Evaluating risk

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Sally Clark

- Solicitor; born 1964.
- Christopher died at 11 weeks.
- One year later, Harry died at 8 weeks.
- Little to no forensic evidence.
- No evidence she had been a violent or uncaring parent.

Roy Meadows

- Sally convicted of murder, spent three years in prison
- Central to conviction was evidence of expert witness Prof. Roy Meadows:
  - Probability of two cot deaths in the same family was 1 in 73 million
  - Less than once a century in the UK.
- Released on appeal, partly because Prof. Meadows's **risk evaluation** was demonstrably wrong.

Smoking and risk

- "We all gotta die of something"
- $P(death|smoker) = 1$
- $P(death|nonsmoker) = 1$
- How about "smokers die younger?"

Smokers die younger (than non-smokers)

- "I knew a lady who smoked every day, and she lived until she was 93"
- If the claim is "ALL smokers die younger than ALL non-smokers" ...
- ...then this counter-example refutes it.
- Perhaps:
  - "On average, smokers die younger than non-smokers"
  - "Smokers have lower life expectancy"

Smokers have lower life expectancy

- 20% of smokers die before they are 60 years old
- Doll et al.,2004. - Smoking habits of 34000 doctors born 1900-1930.
- Convinced?
- Any other information you need?

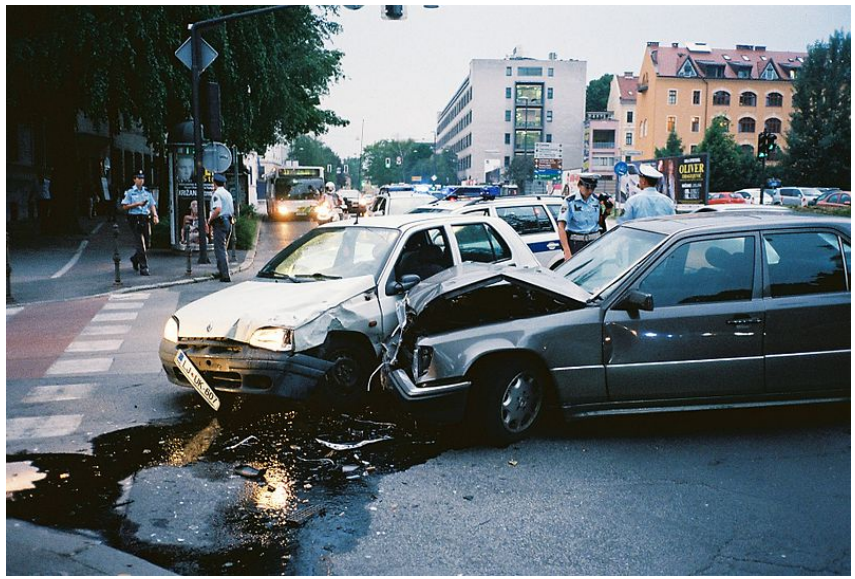
Smokers have lower life expectancy

- Think in terms of *hits* and *false alarms*
- You know $P(DeathBeforeSixty|smoker) = 0.2$ (hit rate)
- You also need to know $P(DeathBeforeSixty|nonsmoker)$ (false alarm)
- $P(DeathBeforeSixty|nonsmoker) = 0.1$ (Doll et al., 2004)

Odds ratio

- $P(DeathBeforeSixty|smoker) = 0.2$
- $P(DeathBeforeSixty|nonsmoker) = 0.1$
- Odds ratio, $OR = 0.2/0.1$
- $OR = 2$
- Smoking doubles the risk of dying before sixty.

Life is risky



- "Yeah, but you could give up smoking and then die in a car accident"
- ...which possibly means...
  - Many activities have some level of risk.
  - It is impossible to avoid all risk.
  - So everything has to be a risk-benefit analysis otherwise you'd never do anything.

Life is risky? Yes, it is!

- Correct. Life is a risk-benefit analysis.
- Benefit is somewhat subjective - what are the benefits of being a smoker? Or a car driver?
- ...but odds ratio can help quantify and compare risk.

Odds ratio

- Mokdad et al. (2004) - USA data
  - Tobacco smoking is the cause of death for about 18% of people.
  - Car accidents are the cause of death for about 0.2% of people.
- $OR = 18/0.2 = 90$
- Smoking is 90 times more likely to kill you than driving a car.
- Much more than that, actually, because only a minority smoke in the US, but most adults drive regularly.

I am an individual, not a statistic!

- Correct.
- These are samples across large numbers of people. They do not *determine* your future cause of death.
- But, risk calculations should inform our decisions. Example...

Russian Roulette

- Playing Russian Roulette once, $P(death) = 0.17$
- After you have played, $P(death) = 1$ or $P(death) = 0$

Inverse Russian Roulette

- Now imagine *inverse* Russian roulette (five bullets)
- Playing Inverse Russian Roulette once, $P(death) = .83$
- Again, after you have played, $P(death) = 1$ or $P(death) = 0$
- If you had to choose between the games, which would you pick ?
- The odds ratio here is $.83/.17 = 5$

Probability

- Probability (by the simplest objective definition) is that property which allows us to calculate the frequency of an event in a very long run of events.

- Fair coin
  - $P(heads) = 0.5, P(tails) = 0.5$
  - Flip a fair coin 1000 times, you get close to 500 heads.
  - The more times you flip the more $heads/flips$ tends towards 0.5.

Probability Exercise 1

- Rolling a six on a six-sided dice.
- Having to stand when 60 passengers board a bus with 40 seats.

Probability Exercise 2

- Of dying during 2022, across everyone living in England or Wales.
- Of getting 4 numbers in the next Lotto game if you buy one ticket.
- Of committing suicide during 2022, if you live in England/Wales, and are aged 5-34 .

GAME SHOW!

"Let's Make A Deal"
with your host, Monty Hall.

Which player would you pass to?

- Player A: Score Score Miss Miss
- Player B: Miss Miss Score Score
- A, B, or doesn't matter?

Roulette

- Red Red Black Red Black Black Black Black
- Bet “red”, bet “black”, or doesn’t matter?

Linda

- Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.
- Which is more probable?
  1. Linda is a shop assistant.
  2. Linda is a feminist shop assistant.

Shared birthdays

- In a class of 30 children, what's the probability that there is a shared birthday in the class?
- More likely there is, or more likely there is not?

Conditional Probability and Randomness

- Probability of some event, given that some other event is known to have occurred.
- $P(heads_t | heads_{t-1}) = 0.5$
- $P(heads_t | tails_{t-1}) = 0.5$
- Events are **independent** if the conditional probabilities are equal to the unconditional probabilities (as close to an adequate definition of "random" as you're ever likely to get).
- Coin flips, roulette wheels, etc. are demonstrably independent.

Gamblers' fallacy

- Red Red Black Red Black Black Black Black
- Bet "red", or bet "black" ?

Hot hand fallacy

- Player A: Score Score Miss Miss
- Player B: Miss Miss Score Score
- A, B, or doesn't matter?
- Gilovich, Vallone & Tversky (1985) - Shots in basketball are independent.

Conjunction fallacy

- Which is more probable?
  1. Linda is a shop assistant.
  2. Linda is a shop assistant and is active in the feminist movement.

The conjunction rule

The probability of two *independent* events both occurring is the product of their individual probabilities.

- $P(heads_{time1}) = 0.5$
- $P(heads_{time2}) = 0.5$
- $P(heads_{times1and2}) = P(heads_{time1}) \times P(heads_{time2}) = 0.5 \times 0.5 = 0.25$
- $P(assistant) = .05, P(feminist) = .95$
- $P(assistant + feminist) = .05 \times .95 = .0475$
- $P(assistant + feminist) < P(feminist)$

Shared birthdays

- In a class of 30 children, what's the probability that there is a shared birthday in the class?
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More high-school maths

- Number of pairs: $n(n-1)/2$
- This gets very large quite quickly.
- Pairs in a group of 2: $2(1)/2 = 1$
- Pairs in a group of 5: $5(4)/2 = 10$
- Pairs in a group of 10: $10(9)/2 = 45$
- Pairs in a group of 20: $20(19)/2 = 190$
- Pairs of children in a class of 30: $30(29)/2 = 435$
- Pairs in Year 1 psychology, approx: $200(199)/2 = 19900$

Birthday example

- 365 days in the year (ignore Feb 29th).
- So, the chance of one pair of kids sharing a birthday is $1/365 = .003$
- Thus, chance of not sharing is .997
- If no pair of kids share a birthday, then there is no shared birthday in the class.
- How many pairs in the class?
- $n(n-1)/2 = 30 \times 29/2 = 435$.
- Under conjunction rule, $p = .997^{435} = .17$
- Thus, probability of a shared birthday is 1-.17 = .83

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Roy Meadows - expert witness

- Chances of a randomly chosen baby dying of cot death are 1 in 1303, $p = .0008$
- If the family is affluent, and the mother is over 26, then the chances are even lower; 1 in 8500, $p = .0001$
- Through the conjunction rule, the probability of two cot deaths in the same family is $.0001 \times .0001 = 1 \times 10^{-8}$
- 1 in 73 million
- Less than once a century in the UK.
- The idea that these deaths were by natural causes can be ruled out beyond *reasonable doubt*.

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- If the family is affluent, and the mother is over 26, then the chances are even lower; 1 in 8500, $p = .0001$
- Through the **conjunction rule**, the probability of two cot deaths in the same family is $.0001 \times .0001 = 1 \times 10^{-8}$ ... **COT DEATHS WITHIN THE SAME FAMILY ARE HIGHLY UNLIKELY TO BE INDEPENDENT EVENTS**.
- 1 in 73 million
- Less than once a century in the UK.
- The idea that these deaths were by natural causes can be ruled out beyond *reasonable doubt*.

Further Reading

Helpful background, only lecture content on these topics is examinable).

- Paulos (1988/2000). *Innumeracy*. Penguin.
- `http://en.wikipedia.org/wiki/Conjunction_fallacy`
- `http://en.wikipedia.org/wiki/Sally_Clark`
- `http://en.wikipedia.org/wiki/Probability`