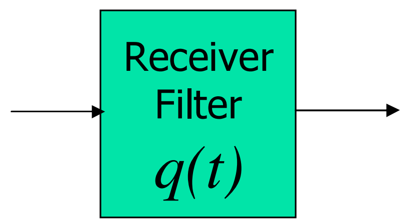
NYQUIST PULSES

Receiver Filter

- Baseband output is a superposition of filtered pulses plus filtered noise

$$aA\sum_{n} x_n p(t - nT_S) + n(t)$$



$$aA\sum_{n} x_n \tilde{p}(t - nT_S) + v(t)$$

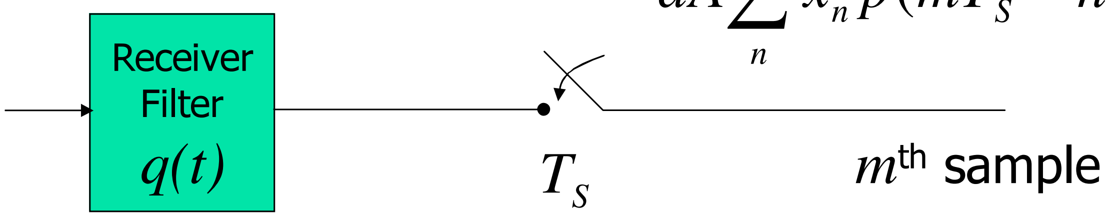
$$\tilde{p}(t) = \frac{1}{2}\int_{-\infty}^{+\infty} p(t-\tau)q(\tau)d\tau$$

Sampling in the Receiver

- The baseband representation of the receiver in additive Gaussian white noise (AWGN) :

$$aA\sum_{n} x_n \tilde{p}(t - nT_S) + v(t)$$

$$aA\sum_{n} x_n \tilde{p}(mT_S - nT_S) + v_m$$

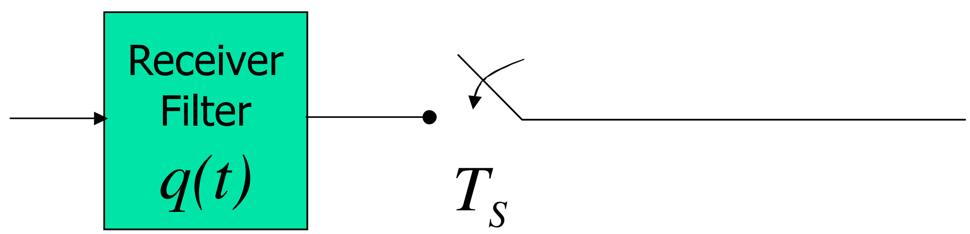


zero-mean
Gaussian RVs

Ideal Situation

- Ideally, the m^{th} sample depends on only the m^{th} symbol and the noise

$$aA\sum_n x_n \tilde{p}(mT_S - nT_S) + v_m = aAx_m + v_m$$



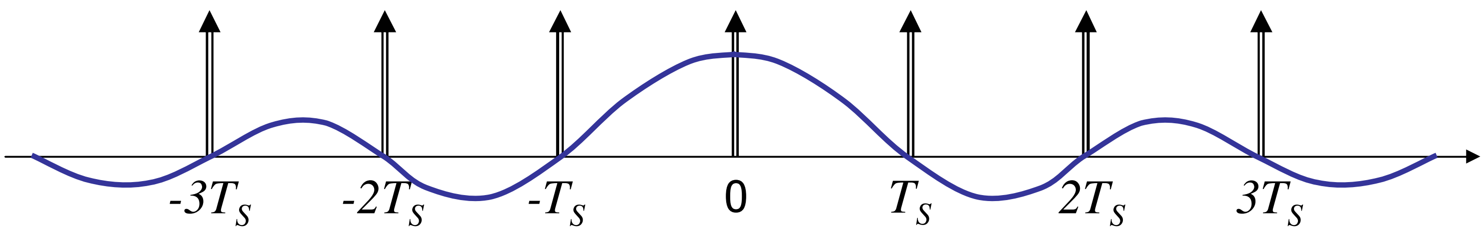
Necessary Condition

- To have this ideal situation, we must have

$$\tilde{p}(mT_S - nT_S) = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$$

- or, alternatively,

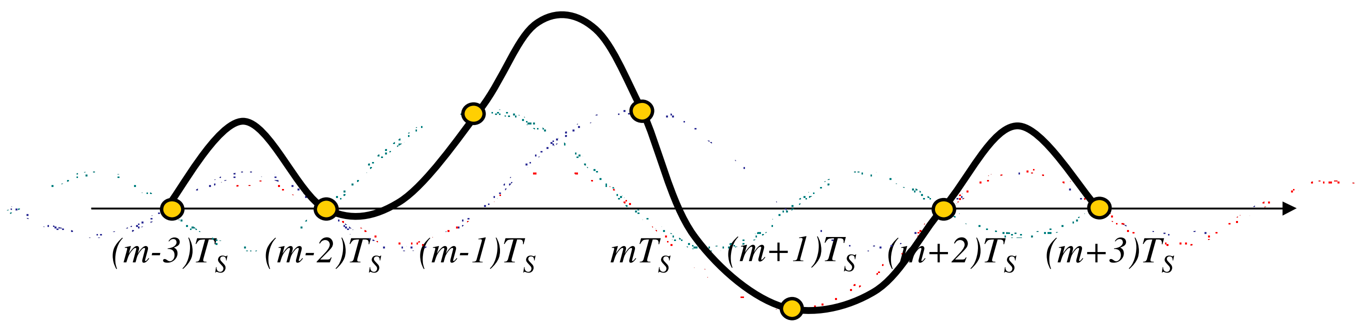
$$\tilde{p}(t) \sum_{n=-\infty}^{+\infty} \delta(t - nT_S) = \delta(t)$$



Received Signal Example

- Suppose the m-1st, m^{th}, and m+1st symbols were +1, +1, -1, respectively

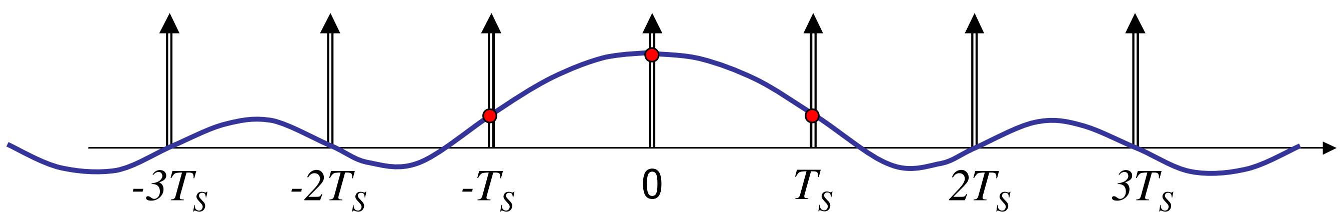
The signal is constrained only at the sample points



Non-Ideal Situation

- Suppose the received pulse did not satisfy the condition, but did this instead:

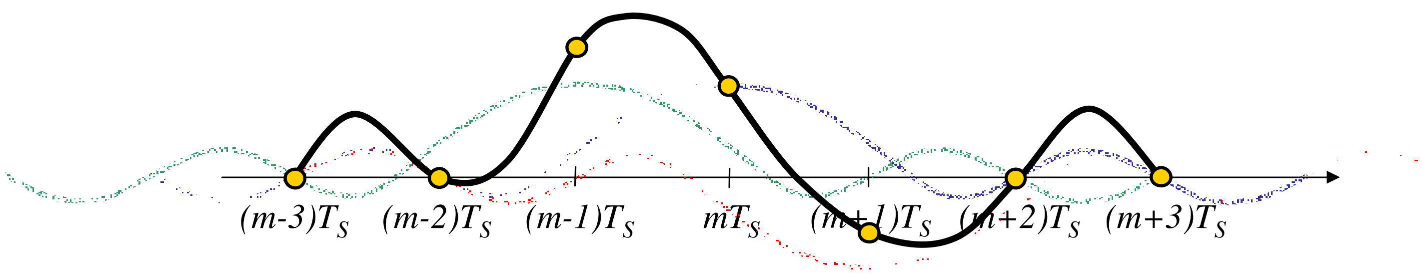
$$\tilde{p}(t)\sum_{n=-\infty}^{+\infty}\delta(t-nT_S)=\delta(t)+0.3\delta(t-T_S)+0.3\delta(t+T_S)$$



Intersymbol Interference (ISI)

- The m-1st, mth, and m+1st received samples become

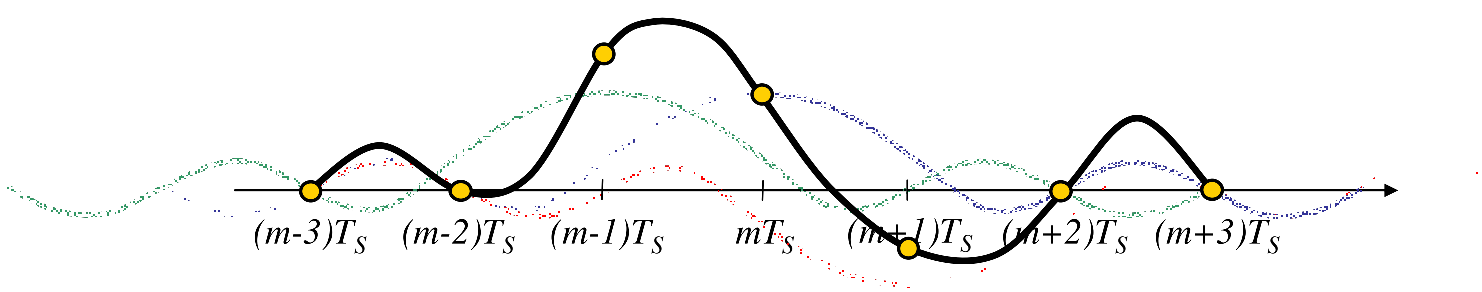
$$1.3 + v_{m-1},\ 1 + v_m,\ -0.7 + v_{m+1}$$



Negative Effects of ISI

- The worst case dominates the probability if bit error

$$1.3 + v_{m-1},\ 1 + v_m,\ 0.7 + v_{m+1}$$



Nyquist Pulses

- Pulses that satisfy the condition for no ISI are called Nyquist Pulses

$$\tilde{p}(t)\sum_{n=-\infty}^{+\infty}\delta(t-nT_S)=\delta(t)$$

Conditions for ISI free transmission

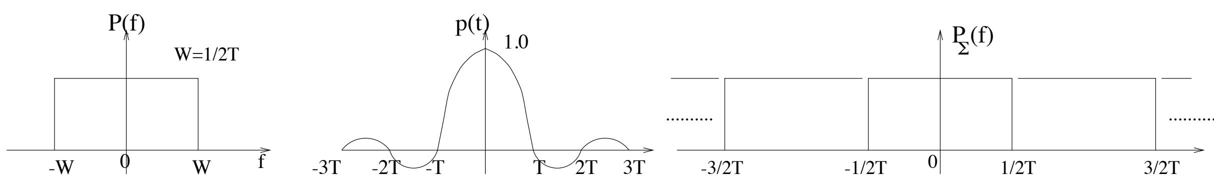
The condition for ISI-free transmission is

$$p_k = \delta_{k0} p_0 = \begin{cases} p_0 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

That is, $p(t)$ must have equally spaced zero crossings, separated by T seconds.
Theorem: The pulse $p(t)$ satisfies $p_k = \delta_{k0} p_0$ iff

$$P_{\sum}(f) \triangleq \frac{1}{T} \sum_{n=-\infty}^{\infty} P(f + n/T) = p_0$$

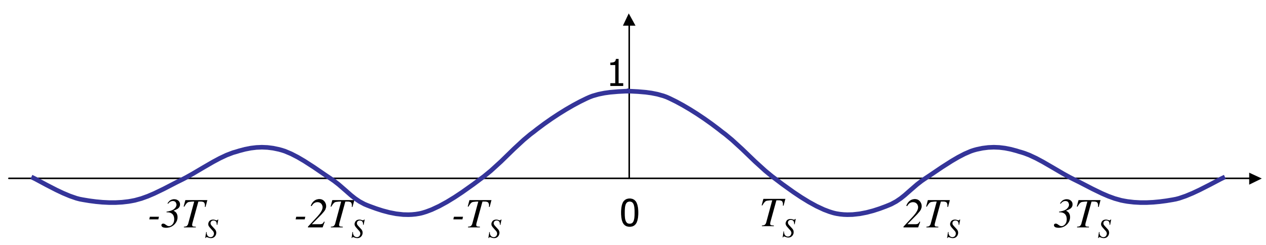
That is the folded spectrum $P_{\sum}(f)$ is flat.



Sinc Pulse

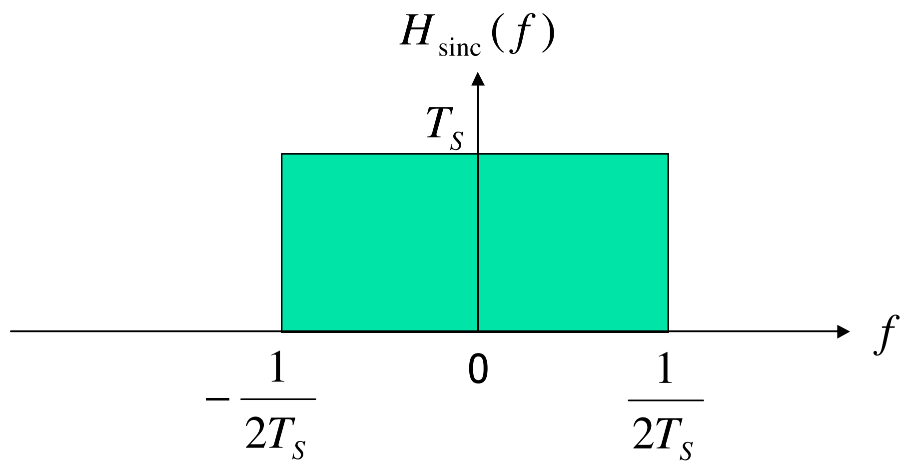
- The sinc pulse is a Nyquist pulse

$$\operatorname{sinc}(t/T_S) = \frac{\sin(\pi t/T_S)}{\pi t/T_S}$$

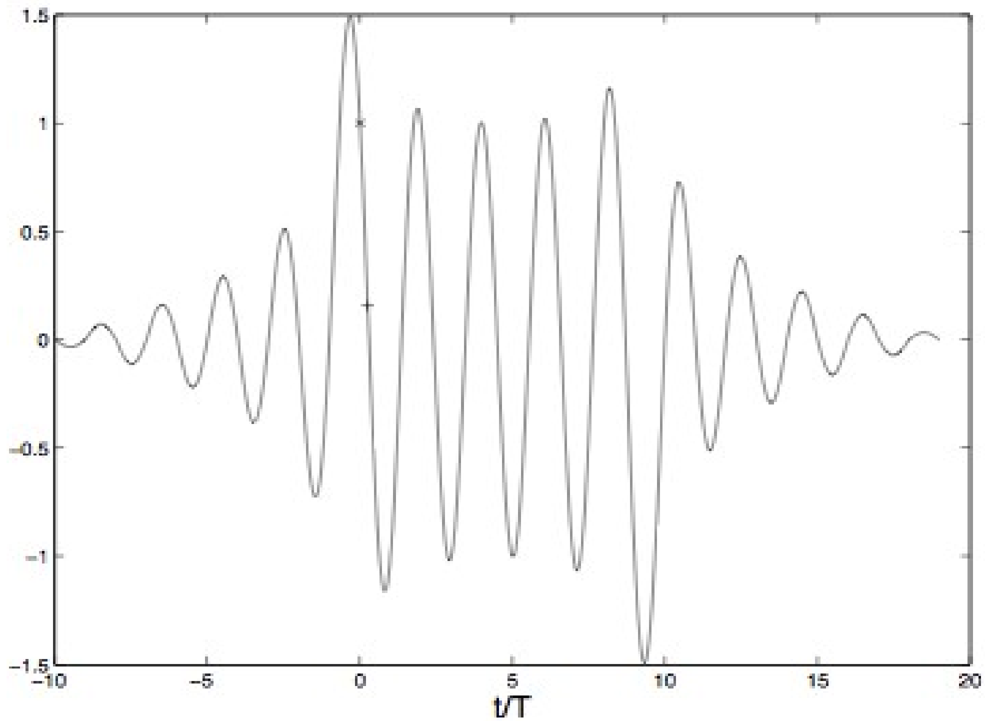


Fourier Transform of Sinc Pulse

- The F.T. of the sinc pulse is the "brick wall" characteristic



Is SINC Pulse Sufficient?



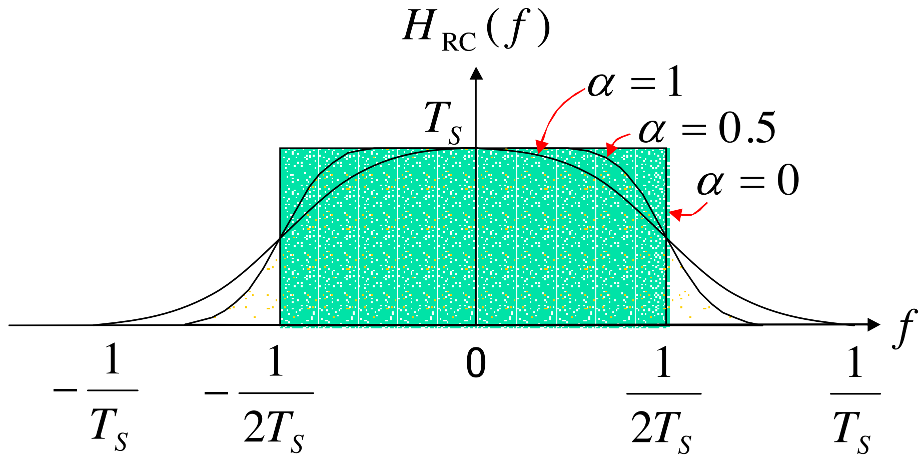
Transmission of 10 BPSK symbols. The point marked 'x' equal +1 (no ISI). However, if sampling time is offset by 0.25T, the sample value, marked '+', becomes much smaller.

Pros and Cons of Sinc

- The F.T. has the narrowest possible bandwidth of all Nyquist pulses
- Noncausal
- Roll-off too gradual

Popular Alternative: Raised Cosine

- Single parameter α



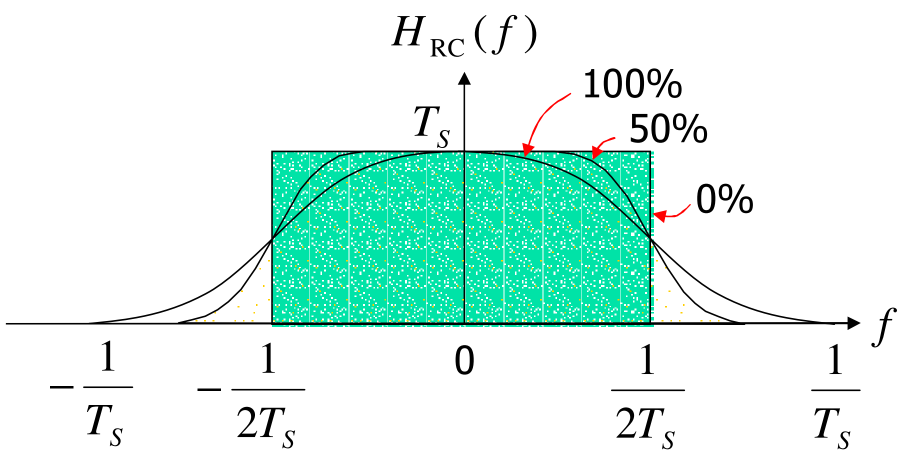
Raised Cosine Formula

$$H_{\mathrm{RC}}(f) = \begin{cases} T_S & 0 \le |f| \le (1-\alpha)/2T_S \\ \dfrac{T_S}{2}\left[1 - \sin\dfrac{\pi T_S}{\alpha}\left(|f| - \dfrac{1}{2T_S}\right)\right] & (1-\alpha)/2T_S \le |f| \le (1+\alpha)/2T_S \\ 0 & (1+\alpha)/2T_S \le |f| \end{cases}$$

$$p_{\mathrm{RC}}(t) = F^{-1}\{H_{RC}(f)\} = \frac{\sin \pi t / T_S}{\pi t / T_S} \frac{\cos \alpha \pi t / T_S}{1 - 4\alpha^2 t^2 / T_S^2}$$

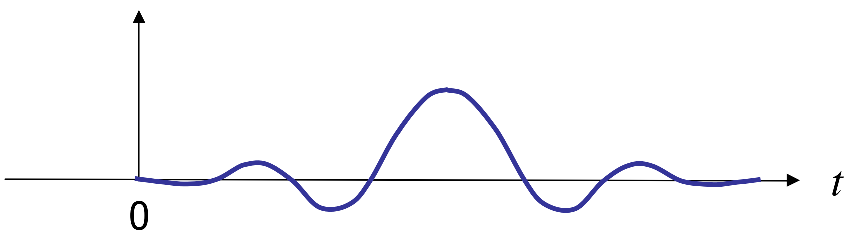
Excess Bandwidth

- The bandwidth of any pulse will be X% greater than that of the sinc Nyquist pulse
- This percentage is the excess bandwidth



Pros of Raised Cosine

- Smooth F.T.
  - Easy to build filters that approximate it
- Falls off as $1/t^3$
- Can approximate a delayed pulse with a causal filter response



Summary

- Intersymbol interference (ISI) can dominate BER
- Nyquist Pulses are pulses that do not cause ISI in an AWGN channel
- Sinc is the Nyquist Pulse with the least bandwidth, but it is impractical
- Raised Cosine is a popular Nyquist Pulse
- Excess Bandwidth indicates how much broader is the bandwidth than that of the sinc Nyquist Pulse