Lecture 6b: PI Curves – Jassby and Platt

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This Theory Accompanies the Following Lab

- Lab 3: PI Curves – Jassby and Platt

1 The Hyperbolic Tangent Model

The hyperbolic tangent model was proposed by Jassby and Platt (1976). It has become one of the most widely used models for describing the relationship between photosynthetic rate and irradiance (light intensity) in aquatic photosynthetic organisms, including algae ranging from kelp to phytoplankton. The model captures the core dynamics of photosynthesis, in which the rate of photosynthesis initially increases with light intensity but eventually saturates as the photosynthetic machinery reaches its maximum efficiency. This is a simple model, but it effective because the biologically meaningful parameters can be directly interpreted to assess plant or algal productivity in various light environments.

The hyperbolic tangent model is expressed as:

$$P(I) = P_{\mathrm{max}} \times \tanh\left(\frac{\alpha I}{P_{\mathrm{max}}}\right)$$

Where:

- $P(I)$ represents the photosynthetic rate at a given irradiance I (light intensity),
- P_{max} is the maximum photosynthetic rate (also referred to as the light-saturated rate),
- α is the initial slope of the curve, which reflects the photosynthetic efficiency at low light levels,
- I is the irradiance (light intensity),

One is also able to determine the saturating irradiance, I_{k}, which is the light intensity at which photosynthesis reaches P_{max}. Simply read this value off the graph where $P(I) = P_{\mathrm{max}}$ (see the lecture slides '6.BDC223_Pigments_Photosynthesis_2024.key.pdf'.

The hyperbolic tangent function $\tanh$ is used to smoothly describe the transition between the linear increase in photosynthesis at low light intensities and the eventual plateau at higher intensities, where photosynthesis becomes light-saturated. The light compensation point, the point at which photosynthesis equals respiration (i.e., net photosynthesis is zero), can also be derived from this model.

The model describes the essential processes of photosynthesis with just two parameters: $P_{\max}$ and α. Both parameters are biologically meaningful and tell us how efficiently an organism can convert light into chemical energy under different light conditions. For example, higher values of $P_{\max}$ indicate a greater potential for photosynthesis under optimal light conditions, while the value of α indicates how quickly photosynthesis responds to low light.

Applications of the hyperbolic tangent model are numerous. It is commonly used to estimate the photosynthetic performance of marine and freshwater algae, seagrasses, and macroalgae under varying environmental conditions. In kelp forests, for instance, we may use this model to assess how different species adapt to light intensities at various depths or how photosynthetic performance shifts in response to seasonal changes in light availability. Looking at phytoplankton, the model helps estimate productivity across different layers of the water column, where light intensity decreases with depth.

Below are a few lines of data taken from a hypothetical P-I experiment. The data are for five replicate experiments with the same light intensities (independent variable), representing conditions typically encountered by kelp at latitudes between −36° and −23°S.

	Replicate	Light (μmol photons m^{-2} s^{-1})	Photosynthesis (mg C m^{-2} h^{-1})
3	1	100	4.59
17	2	200	8.83
53	5	200	8.05
44	4	350	12.27
35	3	500	12.57
54	5	250	9.38
34	3	450	11.90
11	1	500	13.53
27	3	100	4.20

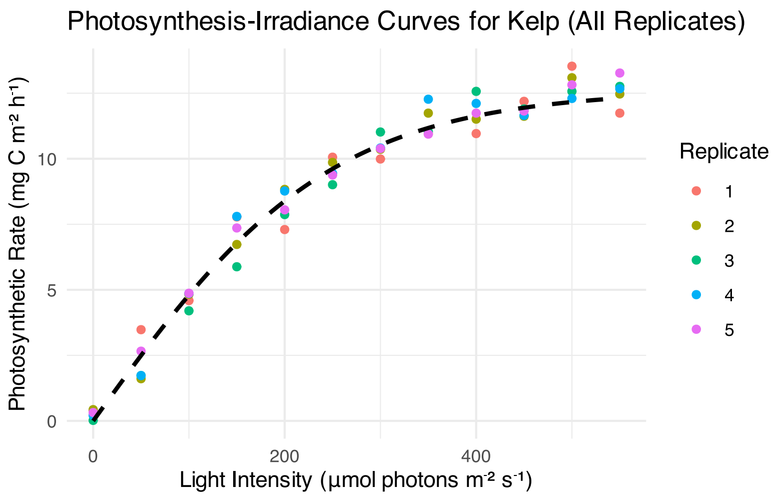

Figure 1: Nonlinear regression Jassby and Platt (1976) model fitted to simulated P-I data for a hypothetical kelp.

After fitting the model to the data, we can determine the values for $P_{\max}$ and α for each replicate and determine the average value across the five fits. The combined plot (Figure 1) displays the observed data points for all replicates and the fitted curve from the first replicate.

The average model fit values of the estimated parameters across all replicates are as follows:

- **$P_{\max}$:** 13.05 mg C m^{-2} h^{-1}
- **α:** 0.05 µmol photons m^{-2} s^{-1}

2 Considering the Light Compensation Point

The **light compensation point** (I_c) is the irradiance level at which the rate of photosynthesis equals the rate of respiration, resulting in a net photosynthetic rate of zero. Below this point, the organism consumes more energy (via respiration) than it produces through photosynthesis, leading to a net loss of energy. Estimating I_c is important for determining the minimum light intensity required for the survival of photosynthetic organisms, after compensation for the effect of cellular respiration.

In the context of the **Jassby and Platt hyperbolic tangent model**, I_c can be estimated by solving for the irradiance I when the net photosynthetic rate $P(I)$ equals zero:

$$0 = P_{\max} \times \tanh\left(\frac{\alpha I_{\mathrm{LCP}}}{P_{\max}}\right)$$

Since $\tanh(0) = 0$, the net photosynthetic rate is zero when $I = 0$. However, due to respiration, the net photosynthesis can be negative at zero light intensity. To account for respiration, we can modify the model to include **dark respiration rate** (R):

$$P(I) = P_{\max} \times \tanh\left(\frac{\alpha I}{P_{\max}}\right) - R$$

Now, I_c is the irradiance at which $P(I) = 0$:

$$0 = P_{\max} \times \tanh\left(\frac{\alpha I_{\mathrm{LCP}}}{P_{\max}}\right) - R$$

We can solve this equation numerically to find I_{LCP}.

	Replicate	Light (µmol photons m^{-2} s^{-1})	Photosynthesis (mg C m^{-2} h^{-1})
3	1	100	3.65
37	4	0	−2.17
5	1	200	6.98
57	5	400	9.43
54	5	250	7.41
60	5	550	10.98
33	3	400	9.66
6	1	250	7.89
49	5	0	−1.86

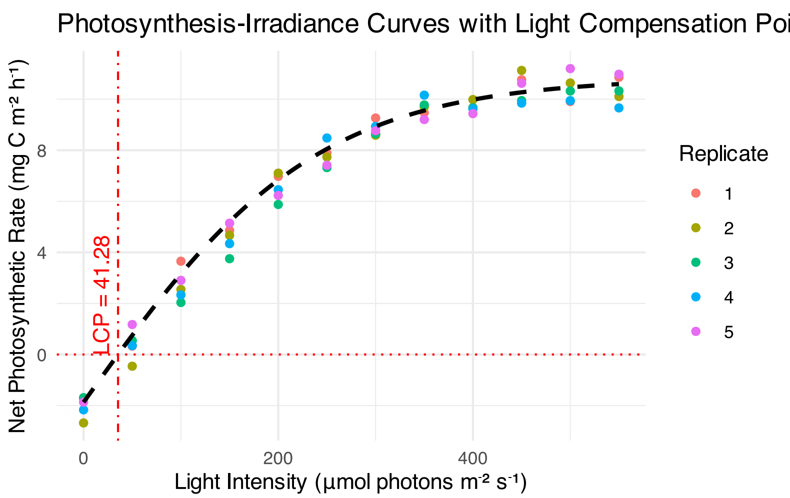

Figure 2: Nonlinear regression Jassby and Platt (1976) model fitted to simulated P-I data for a hypothetical kelp. This model includes the effect of the light compensation point.

The model fit to the data is in Figure 2. The average model fit values of the estimated parameters across all replicates are as follows:

- $\boldsymbol{P_{\max}}$: 13.15 mg C m^{-2} h^{-1}
- $\boldsymbol{\alpha}$: 0.05 µmol photons m^{-2} s^{-1}
- $\boldsymbol{I_c}$: 41.28 µmol photons m^{-2} s^{-1}

3 Platt et al. (1980) Model with Photoinhibition

Let's now look at the **Platt et al. (1980)** model, which incorporates photoinhibition into the photosynthesis-irradiance (P-I) relationship. This model extends the understanding of photosynthesis by accounting for the decrease in photosynthetic efficiency at high light intensities due to photoinhibition—a phenomenon where excessive light damages the photosynthetic apparatus, leading to reduced photosynthetic rates.

The model is expressed mathematically as:

$$P(I) = P_{\max}\left(1 - \exp\left(-\frac{\alpha I}{P_{\max}}\right)\right)\exp\left(-\frac{\beta I}{P_{\max}}\right)$$

Where:

- $P_{\max}$ is the maximum photosynthetic rate in the absence of photoinhibition.
- β is the photoinhibition parameter (rate of decrease in photosynthesis at high light).
- exp denotes the exponential function.

This model combines the positive effect of light on photosynthesis at low irradiance with the negative effect of photoinhibition at high irradiance, providing a comprehensive description of the photosynthetic response across a wide range of light intensities.

	Replicate	Light (µmol photons m^{-2} s^{-1})	Photosynthesis (mg C m^{-2} h^{-1})
45	3	1000	10.05
76	5	700	9.13
34	2	1600	10.16
52	4	0	−2.00
1	1	0	−0.86
77	5	800	9.58
10	1	900	11.26
13	1	1200	11.43
39	3	400	7.98

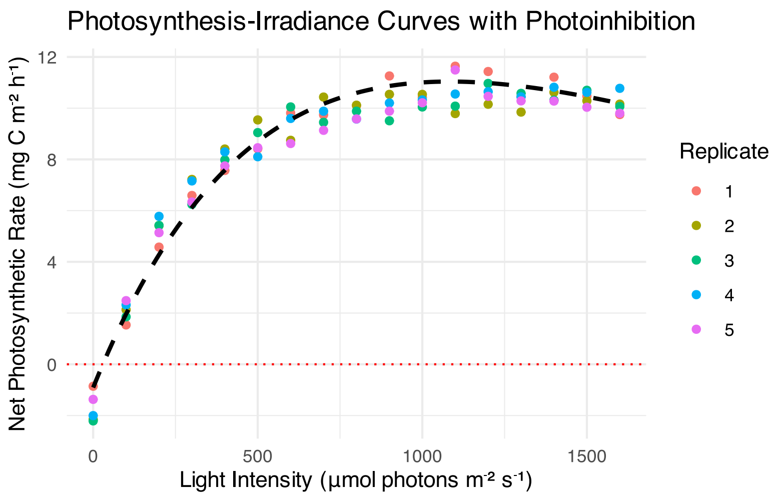

Figure 3: Nonlinear regression Platt et al. (1980) model fitted to simulated P-I data for a hypothetical kelp. This model includes the effect of photoinhibition.

4 References

- **Jassby, A. D., & Platt, T. (1976).** Mathematical formulation of the relationship between photosynthesis and light for phytoplankton. *Limnology and Oceanography*, 21(4), 540-547.
- **Platt, T., Gallegos, C. L., & Harrison, W. G. (1980).** Photoinhibition of photosynthesis in natural assemblages of marine phytoplankton. *Journal of Marine Research*, 38(4), 687-701.

Bibliography