

Evaluating risk

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Roy Meadows



- ▶ Sally convicted of murder, spent three years in prison
- ▶ Central to conviction was evidence of expert witness Prof. Roy Meadows:
 - ▶ Probability of two cot deaths in the same family was 1 in 73 million
 - ▶ Less than once a century in the UK.
- ▶ Released on appeal, partly because Prof. Meadows's **risk evaluation** was demonstrably wrong.



**Smoking
kills**

- ▶ “We all gotta die of something”
- ▶ $P(\text{death}|\text{smoker}) = 1$
- ▶ $P(\text{death}|\text{nonsmoker}) = 1$
- ▶ How about “smokers die younger?”

Smokers die younger (than non-smokers)



- ▶ “I knew a lady who smoked every day, and she lived until she was 93”
- ▶ If the claim is “ALL smokers die younger than ALL non-smokers” ...
- ▶ ...then this counter-example refutes it.
- ▶ Perhaps:
 - ▶ “On average, smokers die younger than non-smokers”
 - ▶ “Smokers have lower life expectancy”

Smokers have lower life expectancy

- ▶ 20% of smokers die before they are 60 years old
- ▶ Doll et al., 2004. - Smoking habits of 34000 doctors born 1900-1930.
- ▶ Convinced?
- ▶ Any other information you need?

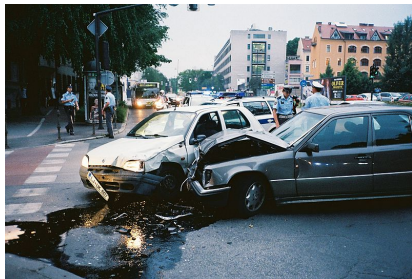
Smokers have lower life expectancy

- ▶ You know $P(\text{DeathBeforeSixty}|\text{smoker}) = 0.2$
- ▶ You also need to know $P(\text{DeathBeforeSixty}|\text{nonsmoker})$
- ▶ $P(\text{DeathBeforeSixty}|\text{nonsmoker}) = 0.1$ (Doll et al., 2004)

Odds ratio

- ▶ $P(\text{DeathBeforeSixty}|\text{smoker}) = 0.2$
- ▶ $P(\text{DeathBeforeSixty}|\text{nonsmoker}) = 0.1$
- ▶ Odds ratio, $OR = 0.2/0.1$
- ▶ $OR = 2$
- ▶ Smoking doubles the risk of dying before sixty.

Life is risky



- ▶ “Yeah, but you could give up smoking and then die in a car accident”
- ▶ ...which possibly means...
 - ▶ Many activities have some level of risk.
 - ▶ It is impossible to avoid all risk.
 - ▶ So everything has to be a risk-benefit analysis otherwise you'd never do anything.

Life is risky? Yes, it is!

- ▶ Correct. Life is a risk-benefit analysis.
- ▶ Benefit is somewhat subjective - what are the benefits of being a smoker? Or a car driver?
- ▶ ...but odds ratio can help quantify and compare risk.

Odds ratio

- ▶ Mokdad et al. (2004) - USA data
 - ▶ Tobacco smoking is the cause of death for about 18% of people.
 - ▶ Car accidents are the cause of death for about 0.2% of people.
- ▶ $OR = 18/0.2 = 90$
- ▶ Smoking is 90 times more likely to kill you than driving a car.
- ▶ Much more than that, actually, because only a minority smoke in the US, but most adults drive regularly.

I am an individual, not a statistic!



- ▶ Correct.
- ▶ These are samples across large numbers of people. They do not *determine* your future cause of death.
- ▶ But, risk calculations should inform our decisions. Example...

Russian Roulette



- ▶ Playing Russian Roulette once, $P(\text{death}) = 0.17$
- ▶ After you have played, $P(\text{death}) = 1$ or $P(\text{death}) = 0$

Inverse Russian Roulette



- ▶ Now imagine *inverse* Russian roulette (five bullets)
- ▶ Playing Inverse Russian Roulette once, $P(\text{death}) = .83$
- ▶ Again, after you have played, $P(\text{death}) = 1$ or $P(\text{death}) = 0$
- ▶ If you had to choose between the games, which would you pick ?
- ▶ The odds ratio here is $.83/.17 = 5$

Probability

- ▶ Probability (by the simplest objective definition) is that property which allows us to calculate the frequency of an event in a very long run of events.



- ▶ Fair coin
 - ▶ $P(\text{heads}) = 0.5, P(\text{tails}) = 0.5$
 - ▶ Flip a fair coin 1000 times, you get close to 500 heads.
 - ▶ The more times you flip the more *heads/flips* tends towards 0.5.

Probability Exercise 1



- ▶ Rolling a six on a six-sided dice.
- ▶ Having to stand when 60 passengers board a bus with 40 seats.



Probability Exercise 2

- ▶ Of dying during 2022, across everyone living in England or Wales.
- ▶ Of getting 4 numbers in the next Lotto game if you buy one ticket.
- ▶ Of committing suicide during 2022, if you live in England/Wales, and are aged 5-34 .

GAME SHOW!



“Let’s Make A Deal”
with your host, Monty Hall.

Which player would you pass to?



- ▶ Player A: Score Score Miss Miss
- ▶ Player B: Miss Miss Score Score
- ▶ A, B, or doesn't matter?

Roulette



- ▶ Red Red Black Red Black Black Black Black
- ▶ Bet “red”, bet “black”, or doesn't matter?

Conditional Probability and Randomness

- ▶ Probability of some event, given that some other event is known to have occurred.
- ▶ $P(\text{heads}_t | \text{heads}_{t-1}) = 0.5$
- ▶ $P(\text{heads}_t | \text{tails}_{t-1}) = 0.5$
- ▶ Events are **independent** if the conditional probabilities are equal to the unconditional probabilities (as close to an adequate definition of “random” as you’re ever likely to get).
- ▶ Coin flips, roulette wheels, etc. are demonstrably independent.

Gamblers' fallacy



- ▶ Red Red Black Red Black Black Black Black
- ▶ Bet “red”, or bet “black” ?

Hot hand fallacy

- ▶ Player A: Score Score Miss Miss
- ▶ Player B: Miss Miss Score Score
- ▶ A, B, or doesn't matter?
- ▶ Gilovich, Vallone & Tversky (1985) - Shots in basketball are independent.

Linda

- ▶ Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.
- ▶ Which is more probable?
 1. Linda is a shop assistant.
 2. Linda is a feminist shop assistant.

Shared birthdays

- ▶ In a class of 30 children, what's the probability that there is a shared birthday in the class?
- ▶ More likely there is, or more likely there is not?

Conjunction fallacy

- ▶ Which is more probable?
 1. Linda is a shop assistant.
 2. Linda is a shop assistant and is active in the feminist movement.

The conjunction rule

The probability of two *independent* events both occurring is the product of their individual probabilities.

- ▶ $P(heads_{time1}) = 0.5$
- ▶ $P(heads_{time2}) = 0.5$
- ▶ $P(heads_{times1and2}) = P(heads_{time1}) \times P(heads_{time2}) = 0.5 \times 0.5 = 0.25$
- ▶ $P(assistant) = .05, P(feminist) = .95$
- ▶ $P(assistant + feminist) = .05 \times .95 = .0475$
- ▶ $P(assistant + feminist) < P(feminist)$

Shared birthdays

- ▶ In a class of 30 children, what's the probability that there is a shared birthday in the class?
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More high-school maths

- ▶ Number of pairs: $n(n - 1)/2$
- ▶ This gets very large quite quickly.
- ▶ Pairs in a group of 2: $2(1)/2 = 1$
- ▶ Pairs in a group of 5: $5(4)/2 = 10$
- ▶ Pairs in a group of 10: $10(9)/2 = 45$
- ▶ Pairs in a group of 20: $20(19)/2 = 190$
- ▶ Pairs of children in a class of 30: $30(29)/2 = 435$
- ▶ Pairs in Year 1 psychology, approx: $200(199)/2 = 19900$

Birthday example

- ▶ 365 days in the year (ignore Feb 29th).
- ▶ So, the chance of one pair of kids sharing a birthday is $1/365 = .003$
- ▶ Thus, chance of not sharing is .997
- ▶ If no pair of kids share a birthday, then there is no shared birthday in the class.
- ▶ How many pairs in the class?
- ▶ $n(n-1)/2 = 30 \times 29/2 = 435$.
- ▶ Under conjunction rule, $p = .997^{435} = .17$
- ▶ Thus, probability of a shared birthday is $1-.17 = .83$

Roy Meadows - expert witness

- ▶ Chances of a randomly chosen baby dying of cot death are 1 in 1303, $p = .0008$
- ▶ If the family is affluent, and the mother is over 26, then the chances are even lower; 1 in 8500, $p = .0001$
- ▶ Through the conjunction rule, the probability of two cot deaths in the same family is $.0001 \times .0001 = 1 \times 10^{-8}$
- ▶ 1 in 73 million
- ▶ Less than once a century in the UK.
- ▶ The idea that these deaths were by natural causes can be ruled out beyond *reasonable doubt*.

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- ▶ Through the **conjunction rule**, the probability of two cot deaths in the same family is $.0001 \times .0001 = 1 \times 10^{-8}$...
COT DEATHS WITHIN THE SAME FAMILY ARE HIGHLY UNLIKELY TO BE INDEPENDENT EVENTS.
- ▶ 1 in 73 million
- ▶ Less than once a century in the UK.
- ▶ The idea that these deaths were by natural causes can be ruled out beyond *reasonable doubt*.

Further Reading

Helpful background, only lecture content on these topics is examinable).

- ▶ Paulos (1988/2000). *Innumeracy*. Penguin.
- ▶ http://en.wikipedia.org/wiki/Conjunction_fallacy
- ▶ http://en.wikipedia.org/wiki/Sally_Clark
- ▶ <http://en.wikipedia.org/wiki/Probability>