

10/5/22

# Branch-and-Bound (using BFS).

15-Puzzle ~~test~~ problem →  
(Initial arrangement).

If any node is  
same as parent,  
then it is not legal  
move.

1	2	3	4
5	6		8
9	10	7	11
13	14	15	12

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

up

down

left

right

(Goal position).

1	2		4
5	6	3	8
9	10	7	11

L

R

1	2	3	4
5	6	7	8
9	10		11
13	14	15	12

D

R.

1	2	3	4
5	6	7	8
9	10	15	11
13	14		12

5	6	7	8
9	10	11	
13	14	15	12

$\hat{c}(x)$  is the estimated min cost to reach the goal node.

$f(x)$  is the length of the path from the root to the node  $x$ .

~~$\hat{g}(x)$  is the path from the root to~~  
 $\hat{g}(x)$  is the estimate of length of a shortest path from  $x$  to goal node in the sub-tree with root  $x$ .

$$\hat{c}(x) = f(x) + \hat{g}(x).$$

$$\hat{c}(1) = 0 + 3 = 3.$$

$$\hat{c}(2) = 1 + 4 = 5.$$

$$\hat{c}(3) = 3 + 0 = 3 \quad \hat{c}(8) = 2 + 1 = 3.$$

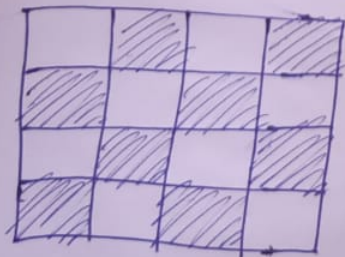
Conditions for checking  $\rightarrow$

1	2	3	4
5	6		8
9	10	7	11
13	14	15	12

initial  
fig 1(a)

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

goal  
fig 1(b)



The goal node of fig 1(b) is reachable from the initial state iff  $\sum_{i=1}^{16} \text{less}(i) + x$  is even.

Let position( $i$ ) be the position number in the initial state of the teles number  $i$ .  
 For any state, let less( $i$ ) be the no. of times  $j$  such that  $j < i$  and position( $j$ ) >

~~xxxx~~  $\boxed{x=1}$  if in the initial state the empty spot is at one of the shaded position of figure 1(c),  
 else  $x=0$  if empty spot is initially in any of the non-shaded position.

less( $i$ )  $\rightarrow$  we calculate less(1) ... less(16).

1  $\rightarrow$  0    2  $\rightarrow$  0    3  $\rightarrow$  0    4  $\rightarrow$  0    5  $\rightarrow$  0    6  $\rightarrow$  0    7  $\rightarrow$  0.

8  $\rightarrow$  1    9  $\rightarrow$  1    10  $\rightarrow$  1    11  $\rightarrow$  0    12  $\rightarrow$  0.

13  $\rightarrow$  1    14  $\rightarrow$  1    15  $\rightarrow$  1    16  $\rightarrow$  9.

Total  $\sum_{i=1}^{16} \text{less}(i) = 1+1+1+1+1+1+9 = 15$ .

$\therefore \Sigma \text{less}(i) + x = 15 + 1 = \text{even}$ . So goal node is reachable.

1 <sub>1</sub>	3 <sub>2</sub>	4 <sub>3</sub>	15 <sub>4</sub>
2 <sub>5</sub>		5 <sub>7</sub>	12 <sub>8</sub>
7 <sub>9</sub>	6 <sub>10</sub>	11 <sub>11</sub>	14 <sub>12</sub>
8 <sub>13</sub>	9 <sub>14</sub>	10 <sub>15</sub>	13 <sub>16</sub>

$i = 14$   
 $j = 6$      $j < i$

$x = 0$ .

1  $\rightarrow$  0

3  $\rightarrow$  1.

4  $\rightarrow$  1.

15  $\rightarrow$  11

2  $\rightarrow$  0

5  $\rightarrow$  0

12  $\rightarrow$  6

7  $\rightarrow$  1

6  $\rightarrow$  0

11  $\rightarrow$  3

14  $\rightarrow$  4.

8  $\rightarrow$  0

9  $\rightarrow$  0

10  $\rightarrow$  0

13  $\rightarrow$  0

16  $\rightarrow$  10

~~xx~~  $T = 37$ .