Hadron Spectroscopy

Andrew W. Jackura

Old Dominion University & Jefferson Lab

RPI Computational Summer School

June 20, 2022

Outline

Hadrons, Quarks, & QCD

Hadron Spectroscopy

The Quark Model

Quantum Chromodynamics

Scattering Theory & the Hadron Spectrum

Scattering Amplitudes

Bound & Resonant States

Lattice QCD & Hadron Spectroscopy

Lattice QCD

Lüscher & the Finite-Volume

Some Reviews

arXiv:1706.06223 (2017)

Scattering processes and resonances from lattice QCD

Raúl A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] and Ross D. Young[3,‡]

[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

[2] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*

[3] *Special Research Center for the Subatomic Structure of Matter (CSSM), Department of Physics, University of Adelaide, Adelaide 5005, Australia*

(Dated: June 21, 2017)

The vast majority of hadrons observed in nature are not stable under the strong interaction, rather they are *resonances* whose existence is deduced from enhancements in the energy dependence of scattering amplitudes. The study of hadron resonances offers a window into the workings of quantum chromodynamics (QCD) in the low-energy non-perturbative region, and in addition, many probes of the limits of the electroweak sector of the Standard Model consider processes which feature hadron resonances. From a theoretical standpoint, this is a challenging field: the same dynamics that binds quarks and gluons into hadron resonances also controls their decay into lighter hadrons, so a complete approach to QCD is required. Presently, lattice QCD is the only available tool that provides the required non-perturbative evaluation of hadron observables. In this article, we review progress in the study of few-hadron reactions in which resonances and bound-states appear using lattice QCD techniques. We describe the leading approach which takes advantage of the periodic finite spatial volume used in lattice QCD calculations to extract scattering amplitudes from the discrete spectrum of QCD eigenstates in a box. We explain how from explicit lattice QCD calculations, one can rigorously garner information about a variety of resonance properties, including their masses, widths, decay couplings, and form factors. The challenges which currently limit the field are discussed along with the steps being taken to resolve them.

CONTENTS

Some Reviews

arXiv:1706.06223 (2017)

Scattering processes and resonances from lattice QCD

Raúl A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] and Ross D. Young[3,‡]

[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

[2] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*

[3] *Special Research Center for the Subatomic Structure of Matter (CSSM), Department of Physics, University of Adelaide, Adelaide 5005, Australia*

(Dated: June 21, 2017)

The vast majority of hadrons observed in nature a
action, rather they are *resonances* whose existen
the energy dependence of scattering amplitudes.
fers a window into the workings of quantum chrom
non-perturbative region, and in addition, many pr
sector of the Standard Model consider processes wh
a theoretical standpoint, this is a challenging field:
and gluons into hadron resonances also controls t
complete approach to QCD is required. Presently,
that provides the required non-perturbative evalu
article, we review progress in the study of few-hadr
bound-states appear using lattice QCD techniques
which takes advantage of the periodic finite spatial
tions to extract scattering amplitudes from the disc
a box. We explain how from explicit lattice QCD
ner information about a variety of resonance prope
decay couplings, and form factors. The challenge
discussed along with the steps being taken to resol

CONTENTS

arXiv:2112.13436 (2021)

Novel approaches in Hadron Spectroscopy

Miguel Albaladejo[a,b], Łukasz Bibrzycki[c], Sebastian M. Dawid[d,e], César Fernández-Ramírez[f,g], Sergi Gonzàlez-Solís[d,e,h], Astrid N. Hiller Blin[a], Andrew W. Jackura[a,i], Vincent Mathieu[j,k], Mikhail Mikhasenko[l,m], Victor I. Mokeev[a], Emilie Passemar[a,d,e], Alessandro Pilloni[n,o,*], Arkaitz Rodas[a,p], Jorge A. Silva-Castro[f], Wyatt A. Smith[d], Adam P. Szczepaniak[a,d,e], Daniel Winney[d,e,q,r],

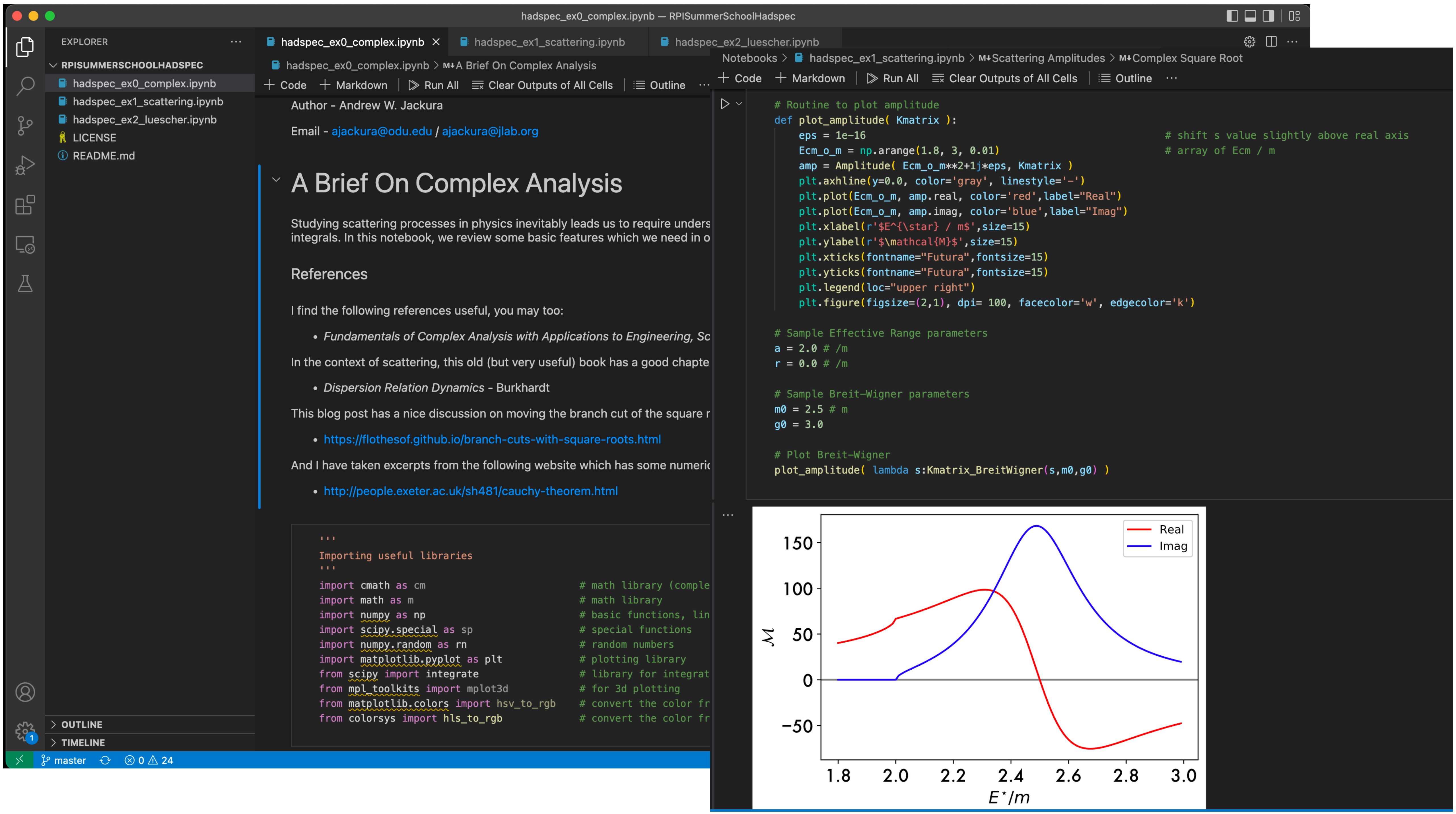
(Joint Physics Analysis Center)

[a] *Theory Center and Physics Division, Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA*
[b] *Instituto de Física Corpuscular (IFIC), Centro Mixto CSIC-Universidad de Valencia, E-46071 Valencia, Spain*
[c] *Pedagogical University of Krakow, 30-084 Kraków, Poland*
[d] *Department of Physics, Indiana University, Bloomington, IN 47405, USA*
[e] *Center for Exploration of Energy and Matter, Indiana University, Bloomington, IN 47403, USA*
[f] *Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Ciudad de México 04510, Mexico*
[g] *Departamento de Física Interdisciplinar, Universidad Nacional de Educación a Distancia (UNED), Madrid E-28040, Spain*
[h] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*
[i] *Department of Physics, Old Dominion University, Norfolk, VA 23529, USA*
[j] *Departament de Física Quàntica i Astrofísica and Institut de Ciències del Cosmos, Universitat de Barcelona, E-08028, Spain*
[k] *Departamento de Física Teórica, Universidad Complutense de Madrid and IPARCOS, E-28040 Madrid, Spain*
[l] *ORIGINS Excellence Cluster, 80939 Munich, Germany*
[m] *Ludwig-Maximilian University of Munich, Germany*
[n] *Dipartimento di Scienze Matematiche e Informatiche, Scienze Fisiche e Scienze della Terra, Università degli Studi di Messina, I-98122 Messina, Italy*
[o] *INFN Sezione di Catania, I-95123 Catania, Italy*
[p] *College of William & Mary, Williamsburg, VA 23187, USA*
[q] *Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China*
[r] *Guangdong-Hong Kong Joint Laboratory of Quantum Matter, Southern Nuclear Science Computing Center, South China*

Python Notebooks

I have created a few Jupyter notebooks to accompany the lectures

- All notebooks use ***python***, and the modules that come with the [Anaconda package manager]
- Includes introductory and advanced exercises
- https://github.com/ajackura/RPISummerSchoolHadspec



Outline

Hadrons, Quarks, & QCD

Hadron Spectroscopy

The Quark Model

Quantum Chromodynamics

Scattering Theory & the Hadron Spectrum

Scattering Amplitudes

Bound & Resonant States

Lattice QCD & Hadron Spectroscopy

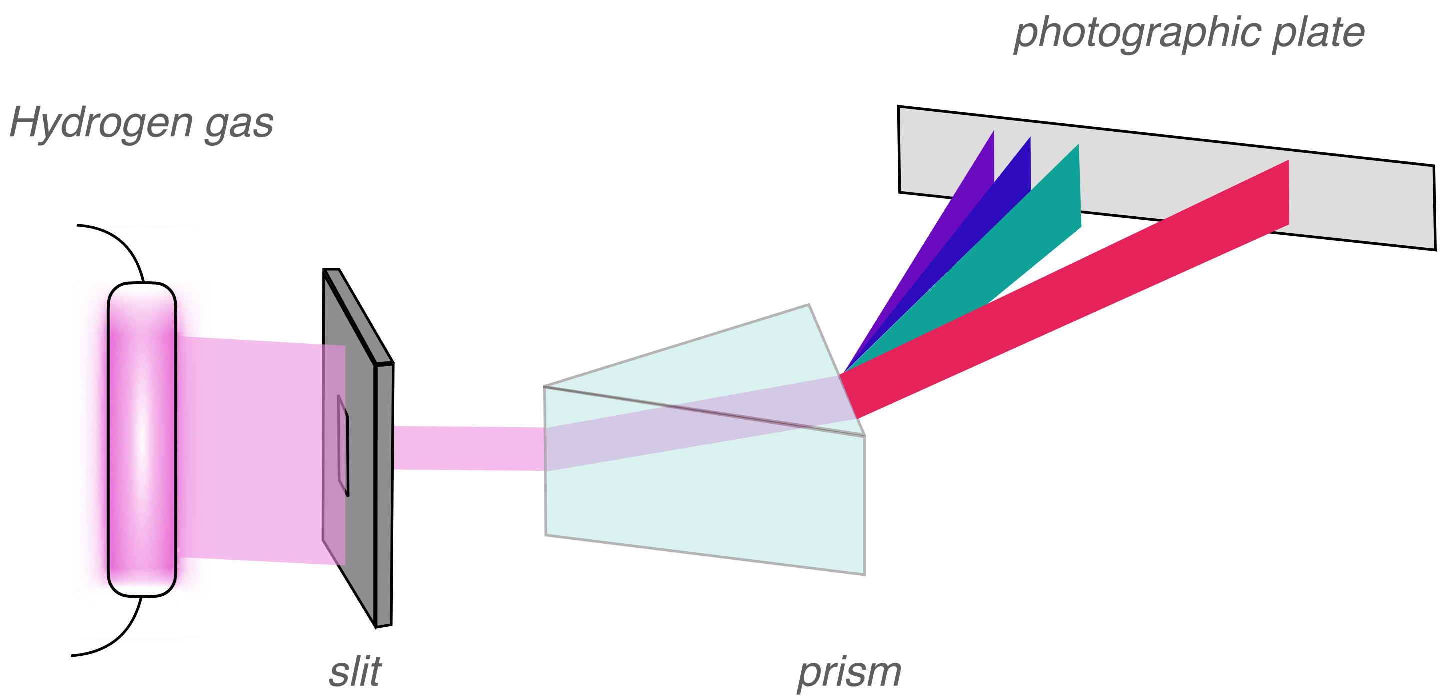
Lattice QCD

Lüscher & the Finite-Volume

Spectroscopy

Studying the ***spectrum*** is key to understanding physical phenomena

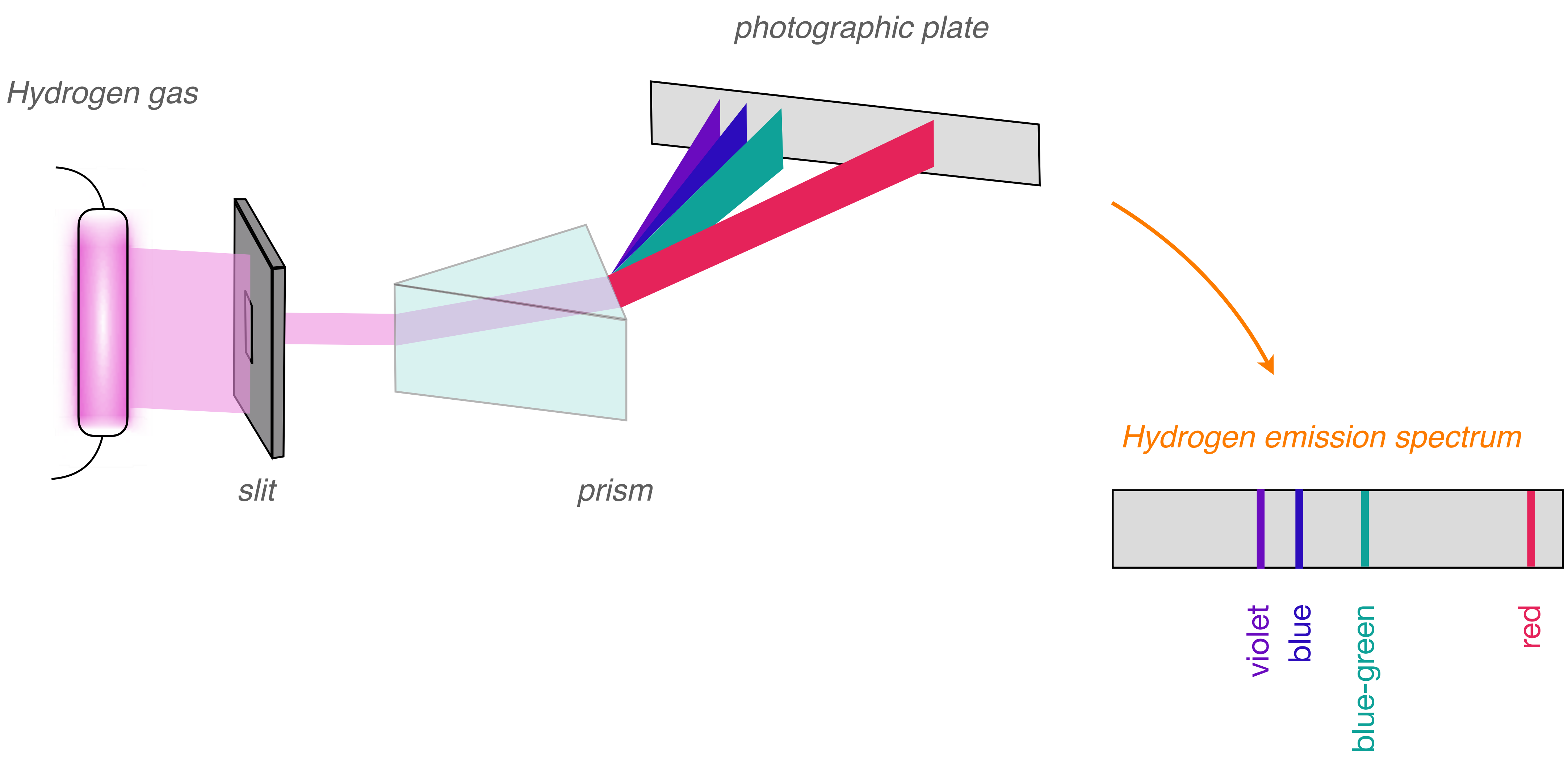
- e.g., the Hydrogen atom led to the discovery of Quantum Mechanics



Spectroscopy

Studying the ***spectrum*** is key to understanding physical phenomena

- e.g., the Hydrogen atom led to the discovery of Quantum Mechanics

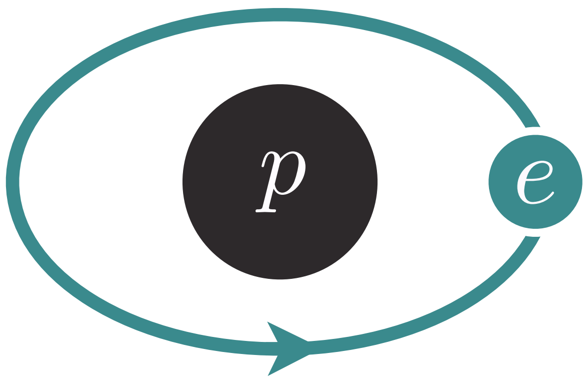


Quantized spectrum

Spectroscopy

Studying the ***spectrum*** is key to understanding physical phenomena

- e.g., the Hydrogen atom led to the discovery of Quantum Mechanics
- Precision spectroscopy paved the way for modern particle physics



$$\widehat{\mathrm{H}} \left|\psi\right\rangle = E_n \left|\psi\right\rangle$$

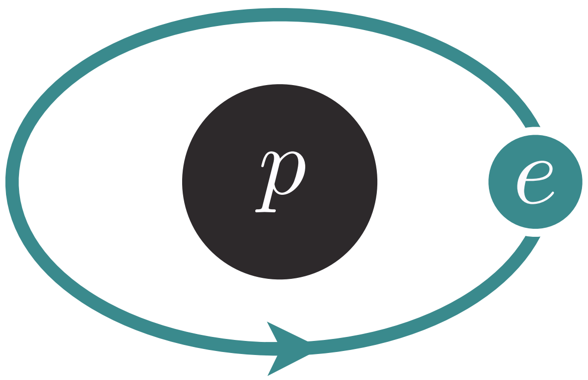
Schrödinger equation

Spectroscopy

$\hbar = c = 1$

Studying the ***spectrum*** is key to understanding physical phenomena

- e.g., the Hydrogen atom led to the discovery of Quantum Mechanics
- Precision spectroscopy paved the way for modern particle physics

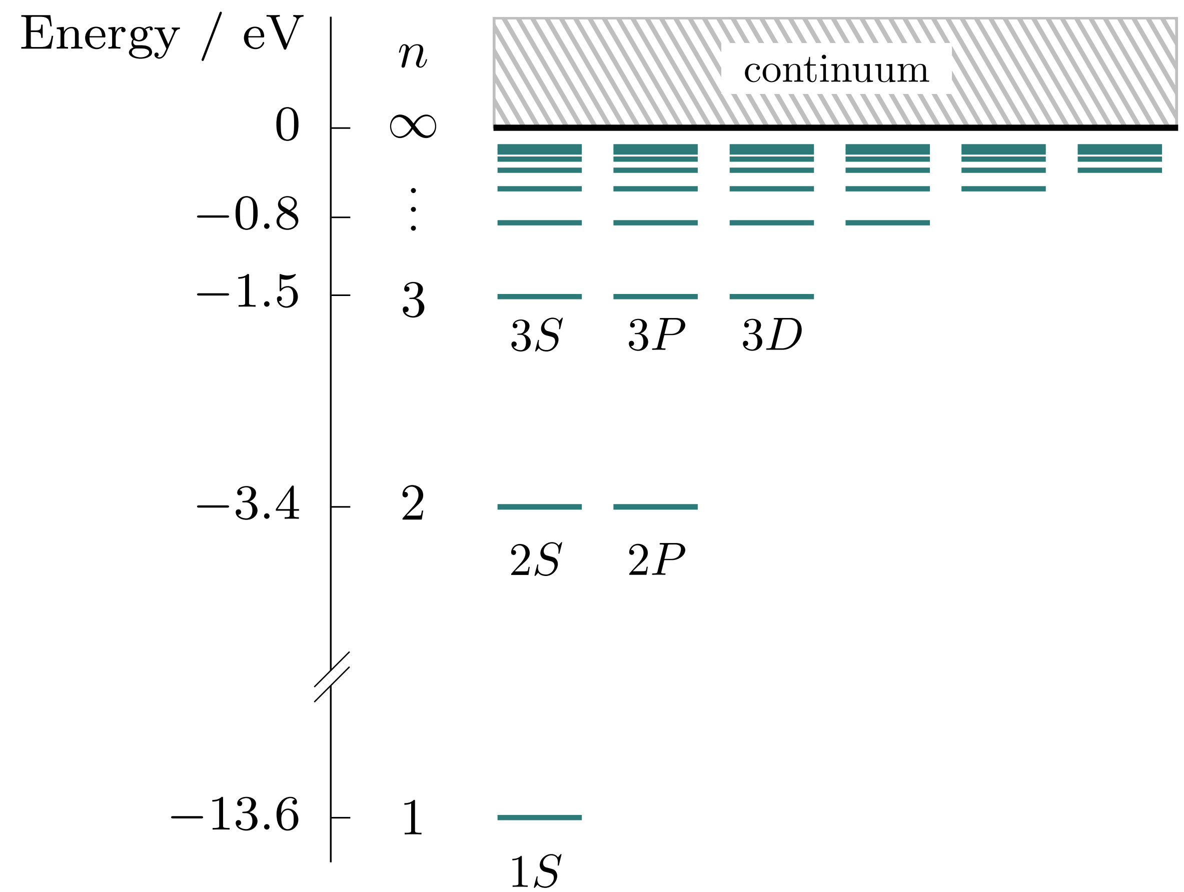


$$\widehat{\mathrm{H}}\,|\psi\rangle = E_n\,|\psi\rangle$$

Schrödinger equation

spectrum

$$E_n = -\frac{m\alpha^2}{2n}$$

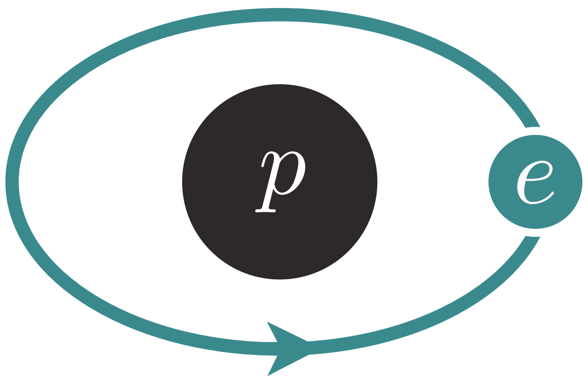


$$\alpha = \frac{e^2}{4\pi}$$

Spectroscopy

Studying the ***spectrum*** is key to understanding physical phenomena

- e.g., the Hydrogen atom led to the discovery of Quantum Mechanics
- Precision spectroscopy paved the way for modern particle physics



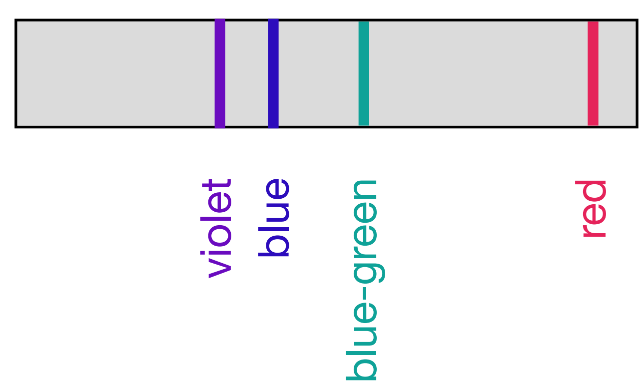
$$\widehat{\mathrm{H}}\,|\psi\rangle = E_n\,|\psi\rangle$$

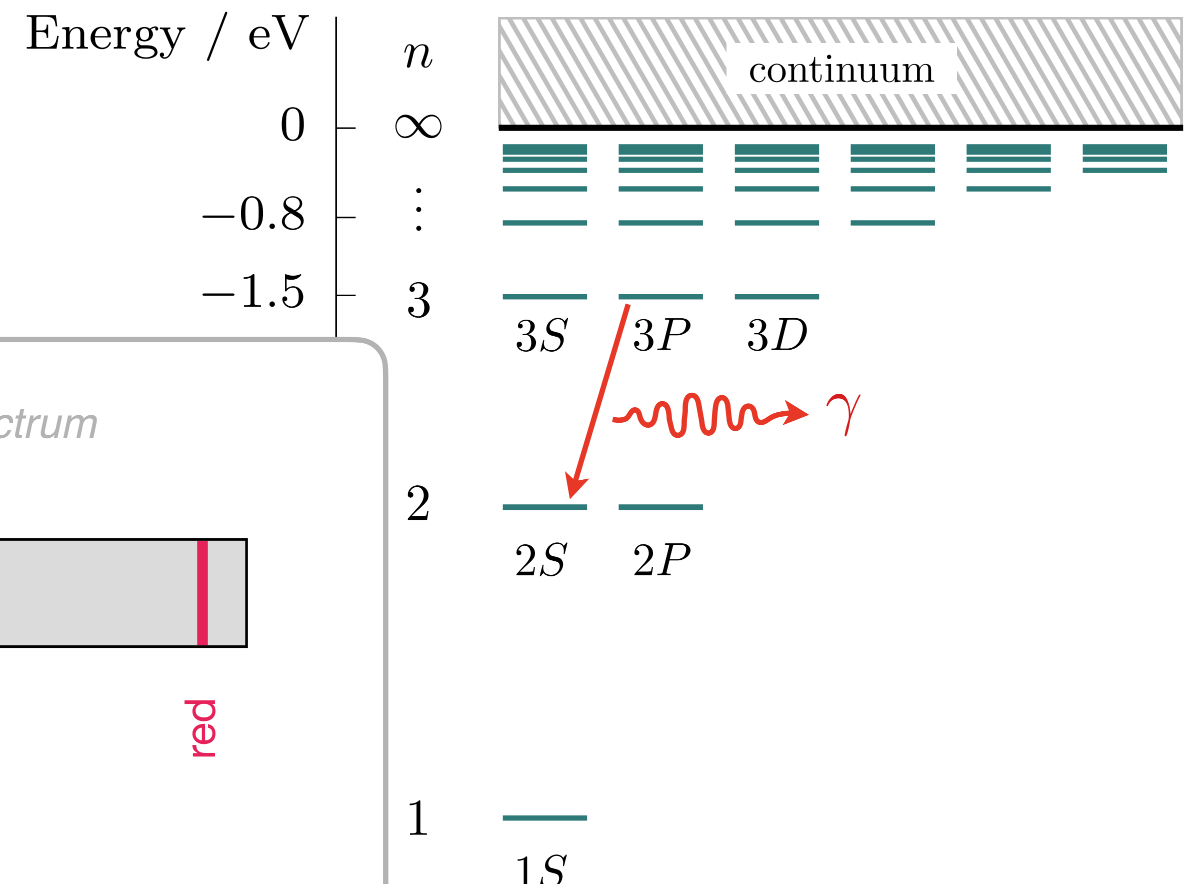
spectrum

$$E_n = -\frac{m}{2}$$

$$\alpha = \frac{e^2}{4\pi}$$

Hydrogen emission spectrum

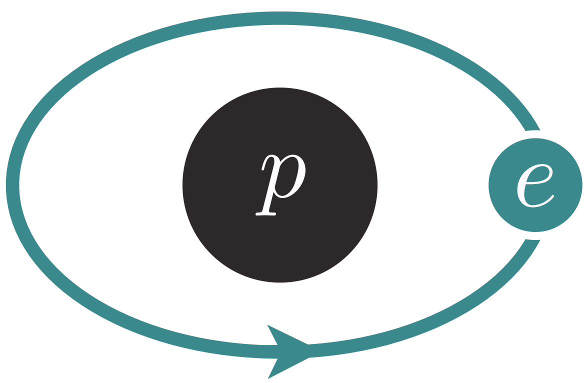




Spectroscopy

Studying the ***spectrum*** is key to understanding physical phenomena

- e.g., the Hydrogen atom led to the discovery of Quantum Mechanics
- Precision spectroscopy paved the way for modern particle physics



$$\widehat{\mathrm{H}}\,|\psi\rangle = E_n\,|\psi\rangle$$

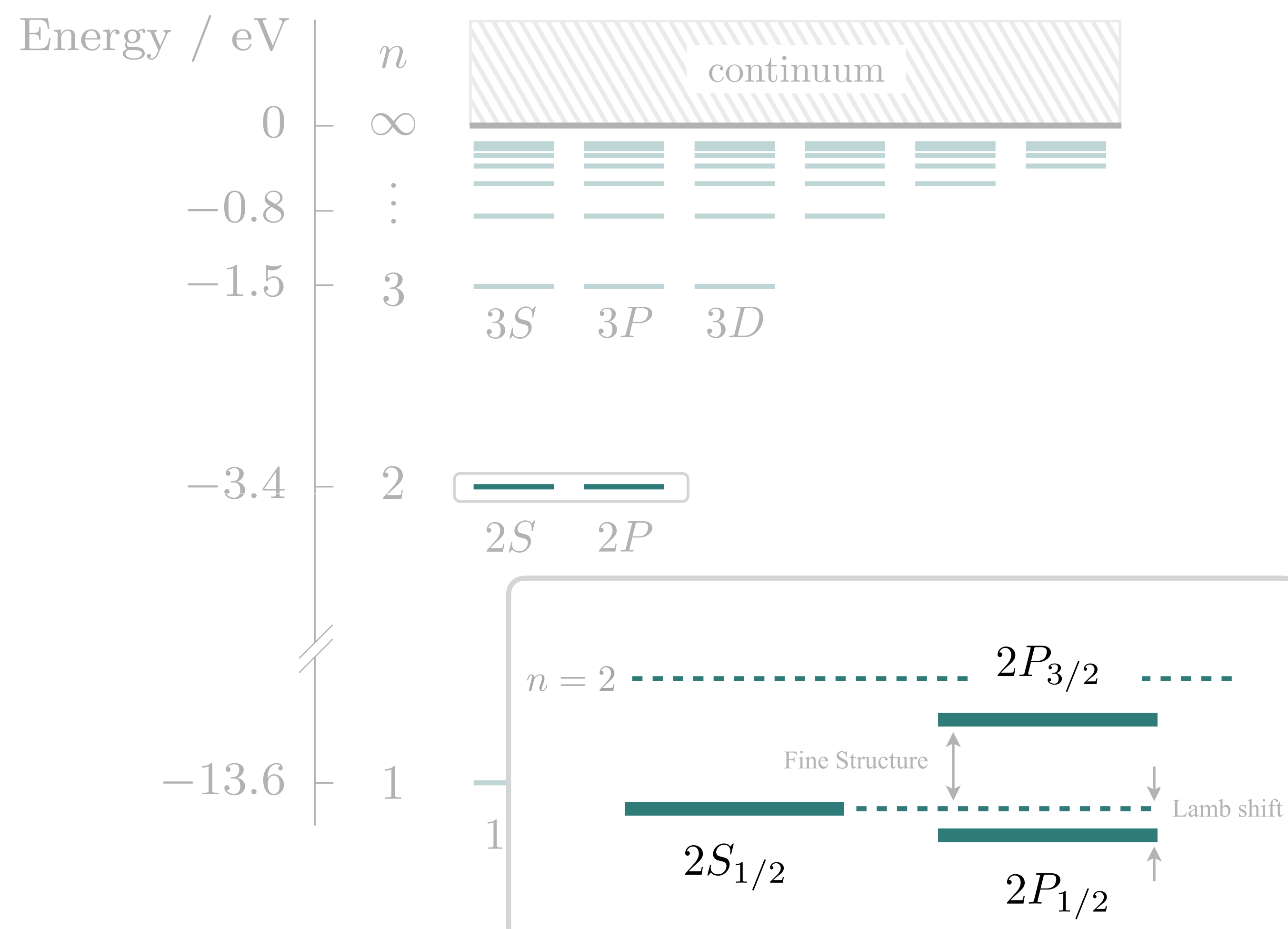
Schrödinger equation

spectrum

$$E_n = -\frac{m\alpha^2}{2n} + \mathcal{O}\left(\alpha^4\right)$$

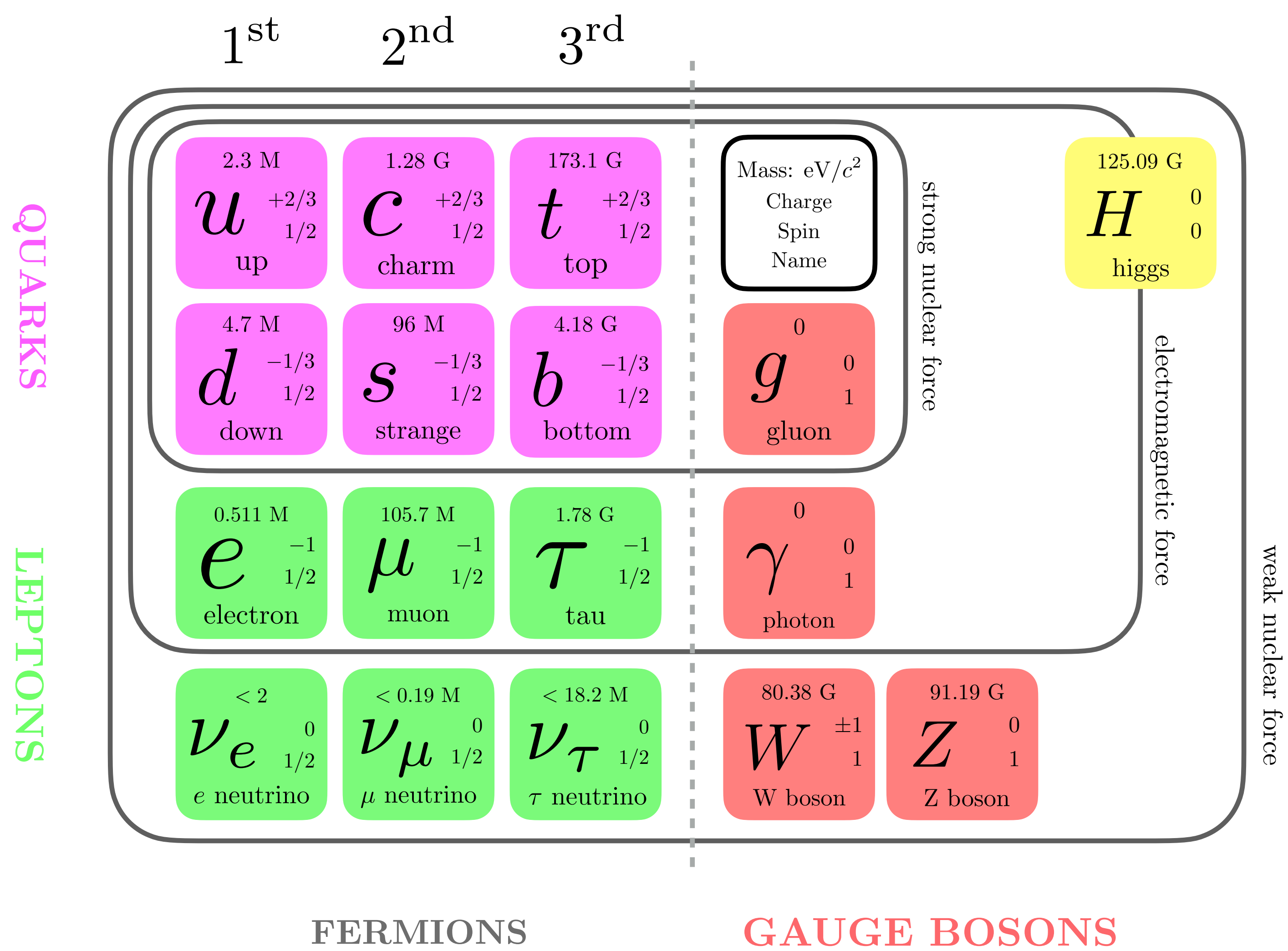
$$\alpha = \frac{e^2}{4\pi}$$

Relativistic effects, QED, …



The Standard Model of Particle Physics

The Standard Model is a remarkably *simple** theory



* *simple = An anomaly-free **relativistic quantum gauge field theory**, invariant under the gauge group* $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ *which spontaneously breaks via a scalar field to* $SU(3)_C \otimes U(1)_Q$

Quantum ElectroDynamics

Theory of electron-photon interactions

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\left(i\not{D} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Quantum Electrodynamics

Quantum ElectroDynamics

Theory of electron-photon interactions

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\left(i\not{D} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Quantum Electrodynamics

electron-positron field

photon field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$D_\mu = \partial_\mu + ieA_\mu$$

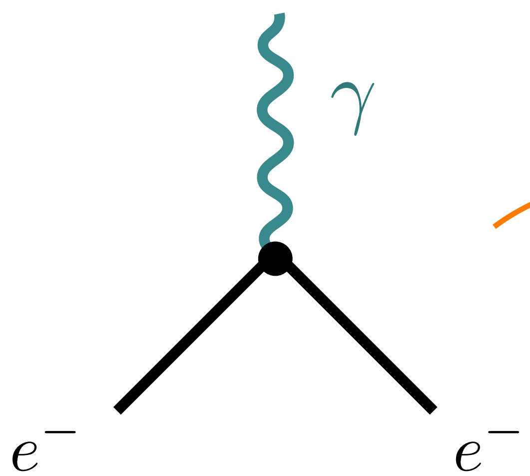
Quantum ElectroDynamics

Theory of electron-photon interactions

$$\mathcal{L}_{\text{QED}} = \bar{\psi}\left(i\not{D} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

electron-positron field

Quantum Electrodynamics



electromagnetic coupling

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

photon field

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

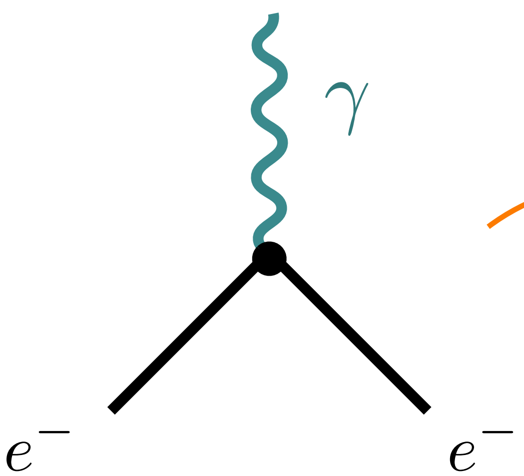
$$D_\mu = \partial_\mu + \boxed{ieA_\mu}$$

Quantum ElectroDynamics

Theory of electron-photon interactions

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\left(i\not{D} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

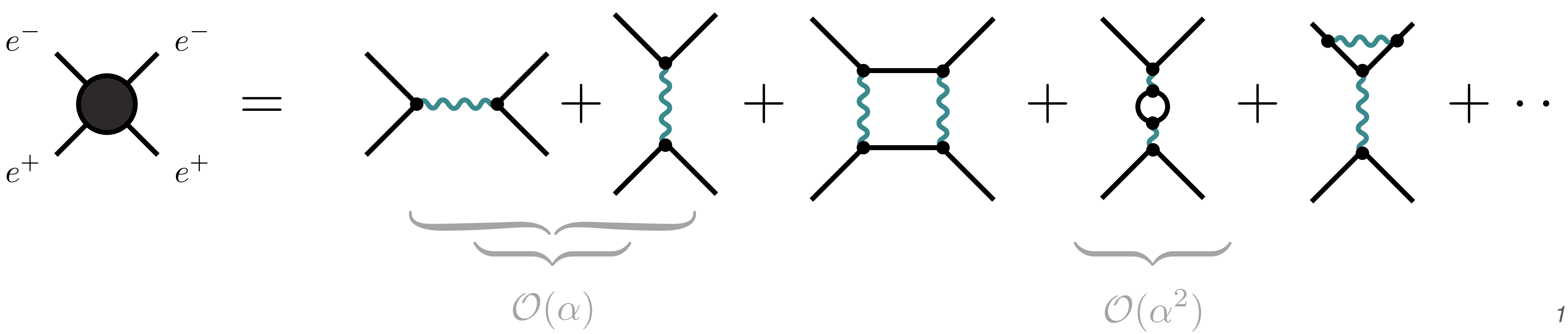
Quantum Electrodynamics



electromagnetic coupling

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

Can compute observables ***perturbatively*** in electric coupling!

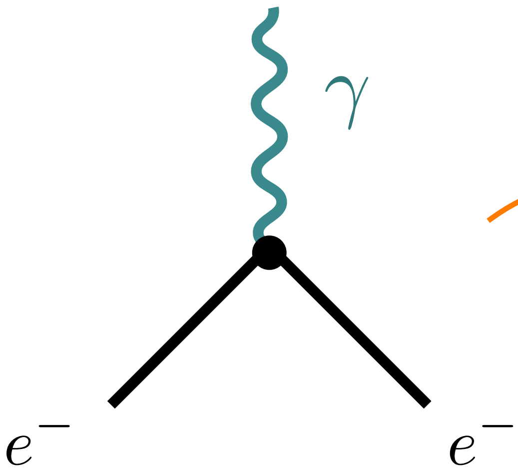


Quantum ElectroDynamics

Theory of electron-photon interactions

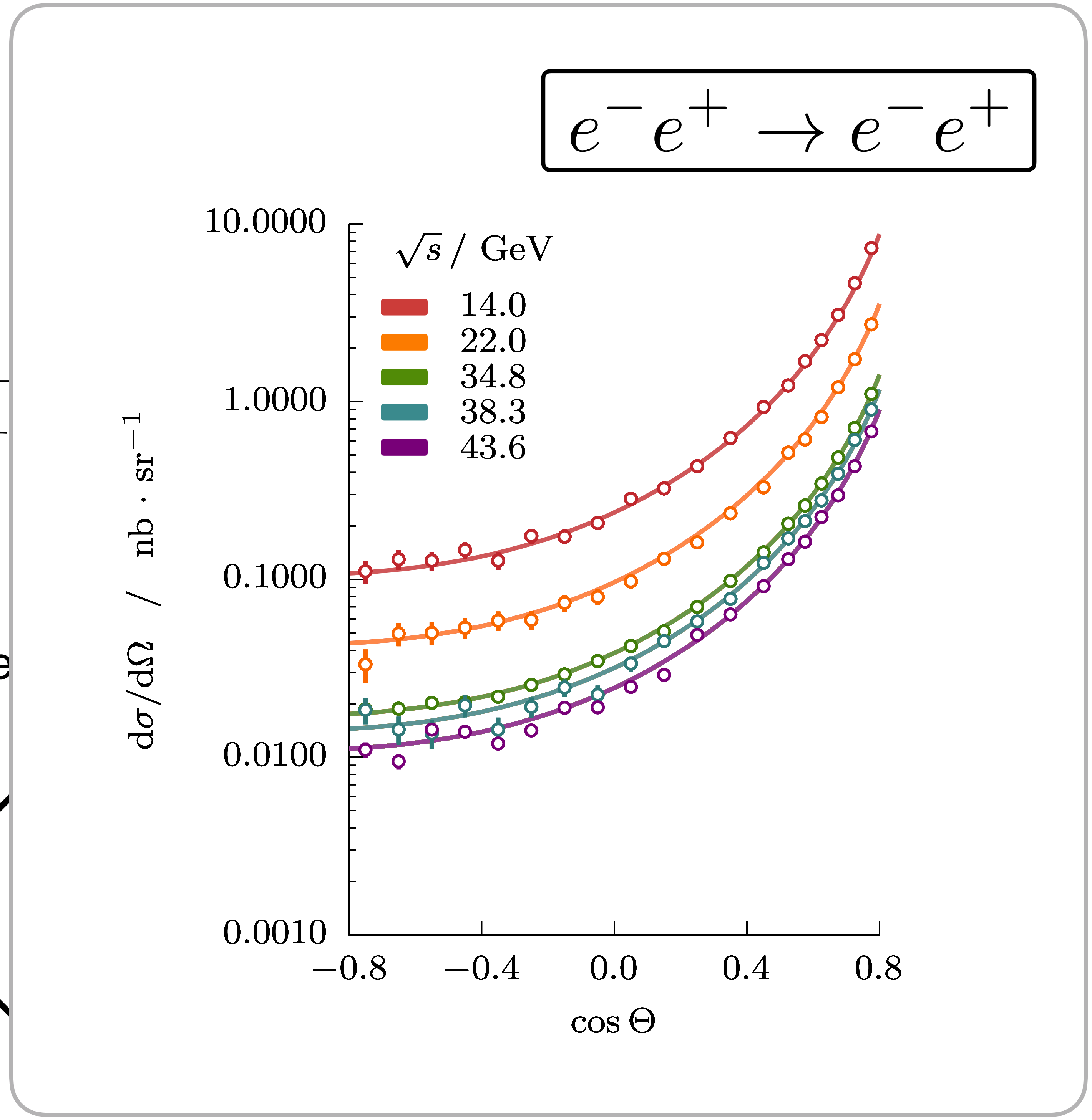
$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\left(i\not{D} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Quantum Electrodynamics

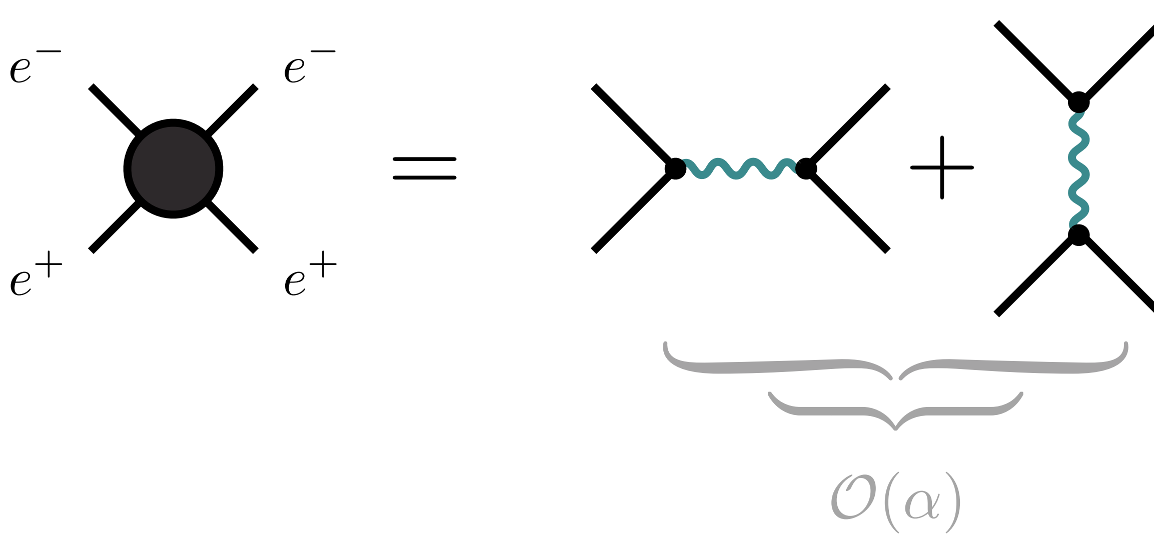


electromagnetic coupling

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{\ldots}$$

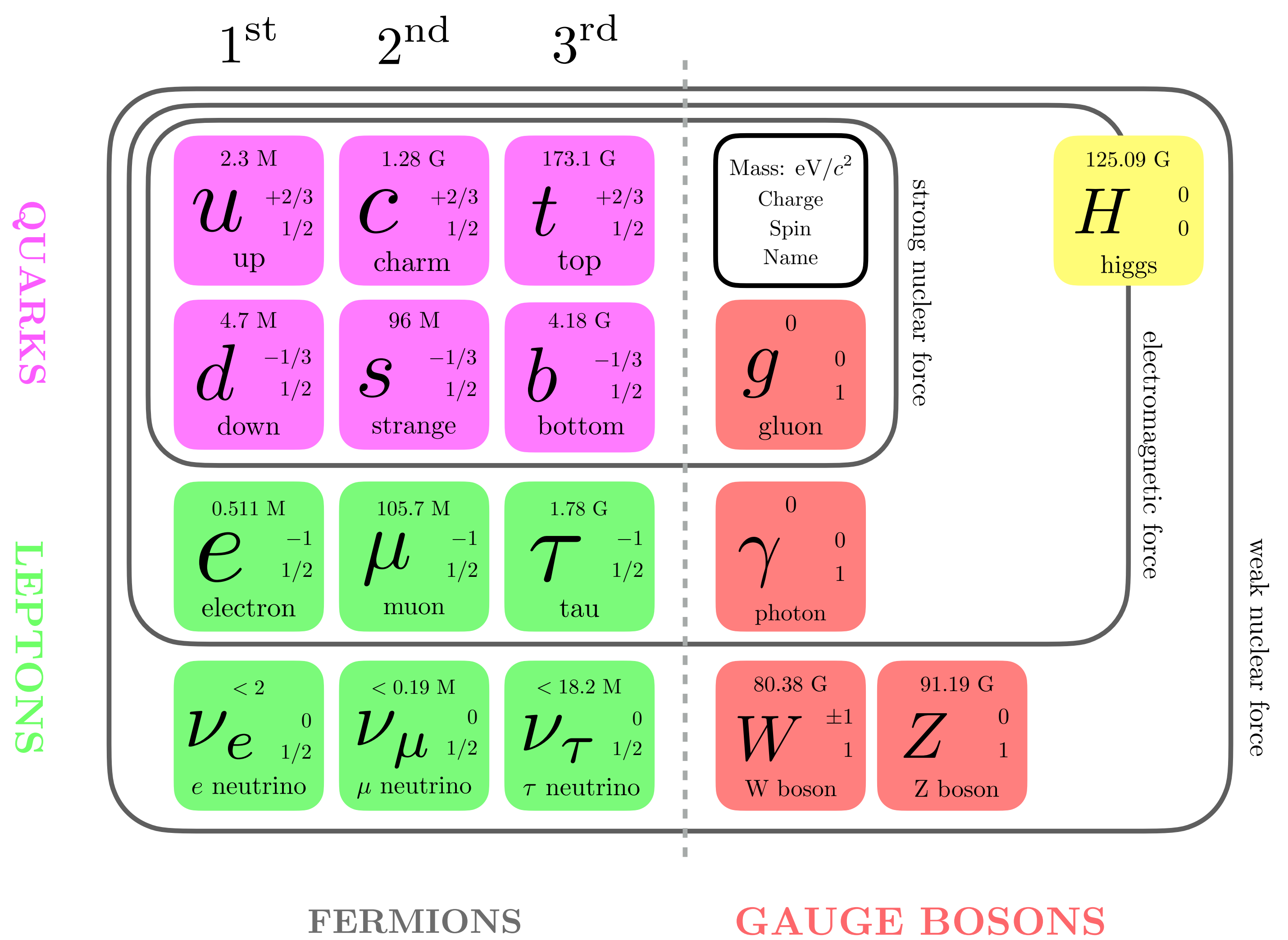


Can compute observables ***perturbatively*** in e…



The Standard Model of Particle Physics

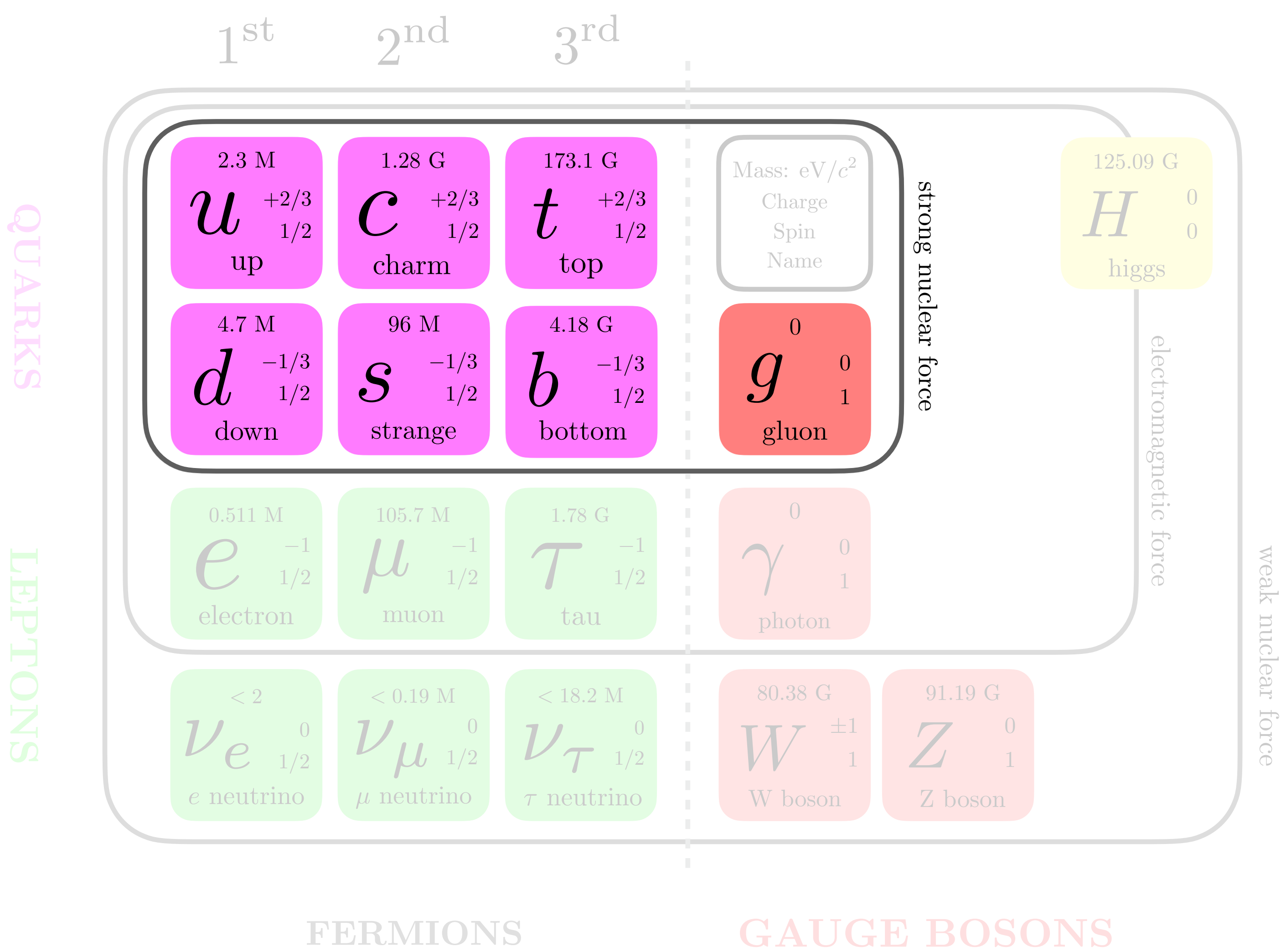
The Standard Model is a remarkably *simple** theory



* *simple = An anomaly-free* ***relativistic quantum gauge field theory****,*
invariant under the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
which spontaneously breaks via a scalar field to $SU(3)_C \otimes U(1)_Q$

The Standard Model of Particle Physics

The Standard Model is a remarkably *simple** theory



Quantum ChromoDynamics

* *simple = An anomaly-free* ***relativistic quantum gauge field theory****,*
invariant under the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
which spontaneously breaks via a scalar field to $SU(3)_C \otimes U(1)_Q$

Quantum ChromoDynamics (QCD)

Theory of quark-gluon interactions

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\boldsymbol{\psi}}_f \left(i\not{\mathbf{D}} - m_f\right) \boldsymbol{\psi}_f - \frac{1}{2}\mathrm{tr}\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right)$$

Quantum Chromodynamics

Quantum ChromoDynamics (QCD)

Theory of quark-gluon interactions

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\boldsymbol{\psi}}_f \left(i\not{\mathbf{D}} - m_f\right) \boldsymbol{\psi}_f - \frac{1}{2}\mathrm{tr}\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right)$$

Quantum Chromodynamics

quark fields

$$\boldsymbol{\psi}_f = \begin{pmatrix} \psi_f \\ \psi_f \\ \psi_f \end{pmatrix}$$

$$f = u, d, s, c, b, t$$

Quantum ChromoDynamics (QCD)

Theory of quark-gluon interactions

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\boldsymbol{\psi}}_f \left(i\not{\mathbf{D}} - m_f\right) \boldsymbol{\psi}_f - \frac{1}{2}\mathrm{tr}\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right)$$

Quantum Chromodynamics

quark fields

$$\boldsymbol{\psi}_f = \begin{pmatrix} \psi_f \\ \psi_f \\ \psi_f \end{pmatrix}$$

$$f = u, d, s, c, b, t$$

gluon fields

$$\mathbf{G}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + ig_s \left[\mathbf{A}_\mu, \mathbf{A}_\nu\right]$$

$$\mathbf{D}_\mu = \partial_\mu + ig_s \mathbf{A}_\mu$$

Quantum ChromoDynamics (QCD)

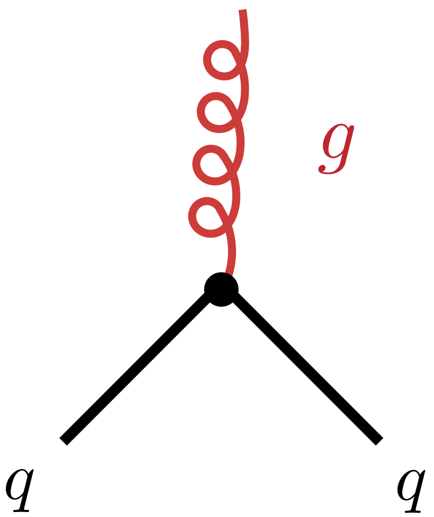
Theory of quark-gluon interactions

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\boldsymbol{\psi}}_f \left(i\not{\mathbf{D}} - m_f\right) \boldsymbol{\psi}_f - \frac{1}{2}\mathrm{tr}\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right)$$

Quantum Chromodynamics

quark fields

$$\boldsymbol{\psi}_f = \begin{pmatrix} \psi_f \\ \psi_f \\ \psi_f \end{pmatrix}$$



strong coupling

$$\alpha_s = \frac{g_s^2}{4\pi}$$

$$f = u, d, s, c, b, t$$

gluon fields

$$\mathbf{G}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + i g_s \left[\mathbf{A}_\mu, \mathbf{A}_\nu\right]$$

$$\mathbf{D}_\mu = \partial_\mu + \boxed{i g_s \mathbf{A}_\mu}$$

Quantum ChromoDynamics (QCD)

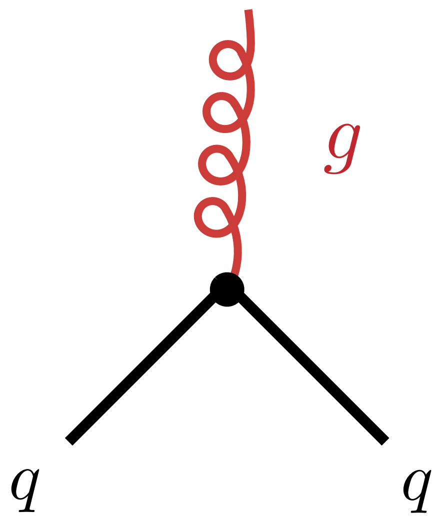
Theory of quark-gluon interactions

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f\right)\psi_f - \frac{1}{2}\mathrm{tr}\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right)$$

Quantum Chromodynamics

quark fields

$$\boldsymbol{\psi}_f = \begin{pmatrix} \psi_f \\ \psi_f \\ \psi_f \end{pmatrix}$$

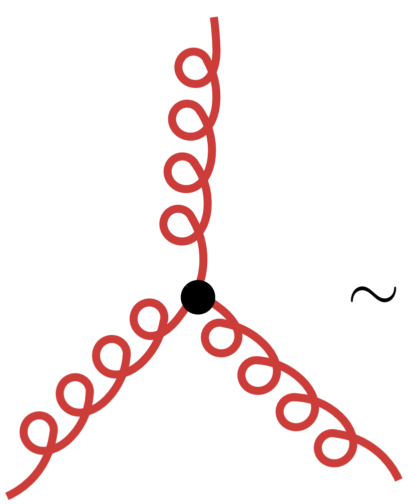


strong coupling

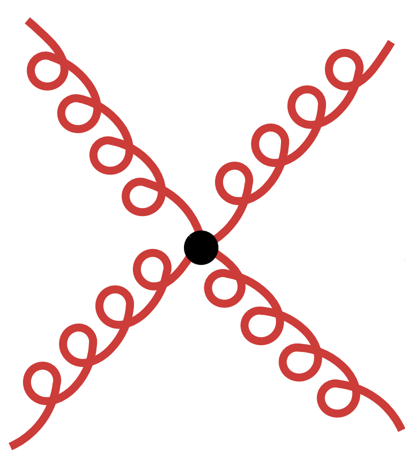
$$\alpha_s = \frac{g_s^2}{4\pi}$$

$$f = u, d, s, c, b, t$$

gluon fields



$\sim g_s$



$\sim g_s^2$

$$\mathbf{G}_{\mu\nu} = \partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu + \boxed{ig_s\left[\mathbf{A}_\mu, \mathbf{A}_\nu\right]}$$

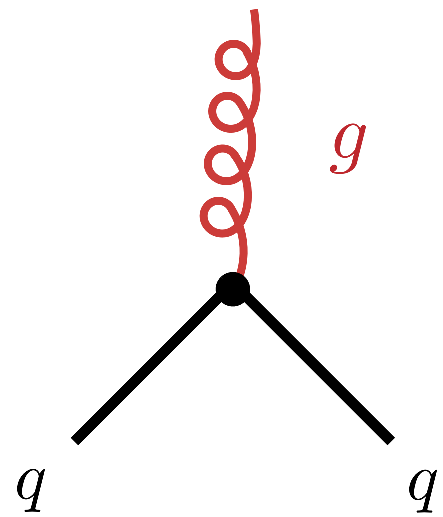
$$\mathbf{D}_\mu = \partial_\mu + \boxed{ig_s \mathbf{A}_\mu}$$

Quantum ChromoDynamics (QCD)

Theory of quark-gluon interactions

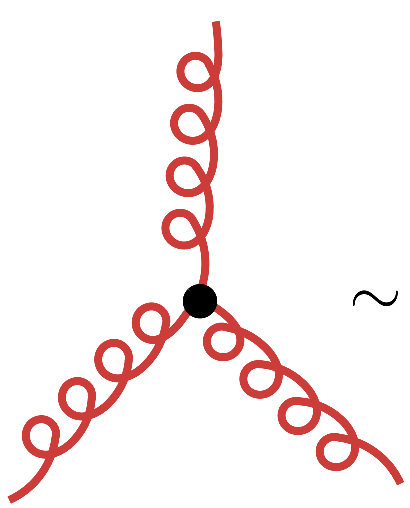
$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f\right) \psi_f - \frac{1}{2}\mathrm{tr}\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right)$$

Quantum Chromodynamics

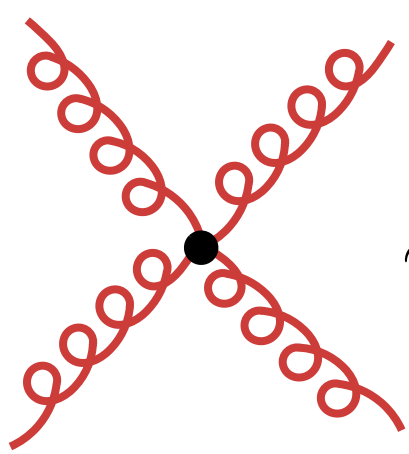


strong coupling

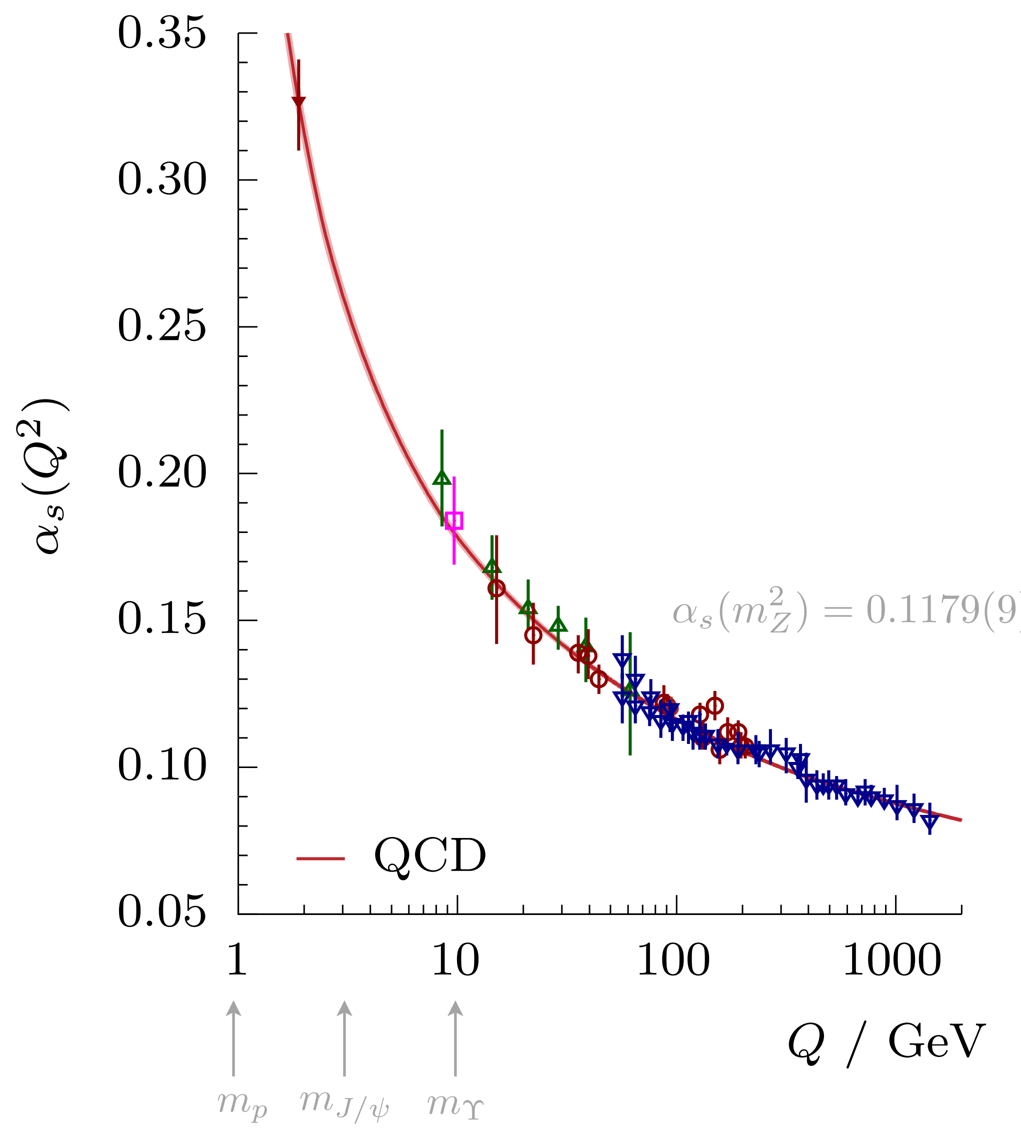
$$\alpha_s = \frac{g_s^2}{4\pi}$$



$\sim g_s$



$\sim g_s^2$



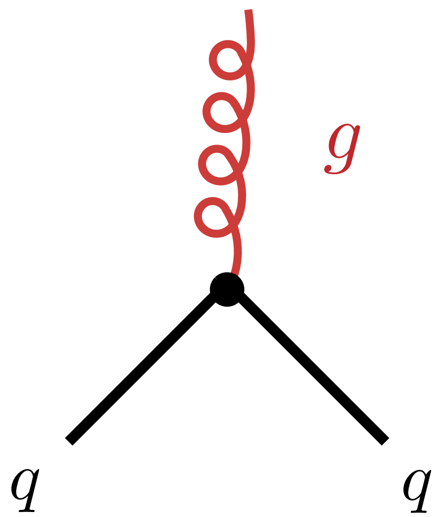
Quantum ChromoDynamics (QCD)

Theory of quark-gluon interactions

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f \left(i\not{\mathbf{D}} - m_f\right) \psi_f - \frac{1}{2}\mathrm{tr}\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right)$$

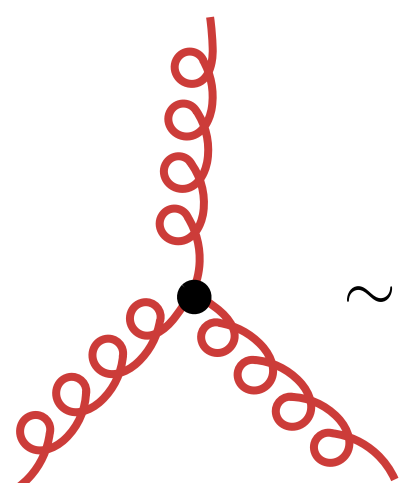
Quantum Chromodynamics

Don't observe free quarks, observe composite hadrons

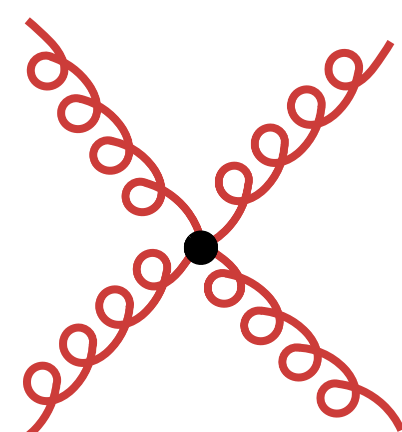


strong coupling

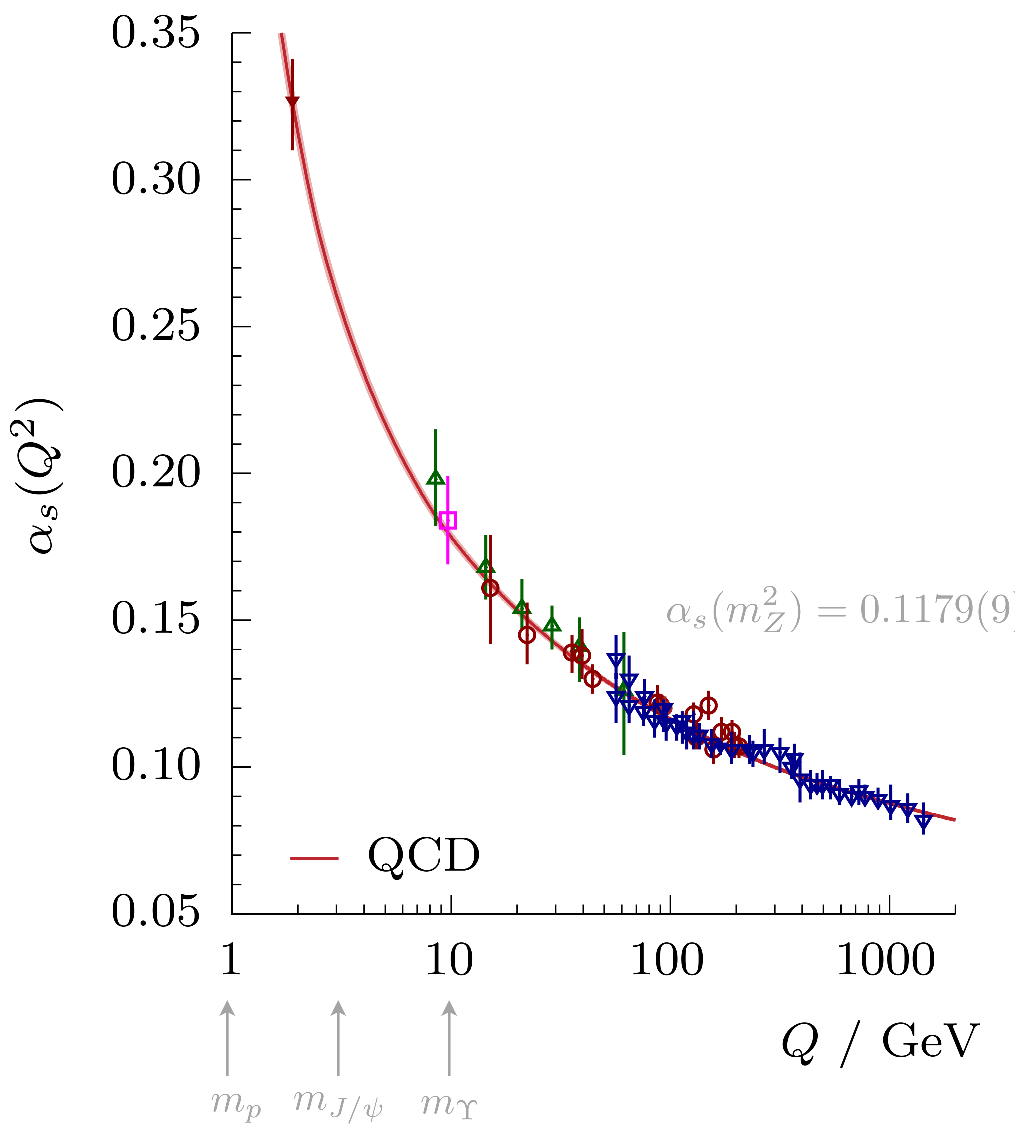
$$\alpha_s = \frac{g_s^2}{4\pi}$$



$\sim g_s$



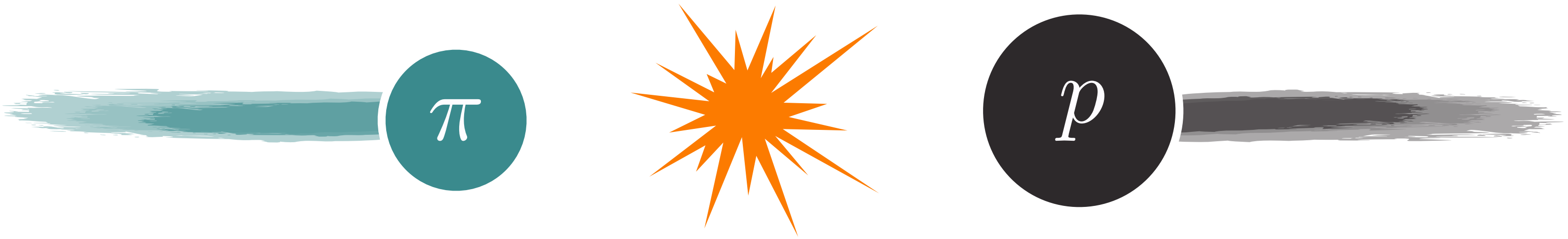
$\sim g_s^2$



Hadron Spectroscopy

The strong interaction spectrum, composed of ***hadrons***, contains many interesting features

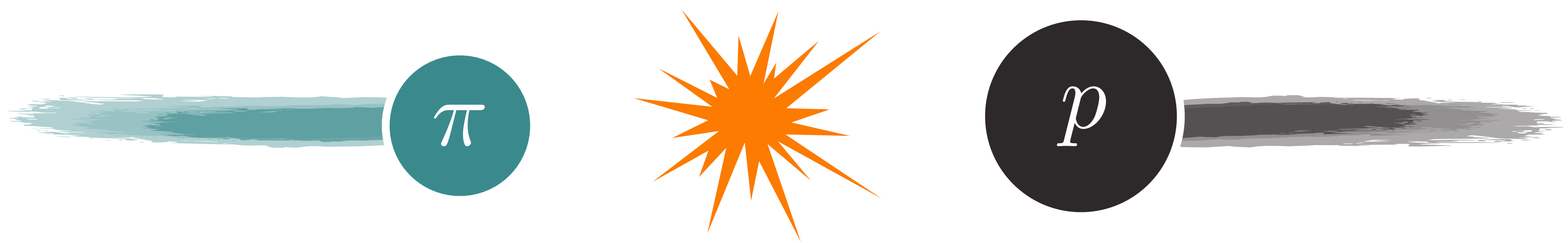
e.g. πN spectrum

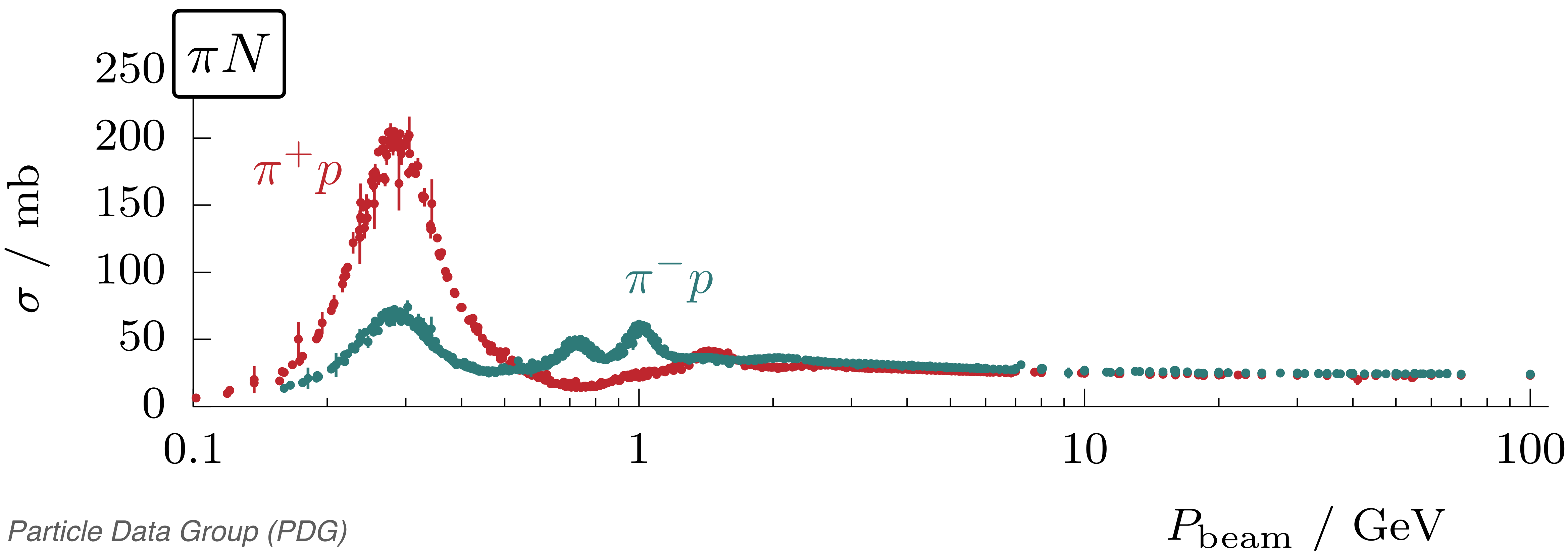


Hadron Spectroscopy

The strong interaction spectrum, composed of ***hadrons***, contains many interesting features

e.g. πN spectrum



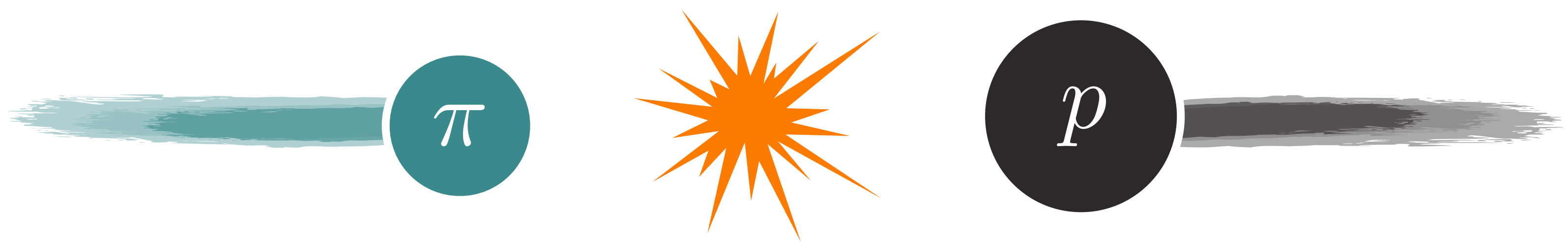


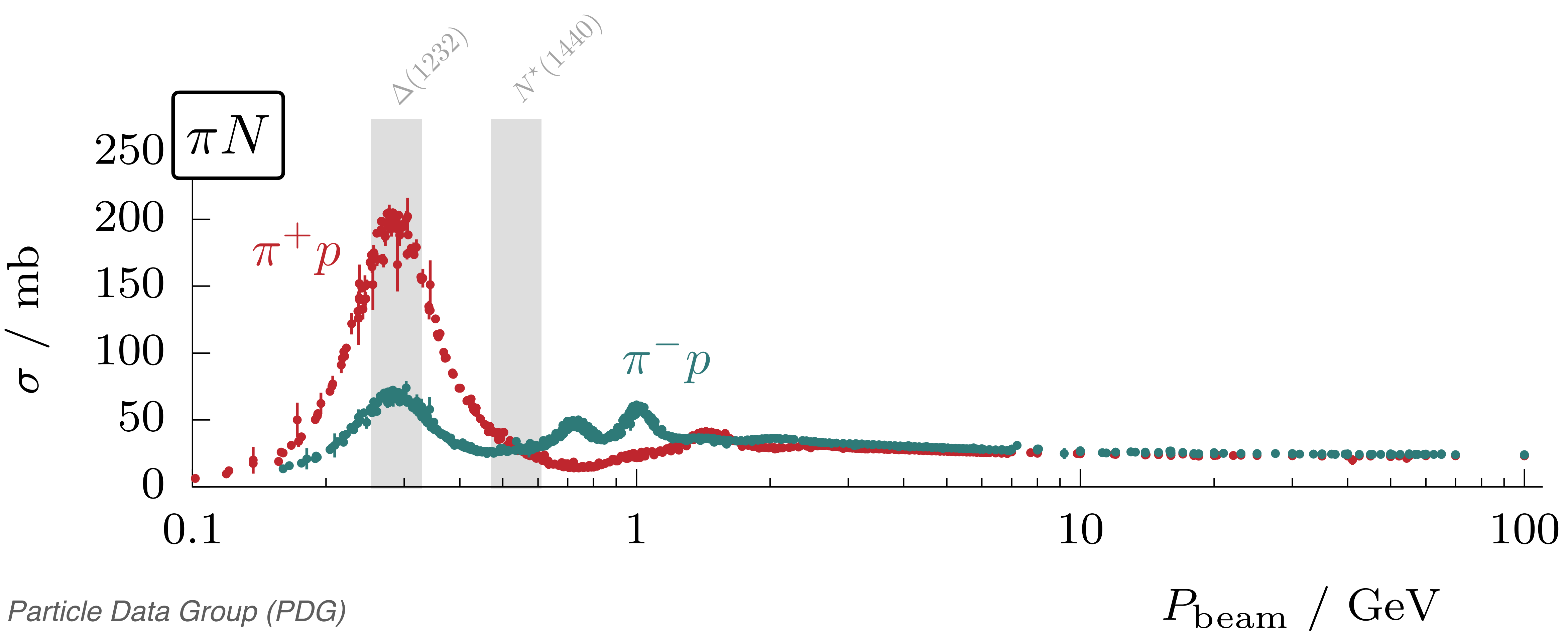
Particle Data Group (PDG)
The Review of Particle Physics (RPP)

Hadron Spectroscopy

The strong interaction spectrum, composed of ***hadrons***, contains many interesting features

e.g. πN spectrum



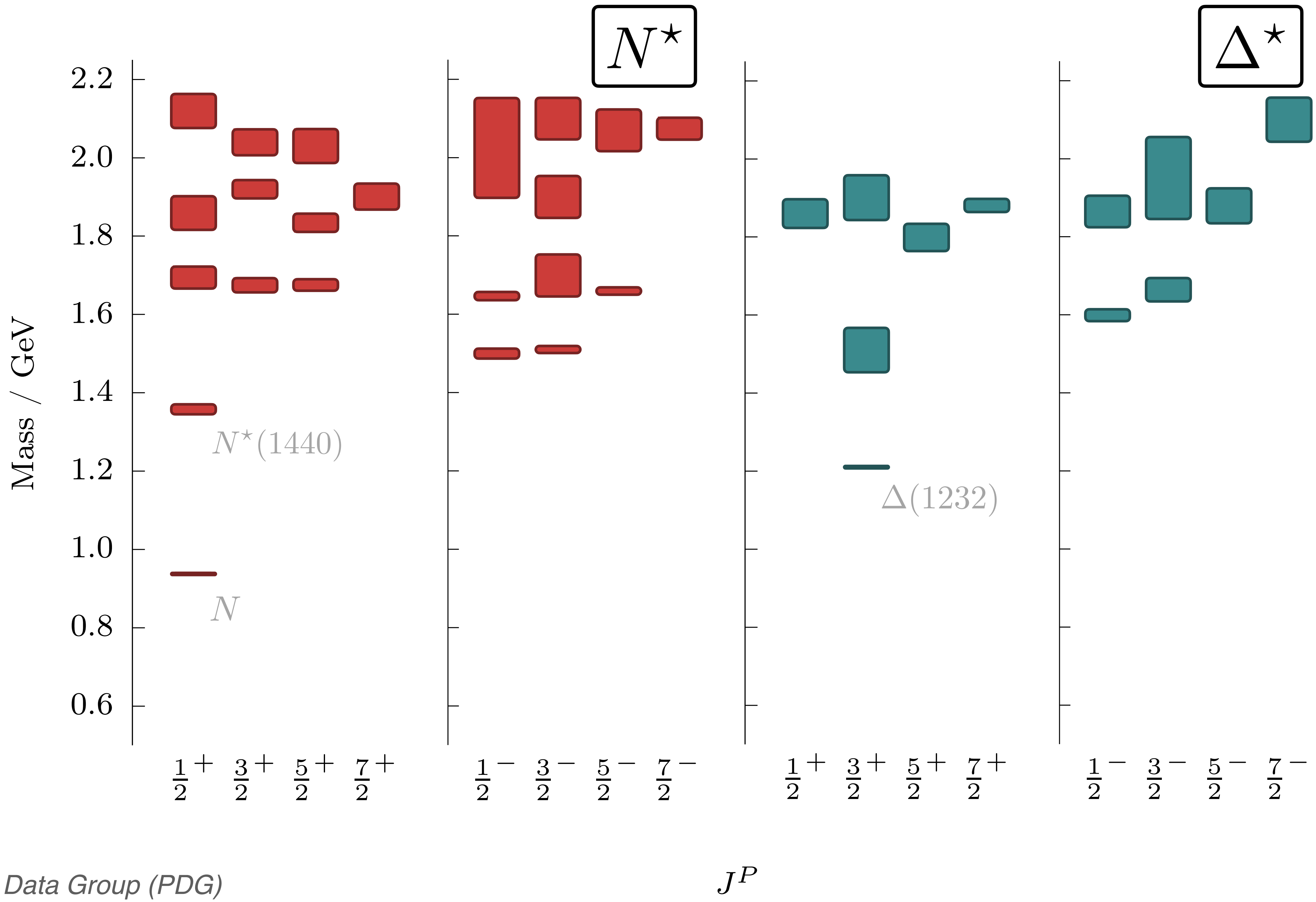


Particle Data Group (PDG)
The Review of Particle Physics (RPP)

Hadron Spectroscopy

The strong interaction spectrum, composed of ***hadrons***, contains many interesting features

e.g. πN spectrum

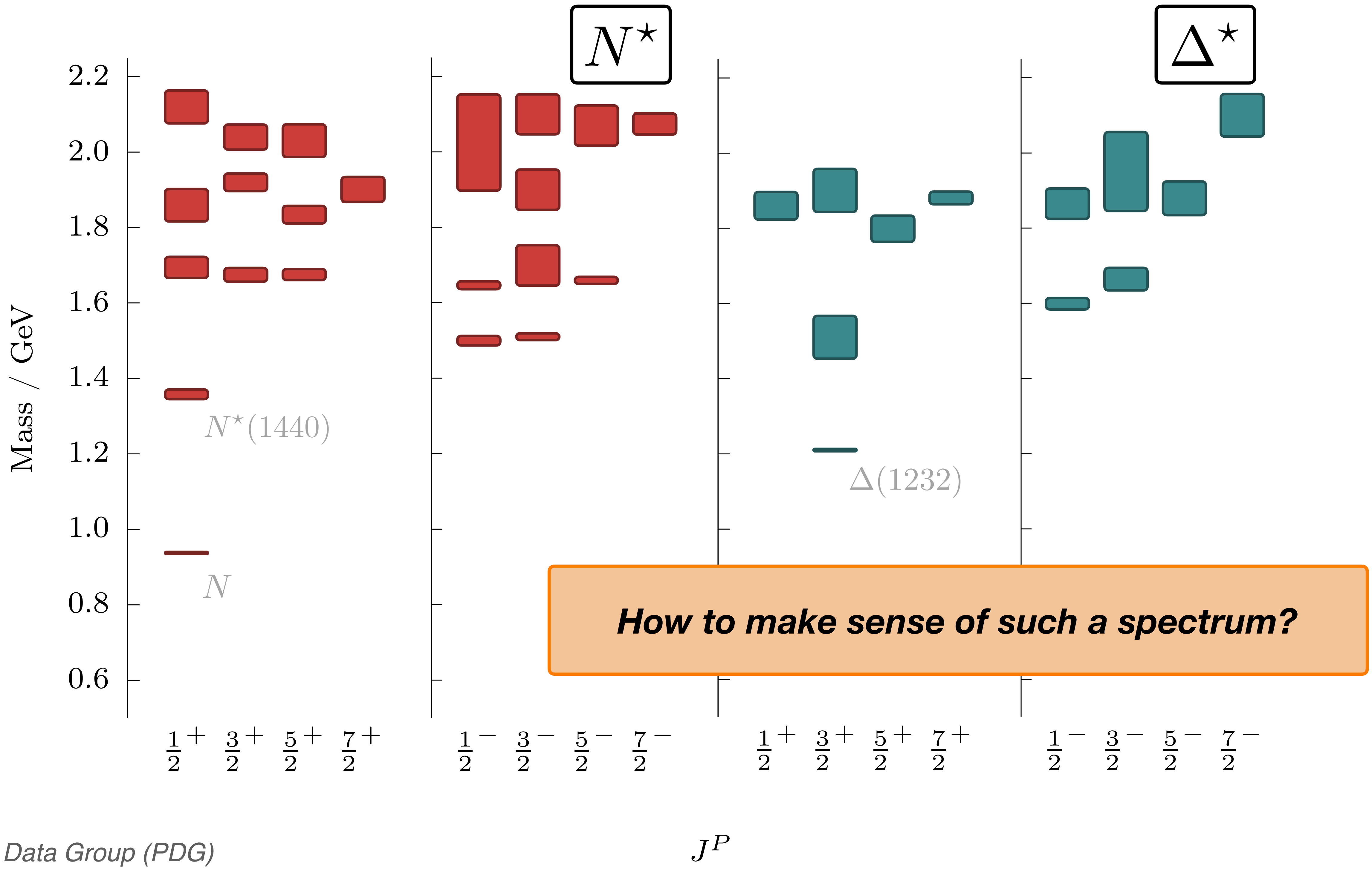


Particle Data Group (PDG)
The Review of Particle Physics (RPP)

Hadron Spectroscopy

The strong interaction spectrum, composed of ***hadrons***, contains many interesting features

e.g. πN spectrum



How to make sense of such a spectrum?

Particle Data Group (PDG)
The Review of Particle Physics (RPP)

The Hadron Spectrum

Hadrons are classified by their ***conserved quantum numbers***

- *spin* (J)
- *parity* (P)
- *charge-conjugation* (C)
- *isospin* (I)
- *strangeness* (S)
- ...

Hadrons

Mesons

$J = 0, 1, 2, \dots$

$I \leq 1$

$|S| \leq 1$

$\pi, K, \eta, \dots$

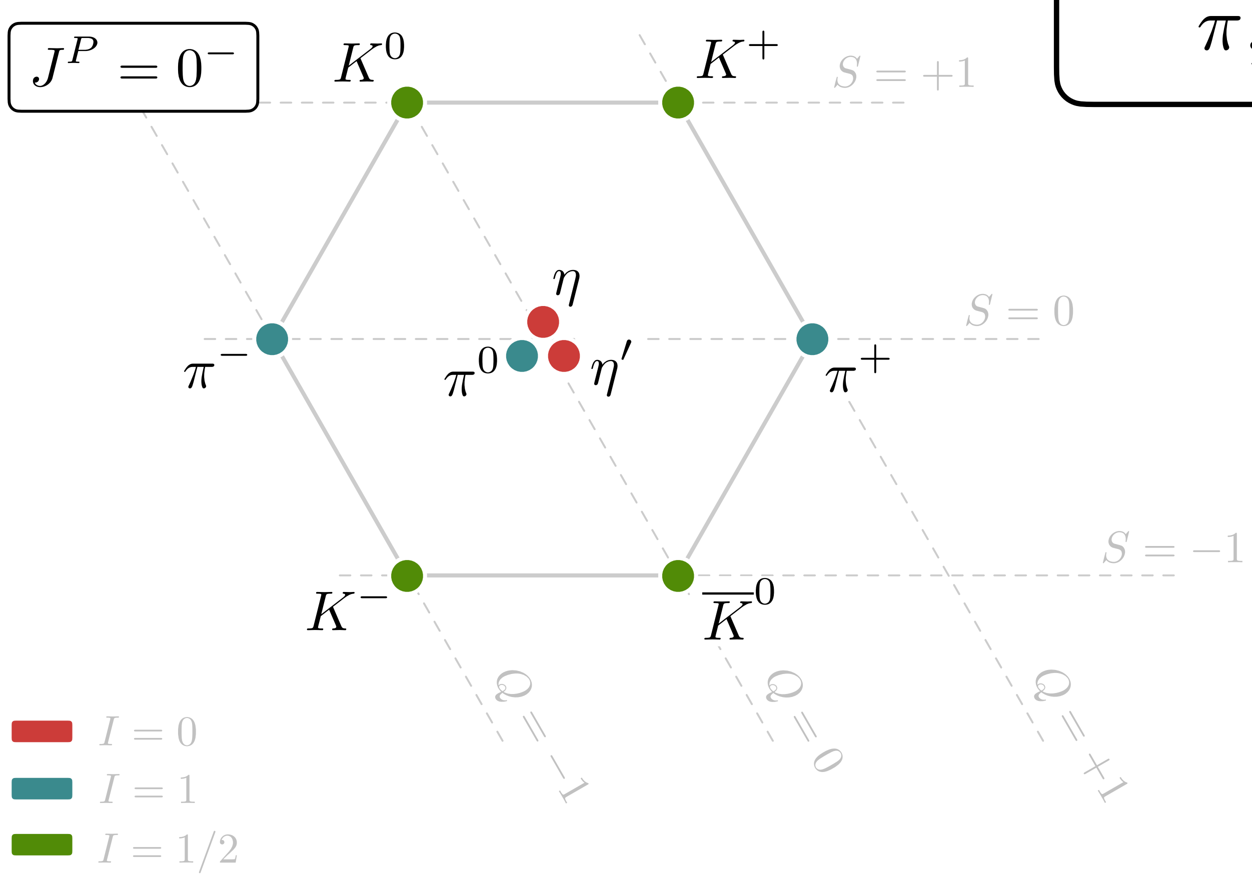
Baryons

$J = 1/2\ , 3/2\ , 5/2\ , \dots$

$I \leq 3/2$

$|S| \leq 3$

$p, n, \Delta, \dots$

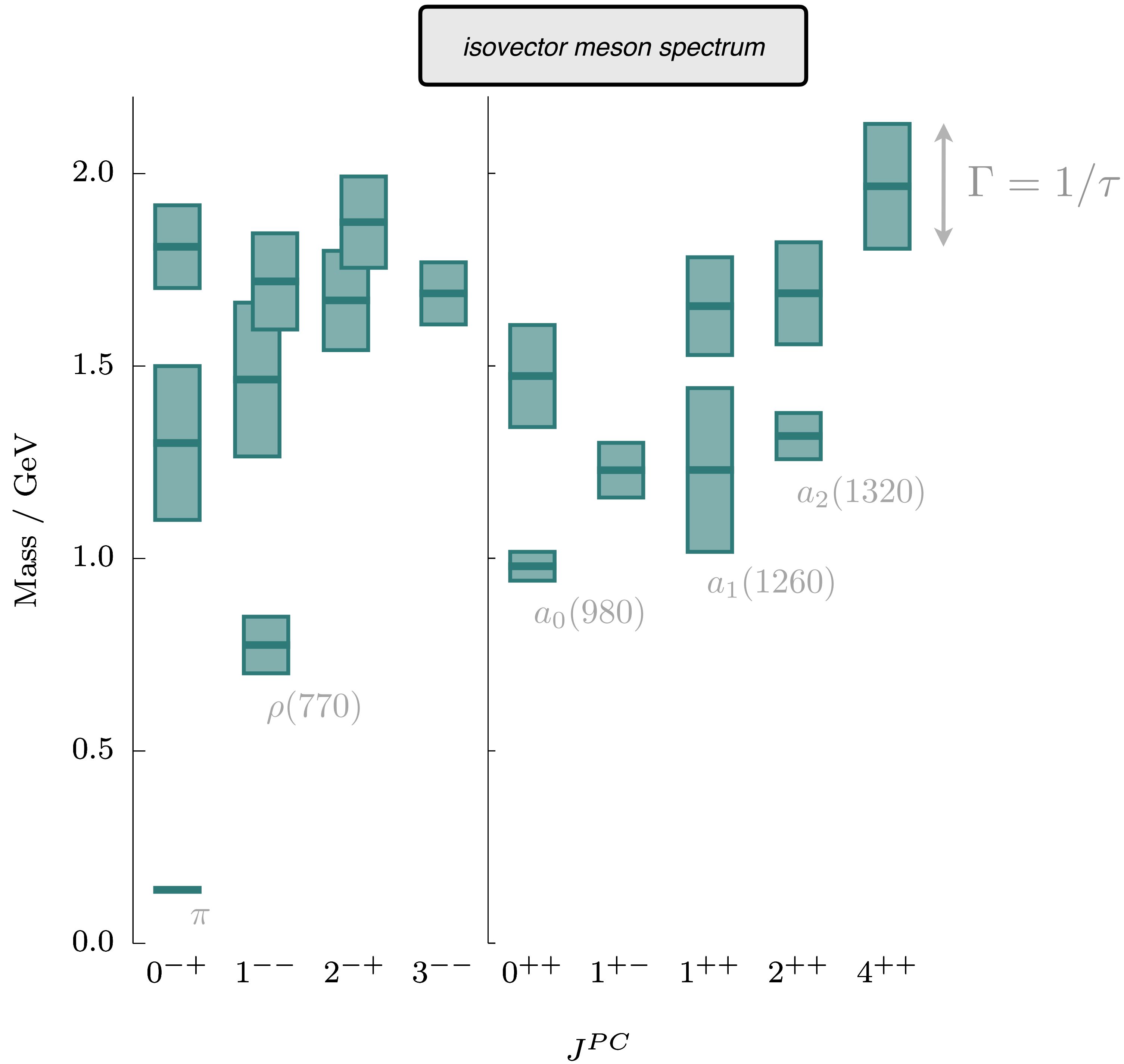


The Hadron Spectrum

Hadrons are classified by their ***conserved quantum numbers***

- Introduce *quarks* as building blocks: ***constituent quark model***

e.g. light isovector mesons

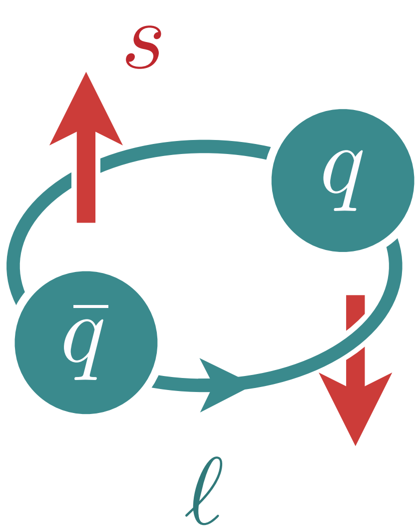


The Hadron Spectrum

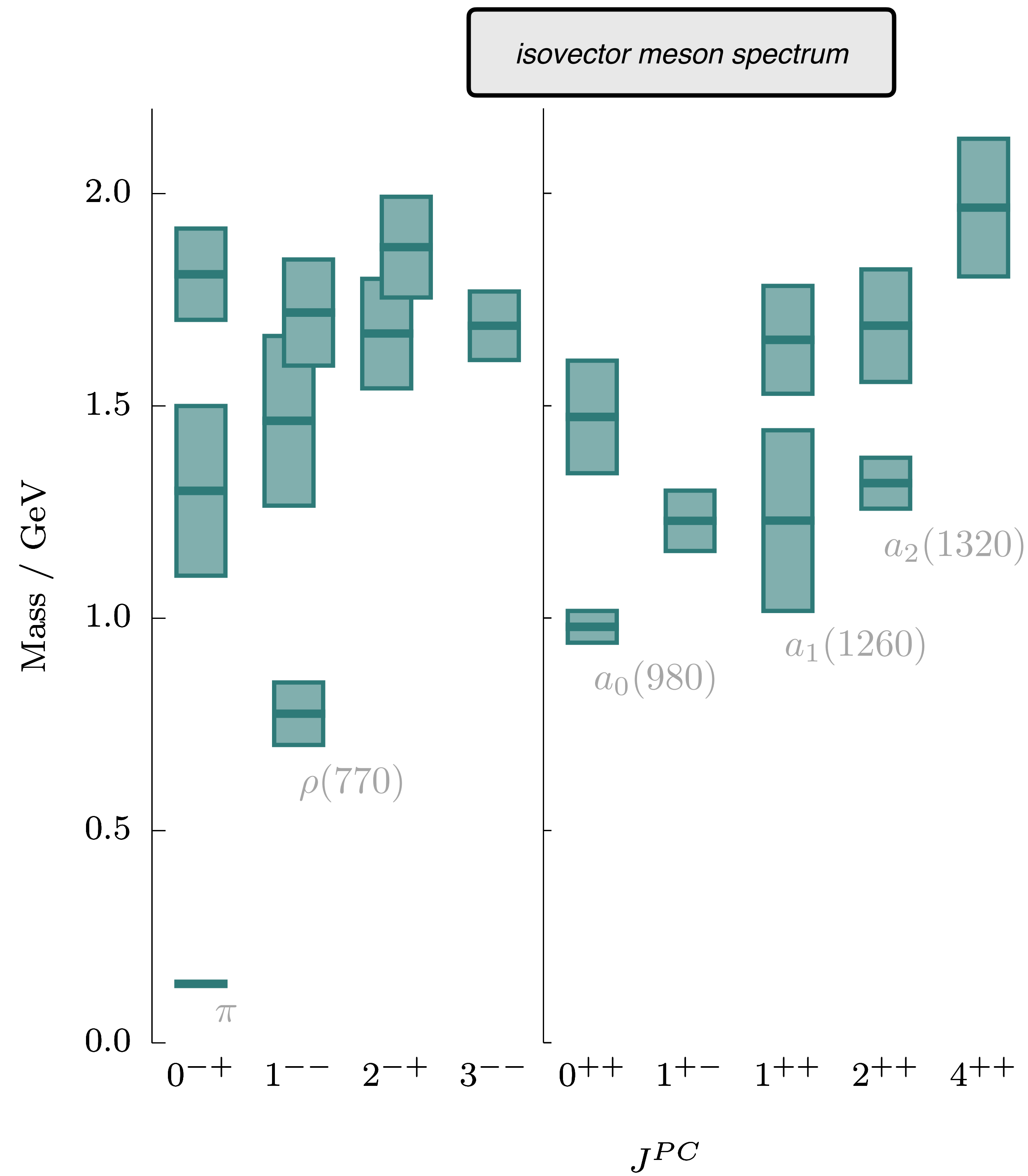
Hadrons are classified by their ***conserved quantum numbers***

- Introduce *quarks* as building blocks: ***constituent quark model***

e.g. light isovector mesons



$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

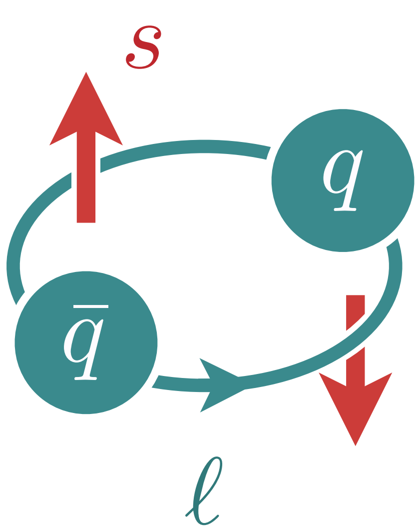


The Hadron Spectrum

Hadrons are classified by their ***conserved quantum numbers***

- Introduce *quarks* as building blocks: ***constituent quark model***

e.g. light isovector mesons



$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$

$$|n^{2s+1}\ell_J, m_J\rangle_{q\bar{q}} = \sum_{m_\ell, m_s} \langle \ell m_\ell; s m_s | J m_J\rangle \sum_{\sigma,\bar{\sigma}} \langle \tfrac{1}{2}\sigma; \tfrac{1}{2}\bar{\sigma} | s m_s\rangle$$

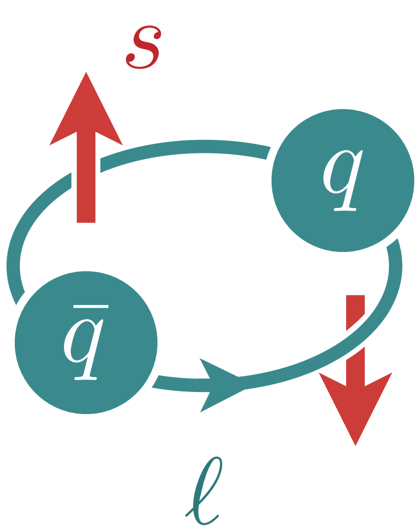
$$\times \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\, \varphi_{n,\ell}(k) Y_{\ell m_\ell}(\hat{\mathbf{k}})\, |q_\sigma(\mathbf{k})\bar{q}_{\bar{\sigma}}(-\mathbf{k})\rangle$$

The Hadron Spectrum

Hadrons are classified by their ***conserved quantum numbers***

- Introduce *quarks* as building blocks: ***constituent quark model***

e.g. light isovector mesons



$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

$$|n^{2s+1}\ell_J, m_J\rangle_{q\bar{q}} = \sum_{m_\ell, m_s} \langle \ell m_\ell; s m_s | J m_J \rangle \sum_{\sigma,\bar{\sigma}} \langle \frac{1}{2}\sigma; \frac{1}{2}\bar{\sigma} | s m_s \rangle$$

$$\times \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\, \varphi_{n,\ell}(k) Y_{\ell m_\ell}(\hat{\mathbf{k}})\, |q_\sigma(\mathbf{k})\bar{q}_{\bar{\sigma}}(-\mathbf{k})\rangle$$

Quantum Numbers

$$\ell = 0, 1, 2, \ldots$$

$$s = \begin{cases} 0 & \text{if antialigned} \\ 1 & \text{if aligned} \end{cases}$$

$$C = (-1)^{\ell+s}$$

$$|\ell - s| \leq J \leq |\ell + s|$$

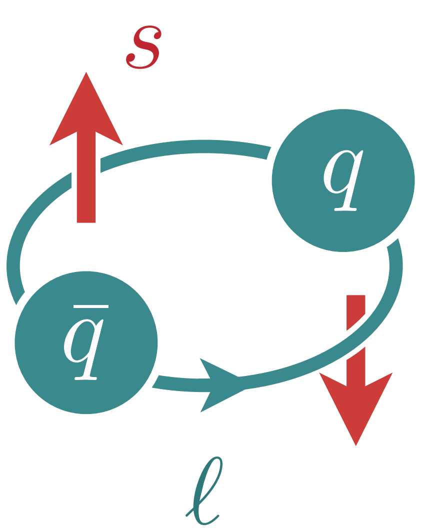
$$P = (-1)^{\ell+1}$$

The Hadron Spectrum

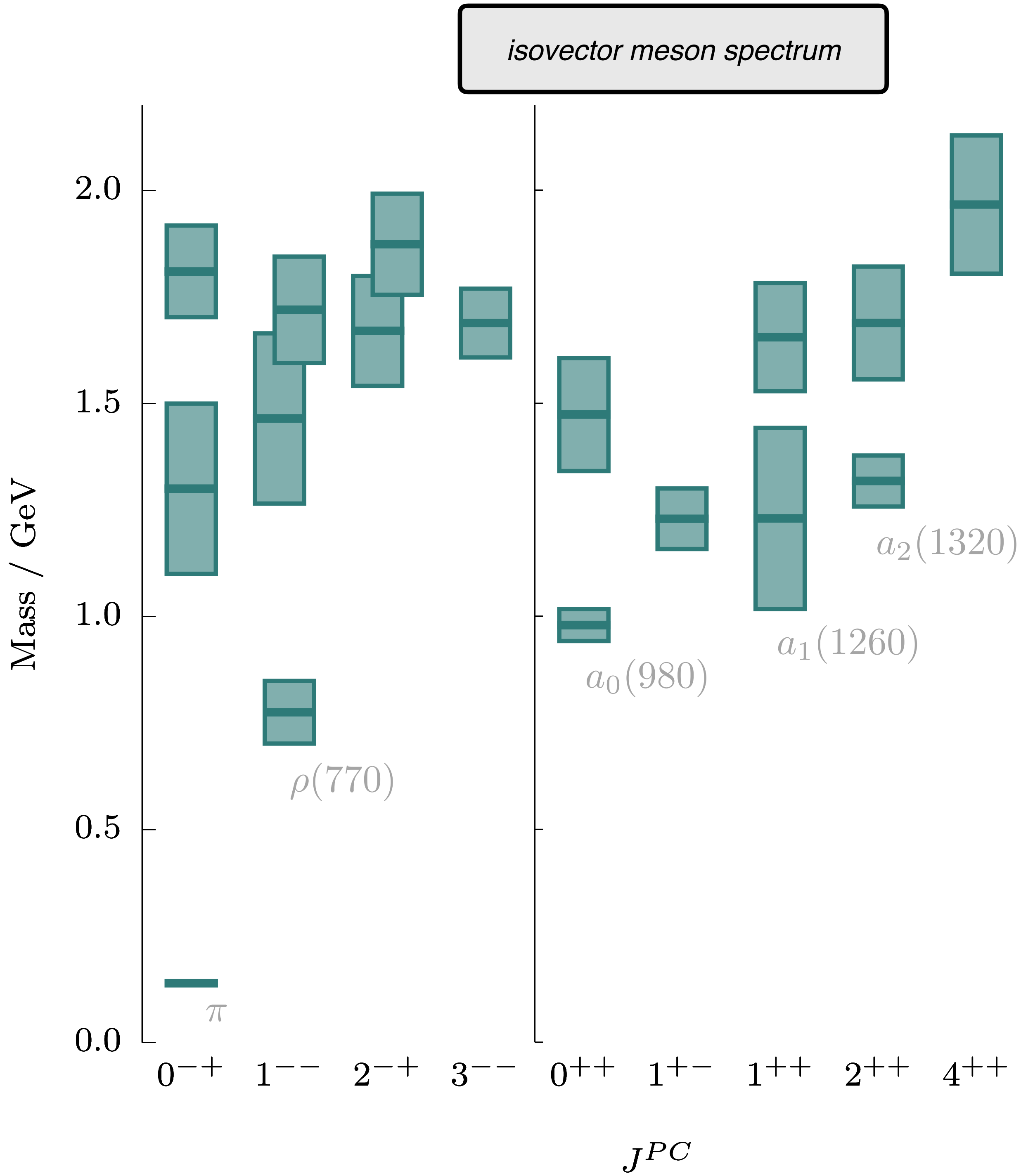
Hadrons are classified by their ***conserved quantum numbers***

- Introduce *quarks* as building blocks: ***constituent quark model***

e.g. light isovector mesons



$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

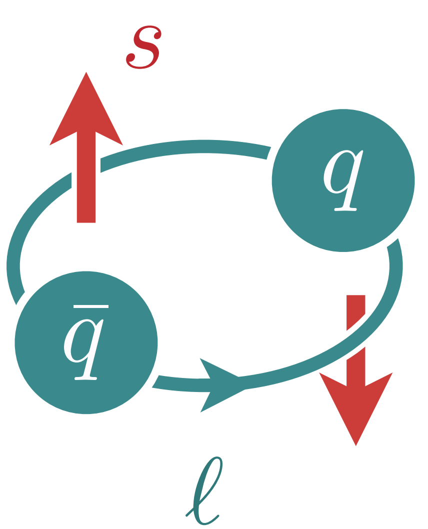


The Hadron Spectrum

Hadrons are classified by their ***conserved quantum numbers***

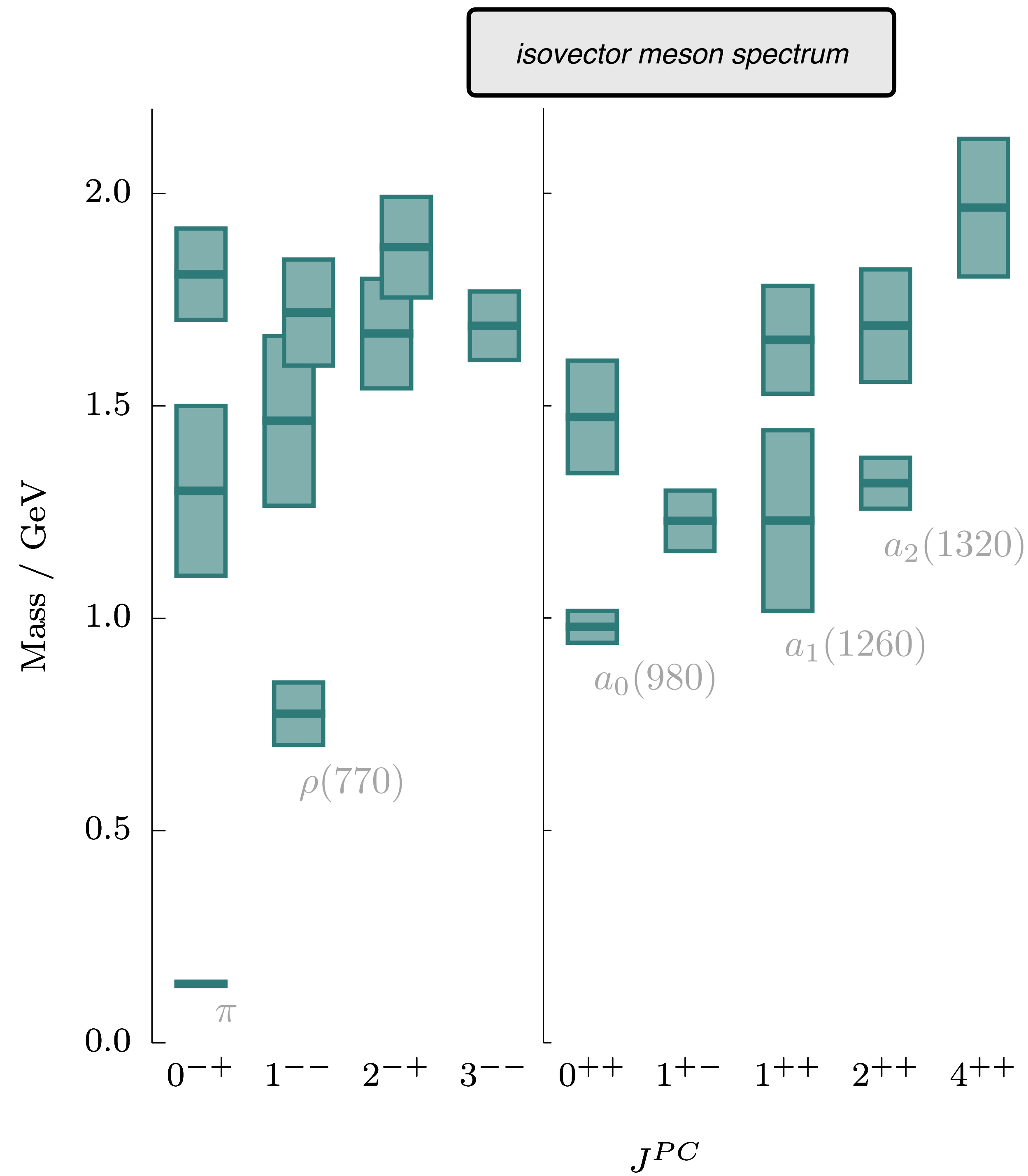
- Introduce *quarks* as building blocks: ***constituent quark model***

e.g. light isovector mesons



$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

	J^{PC} singlet	J^{PC} triplet
$\ell = 0$	0^{-+}	1^{--}
$\ell = 1$	1^{+-}	$(0,1,2)^{++}$
$\ell = 2$	2^{-+}	$(1,2,3)^{--}$
$\vdots$	$\vdots$	$\vdots$

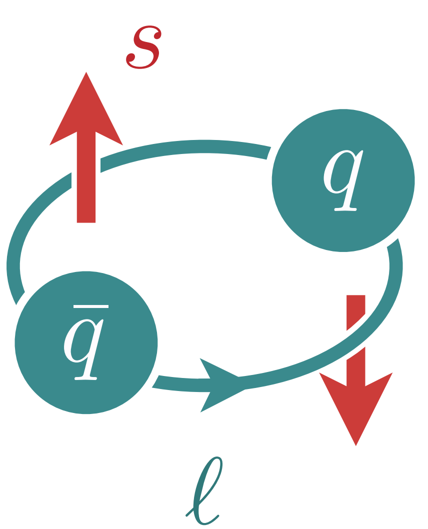


The Hadron Spectrum

Hadrons are classified by their ***conserved quantum numbers***

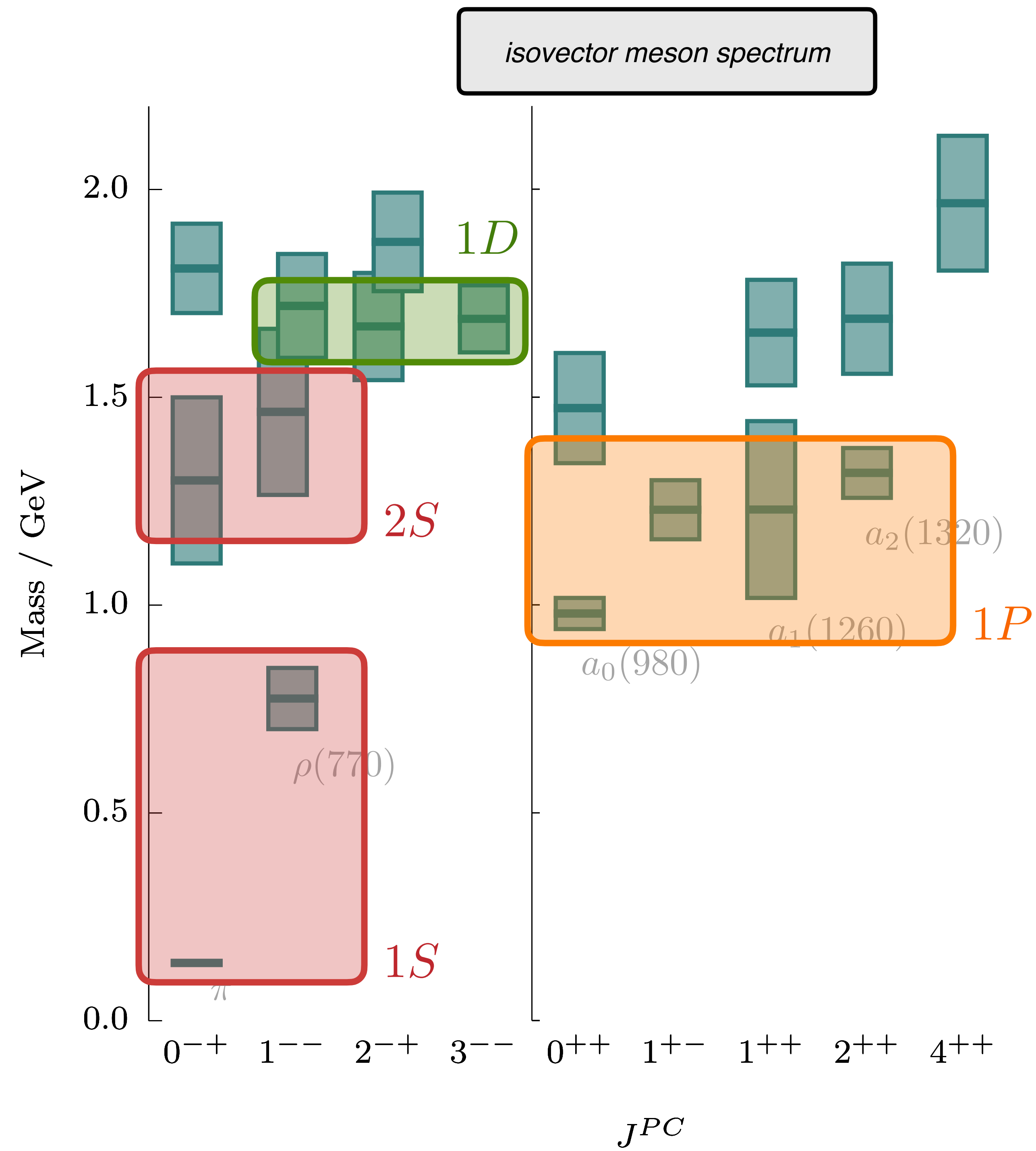
- Introduce *quarks* as building blocks: ***constituent quark model***

e.g. light isovector mesons



$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

	J^{PC} singlet	J^{PC} triplet
$\ell = 0$	0^{-+}	1^{--}
$\ell = 1$	1^{+-}	$(0,1,2)^{++}$
$\ell = 2$	2^{-+}	$(1,2,3)^{--}$
$\vdots$	$\vdots$	$\vdots$

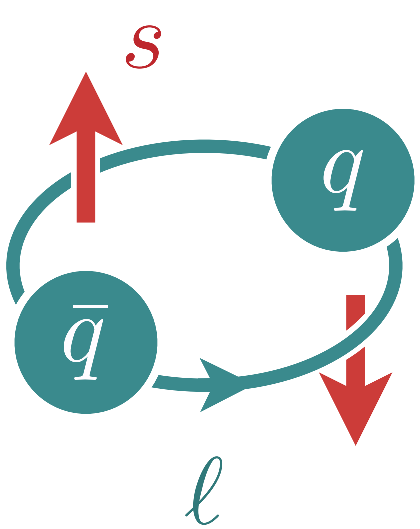


The Hadron Spectrum

Hadrons are classified by their ***conserved quantum numbers***

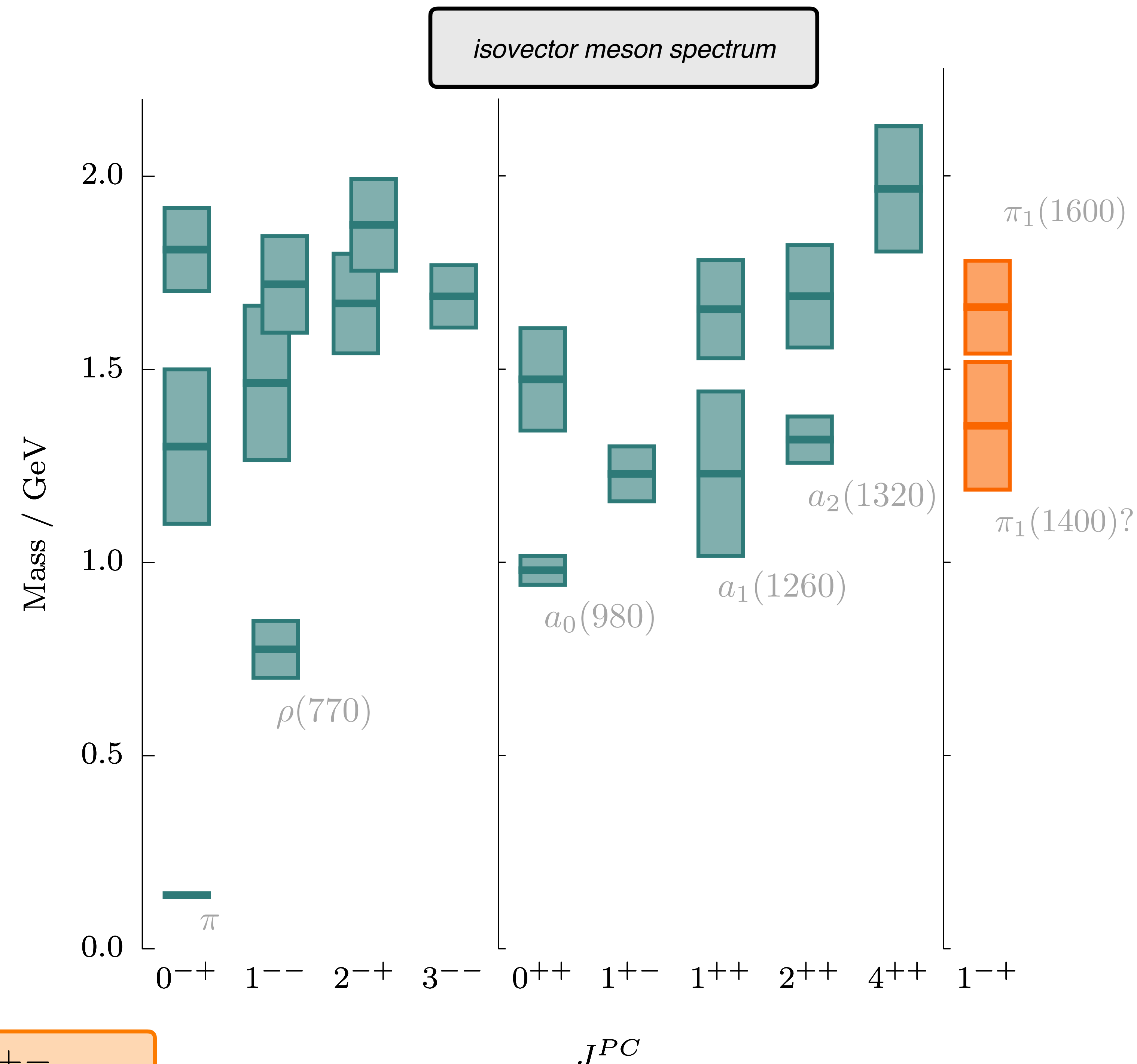
- Introduce *quarks* as building blocks: ***constituent quark model***

e.g. light isovector mesons



$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

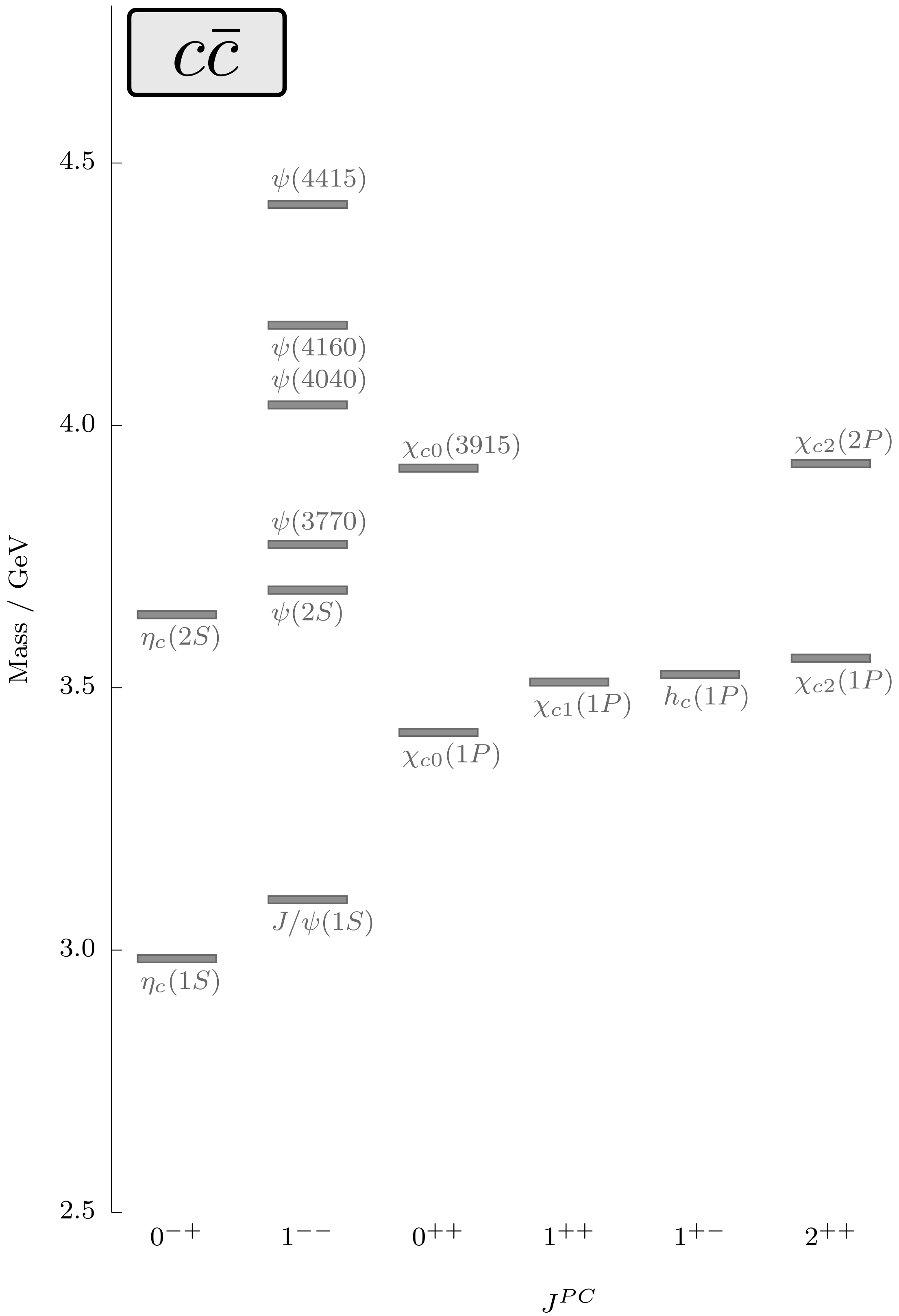
	J^{PC} singlet	J^{PC} triplet
$\ell = 0$	0^{-+}	1^{--}
$\ell = 1$	1^{+-}	$(0,1,2)^{++}$
$\ell = 2$	2^{-+}	$(1,2,3)^{--}$
$\vdots$	$\vdots$	$\vdots$



Forbidden quantum numbers : $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$

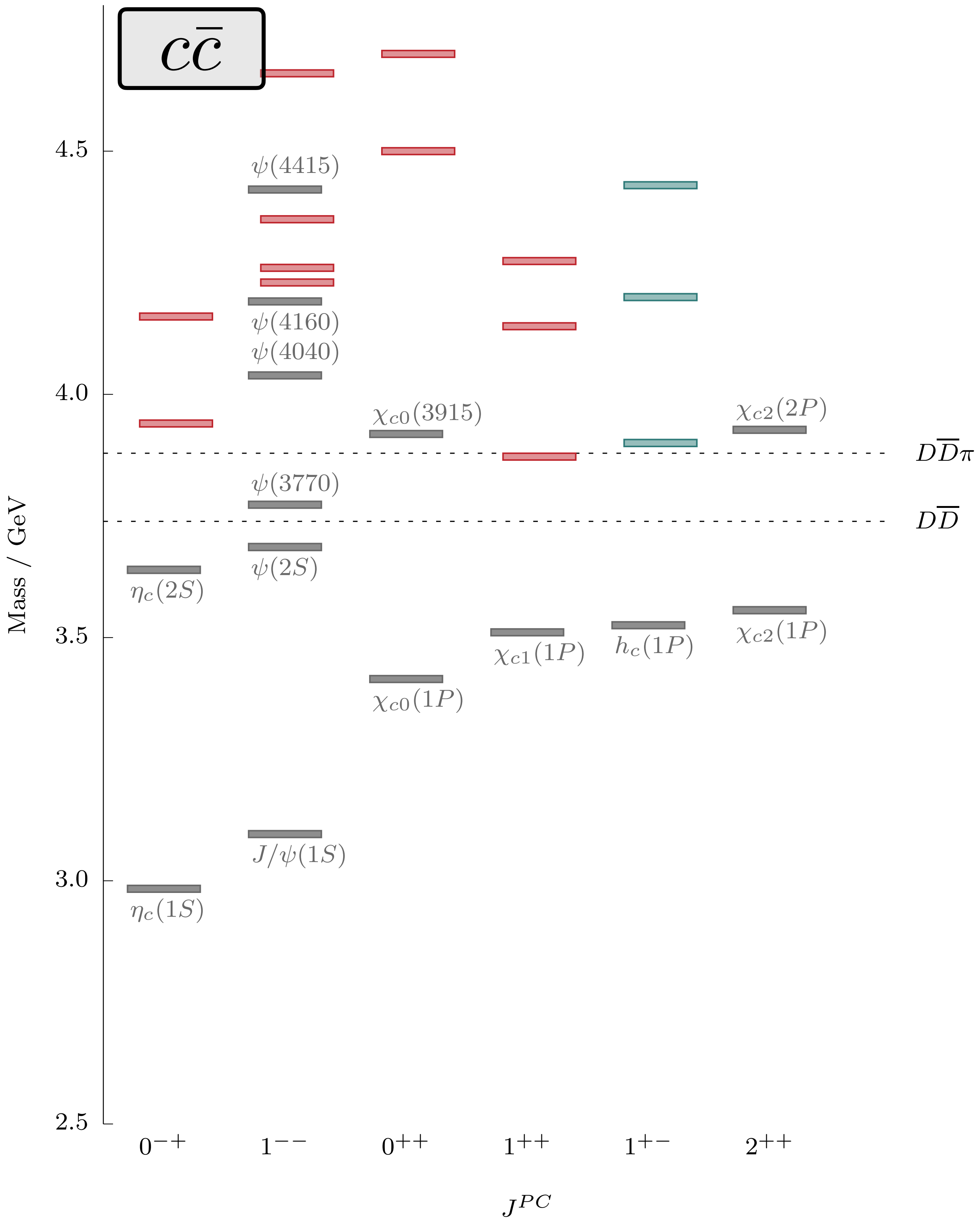
A New Hadron Spectroscopy

Modern experiments have been finding new states which don't fit the conventional quark models



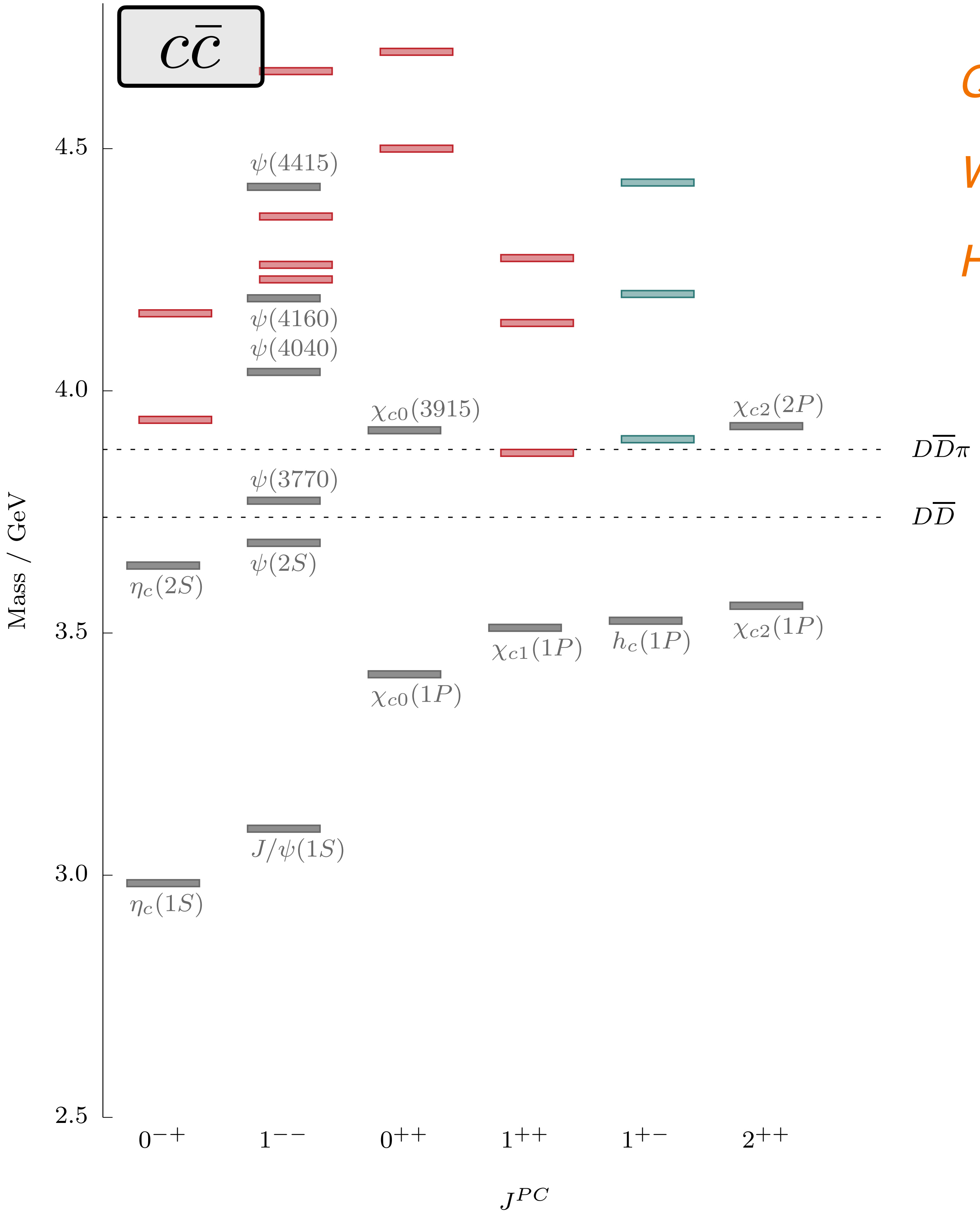
A New Hadron Spectroscopy

Modern experiments have been finding new states which don't fit the conventional quark models



A New Hadron Spectroscopy

Modern experiments have been finding new states which don't fit the conventional quark models



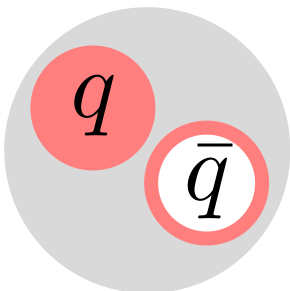
QCD allows for non-quark model hadrons

What is the nature of these states?

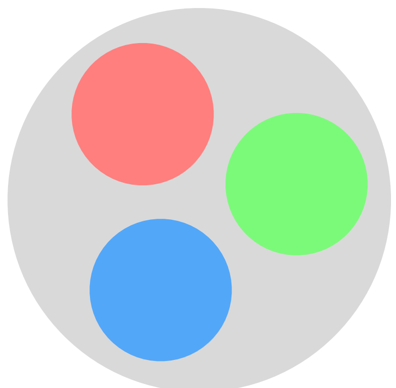
How do we being to find a global explanation?

Hadrons and QCD

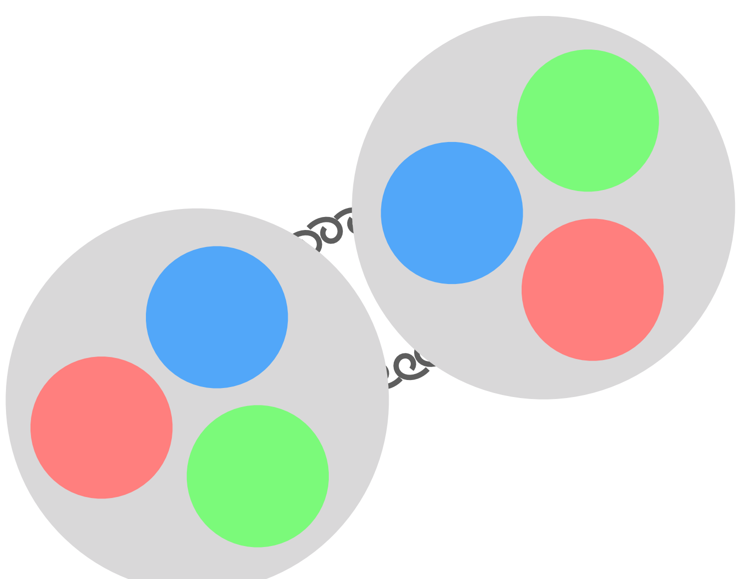
QCD allows for more exotic states — Do we understand QCD?



Meson



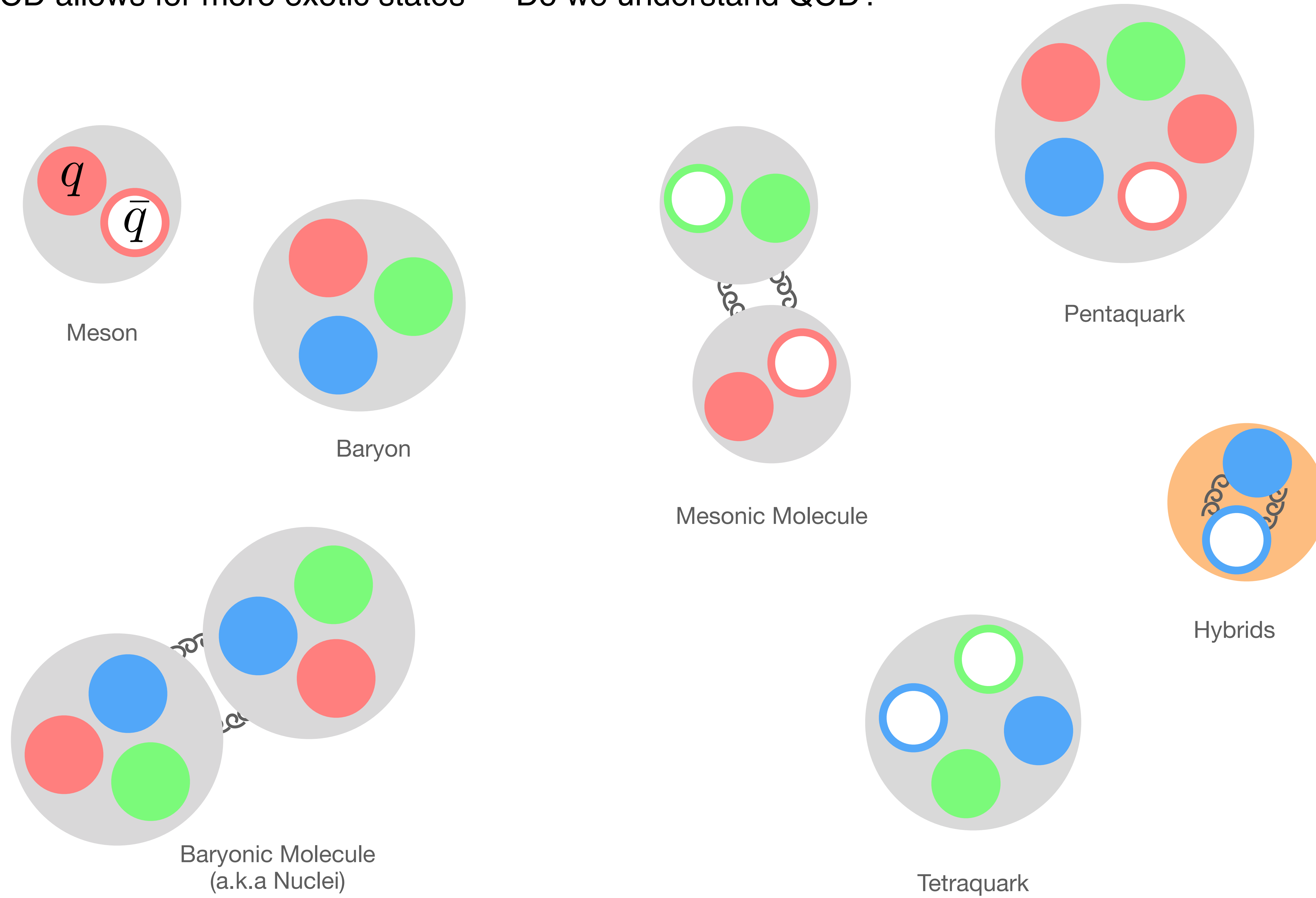
Baryon



Baryonic Molecule
(a.k.a Nuclei)

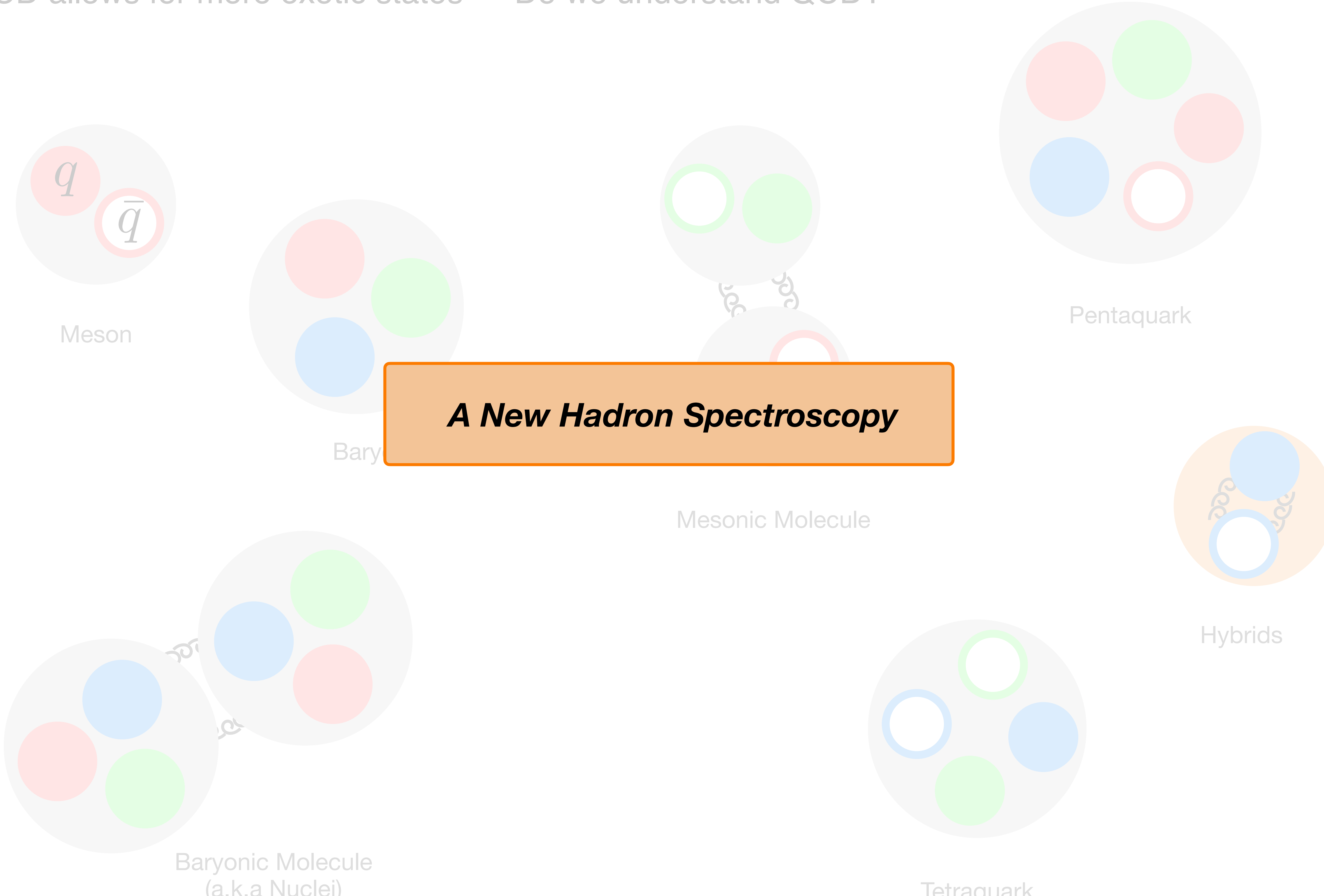
Hadrons and QCD

QCD allows for more exotic states — Do we understand QCD?



Hadrons and QCD

QCD allows for more exotic states — Do we understand QCD?



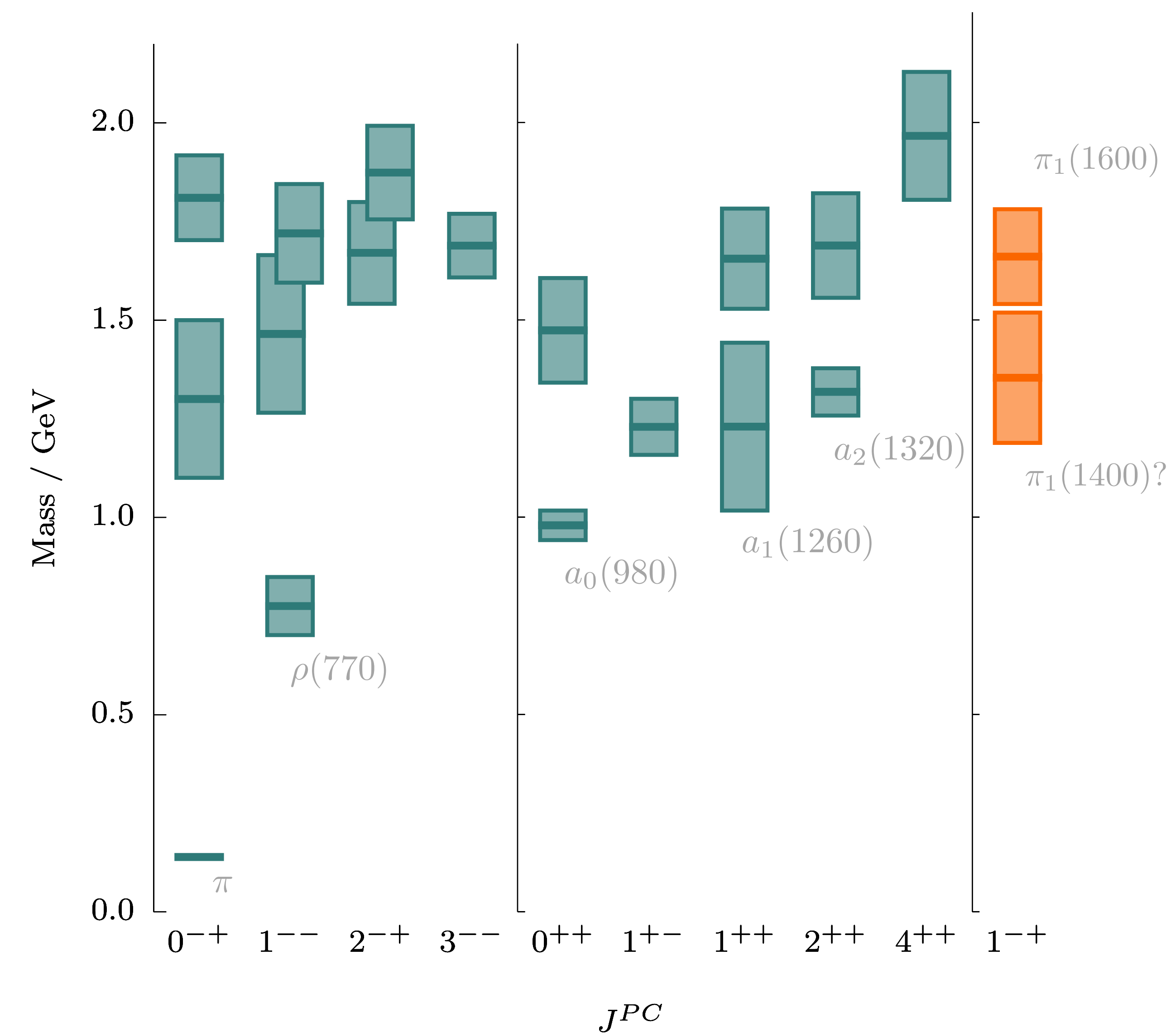
A New Hadron Spectroscopy

Hadrons and QCD

QCD allows for more exotic states — Do we understand QCD?

- How to connect QCD to hadrons?
- Need to understand how to quantify what the hadrons are in nature
- Need to find non-perturbative approach to access these hadrons ***rigorously***

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f \right) \psi_f - \frac{1}{2} \mathrm{tr}\left(\mathbf{G}_{\mu\nu} \mathbf{G}^{\mu\nu} \right)$$



Outline

Hadrons, Quarks, & QCD

- Hadron Spectroscopy
- The Quark Model
- Quantum Chromodynamics

Scattering Theory & the Hadron Spectrum

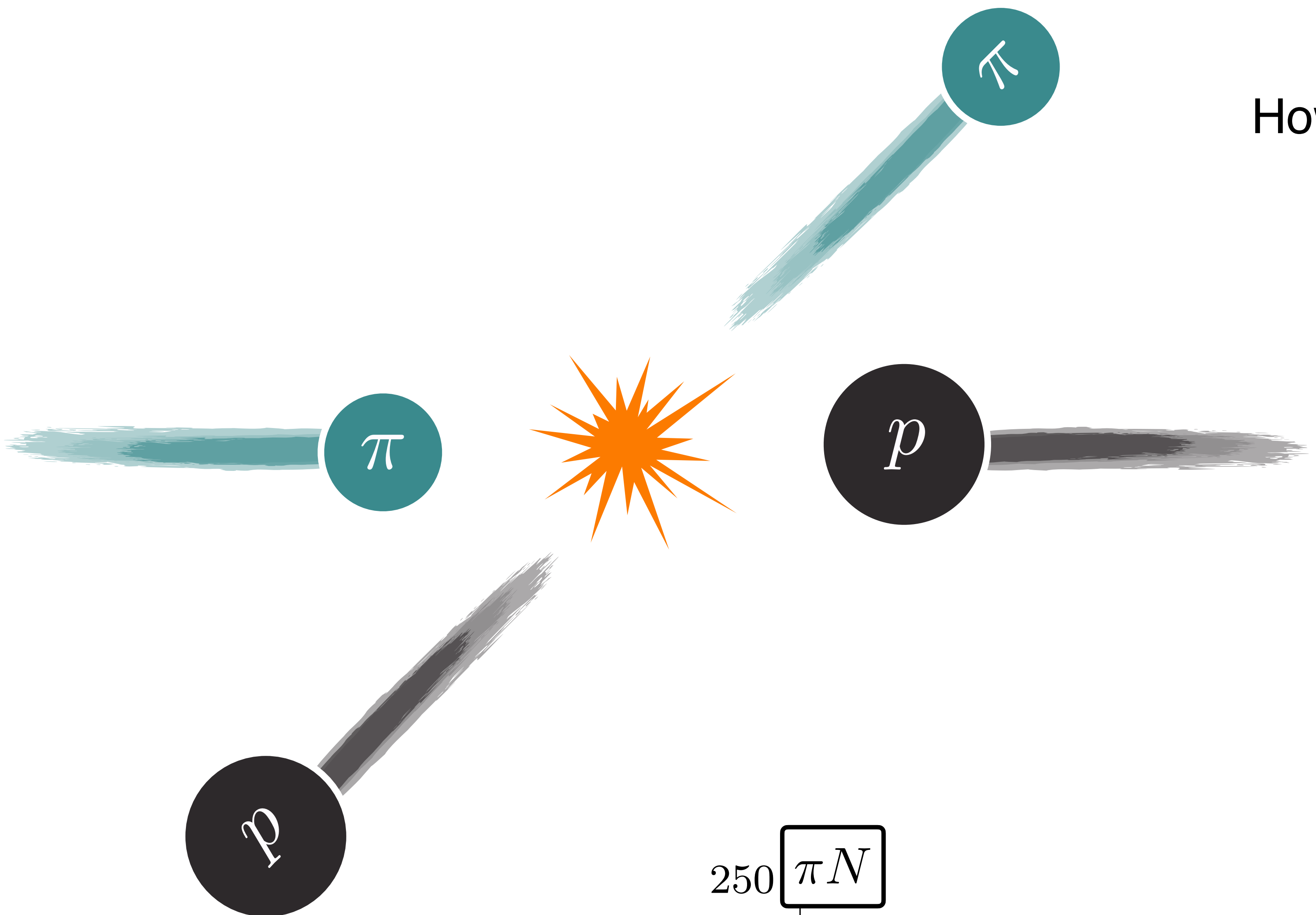
- Scattering Amplitudes
- Bound & Resonant States

Lattice QCD & Hadron Spectroscopy

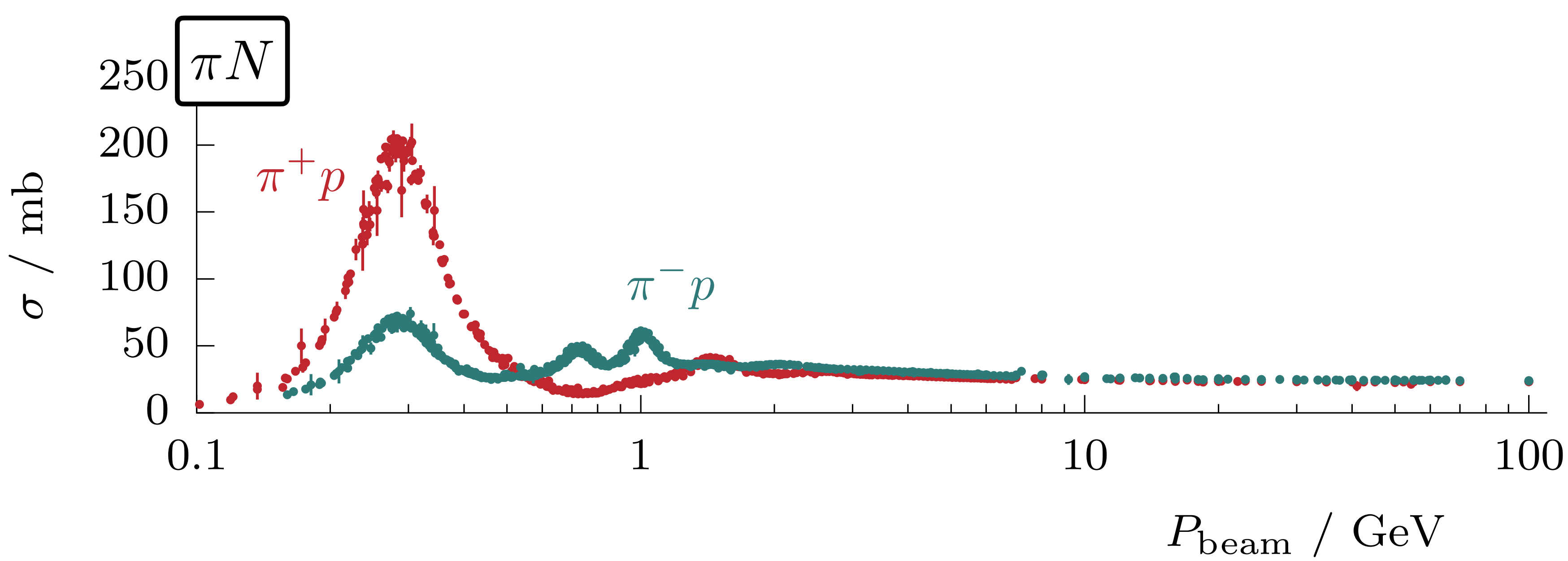
- Lattice QCD
- Lüscher & the Finite-Volume

Accessing the Hadron Spectrum

All our knowledge of the hadrons comes from ***scattering experiments***



How to quantify such a process?



Accessing the Hadron Spectrum

All our knowledge of the hadrons comes from ***scattering experiments***

- In quantum mechanics, want matrix elements of scattering operator — the ***S-matrix***

$$\mathcal{S}_{\alpha\beta}(E) = \langle \beta, \mathrm{out} |\, S \,| \alpha, \mathrm{in} \rangle$$

Outgoing particles

Incoming particles

Probability amplitude

$$\mathrm{Prob}(\alpha \to \beta) = |S_{\alpha\beta}|^2$$

Interaction potential

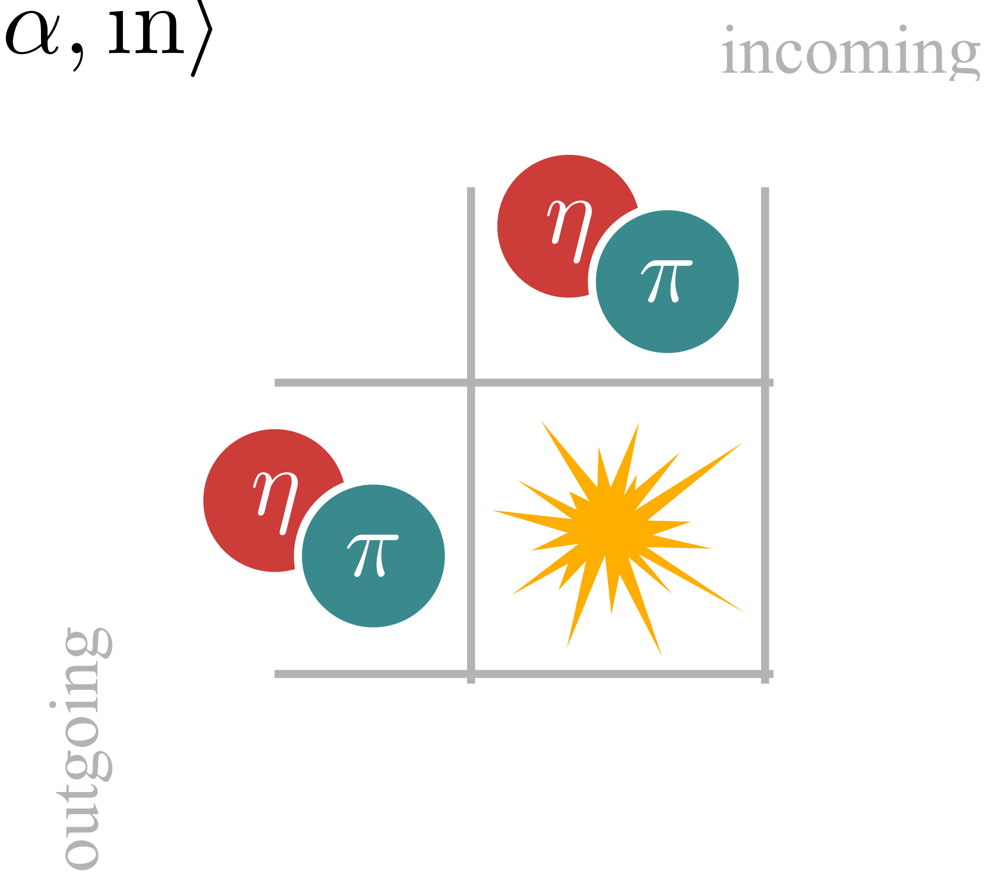
$$S = T\,\exp\left(-i \int_{-\infty}^{\infty} \mathrm{d}t\, V(t) \right)$$

Accessing the Hadron Spectrum

All our knowledge of the hadrons comes from ***scattering experiments***

- In quantum mechanics, want matrix elements of scattering operator — the ***S-matrix***
- Goal: fill in elements of this matrix!

$$\mathcal{S}_{\alpha\beta}(E) = \langle \beta, \mathrm{out}| \, S \, |\alpha, \mathrm{in}\rangle$$

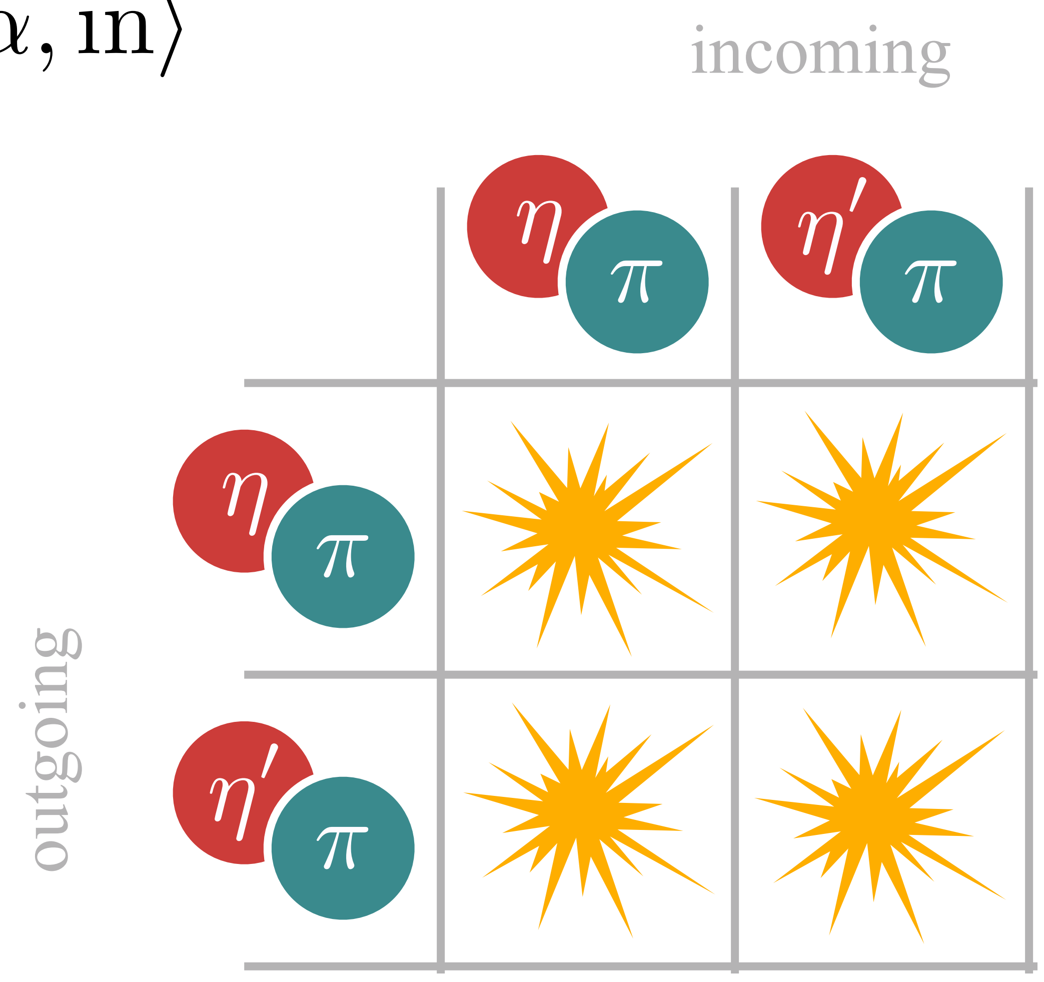


Accessing the Hadron Spectrum

All our knowledge of the hadrons comes from ***scattering experiments***

- In quantum mechanics, want matrix elements of scattering operator — the ***S-matrix***
- Goal: fill in elements of this matrix!

$$\mathcal{S}_{\alpha\beta}(E) = \langle \beta, \mathrm{out} |\, S \,| \alpha, \mathrm{in} \rangle$$

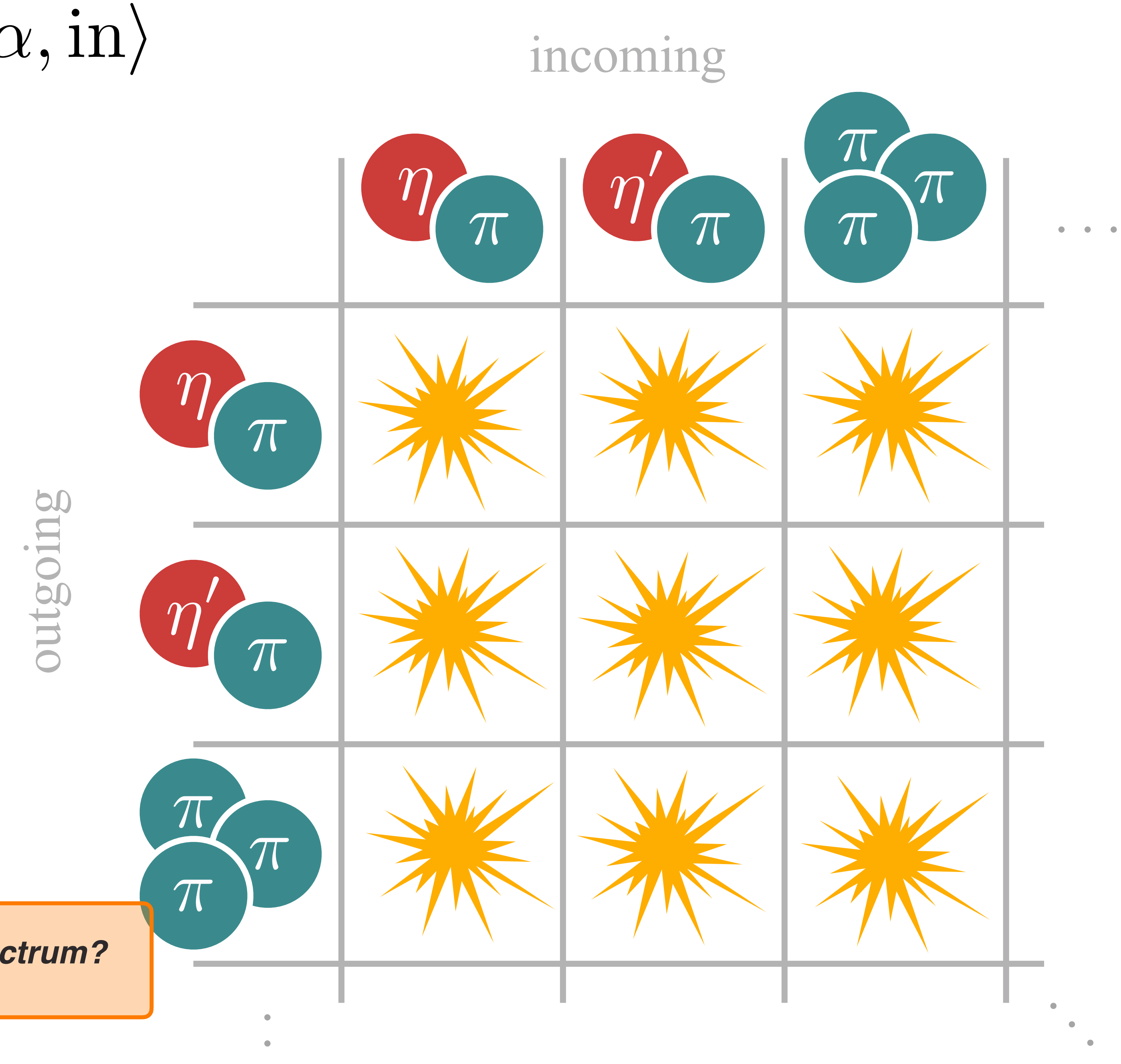


Accessing the Hadron Spectrum

All our knowledge of the hadrons comes from ***scattering experiments***

- In quantum mechanics, want matrix elements of scattering operator — the ***S-matrix***
- Goal: fill in elements of this matrix!

$$\mathcal{S}_{\alpha\beta}(E) = \langle \beta, \mathrm{out}|\, S\, |\alpha, \mathrm{in}\rangle$$



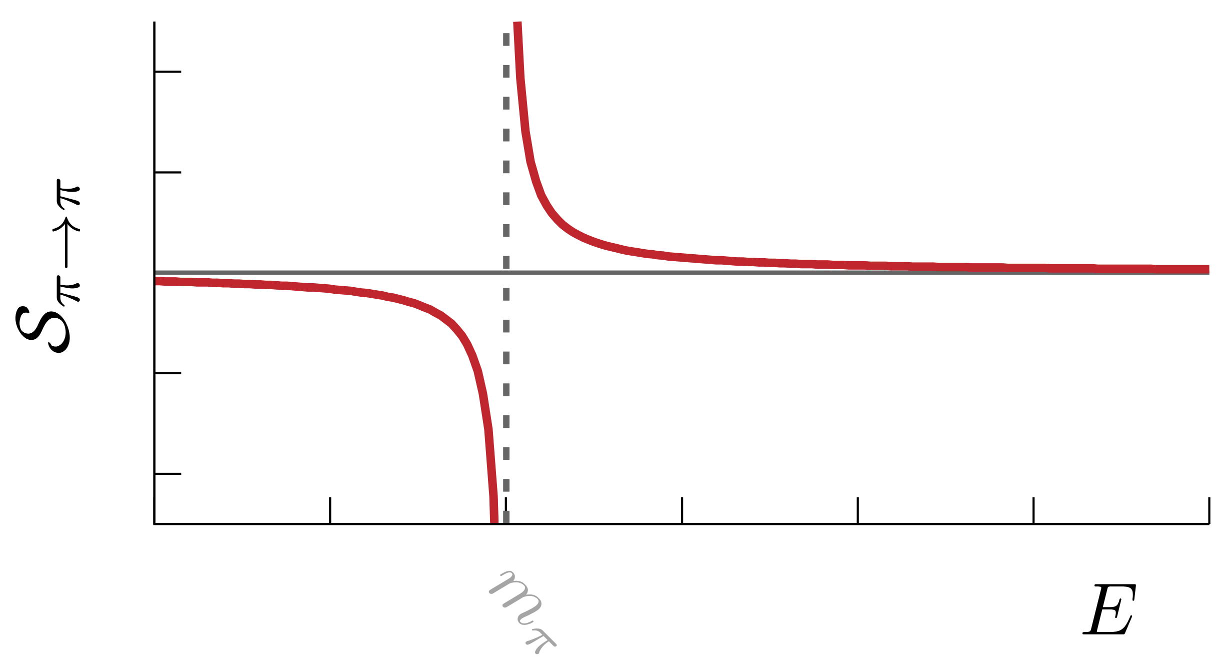
Once we have amplitudes, how to get spectrum?

Amplitudes & Particles

Key idea: Particles are associated with ***pole singularities*** of amplitudes

e.g. amplitude for a free propagating pion

$$\mathcal{S}_{\pi\to\pi}(E) \sim \frac{1}{E^2 - m_\pi^2}$$



Note: sweeping many details under-the-rug

Amplitudes & Particles

Key idea: Particles are associated with ***pole singularities*** of amplitudes

- For unstable particles, ***resonances***, poles are located in ***complex energy plane***

e.g pion-pion scattering producing the rho-meson

$$\mathcal{S}_{\pi\pi\to\rho\to\pi\pi}(E) \sim \frac{1}{E^2 - m_\rho^2 + i m_\rho \Gamma_\rho}$$

Amplitudes & Particles

Key idea: Particles are associated with ***pole singularities*** of amplitudes

- For unstable particles, ***resonances***, poles are located in ***complex energy plane***

e.g pion-pion scattering producing the rho-meson

$$\mathcal{S}_{\pi\pi\to\rho\to\pi\pi}(E) \sim \frac{1}{E^2 - m_\rho^2 + i m_\rho \Gamma_\rho}$$

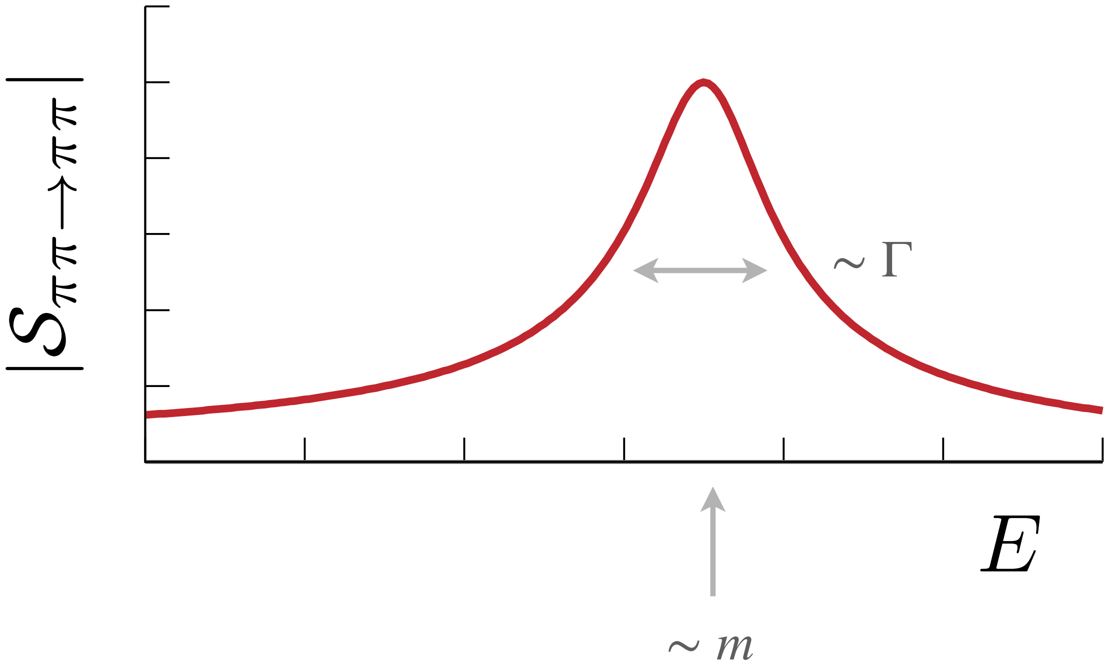
Amplitudes & Particles

Key idea: Particles are associated with ***pole singularities*** of amplitudes

- For unstable particles, ***resonances***, poles are located in ***complex energy plane***

e.g pion-pion scattering producing the rho-meson

$$\mathcal{S}_{\pi\pi\to\rho\to\pi\pi}(E) \sim \frac{1}{E^2 - m_\rho^2 + i m_\rho \Gamma_\rho}$$



Note: sweeping many details under-the-rug

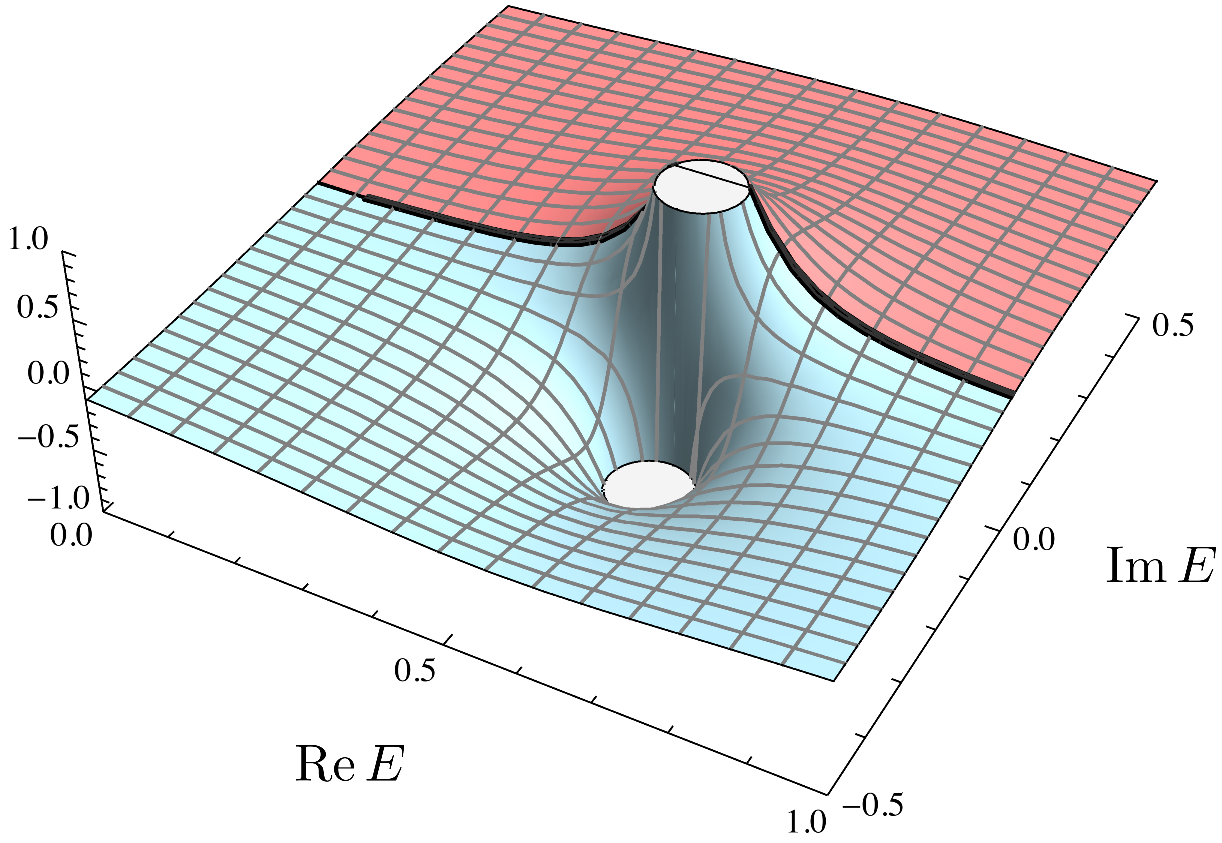
Amplitudes & Particles

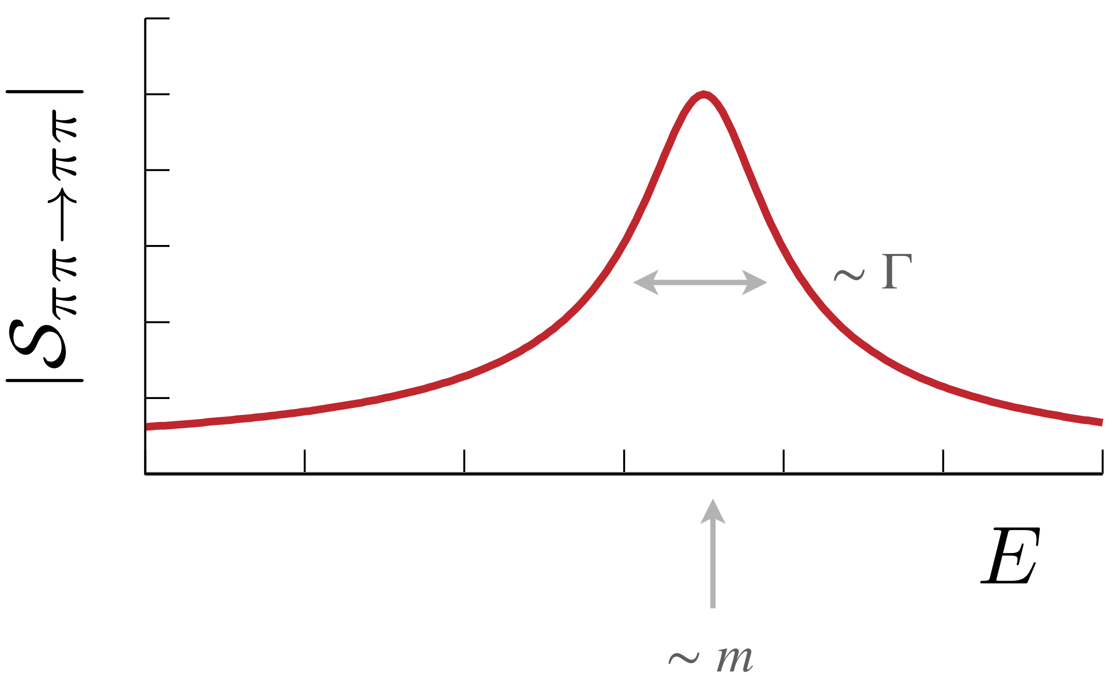
Key idea: Particles are associated with ***pole singularities*** of amplitudes

- For unstable particles, ***resonances***, poles are located in ***complex energy plane***

e.g pion-pion scattering producing the rho-meson

$$\mathcal{S}_{\pi\pi\to\rho\to\pi\pi}(E) \sim \frac{1}{E^2 - m_\rho^2 + i m_\rho \Gamma_\rho}$$





Note: sweeping many details under-the-rug

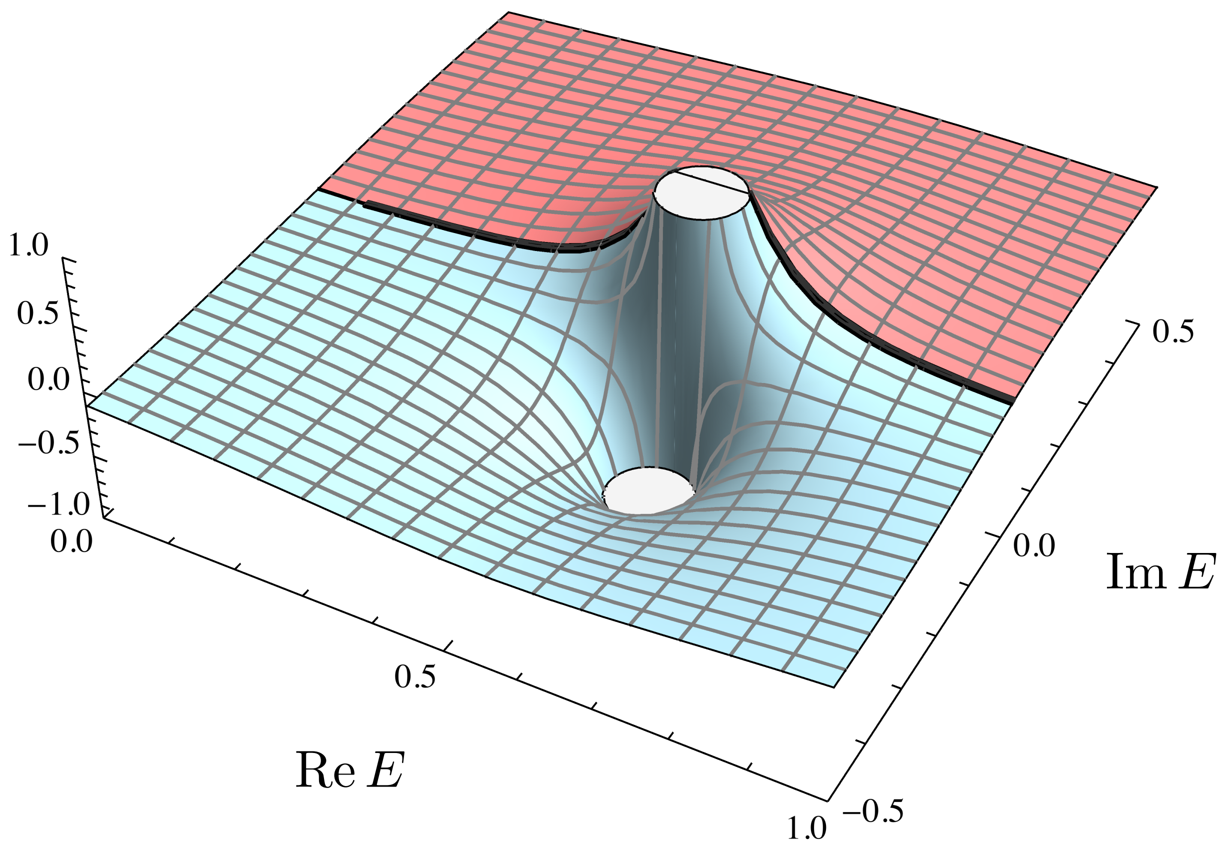
Amplitudes & Particles

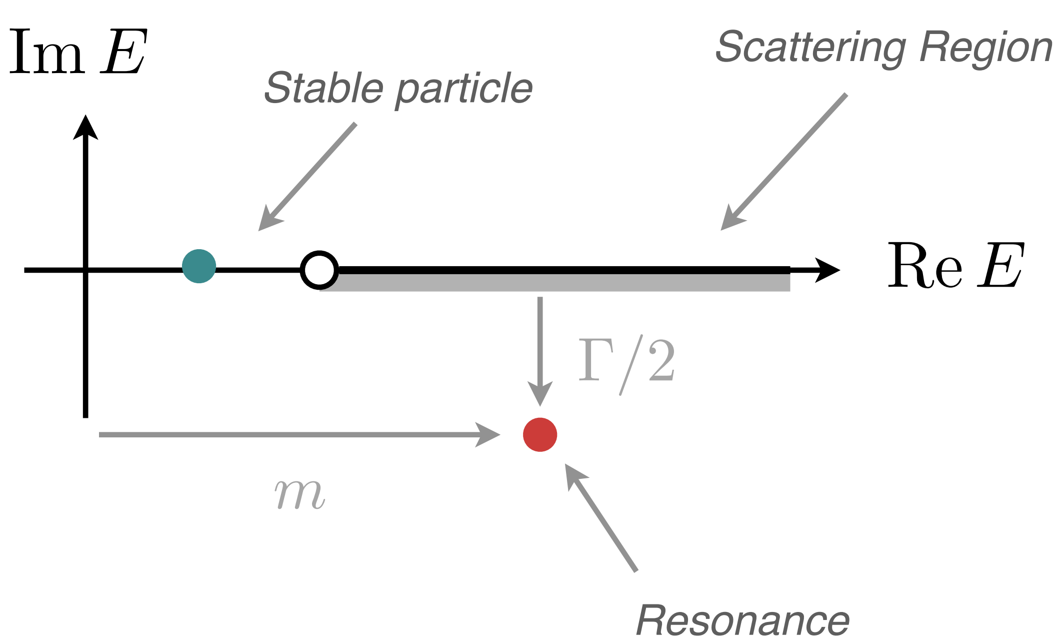
Key idea: Particles are associated with ***pole singularities*** of amplitudes

- For unstable particles, ***resonances***, poles are located in ***complex energy plane***

e.g pion-pion scattering producing the rho-meson

$$\mathcal{S}_{\pi\pi\to\rho\to\pi\pi}(E) \sim \frac{1}{E^2 - m_\rho^2 + i m_\rho \Gamma_\rho}$$





Note: sweeping many details under-the-rug

Amplitudes & Particles

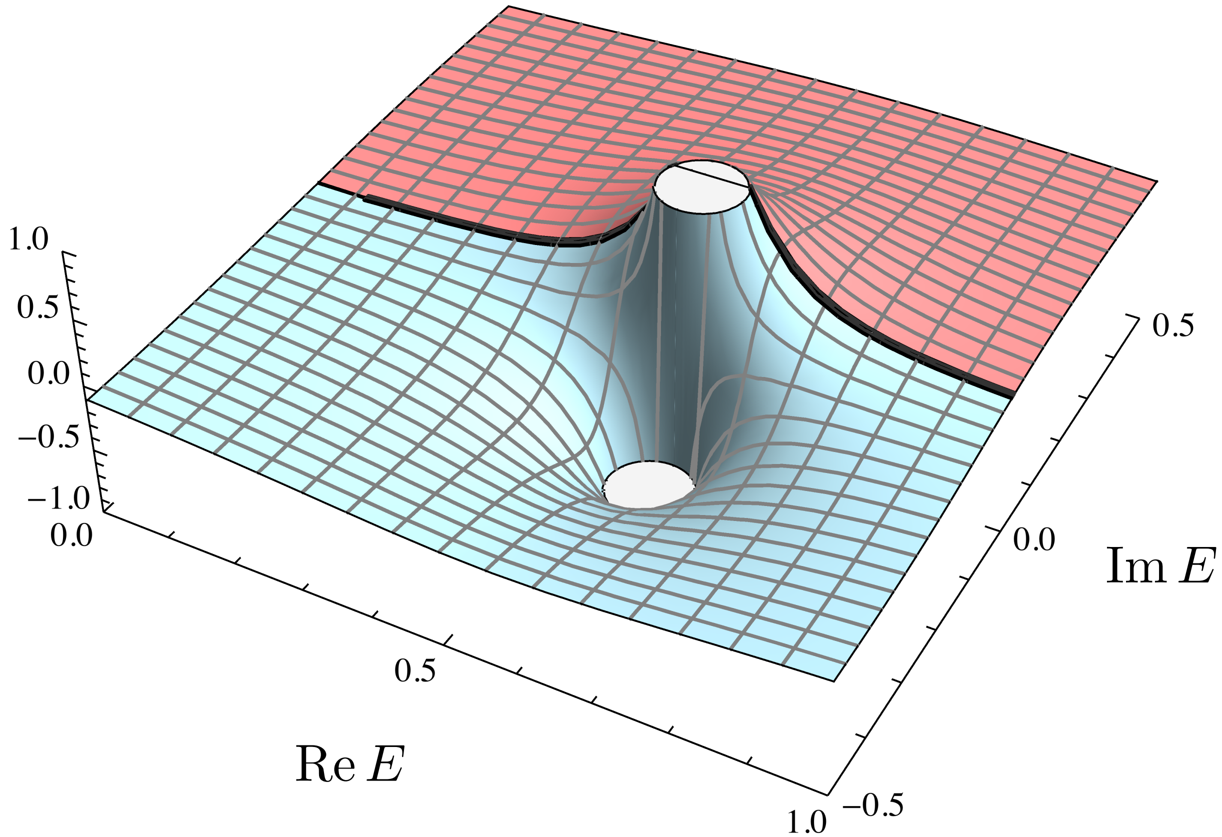
Key idea: Particles are associated with ***pole singularities*** of amplitudes

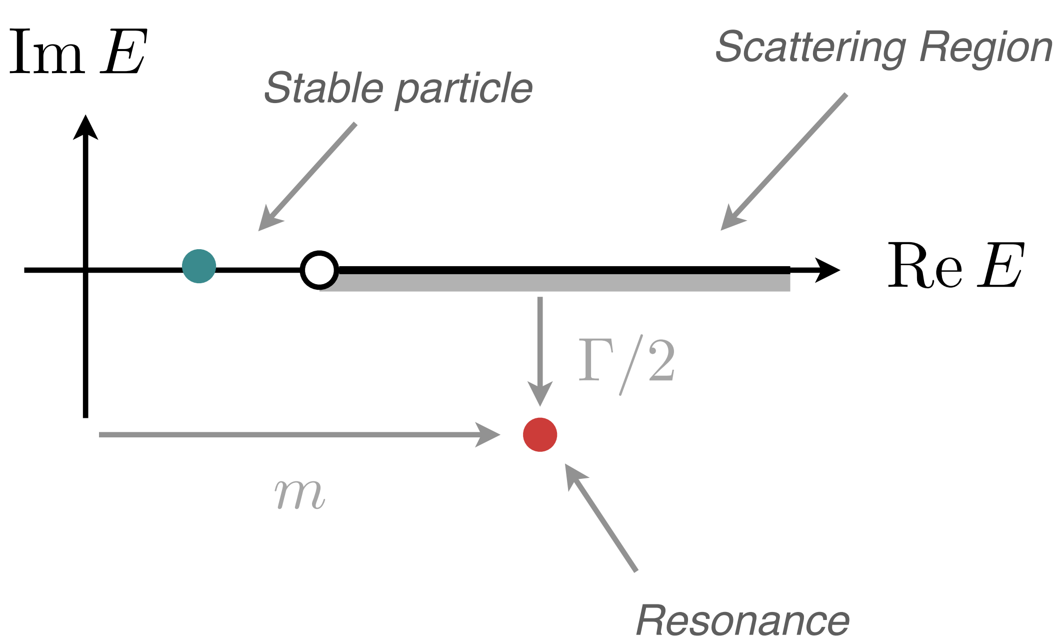
- For unstable particles, ***resonances***, poles are located in ***complex energy plane***

e.g pion-pion scattering producing the rho-meson

Warning! More complicated processes have subtle features

$$\mathcal{S}_{\pi\pi\to\rho\to\pi\pi}(E) \sim \frac{1}{E^2 - m_\rho^2 + i m_\rho \Gamma_\rho}$$



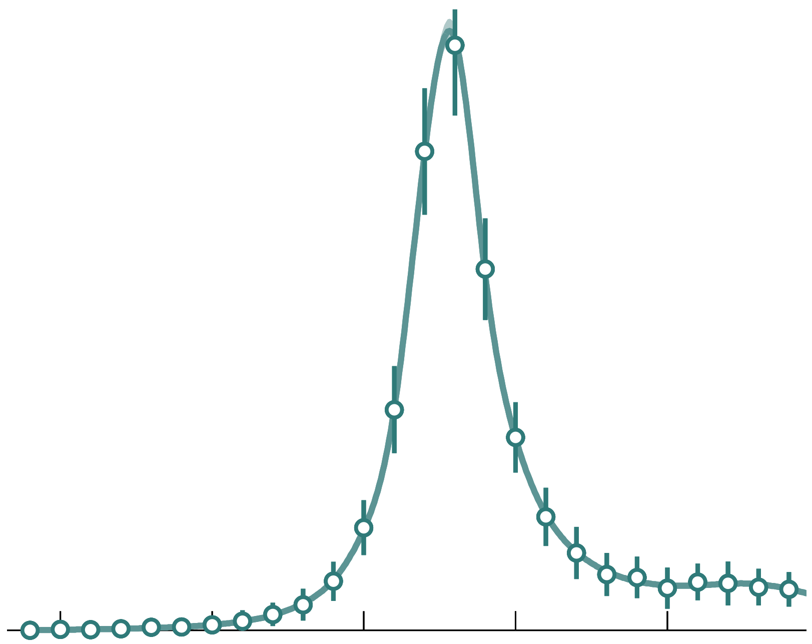


Note: sweeping many details under-the-rug

From Data to Particles

Path to extract physical content from data:

Scattering experiment data

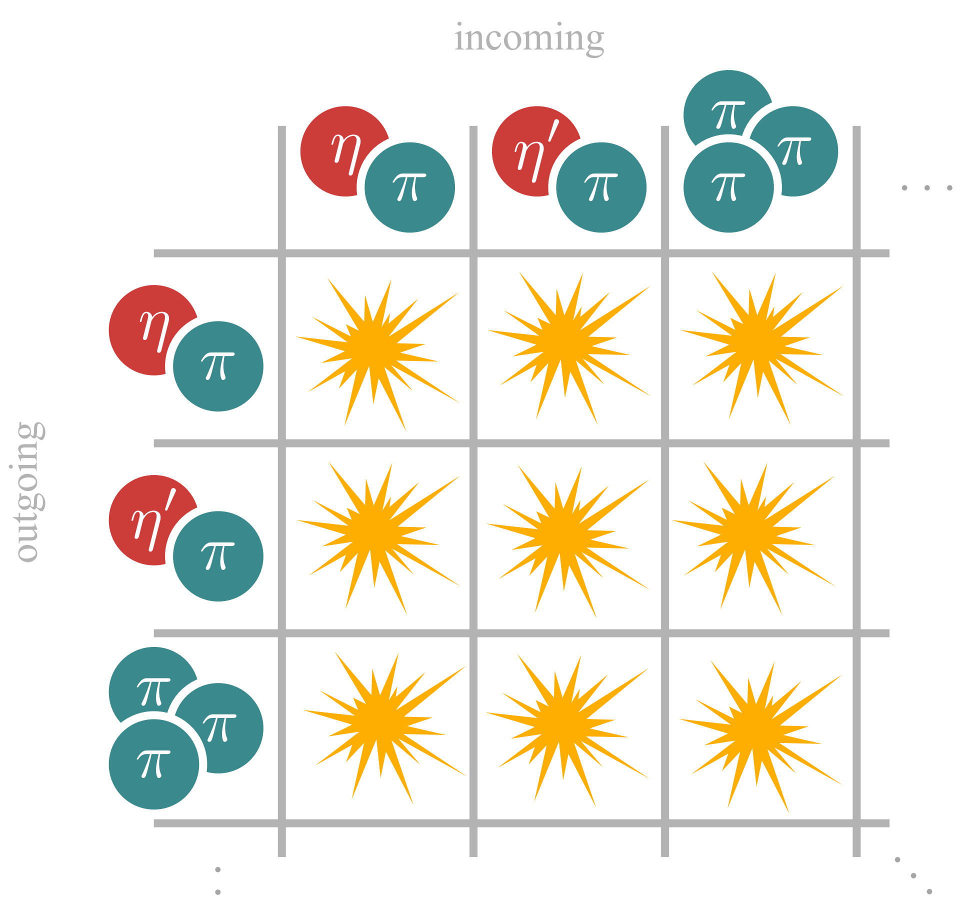


From Data to Particles

Path to extract physical content from data:

Scattering experiment data

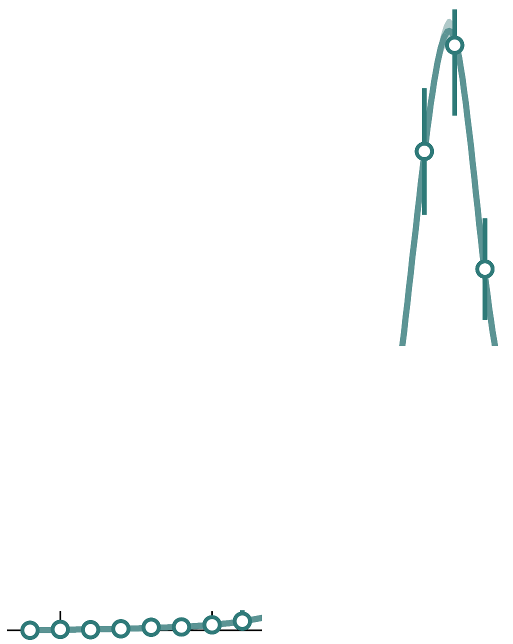
S-matrix elements



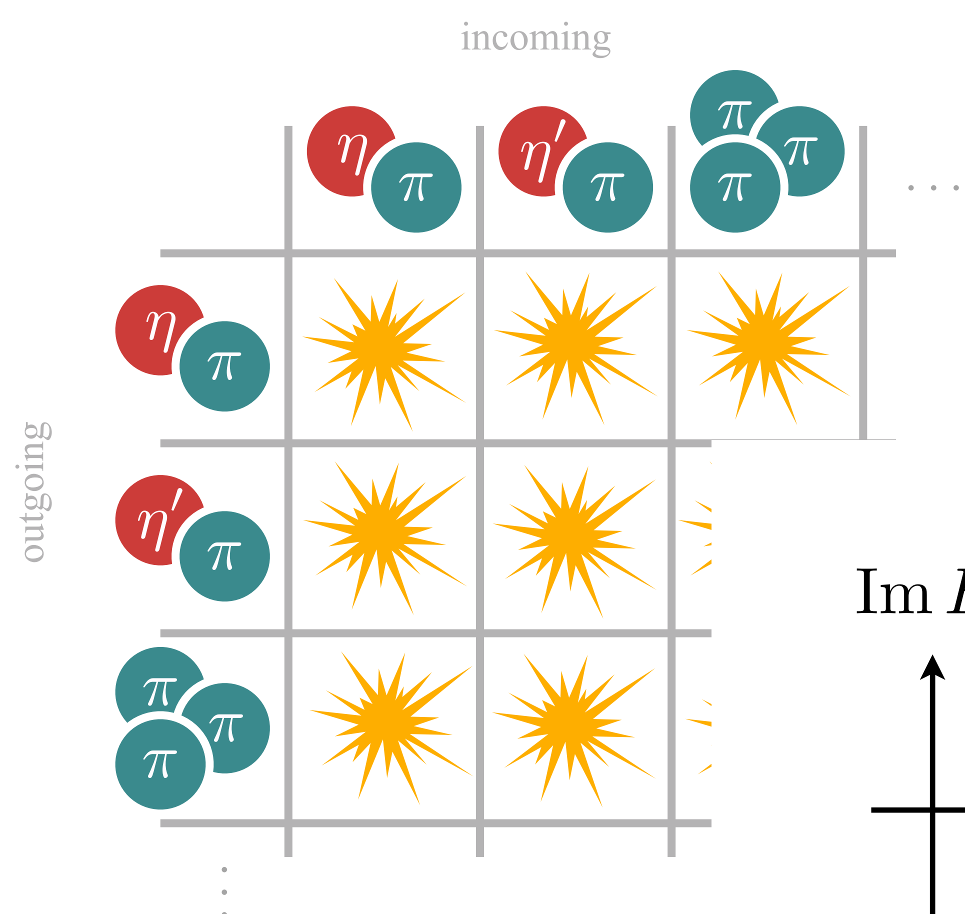
From Data to Particles

Path to extract physical content from data:

Scattering experiment data



S-matrix elements



Spectrum