17/05/22

Discrete Mathematics

(1) Every square integer is of the form $5k, 5k+1$ for some $k \in \mathbb{Z}$

We know from the theorem every integer is of the following form $5k, 5k \pm 1, 5k \pm 2$

$$(5k)^2 = 5 \times 5k^2 = 5 \times p,\ p \in \mathbb{Z}$$

$$(5k \pm 1)^2 = (5k)^2 \pm 2 \times 5k \times 1 + 1^2$$
$$= 25k^2 \pm 10k + 1 = 5(5k^2 \pm 2k) + 1$$
$$= 5p + 1$$
where $p = 5k^2 \pm 2k$ and $k \in \mathbb{Z}$

So, it is of the form $5k+1$ form

$\cancel{(5k-1)^2 = (5k)^2}$

$$(5k \pm 2)^2 = (5k)^2 \pm 2 \cdot 2 \cdot 5k + 2^2$$
$$= 25k^2 \pm 20k + 4$$
$$= 25k^2 \pm 20k + 5 - 1$$
$$= 5(5k^2 \pm 4k + 1) - 1$$
$$= 5p - 1 \text{ where } p = 5k^2 \pm 4k + 1 \in \mathbb{Z}$$

(2) Show that every odd integer is of any of the forms (i) $2p-1$ (ii) $2p+1$ (iii) $4p \pm 1$ (iv) $\pm(4p+1)$ where $p \in \mathbb{Z}$

$\longrightarrow$

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Soln: Since $2p$ is an even integer, therefore $(2p-1)$ and $(2p+1)$ are odd integers, also by the previous theorem(ii) we know that every integer has one of the forms $4p$, $(4p\pm1)$, $(4p\pm2)$ of which $4p$ and $4p\pm2$ are even integers, p being an integer

$\therefore (4p\pm1)$ are odd integers

Now, $(4p-1) = -(-4p+1) = -[4(-p)+1]$

$\therefore \pm(4p+1)$ are odd integers

Thus every odd integer is of the $2p-1$, or $2p+1$ or $4p\pm1$ or $\pm(4p+1)$

(3) Show that one of every three consecutive integers is divisible by 3.

Soln: Let $a, (a+1), (a+2)$ be any consecutive integers. Then by theorem(i) a is of the form $3p$, $3p+1$, $3p-1$, $p\in z$

If $a = 3p$ then a is divisible by 3

If $a = 3p+1$ then $a+2 = 3p+1+2 = 3(p+1)$ is divisible by 3

If $a = 3p-1$ then $a+1 = 3p\cancel{-1}\cancel{+1} = 3p$ is divisible by 3

Thus, one of every 3 consecutive integer is divisible by 3.