Homework-2 Solution

Problem-1

Given:-

Transmitter power, $P_t = 50W$

Carrier Frequency, $f_c = 6\,GHz$, $\lambda = \frac{0.3\times10^9}{6\times10^9} = \frac{3}{60} = \frac{1}{20}$

Distance of Receiver, $d = 10\,Km$

Transmitter antenna Gain, $G_t = 1$

Receiver antenna Gain, $G_r = 1$

a) Power at receiver = $P_r(d) = ?$

$$P_r(d) = 10\log\left[\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2}\right] = 10\log\left[\frac{50\times1\times1\times(1/20)^2}{(4\pi)^2\, 10000^2}\right]$$

$$= 10\log(7.9\times10^{-12}) = -111\ dB$$

b) Magnitude of E-Field?

$$|E| = \sqrt{\frac{P_r(d)\,120\pi}{A_e}} \quad \text{where } A_e = \frac{G_r\lambda^2}{4\pi} = \frac{1}{1600\pi}$$

$$= \sqrt{\frac{7.9\times10^{-12}\times120\pi}{1/1600\pi}} = 0.00387 = 0.0039\,V/m$$

c) Receiver Power in dBm?

$$P_r(d)\ dBm = 10\log(7.9\times10^{-12}\times10^{3})$$

$$= -81\ dBm.$$

d) If Receiver Sensitivity $S_{Rx} = -96\,dBm$

As $P_r(d) > S_{Rx}$ hence receiver will be able to decode this signal.

Problem-2

$$L_p = 25\text{ dB} + 10\log_{10} d^{2.8}$$

$$= 25 + 28\log_{10} d \quad (\text{dB})$$

Also $\Omega_{t\,(\text{dBm})} = S_{Rx\,(\text{dBm})} + L_{p\,(\text{dB})}$

a) For $d = 10{,}000$ m , $S_{Rx\,(\text{dBm})} = -95$

$L_p = 137$ dB

$\therefore \quad \Omega_t = -95 + 137 = 42$ dBm

b) If $\beta = 3.1$, then

$$L_p = 25 + 31\log_{10} d \quad (\text{dB})$$

For $d = 10{,}000$m

$L_p = 149$

Hence,

$\Delta\Omega_{t\,(\text{dB})} = 149 - 137 = 12$ dB.

c) We must have $Q\left(\frac{M_{shad}}{\sigma_\Omega}\right) = 0.1$; $Q^{-1}(0.1) = 1.28$

$\Rightarrow M_{shad} = 1.28 \times 8 = 10.24$ dB.

We need to increase Tx power by 10.24 dB.

d) From graph, it is almost 98% for sigma/n=2.86

Problem-3

a)

Since the multipath is resolved, all we need to do is take the magnitude squared of each impulse's area and then average the squares that share the same delay. Therefore, the first impulse in the power delay profile (PDP) will be at zero ns, and will have the value

$$P(0) = \frac{0.8^2 + 0.9^2 + 0.7^2}{3} = 0.647$$

$$P(1) = \frac{0.4^2 + 0.5^2 + 0.3^2}{3} = 0.167$$

$$P(3) = \frac{0.7^2 + 0.75^2 + 0.6^2}{3} = 0.47$$

b)

Using PDP, first compute the mean excess delay

$$\bar{\tau} = \frac{0 * 0.647 + 1 * 0.167 + 3 * 0.47}{0.647 + 0.167 + 0.47}$$

$$= 1.23\text{us}$$

Next compute the mean delay spread

$$\overline{\tau^2} = \frac{0^2 * 0.647 + 1^2 * 0.167 + 3^2 * 0.47}{0.647 + 0.167 + 0.47} = 3.424\ \mu s^2$$

Then the rms delay spread is

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2}$$

$$= 1.38\text{us}$$

Problem-4

a. Compute the mean excess delay of the channel.

The mean delay is given by $\bar{\tau} = \frac{\int_0^\infty \tau p(\tau) d\tau}{\int_0^\infty p(\tau) d\tau} = \frac{0.5(300ns)}{1.5} = 100ns$

b. Give the rms delay spread of the channel.

The rms delay spread is given by $\sigma_\tau = \sqrt{\frac{\int_0^\infty (\tau-\bar{\tau})^2 \, p(\tau) d\tau}{\int_0^\infty p(\tau) d\tau}}$

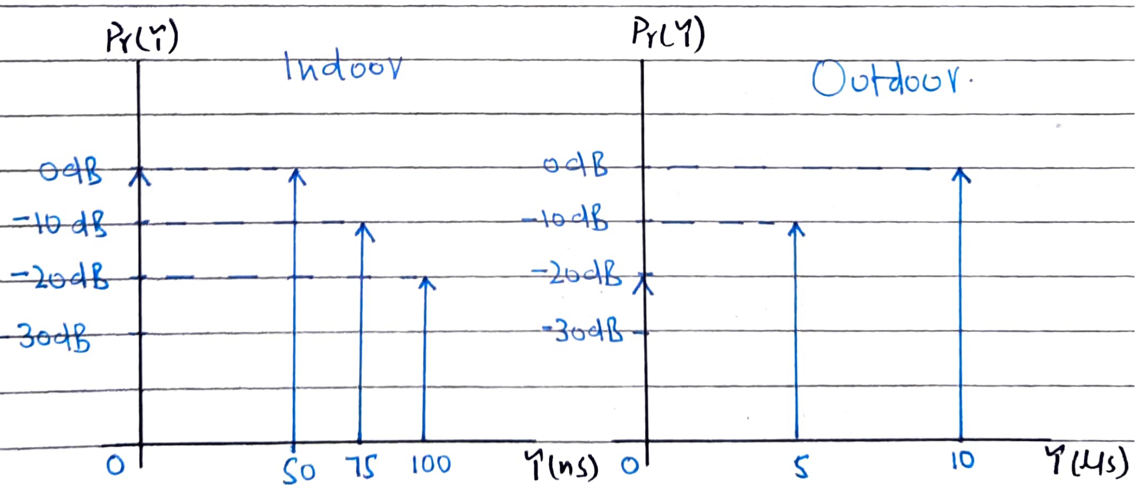
$$\sigma_\tau^2 = \frac{(-100ns)^2 + 0.5(200ns)^2}{1.5} = \frac{10^4 + 0.5(4 \times 10^4)}{1.5}(ns)^2$$
$$= 20{,}000 \, ns^2$$

$$\sigma_\tau = 141.42 \, ns$$

c. Classify this channel in terms of its type of fading.

The classification depends on how the symbol period compares with the rms delay spread. The symbol period is $10^{-5}s = 10^4 \, ns$, which is orders of magnitude larger than the delay spread, so this channel is flat fading. The channel is described as "static" so it is also slow fading.

Problem-5



(a) To calculate 90% and 50% correlation coherence bandwidths of the power delay profiles.

↳ **Indoor:**

For mean excess delay :

$$\bar{\tau} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau)_k}$$

$$\bar{\tau} = \frac{0(1) + 50(1) + 75(0.1) + 100(0.01)}{1+1+0.1+0.01}$$

$$\boxed{\bar{\tau} = 27.72 \text{ ns}}$$

Now,

$$\bar{\tau}^2 = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

$$\bar{\tau}^2 = \frac{0(1) + 50^2(1) + 75^2(0.1) + 100^2(0.01)}{1+1+0.1+0.01}$$

$$\boxed{\bar{\tau}^2 = 1498.8 \text{ (ns)}^2}$$

For RMS delay:

$$\sigma_T = \sqrt{\bar{\tau}^2 - (\bar{\tau})^2}$$

$$\sigma_T = \sqrt{1498.8 - (27.73)^2}$$

$$\boxed{\sigma_T = 27.01 \text{ ns}}$$

For 90% coherence:

$$B_C = \frac{1}{50\sigma_T} = \frac{1}{50(27.01 \times 10^{-9})}$$

thus, $\boxed{B_{C}, 90\% = 740.466\ KHz}$

50 % coherence:

$$B_{C}, 50\% = \frac{1}{5(\bar{\sigma}_{\tau})} = \frac{1}{5 \times 27.01 \times 10^{-9}}$$

$$\boxed{B_{C}, 50\% = 7404.664\ KHz}$$

↳ **Outdoor**

For mean excess delay

$$\bar{\tau} = \frac{\sum_{K} P(\tau_K)\,\tau_K}{\sum_{K} P(\tau_K)}$$

$$\bar{\tau} = \frac{0(0.01) + 5(0.1) + 10(1)}{0.01 + 0.1 + 1}$$

$$\boxed{\bar{\tau} = 9.45\ \mu s}$$

Now, $$\bar{\tau^2} = \frac{\sum_{K} P(\tau_K)\,\tau_K^2}{\sum_{K} P(\tau_K)}$$

$$\bar{\tau^2} = \frac{0(0.01) + 5^2(0.1) + 10^2(1)}{0.01 + 0.1 + 1}$$

$$\boxed{\overline{\tau^2} = 92.34\ (\mu s)^2}$$

For RMS delay we have

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$\sigma_\tau = \sqrt{92.34 - (9.45)^2}$$

$$\boxed{\sigma_\tau = 1.74\ \mu s}$$

For 90 % coherence

$$B_{c,90\%} = \frac{1}{50\sigma_\tau} = \frac{1}{50(1.74\times10^{-6})}$$

$$\boxed{B_{c,90\%} = 11.49\ \text{KHz}}$$

For 50 % coherence

$$B_{c,50\%} = \frac{1}{5\sigma_\tau} = \frac{1}{5(1.74\times10^{-6})}$$

$$\boxed{B_{c,50\%} = 114.942\ \text{KHz}}$$

(b) For indoor channel.

For the indoor channel the 50% coherence was calculated as

$$B_{c,50\%} = 7404.664 \text{ KHz}$$

$$= 7.4 \text{ MHz}$$

now, as $BW_{LTE} > B_{c,50\%}$

and,

$BW_{GSM, CDMA \text{ and } WCDMA} < B_{c,50\%}$

Standards suitable without equalizer	Standards suitable with equalizer.
↳ GSM	↳ LTE
↳ CDMA	
↳ WCDMA.	

For Outdoor Channel:

$$B_{c,50\%} = 114.942 \text{ KHz.}$$

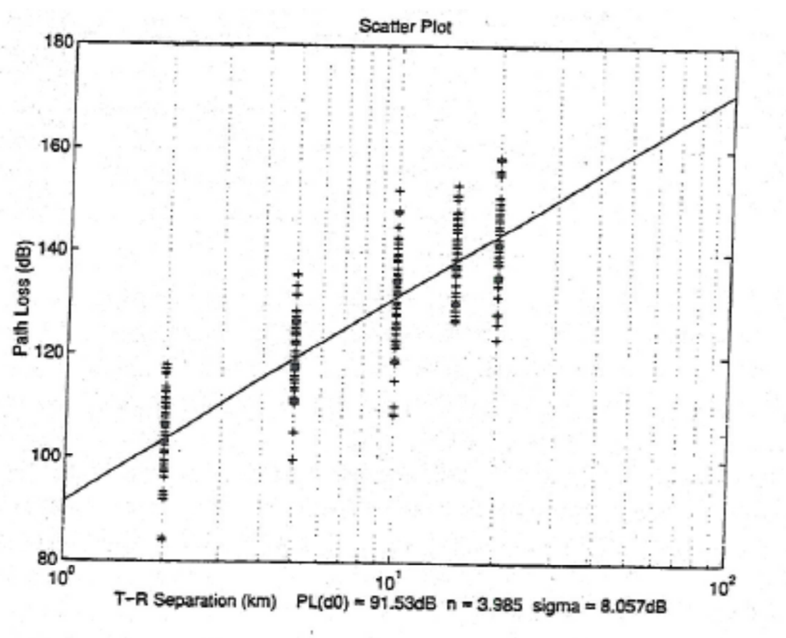
For all standards $BW > B_{c,50\%}$.

This means that the channel will require equalizer for all standards.

Problem 6

Design and create a computer program that produces an arbitrary number of samples of propagation pathloss using a d^n pathloss model with lognormal shadowing. Your program is a radio propagation simulator, and should use, as inputs, the T-R separation, frequency, the pathloss exponent, the standard deviation of the log-normal shadowing, the close-in-reference distance, and the number of desired predicted samples. Your program should provide a check that insures that the input T-R separation is equal to or exceeds the specified input close-in-reference distance, and should provide a graphical output of the produced samples as a function of pathloss and distance (this is called a scatter plot).

Verify the accuracy of your computer program by running it for 50 samples at each of 5 different T-R separation distances (a total of 250 predicted pathloss values), and determine the best fit pathloss exponent and the standard deviation about the mean pathloss exponent of the predicted data using the techniques as described in example in the class. Draw the best fit mean pathloss model on the scatter plot to illustrate the fit of the model to the predicted values. You will know your simulator is working if the best fit pathloss model and the standard deviation for your simulated data is equal to the parameters you specified as inputs to your simulators.



Problem 7:

Code:

```
%-----part a
X=1/sqrt(2)*randn(1,200);
Y=1/sqrt(2)*randn(1,200);
R=abs(X+1j*Y);




stem(R);
title('Stem plot for part a')

%-----part b
rms=sqrt(mean(R.^2))

%-----part c
fraction_R=sum(R<10^(-10/20)*rms)/200

%-----part d
X_hat=X+5*cos(pi/3);
Y_hat=Y+5*sin(pi/3);
R_hat=abs(X_hat+1j*Y_hat);
figure
stem(R_hat)
title('Stem plot for part d')
%The K factor for this Rician random variable is K=25

%-----part e
X_hat1=X+5*cos(pi/6);
Y_hat1=Y+5*sin(pi/6);
R_hat1=abs(X_hat1+1j*Y_hat1);
figure
stem(R_hat1)
title('Stem plot for part e')

%-----part f
rms_hat=sqrt(mean(R_hat.^2))
Yn=R_hat/rms_hat;
figure
stem(Yn)
title('Stem plot for part f')
rms_Yn=sqrt(mean(Yn.^2))
fraction_Yn=sum(Yn<10^(-10/20)*rms_Yn)/200
```

Sample result:

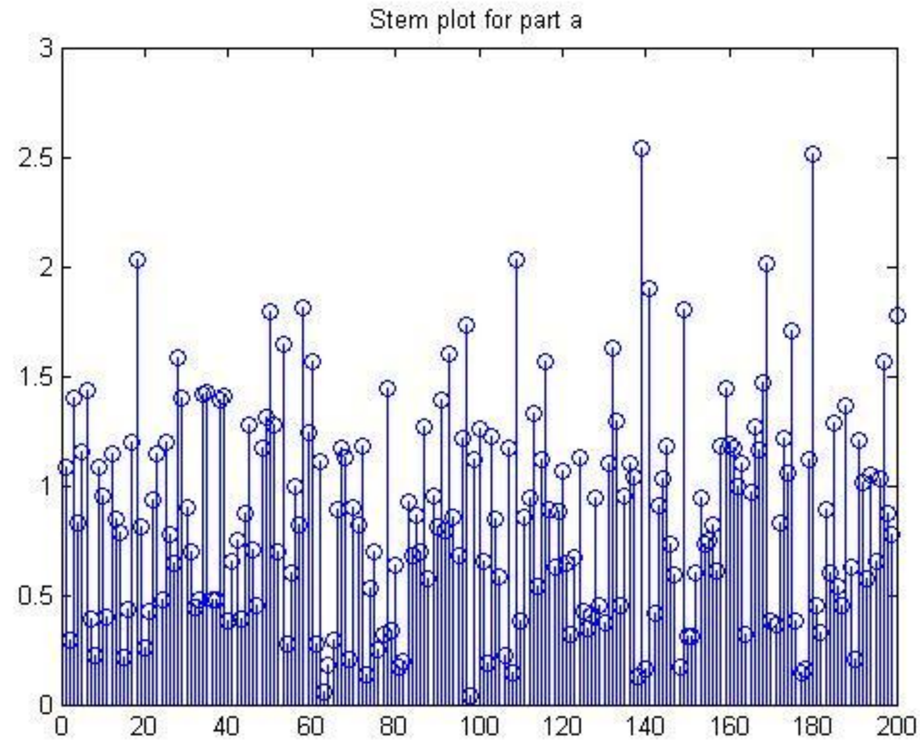
rms = 0.9975

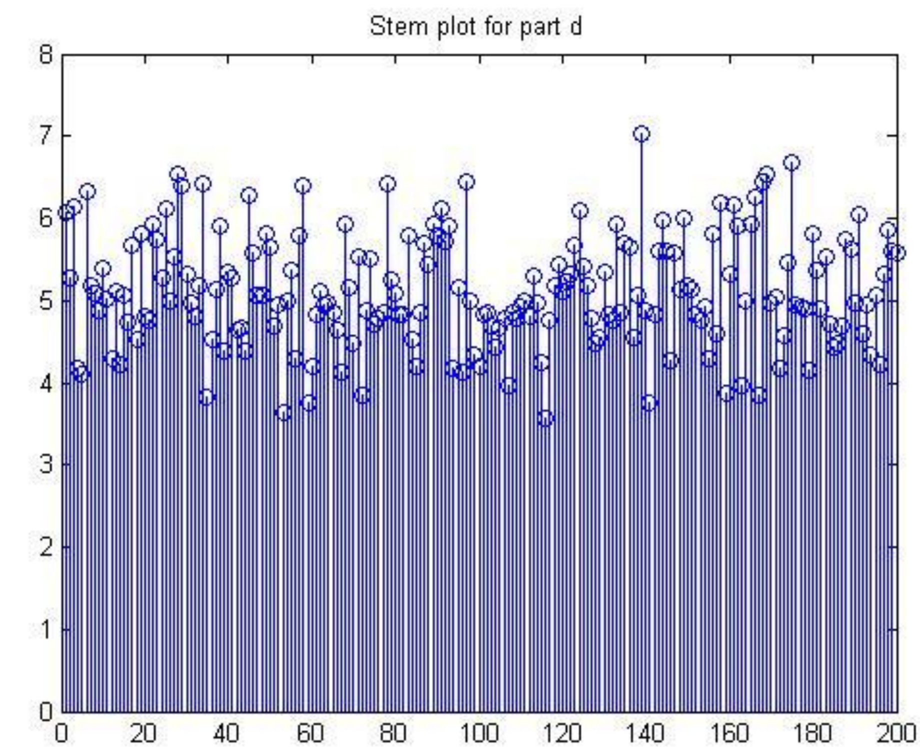
fraction_R = 0.1300

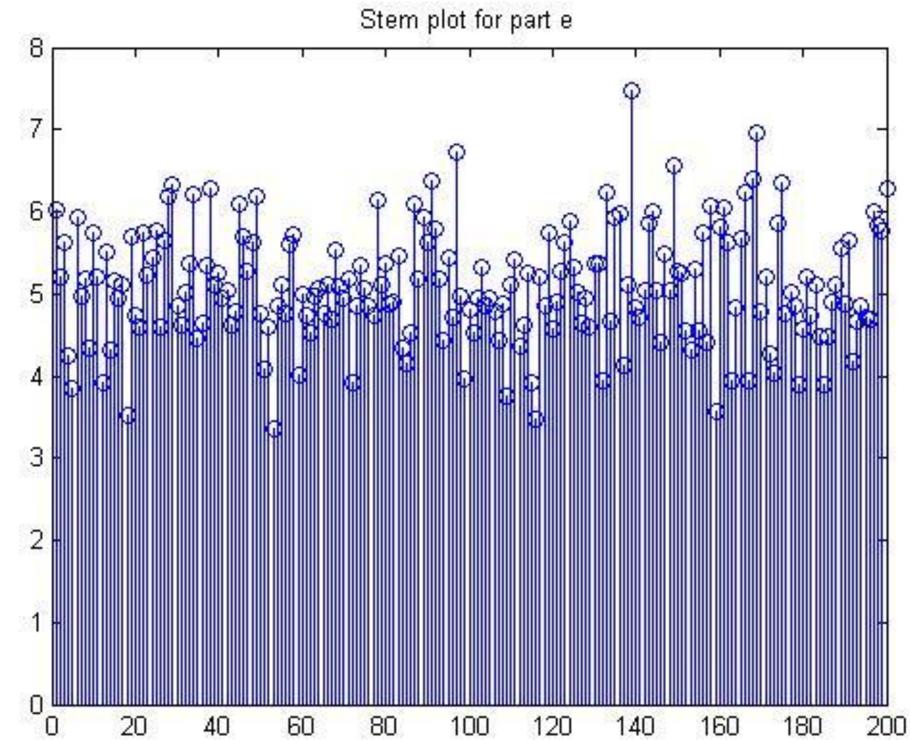
rms_hat = 5.1567

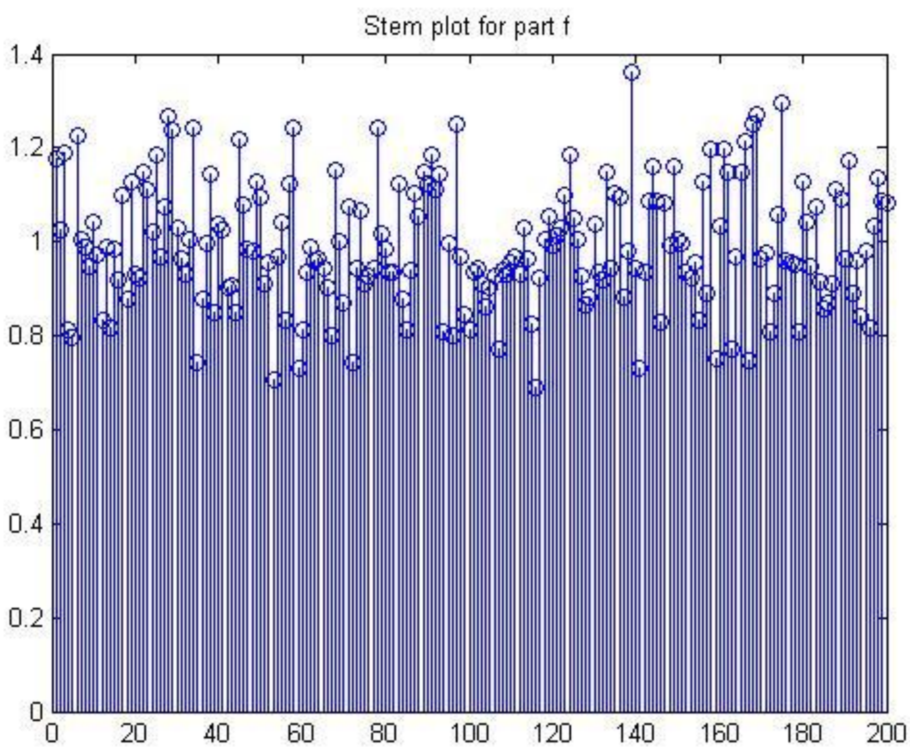
rms_Yn = 1.0000

fraction_Yn = 0


Stem plot for part a
0
0.5
1
1.5
2
2.5
3
0
20
40
60
80
100
120
140
160
180
200


Stem plot for part d
0
1
2
3
4
5
6
7
8
0
20
40
60
80
100
120
140
160
180
200





We can see that Rician fading has less fluctuation than Rayleigh fading.