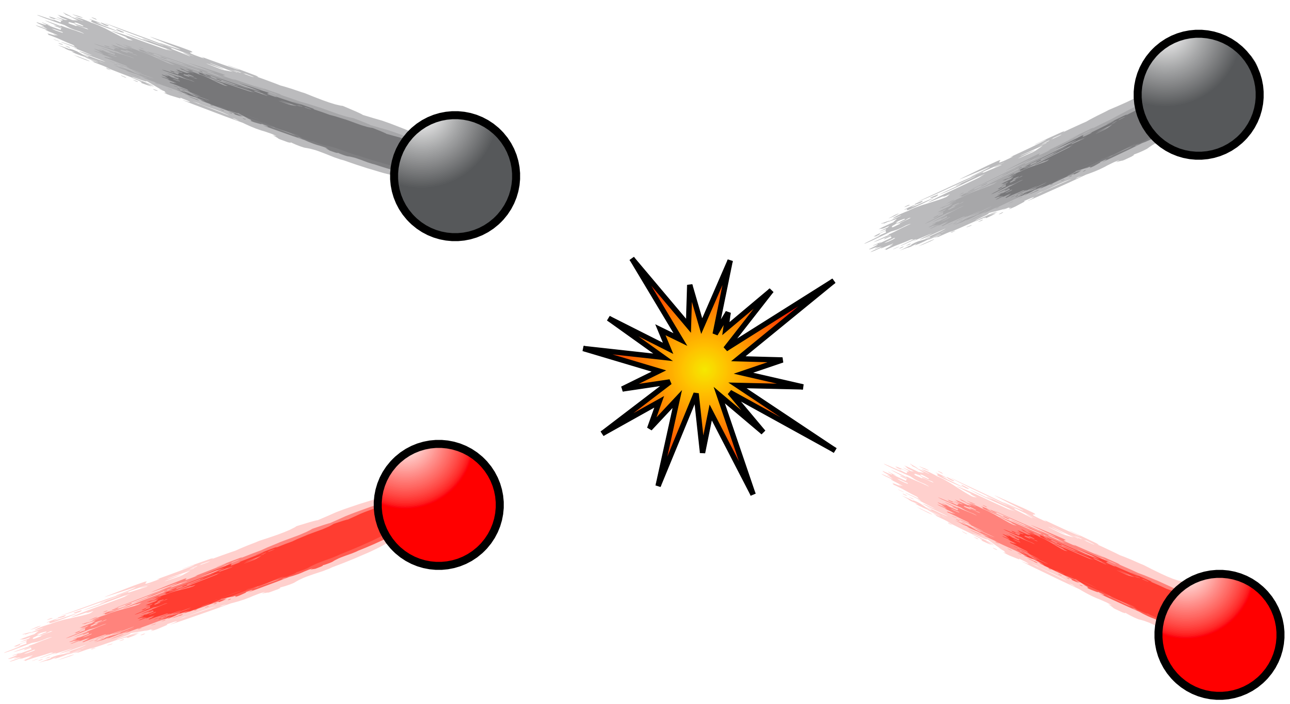
The Deuteron:
Connecting scattering to nuclear binding

Andrew W. Jackura

2021 REYES Nuclear Theory Mentorship



Old Dominion University
Jefferson Lab
Thomas Jefferson National Accelerator Facility

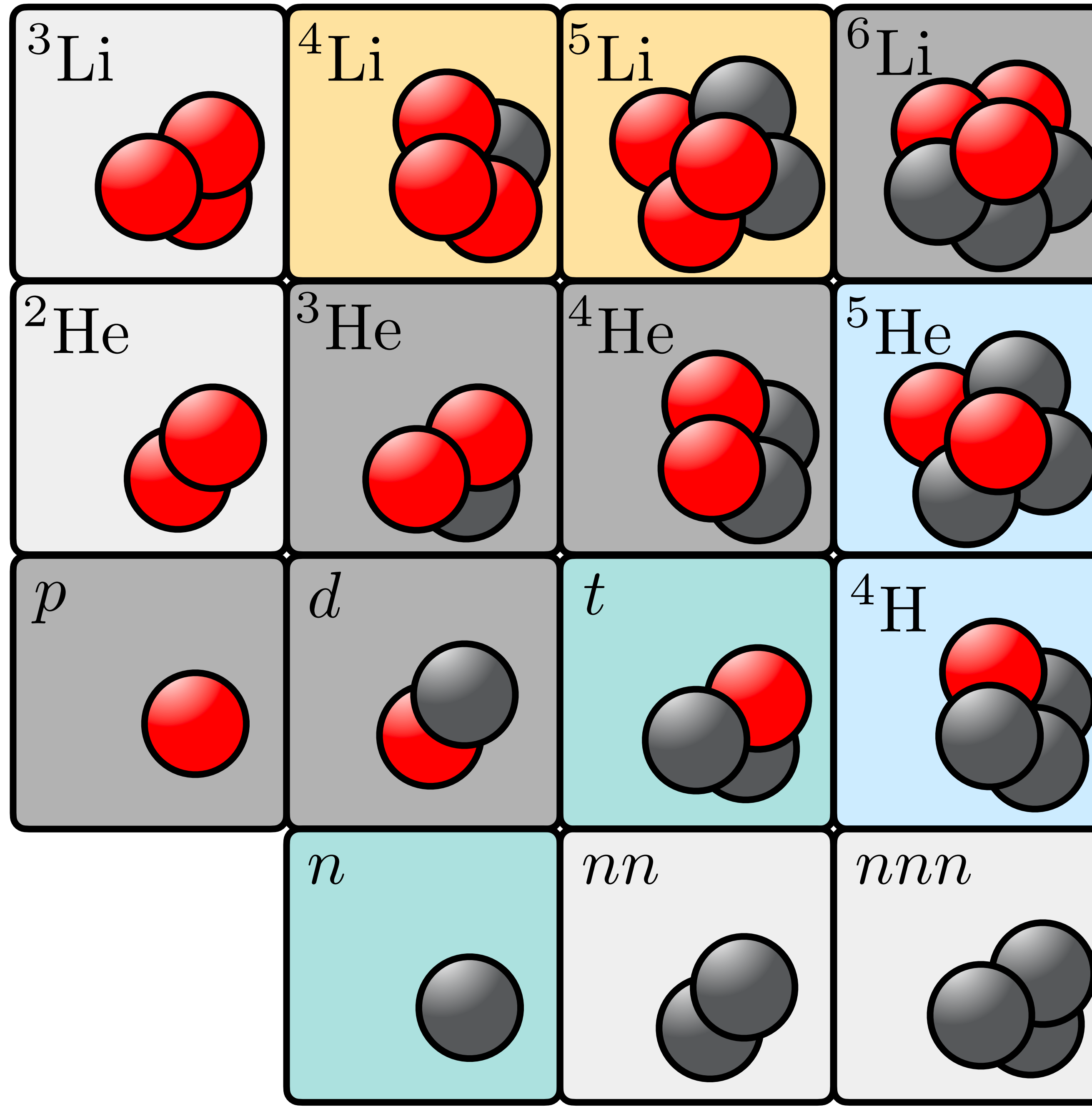
The Deuteron

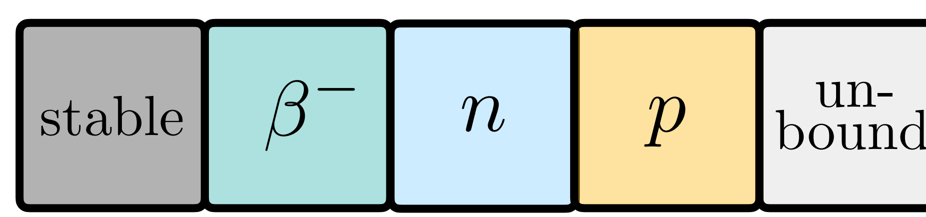
Goal: Understand nuclear interactions from QCD

Focus on simplest interacting case:

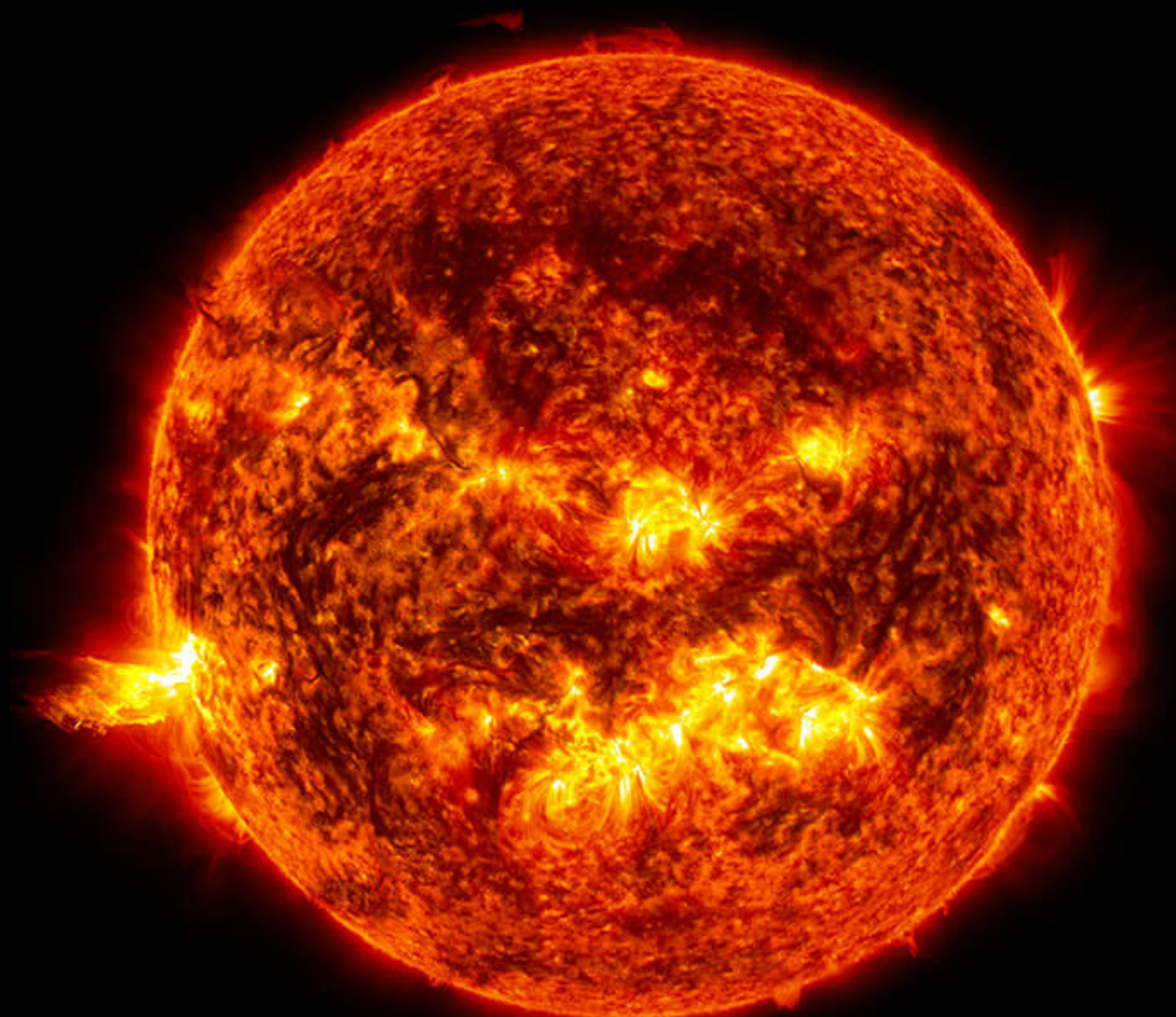
The Deuteron

The deuteron plays a role in stellar fusion of heavier elements



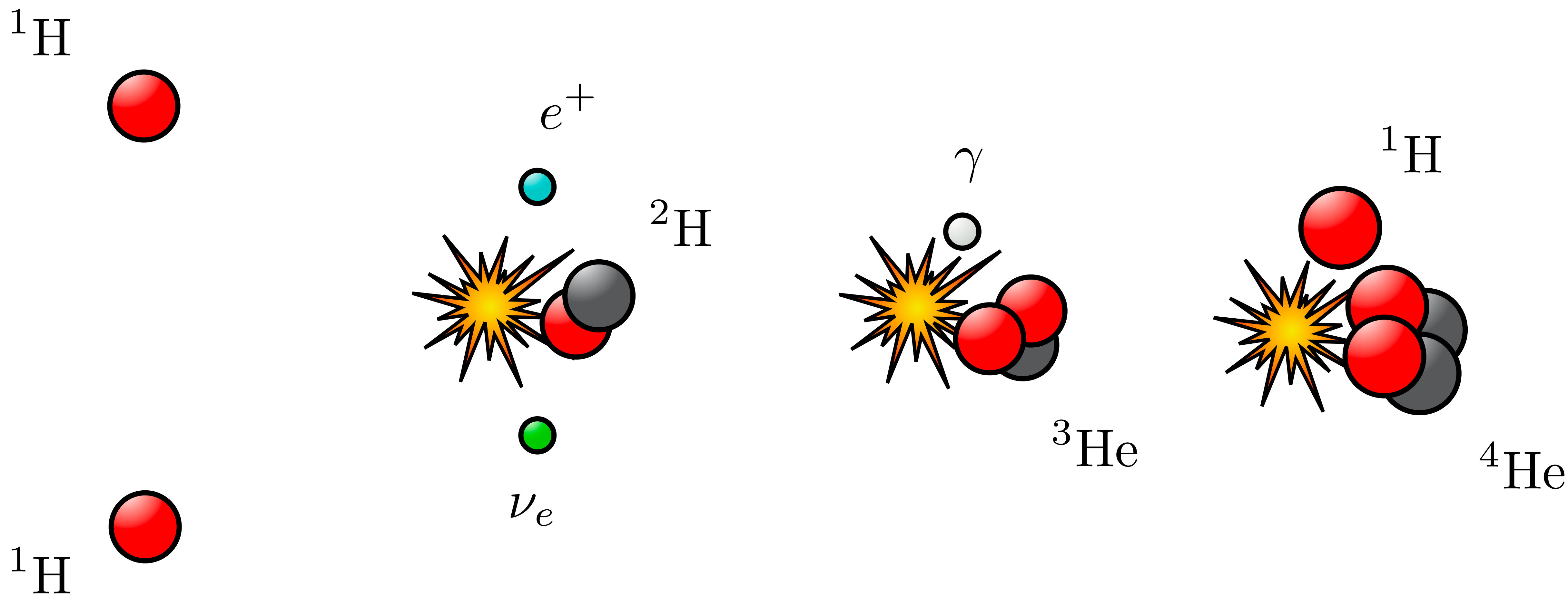


Stellar Nucleosynthesis



https://spaceplace.nasa.gov/gallery-sun/en/

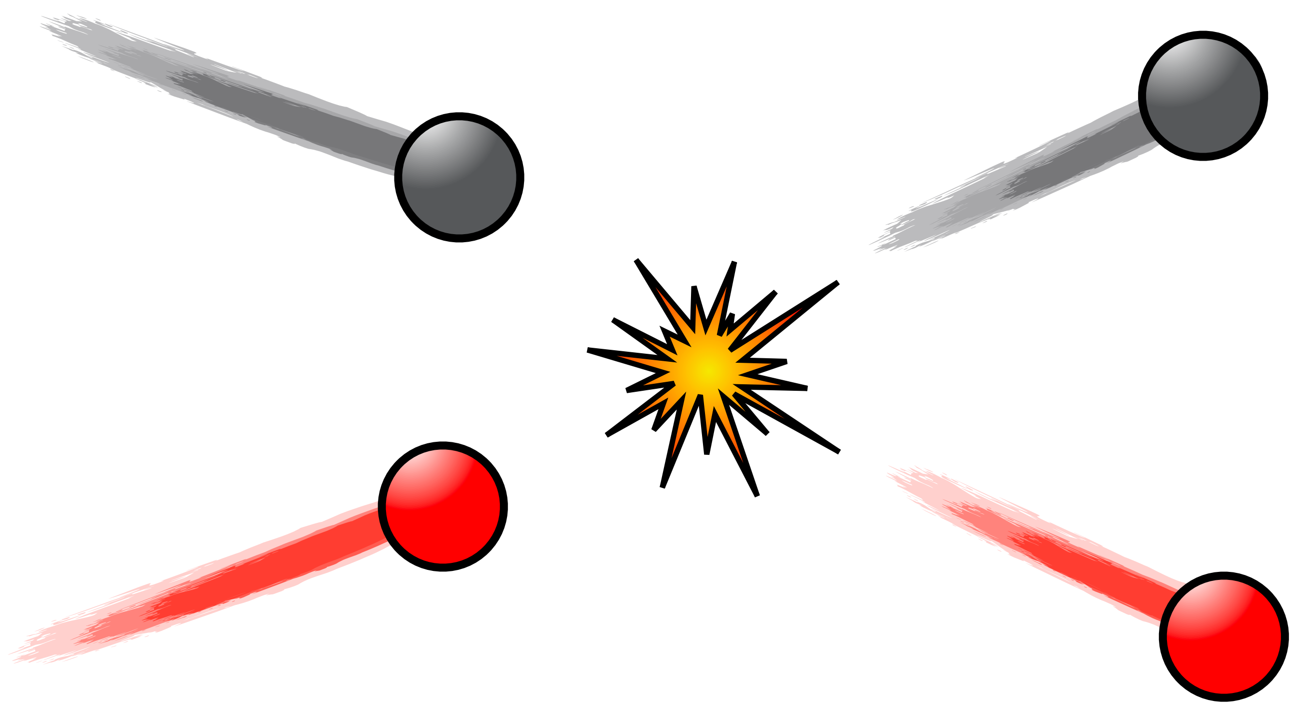
The Deuteron — Proton-Proton Fusion



The Deuteron — Nucleon-Nucleon Scattering

In order to understand complication nuclear interactions, must understand deuteron first

Focus on proton-neutron (NN) scattering



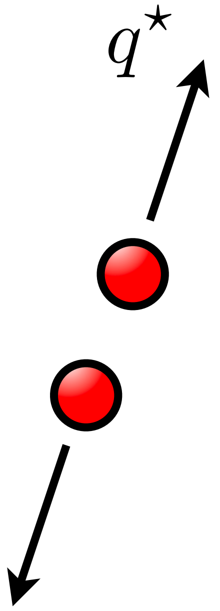
The Deuteron — Nucleon-Nucleon Scattering Amplitude

Let us consider NN scattering, assuming equal mass proton and neutron (*isospin limit*)

- Ignoring some details, the scattering amplitude has the form*

$$\mathcal{M} = \frac{1}{\mathcal{K}^{-1} - i\rho}$$

$$= \frac{8\pi E^\star/\xi}{q^\star \cot\delta - iq^\star}$$



$$m = m_N \approx 940 \text{ MeV}$$

$$q^\star = \frac{1}{2}\sqrt{E^{\star\,2} - 4m^2}$$

$$\xi = \frac{1}{2}$$

* *Technically, we focus on the* $^3S_1$ *amplitude*

The Deuteron — Nucleon-Nucleon Scattering Amplitude

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We focus on a specific representation for the *K* matrix/ phase shift

Effective Range Expansion

$$q^\star \cot\delta = -\frac{1}{a} + \frac{1}{2} r q^{\star\,2} + \mathcal{O}(q^{\star\,4})$$

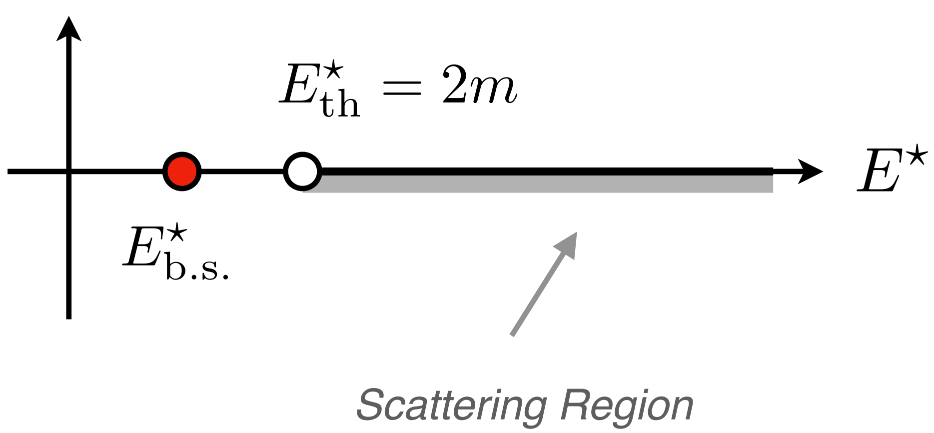
a, scattering length

r, effective range

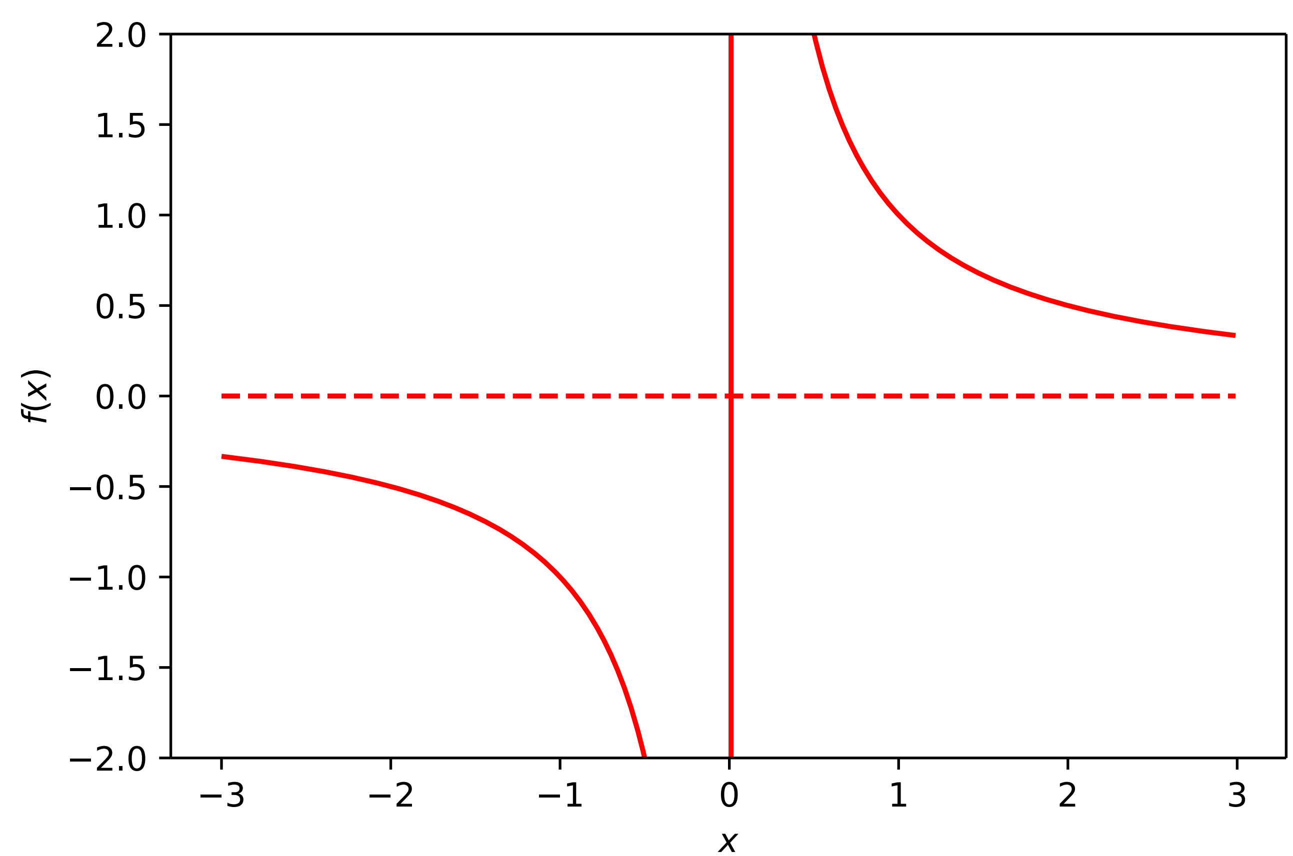
The Deuteron — Nucleon-Nucleon Scattering Amplitude

Let us consider NN scattering, assuming equal mass proton and neutron (*isospin limit*)

- Proton-Neutron scattering — attractive interactions
- Can bind to form the Deuteron (D or $^2\mathrm{H}$) — simplest nucleus
  - **Question**: How do bound states appear in scattering amplitudes?
  - **Answer**: They appear as *pole singularities* below threshold



$$f(x) = \frac{1}{x}$$



Bound state physics

Bound states appear as *pole singularities* below threshold

$$q^\star = i\kappa$$

Binding momentum

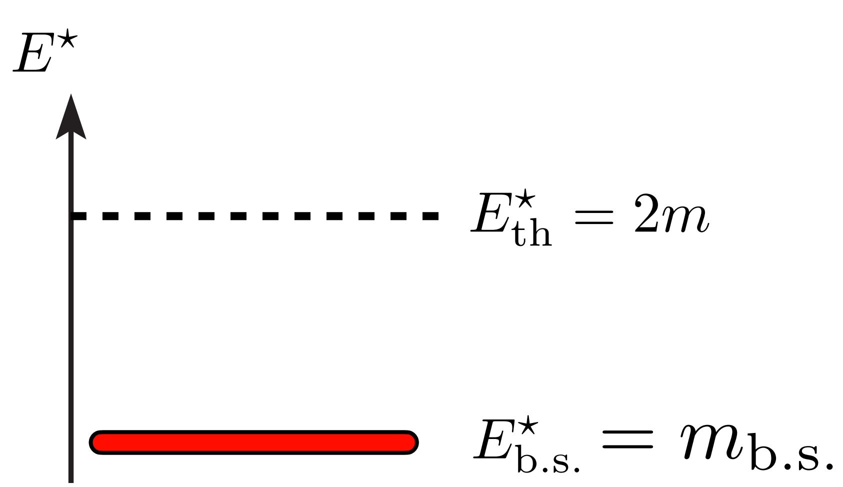
Mass of bound state

$$E^\star_{\text{b.s.}} = 2\sqrt{m^2 - \kappa^2} \quad = m_{\text{b.s.}}$$

$$m_{\text{b.s.}} < E^\star_{\text{th}}$$

Bound state mass

Binding energy



$$E^\star_{\text{binding}} = E^\star_{\text{th}} - E^\star_{\text{b.s.}}$$

$$E^\star = 2\sqrt{m^2 + q^{\star\,2}}$$

The Deuteron — Nucleon-Nucleon Scattering Amplitude

Let us consider NN scattering, assuming equal mass proton and neutron (*isospin limit*)

- Proton-Neutron scattering — attractive interactions
- Can bind to form the Deuteron (D or $^2\mathrm{H}$) — simplest nucleus
  - **Question**: Given the scattering amplitude, what is the deuteron mass?

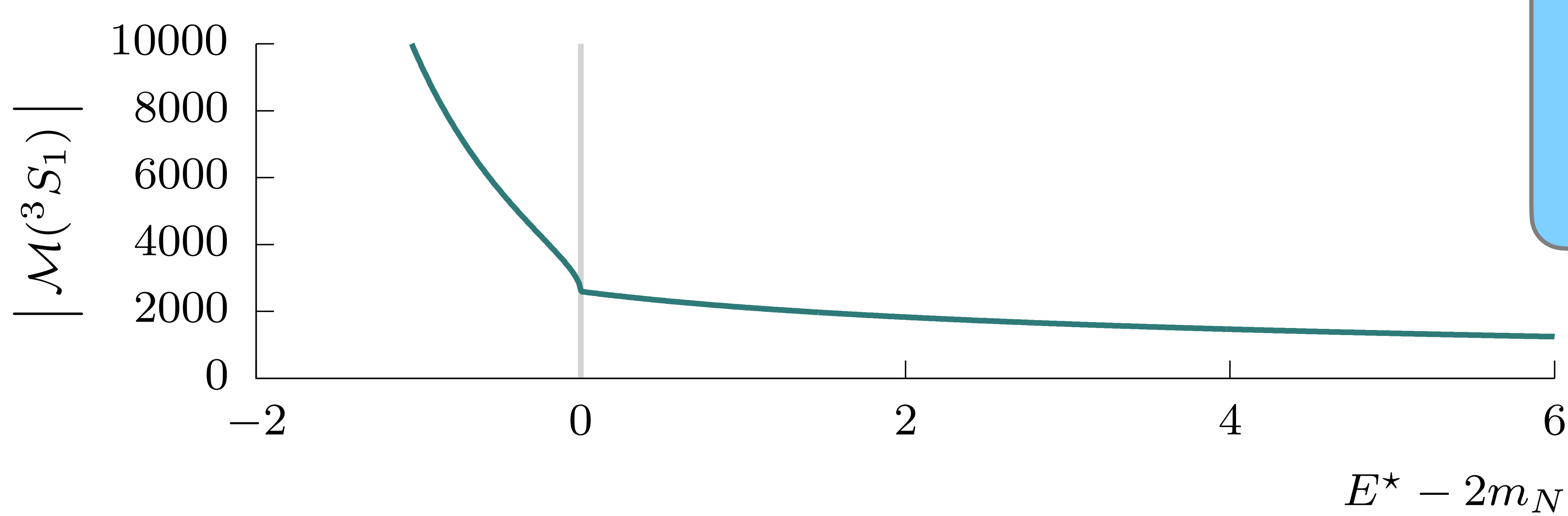
    What is its binding energy?

    How do we get these properties from the scattering amplitude?
  - **Answer**: Compute the position of pole!

    Need scattering parameters (experimentally or theoretically)

$$m_N = 940 \text{ MeV}$$
$$a = 5.425 \text{ fm}$$
$$r = 1.749 \text{ fm}$$

$$1 \text{ fm} \approx \frac{1}{200 \text{ MeV}}$$



The Deuteron — Nucleon-Nucleon Scattering Amplitude

Let us find pole position

$$\mathcal{M} = \frac{8\pi E^\star/\xi}{q^\star \cot\delta - iq^\star}$$

$$q^\star \cot\delta = -\frac{1}{a} + \frac{1}{2} r q^{\star\,2} + \mathcal{O}(q^{\star\,4})$$

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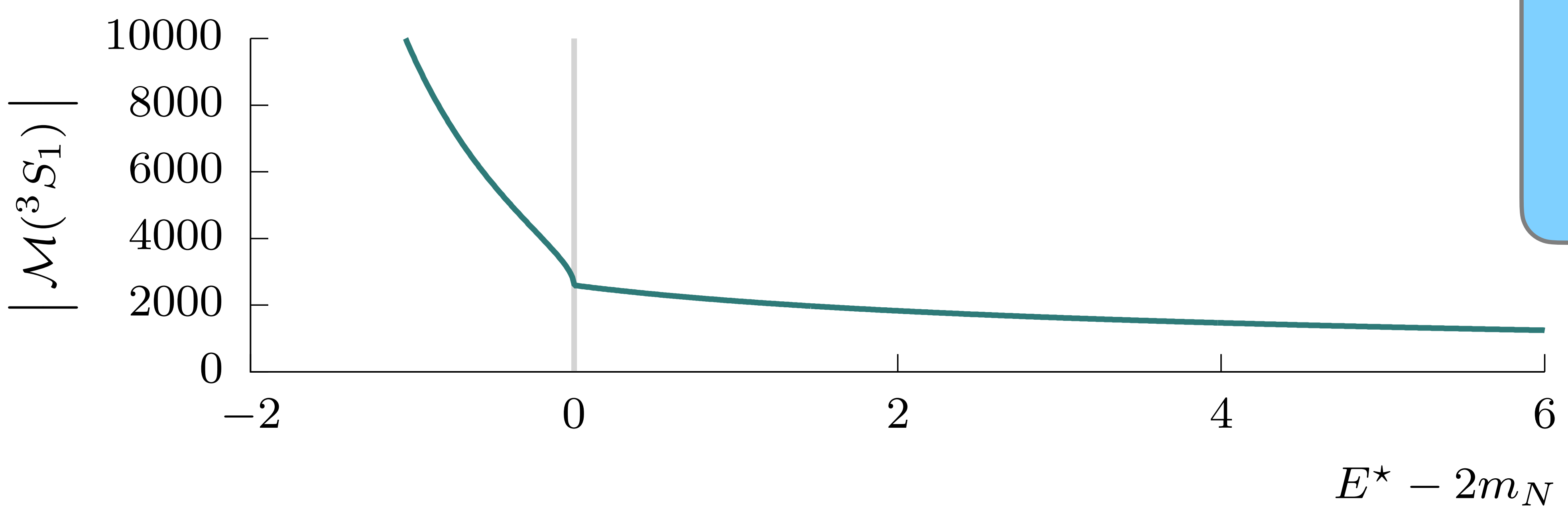

The Deuteron — Nucleon-Nucleon Scattering Amplitude

Let us find pole position

$$\mathcal{M} = \frac{8\pi E^\star/\xi}{q^\star \cot\delta - iq^\star}$$

First assume only scattering length $q^\star \cot\delta \approx -\dfrac{1}{a}$

$$\mathcal{M} \approx \frac{8\pi E^\star/\xi}{-\frac{1}{a} - iq^\star}$$

Pole occurs at $-\dfrac{1}{a} - iq^\star = 0 \quad \Longrightarrow \quad q^\star = \dfrac{i}{a}$

The Deuteron — Nucleon-Nucleon Scattering Amplitude

Binding momentum of deuteron

$$\kappa = \frac{1}{a} \approx 37 \text{ MeV}$$

Mass of deuteron

$$m_{\text{b.s.}} = 2\sqrt{m^2 - \kappa^2} \approx 1878 \text{ MeV}$$

Binding energy

$$E^{\star}_{\text{binding}} = E^{\star}_{\text{th}} - E^{\star}_{\text{b.s.}}$$

$$\approx 2 \text{ MeV}$$

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The deuteron binding energy

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Abstract

The ^{1}H(n, γ)^{2}H γ-ray energy has been measured relative to ^{48}V and ^{144}Ce γ-rays, which are both based on the gold standard for γ-ray energies. The ensuing deut binding energy, B(^{2}H= 2224575 ± 9eV, confirms (with higher accuracy) the value from one of two conflicting recent precision measurements. This value has been used to recalculate the energies of γ-rays from thermal-neutron capture in ^{2}H, ^{1}

Suggested Exercise

(a) For the NN scattering parameters, plot the scattering amplitude

(b) Include the effective range (r) into the calculation for the deuteron

1. Find the pole position $q^\star = q^\star(a, r)$
2. Find the binding momentum κ (recall, $q^\star = i\kappa$)
3. Find the mass of the deuteron
4. Find the binding energy

(c) Verify numerically the pole location agrees with the plot

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