# Filtering
## CS-477 Computer Vision

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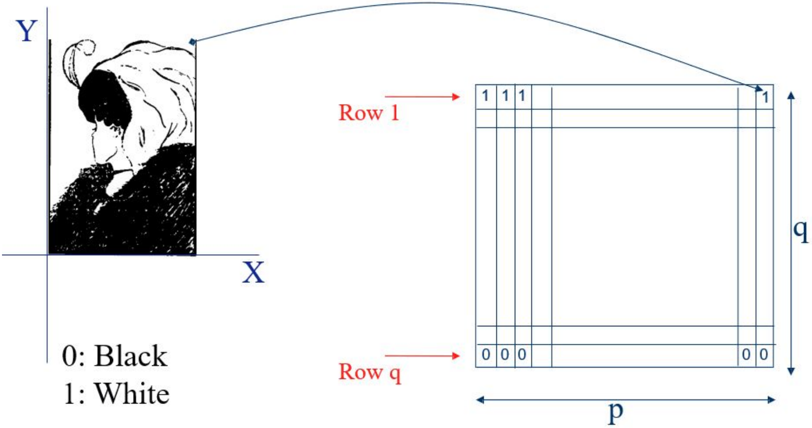
Binary

Gray Scale

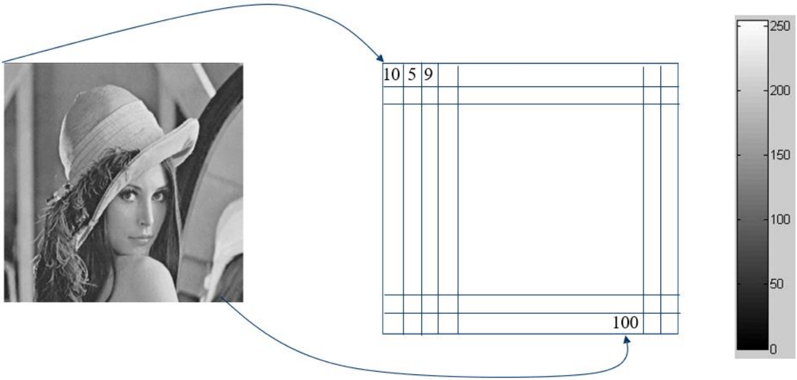
Color

# Binary image

Image is a matrix of numbers and is represented as.

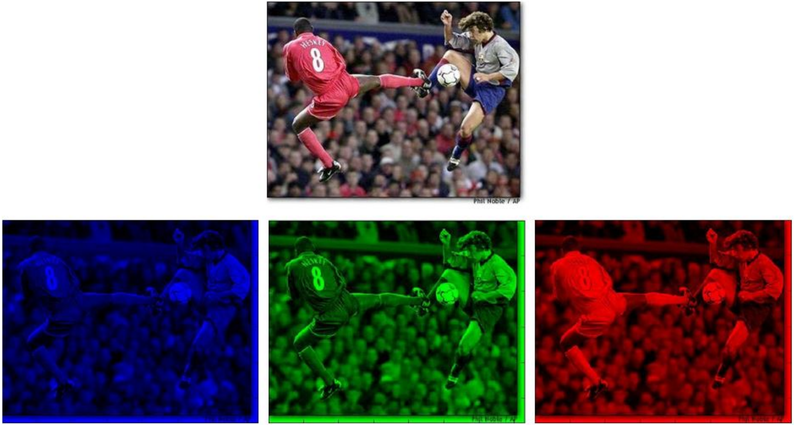


# Gray scale image



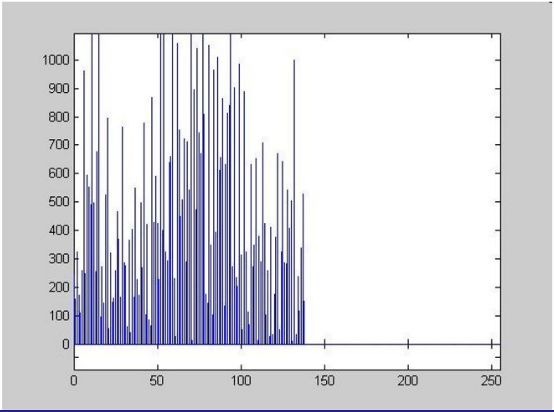
# Gray scale image

# Color image - Red, Green and Blue channels

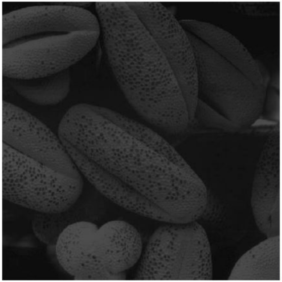


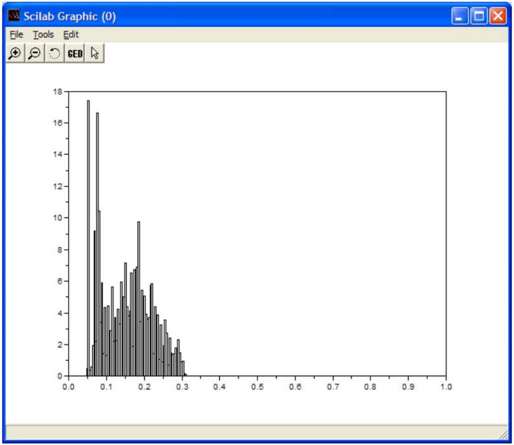
## Image histogram

An important property/feature of an image is Histogram. The histogram of an image shows us the distribution of grey levels in the image i.e., what are the different grey levels in the image and how many times they occur. Massively useful in image processing, especially in segmentation

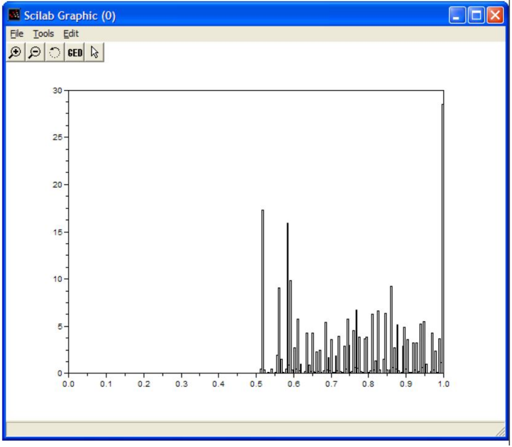


# Image histogram

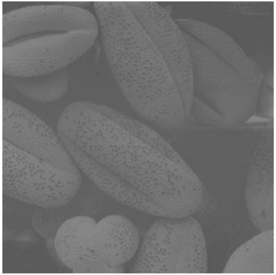


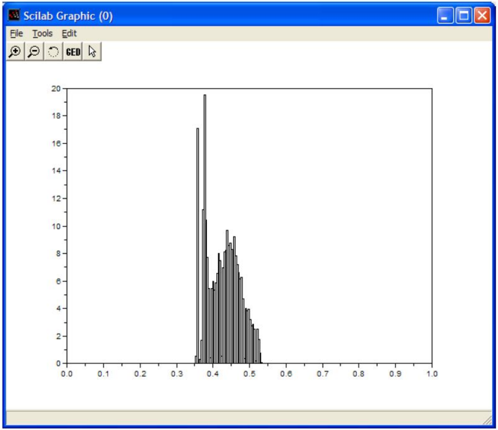


# Image histogram

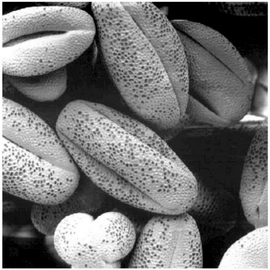


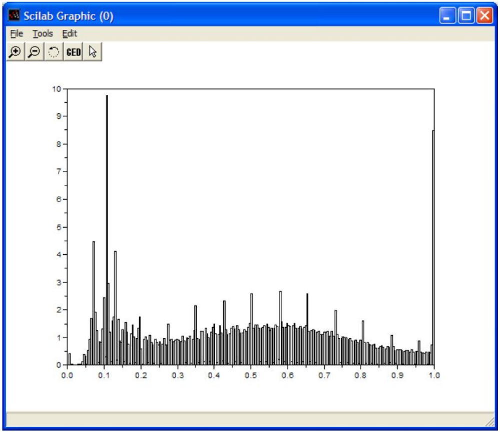
# Image histogram





# Image histogram

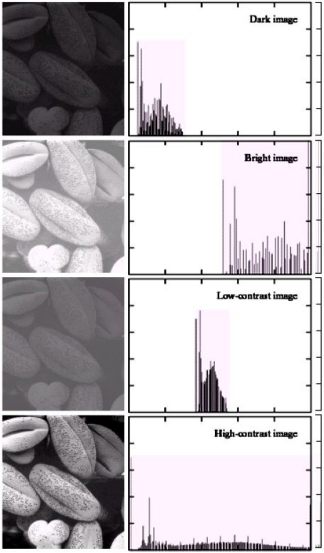




## Image histogram

- A selection of images and their histograms
- Notice the relationships between the images and their histograms
- Note that the high contrast* image has the most evenly spaced histogram

*Contrast describes tones, specifically the relationship between the darkest and brightest parts of an image. If the difference between darkest and lightest portions of an image is vast - for example, if the shadows are very dark and the highlights are very bright - an image is said to have high contrast.



## Image noise

The images may contain noise. The noise can be due to following reasons (but not limited to).

- Light variations
- Camera electronics
- Surface reflectance
- Lens

The goal is to remove/reduce the noise from an image

## Image noise

Let $I(x, y)$ be the true pixel values and $n(x, y)$ be the noise at pixel $(x, y)$. Mathematically,

$$\hat{I}(x, y) = I(x, y) + n(x, y) \tag{1}$$

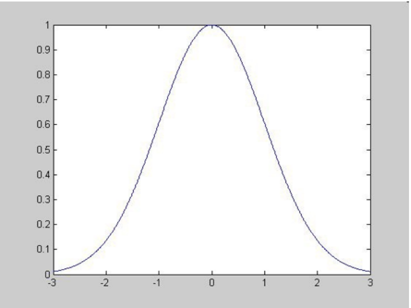
# Gaussian Noise

- It is a random noise generated using a Gaussian distribution, also known as the normal distribution.
- Adding Gaussian noise to images can be used for various purposes, such as simulating real-world noise in images or for image denoising algorithms.
- It is represented as pixel values that follow a Gaussian distribution with a mean $\mu$ and standard deviation $\sigma$. The distribution is symmetric and bell-shaped.

# Gaussian Noise

Mathematically,

$$n(x) = e^{\frac{-x^2}{2\sigma^2}} \tag{2}$$



# Definitions

In computer vision, the important operations to perform on images are Differentiation and averaging.

- Derivative: Rate of change
  - *Speed* is a rate of change of *distance*
  - *Acceleration* is a rate of change of *speed*
- Average (Mean)
  - Dividing the sum of $N$ values by $N$

**Derivative:**

$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x} = f'(x) = f_x$$

**Discrete derivative:**

- As images are discrete, we have to come up with approximation of derivative in the discrete domain.
- The rows and columns are represented as integers (numbers) and not as floating point numbers.
- What is the smallest $\Delta x$ that can be selected?

$$\frac{df}{dx} = \frac{f(x) - f(x-1)}{1} = f'(x)$$

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$

# Discrete derivative - Finite difference

**Backward difference:**

$$\frac{df}{dx} = f(x) - f(x-1) = f'(x)$$

**Forward difference:**

$$\frac{df}{dx} = f(x) - f(x+1) = f'(x)$$

**Central difference:**

$$\frac{df}{dx} = f(x+1) - f(x-1) = f'(x)$$

# Example

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $f(x)$: | 10 | 15 | 10 | 10 | 25 | 20 | 20 | 20 |
| $f'(x)$: | 0 | 5 | -5 | 0 | 15 | -5 | 0 | 0 |
| $f''(x)$: | 0 | 5 | -10 | 5 | 15 | -20 | 5 | 0 |

**Derivative masks:**

- Backward difference: $[-1 \quad 1]$
- Forward difference: $[1 \quad -1]$
- Central difference: $[-1 \quad 0 \quad 1]$

# Derivatives in 2-Dimensions

As images are in 2-dimensions because it consists of rows and columns. The function will now consist of $x$ and $y$. Given function: $f(x, y)$

**Gradient vector:** $\nabla f(x, y) = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \end{bmatrix}$

**Gradient magnitude:** $|\nabla f(x, y)| = \sqrt{f_x^2 + f_y^2}$

**Gradient direction:** $\theta = \tan^{-1} \frac{f_y}{f_x}$

# Derivatives of images

For averaging, extending the idea of central difference. The row above and below (neighboring rows) will be considered as well.
**Derivative masks:**

$$f_x = \frac{1}{3}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix} \qquad f_y = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$$I = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix} \qquad I_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 10 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

# Derivatives of images

$$I = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix} \qquad I_y = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

# Correlation

$$f \otimes h = \sum_k \sum_l f(k,l)h(k,l)$$
$$f = \text{Image}$$
$$h = \text{Kernel}$$

$f$

| | | |
|---|---|---|
| $f_1$ | $f_2$ | $f_3$ |
| $f_4$ | $f_5$ | $f_6$ |
| $f_7$ | $f_8$ | $f_9$ |

$\otimes$

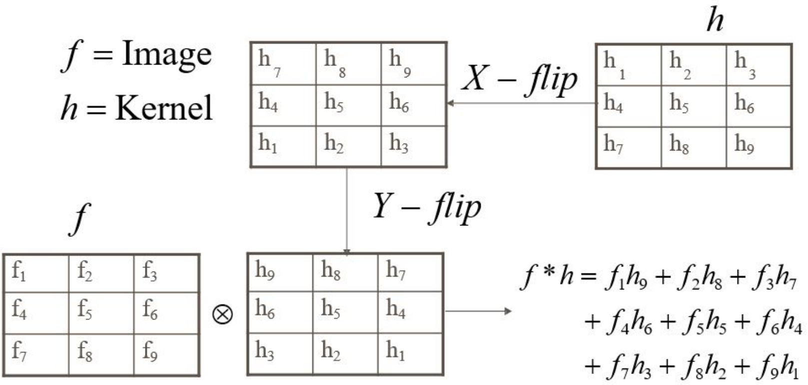
$h$

| | | |
|---|---|---|
| $h_1$ | $h_2$ | $h_3$ |
| $h_4$ | $h_5$ | $h_6$ |
| $h_7$ | $h_8$ | $h_9$ |

$$\begin{aligned} f * h = {} & f_1h_1 + f_2h_2 + f_3h_3 \\ & + f_4h_4 + f_5h_5 + f_6h_6 \\ & + f_7h_7 + f_8h_8 + f_9h_9 \end{aligned}$$

# Convolution

$$f * h = \sum_k \sum_l f(k,l)h(i-k, j-l)$$



1. Types of images
2. Image derivatives and averages
3. Filters

# Image filtering

1. Low pass
   - Block high frequencies
   - Used for smoothing
   - Removing noise
2. High pass
   - Block low frequencies
   - Used for sharpening the image
   - Detecting edges
   - Remove background

# Nature of filtering

1. Linear filtering
   - Mean/Box filter
   - Weighted average filter
   - Gaussian Filter
2. Non-linear filtering
   - Median filter
   - Min filter
   - Max filter

- A linear spatial filter performs a sum-of-products operation between an image $f$ and a filter kernel, $h$.
- The kernel is an array whose size defines the neighbourhood of operation, and whose coefficients determine the nature of the filter.
- Other terms used for kernel are:
  - Mask
  - Template
  - Window

# Averages

**Mean:**

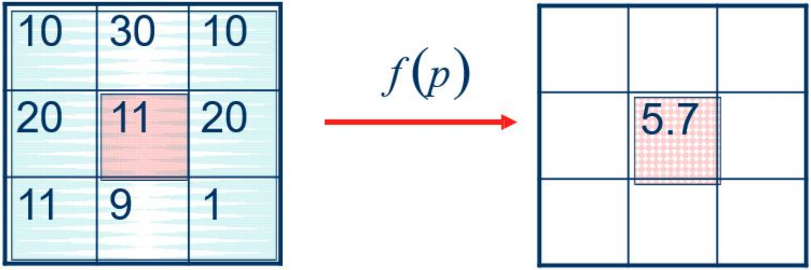
$$I = \frac{I_1 + I_2 + \ldots, I_n}{n} = \frac{\sum_{i=1}^{n} I_i}{n}$$

**Weighted mean:**

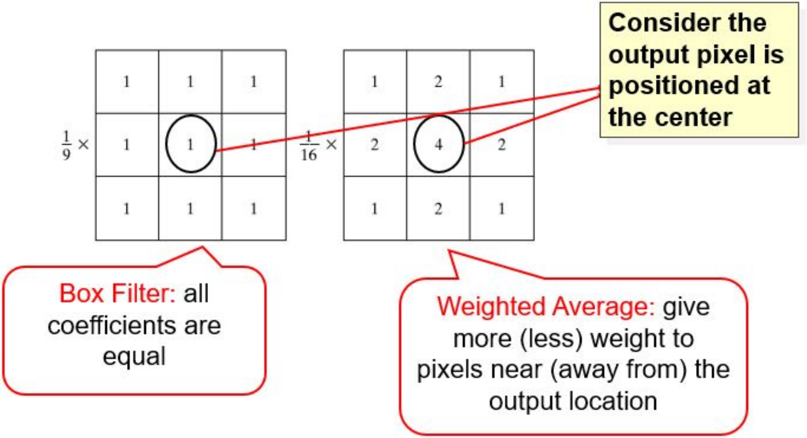
$$I = \frac{w_1 I_1 + w_2 I_2 + \ldots, w_n I_n}{n} = \frac{\sum_{i=1}^{n} w_i I_i}{n}$$

# Filtering

Modify pixels based on some function of the neighborhood



# Linear filter

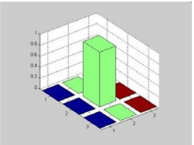


## Linear filter

The output is the linear combination of the neighborhood pixels

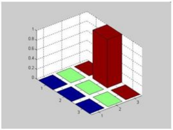
$$\underset{\text{Image}}{\begin{bmatrix} 1 & 3 & 0 \\ 2 & 10 & 2 \\ 4 & 1 & 1 \end{bmatrix}} \otimes \underset{\text{Kernel}}{\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0.1 & -1 \\ 1 & 0 & -1 \end{bmatrix}} = \underset{\text{Filter Output}}{\begin{bmatrix} & & \\ & 5 & \\ & & \end{bmatrix}}$$

# Linear filter



$$* \begin{array}{|c|c|c|} \hline 0 & 0 & 0 \\ \hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} =$$

# Linear filter



$*$

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|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

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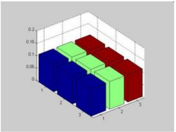
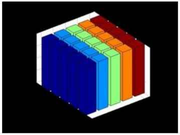
# Linear filter




$$* \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} =$$

# Linear filter



$$* \frac{1}{25} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} =$$

# Linear filter

$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

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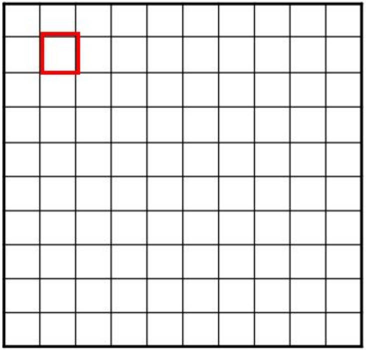
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |



# Linear filter

$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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# Linear filter

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
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| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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# Linear filter

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| 1 | 1 | 1 |
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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
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| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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# Linear filter

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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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# Linear filter

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| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

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# Linear filter

$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

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|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
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# Linear filter

$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

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|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | | | | | |
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# Linear filter

$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
| | | | | | | | | | |
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# Linear filter

$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
| | | | | | | | | | |
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| | | | | | | | | | |
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# Linear filter

$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | ? | | | |
| | | | | | | | | | |
| | | | | 50 | | | | | |
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| | | | | | | | | | |

# Linear filter

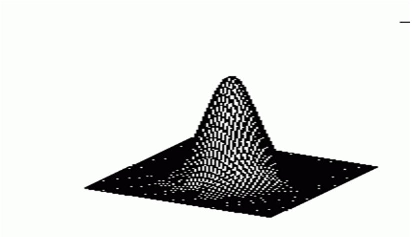
$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | 60 | 90 | 90 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 10 | 20 | 30 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 0 | 0 | 0 | 0 | 0 | |
| | | | | | | | | | |

# Gaussian filter

$$g(x) = e^{\frac{-x^2}{2\sigma^2}}$$



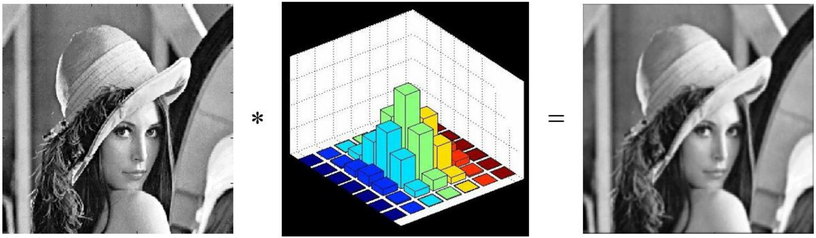
$$g(x,y) = e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

$$g(x) = \begin{bmatrix} .011 & .13 & .6 & 1 & .6 & .13 & .011 \end{bmatrix} \qquad \sigma = 1$$

# Properties of Gaussian

- Most common natural model
- Smooth function, it has infinite number of derivatives
- Fourier Transform of Gaussian is Gaussian
- Convolution of a Gaussian with itself is a Gaussian
- There are cells in eye that perform Gaussian filtering

# Filtering - Gaussian



# Gaussian vs. Averaging

Gaussian smoothing

Smoothing by averaging

# Noise filtering

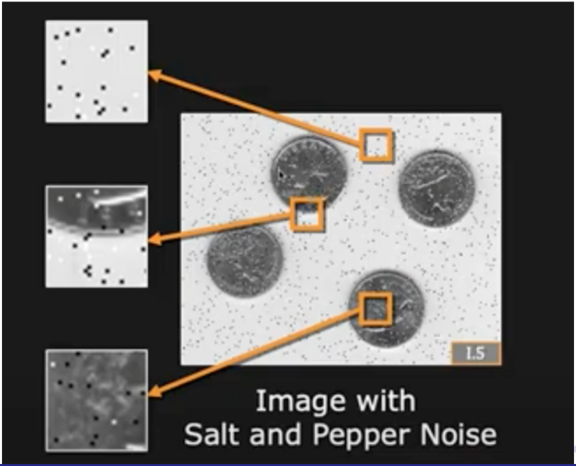
Gaussian noise

After averaging

After Gaussian smoothing

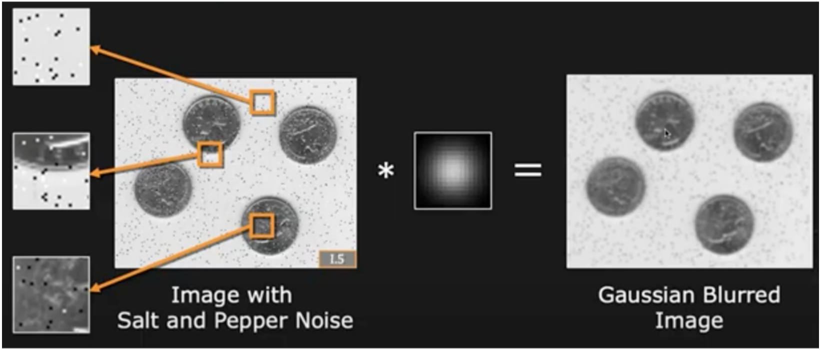
## Non-linear filtering

- A very famous example of salt and pepper noise in an image
- Goal is to remove noise while preserving the other contents in the image



## Non-linear filtering

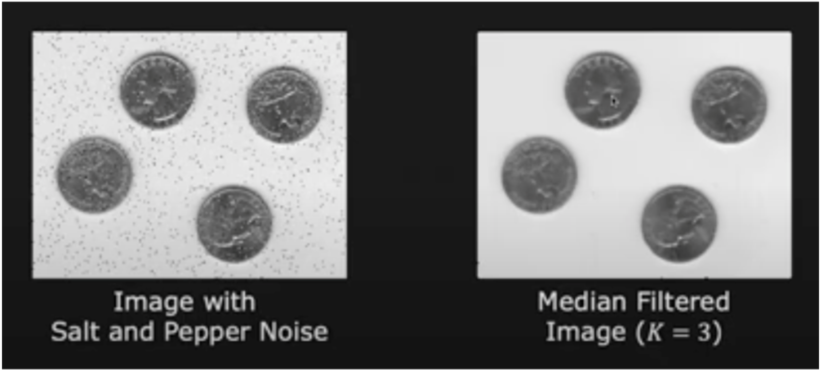
Let us see, what will happen if we apply any linear filter




- It did not remove the outliers
- It blurred the edges

# Non-linear filtering

While applying the non-linear filter (Median Filtering)



# Max and Min filters

- **Min filter** replace the corresponding value with minimum value in the masked region (helpful in removing salt noise)
- **Max filter** replace the corresponding value with maximum value in the masked region (helpful in removing pepper noise)

# Median filtering

- Define the kernel/filter size
- Select the pixel on which we have apply the median filter
- Select the neighboring pixels equal to the size of the kernel
- Arrange all the pixels in ascending order
- Find the median and assign that value to the selected pixel

# Median filtering

**Input**

| | | | | | |
|---|---|---|---|---|---|
| 1 | 4 | 0 | 1 | 3 | 1 |
| 2 | 2 | 4 | 2 | 2 | 3 |
| 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 2 | 1 | 0 | 2 | 2 |
| 2 | 5 | 3 | 1 | 2 | 5 |
| 1 | 1 | 4 | 2 | 3 | 0 |

Sorted:0,0,1,1,1,2,2,4,4

**Output**

| | | | | | |
|---|---|---|---|---|---|
| 1 | 4 | 0 | 1 | 3 | 1 |
| 2 | 1 | 1 | 1 | 1 | 3 |
| 1 | 1 | 1 | 1 | 2 | 0 |
| 1 | 1 | 1 | 1 | 1 | 2 |
| 2 | 2 | 2 | 2 | 2 | 5 |
| 1 | 1 | 4 | 2 | 3 | 0 |

# Median filtering

- It works well for impulse noise (e.g. salt and pepper).
- It requires sorting of the image values.
- It preserves the edges better than an average filter in the case of impulse noise.

# Median filtering vs. averaging

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Example | x[n] | 1 | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | 2 |

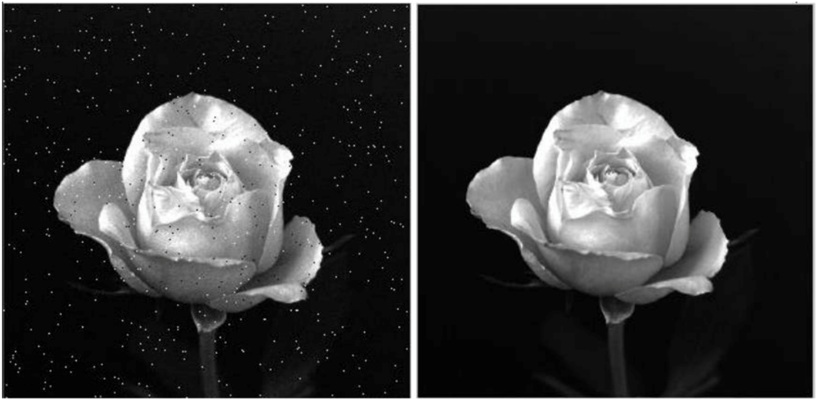
edge

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Impulse noise | x[n] | 1 | 3 | 1 | 1 | 1 | 2 | 3 | 2 | 2 | 3 |
| Median (N=3) | x[n] | - | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 | - |
| Average (N=3) | x[n] | - | 1.7 | 1.7 | 1 | 1.3 | 2 | 2.3 | 2.3 | 2.2 | - |

The edge is smoothed

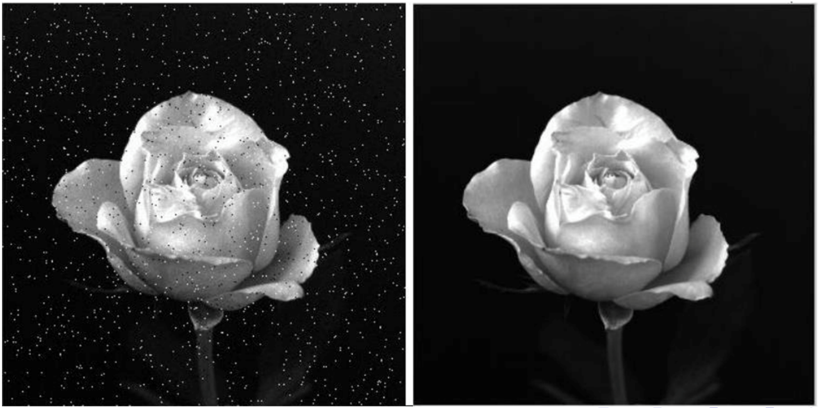
# Median filtering

For noise density = 0.01



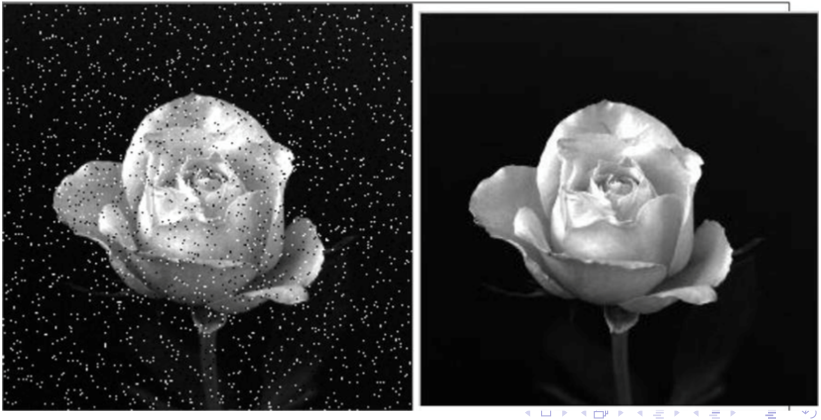
# Median filtering

For noise density = 0.02



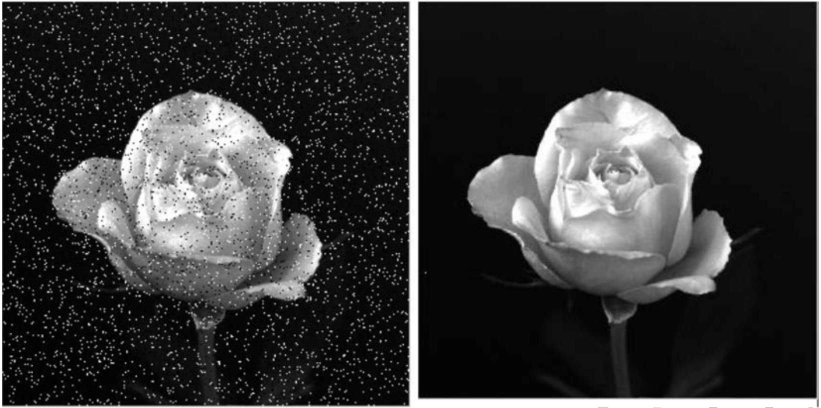
# Median filtering

For noise density = 0.05



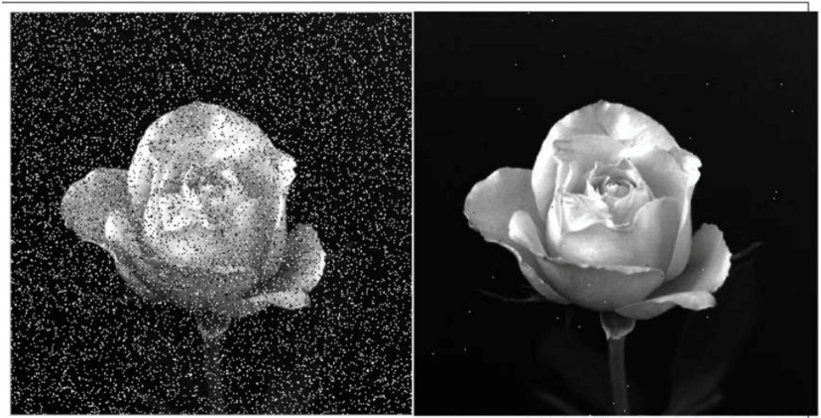
# Median filtering

For noise density = 0.07



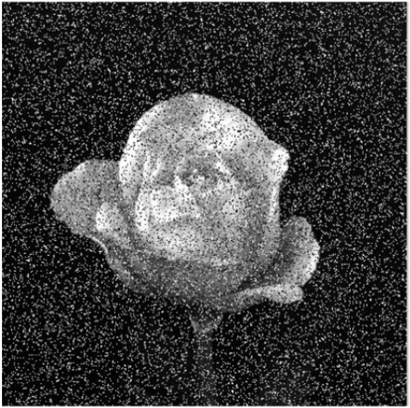
# Median filtering

For noise density = 0.15



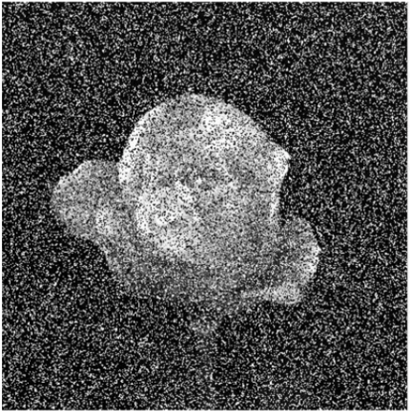
# Median filtering

For noise density = 0.2



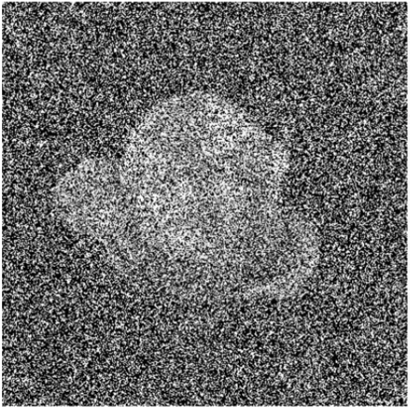
# Median filtering

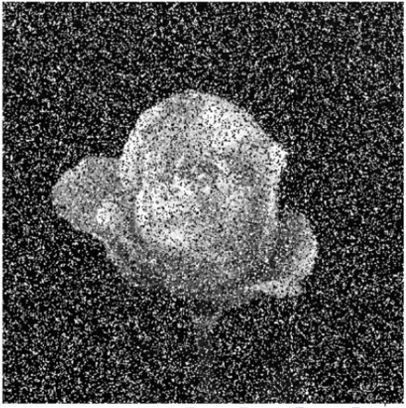
For noise density = 0.4



# Median filtering

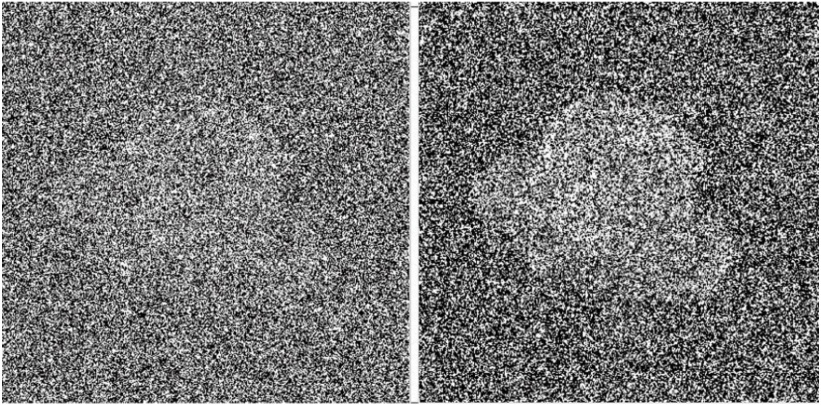
For noise density = 0.7





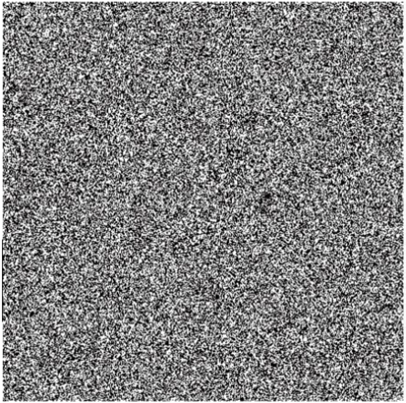
# Median filtering

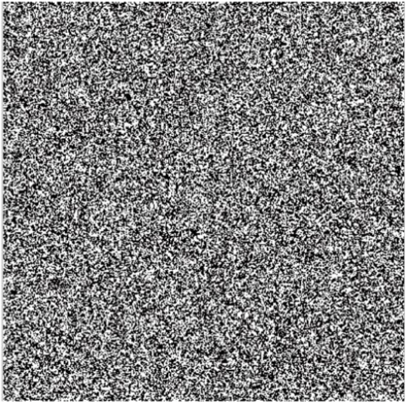
For noise density = 0.9



# Median filtering

For noise density = 1.0





# Median filtering

- Median filtering has limitations
- It is not robust against more realistic noise (especially image acquired under low lighting conditions)