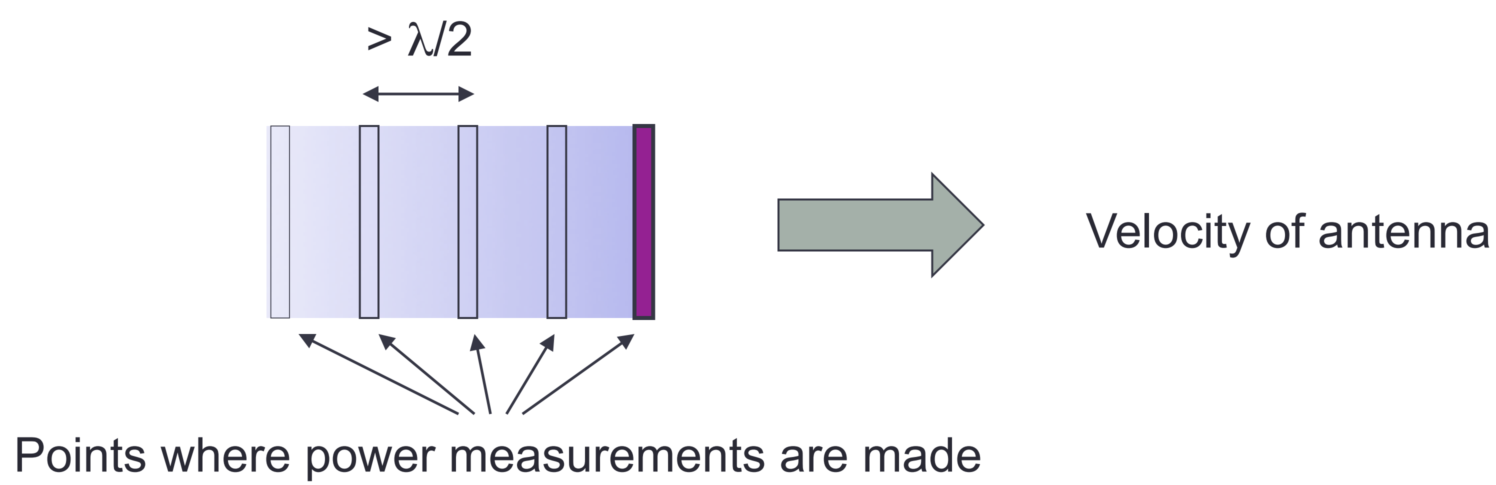
LOG-NORMAL SHADOWING

Shadowing

- Also called *slow-fading*
- Accounts for random variations in received power observed over distances comparable to the widths of buildings
- Extra transmit power (*a fading margin*) must be provided to compensate for these fades

Local Average Power Measurements

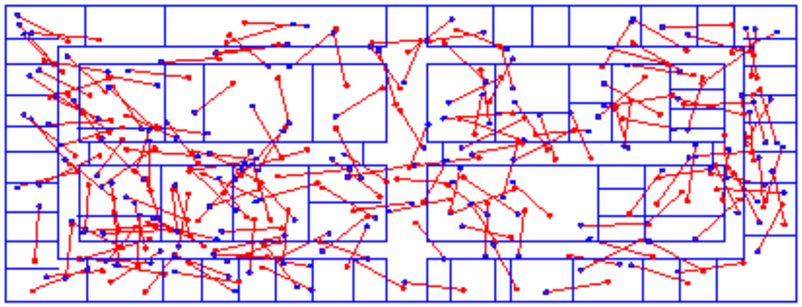
- Take power measurements in Watts as the antenna is moved in a on the order of a few wavelengths
- Average these measurements to give a local average power measurement



Same-Distance Measurements

- Local averages are made for many different locations, keeping the same transmitter-receiver distance
- These local averages will vary randomly with location

Example Tx-Rx
locations within
a floor of a building



Repeat for Multiple Distances

- Similar collections of average powers are made for other Tx-Rx distances

Likelihood of Coverage

- At a certain distance, d, what is the probability that the local average received power is below a certain threshold γ?

$$P(P_r(d) < \gamma)$$

$$P_r(d) = \frac{P_t G_t G_r}{L_t \underbrace{L(d)}_{\text{Path Loss}} \underbrace{X_\sigma}_{\text{Shadowing}} L_r}$$

Likelihood of Coverage, cont'd

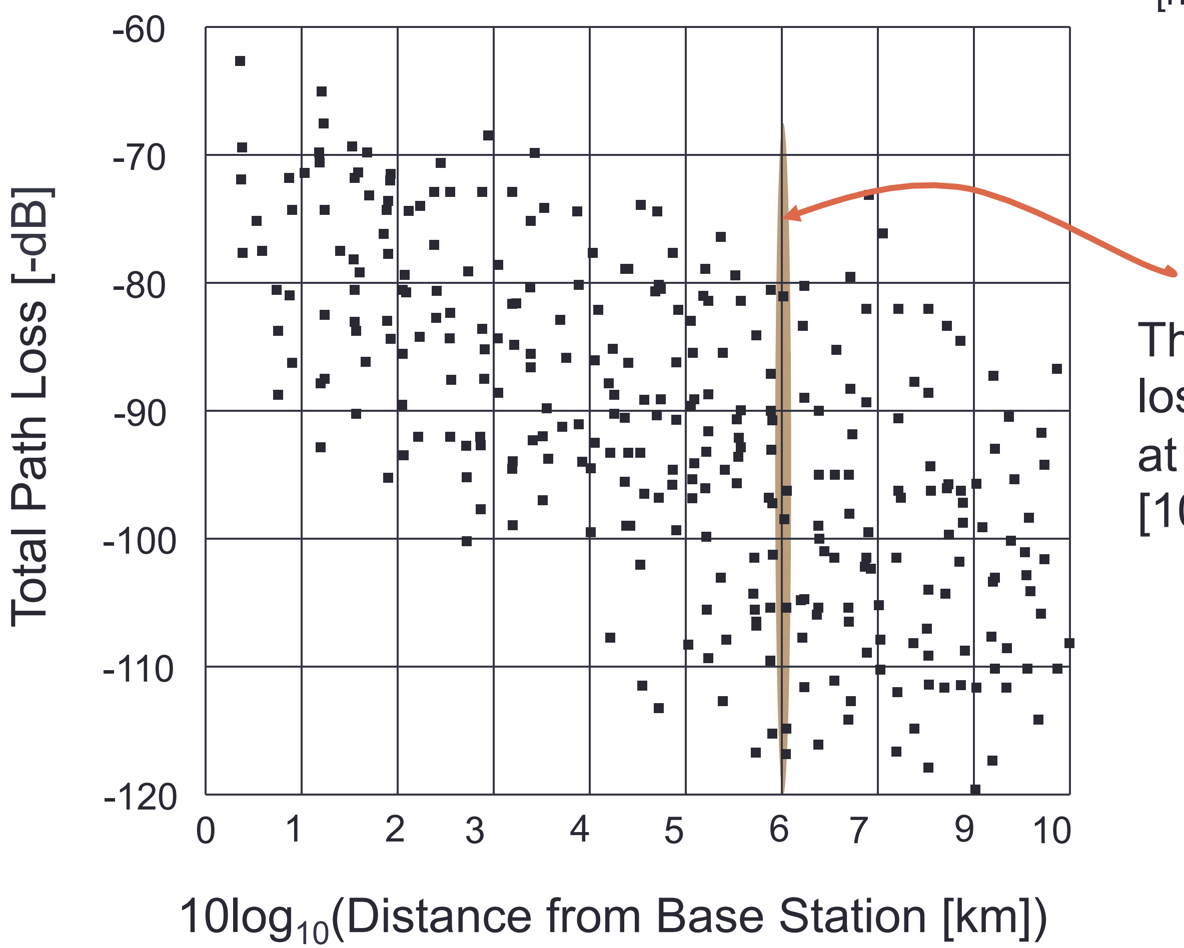
- Since only X_σ is random, the probability can be expressed as a probability involving it:

$$P(P_r(d) > \gamma) = P(X_\sigma > \beta)$$

Chosen to give a desired quality of service

Typical Macrocell Characteristics

[not real data]



The set of average losses measured at about 4km [$10\log_{10}(4)=6$]

Path Loss Assumptions

- The mean loss in dB follows the *power law* :

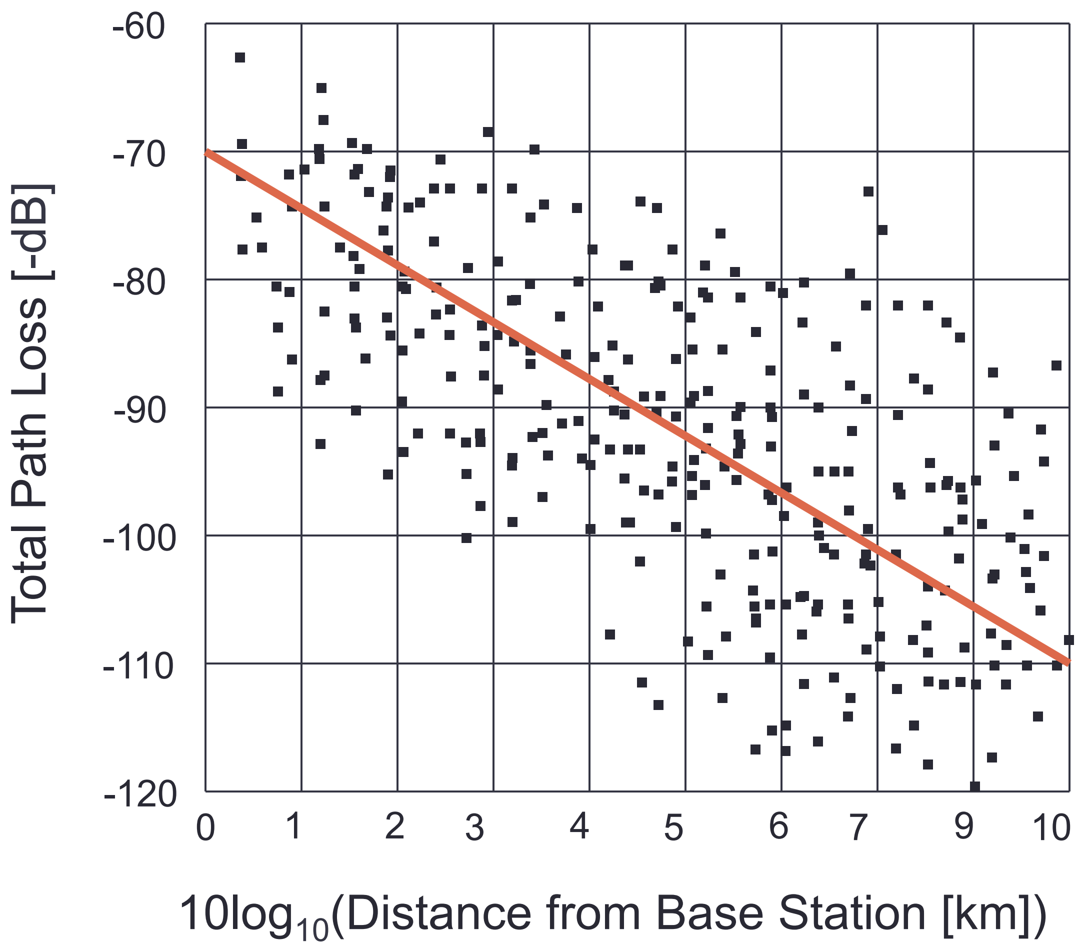
$$\overline{L}(d) = \overline{L}(d_o) + 10n\log_{10}\left(\frac{d}{d_o}\right)$$

- The measured loss in dB varies about this mean according to a zero-mean Gaussian RV, X_σ, with standard deviation σ

$$L(d) = \overline{L}(d_o) + 10n\log_{10}\left(\frac{d}{d_o}\right) + X_\sigma$$

Typical Data Characteristics

[not real data]



Best Fit Path Loss Exponent: n=4

σ is usually 5-12 dB for mobile comm

Probability Calculation

- Since X_σ is Gaussian, we need to know how to calculate probability involving Gaussian RVs

$$P(X_\sigma > \beta)$$

Q Function

- If X is a Gaussian RV with mean α and standard deviation σ, then

$$P(X > b) = Q\left(\frac{b-\alpha}{\sigma}\right)$$

where Q is a function defined as

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_{z}^{+\infty} \exp\left(-\frac{x^2}{2}\right) dx$$

The Problem with Q

- The integrand of Q has no antiderivative
- Q is found tabulated in books
- Q can be calculated using numerical integration

What is Log-normal Shadowing?

- If Y is a Gaussian RV and Z is defined such that Y=logZ, then Z is a log-normal RV
- Shadowing is log-normal shadowing when the path loss in dB is Gaussian; this means that the path loss expressed as a ratio is log-normal

Inverse Q Problems

- Sometimes, the probability is specified and we must find one of the parameters in the argument of Q

$$P(X > b) = Q\left(\frac{b - \alpha}{\sigma}\right)$$

- Suppose the value of $P(X > b)$ is given, along with values of b and α. Solve for σ
- Must look up the argument of Q that gives the specified value.

Example Inverse Q Problem

- Suppose the mean of the local average received powers at a certain distance is -30dBm, that the standard deviation of shadow fading is 9 dB, and that the observed received power is above the threshold 95% of the time. What is the threshold power?

- Q is usually tabulated for arguments of 0.5 and less, so we must use the fact that

$$P(P_r > b) = Q\left(\frac{b-(-30)}{9}\right) = 0.95$$

- The argument of Q that yields 0.05 is about 1.65

$$Q(z) = 1 - Q(-z)$$

$$-\frac{b+30}{9} = 1.65, \text{ and } b = -44.85 \text{ dBm}$$

Boundary Coverage

- Suppose that a cell has radius R and γ is the minimum acceptable received power level
- Then $P(P_r(R) > \gamma)$ is the *"likelihood of coverage"* at the boundary of the cell
- $P(P_r(R) > \gamma)$ is also the *"fraction of time"* that a mobile's signal is acceptable at a distance R from the transmitter, assuming the car moves around that circle

Percentage of Useful Service Area

- By integrating these probabilities over all the circles within a disk, one can compute the fraction of the area within the cell that will have acceptable power levels

$$U(\gamma) = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^{R} P(P_r(r) > \gamma)\, r\, dr\, d\theta$$

Integral Evaluated

- Assuming log-normal shadowing and the power path loss model, the fraction of useful service area is

$$U(\gamma) = \frac{1}{2}\left(1 - \operatorname{erf}(a) + \exp\left(\frac{1-2ab}{b^2}\right)\left[1 - \operatorname{erf}\left(\frac{1-ab}{b}\right)\right]\right)$$

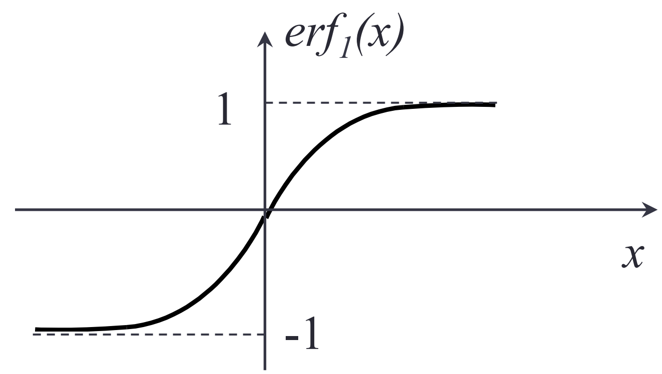
where

$$a = \frac{\gamma - \overline{P_r(R)}}{\sigma\sqrt{2}} \quad \text{and} \quad b = \frac{10n\log_{10} e}{\sigma\sqrt{2}}$$

The Error Function

- erf(x) is another form of the Gaussian integral (like $Q(x)$)
- erf(x) has odd symmetry, with extreme values ±1.

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$



- Note that some authors may define erf differently

erf and Q

- The erf function and Q are related:

$$\operatorname{erf}(z) = 1 - 2Q\left(\sqrt{2}z\right)$$

When the Average Boundary Power is Acceptable

- Suppose $P_r(R) = \gamma$ Then we may use this graph from [Rappaport ' 96] to figure the percent useful service area

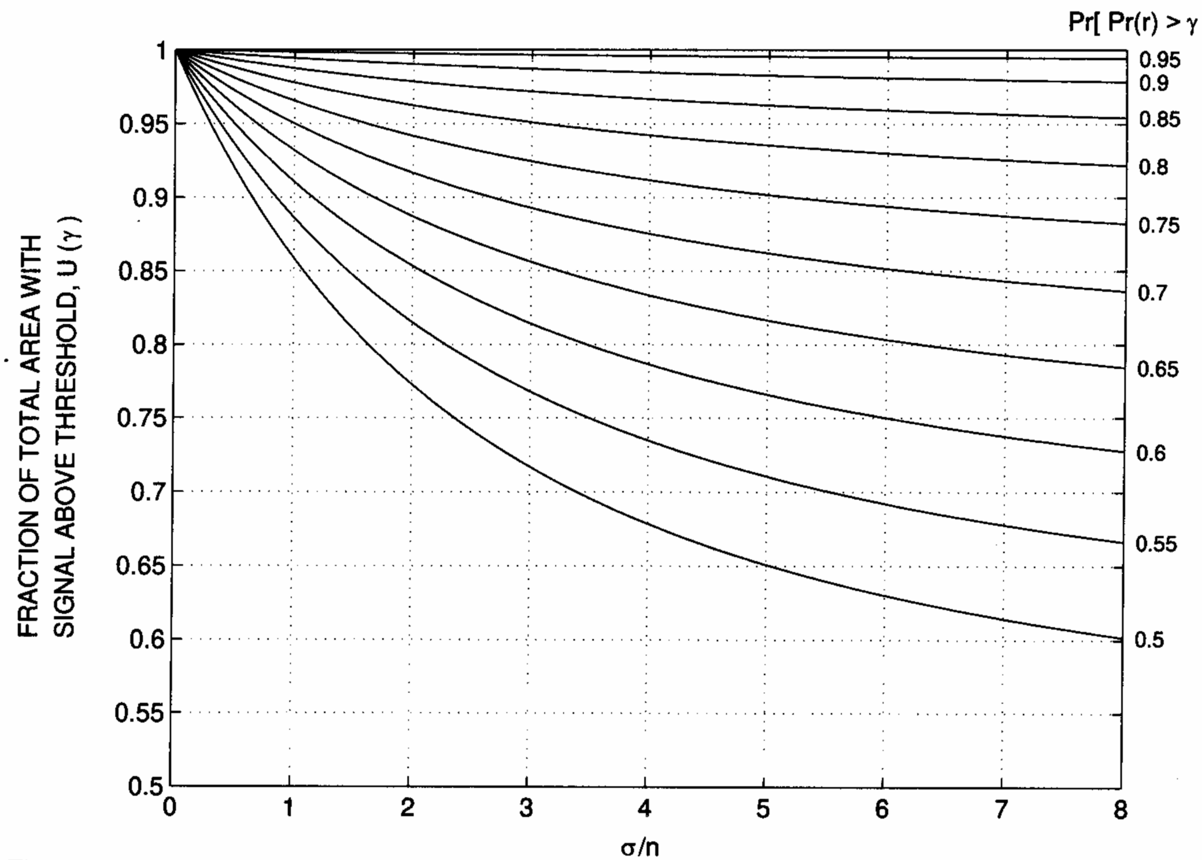


Figure 3.18
Family of curves relating fraction of total area with signal above threshold, $U(\gamma)$ as a function of probability of signal above threshold on the cell boundary.

Summary

- The logs of local averages of received power (or path loss) tend to be Gaussian when the ensemble is all Tx-Rx locations with the same distance in the same type of environment
- The mean local average path loss follows the standard power model (proportional to 10logd^n)
- Can use Q or erf to calculate the likelihood of boundary coverage or the percent of useful service area

References

- [Rapp, ’96] T.S. Rappaport, *Wireless Communications*, Prentice Hall,
- [Saunders,`99] Simon R. Saunders, *Antennas and Propagation for Wireless Communication Systems*, John Wiley and Sons, LTD, 1999.