

Binary Composition (BC) Let A be a non-empty set A Binary Composition on

set A is a mapping of : AXA - A A is generally denoted by o For a pair of elements a, b in A, the image of their (a,b) under the binary composition o' is

denoted by a o b Eg - on the set of tetino stand for the binary compas-

- Ition 'addition', If we write 203 = 5, 6042= 107 If binary composition 'o' represent 'multiplication'.

(dos vidas - and an si A oronw) 8 - 2037 = 6 nigtom A Adx x = (x)4 di A gos po poiggoso 4 = 24 pi no ad 'subtraction' is not a binory composition on the set N.

A Binary Composition o is said to be defined on a nonempty set A if a ob & A Va, b & A. Type proposed of A or base a + A: + privation out In this case the ricet A is usaid to be closed under the binary composition 'o' . to mornet lours and

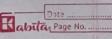
Let 'o' be the Binary composition on a set A . O' is said to be commutative if a ob = boa Vo, b & A It is said to be associative if a (boi) = (a . b) oc.

(1) a o b = boa, y a, b & A. (commutative) = p (2) a. (b.c) = (a.b) oc, Va, b, C & (Associative)

pioguores

Let be a non- empty set

Va,b,c EA.



let G be a non- empty set on which a Binary Composition to is defined some algebraic structure is imposed on by the composition . Then ((9,0) becomes a algebraic system. This (6,0) is said to be groupoid Let e be the 1est identity and f be the right in the (z,+), (z,-) are examples ion gralipoid duag sit in (g, f), r = p q # 0 le Attodord (1) 0 = 008 (B) A groupoid (G13 0) is said to be commutative groupoid if the Binary composition 'o' is commutative. Now. Est = f (by broperty of C) An element e' in set Gras said to an identity element in the groupoid (G, 0) if lave = eoa = 0; = a & G, unique identity element one in the groupoid on ● An element e in G is said to a right identity element in the groupoid (G, o) it are = a, YaeG . Let (et a) pe a drontoin containing his identity element Eg = (Z; +) +0 is a right - identity element a - 0 = 0. invertible if there exists on element a in G such that (An element secons God's said to be a deft identity element in the groupoid (G, o) if eoa = a, Vae G 5+(-5) = 0 . a= 10 - (2-)+3 Theorem - If a groupoid (G, o) contains on identity element then and the element is unique si De of themeson A (a) exists in on element to a such that boo = e Proof - If possible let there be two identity elements e and fin (1) He be inst a lest identify (on 2) & bioquorist in H (6) Hovales AThenad are price asi = and and homels as not the there exists the common of it is be be a see and han town many it alditional - a triple and of pine of o (Now, leaf sole by property of if fibre party of themes of bioseofi = fr liby property of edit - 1910 ad of e = fbe a right e-javerse of a

Theorem - If a groupoid contains a left identity as well equal as a right-identity then they are equal and the element is the identity element Let e be the left identity and f be the right identity in the groupoid (G1,0) then grows out (- 5) e a = a (by property of e) and i bigging withdarf = a of by property of f) biognost systeman si 's' garlendines grand Now, eof = f (by property of e) moto vient = e (by property of f) is formate and poperformes and filant bicquis This proves theat 'e' or 'f' are the same and the unique identity element in the groupoid or. An element e in G is said to a right identity element in the DO T DE BOD TI (O, D) hickword · let (G, 0) be a groupoid containing the identity element An element a inter- la & Go) is could to be inverted invertible if there exists an element a"in G, such that at at tomorphio apprinto of = e of loinis the inverse of as). aroughoid (God) is con = a. Vac Go $F_0 - (Z, +) i e = 0$. (x) An element a e G is said to be left-invertible if there exists in an element be G such that boa = e Proof - It projected they be two identity elements It e be just a left identity in the groupoid in H (G,0) then on element & a in of is said to be test e-invertible if there exists an element b in Go boa = e, and a is said to be right e-invertible if there exist and element c in Gy such that movey = ed . Here 187 is useled

to be a left e-inverse viole of and 'c' is said to

be a right e-inverse of a

g) Let (Z,*) be a groupoid where * is defined by a * b = a + 2b, a, b & Z Does any identity element exists in the groupoid 0' = -1262 [: n = 6 is a 1864 - resentity a* 0 = 1 a + 240 = a. . (A :. 0 * e = a . Honort Without - Honor 0 * b = 0 + 2b = 2b * b. .. a* b ± b. (e* b = b) [: def to b Left Identity] De 01 0 = 0 € 0 € 0 only right identity element is present ite of a comment a de-3 62 1. U= . 8 e is a kight tovertible 5 & Z = (2 +), * is defined by a * b = a + 2b, a, b & Z. 6 15 5 left 0 invertible element? 15 5 right 0 invertible element O Invertible Ans) Thet and left and south to so to so tinguous a tol (c briden ni svitoriosso si o neipognes menni (17.4), (2.4), (8.4), (8.4) and all examples of a' + 2a = 0 . . Here a = 5 (3) 164 (4,5) = 0. Ann guran imp2 a si (0, p) 191 (8) = 01 = 010 (62). a construction a left Ivertible clemen Det not Right Invertible as about and an enidered good this Bodod & Bodod & G. a + a' = e. as And the 28' = 00 o (a = 5) way largeter out of € 5 + 20' = 0 . : 2 MONG ⇒ . 20' 50, −5 °.0 , 000 € 0 , 0 € 10 =) 0' = -5 K Z nin a = 5 is not a Right Invertible Element, If a = 6. Left Invertible 0' * a = e.

Date . Habita Page No. de- n - 1 + 201 = e [e=0 & 0,= 6] . d (= 1)

. d ± d * 0

trag net of the 12 50 0- remote widoshi you so finertible?

= 0' = -12 & Z [: 0 = 6 is a Left - redontity 0 = OFC + O Flemen HJ.

Right - Identity Element.

, d + dc = ds +0 = d +0

[+1400 + 1 +101 0 * 01 = 1e (d [e = 0]) ⇒ a + 20' = 0 [0 = 6]

=) 6 + 20' =0 insert i taggards editorti tagia ulan

≥ a'=-3 € Z 1: a= · B 6 is a Right Invertible

5 3 d o do + 0 = d + 0 ud banish si & (* Etement Ja :

bys steet a investible element? Semigroup to small additional O yack & al

1) let a groupoid (G, O) is said to a cornigroup if the binary composition o' is associative in nature (2,+), (Z,*), (R,+), (R,*) are an examples of

semigroup. . = = 0 = 10H . . 0 = 05 + 0

(1) let (G, 0) is a semigroup and a E G then a a E G. remonde elle (a o (a o a) = (a o a) o a, as to is associative.

> Dropping the parenthesis, each of them is written to acaca thus acacath, acacath,...

The tre integral powers of a cor are defined as $a' = a, a^2 = a \cdot a, a^3 = a \cdot a \cdot a, . = .$ follows:

eldtrout deigant =matio a D Vn ENX 2- = 0 6 an = an-1 o a.

1 of the Travertible

Habita Page No. tet (15, 11) be a groupoid let (S, 0) be a (groupoid) semigraup & a & 5. Then am+n = amoan y m, n EN. on and votion yours drider of doors or i decin toll $a^{m+n} = a^m \cdot a^n$ = a · a · a · a · (m+n times)

(As the Binary Composition
is associative ← m times → ← n times → $a^{m+n} = a^m \circ a^n$ decipo rolligos Monoid socie jo remina municom anotano deno ristamos A An algebraic system (G, o) is said to be a monoid if i) (a o b) o c = a o (b o c), Vaib, C & G & p 99 mg ii) There exist an element e in G such that . e. al = lace = act at Gi, on usobs to radmuh regarden(Z, +) oran(Z, *) of bobbs is some netwo and it deput platais o morning at al time Theorem - If In a monoid (G, 0) is any element a be invertible then it has an unique laverse How many maximum distributions from the simble Proof - If possible let there be two inverses (0) and (a") EG then a o a' = e = a' o a and a o a" = a" o a = e. where e being the identity element Now (a'o a) o a" = a'o (a o a") [: since o is associative] a' o (a o a') of la' o e Hair stolarios word Vice Versa may or maynot be trule $\cdot \cdot \cdot \cdot \cdot \alpha' = \alpha'' \cdot \cdot$

Leff itiventible 05

In a monoid (G,0) if an element a is jeft invertible as well as right invertible then a is invertible and has the linique invertible in the monoid.

Theorem

Proof - let e be the identity element and b be a left inverse and c be a right inverse of the element a.

Then we can write, the second of the element a

acc = e . from the food or forth word on A

GROUP - A non-empty set Gris said to form a group with respect

i) Gives cotosed used the binary composition of

iii) Binary Composition o' is associative.

that a'oa = a o a' = e. How exists an element a'in G such

Proof - Let therebe two identity elements e, f & Gf, where

Gisa group (G, 0) (e + RI, f - LI).

Let a be an element of (G, 0) then,

fo a = a (by property of file)

then,

e = f . [: 1 There is only one identity element].

foe = f (By property of e) ...

Prove that in a group only one unique inverse is present for in given relement to addition of a most addressed addition consider a group (G1,0) and an element a EGI Let a' and a" be two inverses of the element a then, By the property of inverse trans all a dat - dans a tramale and to seraval them a ad a har a'oa= aoa'=e and, .- nu aco nu mat a" o a = a o a" = e, where is is the identity element. As, we know that 'o' is associative in nature in a .: (a'oa)oa" = a'o(aoa"). = 309 => e o a" = a' o e duct) a" # à of Hoff 10 452 videns - nou A - 20090 31 meripodinos unanid o of Theorem - In a Group (G, 0) i) a o b ma o c implies b = c (left concellation Biomy (wolosition o' is associative li) boo a = co a nimplies b= e light cancellation of is for each element o in a thou exists on element a" Y a, b, c & G1. 9 = 0 0 0 = D 0 0 + DOH Proof a) let (G, o) be a group and a, b, Cof G, Let a & G be the inverse of element a thousand Gisc group (G.o) (earl of - 11) a ob = a oc - from this we can write -€000b= €0 We have the place and b = 500 > (aoa) ob + (aoa) o c [: aoa' = e] e obt= ceoclory VEli: eon = n & whow neg] [+ no mo | v+ + no b = oco [Hence Proved]], 1= 9

let the Be (G, 0) be a group and a, b, c & G1.

= b = c

+ (POP,) OC = 1

=> b = (a = o') = c = (a = a') [::

= boe = coe