QCD Spectroscopy
An Introduction

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Department of Physics

RPI Computational Summer School

June 19th - July 7th 2023

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Outline

Hadrons, Quarks, & QCD

Hadron Spectroscopy

The Quark Model

Quantum Chromodynamics

Scattering Theory & the Hadron Spectrum

Scattering Amplitudes

Bound & Resonant States

Lattice QCD & Hadron Spectroscopy

If time permits…

Lattice QCD

Lüscher & the Finite-Volume

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Some Reviews

arXiv:1706.06223 (2017)

Scattering processes and resonances from lattice QCD

Raúl A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] and Ross D. Young[3,‡]

[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

[2] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*

[3] *Special Research Center for the Subatomic Structure of Matter (CSSM), Department of Physics, University of Adelaide, Adelaide 5005, Australia*

(Dated: June 21, 2017)

The vast majority of hadrons observed in nature are not stable under the strong interaction, rather they are *resonances* whose existence is deduced from enhancements in the energy dependence of scattering amplitudes. The study of hadron resonances offers a window into the workings of quantum chromodynamics (QCD) in the low-energy non-perturbative region, and in addition, many probes of the limits of the electroweak sector of the Standard Model consider processes which feature hadron resonances. From a theoretical standpoint, this is a challenging field: the same dynamics that binds quarks and gluons into hadron resonances also controls their decay into lighter hadrons, so a complete approach to QCD is required. Presently, lattice QCD is the only available tool that provides the required non-perturbative evaluation of hadron observables. In this article, we review progress in the study of few-hadron reactions in which resonances and bound-states appear using lattice QCD techniques. We describe the leading approach which takes advantage of the periodic finite spatial volume used in lattice QCD calculations to extract scattering amplitudes from the discrete spectrum of QCD eigenstates in a box. We explain how from explicit lattice QCD calculations, one can rigorously garner information about a variety of resonance properties, including their masses, widths, decay couplings, and form factors. The challenges which currently limit the field are discussed along with the steps being taken to resolve them.

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arXiv:2112.13436 (2021)

Novel approaches in Hadron Spectroscopy

Miguel Albaladejo[a,b], Łukasz Bibrzycki[c], Sebastian M. Dawid[d,e], César Fernández-Ramírez[f,g], Sergi Gonzàlez-Solís[d,e,h], Astrid N. Hiller Blin[a], Andrew W. Jackura[a,i], Vincent Mathieu[j,k], Mikhail Mikhasenko[l,m], Victor I. Mokeev[a], Emilie Passemar[a,d,e], Alessandro Pilloni[n,o,*], Arkaitz Rodas[a,p], Jorge A. Silva-Castro[f], Wyatt A. Smith[d], Adam P. Szczepaniak[a,d,e], Daniel Winney[d,e,q,r],

(Joint Physics Analysis Center)

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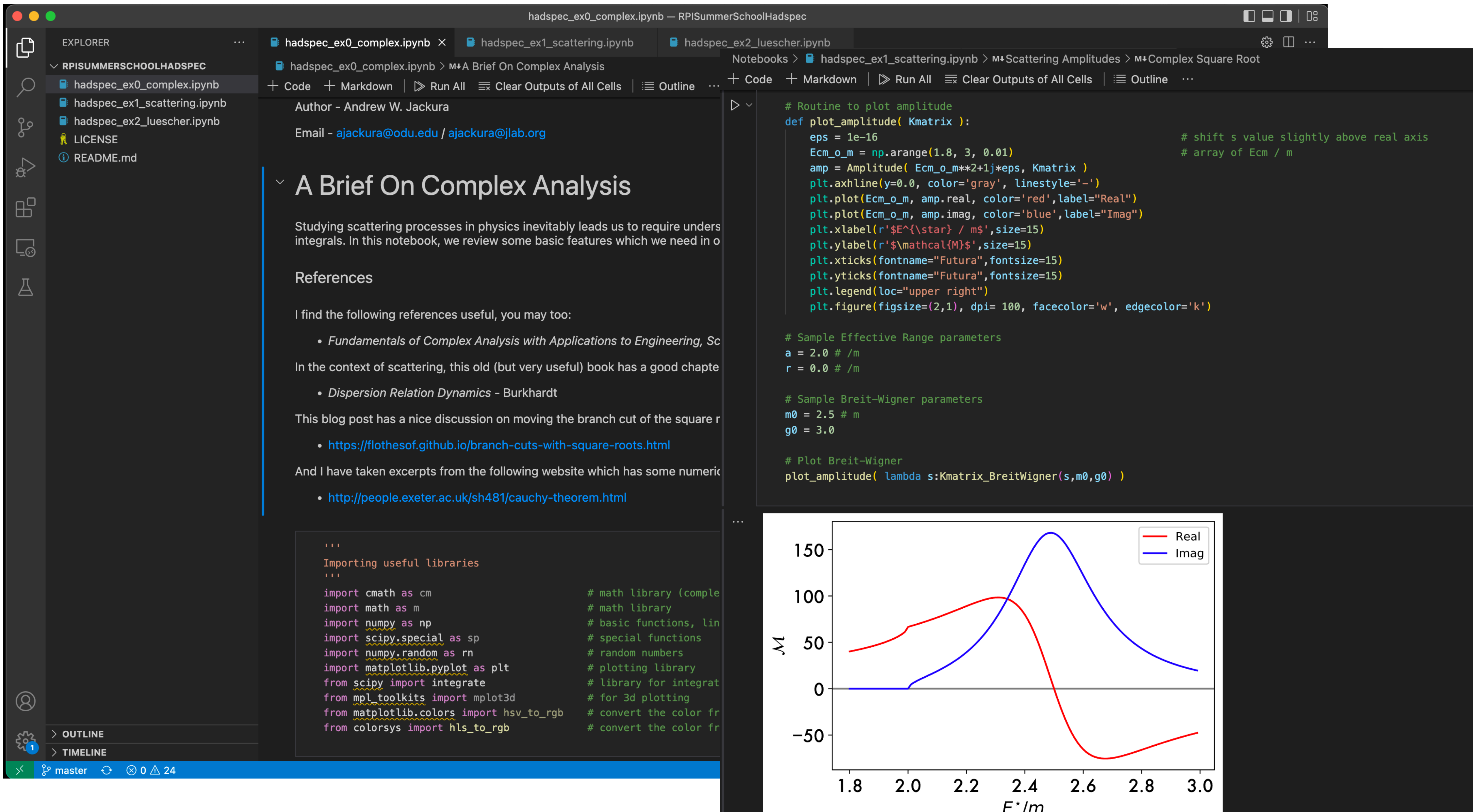
[b] *Instituto de Física Corpuscular (IFIC), Centro Mixto CSIC-Universidad de Valencia, E-46071 Valencia, Spain*
[c] *Pedagogical University of Krakow, 30-084 Kraków, Poland*
[d] *Department of Physics, Indiana University, Bloomington, IN 47405, USA*
[e] *Center for Exploration of Energy and Matter, Indiana University, Bloomington, IN 47403, USA*
[f] *Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Ciudad de México 04510, Mexico*
[g] *Departamento de Física Interdisciplinar, Universidad Nacional de Educación a Distancia (UNED), Madrid E-28040, Spain*
[h] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*
[i] *Department of Physics, Old Dominion University, Norfolk, VA 23529, USA*
[j] *Departament de Física Quàntica i Astrofísica and Institut de Ciències del Cosmos, Universitat de Barcelona, E-08028, Spain*
[k] *Departamento de Física Teórica, Universidad Complutense de Madrid and IPARCOS, E-28040 Madrid, Spain*
[l] *ORIGINS Excellence Cluster, 80939 Munich, Germany*
[m] *Ludwig-Maximilian University of Munich, Germany*
[n] *Dipartimento di Scienze Matematiche e Informatiche, Scienze Fisiche e Scienze della Terra, Università degli Studi di Messina, I-98122 Messina, Italy*
[o] *INFN Sezione di Catania, I-95123 Catania, Italy*
[p] *College of William & Mary, Williamsburg, VA 23187, USA*
[q] *Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China*
[r] *Guangdong-Hong Kong Joint Laboratory of Quantum Matter, Southern Nuclear Science Computing Center, South China*

Python Notebooks

I have created a few Jupyter notebooks to accompany the lectures

- All notebooks use ***python***, and the modules that come with the [Anaconda package manager]()
- Includes introductory and advanced exercises
- [https://github.com/ajackura/RPISummerSchoolHadspec]()



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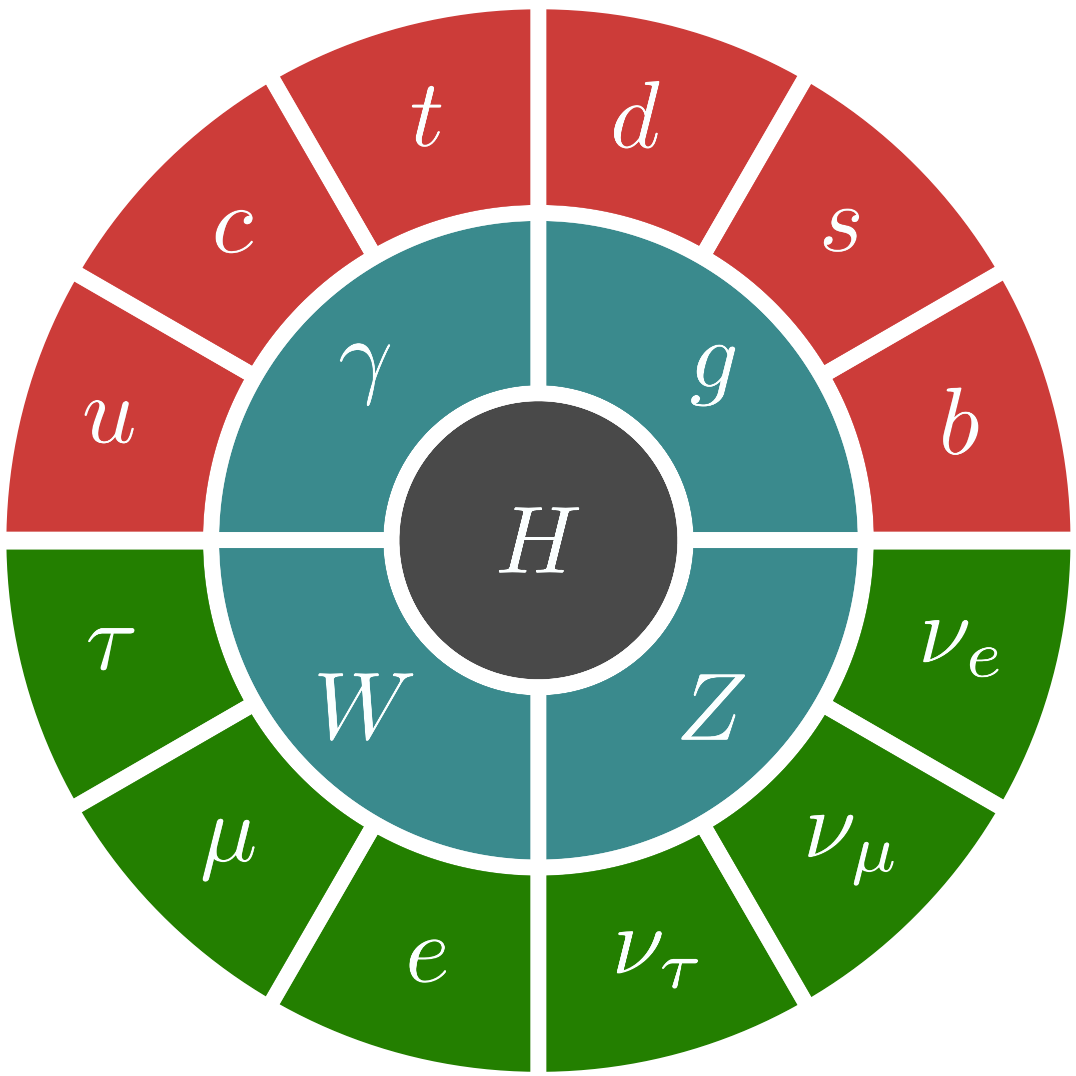
The Standard Model of Particle Physics

Nature can be described by a remarkably *simple** theory



Matter		*Force Carriers*	
	Quarks		**Gauge Bosons**
	Leptons		**Higgs Boson**

* *simple = An anomaly-free renormalizable relativistic quantum gauge field theory, invariant under the gauge group* $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ *which spontaneously breaks via a scalar field to* $SU(3)_C \otimes U(1)_Q$

The Standard Model of Particle Physics

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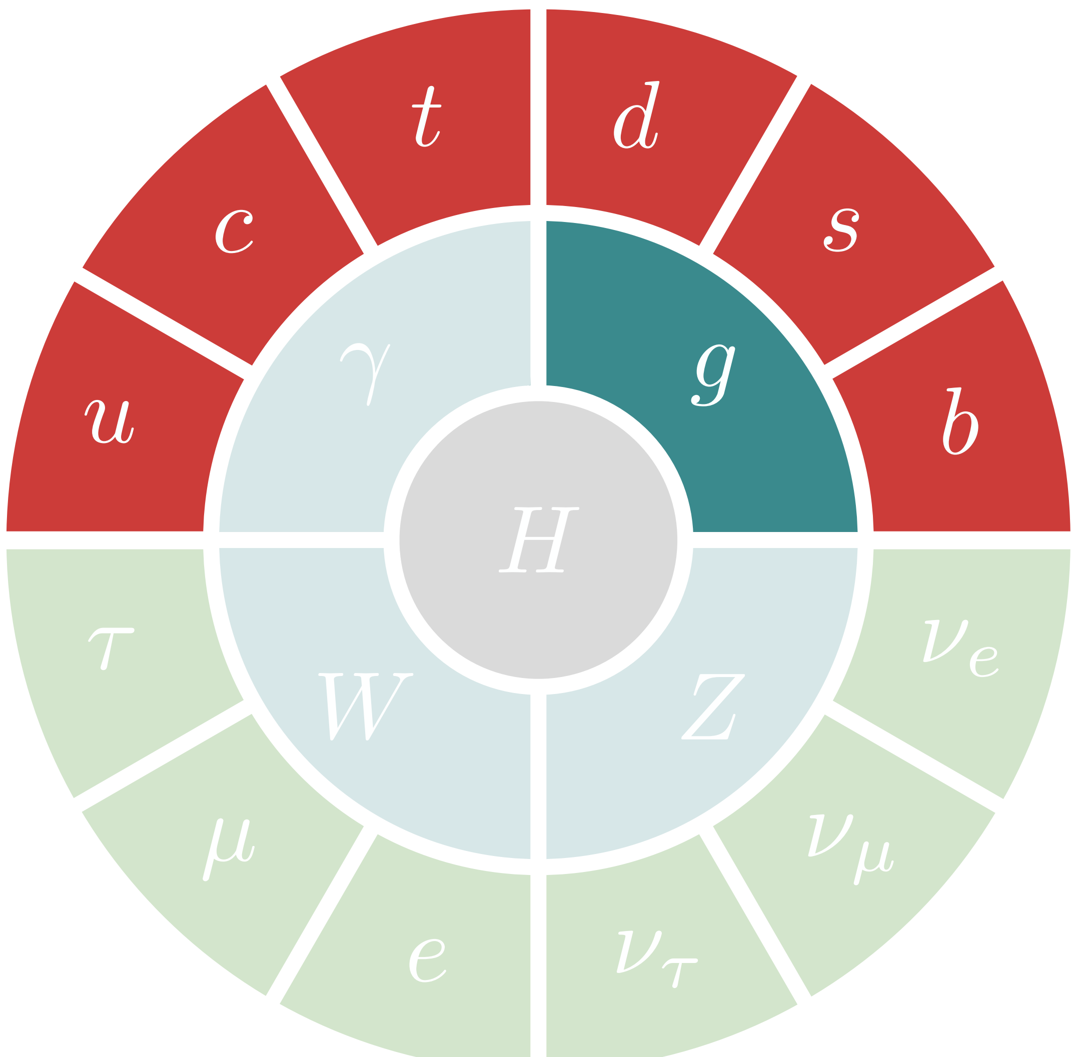
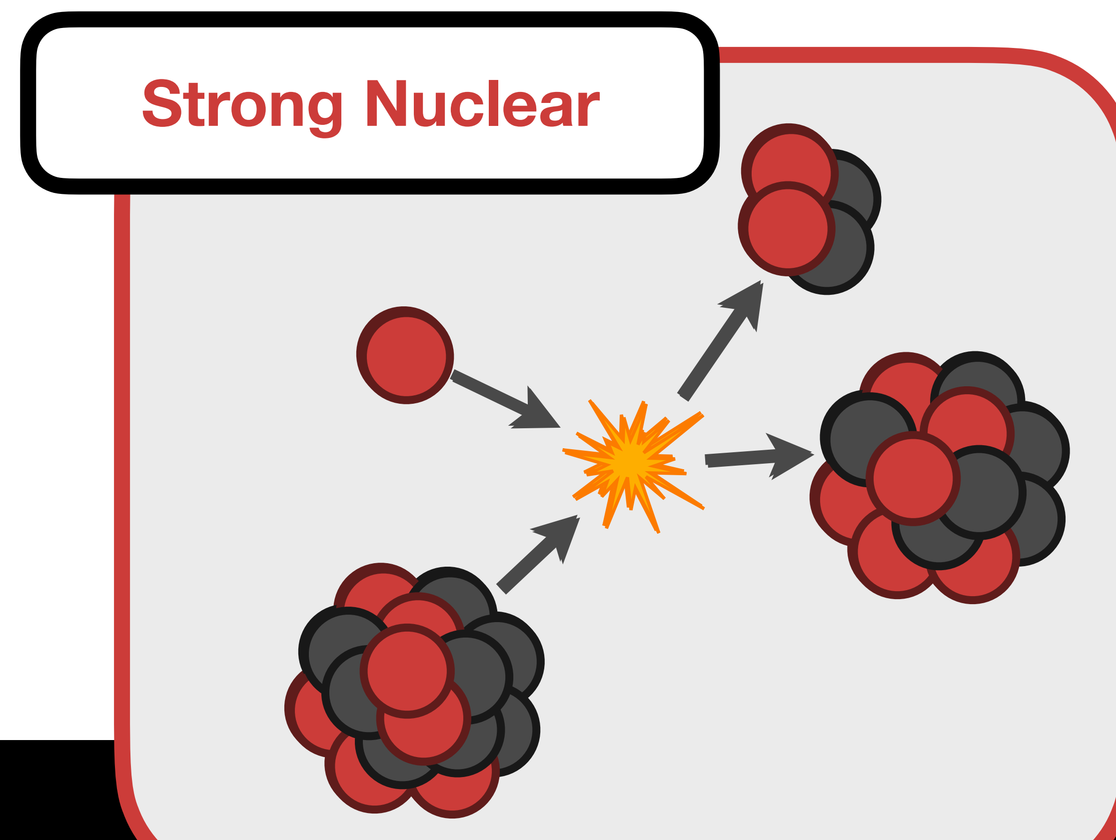


Matter | *Force Carriers*

Quarks | Gauge Bosons

Leptons | Higgs Boson



Quantum ChromoDynamics (QCD)

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invariant under the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
which spontaneously breaks via a scalar field to $SU(3)_C \otimes U(1)_Q$

Quantum ElectroDynamics (QED)

Theory of electron-photon interactions

photon fields

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\left(i\not{D} - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

lepton fields

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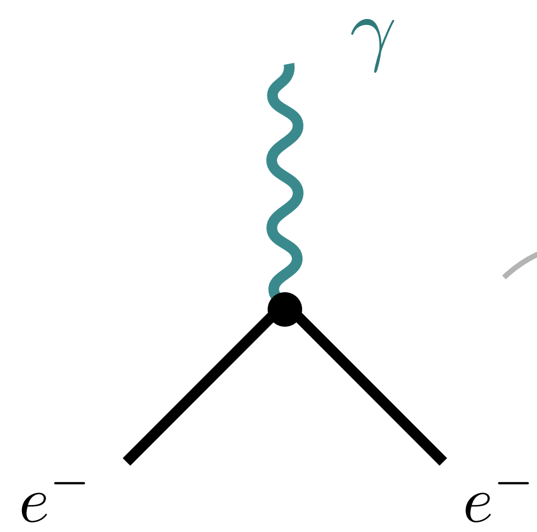

electromagnetic coupling

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

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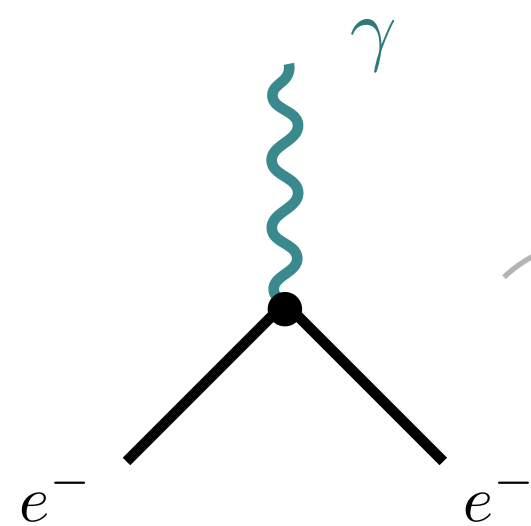

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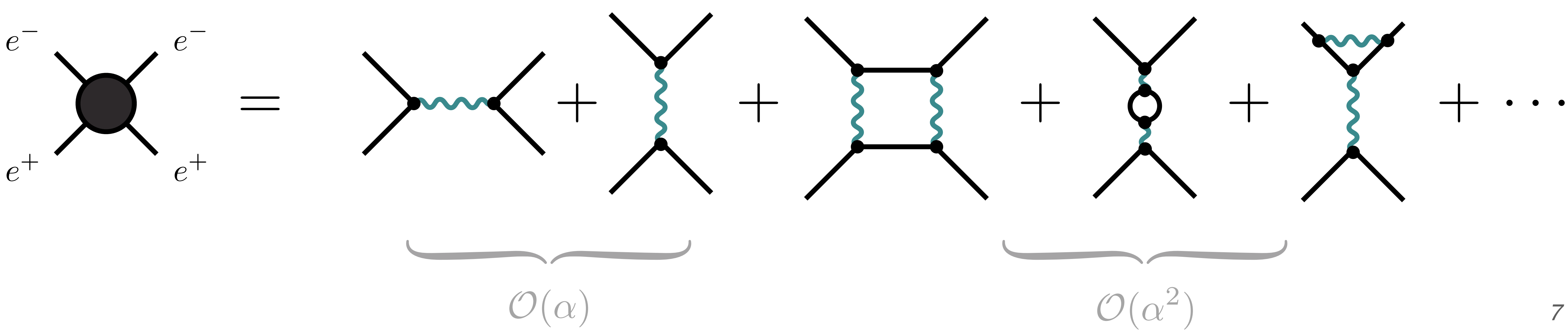
Can compute observables ***perturbatively*** in electric coupling



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e^-

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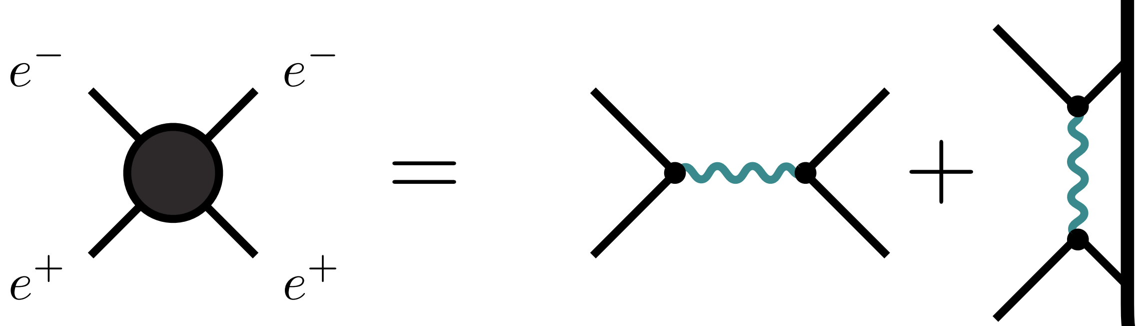
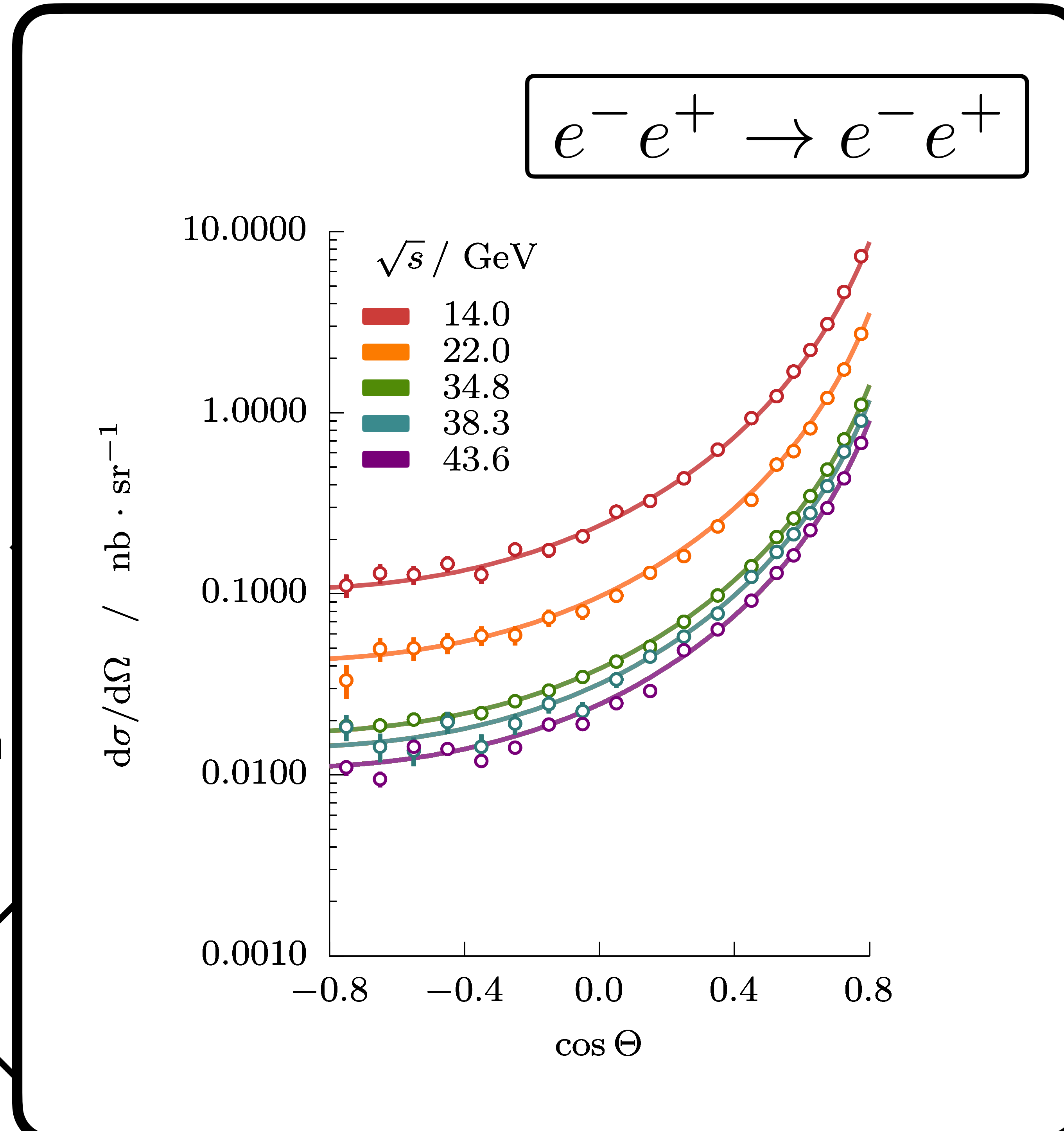




$\mathcal{O}(\alpha)$ $\mathcal{O}(\alpha^2)$

Quantum ChromoDynamics (QCD)

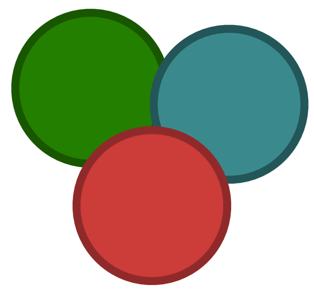
Theory of quark-gluon interactions

gluon fields

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\boldsymbol{\psi}}_f \left(i\not{\mathbf{D}} - m_f\right) \boldsymbol{\psi}_f - \frac{1}{2}\mathrm{tr}\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right)$$

quark fields

color charge



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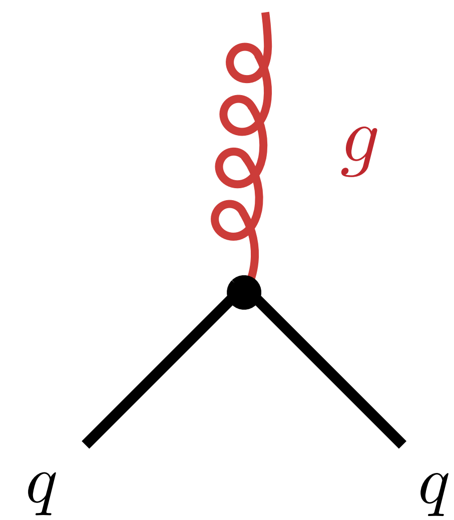
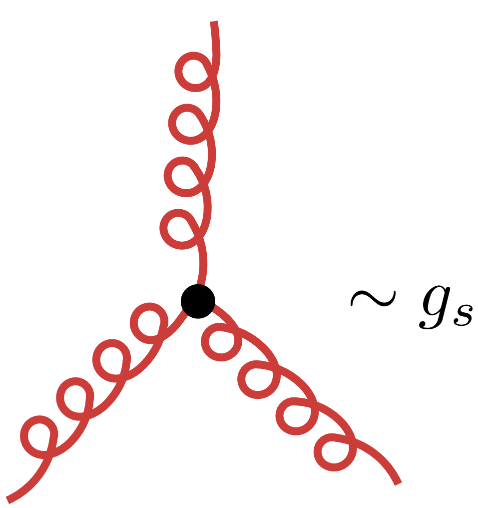
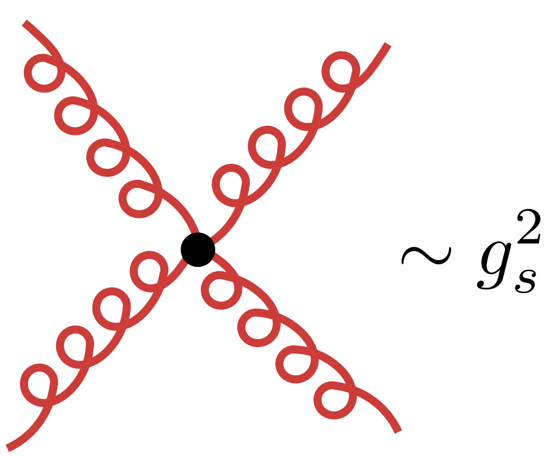


strong coupling

$$\alpha_s = \frac{g_s^2}{4\pi}$$





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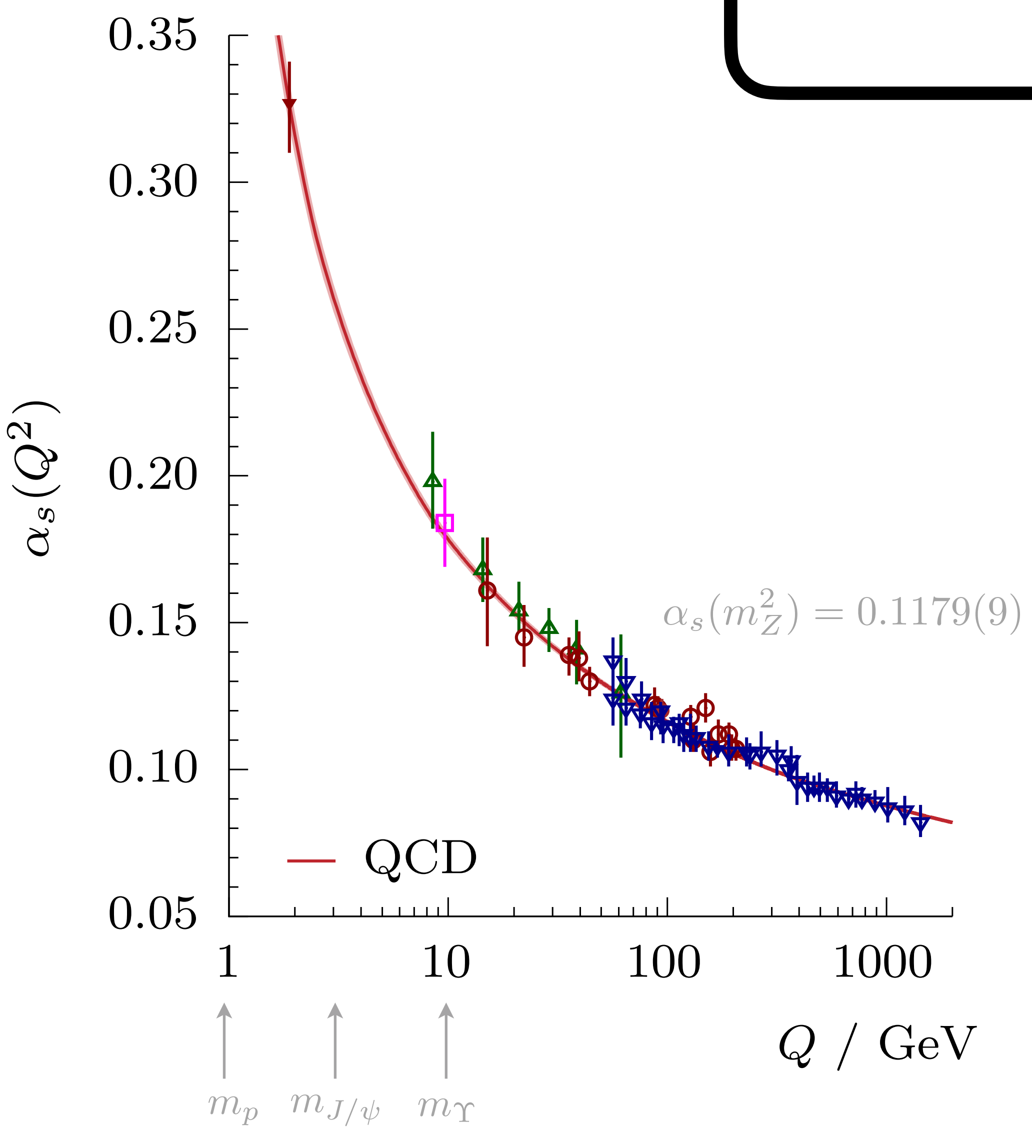
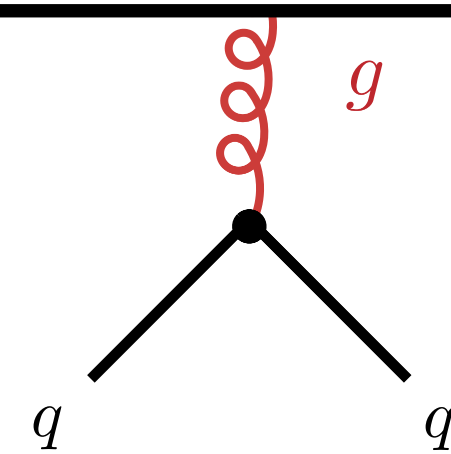
At low-energy we don't observe free quarks…
…we observe composite ***hadrons***





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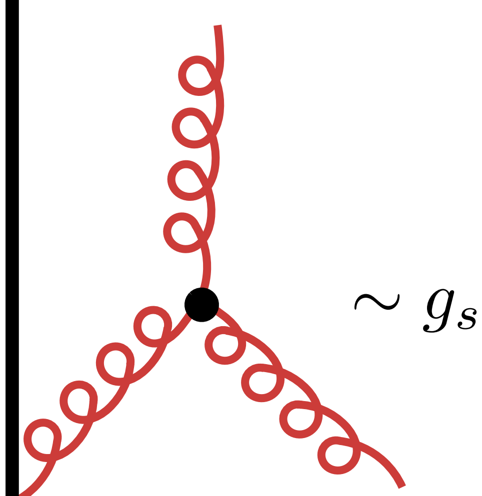
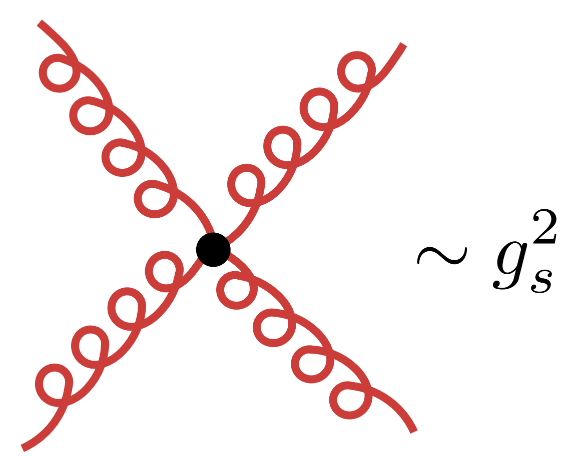
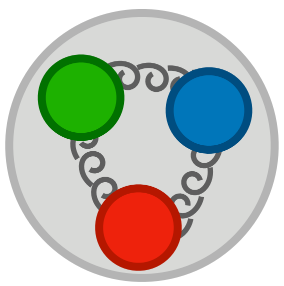




Quantum ChromoDynamics (QCD)

Hadrons are the *color-neutral* bound states of quarks & gluons (***confinement***)

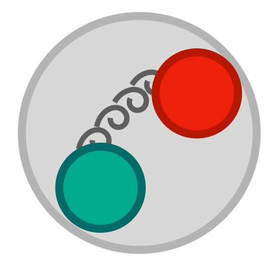
Baryons (fermions)



$$J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$$

$$p, n, \Delta, \ldots$$

Mesons (bosons)



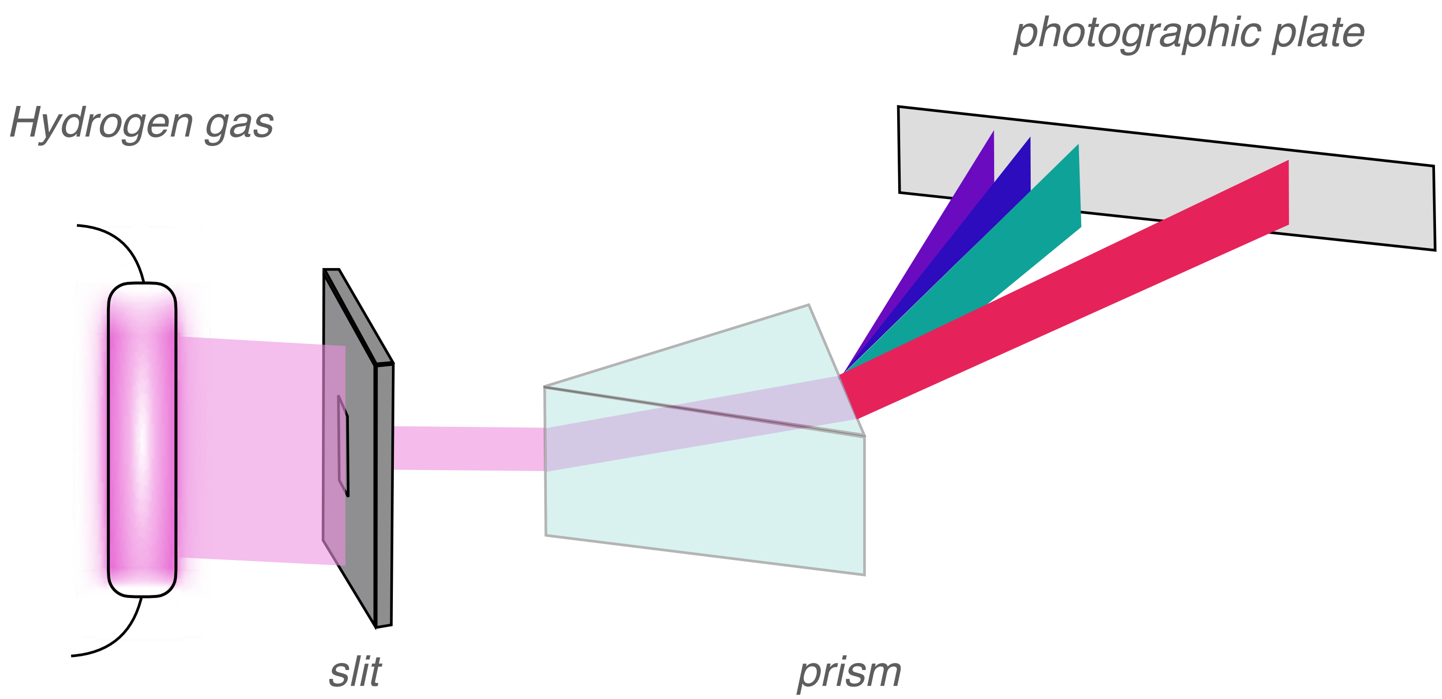
$$J = 0, 1, 2, \ldots$$

$$\pi, K, \eta, \ldots$$

Spectroscopy

Spectroscopy provides vital information towards understanding physical phenomena

- Example — the Hydrogen atom led to the discovery of Quantum Mechanics



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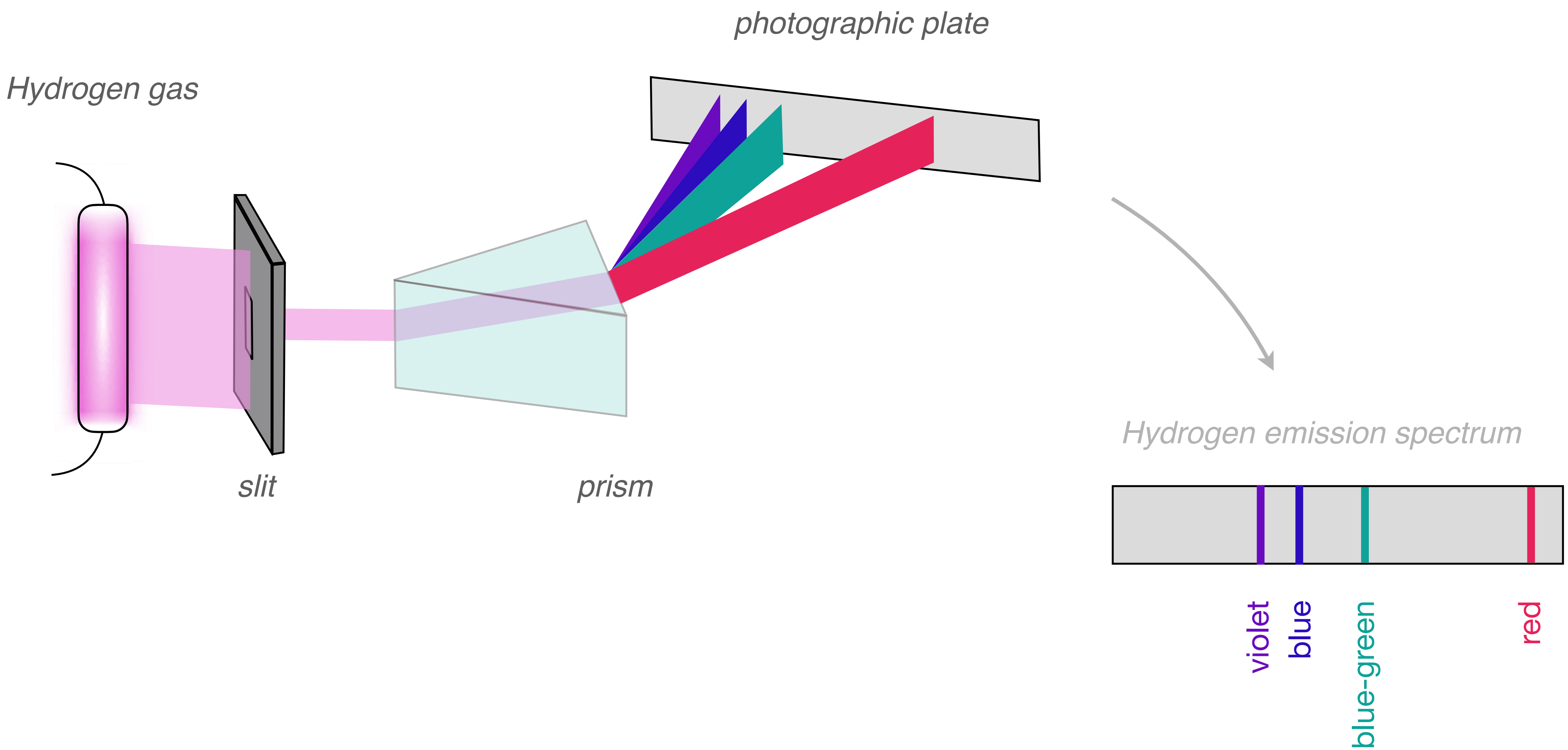

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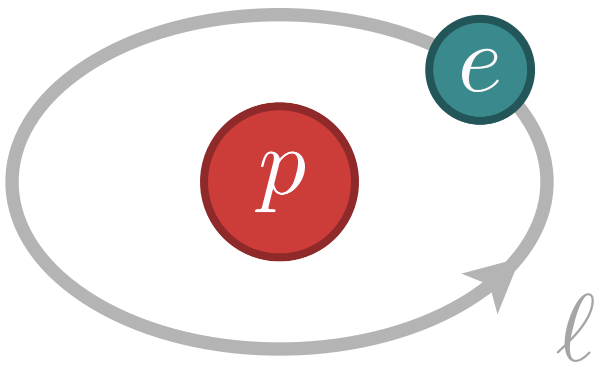

$$\widehat{\mathrm{H}}\,|\psi\rangle = E_n\,|\psi\rangle$$

Schrödinger equation

Spectroscopy

$\hbar = c = 1$

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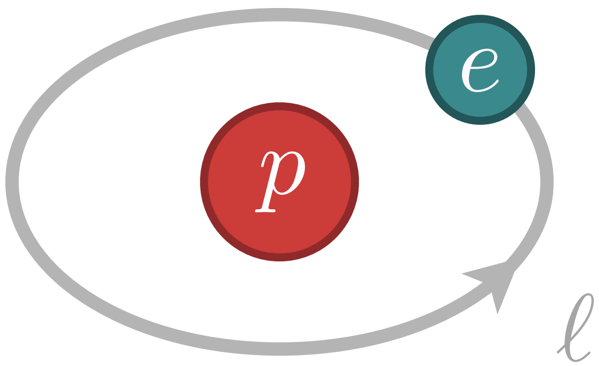
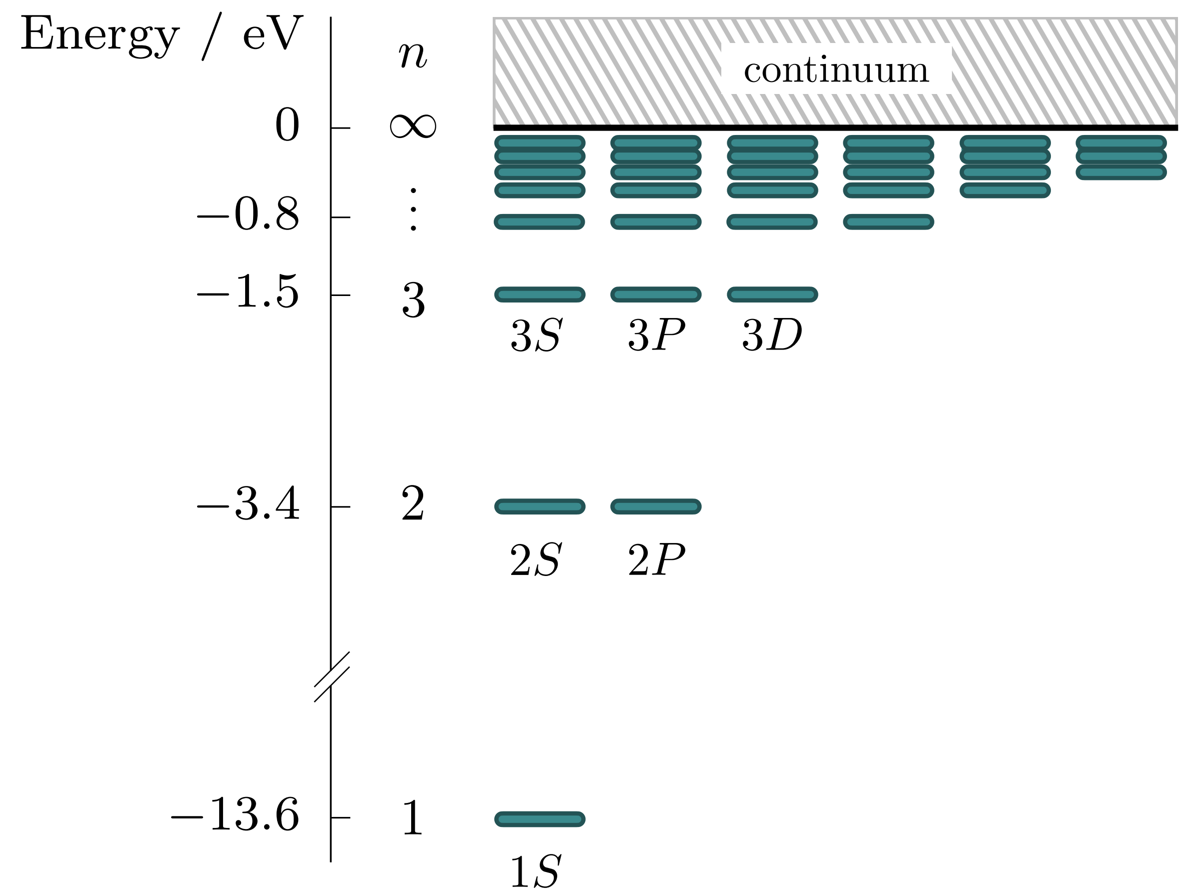


$$\widehat{\mathrm{H}}\,|\psi\rangle = E_n\,|\psi\rangle$$

spectrum

$$E_n = -\frac{m\alpha^2}{2n}$$



$\alpha = \dfrac{e^2}{4\pi}$

$\ell = 0\,(S),\; 1\,(P),\; 2\,(D),\; \ldots$

Spectroscopy

$\hbar = c = 1$

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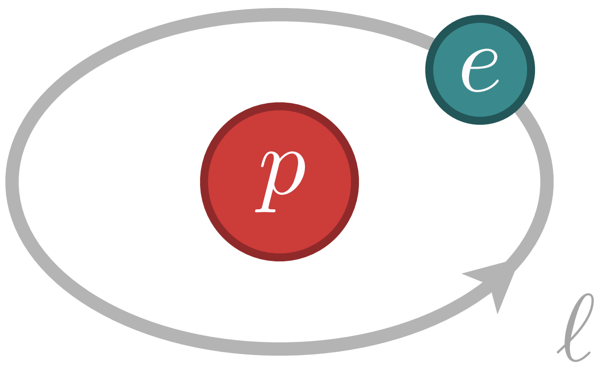

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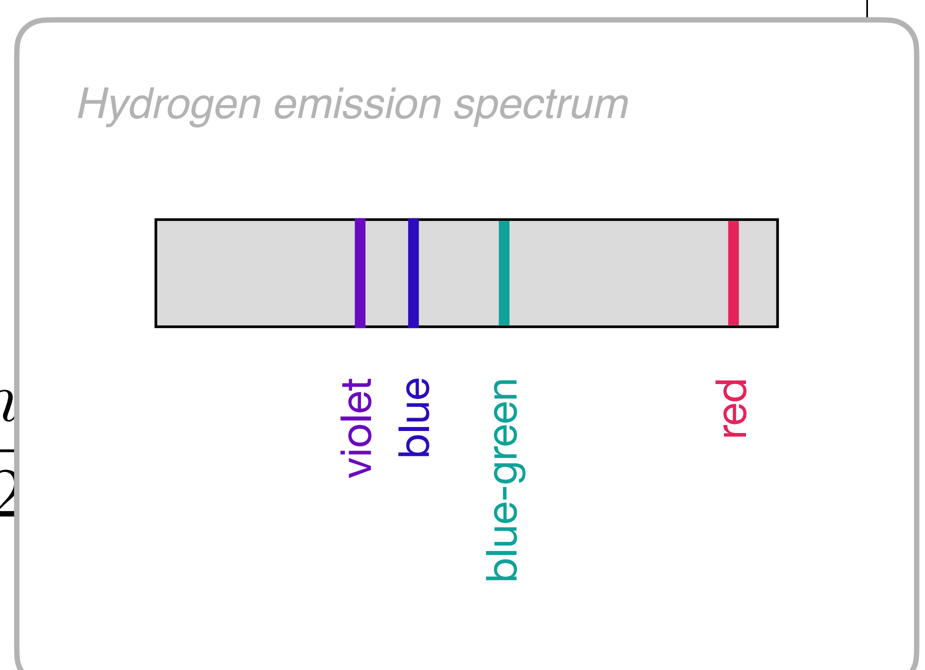

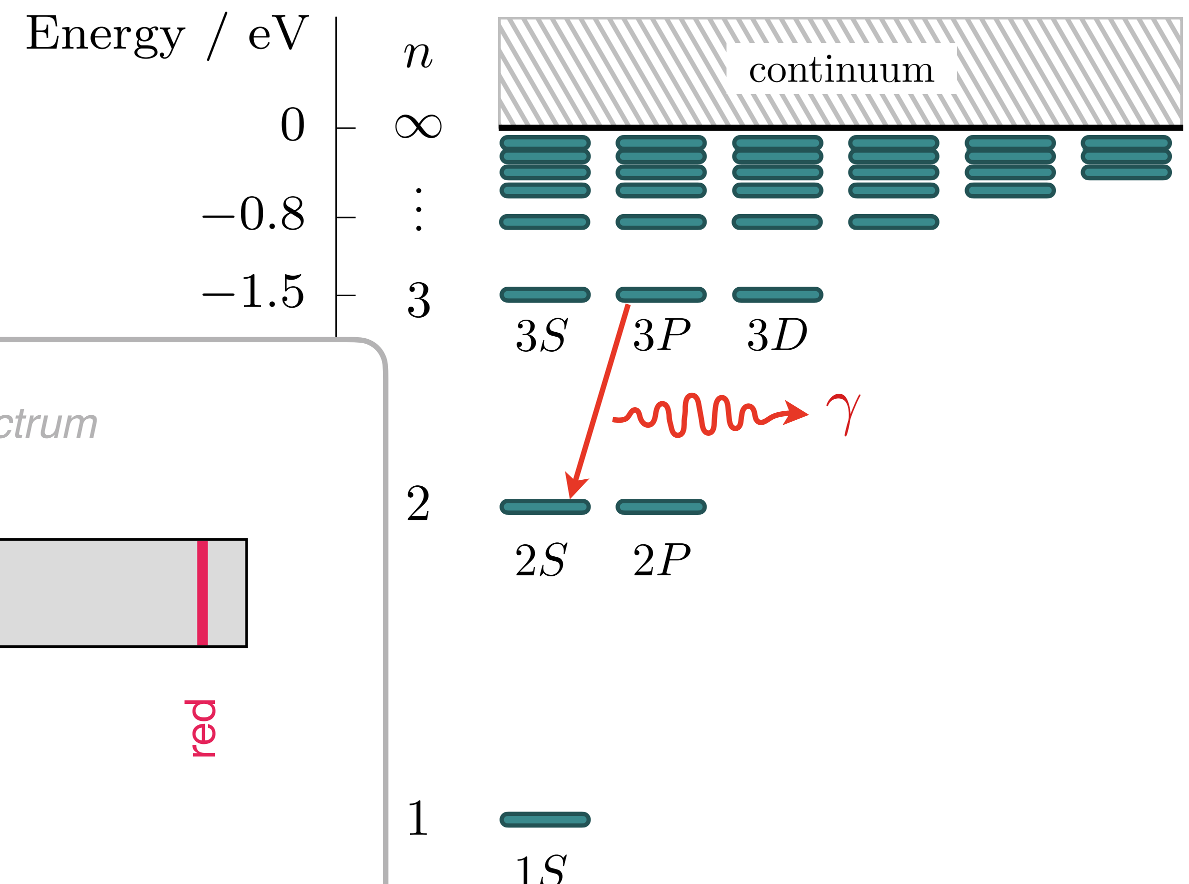

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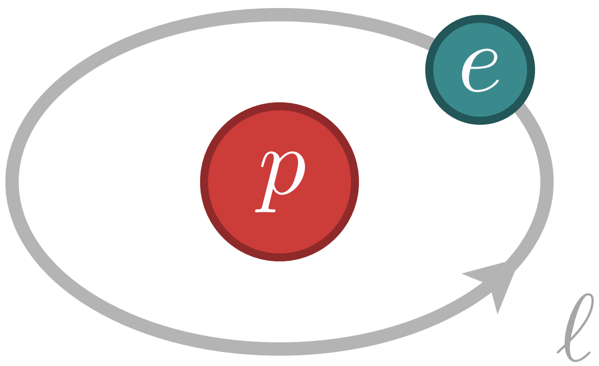
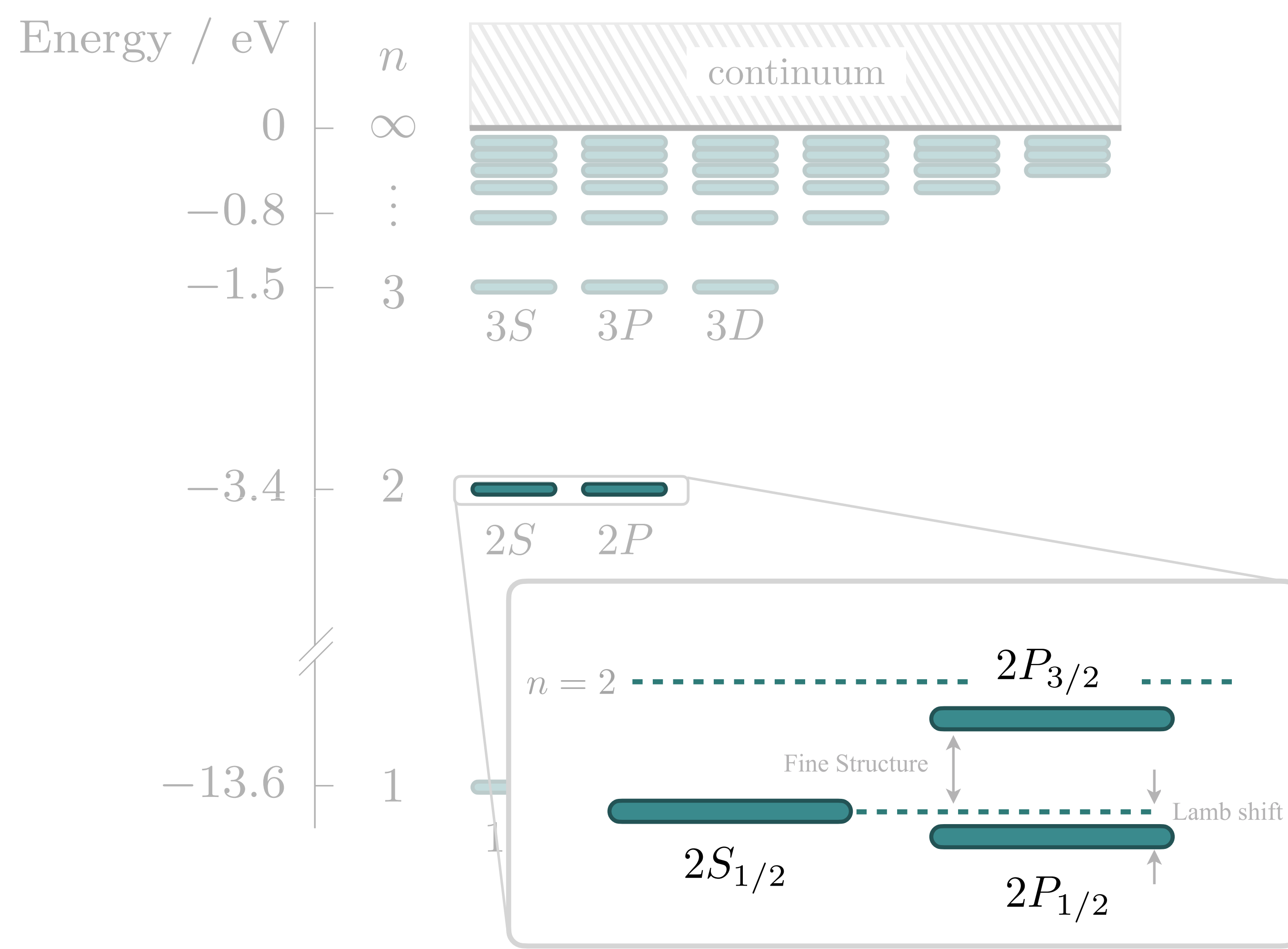


$$\widehat{\mathrm{H}}\,|\psi\rangle = E_n\,|\psi\rangle$$

$$E_n = -\frac{m\alpha^2}{2n} + \mathcal{O}\left(\alpha^4\right)$$

$$\alpha = \frac{e^2}{4\pi}$$

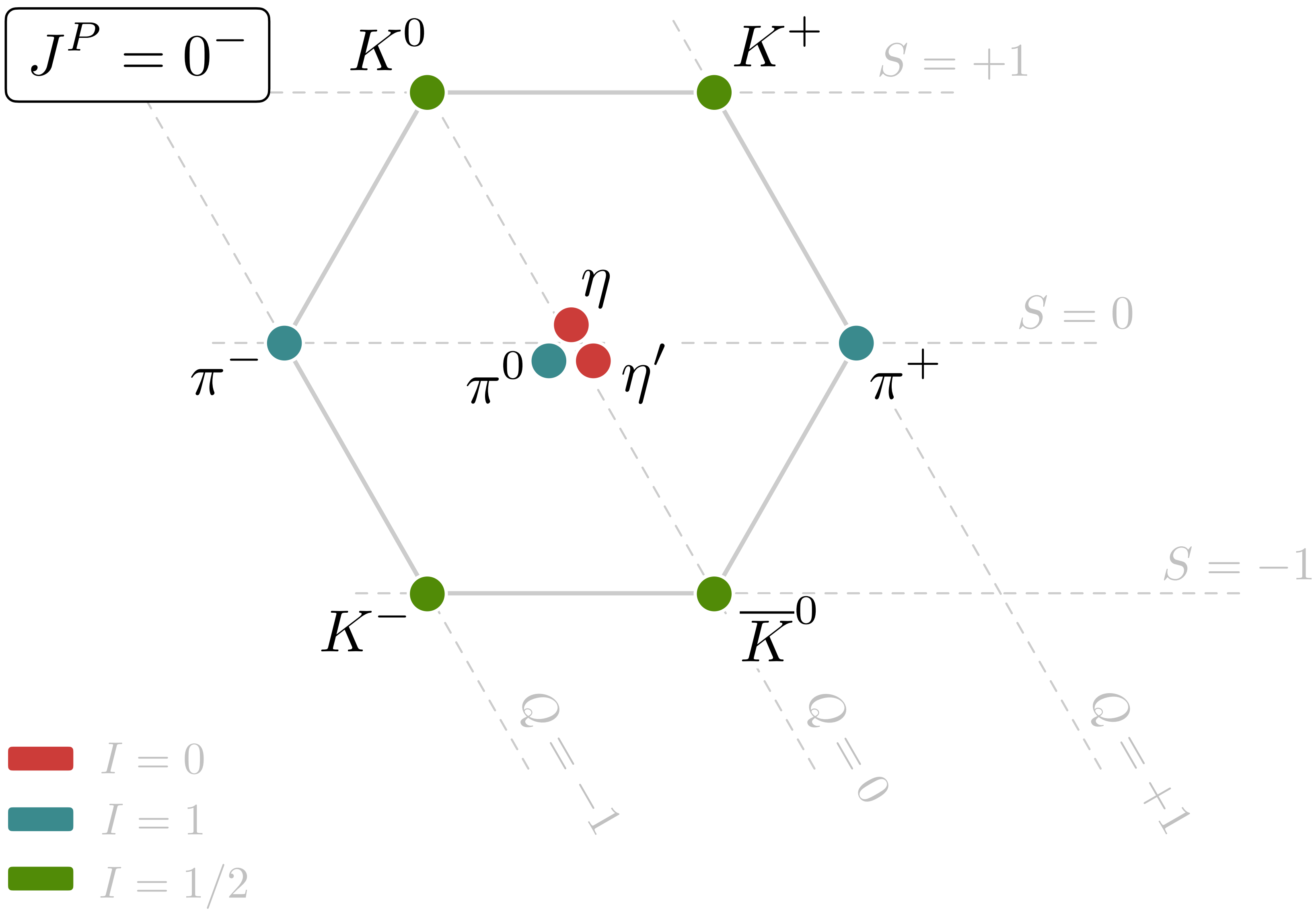


Relativistic effects, QED, …

QCD Spectroscopy

Hadrons are classified by their ***conserved quantum numbers***

- spin (J)
- parity (P)
- charge-conjugation (C)
- isospin (I)
- strangeness (S)
- ...

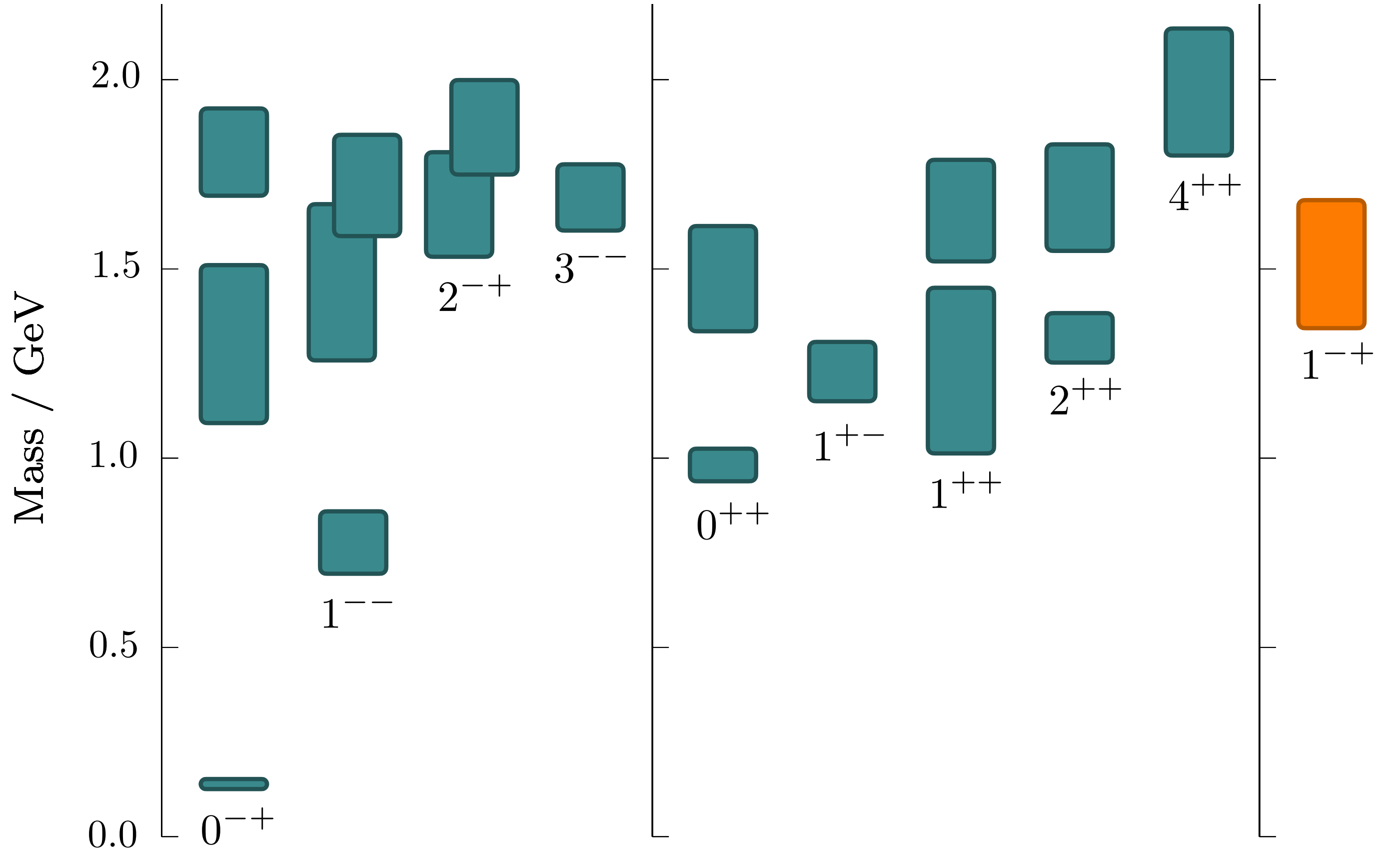


QCD Spectroscopy

Hadrons are classified by their ***conserved quantum numbers***

- Constituent quark model gives gross structure

light isovector mesons



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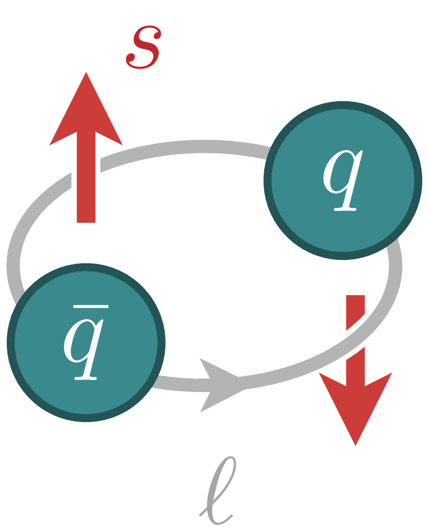
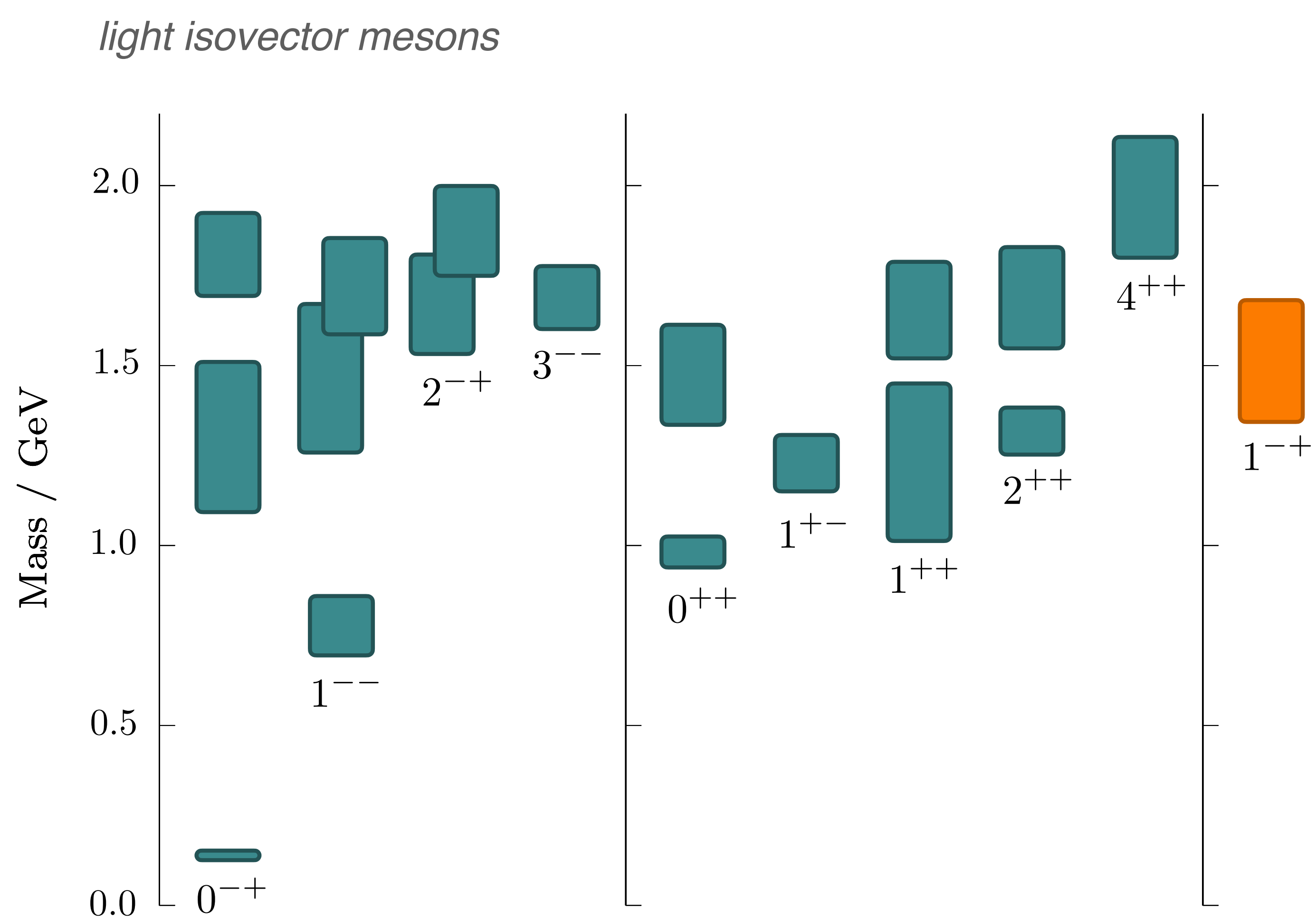


$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

	J^{PC}	
	singlet	triplet
$\ell = 0$	0^{-+}	1^{--}
$\ell = 1$	1^{+-}	$(0,1,2)^{++}$
$\ell = 2$	2^{-+}	$(1,2,3)^{--}$
$\vdots$	$\vdots$	$\vdots$



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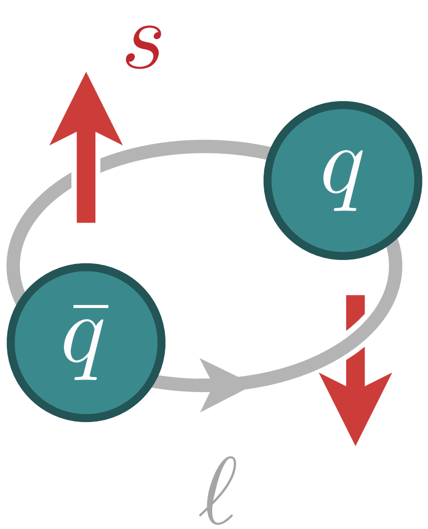

$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

	J^{PC} singlet	J^{PC} triplet
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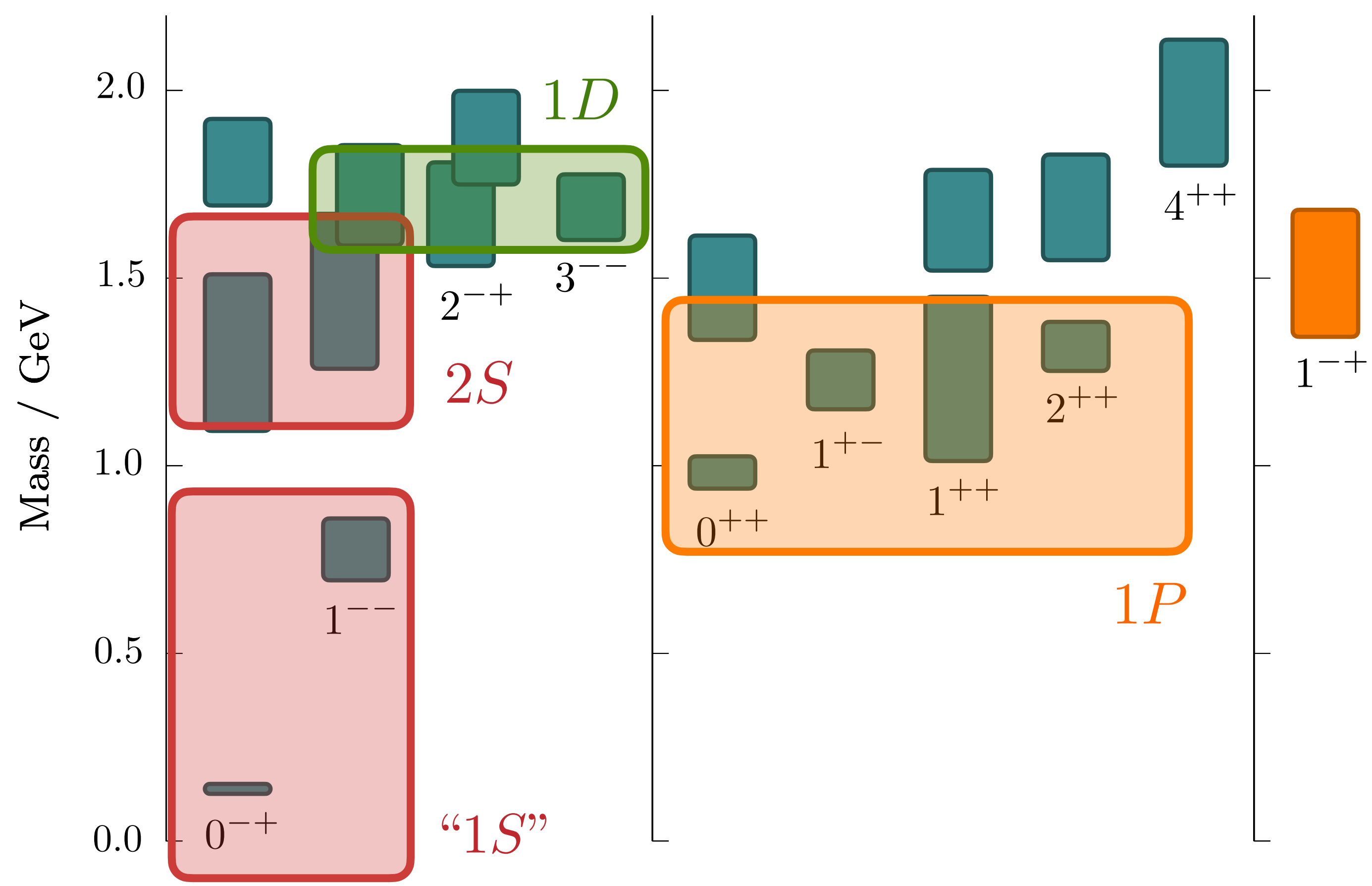
light isovector mesons



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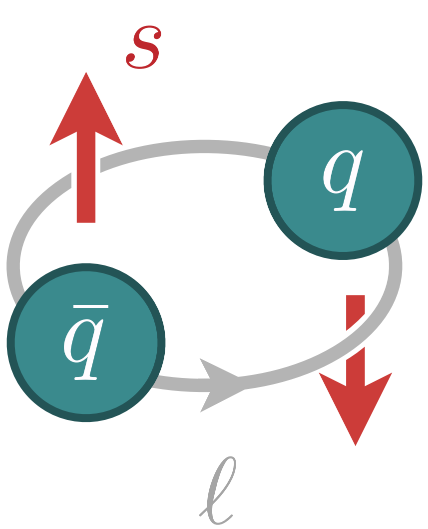

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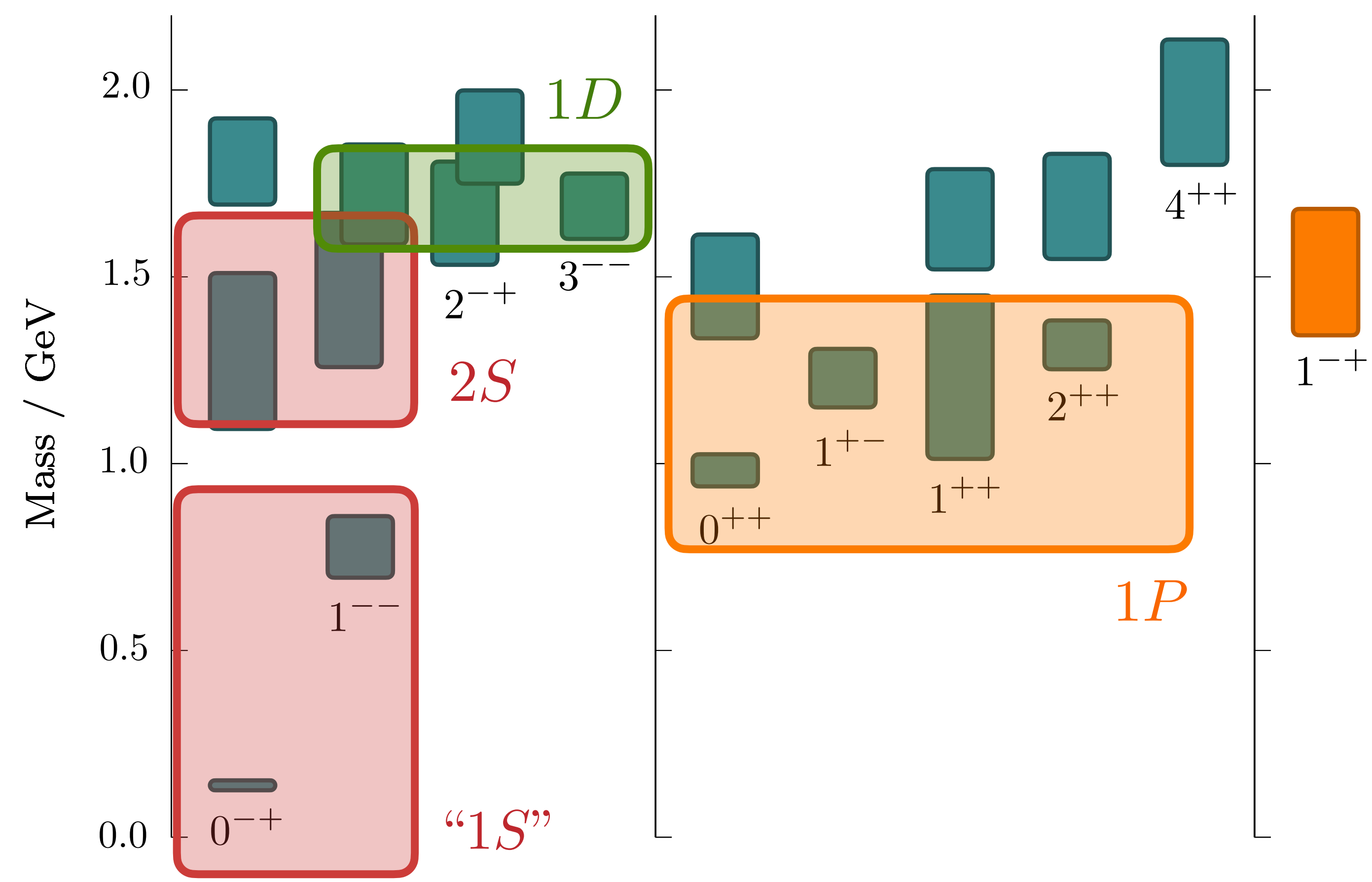


Forbidden quantum numbers : $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$

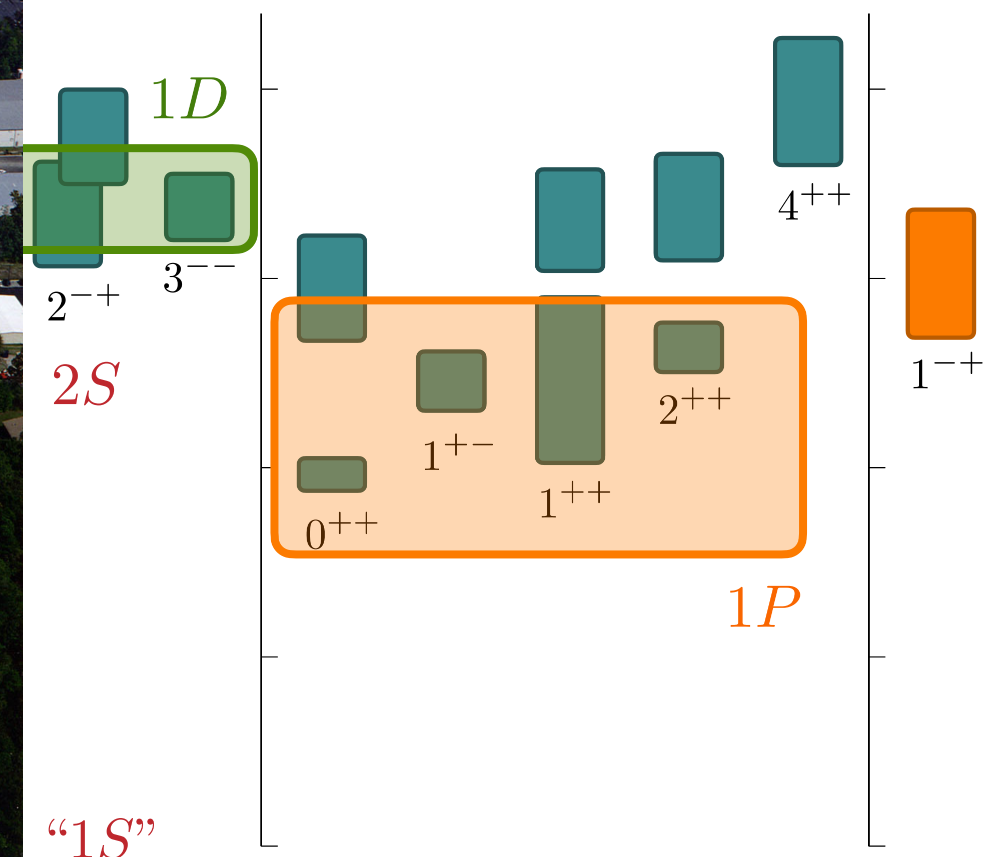
QCD Spectroscopy

Hadrons are classified by their ***conserved quantum numbers***

- Constituent quark model gives gross structure

GlueX

Jefferson Lab
Thomas Jefferson National Accelerator Facility

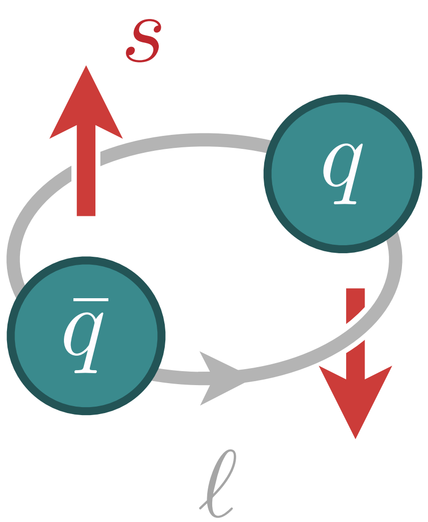


Forbidden quantum numbers : $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, \ldots$

QCD Spectroscopy

Hadrons are classified by their ***conserved quantum numbers***

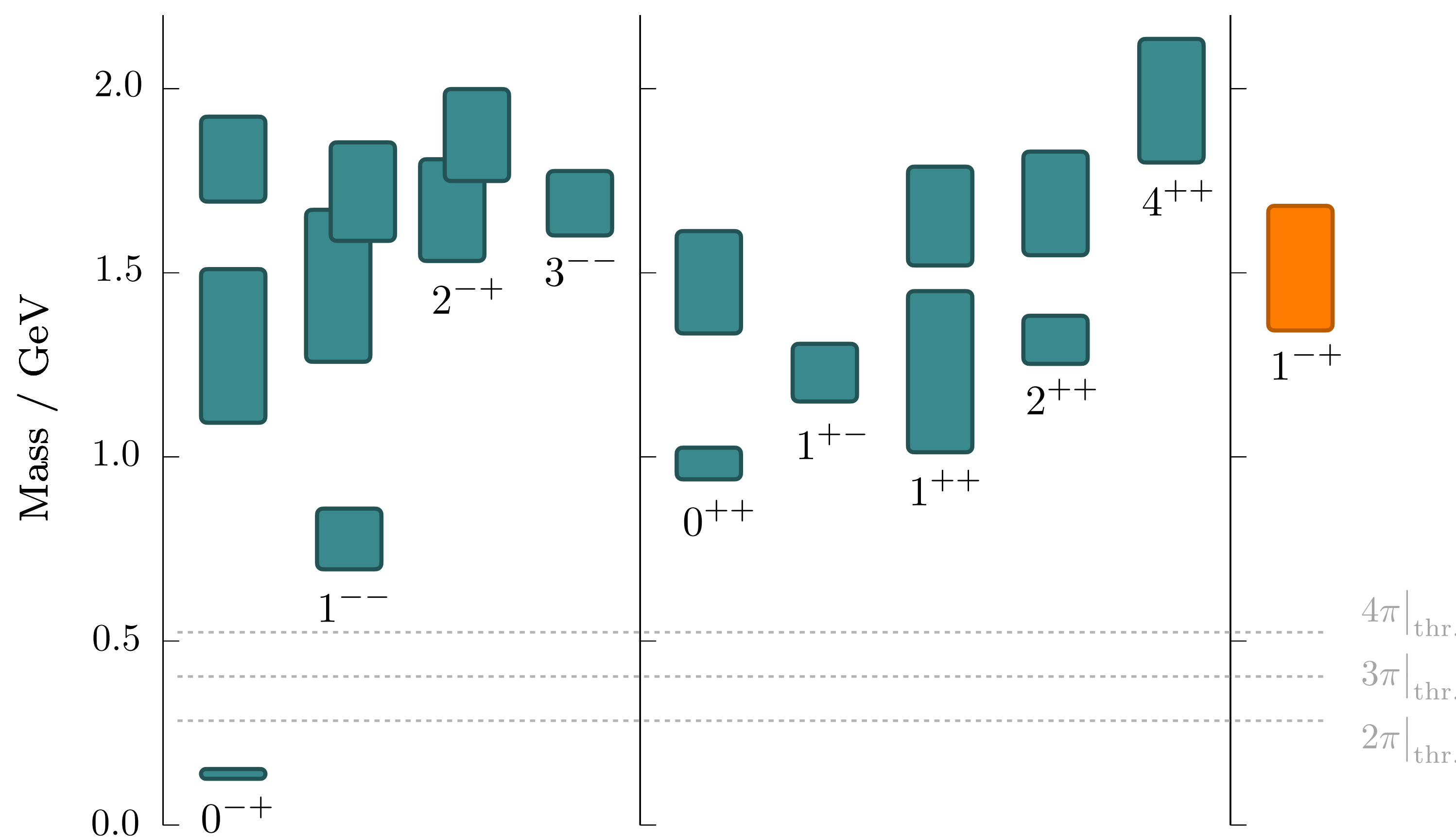
- Constituent quark model gives gross structure



$$q\bar{q}\ \left(n^{2s+1}\ell_J\right)$$

	J^{PC} singlet	J^{PC} triplet
$\ell = 0$	0^{-+}	1^{--}
$\ell = 1$	1^{+-}	$(0,1,2)^{++}$
$\ell = 2$	2^{-+}	$(1,2,3)^{--}$
$\vdots$	$\vdots$	$\vdots$

light isovector mesons



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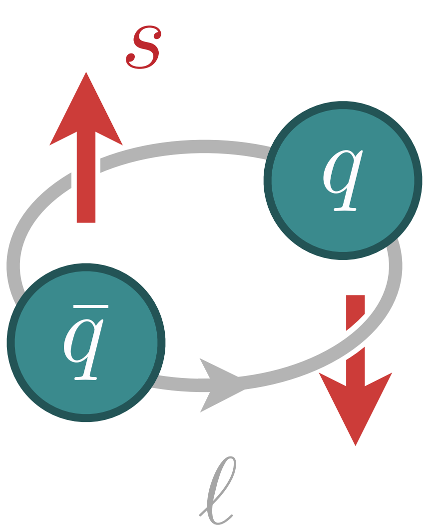

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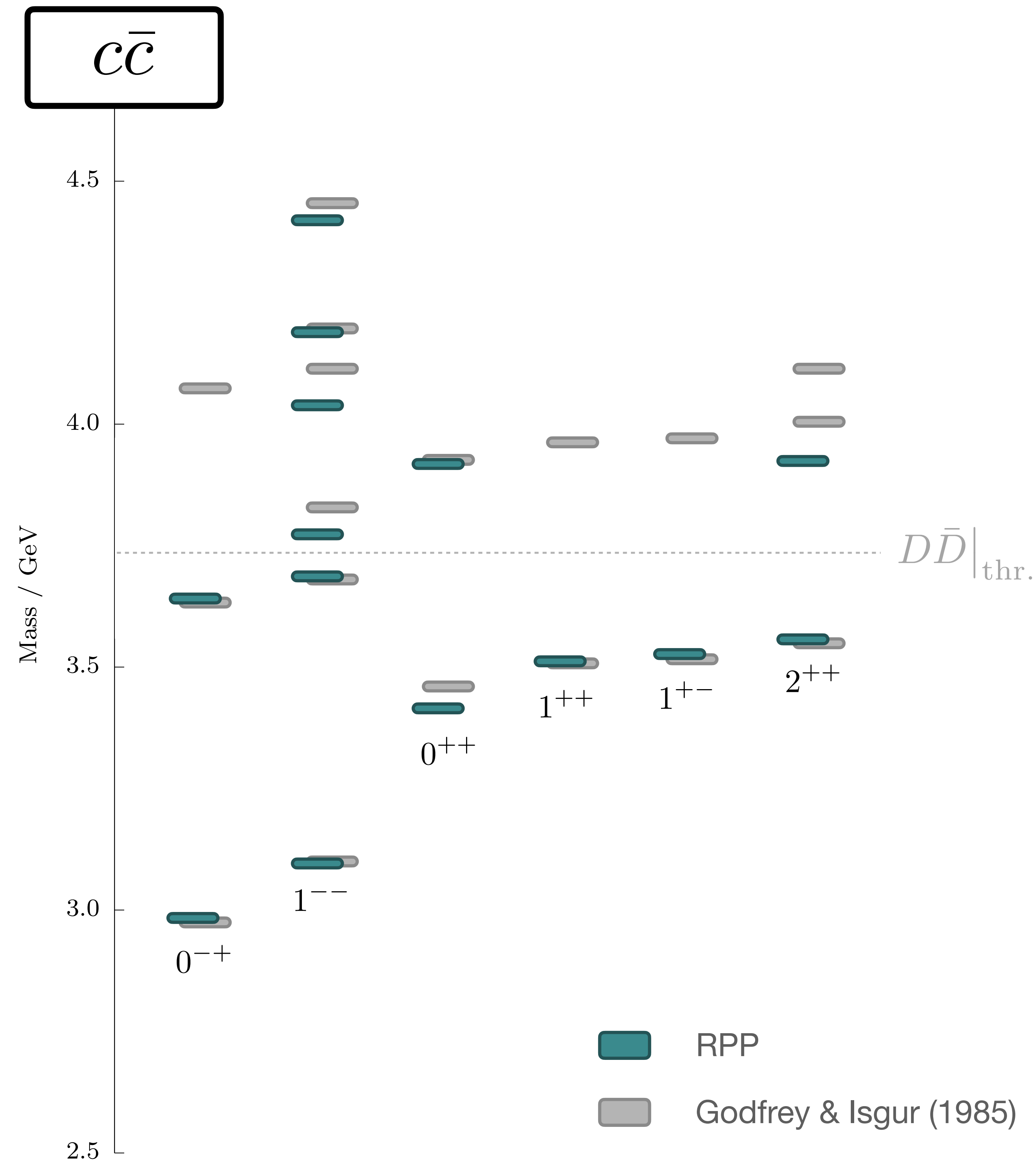

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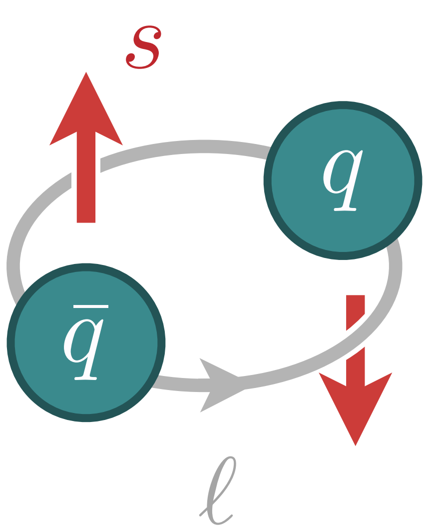

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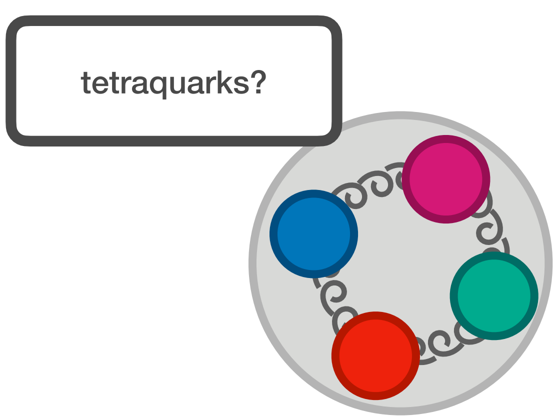
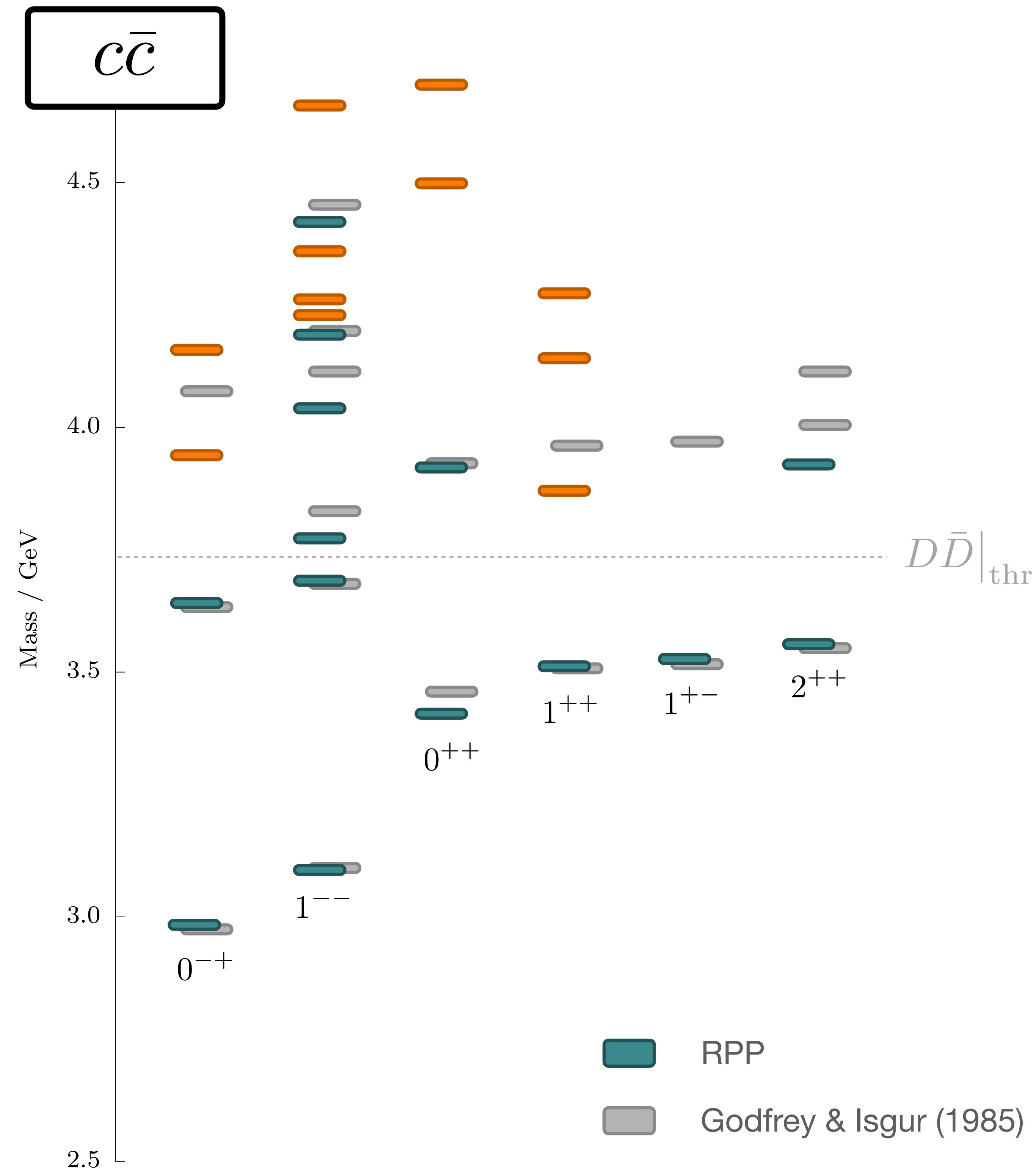
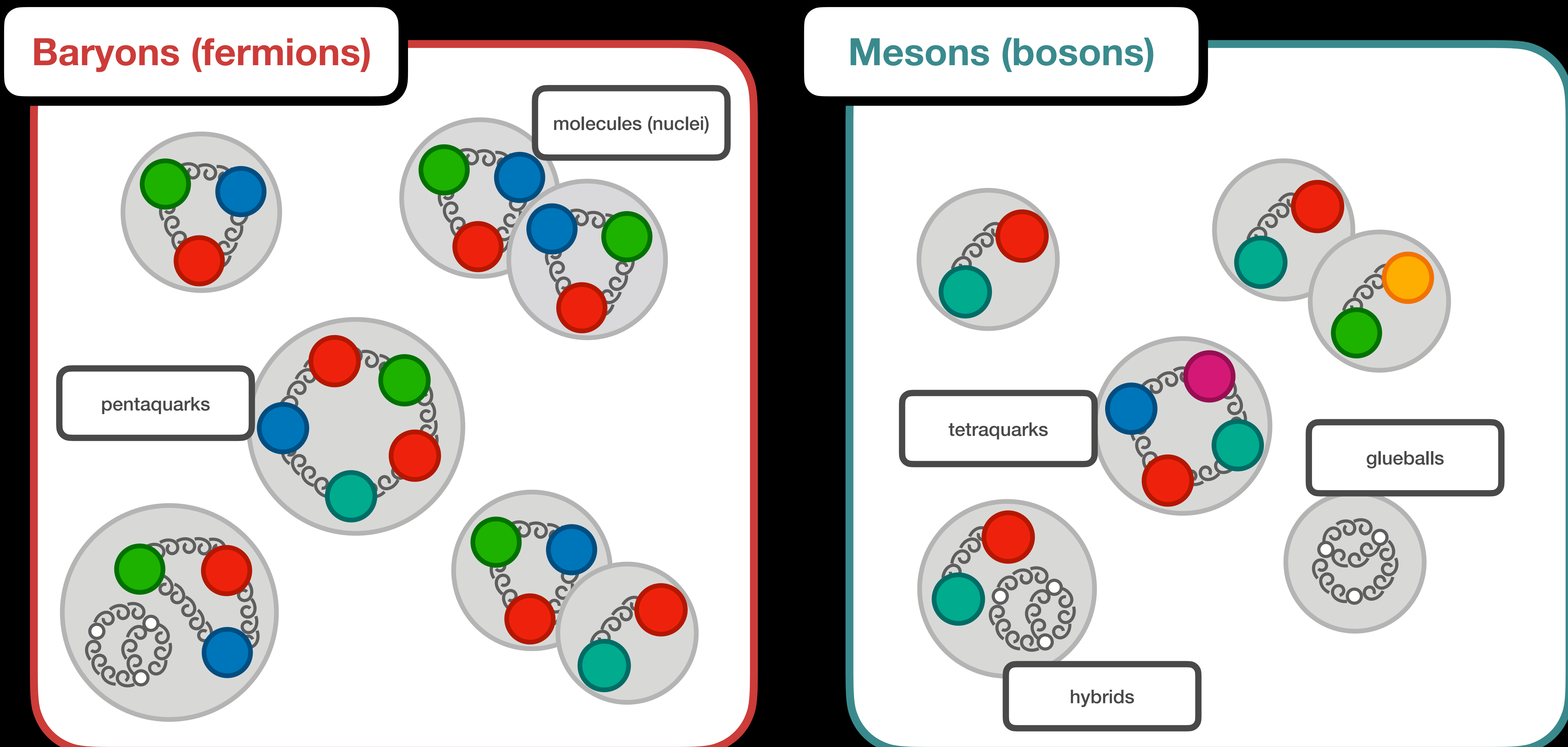




A new QCD Spectroscopy

Can we understand these hadrons from QCD?



QCD Spectroscopy

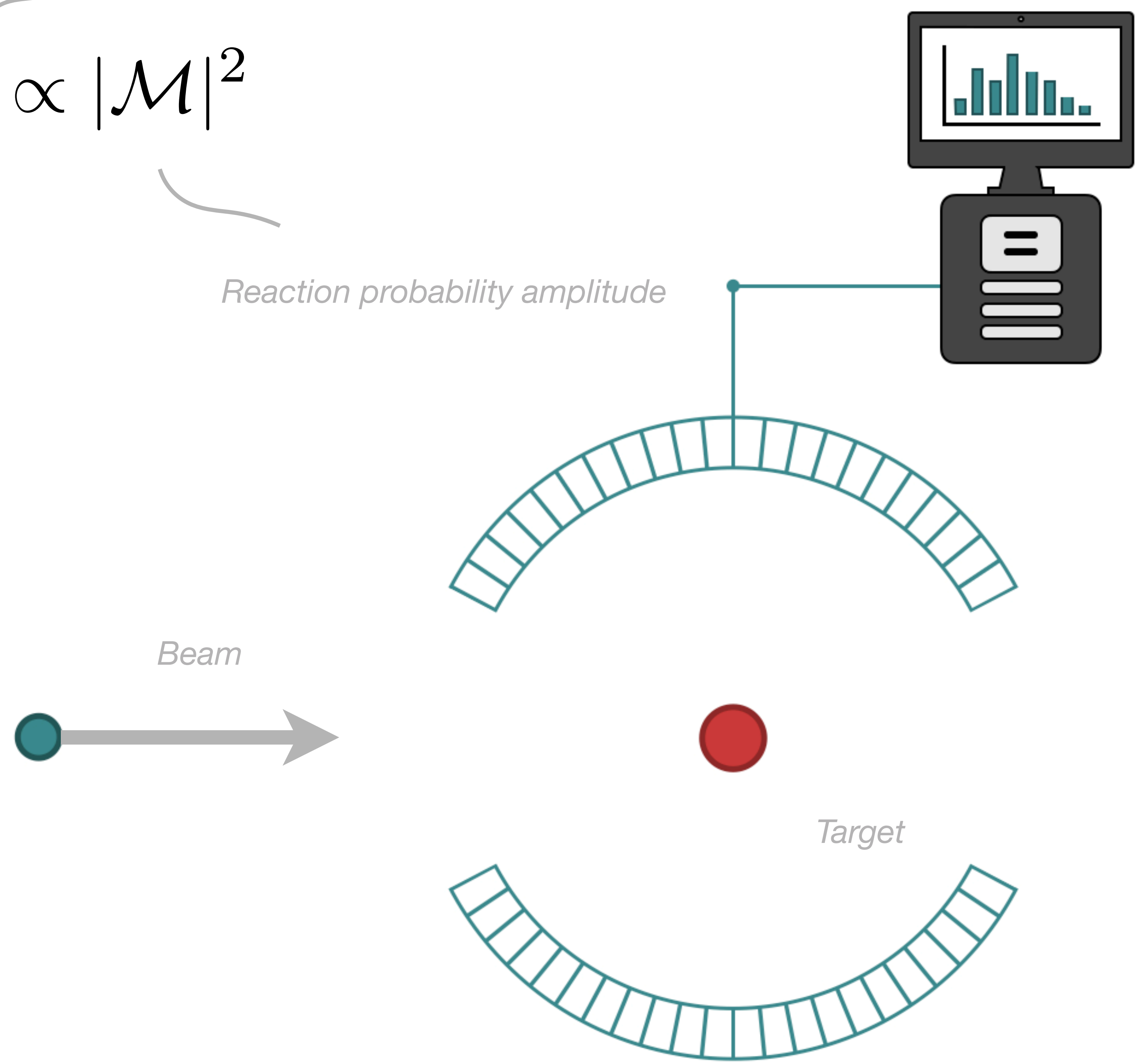
We observe strongly interacting hadrons, through reactions in accelerators/colliders

Cross-section

$$N \propto \sigma \propto |\mathcal{M}|^2$$

Events experimentally measured

Reaction probability amplitude



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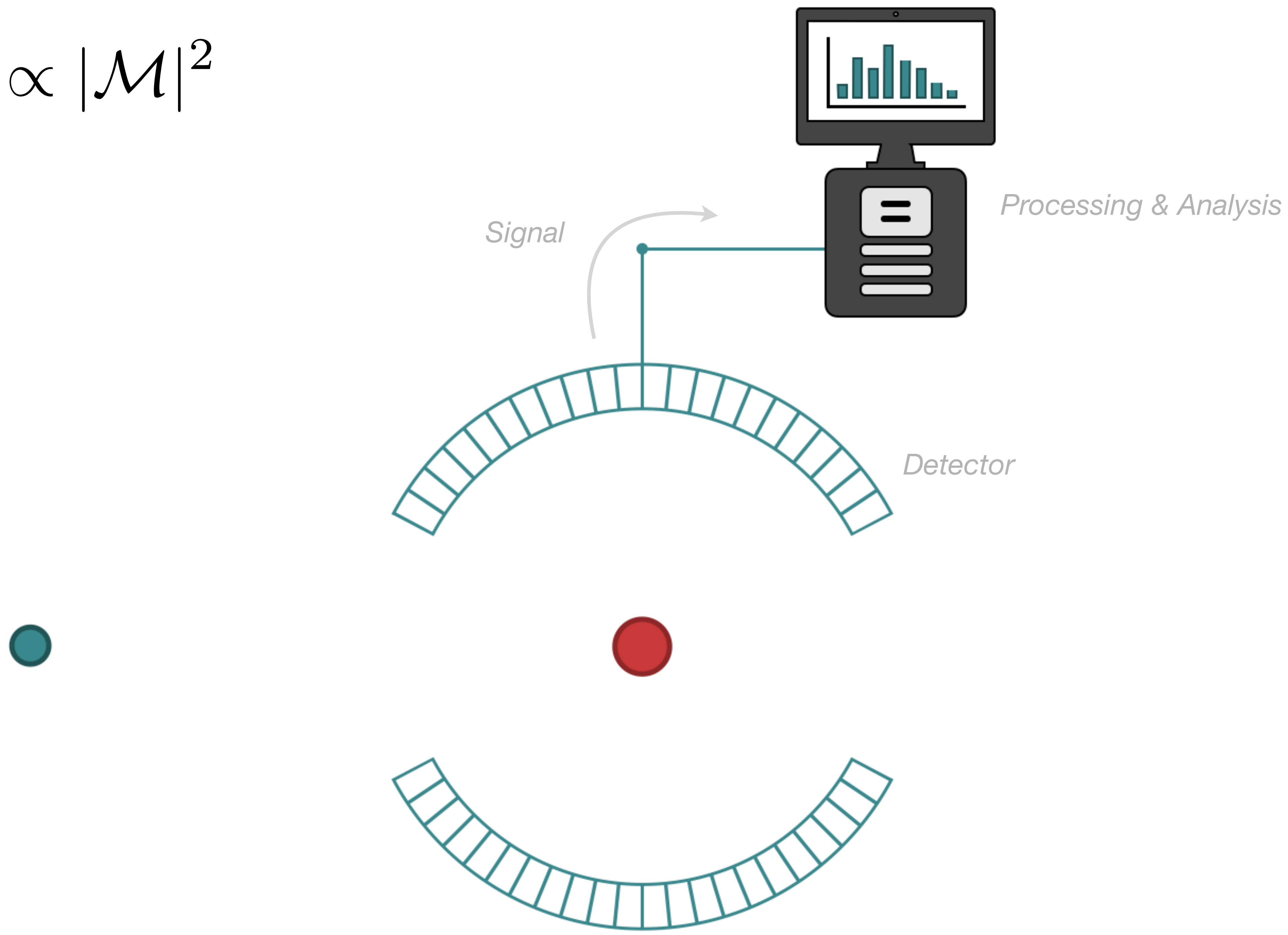

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We observe strongly interacting hadrons, through reactions in accelerators/colliders

- If the interaction is sufficiently attractive, particles can form a ***resonance***

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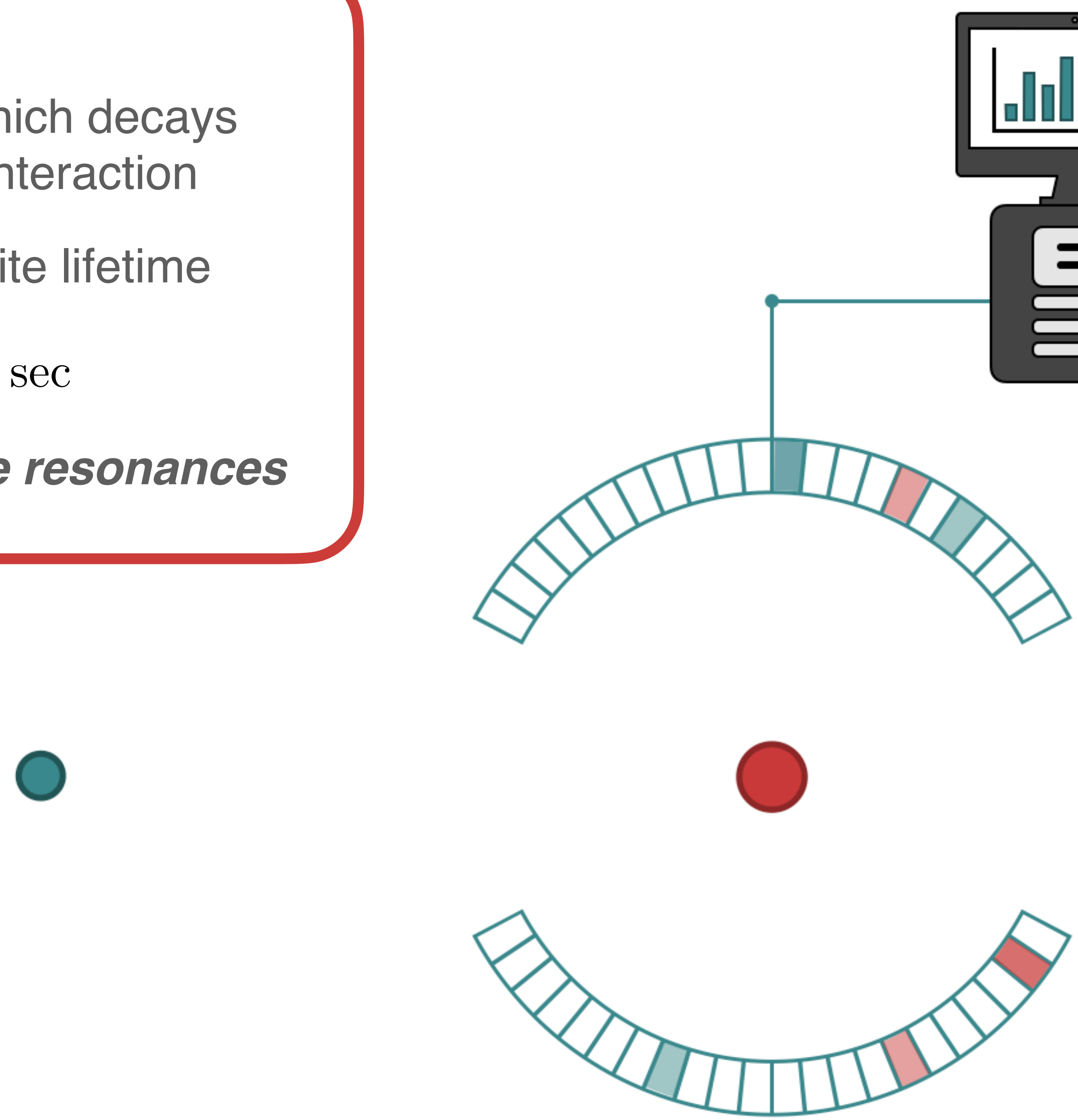
Resonance

Unstable hadron which decays via strong nuclear interaction

Has a mass *and* finite lifetime

$$\tau \sim 10^{-23}\,\mathrm{sec}$$

Most hadrons are resonances



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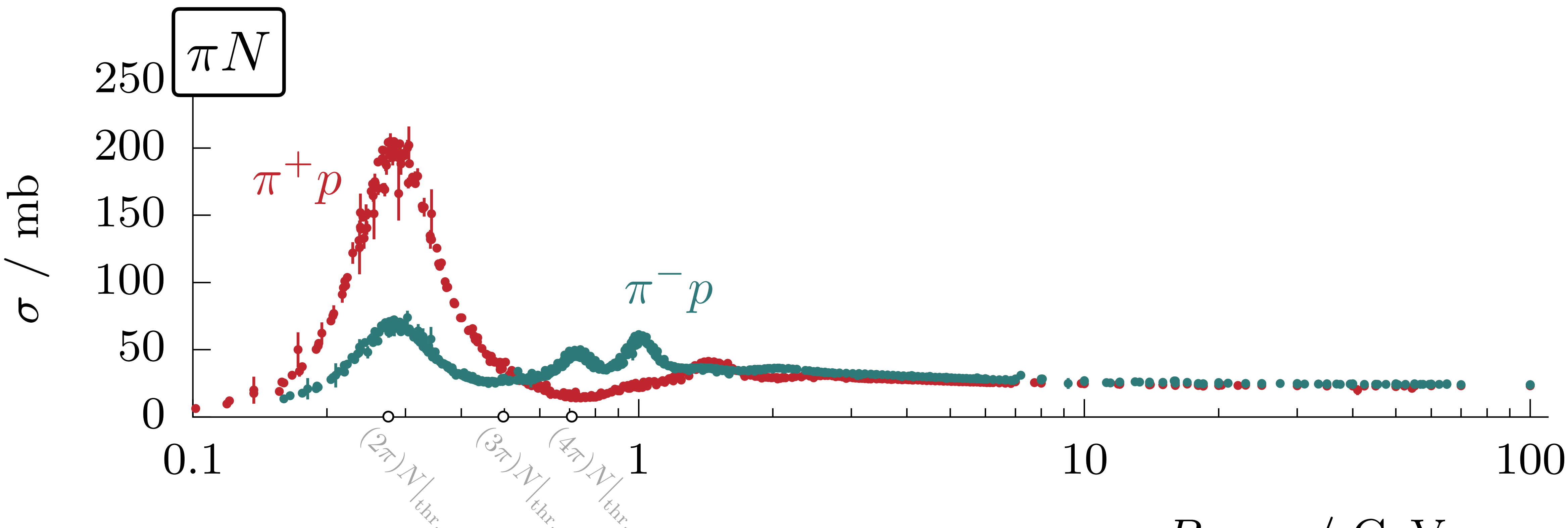
- Resonances can appear as enhancements in the cross-section / amplitude



Particle Data Group (PDG)
The Review of Particle Physics (RPP)

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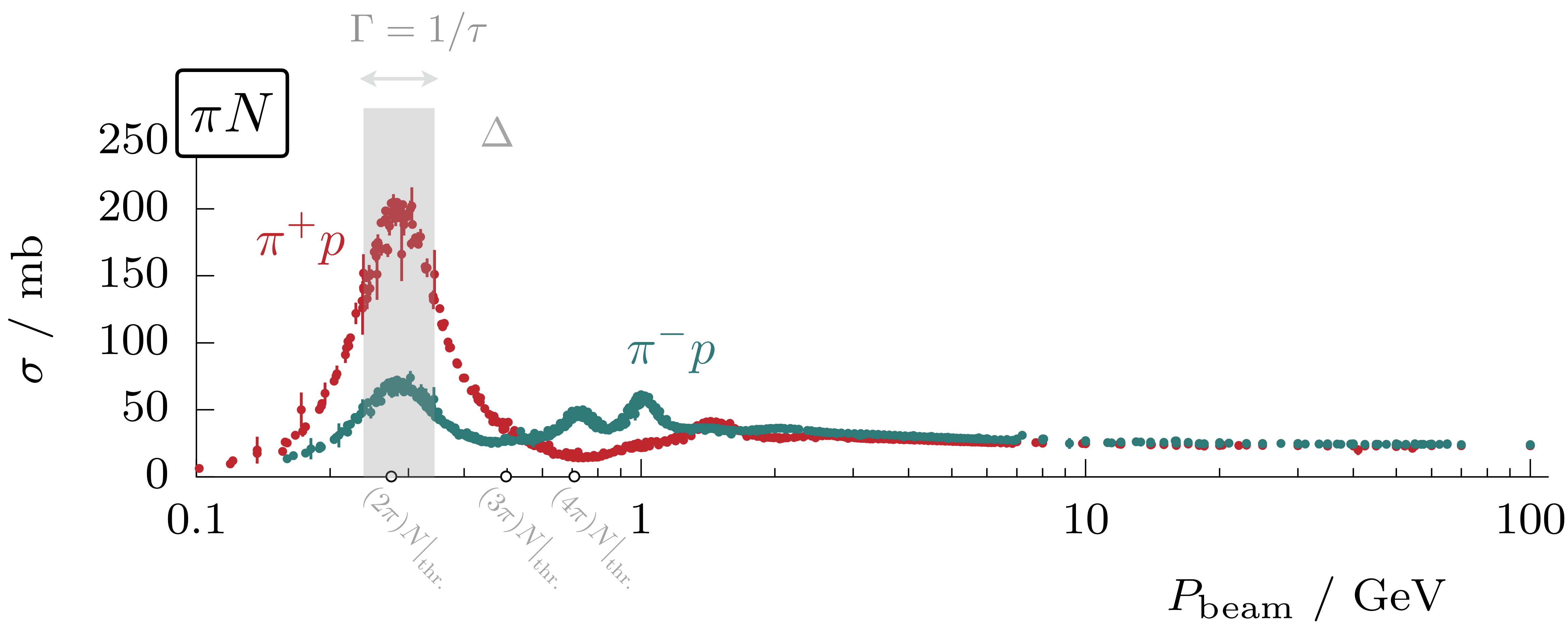

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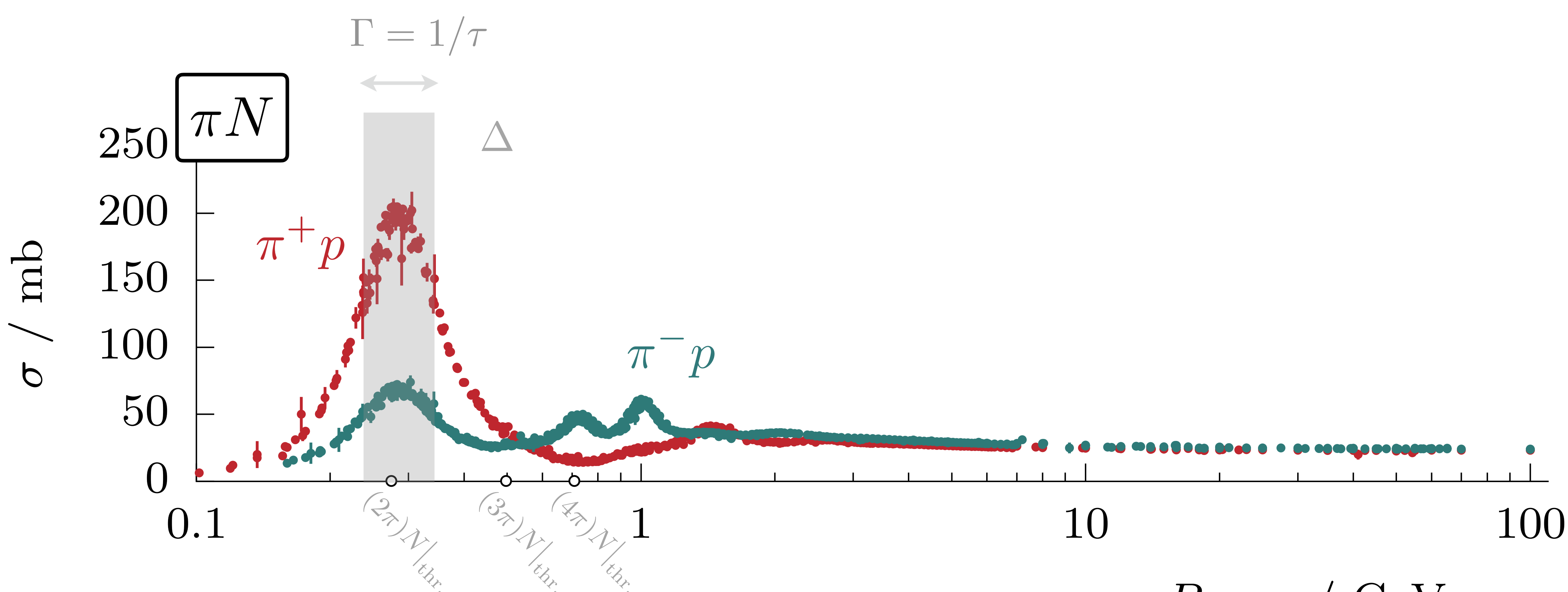
To get properties of resonances…
…study structure of reaction amplitude $\mathcal{M}$



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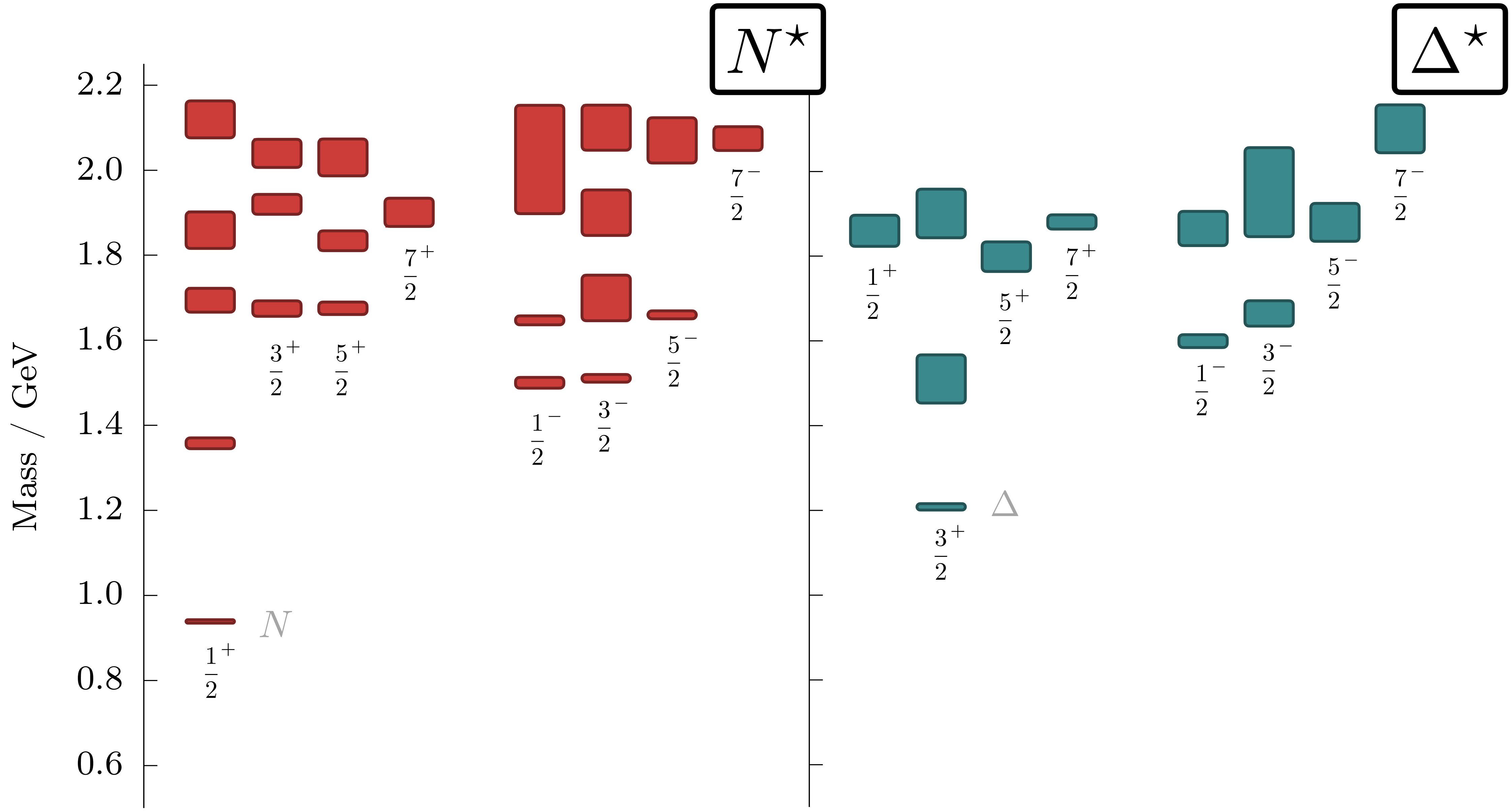

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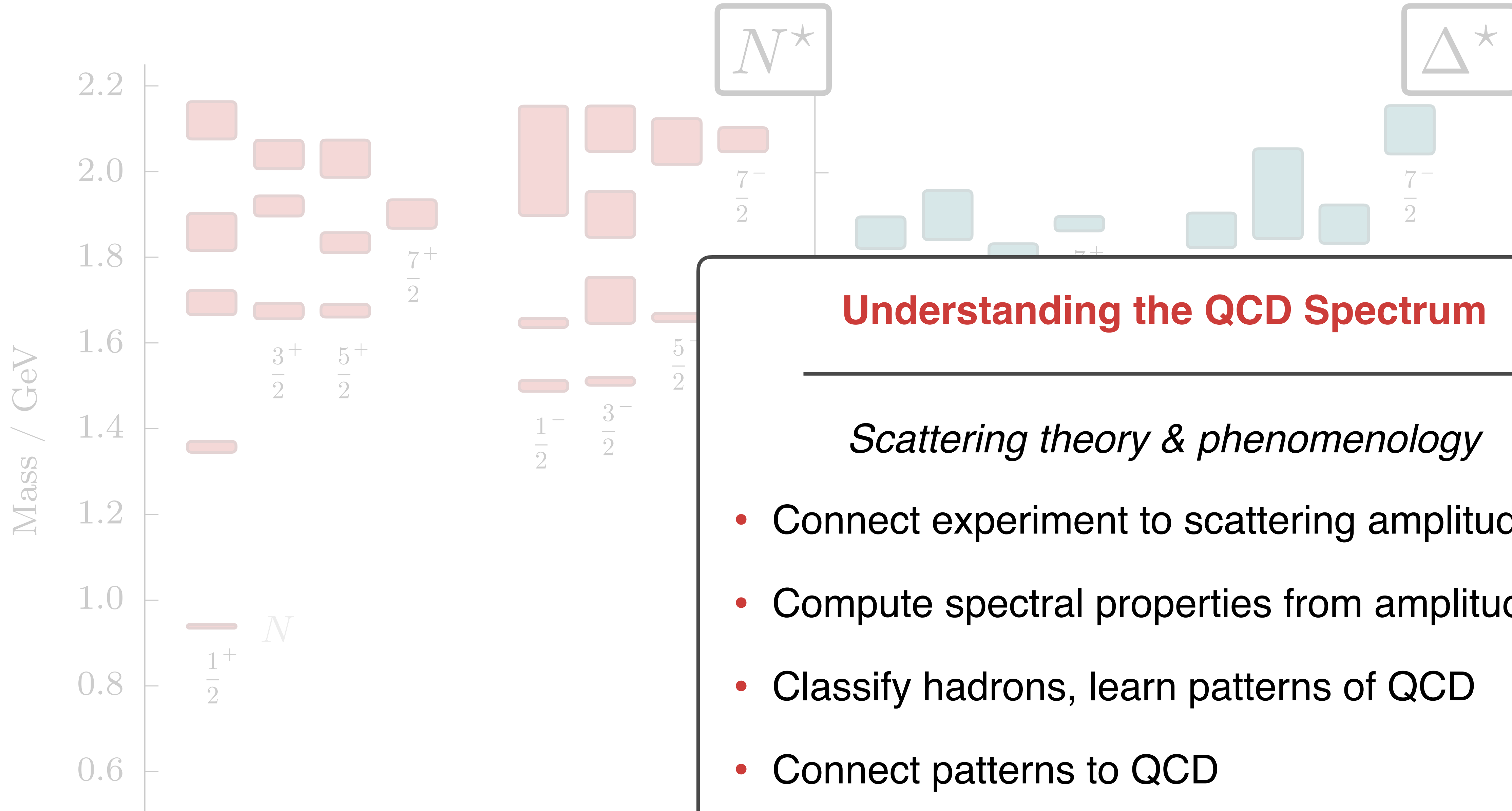

Understanding the QCD Spectrum

Scattering theory & phenomenology

- Connect experiment to scattering amplitudes
- Compute spectral properties from amplitudes
- Classify hadrons, learn patterns of QCD
- Connect patterns to QCD

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Outline

Hadrons, Quarks, & QCD

Hadron Spectroscopy

The Quark Model

Quantum Chromodynamics

Scattering Theory & the Hadron Spectrum

Scattering Amplitudes

Bound & Resonant States

Lattice QCD & Hadron Spectroscopy

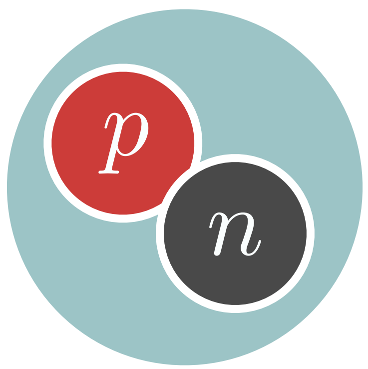
Lattice QCD

Lüscher & the Finite-Volume

Case study — *the deuteron*

As an example of connecting scattering physics to hadron properties, consider ***the deuteron***

- Simplest nucleus, bound state of ***proton*** and ***neutron***
- "Hydrogen atom of nuclear physics" — study nature of two-body nuclear forces



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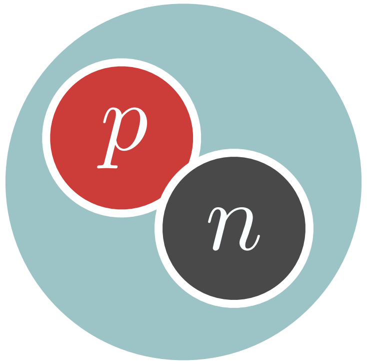

Deuteron

Spectral properties checklist

- [ ] Mass
- [ ] Lifetime
- [ ] Spin
- [ ] Parity
- [ ] Radius
- [ ] Charge

…

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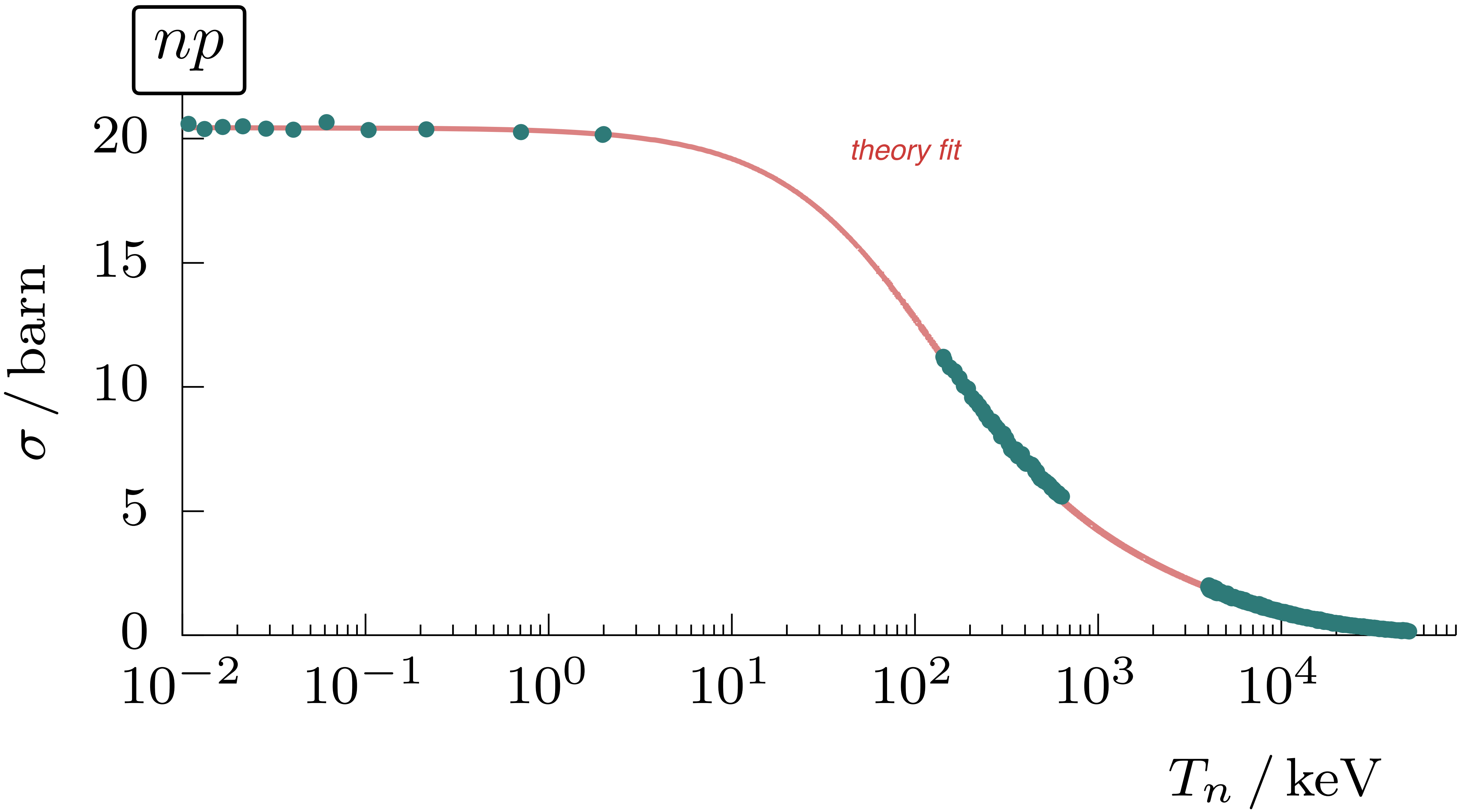

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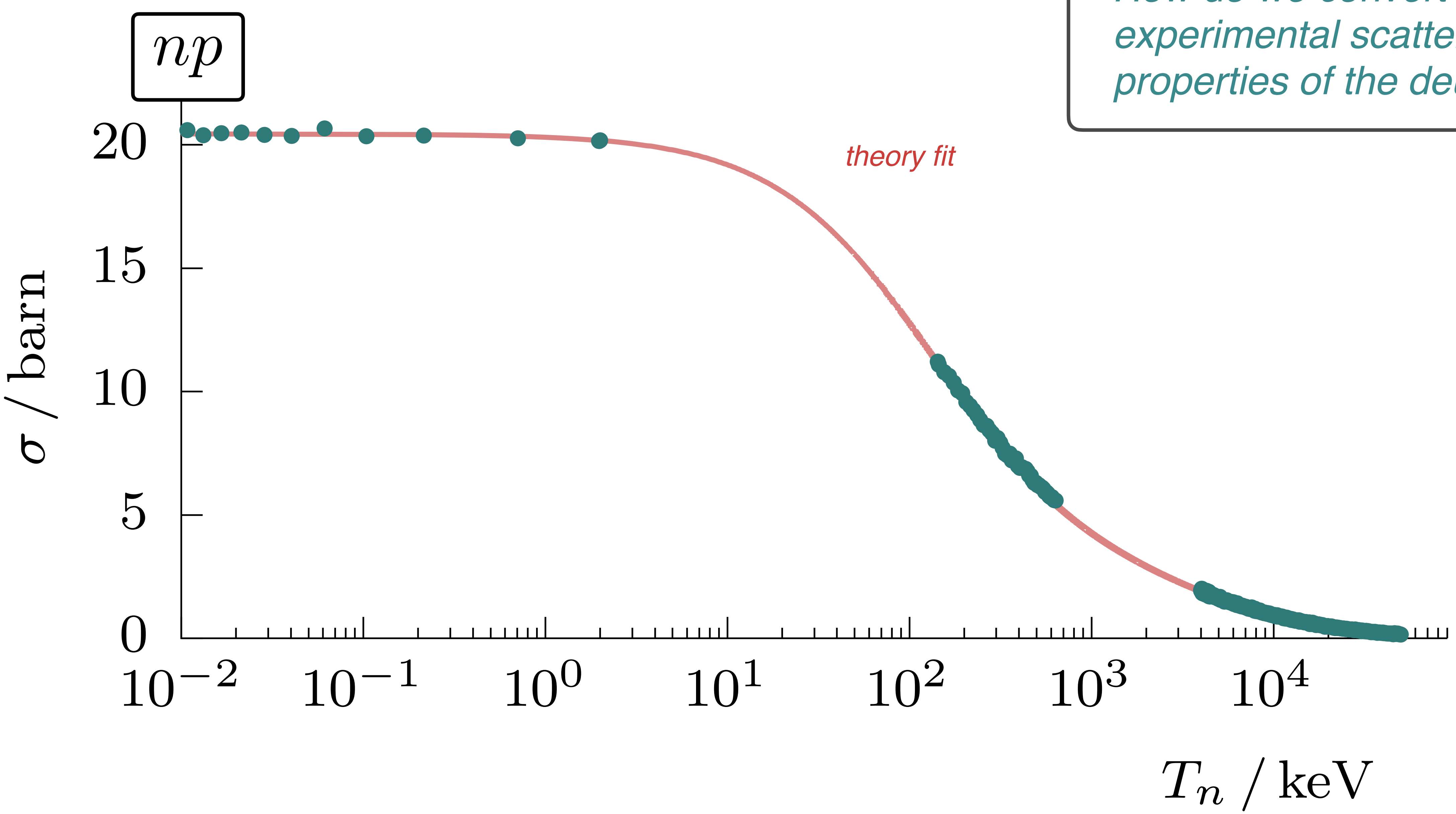

How do we convert experimental scattering data to properties of the deuteron?

Kinematics — Energy & Momentum

Let us define the kinematics of protons and neutrons

- proton and neutron masses are nearly equal — different isospin projections of Nucleon

$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$

Nucleon

$$m_p \approx m_n$$

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Nucleon

$$m_p \approx m_n$$

Nucleon mass

First approximation — Isospin limit

$$m_p = m_n \equiv m$$

$$\approx 940 \text{ MeV}$$

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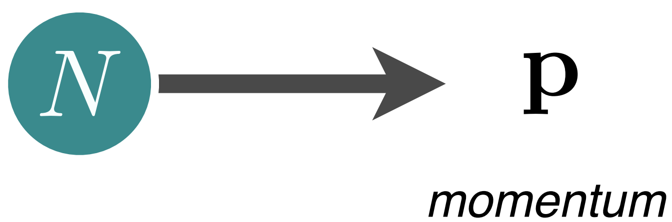
Let us define the kinematics of protons and neutrons

- proton and neutron masses are nearly equal — different isospin projections of Nucleon
- Relativistic nucleon kinematics



energy

$$\omega_{\mathbf{p}} = \sqrt{m^2 + \mathbf{p}^2}$$

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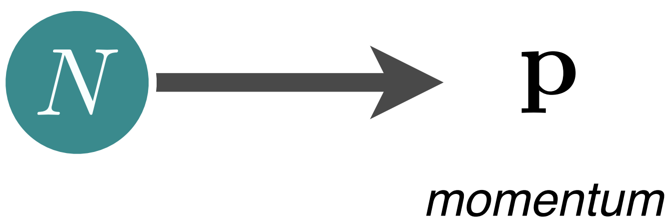

energy

$$\omega_{\mathbf{p}} = \sqrt{m^2 + \mathbf{p}^2}$$

$$= m + \frac{\mathbf{p}^2}{2m} + \mathcal{O}\left(\mathbf{p}^4\right)$$

non-relativistic limit, $p/m \ll 1$

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$\mathbf{p}$

momentum

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kinetic energy

$$T_{\mathbf{p}} = \omega_{\mathbf{p}} - m$$

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Exercise

Prove the non-relativistic expansion

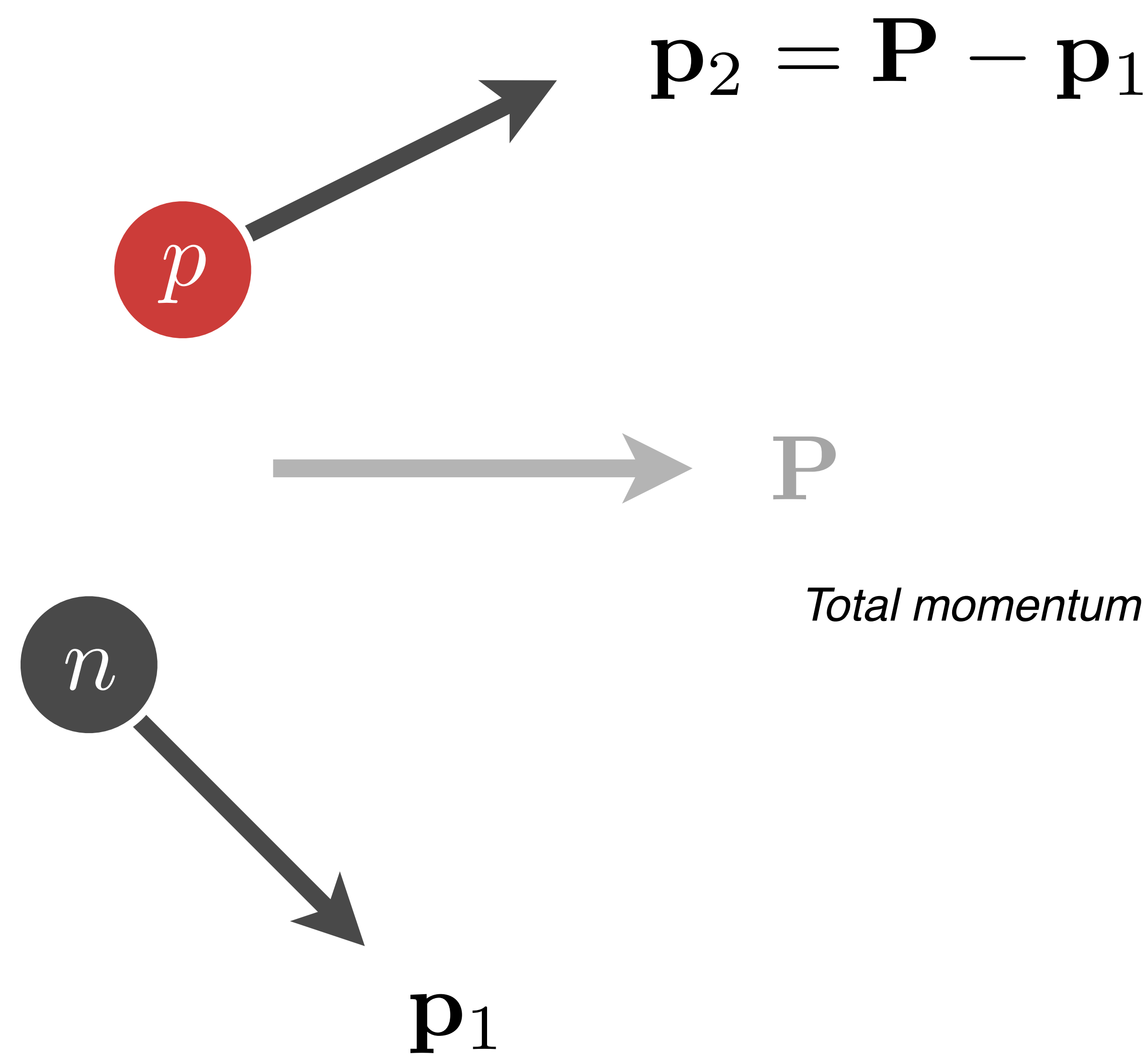
Kinematics — Energy & Momentum

Let us define the kinematics of protons and neutrons

- proton and neutron masses are nearly equal — different isospin projections of Nucleon
- Relativistic nucleon kinematics
- Two-nucleon kinematics

Total energy

$$E = \sqrt{m^2 + \mathbf{p}_1^2} + \sqrt{m^2 + \mathbf{p}_2^2}$$



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Let us define the kinematics of protons and neutrons

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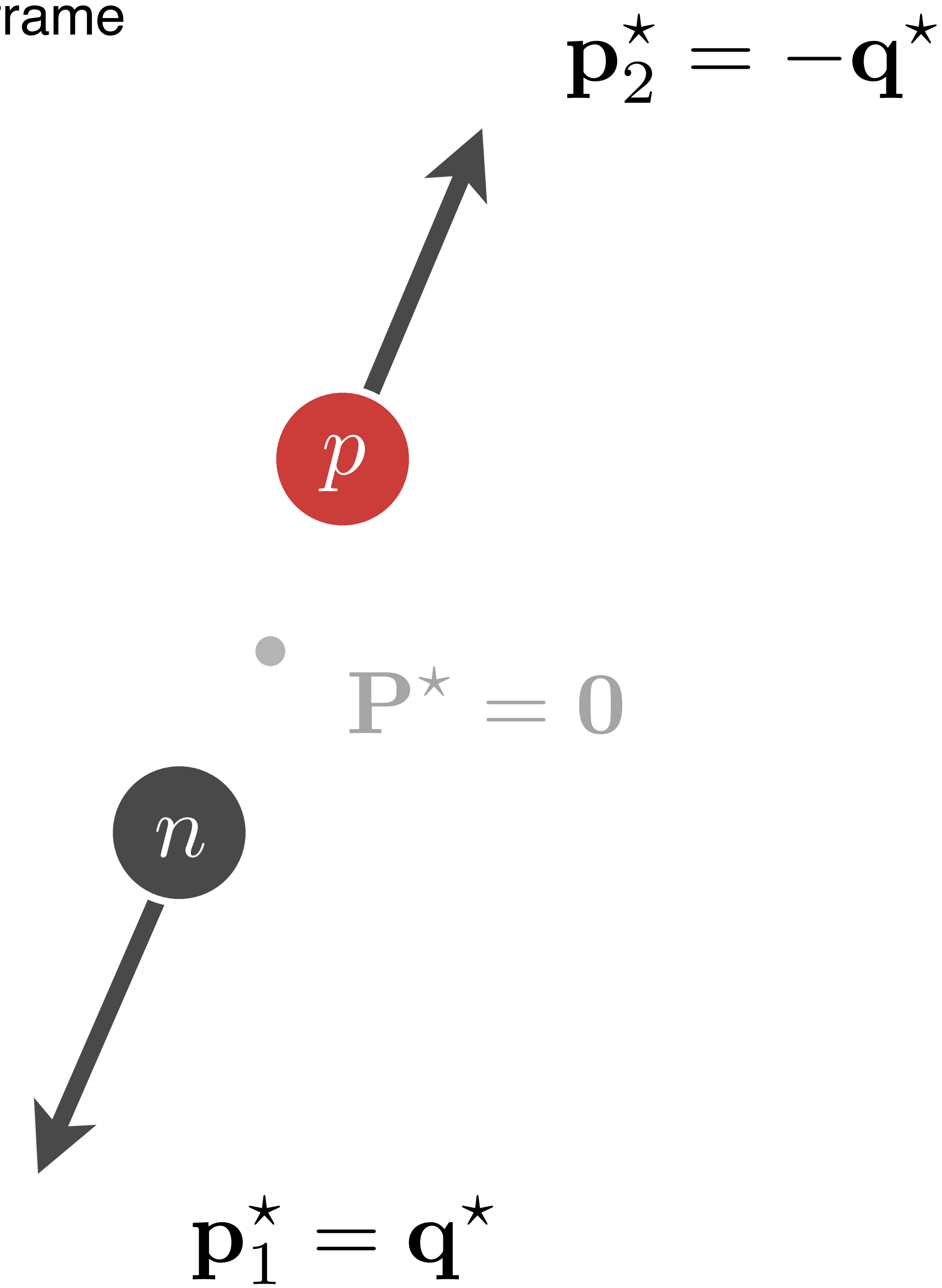
- Relativistic nucleon kinematics
- Two-nucleon kinematics — choose *center-of-momentum* frame

Total CM energy

$$E^\star = 2\sqrt{m^2 + q^{\star\,2}}$$



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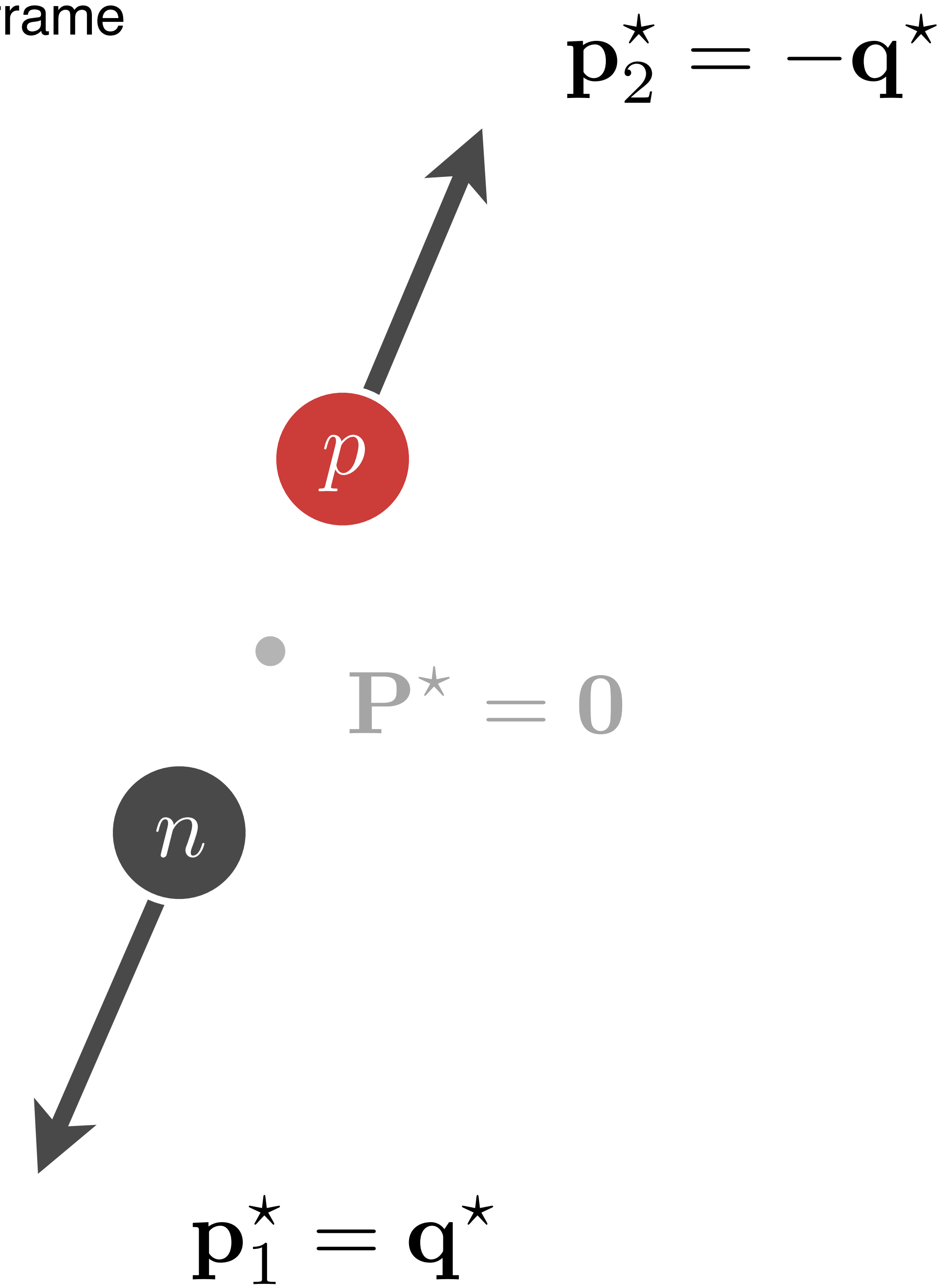

Total CM energy

$$E^\star = 2\sqrt{m^2 + q^{\star\,2}}$$

$$\Longrightarrow \qquad q^\star = \frac{1}{2}\sqrt{E^{\star\,2} - 4m^2}$$

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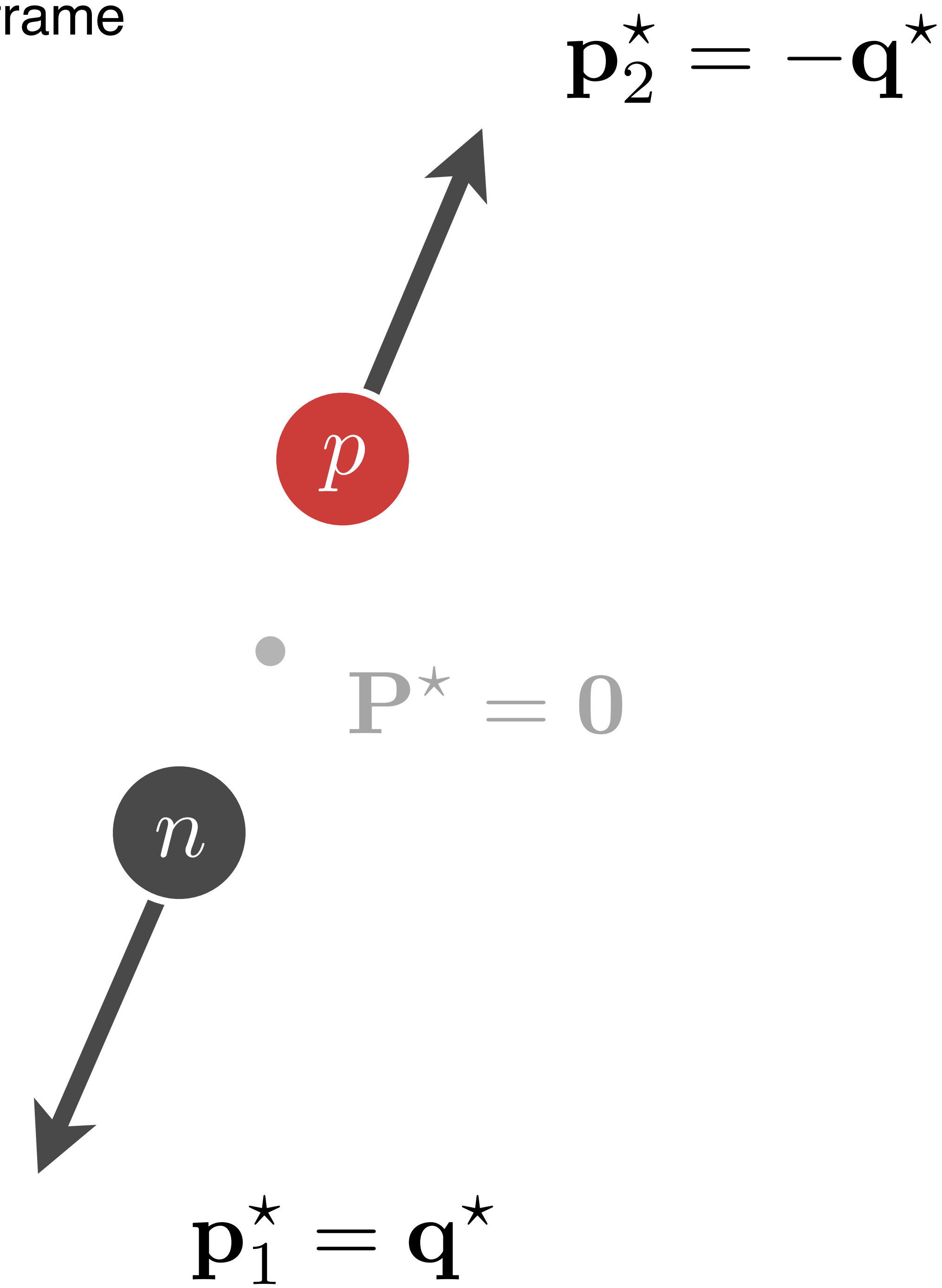

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$$\implies \qquad q^\star = \frac{1}{2}\sqrt{E^{\star\,2} - 4m^2}$$

Exercise

(a) *Show that* $q^\star = \sqrt{E^{\star\,2}/4 - m^2}$

(b) *Plot* $q^\star$ *as a function of* $E^\star$

Kinematics — Phase Space

Two-nucleon energy is restricted by energy- and momentum-conservation

- Kinematic *phase space* — available space of kinematic configurations

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Particle 2 distribution

Lorentz invariant phase space

$$\mathrm{d}\Phi_2(P \to p_1 + p_2) = (2\pi)^4 \delta(E - \omega_{\mathbf{p}_1} - \omega_{\mathbf{p}_1})\, \delta^{(3)}(\mathbf{P} - \mathbf{p}_1 - \mathbf{p}_2)\, \frac{\mathrm{d}^3\mathbf{p}_1}{(2\pi)^3\, 2\omega_{\mathbf{p}_1}} \frac{\mathrm{d}^3\mathbf{p}_2}{(2\pi)^3\, 2\omega_{\mathbf{p}_2}}$$

Energy conservation
$E = \omega_{\mathbf{p}_1} + \omega_{\mathbf{p}_2}$

Momentum conservation
$\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$

Particle 1 distribution

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$$= \cdots$$

$$= 2\rho\, \Theta(E^\star - 2m)\, \frac{\mathrm{d}\Omega^\star_{\mathbf{q}}}{4\pi}$$

Orientation (not fixed)

Energy greater than threshold
$E^\star \geq 2m$

$$E^\star_{\mathrm{thr.}} = 2m \quad \Longleftrightarrow \quad q^\star_{\mathrm{thr.}} = 0$$

$$m \approx 940\,\mathrm{MeV}$$

$$E^\star_{\mathrm{thr.}} \approx 1880\,\mathrm{MeV}$$

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Two-body kinematic phase space factor

$$\rho = \frac{q^\star}{8\pi E^\star}$$

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Exercise

Plot ρ as a function of $E^\star$

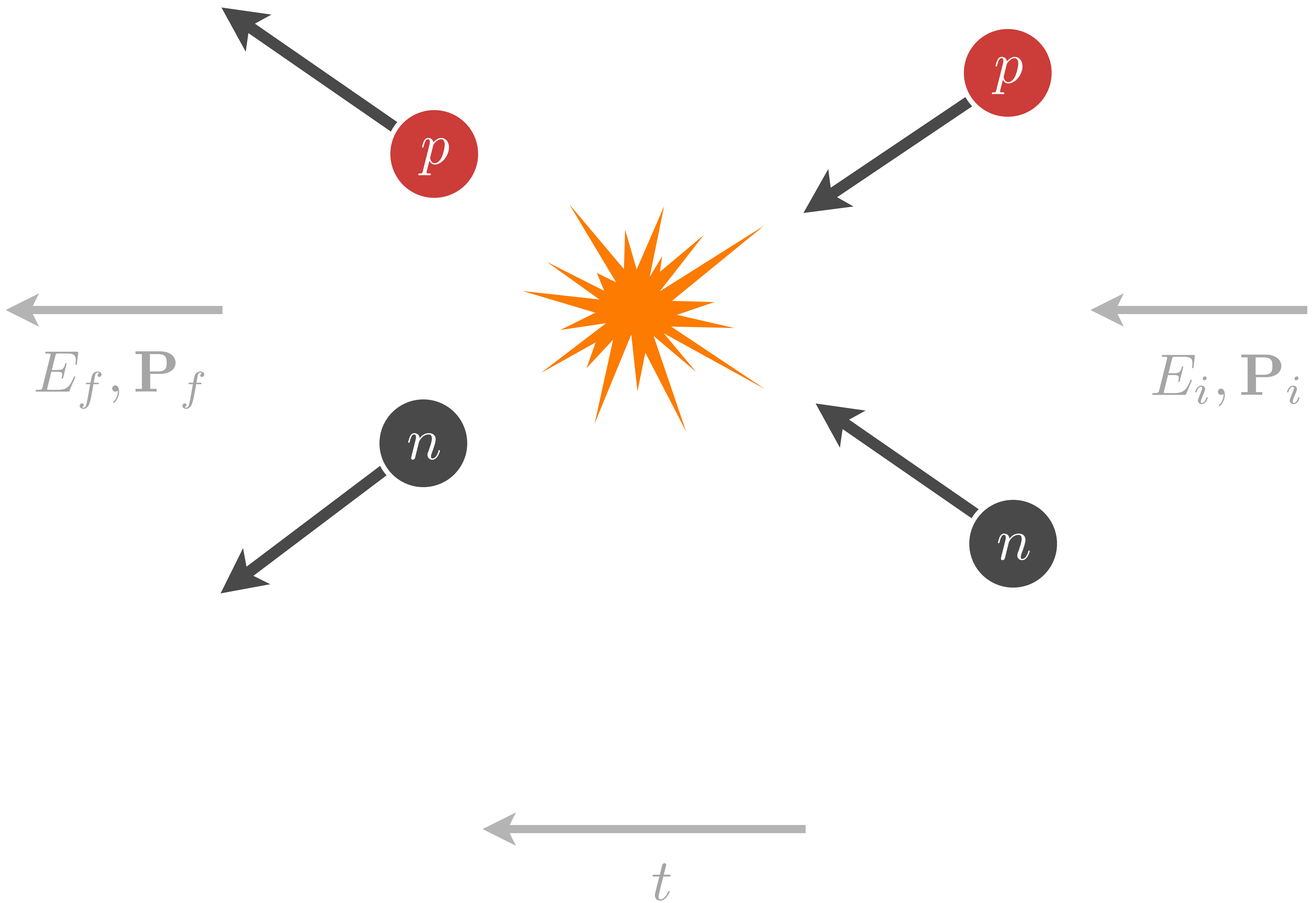
** Derive ρ*

Two-body kinematic phase space factor

$$\rho = \frac{q^\star}{8\pi E^\star}$$

Kinematics — Binary Reactions

np-elastic scattering



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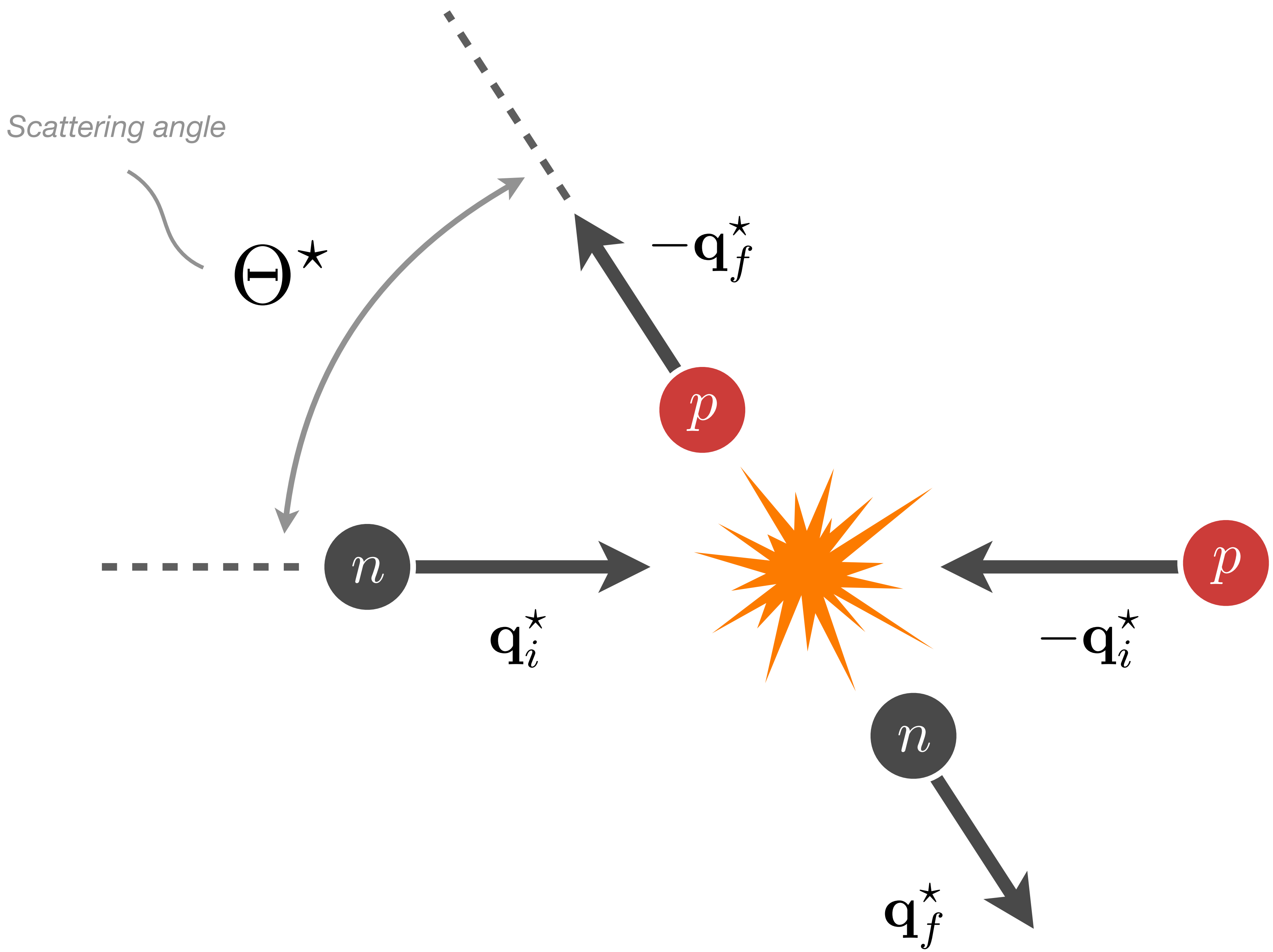
np-elastic scattering

- Total energy-momentum is *conserved* — $E_i = E_f \equiv E\,,\ \mathbf{P}_i = \mathbf{P}_f \equiv \mathbf{P}$
- Analyze in *center-of-momentum* (CM) frame — $\mathbf{P}^\star = \mathbf{0}$
- Elastic, equal mass scattering — $q_i^\star = q_f^\star \equiv q^\star$



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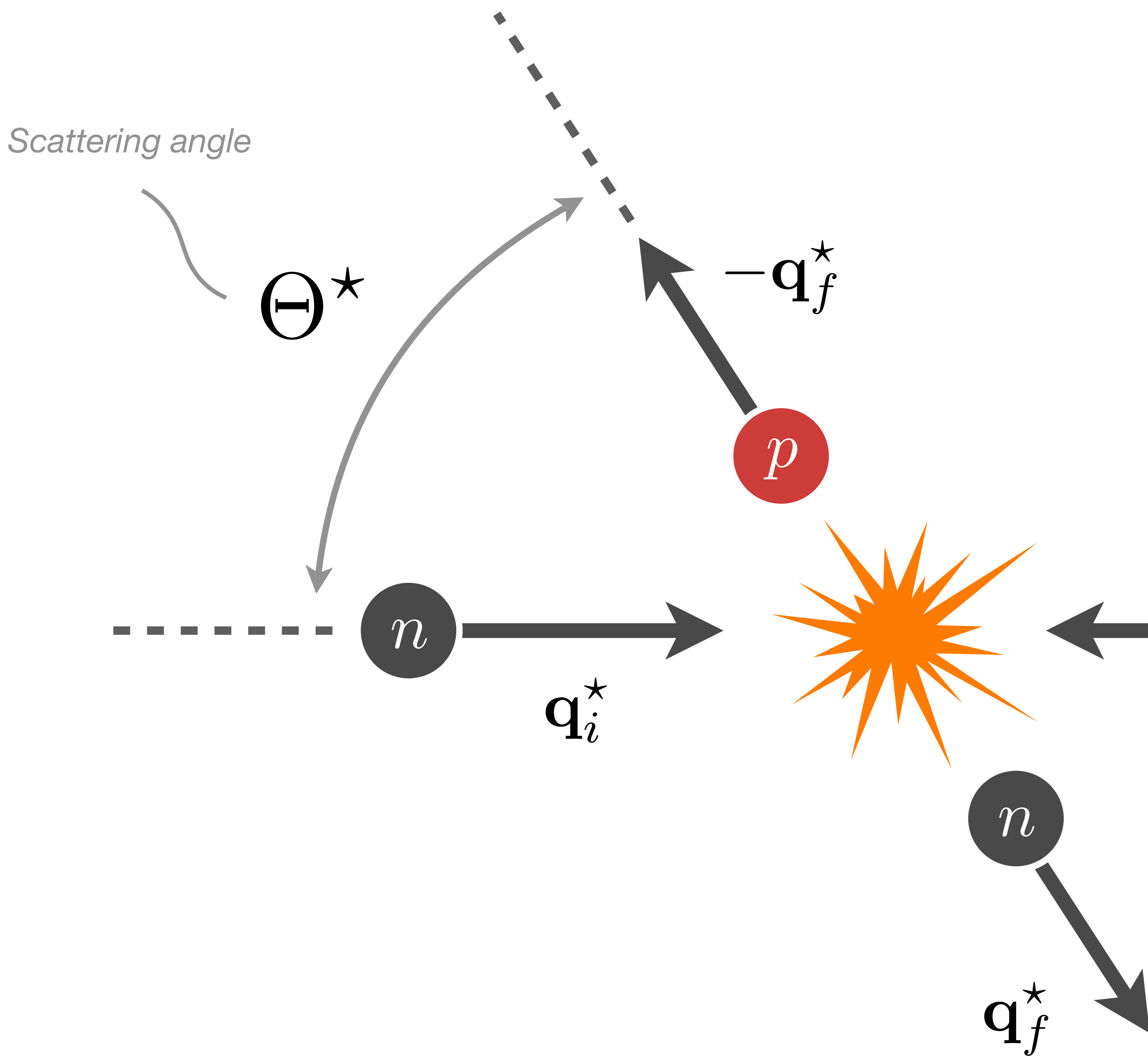

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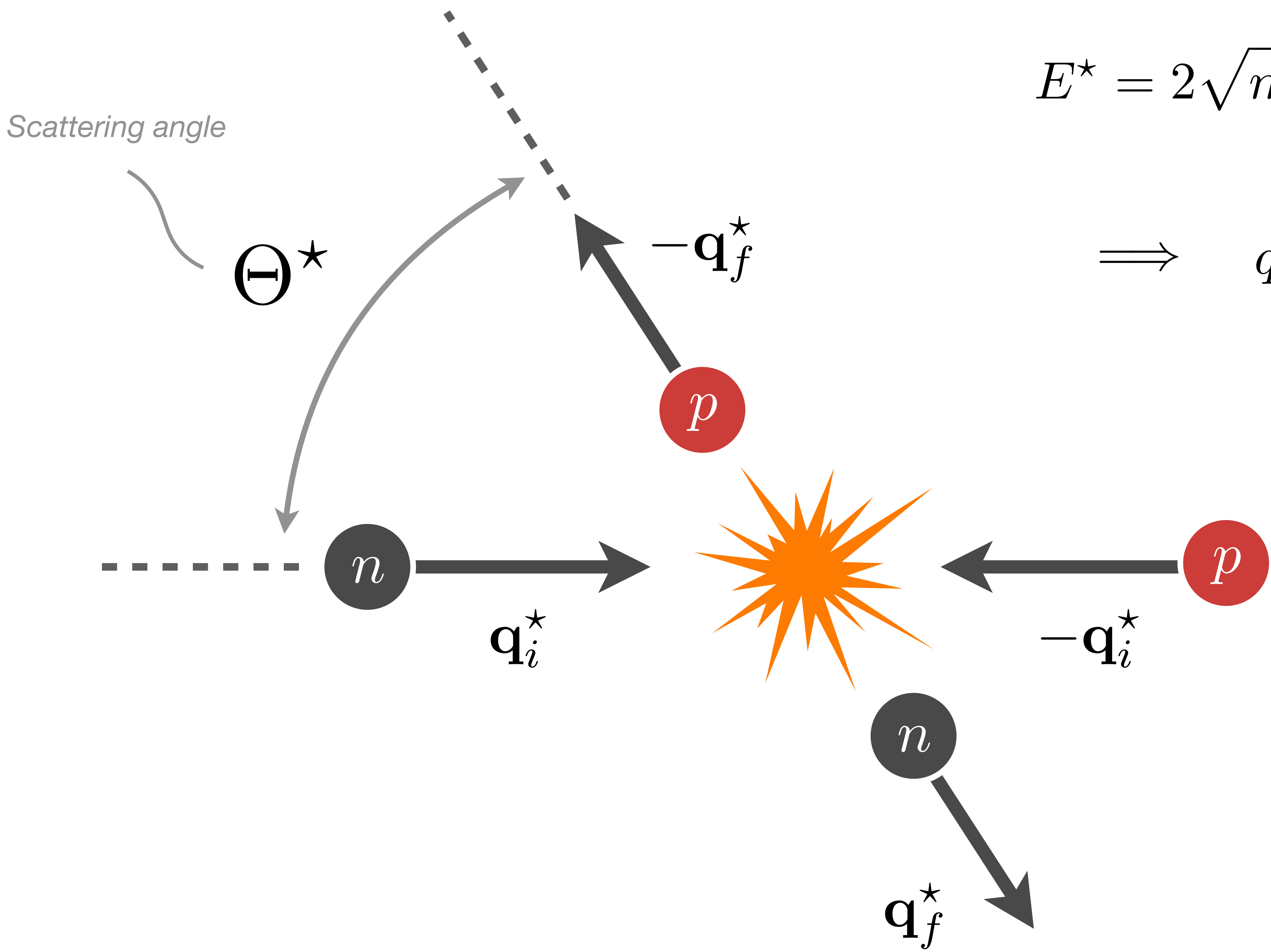

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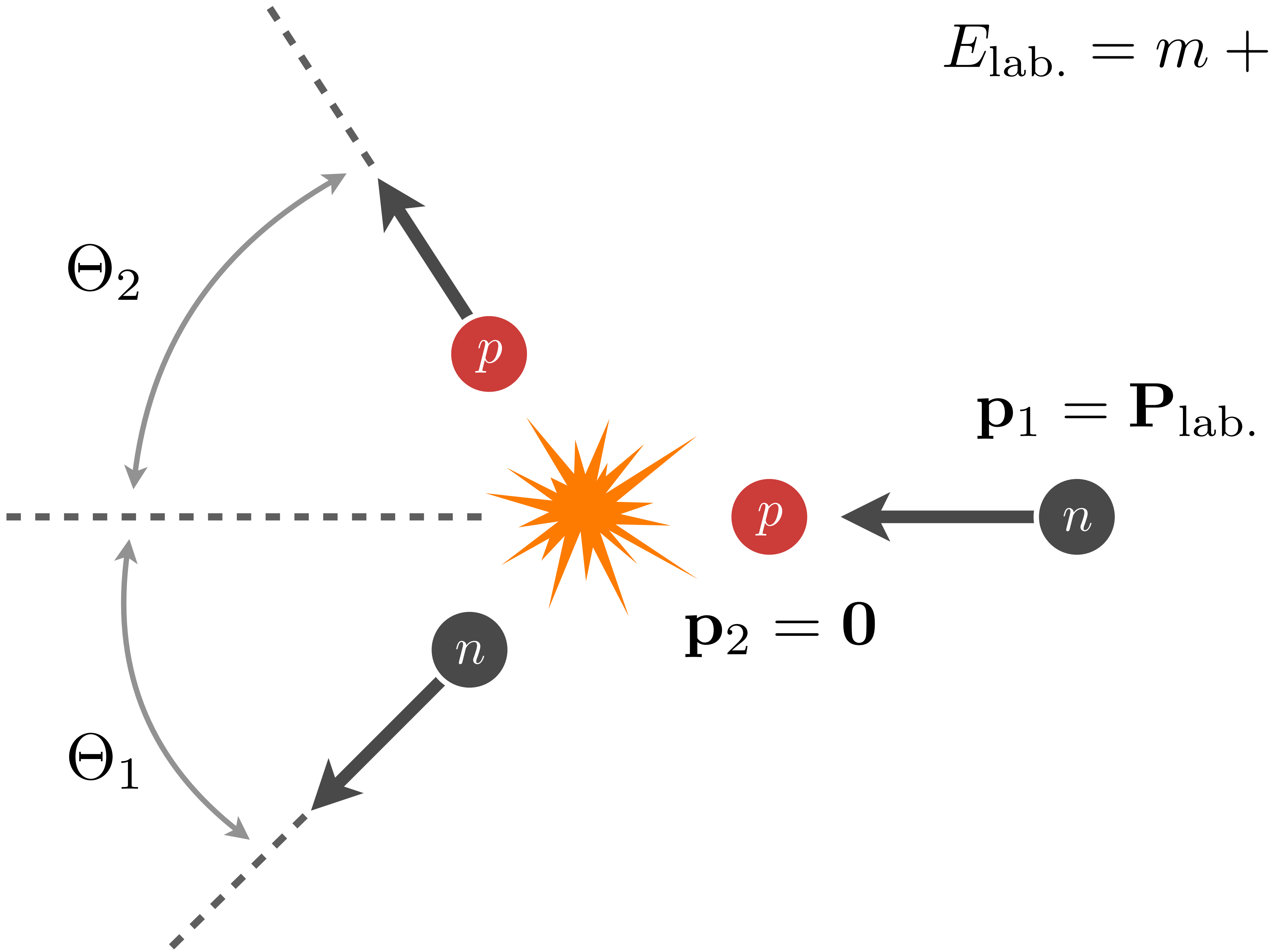
Exercise

Show these results

np-elastic scattering

- Experiments often use *fixed-target* of *lab* frame

$$E_{\text{lab.}} = m + \sqrt{m^2 + \mathbf{P}^2_{\text{lab.}}}$$



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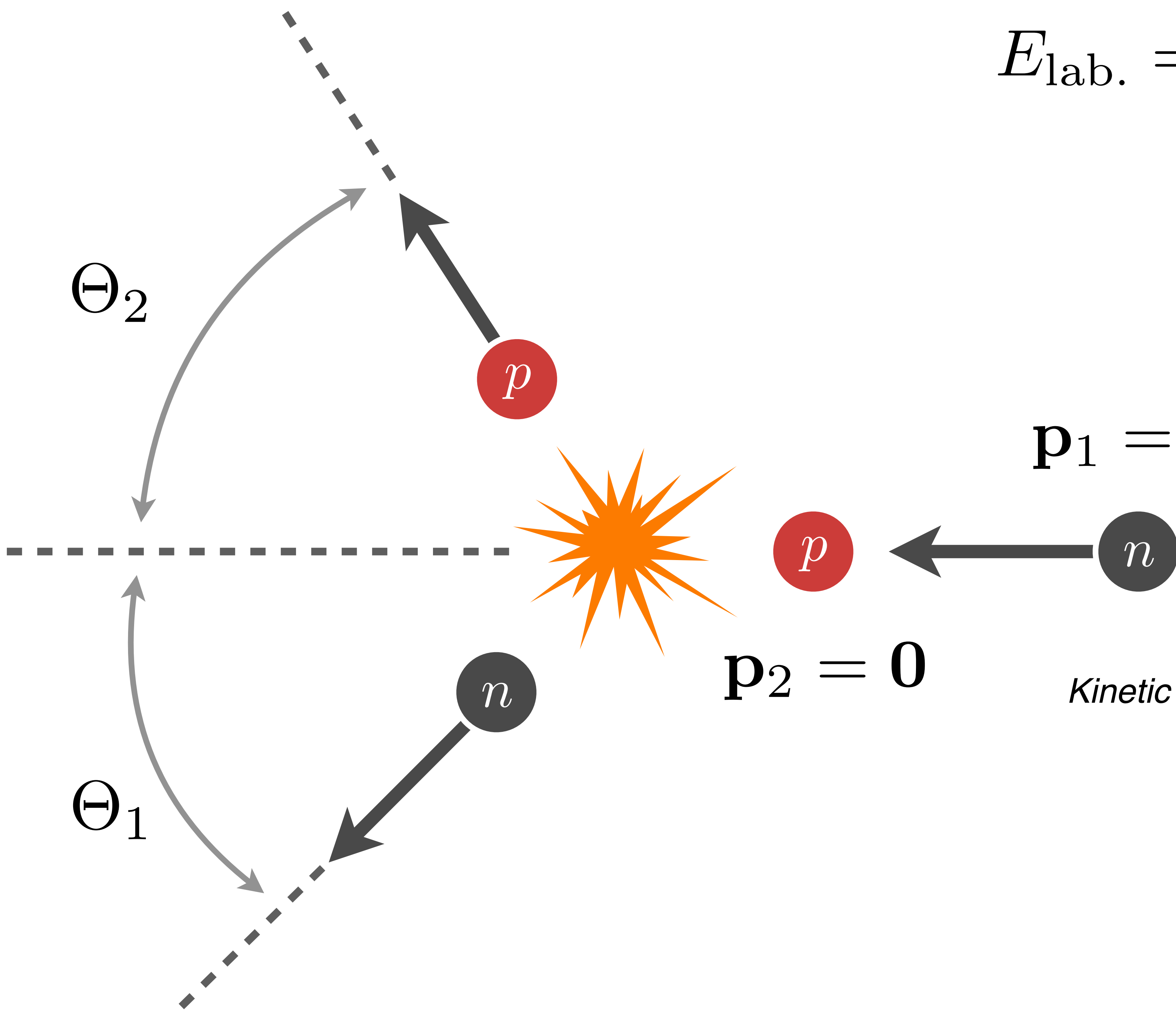

$$\mathbf{p}_1 = \mathbf{P}_{\text{lab.}}$$

Kinetic energy

$$T_n = \sqrt{m^2 + \mathbf{P}^2_{\text{lab.}}} - m$$

$$q^{\star\,2} = \frac{1}{2} m T_n$$

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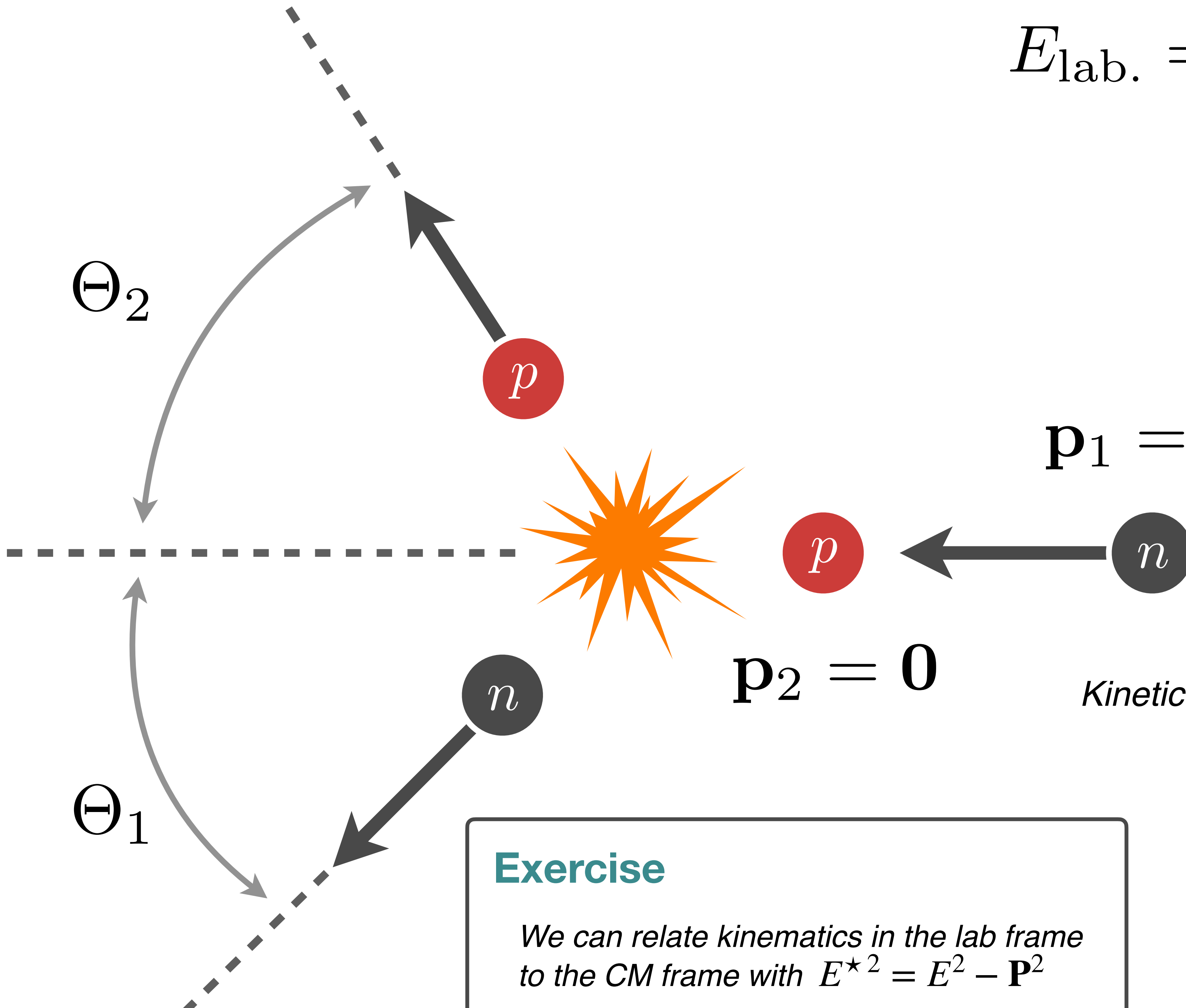

$$\mathbf{p}_1 = \mathbf{P}_{\text{lab.}}$$

$$\mathbf{p}_2 = \mathbf{0}$$

$$E_{\text{lab.}} = m + \sqrt{m^2 + \mathbf{P}^2_{\text{lab.}}}$$

Kinetic energy

$$T_n = \sqrt{m^2 + \mathbf{P}^2_{\text{lab.}}} - m$$

$$q^{\star\,2} = \frac{1}{2} m T_n$$

Exercise

We can relate kinematics in the lab frame to the CM frame with $E^{\star\,2} = E^2 - \mathbf{P}^2$

Show the relation between $q^\star$ *and* T_n

QCD Spectroscopy
An Introduction

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June 19th - July 7th 2023

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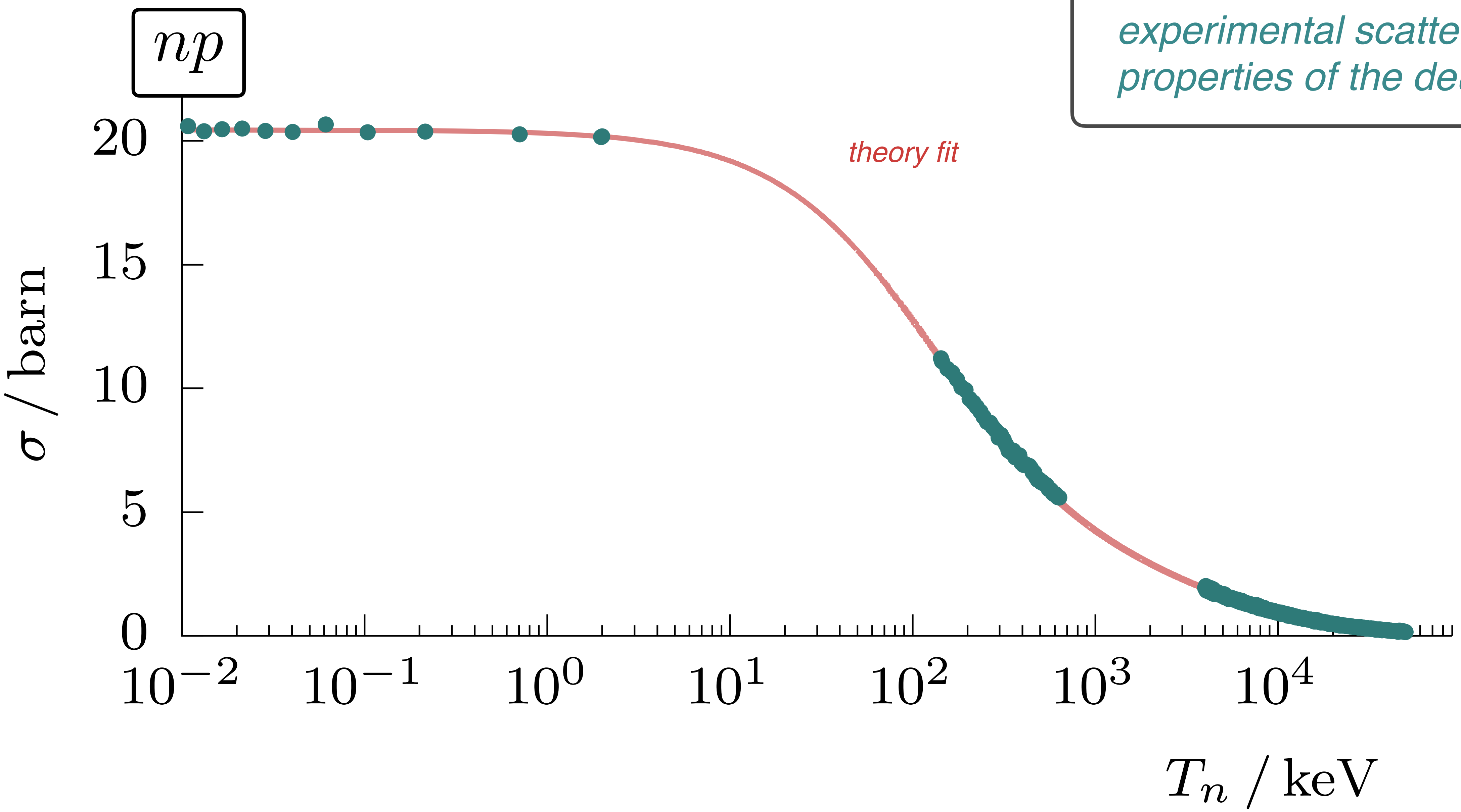
Case study — *the deuteron*

As an example of connecting scattering physics to hadron properties, consider ***the deuteron***

- Simplest nucleus, bound state of ***proton*** and ***neutron***
- "Hydrogen atom of nuclear physics" — study nature of two-body nuclear forces
- Examine low-energy *neutron-proton elastic scattering*

How do we convert experimental scattering data to properties of the deuteron?



Scattering Theory — Amplitudes

How do we dynamically describe reactions?

- In quantum mechanics, reaction dynamics encoded in *scattering probability amplitudes*

$$|\psi_f\rangle = \mathcal{S}\,|\psi_i\rangle$$

Initial state

Final state

Energy dependent amplitude $\mathcal{S} = \mathcal{S}(E^\star, \Theta^\star)$

Probability for initial state to evolve to final state

$$\mathrm{Prob}(i \to f) = |\mathcal{S}|^2$$

Scattering Theory — Amplitudes

How do we dynamically describe reactions?

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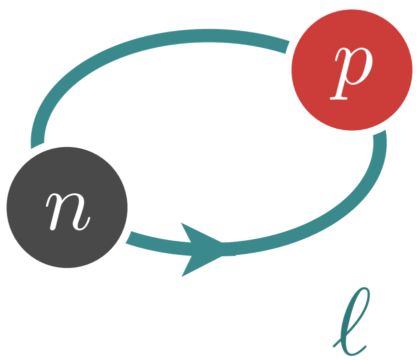
- Rotational symmetry — Break up interaction into angular momentum orbitals



Angular momentum quantized

$$\ell = 0, 1, 2, \ldots$$

Scattering Theory — Amplitudes

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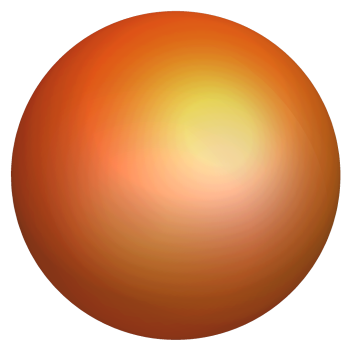
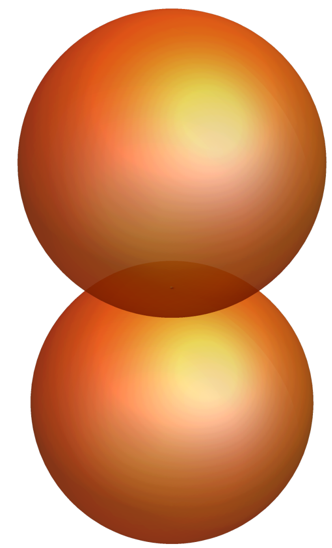
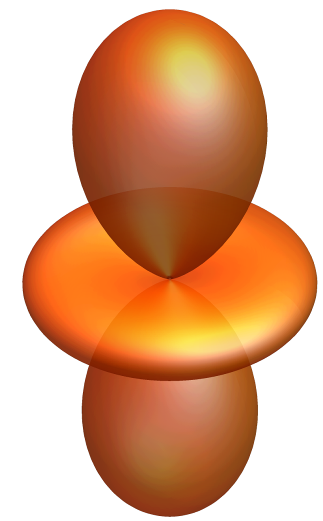
$\mathcal{S}(E^\star, \Theta^\star) =$  $+$  $+$  $+ \cdots$

$\mathcal{S}_0(E^\star)$ $\mathcal{S}_1(E^\star)$ $\mathcal{S}_2(E^\star)$

S-wave *P-wave* *D-wave*

$$\mathcal{S}(E^\star, \Theta^\star) = \sum_\ell (2\ell + 1)\, \mathcal{S}_\ell(E^\star) P_\ell(\cos\Theta^\star)$$

Scattering Theory — Amplitudes

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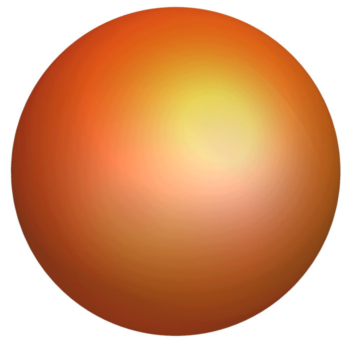
$\mathcal{S}(E^\star, \Theta^\star) =$  $+$ $+$ $+ \cdots$

$\mathcal{S}_0(E^\star)$ $\mathcal{S}_1(E^\star)$ $\mathcal{S}_2(E^\star)$

S-wave *P-wave* *D-wave*

At low energies, isotropic interaction dominates

Ignore subscript $\mathcal{S}_0 \to \mathcal{S}$

$$\mathcal{S}(E^\star, \Theta^\star) = \sum_\ell (2\ell + 1)\, \mathcal{S}_\ell(E^\star) P_\ell(\cos\Theta^\star)$$

Scattering Theory — Amplitudes

How do we dynamically describe reactions?

- In quantum mechanics, reaction dynamics encoded in *scattering probability amplitudes*
- Rotational symmetry — Break up interaction into orbitals
- Remove trivial case where particles do not interact — define ***scattering amplitude***

$$\mathcal{S} = 1 + 2i\,\rho\,\mathcal{M}$$

Scattering amplitude — encodes ALL dynamics

No scattering

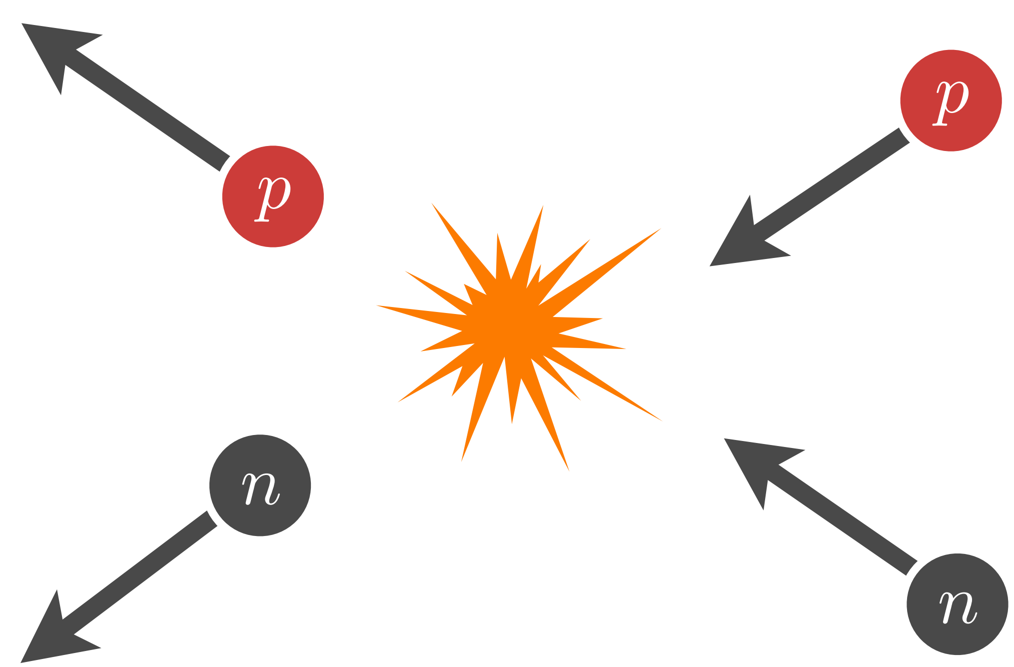
Available phase space

$$\rho = \frac{q^\star}{8\pi E^\star}$$

Relation to cross-section

$$\sigma \propto |\mathcal{M}|^2$$

$$\mathcal{M} =$$



Scattering Theory — Unitarity

Conservation of probability imposes constraints on the scattering amplitude

$$\sum_f \mathrm{Prob}(i \to f) = 1$$

Probability must be conserved!

Elastic scattering — only one final state

$$\implies \quad |\mathcal{S}|^2 = \mathcal{S}^* \mathcal{S} = 1$$

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Unitarity condition for scattering amplitude

$$\mathcal{S} = 1 + 2i\,\rho\mathcal{M}$$

$$\implies \quad \mathrm{Im}\,\mathcal{M} = \rho|\mathcal{M}|^2$$

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$$\mathcal{S} = 1 + 2i\,\rho\mathcal{M}$$

$$\Longrightarrow \quad \mathrm{Im}\,\mathcal{M} = \rho|\mathcal{M}|^2$$

Exercise

Derive the unitarity condition for the scattering amplitude

$$\mathrm{Prob}(i \to f) = |\mathcal{S}|^2$$

Scattering Theory — Unitarity

Conservation of probability imposes constraints on the scattering amplitude

- At a fixed energy, amplitude needs *two* real numbers

$$\mathcal{M} = \mathrm{Re}\,\mathcal{M} + i\,\mathrm{Im}\,\mathcal{M}$$

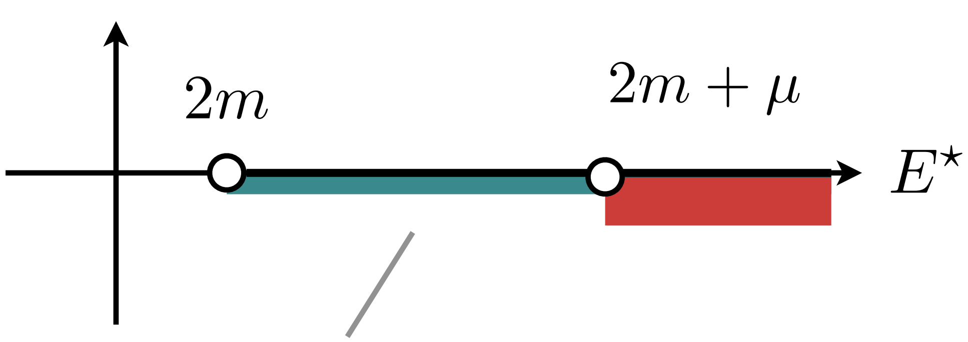
Unitarity relates real and imaginary parts

$$\mathrm{Im}\,\mathcal{M} = \rho|\mathcal{M}|^2$$

For $2m \leq E^\star < 2m + \mu$

Available phase space

$$\rho = \frac{q^\star}{8\pi E^\star}$$



Elastic scattering region

Scattering Theory — Unitarity

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Imaginary part fixed by kinematics

Can rewrite as

$$\mathrm{Im}\,\mathcal{M}^{-1} = -\rho$$

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Only need one real number to completely specify scattering

$$\Longrightarrow \quad \mathcal{M}^{-1} = \mathcal{K}^{-1} - i\rho$$

Real function — encodes ALL dynamics!

Scattering Theory — Unitarity

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Exercise

Show that $\mathrm{Im}\,\mathcal{M}^{-1} = -\rho$

Only need one real number to completely specify scattering

$$\Longrightarrow \quad \mathcal{M}^{-1} = \mathcal{K}^{-1} - i\rho$$

Real function — encodes ALL dynamics!

Scattering Theory — Unitarity

Unitarity implies only one real dynamical function needed to describe scattering

- Theoretically, want to compute *K* matrix
- Can infer *K* matrix by fitting experimental data

$$\mathcal{M}^{-1} = \mathcal{K}^{-1} - i\rho$$

Know this function

Want to determine this function

Scattering Theory — Phase shifts

Alternatively, can cast amplitude in terms of phase shifts

- Unitarity is starting point

$$\mathrm{Im}\,\mathcal{M} = \rho|\mathcal{M}|^2$$

Use polar representation

$$\mathcal{M} = |\mathcal{M}|\,e^{i\delta}$$

Scattering Theory — Phase shifts

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Impose unitarity, can show

$$\implies \quad \mathcal{M} = \frac{1}{\rho}\, e^{i\delta}\, \sin\delta$$

Phase shift, a real function of energy

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Phase shift, a real function of energy

Exercise

(a) Derive the phase shift representation

(b) Show that $\mathcal{K}^{-1} = \rho\cot\delta$

Scattering Theory — Effective Range Expansion

Can use either *K* matrix or phase shifts to describe reaction (often use both)

- They are real functions, can Taylor expand in real variables — assign physical meaning

$$\mathcal{K}^{-1} = \rho \cot \delta$$

$$\propto q^\star \cot \delta = -\frac{1}{a} + \frac{1}{2} r q^{\star\,2} + \mathcal{O}(q^{\star\,4})$$

Effective range expansion

Scattering length — "strength of interaction"

Effective range — "range of interaction"

$$\rho = \frac{q^\star}{8\pi E^\star}$$

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Effective range expansion

Scattering length — "strength of interaction"

Effective range — "range of interaction"

Amplitude at threshold is constant, fixed by strength of interaction

$$\mathcal{M}_{\text{thr.}} \equiv \mathcal{M}(E^\star_{\text{thr.}}) = -16\pi\, m a$$

$$\rho = \frac{q^\star}{8\pi E^\star}$$

Scattering Theory — Effective Range Expansion

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Effective range expansion

Effective range — "range of interaction"

Scattering length — "strength of interaction"

Amplitude at threshold is constant, fixed by strength of interaction

$$\mathcal{M}_{\text{thr.}} \equiv$$

Exercise

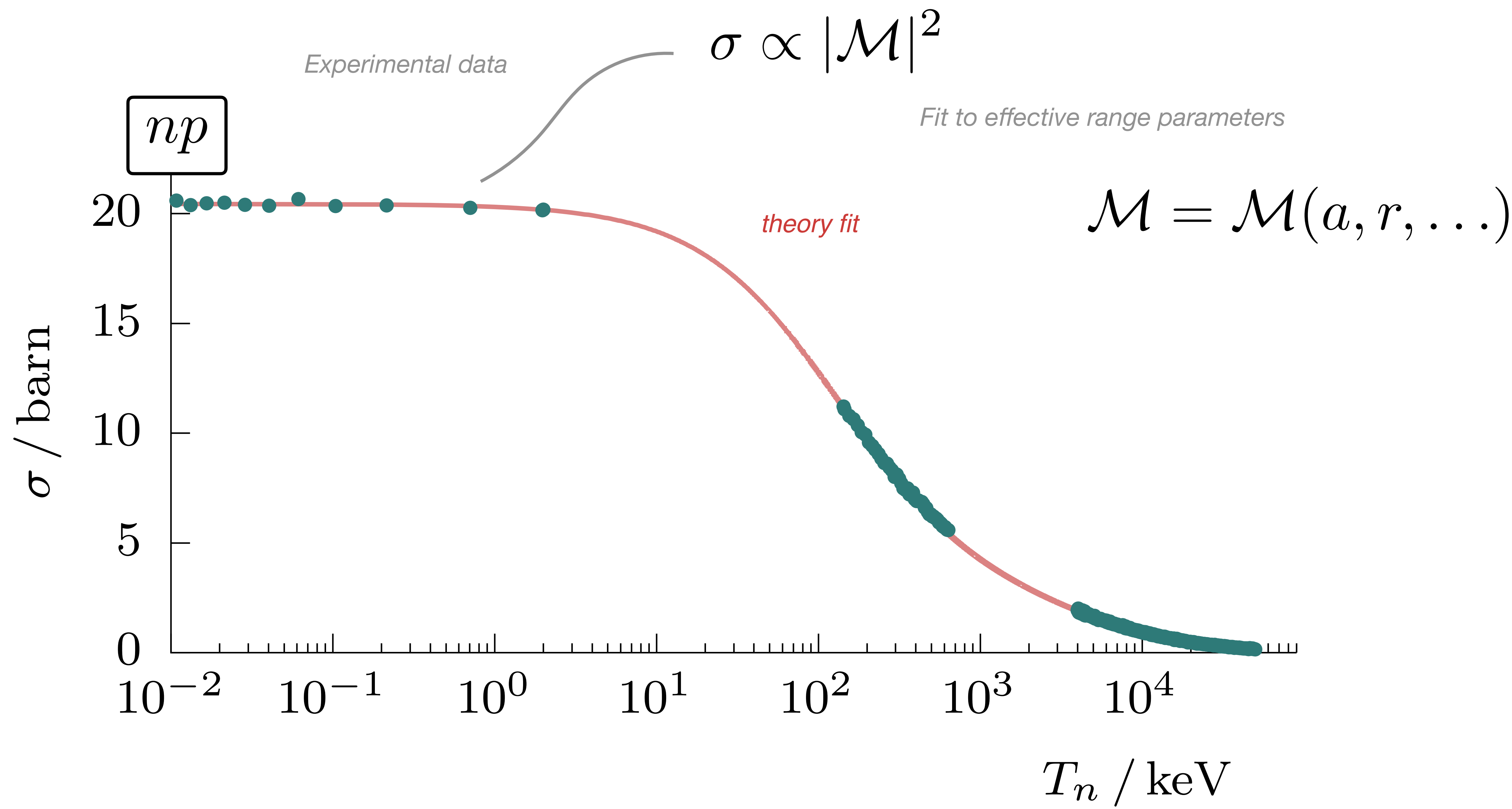
Show that at threshold the scattering amplitude is constant and fixed by the scattering length

$$\rho = \frac{q^\star}{8\pi E^\star}$$

Scattering Theory — Effective Range Expansion

Can use either K matrix or phase shifts to describe reaction (often use both)

- They are real functions, can Taylor expand in real variables — assign physical meaning
- These are fit parameters — can learn about the interaction



Spectroscopy — Bound states

Once we have the amplitude, we can now search for the ***spectrum***

- Bound states are enhancements in the amplitude — they are pole singularities

Scattering $\Longleftrightarrow$ Bound States

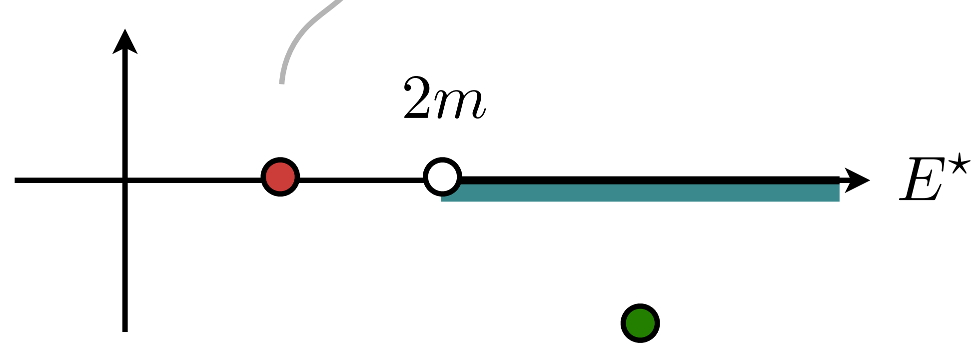
$$\mathcal{M} \propto \frac{1}{E^\star - M - i\Gamma/2}$$

Hadron Mass

$$\tau = \frac{1}{\Gamma}$$

Hadron Lifetime

Stable bound states ($\tau \to \infty$)



Unstable states (resonances)

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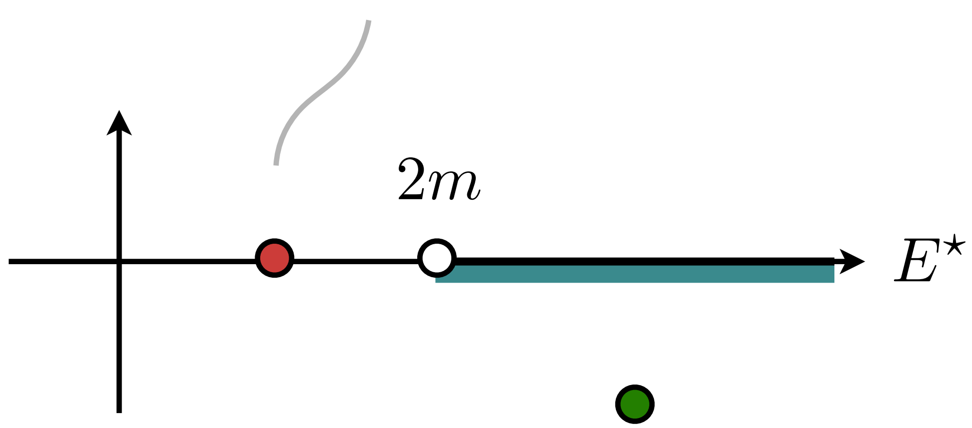

Unstable states (resonances)

> *Search for zeroes of denominator*
>
> $$\mathcal{M}^{-1}(E_b^\star) = 0$$

Spectroscopy — Bound states

Once we have the amplitude, we can now search for the ***spectrum***

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$$\mathcal{M} = \frac{8\pi E^\star}{q^\star \cot\delta - iq^\star}$$

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$$\mathcal{M}^{-1} = 0 \implies q^\star \cot\delta = iq^\star$$

Assume leading order effective range, $a > 0$

$$q^\star \cot\delta = -\frac{1}{a} \qquad \implies \qquad q^\star = \frac{i}{a} \equiv i\kappa$$

Binding momentum

$$\begin{aligned} E^\star &= 2\sqrt{m^2 + q^{\star\,2}} \\ &= 2\sqrt{m^2 - \kappa^2} \end{aligned}$$

Spectroscopy — Bound states

Once we have the amplitude, we can now search for the ***spectrum***

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$$\mathcal{M} = \frac{8\pi E^\star}{q^\star \cot\delta - i q^\star}$$

$$\mathcal{M}^{-1} = 0 \implies q^\star \cot\delta = i q^\star$$

Compare to

$$\mathcal{M} \propto \frac{1}{E^\star - M - i\Gamma/2}$$

$$\equiv i\kappa$$

Binding momentum

$$\implies \quad M = 2\sqrt{m^2 - \frac{1}{a^2}} \qquad \Gamma = 0$$

Spectroscopy — Bound states

Once we have the amplitude, we can now search for the ***spectrum***

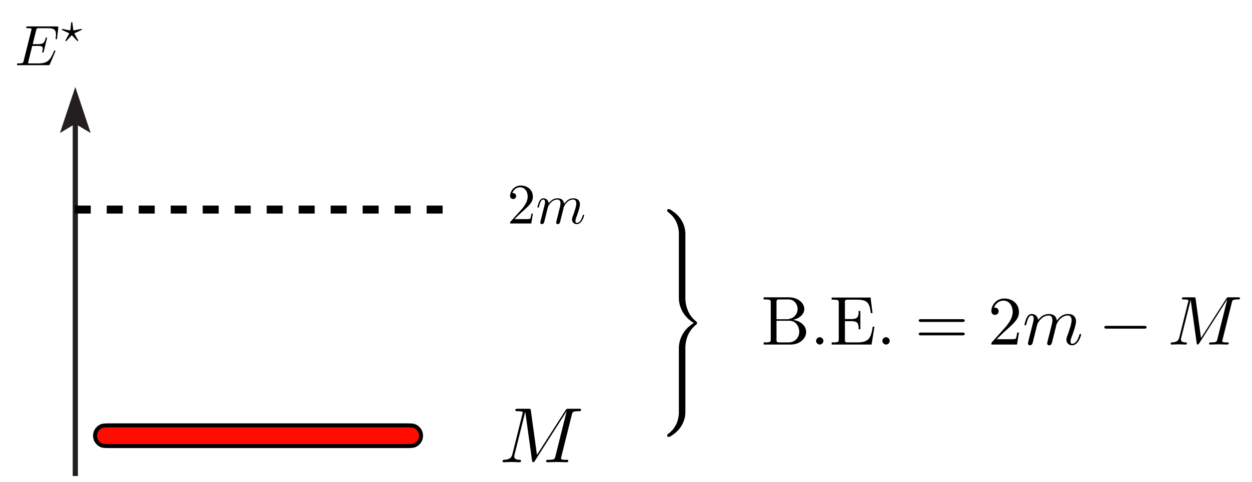
- Bound states are enhancements in the amplitude — they are pole singularities

For $a > 0$, find $M < 2m$ — Stable bound state

$$M = 2\sqrt{m^2 - \frac{1}{a^2}}$$

$$\Gamma = 0$$

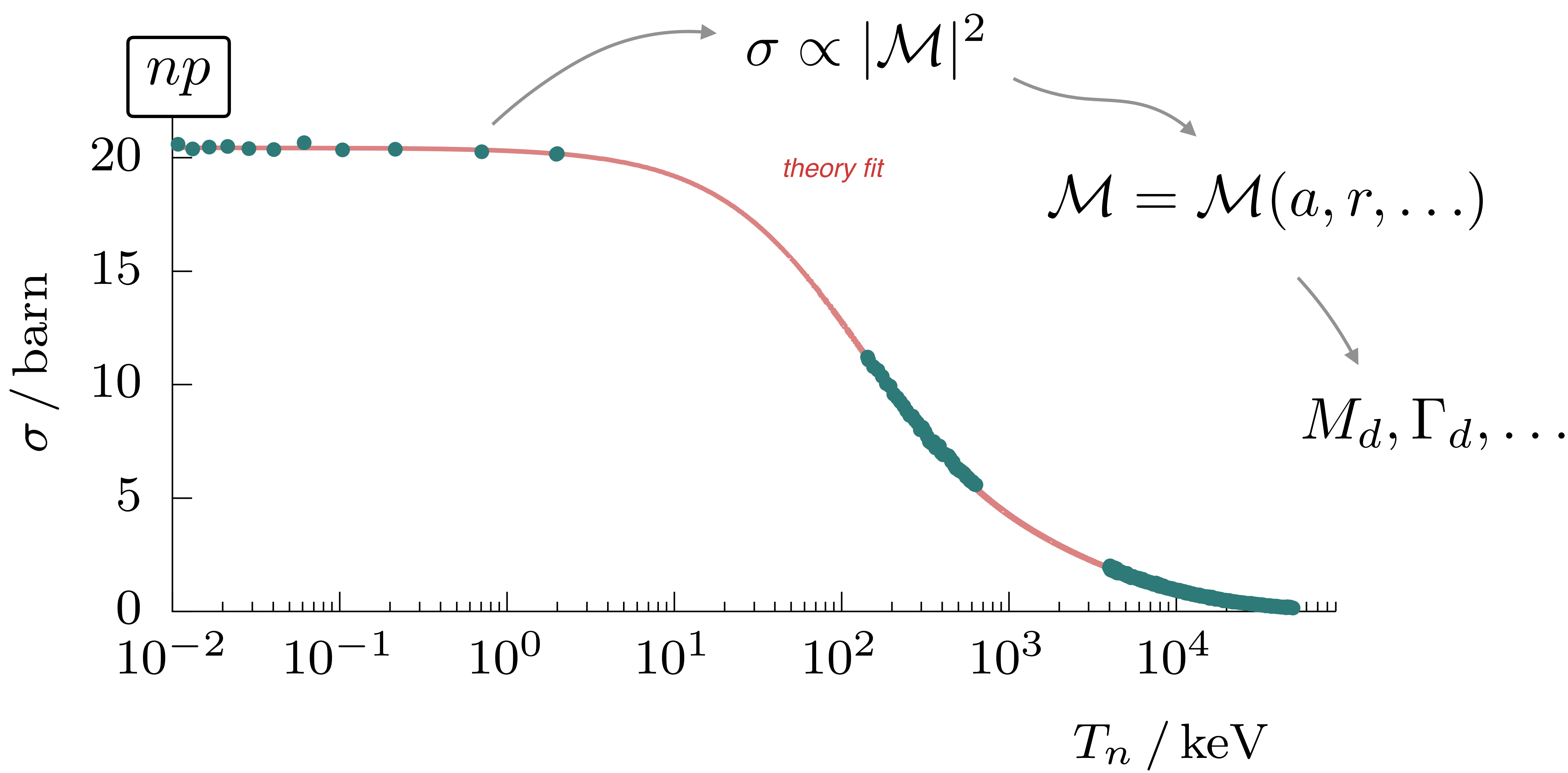
Binding energy



Neutron-Proton Scattering — Spin Effects

Goal — analyze low-energy neutron-proton scattering data

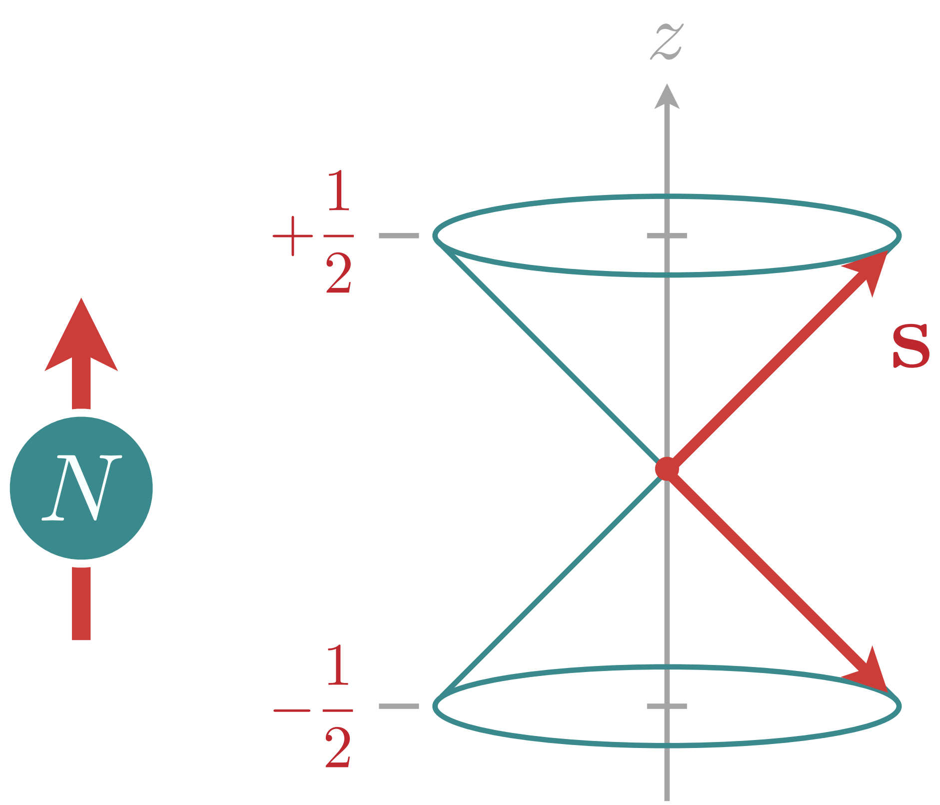
- Have a kinematic description of reaction
- Know how to parameterize amplitude in terms of physical parameters
- Can fit amplitude with unknown parameters to data
- Once we have scattering parameters, can determine bound state spectrum



Neutron-Proton Scattering — Spin Effects

So far, assumed nucleon was structure-less

- Nucleons have intrinsic *spin* angular momentum — spin-1/2 fermions
- The strong interactions can (and does) depend on the spin degrees-of-freedom
- Assume still in low-energy regime (no angular dependence)



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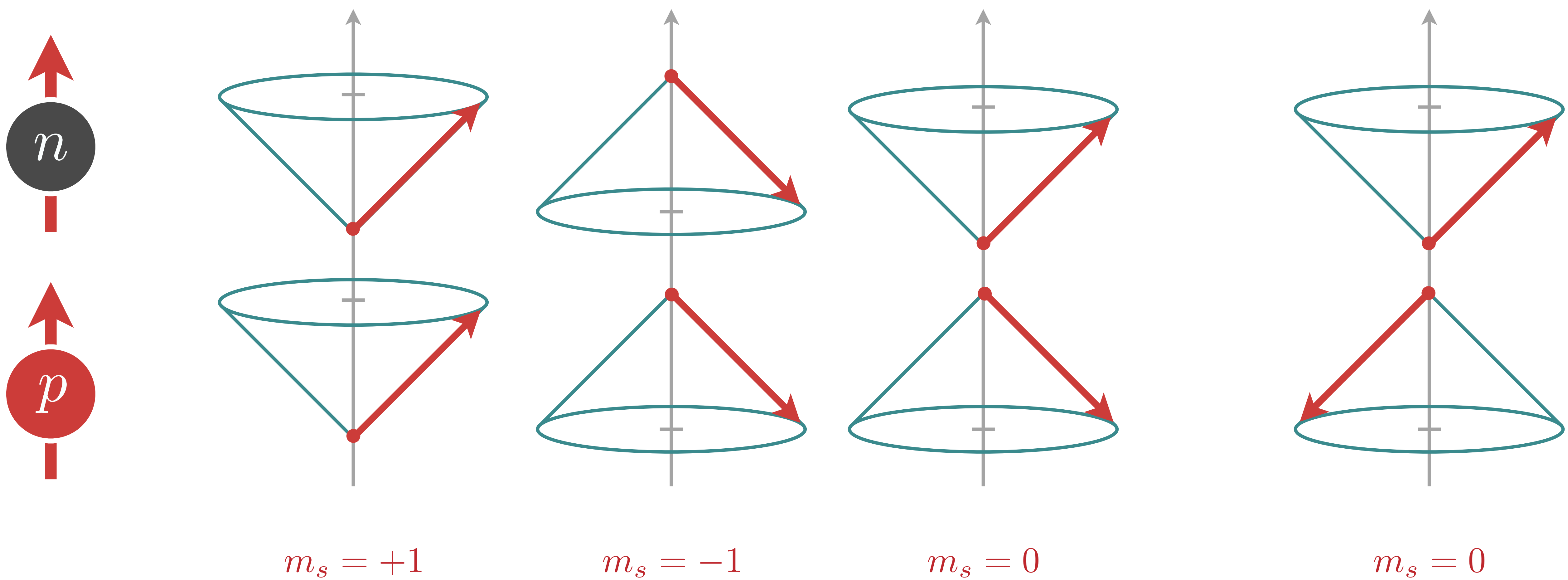
Total spin of neutron-proton system $$m_s = s_n + s_p$$

$$\implies \quad s = 0 \text{ or } 1$$



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- Nucleons have intrinsic *spin* angular momentum — spin-1/2 fermions
- The strong interactions can (and does) depend on the spin degrees-of-freedom
- Assume still in low-energy regime (no angular dependence)
- np scattering can occur in the singlet or triplet state
- Total cross section is weighted sum of singlet and triplet cross sections

$$\sigma = \frac{1}{4}\sigma_s + \frac{3}{4}\sigma_t$$

Triplet $(s = 1)$

Singlet $(s = 0)$

Total number of spin states = 4

No spin-orbit interactions

$$\ell = 0, s = 0 \text{ or } 1 \implies J = s$$

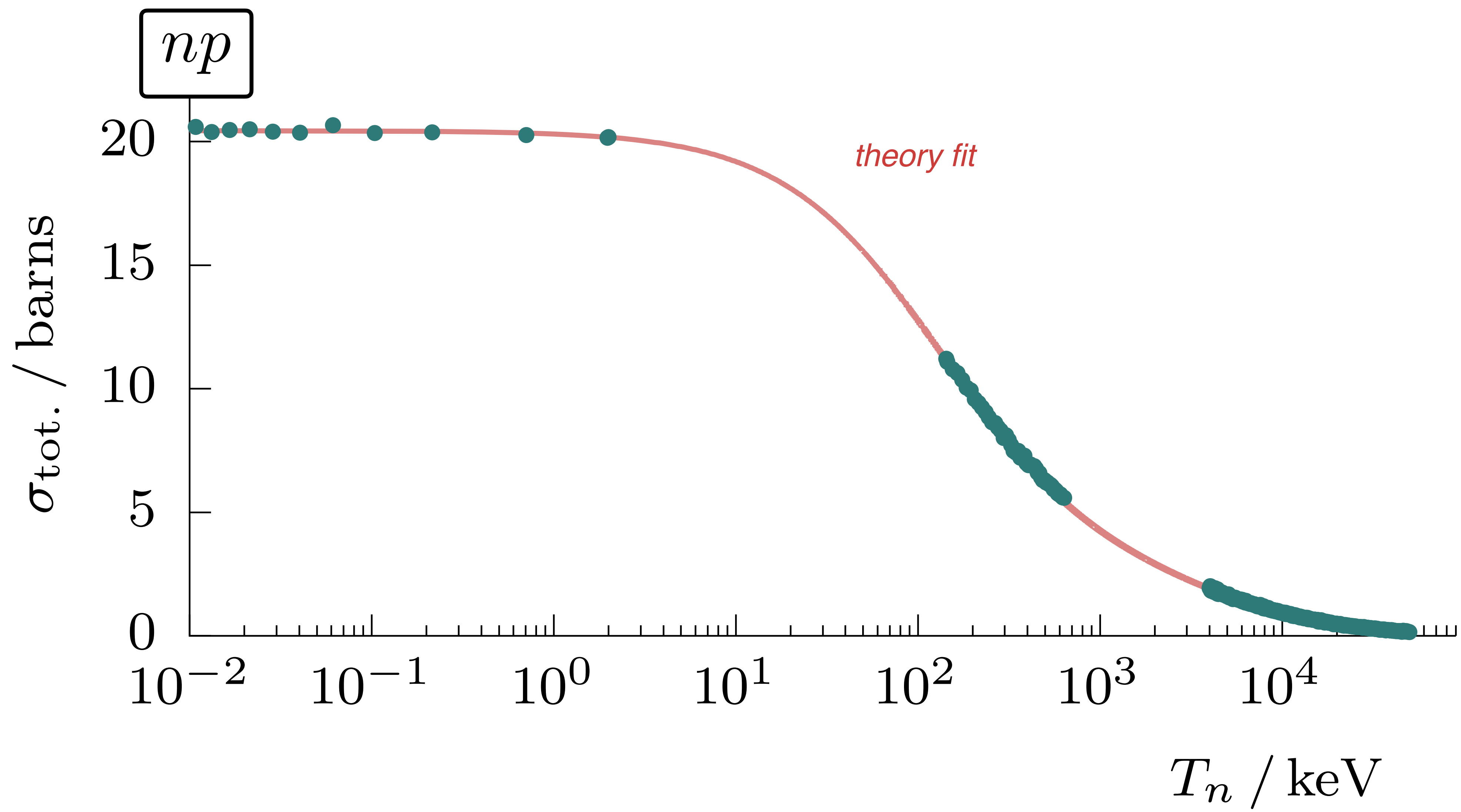
Neutron-Proton Scattering — Cross-section

$$\sigma_x = \frac{|\mathcal{M}_x|^2}{16\pi E^{\star\,2}}$$

$$\mathcal{M}_x = \frac{8\pi E^\star}{q^\star \cot\delta_x(q^\star) - iq^\star}$$

Have enough tools to analyze data

$$\sigma = \frac{1}{4}\,\sigma_s + \frac{3}{4}\,\sigma_t$$



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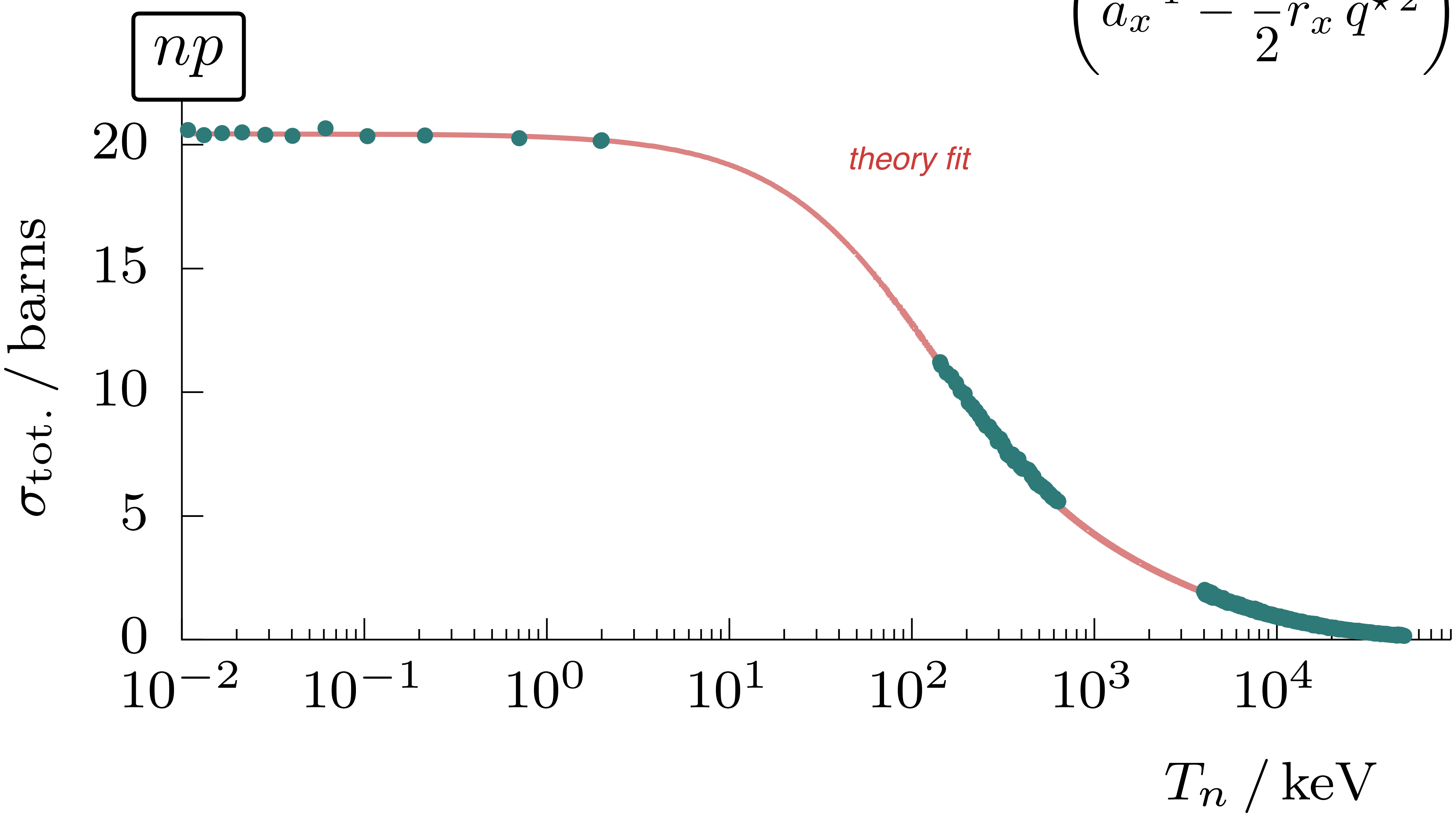
$$\sigma = \frac{1}{4}\sigma_s + \frac{3}{4}\sigma_t$$

Low-energy shape-independent approximation

$$\sigma_x = \frac{4\pi}{\left(a_x^{-1} - \frac{1}{2} r_x\, q^{\star\,2}\right)^2 + q^{\star\,2}}$$

$$q^{\star\,2} = \frac{1}{2} m T_n$$



Neutron-Proton Scattering — Cross-section

$m \approx 940$ MeV

$1\,\text{fm} \approx \dfrac{1}{200\ \text{MeV}}$

$1\,\text{b} = 100\,\text{fm}^2\ (= 10^{-28}\,\text{m}^2)$

Have enough tools to analyze data

$$\chi^2 = \sum \left(\frac{\sigma_{\text{exp.}} - \sigma_{\text{th}}}{\Delta\sigma_{\text{exp.}}} \right)^2$$

My simple fit

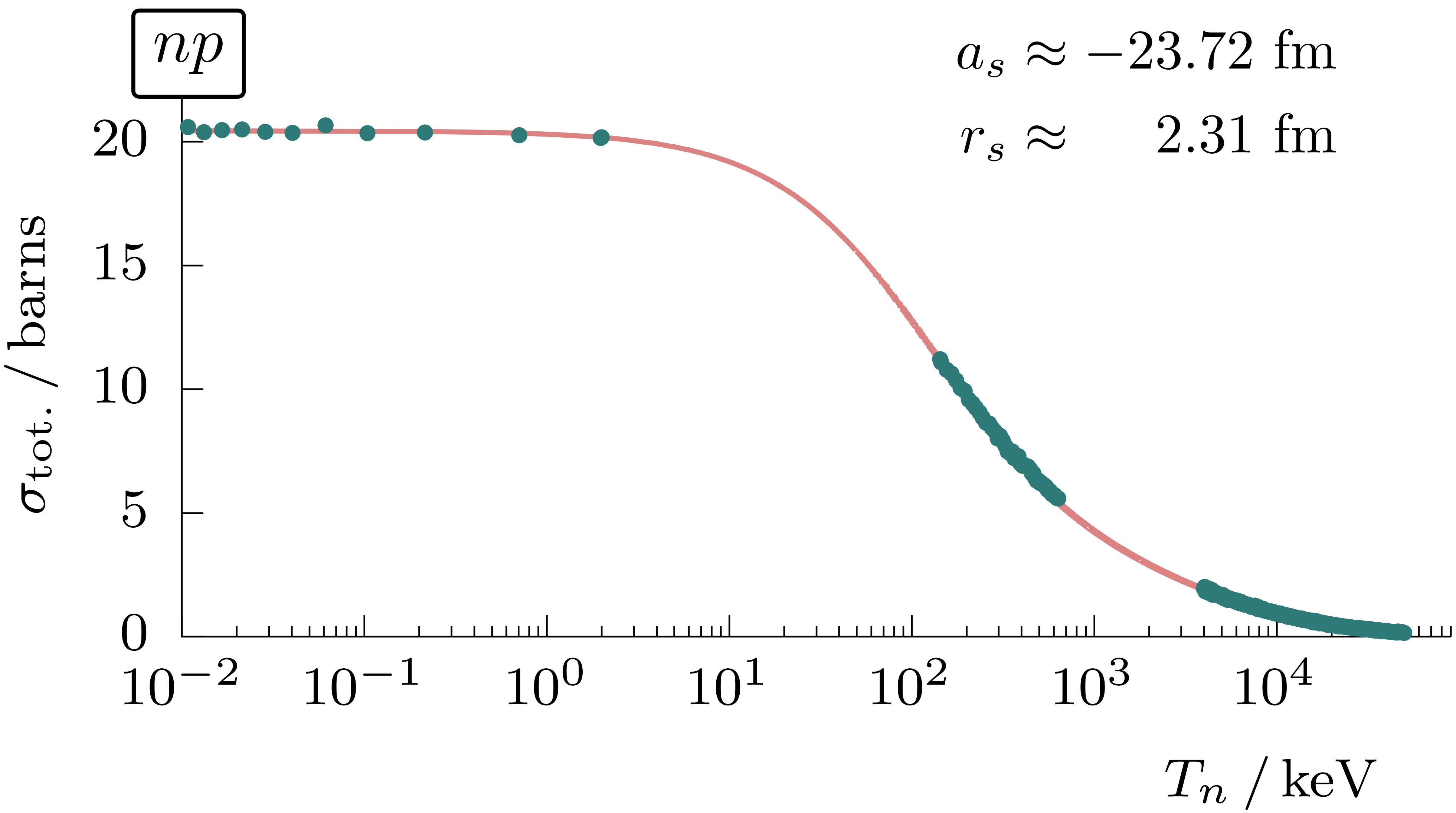
$$\chi^2/\nu = 472.9/(486-4) \approx 0.98$$

$$a_t \approx 5.41\ \text{fm}$$

$$r_t \approx 1.62\ \text{fm}$$

$$a_s \approx -23.72\ \text{fm}$$

$$r_s \approx 2.31\ \text{fm}$$



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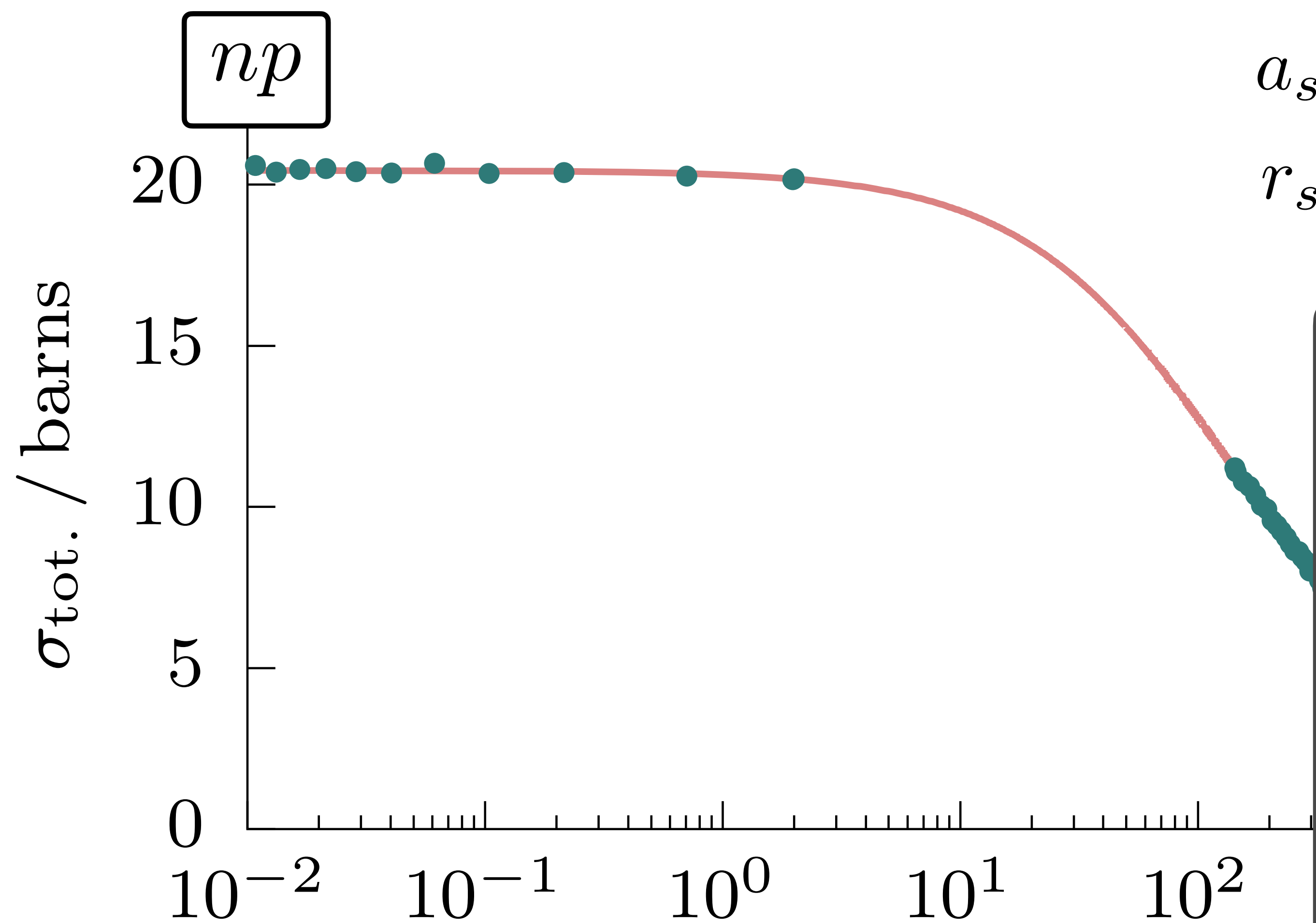

Compare to Hackenberg (2006)

$$\begin{aligned} a_t &= 5.4114 \pm 0.0015\,\mathrm{fm} \\ r_t &= 1.7468 \pm 0.0019\,\mathrm{fm} \\ a_s &= -23.7153 \pm 0.0043\,\mathrm{fm} \\ r_s &= 2.754 \pm 0.059\ \mathrm{fm} \end{aligned}$$

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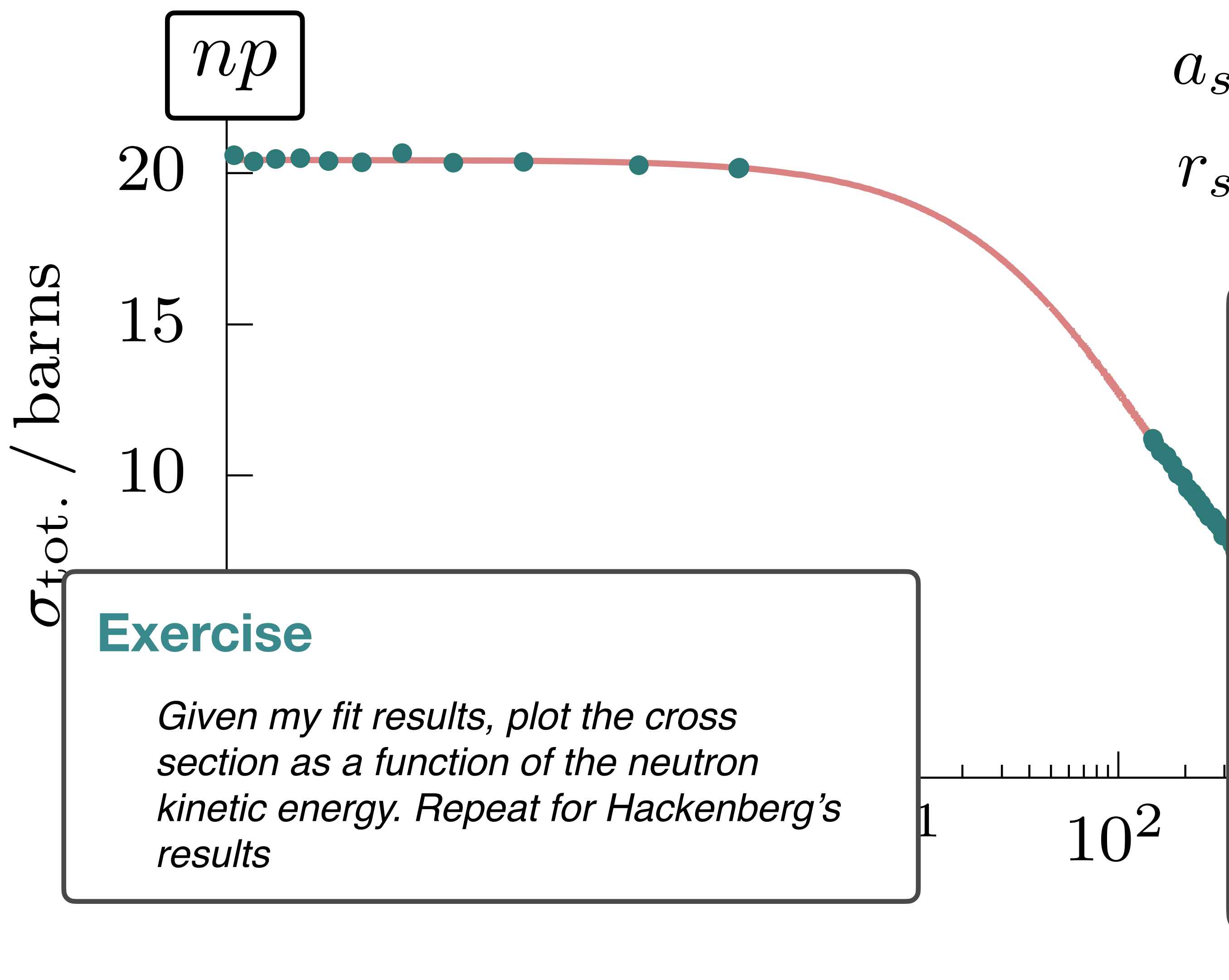

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Exercise

Given my fit results, plot the cross section as a function of the neutron kinetic energy. Repeat for Hackenberg's results

Neutron-Proton Scattering — the Deuteron

$$m \approx 940 \text{ MeV}$$

$$1 \text{ fm} \approx \frac{1}{200 \text{ MeV}}$$

Examine bound-state spectrum

- Focus on spin-triplet results

My simple fit

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My simple fit

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Search for poles of amplitude

$$\mathcal{M}^{-1} = 0 \qquad \Longrightarrow \qquad -\frac{1}{a} + \frac{1}{2} r q^{\star\,2} - i q^{\star} = 0$$

Neutron-Proton Scattering — the Deuteron

$m \approx 940 \text{ MeV}$

$1 \text{ fm} \approx \dfrac{1}{200 \text{ MeV}}$

Examine bound-state spectrum

- Focus on spin-triplet results

My simple fit

$$a_t \approx 5.41 \text{ fm}$$

$$r_t \approx 1.62 \text{ fm}$$

Search for poles of amplitude

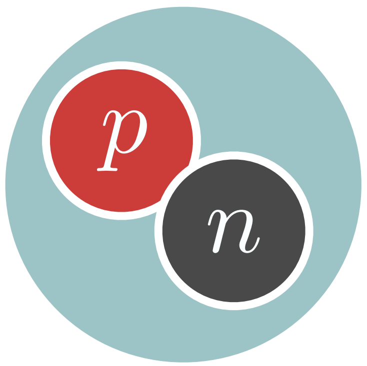
$$\mathcal{M}^{-1} = 0 \qquad \Longrightarrow \qquad -\frac{1}{a} + \frac{1}{2} r q^{\star\,2} - i q^{\star} = 0$$

Find a shallow bound state (with my numbers) — the deuteron

$$M_d \approx 1878 \text{ MeV}$$

$$\Gamma_d = 0$$

$$\text{B.E.} = 2.1 \text{ MeV}$$



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Exercise

Given my fit results, compute the deuteron mass. Repeat the exercise for Hackenberg's results

Search for poles of amplitude

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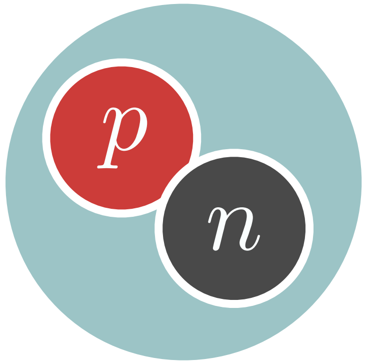
Neutron-Proton Scattering — the Deuteron

Bound state spectrum determined from scattering analysis

The deuteron is a shallow bound state of triplet S-wave np scattering



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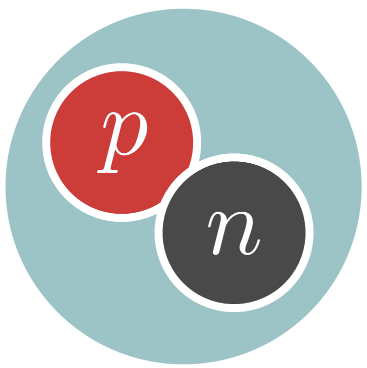

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Spectral properties checklist

- [x] Mass
- [x] Lifetime
- [x] Spin
- [x] Parity
- [x] Charge

...

Outline

Hadrons, Quarks, & QCD

Hadron Spectroscopy

The Quark Model

Quantum Chromodynamics

Scattering Theory & the Hadron Spectrum

Scattering Amplitudes

Bound & Resonant States

Lattice QCD & Hadron Spectroscopy

Lattice QCD

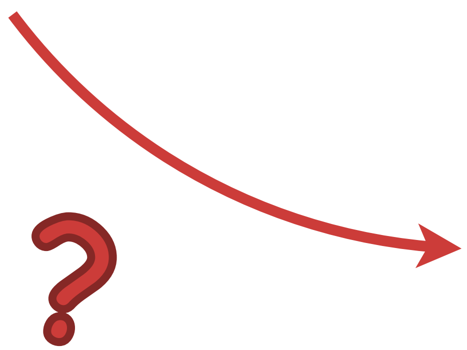
Lüscher & the Finite-Volume

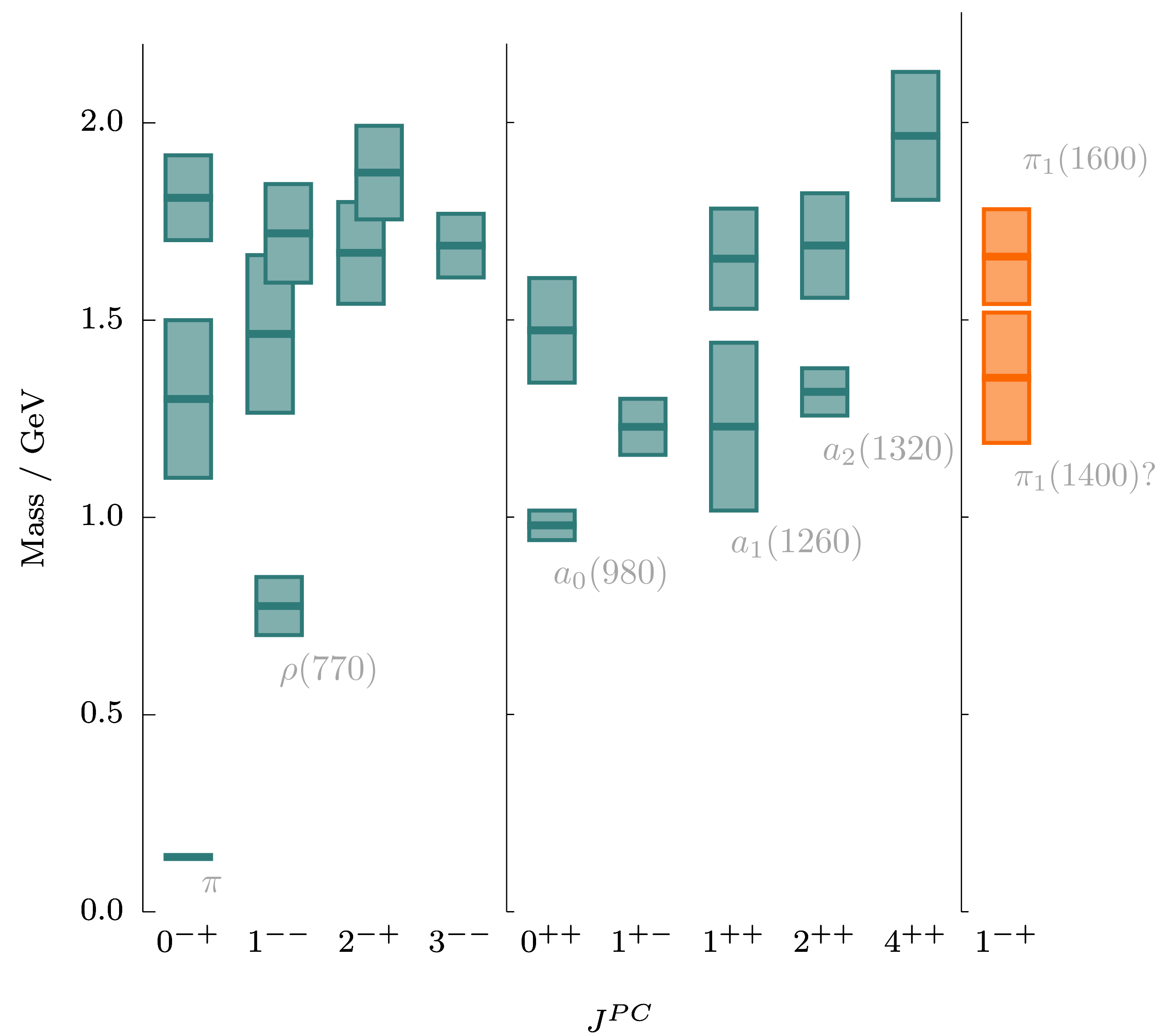
Hadrons and QCD

QCD allows for more exotic states — Do we understand QCD?

- How to connect QCD to hadrons?
- Need to understand how to quantify what the hadrons are in nature
- Need to find non-perturbative approach to access these hadrons ***rigorously***

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f\right) \psi_f - \frac{1}{2}\mathrm{tr}\left(\mathbf{G}_{\mu\nu}\mathbf{G}^{\mu\nu}\right)$$

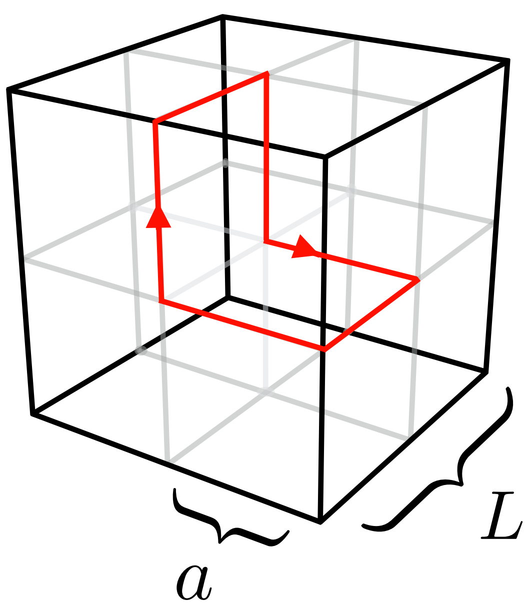




Lattice QCD

Lattice QCD offers a systematic approach to compute hadrons from QCD

- Numerically evaluate QCD path integral via Monte Carlo sampling



$$Z_{\mathrm{QCD}} = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A_\mu \, e^{iS_{\mathrm{QCD}}(\psi,\bar{\psi},A_\mu)}$$

- *Euclidean spacetime,* $t \to -i\tau$
- *Finite volume,* L
- *Discrete spacetime,* a
- *Heavier than physical quark mass,* $m > m_{\mathrm{phys.}}$

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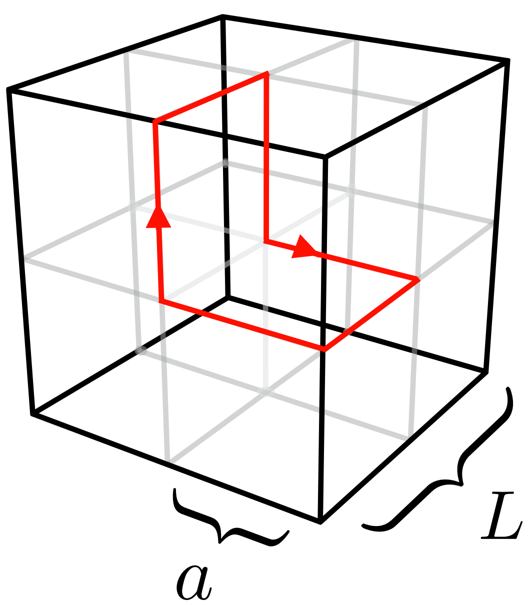

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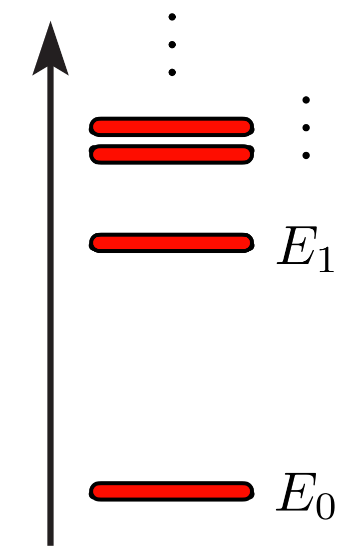
- *Heavier than physical quark mass,* m

Correlation functions yield discrete spectrum

$$\langle \mathcal{O}(\tau) \mathcal{O}^\dagger(0) \rangle = \sum_{\mathfrak{n}} \left| \langle 0 | \, \mathcal{O} \, | \mathfrak{n} \rangle \right|^2 \, e^{-E_{\mathfrak{n}} \tau}$$



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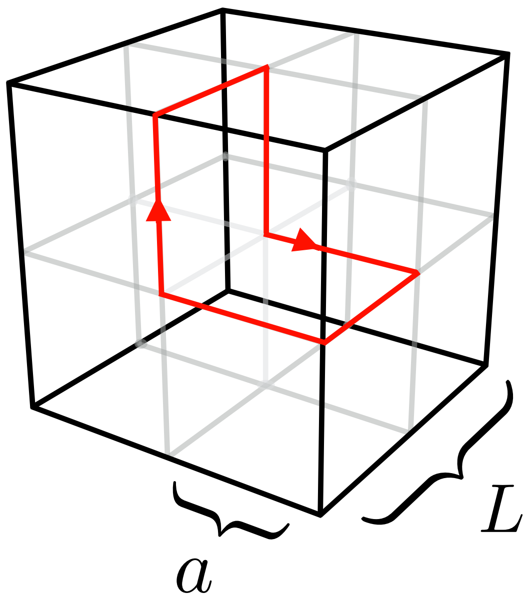

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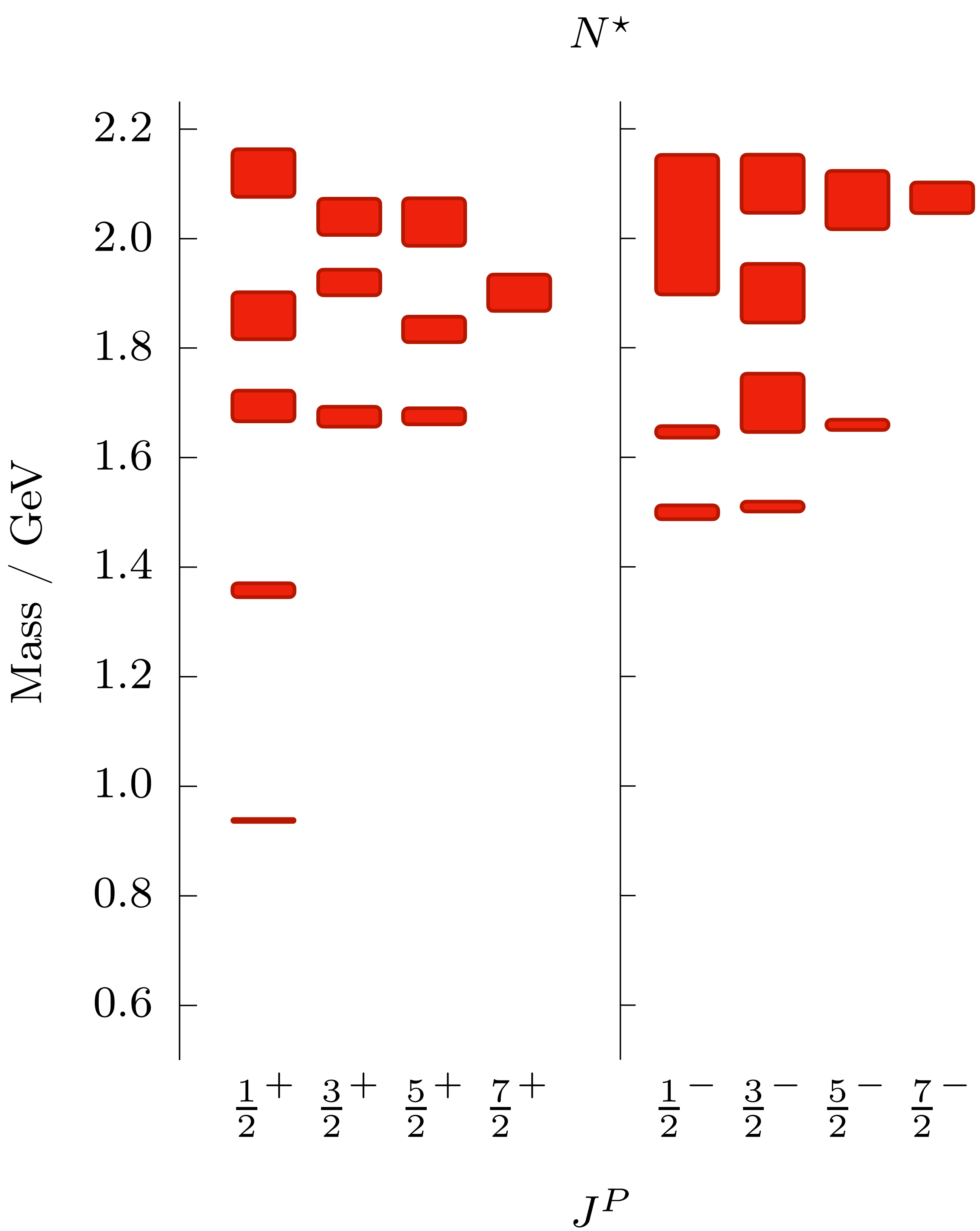

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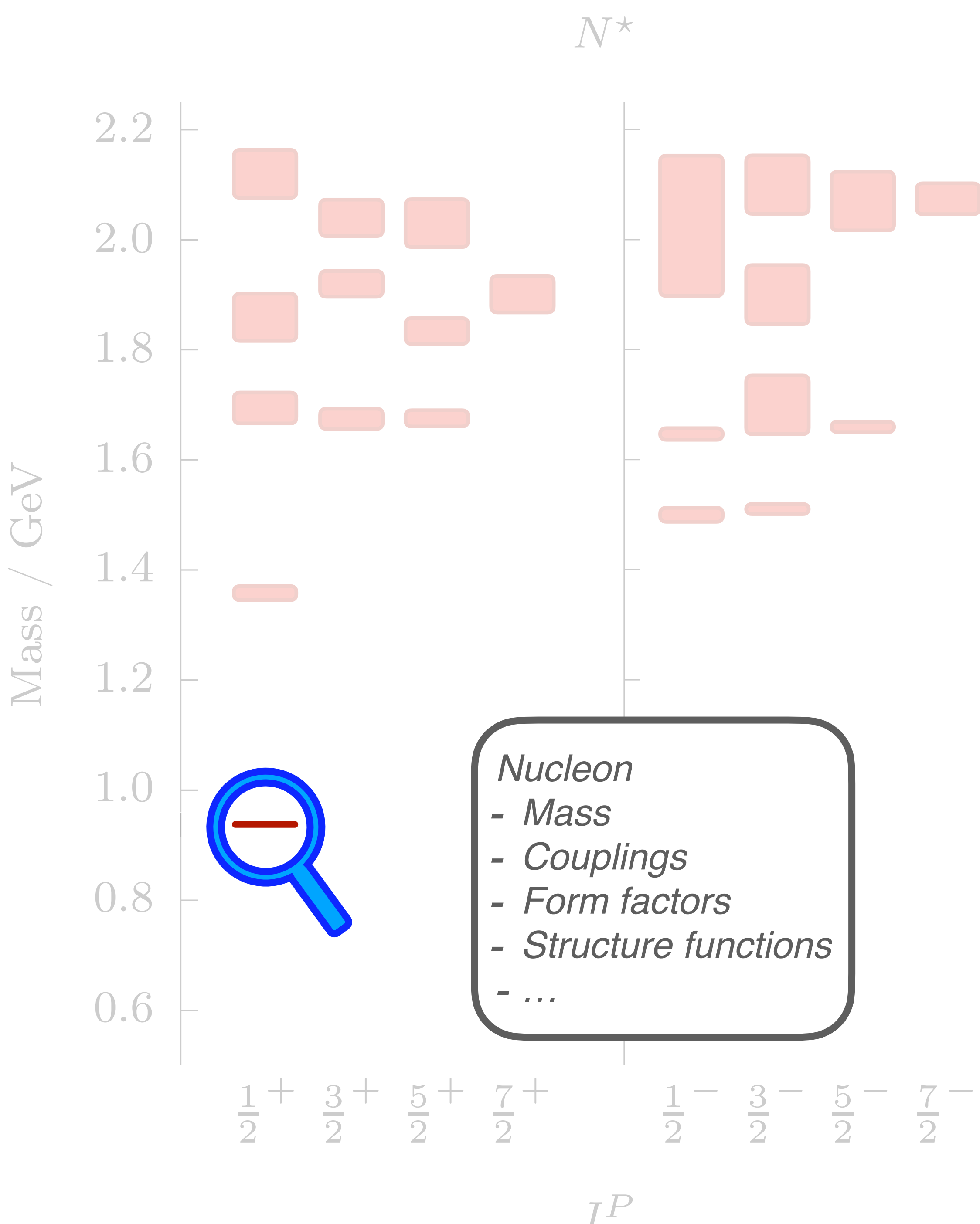
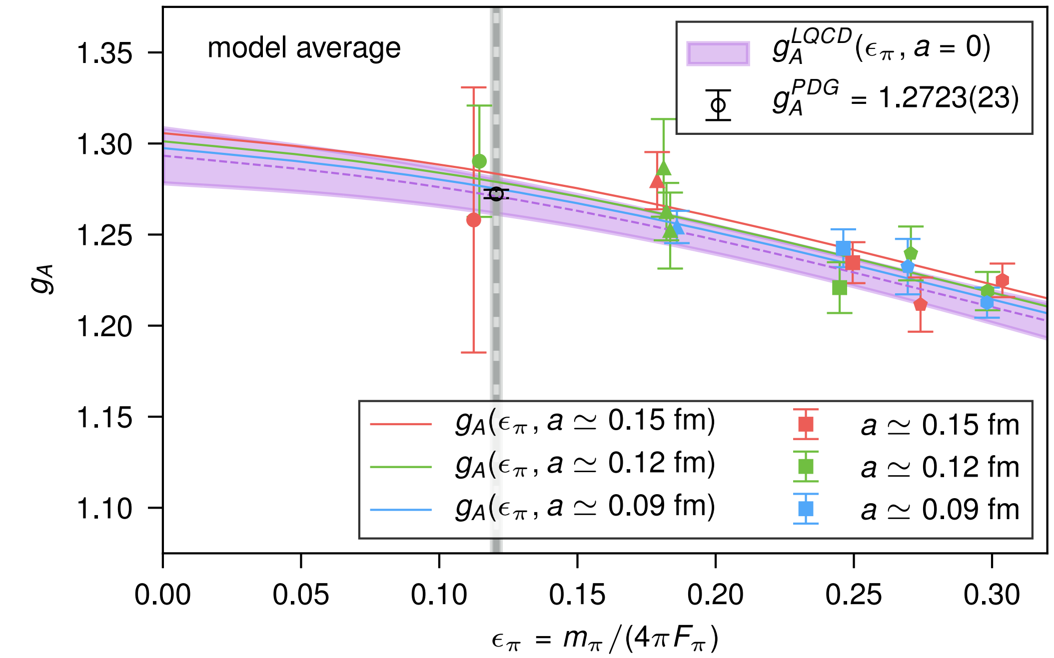
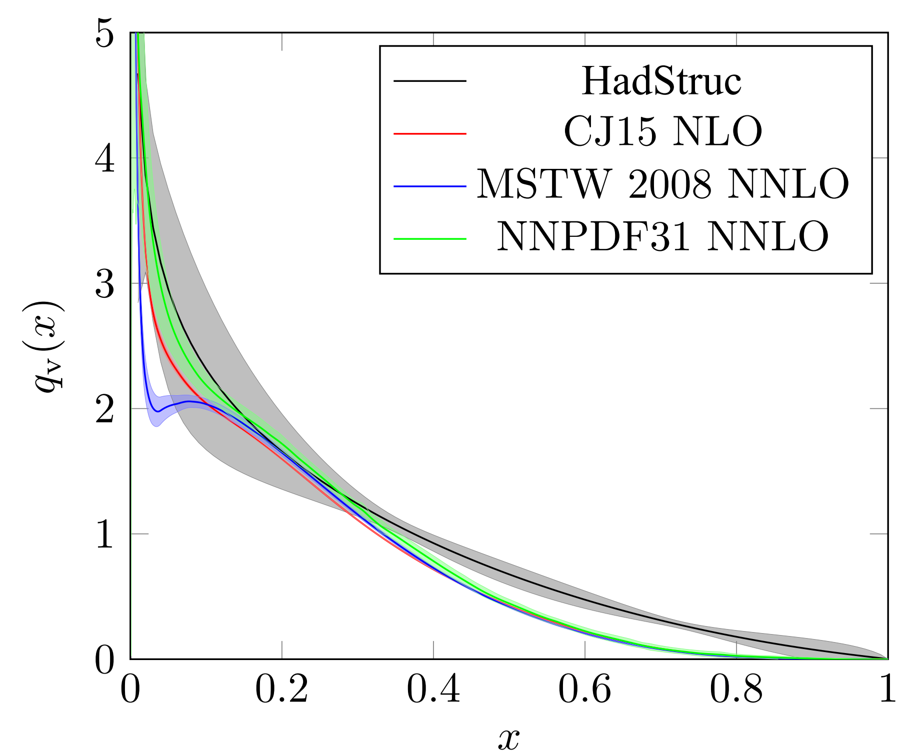




C. Chang et al.
Nature **558**, (2018) 7708



B. Joó et al. [HadStruc]
Phys. Rev. Lett. **125**, (2020) 232003

PDG listings

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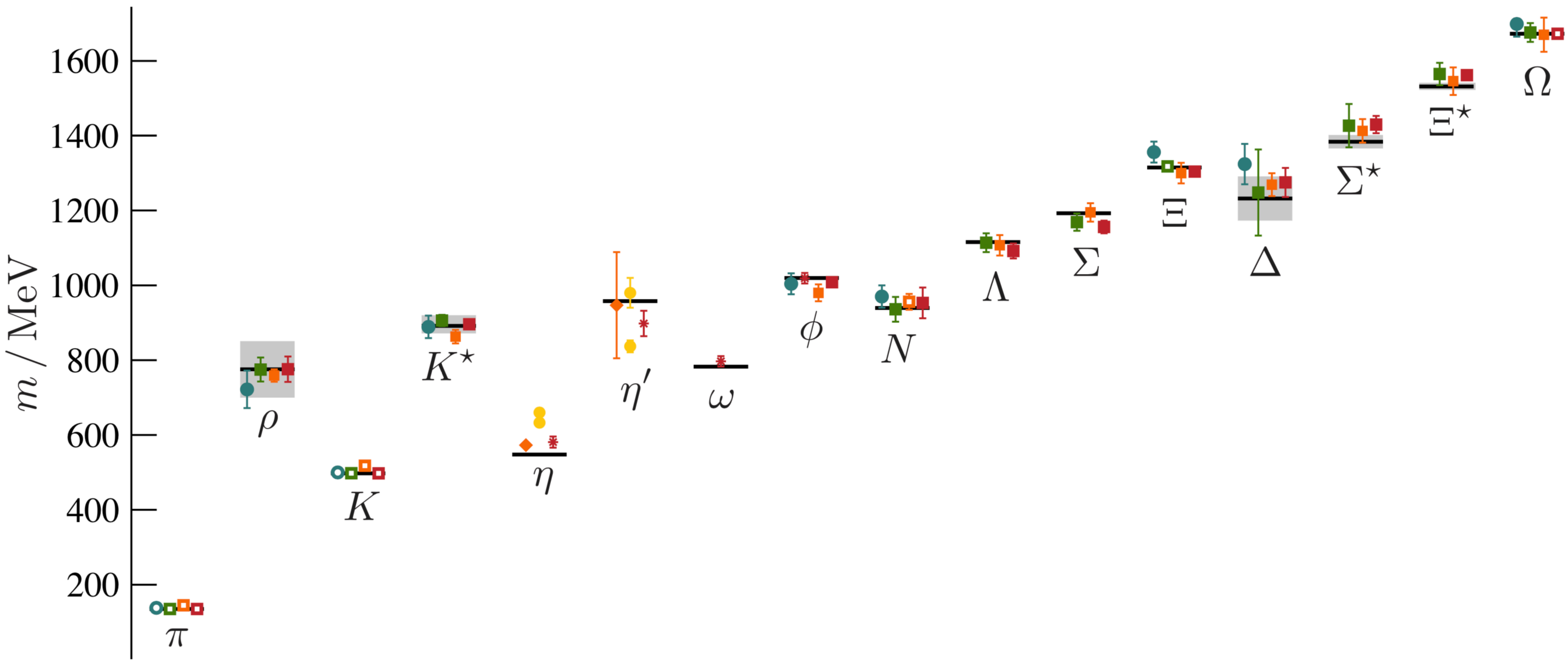

summary compiled by Andreas Kronfeld
Ann. Rev. Nucl. Part. Sci 62 265 (2012)

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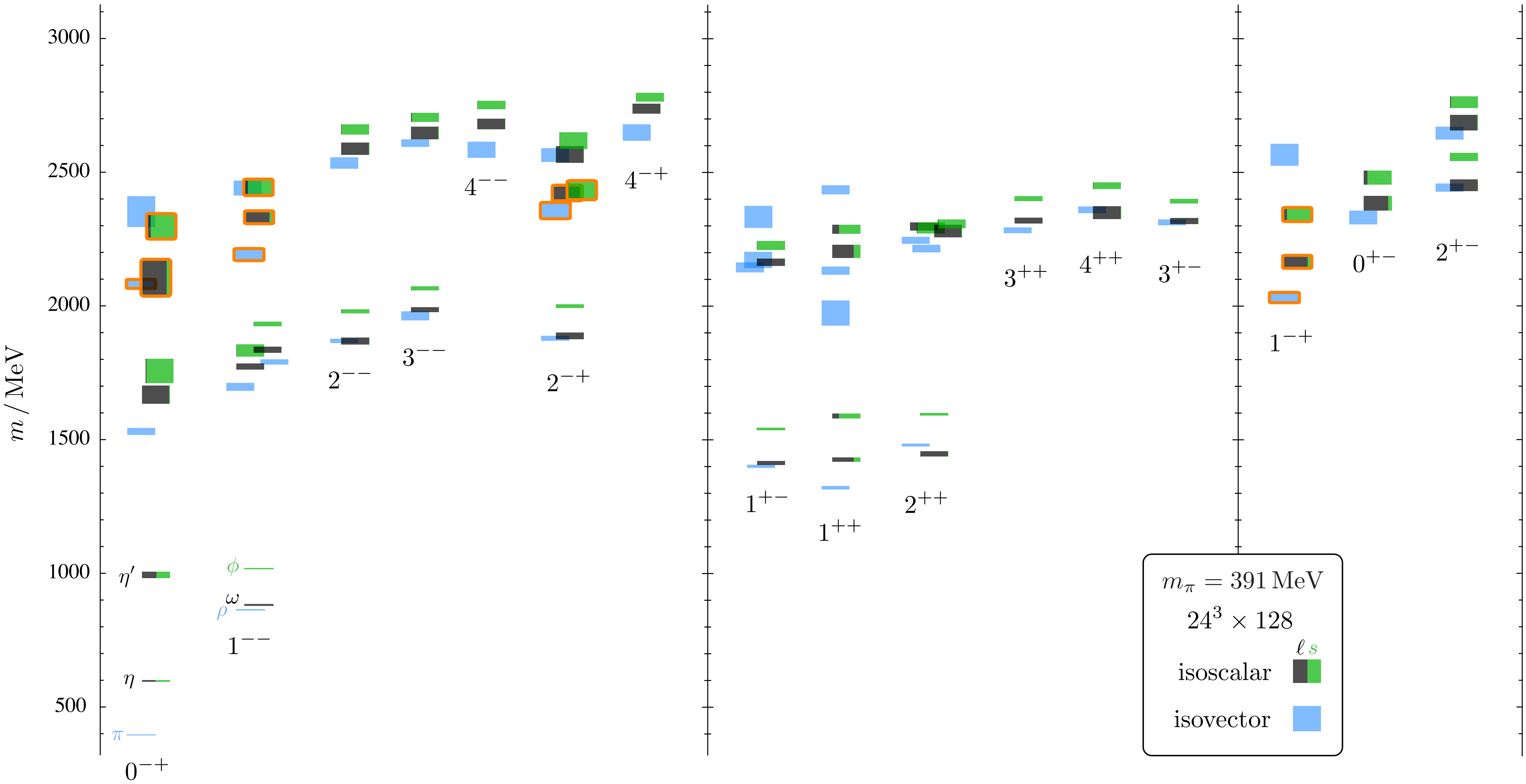

J. Dudek, R. Edwards, P. Guo, and C. Thomas
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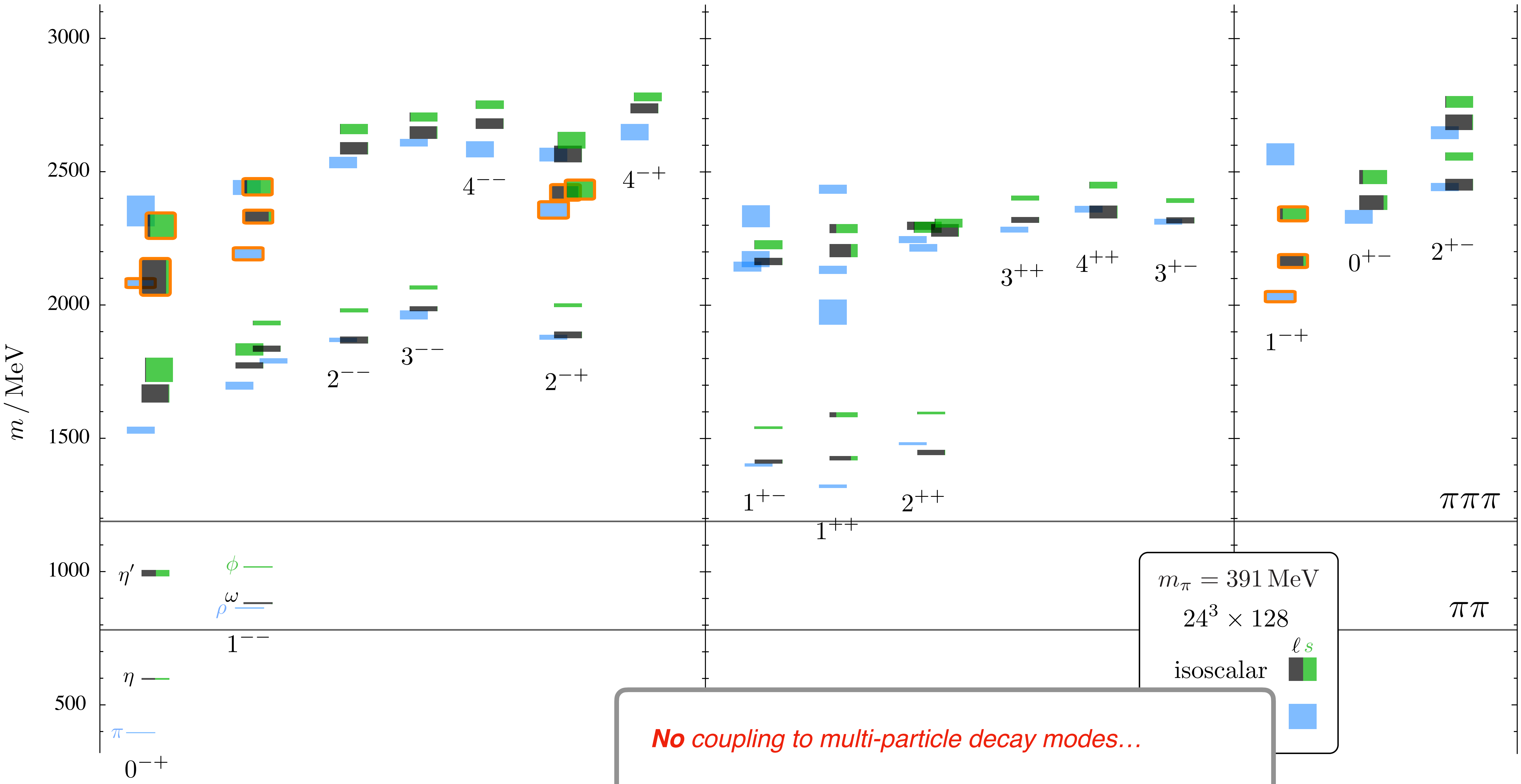


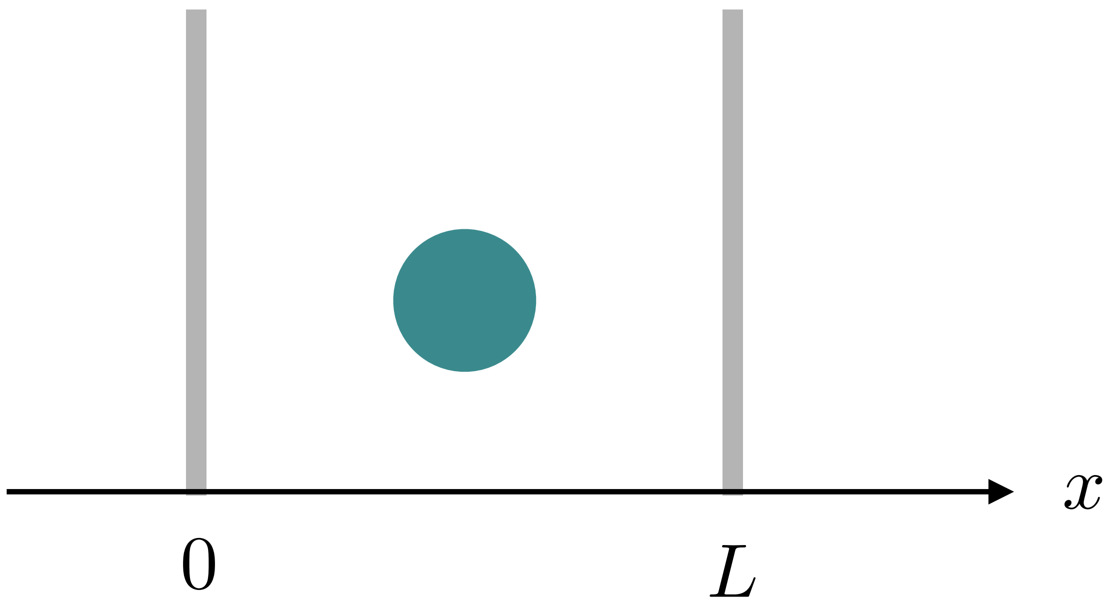
***No** coupling to multi-particle decay modes…*

…An incomplete picture of QCD spectrum

J. Dudek, R. Edwards, P. Guo, and C. Thomas
Phys.Rev.D **88**, 094505 (2013)

Particles in a box

Consider a particle, confined in a 1d box



Schrödinger equation

$$-\frac{1}{2m}\frac{\mathrm{d}^2\psi(x)}{\mathrm{d}x^2} = E\psi(x)$$

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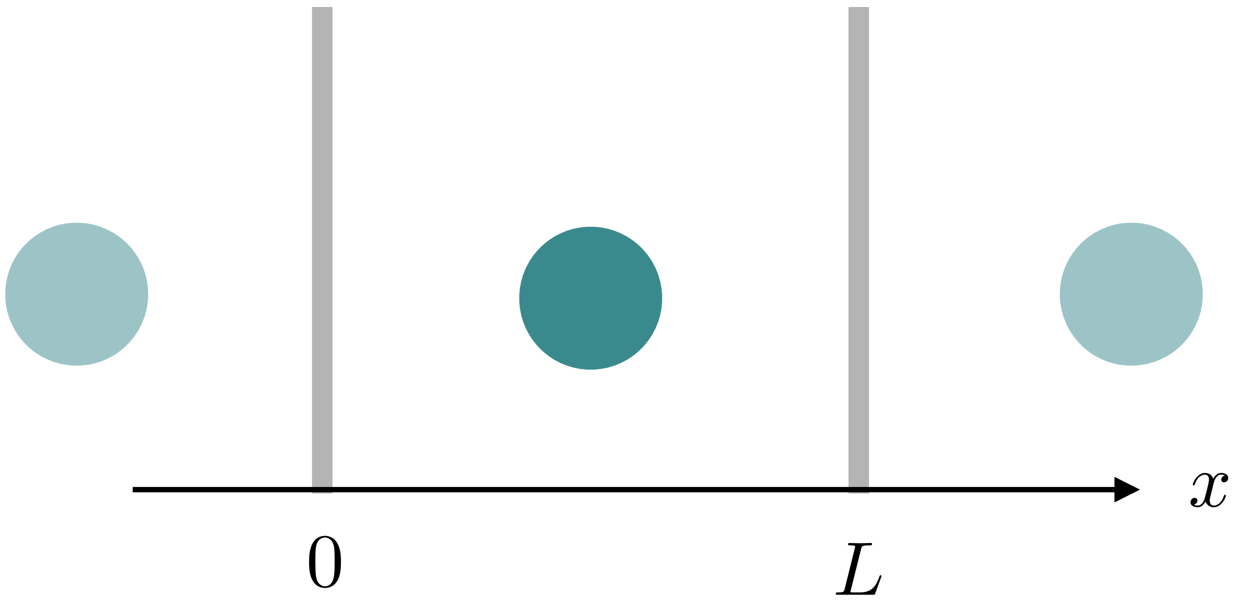

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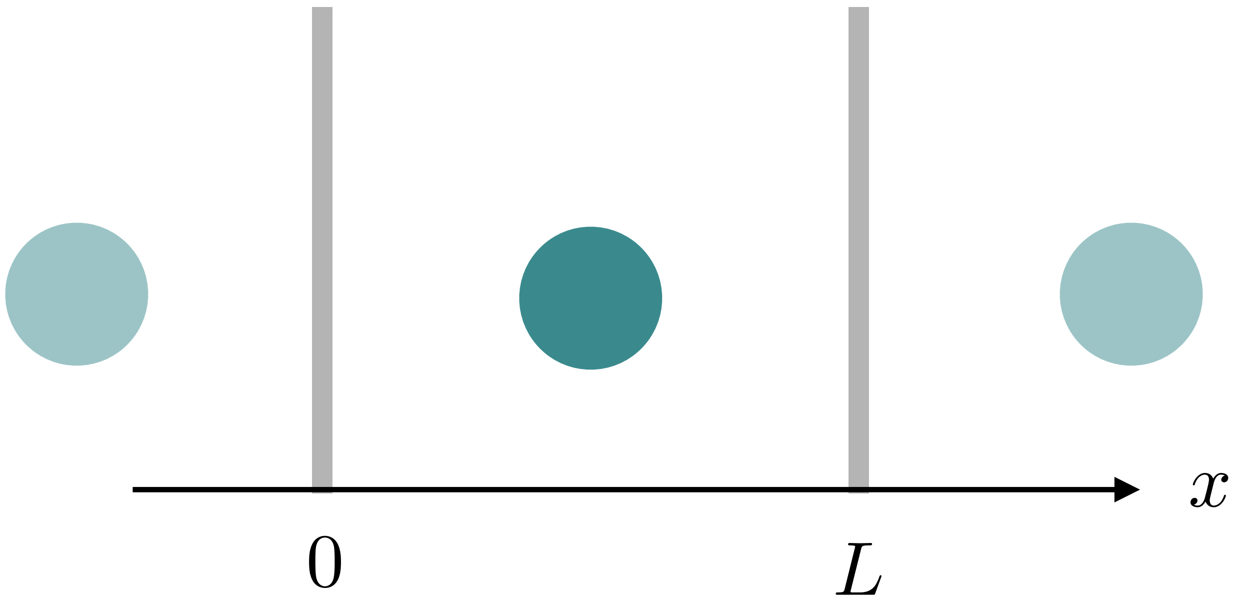

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$$p_n = \frac{2\pi}{L}n \ , \qquad n = 0, \pm 1, \pm 2, \ldots \qquad \Longrightarrow \quad E_n = \frac{p_n^2}{2m} = \frac{2\pi^2}{mL^2}n^2$$

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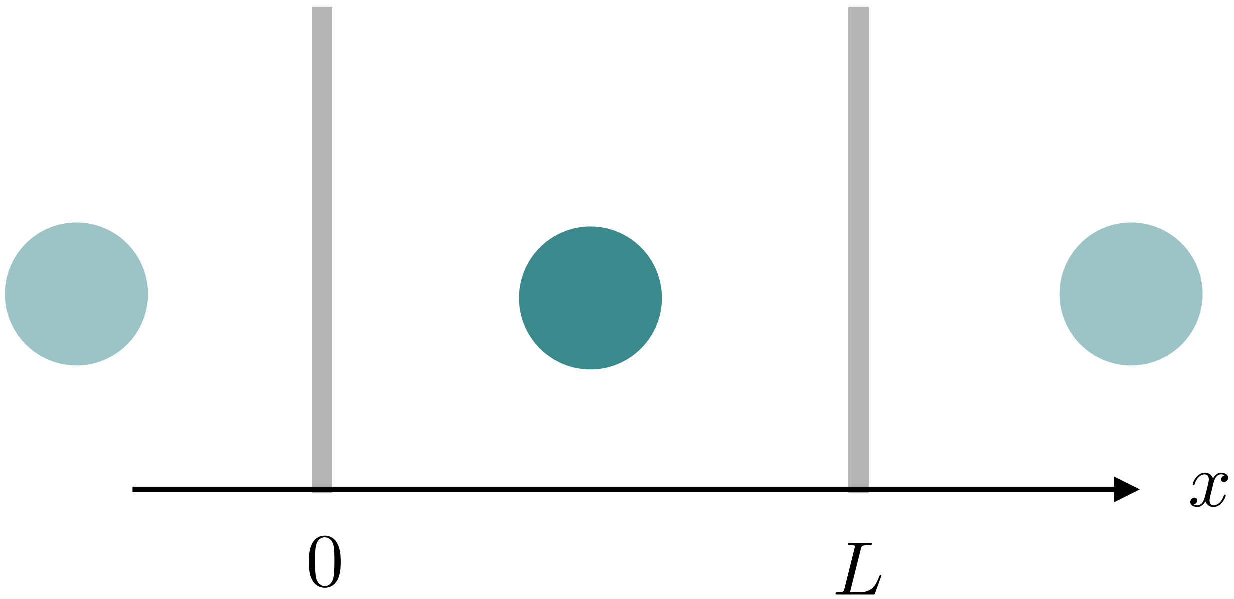

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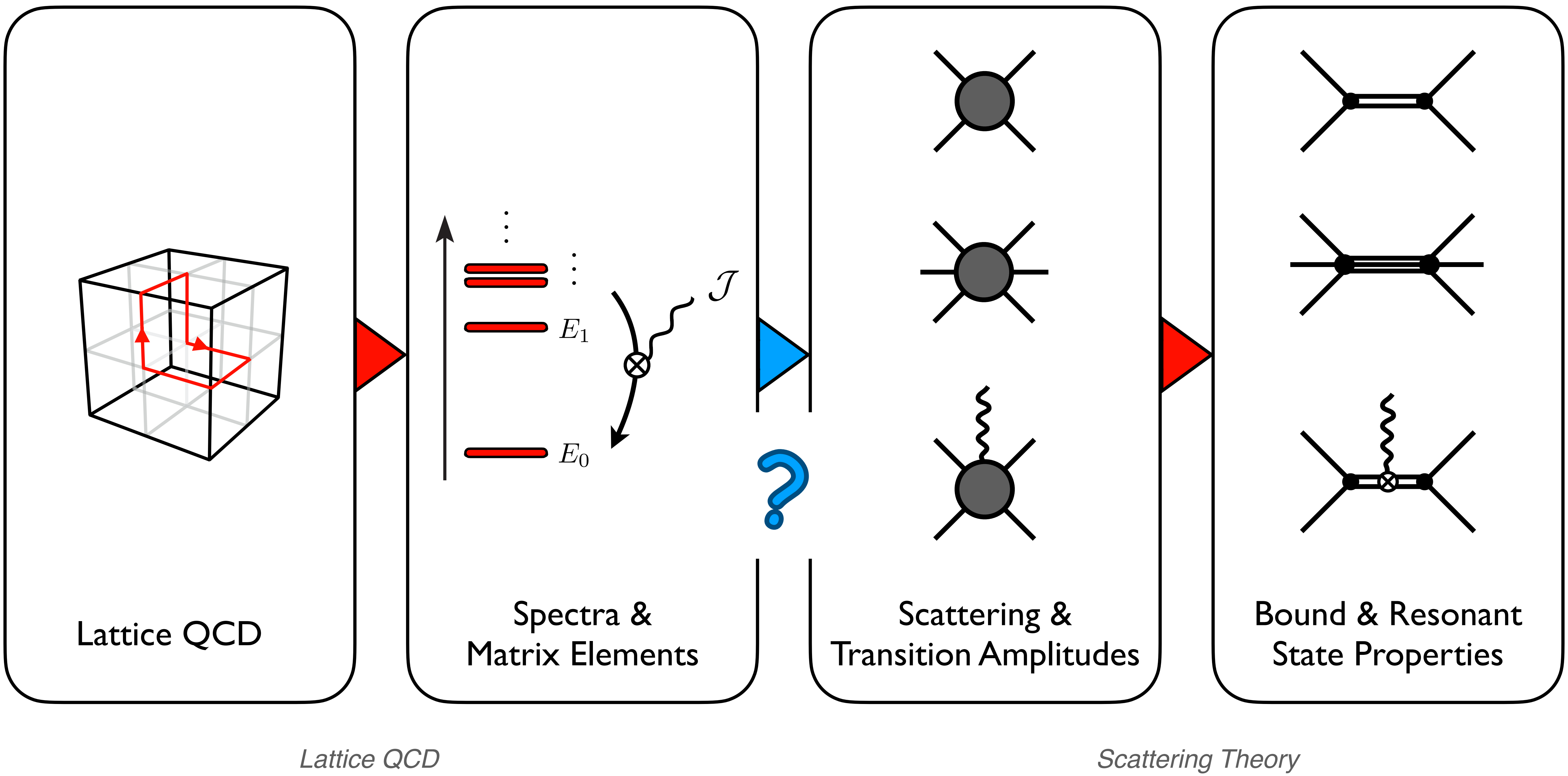
Connecting Scattering Physics to QCD

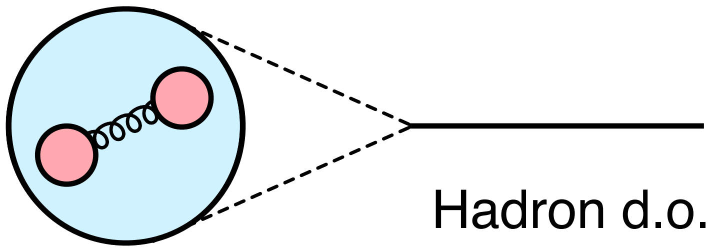
Path to few-body physics from QCD

- Link finite-volume spectra and matrix elements to scattering amplitudes
- Tools: *Lattice QCD*, *Scattering Theory*, & *Effective Field Theory*



Connecting Scattering Physics to QCD

Use generic Effective Field Theory to generate connection



Connecting Scattering Physics to QCD

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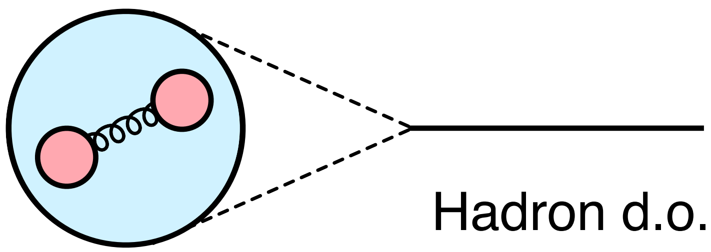
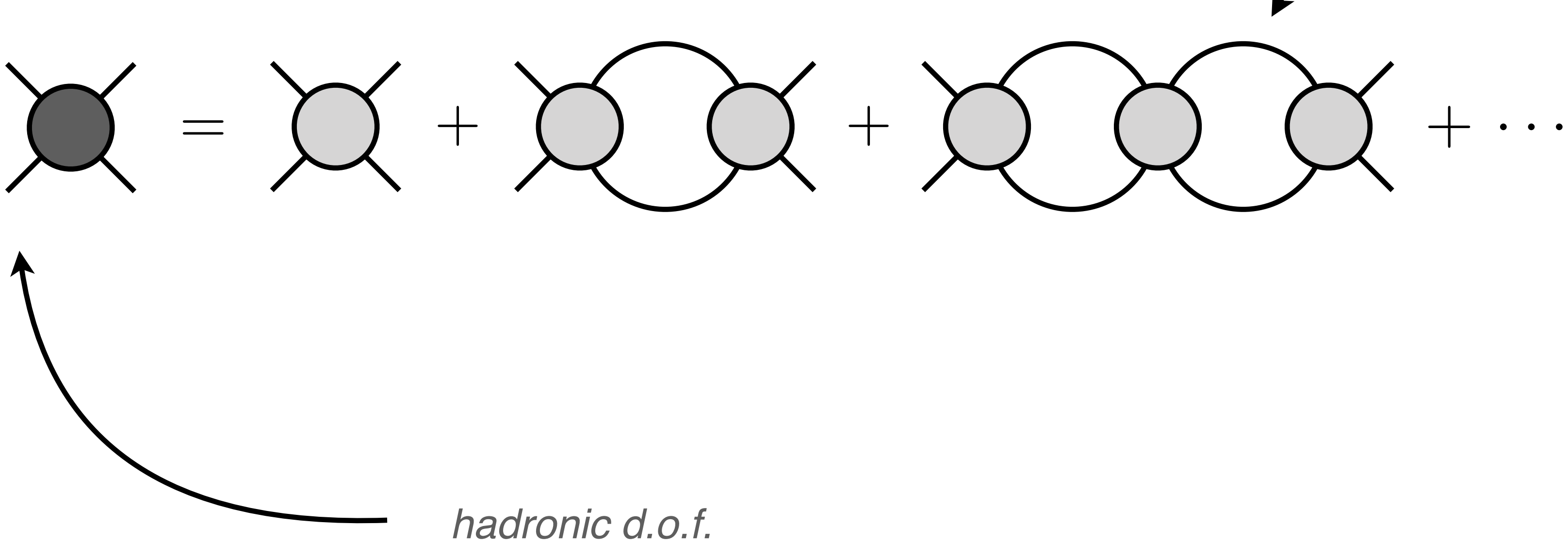
Quarks and gluons at low energy



Hadron d.o.f., e.g. $\pi, K, \ldots$

e.g., $\mathbf{2} \rightarrow \mathbf{2}$ scattering within some generalized EFT

Fully dressed propagator



hadronic d.o.f.

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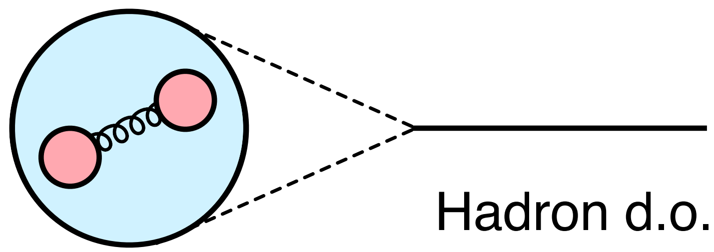

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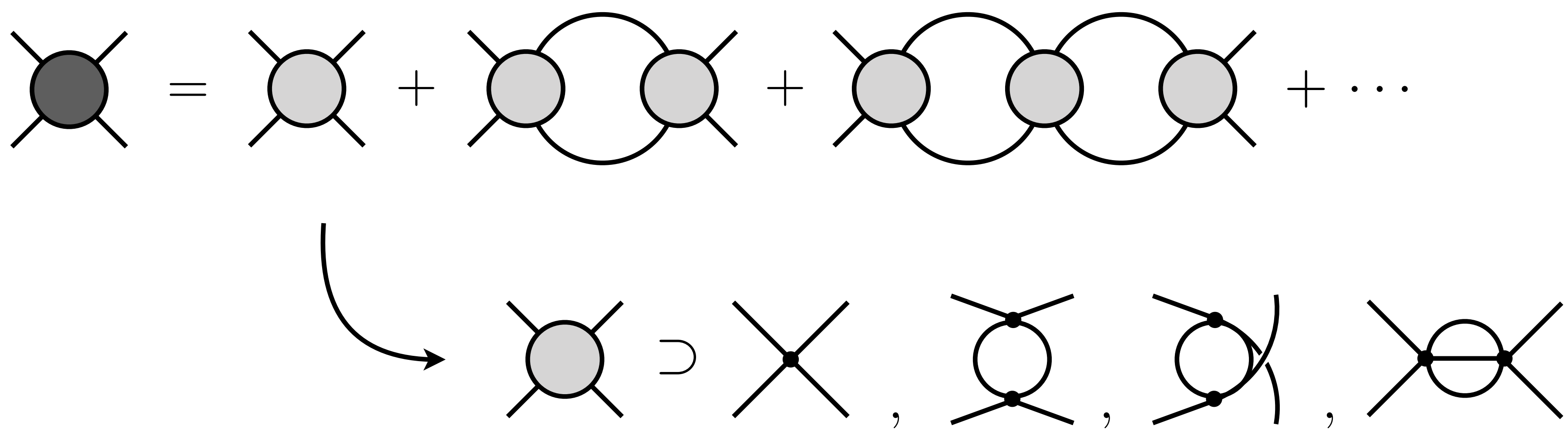


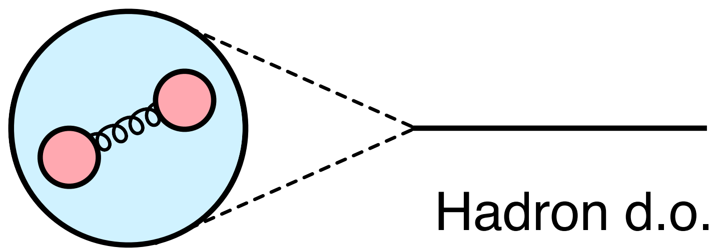
Bethe-Salpeter kernels

All 2PI diagrams - left hand cuts & higher multi-particle thresholds

Connecting Scattering Physics to QCD

Use generic Effective Field Theory to generate connection

Quarks and gluons
at low energy



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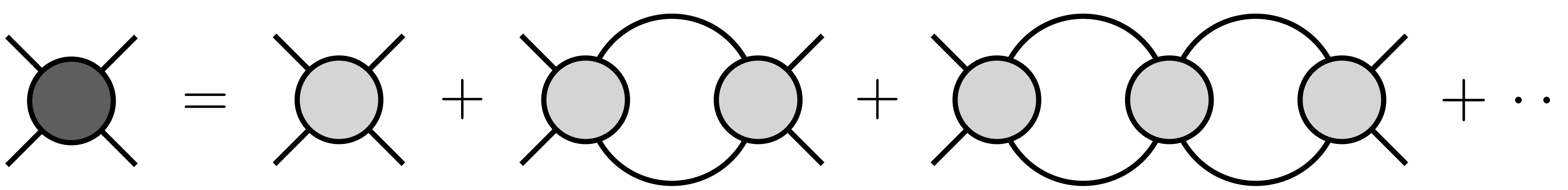
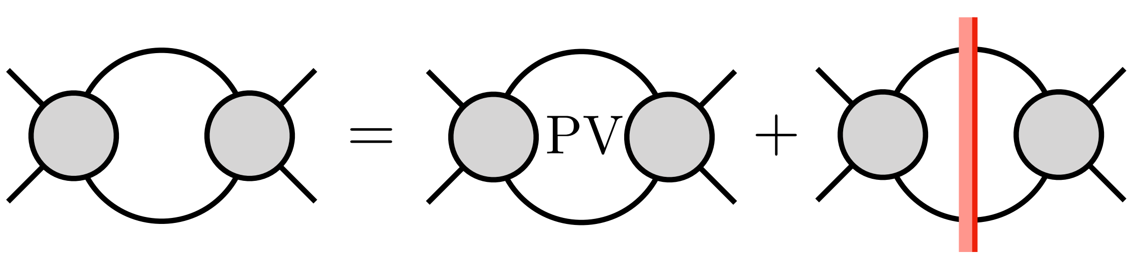


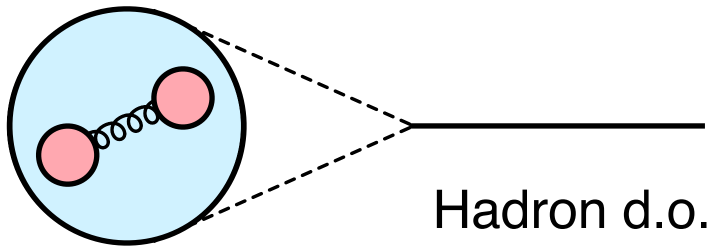


$$\rho = \frac{q}{8\pi E} \sim \sqrt{s - s_{\mathrm{th}}}$$

Connecting Scattering Physics to QCD

Use generic Effective Field Theory to generate connection

Quarks and gluons
at low energy



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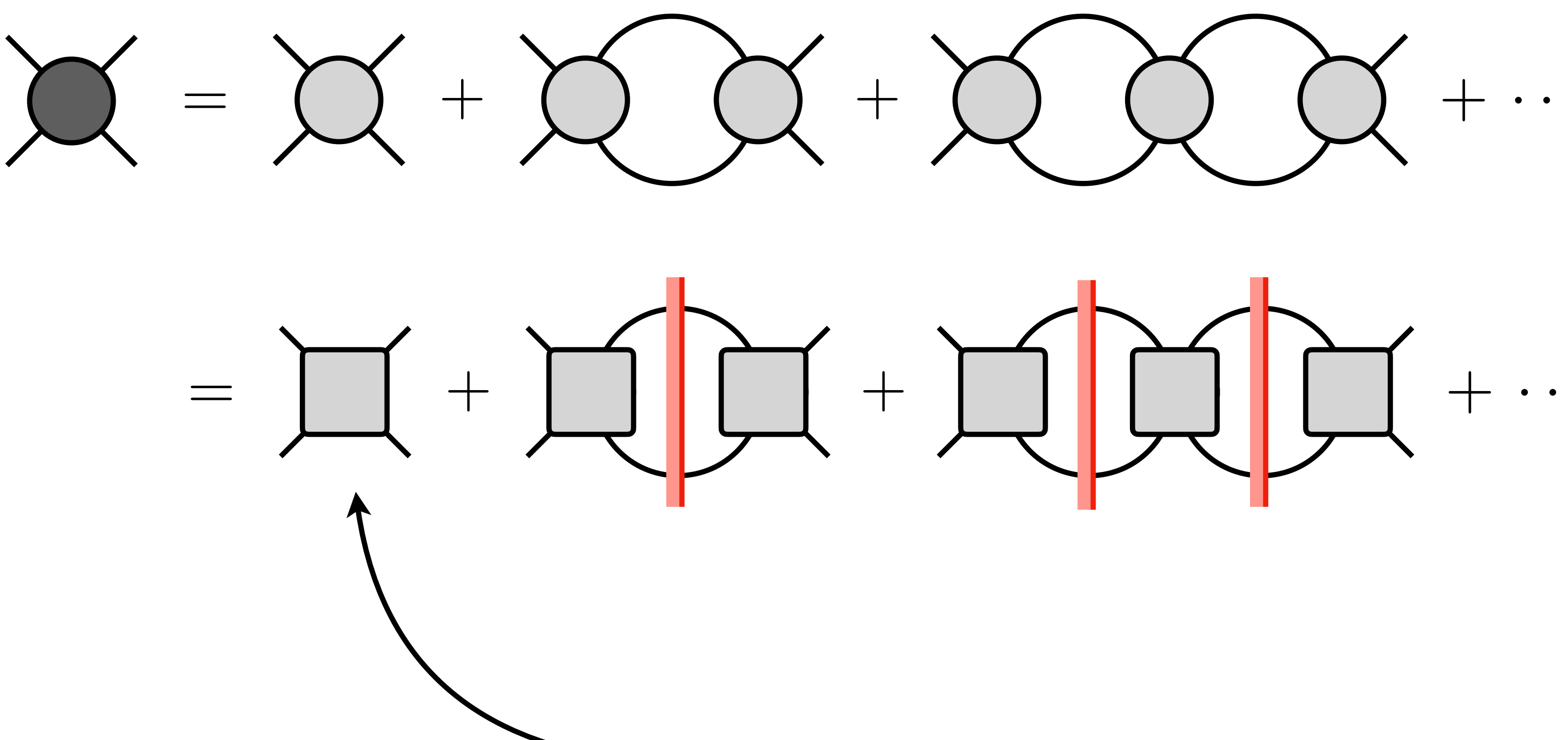


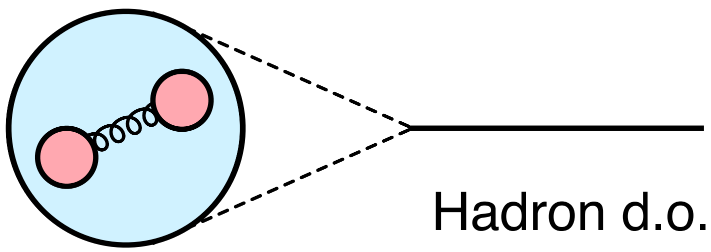
K-matrix — All short-distance physics which cannot go on-shell

— Unknown! — theory specific

Connecting Scattering Physics to QCD

Use generic Effective Field Theory to generate connection

Quarks and gluons at low energy



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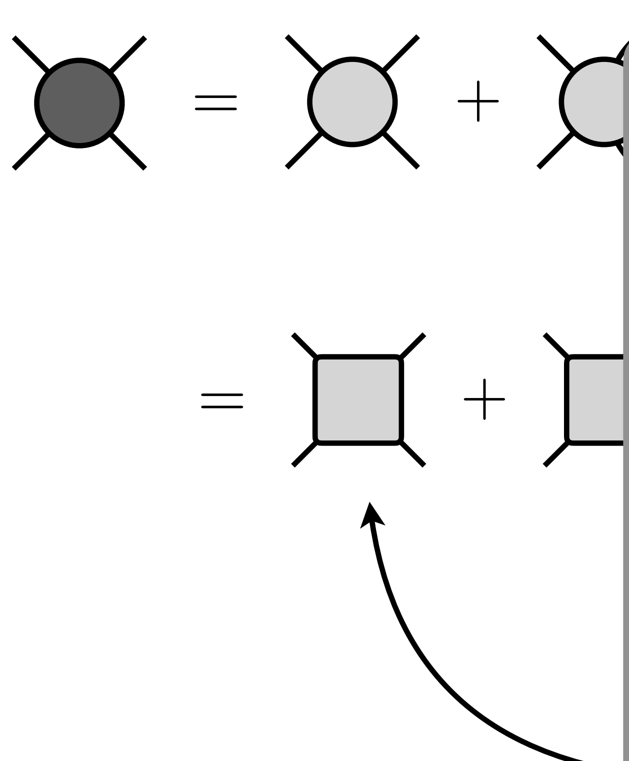
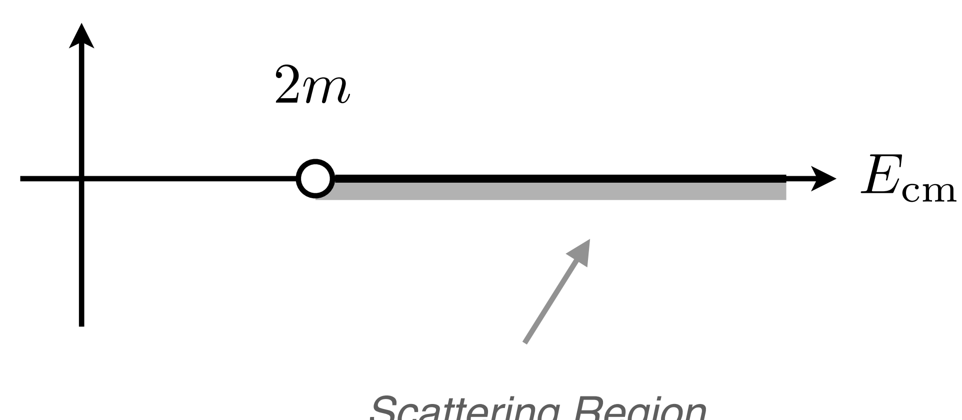


On-shell representation of scattering amplitude

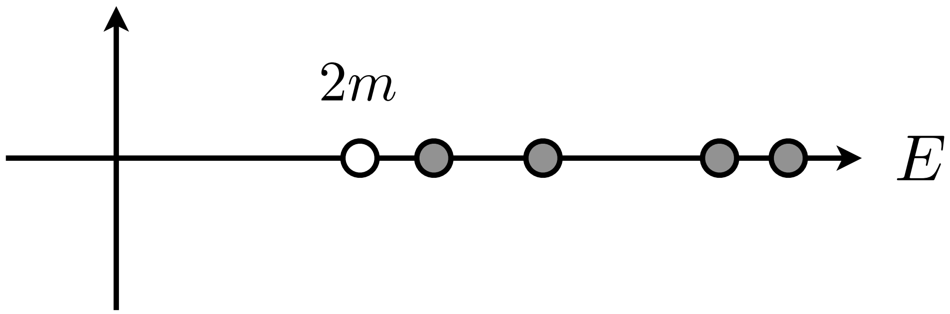
$$\mathcal{M} = \mathcal{K}\,\frac{1}{1 - i\rho\mathcal{K}}$$



Connecting Scattering Physics to QCD

Q: How do we connect a finite-volume spectrum computed from QCD…

$$\int_L \mathrm{d}^4x \, e^{iP\cdot x} \, \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \sum_{\mathfrak{n}} \frac{i\,|\langle 0|\,\mathcal{O}\,|\mathfrak{n}\rangle|^2}{E - E_{\mathfrak{n}}}$$



…to infinite-volume scattering amplitudes?

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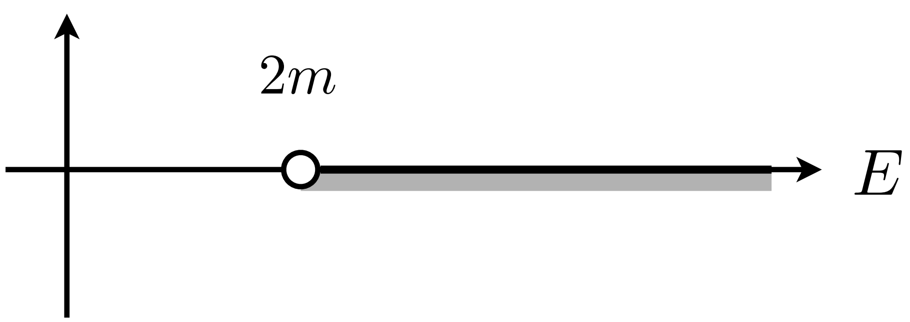

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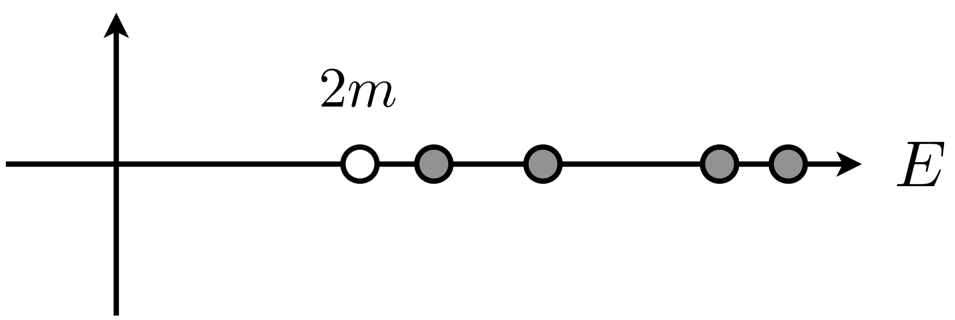

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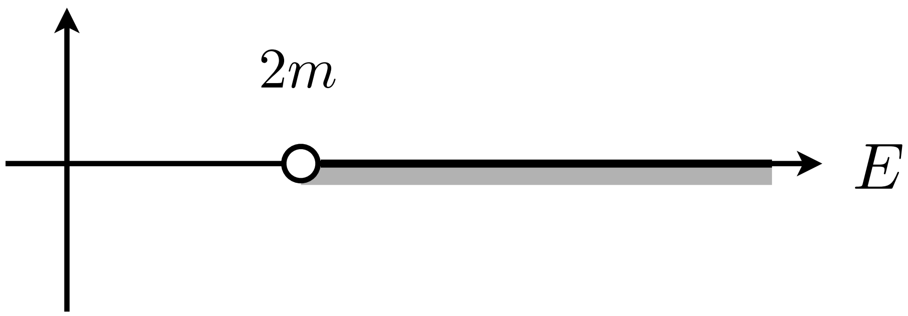
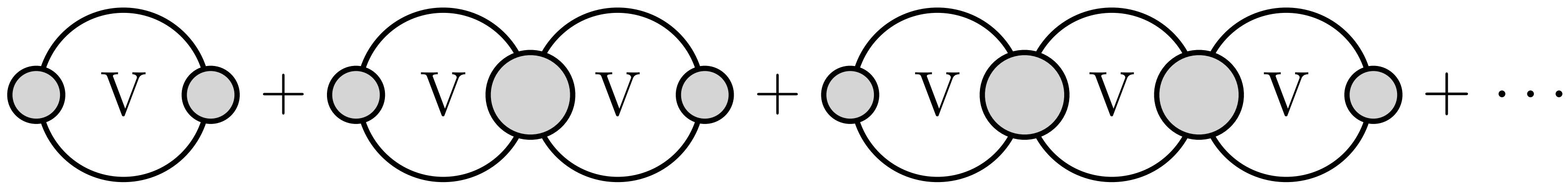


A: Correct analytic structure of finite-volume correlators

Connecting Scattering Physics to QCD

Two-point correlator to all-orders

$$C_L(P) =$$



Connecting Scattering Physics to QCD

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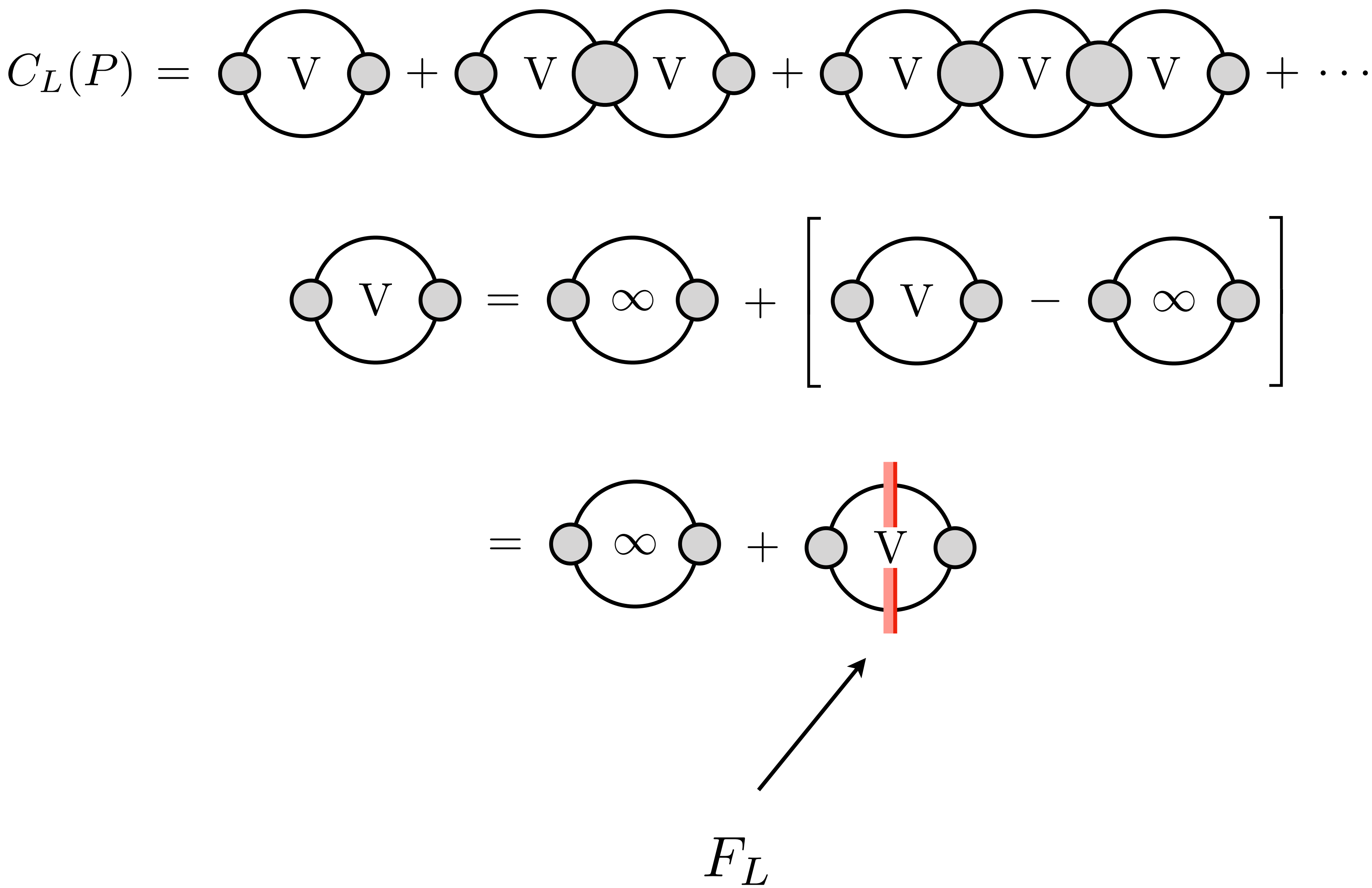

Geometric function — characterizes finite-volume distortions

Connecting Scattering Physics to QCD

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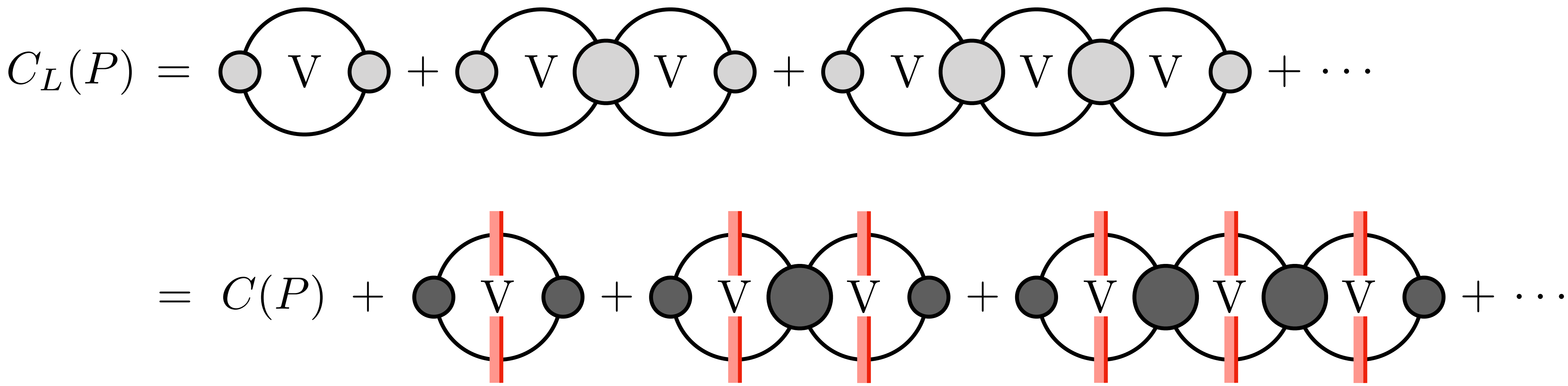

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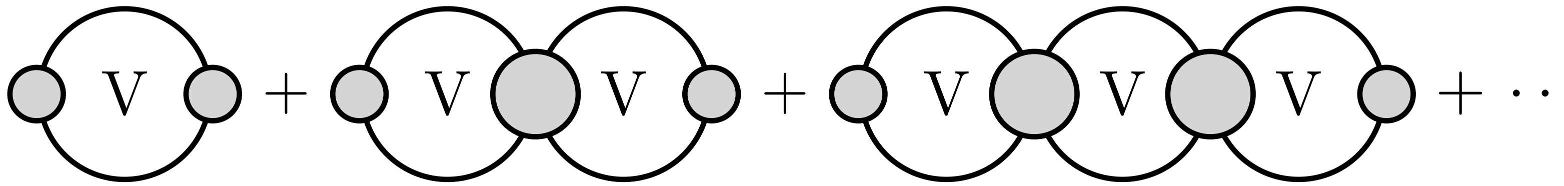
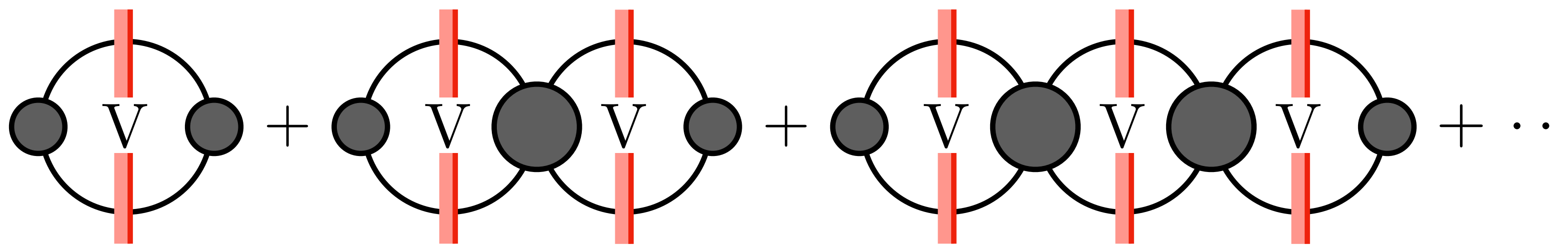


$= C(P) +$

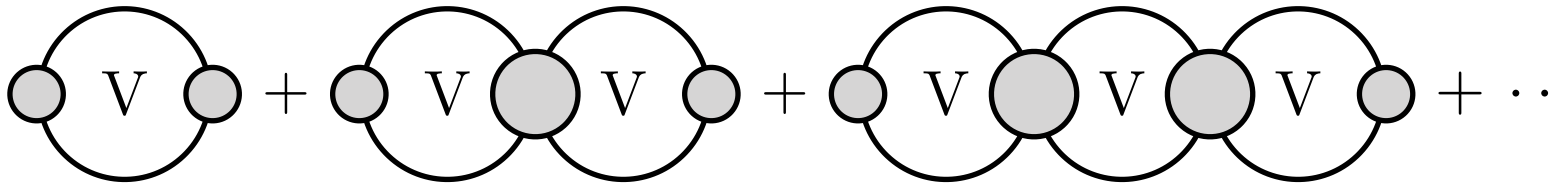


$$= C(P) + i\mathcal{A}\,\frac{i}{\mathcal{M}^{-1} + F_L}\,F_L\,i\mathcal{A}$$

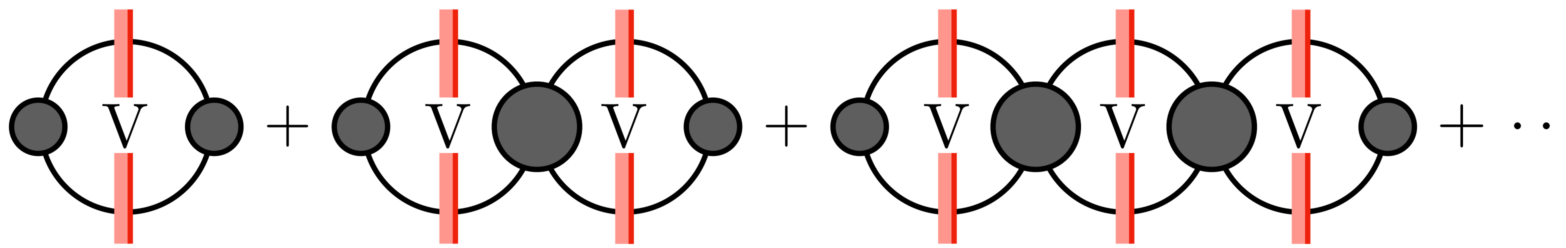
Connecting Scattering Physics to QCD

Two-point correlator to all-orders

$C_L(P) =$



$= C(P) +$



$$= C(P) + i\mathcal{A}\,\frac{i}{\mathcal{M}^{-1} + F_L}\,F_L\, i\mathcal{A}$$

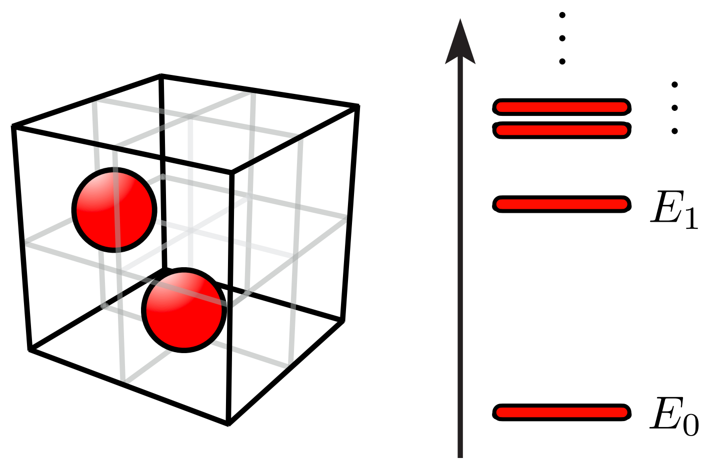
Finite-Volume poles must match!

$$\det\left(\mathcal{M}^{-1} + F_L\right)_{E=E_\mathfrak{n}} = 0$$

Lüscher quantization condition

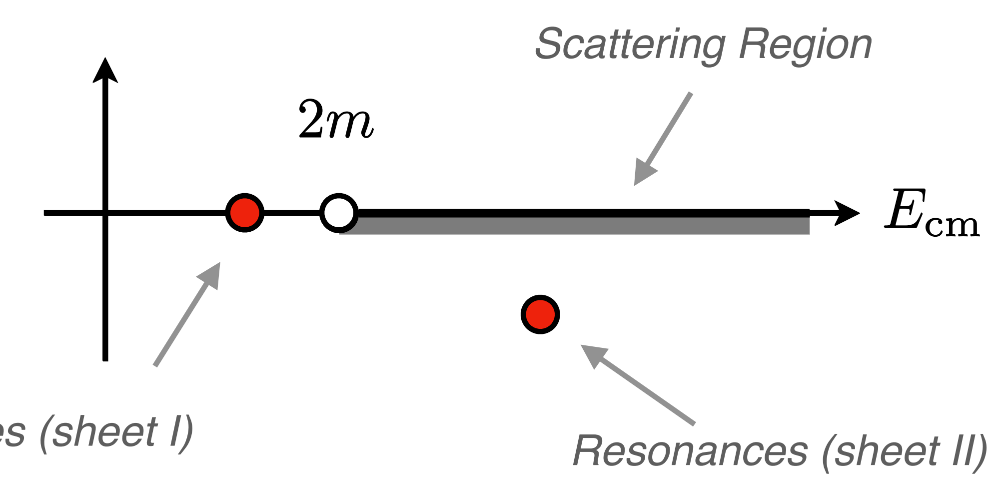
Few-Body Physics & QCD

Employing scattering theory and EFTs to all-orders connects lattice QCD to scattering observables



$$\det\left(\mathcal{M}^{-1} + F_L\right)_{E=E_\mathfrak{n}} = 0$$

$$\mathcal{M} = \mathcal{K}\,\frac{1}{1 - i\rho\mathcal{K}}$$



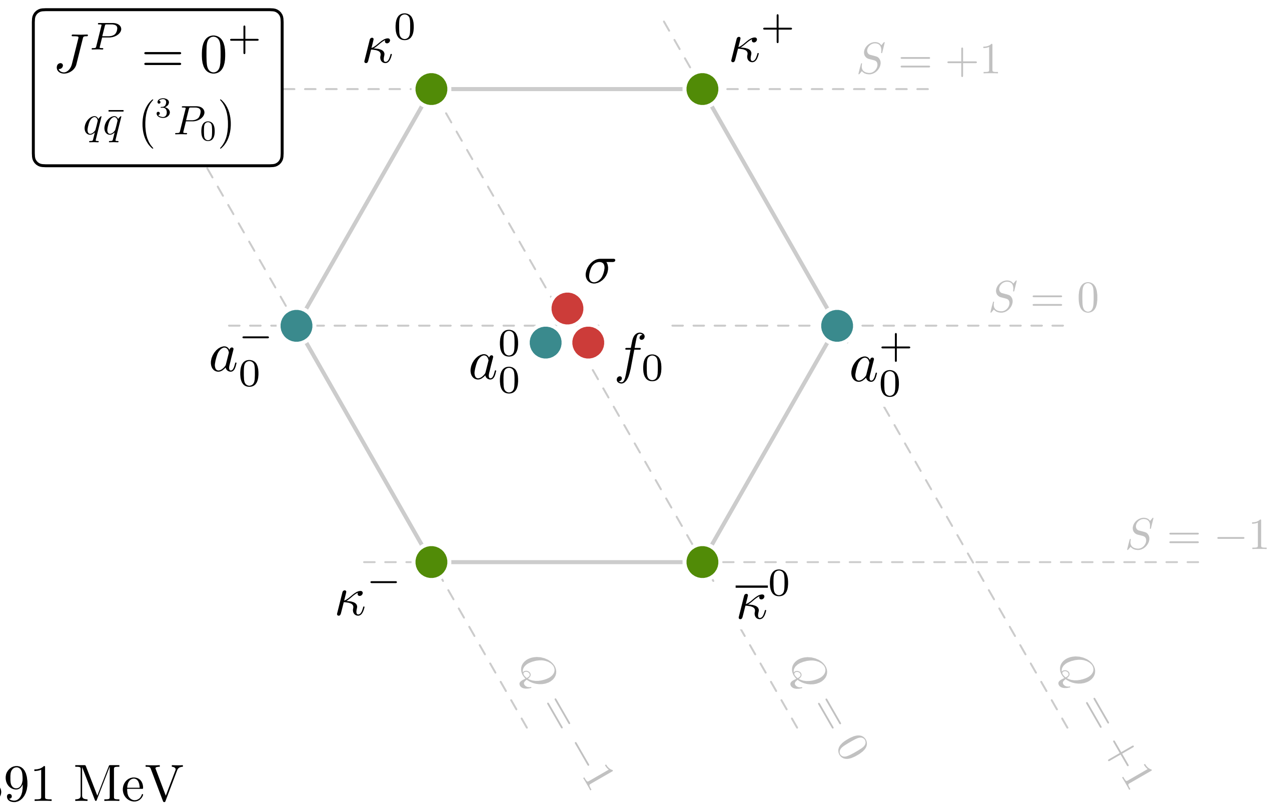
M. Lüscher
Commun.Math.Phys. **105**, 153 (1986)
Nucl.Phys. **B354**, 531 (1991)

Many others…

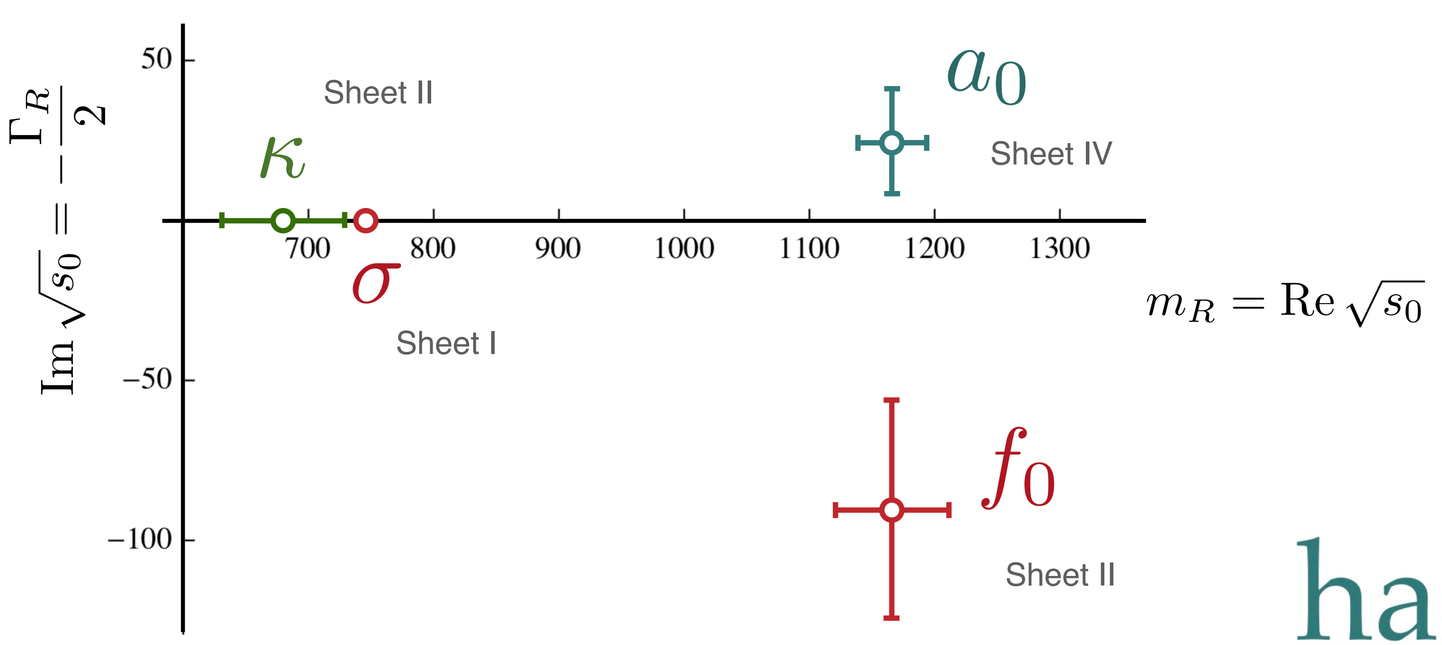
Connecting Resonances to QCD

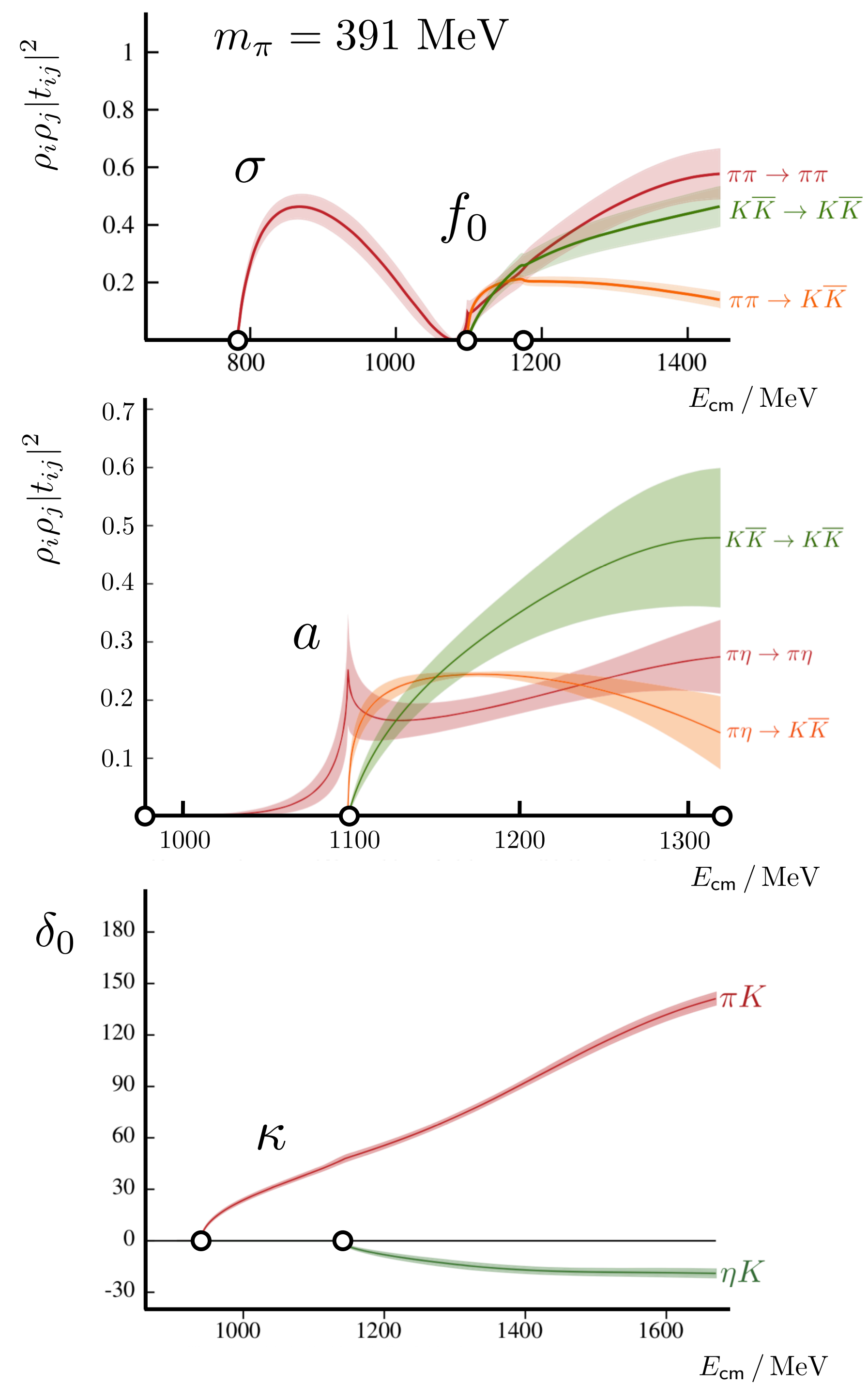
Can we directly connect resonances to QCD?

- Yes! Lattice QCD offers pathway



$m_\pi = 391$ MeV





hadspec

R.A. Briceño et al. [HadSpec]
Phys. Rev. **D97**, 054513 (2018)

J.J. Dudek et al. [HadSpec]
Phys. Rev. **D93**, 094506 (2016)

J.J. Dudek et al. [HadSpec]
Phys. Rev. Lett. **113**, 182001 (2014)

Few-Body Physics & QCD

Path to few-body physics from QCD

- Link finite-volume spectra and matrix elements to scattering amplitudes
- Tools: *Lattice QCD*, *Scattering Theory*, & *Effective Field Theory*