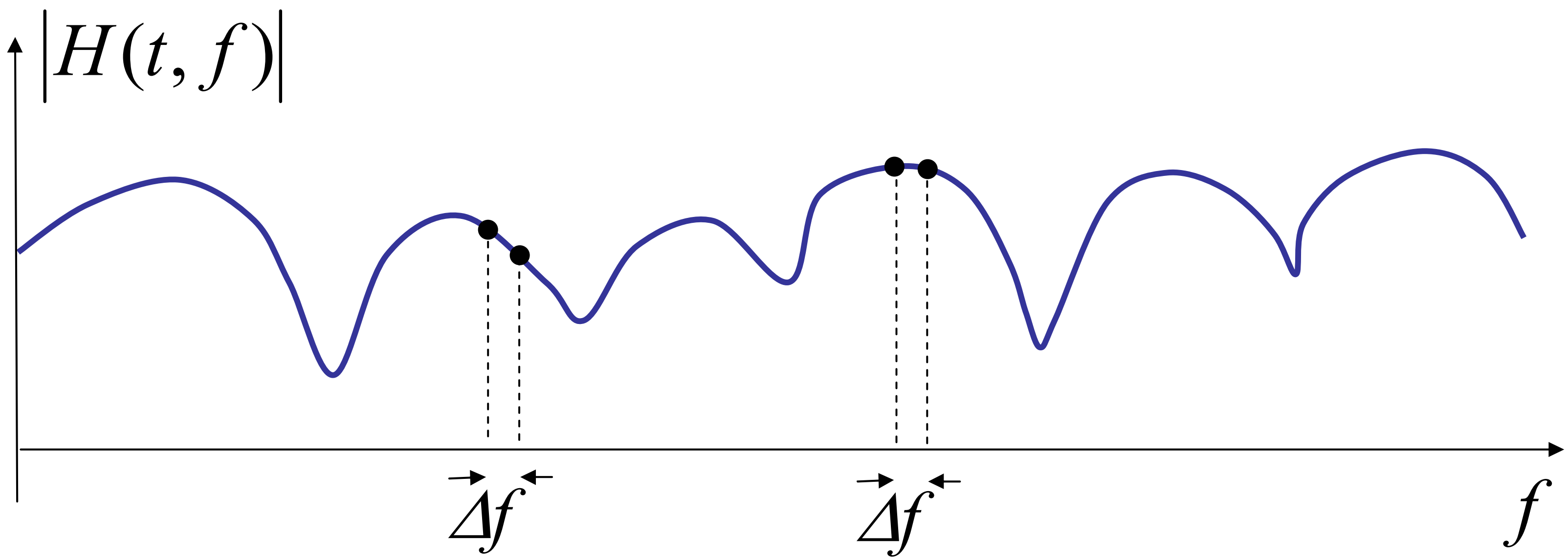
Coherence Bandwidth and Coherence Time

COHERENCE BANDWIDTH

Correlation In Frequency

- We can view the frequency response of a channel as a Random Process as a function of f
- We can ask, "What is the correlation between responses at different frequencies?"



Review: Correlation Coefficient

- Suppose X and Y are two complex RVs
- Their correlation coefficient is defined

$$\rho_{XY} = \frac{E\left\{(X - m_X)(Y - m_Y)^*\right\}}{\sigma_X \sigma_Y}$$

- This is a normalized covariance; it varies between +1 and -1

Correlation Coefficient for Random Processes

- Now suppose $H(f)$ is a RP wrt f
- That means that for any fixed f, $H(f)$ is a RV
- Consider two frequencies f_1 and f_2. The correlation coefficient becomes

$$\rho_{H(f_1)H(f_2)} = \frac{E\left\{\left(H(f_1) - m_{H(f_1)}\right)\left(H(f_2) - m_{H(f_2)}\right)^*\right\}}{\sigma_{H(f_1)}\sigma_{H(f_2)}}$$

Wide-Sense Stationary Uncorrelated Scattering (WSSUS)

- Assumes that path gains at different delays are uncorrelated
- Assumes correlations between frequency responses depends only on the frequency difference Δf

$$\rho_{\Delta f} = \frac{E\left\{\left(H(f)-m_H\right)\left(H(f+\Delta f)-m_H\right)^*\right\}}{\sigma_H^2}$$

Mean is Zero

- Because the phase is uniformly distributed over [0,2π], m_H=0, so the correlation coefficient becomes

$$\rho_{\Delta f} = \frac{E\left\{H(f)H(f+\Delta f)^*\right\}}{E\left\{\left|H(f)\right|^2\right\}}$$

Coherence Bandwidth

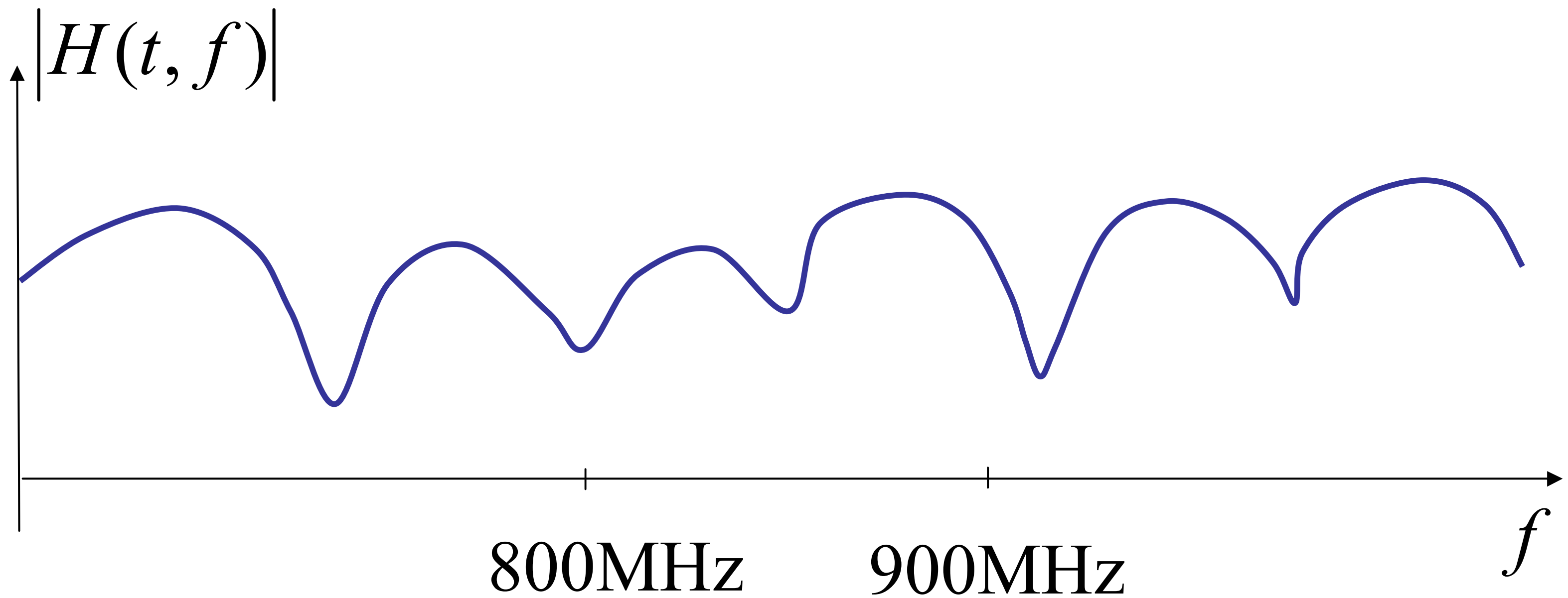
- The X% coherence bandwidth is that value of Δf such that

$$\rho_{\Delta f} = \frac{X}{100}$$

- If the 90% coherence bandwidth is 30KHz, then responses for frequencies separated by 30KHz or less will be nearly equal

Example

- Do you think the 90% coherence bandwidth is > or < 100MHz?



Relation to RMS Delay Spread

- The 90% coherence bandwidth is approximately

$$B_{C,90} = \frac{1}{50\sigma_\tau}$$

- The 50% coherence bandwidth is approximately

$$B_{C,50} = \frac{1}{5\sigma_\tau}$$

Need for Equalization

- If a transmitted signal's bandwidth is greater than the 50% coherence bandwidth, then the channel is frequency selective
- An equalizer (adaptive tapped delay filter) will be needed in the receiver
- Flat-fading channels do not require equalization

Time Dispersion Relationship

Flat Fading

$$B_{C,50} > B_S$$

$$\sigma_\tau < 0.2T_S$$

Frequency Selective Fading

$$B_{C,50} < B_S$$

$$\sigma_\tau > 0.2T_S$$

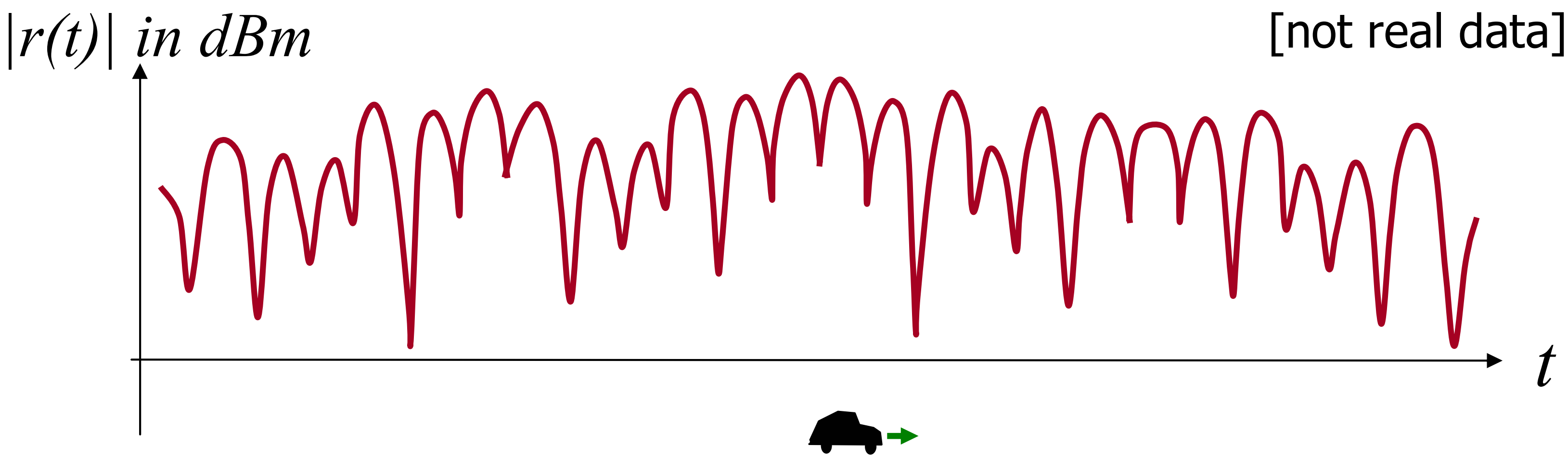
Summary

- Delay spread and coherence bandwidth are inversely related and quantify the effects of multipath delays
- They can be used to estimate the maximum data rate that can be supported without the use of an equalizer

COHERENCE TIME

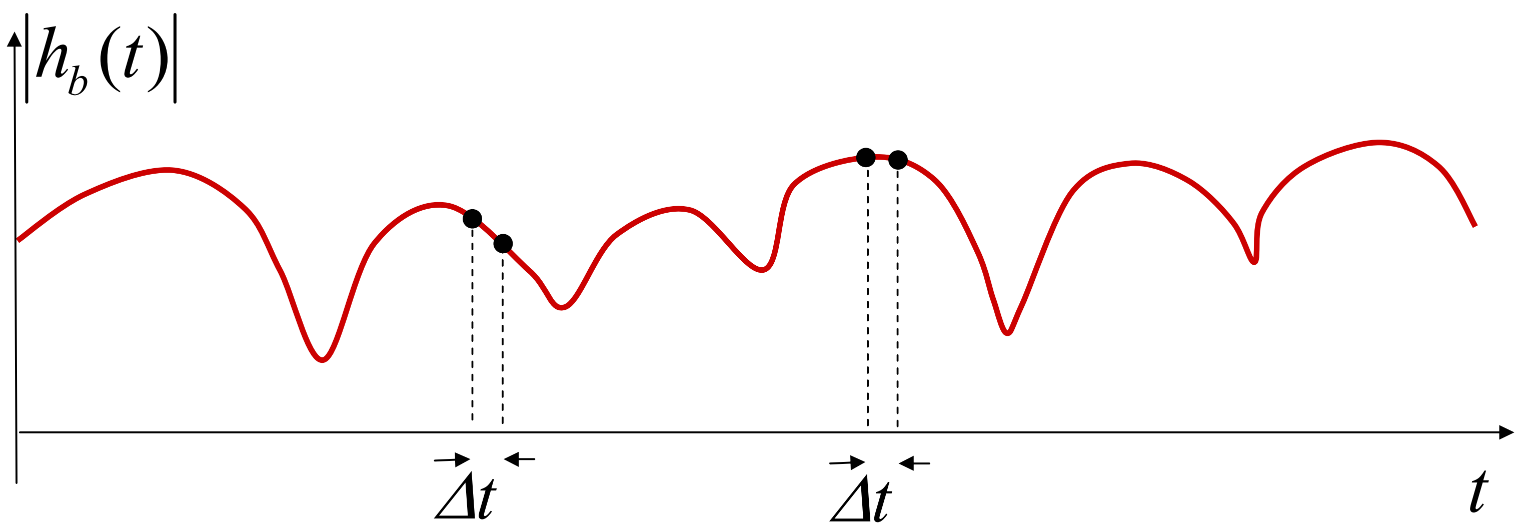
Recall Flat Fading Channels

- The response to a CW (or long pulse) probe signal will fluctuate in time as the vehicle moves



Correlation in Time

- We can view the time fluctuation of a channel as a Random Process as a function of t
- We can ask, "What is the correlation between responses at different times?"



Correlation Coefficient

- Assumes correlations between flat fades at different times depends only on the time difference Δt

$$\rho_{\Delta t} = \frac{E\left\{h_b(t)\, h_b(t+\Delta t)^*\right\}}{E\left\{\left|h_b(t)\right|^2\right\}}$$

Coherence Time

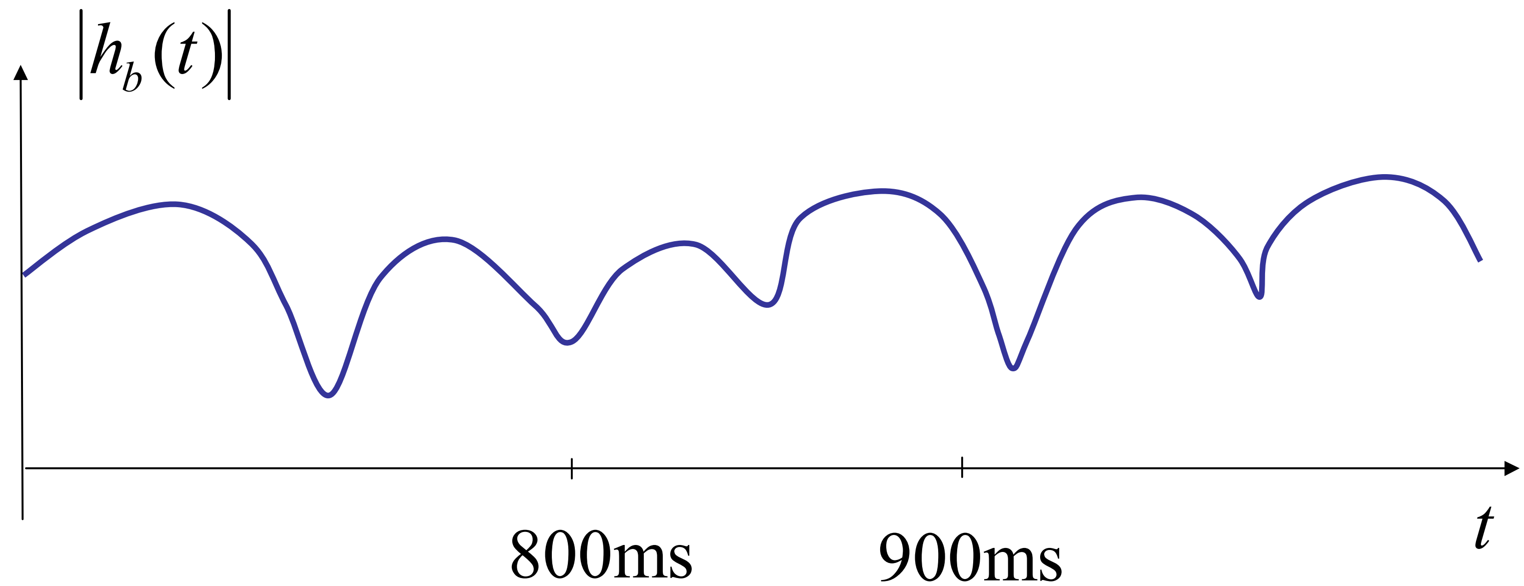
- The X% coherence time is that value of $\Delta\tau$ such that

$$\rho_{\Delta t} = \frac{X}{100}$$

- If the 90% coherence time is 3ms, then the response to a pulse of length 100μs will have a nearly constant envelope

Example

- Do you think the 90% coherence time is > or < 100ms?



Doppler Effect

- Suppose a mobile transmitting at carrier frequency f_o approaches a stationary receiver directly at a speed of v

- The carrier frequency of the received signal will be

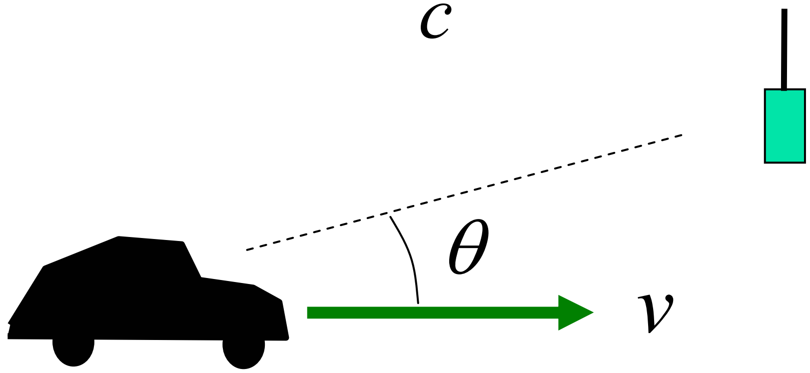
$$f_{rec} = f_o + f_d, \qquad \text{where} \qquad f_d = f_o \frac{v}{c}$$

Maximum Doppler Shift

Non-Direct Approach

- If the velocity of the vehicle is at an angle θ to the receiver, then the Doppler shift is

$$f_d = f_o \frac{v}{c} \cos(\theta)$$



Fast Fading

- A channel is ***fast-fading*** if the symbol period is longer than the coherence time

$$T_S > T_C$$

- Alternatively, the condition can be expressed in terms of the signal bandwidth and the ***Doppler spread***, which is inversely related to the coherence time

$$B_S < B_D = \frac{k}{T_C}$$

Slow Fading

- Slow-fading is the (conservative) opposite of fast-fading
- Safe to assume $h_b(t)$ is constant during at least one symbol period

Doppler Related Relationships

Fast Fading

$$B_S < B_D$$

$$T_S > T_C$$

Slow Fading

$$B_S >> B_D$$

$$T_S << T_C$$

Combination of Channels

- Channels can be
  - Slow and Flat
  - Fast and Flat
  - Slow and Frequency Selective
  - Fast and Frequency Selective