Functional Programming
Lecture 2

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Lists

Pairs

Allow to construct hierarchical data structures.

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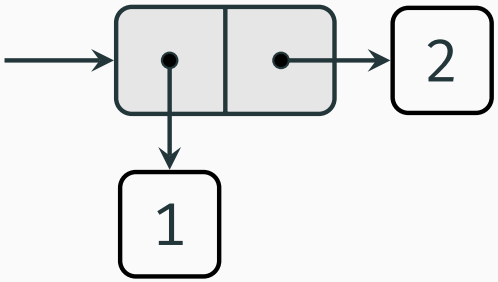


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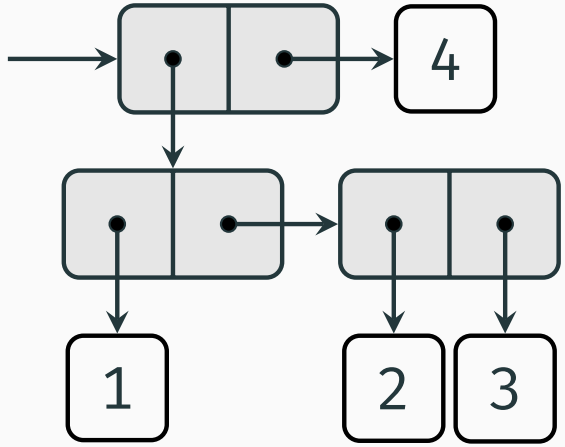


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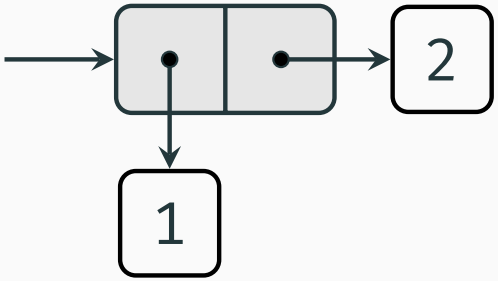

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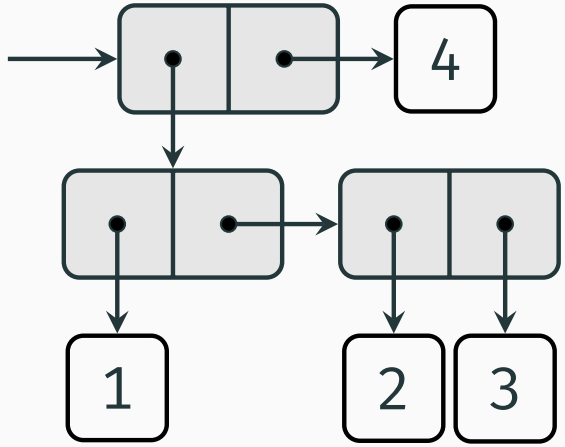

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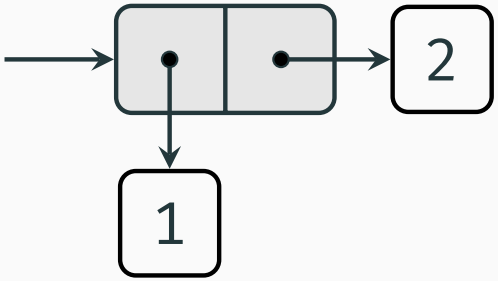

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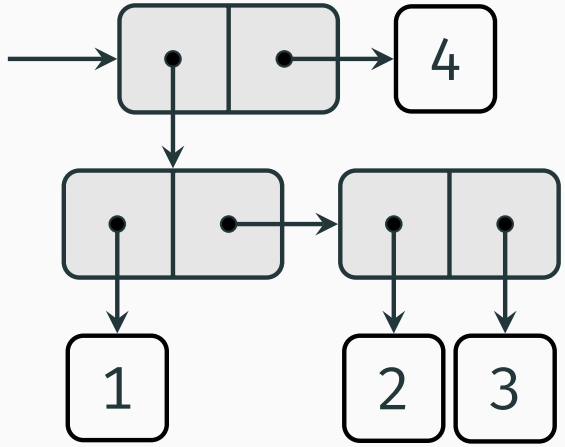

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We can create pairs compound of any data types (not only basic data types).

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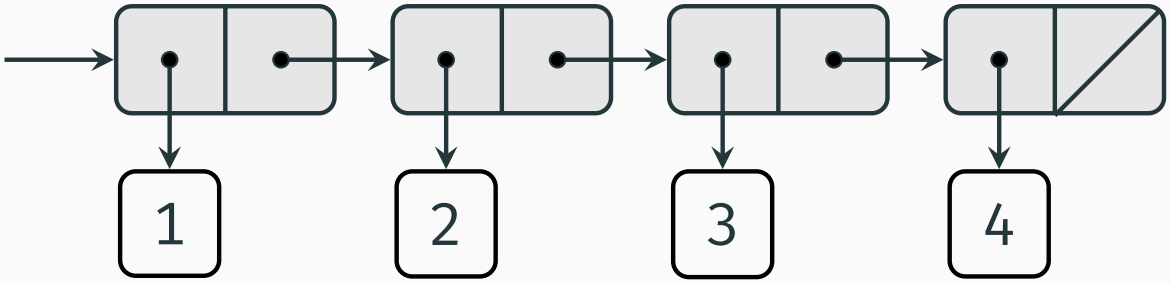

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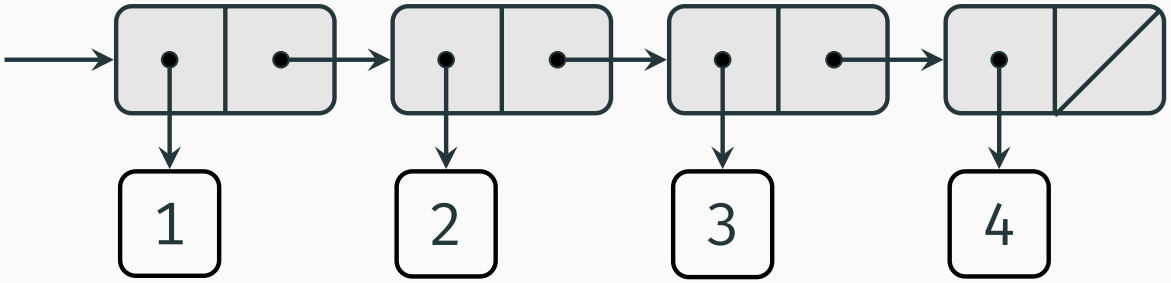


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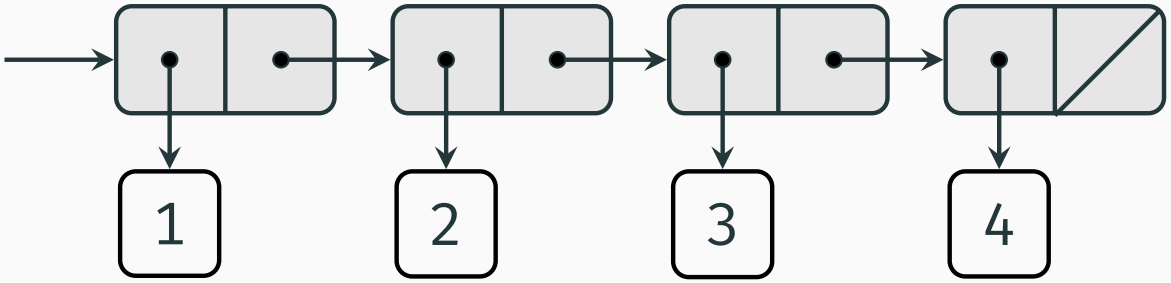


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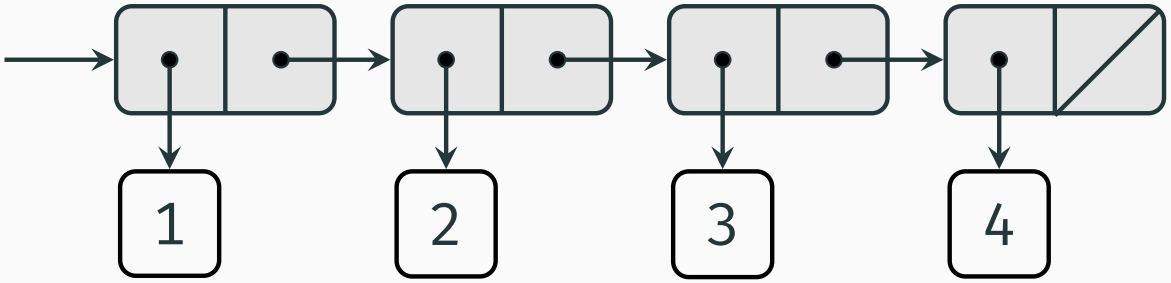


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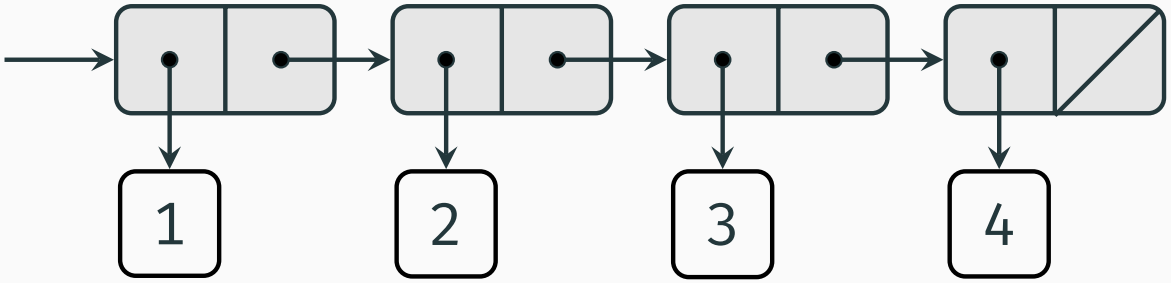


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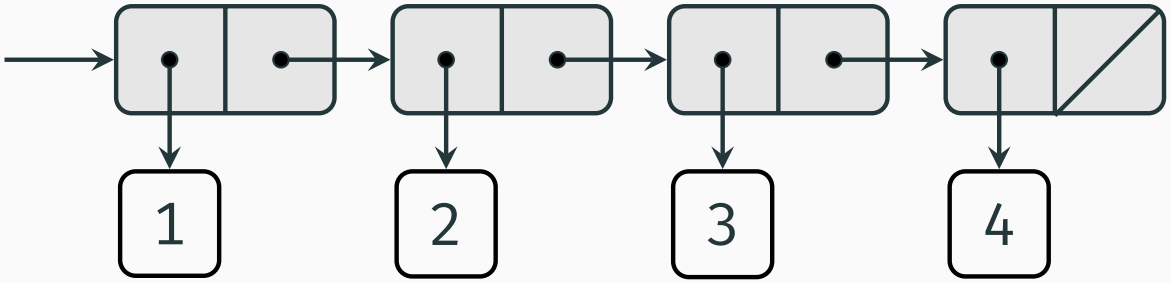


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Note that S-expressions are lists.

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Quoting - no evaluation of arguments

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Appending - copies first `n - 1` lists

```
(append '(1) '(2) '(3 4)) => '(1 2 3 4)
```

Equalities

Function “=” is only for numbers - numerical equivalence

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Equivalence of objects’ identities (typically comparing pointers)

```
(eq? 'symbol (string->symbol "symbol")) ; #t
(eq? 42 (* 6 7)) ; #t
(eq? "foobar" "foobar") ; #t
(eq? "foobar" (string-append "foo" "bar")) ; #f
(eq? '(1 2) '(1 2)) ; #f
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Equivalence of primitive values (“cheap”)

```
(eq?  (cos 1) (cos 1)) => #f
(eqv? (cos 1) (cos 1)) => #t
(eqv? 1 1.0) => #f ; different exactness
(eqv? '(a b) '(a b)) => #f
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```

Recursive, structural equivalence

```
(equal? '(a 1) '(a 1)) => #t
```

Example — filtering lists

Suppose we want to define a function filtering a given value out of a given list.

```
(my-filter val lst)
```

```
(my-filter 'a '(1 a 2 a)) => '(1 2)
```

Example — Filter

```
(define (my-filter-3 pred lst [acc '()])
  (cond
    [(null? lst) (reverse acc)]
    [(pred (car lst))
     (my-filter-3 pred (cdr lst)
                  (cons (car lst) acc))]
    [else
     (my-filter-3 pred (cdr lst) acc)]))
```

Lambda abstraction

A construction for creating anonymous functions

```
(lambda (arg1 ... argN) <exp>)
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Define for functions is an abbreviation

```
(define (square x) (* x x))
```

is the same as

```
(define square (lambda (x) (* x x)))
```

Examples

```
(filter (lambda (x) (< x 5))
        '(1 7 3 8)) => '(7 8)
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        '((a b) (5) ())) => '((a b) (5))
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Another useful function iterating through a list is `map`.
`(map f lst)` applies `f` to each element of `lst`:

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Another useful function iterating through a list is `map`.
`(map f lst)` applies `f` to each element of `lst`:

```
(map (lambda (x) (* x x)) '(1 2 3)) => '(1 4 9)
(map cdr '((a b c) (1))) => '((b c) ())
```

Bad maxlist

```
(define (bad-maxlist lst)
  (if (null? lst)
      -inf.0
      (if (> (car lst) (bad-maxlist (cdr lst)))
          (car lst)
          (bad-maxlist (cdr lst)))))
```

Inefficient tree recursive function making two recursive calls!

Local definitions

Reuse of a computation/result is often required

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(let ([<var1> <exp1>]
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(let ([<var1> <exp1>]
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(let* ([<var1> <exp1>]
       [<var2> <exp2-using-var1>])
  <body-using-var1-var2>)
```

Better maxlist

```
(define (better-maxlist lst)
  (if (null? lst)
      -inf.0
      (let ([m (better-maxlist (cdr lst))])
        (if (> (car lst) m) (car lst) m))))
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```

Tail recursive version is the best:

```
(define (best-maxlist lst [acc -inf.0])
  (cond
    [(null? lst) acc]
    [(> (car lst) acc)
     (best-maxlist (cdr lst) (car lst))]
    [else (best-maxlist (cdr lst) acc)]))
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Trees

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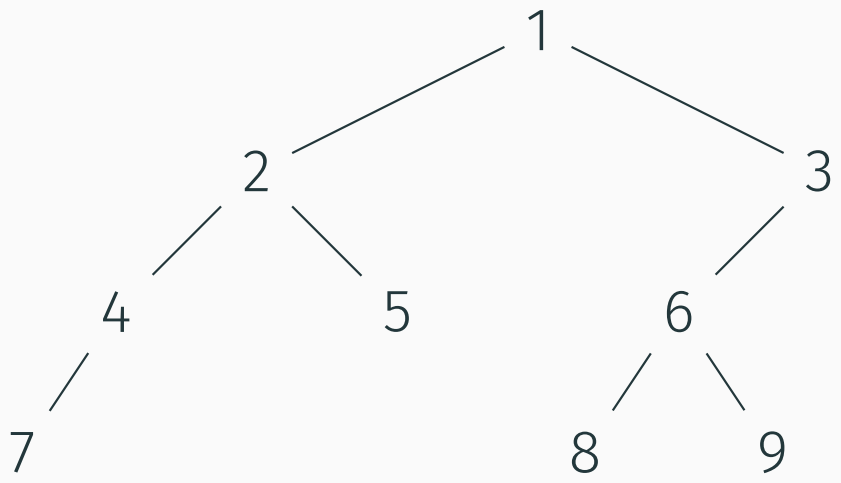

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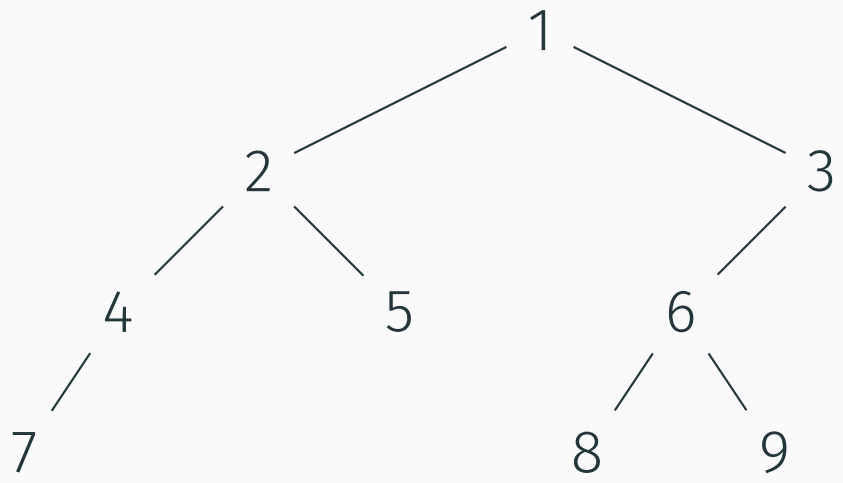


```
'(1 (2 (4 (7 #f #f) #f)
       (5 #f #f))
    (3 (6 (8 #f #f)
          (9 #f #f))
       #f))
```

Tree recursion

```
(define get-data car)
(define get-left cadr)
(define get-right caddr)

(define (find pred tree)
(if tree
    (let* ([data (get-data tree)]
           [left (find pred (get-left tree))]
           [right (find pred (get-right tree))]
           [both (append left right)])
      (if (pred data)
          (cons data both)
          both))
    '()))
```

Evaluate an algebraic expression

Expression - a number or a list with an operator followed by arguments, which are expressions

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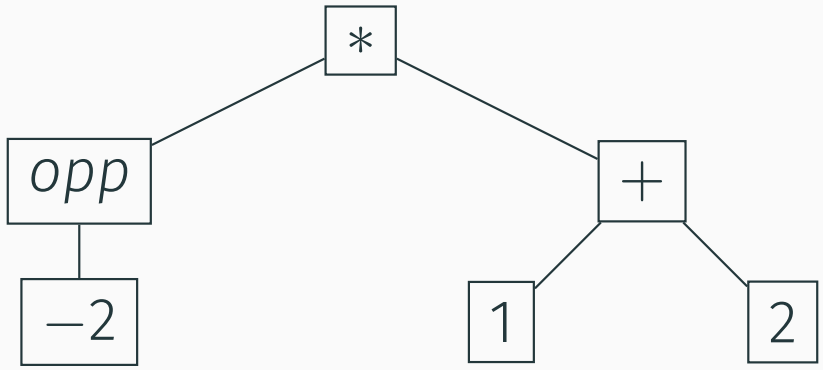

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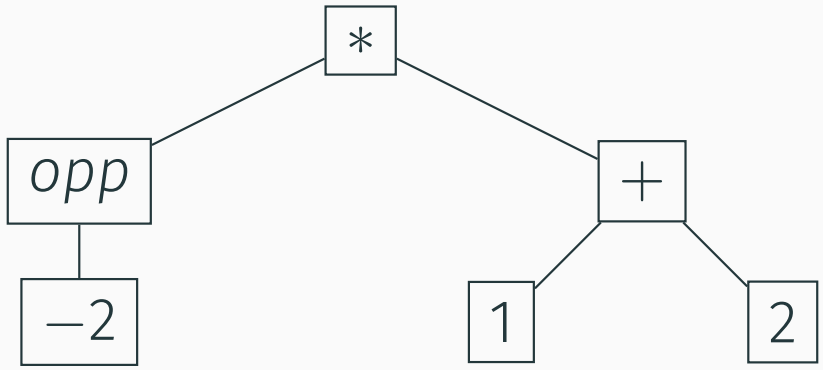


```
(eval-expr '(* (opp -2) (+ 1 2)))
=> (eval-expr '(* 2 3))
=> (eval-expr 6)
=> 6
```

Expression evaluation

```
(define (eval-expr e)
  (if (number? e)
      e
      (let ([op (car e)]
            [children (map eval-expr (cdr e))])
        (cond
          [(eq? op '+) (apply + children)]
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`apply` takes a function and a list of arguments, “unwraps” the arguments - `(apply + '(1 2 3)) => (+ 1 2 3)`

Unit testing

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Checks check conditions, report failure if not met

Test cases are named collections of sequential checks

Test suites are named collection of test cases

Checks

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; lec02.rkt:6:0
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message:    "optional message"
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Consult the documentation for the rest

Test cases

`test-case` starts a block, if any check fails, the rest are not run

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```
(test-case "filter tests"
  (check-equal? (my-filter-tr 2 '(1 2 3)) '(1 2 3))
  (check-equal? (my-filter-tr 1 '(1 2 3)) '(1 2 3)))
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```
--------------------
filter tests
; FAILURE
; lec02.rkt:33:0
name:       check-equal?
location:   lec02.rkt:33:4
actual:     '(1 3)
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actual:     '(1 3)
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```

Shortcuts available - `test-equal?` is like `check-equal?` with an extra string argument for the name

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`module+` defines a module “by parts”, while inheriting identifiers from parent module

module+ example

```
(define (add a b) (+ a b))
(module+ test
  ; only test submodule will depend on rackunit
  (require rackunit)
  ; not running during module import
  (check-equal? (add 1 2) 3))

(define (sub a b) (- a b))
; tests are very close to code!
(module+ test
  (check-equal? (sub 1 2) -1))
```

Running tests

From DrRacket: “Run” button (`Ctrl+R`) automatically runs the submodule `test`, if defined

From terminal: `raco test <file>.rkt`

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- There are different kinds of equalities.
- Abstract functions are defined by lambda.
- Local definitions are useful for reusing of a computation.
- Tests go into their own module, our tools run them for us.