Hadron Spectroscopy

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Old Dominion University & Jefferson Lab

RPI Computational Summer School

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OLD DOMINION UNIVERSITY

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Outline

Hadrons, Quarks, & QCD

Hadron Spectroscopy

The Quark Model

Quantum Chromodynamics

Scattering Theory & the Hadron Spectrum

Scattering Amplitudes

Bound & Resonant States

Lattice QCD & Hadron Spectroscopy

Lattice QCD

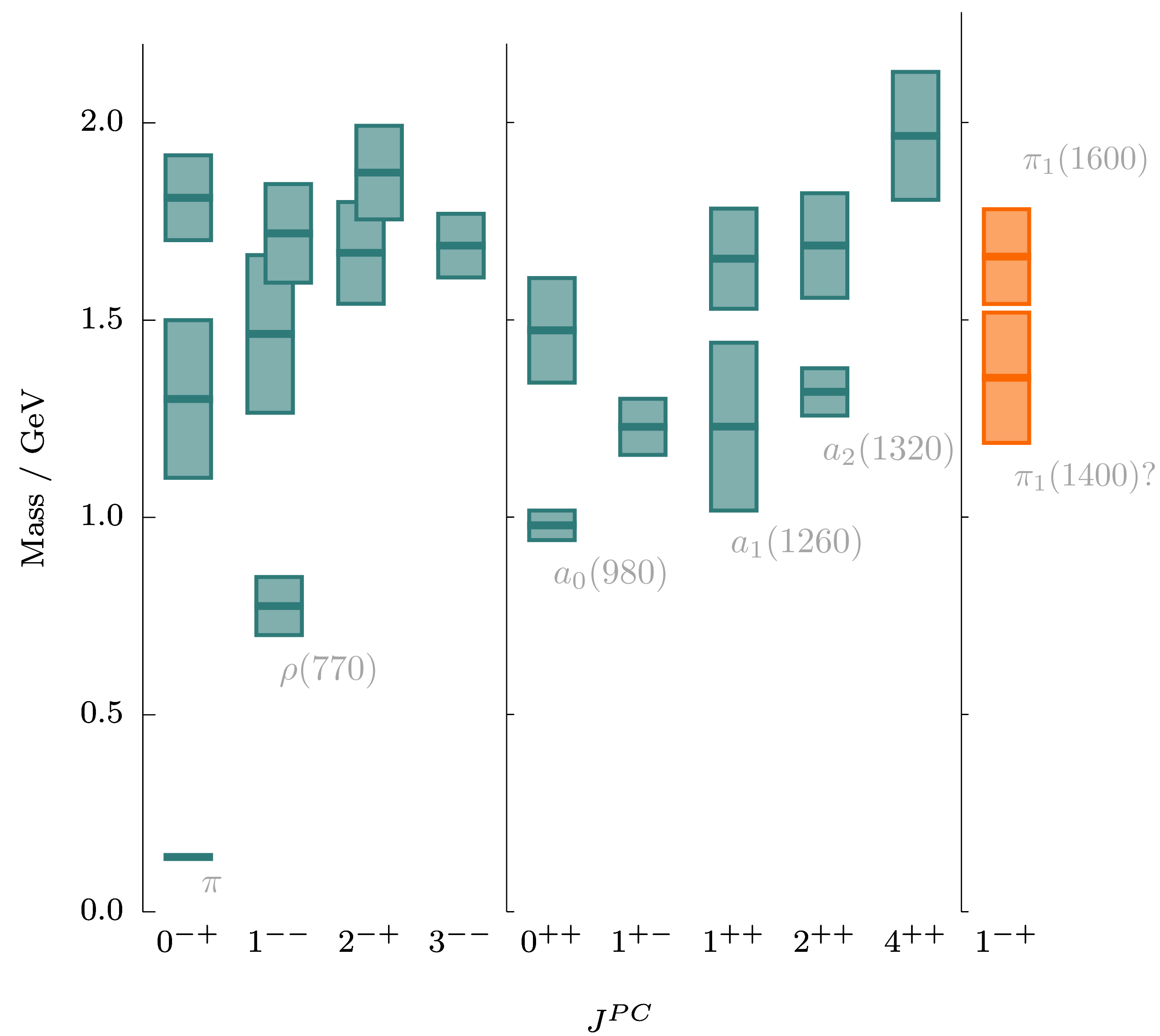
Lüscher & the Finite-Volume

Hadrons and QCD

QCD allows for more exotic states — Do we understand QCD?

- How to connect QCD to hadrons?
- Need to understand how to quantify what the hadrons are in nature
- Need to find non-perturbative approach to access these hadrons ***rigorously***

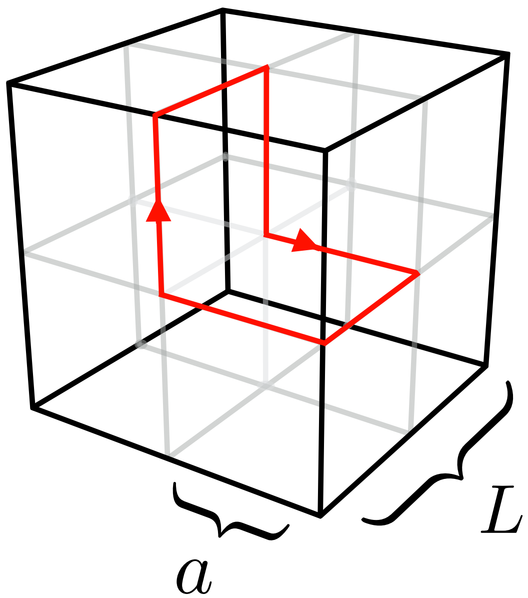
$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f \right) \psi_f - \frac{1}{2}\mathrm{tr}\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu} \right)$$



Lattice QCD

Lattice QCD offers a systematic approach to compute hadrons from QCD

- Numerically evaluate QCD path integral via Monte Carlo sampling



$$Z_{\mathrm{QCD}} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_{\mu}\, e^{iS_{\mathrm{QCD}}(\psi,\bar{\psi},A_{\mu})}$$

- *Euclidean spacetime,* $t \to -i\tau$
- *Finite volume,* L
- *Discrete spacetime,* a
- *Heavier than physical quark mass,* $m > m_{\mathrm{phys.}}$

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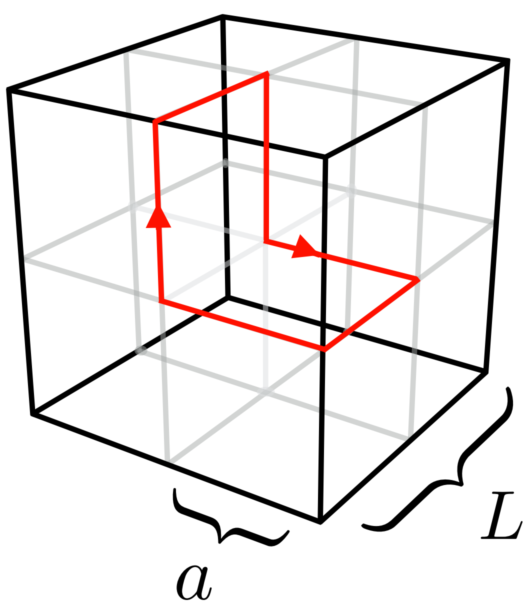

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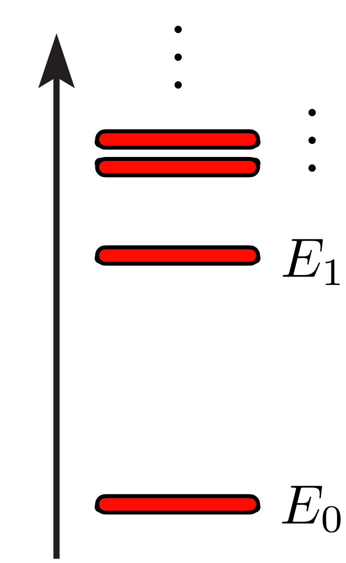
Correlation functions yield discrete spectrum

$$\langle \mathcal{O}(\tau) \mathcal{O}^\dagger(0) \rangle = \sum_{\mathfrak{n}} \left| \langle 0 | \, \mathcal{O} \, | \mathfrak{n} \rangle \right|^2 \, e^{-E_{\mathfrak{n}} \tau}$$



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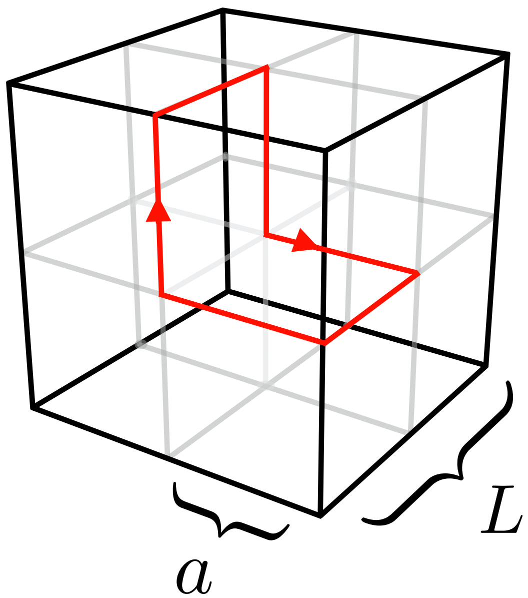

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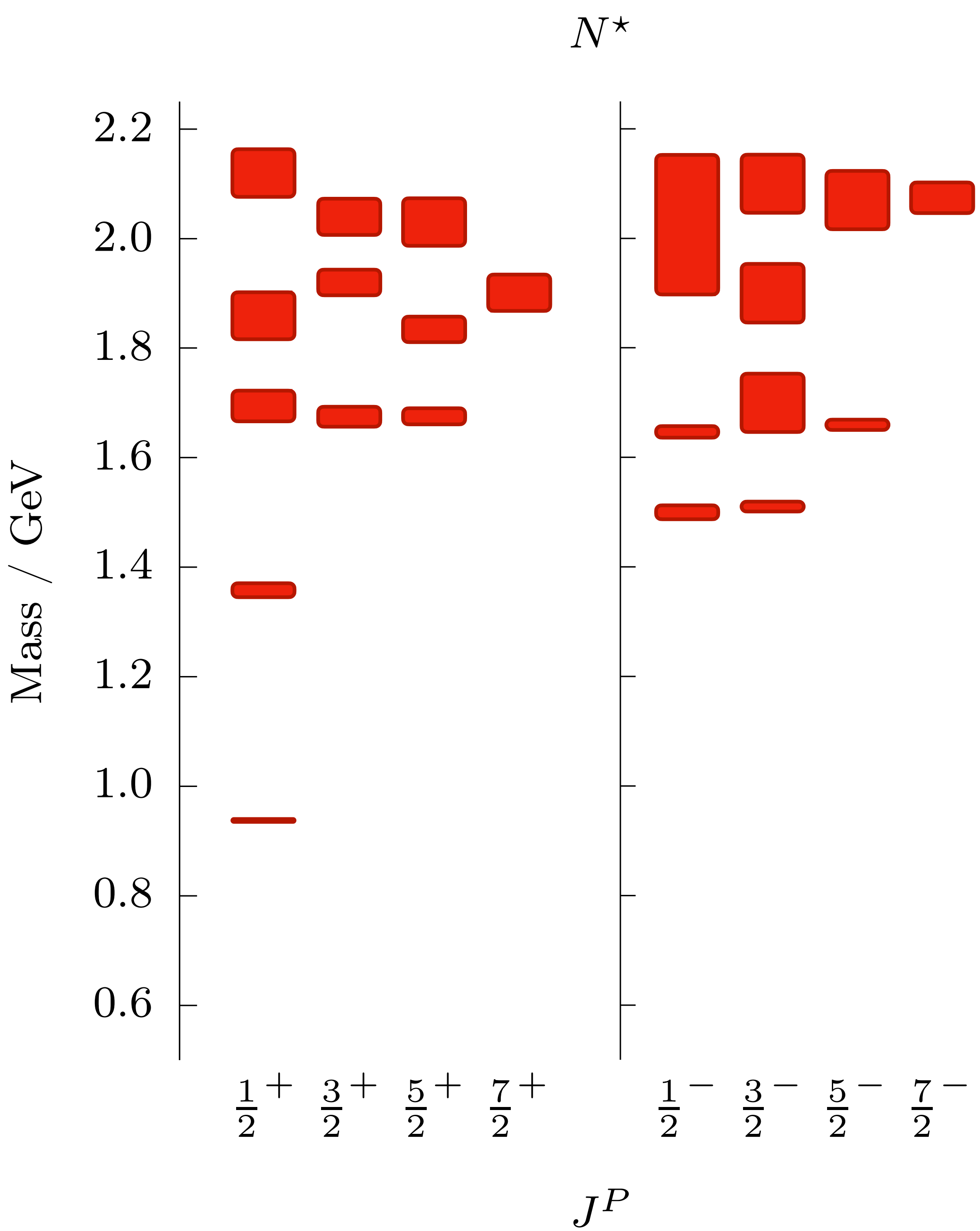

PDG listings

Lattice QCD

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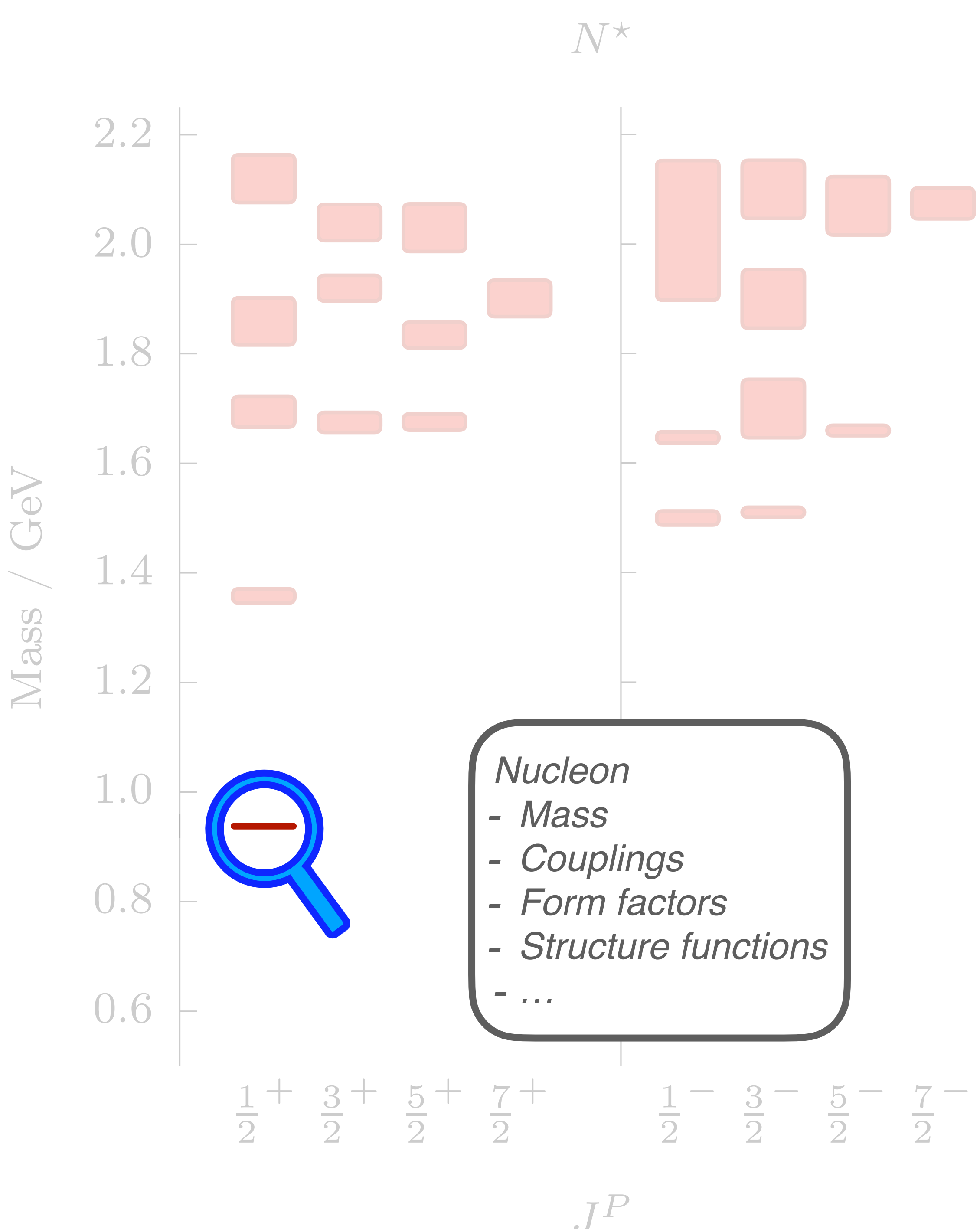
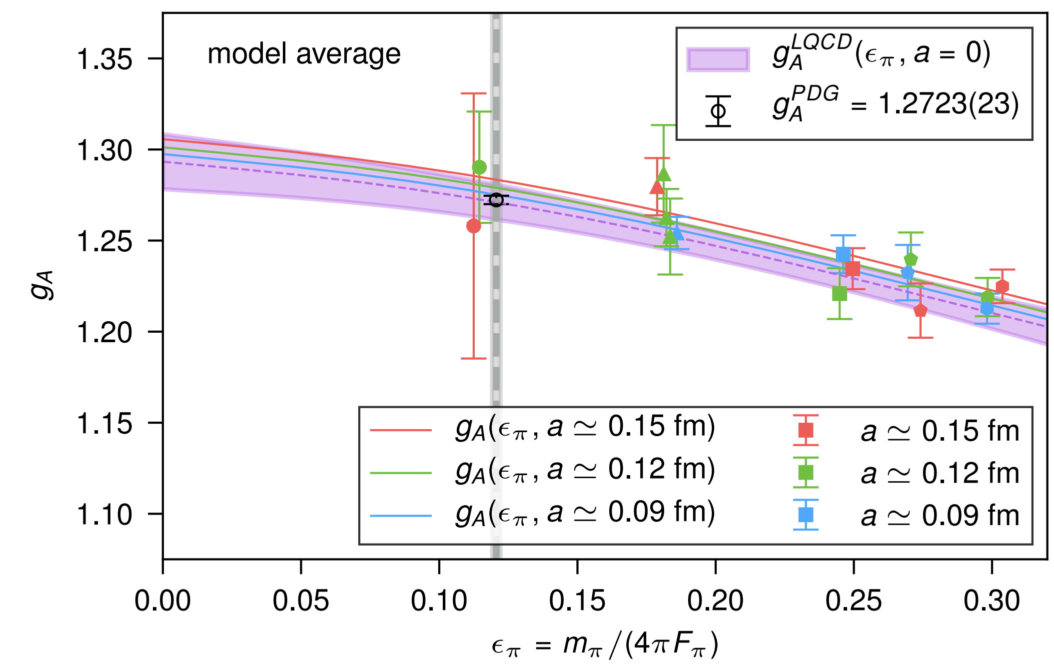
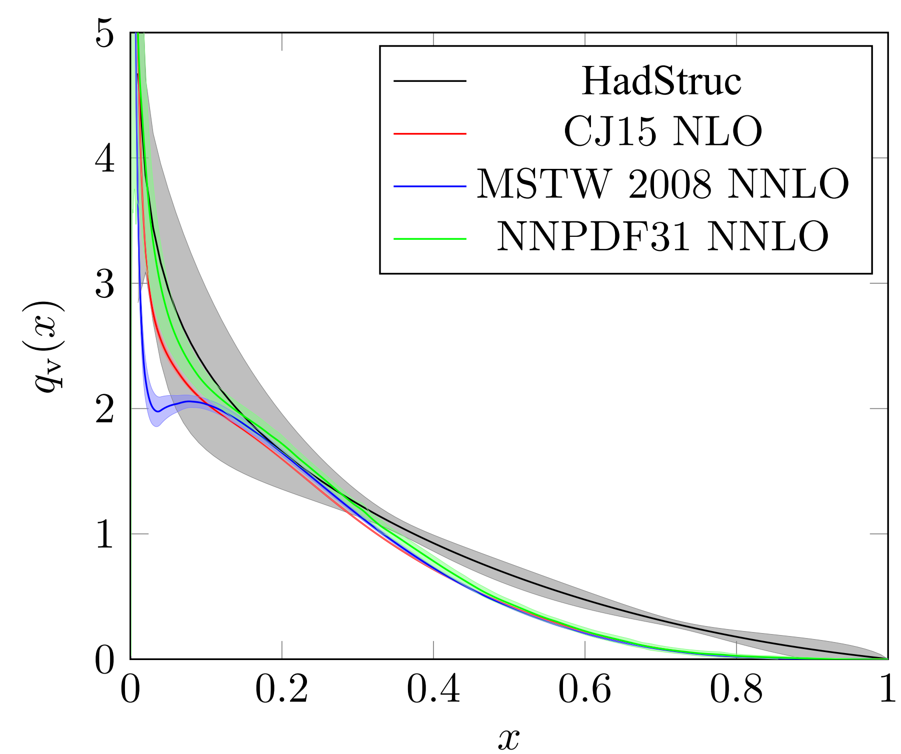




C. Chang et al.
Nature **558**, (2018) 7708



B. Joó et al. [HadStruc]
Phys. Rev. Lett. **125**, (2020) 232003

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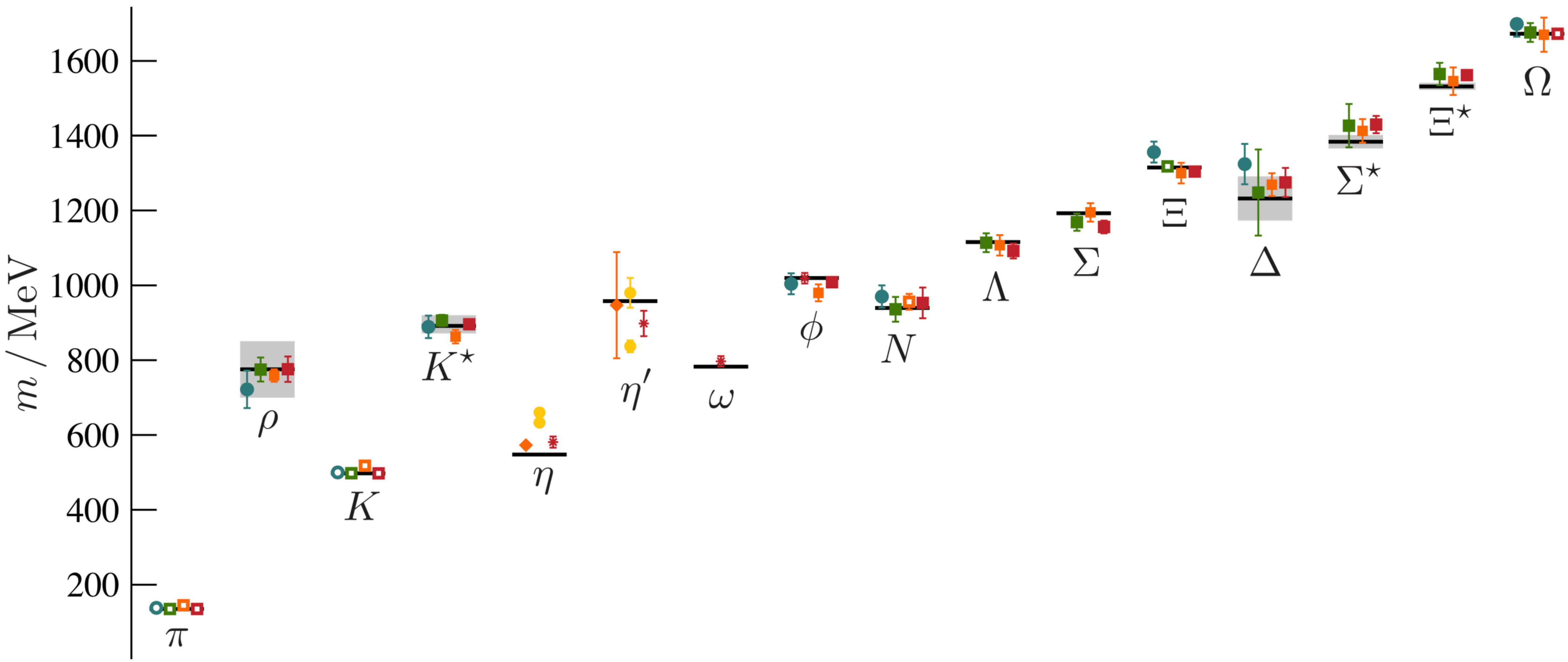

summary compiled by Andreas Kronfeld
Ann. Rev. Nucl. Part. Sci 62 265 (2012)

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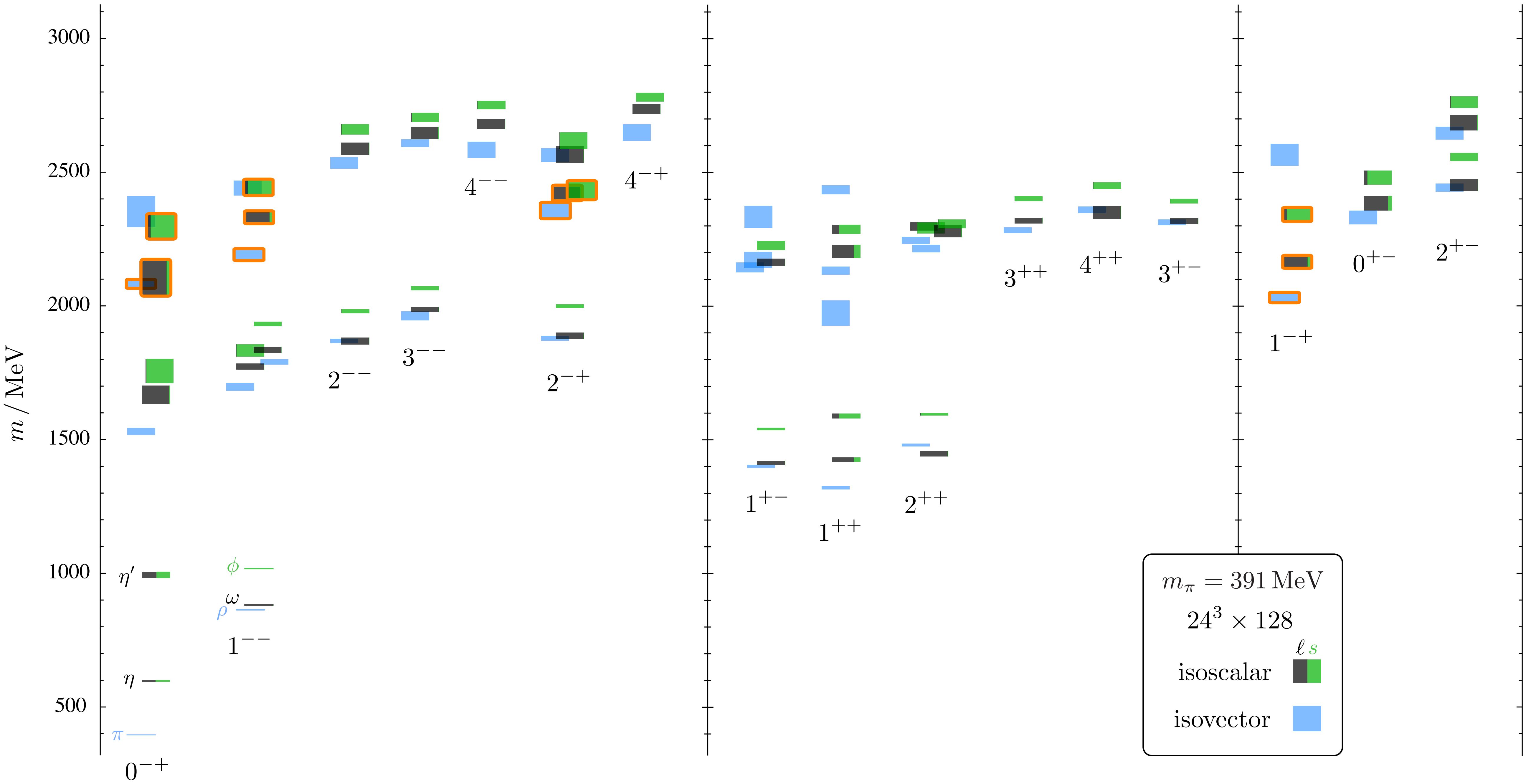

J. Dudek, R. Edwards, P. Guo, and C. Thomas
Phys.Rev.D **88**, 094505 (2013)

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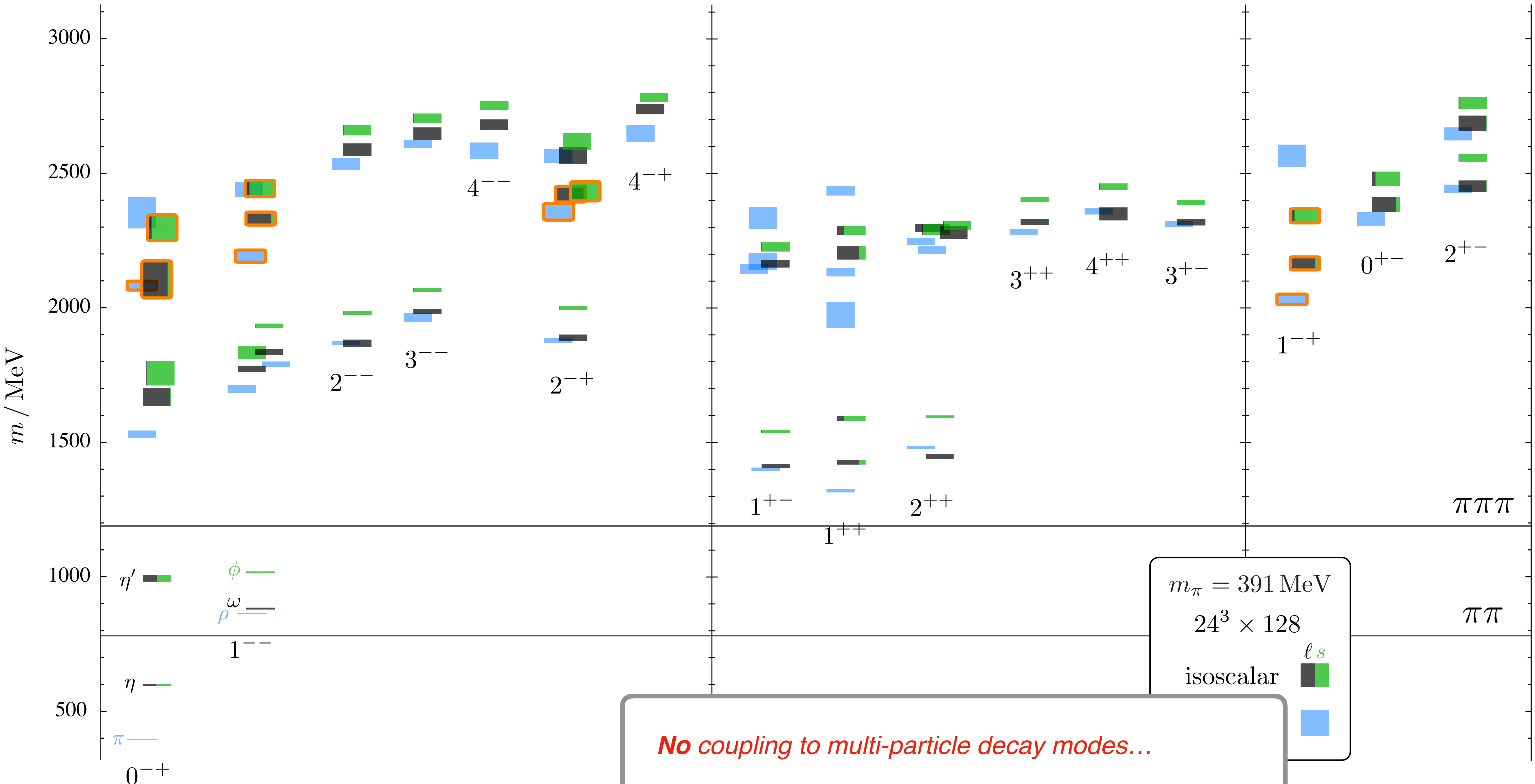

***No** coupling to multi-particle decay modes…*

…An incomplete picture of QCD spectrum

J. Dudek, R. Edwards, P. Guo, and C. Thomas
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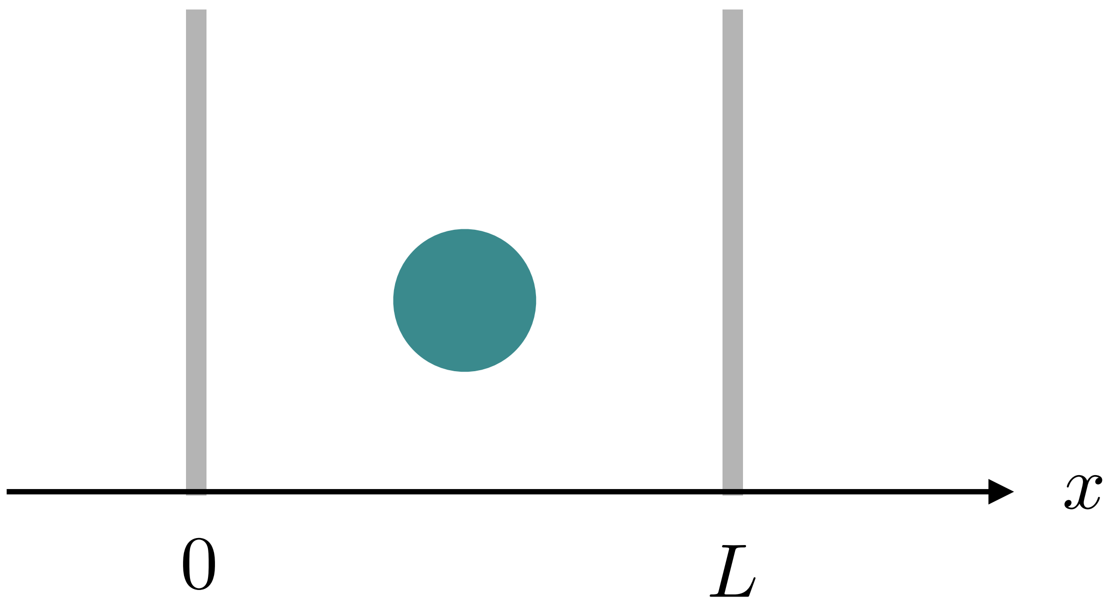
Particles in a box

Consider a particle, confined in a 1d box



Schrödinger equation

$$-\frac{1}{2m}\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} = E\psi(x)$$

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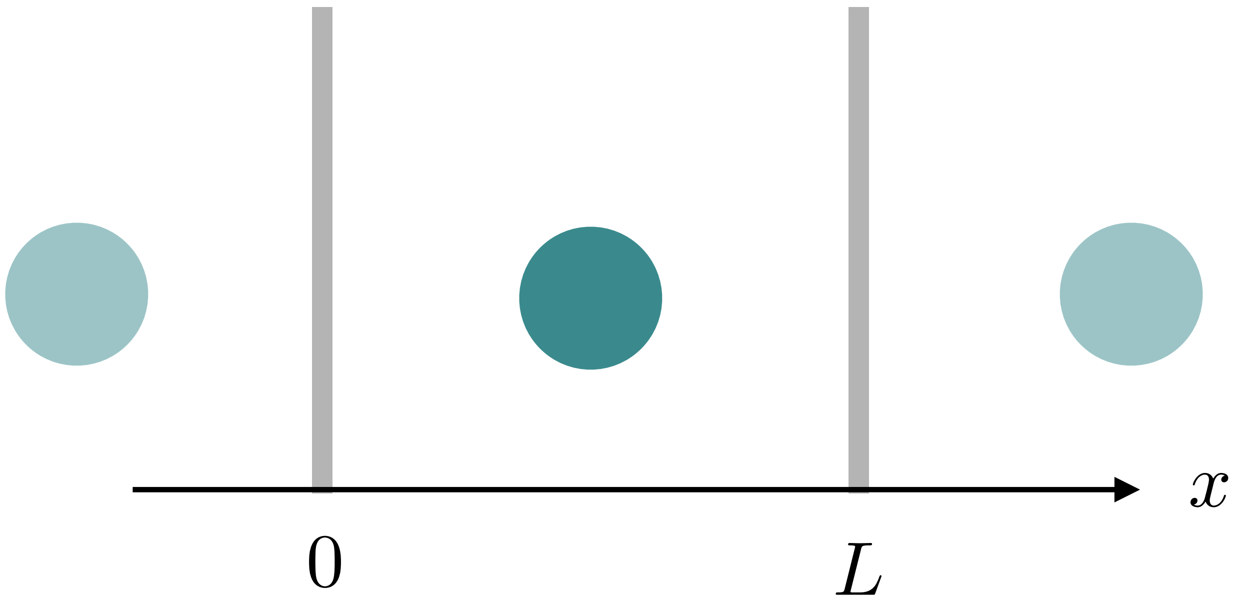

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Periodic boundary conditions

$$\psi(0) = \psi(L)$$

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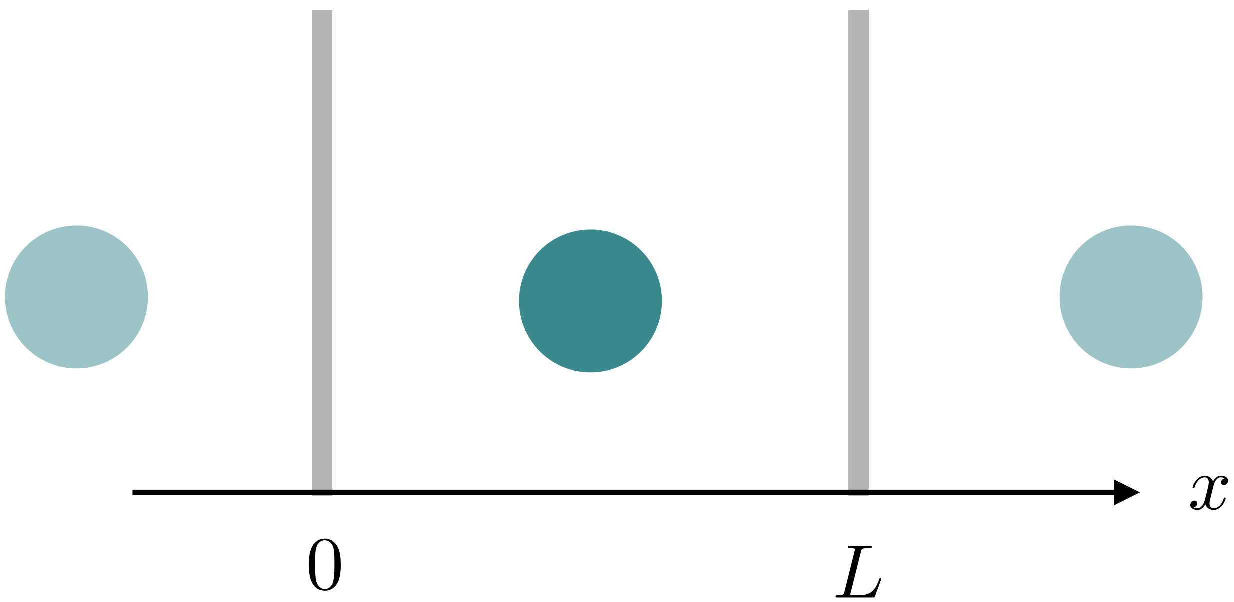

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Periodicity condition

$$\psi(x+L) = \psi(L)$$

Wave functions

$$\psi_n(x) \sim e^{ip_n x}$$

Momentum quantization

$$p_n = \frac{2\pi}{L}n \ , \quad n = 0, \pm 1, \pm 2, \ldots \quad \Longrightarrow \quad E_n = \frac{p_n^2}{2m} = \frac{2\pi^2}{mL^2}n^2$$

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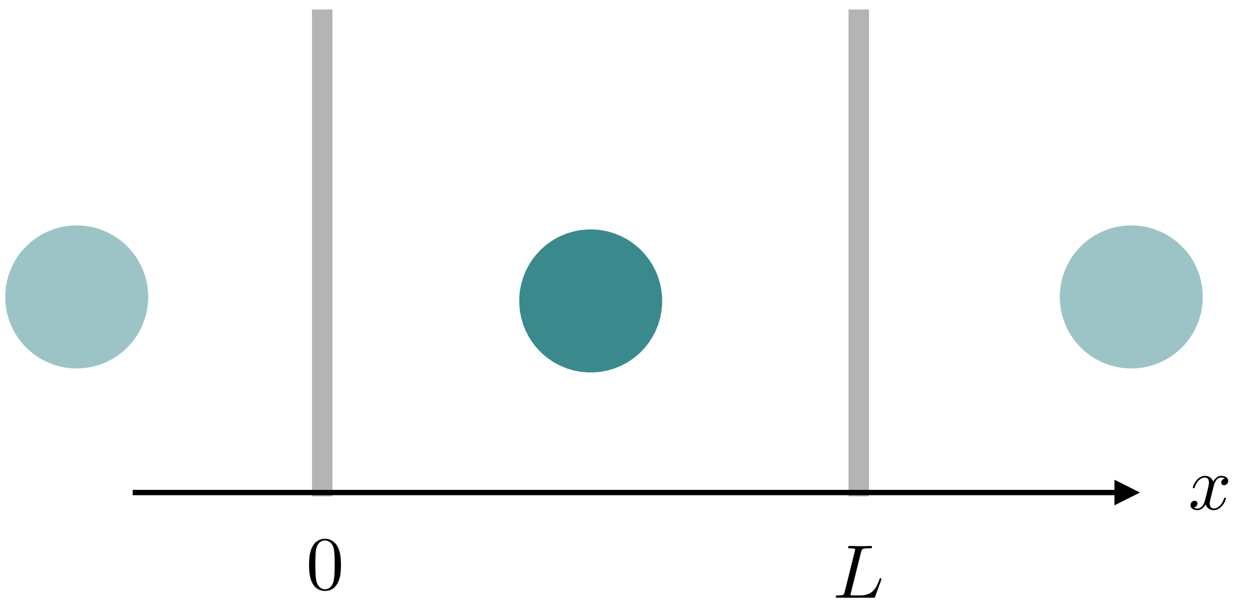

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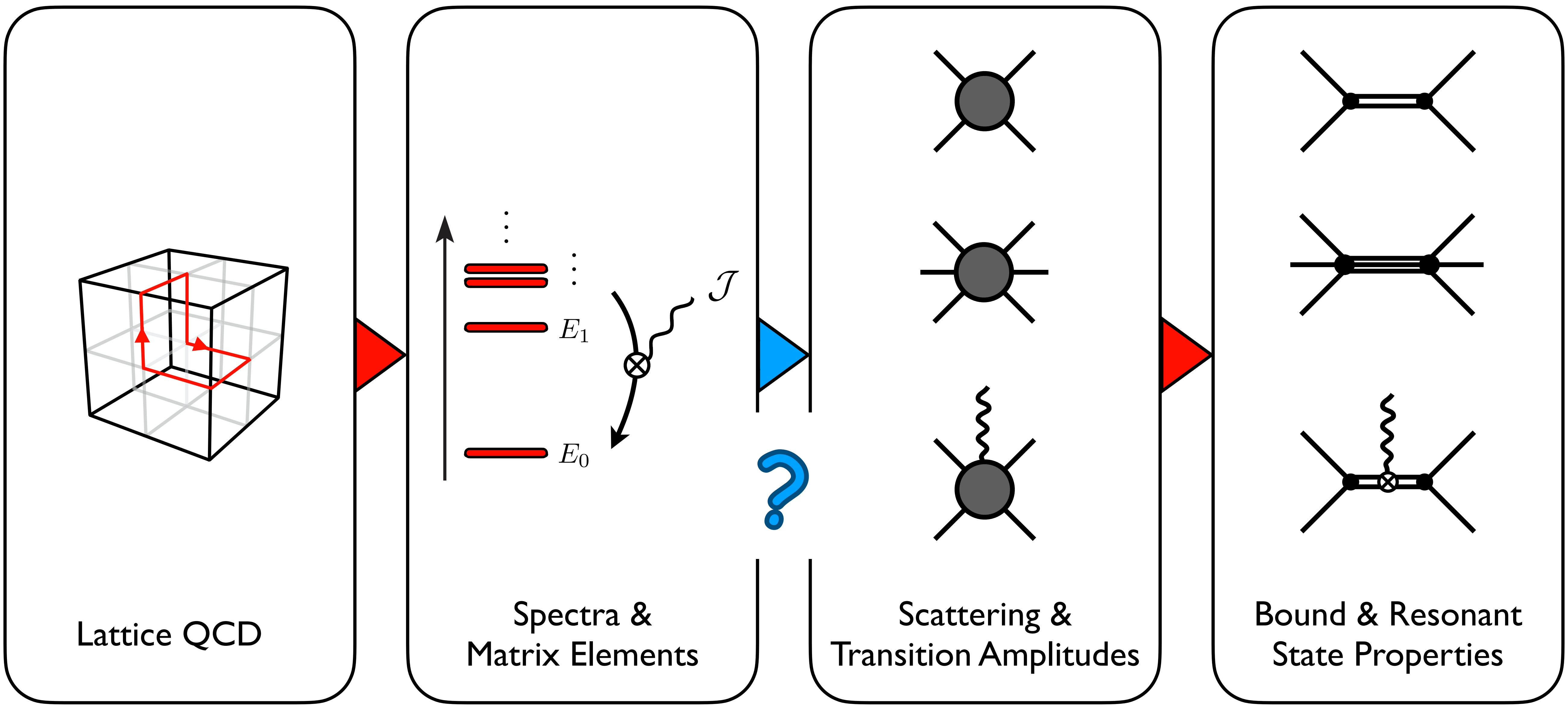
Connecting Scattering Physics to QCD

Path to few-body physics from QCD

- Link finite-volume spectra and matrix elements to scattering amplitudes
- Tools: *Lattice QCD*, *Scattering Theory*, & *Effective Field Theory*

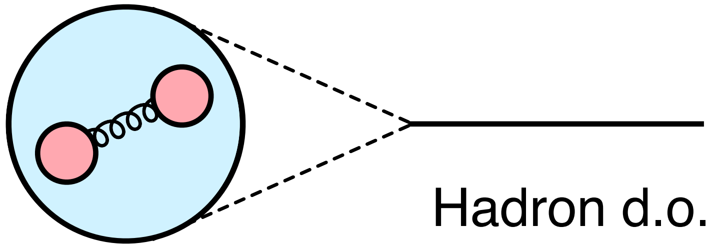


Lattice QCD *Scattering Theory*

Connecting Scattering Physics to QCD

Use generic Effective Field Theory to generate connection

Quarks and gluons at low energy



Hadron d.o.f., e.g. $\pi, K, \ldots$

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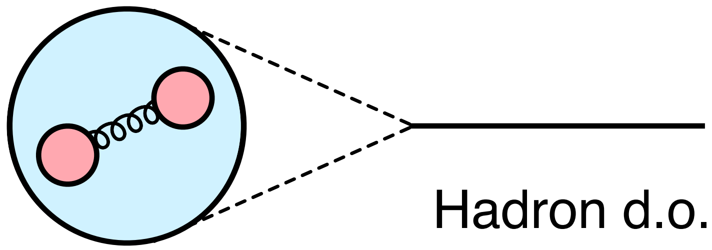
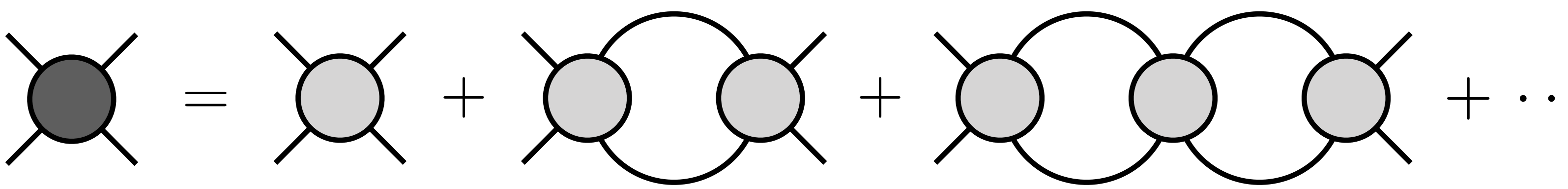


Hadron d.o.f., e.g. $\pi, K, \ldots$

e.g., $\mathbf{2} \to \mathbf{2}$ scattering within some generalized EFT

Fully dressed propagator



hadronic d.o.f.

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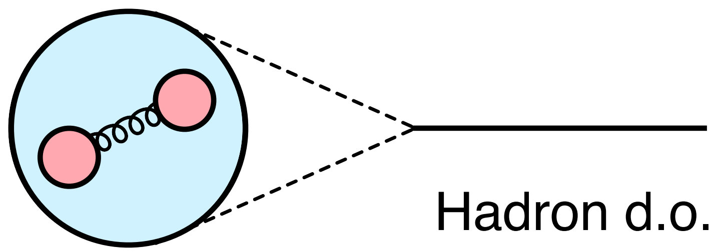

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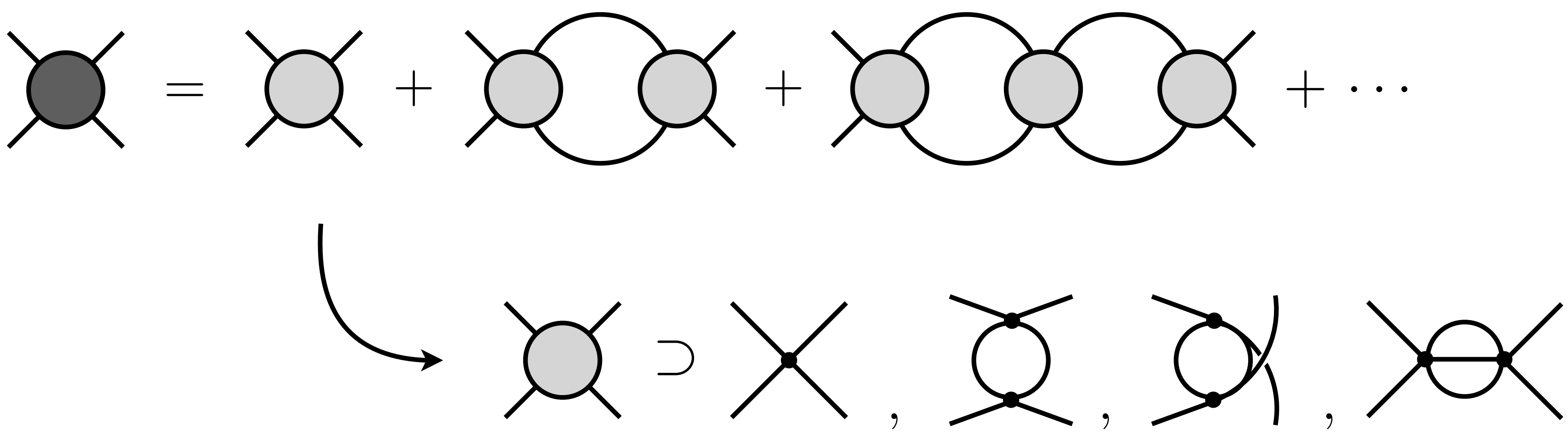

Bethe-Salpeter kernels

All 2PI diagrams - left hand cuts & higher multi-particle thresholds

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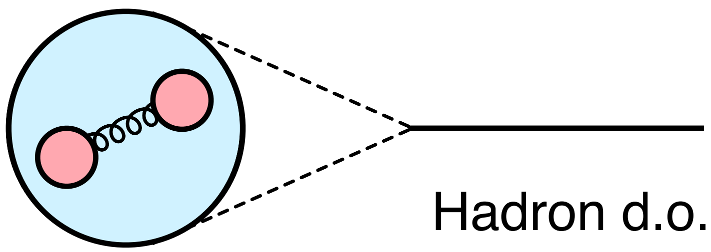

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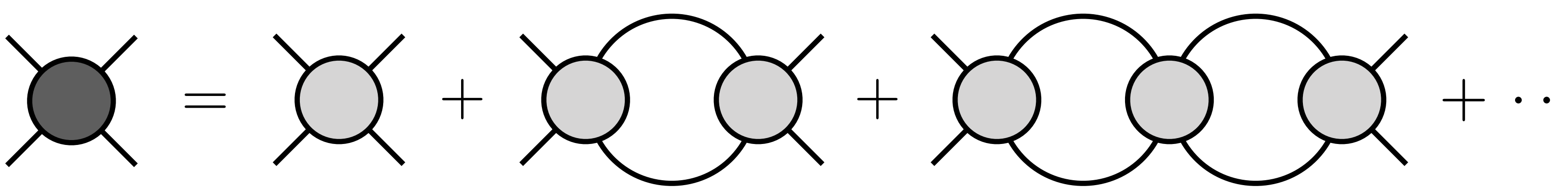
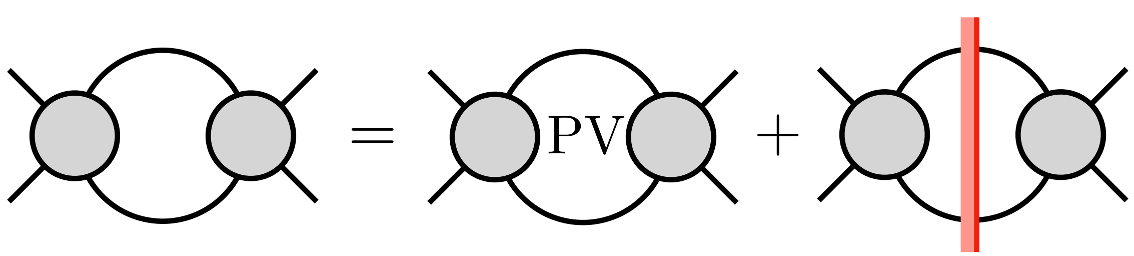


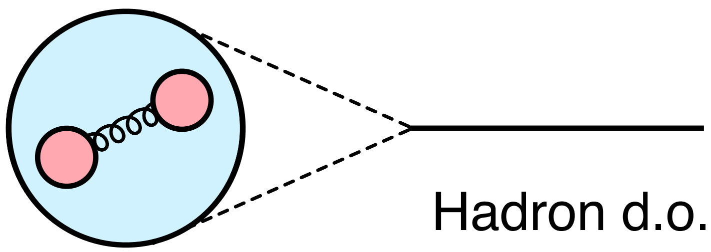


$$\rho = \frac{q}{8\pi E} \sim \sqrt{s - s_{\rm th}}$$

Connecting Scattering Physics to QCD

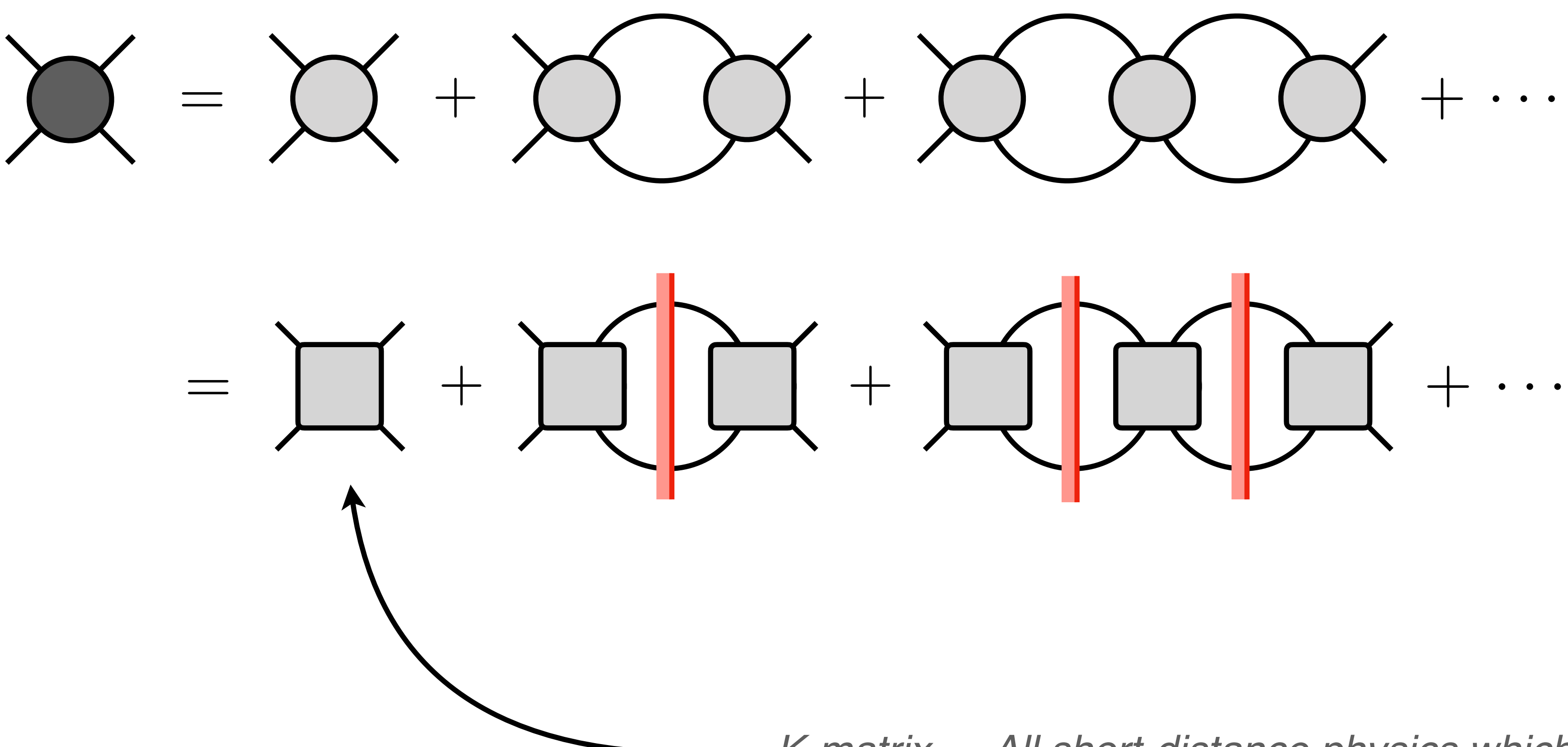
Use generic Effective Field Theory to generate connection

Quarks and gluons
at low energy



Hadron d.o.f., e.g. $\pi, K, \ldots$

e.g., $\mathbf{2} \to \mathbf{2}$ *scattering within some generalized EFT*



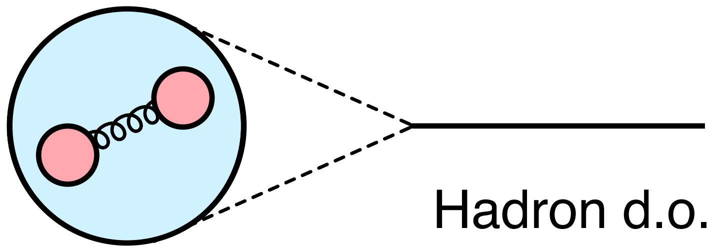
K-matrix — All short-distance physics which cannot go on-shell

— Unknown! — theory specific

Connecting Scattering Physics to QCD

Use generic Effective Field Theory to generate connection

Quarks and gluons at low energy



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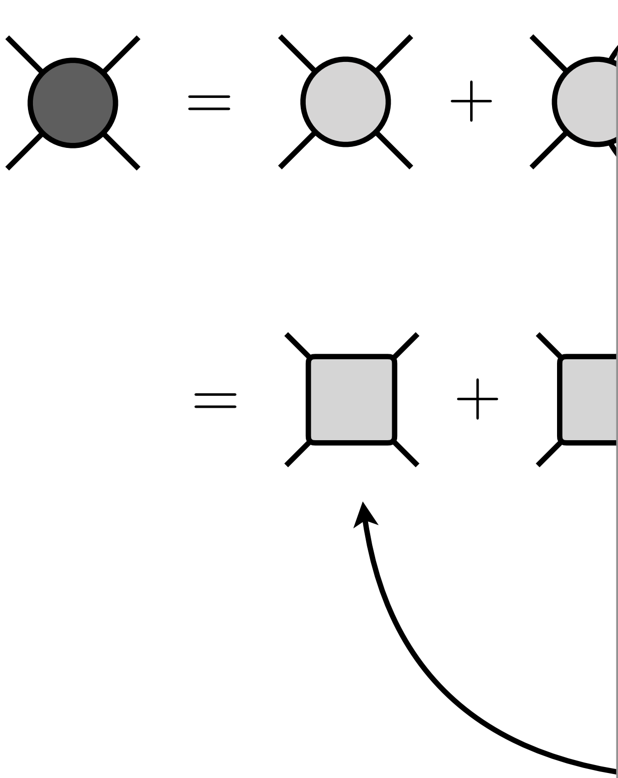
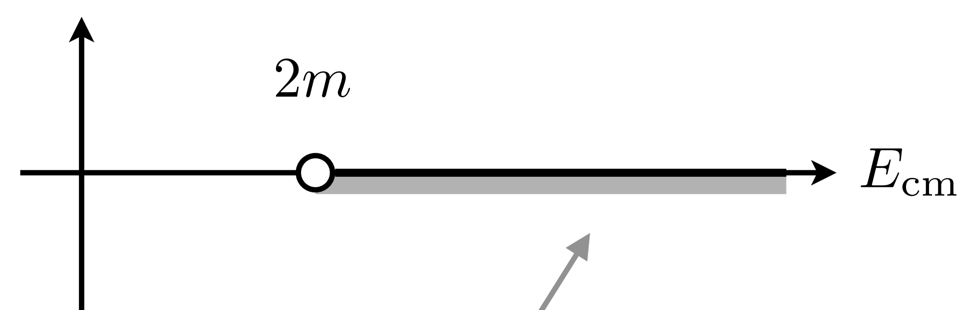


On-shell representation of scattering amplitude

$$\mathcal{M} = \mathcal{K}\,\frac{1}{1 - i\rho\mathcal{K}}$$

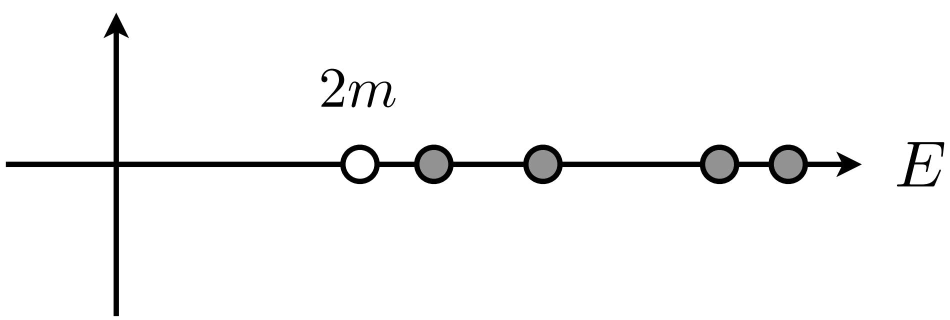


Scattering Region

Connecting Scattering Physics to QCD

Q: How do we connect a finite-volume spectrum computed from QCD…

$$\int_L \mathrm{d}^4x\, e^{iP\cdot x}\, \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_{\mathfrak{n}} \frac{i\,|\langle 0|\,\mathcal{O}\,|\mathfrak{n}\rangle|^2}{E - E_{\mathfrak{n}}}$$



…to infinite-volume scattering amplitudes?

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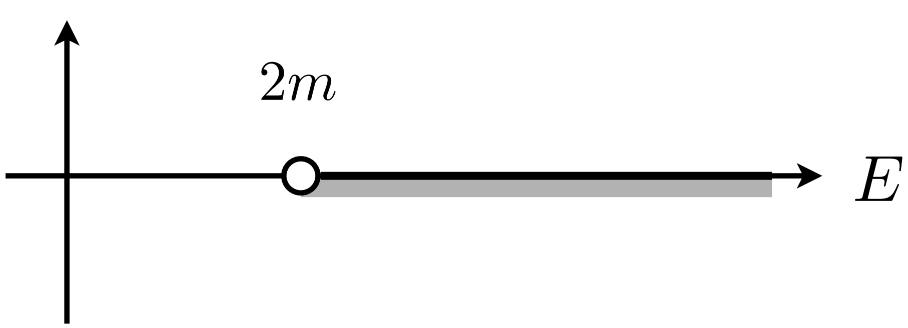

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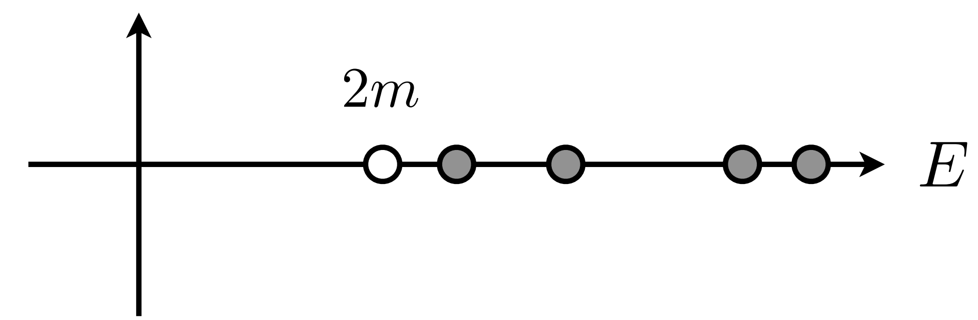

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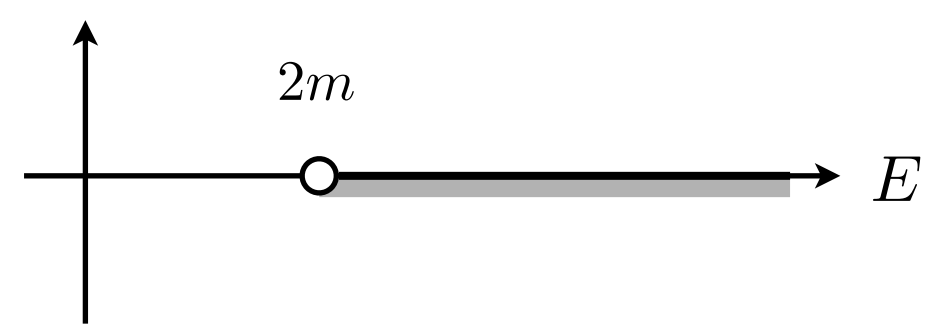
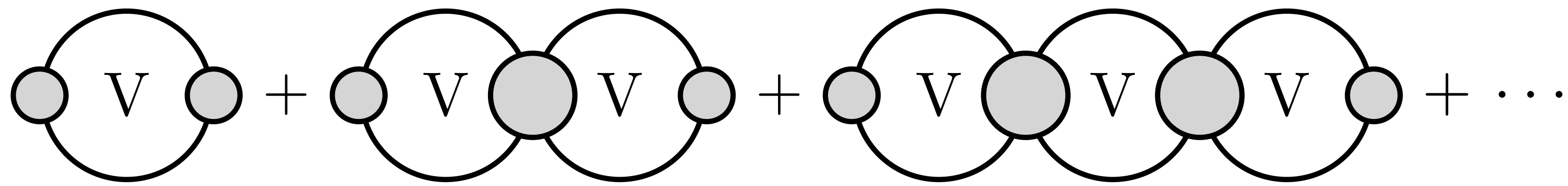


A: Correct analytic structure of finite-volume correlators

Connecting Scattering Physics to QCD

Two-point correlator to all-orders

$C_L(P) =$



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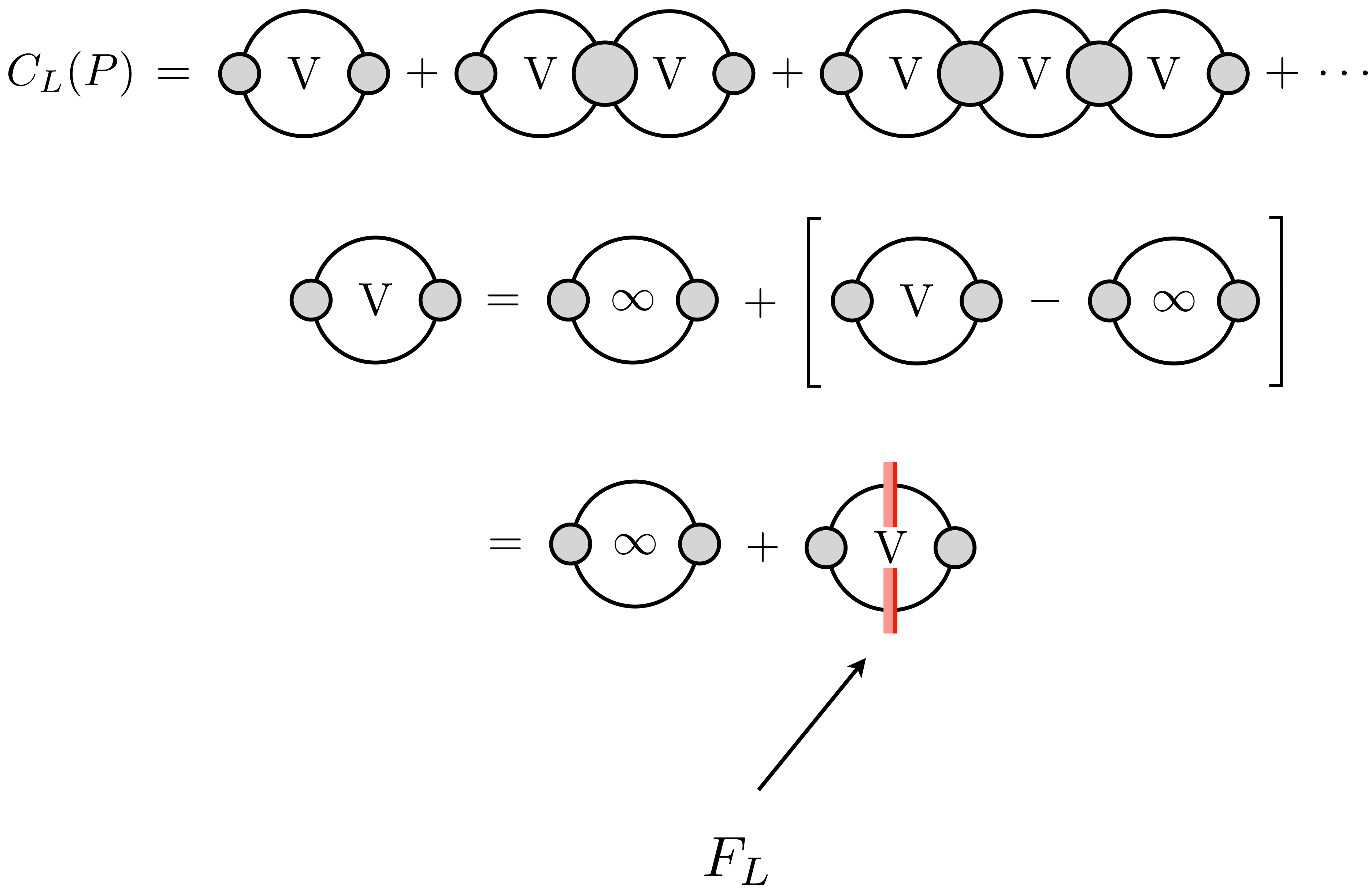

Geometric function — characterizes finite-volume distortions

Connecting Scattering Physics to QCD

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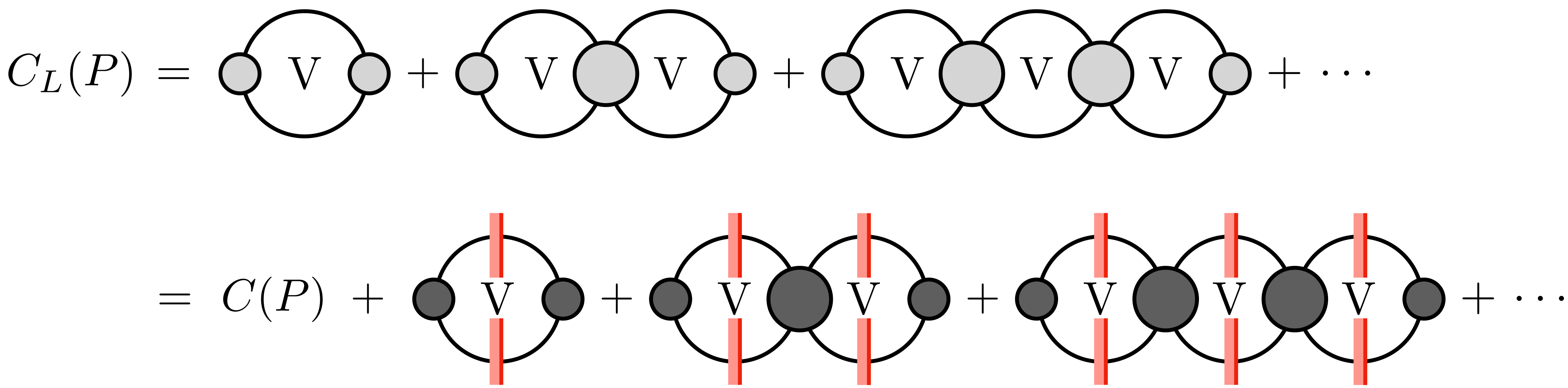

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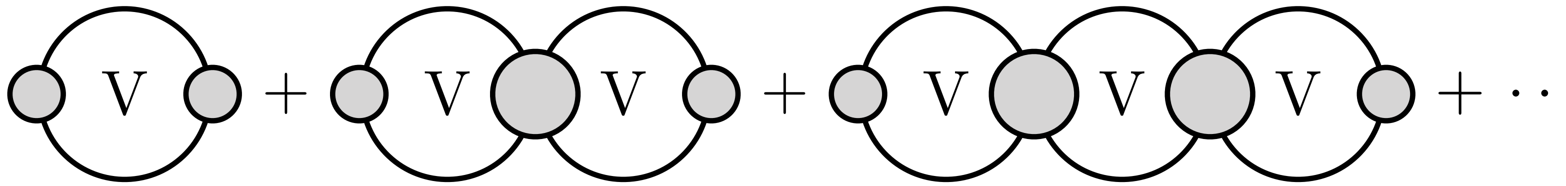
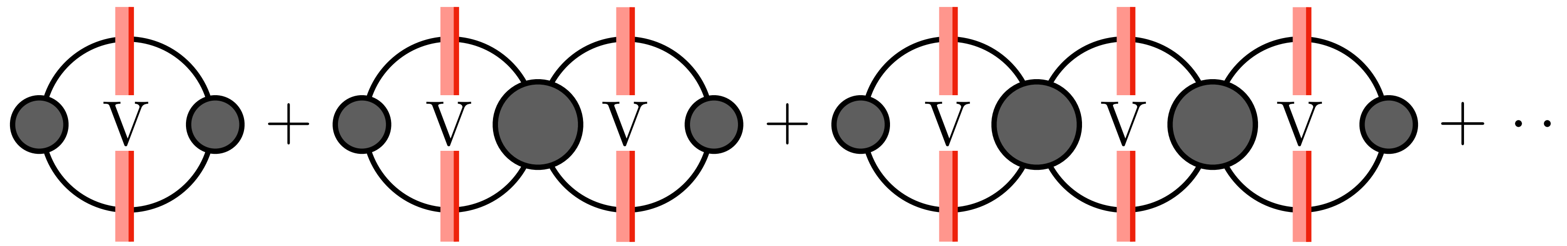


$= C(P) +$



$$= C(P) + i\mathcal{A}\,\frac{i}{\mathcal{M}^{-1} + F_L}\,F_L\,i\mathcal{A}$$

Connecting Scattering Physics to QCD

Two-point correlator to all-orders

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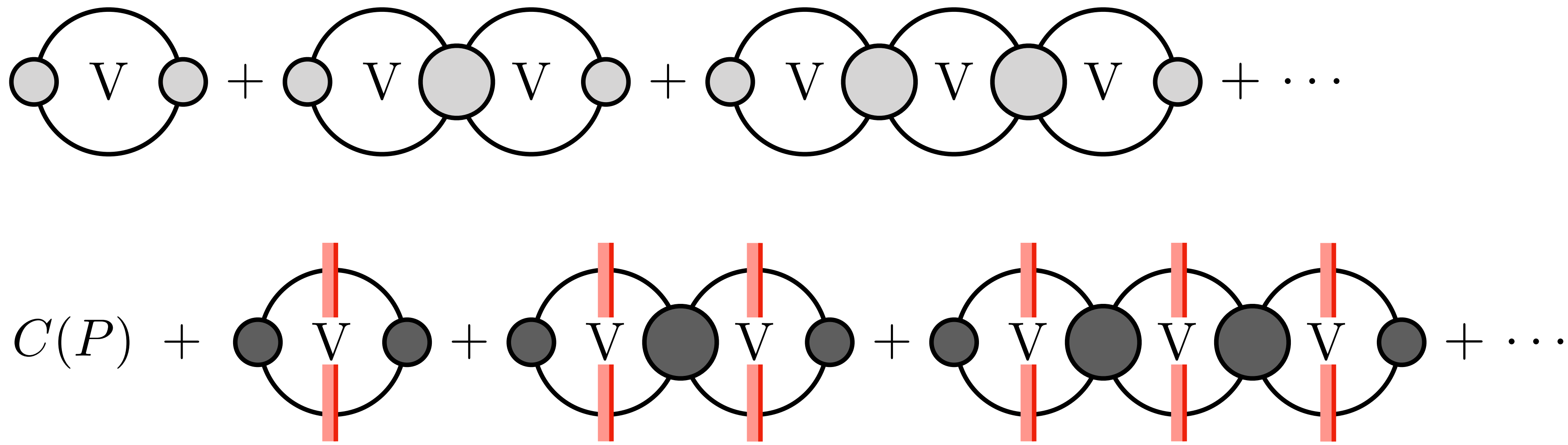
$= C(P) +$



$$= C(P) + i\mathcal{A}\,\frac{i}{\mathcal{M}^{-1} + F_L}\,F_L\,i\mathcal{A}$$

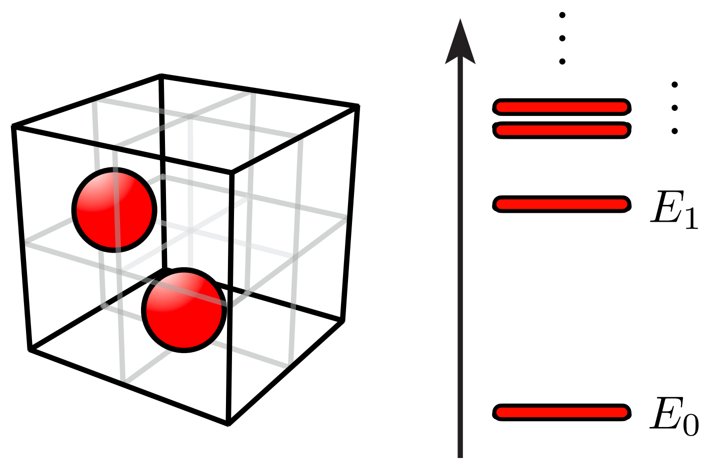
Finite-Volume poles must match!

$$\det\left(\mathcal{M}^{-1} + F_L\right)_{E=E_\mathfrak{n}} = 0$$

Lüscher quantization condition

Few-Body Physics & QCD

Employing scattering theory and EFTs to all-orders connects lattice QCD to scattering observables



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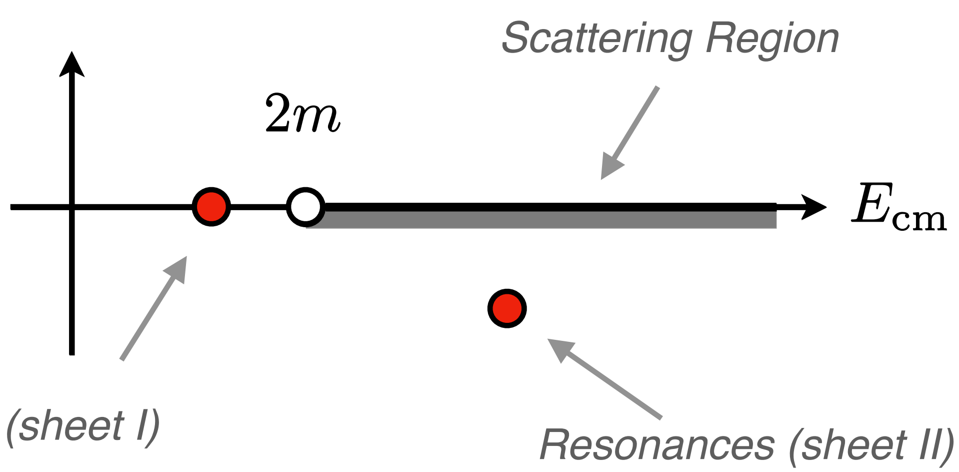


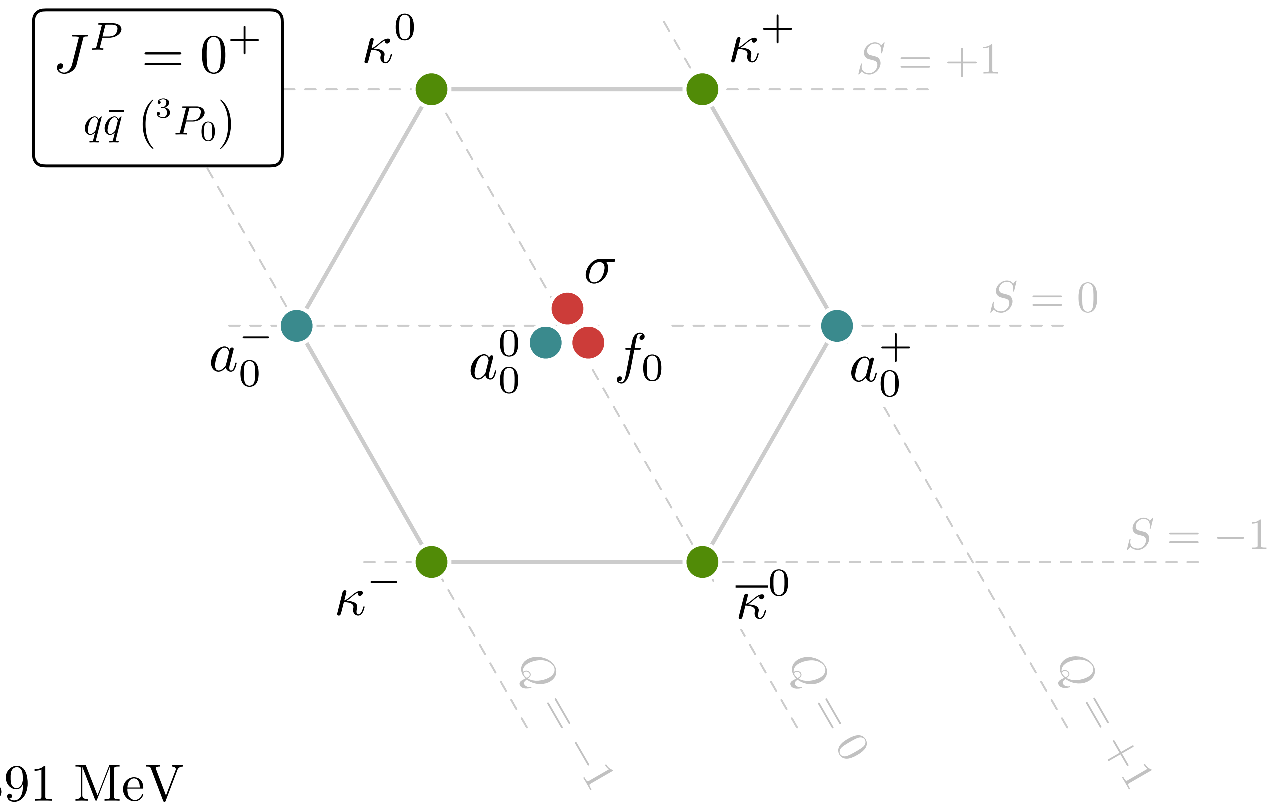
M. Lüscher
Commun.Math.Phys. **105**, 153 (1986)
Nucl.Phys. **B354**, 531 (1991)

Many others…

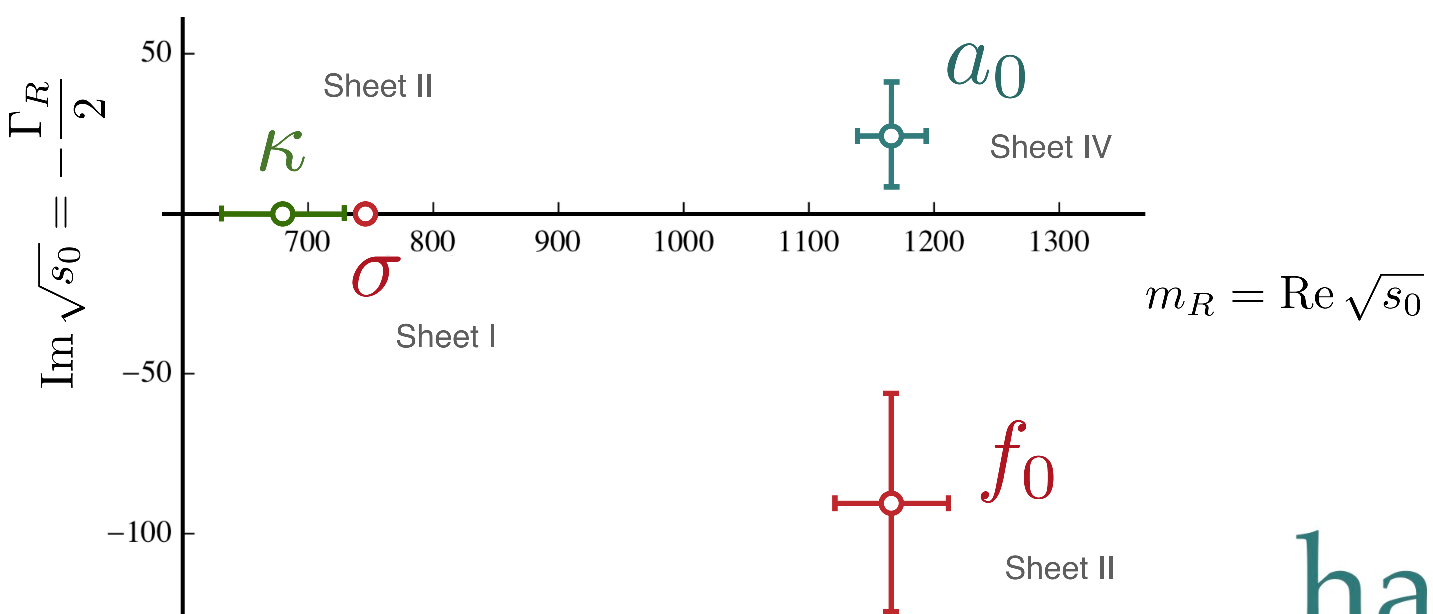
Connecting Resonances to QCD

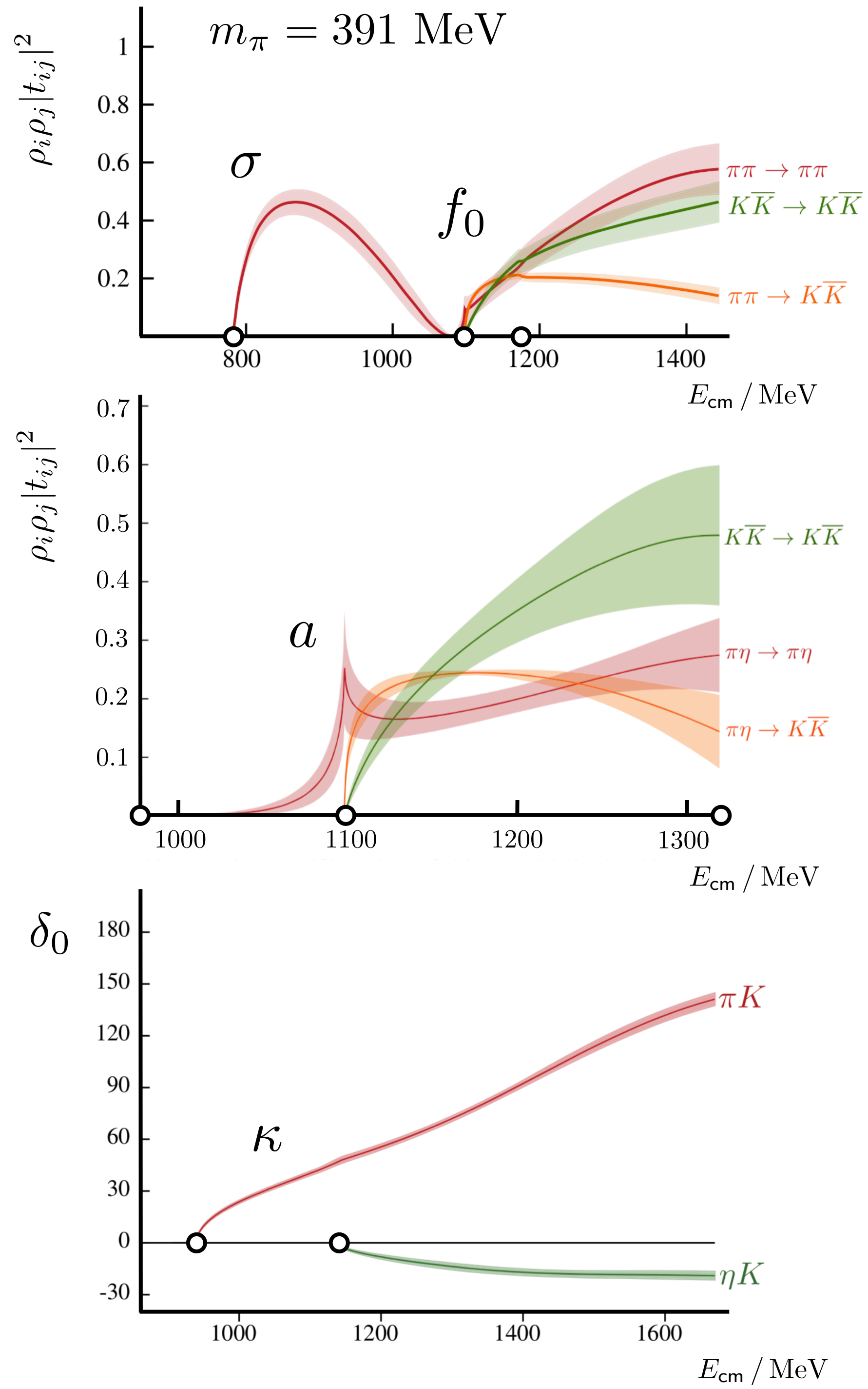
Can we directly connect resonances to QCD?

- Yes! Lattice QCD offers pathway



$m_\pi = 391$ MeV





R.A. Briceño et al. [HadSpec]
Phys. Rev. **D97**, 054513 (2018)

J.J. Dudek et al. [HadSpec]
Phys. Rev. **D93**, 094506 (2016)

J.J. Dudek et al. [HadSpec]
Phys. Rev. Lett. **113**, 182001 (2014)

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