Functional Programming
Lecture 4

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Pattern matching

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Similar to **cond** matching the expression exp against the patterns.

The patterns might include variables. If a pattern matches exp, its variables are bound to the corresponding values in exp.

Matching literals and predicates

```
(struct point (x y))

(match exp
  [0 'zero]
  [1 'one]
  [2 'two]
  ["abc" 'abc]
  [(point 0 0) 'point]
  [(? string?) 'string]
  [(and (? number? x) (? positive?))
   (format "positive num ~a" x)]
  [_ 'other])
```

Matching lists

```
(match lst
  [(list) 'empty]
  [(list x) (format "singleton (~a)" x)]
  [(list 'fn ys ...)
   (format "fn and rest ~a" ys)]
  [(list (list 'fn args ...) ys ...)
   (format "fn with ~a and rest ~a" args ys)]
  [(list 1 ys ... z)
   (format "1, rest ~a and last ~a" ys z)]
  [(list x ys ...)
   (format "~a and rest ~a" x ys)]
  [_ 'other])
```

Lazy evaluation

Postponing evaluation

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Can we force the interpreter to postpone the argument evaluations?

```
(define (my-if c a b) (if c a b))
(my-if (< 0 1) 0 (/ 1 0)) => /: division by zero
```

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  `(thread (thunk exp))`
- Programs can use potentially large or infinite data structures, e.g. streams. This can make the code more efficient or improve its modularity.

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```
(define (ints-from n)
  (cons n (delay (ints-from (+ n 1)))))

(define nats (ints-from 0))
(force (cdr nats)) => (1 . #<promise:...
```

Functions for streams

Functions creating and manipulating streams are implemented in Racket (not Scheme)

streams	lists
`stream-cons`	`cons`
`stream`	`list`
`stream-first`	`car`
`stream-rest`	`cdr`
`stream-empty?`	`null?`
`stream-filter`	`filter`
`stream-map`	`map`
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Any list can be used as a stream. Finite stream can be converted to a list by `stream->list`.

Streams defined explicitly

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We can construct an infinite stream for any function *f* computing the next element from the current one:

```
(define (repeat f a0)
  (stream-cons a0 (repeat f (f a0))))
```

Streams defined implicitly

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(define stream-days
        (stream* 'mon 'tue 'wed
                 'thu 'fri 'sat 'sun
                 stream-days))
```

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```

	1	1	1	1	1
+	0	1	2	3	4
0	1	2	3	4	5

Applications of streams

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\begin{aligned}
2g_k &= g_k + n/g_k \\
g_k &= n/g_k \\
g_k &= \sqrt{n}
\end{aligned}
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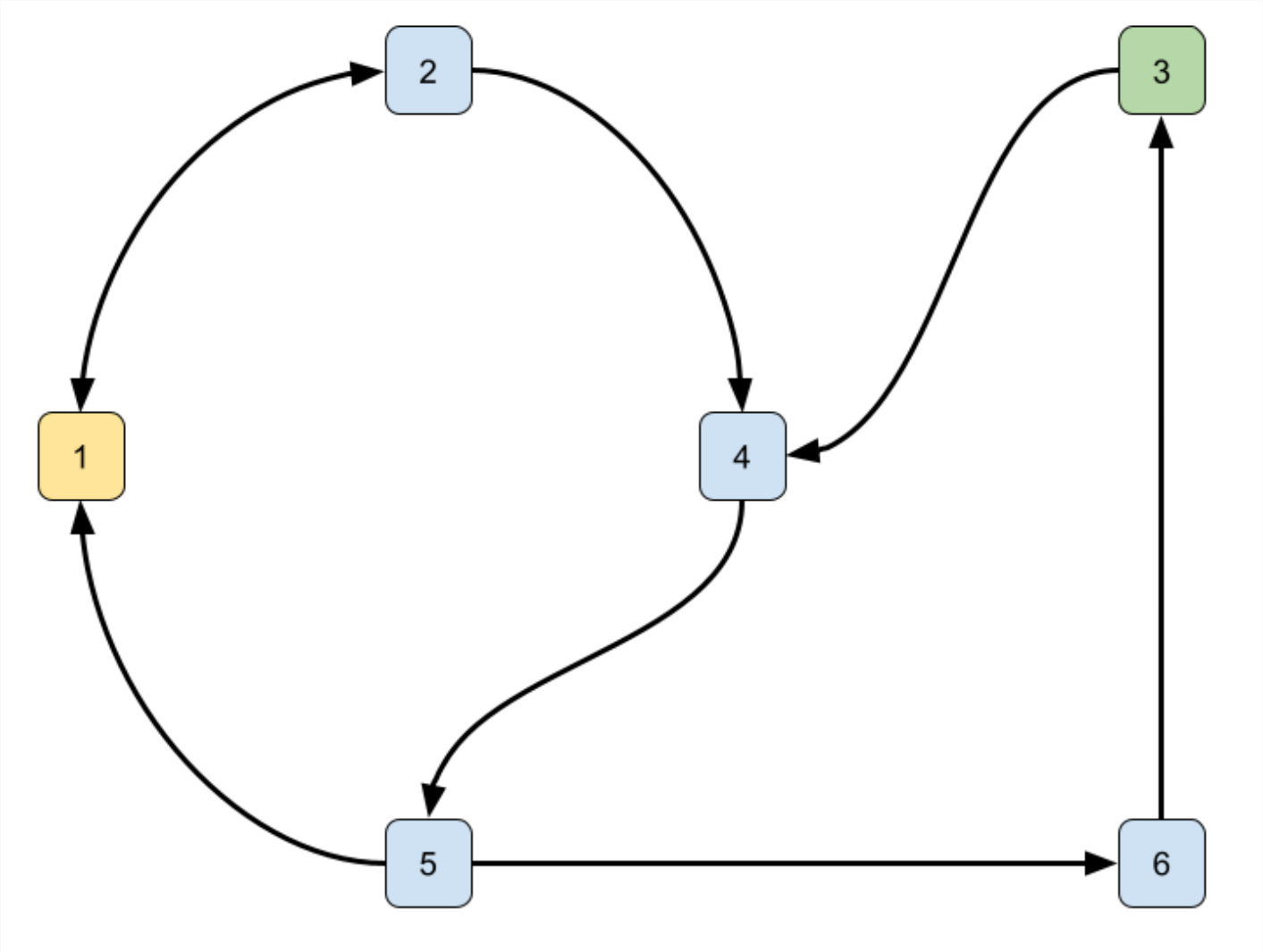
$$\begin{aligned} 2g_k &= g_k + n/g_k \\ g_k &= n/g_k \\ g_k &= \sqrt{n} \end{aligned}$$

In practice, we test $|1 - \frac{g_k}{g_{k+1}}| \leq \varepsilon$

Lazy tree



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- Streams are lazy lists which can be even infinite.
- Lazily evaluated structures provide better modularity.