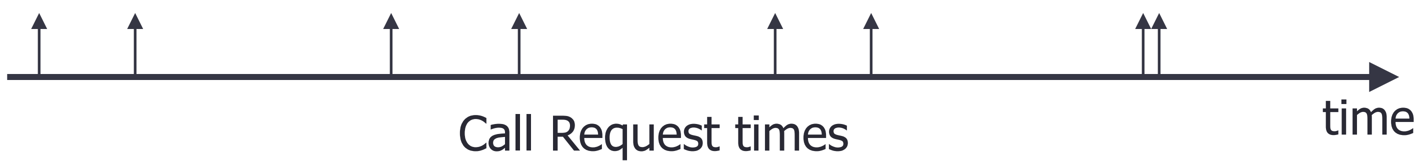
TRUNKING THEORY

An Old Idea

- Trunking theory tells the size of a population that can be served by a limited number of servers with a specified grade-of-service (GOS)
- In the simple case, the GOS is the blocking probability
- Developed in the late 1800s by Erlang

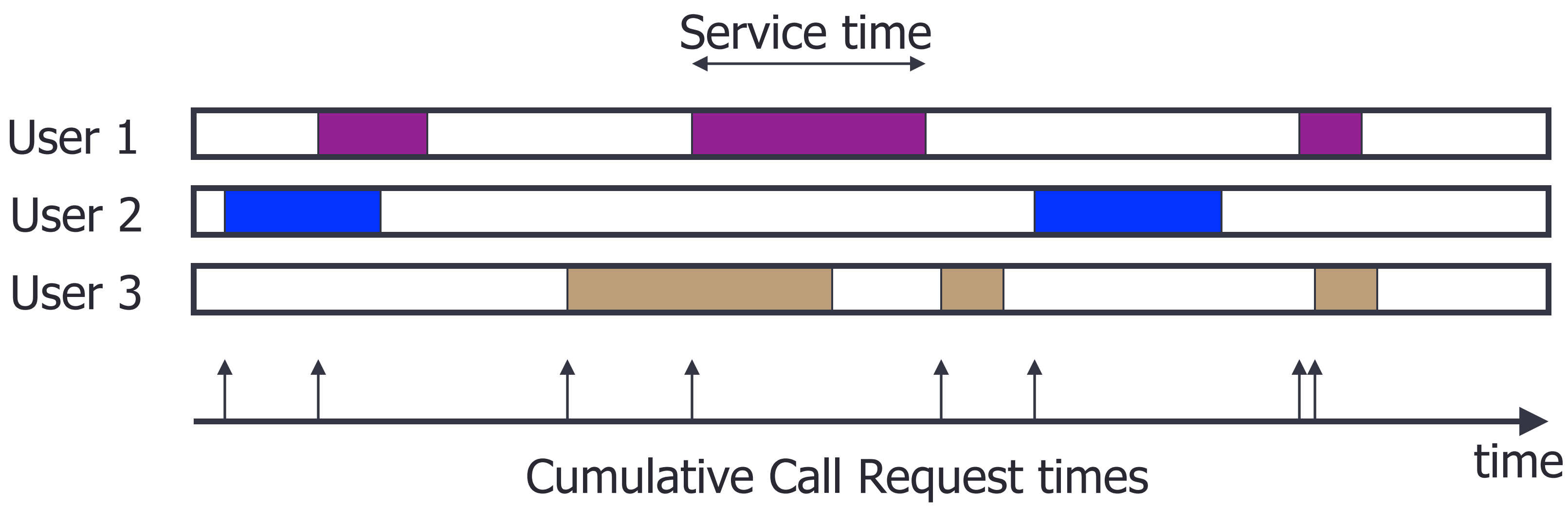
Request Model

- Assume that customers request service at random times at a certain cumulative average rate, λ_{cum}
  - e.g. λ_{cum} =13 requests per hour
- Times between consecutive requests are independent exponential random variables (RVs) with parameter λ_{cum}



Server Model

- The durations of service (i.e. the lengths of the calls or “holding times”) are independent exponential RVs with expected value H



Measuring Traffic Intensity

- An “Erlang” is the average fraction of time that a channel is occupied
- One continuous call is an example of traffic intensity of 1 Erlang
- A channel that carries traffic only half the time carries 0.5 Erlangs of traffic
- For the request and server models in previous slides, the traffic intensity is λH

Single User vs. Total

- Suppose each user generates a traffic intensity of A_u Erlangs
- Suppose there are U users
- Then the total traffic intensity in Erlangs is

$$A = A_u U$$

- Let λ be the rate of call requests per user. Then the traffic intensity per user can be expressed

$$A_u = \lambda H$$

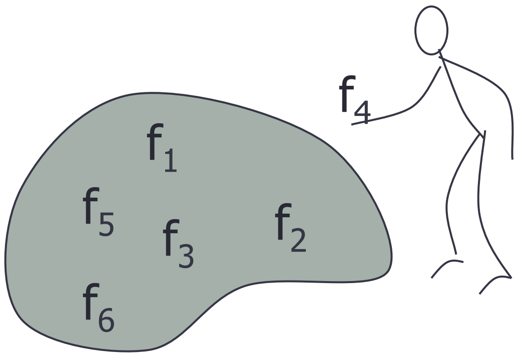
Channels

- Each call requires a channel
- One approach is to dedicate a channel to each user
  - A user's call request is never denied
  - A channel sits idle when its user is not making a call

Not an efficient use of resources!

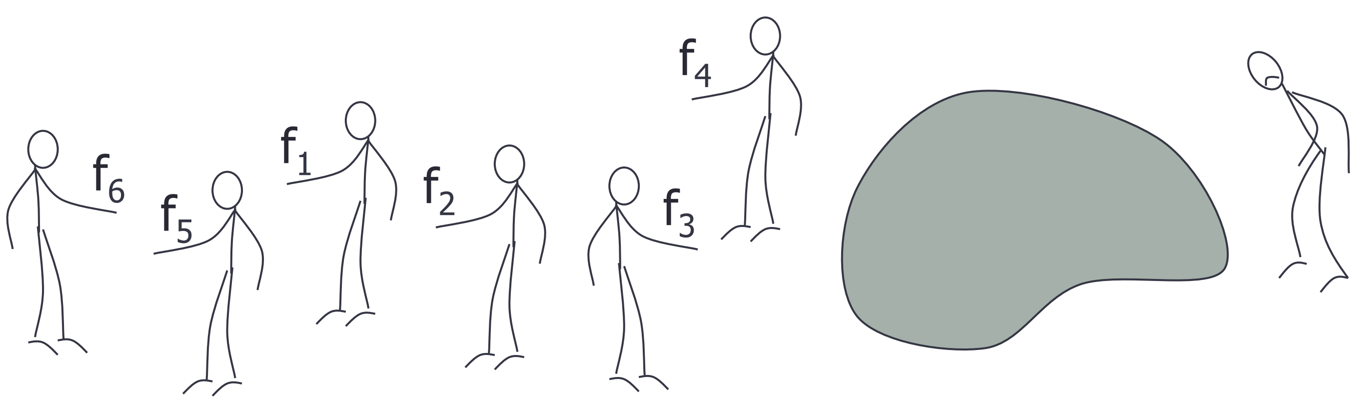
Trunked System

- Channels are "pooled"
- No user has a fixed channel
- A new user is assigned some channel from the pool
- When a call is finished, the channel is released back into the pool



“Block Calls Cleared”

- In a Block Calls Cleared type of system, a call request is simply denied if all channels in the pool are in use
- The blocked caller is free to make a new request
- Mobile Cellular systems are Block Calls Cleared systems



Probability of Blocking

- The GOS measure for Block Calls Cleared systems is the probability that a user's call request is blocked
- The Erlang B formula determines the blocking probability, p, given a certain total offered traffic intensity, A, and a certain number of channels C in the pool

$$p = B(A, C)$$

Assumptions

- There are memory-less arrivals of requests
  - call can be made anytime
- Probability of user occupying a channel is exponentially distributed
  - Longer calls less likely
- Finite number of channels are available

Erlang B Formula

- A is the total *offered* traffic
- Because some calls are blocked, A is not the traffic carried by the system

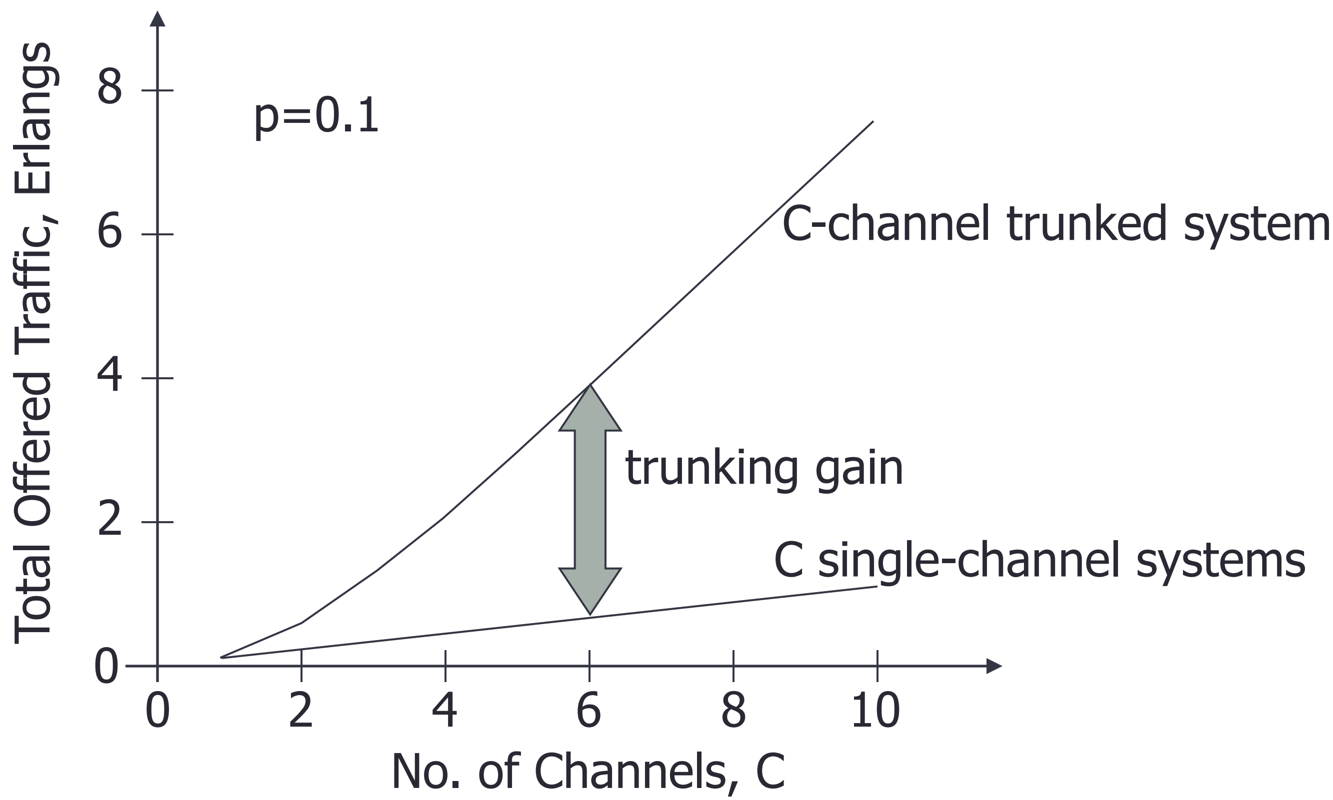
$$p = B(A,C) = \frac{\dfrac{A^C}{C!}}{\displaystyle\sum_{k=0}^{C} \frac{A^k}{k!}}$$

Trunking Gain

- Trunking gain is the improvement in offered traffic intensity that is obtained when sets of channels are merged into trunk pools
- In the next slide, the offered traffic intensities for a 10% blocking probability are compared for a C-channel trunked system and C fixed, single-channel systems

Graphical Comparison

- Sketched from [Hernando and Pérez-Fontán,`99]



Summary

- In a trunked system, channels are pooled for common use on an as-needed basis
- In a Block Calls Cleared system, a new request is simply denied if all channels are busy
- The more channels in the pool, the higher the offered traffic can be for a given probability of blocking

BLOCKED CALLS DELAYED

Blocked (or Lost) Calls Delayed

- In a previous module, we considered Blocked Calls Cleared, and the grade-of-service (GOS) was probability of blocking
- In a Blocked Calls Delayed system, if all channels are busy, a new user goes into a queue to wait for service
- Private mobile radio (PMR) systems are often the Blocked Calls Delayed type [Hernando and F. Pérez-Fontán, 1999]

Erlang C Formula

- The GOS is given by $P(delay>t)$, where t is some specified time limit, like 20 seconds

$$P(delay > t) = P(delay > t \mid delay > 0)P(delay > 0)$$

- $P(delay>0)$ shall be denoted $C(A,N)$, and is given by the Erlang C formula:

$$C(A,N) = \frac{\left(\frac{A^N}{N!}\right)\left(\frac{N}{N-A}\right)}{\sum_{k=0}^{N-1} \frac{A^k}{k!} + \left(\frac{A^N}{N!}\right)\left(\frac{N}{N-A}\right)}$$

Alternative Expression for Erlang C

- $C(A,N)$ can be expressed in terms of the blocking probability $B(A,N)$:

$$C(A,N) = \frac{NB(A,N)}{N - A\left[1 - B(A,N)\right]}$$

Full GOS Formula

- The final expression for the GOS is

$$P(delay > t) = C(A,N)\exp\left[-(N-A)\frac{t}{H}\right]$$

where H is the average call duration.

Summary

- In Blocked Calls Cleared, if all channels are busy, the new user is denied service
- In Blocked Calls Delayed, if all channels are busy, the new user waits for service
- Erlang B is used for the former, Erlang C for the latter

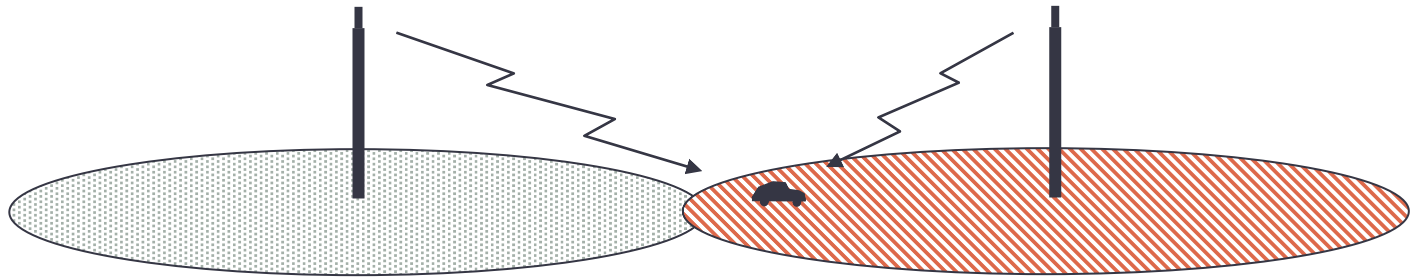
OTHER ISSUES WITH CELLS

Four Topics:

- Hand-offs
- Cell Splitting
- Sectoring
- Power Control

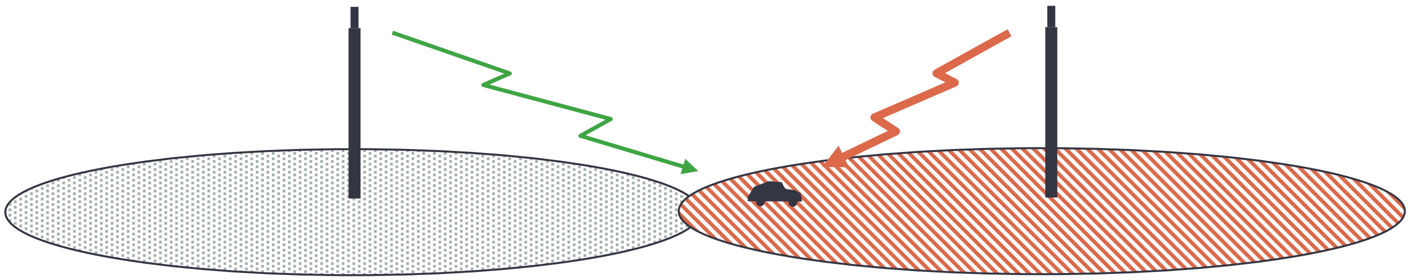
Hand-offs

- As a subscriber moves away from its serving base station, the SIR decreases
- The subscriber compares received powers from the surrounding base stations
- Eventually, the subscriber changes to another serving base station to get a better SIR



Must Change Channels

- Because the adjacent cells use a different set of channels, the carrier frequency must change during a call
- This frequency change is a “hand-off”
- Because of multipath fading, there needs to be a hysteresis to avoid excessive switching

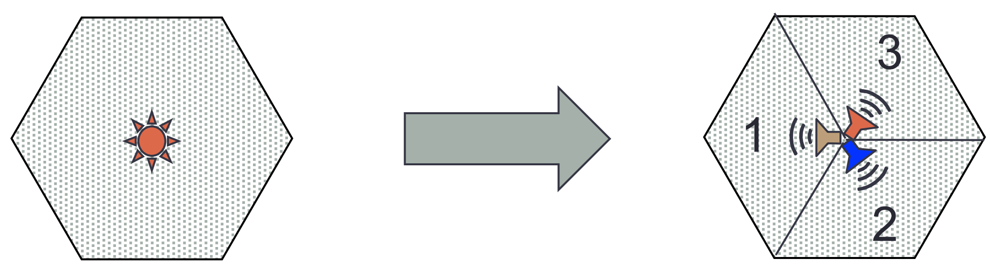


Hand-offs Have Priority

- Since it is bad for an existing call to be dropped, hand-offs have priority over new callers for channel assignments
- For this reason, the Erlang B formula is not quite applicable, but it still provides insight

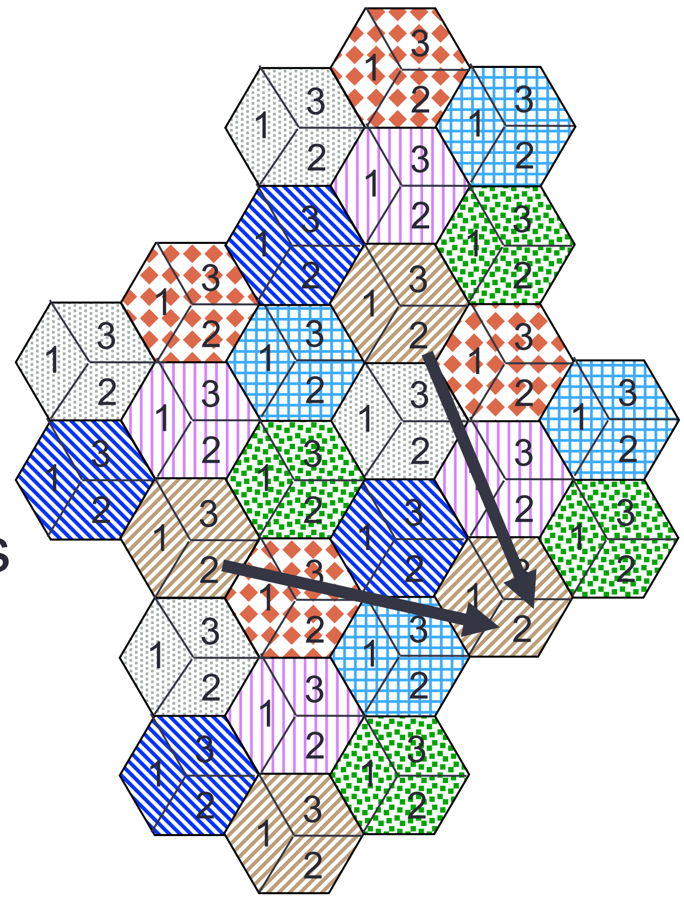
Sectoring

- Sectoring is a way to increase the SIR
- The omni antennas are replaced with 3 or 6 directional antennas
- The channel pool in the hexagonal cell is split into three disjoint pools
- Hand-offs now occur between sectors



Interference Reduced

- The significant interference now comes from only 2 sources instead of 6
- Cluster sizes can be reduced (e.g. from 12 to 7), increasing the number of channels per hexagon
- However, since the channels of a hexagon are split into 3 sectors, there is an overall loss in trunking efficiency



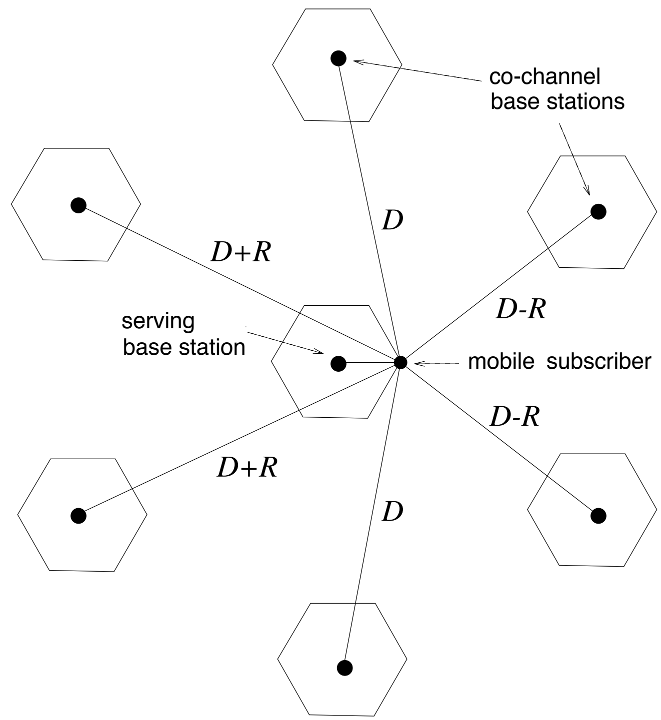
Power Control

- Base stations monitor the power received by subscribers
- The Base Station commands the subscriber to use the lowest power necessary to maintain GOS
- This minimizes interference from other subscribers and is important for CDMA

Summary

- A hand-off is a channel change as a subscriber moves to a new serving base station or to a new sector
- Cell-splitting is a way for a part of a network to accommodate growth
- Sectoring reduces interference and trunking efficiency
- Power control is used to minimize interference on the uplink (from subscriber to BS)

Co-channel Interference



Worst case co-channel interference on the forward channel.

Worst Case Co-Channel Interference

- **There are six co-channel base-stations, two at distance $D-R$, two at distance D, and two at distance of $D+R$.**

- **The worst case carrier-to-interference ratio is**

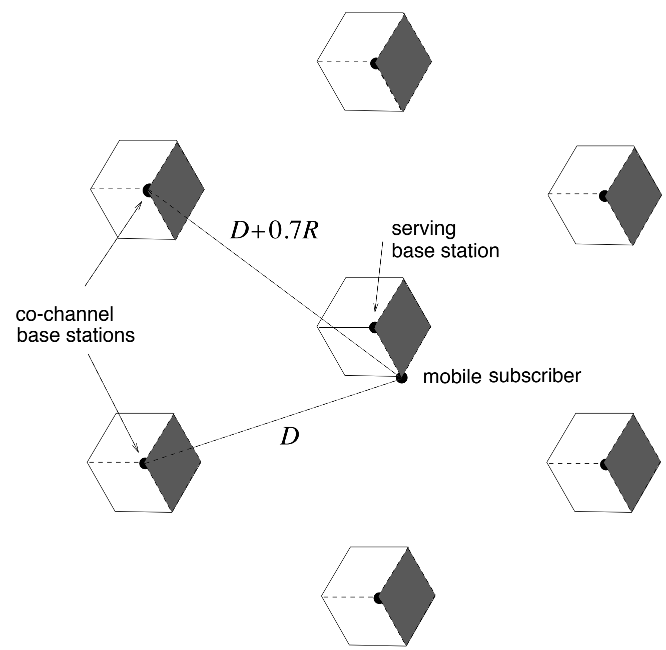
$$\begin{aligned}
\Lambda &= \frac{1}{2}\,\frac{R^{-\beta}}{(D-R)^{-\beta}+D^{-\beta}+(D+R)^{-\beta}} \\
&= \frac{1}{2}\,\frac{1}{\left(\frac{D}{R}-1\right)^{-\beta}+\left(\frac{D}{R}\right)^{-\beta}+\left(\frac{D}{R}+1\right)^{-\beta}} \\
&= \frac{1}{2}\,\frac{1}{\left(\sqrt{3N}-1\right)^{-\beta}+\left(\sqrt{3N}\right)^{-\beta}+\left(\sqrt{3N}+1\right)^{-\beta}}
\end{aligned}$$

- **Hence, for $\beta = 3.5$**

$$\Lambda_{(\mathrm{dB})} = \begin{cases} 14.3 \text{ dB} & \textbf{for } N=7 \\ 9.2 \text{ dB} & \textbf{for } N=4 \\ 6.3 \text{ dB} & \textbf{for } N=3 \end{cases}$$

  - **Shadows will introduce variations in the worst case C/I.**

Cell Sectoring



Worst case co-channel interference on the forward channel with 120° cell sectoring.

- 120° **cell sectoring reduces the number of co-channel base stations from six to two. The co-channel base stations are at distances** D **and** $D + 0.7R$**.**

- **The carrier-to-interference ratio becomes**

$$\begin{aligned}\Lambda &= \frac{R^{-\beta}}{D^{-\beta} + (D+0.7R)^{-\beta}} \\ &= \frac{1}{\left(\frac{D}{R}\right)^{-\beta} + \left(\frac{D}{R}+0.7\right)^{-\beta}} \\ &= \frac{1}{\left(\sqrt{3N}\right)^{-\beta} + \left(\sqrt{3N}+0.7\right)^{-\beta}}\end{aligned}$$

- **Hence**

$$\Lambda_{(\mathrm{dB})} = \begin{cases} 21.1 \text{ dB} & \textbf{for } N = 7 \\ 17.1 \text{ dB} & \textbf{for } N = 4 \\ 15.0 \text{ dB} & \textbf{for } N = 3 \end{cases}$$

- **For** $N = 7$, 120° **cell sectoring yields a 6.8 dB** C/I **improvement over omni-cells.**

- **The minimum allowable cluster size is determined by the minimum** C/I **requirement of the radio receiver. For example, if the radio receiver can operate at** $\Lambda = 15.0$ **dB, then a 3/9 reuse cluster can be used (3/9 means 3 cells or 9 sectors per cluster).**