CIS Software Manual

Release v4.0

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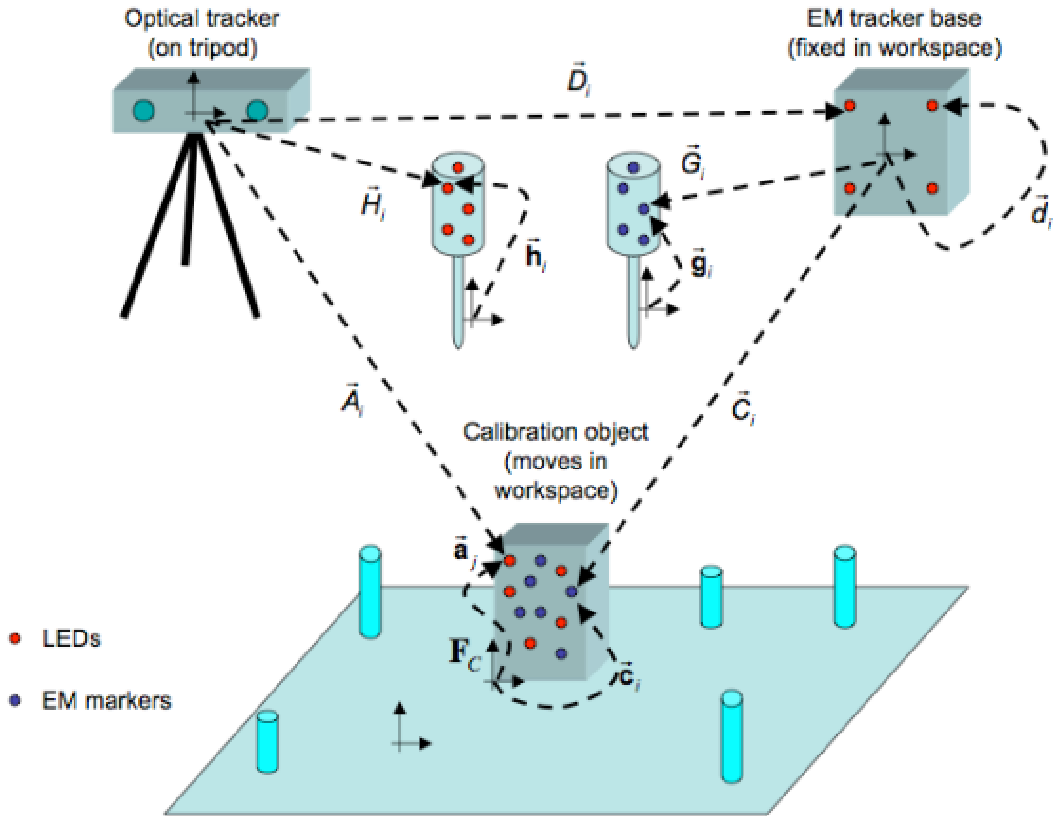
Contents

1 Introduction

1.1 PA1

The purpose of PA1 was to develop an algorithm for a 3D point set to 3D point set registration and a pivot calibration. The problem involved a stereo tactic navigation system and an electromagnetic positional tracking device. Tracking markers were placed on objects so the optical tracking device and an electromagnetic tracking device could measure the 3D positions of objects in space relative to measuring base units. These objects were then registered so that they could be related in the same coordinate frames. Pivot calibration posts were placed in the system so pivot calibration could be performed and the 3D position of two different probes could be tracked throughout the system. The diagram below from the assignment document gives a visual description of the system.

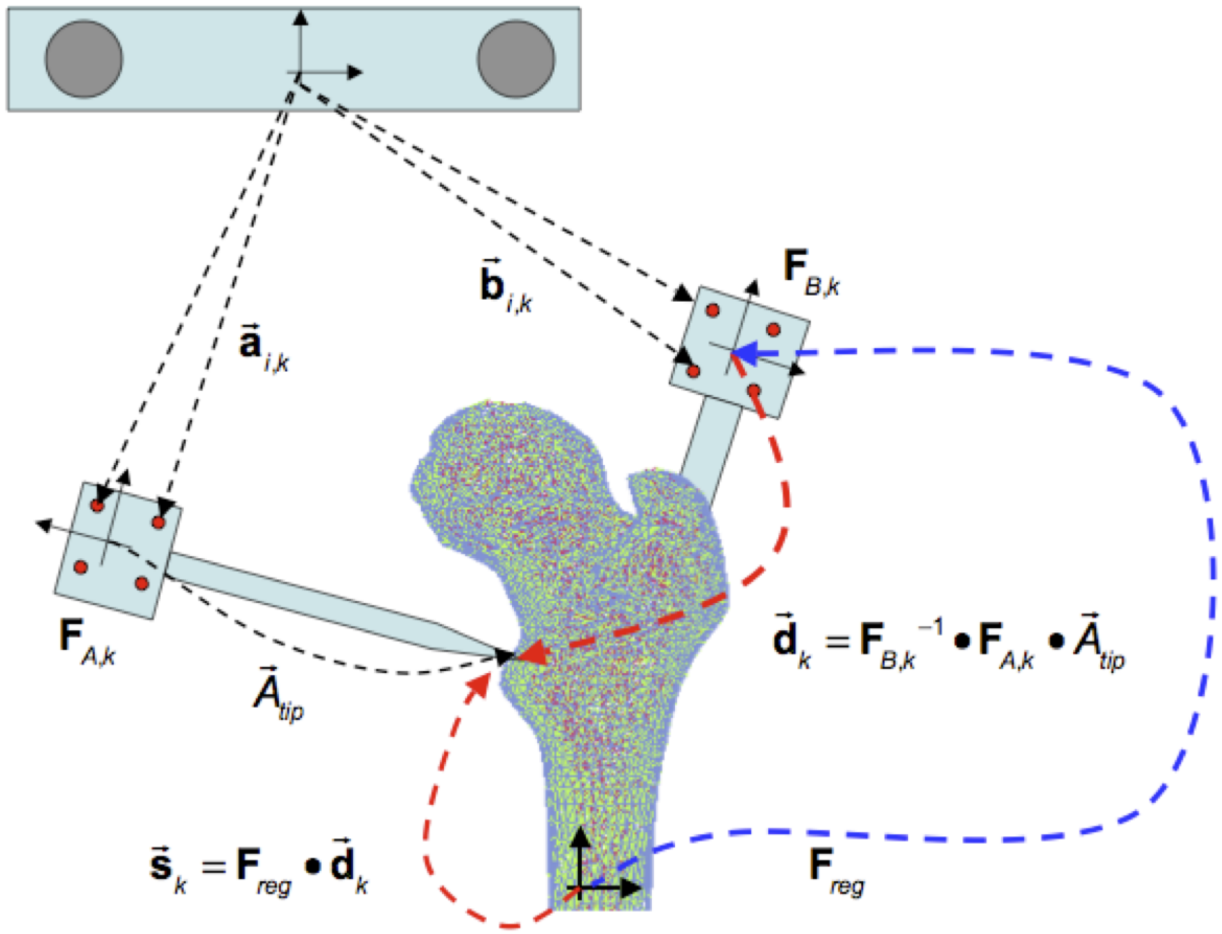


1.2 PA2

In addition to all of the steps outlined in PA1, the core purpose of PA2 was to develop an algorithm for distortion correction and to implement it for use with the stereo tactic navigation system of PA1. In addition to distortion correction, we performed registration of the device coordinate frames to prior CT coordinate frames.

1.3 PA3

The purpose of PA3 was to develop the find closest point step of an iterative-closest point registration algorithm. The problem involved a 3D triangular surface mesh of a bone found in CT coordinates and two rigid bodies. One rigid body is rigidly attached to the bone and one to be used as a pointer. LED markers were attached to the two rigid bodies so that the coordinates could be determined in optical coordinates. The closest point on the triangular mesh to a number of points where the tip of the pointer contacted the bone was found using our find closest point algorithm. The diagram below from the assignment document gives a visual description of the system.


$\vec{b}_{i,k}$
$\mathbf{F}_{B,k}$
$\vec{a}_{i,k}$
$\mathbf{F}_{A,k}$
$\vec{A}_{tip}$
$\vec{d}_k = \mathbf{F}_{B,k}^{-1} \bullet \mathbf{F}_{A,k} \bullet \vec{A}_{tip}$
$\vec{s}_k = \mathbf{F}_{reg} \bullet \vec{d}_k$
$\mathbf{F}_{reg}$

1.4 PA4

The purpose of PA4 was to build off the find closest points algorithm from PA3 and implement an iterative-closest point registration (ICP) algorithm. The closest point on the triangular mesh to the points where the tip of the pointer contacted the bone was found iteratively until the error threshold was reached. In this way, the error between these points could be minimized.

2 Mathematical Approach

2.1 Distortion Correction

A number of distortion correction methods are available to correct for inaccuracies among various sensor coordinate systems and the real physical dimensions of the world. We selected Bernstein polynomials for our implementation due to their numerical stability and accuracy for the specific electromagnetic distortion problem we encounter. The basic idea in this case is to construct a 3D polynomial representing the spatial flexing caused by distortions in measurements.

To reap the best of the numerical stability properties of the Bernstein polynomial we scale the input values to the range from 0 to 1. Therefore we scale the values to within the range [0,1] in each dimension, utilizing the minimum bounding rectangle (MBR) to determine the scale factor. Then, we construct a 5th degree Bernstein polynomial for each point using the polynomial function outlined in slide 18 of the InterpolationReview.pdf lecture notes pictured below.

$$B_{N,k}(v) = \binom{N}{k}(1-v)^{N-k}v^k$$

We then stack the polynomials to form the F Matrix, although this polynomial can be increased for higher precision or decreased for higher performance as needed. Once we have these polynomials stacked as a large matrix we solve the least squares problem utilizing SVD against the ground truth data, as outlined on slide 43 of the lecture notes pictured below.

$$\begin{bmatrix} & \vdots & \\ F_{000}(\vec{\mathbf{u}}_s) & \cdots & F_{555}(\vec{\mathbf{u}}_s) \\ & \vdots & \end{bmatrix}\begin{bmatrix} c^x_{000} & c^y_{000} & c^z_{000} \\ \vdots & \vdots & \vdots \\ c^x_{555} & c^y_{555} & c^z_{555} \end{bmatrix} \cong \begin{bmatrix} & \vdots & \\ p^x_s & p^y_s & p^z_s \\ & \vdots & \end{bmatrix}$$

The output of that equation is the calibration coefficient matrix which can then be multiplied by the stacked F matrix of a distorted point set to generate the final corrected and undistorted point set. Multiplication of a stacked matrix is a more efficient alternative to the loop of sums from the slides.

Tradeoffs

One of the particular advantages of Bernstein polynomials is the ability to select the degree of the polynomial. The polynomial degree presents an interesting trade off, because a higher degree polynomial allows more precise representation of distortions and lower error. This benefit comes at the cost of an exponential increase in computation time for each additional polynomial degree.

2.2 Point Cloud Registration

A number of least squares methods could be used to determine a transformation matrix for a 3D point set registration. We selected Horn's method because a rotation matrix is always found and no iterative approximation is involved. The first step is to find the centroid of the point clouds in the two different coordinate systems. Then the centroid is subtracted from each point measurement of the separate point clouds so the points will be relative to the centroid. Next an H matrix is created which is the sum of the products of each corresponding point in the two frames. A real symmetric G matrix is then created from the sums and differences of the elements in the H matrix which was previously created. Next, the eigenvalues and corresponding eigenvectors of the G matrix were calculated. The eigenvector corresponding to the most positive eigenvalue represents the unit quaternion of the matrix. Once the quaternion is known, the rotation matrix can be found using Rodriguez's formula. The translation between the two coordinate systems is next found from the difference between the centroid of the known point cloud and the scaled centroid of the unknown point cloud. Finally a homogeneous transformation matrix could be made to know the frame transformation between the two point clouds.

2.3 Pivot Calibration

For the pivot calibration, singular value decomposition was used to estimate the orientation of the probe by finding the positions of the centroid of the tracked markers on the probe and the tip of the probe. First, a matrix was created which consisted of the rotation matrices calculated in each frame and the negative identity matrix found using Horn's method.[1] Next, a vector was created which consisted of the stack of the translation vectors in each frame also found using Horn's method. The singular value decomposition of the matrix was performed to split the matrix into the matrices containing the singular values, the left-singular vectors, and the right singular vectors. Once this was done, the vector between the centroid of the tracked markers on the probe and the probe tip could be approximated using the SVD matrices and the translation vectors of each frame.

- [1] Horn, Closed-form solution of absolute orientation using unit quaternions, Optical Society of America (1987)

2.4 Finding the Closest Point on a Triangle

If the vertices of a triangle are know and there is a point in space, then the closest point that lies on the triangle to the point in space can be found. This is done by using the equations given below:[2]

$$\mathbf{u} = P2 - P1$$

$$\mathbf{v} = P3 - P1$$

$$\mathbf{w} = P - P1$$

$$\mathbf{n} = \mathbf{u} \times \mathbf{v}$$

Where P is the point in space, P1, P2, and P3 are the vertices of the triangles. Then the closest point on the triangle P' can be found using Barycentric coordinates listed below:[2]

$$\gamma = [(\mathbf{u} \times \mathbf{w}) \cdot \mathbf{n}]/\mathbf{n}^2$$

$$\beta = [(\mathbf{w} \times \mathbf{v}) \cdot \mathbf{n}]/\mathbf{n}^2$$

$$\alpha = 1 - \gamma - \beta$$

$$P' = \alpha P_1 + \beta P_2 + \gamma P_3$$

If alpha, beta, and gamma are all between zero and one, then the P' is the closest point and lies within the triangle's boundaries. If not, the closest point lies on the boundaries of the triangle and the point P must be projected onto every side of the triangle. The equations (from the Finding point-pairs lecture page 8) below how this is implemented:

$$\lambda = \frac{(P - X) \cdot (Y - X)}{(Y - X) \cdot (Y - X)}$$

$$\lambda^* = Max(0, Min(\lambda, 1))$$

Where X and Y are the two end points (or vertices) of the line segment, P is the point to be projected on the line segment, P* is the projected point on the line segment, and lambda* is the ratio of normalized length from X to P*. If two of the three P* projections lie on the same vertex, then the closest point on the triangle is that vertex. Otherwise, the closest point will be the P* projection on the side whose value for lambda* satisfies the conditions of being between one and zero. Then the equation (from the Finding point-pairs lecture page 8) below is implemented to find the closest point P*:

$$P^* = P + \lambda^*(Y - X)$$

- [2] W. Heidrich, Journal of Graphics, GPU, and Game Tools,Volume 10, Issue 3, 2005.

2.5 ICP

For every point on the bone where the probe tip was placed, the find the closest point method was implemented for every triangle on the mesh. Once the closest point on the mesh was found, the error between the two points was computed by taking the norm. Then the point on the triangle with the smallest error was said to be the closest point on the mesh. Next, a new transformation between the points on the bone and the point on the mesh is found using the Point Cloud Registration method. Then the process is repeated using new points from the new transformation to minimize the error between the points on the bone and the points on the mesh.

3 Algorithmic Approach:

3.1 Parsing

We developed our algorithm using C++. The Eigen library was used as a Cartesian math package for 3D points, rotations, and frame transformations. The Boost library was also used to write a parser file and develop various aspects of our algorithms. The first step was to write parser code that could interpret the given data. The parser needed to interpret which data set was being entered, the number of frames in each data set, and which markers were being tracked in the data set. The parser would store the data as Eigen matrices to be easily used for our algorithms.

3.2 Transforms

Once the data was parsed, two matrices containing marker positions in different coordinate frames was put in the function hornRegistration to determine the corresponding transformation matrix between the two frames. The first step of the hornRegistration was to find the two centroids of two 3D marker positions and subtract it from each marker position using functions in the Eigen library. The next step was to put these values in a function that would create a 3x3 H matrix. Once this was done, the H matrix could be put in a separate function that would calculate the 4x4 G matrix. The eigenvalues and the corresponding eigenvectors of the G matrix were next calculated by using functions of the Eigen library. A vector of each eigenvalue and the corresponding eigenvector was then created so that the eigenvalues could be sorted to find the most positive eigenvalue and its corresponding eigenvector which represented the unit quaternion of the rotation. Next, the 3x3 rotation matrix was created by an Eigen function that converted a unit quaternion into the corresponding rotation matrix. Finally, the translation vector between the two centroids was calculated and a 4x4 homogeneous transformation matrix was created by using another function that takes a rotation matrix and a translation vector and outputs the corresponding transformation matrix.

3.3 Pivot Calibration

Next a pivot calibration algorithm was created which used both the parser and hornRegistration algorithms mentioned above. First, the tracker data was parsed into separate matrices which corresponded to each frame of tracked data. Each matrix of frame data was compared to the base matrix frame using the hornRegistration function described above and the corresponding homogeneous transformation from the base frame to the current frame was found. The rotational component of each frame was put into an Eigen matrix and the translational component of each frame was put into an Eigen vector with the form described in the mathematical approach above. The function of JacobiSVD of the Eigen library was then used to solve the least squares vector between the rotational matrix and translation vectors. The least squares vector contained approximated orientation of the probe and the position of the probe tip.

3.4 Distortion Calibration

Next we create a distortion calibration algorithm, which followed the mathematical procedure outlined above. First, the data was parsed and stored in a large vector so the the maximum and minimum values could be obtained in the X, Y, and Z dimensions of the data set. Then the values of the data set were scaled to between [0 1] to create a minimum bounding box. We calculate Bernstein polynomials for each point and stack them into the F matrix. The Eigen library is utilized to calculate the SVD of Fc=p, where F is the F matrix of Bernstein Polynomials, c is the calibration coefficient matrix, and p is the undistorted points matrix that you compare the distorted points to. A separate set of points can be scaled according to the same distortion parameters from above and the distortion associated with their measurement can be corrected.

Once this is done, an the same Fmatrix calculated above can be calculated for this new set of points. Then the c calibration coefficient matrix was multiplied by the Fmatrix to find the undistorted points in the new coordinate system. Once the data was undistorted, we were able to run a new pivot calibration with the undistorted points to see how well our undistortion works. The next step was to use the distortion matrix to find CT fiducial in the EM frame. The new data was scaled by the same scaling function as above and put into a new Fmatrix. In this way, a new Fmatrix could be found and the measured points could be undistorted. Then these values were used with known values of points measured in the CT frame to find a transformation matrix Freg that would take you from the EM frame to the CT frame. Finally the tip of the EM probe could be measured in the CT frame.

3.5 Find Nearest Point

Next a Find Nearest Point algorithm was created which used both the parser and hornRegistration algorithms mentioned above. First, each point of tracker data was parsed into Eigen vectors of (x,y,z) coordinates which corresponded to the position of the trackers attached to the rigid bodies, A and B, in optical coordinates. Next, another set of tracker data was parsed into Eigen vectors of (x,y,z) coordinates which corresponded to the position of the trackers attached to the rigid bodies in their body coordinates. The transformation matrix from the body frame to the optical tracker frame was then computed using the hornRegistration function described above. Then the coordinates tip of the rigid body A with respect to rigid body B was found by multiplying the vector of the tip in body A coordinates by the transformations previously found.

Next, the mesh data was parsed so the vertices of each triangle was known. Then ICP registration could be used to find the point on the mesh that was closest to the tip of rigid body A. First, the transformation from the CT mesh coordinates to the rigid body B coordinates was assumed to be the identity matrix. Once this assumption was made, sample points were found by multiply the transformation from CT mesh coordinates to the rigid body B coordinates by the tip of the pointer A in rigid body A coordinates. Now these sample points were used to which points on the CT mesh they were closest to with the given transformation. The simplest FindNearestPoint function was implemented in which the the nearest point to the sample points on the CT mesh was calculated for every triangle in the mesh. The error between the two points for each triangle was calculated by taking the norm between the points and the smallest error corresponded to the nearest point on the mesh to the pointer tip A.

3.6 ICP

The ICP algorithm continuously ran the Find Nearest point algorithm until the error criteria was reached. On the first iteration, the transformation from the CT mesh coordinates and the rigid body B coordinates was assumed to be the identity matrix. Then on subsequent iterations, a new estimate of the transformation was found using a hornRegistration from the rigid body B coordinates to the closest points found on the CT mesh found in the previous iteration. Once the transformation was found, the sample points were computed the same way as in the Find Nearest Point Algorithm and the steps were continued until new closest points on the mesh were found. The error calculated in each iteration was compared to the error criteria and if the error calculated was below the criteria, then ICP algorithm was stopped and the new closest points and transformation were assumed to be a good estimate.

3.7 Spatial Indexing

Additionally, an alternative ICP algorithm was implemented using the Boost.Geometry Spatial Index library , which contains an r-tree implementation. The preferred mechanism to accelerate search using a SpatialIndex is to insert each triangle into the index, then query the r-tree for the nearest triangle to a given point. This substantially speed up all data access by reducing access time for an individual element from O(n) to O(M log_M(n)), where M is the number of entries in a node.

However, this data structure does not yet have direct polygon insertion implemented. Instead the minimum bounding rectangle, or bounding box, of each triangle is inserted into the index with a reference to the underlying triangle. From this, the nearest bounding boxes are queried and visited in order from nearest to furthest, and the underlying polygon distance is checked. As soon as a bounding box distance is reached that is entirely further away than the nearest polygon, the search is stopped and the ICP algorithm proceeds as normal. This is necessary because the bounding boxes of different triangles can overlap, returning results that are near to but not the actual closest triangle to the point. ultimately this is a substantially accelerated lookup of the closest point on the mesh.

4 Structure of the Program

The software is structured as a set of header only libraries in the include folder, which are utilized by the unit tests, main, and any external libraries that choose to use these utilities.

The most important files include:

File name	Description
IterativeClosestPoint.hpp	Algorithm for finding ICP registration.
hornRegistration.hpp	Horn's method of Point Cloud to Point Cloud registration.
DistortionCalibration.hpp	Bernstein Polynomial method of distortion correction.
PivotCalibration.hpp	Pivot Calibration.
PA2.hpp	**fiducialPointInEMFrame**() and **probeTipPointinCTFrame**() PA2 #4,6
cisHW1test.cpp	An extensive set of unit tests for the library relevant to PA1.
cisHW2test.cpp	An extensive set of unit tests for the library relevant to PA2.
cisHW3test.cpp	An extensive set of unit tests for the library relevant to PA3.
cisHW1-2.cpp	Main executable source, contains cmdline parsing code and produces output data.
cisHW3-4.cpp	Main executable source, contains cmdline parsing code and produces output data.
parseCSV...	File parsing functions are in **parseCSV_CIS_pointCloud.hpp**.

4.1 Important Functions and Descriptions

Each function includes substantial doxygen documentation explaining its purpose and usage. This documentation can be viewed inline with the source code, or via a generated html sphinx + doxygen website generated using CMake. Here is a list of the most important functions used in the program is a brief description of each of them.

PA 1

EigenMatrix()

Computes the eigenvalues and corresponding eigenvectors from a given G matrix. It outputs a rotation matrix corresponding to the unit quaternion of the largest positive eigenvalue

homogeneousmatrix()

Creates a 4x4 homogeneous matrix from a derived rotational matrix and translational vector

hornRegistration()

Computes the homogeneous transformation matrix F given a set of two cloud points. It is comprised of the various functions listed above

homogeneousInverse()

Computes the inverse of a given homogeneous matrix

registrationToFirstCloud()

Parses the data and runs the hornRegistration function for pivot calibration

transformToRandMinusIandPMatrices()

Creates the A and b components of the form Ax=b for singular value decomposition. A is of the form [R|-I] while b is of the form [-p] where R is the stack of rotational matrices of the F transformation matrices, I is stack of 3x3 identity matrices, and p is the stack of the translational vectors of the F transformation matrices.

SVDSolve()

Computes the x of the least squares problem Ax=b using singular value decomposition when the stack of matrices in given

Hmatrix()

Computes a sum of the products H matrix given a set of two cloud points

Gmatrix()

Computes a sum of the differences of the given H matrix

pivotCalibration()

Computes the pivot point position from tracking data using the SVDSolve(), registrationToFirstCloud(), and transformToRandMinusIandPMatrices() functions

PA 2

CorrectDistortion()

Correct distortions in one point cloud by utilizing distorted and undistorted versions of a second point cloud. Bernstein Polynomials are utilized to perform the correction.

BernsteinPolynomial()

Find the solution to the Bernstein polynomial when at varying degrees and points depending on the input.

Fmatrix()

Multiplies the Bernstein polynomial into a matrix so that a function of every degree of i, j, and k are found and a distortion calibration can be done using the matrix.

ScaleToUnitBox()

Calculates maximum and minimum values in the X,Y, and z coordinates of a point cloud and then normalizes the value of every single point.

probeTipPointinCTF()

Uses measured positions of EM tracker points on the EM probe in the EM frame when the tip is in a CT fiducial and returns the point of the fiducial dimple (solves problem 5).

fiducialPointInEMFrame()

Uses measured positions of EM tracker points on the EM probe in the EM frame when the tip is in a CT fiducial and returns points of the CT fiducial locations in EM frame.

PA 3

ICPwithSimpleSearchStep()

Primary function implementing the first iteration of the simple search ICP algorithm.

ICPwithSpatialIndexStep()

Primary function implementing the first iteration of the spatial index ICP algorithm to increase efficiency.

dkKnownMeshPointsBaseFrame()

Finds the location of of Atip in fiducial body B coordinates (dk) using hornRegistration.

FindClosestPoint()

Finds the closest point on the triangle to a point in space. If the closest point lies with in triangle, then the function finds the nearest point internally. Else if the closest point lies on an edge or vertex, the function OutsideOfTriangle() is called to find the nearest point.

OutsideOfTriangle()

Finds the closest point on the triangle to a point in space if the closest point lies on an edge or vertex

ProjectOnSegment()

Finds the nearest point on a line segment to a point in space. Called by the function OutsideOfTriangle() to determine where the nearest point is to each side of the triangle.

PointEqualityCheck()

Determines if two points are equal. Used by the function OutsideOfTriangle to determine if the nearest point on the triangle lies on a vertex

PA 4

ICPwithSimpleSearch()

ICP algorithm that iterates the simple search of the FindClosestPoint algorithm until an error threshold is reached.

ICPwithSpatialIndex()

ICP algorithm that iterates the spatial index of the FindClosestPoint algorithm until an error threshold is reached. The spatial index is used to increase efficiency of the ICP algorithm.

TerminationCriteria

TerminationCriteria is a C++ Class implemented to evaluate if the current iterations of the ICP algorithm is within our desired error threshold, and to collect statistics on the error as ICP progresses.

5 Results and Discussion

5.1 Validation

We took several approaches to the validation of our software. These include manual and automatic execution of the supplied test data, the implementation of unit tests to verify the data, and initial integration of continuous integration software to catch errors early. We implemented a battery of unit tests to verify the basic functions and ensure they are running correctly.

Point Cloud Registration

We have been able to ensure that point cloud to point cloud registration is working correctly by finding the transformation of one point cloud to another and then the opposite. Multiplying these two transformation matrices together resulted in an identity matrix which would be expected. We tested the input data set as well, ensuring that we were within the given tolerance range. This shows the strength of Horn's method and since it requires no special case exceptions for a solution, we concluded it was the best method of the one's taught in class.

Calibration

The position of the tip of the probe when calibrate by EM also gave us results well within our tolerance levels. Our results were less accurate when error was introduced, but not to an unreasonable degree.

Finding the Closest Point on a Triangle

We have also been able to ensure that finding the closest point on a triangle algorithm is working correctly by assigning vertices to an arbitrary triangle and then testing points in space where we knew what the closest point on the triangle was. We tested the different special cases of the problem as the closest point lying within the boundaries of the triangle, on one of the sides of the triangle, and on one of the vertices of the triangle. Our algorithm was able to return the nearest point for every case.

6 Status of results

6.1 PA 2

We have encountered errors in our software that we have narrowed down to points after the EM distortion calibration steps, because we have been able to verify our Bernstein functions using unit tests and debug data. However, a bug remains in either the steps for calculating Freg or finding each of the CT fiducial. Since the underlying components are largely well tested, we expect the bug to be in the transform or data flow steps of the generateOutputFile() function in cisHW1-2.cpp or the function definitions in PA2.hpp.

6.2 PA 3

Our initial algorithm passed our unit tests and gave us similar errors to the output data files. We concluded that we were able to successfully implement the first iteration of an ICP registration. Although the simplest linear method was implemented, our program was able to run the program of all debug and unknown data sets in less than thirty seconds even in debug mode. Adding threads and running in release mode made our program run even faster. A data structure was not needed for this assignment because we concluded that there was no problem with the speed of the program.

For PA 4, we will re-evaluate this conclusion as the ICP will need to iterate multiple times, greatly increasing our runtime.

Tabular Summary of PA 3 Unknown Data Results

Point Number	Error in G Data	Error in H Data	Error in I Data
1	0.14811	0.77494	2.84084
2	0.60432	0.96266	0.46215
3	2.24293	0.67760	4.43498
4	0.76945	0.24216	1.77055
5	1.45001	0.51152	0.40509
6	0.80641	1.54260	0.10251
7	0.86017	0.24527	0.30031
8	2.61080	0.72909	0.79210
9	2.10517	0.48146	2.54912
10	0.96220	0.30248	2.07283
11	0.00198	1.43465	5.54007
12	2.45932	0.01729	1.06041
13	1.47099	1.38414	2.93195
14	1.34390	0.42595	1.44610
15	0.41964	2.43422	1.83156
16	1.44879	0.13935	0.57452
17	0.01526	0.29749	0.47596
18	1.96765	0.88617	0.87031
19	1.30039	0.65453	3.82845
20	1.02220	0.18214	1.17058
Average Error	**1.20048**	**0.71629**	**1.77302**
Max Error	**2.61080**	**2.43422**	**5.54007**

The error in debug data sets is consistent with the error in the output files. On the unknown output, the overall error bounds seem reasonable when compared to the debug data sets. We expect our algorithms are not the most substantial source of error in these results and instead are attributed to noise or other sources of error.

6.3 PA 4

Performance

We took several approaches to performance optimization. First, we ensured there are instructions for building and executing the code in release mode with high performance settings for fast execution. We also ensured all of our functions were implemented with high performance in mind, utilizing the Eigen C++ library functions and other cases where vectorization of data allows higher performance. Then, we implemented threading so that all data sets can be executed simultaneously, which resulted in an approximately 8 fold speed up for computers with multiple processors. These performance criteria allow the application to execute all the data sets in 35.5 seconds on a 2014 Intel Core i7 processor at 2.5 Ghz. This performance was achieved with simple search. Spatial Indexing provides a substantial additional performance boost on top of optimized simple search, executing all the data sets in 2.41s on a 2014 Intel Core i7 processor at 2.5 Ghz.

Stopping Error Criteria

We implemented several different stopping criteria for our ICP algorithm. The first was a minimum iteration so that the ICP had to run a minimum number of iterations before it could be stopped. The second was a mean error criteria so that the ICP would not stop iterating unless the mean error was sufficiently low. The third was a max error criteria so that the ICP would not stop iterating unless the maximum error criteria was sufficiently low. The next criteria computes a rolling variance in the mean error at each iteration. Effectively, this allows the algorithm to stop correctly for a variety of data sets when error improvements at each iteration have diminished, regarless of what ideal minimum is possible. This criteria produced the best results for stopping correctly across a variety of data sets. The final criteria was a maximum iteration so that the ICP would not just keep running forever if the error never met any of the previous criteria. Each error criteria is checked after each iteration by the TerminationCriteria object to determine if the ICP should be stopped.

Tabular Summary of PA 4 Results

File	Error Mean	Error Max	Error Variance
PA4-A-Debug	0.00197113	0.00717058	2.23062e-06
PA4-B-Debug	0.00186124	0.00787022	2.47282e-06
PA4-C-Debug	0.00176908	0.0069919	2.21001e-06
PA4-D-Debug	0.00324727	0.0133826	6.58918e-06
PA4-E-Debug	0.0668846	0.275337	0.0031848
PA4-F-Debug	0.0593784	0.256141	0.00257083
PA4-G-Unknown	0.00332878	0.0149228	7.39938e-06
PA4-H-Unknown	0.00364877	0.0118835	7.24306e-06
PA4-J-Unknown	0.0650882	0.337438	0.00308615
PA4-K-Unknown	0.0665567	0.259723	0.00288765

The mean, max, and variance of the errors found at the end of the ICP algorithm were low for all data sets. Also our error results from the debug data sets were similar to those of Dr. Taylor's output files. From these observations, we concluded that our final Freg found was a very reasonable transformation from the points on the bone to the points on the mesh and that we chose our error stopping criteria well.

In the tabular summary above, it can be seen that the mean and max error is an order of magnitude higher in data sets E, F, J, and K than in the rest of the data sets. This is due to error propagation in these data sets which are discussed below. In the two debug sets E and F, our error results were still similar to Dr. Taylor's output files.

6.4 Error Propagation

Barring errors due to software bugs, error propagation can occur based on several sources. If there is systemic biased measurement in a single direction, this can offset error and cause it to propagate along transform chains and even amplify error.

PA 2

Error sources and propagation can come from a variety of sources, including EM distortion, EM Noise, and OT jiggle. We were able to account for the EM distortion through our distortion calibration functions. It is expected that some amount of EM Noise, distortion, and jiggle will be propagated throughout the system that we are unable to account for.

One example of how error can propagate is if both the optical tracker and EM tracker are off with a common distortion component, it is possible for this information to cause the Bernstein curve to misestimate the actual curve, and consequently cause the registration between the CT scan and the other sensors to have a higher error. In this way errors can propagate through the whole system. This particular example can be mitigated through the use of fixed physical structures that are known in advance that can be used to estimate and account for such systemic errors.

Additionally, inaccurate sensors due to large random variation are an example of error which cannot be removed through distortion calibration.

PA 3 and 4

Possible error sources and propagation were due to simulated noise. The early debug sets had the least amount of simulated noise corresponding to very low error while the later debug sets and unknown sets had a greater simulated noise corresponding to a slightly larger error. The simulated noise could have been attributed to a number of sources including Optical distortion, Optical Noise, Optical jiggle, CT distortion, or CT Noise.

6.5 Results Metric

PA 2

We know that our distortion is correct and we can measure its accuracy because we can compare the old values of EM pivot to the newly undistorted values that we encounter. By comparing to prior ground truth values we can assess the accuracy of our calibration.

Our metric for error is the distance difference between our calculations and the debug outputs. This can be measured as an average, or with other statistical tools. We can also detect certain sources of error by specifying our own test functions. We also utilize the **BOOST_VERIFY** macro and the checkWitinTolerances() function to verify that functions are being called and returning values that or correct to within certain tolerances, considering the limits of the particular algorithms we are using.

PA 3

Our metric for error is the norm between the sample points and the nearest points on the CT mesh. The norm was small in most cases so we concluded that our implementation of the first step of ICP registration was successful. In PA 4, we will expand on our current ICP by iterating and setting an error bound to obtain more accurate results.

PA 4

Our metric for error is the norm between the sample points and the nearest points on the CT mesh. An ICP was run until the error was minimized below our error criteria. Our ICP algorithm produced low errors so we concluded that our transformation matrix, Freg, was a good representation of mapping points on the bone to points on the CT mesh. In addition to our low errors, our ICP also ran quickly so we concluded that our error bounds used as stopping criteria was a good choices for our program.

Andrew and Alex spent approximately equal time on the assignment, with significant amounts of time spent pair programming. Both contributed equally to the implementation and debugging of functions.

7 Additional Information

7.1 Features

Horn Registration

- Point cloud to point cloud transformations .

Pivot calibrations

- Pivot calibrations allow a coordinate system to be established with respect to existing data and the world frame.

Distortion Correction

- Bernstein Polynomial based distortion correction.

Iterative Closest Point

- Solver for finding the closest matching set of points to a mesh.

7.2 People

Software Development

- Andrew Hundt
- Alex Strickland

7.3 Quick Start

First Steps

The following steps will show you how to

- download and install CIS on your system.
- use the installation to create an example.
- build and test the example project.

You need to have a Unix-like operating system such as Linux or Mac OS X installed on your machine in order to follow these steps. At the moment, there is no separate tutorial available for Windows users, but you can install CygWin as an alternative. Note, however, that CIS can also be installed and used on Windows.

Install CIS

Get a copy of the source code Clone the Git repository from GitHub as follows:

```
mkdir -p ~/local/src
cd ~/local/src
git clone https://github.com/ahundt/cis
cd cis
```

or *Download* a pre-packaged `.tar.gz` of the latest release and unpack it using the following command:

```
mkdir -p ~/local/src
cd ~/local/src
tar xzf /path/to/downloaded/cis-$version.tar.gz
cd cis-$version
```

Configure the build Configure the build system using CMake 3.0.0 or a more recent version:

```
mkdir build && cd build
ccmake ..
```

- Press `c` to configure the project.
- Change `CMAKE_INSTALL_PREFIX` to `~/local`.
- Set option `BUILD_EXAMPLE` to `ON`.
- Make sure that option `BUILD_PROJECT_TOOL` is enabled.

- Press `g` to generate the Makefiles.

Note that if you do not have the **BASIS** dependency, an error will appear. Simply running the CMake steps a second time will cause BASIS to automatically download and install.

Build and install CIS CMake has generated Makefiles for GNU Make. The build is thus triggered by the make command:

```
make
```

To install CIS after the successful build, run the following command:

```
make install
```

As a result, CMake copies the built files into the installation tree as specified by the `CMAKE_INSTALL_PREFIX` variable.

Set up the environment For the following tutorial steps, set up your environment as follows. In general, however, only the change of the `PATH` environment variable is recommended. The other environment variables are only needed for the tutorial sessions.

Using the C or TC shell (csh/tcsh):

```
setenv PATH "~/local/bin:${PATH}"
setenv CIS_EXAMPLE_DIR "~/local/share/cis/example"
```

Using the Bourne Again SHell (bash):

```
export PATH="~/local/bin:${PATH} "
export CIS_EXAMPLE_DIR="~/local/share/basis/example"
```

Test the Example The following is an example of how to run the main cisHW1-2 executable, cisHW3-4 is similar.

```
./cisHW1-2 --dataFilenamePrefix pa1-debug-a --dataFolderPath /path/to/cis/data/PA1-2/


PivotCalibration result for pa1-debug-a-empivot.txt:

197.115
192.677
192.437
197.113
192.677
192.434
```

The following is an example of how to run the main cisHW3-4 executable.

```
./cisHW3-4 --dataFolderPath /path/to/cis/data/PA3-5/ --threads --minIterationCount 500
```

Command Line Format The command line format follows standard conventions, plus the ability to store a response file, typically named *.rsp, which saves additional command line parameters for future use and convenience. The available command line parameters and descriptions for the primary **cisHW1-2** executable file are below.

```
./cisHW1-2

General Options:
  --responseFile arg                    File containing additional command line
```

```
                                        parameters
  --help                                produce help message
  --debug                               enable debug output
  --debugParser                         display debug information for data file
                                        parser

Algorithm Options:
  --threads                             run each source data file in a separate
                                        thread

Data Options:
  --pa1                                 set automatic programming assignment 1
                                        source data parameters, overrides
                                        DataFilenamePrefix, exclusive of pa1
  --pa2                                 set automatic programming assignment 2
                                        source data parameters, overrides
                                        DataFilenamePrefix, exclusive of pa2
  --dataFolderPath arg (=/Users/athundt/source/cis/build/bin)
                                        folder containing data files, defaults
                                        to current working directory
  --outputDataFolderPath arg (=/Users/athundt/source/cis/build/bin)
                                        folder for output data files, defaults
                                        to current working directory
  --dataFilenamePrefix arg              constant prefix of data filename path.
                                        Specify this multiple times to run on
                                        many data sources at once
  --dataFileNameSuffix_calbody arg (=-calbody.txt)
                                        suffix of data filename path
  --dataFileNameSuffix_calreadings arg (=-calreadings.txt)
                                        suffix of data filename path
  --dataFileNameSuffix_empivot arg (=-empivot.txt)
                                        suffix of data filename path
  --dataFileNameSuffix_optpivot arg (=-optpivot.txt)
                                        suffix of data filename path
  --dataFileNameSuffix_output1 arg (=-output1.txt)
                                        suffix of data filename path
  --dataFileNameSuffix_ct_fiducials arg (=-ct-fiducials.txt)
                                        suffix of data filename path
  --dataFileNameSuffix_em_fiducials arg (=-em-fiducialss.txt)
                                        suffix of data filename path
  --dataFileNameSuffix_em_nav arg (=-EM-nav.txt)
                                        suffix of data filename path
  --dataFileNameSuffix_output2 arg (=-output2.txt)
                                        suffix of data filename path
  --calbodyPath arg                     full path to data txt file, optional
                                        alternative to prefix+suffix name
                                        combination
  --calreadingsPath arg                 full path to data txt file, optional
                                        alternative to prefix+suffix name
                                        combination
  --empivotPath arg                     full path to data txt file, optional
                                        alternative to prefix+suffix name
                                        combination
  --optpivotPath arg                    full path to data txt file, optional
                                        alternative to prefix+suffix name
                                        combination
  --output1Path arg                     full path to data txt file, optional
                                        alternative to prefix+suffix name
                                        combination
  --ct_fiducialsPath arg                full path to data txt file, optional
                                        alternative to prefix+suffix name
                                        combination
  --em_fiducialsPath arg                full path to data txt file, optional
                                        alternative to prefix+suffix name
                                        combination
  --em_navPath arg                      full path to data txt file, optional
                                        alternative to prefix+suffix name
                                        combination
  --output2Path arg                     full path to data txt file, optional
                                        alternative to prefix+suffix name
                                        combination
```

The available command line parameters and descriptions for the primary **cisHW3-4** executable file are below.

```
./cisHW3-4

General Options:
  --responseFile arg                    File containing additional command line
                                        parameters
  --help                                produce help message
  --debug                               enable debug output
  --debugParser                         display debug information for data file
                                        parser

Algorithm Options:
  --threads                             run each source data file in a separate
                                        thread. May speed up execution
                                        dramatically.
  --icpAlgorithm arg (=spatialIndex)    Options: spatialIndex, simpleSearch.
                                        Selects the ICP algorithm version to
                                        use. spatialIndex provides a
                                        substantial performance boost for large
                                        data sets.
  --meanErrorThreshold arg (=0.01)      stop ICP when mean error drops below
                                        this level
  --maxErrorThreshold arg (=0.10000000000000001)
                                        stop ICP when max error drops below
                                        this level
  --minVarianceInMeanErrorBetweenIterations arg (=1e-10)
                                        stop ICP when mean error no longer
                                        varies between iterations
  --minIterationCount arg (=10)         Do not stop ICP unless this many
                                        iterations have run
  --maxIterationCount arg (=300)        Stop ICP when the maximum iteration
                                        count threshold is reached, superceded
                                        by minIterationCount

Data Options:
  --pa3                                 set automatic programming assignment 3
                                        source data parameters, overrides
                                        DataFilenamePrefix, exclusive of pa4
  --pa4                                 set automatic programming assignment 4
                                        source data parameters, overrides
                                        DataFilenamePrefix, exclusive of pa3
  --dataFolderPath arg (=/Users/athundt/source/cis/build/bin)
                                        folder containing data files, defaults
                                        to current working directory
  --outputDataFolderPath arg (=/Users/athundt/source/cis/build/bin)
                                       folder for output data files, defaults
                                       to current working directory
  --dataFilenamePrefix arg             constant prefix of data filename path.
                                       Specify this multiple times to run on
                                       many data sources at once
  --dataFilenameProblemPrefix arg (=Problem4)
                                       constant prefix of data typically
                                       starting with "Problem" filename path.
                                       Specify this multiple times to run on
                                       many data sources at once
  --suffixAnswer arg (=-Answer.txt)     suffix of data filename path
  --suffixOutput arg (=-Output.txt)     suffix of data filename path
  --suffixSample arg (=-SampleReadingsTest.txt)
                                       suffix of data filename path
  --suffixMesh arg (=MeshFile.sur)      suffix of data filename path
  --suffixBodyA arg (=-BodyA.txt)       suffix of data filename path
  --suffixBodyB arg (=-BodyB.txt)       suffix of data filename path
  --AnswerPath arg                     full path to data txt file, optional
                                       alternative to prefix+suffix name
                                       combination
  --OutputPath arg                     full path to data txt file, optional
                                       alternative to prefix+suffix name
                                       combination
  --SamplePath arg                     full path to data txt file, optional
                                       alternative to prefix+suffix name
                                       combination
  --MeshPath arg                       full path to data txt file, optional
                                       alternative to prefix+suffix name
                                       combination
  --BodyAPath arg                      full path to data txt file, optional
                                       alternative to prefix+suffix name
                                       combination
  --BodyBPath arg                      full path to data txt file, optional
                                       alternative to prefix+suffix name
                                       combination
  --em_fiducialsPath arg               full path to data txt file, optional
                                       alternative to prefix+suffix name
                                       combination
  --em_navPath arg                     full path to data txt file, optional
                                       alternative to prefix+suffix name
                                       combination
  --output2Path arg                    full path to data txt file, optional
                                       alternative to prefix+suffix name
                                       combination
```

Unit Test The easiest way to run the unit test is to build the software, then symlink the data folder "PA1-2" from "data/PA1-2" into the same directory as the unit tests. In other words, the unit tests expect the directory "PA1-2" to be in the same directory as the unit test executable when it is run. The same should be done for the OUTPUT folder to PA1-2-OUTPUT for comparison of debug output files for identifying problems in the system.

```
ln -s /path/to/cis/data/PA1-2
ln -s /path/to/cis/OUTPUT PA1-2-OUTPUT
./cisHW1test
```

Next Steps

Congratulations! You just finished your first CIS tutorial.

Now check out the *Advanced Information* for more details regarding each of the above steps and in-depth information about the used commands if you like, or move on to the various `How-to Guides`.

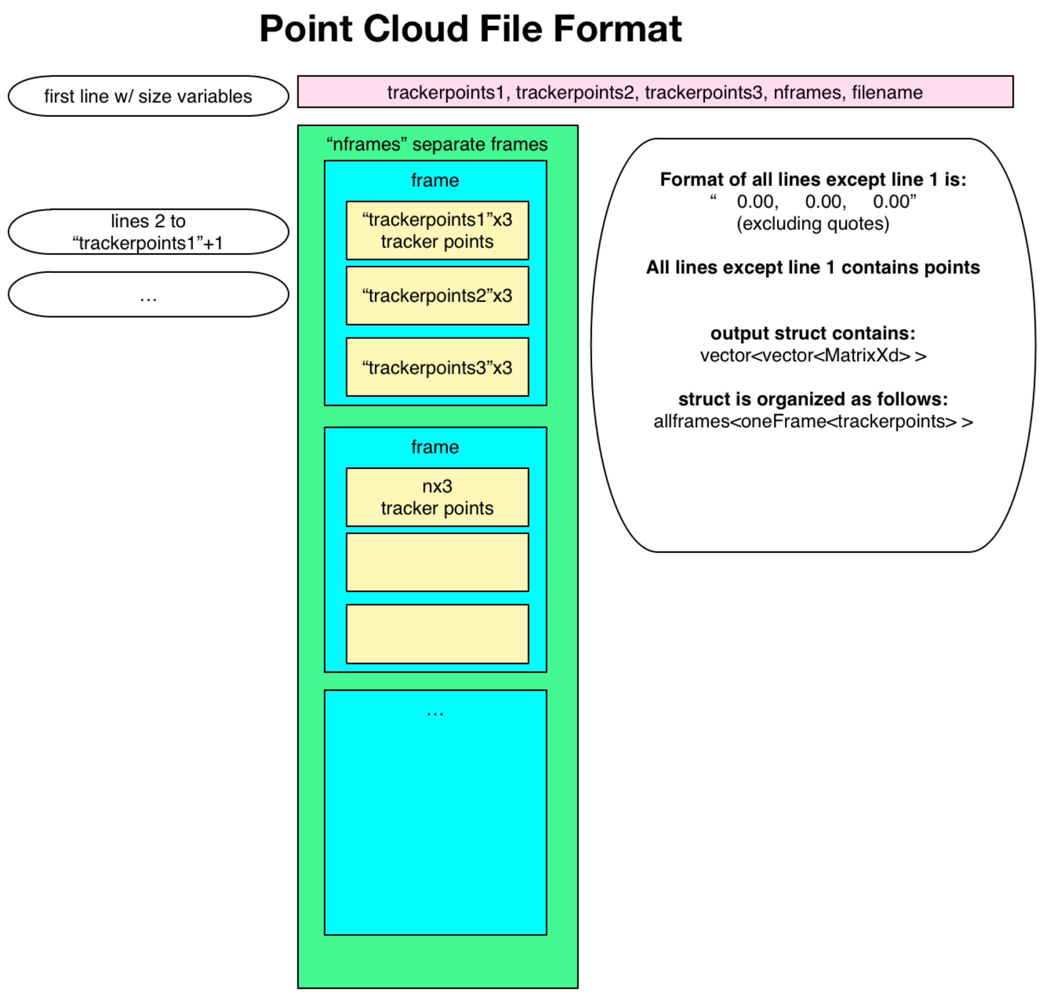
Advanced Information

For advanced documentation, please see the doxygen API documentation, unit tests, and software manual. If you cannot view these files and documents, they are visible as inline source code documentation and and restructured text files found in the /doc folder.

For a less comprehensive tutorial-like introduction, please refer to the *First Steps* above.

7.4 Read the Data format

The following image illustrates the basics of the input testing data format.


Point Cloud File Format
first line w/ size variables
trackerpoints1, trackerpoints2, trackerpoints3, nframes, filename
lines 2 to "trackerpoints1"+1
…
"nframes" separate frames
frame
"trackerpoints1"x3 tracker points
"trackerpoints2"x3
"trackerpoints3"x3
frame
nx3 tracker points
…
Format of all lines except line 1 is:
" 0.00, 0.00, 0.00"
(excluding quotes)
All lines except line 1 contains points
output struct contains:
vector<vector<MatrixXd> >
struct is organized as follows:
allframes<oneFrame<trackerpoints> >

7.5 Getting Help

Please report any issues with CIS, including bug reports, feature requests, or support questions, on GitHub.

7.6 Reference

Source Package

config/	Package configuration files.
data/	Data files required by the software.
doc/	Documentation source files.
example/	Example files for users to try out the software.
include/	Header files of the public API of libraries.
lib/	Module files for scripting languages.
modules/	Project modules (i.e., subprojects).
src/	Source code files.
test/	Implementations of unit and regression tests.
AUTHORS.md	A list of the people who contributed to this sofware.
BasisProject.cmake	Sets basic project information and lists external dependencies.
CMakeLists.txt	Root CMake configuration file.
COPYING.txt	The copyright and license notices.
INSTALL.md	Build and installation instructions.
README.md	Basic summary and references to the documentation.

7.7 Download

Source Code

The source code of the CMake BASIS package is hosted on GitHub from which all releases and latest development versions can be downloaded. See the `changelog` for a summary of changes in each release.

Either clone the Git repository:

```
git clone https://github.com/ahundt/cis.git
```

or download a pre-packaged `.zip` of the latest CIS release:

- Download CIS v1.0.0 as .zip

See also:

The *Quick Start Guide* can help you get up and running.

System Requirements

Operating System: Linux, Mac OS X, Microsoft Windows

Software License

Documentation

CIS Manual: Online version of this manual

7.8 Installation

See the BASIS guide on software installation for a complete list of build tools and detailed installation instructions.

Prerequisites

Dependency	Version	Description
BASIS	3.1.0	Utility to automate and standardize creating, documenting, and sharing software.
Boost	1.56.0+	C++ Library collection for general use
CMake	3.0.0	Build Tools.
Eigen	3.2.0	Linear Algebra Library.

Configure

1. Extract source files:

   ```
   tar -xzf cis-1.0.0-source.tar.gz
   ```

2. Create build directory:

   ```
   mkdir cis-1.0.0-build
   ```

3. Change to build directory:

   ```
   cd cis-1.0.0-build
   ```

4. Run CMake to configure the build tree:

   ```
   ccmake -DBASIS_DIR:PATH=/path/to/basis ../cis-1.0.0-source
   ```

   - Press `c` to configure the build system and `e` to ignore warnings.
   - Set `CMAKE_INSTALL_PREFIX` and other CMake variables and options.
   - Continue pressing `c` until the option `g` is available.
   - Then press `g` to generate the GNU Make configuration files.

Build

After the configuration of the build tree, the software can be build using GNU Make:

```
make
```

Test

After the build of the software, optionally run the tests using the command:

```
make test
```

In case of failing tests, re-run the tests, but this time by executing CTest directly with the `-V` option to enable verbose output and redirect the output to a text file:

```
ctest -V >& cis-test.log
```

and attach the file `cis-test.log` to the issue report.

Install

The final installation copies the built files and additional data and documentation files to the installation directory specified using the `CMAKE_INSTALL_PREFIX` option during the configuration of the build tree:

```
make install
```

After the successful installation, the build directory can be removed again.