Evaluating risk

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Smoking and risk

- "We all gotta die of something"
- $P(death|smoker) = 1$
- $P(death|nonsmoker) = 1$
- How about "smokers die younger?"

Smokers die younger (than non-smokers)

- "I knew a lady who smoked every day, and she lived until she was 93"
- If the claim is "ALL smokers die younger than ALL non-smokers"...
- ...then this counter-example refutes it.
- Perhaps:
  - "On average, smokers die younger than non-smokers"
  - "Smokers have lower life expectancy"

Smokers have lower life expectancy

- 20% of smokers die before they are 60 years old
- Doll et al.,2004. - Smoking habits of 34000 doctors born 1900-1930.
- Convinced?
- Any other information you need?

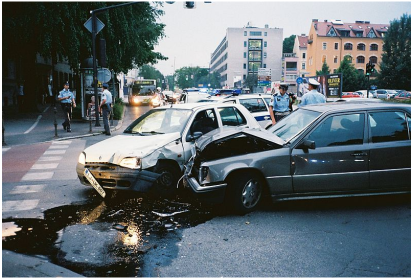
Smokers have lower life expectancy

- Think in terms of *hits* and *false alarms*
- You know $P(DeathBeforeSixty|smoker) = 0.2$ (hit rate)
- You also need to know $P(DeathBeforeSixty|nonsmoker)$ (false alarm)
- $P(DeathBeforeSixty|nonsmoker) = 0.1$ (Doll et al., 2004)

Odds ratio

- $P(DeathBeforeSixty|smoker) = 0.2$
- $P(DeathBeforeSixty|nonsmoker) = 0.1$
- Odds ratio, $OR = 0.2/0.1$
- $OR = 2$
- Smoking doubles the risk of dying before sixty.

Life is risky



- "Yeah, but you could give up smoking and then die in a car accident"
- ...which possibly means...
  - Many activities have some level of risk.
  - It is impossible to avoid all risk.
  - So everything has to be a risk-benefit analysis otherwise you'd never do anything.

Life is risky? Yes, it is!

- Correct. Life is a risk-benefit analysis.
- Benefit is somewhat subjective - what are the benefits of being a smoker? Or a car driver?
- ...but odds ratio can help quantify and compare risk.

Odds ratio

- Mokdad et al. (2004) - USA data
  - Tobacco smoking is the cause of death for about 18% of people.
  - Car accidents are the cause of death for about 0.2% of people.
- $OR = 18/0.2 = 90$
- Smoking is 90 times more likely to kill you than driving a car.
- Much more than that, actually, because only a minority smoke in the US, but most adults drive regularly.

I am an individual, not a statistic!

- Correct.
- These are samples across large numbers of people. They do not *determine* your future cause of death.
- But, risk calculations should inform our decisions. Example...

Russian Roulette

- Playing Russian Roulette once, $P(death) = 0.17$
- After you have played, $P(death) = 1$ or $P(death) = 0$

Inverse Russian Roulette

- Now imagine *inverse* Russian roulette (five bullets)
- Playing Inverse Russian Roulette once, $P(death) = .83$
- Again, after you have played, $P(death) = 1$ or $P(death) = 0$
- If you had to choose between the games, which would you pick ?
- The odds ratio here is $.83/.17 = 5$

Probability

- Probability (by the simplest objective definition) is that property which allows us to calculate the frequency of an event in a very long run of events.

- Fair coin
  - $P(heads) = 0.5, P(tails) = 0.5$
  - Flip a fair coin 1000 times, you get close to 500 heads.
  - The more times you flip the more $heads/flips$ tends towards 0.5.

Probability Exercise 1

- Rolling a six on a six-sided dice.
- Having to stand when 60 passengers board a bus with 40 seats.

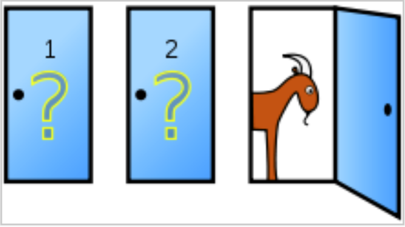
Probability Exercise 2

- Of dying during 2018, across everyone living in England or Wales.
- Of getting 4 numbers in the next Lotto game if you buy one ticket.
- Of committing suicide if you live in England/Wales, and are aged 5-34 .

GAME SHOW!

“Let’s Make A Deal”
with your host, Monty Hall.

Monty Hall problem



- Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You want to win the car. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? (vos Savant, 1990).
  - Better to switch?
  - Better to stick?
  - Doesn't matter, stick or switch equally likely to win?

Working out the Monty Hall problem

- The host is not going to open the door with the car behind it.
- On two-thirds of occasions, your first choice will be a goat.
  - The host will then reveal the other goat.
  - So, the car is behind the other door.
  - If you switch, you will win the car
- On one-third of occasions, your first choice will be the car.
  - If you switch, you will not win the car.
- So, on two-thirds of occasions, switching will win the car.
- You should always switch.

Which player would you pass to?

- Player A: Score Score Miss Miss
- Player B: Miss Miss Score Score
- A, B, or doesn't matter?

Roulette

- Red Red Black Red Black Black Black Black
- Bet “red”, bet “black”, or doesn’t matter?

Conditional Probability and Randomness

- Probability of some event, given that some other event is known to have occurred.
- $P(heads_t | heads_{t-1}) = 0.5$
- $P(heads_t | tails_{t-1}) = 0.5$
- Events are **independent** if the conditional probabilities are equal to the unconditional probabilities (as close to an adequate definition of "random" as you're ever likely to get).
- Coin flips, roulette wheels, etc. are demonstrably independent.

Gamblers' fallacy

- Red Red Black Red Black Black Black Black
- Bet "red", bet "black", or doesn't matter?
- Common answer: "bet red"
- Rational answers
  - If the wheel is known to be unbiased, $P(red) = P(black)$, and it doesn't matter.
  - From the sample above estimates are that $P(red) < P(black)$, so bet black.

Hot hand fallacy

- Player A: Score Score Miss Miss
- Player B: Miss Miss Score Score
- A, B, or doesn't matter?
- Common answer: "Pass to B" - Hot Hand fallacy
- Gilovich, Vallone & Tversky (1985) - Shots in basketball are independent.
- Rational answer: Doesn't matter
- Things to note:
  - Basketball experts and players exhibit a hot hand fallacy
  - Hot Hand and Gamblers' Fallacy are **opposite** beliefs about independent events. What drives this?

Independence - Summary

- Events are independent if the probabilities conditional on previous events are equal to the unconditional probabilities:
- For example:
  $P(heads_t|heads_{t-1}) = P(heads_t|tails_{t-1}) = P(heads) = 0.5$
- Failure to understand independence leads to Gamblers' Fallacy and Hot Hand Fallacy.

Linda

- Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.
- Which is more probable?
  1. Linda is a bank teller.
  2. Linda is a feminist bank teller.

Conjunction fallacy

- Which is more probable?
  1. Linda is a bank teller
  2. Linda is a bank teller and is active in the feminist movement.
- The probability of two events both happening can NEVER be higher than the probability of just one happening.

The conjunction rule

The probability of two *independent* events both occurring is the product of their individual probabilities.

- $P(heads_{time1}) = 0.5$
- $P(heads_{time2}) = 0.5$
- $P(heads_{times1and2}) = P(heads_{time1}) \times P(heads_{time2}) = 0.5 \times 0.5 = 0.25$
- $P(teller) = .05, P(feminist) = .95$
- $P(teller + feminist) = .05 \times .95 = .0475$
- $P(teller + feminist) < P(feminist)$

Sally Clark

- Solicitor; born 1964.
- Christopher died at 11 weeks.
- One year later, Harry died at 8 weeks.
- Little to no forensic evidence.
- No evidence she had been a violent or uncaring parent.

Roy Meadows

- Sally convicted of murder, spent three years in prison
- Central to conviction was evidence of expert witness Prof. Roy Meadows:
  - Probability of two cot deaths in the same family was 1 in 73 million
  - Less than once a century in the UK.
- Released on appeal, partly because Prof. Meadows's **probabilistic inference** was demonstrably wrong.

Roy Meadows - expert witness

- Chances of a randomly chosen baby dying of cot death are 1 in 1303, $p = .0008$
- If the family is affluent, and the mother is over 26, then the chances are even lower; 1 in 8500, $p = .0001$
- Through the multiplicative law, the probability of two cot deaths in the same family is $.0001 \times .0001 = 1 \times 10^{-8}$
- 1 in 73 million
- Less than once a century in the UK.
- The idea that these deaths were by natural causes can be ruled out beyond *reasonable doubt*.

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- Through the **multiplicative law**, the probability of two cot deaths in the same family is $.0001 \times .0001 = 1 \times 10^{-8}$ ... **COT DEATHS WITHIN THE SAME FAMILY ARE HIGHLY UNLIKELY TO BE INDEPENDENT EVENTS**.
- 1 in 73 million
- Less than once a century in the UK.
- The idea that these deaths were by natural causes can be ruled out beyond *reasonable doubt*.

Shared birthdays

- In a class of 30 children, what's the probability that there is a shared birthday in the class? (Assume independence, and equiprobability of birth date).
- More likely there is, or more likely there is not?

More high-school maths

- Number of pairs: $n(n-1)/2$
- This gets very large quite quickly.
- Pairs in a group of 2: $2(1)/2 = 1$
- Pairs in a group of 5: $5(4)/2 = 10$
- Pairs in a group of 10: $10(9)/2 = 45$
- Pairs in a group of 20: $20(19)/2 = 190$
- Pairs of children in a class of 30: $30(29)/2 = 435$
- Pairs in Year 1 psychology, approx: $300(299)/2 = 44850$

Birthday example

- 365 days in the year (ignore Feb 29th).
- So, the chance of one pair of kids sharing a birthday is $1/365 = .003$
- Thus, chance of not sharing is .997
- If no pair of kids share a birthday, then there is no shared birthday in the class.
- How many pairs in the class?
- $n(n-1)/2 = 30 \times 29/2 = 435.$
- Under conjunction rule, $p = .997^{435} = .17$
- Thus, probability of a shared birthday is 1-.17 $=$ .83

Summary

- Conjunction rule (Linda)
- Events are not always independent (Sally Clarke)
- If a lot of things all have to happen, this is quite unlikely, even if each event is individually very likely (avoiding hurricanes for 100 years; all non-shared birthdays in a class of 30).
- The number of pairs in a large group is much bigger than you think it is. This can often lead us to make incorrect predictions about large groups.

Further Reading

Helpful background, only lecture content on these topics is examinable).

- Paulos (1988/2000). *Innumeracy*. Penguin.
- http://en.wikipedia.org/wiki/Conjunction_fallacy
- http://en.wikipedia.org/wiki/Sally_Clark
- http://en.wikipedia.org/wiki/Probability