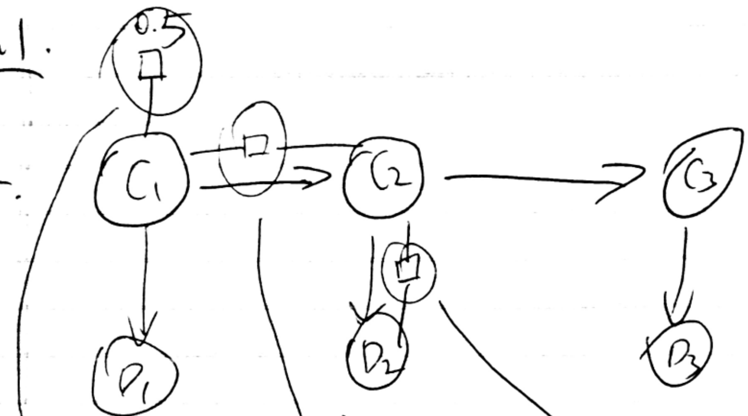
Homework 7

Problem 1.

1a.



$$P(C_2=1 \mid D_2=0)$$

$$= \sum_{C_1} P(C_1)\, P(C_2=1 \mid C_1)\, P(D_2=0 \mid C_2=1)$$

$$= P(C_1=0)\, P(C_2=1 \mid C_1=0)\, P(D_2=0 \mid C_2=1)$$

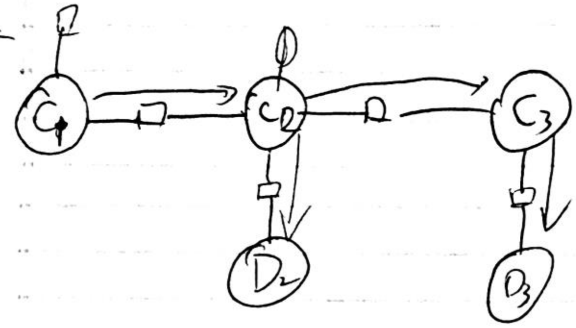
$$+ P(C_1=1)\, P(C_2=1 \mid C_1=1)\, P(D_2=0 \mid C_2=1)$$

$$= 0.5\,\epsilon\,\eta + 0.5(1-\epsilon)\,\eta = 0.5\eta$$

$$P(C_2=0 \mid D_2=0) = 0.5(1-\epsilon)(1-\eta) + 0.5\,\epsilon\,(1-\eta) = 0.5(1-\eta)$$

$$\Rightarrow P(C_2=1 \mid D_2=0) = \frac{0.5\eta}{0.5\eta + 0.5(1-\eta)} = \underline{\underline{\eta}}$$

1.b



$P(C_2=1 \mid D_2=0, D_3=1)$

multiply by

$$= \sum_{C_1} P(C_1)\, P(C_2=1 \mid C_1)\, P(D_2=1 \mid C_2=1) \; *$$

$$\sum_{C_3} P(C_3 \mid C_2=1)\, P(D_3=1 \mid C_3)$$

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$$= (0.5\varepsilon\eta + 0.5(1-\varepsilon)\eta)\left(\cancel{\varepsilon(1-\eta)}^{\,\varepsilon\eta} + (1-\varepsilon)(1-\eta)\right)$$

$$= 0.5\eta\left(\varepsilon\eta + (1-\varepsilon)(1-\eta)\right) \quad \text{— ①}$$

$P(C_2=0 \mid D_2=0, D_3=1) = (\text{Same formula})$

$$= 0.5(1-\eta)\left((1-\varepsilon)\eta + \varepsilon(1-\eta)\right) \quad \text{— ②}$$

$$\Rightarrow P(C_2=1 \mid D_2=0, D_3=1) = \frac{①}{① + ②} = \frac{\eta\left(\varepsilon\eta + (1-\varepsilon)(1-\eta)\right)}{(1-\eta)\left((1-\varepsilon)\eta + \varepsilon(1-\eta)\right) + \eta\left(\varepsilon\eta + (1-\varepsilon)(1-\eta)\right)}$$

1.C

i. Plug in 1.a and 1.b, we get:

$P(C_2=1 \mid D_2=0) = 0.2$

$P(C_2=1 \mid D_2=0, D_3=1) = 0.4157$

(ii) The probability (posterior) increased for $C_2=1$ after adding $C_3=1$.

This is because knowing the car's (estimated) position at one more timestep helps us infer the car's position in another timestep.

(iii) From expression (1)/(2) in 1.b,

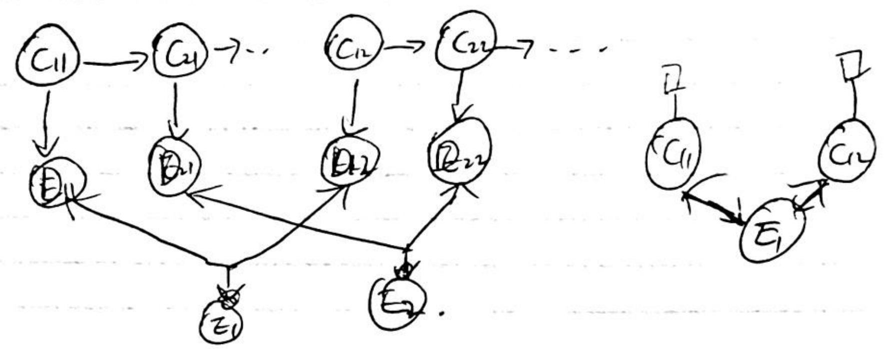
$$0.5\eta^2\varepsilon + 0.5\eta(1-\eta)\cancel{(1-\varepsilon)} = 0.5\cancel{(1-\eta)(1-\varepsilon)}\eta + 0.5(1-\eta)^2\varepsilon$$

Only possible if $\varepsilon = 0$.

i.e. the car does not move.

Problem 5.

5.a



$$P(C_{11}, C_{12} \mid e_1 = E_1) =$$

$$\underbrace{\frac{1}{2!}}_{=2} \underbrace{P(C_{11})}_{=C_{11}} \underbrace{P(C_{12})}_{=C_{12}} \Big[P_N(e_{11}; \|a_1 - C_{11}\|, \sigma^2) P_N(e_{12}; \|a_1 - C_{12}\|, \sigma^2) +$$
$$P_N(e_{11}; \|a_1 - C_{12}\|, \sigma^2) P_N(e_{12}; \|a_1 - C_{11}\|, \sigma^2) \Big]$$

5.b.

Obtain a general expr:

$$P(C_{11} = C_{11}, C_{12} = C_{12} \cdots \mid E_1 = e_1) = \frac{1}{k!} \overbrace{P(C_{11} = C_{11}) P(C_{12} = C_{12}) \cdots P(C_{1k} = C_{1k})}^{1} *$$

$$\sum_{\substack{\text{All permutations} \\ \text{of } C_i}} \Big[P(e_{11}; \|a_1 - C_{11}\|, \sigma^2) P(e_{12}; \|a_1 - C_{12}\|, \sigma^2) \cdots P(e_{1k}; \|a_1 - C_{1k}\|, \sigma^2) \Big]$$

$\Rightarrow$ ~~As~~ To maximize, we need to maximize 1.

However, if ① achieves maximum with $\vec{c}_{11}$, then all permutations of $\vec{c}_{11}$ $\left(\{c_{11}, c_{12}, c_{13} \dots\}\right)$ will have the same product $P(C_{11}=c_{1})P(C_{12}=c_{2}) \cdots P(C_{1K}=c_{1K})$. Since $\vec{C}_{11}$ has $K!$ permutations, this proves ~~Out~~ ~~the~~ that we have at lease $k!$ assignments to achieve max probability.

5-c

The arity of this probability is K. Thus, the treewidth is also K.

5-d