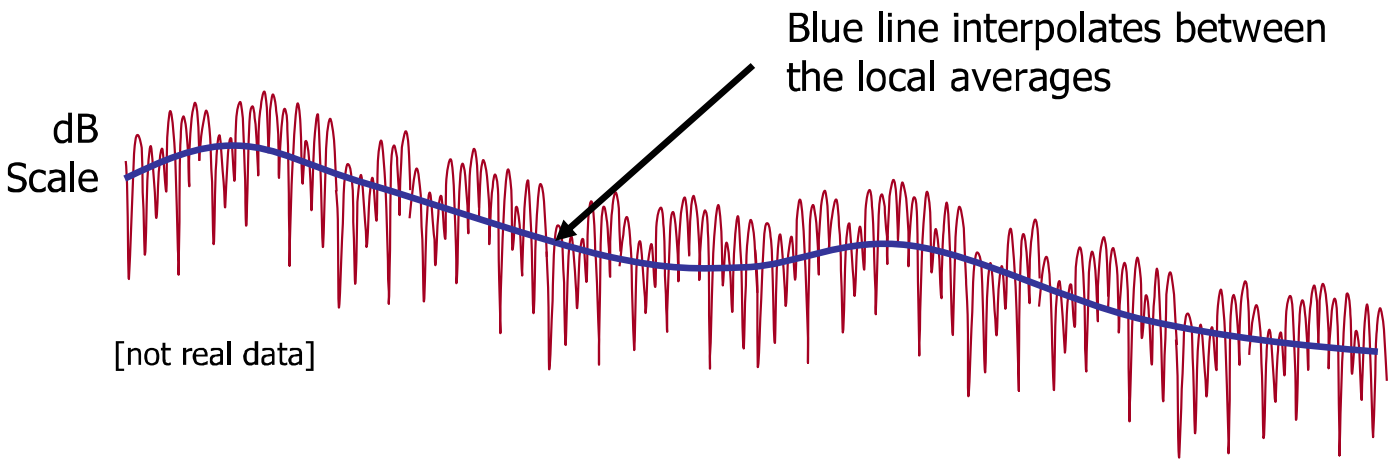
Small Scale Fading Distributions

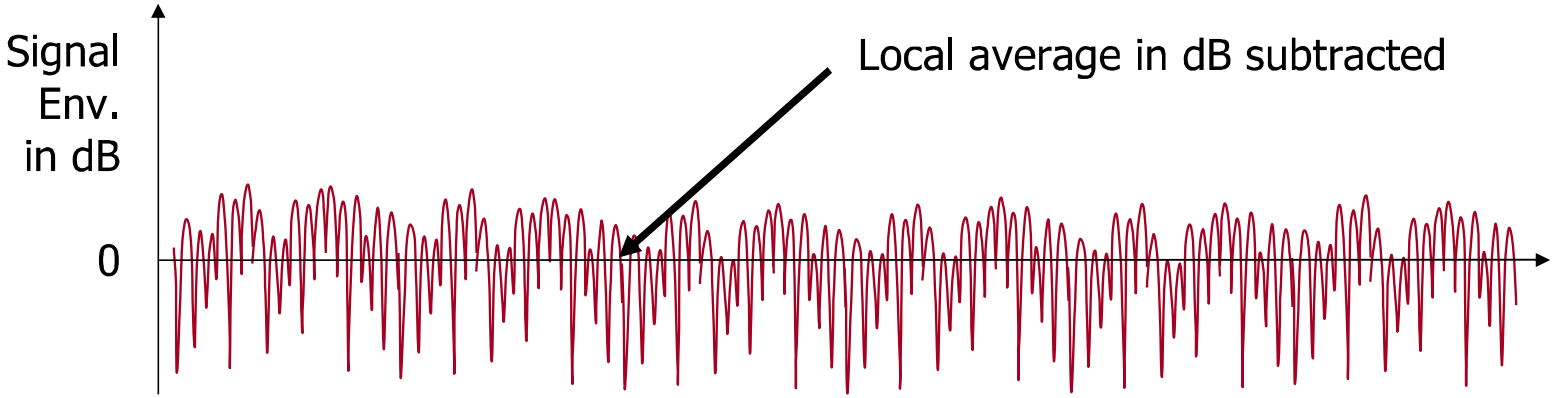
Small-Scale Fading

- Describes the fluctuations in the received signal envelope relative to the local average used for path loss



Types of Statistics

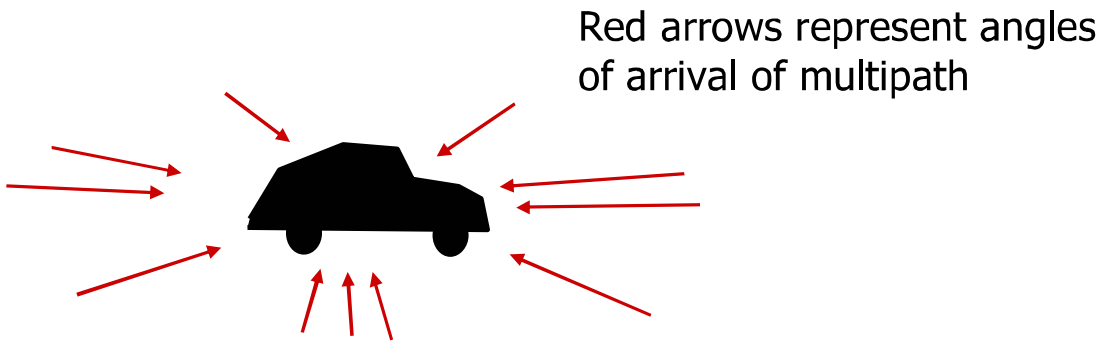
- PDF or CDF describing the values ⇐ In this Module
- Level crossings
- Fade durations



[not real data]

Physical Motivation

- In a typical non-line-of-sight (NLOS) channel, many paths of comparable strength combine at the receiver
- The Central Limit Theorem predicts that both the real and imaginary parts of the resulting waveform have Gaussian statistics



The Envelope

- Suppose $\alpha = X + jY$, where X and Y are independent Gaussian RVs, both zero mean and variance σ^2
- Let $R = |\alpha|$, or

$$R = \sqrt{X^2 + Y^2}$$

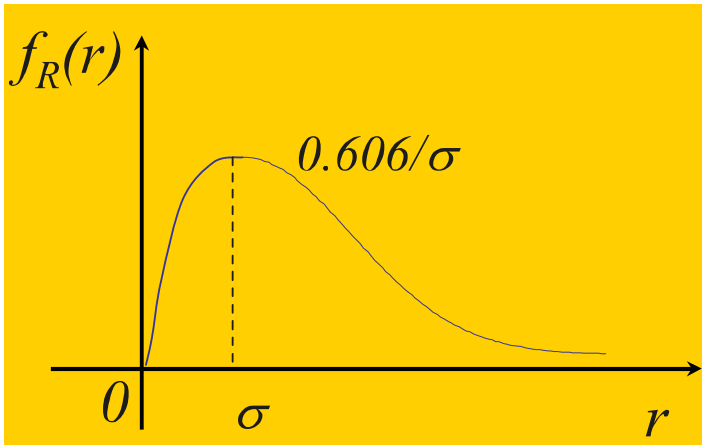
- Then, R has a Rayleigh Distribution

Rayleigh Disribution

- The PDF for the Rayleigh RV is

$$f_R(r) = \begin{cases} \dfrac{r}{\sigma^2}\exp\left\{-\dfrac{r^2}{2\sigma^2}\right\} & r \geq 0 \\ 0 & r < 0 \end{cases}$$

[Rappaport, 1996]



Rayleigh Moments

- The general formula for moments is [Papoulis, 1984]

$$E\{R^n\} = \begin{cases} 1 \cdot 3 \cdots n \sigma^n \sqrt{\pi/2} & n = 2k+1 \\ 2^k k! \sigma^{nk} & n = 2k \end{cases}$$

- In particular,

$$E\{R\} = \sigma\sqrt{\pi/2}$$

$$E\{R^2\} = 2\sigma^2$$

$$Var\{R\} = 2\sigma^2 - \sigma^2\pi/2$$

LOS Multipath Channel

- Now, suppose that in addition to the many non-LOS paths, there is a LOS path with peak amplitude A
- Let $$K = \left|A\right|^2 \Big/ 2\sigma^2$$ be the ratio of deterministic signal power $|A|^2/2$ to the average power of the rest of the signal, σ^2
- This is the "K factor" or "Ricean Factor"
- We don't expect such deep fades if K>1

LOS Envelope

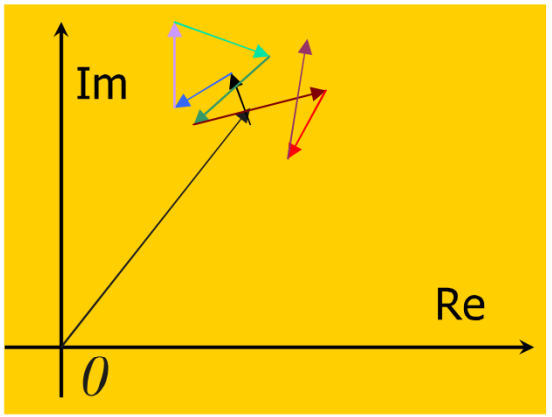
- Let $\alpha = X + jY + A$, where X and Y are as before, and A is non-random
- Again, let $R = |\alpha|$, or

$$R = \sqrt{\left(X + \mathrm{Re}(A)\right)^2 + \left(Y + \mathrm{Im}(A)\right)^2}$$

- Then R has a Ricean Distribution

Rice Distribution

- This PDF describes the magnitude of the sum of one deterministic vector with a lot of iid random vectors

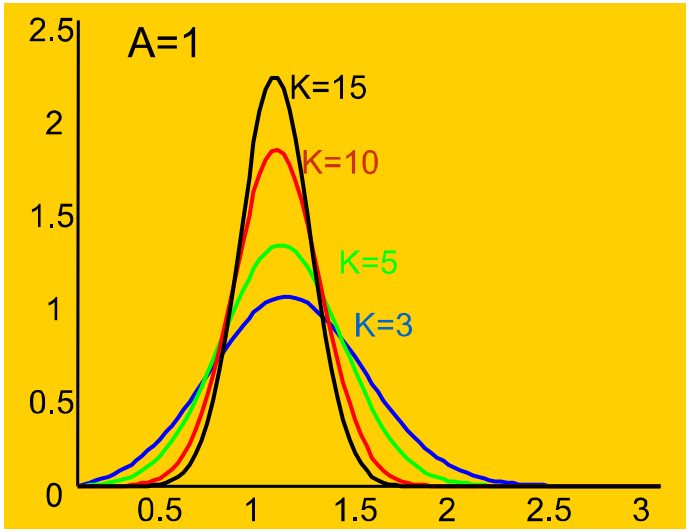


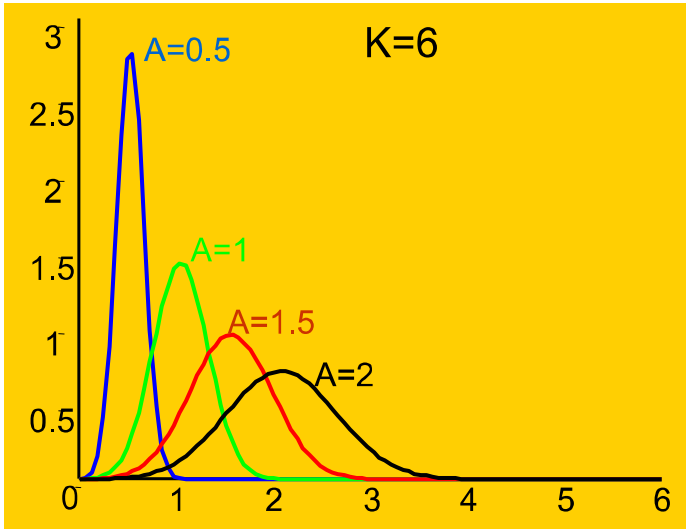
$$f_R(r) = \begin{cases} \dfrac{r}{\sigma^2}\exp\left\{-\dfrac{r^2+A^2}{2\sigma^2}\right\}\cdot \mathrm{I}_0\left(\dfrac{\mathrm{Ar}}{\sigma^2}\right) & A \geq 0, r \geq 0 \\ 0 & r < 0 \end{cases}$$

Modified Bessel function of the first kind and zero order

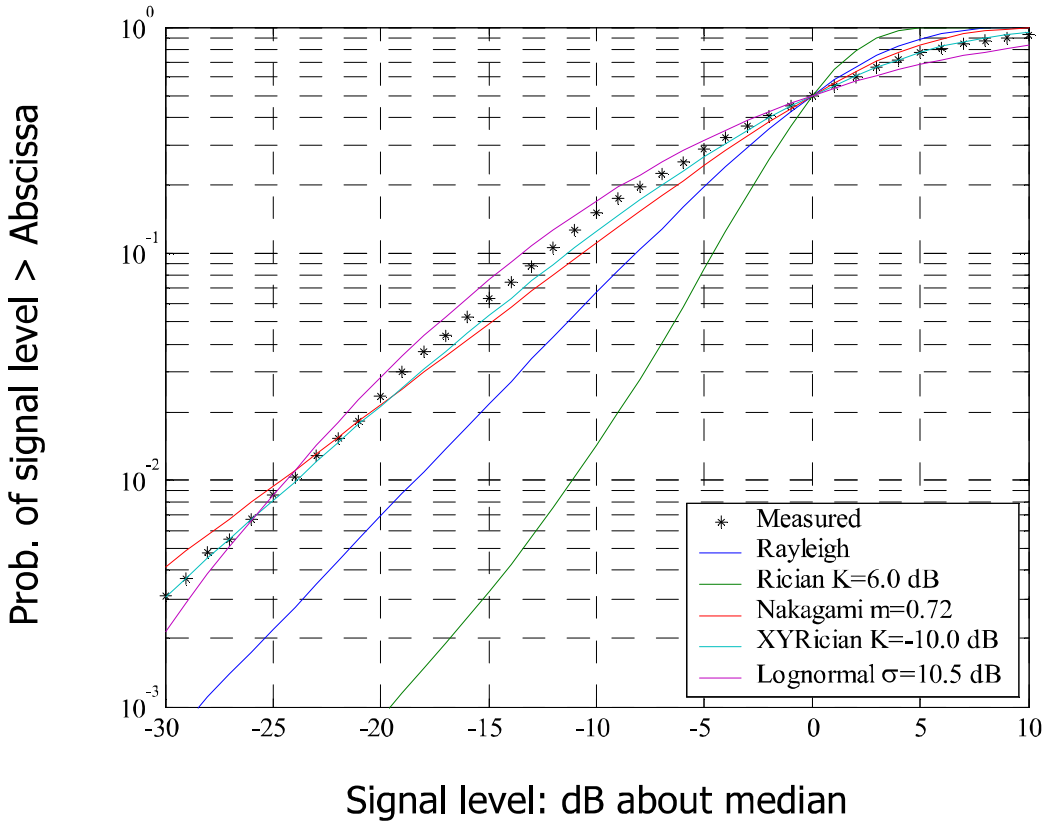
Ricean Examples

- Left: the PDF for fixed A and various K
- Right: the PDF for fixed K and various A





Example CDFs



Back to Rayleigh

- The Rayleigh Distribution is a special case of the Ricean Distribution when K=0

Summary

- Two popular distributions for describing small-scale fading are the Rayleigh and Ricean distributions
- Rayleigh for non-LOS channels
- Ricean for LOS channels
- The K factor is the key parameter