

EE451 Mobile Communication Systems

Homework 1 (CLO-1 & CLO-2)

Problem 1 (CLO-1):

- (a) Compute the maximum system capacity in Erlangs when providing a 1% blocking probability with 4 channels, with 20 channels, with 40 channels?
- (b) How many users can be supported with 40 channels at 1% blocking? Assume $H=120s$, $A=2$ call/hour.
- (c) Using the traffic intensity per channel calculated in part(a), demonstrate the grade of service in a lost call delayed system for the case of delays being greater than 30 seconds. Assume that $H=120s$, and determine the GOS for 4 channels, for 20 channels, and for 40 channels.

(a) Maximum system capacity in Erlangs A?

Blocking Probability = 0.01 (1%)

Channels 4,	using erlang B chart,	A = 0.89
Channels 20,	using erlang B chart,	A = 12
Channels 40,	using erlang B chart,	A = 29

(b) Users supported with 40 Channels. With $P_b = 0.01$, $H = 120s$ and $A = 2$ Calls/Hour

$$H = 120s = 120/3600 = 1/30$$

$$A_u = \lambda H = (2) * (1/30) = 1/15$$

$$\text{Users} = U = A/A_u = 29 / (1/15) = 435 \text{ users}$$

(c) GOS = P (delay > 30sec) ?

We know that.

$$P(\text{delay} > 20\text{sec}) = P(\text{delay} > 30 | \text{delay} > 0) P(\text{delay} > 0) \quad (1)$$

$$\text{Where } P(\text{delay} > 0) = C(C, A) \quad (2)$$

$$P(\text{delay} > t | \text{delay} > 0) = e^{-\{(C-A) t / H\}} \quad (3)$$

$$C(A, N) = N B(A, N) / N - A [1 - B(A, N)]$$

Channels = 4

$$C(4, 0.89) = 4 * (0.01) / 4 - 0.89[1 - 0.01] = 0.0128$$

$$GOS = C(A, N) e \{-(C-A) t/H\} = (0.0128) * e \{-(4-0.89) 30/120\} = 5.38 \times 10^{-3}$$

Channels = 20

$$C(20, 12) = 20 * (0.01) / 20 - 12[1 - 0.01] = 0.025$$

$$GOS = C(A, N) e \{-(C-A) t/H\} = (0.025) * e \{-(20-12) 30/120\} = 3.38 \times 10^{-3}$$

Channels = 40

$$C(40, 29) = 40 * (0.01) / 40 - 29[1 - 0.01] = 0.035$$

$$GOS = C(A, N) e \{-(C-A) t/H\} = (0.035) * e \{-(40-0.035) 30/120\} = 2.24 \times 10^{-3}$$

Problem 2 (CLO-1):

Co-channel interference	Adjacent channel interference
Definition Interference received at home cell from transmissions from other cells using same set of frequencies is called cochannel interference.	Definition Interference received from the transmissions from adjacent cells (normally using different set of frequencies).
Reduction Techniques (i) By increasing spatial separation between co-channel cells. In other words, cochannel reuse ratio $Q = D/R$ may be increased. (ii) (ii) Using sectoring technique in the cells improves Co-channel interference.	Reduction Techniques (i) Improve filters in the receivers. (ii) (ii) Assigning frequencies to adjacent cells to be as far as possible. (iii) (iii) Using advanced signal processing techniques at the receiver end.

Problem 3 (CLO-1)

Suppose a Cellular Service Provider has a total of 78 channels. Suppose each user contributes 0.8 Erlangs of traffic.

- (a) Compute the maximum number of users in one cell that can be supported with a 0.01 probability of blocking if 4-cell clusters are used?

$$\text{No. of channels for each cell} = \frac{78}{4} = 19.5 \approx 19 \text{ channels (rounded-down)}$$

$$N = 19, P_B = 0.01; \quad A_T = 11.2 \text{ Er (From Erlang-B graph)}$$

$$\text{No. of Users} = \frac{A_T}{A_u} = \frac{11.2}{0.8} = \boxed{14 \text{ users}}$$

- (b) Compute the maximum number of users in one cell that can be supported with a 0.01 probability of blocking if 12-cell clusters are used?

$$\text{No. of channels for each cell} = \frac{78}{12} = 6.5 \approx 6 \text{ channels (rounded-down)}$$

$$N = 6, P_B = 0.01; \quad A_T = 1.9 \text{ Er (From Erlang-B graph)}$$

$$\text{No. of Users} = \frac{A_T}{A_u} = \frac{1.9}{0.8} = 2.375 \approx \boxed{2 \text{ users}}$$

Problem 4 (CLO-2)

A cellular service provider decides to design a system on the forward channel that can tolerate 15dB of worst-case carrier-to-interference ratio. Compute the optimal value of N for a) omni-directional antennas b) for 120-degree cell sectoring and c) for 60-degree cell sectoring. Should sectoring be used? If so, what case b) or c) should be used. Assume path loss exponent of 4.

$$\text{Tolerance} = 15 \text{ dB}, \quad \beta = 4$$

(a) Omnidirectional Antenna: (6 co-channel interference)

$$\frac{C}{I} = \frac{R^{-\beta}}{2(D-R)^{-\beta} + 2D^{-\beta} + 2(D+R)^{-\beta}} = \frac{1}{2} \cdot \frac{1}{\left(\frac{D}{R} - 1\right)^{-\beta} + \left(\frac{D}{R}\right)^{-\beta} + \left(\frac{D}{R} + 1\right)^{-\beta}}$$

$$\frac{C}{I} = \frac{1}{2} \cdot \frac{1}{(\sqrt{3N} - 1)^{-\beta} + (\sqrt{3N})^{-\beta} + (\sqrt{3N} + 1)^{-\beta}} \quad \left(\because Q = \frac{D}{R} = \sqrt{3N}\right)$$

$$\frac{C}{I} \text{ (dB)} = \begin{cases} 17.27 \text{ dB} & ; N = 7 \checkmark \text{ (optimal)} \\ 11.2 \text{ dB} & ; N = 4 \end{cases}$$

(b) 120° cell sectoring: (2 co-channel interference)

$$\frac{C}{I} = \frac{R^{-\beta}}{D^{-\beta} + (D + 0.7R)^{-\beta}} = \frac{1}{\left(\frac{D}{R}\right)^{-\beta} + \left(\frac{D}{R} + 0.7\right)^{-\beta}}$$

$$\frac{C}{I} = \frac{1}{(\sqrt{3N})^{-\beta} + (\sqrt{3N} + 0.7)^{-\beta}}$$

$$\frac{C}{I} \text{ (dB)} = \begin{cases} 17.52 \text{ dB} & ; N = 3 \checkmark \text{ (optimal)} \\ 9.55 \text{ dB} & ; N = 1 \end{cases}$$

(c) 60° cell sectoring:

For $N=3$, 2 co-channel interference should be catered.

$$\frac{C}{I} = \frac{R^{-\beta}}{(D)^{-\beta} + (D+0.7R)^{-\beta}} = \frac{1}{(\sqrt{3}N)^{-\beta} + (\sqrt{3}N + 0.7)^{-\beta}}$$

For $N=4$ to 9, 1 co-channel interference should be catered.

$$\frac{C}{I} = \frac{R^{-\beta}}{(D + 0.7R)^{-\beta}} = \frac{1}{(\sqrt{3}N + 0.7)^{-\beta}}$$

$$\frac{C}{I} \text{ (dB)} = \begin{cases} 24.79 \text{ dB} & ; N=4 \\ 17.52 \text{ dB} & ; N=3 \checkmark \text{ (optimal)} \end{cases}$$

Result: Yes, sectoring should be used. 120° sectoring is the optimal scheme in comparison to 60° sectoring because in both cases, N -value is same but in 120° sectoring, trunking efficiency will be higher.

Problem 5 (C10-2)

If a measurement campaign shows that the path loss exponent is 3 instead of 4 in problem 2, how does your design change? Compare it.

$$\beta = 3$$

(a) Omnidirectional Antenna:

$$\frac{C}{I} = \frac{1}{2} \cdot \frac{1}{(\sqrt{3}N-1)^{-\beta} + (\sqrt{3}N)^{-\beta} + (\sqrt{3}N+1)^{-\beta}}$$

$$\frac{C}{I} \text{ (dB)} = \begin{cases} 15.07 \text{ dB}, N=12 \checkmark \text{ (optimal)} \\ 13.03 \text{ dB}, N=9 \end{cases}$$

(b) 120° cell sectoring:

$$\frac{C}{I} = \frac{1}{(\sqrt{3}N)^{-\beta} + (\sqrt{3}N + 0.7)^{-\beta}}$$

$$\frac{C}{I} \text{ (dB)} = \begin{cases} 17.65 \text{ dB}, N=7 \checkmark \text{ (optimal)} \\ 14.21 \text{ dB}, N=4 \end{cases}$$

(c) 60° cell sectoring:

For $N=3$, 2 co-channel interference should be catered.

$$\frac{C}{I} = \frac{R^{-\beta}}{(D)^{-\beta} + (D+0.7R)^{-\beta}} = \frac{1}{(\sqrt{3}N)^{-\beta} + (\sqrt{3}N + 0.7)^{-\beta}}$$

For $N=4$ to 9, 1 co-channel interference should be catered.

$$\frac{C}{I} = \frac{R^{-\beta}}{(D + 0.7R)^{-\beta}} = \frac{1}{(\sqrt{3}N + 0.7)^{-\beta}}$$

$$\frac{C}{I} \text{ (dB)} = \begin{cases} 19.58 \text{ dB} & ; N=4 \checkmark \text{ (optimal)} \\ 12.46 \text{ dB} & ; N=3 \end{cases}$$

Result: Yes, sectoring should be used. In this case, 60° sectoring is optimal scheme because cluster size is lowest in this case which in turn increases number of channels per cell.