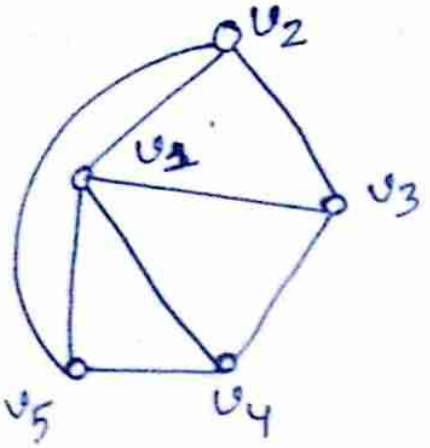
(Descrete Math)
(Graph part)

1) 

chromatic number = 3

chromatic polynomial

$= C_1 \times C_1 + C_2 \times C_2 + \cdots + C_n \times C_n$

$C_1 = 0 \quad C_2 = 0$

$C_3 = 3! = 6$

$C_4 = 4! \times 2! = 48$

$C_5 = 5! = 120$

$$f(G, x) = \left(0\, x\, {}^{x}C_1 + 0\, x\, {}^{x}C_2 + 6x\, {}^{x}C_3 + 48\, {}^{x}C_4 + 120 \times {}^{x}C_5\right)$$

$$\left[\frac{x!}{3!(x-3)!}\right] = \left[\frac{x(x-1)(x-2)}{3!}\right]$$

$$= \left[\cancel{6} \times \frac{x(x-1)(x-2)}{\cancel{6}} + \cancel{48}^{\,2} \times \frac{x(x-1)(x-2)(x-3)}{\cancel{24}} + \cancel{120} \times \frac{x(x-1)(x-2)(x-3)(x-4)}{\cancel{120}}\right]$$

$$= \left[x(x-1)(x-2) + 2x(x-1)(x-2)(x-3) + x(x-1)(x-2)(x-3)(x-4)\right]$$

(Ans)

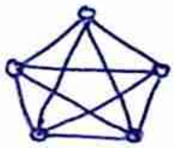
2) (Chromatic Polynomial of graph:)

Def: The chromatic polynomial is a graph polynomial studied in algebraic graph. theory, a branch of math. It counts the num of graph coloring as a function of the num of colors and was originally defined by George David Birkhoff to study the four color problem

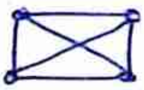
Statement: The chromatic polynomial of a graph with n vertices and 1- edges in $x^n - x^{n-1}$. So, our statement is true for such a graph. =

$$(x^n - a_{n-1}x^{n-1} + a_{n-2}x^{n-2} - + \dots) - (x^{n-1} - b_{n-2}x^{n-2} + b_{n-3}x^{n-3} - + \dots).$$

diagram:



$C(K_5) = \lambda(\lambda-1)(\lambda-2)(\lambda-3)$



$C(K_4) = \lambda(\lambda-1)(\lambda-2)(\lambda-3)$

formula: If $G \leftarrow$ Simple graph

then, $PG(k) \leftarrow$ as the num of ways we can achieve a proper coloring on vertices of G given k colors

& PG called chromatic function

If $k < \chi(G)$

then $PG(k) = 0$