Université de Montpellier – Second year Masters degree
Cosmos, Champs & Particules

Centre de Physique des Particules de Marseille

Probing Dark Energy using Photometric Measurements of Supernovae

---

Florian Kéruzoré

Supervisors :
Dr. Johanna Pasquet (CPPM),
Dr. Dominique Fouchez (CPPM)

Abstract

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

March – July 2018

Introduction

intro

	α	β	Ω_m	M_B	Δ_M
Full JLA (Bet14)	0.140 ± 0.006	3.139 ± 0.072	0.289 ± 0.018	-19.04 ± 0.01	-0.060 ± 0.012
Full JLA (us)	0.143 ± 0.006	3.143 ± 0.078	0.282 ± 0.033	-19.06 ± 0.02	-0.07 ± 0.022
55 SNIa z_{spec}	0.133 ± 0.020	3.111 ± 0.259	0.153 ± 0.169	-19.2 ± 0.10	0.111 ± 0.093
55 SNIa z_{phot}	0.085 ± 0.026	5.254 ± 0.321	0.318 ± 0.26	-19.16 ± 0.11	0.149 ± 0.106
55+low-z z_{spec}	0.138 ± 0.011	2.918 ± 0.145	0.381 ± 0.125	-19.12 ± 0.05	0.016 ± 0.054
55+low-z z_{phot}	0.12 ± 0.012	3.96 ± 0.149	0.315 ± 0.142	-19.06 ± 0.05	-0.026 ± 0.057

Contents

I – Theory (temp title)

1.1 Elements of cosmology

1.1.1 Elements of general relativity

The theory of general relativity, as published by Albert Einstein in 1915 [1], is curently our best working theory of gravitation. It stipulates that gravity is a consequence of our universe's shape, and that this shape is modeled by the universe's contents through *Einstein's field equations* :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R} = -\frac{8\pi G}{c^4} T_{\mu\nu} \tag{1.1}$$

where here, $R_{\mu\nu}$ is the Ricci tensor, $\mathcal{R}$ is the Riemann curvature scalar, $T_{\mu\nu}$ is the stress-energy tensor, and $g_{\mu\nu}$ is the metric tensor, encoding the global shape of the universe.

To transcribe these equations in more physical terms, some assumptions need to be made. The *cosmological principle* is the best accepted of these assumptions : it presumes an homogeneous and isotropic distribution of the content across the universe. In other words, the cosmological principle is an hypothesis that there is no favoured place in our universe. In this framework, the simplest non-static solution to (1.1) gives the metric tensor $g_{\mu\nu}$:

$$g_{\mu\nu}\, \mathrm{d}x^\mu \mathrm{d}x^\nu = c^2 \mathrm{d}t^2 - \frac{a^2(t)}{\left(1 + \frac{kr^2}{4}\right)^2} \left(\mathrm{d}r^2 + r^2 \mathrm{d}\theta^2 + r^2 \sin^2(\theta) \mathrm{d}\varphi^2\right) \tag{1.2}$$

where $a(t)$ is the scale factor of the universe, characterizing its expansion ; (t, r, θ, φ) are the four-dimensional spherical coordinates, and k describes the curvature of the universe[1]. Eq.(1.2) is commonly refered to as the *Robertson-Walker* (RW) metric, and can describe three global four-dimensional shapes, depending on the curvature k :

$$\begin{cases} k = 1 & : \text{spherical (closed) universe,} \\ k = -1 & : \text{hyperbolic (open) universe,} \\ k = 0 & : \text{flat universe} \end{cases} \tag{1.3}$$

1.1.2 From relativity to cosmology

As of today, our understanding of the observed accelerated expansion of the universe is based upon the Friedman-Lemaître-Robertson-Walker (FLRW) cosmology, which gives one of the simplest solutions to Einstein's general relativity applied to the whole universe. In this approach, one treats the universe's content as a combination of independant comoving perfect fluids, of energy density ρ and pressure p, such as their stress-energy tensor $T^{\mu\nu}$ is :

$$T^{\mu\nu} = (p + \rho)\, u^\mu u^\nu + p\, g^{\mu\nu} \tag{1.4}$$

where u is the 4-velocity of the fluid. Gives the Friedman equations :

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum \rho - \frac{k}{a^2} \tag{1.5}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left[\sum \rho + 3 \sum p\right] \tag{1.6}$$

and the conservation of energy :

$$\dot{\rho} = -3H\,(\rho + p) \tag{1.7}$$

[1] We choose the natural system of units, in which the fundamental constants c, $\hbar$ and k_B are set to 1 (without dimension).

A simpler way to write eq. (1.5) is by introducing the Hubble parameter H :

$$H(t) \equiv \frac{\dot{a}}{a} = H_0\, h(t) \tag{1.8}$$

where H_0 is the current value of the Hubble parameter. Then we can write the cosmological abundances :

$$\Omega \equiv \frac{\rho}{\rho_c} \quad \text{with} \quad \rho_c \equiv \frac{3H_0^2}{8\pi G} \tag{1.9}$$

and the first Friedman equation becomes :

$$H^2 = H_0^2 \sum \Omega + \Omega_k \quad \text{with} \quad \Omega_k \equiv -\frac{k^2}{H_0^2 a^2} \tag{1.10}$$

or, even more simply :

$$H^2 = H_0^2\, h^2(z) \quad \text{with} \quad h^2(z) \equiv \sum \Omega + \Omega_k \tag{1.11}$$

1.1.3 Cosmological distances

An amusing consequence of a non-euclidean space is that different ways of measuring distances do not necesserally give the same result. With this in mind, we need to define different physical quantities for the different results one can get when trying to estimate the distance to a very far object.

Let us first introduce the *redshift*, z. Visualizing an expanding universe as a stretched canvas, it is easy to understand that lengths are dilated. The wavelength of photons is no exception : it increases as they propagate through the universe. Therefore, the light we receive from astrophysical sources is shifted. This effect is quantifiable by the redshift z defined as :

$$1 + z \equiv \frac{\lambda_{\text{received}}}{\lambda_{\text{emitted}}} = \frac{1}{a(t)} \tag{1.12}$$

Comoving distance $\chi(z)$ The comoving distance $\chi(z)$ is the distance one would measure if they were able to set a ruler between the astrophysical source and them. Parametrizing the Hubble parameter as a function of redshift :

$$H(t) = \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{1+z}\right) = -\frac{1}{1+z}\frac{\mathrm{d}z}{\mathrm{d}t} \tag{1.13}$$

We can then deduce :

$$\mathrm{d}t = -\frac{\mathrm{d}z}{(1+z)H(z)} \tag{1.14}$$

We are looking for an expression of the comoving distance to an object that emitted at a redshift z, considering a flat universe[2]. In such a universe, the spacetime metric can be deduced from equation (1.2) as :

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\mathrm{d}\chi^2 - a^2(t)\chi^2\mathrm{d}\Omega^2 \tag{1.15}$$

where χ is a radial coordinate, and $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\varphi^2$ is the infinitesimal solid angle. We consider a light ray ; the comoving distance between the object, at a redshift z, and an observer at $\chi = 0$ is obtained by integrating $d\chi$ along a null geodesic, i.e. $ds^2 = 0$. Then, from ((1.15)), we deduce :

$$\mathrm{d}\chi = \frac{\mathrm{d}t}{a} = -\frac{\mathrm{d}z}{(1+z)a(z)H(z)} \tag{1.16}$$

Integrating this element from the emitting object to the receptor, we can find the comoving distance corresponding to a given redshift z :

$$\chi(z) = -\int_z^0 \frac{\mathrm{d}z}{(1+z)a(z)H(z)} = \int_0^z \frac{\mathrm{d}z}{H(z)} = \frac{1}{H_0}\int_0^z \frac{\mathrm{d}z}{h(z)} \tag{1.17}$$

[2] From here on, unless specified otherwise, we will work in the simpler case of a flat universe. **[motiv?]**

Luminosity distance $d_L(z)$ Knowing the luminosity of the object L_S and measuring the flux coming from it ϕ_R, we can deduce a distance measurement, the luminosity distance :

$$d_L^2 = \frac{L_S}{4\pi\,\phi_R} \tag{1.18}$$

It is related to the comoving distance χ by the relation :

$$d_L(z) = (1+z)\chi(z) = \frac{1+z}{H_0}\int_0^z \frac{\mathrm{d}z}{h(z)} \tag{1.19}$$

It is commonly expressed as the distance modulus μ :

$$\mu(z) \equiv m - M = 5\log\left(\frac{d_L(z)}{10pc}\right) \tag{1.20}$$

which is an interesting physical quantity, as it can be measured ; we will develop this interest in [REF NEEDED], where we will use surveys of supernovae observations in which the distance to objects is measured as the distance modulus.

Angular diameter distance $d_A(z)$ As in a euclidean space, the angular diameter distance between a source and an observer is defined as the ratio of the diameter of said source and its apparent angle for the observer :

$$d_A(z) = \frac{d}{\theta_d} \tag{1.21}$$

It can also be related to the comoving distance by the relation :

$$d_A(z) = \frac{\chi(z)}{1+z} = \frac{1}{H_0(1+z)}\int_0^z \frac{\mathrm{d}z}{h(z)} \tag{1.22}$$

This expression of distance can be used as a way to measure the distance to an object when the observable is its angular size.

These three distances measurement are equal in a euclidean space, and are very close for small redshifts (i.e. close objects), when $1 + z \to 1$, explaining why there is no observable difference between them when measuring distances on earth or in the solar system.

1.2 Probing cosmology

These distance measurements all share an interesting property : they depend on the cosmology through the integral of $h^{-1}(z)$. This dependance is what allows us to set cosmological constraints. Indeed, given a measurement of the redshift of an object and any measurement of its distance from us, we can compute the cosmological model $h(z)$ that gives us the best adequation between the measured distance and the distance infered from the redshift measurement. For example, say we know the intrinsic luminosity of an object ; we then know the luminosity distance to it $d_{L,\text{meas.}}$. If we are also able to have a measurement of its redshift $z_{\text{meas.}}$, we can compute another measurement of the same luminosity distance :

$$d_L(z_{\text{meas.}}) = \frac{1+z_{\text{meas.}}}{H_0}\int_0^{z_{\text{meas.}}} \frac{\mathrm{d}z}{h(z)} \tag{1.23}$$

The only way to have these two distance measurements equal is to adjust the cosmological model $h(z)$; for example, via a chi-squared adjustment, where adjusting the cosmological parameters in order to minimize the chi-squared value :

$$\chi^2 = \sum_{sample} \frac{\left[d_L(z_{\text{meas.}}) - d_{L,\text{meas.}}\right]^2}{\Delta d_{L,\text{meas.}}} \tag{1.24}$$

will give us the cosmological parameters that best fit our observations.

1.3 Type Ia supernovae

Type Ia supernovae (SNIa) are an interesting type of transient object. They are created when a white dwarf inside a binary system accumulates enough mass to ignite carbon fusion, resluting in a violent explosion when the star reaches the Chandrasekar mass $M \sim 1.44\, M_{\odot}$.

Although the details of this process are not yet fully understood, it gives SNIa some very satisfying properties, epecially a low luminosity variability. This means that the observed magnitudes of supernovae at a given redshift are all quite similar, with a dispersion of $\sim 30\%$.

For this peculiarity, SNIa are often refered to as "standard candles". However, this term is not quite correct, as perfect standard candles would have a near 0 dispersion. Instead, SNIa are better described as "standardisable candles", meaning that a very low dispersion in luminosities can be achieved with a few empirical modeling.

1.3.1 Observational properties of SNIa

As we mentionned before, the physical mechanism during which a star dies to give birth to a supernova is not entirely mastered. Therefore, SNIa are often described with respect to their observed properties, rather than with the physics of the explosion.

Among these properties, SNIa present strong spectroscopic signatures. **[insert more about that here]**

Besides their spectroscopy, SNIa are also identifiable by their photometric features. **[insert more about that here]**

1.3.2 The SALT2 model

We said earlier that SNIa luminosities could be empirically modeled in order to lower their dispersion in luminosity. The current favored model is SALT2 **[ref]**, which models the SNIa lightcurve using two

Bibliography

[1] A. Einstein, “Zur allgemeinen relativitätstheorie – the theory of general relativity,” *Preussische Akademie der Wissenschaften*, pp. 778–786, 799–801, 1915.

[2] A. G. Riess *et al.*, “Observational evidence from supernovae for an accelerating universe and a cosmological constant,” *Astron. J.*, vol. 116, pp. 1009–1038, 1998.

[3] S. Perlmutter *et al.*, “Measurements of Ω and Λ from 42 high redshift supernovae,” *Astrophys. J.*, vol. 517, pp. 565–586, 1999.

[4] M. Betoule *et al.*, “Improved cosmological constraints from a joint analysis of the SDSS-II and SNLS supernova samples,” *Astron. Astrophys.*, vol. 568, p. A22, 2014.

[5] R. Beck *et al.*, “Photometric redshifts for the sdss data release 12,” *Monthly Notices of the Royal Astronomical Society*, vol. 460, no. 2, pp. 1371–1381, 2016.