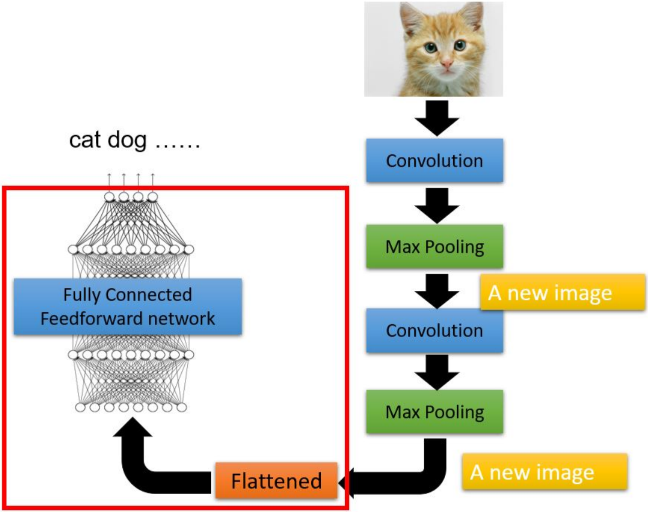
The whole CNN



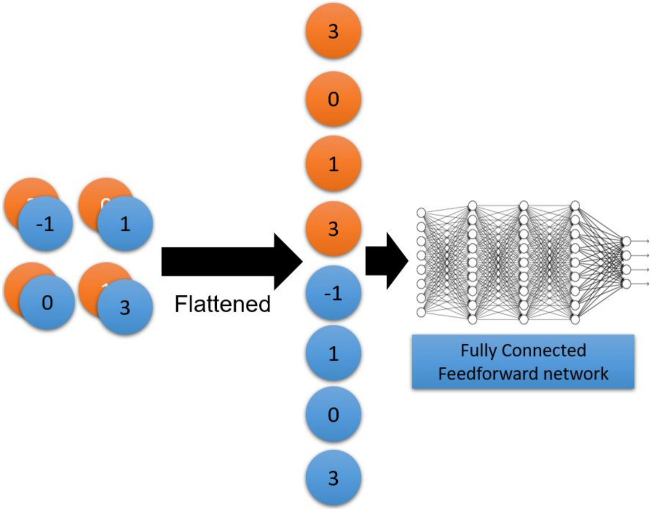
Smaller than the original image

The number of channels is the number of filters

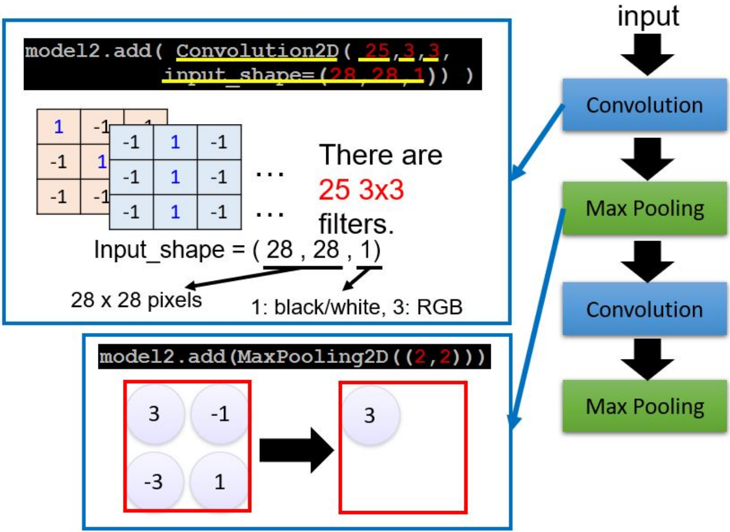
The whole CNN



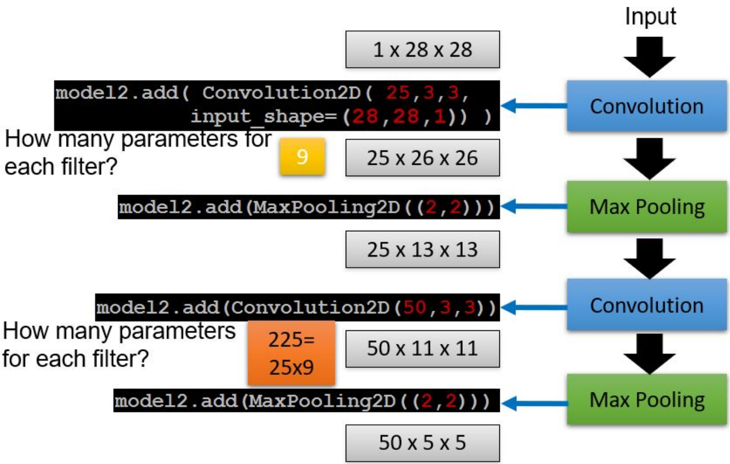
Flattening


-1
1
0
3
Flattened
3
0
1
3
-1
1
0
3
Fully Connected
Feedforward network

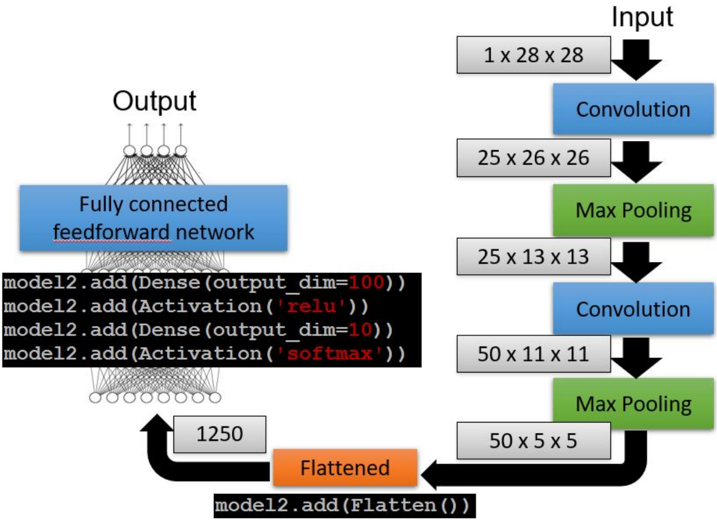
CNN in Keras

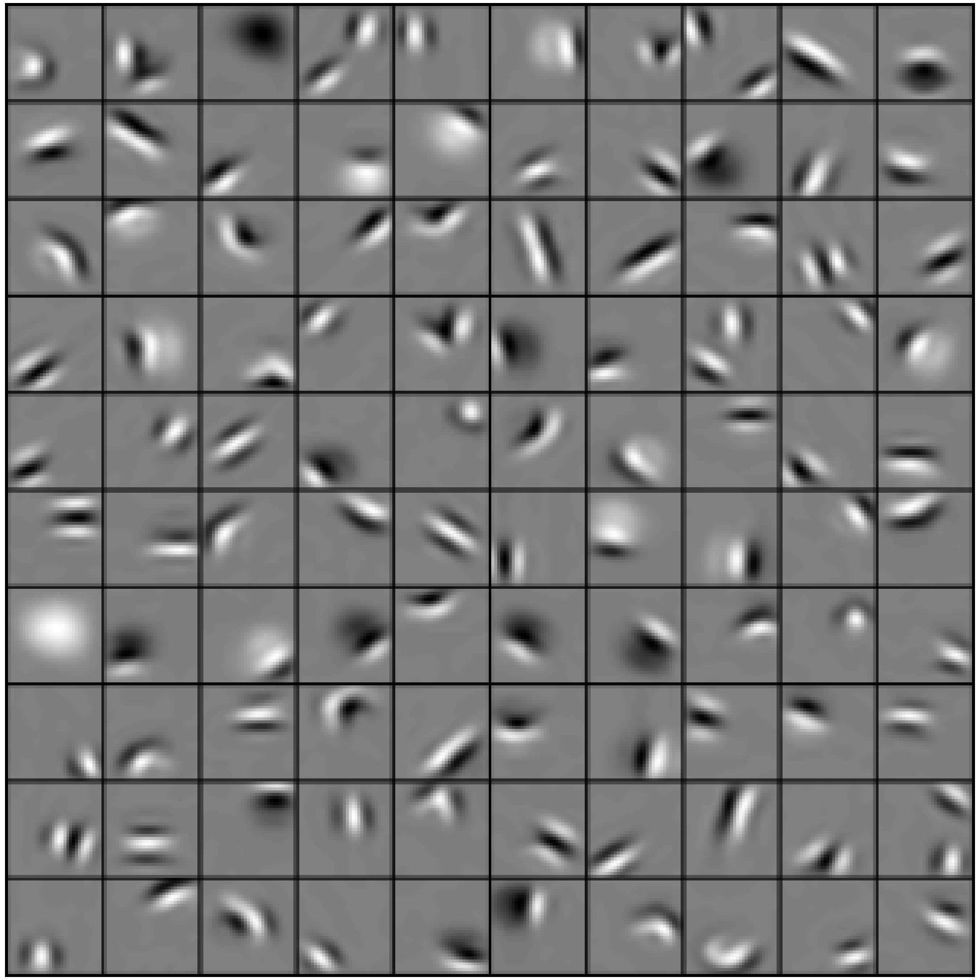

model2.add(Convolution2D(25,3,3,
input_shape=(28,28,1)))
1 -1 -1
-1 1 -1
-1 -1 1
-1 1 -1
-1 1 -1
-1 1 -1
...
...
There are
25 3x3
filters.
Input_shape = (28 , 28 , 1)
28 x 28 pixels
1: black/white, 3: RGB
model2.add(MaxPooling2D((2,2)))
3 -1
-3 1
3
input
Convolution
Max Pooling
Convolution
Max Pooling

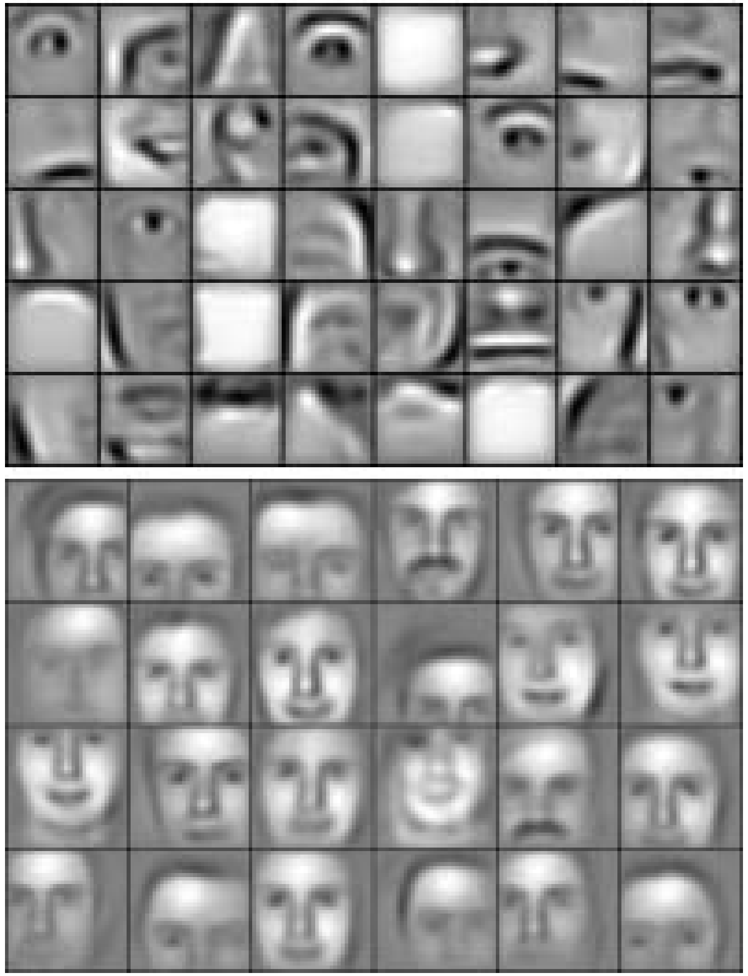
CNN in Keras

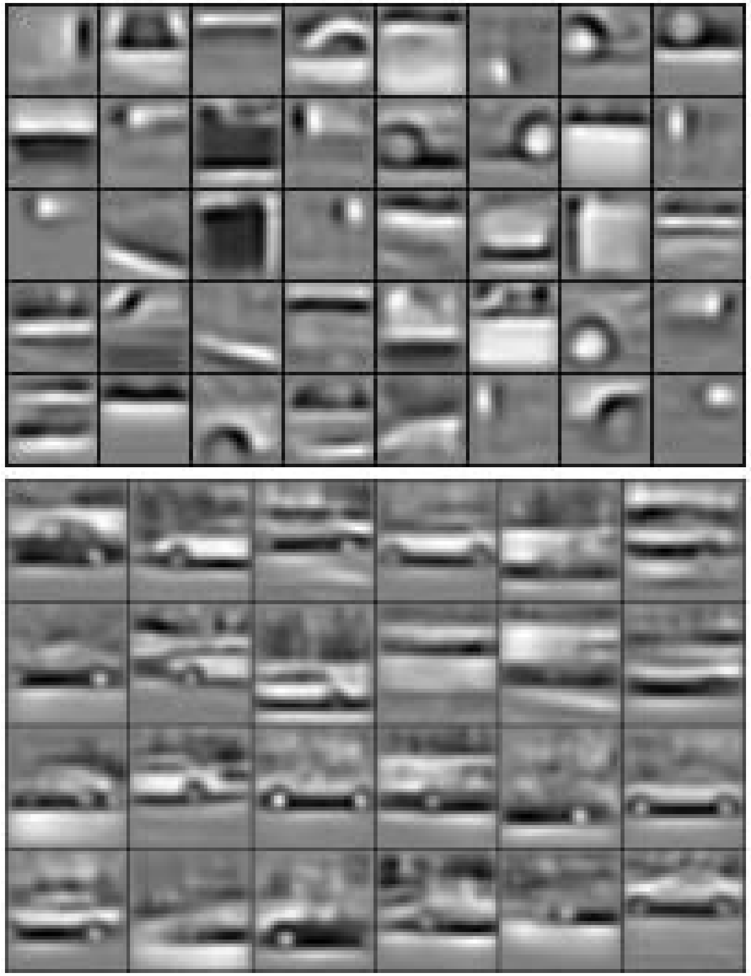

Input
1 x 28 x 28
model2.add(Convolution2D(25,3,3,
input_shape=(28,28,1)))
Convolution
How many parameters for
each filter?
9
25 x 26 x 26
model2.add(MaxPooling2D((2,2)))
Max Pooling
25 x 13 x 13
model2.add(Convolution2D(50,3,3))
Convolution
How many parameters
for each filter?
225=
25x9
50 x 11 x 11
model2.add(MaxPooling2D((2,2)))
Max Pooling
50 x 5 x 5

CNN in Keras


Input
1 x 28 x 28
Convolution
25 x 26 x 26
Max Pooling
25 x 13 x 13
Convolution
50 x 11 x 11
Max Pooling
50 x 5 x 5
Flattened
model2.add(Flatten())
1250
Fully connected
feedforward network
model2.add(Dense(output_dim=100))
model2.add(Activation('relu'))
model2.add(Dense(output_dim=10))
model2.add(Activation('softmax'))
Output







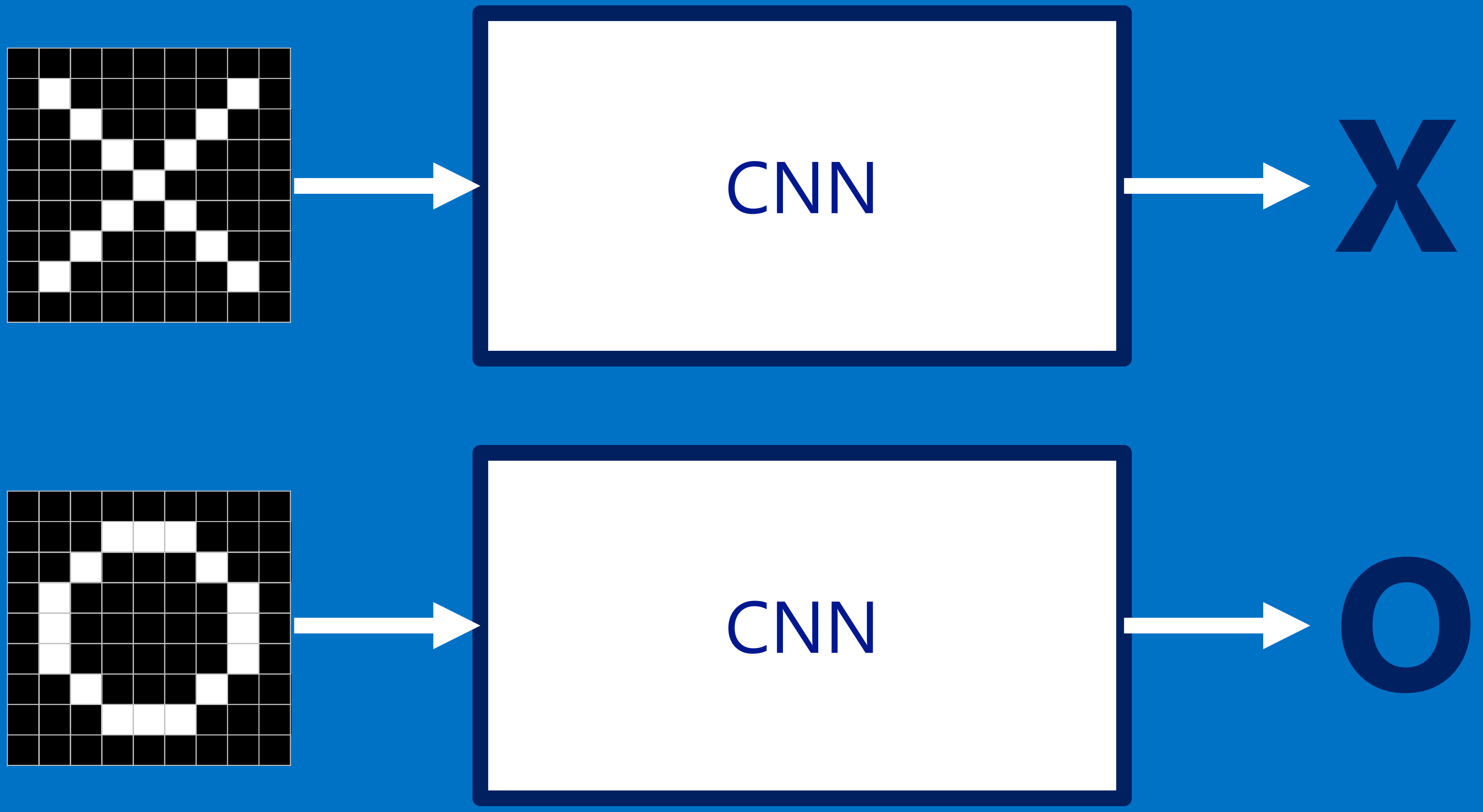
[Convolutional Deep Belief Networks for Scalable Unsupervised Learning of Hierarchical Representations Honglak Lee, Roger Grosse, Rajesh Ranganath, Andrew Y. Ng]()

A toy ConvNet: X's and O's

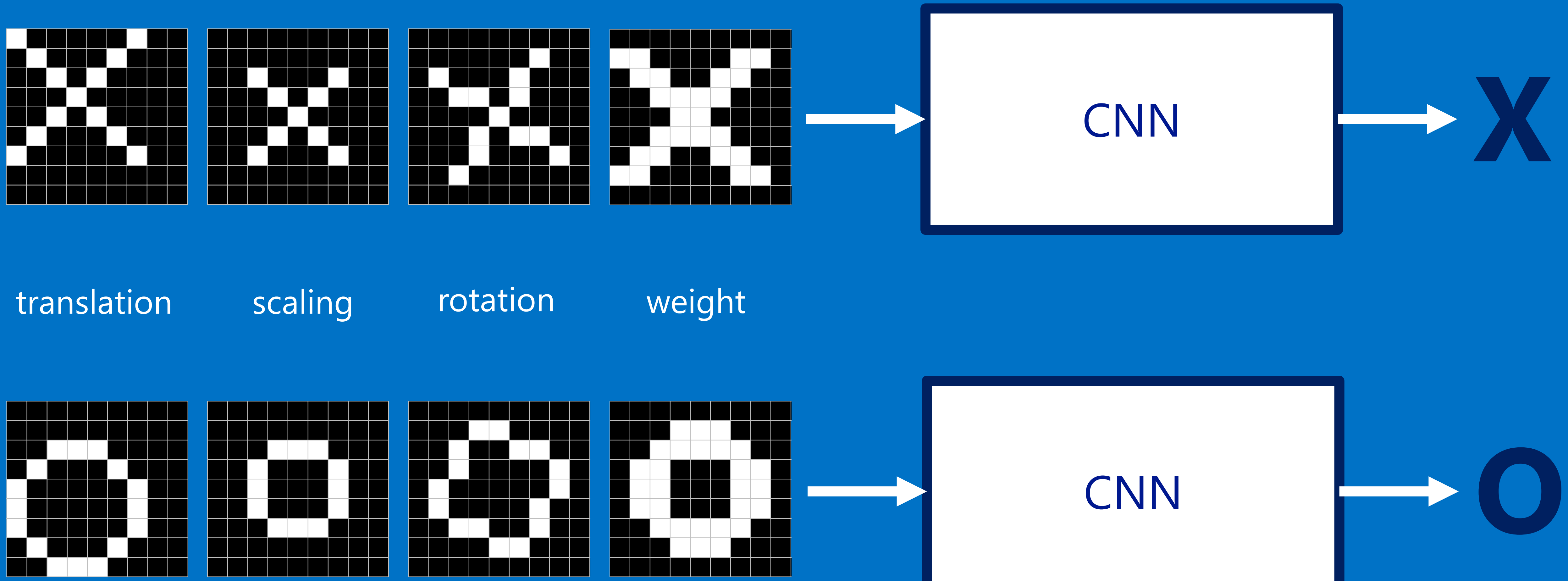
Says whether a picture is of an X or an O

A two-dimensional
array of pixels

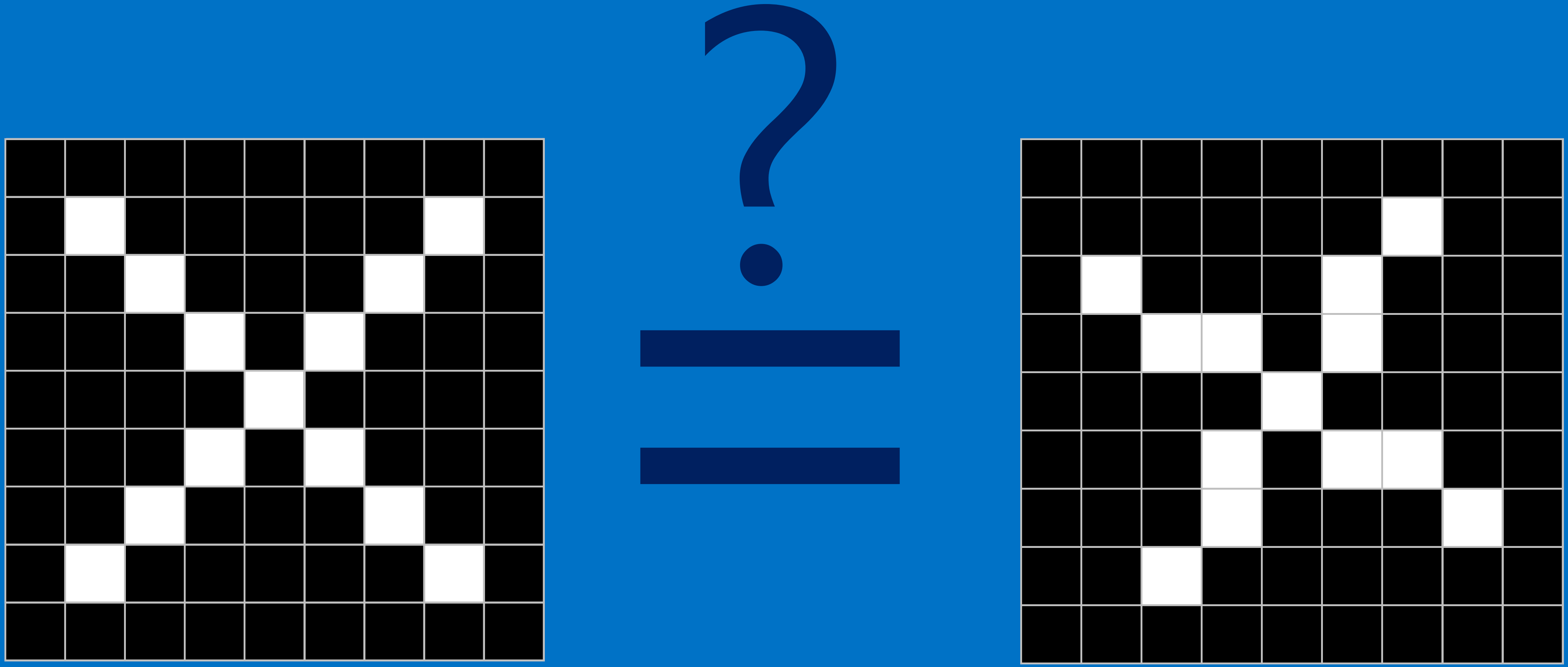
For example



Trickier cases



Deciding is hard



What computers see

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

$\stackrel{?}{=}$

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	1	-1	-1	-1
-1	-1	1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	1	-1	-1
-1	-1	-1	1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

What computers see

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	X	-1	-1	-1	-1	X	X	-1
-1	X	X	-1	-1	X	X	-1	-1
-1	-1	X	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	X	-1	-1
-1	-1	X	X	-1	-1	X	X	-1
-1	X	X	-1	-1	-1	-1	X	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

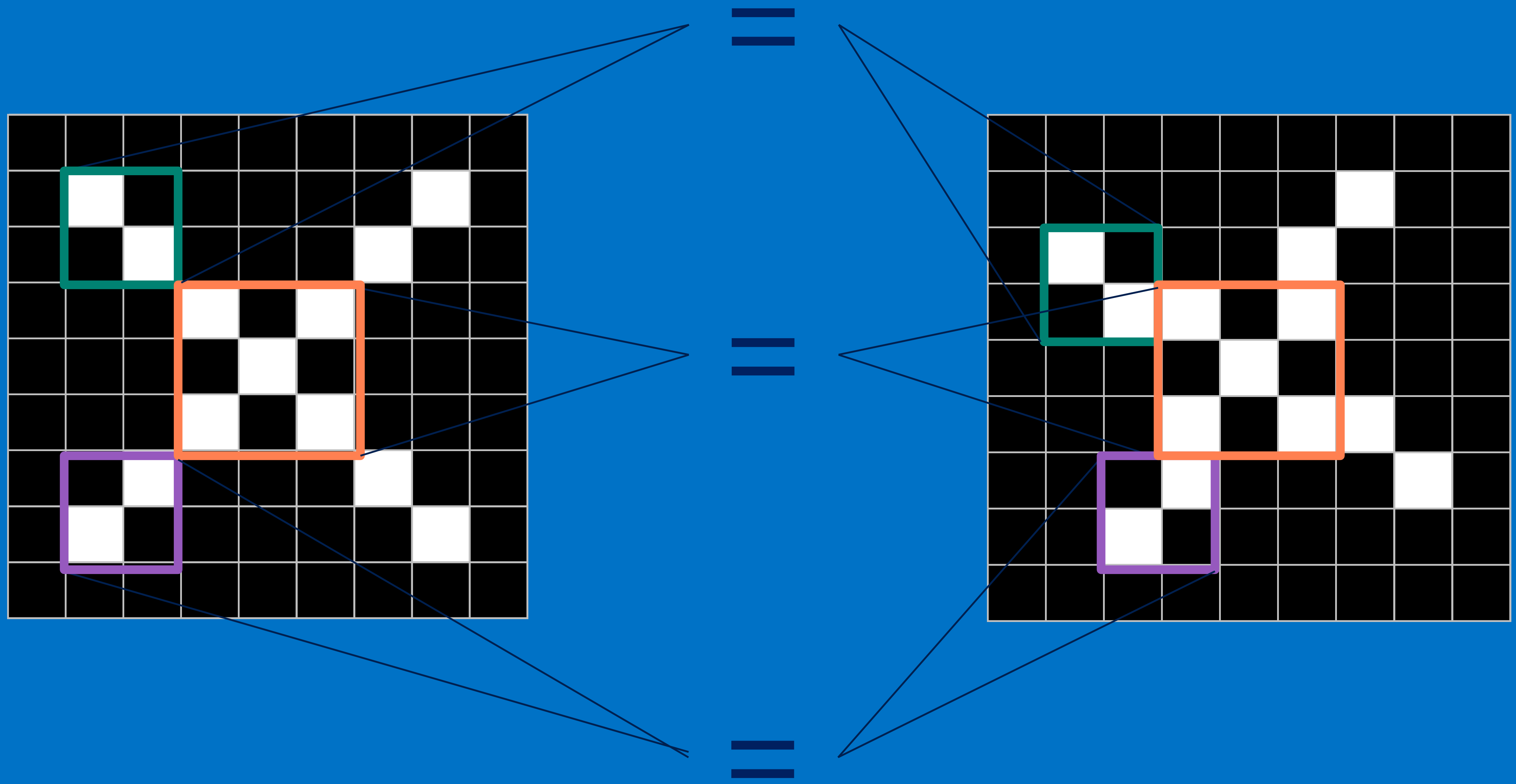
Computers are literal

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

$\neq$

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	1	-1	-1	-1
-1	-1	1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	1	-1	-1
-1	-1	-1	1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	-1	-1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

ConvNets match pieces of the image

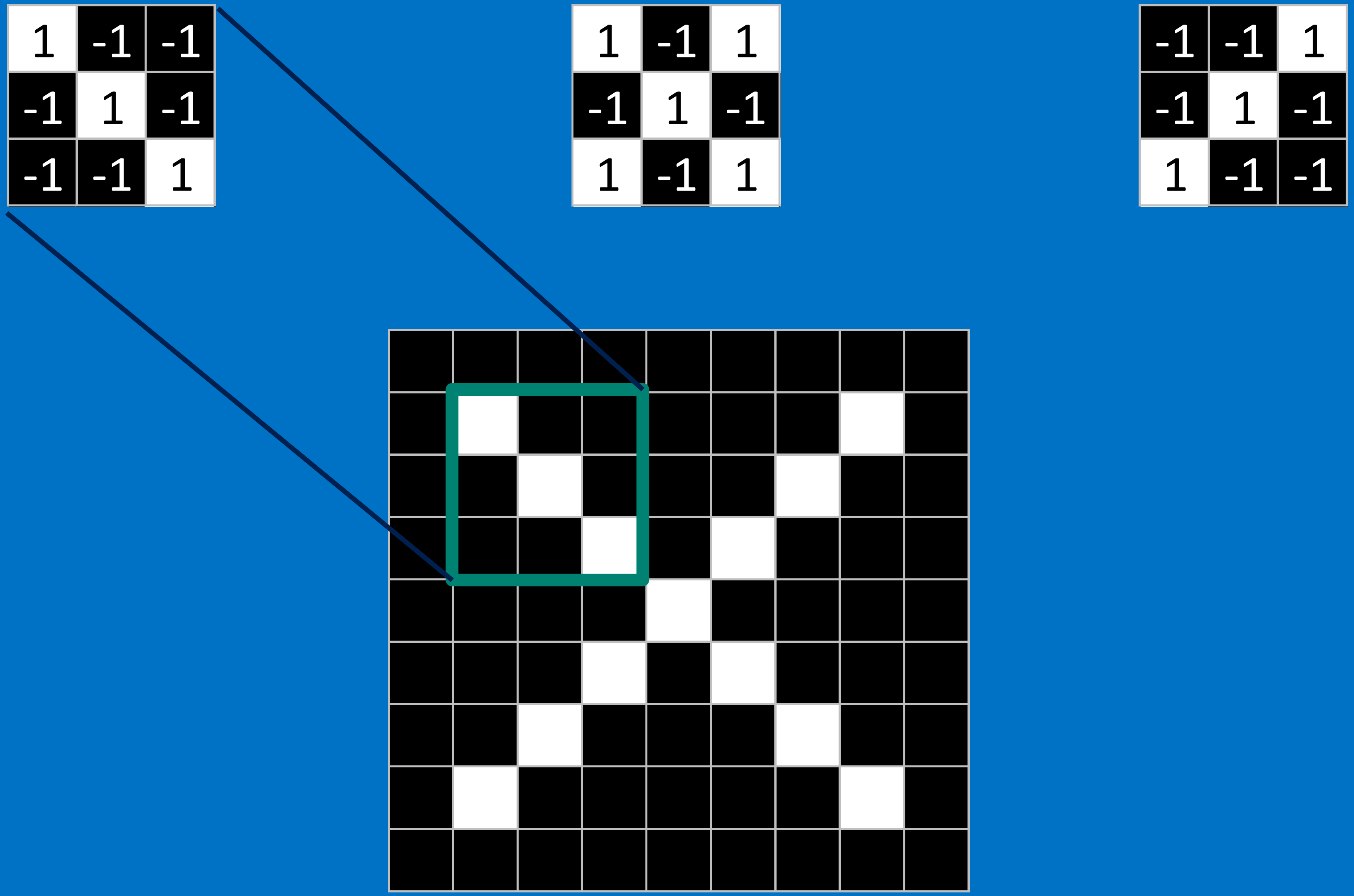


Features match pieces of the image

1	-1	-1
-1	1	-1
-1	-1	1

1	-1	1
-1	1	-1
1	-1	1

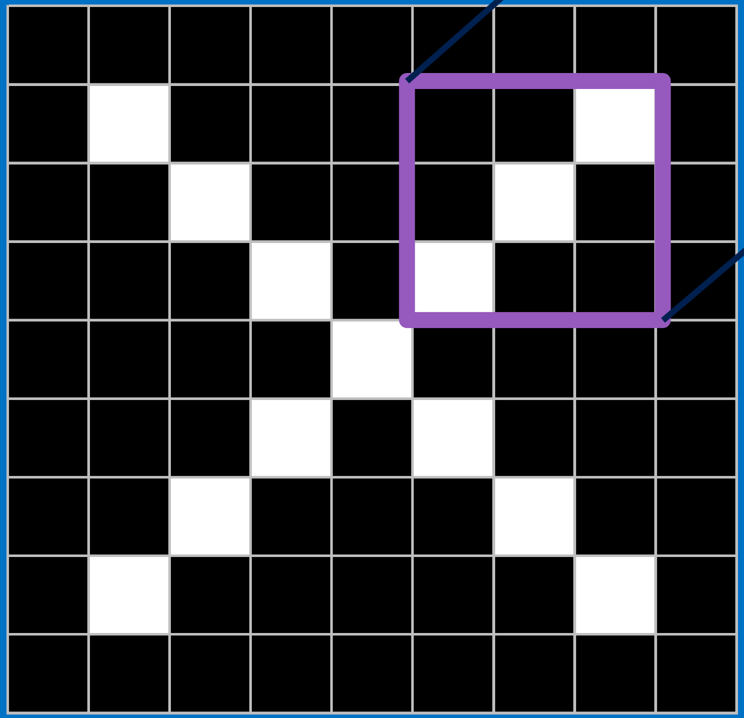
-1	-1	1
-1	1	-1
1	-1	-1

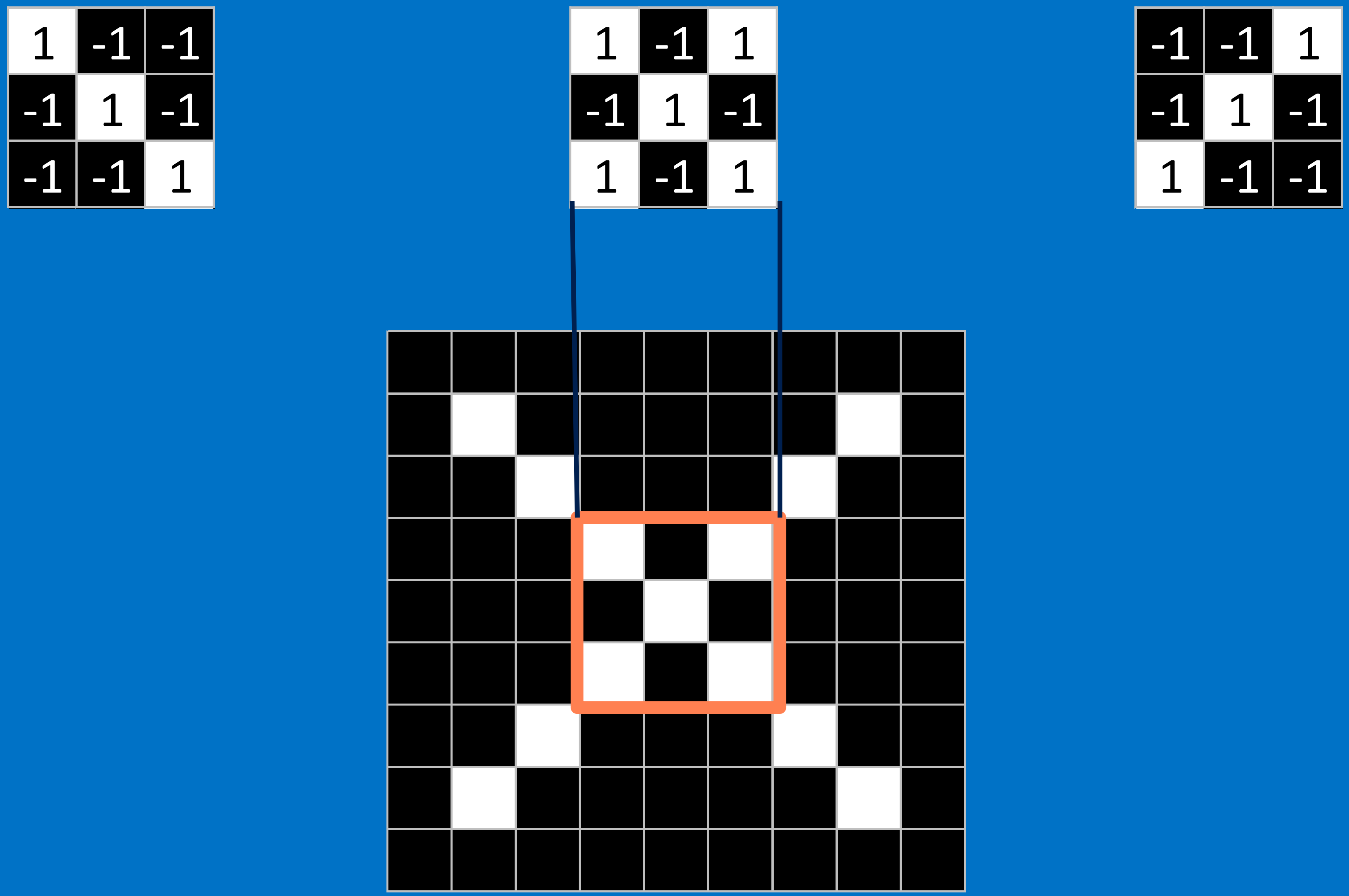

1 -1 -1
-1 1 -1
-1 -1 1
1 -1 1
-1 1 -1
1 -1 1
-1 -1 1
-1 1 -1
1 -1 -1

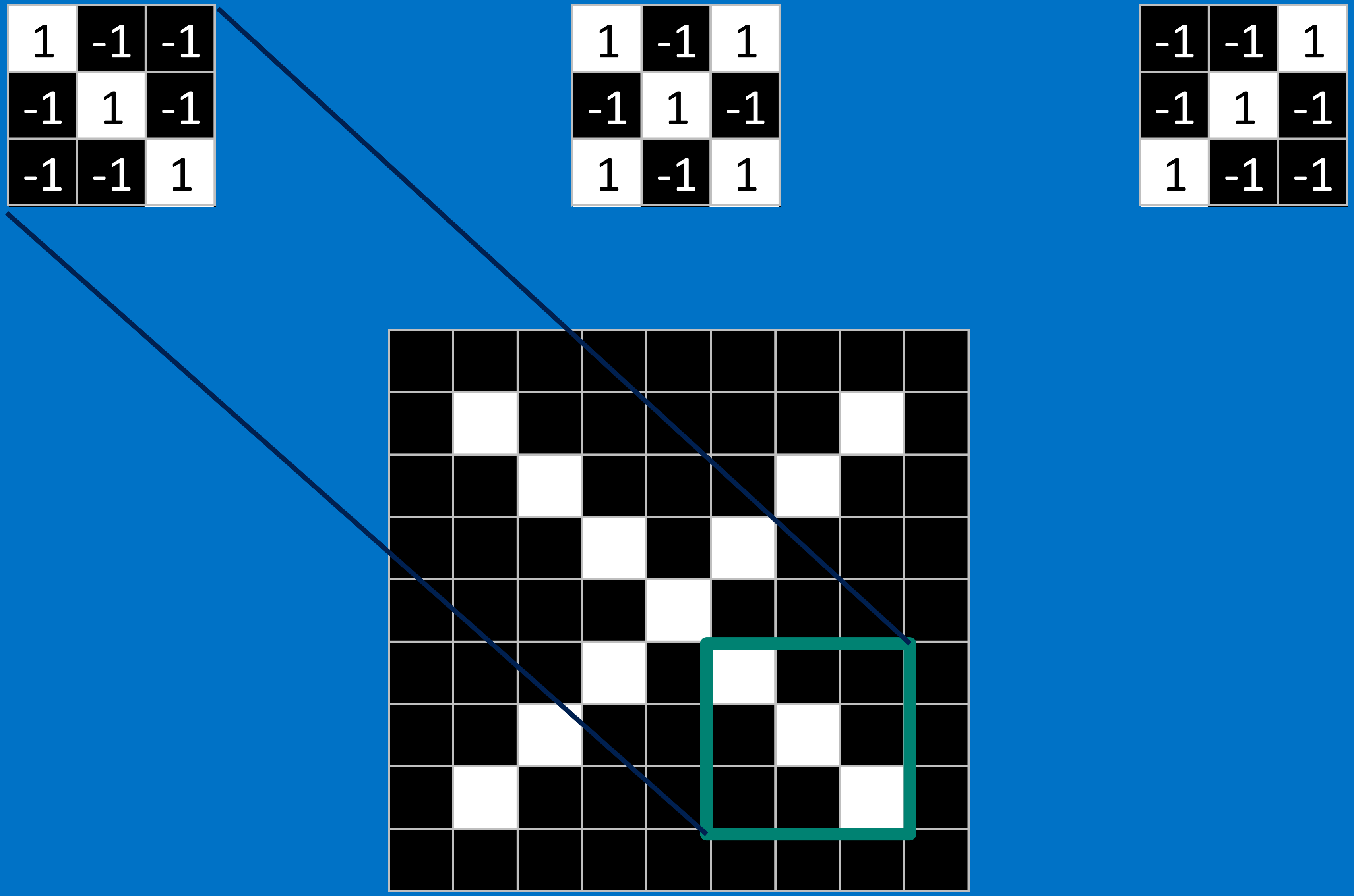
1	-1	-1
-1	1	-1
-1	-1	1

1	-1	1
-1	1	-1
1	-1	1

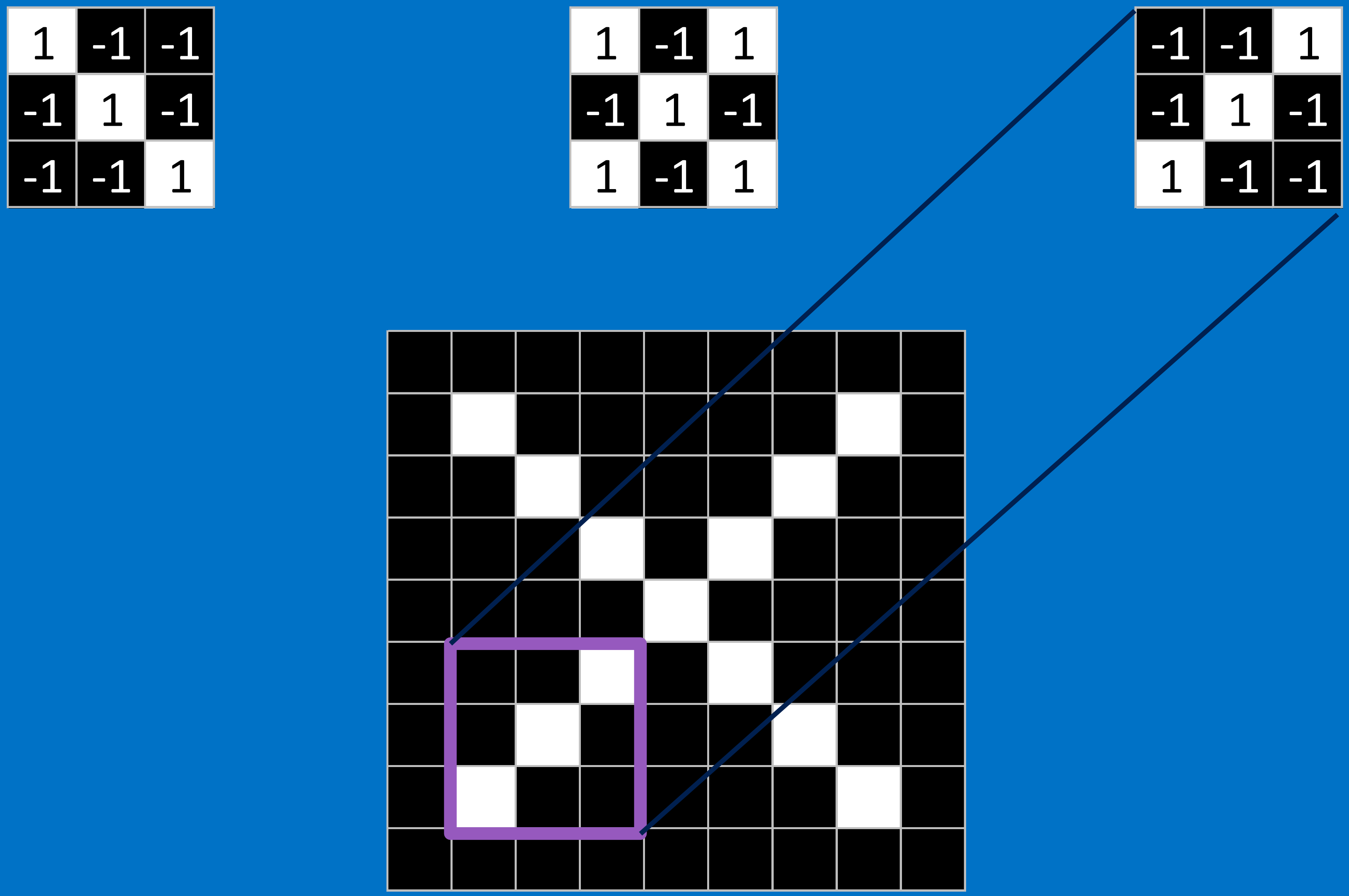
-1	-1	1
-1	1	-1
1	-1	-1




1 -1 -1
-1 1 -1
-1 -1 1
1 -1 1
-1 1 -1
1 -1 1
-1 -1 1
-1 1 -1
1 -1 -1



1 -1 -1
-1 1 -1
-1 -1 1
1 -1 1
-1 1 -1
1 -1 1
-1 -1 1
-1 1 -1
1 -1 -1



1 -1 -1
-1 1 -1
-1 -1 1
1 -1 1
-1 1 -1
1 -1 1
-1 -1 1
-1 1 -1
1 -1 -1

Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

Filtering: The math behind the match

1. Line up the feature and the image patch.
2. Multiply each image pixel by the corresponding feature pixel.
3. Add them up.
4. Divide by the total number of pixels in the feature.

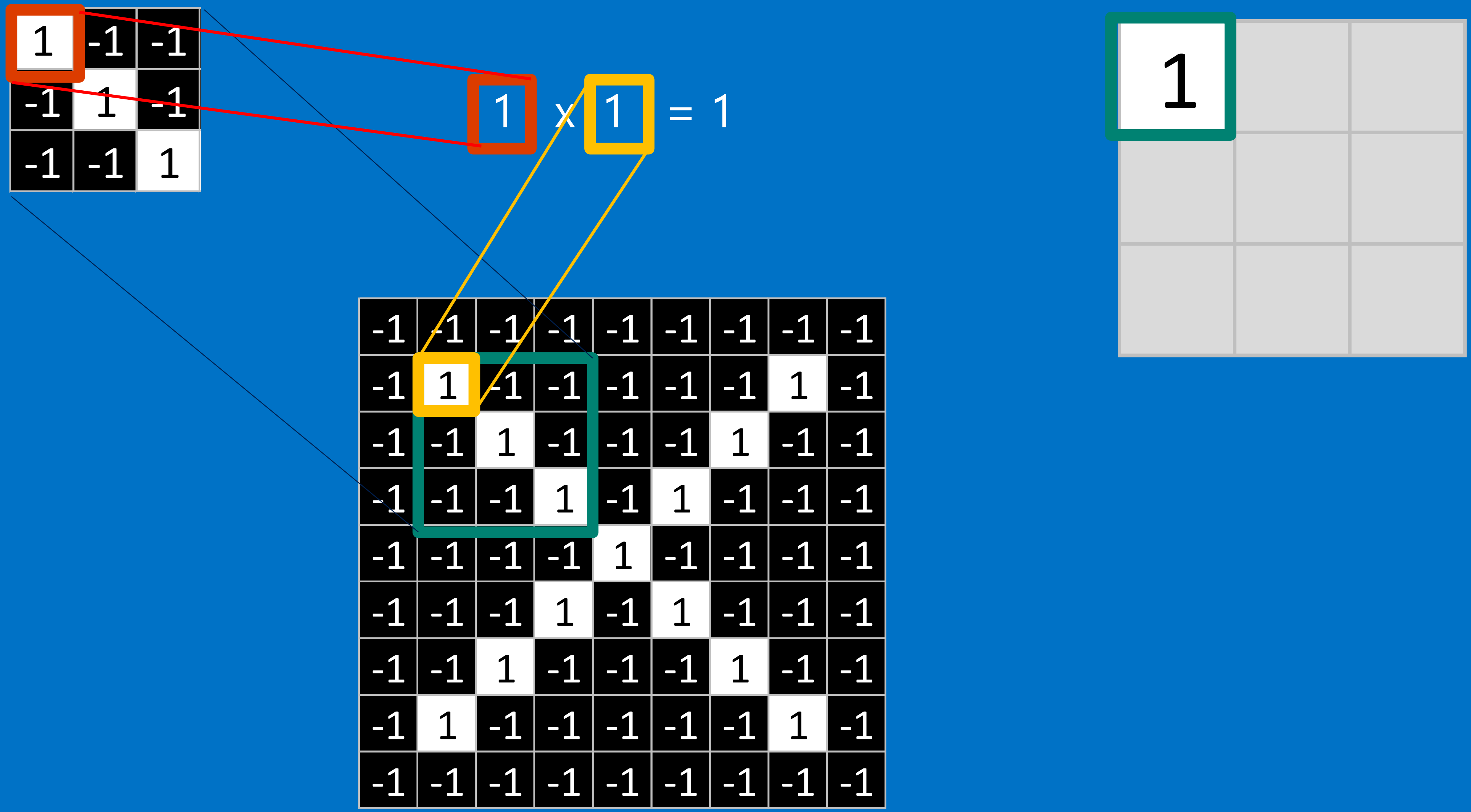
Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

$\boxed{1} \times \boxed{1} = 1$

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

Filtering: The math behind the match



Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

$-1 \times -1 = 1$

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

1	1	

Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

-1 x -1 = 1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

1	1	1

Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

$-1 \times -1 = 1$

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

1	1	1
1		

Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

1 x 1 = 1

1	1	1
1	1	

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

-1 x -1 = 1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

1	1	1
1	1	1

Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

-1 x -1 = 1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

1	1	1
1	1	1
1		

Filtering: The math behind the match

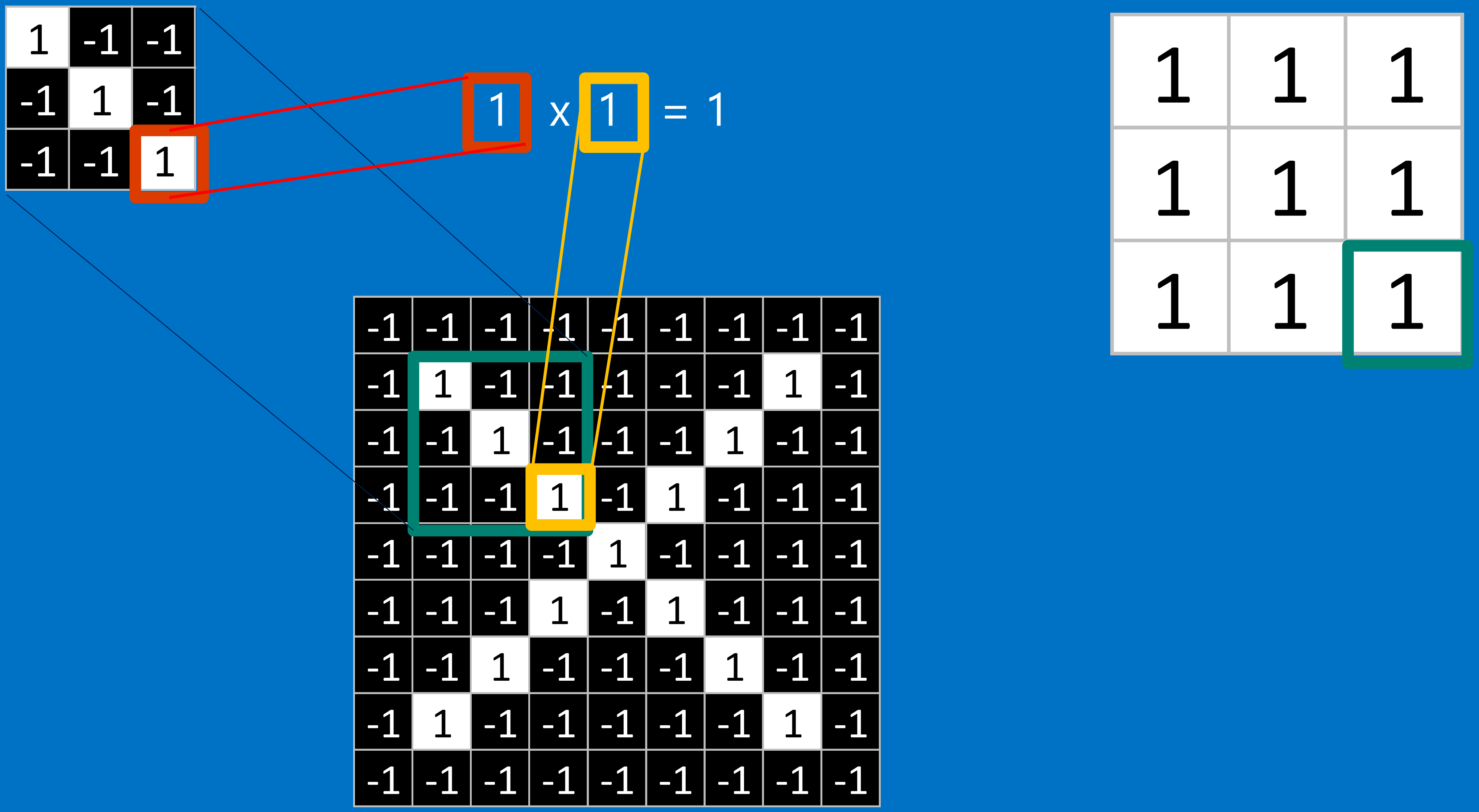
1	-1	-1
-1	1	-1
-1	-1	1

-1 x -1 = 1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

1	1	1
1	1	1
1	1	

Filtering: The math behind the match



Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

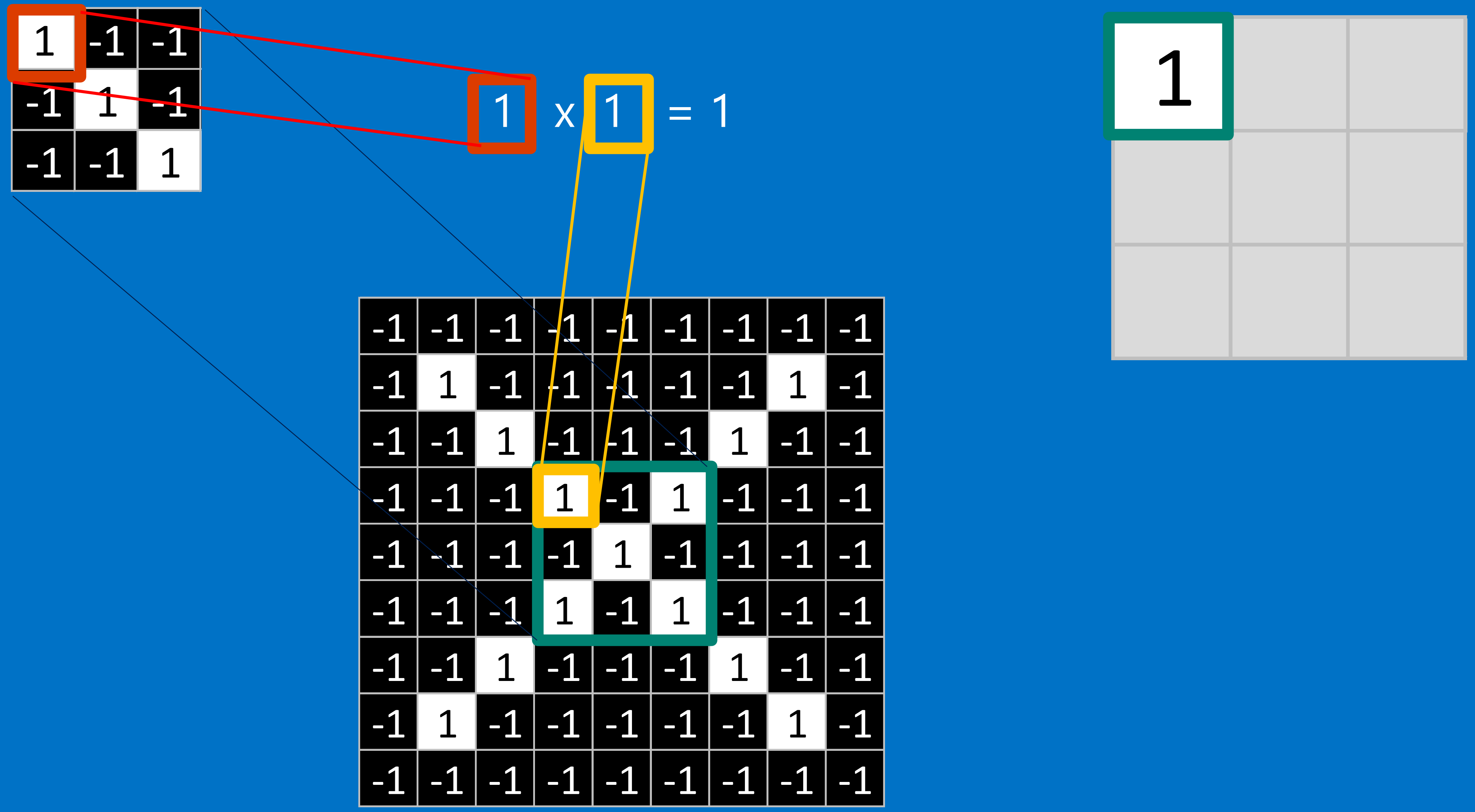
1	1	1
1	1	1
1	1	1

$$\frac{1+1+1+1+1+1+1+1+1}{9} = \boxed{1}$$

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

		1						

Filtering: The math behind the match



Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

-1 x 1 = -1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

1	1	-1

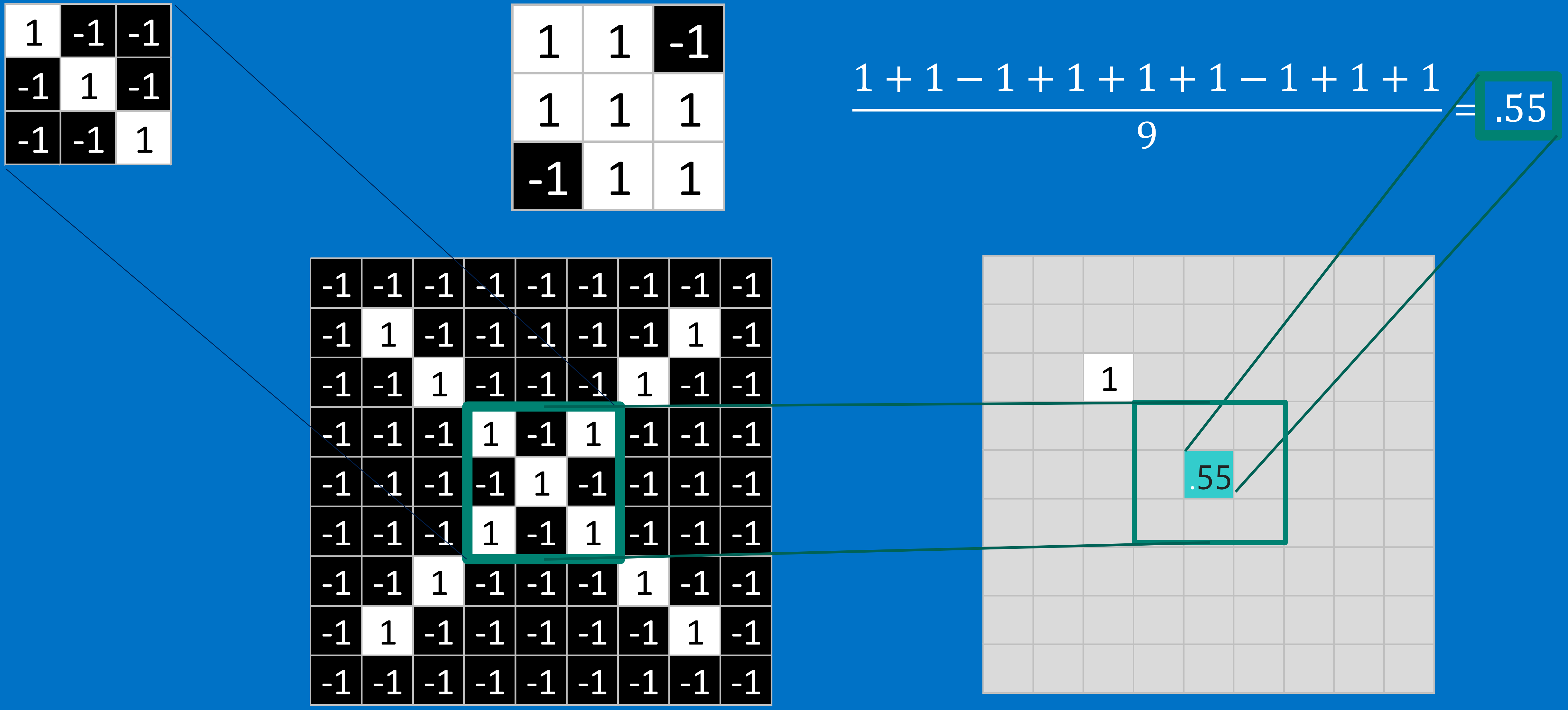
Filtering: The math behind the match

1	-1	-1
-1	1	-1
-1	-1	1

1	1	-1
1	1	1
-1	1	1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

Filtering: The math behind the match



$$\frac{1+1-1+1+1+1-1+1+1}{9} = .55$$

Convolution: Trying every possible match

1	-1	-1
-1	1	-1
-1	-1	1

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

Convolution: Trying every possible match

$$
\begin{bmatrix}
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \\
-1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\
-1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\
-1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 \\
-1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\
-1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\
-1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1
\end{bmatrix}
\otimes
\begin{bmatrix}
1 & -1 & -1 \\
-1 & 1 & -1 \\
-1 & -1 & 1
\end{bmatrix}
=
\begin{bmatrix}
0.77 & -0.11 & 0.11 & 0.33 & 0.55 & -0.11 & 0.33 \\
-0.11 & 1.00 & -0.11 & 0.33 & -0.11 & 0.11 & -0.11 \\
0.11 & -0.11 & 1.00 & -0.33 & 0.11 & -0.11 & 0.55 \\
0.33 & 0.33 & -0.33 & 0.55 & -0.33 & 0.33 & 0.33 \\
0.55 & -0.11 & 0.11 & -0.33 & 1.00 & -0.11 & 0.11 \\
-0.11 & 0.11 & -0.11 & 0.33 & -0.11 & 1.00 & -0.11 \\
0.33 & -0.11 & 0.55 & 0.33 & 0.11 & -0.11 & 0.77
\end{bmatrix}
$$

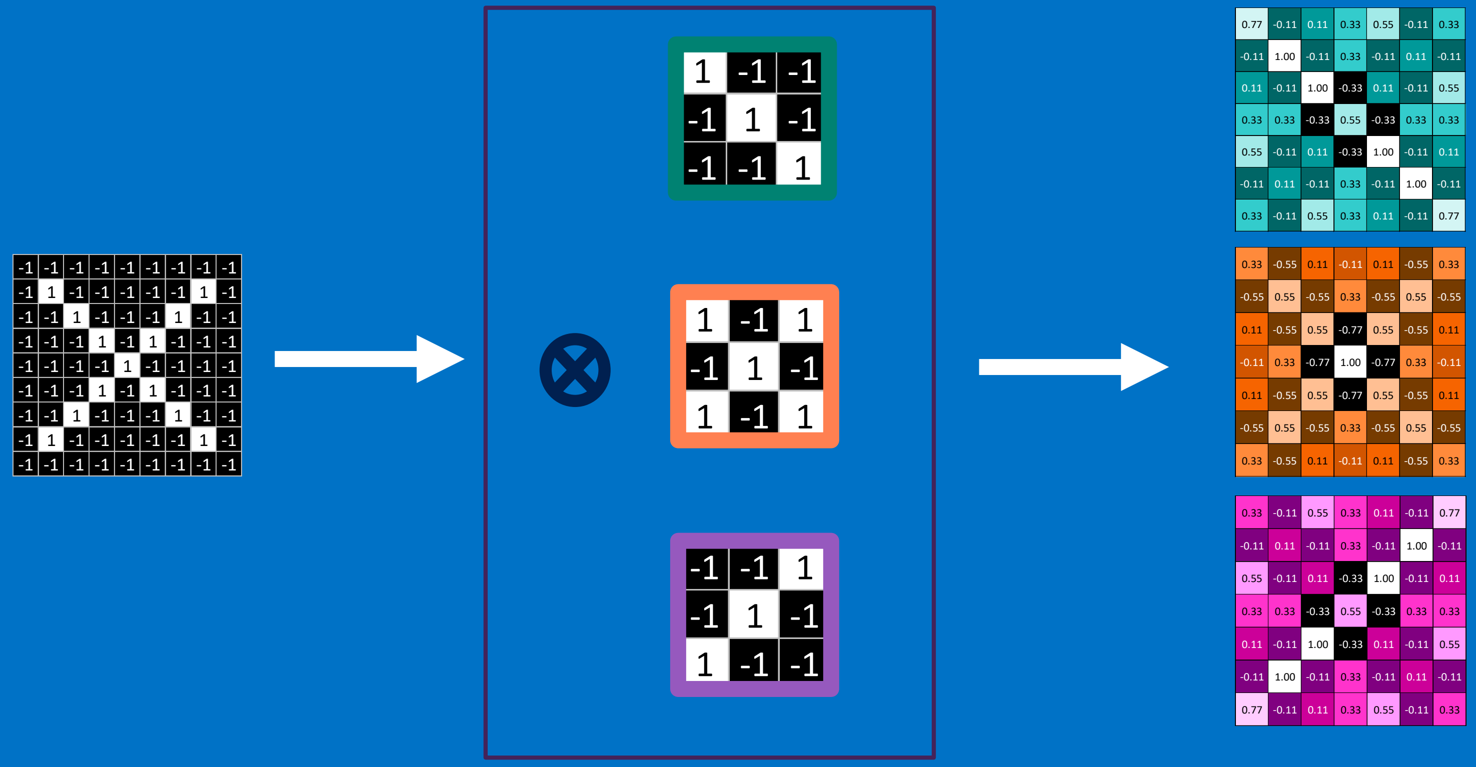
$$\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\ -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0.77 & -0.11 & 0.11 & 0.33 & 0.55 & -0.11 & 0.33 \\ -0.11 & 1.00 & -0.11 & 0.33 & -0.11 & 0.11 & -0.11 \\ 0.11 & -0.11 & 1.00 & -0.33 & 0.11 & -0.11 & 0.55 \\ 0.33 & 0.33 & -0.33 & 0.55 & -0.33 & 0.33 & 0.33 \\ 0.55 & -0.11 & 0.11 & -0.33 & 1.00 & -0.11 & 0.11 \\ -0.11 & 0.11 & -0.11 & 0.33 & -0.11 & 1.00 & -0.11 \\ 0.33 & -0.11 & 0.55 & 0.33 & 0.11 & -0.11 & 0.77 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\ -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix} \otimes \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 0.33 & -0.55 & 0.11 & -0.11 & 0.11 & -0.55 & 0.33 \\ -0.55 & 0.55 & -0.55 & 0.33 & -0.55 & 0.55 & -0.55 \\ 0.11 & -0.55 & 0.55 & -0.77 & 0.55 & -0.55 & 0.11 \\ -0.11 & 0.33 & -0.77 & 1.00 & -0.77 & 0.33 & -0.11 \\ 0.11 & -0.55 & 0.55 & -0.77 & 0.55 & -0.55 & 0.11 \\ -0.55 & 0.55 & -0.55 & 0.33 & -0.55 & 0.55 & -0.55 \\ 0.33 & -0.55 & 0.11 & -0.11 & 0.11 & -0.55 & 0.33 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\ -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\ -1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 \\ -1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\ -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 \\ -1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & -1 \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix} \otimes \begin{bmatrix} -1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix} = \begin{bmatrix} 0.33 & -0.11 & 0.55 & 0.33 & 0.11 & -0.11 & 0.77 \\ -0.11 & 0.11 & -0.11 & 0.33 & -0.11 & 1.00 & -0.11 \\ 0.55 & -0.11 & 0.11 & -0.33 & 1.00 & -0.11 & 0.11 \\ 0.33 & 0.33 & -0.33 & 0.55 & -0.33 & 0.33 & 0.33 \\ 0.11 & -0.11 & 1.00 & -0.33 & 0.11 & -0.11 & 0.55 \\ -0.11 & 1.00 & -0.11 & 0.33 & -0.11 & 0.11 & -0.11 \\ 0.77 & -0.11 & 0.11 & 0.33 & 0.55 & -0.11 & 0.33 \end{bmatrix}$$

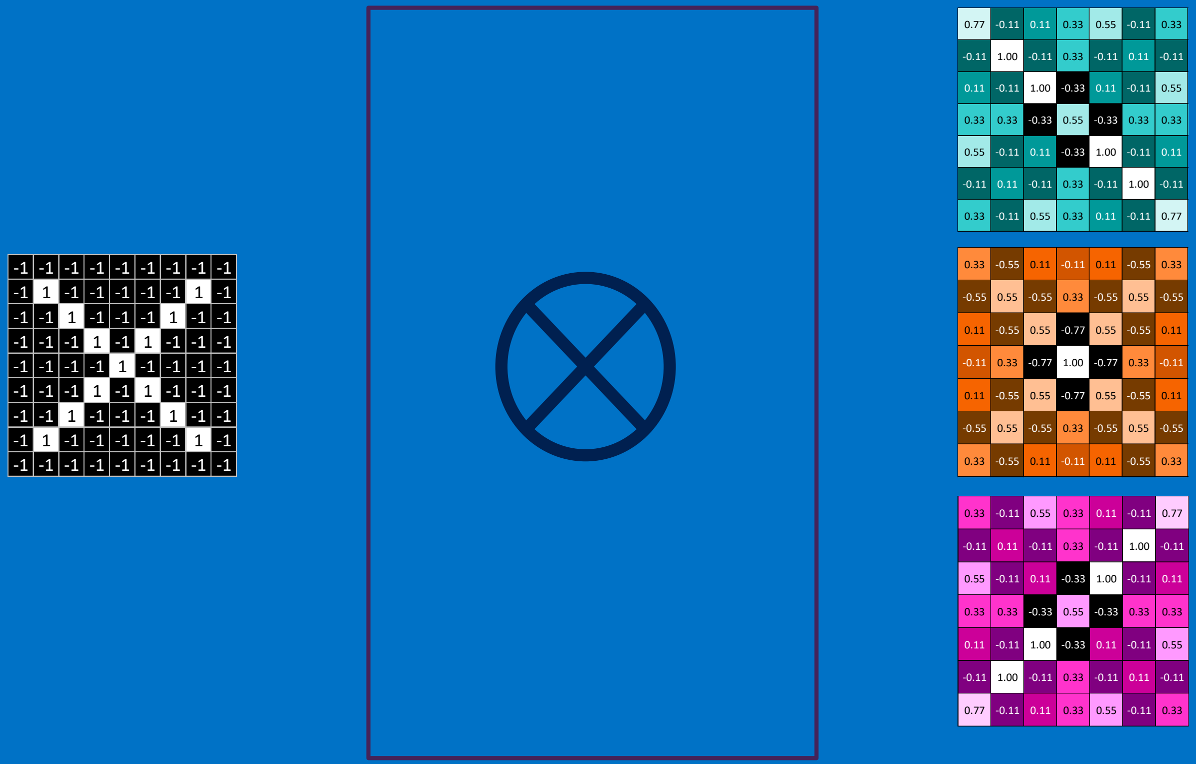
Convolution layer

One image becomes a stack of filtered images



Convolution layer

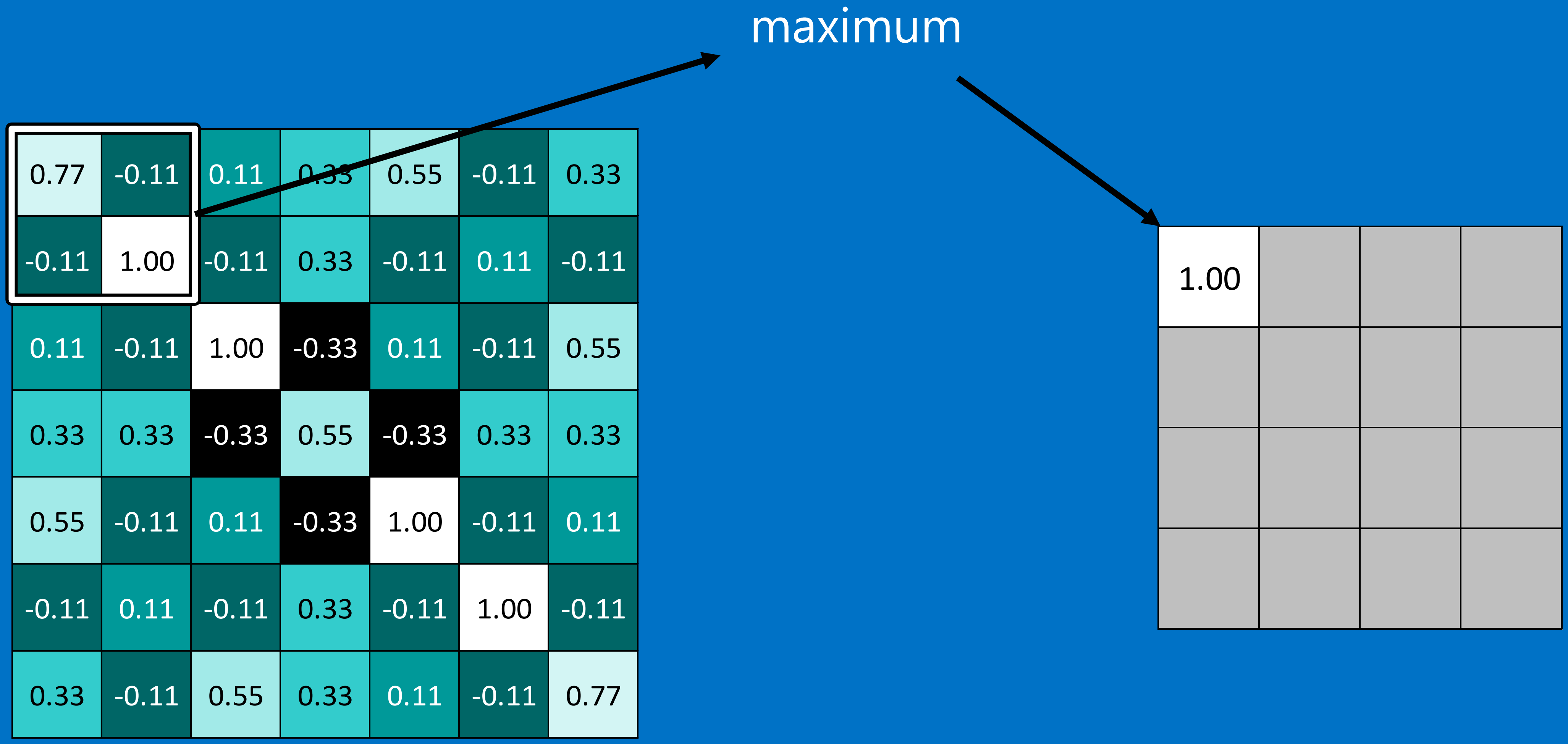
One image becomes a stack of filtered images



Pooling: Shrinking the image stack

1. Pick a window size (usually 2 or 3).
2. Pick a stride (usually 2).
3. Walk your window across your filtered images.
4. From each window, take the maximum value.

Pooling


maximum
0.77 -0.11 0.11 0.33 0.55 -0.11 0.33
-0.11 1.00 -0.11 0.33 -0.11 0.11 -0.11
0.11 -0.11 1.00 -0.33 0.11 -0.11 0.55
0.33 0.33 -0.33 0.55 -0.33 0.33 0.33
0.55 -0.11 0.11 -0.33 1.00 -0.11 0.11
-0.11 0.11 -0.11 0.33 -0.11 1.00 -0.11
0.33 -0.11 0.55 0.33 0.11 -0.11 0.77
1.00

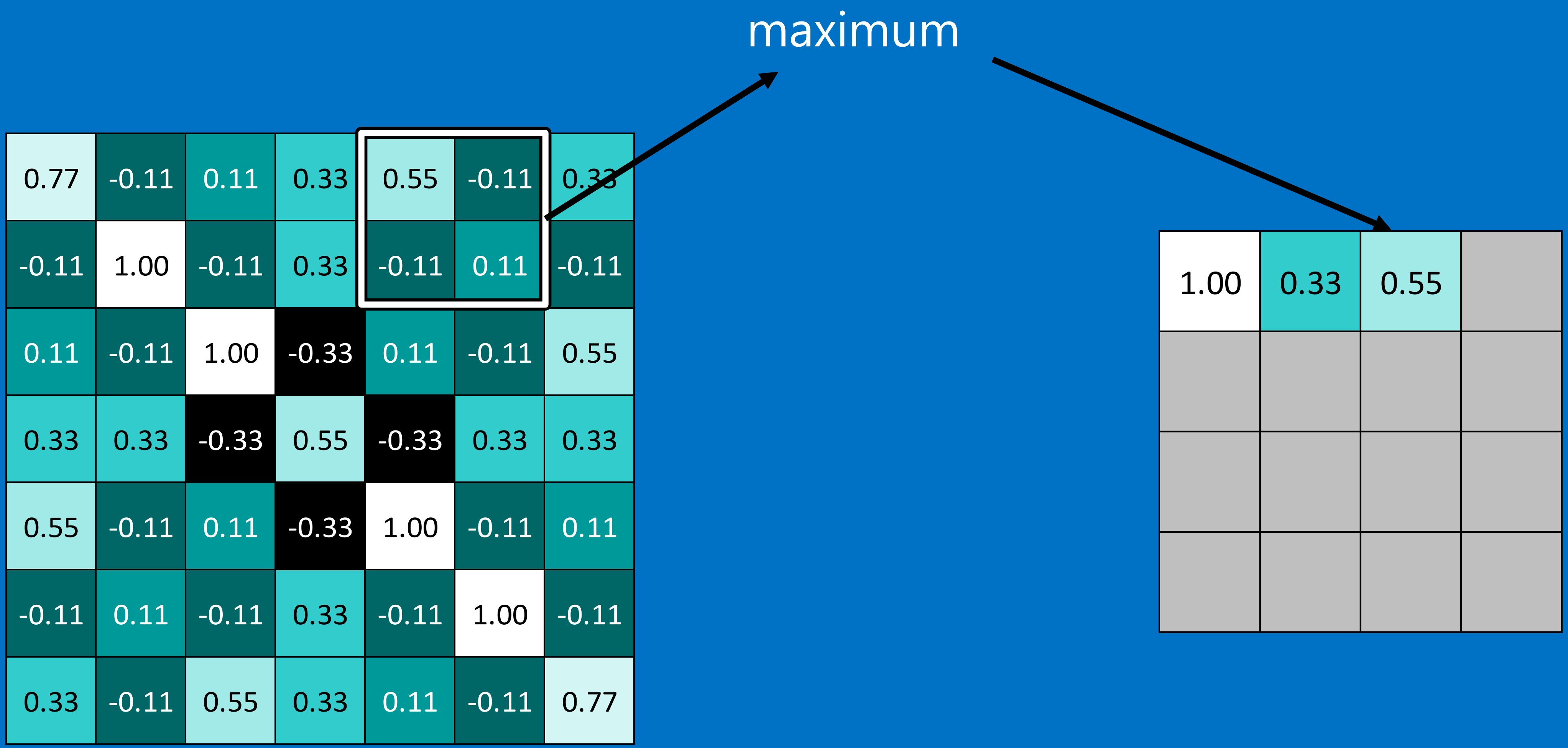
Pooling

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

maximum

1.00	0.33		

Pooling


maximum
0.77 -0.11 0.11 0.33 0.55 -0.11 0.33
-0.11 1.00 -0.11 0.33 -0.11 0.11 -0.11
0.11 -0.11 1.00 -0.33 0.11 -0.11 0.55
0.33 0.33 -0.33 0.55 -0.33 0.33 0.33
0.55 -0.11 0.11 -0.33 1.00 -0.11 0.11
-0.11 0.11 -0.11 0.33 -0.11 1.00 -0.11
0.33 -0.11 0.55 0.33 0.11 -0.11 0.77
1.00 0.33 0.55

Pooling

maximum

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

1.00	0.33	0.55	0.33

Pooling

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

maximum

1.00	0.33	0.55	0.33
0.33			

Pooling

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

max pooling

1.00	0.33	0.55	0.33
0.33	1.00	0.33	0.55
0.55	0.33	1.00	0.11
0.33	0.55	0.11	0.77

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

1.00	0.33	0.55	0.33
0.33	1.00	0.33	0.55
0.55	0.33	1.00	0.11
0.33	0.55	0.11	0.77

0.33	-0.55	0.11	-0.11	0.11	-0.55	0.33
-0.55	0.55	-0.55	0.33	-0.55	0.55	-0.55
0.11	-0.55	0.55	-0.77	0.55	-0.55	0.11
-0.11	0.33	-0.77	1.00	-0.77	0.33	-0.11
0.11	-0.55	0.55	-0.77	0.55	-0.55	0.11
-0.55	0.55	-0.55	0.33	-0.55	0.55	-0.55
0.33	-0.55	0.11	-0.11	0.11	-0.55	0.33

0.55	0.33	0.55	0.33
0.33	1.00	0.55	0.11
0.55	0.55	0.55	0.11
0.33	0.11	0.11	0.33

0.33	-0.11	0.55	0.33	0.11	-0.11	0.77
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.77	-0.11	0.11	0.33	0.55	-0.11	0.33

0.33	0.55	1.00	0.77
0.55	0.55	1.00	0.33
1.00	1.00	0.11	0.55
0.77	0.33	0.55	0.33

Pooling layer

A stack of images becomes a stack of smaller images.

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

0.33	-0.55	0.11	-0.11	0.11	-0.55	0.33
-0.55	0.55	-0.55	0.33	-0.55	0.55	-0.55
0.11	-0.55	0.55	-0.77	0.55	-0.55	0.11
-0.11	0.33	-0.77	1.00	-0.77	0.33	-0.11
0.11	-0.55	0.55	-0.77	0.55	-0.55	0.11
-0.55	0.55	-0.55	0.33	-0.55	0.55	-0.55
0.33	-0.55	0.11	-0.11	0.11	-0.55	0.33

0.33	-0.11	0.55	0.33	0.11	-0.11	0.77
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.77	-0.11	0.11	0.33	0.55	-0.11	0.33

1.00	0.33	0.55	0.33
0.33	1.00	0.33	0.55
0.55	0.33	1.00	0.11
0.33	0.55	0.11	0.77

0.55	0.33	0.55	0.33
0.33	1.00	0.55	0.11
0.55	0.55	0.55	0.11
0.33	0.11	0.11	0.33

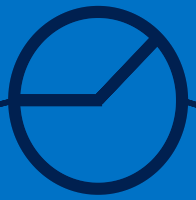
0.33	0.55	1.00	0.77
0.55	0.55	1.00	0.33
1.00	1.00	0.11	0.55
0.77	0.33	0.55	0.33

Normalization

Keep the math from breaking by tweaking each of the values just a bit.

Change everything negative to zero.

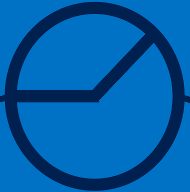
Rectified Linear Units (ReLUs)



0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

0.77						

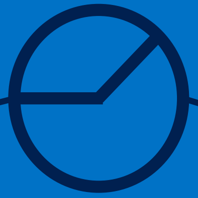
Rectified Linear Units (ReLUs)



0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

0.77	0					

Rectified Linear Units (ReLUs)

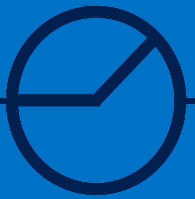


0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

0.77	0	0.11	0.33	0.55	0	0.33

Rectified Linear Units (ReLUs)

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77



0.77	0	0.11	0.33	0.55	0	0.33
0	1.00	0	0.33	0	0.11	0
0.11	0	1.00	0	0.11	0	0.55
0.33	0.33	0	0.55	0	0.33	0.33
0.55	0	0.11	0	1.00	0	0.11
0	0.11	0	0.33	0	1.00	0
0.33	0	0.55	0.33	0.11	0	0.77

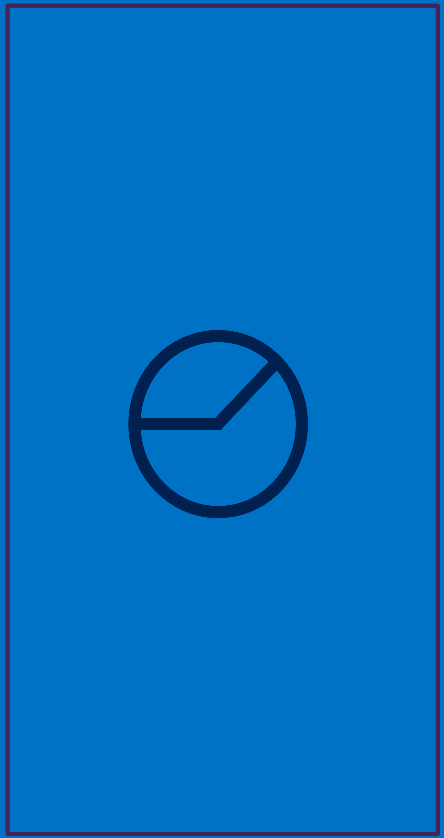
ReLU layer

A stack of images becomes a stack of images with no negative values.

0.77	-0.11	0.11	0.33	0.55	-0.11	0.33
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.33	-0.11	0.55	0.33	0.11	-0.11	0.77

0.33	-0.55	0.11	-0.11	0.11	-0.55	0.33
-0.55	0.55	-0.55	0.33	-0.55	0.55	-0.55
0.11	-0.55	0.55	-0.77	0.55	-0.55	0.11
-0.11	0.33	-0.77	1.00	-0.77	0.33	-0.11
0.11	-0.55	0.55	-0.77	0.55	-0.55	0.11
-0.55	0.55	-0.55	0.33	-0.55	0.55	-0.55
0.33	-0.55	0.11	-0.11	0.11	-0.55	0.33

0.33	-0.11	0.55	0.33	0.11	-0.11	0.77
-0.11	0.11	-0.11	0.33	-0.11	1.00	-0.11
0.55	-0.11	0.11	-0.33	1.00	-0.11	0.11
0.33	0.33	-0.33	0.55	-0.33	0.33	0.33
0.11	-0.11	1.00	-0.33	0.11	-0.11	0.55
-0.11	1.00	-0.11	0.33	-0.11	0.11	-0.11
0.77	-0.11	0.11	0.33	0.55	-0.11	0.33



0.77	0	0.11	0.33	0.55	0	0.33
0	1.00	0	0.33	0	0.11	0
0.11	0	1.00	0	0.11	0	0.55
0.33	0.33	0	0.55	0	0.33	0.33
0.55	0	0.11	0	1.00	0	0.11
0	0.11	0	0.33	0	1.00	0
0.33	0	0.55	0.33	0.11	0	0.77

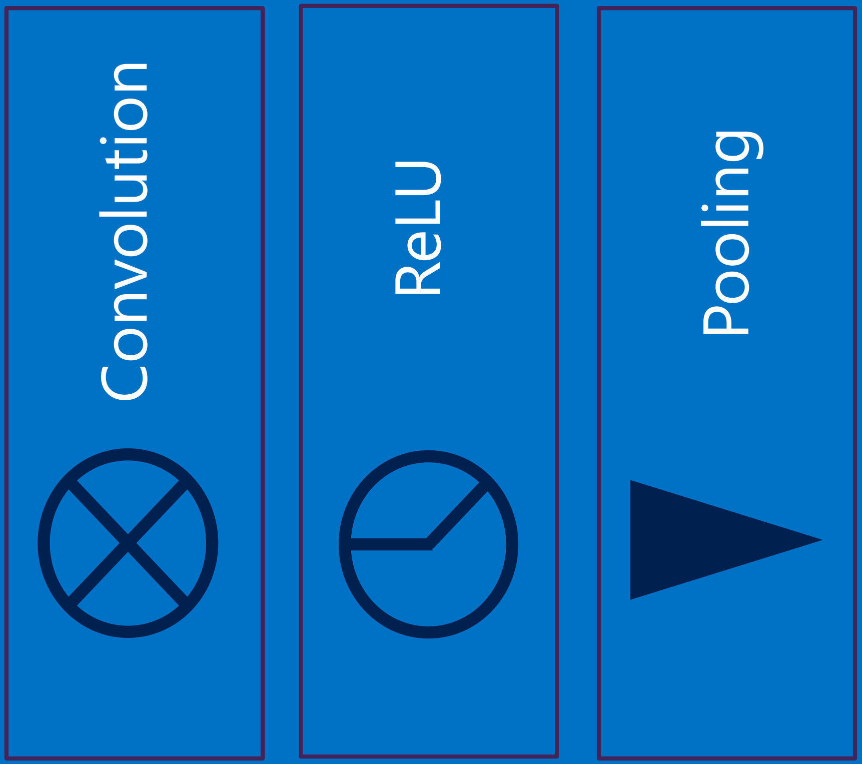
0.33	0	0.11	0	0.11	0	0.33
0	0.55	0	0.33	0	0.55	0
0.11	0	0.55	0	0.55	0	0.11
0	0.33	0	1.00	0	0.33	0
0.11	0	0.55	0	0.55	0	0.11
0	0.55	0	0.33	0	0.55	0
0.33	0	0.11	0	0.11	0	0.33

0.33	0	0.55	0.33	0.11	0	0.77
0	0.11	0	0.33	0	1.00	0
0.55	0	0.11	0	1.00	0	0.11
0.33	0.33	0	0.55	0	0.33	0.33
0.11	0	1.00	0	0.11	0	0.55
0	1.00	0	0.33	0	0.11	0
0.77	0	0.11	0.33	0.55	0	0.33

Layers get stacked

The output of one becomes the input of the next.

-1	-1	-1	-1	-1	-1	-1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	-1	-1	1	-1	-1	-1	-1
-1	-1	-1	1	-1	1	-1	-1	-1
-1	-1	1	-1	-1	-1	1	-1	-1
-1	1	-1	-1	-1	-1	-1	1	-1
-1	-1	-1	-1	-1	-1	-1	-1	-1



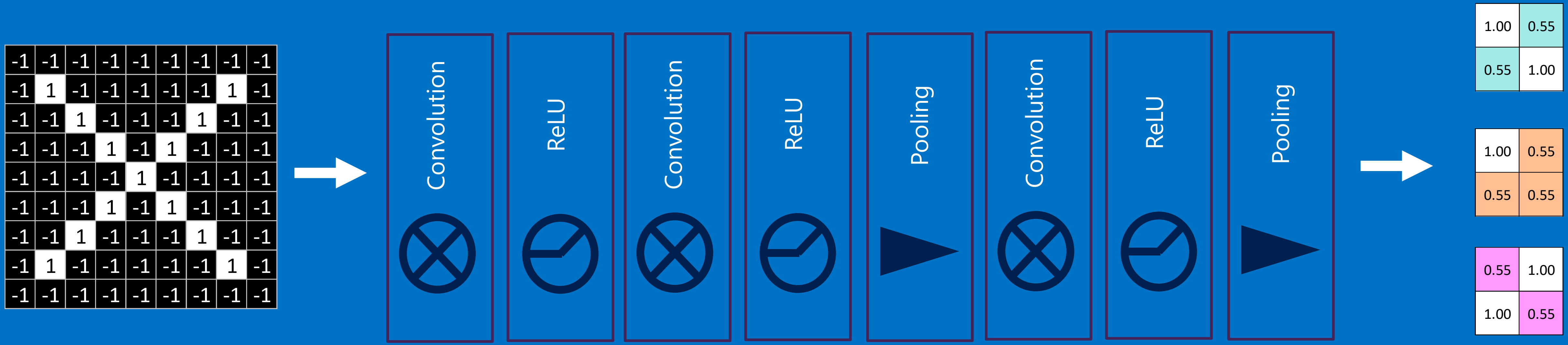
1.00	0.33	0.55	0.33
0.33	1.00	0.33	0.55
0.55	0.33	1.00	0.11
0.33	0.55	0.11	0.77

0.55	0.33	0.55	0.33
0.33	1.00	0.55	0.11
0.55	0.55	0.55	0.11
0.33	0.11	0.11	0.33

0.33	0.55	1.00	0.77
0.55	0.55	1.00	0.33
1.00	1.00	0.11	0.55
0.77	0.33	0.55	0.33

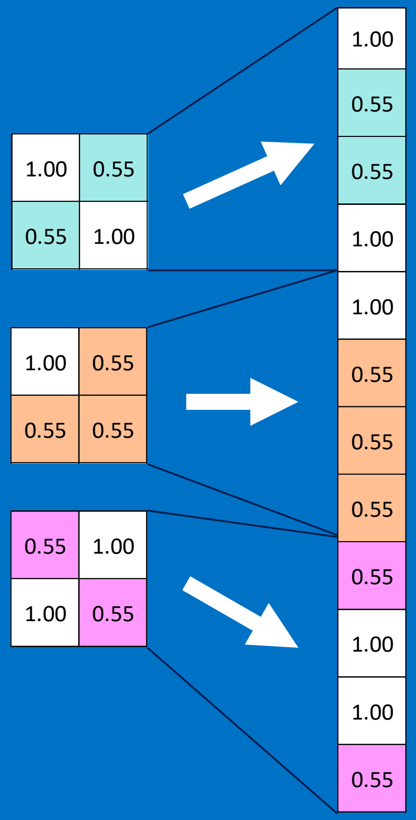
Deep stacking

Layers can be repeated several (or many) times.



Fully connected layer

Every value gets a vote

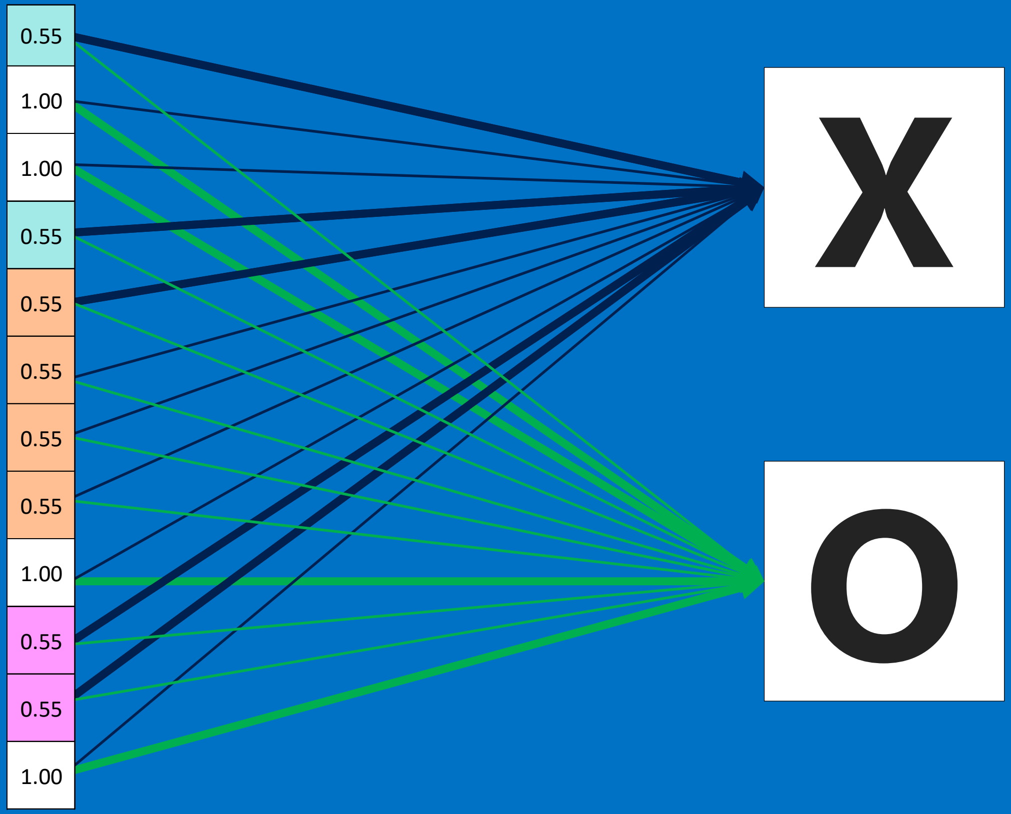


Fully connected layer

Vote depends on how strongly a value predicts X or O

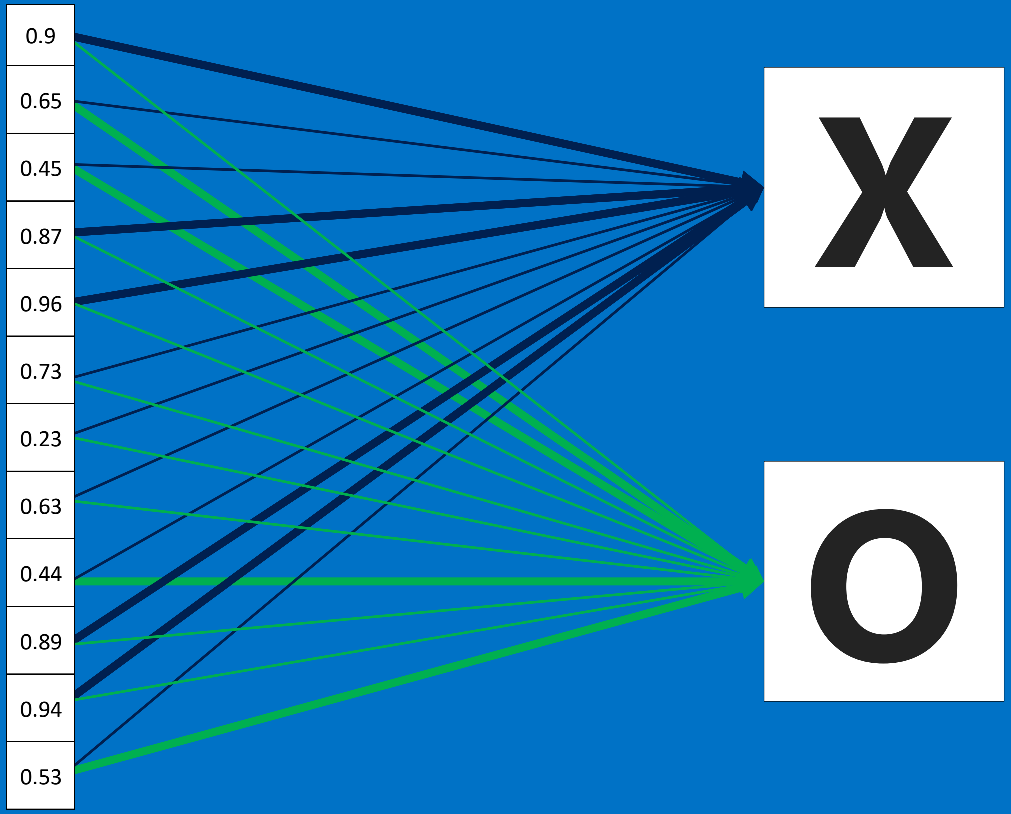
Fully connected layer

Vote depends on how strongly a value predicts X or O



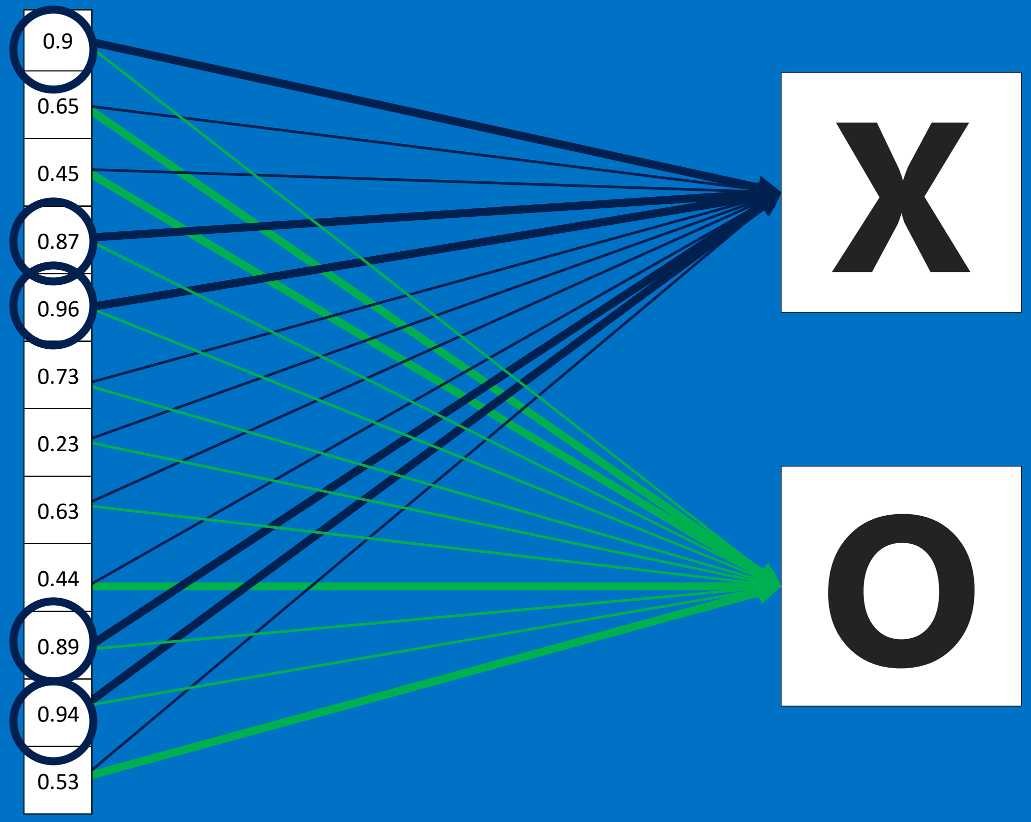
Fully connected layer

Future values vote on X or O



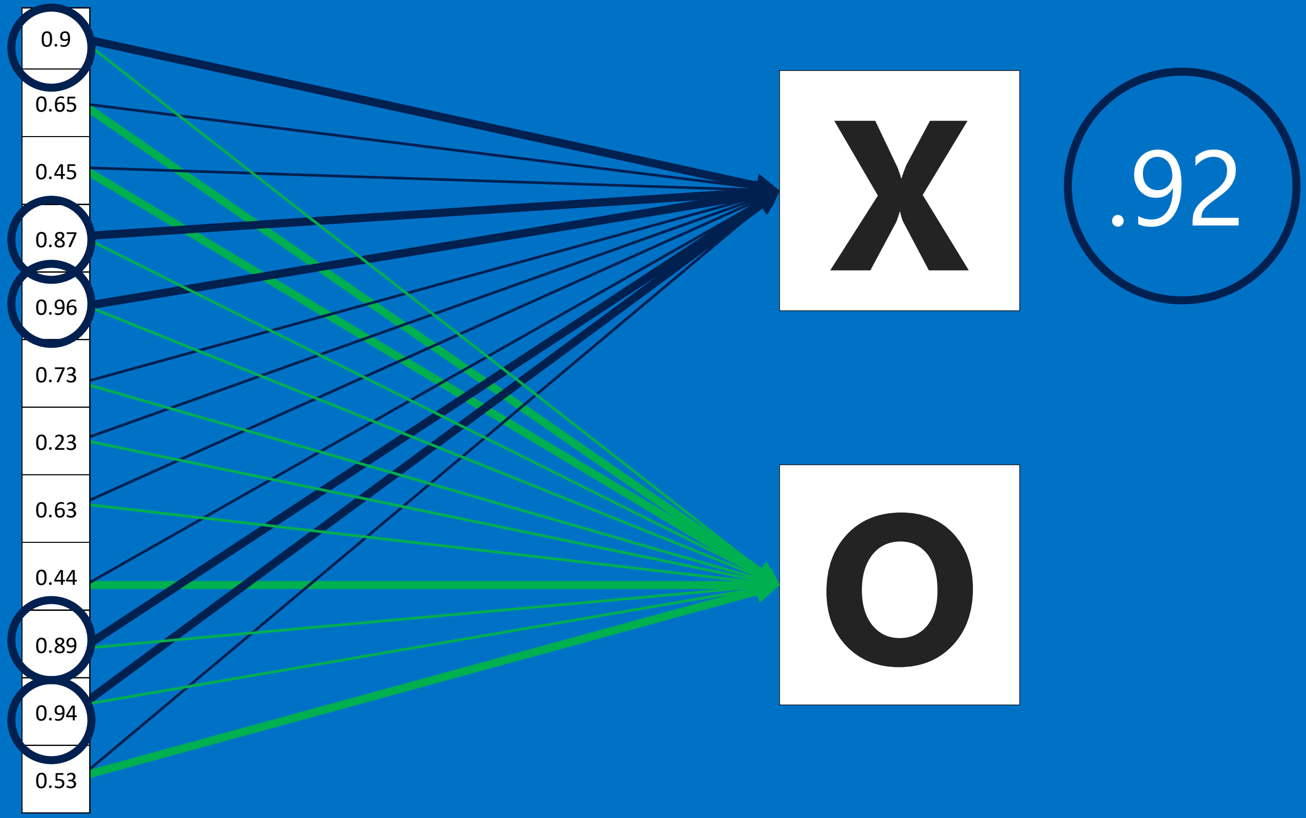
Fully connected layer

Future values vote on X or O



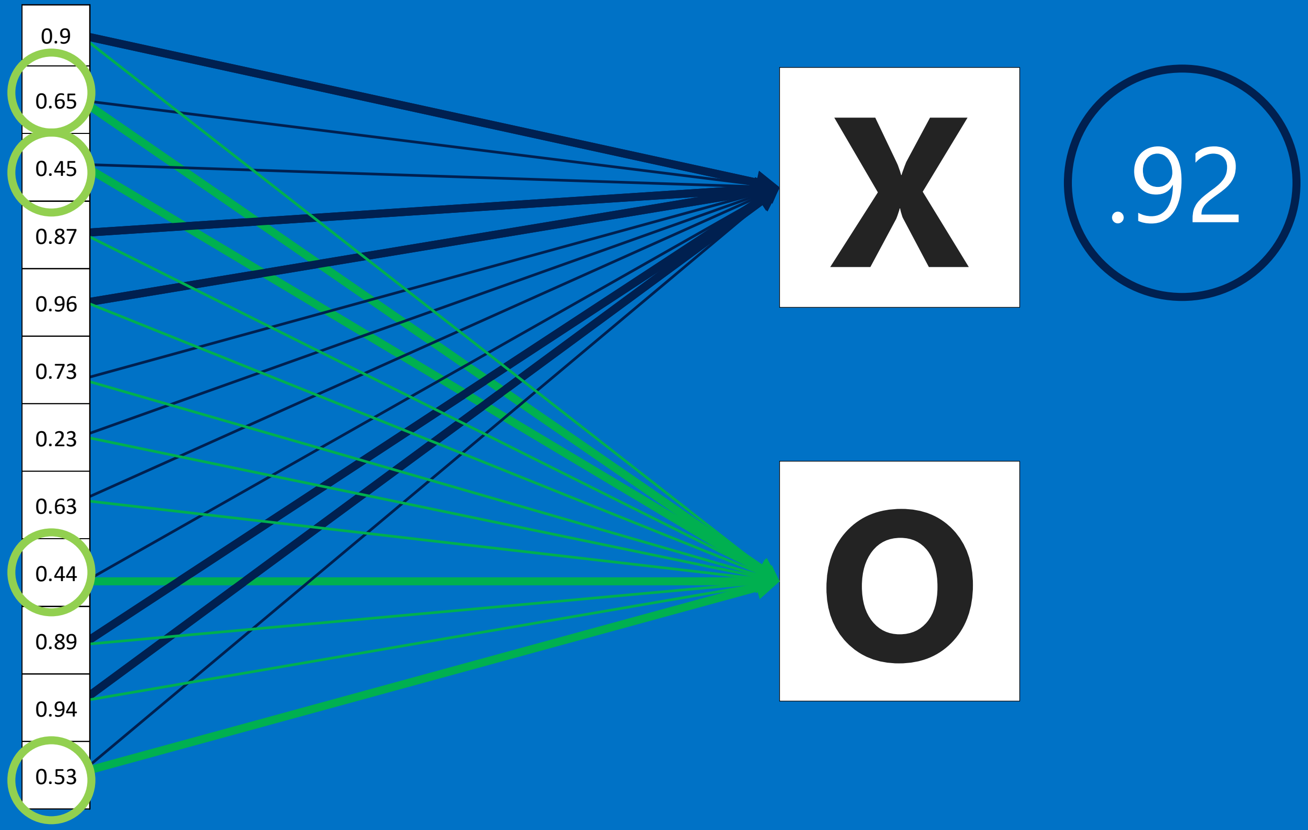
Fully connected layer

Future values vote on X or O



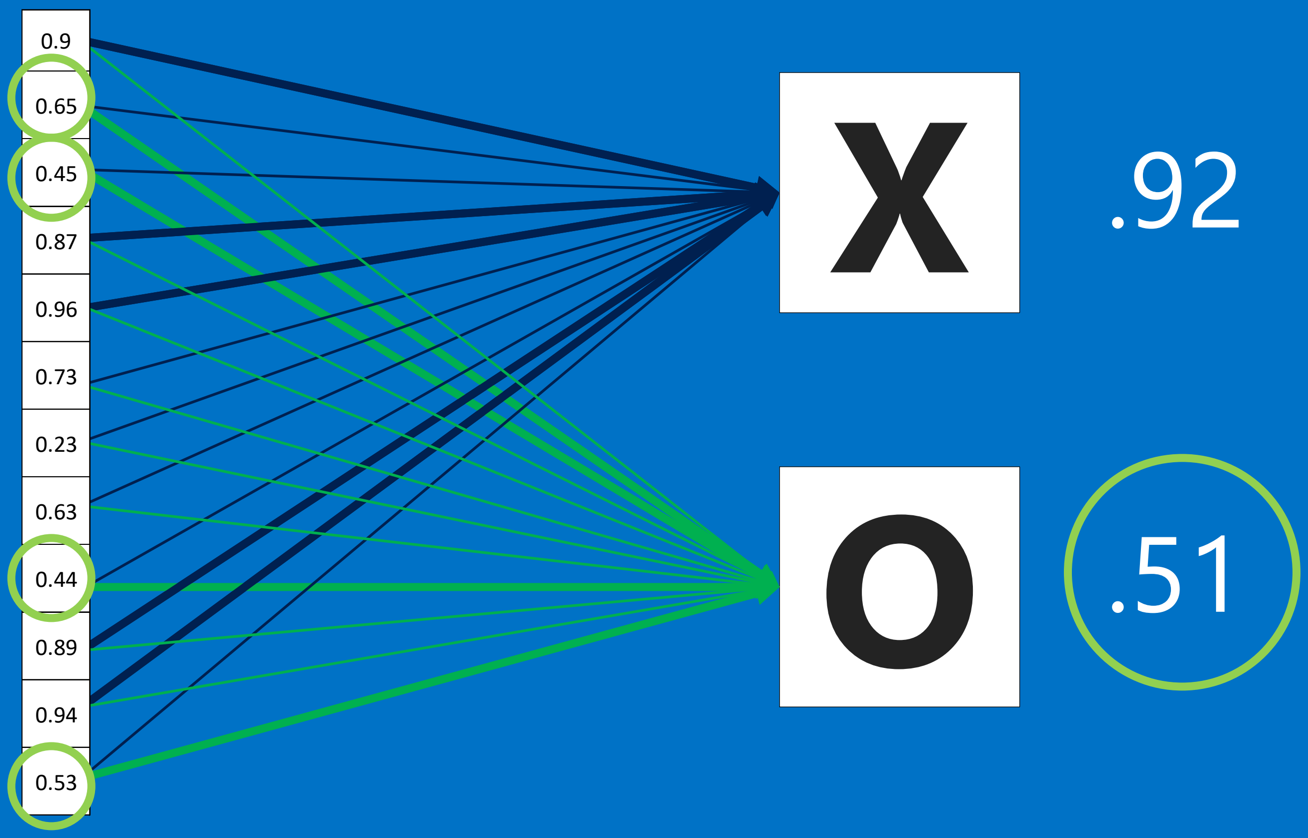
Fully connected layer

Future values vote on X or O



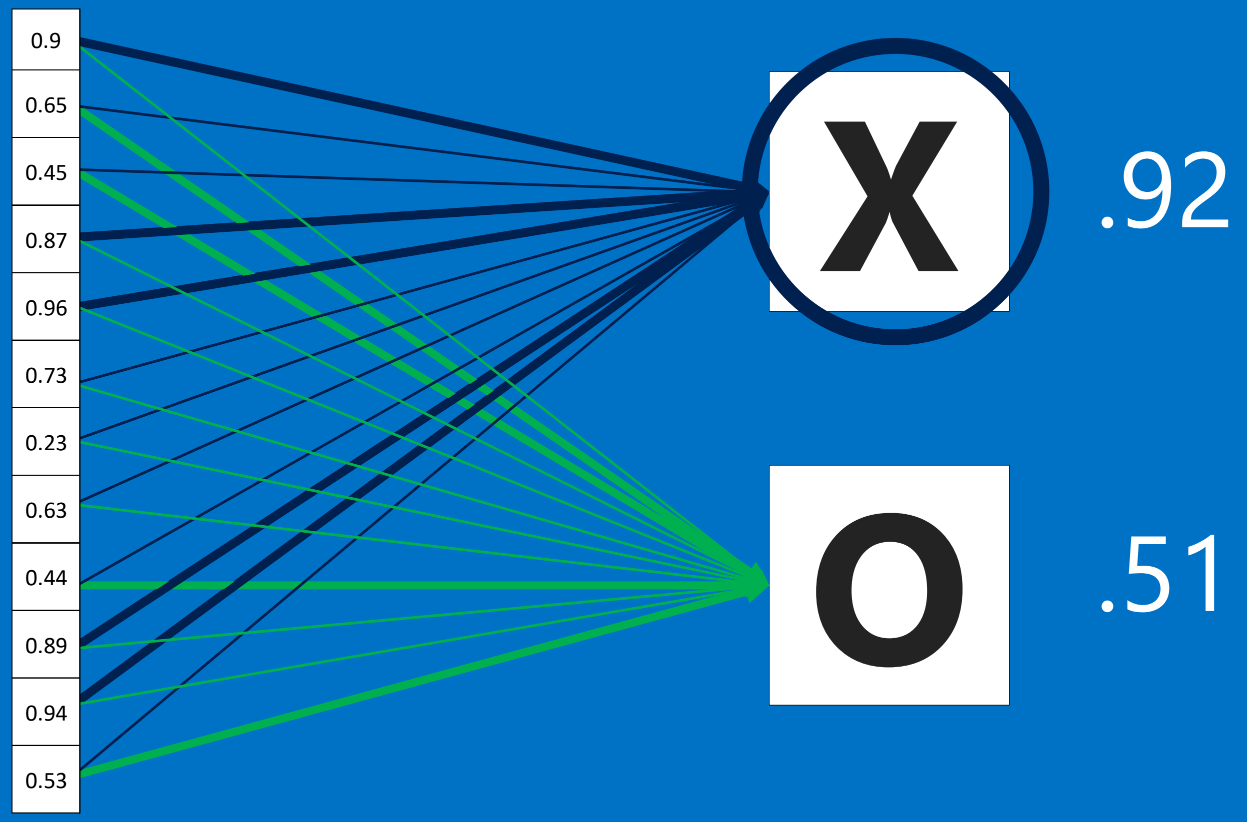
Fully connected layer

Future values vote on X or O



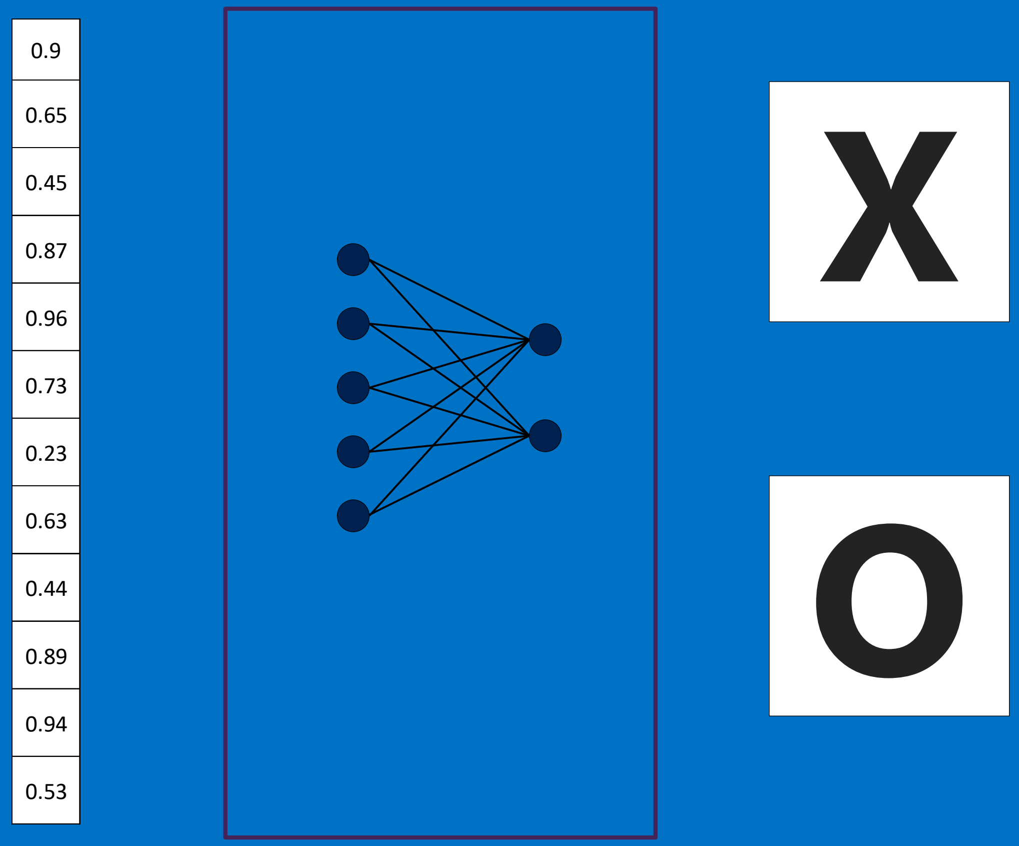
Fully connected layer

Future values vote on X or O



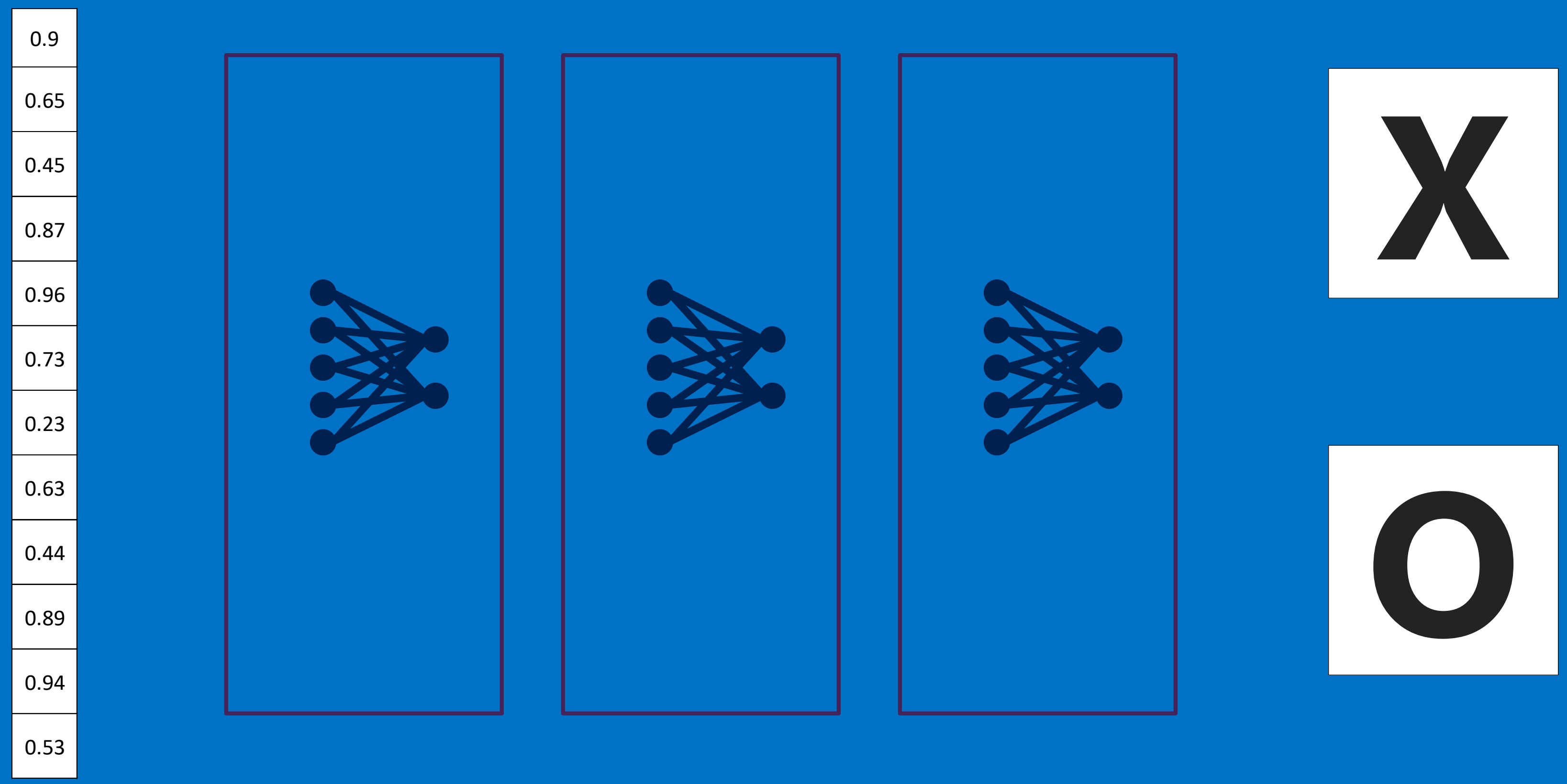
Fully connected layer

A list of feature values becomes a list of votes.



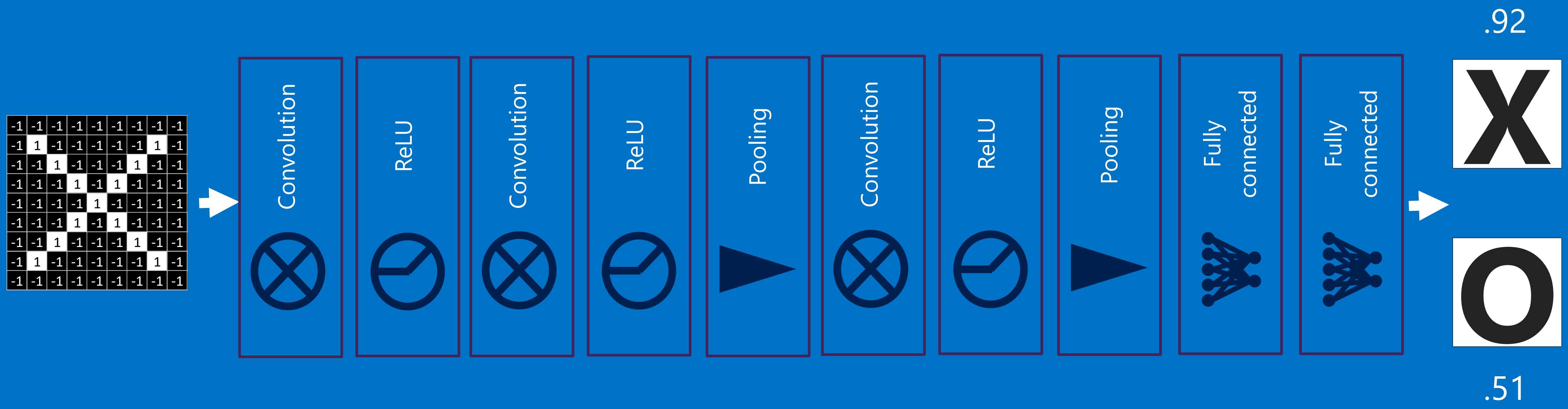
Fully connected layer

These can also be stacked.



Putting it all together

A set of pixels becomes a set of votes.



Learning

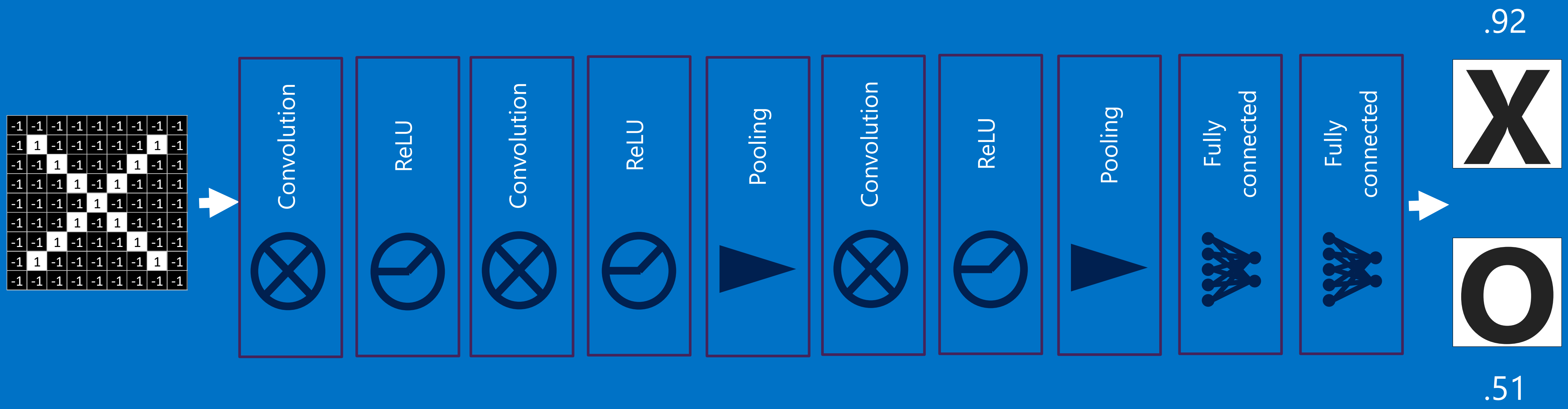
Q: Where do all the magic numbers come from?

- Features in convolutional layers
- Voting weights in fully connected layers

A: Backpropagation

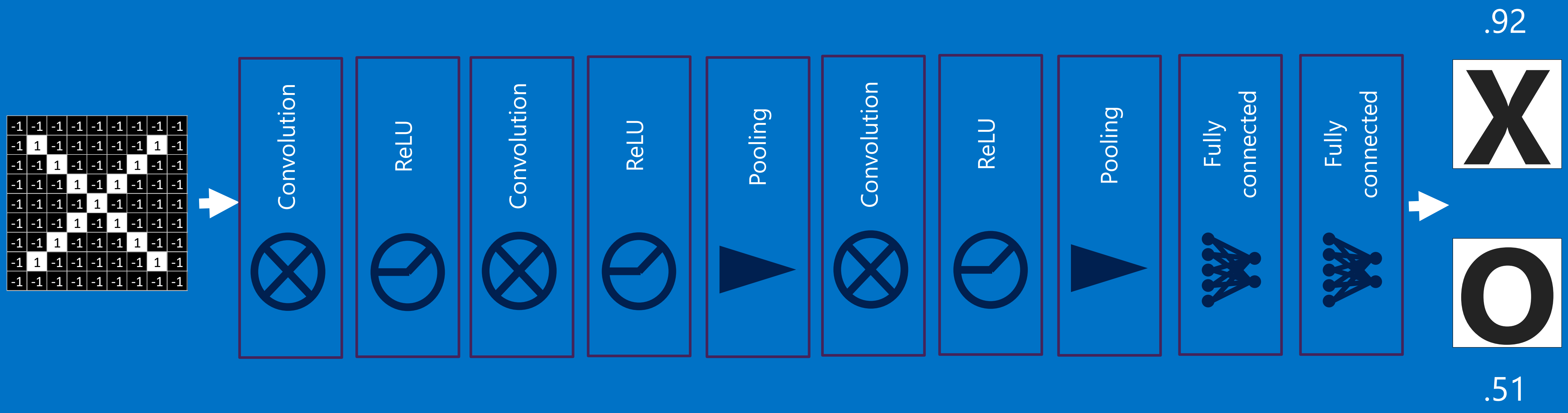
Backprop

Error = right answer – actual answer



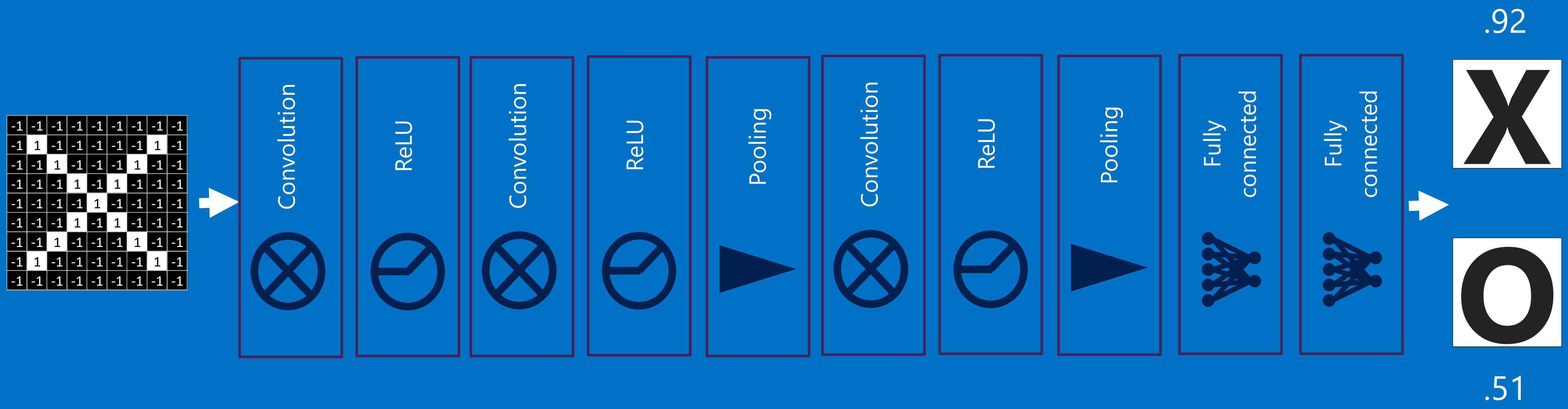
Backprop

	Right answer	Actual answer	Error
X	1		
O			



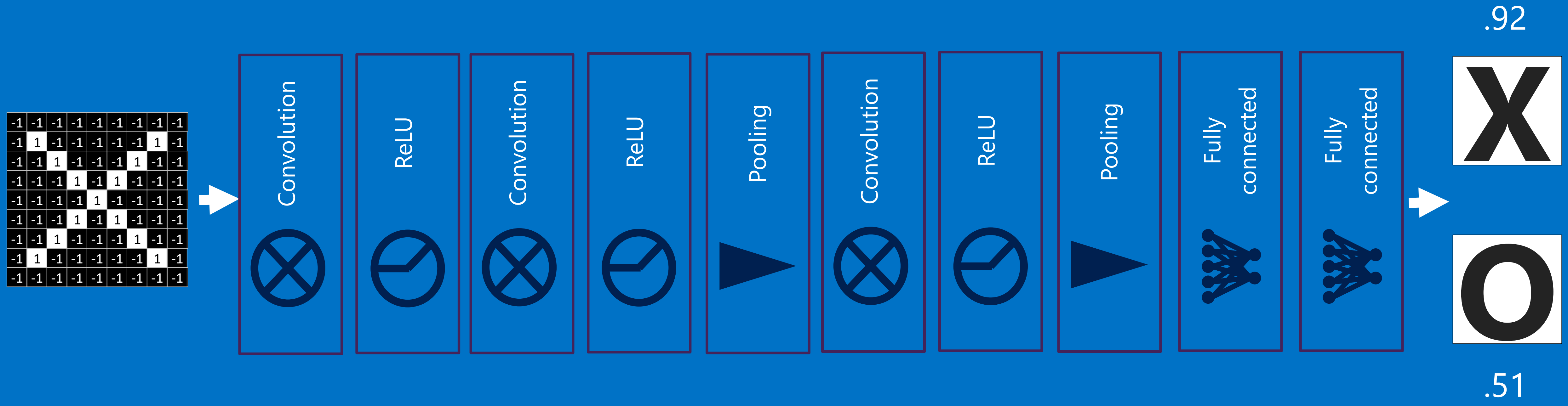
Backprop

	Right answer	Actual answer	Error
X	1	0.92	
O			



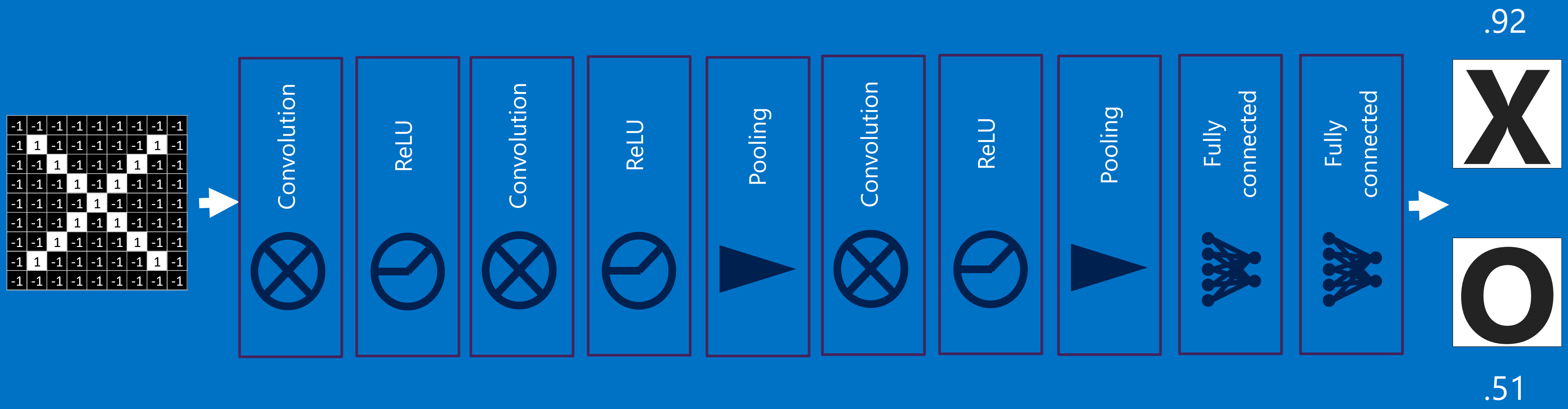
Backprop

	Right answer	Actual answer	Error
X	1	0.92	0.08
O			



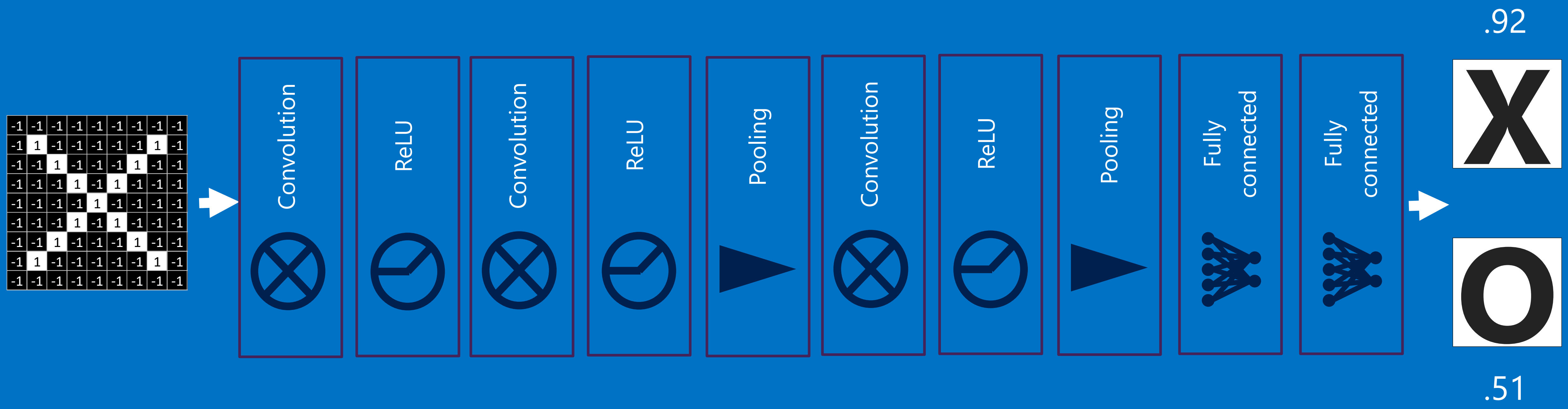
Backprop

	Right answer	Actual answer	Error
X	1	0.92	0.08
O	0	0.51	0.49



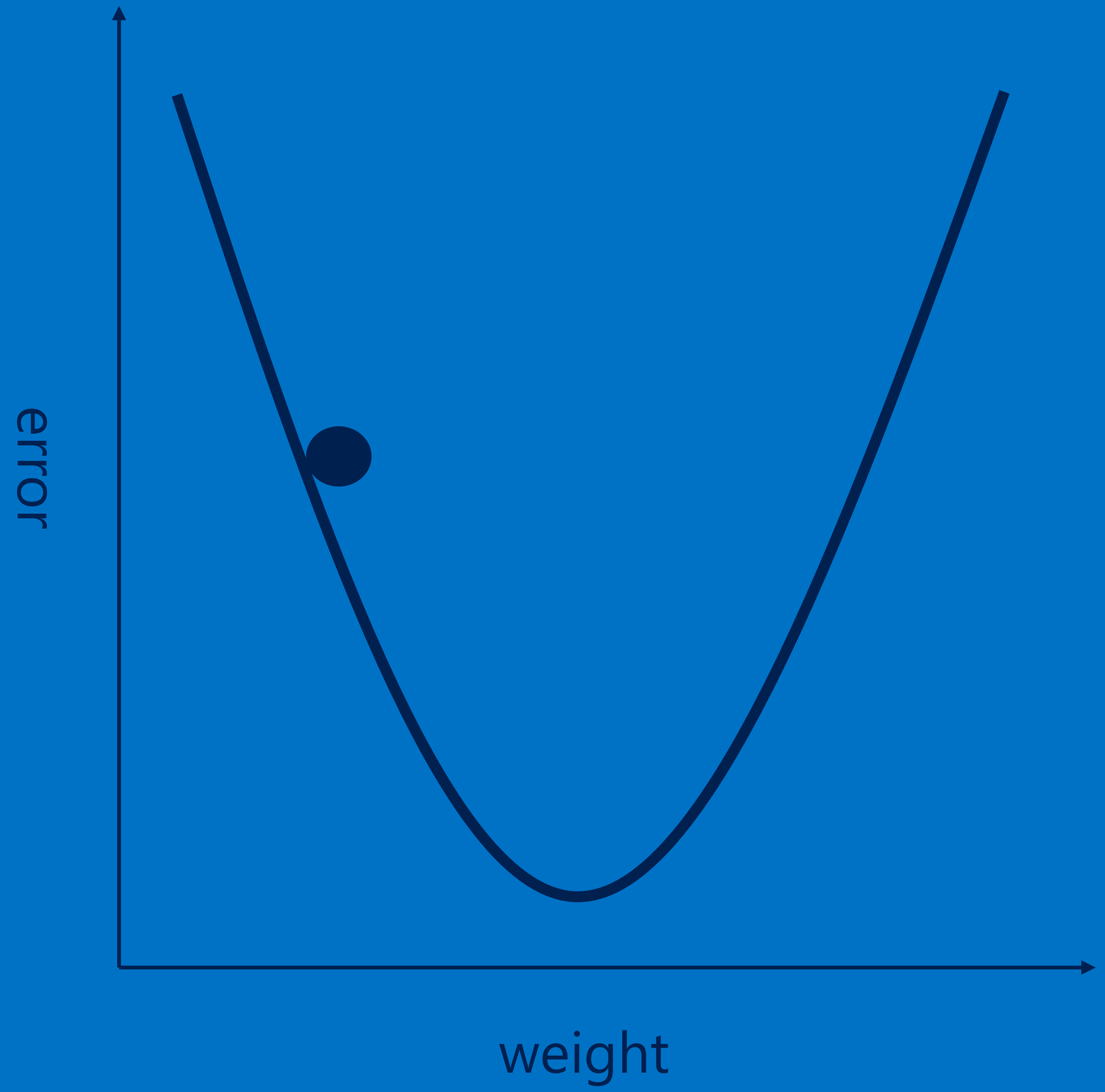
Backprop

	Right answer	Actual answer	Error
X	1	0.92	0.08
O	0	0.51	0.49
		Total	0.57



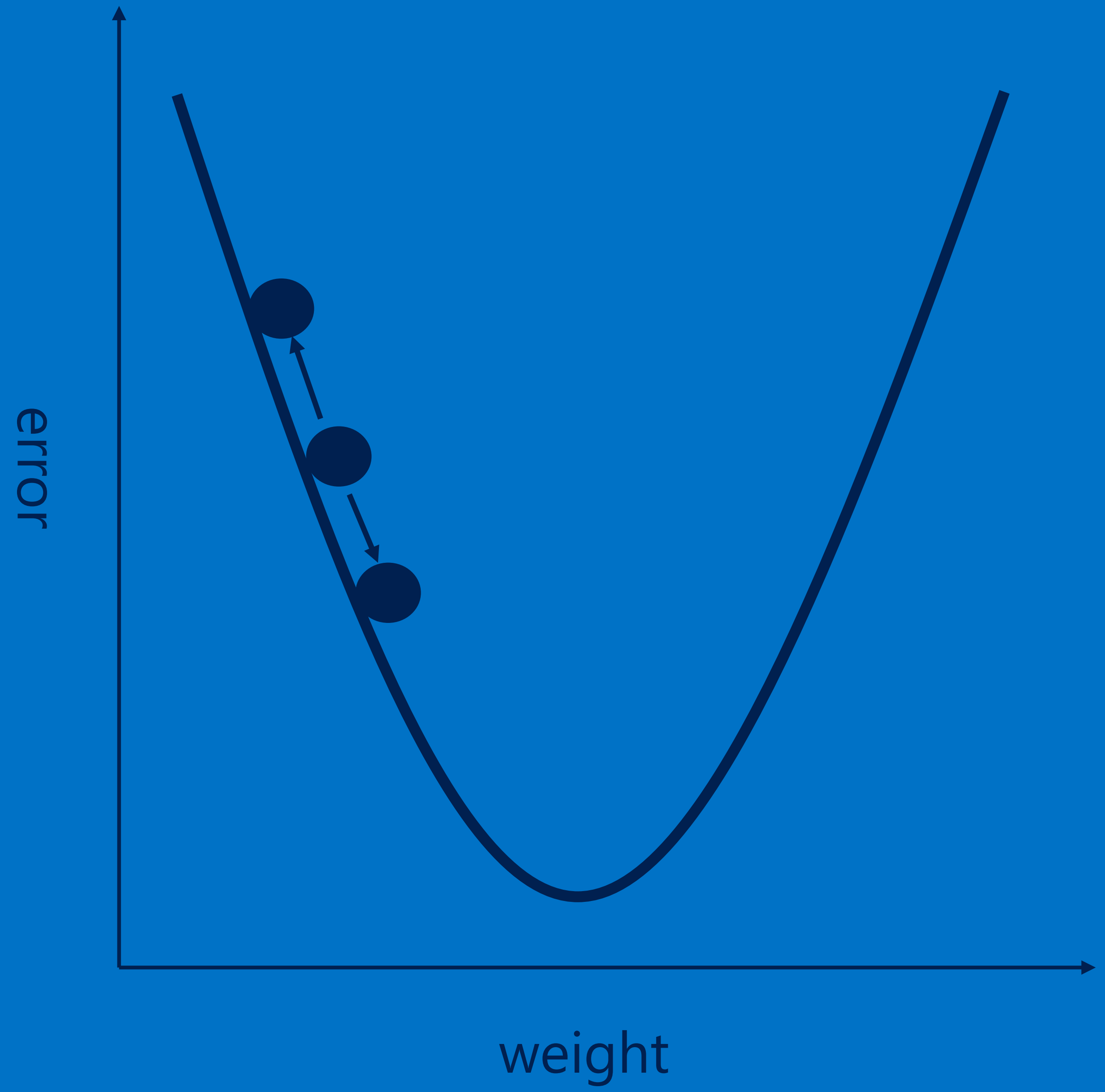
Gradient descent

For each feature pixel and voting weight, adjust it up and down a bit and see how the error changes.



Gradient descent

For each feature pixel and voting weight, adjust it up and down a bit and see how the error changes.



Hyperparameters (knobs)

- Convolution
  - Number of features
  - Size of features
- Pooling
  - Window size
  - Window stride
- Fully Connected
  - Number of neurons

Architecture

How many of each type of layer?

In what order?

In a nutshell

ConvNets are great at finding patterns and using them to classify images.

Some ConvNet/DNN toolkits

- Caffe (Berkeley Vision and Learning Center)
- CNTK (Microsoft)
- Deeplearning4j (Skymind)
- TensorFlow (Google)
- Theano (University of Montreal + broad community)
- Torch (Ronan Collobert)
- Many others