Homework Assignment 3 – 600.445/645 Fall 2014

Instructions and Score Sheet (hand in with answers)

Name	
Email	
Other contact information (optional)	
Signature (required)	I have followed the rules in completing this assignment __________________________________
Name	
Email	
Other contact information (optional)	
Signature (required)	I have followed the rules in completing this assignment __________________________________

- Please circle whether you are in 600.445 or 600.645
- Remember to put your name(s) on <u>every</u> sheet you hand in

Question	Points	Total	Points	Total
1	5			
2	15			
3	20			
4	20			
5	20			
6a	5			
6b	5			
6c	5			
6d	5			
Total		100		

1. Remember that this is a graded homework assignment.
2. You are to work alone or in a team of two people and are not to discuss the problems with anyone other than the TAs or the instructor.
3. Put your names and email address on each sheet and number the sheets.
4. You are encouraged to make free use of any published materials, the web, etc. in developing your answer but a) you must give full and proper citations to any references consulted and b) you may not consult, discuss, or otherwise communicate about this assignment with any human being except your lab partner, the course instructor, or the TAs. The one exception is that you should not refer to previous years' homework.
5. Please refer to the course organizational notes for a fuller listing of all the rules. I am not reciting them all here, but they are still in effect.
6. Unless I say otherwise in class, it is due before the start of class on the due date posted on the web.
7. Sign and hand in the score sheet as the first sheet of your assignment
8. Remember to include a sealable 8 ½ by 11 inch self-addressed envelope if you want your assignment.
9. Attach the grade sheet as the first sheet and attach all sheets together.
10. You must include a self-addressed, seal-able 8 ½ x 11 inch envelope if you expect to the homework to be returned (per JHU's interpretation of FERPA).

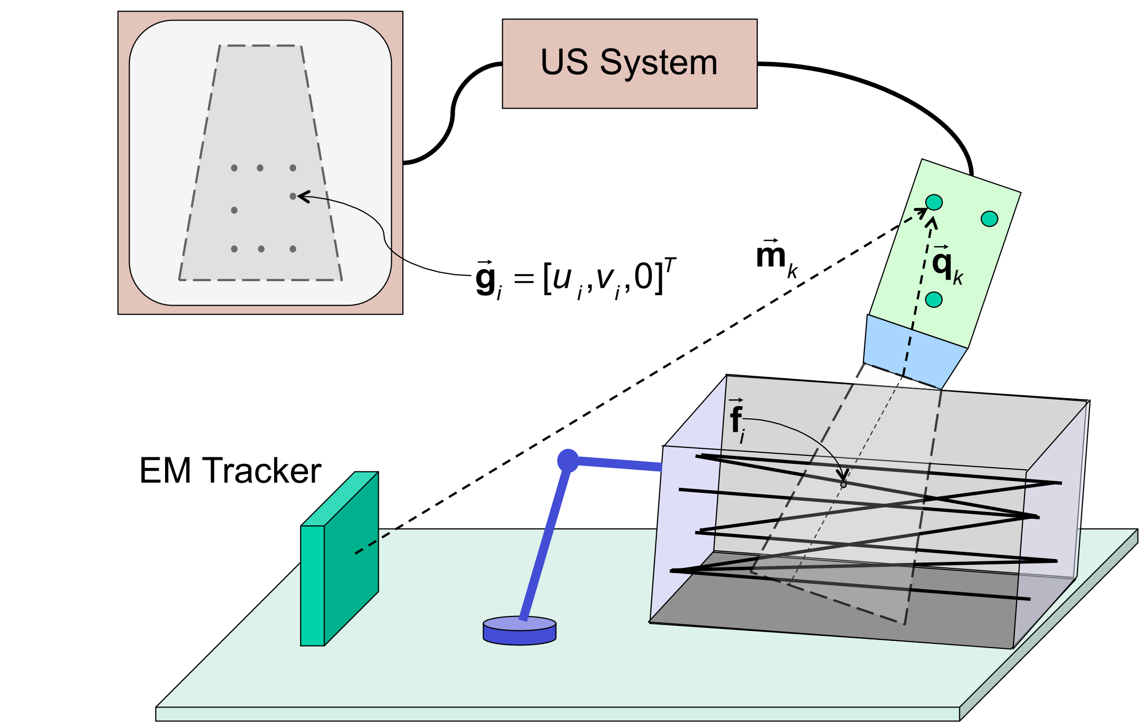

Figure 1

Scenario

Consider the ultrasound system show in Figure 1. The system consists of an ultrasound system equipped with a B-mode probe, an EM tracker system, and a calibration object. There are three EM markers mounted on the ultrasound probe at unknown locations relative to the probe. The EM tracker system is, however, able to measure the 3D position $\vec{\mathbf{m}}_k$ of each marker *k*. The calibration object is held in a fixed, but unknown, pose $\mathbf{F}_C$ relative to the EM tracker. Inside the calibration object are eight wires or thin rods whose endpoints are as indicated in Table 1. The center of the image of each wire *i* in ultrasound image coordinates is given by $\vec{\mathbf{g}}_i = [u_i, v_i, 0]$. This position corresponds to an (unknown) position $\vec{\mathbf{f}}_i$ in calibration object coordinates where the ultrasound B-mode scan plane crosses wire *i*. The probe is positioned so that all eight wires are visible in the ultrasound image and that the image processing software is able to determine which wire is which, so that the $\vec{\mathbf{g}}_i$ are known unambiguously.

Table 1

i	Endpoints relative to object	
1	[0,0,0]	[a,0,c]
2	[0,b,0]	[a,b,c]
3	[0,0,0]	[0,b,c]
4	[a,0,0]	[a,b,c]
5	[0,0,0]	[0,0,c]
6	[a,0,0]	[a,0,c]
7	[0,b,0]	[0,b,c]
8	[a,b,0]	[a,b,c]

Questions

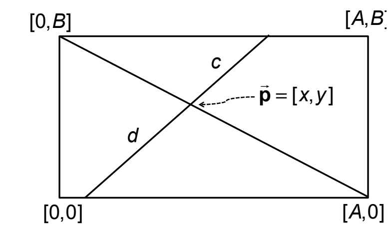

Figure 2

1. Consider the rectangle in Figure 2. A line cuts across the middle of the rectangle and intersects the $[0,B]-[A,0]$ diagonal at an unknown point $\vec{\mathbf{p}} = [x, y]$. The lengths of the two segments of this line are c and d, as shown. Give a formula for computing $\vec{\mathbf{p}}$.

2. Describe a method for computing the transformation $\mathbf{F}_{CU}$ between calibration object coordinates and ultrasound image coordinates. Give all pertinent formulas in sufficient detail so that it would be possible to implement your method. **Hint:** you might want to consider how you might find the values of some of the $\vec{\mathbf{f}}_i$.

3. Suppose that the image processing software is accurate to within an amount δ. I.e., a measured location $\vec{\mathbf{g}}_i$ will differ from its true value $\vec{\mathbf{g}}_i^*$ with a bound $\left\|\vec{\mathbf{g}}_i^* - \vec{\mathbf{g}}_i\right\| \le \delta$. This will introduce some error into your estimation of the transformation $\mathbf{F}_{CU}$. If the actual value is $\mathbf{F}_{CU}^*$, then we have $\mathbf{F}_{CU}^* = \mathbf{F}_{CU}\Delta\mathbf{F}_{CU} \approx \mathbf{F}_{CU} \bullet [\mathbf{I} + skew(\vec{\alpha}), \vec{\varepsilon}]$. Develop a set of constraints for estimating the limits on $\vec{\alpha}$ and $\vec{\varepsilon}$. **Hint:** you might want to consider the effects of δ on how accurately you can find $\vec{\mathbf{f}}_i$. Also, if you find it more convenient, you may use the L1 norm $\left|\vec{\mathbf{g}}_i^* - \vec{\mathbf{g}}_i\right| \le \delta$ instead of the L2 norm given above.

4. Suppose that δ is small enough to be ignored, and (for the moment) that the EM tracker errors are relatively small and that there is no systematic distortion. Describe a method for determining the positions $\vec{\mathbf{q}}_k$ relative to ultrasound image coordinates of each of the EM markers *k*. Give all pertinent formulas in sufficient detail so that it would be possible to implement your method.

5. Suppose that your calibration of the $\vec{\mathbf{q}}_k$ has been successful. Suppose, now that the calibration object is removed from the workspace, and a patient is introduced, as shown in Figure 3. A CT scan of the patient is available, and a suspected tumor has been found in the CT image. A planning system has determined that a good place to biopsy this tumor is at location $\vec{\mathbf{p}}_{CT}$ relative to CT coordinates. In order to biopsy the tumor, the target coordinates $\vec{\mathbf{p}}_{et}$ relative to the EM tracker must be determined, which, in turn requires the computation of a registration transformation $\mathbf{F}_{reg}$ such that $\vec{\mathbf{p}}_{et} = \mathbf{F}_{reg}\vec{\mathbf{p}}_{ct}$. The tumor is not visible in ultrasound images, but some nearby structures are visible in both

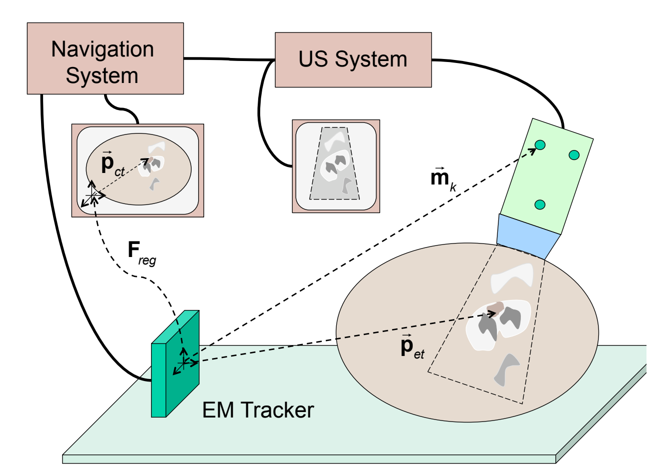

Figure 3

CT and ultrasound images. Further segmentation software is available to determine the exterior surfaces of these structures. Describe a method for determining $\mathbf{F}_{reg.}$. Provide sufficient step-by-step detail and pertinent formulas so that your method is clear.

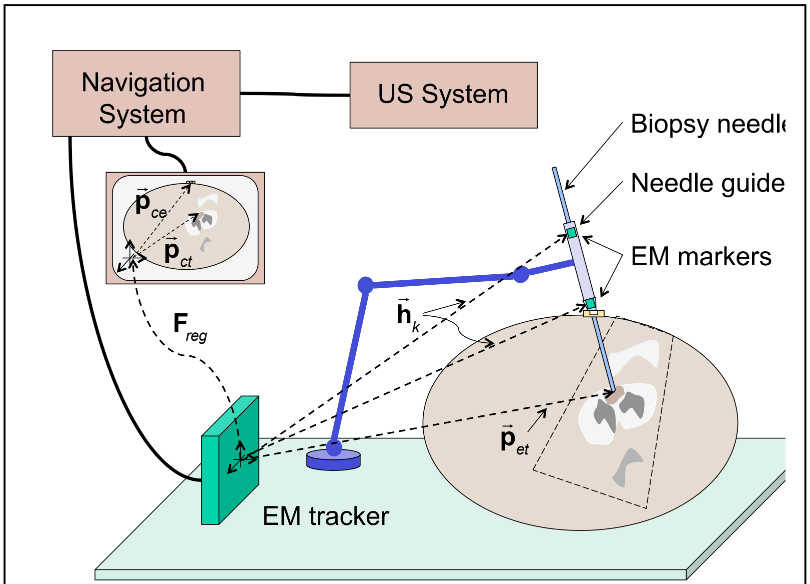

Figure 4

6. Suppose that your method has succeeded in computing $\mathbf{F}_{reg.}$ to very high accuracy. A tracked needle guide is available for assisting in placing the biopsy needle, as shown in Figure 4, The biopsy tool passes through a cannula in the needle guide. The guide is equipped with two miniature EM marker coils located coaxially with the cannula. One of these is located at the distal end of the cannula, where the biopsy needle emerges, and the other is located at a distance d further up the cannula. The EM tracker is able to read the positions $\vec{\mathbf{h}}_1$ and $\vec{\mathbf{h}}_2$ of these two coils relative to the EM tracker base. The EM tracker is subject both to random and systematic errors. In particular, if $\vec{\mathbf{h}}_i^* = \vec{\mathbf{h}}_i + \Delta\vec{\mathbf{h}}_i$ is the actual value of associated with coil i at time t, then the error $\Delta\vec{\mathbf{h}}_i = \vec{\varepsilon}_N(t) + \vec{\varepsilon}_S(\vec{\mathbf{h}}_j)$, where $\left\|\vec{\varepsilon}_N(t)\right\| \le \nu$ and $\left\|\vec{\varepsilon}_S(\vec{\mathbf{h}}_1) - \vec{\varepsilon}_S(\vec{\mathbf{h}}_2)\right\| \le \eta \left\|\vec{\mathbf{h}}_1 - \vec{\mathbf{h}}_2\right\|^2$. The biopsy needle is marked along its length, so that the amount that the needle extends beyond to distal end of the cannula is accurately known. In the proposed biopsy procedure, the cannula distal end will be placed onto an externally defined landmark on the surface of the patient at a location $\vec{\mathbf{p}}_{ce}$ in CT coordinates. The needle guide will then be aimed so that the cannula is coaxial with the line through the points $\mathbf{F}_{reg}\vec{\mathbf{p}}_{ce}$ and $\vec{\mathbf{p}}_{et} = \mathbf{F}_{reg}\vec{\mathbf{p}}_{ct}$, and the biopsy needle will be inserted to the desired depth.

   a. If $\left\|\vec{\mathbf{p}}_{ce} - \vec{\mathbf{p}}_{ct}\right\| = D$, give a formula for computing the worst case aiming error. I.e., give a bound on the distance $\left\|\vec{\mathbf{e}}\right\|$ between the biopsy tip and the target when the needle is inserted a distance D.

   b. Give a formula for computing the distance d between the two EM markers that will enable the cannula to be aimed with the greatest accuracy. I.e., what value for d will minimize the bound on $\left\|\vec{\mathbf{e}}\right\|$?

   c. Suppose $D = 100$ mm, ν = 0.25 mm, η = 0.5×10^{-4} mm^{-1} . What range of values of d will guarantee that $\left\|\vec{\mathbf{e}}\right\| \le 2$ mm?

d. For D = 100 mm, ν = 0.25 mm, what is the largest value for η that will still permit a needle guide to be designed for which $\left\|\vec{\mathbf{e}}\right\| \leq 2\text{ mm}$? I.e., for which a feasible value of d can be found?