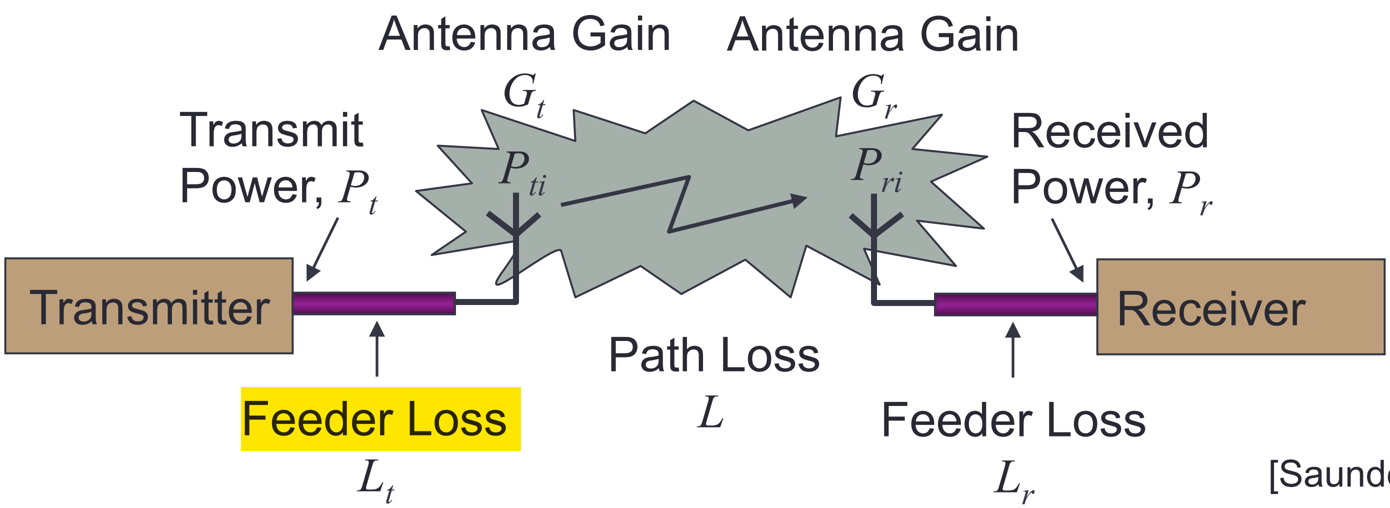
# PATH LOSS

# Definition of Path Loss

- Path loss includes all of the lossy effects associated with distance




[Saunders,`99]

# Motivation

- Need path loss to determine range of operation (using a link budget)
- This module considers two cases,
  - Free space
  - Flat earth

# Received Power

- The power appearing at the receiver input terminals is

$$P_r = \frac{P_t G_t G_r}{L_t L L_r}$$

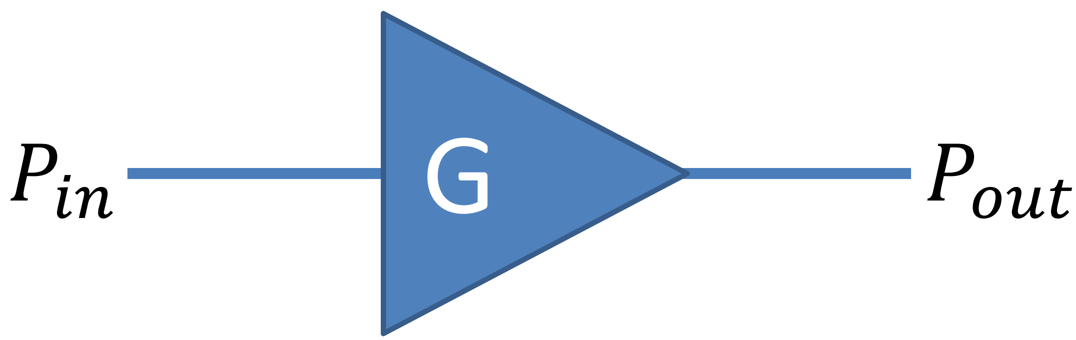
- All gains $G$ and losses $L$ are expressed as power ratios and the powers are in Watts

# dBm and dBW

- Powers may also be expressed in
  - dBm, the number of dB the power exceeds 1 milliwatt
  - dBW, the number of dB the power exceeds 1 Watt.

$$P_r \text{ (in dBm)} = 10 \log_{10} \frac{P_r \text{ (in Watts)}}{10^{-3}\, Watts}$$

# Assume …



# Definitions of Units

- dB [Ratio]
  - $G(dB) = 10log\frac{P_{out}}{P_{in}}$
- dBm [Unit conversion]
  - $P_{out}(dBm) = 10log\frac{P_{out}}{1mW}$
- dBW [Unit conversion]
  - $P_{out}(dBW) = 10log\frac{P_{out}}{1W}$

# dBm and dBW

$$1W = 10log\frac{1W}{1W}(dBW) = 10log1dBW = 10 \times 0dBW$$
$$= 0dBW$$

$$1mW = 10log\frac{1mW}{1mW}(dBm) = 10log1dBm = 10 \times 0dBm$$
$$= 0dBm$$

# dBm ↔ dBW

$$1W = 10log\frac{1W}{1mW}(dBm) = 10log10^3 dBm = 3 \times 10 \times 1dBm$$
$$= 30dBm = 0dBW$$

$$1mW = 10log\frac{1mW}{1W}(dBW) = 10log10^{-3} dBW = -3 \times 10 \times 1dBW$$
$$= -30dBW = 0dBm$$

# Adding/Subtracting dB and dBM

- **Adding** dB values is the same as **multiplying** with regular numbers. So if you add 10dB to a decibel value it is the same as multiplying a regular number by 10.

- **Subtracting** dB values is the same as **dividing** with regular numbers.

- **<u>It is OK to add dB values to an initial dBm value.</u>** This is the same as starting with an input power level and adding amplification or subtracting attenuation from that power level. The final answer will be your output power level in dBm.

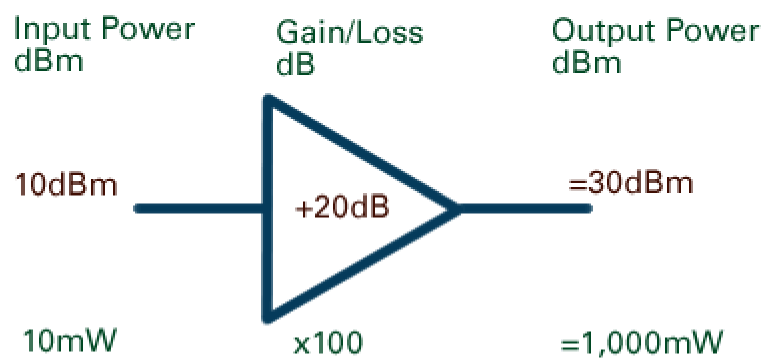
$$\begin{array}{ccccc} P_{out} & = & P_{in} & \times & G \\ (W) & = & (W) & \times & (\,) \end{array}$$

$$\begin{array}{ccccc} P_{out} & = & P_{in} & + & G \\ (dBm) & = & (dBm) & + & (dB) \end{array}$$

Reference: BesserNet, http://www.bessernet.com/articles/dbm/Convert020.html

# Adding/Subtracting dB and dBM

**Example:**




- In the figure above we have an input power level of 10dBm to which we add 20dB of amplification. The result is an output power of 30dBm.
- This is the same as starting with 10mW of input power and multiplying that by a factor of 100, giving an output power of 1,000 mW.

Reference:

BesserNet, http://www.bessernet.com/articles/dbm/Convert020.html

# EIRP

- The effective isotropic radiated power (EIRP) is

$$P_{ti} = \frac{P_t G_t}{L_t}$$

- The effective isotropic *received* power is

$$P_{ri} = \frac{P_r L_r}{G_r} = \frac{EIRP}{L}$$

# Antenna Gains

- Antenna gain may be expressed in dBi or dBd
  - dBi:  maximum radiated power relative to an isotropic antenna
  - dBd: maximum radiated power relative to a half-wave dipole antenna
    - A half-wave dipole has a peak gain of 2.15 dBi

# Path Loss

- The path loss is the ratio of the EIRP to the effective isotropic received power

$$L = \frac{P_{ti}}{P_{ri}}$$

- Path loss is independent of system parameters except for the antenna radiation pattern
  - The pattern determines which parts of the environment are illuminated

# Free-Space Path Loss

- In the far-field of the transmit antenna, the free-space path loss is given by

$$L = \frac{(4\pi)^2 d^2}{\lambda^2}$$

- The far-field is any distance $d$ from the antenna, such that

$$d >> \frac{2D^2}{\lambda}, \quad d >> D, \quad \text{and } d >> \lambda$$

where $D$ is the largest dimension of the antenna.

# Power and Electric Field

- The peak power flux density (W/m$^2$) in free space:

$$P_d = \frac{EIRP}{4\pi d^2} = \frac{P_t G_t}{L_t 4\pi d^2} = \frac{|E|^2}{\eta}$$

$$= \frac{|E|^2}{120\pi\Omega} = \frac{|E|^2}{377\Omega}$$

where $|E|$ = envelope of the electric field in V/m

- This holds in the neighborhood (but far field) of transmitters on towers

# Effective Aperture

- Antenna gain may be expressed in terms of effective aperture, $A_e$

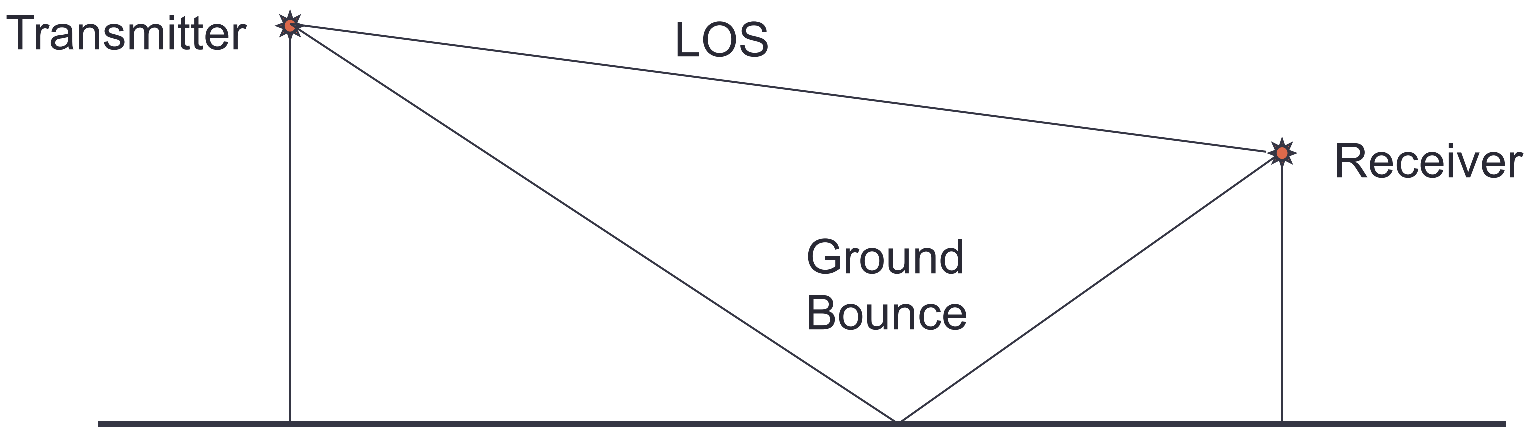
$$G = \frac{4\pi A_e}{\lambda^2}$$

- For aperture antennas, such as dish antennas, $A_e = A\eta$, where $\eta$ is the antenna efficiency and $A$ is the area of the aperture
- The aperture intercepts the power flux density

$$P_{ri} = P_d A_e$$

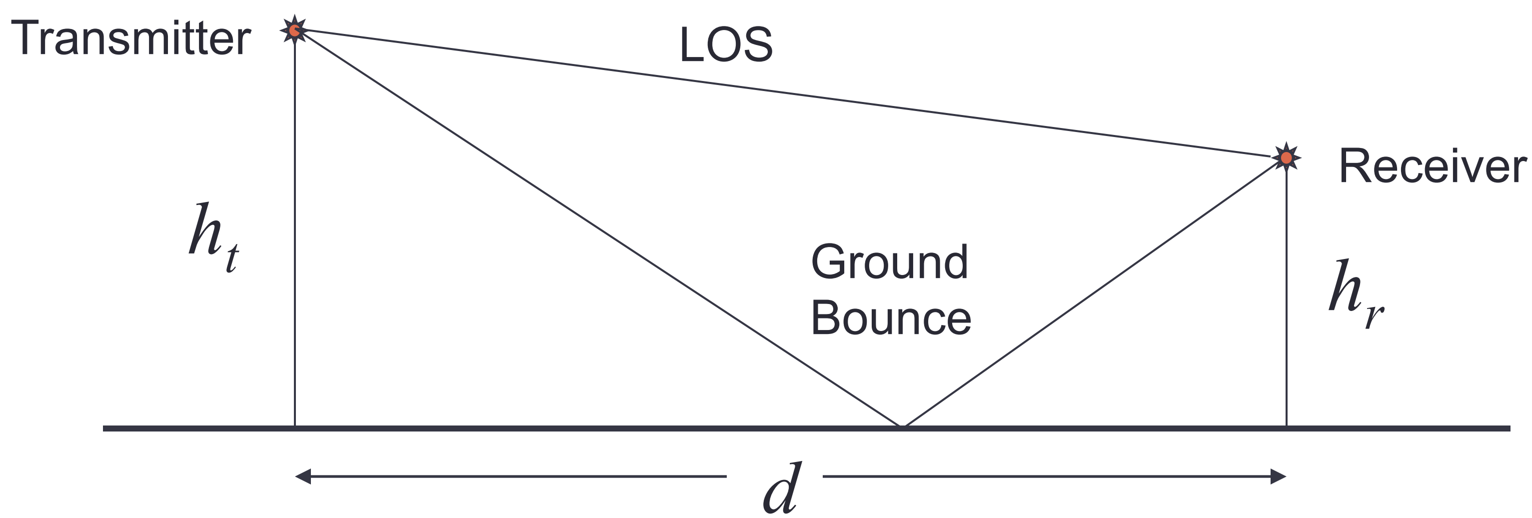
# Flat Earth (2-Ray) Model

- If there is a line-of-sight (LOS) path, then the second strongest path is the ground bounce



# Typical Relative Dimensions

- $d >> h_t$, $d >> h_r$ for a typical mobile communications geometry



# Field Near Transmitter

- Let the field at a distance $d_o$ in the neighborhood of, but also in the far field of, the transmit antenna be $E(d_o,t)$ , and its envelope be $E_o$
- Assuming the transmitter is high enough,

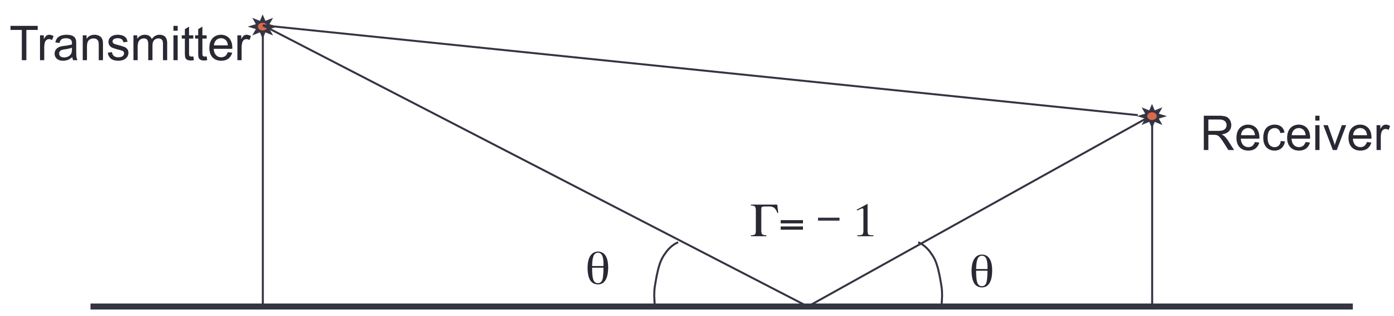
$$\frac{P_t G_t}{L_t 4\pi d_o^{\ 2}} = \frac{E_o^{\ 2}}{120\pi}$$

- The field at some other distance $d>d_o$ is

$$E(d,t) = \frac{E_o d_o}{d} \cos\left( \omega_c \left[ t - \frac{d}{c} \right] \right)$$

# Low Grazing Angle

- At such a low (grazing) angle of incidence ($\theta$=a few degrees), the reflection coefficient is -1 for horizontal polarization

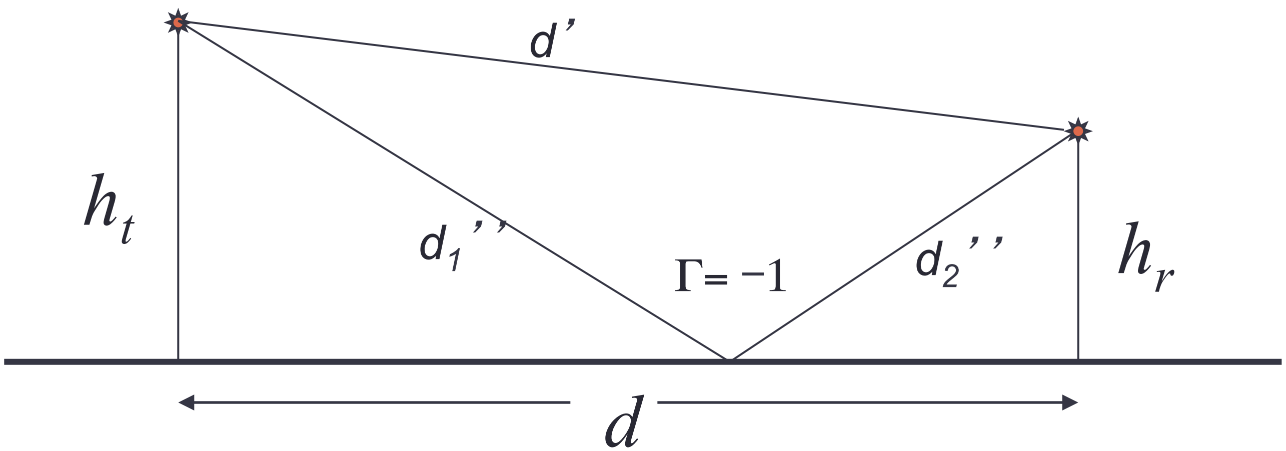


# Field at Receiver

- The direct and bounce paths add coherently

$$E_{TOT}(d,t) = E(d',t) - E(d'',t)$$

$$d'' = d_1'' + d_2''$$



# Long Baseline Effects

- Since $d$ is so large,

$$\frac{1}{d'} \approx \frac{1}{d''} \approx \frac{1}{d}$$

$$E(d,t) = \frac{E_o d_o}{d'} \mathrm{Re}\left\{ e^{j\omega_c\left[t-\frac{d'}{c}\right]} \right\} - \frac{E_o d_o}{d''} \mathrm{Re}\left\{ e^{j\omega_c\left[t-\frac{d''}{c}\right]} \right\}$$

$$\approx \frac{E_o d_o}{d} \mathrm{Re}\left\{ e^{j\omega_c\left[t-\frac{d'}{c}\right]} - e^{j\omega_c\left[t-\frac{d''}{c}\right]} \right\}$$

$$= \frac{E_o d_o}{d} \mathrm{Re}\left\{ e^{j\omega_c\left[t-\frac{d''}{c}\right]} \left( e^{j\omega_c\left[\frac{d''-d'}{c}\right]} - 1 \right) \right\}$$

# A Trick

- Pull an exponential with half the phase out to make a sine

$$\frac{E_o d_o}{d}\mathrm{Re}\left\{e^{j\omega_c\left[t-\frac{d''}{c}\right]}e^{j\omega_c\left[\frac{d''-d'}{2c}\right]}2j\left(\frac{e^{j\omega_c\left[\frac{d''-d'}{2c}\right]}-e^{-j\omega_c\left[\frac{d''-d'}{2c}\right]}}{2j}\right)\right\}$$

$$=\frac{2E_o d_o}{d}\mathrm{Re}\left\{e^{j\omega_c\left[t-\frac{d''}{c}\right]}e^{j\omega_c\left[\frac{d''-d'}{2c}\right]}j\sin\left(\omega_c\left[\frac{d''-d'}{2c}\right]\right)\right\}$$

# Field Envelope at Receiver

- Recall d’ ’ >d’
- The envelope of the field is then

$$\left|E_{TOT}\right| = \frac{2E_o d_o}{d}\sin\left(\omega_c\left[\frac{d''-d'}{2c}\right]\right)$$

- Can show that $d''-d' \approx \frac{2h_t h_r}{d}$, and

$$\sin\left(\omega_c\left[\frac{d''-d'}{2c}\right]\right) \approx \omega_c\left[\frac{d''-d'}{2c}\right]$$

# Power Received

- Making the substitutions yields

$$\left|E_{TOT}\right| = \frac{2E_o d_o}{d}\frac{2\pi h_t h_r}{\lambda d}$$

- The power received is

$$P_{ri} = P_d A_e = \left(\frac{\left|E_{TOT}\right|^2}{120\pi}\right)\left(\frac{G_r \lambda^2}{4\pi}\right)$$

# Flat Earth Path Loss

- Recalling

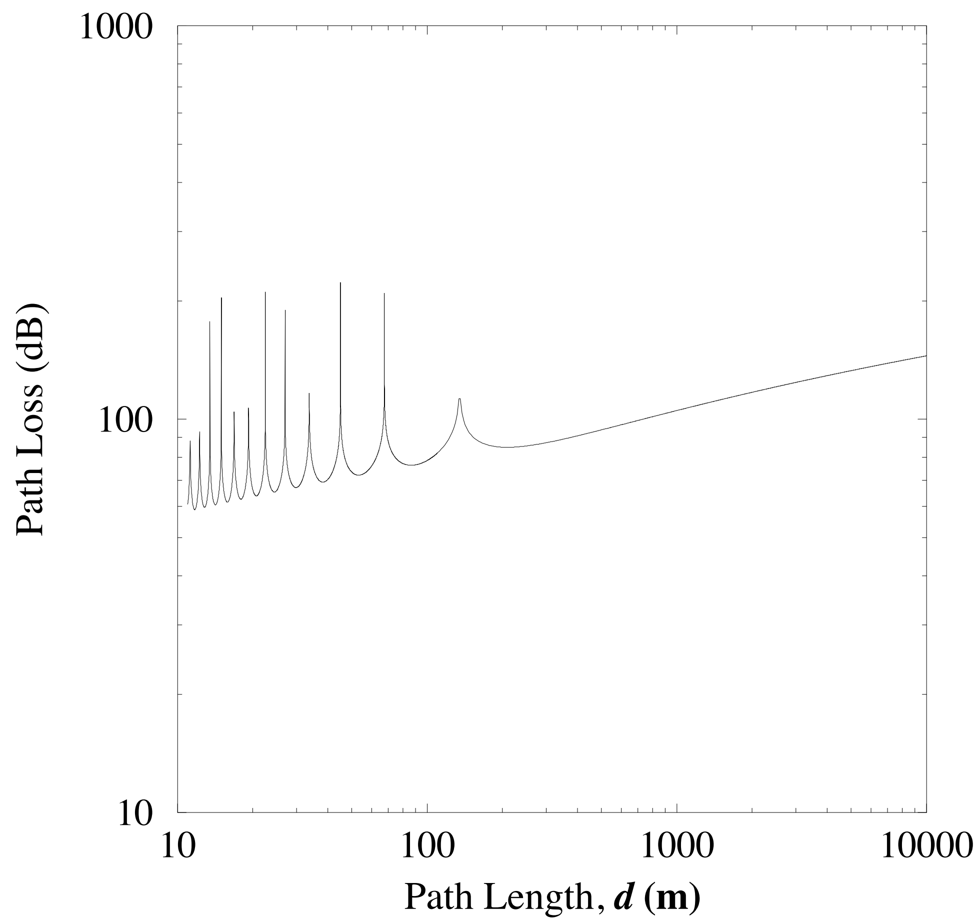
$$\frac{P_t G_t}{L_t 4\pi d_o^{\,2}} = \frac{E_o^{\,2}}{120\pi}$$

gives

$$P_{ri} = \frac{P_t G_t G_r h_t^2 h_r^2}{L_t d^4}$$

- The flat earth path loss is therefore

$$L = \frac{d^4}{h_t^2 h_r^2}$$




*Propagation path loss $L_{p\ (\mathrm{dB})}$ with distance over a flat reflecting surface;*
*$h_b = 7.5\ m$, $h_m = 1.5\ m$, $f_c = 1800\ MHz$.*

$$L_r = \left[ \left( \frac{\lambda_c}{4\pi d} \right)^2 4 \sin^2 \left( \frac{2\pi h_b h_m}{\lambda_c d} \right) \right]^{-1}$$

- **In reality, the earth's surface is curved and rough, and the signal strength typically decays with the inverse $\beta$ power of the distance, and the received power is**

$$\Omega_p = k\frac{\Omega_t}{d^\beta}$$

**where $k$ is a constant of proportionality. Expressed in units of dBm, the received power is**

$$\Omega_{p\ (\mathrm{dBm})} = 10\log_{10}(k) + \Omega_{t\ (\mathrm{dBm})} - 10\beta\log_{10}(d)$$

- **$\beta$ is called the path loss exponent. Typical values of $\beta$ are have been determined by empirical measurements for a variety of areas**

| Terrain | $\beta$ |
|---|---|
| Free Space | 2 |
| Open Area | 4.35 |
| North American Suburban | 3.84 |
| North American Urban (Philadelphia) | 3.68 |
| North American Urban (Newark) | 4.31 |
| Japanese Urban (Tokyo) | 3.05 |

# Using a Reference Power Measurement

- Suppose that a reference measurement of received power, $P_o$, is taken at some point in the far field of the antenna
- Then the power taken at some more distant point may be expressed relative to the reference power:

$$P_{ri} = P_o \left( \frac{d_o}{d} \right)^n$$

# Summary

- Free space path loss depends only on distance and wavelength, and falls off as $1/d^2$
- Flat earth path loss
  - depends also on the antenna heights, and falls off as $1/d^4$
  - Has a pretty good fit to urban and suburban environments, even though it is an idealization, derived only for horizontal polarization
- The power of d is called the path loss exponent
- For mobile comm, this exponent is typically between 3.5 and 4

# References


- [Saunders,`99] Simon R. Saunders, *Antennas and Propagation for Wireless Communication Systems*, John Wiley and Sons, LTD, 1999.
- [Rapp, ’96] T.S. Rappaport, *Wireless Communications*, Prentice Hall, 1996
- [Lee, ’98] W.C.Y. Lee, *Mobile Communications Engineering*, McGraw-Hill, 1998